

Clean versus Dirty Economic Growth

Tapio Palokangas
University of Helsinki, HECER and IIASA

Discussion Paper No. 649:2012
ISBN 978-952-10-7225-3, ISSN 1459-3696

June 6, 2012

Abstract

This document considers an economy with many regions and two engines of growth: *horizontal R&D*, which increases the number of polluting product lines; and *vertical R&D*, which improves productivity in these lines. Pollution in any region decreases welfare in all regions. Any group of regions can form a jurisdiction where a common policy maker controls pollution. Large jurisdictions, which can better internalize externality through pollution, perform vertical R&D. Because jurisdictions face decreasing unit costs of administration, they expand, performing first horizontal and then vertical R&D. This generates an *environmental Kuznets curve (EKC)* on which pollution first aggravates and then alleviates.

Journal of Economic Literature: 044, Q55, Q56, Q58

Keywords: Pollutants, growth and environment, endogenous public policy, economic integration, environmental Kuznets curve

1 Introduction

In this document, I examine an economy with two engines of growth: *horizontal R&D*, which creates new polluting product lines, degrading environmental quality; and *vertical R&D*, which improves productivity in these lines, improving environmental quality. In this setting, I consider the patterns of pollution and economic growth when public policy is endogenous.

According to Kijima et al. (2010), there exists an *environmental Kuznets curve (EKC)* as follows: “In early stages of industrialization, pollution grows rapidly, because high priority is given to increasing material output, and people are more interested in income than environment. In the later stage, however, as income rises, the willingness to pay for a clean environment increases by a greater proportion than income, regulatory institutions become more effective for the environment, and pollution level starts declining.” I specify the regulatory institutions mentioned in this reference as local policy makers. Some authors introduce taxes/subsidies as instruments of environmental policy.¹ In contrast, I assume simply that local policy makers have the authority to prevent firms from establishing new polluting product lines.

Some papers focus on the allocation of resources between abatement and other activities.² In this document, I specify abatement as vertical R&D which alleviates pollution. Some others use a model of capital accumulation in their analysis.³ In this document, I take rather a R&D-based model of endogenous growth as a starting point. Smulders et al. (2012) model the transition in the pollution pattern as a change in general purpose technology and investigate how it interferes with economic growth driven by quality improvements. In contrast, I assume that horizontal R&D creates new pol-

¹Cf. Jones and Manuelli (2001), Egli and Steger (2007), and Smulders et al. (2012).

²Cf. Selden and Song (1995), Andreoni and Levinson (2001), Lieb (2002), Dinda (2005), Egli and Steger (2007), Brock and Taylor (2010).

³Cf. John and Pechenino (1994), Stokey (1998), Andreoni and Levinson (2001), Jones and Manuelli (2001), Tahvonen and Salo (2001), Egli and Steger (2007), Lehmijoki and Palokangas (2010), and Brock and Taylor (2010).

luting product lines, while vertical R&D creates less-polluting versions of the old products. I construct a model where the economy keeps on its balanced-growth path even with changes in environmental policy.

Concerning the relationship between income and environmental degradation, empirical studies have shown the following:⁴ On the one hand, some indicators of environmental degradation (e.g. carbon dioxide emissions and municipal solid wastes) monotonically increase with respect to income, other indicators such as the lack of safe water and urban sanitation fall monotonically as income rises. On the other hand, many indicators (e.g. sulfur dioxide and nitrous oxide emissions) show an inverted-U relationship with respect to income. Some indicators (e.g. CO₂ and throughput³) exhibit an N shape, meaning that the environmental degradation starts increasing again after a decrease to a certain level.⁵ These empirical studies have been criticized on two grounds (Kijima et al. 2010) (*i*) A variety of time series, cross-section and panel data analyses indicate that the empirical results are sensitive to the sample of countries chosen and to the time period considered.⁶ (*ii*) The choice of scaling factors in the regression model affects the empirical results. In this document, I construct a model that explains an inverted-U relationship with respect to stages of development.

In the model of this document, the economy consists of a number of regions. Any subset of regions can establish a jurisdiction where a common policy maker is authorized to run environmental policy and accept new members into the jurisdiction. The policy maker's response then generates the patterns of pollution, economic growth and the extent of the jurisdiction. The remainder of this document is organized as follows. Section 2 presents the structure of the economy. The microfoundations of regions, jurisdictions and policy makers are examined in Sections 3, 4 and 5. An equilibrium with an

⁴Cf. Shafik and Bandyopadhyay (1992), and Kijima et al. (2010).

⁵Cf. de Bruyn and Opschoor (1997), and Sengupta (1997).

⁶Grossman and Krueger (1993), Selden and Song (1994), and Hill and Magnani (2002).

EKC (or an inverted-U relationship) is considered in Section 6.

2 The economy as a whole

The economy consists of a number of regions which are placed evenly within the limit $[0, 1]$. All regions produce the same final good, the price of which is normalized at unity. There are three primary inputs: pollutants, skilled labor and unskilled labor. Each region supplies inelastically one unit of both skilled and unskilled labor. The use of pollutants anywhere decreases welfare everywhere. There are no extraction costs of pollutants.⁷

I assume that research and development (R&D) employs only skilled labor, and skilled labor is used only in R&D, for simplicity.⁸ The final good is produced from unskilled labor and a number of intermediate goods. I assume that each intermediate good is produced only from pollutants, for simplicity.⁹ There are two types of R&D: *horizontal R&D*, which increases the number of product lines that produce intermediate goods, generating *dirty growth* (i.e. growth with pollution); and *vertical R&D*, which improves the level of productivity in the already existing product lines, generating *clean growth* (i.e. growth without pollution). The supplier of pollutants discriminates between product lines. Finally, I assume that a product line faces increasing returns to scale since otherwise, there would be no dynamics of pollution: it would be all the same whether the number of polluting product lines or the quantity of pollutants in each product line is increased.

⁷Extraction costs would make product lines interdependent, in which case analytical solution of the dynamic optimization in section 5 were extremely complicated.

⁸The input of skilled labor to manufacturing would excessively complicate the model, without having any qualitative impact on the results.

⁹Inputting unskilled labor or the final good into the production of intermediate goods would excessively complicate the analysis, with no qualitative impact on the results.

2.1 Regulatory institutions

Any subset of regions, $\Gamma_k \subset [0, 1]$, can form a jurisdiction k . Consequently, all regions $i \in [0, 1]$ are organized into m jurisdictions as follows:

$$\bigcup_{k=1}^m \Gamma_k = [0, 1], \quad \Gamma_k \cap \Gamma_\ell = \emptyset \text{ for } k \neq \ell, \quad n_k \doteq \int_{i \in \Gamma_k} di \in [0, 1], \quad (1)$$

where n_k is the proportion of regions organized in jurisdiction $k \in [0, m]$. Each jurisdiction k has a benevolent policy maker (hereafter labeled k) that controls the establishment of new polluting product lines and decides on new members to the jurisdiction. Policy makers can respond to environmental degradation by extending the scope of the jurisdiction. Because this response is slow due to the adjustment costs of the scope, the economy will switch (possibly several times) between clean and dirty growth.

2.2 Welfare

Pollution P depends on the total quantity of pollutants, $\int_0^m X_k dk$, and time t as follows:

$$P \doteq \left(\int_0^m X_k dk \right) e^{-\delta t} \text{ with } 0 < \delta < 1, \quad (2)$$

where δ is the constant rate of abatement: if there is no use of pollutants, $\int_0^m X_k dk = 0$, then the nature absorbs pollution at the rate δ .

Temporary utility in jurisdiction k increases with consumption per region in that jurisdiction, C_k , and decreases with economy-wide pollution P :

$$u_k \doteq C_k^\epsilon P^{-\gamma}, \quad \epsilon > \gamma > 0,$$

where γ and ϵ are parameters. The expected inter-temporal utility in jurisdiction k starting at time T is then given by

$$E \int_T^\infty u_k e^{-\zeta(t-T)} dt = E \int_T^\infty C_k^\epsilon P^{-\gamma} e^{-\zeta(t-T)} dt, \quad (3)$$

where t is time, $\zeta > 0$ the constant rate of time preference and E the expectation operator.

3 Regions

There are four sectors in each region $i \in [0, 1]$: the *manufacturing sector*, in which competitive firms make the final good from unskilled labor and the existing intermediate goods; the *intermediate-goods sector*, in which monopolistic firms make intermediate goods for the final-good firms only from pollutants; the *horizontal R&D sector*, in which competitive R&D firms produce blueprints for new intermediate goods only from skilled labor; and the *vertical R&D sector*, in which competitive R&D firms produce blueprints for better versions of old intermediate goods only from skilled labor. Because each blueprint authorizes to produce a different intermediate good, the number of intermediate goods is equal to that of blueprints in each region.

The administration of any jurisdiction $k \in [0, m]$ is subject to increasing returns to scale:¹⁰ it employs $f(n_k)$ units of unskilled labor per region, where n_k is the size of the jurisdiction and $df/dn_k < 0$. Furthermore, the integration of new members in any jurisdiction $k \in [0, m]$ involves adjustment costs in terms of unskilled labor. Thus, there is a convex and linearly homogenous adjustment cost function with respect to the number of old members, n_k , and the number of new members, $\dot{n}_k \doteq \frac{dn_k}{dt}$, as follows (cf. Fig. 1):

$$\theta(g_k)n_k, \quad g_k \doteq \frac{\dot{n}_k}{n_k}, \quad \theta(0) = 0, \quad \theta'' > 0, \quad (4)$$

where g_k is the growth rate of jurisdiction $k \in [0, m]$ and $\theta(g_k)$ adjustment costs per region in that jurisdiction.

Because region $i \in \Gamma_k$ possesses one unit of both skilled and unskilled labor, its labor markets are balanced, if

$$l_i + f(n_k) + \theta(g_k) = 1 \text{ and } h_i + z_i = 1 \text{ with } f'(n_k) < 0 \text{ for } k \in \Gamma_k, \quad (5)$$

where l_i (f) is unskilled labor devoted to manufacturing (administration), θ unskilled labor devoted to integrating new members and h_i (z_i) skilled labor devoted to horizontal (vertical) R&D in region i .

¹⁰With this assumption, jurisdictions have incentives to expand.

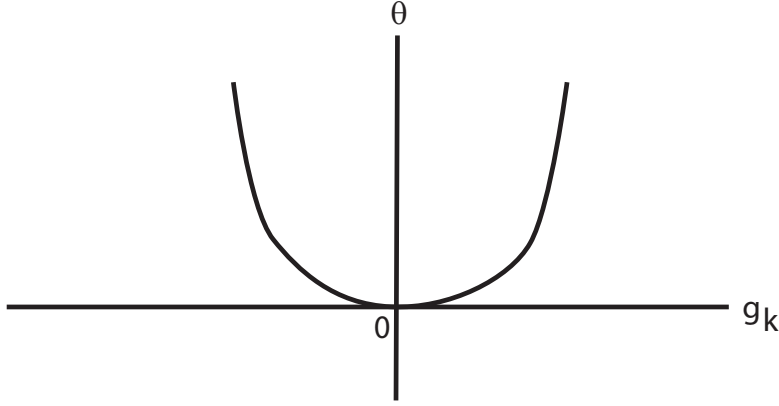


Figure 1: The adjustment cost function $\theta(g_k)$.

3.1 The manufacturing sector

A number b_i of product lines provides specific intermediate inputs for manufacturing in region $i \in [0, 1]$. The representative competitive firm produces the quantity Y_i of the final good from unskilled labor l_i and intermediate inputs, x_{ij} for $j \in [0, b_i]$, according to [cf. (5)]

$$Y_i = l_i^\alpha \int_0^{b_i} a_{ij}(x_{ij} - \phi)^{1-\alpha} dj = [1 - f(n_k) - \theta(g_k)]^\alpha \int_0^{b_i} a_{ij}(x_{ij} - \phi)^{1-\alpha} dj, \quad (6)$$

where a_{ij} is the level of productivity in product line $j \in [0, b_i]$ in region $i \in [0, 1]$, $\alpha \in (0, 1)$ is a parameter and $\phi > 0$ a fixed cost in a product line.

3.2 The intermediate-goods sector

In equilibrium, the price p_{ij} for intermediate input x_{ij} is equal to the marginal product of that input, $\partial Y_i / \partial x_{ij}$. Noting (6), this implies

$$p_{ij} = \frac{\partial Y_i}{\partial x_{ij}} = (1 - \alpha) l_i^\alpha (x_{ij} - \phi)^{-\alpha} a_{ij}. \quad (7)$$

Because the supply of pollutants involves no costs, the supplier of pollutants in product line j maximizes its profit [cf. (7)]

$$\Pi_{ij} = p_{ij} x_{ij} = (1 - \alpha) l_i^\alpha (x_{ij} - \phi)^{-\alpha} x_{ij} a_{ij}$$

by its supply x_{ij} to product line j , given total labor devoted to production in region i , l_i , and productivity in that product line, a_{ij} . The first-order condition of this maximization, $\partial\Pi_{ij}/\partial x_{ij} = 0$, implies

$$x_{ij} = x \doteq \phi/(1 - \alpha) = \text{constant for all } i \text{ and } j. \quad (8)$$

3.3 The vertical R&D sector

In product line $j \in [0, b_i]$ in region i , competitive firms employ z_{ij} units of skilled labor to produce blueprints for better versions of intermediate input $j \in [0, b_i]$. Then, the total quantity of skilled labor devoted to vertical R&D in all product lines $j \in [0, b_i]$ in region i is

$$z_i \doteq \int_0^{b_i} z_{ij} dj. \quad (9)$$

The invention of a new technology raises the a serial number of technology, τ_{ij} , by one and the level of productivity, a_{ij} , by constant $\mu > 1$ in product line j . Thus, the level of productivity in product line j is

$$a_{ij} \doteq \mu^{\tau_{ij}} = e^{(\log \mu)\tau_{ij}} \text{ with } \mu > 1. \quad (10)$$

The improvement of technology in product line j (i.e. the increase τ_{ij} by one) is in fixed proportion λ to labor z_{ij} devoted to R&D in product line j . I assume that in a small period of time dt ,

- the probability that R&D leads to development of a new technology is given by $\lambda z_{ij} dt$, where $\lambda > 0$ is a constant,
- the probability that R&D remains without success is given by $1 - \lambda z_{ij} dt$.

This defines a Poisson process q_{ij} with

$$dq_{ij} = \begin{cases} 1 & \text{with probability } \lambda z_{ij} dt, \\ 0 & \text{with probability } 1 - \lambda z_{ij} dt, \end{cases} \quad (11)$$

where dq_{ij} is the increment of the process q_{ij} .

3.4 The horizontal R&D sector

The average input to horizontal R&D in jurisdiction k , H_k , and that in the other jurisdictions $\ell \neq k$, H_{-k} , are defined by

$$H_k \doteq \frac{1}{n_k} \int_{i \in \Gamma_k} h_i di, \quad H_{-k} \doteq \frac{1}{1 - n_k} \int_{i \notin \Gamma_k} h_i di, \quad (12)$$

where $n_k = \int_{i \in \Gamma_k} di$ and $1 - n_k = \int_{i \notin \Gamma_k} di$ [cf. (1)]. Technological knowledge in horizontal R&D in region $i \in [0, 1]$ is measured by the stock of blueprints in that region, b_i . Correspondingly, technological knowledge in the whole economy is given by

$$\int_0^1 b_i di = n_k B_k + (1 - n_k) B_{-k}, \quad (13)$$

where B_k is the average technological knowledge in jurisdiction k , and B_{-k} that elsewhere in the economy:

$$B_k \doteq \frac{1}{n_k} \int_{i \in \Gamma_k} b_i di, \quad B_{-k} \doteq \frac{1}{1 - n_k} \int_{i \notin \Gamma_k} b_i di = \frac{1}{1 - n_k} \int_{\ell \neq k} n_\ell B_\ell d\ell. \quad (14)$$

Because firms performing horizontal R&D adopt technology from the ‘old’ product lines in the economy, then, in each region i , new blueprints for new product lines, $\dot{b}_i \doteq \frac{db_i}{dt}$, are produced from labor devoted to horizontal R&D in that region, h_i , so that productivity is in fixed proportion ξ to technological knowledge in the whole economy, (13):

$$\dot{b}_i = \xi [n_k B_k + (1 - n_k) B_{-k}] h_i \text{ for } i \in [0, 1]. \quad (15)$$

The parameter ξ characterizes *technology spillover*: the higher ξ , the easier it is to adopt technology from the ‘old’ product lines. Noting (12), (14) and (15), the average production of new blueprints for horizontal R&D in jurisdiction k is given by

$$\dot{B}_k = \frac{1}{n_k} \int_{i \in \Gamma_k} \dot{b}_i di = \xi [n_k B_k + (1 - n_k) B_{-k}] H_k. \quad (16)$$

4 Jurisdictions

4.1 Pollution

The use of pollutants in jurisdiction $k \in [0, m]$, X_k , is equal to that throughout the regions $i \in \Gamma_k$ of that jurisdiction. Noting (8) and (14), this implies

$$X_k \doteq \int_{i \in \Gamma_k} \left(\int_0^{b_i} x_{ij} dj \right) di = x \int_{i \in \Gamma_k} \left(\int_0^{b_i} dj \right) di = x \int_{i \in \Gamma_k} b_i di = x n_k B_k. \quad (17)$$

Correspondingly, pollution in other jurisdictions X_{-k} , is equal to the sum of pollutants throughout regions $i \notin \Gamma_k$. Noting (14) and (17), this implies

$$X_{-k} \doteq \int_{\ell \neq k} X_\ell d\ell = x \int_{\ell \neq k} n_\ell B_\ell d\ell = x(1 - n_k) B_{-k}. \quad (18)$$

Noting (2), (17) and (18), total pollution P evolves according to:

$$P \doteq \left(\int_0^m X_k dk \right) e^{-\delta t} = x [n_k B_k + (1 - n_k) B_{-k}] e^{-\delta t} \quad (19)$$

4.2 Welfare

Because the inventor of a new product line b_i adopts technological knowledge from the old product lines $j \in [0, b_i)$, the initial productivity in product line b_i , a_{ib_i} , is determined by the average productivity of the latter, a_i :

$$a_{ib_i} = a_i \doteq \frac{1}{b_i} \int_0^{b_i} a_{ij} dj. \quad (20)$$

Noting (14) and (20), the average productivity in jurisdiction k , A_k , is

$$A_k \doteq \frac{1}{n_k B_k} \int_{i \in \Gamma_k} \left(\int_0^{b_i} a_{ij} dj \right) di = \frac{1}{n_k B_k} \int_{i \in \Gamma_k} a_i b_i di, \quad (21)$$

where $n_k B_k \doteq \int_{i \in \Gamma_k} b_i di$ is the number of product lines and $\int_{i \in \Gamma_k} \left(\int_0^{b_i} a_{ij} dj \right) di$ is the sum of productivity parameters a_{ij} in jurisdiction k . Because there is only one final good in the economy, noting (6), (8) and (21), consumption

per region in jurisdiction k , C_k , is equal to the average output of the regions $i \in \Gamma_k$ of the jurisdiction as follows:

$$\begin{aligned} C_k &\doteq \frac{1}{n_k} \int_{i \in \Gamma_k} Y_i di = \frac{(x - \phi)^{1-\alpha}}{n_k} [1 - f(n_k) - \theta(g_k)]^\alpha \int_{i \in \Gamma_k} \left(\int_0^{b_i} a_{ij} dj \right) di \\ &= (x - \phi)^{1-\alpha} [1 - f(n_k) - \theta(g_k)]^\alpha A_k B_k, \end{aligned} \quad (22)$$

where n_k is the number of regions and $\int_{i \in \Gamma_k} Y_i di$ the total output of the consumption good in jurisdiction k . Noting (19) and (22), the expected utility in jurisdiction k , (3), becomes

$$\begin{aligned} E \int_T^\infty C_k^\epsilon P^{-\gamma} e^{-\zeta(t-T)} dt \\ = \frac{(x - \phi)^{(1-\alpha)\epsilon}}{x^\gamma} \int_T^\infty \frac{[1 - f(n_k) - \theta(g_k)]^{\alpha\epsilon} A_k^\epsilon B_k^\epsilon e^{-\rho(t-T)}}{[n_k B_k + (1 - n_k) B_{-k}]^\gamma} dt, \end{aligned} \quad (23)$$

where $\rho \doteq \zeta - \delta\gamma$ is a constant. It is assumed $\rho > 0$, since otherwise the integral (23) would not converge.

4.3 Clean technological change

Given (20), the average productivity, a_i , is independent of the number of product lines, b_i , in each region $i \in [0, 1]$:

$$\frac{\partial a_i}{\partial b_i} = \frac{a_i b_i}{b_i} - \frac{1}{b_i^2} \int_0^{b_i} a_{ij} dj = \frac{a_i b_i}{b_i} - \frac{a_i}{b_i} = 0. \quad (24)$$

This implies that the average productivity, A_k , is independent of the average number of product lines, B_k , in each jurisdiction k :

$$\partial A_k / \partial B_k = 0. \quad (25)$$

The index of clean technology for jurisdiction k , τ_k , is given by

$$\tau_k \doteq \frac{\log A_k}{\log \mu} \quad \text{or} \quad A_k = e^{(\log \mu) \tau_k} = \mu^{\tau_k}. \quad (26)$$

Given (10), (21), (25) and (26), the index τ_k depends only on the indices τ_{ij} of clean technology for $j \in [0, b_k)$, with the properties:

$$\begin{aligned}\frac{\partial \tau_k}{\partial \tau_{ij}} &= \frac{1}{\log \mu} \frac{1}{A_k} \frac{\partial A_k}{\partial \tau_{ij}} = \frac{1}{\log \mu} \frac{1}{A_k} \frac{1}{n_k B_k} \int_{i \in \Gamma_k} \left(\int_0^{b_i} \frac{\partial a_{ij}}{\partial \tau_{ij}} dj \right) di \\ &= \frac{1}{n_k A_k B_k} \int_{i \in \Gamma_k} \left(\int_0^{b_i} a_{ij} dj \right) di = 1 \text{ for } j \in [0, b_i).\end{aligned}$$

By this, (1), (5), (9), (11), (12) and the Mean Value Theorem, one obtains

$$\begin{aligned}\mathcal{P}(\tau_k \text{ increases by one}) &= \frac{1}{n_k} \int_{i \in \Gamma_k} \left[\int_0^{b_i} \frac{\partial \tau_k}{\partial \tau_{ij}} \mathcal{P}(\tau_{ij} \text{ increased by one}) dj \right] di \\ &= \frac{1}{n_k} \int_{i \in \Gamma_k} \left(\int_0^{b_i} \frac{\partial \tau_k}{\partial \tau_{ij}} \lambda z_{ij} dj \right) di = \frac{\lambda}{n_k} \int_{i \in \Gamma_k} \left(\int_0^{b_i} z_{ij} dj \right) di = \frac{\lambda}{n_k} \int_{i \in \Gamma_k} z_i di \\ &= \frac{\lambda}{n_k} \int_{i \in \Gamma_k} [1 - h_i] di = \lambda(1 - H_k),\end{aligned}$$

where $\mathcal{P}(\cdot)$ is the probability function. Noting this and (11), one can define a Poisson process q_k for whole jurisdiction k as follows:

$$dq_k = \begin{cases} 1 & \text{with probability } \lambda(1 - H_k)dt, \\ 0 & \text{with probability } 1 - \lambda(1 - H_k)dt, \end{cases} \quad (27)$$

where dq_k is the increment of the process q_k .

5 Policy makers

Policy maker k can take new regions as new members into jurisdiction $k \in [0, m]$ and prevent R&D firms from establishing new polluting product lines in regions $i \in \Gamma_k$ belonging to jurisdiction $k \in [0, m]$. Thus, it controls the growth rate $g_k \doteq \dot{n}_k/n_k$ of that jurisdiction and labor devoted to horizontal R&D in jurisdiction k , H_k .

5.1 The maximization of welfare

Policy maker k maximizes the expected utility of jurisdiction k , (23), by $H_k \in [0, 1]$ and $g_k \doteq \dot{n}_k/n_k$ subject to $n_k \leq 1$ [cf. (1)], the development

of pollution, (19), and technological change in vertical and horizontal R&D, (27) and (16), given technological knowledge elsewhere, B_{-k} .

Assume for a while that $n_k < 1$. Omitting the constant $(x - \phi)^{(1-\alpha)\epsilon} x^{-\gamma}$, the value of the optimal program of policy maker $i \in [0, n]$ starting at time T becomes

$$\Gamma(\tau_k, n_k, B_k, T) \doteq \max_{H_k \in [0,1], g_k} \int_T^\infty \frac{[1 - f(n_k) - \theta(g_k)]^{\alpha\epsilon} A_k^\epsilon B_k^\epsilon e^{-\rho(t-T)}}{[n_k B_k + (1 - n_k) B_{-k}]^\gamma} dt. \quad (28)$$

Let us denote

$$\Gamma_k = \Gamma(\tau_k, n_k, B_k, T), \quad \widetilde{\Gamma}_k = \Gamma(\tau_k + 1, n_k, B_k, T). \quad (29)$$

The Bellman equation corresponding to the optimal program (28) is then

$$\rho\Gamma = \max_{H_k \in [0,1], g_k} \Psi(H_k, g_k, n_k, \tau_k, B_k, T), \quad (30)$$

where, by (4) and (28),

$$\begin{aligned} & \Psi(H_k, g_k, n_k, \tau_k, B_k, T) \\ & \doteq \frac{[1 - f(n_k) - \theta(g_k)]^{\alpha\epsilon} A_k^\epsilon B_k^\epsilon}{[n_k B_k + (1 - n_k) B_{-k}]^\gamma} + \lambda(1 - H_k)(\widetilde{\Gamma}_k - \Gamma_k) + \frac{\partial \Gamma_k}{\partial B_k} \dot{B}_k + \frac{\partial \Gamma_k}{\partial n_k} \dot{n}_k \\ & = \frac{[1 - f(n_k) - \theta(g_k)]^{\alpha\epsilon} A_k^\epsilon B_k^\epsilon}{[n_k B_k + (1 - n_k) B_{-k}]^\gamma} + \lambda(1 - H_k)(\widetilde{\Gamma}_k - \Gamma_k) \\ & \quad + \frac{\partial \Gamma_k}{\partial B_k} \xi [n_k B_k + (1 - n_k) B_{-k}] H_k + \frac{\partial \Gamma_k}{\partial n_k} g_k n_k. \end{aligned} \quad (31)$$

The Lagrangean for the maximization in (30) is

$$\mathcal{L}_k = \Psi(H_k, g_k, n_k, \tau_k, B_k, B_{-k}, T) + \underline{\nu}_k H_k + \bar{\nu}_k (1 - H_k), \quad (32)$$

where the multipliers $\underline{\nu}_k$ and $\bar{\nu}_k$ satisfy the Kuhn-Tucker conditions

$$\underline{\nu}_k H_k = 0, \quad \underline{\nu}_k, \quad \bar{\nu}_k (1 - H_k) = 0, \quad \bar{\nu}_k \geq 0. \quad (33)$$

Noting (31), one obtains the first-order conditions for the maximization (30):

$$\frac{\partial \mathcal{L}_k}{\partial H_k} = \underline{\nu}_k - \bar{\nu}_k + \frac{\partial \Gamma_k}{\partial B_k} \xi [n_k B_k + (1 - n_k) B_{-k}] - \lambda(\widetilde{\Gamma}_k - \Gamma_k) = 0, \quad (34)$$

$$\frac{\partial \mathcal{L}_k}{\partial g_k} = \frac{\partial \Gamma_k}{\partial n_k} n_k - \alpha\epsilon\theta'(g_k) \frac{[1 - f(n_k) - \theta(g_k)]^{\alpha\epsilon-1} A_k^\epsilon B_k^\epsilon}{[n_k B_k + (1 - n_k) B_{-k}]^\gamma} = 0. \quad (35)$$

5.2 Convergence

To solve the dynamic program, one can try the solution that the value of the program, Γ_k , is in fixed proportion $\vartheta_k > 0$ to temporary utility:

$$\Gamma_k = \Gamma(\tau_k, n_k, B_k, T) = \vartheta_k \frac{[1 - f(n_k) - \theta(g_k)]^{\alpha\epsilon} A_k^\epsilon B_k^\epsilon}{[n_k B_k + (1 - n_k) B_{-k}]^\gamma}. \quad (36)$$

From (26), (29) and (36) it then follows that

$$\begin{aligned} \frac{\tilde{\Gamma}_k}{\Gamma_k} &= \left(\frac{A_k|_{\tau_k+1}}{A_k|_{\tau_k}} \right)^\epsilon = \mu^\epsilon > 1, & \frac{\partial \Gamma_k}{\partial B_k} \frac{B_k}{\Gamma_k} &= \epsilon - \frac{\gamma n_k B_k}{n_k B_k + (1 - n_k) B_{-k}}, \\ \frac{\partial \Gamma_k}{\partial n_k} \frac{n_k}{\Gamma_k} &= -\frac{\alpha \epsilon f'(n_k) n_k}{1 - f(n_k) - \theta(g_k)} - \frac{\gamma (B_k - B_{-k}) n_k}{n_k B_k + (1 - n_k) B_{-k}}. \end{aligned} \quad (37)$$

Inserting (36) and (37) into the Bellman equation (30) and (31) yields

$$\begin{aligned} \rho &= \Psi / \Gamma_k \\ &= \frac{1}{\vartheta_k} + \lambda(1 - H_k) \left(\frac{\tilde{\Gamma}_k}{\Gamma_k} - 1 \right) + \frac{\partial \Gamma_k}{\partial n_k} \frac{n_k}{\Gamma_k} g_k + \frac{\partial \Gamma_k}{\partial B_k} \frac{\xi}{\Gamma_k} [n_k B_k + (1 - n_k) B_{-k}] H_k \\ &= \frac{1}{\vartheta_k} + (\mu^\epsilon - 1) \lambda(1 - H_k) - \left[\frac{\alpha \epsilon f'(n_k) n_k}{1 - f(n_k) - \theta(g_k)} + \frac{\gamma (B_k - B_{-k}) n_k}{n_k B_k + (1 - n_k) B_{-k}} \right] g_k \\ &\quad + \left[\epsilon - \frac{\gamma n_k B_k}{n_k B_k + (1 - n_k) B_{-k}} \right] \xi \left[n_k + (1 - n_k) \frac{B_{-k}}{B_k} \right] H_k. \end{aligned}$$

Solving for $1/\vartheta_k$ yields

$$\begin{aligned} 0 &< 1/\vartheta_k \\ &= \rho + (1 - \mu^\epsilon) \lambda(1 - H_k) + \left[\frac{\alpha \epsilon f'(n_k) n_k}{1 - f(n_k) - \theta(g_k)} + \frac{\gamma (B_k - B_{-k}) n_k}{n_k B_k + (1 - n_k) B_{-k}} \right] g_k \\ &\quad - \left[\epsilon - \frac{\gamma n_k B_k}{n_k B_k + (1 - n_k) B_{-k}} \right] \xi \left[n_k + (1 - n_k) \frac{B_{-k}}{B_k} \right] H_k. \end{aligned}$$

Inserting (36) and (37) into the condition (35) yields

$$\begin{aligned} 0 &= \frac{1}{\Gamma_k} \frac{\partial \mathcal{L}_k}{\partial g_k} = \frac{\partial \Gamma_k}{\partial n_k} \frac{n_k}{\Gamma_k} - \alpha \epsilon \theta'(g_k) \frac{[1 - f(n_k) - \theta(g_k)]^{\alpha\epsilon-1} A_k^\epsilon B_k^\epsilon}{[n_k B_k + (1 - n_k) B_{-k}]^\gamma \Gamma_k} \\ &= -\frac{\alpha \epsilon f'(n_k) n_k}{1 - f(n_k) - \theta(g_k)} - \frac{\gamma (B_k - B_{-k}) n_k}{n_k B_k + (1 - n_k) B_{-k}} - \frac{\alpha \epsilon \theta'(g_k)}{[1 - f(n_k) - \theta(g_k)] \vartheta_k}. \end{aligned}$$

Noting (4) and (5), this implies

$$\theta'(g_k)|_{B_k=B_{-k}} = - \underbrace{\vartheta_k}_{+} \underbrace{f'(n_k)}_{-} \underbrace{n_k}_{+} > 0, \quad g_k > 0, \quad \dot{n}_k > 0,$$

for $n_k < 1$. This result can be rephrased as follows:

Proposition 1 *With enough symmetry among the regions, $B_k \approx B_{-k}$, jurisdiction k has incentives to expand (i.e. $\dot{n}_k > 0$) as long as there is space to expand (i.e. $n_k < 1$).*

Given (4) and Proposition 1, the constraint $n_k \leq 1$ (which has so far been ignored) takes the form $g_k|_{n_k=1} = 0$. In other words, once jurisdiction k contains the whole economy (i.e. $n_k \rightarrow 1$), its expansion stops (i.e. g_k falls discontinuously to zero).

Inserting (36) and (37) into the condition (34) yields

$$\begin{aligned} \frac{1}{\Gamma_k} \frac{\partial \Psi}{\partial H_k} &= \frac{\nu_k - \bar{\nu}_k}{\Gamma_k} + \frac{\partial \Gamma_k}{\partial B_k} \frac{1}{\Gamma_k} \xi [n_k B_k + (1 - n_k) B_{-k}] - \lambda \left(\frac{\tilde{\Gamma}_k}{\Gamma_k} - 1 \right) \\ &= \frac{\nu_k - \bar{\nu}_k}{\Gamma_k} + \left[\epsilon - \frac{\gamma n_k B_k}{n_k B_k + (1 - n_k) B_{-k}} \right] \xi \left[n_k + (1 - n_k) \frac{B_{-k}}{B_k} \right] \\ &\quad - \lambda (\mu^\epsilon - 1) = 0. \end{aligned} \tag{38}$$

From (33) and (38) it follows that

$$\begin{aligned} H_k = 0 &\Leftrightarrow \Phi(B_k/B_{-k}, n_k) \doteq \\ &(\mu^\epsilon - 1)\lambda - \left[\epsilon - \frac{\gamma n_k B_k/B_{-k}}{n_k B_k/B_{-k} + 1 - n_k} \right] \xi \left[n_k + (1 - n_k) \frac{B_{-k}}{B_k} \right] > 0, \\ H_k = 1 &\Leftrightarrow \Phi(B_k/B_{-k}, n_k) < 0. \end{aligned} \tag{39}$$

Because, given (39), it holds true that

$$\frac{\gamma n_k B_k/B_{-k}}{n_k B_k/B_{-k} + \underbrace{1 - n_k}_{+}} < \gamma < \epsilon,$$

$$\begin{aligned} \frac{\partial \Phi}{\partial(B_k/B_{-k})} = & \xi \underbrace{\left[n_k + (1 - n_k) \frac{B_{-k}}{B_k} \right]}_{+} \underbrace{\frac{\gamma(1 - n_k)n_k^2}{n_k B_k/B_{-k} + 1 - n_k}}_{+} \\ & + \underbrace{\left[\epsilon - \frac{\gamma n_k B_k/B_{-k}}{n_k B_k/B_{-k} + 1 - n_k} \right]}_{+} \underbrace{(1 - n_k)\xi}_{+} \left(\frac{B_{-k}}{B_k} \right)^2 > 0, \end{aligned}$$

$H_k = 0$ is likely for high values and $H_k = 1$ for low values of B_k/B_{-k} . This result can be rephrased as follows:

Proposition 2 *There is convergence in the management of pollution: jurisdictions k with a relatively large (relatively small) dirty capital per region, B_k/B_{-k} , are more likely subject to clean (dirty) growth $H_k = 0$ ($H_k = 1$).*

6 The symmetric case $B_k = B_{-k}$

Assume now that all jurisdictions are equally polluted on the average, $B_k = B_{-k}$. Then, conditions (39) become

$$\begin{aligned} H_k = 0 & \Leftrightarrow (\mu^\epsilon - 1)\lambda + (\gamma n_k - \epsilon)\xi > 0 \Leftrightarrow n_k > \underline{n}_k \doteq \frac{1}{\gamma} \left[\epsilon + (1 - \mu^\epsilon) \frac{\lambda}{\xi} \right], \\ H_k = 1 & \Leftrightarrow n_k < \underline{n}_k. \end{aligned} \tag{40}$$

This result can be rephrased as follows:

Proposition 3 *A small jurisdiction (with $n_k < \underline{n}_k$) performs dirty growth, $H_k = 1$, and a big jurisdiction (with $n_k > \underline{n}_k$) clean growth, $H_k = 0$.*

Because jurisdiction k adjusts its allocation of skilled labor, H_k , immediately, but its size n_k only with time (cf. Proposition 1), Proposition 3 leads to the following corollary:

Proposition 4 *Jurisdiction k performs a cycle with dirty growth at early stages $n_k < \underline{n}_k$ and clean growth at later stages $n_k > \underline{n}_k$ of development.*

If the parameter of technology spillover is greater than the constant rate of abatement, $\xi > \delta$, then, noting (2) and (16), pollution (19) evolves in the vicinity of the symmetric equilibrium $B_k = B_{-k}$ according to

$$\begin{aligned} \frac{\dot{P}}{P} \Big|_{B_k=B_{-k}} &= \frac{d \log P}{dt} \Big|_{B_k=B_{-k}} = \frac{d \log B_k}{dt} \Big|_{B_k=B_{-k}} - \delta = \frac{\dot{B}_k}{B_k} \Big|_{B_k=B_{-k}} - \delta \\ &= \xi H_k - \delta = \begin{cases} \xi - \delta > 0 & \text{for } n_k < \underline{n}_k, \\ -\delta < 0 & \text{for } n_k > \underline{n}_k. \end{cases} \end{aligned} \quad (41)$$

This yields an *environmental Kuznets curve* (Fig. 2) on which pollution P first increases and then decreases.

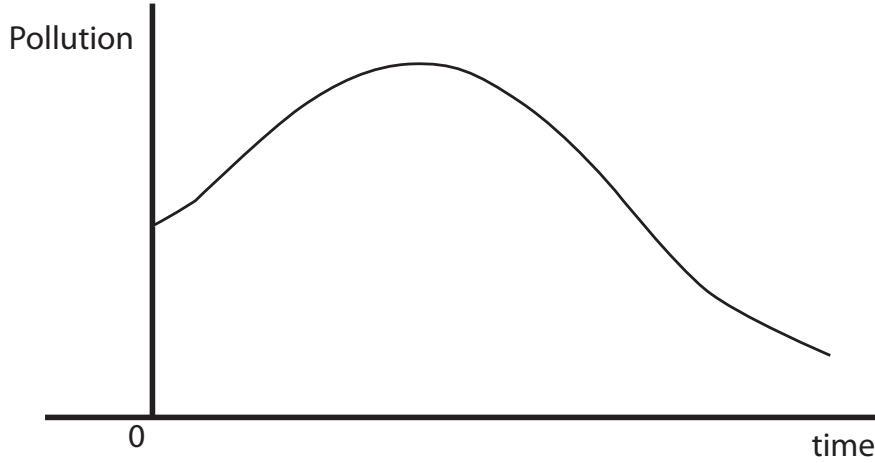


Figure 2: An environmental Kuznets curve.

7 Conclusions

Following the approach represented e.g. by Jones and Manuelli (2001), Egli and Steger (2007), and Smulders et al. (2012), this document attempts to explain the patterns of environmental degradation by the behavior of regulatory institutions. It considers an economy with a large number of regions, so that any subset of regions can establish a jurisdiction that runs environmental policy independently: it has a representative policy maker

that has authority to regulate the establishment of new polluting units and to accept new regions in the jurisdiction.

In the model, final goods are manufactured from (unskilled) labor and a number of intermediate goods. Each intermediate good is produced in a separate product line from pollutants according to increasing returns to scale. Consequently, even when there are no extraction costs for pollutants, there is an equilibrium for a monopoly that provides pollutants for the intermediate-good sector. In this setting, economic growth can appear in two forms: *horizontal R&D* that increases the number of polluting product lines, generating *dirty growth*; and *vertical R&D* that improves the level of productivity in the already existing product lines, generating *clean growth*. In addition to these activities, there is natural abatement that absorbs pollution with time.

Because dirty and clean growth rates are multiplicative, jurisdictions keep on their balanced-growth paths, but so that clean and dirty growth can permute. If the management of a jurisdiction is subject to increasing returns to scale, but if its expansion involves adjustment costs, then the policy makers attempt to expand their jurisdiction slowly, generating first dirty and then clean growth. This generates an environmental Kuznets curve (EKC), on which pollution first aggravates and then alleviates.

While a great deal of caution should be exercised when a highly stylized game-theoretical model is used to derive results on growth and environment, the following conclusion seems to be justified. The spillover of polluting technology associated with policy makers' sluggish response to environmental problems may be the cause for an EKC.

References:

- Andreoni, J., A. Levinson, A. (2001) “The simple analytics of the environmental Kuznets curve.” *Journal of Public Economics* 80(2): 269–286.
- Brock, W.A., Taylor, M.S. (2010) “The green Solow model.” *Journal of Economic Growth* 15(2): 127–153.
- de Bruyn, S., Opschoor, J. (1997) “Developments in the throughput-income relationship: theoretical and empirical observations.” *Ecological Economics* 20(3): 255–268.
- Dinda, S. (2005). “A theoretical basis for the environmental Kuznets curve.” *Ecological Economics* 53(3): 403–413.
- Egli, H., Steger, Th.M. (2007) “A dynamic model of the environmental Kuznets curve: turning point and public policy.” *Environmental and Resource Economics* 36: 15–34.
- Grossman, G.M., Krueger, A.B. (1993) “Environmental impacts of a north American free trade agreement.” In: P.M. Garber (Ed.) *The Mexico-U.S. Free Trade Agreement*. MIT Press, Cambridge, MA: 13–56.
- Hill, R.J., Magnani, E. (2002) “An exploration of the conceptual and empirical basis of the environmental Kuznets curve.” *Australian Economic Papers* 41 (2): 239–254.
- John, A., Pecchenino, J. (1994) “An overlapping generations model of growth and the environment.” *Economic Journal* 104: 1393–1410.
- Jones, L.E., Manuelli, R.E. (2001) “Endogenous policy choice: the case of pollution and growth.” *Review of Economic Dynamics* 4(2): 369–405.
- Kijima, M., Nishide, K., Ohyama, A. (2010) “Economic models for the environmental Kuznets curve: A survey.” *Journal of Economic Dynamics and Control* 34(7): 1187–1201.
- Lehmijoki, U., Palokangas, T. (2010) “Trade, Population Growth, and the Environment in Developing Countries.” *Journal of Population Economics* 23: 1351–1370.
- Lieb, Ch.M. (2002) “The environmental Kuznets curve and satiation: a simple static model.” *Environment and Development Economics* 7(3): 429–448.

- Shafik, N., Bandyopadhyay, S. (1992) *Economic growth and environmental quality: time series and cross-country evidence*. Background Paper for World Development Report. World Bank, Washington, DC.
- Selden, T.M., Song, D. (1994) “Environmental quality and development: Is there a Kuznets curve for air pollution emissions?” *Journal of Environmental Economics and Management* 27: 147–162.
- Selden, T.M., Song, D. (1995). “Neoclassical growth, the J Curve for abatement, and the Inverted U Curve for Pollution.” *Journal of Environmental Economics and Management* 29(2): 162–168.
- Sengupta, R.P. (1997) “CO2 emission-income relationship: policy approach for climate control.” *Pacific Asia Journal of Energy* 7(2): 207–229.
- Smulders, S., Bretscher, L., Egli, H. (2011) “Economic growth and the diffusion of clean technologies: explaining environmental Kuznets curves.” *Journal of Environmental Economics and Management* 49(1): 79–99.
- Stokey, N.L. (1998) “Are there limits to growth?” *International Economic Review* 39: 1–31.
- Tahvonen, O., Salo, S. (2001) “Economic growth and transitions between renewable and nonrenewable energy resources.” *European Economic Review* 45(8): 1379–1398.