Tests for Abnormal Returns under Weak Cross Sectional Dependence
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Key words: Abnormal return, Cross sectional correlation, Event study, Spatial autoregressive model

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Niklas Ahlgren and Jan Antell
Hanken School of Economics
Department of Finance and Statistics

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Tests for Abnormal Returns under Weak Cross Sectional Dependence*

Niklas Ahlgren
Hanken School of Economics, PO Box 479 (Arkadiagatan 22), 00101 Helsingfors, Finland. Telephone: +358 40 3521242. E-mail: niklas.ahlgren@hanken.fi.

Jan Antell
Hanken School of Economics, PO Box 479 (Arkadiagatan 22), 00101 Helsingfors, Finland. Telephone: +358 40 3521384. E-mail: jan.antell@hanken.fi.

Abstract

Tests for abnormal returns which are derived under the assumption of cross sectional independence are invalid if the abnormal returns are cross sectionally correlated. We model the cross sectional correlation by a spatial autoregressive model. The abnormal returns of firms belonging to the same group according to their business activities are correlated, whereas the abnormal returns of firms belonging to different groups are uncorrelated. Tests for abnormal returns corrected for cross sectional correlation are derived. An empirical application to US stock returns around Bear Stearns’ collapse and Lehman Brothers’ bankruptcy in 2008 is provided as an illustration. (JEL C21, C22, G12).
I. Introduction

The event study methodology pioneered by Fama et al. (1969) has been widely applied. Event studies measure the effect of an economic event on the value of a firm. The event’s economic impact is measured using abnormal returns (see Campbell et al. (1997) for a survey of the event study methodology).

In event studies the abnormal returns are assumed to be cross sectionally independent. The independence assumption is valid when the event day is not common to the firms. In the case where the event day is common but the firms are not from the same industry, Brown and Warner (1980, 1985) show that use of the market model to derive the abnormal returns makes the cross sectional correlations close to zero, so that they can be ignored. However, if the event day is common (known as event day clustering) and if the firms are from the same industry, the market model may not remove all cross sectional correlations in the abnormal returns.

Tests for abnormal returns which are derived under the assumption of cross sectional independence are invalid if the abnormal returns are cross sectionally correlated. Cross sectional correlation may lead to severe size distortion of tests for abnormal returns (Kolari and Pynnönen (2010)) and the finding of spurious event effects.

In this paper we model the cross sectional correlation in the abnormal returns by a spatial autoregressive (SAR) model. The framework we assume is that of a common event day with the firms from the same industry. The firms are divided into groups according to their business activities. The abnormal returns of firms belonging to the same group are correlated, whereas the abnormal returns of firms belonging to different groups are uncorrelated. The SAR model formalises weak cross sectional dependence (Pesaran and Tosetti (2007, 2011), Breitung and Pesaran (2008)). Tests for abnormal returns corrected for cross sectional correlation are derived.

A more precise formulation is as follows. The covariance matrix of the SAR model is used to estimate the covariances, and thus the cross sectional correlations, of the abnormal
returns. We derive spatial autocorrelation-consistent standard errors for tests for abnormal returns. These are easy to implement, since they depend only on two parameters that can be estimated consistently, namely the spatial autoregressive parameter and the variance of the error term. The distribution of test statistics corrected for cross sectional correlation is asymptotically normal.

Simulations show that spatial autocorrelation causes tests for abnormal returns to overreject the null hypothesis of no event effect. The tests corrected for cross sectional correlation have the nominal level and nontrivial power in large samples. An empirical application to US stock returns around Bear Stearns’ collapse and Lehman Brothers’ bankruptcy in 2008 is provided to illustrate the use of the tests with real data.

A similar approach to ours has been taken by Froot (1989), where firms in a given industry are assumed to be correlated through a correlation coefficient which is constant across industries. Kolari and Pynnönen (2010) propose a correction to tests for abnormal returns based on the average correlation coefficient. However, for their tests to be valid, the correlations must depend on the sample size and tend to zero with it, a condition that will typically not hold.

The remainder of the paper is organised as follows. Section II introduces models for cross sectional correlation in returns and abnormal returns. Tests for abnormal returns under cross sectional correlation are derived in Section III. Section IV contains a Monte Carlo study and Section V an empirical application to US stock returns around Bear Stearns’ collapse and Lehman Brothers’ bankruptcy in 2008. Conclusions are given in Section VI. The proof of the main result is placed in the Appendix.

The following notation is used. The time line is divided into an estimation period, an event period which includes the event day and a post-event period. We denote the estimation period by \( t = 1, \ldots, T \), the event day by \( \tau \) and the post-event period by \( \tau + 1, \ldots, L \). We adopt the convention \( \tau = 0 \). The number of events is \( i = 1, \ldots, N \).
II. Cross Sectional Correlation in Returns and Abnormal Returns

We consider models for cross sectional correlation in returns and abnormal returns. The expected, or normal, return is defined as the return that would be expected if an event did not take place. The abnormal return is defined as the difference between the return and expected return. The cross sectional correlation in the returns is modelled by a factor model and the remaining cross sectional correlation in the abnormal returns is modelled by a spatial autoregressive model.

A. Models for Returns

We consider a common factors model for the returns:

\[ r_{it} = \gamma_i' f_t + u_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T, \]

where \( r_{it} \) denotes the return of firm \( i \) in time period \( t \), \( f_t = (f_{1t}, \ldots, f_{mt})' \) is an \( m \times 1 \) vector of common factors assumed to be independent of \( u_{it} \), \( \gamma_i = (\gamma_{i1}, \ldots, \gamma_{mi})' \) is an \( m \times 1 \) vector of factor loadings measuring the impact of the common factors on the return and \( u_{it} \) is the abnormal return. The model includes the constant-mean-return model, the market model and the Fama and French (1993) three-factor model as special cases. For example, the market model removes the portion of the return that is related to variation in the market’s return, and hence reduces the variance of the abnormal return.

Stacked over the cross section, the model can be written as

\[ r_t = \Gamma f_t + u_t, \]

where \( r_t = (r_{1t}, \ldots, r_{Nt})' \) is an \( N \times 1 \) vector of returns, \( \Gamma = (\gamma_1, \ldots, \gamma_N)' \) is an \( N \times m \) matrix of factor loadings and \( u_t = (u_{1t}, \ldots, u_{Nt})' \) is an \( N \times 1 \) vector of abnormal returns.

The presence of the common factors \( f_t \) implies that some of the eigenvalues of \( \Sigma_r = \]
cov(\mathbf{r}_t) are \(O(N)\) and thus unbounded, as \(N \to \infty\). The factor model formalises strong cross sectional dependence (Breitung and Pesaran (2008), Pesaran and Tosetti (2007, 2011)).

**B. Models for Abnormal Returns**

Let \( \mathbf{W} \) be an \( N \times N \) spatial weights matrix and let \( \mathbf{u}_t \) be generated according to a spatial autoregressive (SAR) model (see e.g. LeSage and Pace (2009))

\[
\mathbf{u}_t = \rho \mathbf{W} \mathbf{u}_t + \mathbf{\varepsilon}_t, \quad \mathbf{\varepsilon}_t \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_N).
\]

The spatial weights matrix specifies which of the elements of the vector of abnormal returns that are cross sectionally correlated and the spatial autoregressive parameter \( \rho, |\rho| < 1 \), measures the strength of the cross sectional correlation in the abnormal returns. The vector \( \mathbf{\varepsilon}_t = (\varepsilon_{1t}, \ldots, \varepsilon_{Nt})' \) is an \( N \times 1 \) vector of errors assumed to be normal, independent and identically distributed \( N(\mathbf{0}, \sigma^2 \mathbf{I}_N) \).

The spatial autoregressive model has the solution

\[
\mathbf{u}_t = (\mathbf{I}_N - \rho \mathbf{W})^{-1} \mathbf{\varepsilon}_t,
\]

from which it is seen that the model is a special case of a model with non-spherical error terms. The covariance matrix of \( \mathbf{u}_t \) is

\[
\Sigma_u = \text{cov}(\mathbf{u}_t) = \text{E}(\mathbf{u}_t \mathbf{u}_t') = \sigma^2 \left[ (\mathbf{I}_N - \rho \mathbf{W})' (\mathbf{I}_N - \rho \mathbf{W}) \right]^{-1}.
\]

The weights matrix \( \mathbf{W} \) is typically sparse, i.e. it contains only a few non-zero elements. The covariance matrix of the spatial autoregressive model is not sparse because of the matrix powers and products of \( \mathbf{W} \). The off-diagonal elements of \( \Sigma_u \) express the assumed structure of cross sectional correlation in the abnormal returns \( \mathbf{u}_t \).

We make the following general assumption about the weights matrix:
Assumption 1. The weights matrix $W$ has bounded row and column norms, i.e.

$$
\|W\|_r = \max_{1 \leq i \leq N} \sum_{j=1}^{N} |w_{ij}| \quad \text{and} \quad \|W\|_c = \max_{1 \leq j \leq N} \sum_{i=1}^{N} |w_{ij}|
$$

are bounded.

The above assumption guarantees that the eigenvalues of the covariance matrix $\Sigma_u$, $\lambda_1 \geq \cdots \geq \lambda_N$, satisfy $\lambda_1(\Sigma_u) < K$ for some finite constant $K$. The largest eigenvalue of $\Sigma_u$, is $O(1)$ and thus bounded, as $N \to \infty$. The spatial autoregressive model formalises weak cross sectional dependence (Breitung and Pesaran (2008), Pesaran and Tosetti (2007, 2011)).

C. Specification of the Weights Matrix

Following Case (1991), we use a block structure for the weights matrix, by dividing the firms into groups according to their business activities. The abnormal returns of firms belonging to the same group are correlated, whereas the abnormal returns of firms belonging to different groups are uncorrelated.

Let $n_h$ be the number of firms belonging to group $h$, $h = 1 \ldots k$, with $\sum_{h=1}^{k} n_h = N$. We may, without loss of generality, assume that the first $n_1$ firms belong to group 1, the next $n_2$ firms to group 2, and so on. The associated weights matrix $W$ is then of the form (see e.g. Caporin and Paruolo (2009))

(5) \quad W = \text{diag}(W_{n_1}, \ldots, W_{nk}),

where

(6) \quad W_{n_h} = \frac{1}{n_h - 1} (1_{n_h} 1'_{n_h} - I_{n_h}).

Here $1_{n_h}$ is a vector of ones and $I_{n_h}$ is the identity matrix of dimension $n_h$. The row
normalisation of $W_{n_h}$ ensures that the weights matrix has bounded row and column norms. The weights matrix defined in (5)–(6) is therefore a special case covered by our general Assumption 1.

We illustrate the specification of the weights matrix by an example.

**Example 2.** Let $N = 10$, $k = 2$ and $n_h = N/k$ for $h = 1, 2$. Then

$$W_{n_h} = \begin{pmatrix}
0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0
\end{pmatrix}, \quad h = 1, 2,$$

and

$$W = \text{diag}(W_{n_1}, W_{n_2}).$$

Notice that when $n_h$ gets large, the weights get small. In practice, when $N$ is large, it may be better to increase $k$, i.e. divide the firms into more groups.

In the empirical application we use the industry classification measured by the Standard Industrial Classification (SIC) System. Details are deferred to Section V.

**D. Estimation**

The estimation of the models for the returns and abnormal returns proceeds in two steps. In the first step we estimate the model for the normal returns using data from the estimation period. We estimate a factor model for each cross section. Stack the model (1) over time to obtain

$$r_i = f_i\gamma_i + u_i,$$
where \( r_i = (r_{i1}, \ldots, r_{iT})' \) is a \( T \times 1 \) vector of returns, \( f_i \) is a \( T \times m \) matrix of common factors, \( \gamma_i \) is an \( m \times 1 \) parameter vector and \( u_i = (u_{i1}, \ldots, u_{iT})' \) is a \( T \times 1 \) vector of abnormal returns.

Under the assumption that \( u_i \) is independent of the common factors \( f_i \), the factor model can be estimated consistently by least squares (LS). Denote the LS estimate of \( \gamma_i \) by \( \hat{\gamma}_i \). Then an estimate of the abnormal returns is obtained as \( \hat{u}_i = r_t - \hat{\Gamma}f_t \), where \( \hat{\Gamma} = (\hat{\gamma}_1, \ldots, \hat{\gamma}_N)' \).

The factor model removes the cross sectional correlation in the returns which is due to the common factors and therefore is of the strong cross sectional dependence type. The leading case of a common factor is the market factor.

Under the null hypothesis that the event has no impact on the mean and variance of the returns, we have that \( \hat{u}_t \sim N(0, V_i) \), where \( V_i = \sigma_u^2 I + f_i(f_i'f_i)^{-1}f_i'\sigma_u^2 \). The second term, which is the prediction error (PE) correction, is the variance of the abnormal returns \( \hat{u}_t \) due to sampling errors, and tends to zero as \( T \) gets large.

In the second step a SAR model (3) for the event day abnormal returns \( \hat{u}_t \) is estimated by maximum likelihood (ML) to obtain estimates of the spatial autoregressive parameter \( \rho \) and the variance of the error term \( \sigma^2 \). The parameters \( \rho \) and \( \sigma^2 \) may also be estimated on the data from the estimation period, by estimating a spatial panel model similar to (3). The model may be estimated along the same lines as the cross sectional model, provided that all notations are adjusted from one cross section to \( T \) cross sections of \( N \) observations (Elhorst (2011)).

### III. Tests for Abnormal Returns

We derive tests for abnormal returns corrected for cross sectional correlation. Since the focus of the paper is on a single event day which is common to all firms, we may assume that the event day is known with certainty. Consequently, we do not consider tests based on cumulative abnormal returns.
A. Cross Sectional Independence

We review tests for abnormal returns under cross sectional independence.

Let \( \hat{u}_i \) be an estimate of the event day abnormal return \( u_i \) of firm \( i \). For simplicity we omit the time index of the event day. Given a sample of \( N \) events, define the average abnormal return

\[
\bar{u} = \frac{1}{N} \sum_{i=1}^{N} \hat{u}_i.
\]

The cross sectional test (see e.g. Boehmer et al. (1991)) assumes that the abnormal returns are independent and identically distributed \( \mathcal{N}(0, \sigma^2) \). To test the hypothesis of no event effect, we use the statistic

\[
J_0 = \frac{\bar{u}}{\hat{\sigma}/\sqrt{N}},
\]

where \( \sigma^2 \) is estimated by \( \hat{\sigma}^2 \), the cross sectional variance of the abnormal returns. The cross sectional test estimates the variance of the abnormal returns from the event day. It is therefore not sensitive to an event-induced increase in the volatilities of the abnormal returns.

Another test is based on estimating the variance of the abnormal return of event \( i \), or \( \sigma^2_i \), from the estimation period time series of abnormal returns (Campbell et al. (1997)). We refer to the test as the time series test. This test assumes that the variances may be unequal. We can test the null hypothesis using

\[
J_1 = \frac{\bar{u}}{\sqrt{\hat{\sigma}^2}},
\]

where

\[
\hat{\sigma}^2 = \frac{1}{N^2} \sum_{t=1}^{N} \hat{\sigma}_i^2.
\]
An estimator of $\sigma_i^2$ is

$$\hat{\sigma}_i^2 = \frac{1}{T} \tilde{u}_i^t \tilde{u}_i,$$

where $\tilde{u}_i$ is the vector of residuals from the model for the normal returns (7). The time series test estimates the variances of the abnormal returns from the estimation period and is therefore sensitive to an event-induced increase in the volatilities of the abnormal returns.

The third test is due to Brown and Warner (1980, 1985), who suggest a crude dependence adjustment for cross sectional dependence. The variance of the event day average abnormal return $\overline{u}$ is estimated from the estimation period time series of average abnormal returns $\overline{u}_1, \ldots, \overline{u}_T$:

$$\hat{\sigma}^2 = \frac{1}{T-1} \sum_{t=1}^{T} (\overline{u}_t - \overline{u})^2,$$

where

$$\overline{u}_t = \frac{1}{N} \sum_{i=1}^{N} \tilde{u}_{it}$$

and

$$\overline{\overline{u}} = \frac{1}{T} \sum_{t=1}^{T} \overline{u}_t = \frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \tilde{u}_{it}.$$

The test statistic is

$$J_2 = \frac{\overline{\overline{u}}}{\sqrt{\hat{\sigma}^2}}.$$  

Like the time series test, the crude dependence adjustment estimates the variance of the average abnormal return from the estimation period and is therefore not robust against an event-induced increase in the volatilities and cross sectional correlations of the abnormal
returns.

The tests may also be used with standardised abnormal returns $\hat{u}_i/\hat{\sigma}_i$, where $\hat{\sigma}_i^2$ is an estimate of $\sigma_i^2$. Formula (12) may be used for this purpose, possible with a prediction error correction (see below).

Under the assumption of Gaussian errors and cross sectional independence, all test statistics are asymptotically standard normal. The same limiting distribution is obtained if we assume that the abnormal returns are IID. For the time series test with unequal variances, the Lindeberg condition is a necessary and sufficient condition for asymptotic normality.

The abnormal returns $\hat{u}_i$ in (12) are defined as the residuals from the model for the normal return. Therefore, a prediction error (PE) correction is used to estimate $\sigma_i^2$ in the time series test and in the standardisation of the abnormal returns. The standardised $J_1$ statistic is, in addition, corrected using the Patell (1976) correction.

In finite samples the distribution of $J_0$ is approximated by a $t$ distribution with $N - 1$ degrees of freedom, the distribution of $J_1$ is approximated by the standard normal distribution and the distribution of $J_2$ is approximated by a $t$ distribution with $T - 1$ degrees of freedom.

**B. Cross Sectional Correlation**

We derive the limiting distributions of the $J_0$ and $J_1$ statistics when the abnormal returns are spatially autocorrelated. The results are used to obtain spatial autocorrelation-consistent standard errors. Tests for abnormal returns corrected for cross sectional correlation are proposed.

Let $\Sigma_u = (\sigma_{ij})$ be the spatial covariance matrix of the abnormal returns $u_t$ and $R = (\rho_{ij})$ the spatial correlation matrix obtained from $\Sigma_u$. Consider first the cross sectional test. The square of the denominator of the $J_0$ statistic can be written as

$$\text{Var} \left( \sqrt{N \bar{u}} \right) = \sigma^2 \left( 1 + \frac{1}{N} \sum_{i \neq j} \sum \rho_{ij} \right)$$
(see the Appendix). The asymptotic behaviour of the test is determined by the limiting behaviour of \( \sum \sum \rho_{ij} / N \).

Turning to the time series test, the square of the denominator of the \( J_1 \) statistic can be written as

\[
\text{Var} \left( \sqrt{N} \bar{u} \right) = \frac{1}{N} \sum_{t=1}^{N} \sigma_i^2 + \frac{1}{N} \sum_{i \neq j} \sigma_{ij}.
\]

The asymptotic behaviour of the test is determined by the limiting behaviour of \( \sum_{i \neq j} \sigma_{ij} / N \).

The limiting distributions of the \( J_0 \) and \( J_1 \) statistics when the abnormal returns are spatially autocorrelated are given below.

**Theorem 3.** Let the abnormal returns \( u_t \) be generated by a spatial autoregressive model (3) and let Assumption 1 hold. Then under the null hypothesis of no event effect, as \( T \to \infty \) and \( N \to \infty \), the \( J_0 \) statistic in (9) tends to a normal distribution \( N(0, 1 + \gamma_0) \), where

\[
\frac{1}{N} \sum_{i \neq j} \rho_{ij} \to \gamma_0
\]

for some finite \( \gamma_0 \).

The \( J_1 \) statistic in (10) tends to a normal distribution \( N(0, 1 + \gamma_1) \), where

\[
\frac{1}{N} \sum_{i \neq j} \sum \sigma_{ij} \to \gamma_1
\]

for some finite \( \gamma_1 \).

The limits in (17) and (18) exist due to the fact that the largest eigenvalue of the spatial covariance matrix \( \Sigma_u \) is bounded.

The proof is placed in the Appendix.

**Remark 4.** The conclusions of Theorem 3 continue to hold asymptotically without reference to the assumption of Gaussian errors, because the spatial autoregressive model satisfies a central limit theorem (Jenish and Prucha (2009)).
Remark 5. The square of the denominator of the $J_2$ statistic is estimated from the averages of the abnormal returns in the estimation period. The crude dependence adjustment is valid if the cross sectional correlation is the same in the estimation period and on the event day.

The results of Theorem 3 can be used to calculate spatial autocorrelation-consistent standard errors. The formulas for the spatial autocorrelation-consistent standard errors in the case of the cross sectional and time series tests are

\begin{equation}
\sqrt{\frac{\sigma^2}{N} + \frac{1}{N^2} \sum_{i \neq j} \sigma_{ij}}
\end{equation}

and

\begin{equation}
\sqrt{\frac{1}{N^2} \sum_{t=1}^{N} \sigma_t^2 + \frac{1}{N^2} \sum_{i \neq j} \sigma_{ij}},
\end{equation}

respectively. Notice that if the cross sectional correlations are generally positive, the spatial autocorrelation-consistent standard errors will be larger than the standard errors assuming cross sectional independence of the abnormal returns.

The variance of the abnormal returns is estimated from the event day in the cross sectional test and from the estimation period in the time series test. We therefore suggest to estimate \( \rho \) and \( \sigma^2 \) from the event day in the cross sectional test and from the estimation period in the time series test.

Let \( \hat{\Sigma}_u \) be an estimate of \( \Sigma_u \). It is possible to estimate both the variances and covariances in (19) and (20) by the corresponding quantities in \( \hat{\Sigma}_u \). For sensitivity against misspecification of the weights matrix \( W \), we suggest to estimate the variances as in the case when the abnormal returns are cross sectionally independent.
IV. Simulated Data

A. Simulation Design

Data are generated from the following data-generation process (DGP):

\[ r_t = \mu_t + u_t, \quad u_t = \rho W u_t + \varepsilon_t, \quad t = 1, \ldots, T, \ldots, \tau, \ldots L. \]

The observations \( t = 1, \ldots, T \) constitute the estimation period, the event date is \( \tau \) and the remaining observations \( \tau + 1, \ldots, L \) constitute the post-event period. The timing sequence is chosen to be typical of an event study. The number of daily time series observations in the estimation period is \( T = 231 \), the event date \( \tau \) is observation number 242 and \( L = 252 \). The observations in the post-event period are not used by the tests for abnormal returns.

The expected returns \( \mu_t \) are a constant. We set the expected annual returns equal to 10%. Notice that the tests for abnormal returns are invariant to \( \mu_t \).

The model for the abnormal returns \( u_t \) is a spatial autoregressive model. The abnormal returns are cross sectionally independent for \( \rho = 0 \), whereas they are cross sectionally correlated for \( \rho \neq 0 \). The values for \( \rho \) are 0, 0.1, 0.2, 0.5, 0.8. The value \( \rho = 0 \) corresponds to the case with spherical errors. The values 0.1 and 0.2 represent small, the value 0.5 moderate and the value 0.8 large spatial autocorrelation. Notice that the spatial autoregressive parameter is the same in the estimation period and on the event day.

The errors are simulated as \( \varepsilon_t \sim N(\mathbf{0}, \sigma_{\varepsilon}^2 I_N) \). The variance of the abnormal returns \( u_t \) depends on \( \rho \) and \( \sigma_{\varepsilon}^2 \). The value of \( \sigma_{\varepsilon}^2 \) is chosen, given the value of \( \rho \), so that the annual volatility is 36%. Notice that the size of the tests does not depend on the volatility, but the power does.

The number of events is \( N = 25, 50, 100, 200, 500, 1000 \). In event studies a sample size of 200 is considered large. In spatial econometrics sample sizes are typically larger, so that a sample size below 1000 is considered small (Pace et al. (2011)).

The number of firms belonging to group \( h \) is \( n_h = N/k \) for all \( h = 1, \ldots, k \), with \( k = 5 \)
for $N = 25, 50, 100$, $k = 10$ for $N = 200$, $k = 25$ for $N = 500$ and $k = 50$ for $N = 1000$. The value of $k$ depends on the sample size in order to avoid the weights tending to zero, as $N$ gets large. The weights matrix $W$ is in all cases given by (5) and (6).

The abnormal returns are zero under the null hypothesis of no event effect. Under the alternative hypothesis, a constant equal to 0.5, 1 and 2 per cent is added to the event day abnormal returns in order to estimate the power of the tests.

The constant-mean-return model is used to compute the expected returns. The prediction error correction is used to estimate the variance of the abnormal returns in the time series test and in the standardisation of the abnormal returns. The standardised $J_1$ statistic is corrected using the Patell correction.

The computations and simulations are performed using GAUSS 7.0. The Spatial Econometric Toolbox for Matlab by LeSage (version 2010) is used in the estimations of the spatial models. The number of replications is 10000, except for $N = 500, 1000$, for which the number of replications is 2000. The nominal significance level is 5%.

**B. Asymptotic Level**

The asymptotic level of the tests for abnormal returns is

\[
\alpha' = 2 \left( 1 - \Phi \left( \frac{z_{\alpha/2}}{\sqrt{1 + \gamma_0}} \right) \right)
\]

(see e.g. Lehmann (1999), p. 199). Notice that the asymptotic level depends on $k$ and $\rho$ only through $\gamma_0$, where $\gamma_0$ is defined in (17). Due to the row normalisation of the weights matrix, $\gamma_0$ is almost invariant to $k$, except when $\rho$ is large.

Table 1 reports the estimated value of $\gamma_0$ with $N = 1000$ and the asymptotic level $\alpha'$. Notice that $\sqrt{1 + \gamma_0}$ is the ratio of the spatial autocorrelation-consistent standard error to the standard error assuming cross sectional independence. For example, assume that $k = 5$ and the value of the spatial autoregressive parameter is 0.5. Then $\sqrt{1 + \gamma_0} = \sqrt{1 + 2.960} \approx 2$,
Table 1: Asymptotic level of tests for abnormal returns under cross sectional correlation. The table shows the estimated value of $\gamma_0$ with $N = 1000$ and the asymptotic level $\alpha'$. The nominal level is 5%.

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<th>$k$</th>
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<th>0.2</th>
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<th>0.8</th>
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<tr>
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<td>50</td>
<td>$\gamma_0$</td>
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<td>0.232</td>
<td>0.549</td>
<td>2.630</td>
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<tr>
<td></td>
<td>$\alpha'$</td>
<td>0.050</td>
<td>0.077</td>
<td>0.115</td>
<td>0.304</td>
</tr>
</tbody>
</table>

so that the spatial autocorrelation-consistent standard error is larger by a factor of about 2. We find that a nominal 5% level test then has asymptotic level 32.5%. More generally, the results in the table show that cross sectional correlation causes serious size distortions of tests for abnormal returns.

The asymptotic level can be taken as a benchmark to compare with the simulated size of the tests in finite samples.
C. Size in Finite Samples

We simulate the size of the tests for abnormal returns in finite samples. In each simulation we use the estimated parameters ($\hat{\rho}$ and $\hat{\sigma}_z^2$) to estimate the spatial covariance matrix $\Sigma_u$. In the simulations insignificant estimates of $\rho$ are set to zero. The spatial autocorrelation-consistent standard errors are calculated using formulas (19) and (20).

It is worth noting that the estimate of $\rho$ from the event day is biased towards zero. For $N = 100$ observations and $\rho = 0.5$ the mean of $\hat{\rho}$ over 10000 replications is 0.320 and the estimate is statistically significant in about half of the simulations. For $N = 500$ the mean of $\hat{\rho}$ is 0.476 and for $N = 1000$ the mean is 0.487. The estimates are statistically significant in almost all simulations. The sample size required to obtain unbiased ML estimates of $\rho$ is about 500. Estimation of $\rho$ on the data from the estimation period in the spatial panel requires much smaller samples. For $N = 25$ observations and $\rho = 0.5$ the mean of $\hat{\rho}$ in the spatial panel is 0.500 and the estimate is statistically significant in all simulations.

Table 2 reports the rejection probabilities of the tests. The rejection probabilities mirror the asymptotic levels in Table 1. In the case of cross sectional independence ($\rho = 0$) all tests have size close to the nominal significance level 5%. Cross sectional correlation causes the tests to be oversized. The rejection probabilities are about 8% when $\rho = 0.1$ and 12% when $\rho = 0.2$. The tests are severely oversized and obtain rejection probabilities close to 30% when $\rho = 0.5$.

The cross sectional test corrected for cross sectional correlation is oversized if the sample size is not large. For $N = 100$ observations and $\rho = 0.5$ the size of the test is 18%. For $N = 500$ the size is 6.2% and for $N = 1000$ the size is 5.4%. The time series test corrected for cross sectional correlation has size 5% for all sample sizes $N$ and all values of $\rho$. The time series test outperforms the cross sectional test, because it uses a more precise estimate of $\rho$. Finally, the crude dependence adjustment test has size close to the nominal level for all sample sizes $N$ and all values of $\rho$.

In the simulations the volatilities and cross sectional correlations of the abnormal returns
are the same in the estimation period and on the event day. Notice that since the time series test and crude dependence adjustment estimate the standard errors from the estimation period, they are not robust to an event-induced increase in the volatilities and cross sectional correlations.
Table 2: Size of tests for abnormal returns under cross sectional correlation. The number of events is \(N\) and the number of groups is \(k\). The table shows the size of the tests over the size of the tests corrected for cross sectional correlation. The nominal level is 5%.

<table>
<thead>
<tr>
<th>(N)</th>
<th>(k)</th>
<th>(\rho = 0)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>0.8</th>
<th>(\rho = 0)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
<th>0.8</th>
<th>(\rho = 0)</th>
<th>0.1</th>
<th>0.2</th>
<th>0.5</th>
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<tr>
<td></td>
<td></td>
<td>Cross sectional test</td>
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<td>Time series test</td>
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<td>0.078</td>
<td>0.112</td>
<td>0.256</td>
<td>0.404</td>
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<td>0.078</td>
<td>0.112</td>
<td>0.243</td>
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<td>0.052</td>
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<tr>
<td></td>
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<td>0.076</td>
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<td>0.049</td>
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<tr>
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<td>0.081</td>
<td>0.117</td>
<td>0.297</td>
<td>0.519</td>
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<td>0.081</td>
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</table>
D. Power

The power of the tests for abnormal returns is estimated by adding a constant to the event day abnormal returns. The power of the tests depends on the volatility of the abnormal returns. Since the volatility turns out not to be of great interest for the impact of $\rho$ on the power, we fix the volatility to the value used in the previous subsections, i.e. an annual volatility equal to 36%. Table 3 shows the power of the tests for $\rho = 0$ and the power of the tests corrected for cross sectional correlation for $\rho \neq 0$. In order to save space, we only show the power for $\rho = 0$ and 0.5. The nominal significance level is 5%.

The corrected tests have lower power when $\rho = 0.5$ compared with $\rho = 0$. Thus there is a loss in power which goes beyond removing the size distortion of the tests. In large samples the loss in power disappears. There are no discernible differences between the power of the different tests. If cross sectional correlation is the main concern, then the tests are equally powerful. For $N = 100$ observations and an abnormal return equal to 1% typical in applications, the power of the tests is almost 1 when $\rho = 0$, whereas the power of the tests corrected for cross sectional correlation is 63% when $\rho = 0.5$. For the smallest numbers of events $N = 25$ and 50, the power of the tests is low when the abnormal return is 0.5%.
Table 3: Power of tests for abnormal returns under cross sectional correlation. The abnormal return is $u$ and is measured in per cent. The number of events is $N$ and the number of groups is the same as in Table 2. The table shows the power of the tests for $\rho = 0$ and the power of the tests corrected for cross sectional correlation for $\rho = 0.5$. The nominal level is 5\%.

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<th>1000</th>
<th>25</th>
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<th>100</th>
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<td>0.996</td>
<td>1.000</td>
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</tr>
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</table>
V. Real Data

In this section two empirical examples involving data from Bear Stearns’ collapse and Lehman Brothers’ bankruptcy in 2008 are provided to illustrate the use of the tests for abnormal returns corrected for cross sectional correlation. We want to find out whether these events had an effect on the financial sector by testing for abnormal returns on the event days.

Data are retrieved from the Center for Research in Security Prices (CRSP) database. We select all firms with Standard Industrial Classification (SIC) code 6. This industry consists of finance, insurance and real estate. Adopting the convention in Section I, the event day is designated ‘day 0’. For a security to be included in the sample, it must have at least 50 observations in the estimation period (defined to be \(-242\) to \(-11\)) and no missing observations in the 30 days around the event date (defined to be \(-19\) to \(+10\)).

The constant-mean-return model and the market model are used to compute the expected returns. The prediction error correction is used to estimate the variance of the abnormal returns in the time series test and in the standardisation of the abnormal returns. The standardised \(J_1\) statistic is corrected using the Patell correction.

The constant-mean-return model and the market model are two different approaches to calculate the normal return, which is the benchmark in tests for abnormal returns. In the former the return is assumed to be constant through time, whereas the latter relates the return to the market’s return. In this application we suggest to use the market model to compute the abnormal returns. The market model removes the cross sectional correlation in the abnormal returns which is due to the market factor and therefore is of the strong cross sectional dependence type. The remaining cross sectional correlation in the abnormal returns is modelled by a spatial autoregressive (SAR) model. Since the SAR model is a model for weak cross sectional dependence, it cannot be used to model cross sectional dependence which is due to the market factor.

The spatial weights matrix \(W\) is constructed by classifying the firms into major groups.
according to the 2-digit SIC codes. We also experimented with other specifications of the weights matrix. Since they give practically the same results, we report only the results for the specification based on the 2-digit SIC codes.

A. Bear Stearns’ Collapse

The first event day is 17 March 2008, which coincides with Bear Stearns’ collapse caused by a loss of confidence following heavy losses in the US subprime mortgage market. The S&P 500 stock market index ended down 0.91% after JP Morgan’s takeover of Bear Stearns. The S&P 500 Financials lost 1.54%.

The sample covers one year of daily returns from 30 March 2007 to 1 April 2008. The total number of firms is 1037 and the number of firms in the sample is 898. Table 4 shows the SIC codes and number of firms in the different groups. The weights matrix $W$ is constructed using (5)–(6) and the information in the table. The number of time series observations is 253 and the number of observations in the estimation period is 232.

Moran’s $I$ is used to test for spatial autocorrelation in the event day abnormal returns $\hat{u}_r$. More precisely, the following $I$ statistic is used:

$$I = \frac{\hat{u}_r' W \hat{u}_r}{\hat{u}_r' \hat{u}_r},$$

which is asymptotically distributed as a standard normal. The results are shown in Table 5. We find that Moran’s $I$ is statistically significant at the 1% level in the constant-mean-return model and market model for the abnormal returns and standardised abnormal returns. Thus there is cross-sectional correlation in the event day abnormal returns.

The estimated value of the spatial autoregressive parameter is 0.649 in the constant-mean-return model and 0.560 in the market model (see Table 5). The estimates are statistically significant at the 1% level. We therefore have a case with moderate spatial autocorrelation in the abnormal returns. The estimate of the spatial autoregressive parameter
is smaller in the market model than in the constant-mean-return model, which is what we would expect. The market factor removes some but not all cross sectional correlations in the abnormal returns. The asymptotic level $\alpha'$ of a nominal 5% test is estimated to be 48% in the constant-mean-return model and 38% in the market model. The size distortion that would occur if the cross sectional correlation in the abnormal returns is ignored is therefore substantial. Similar results are obtained for the standardised abnormal returns.

The estimate of $\rho$ in the spatial panel from the estimation period is of a similar magnitude and is statistically significant in the constant-mean-return model and market model for the abnormal returns and standardised abnormal returns.

The main results in the form of the tests for abnormal returns are presented in Table 6. The mean abnormal return is estimated to be $-1.456\%$ per cent in the constant-mean-return model and $-0.278\%$ per cent in the market model. The standard errors in the cross sectional test assuming cross sectional independence of the abnormal returns are larger than the standard errors in the time series test, which may be taken as evidence of an event-induced increase in the volatilities of the abnormal returns. Comparing the spatial autocorrelation-consistent standard errors with the standard errors assuming cross sectional independence reveals that the spatial autocorrelation-consistent standard errors are larger by a factor of 2, indicating that the off-diagonal elements of the covariance matrix $\hat{\Sigma}_u$ are generally positive. In this example, accounting for cross sectional correlation in the abnormal returns therefore has a large effect on the standard errors. Incidentally, we note that the crude dependence adjustment standard errors are much larger than the spatial autocorrelation-consistent standard errors. However, the crude dependence adjustment standard errors are grossly inflated by some large outliers. If the outliers are removed, the standard errors are more in line with the standard errors in the other tests.

In the constant-mean-return model all tests except the crude dependence adjustment test reject the null hypothesis that the abnormal returns are zero at the 1% level. In this application the market model is our preferred model for deriving the abnormal returns,
as argued in the introduction to the section. In the market model the time series test rejects, whereas the cross sectional and crude dependence adjustment tests do not reject. The tests corrected for cross sectional correlation do not reject the null hypothesis that the abnormal returns are zero. The standardised tests do not reject the null hypothesis that the abnormal returns are zero. Notice finally that the $p$-values of the tests corrected for cross sectional correlation are much larger than those obtained for the tests assuming cross sectional independence. The statistical evidence points towards the conclusion that there is no event effect caused by Bear Stearns’ collapse, whereas the tests assuming cross sectional independence tend to find a spurious event effect.

In an event study we test the effect of an event over and above a benchmark. The finding that there is no event effect in the market model once we account for the cross sectional correlation in the abnormal returns does not mean that Bear Stearns’ collapse is an insignificant economic event. It means merely that there is no statistically significant event effect after controlling for the market’s return.
Table 4: SIC codes and number of firms belonging to the different groups. The table shows the number of firms $n_h$ belonging to group $h$.

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<th>SIC Group</th>
<th>Number of firms</th>
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</tr>
</tbody>
</table>
Table 5: Spatial autocorrelation in the abnormal returns. The table shows Moran’s $I$ statistic and the $p$-value, the estimate of the spatial autoregressive parameter $\rho$ from the event day, $t$-statistic and $p$-value, the estimate of the spatial autoregressive parameter $\rho$ from the estimation period, $t$-statistic and $p$-value. The asymptotic level $\alpha'$ is estimated using the estimate of the spatial autoregressive parameter from the event day. The nominal level is 5%.

<table>
<thead>
<tr>
<th>Model</th>
<th>$I$</th>
<th>$p$-value</th>
<th>$\hat{\rho}$</th>
<th>$t$-stat</th>
<th>$p$-value</th>
<th>$\hat{\rho}$</th>
<th>$t$-stat</th>
<th>$p$-value</th>
<th>$\alpha'$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bear Stearns</strong></td>
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<tr>
<td>Event day 17 March 2008</td>
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<tr>
<td>Abnormal returns</td>
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<td></td>
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</tr>
<tr>
<td>Constant-mean-return</td>
<td>16.207</td>
<td>0.000</td>
<td>0.649</td>
<td>6.195</td>
<td>0.000</td>
<td>0.763</td>
<td>183.137</td>
<td>0.000</td>
<td>0.482</td>
</tr>
<tr>
<td>Market</td>
<td>9.975</td>
<td>0.000</td>
<td>0.560</td>
<td>4.765</td>
<td>0.000</td>
<td>0.594</td>
<td>83.388</td>
<td>0.000</td>
<td>0.383</td>
</tr>
<tr>
<td>Standardised abnormal returns</td>
<td></td>
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</tr>
<tr>
<td>Constant-mean-return</td>
<td>13.812</td>
<td>0.000</td>
<td>0.653</td>
<td>7.037</td>
<td>0.000</td>
<td>0.794</td>
<td>219.205</td>
<td>0.000</td>
<td>0.487</td>
</tr>
<tr>
<td>Market</td>
<td>9.716</td>
<td>0.000</td>
<td>0.569</td>
<td>4.942</td>
<td>0.000</td>
<td>0.672</td>
<td>116.672</td>
<td>0.000</td>
<td>0.393</td>
</tr>
<tr>
<td><strong>Lehman Brothers</strong></td>
<td></td>
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<tr>
<td>Event day 15 September 2008</td>
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<td>Abnormal returns</td>
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<tr>
<td>Constant-mean-return</td>
<td>13.166</td>
<td>0.000</td>
<td>0.630</td>
<td>6.369</td>
<td>0.000</td>
<td>0.762</td>
<td>182.124</td>
<td>0.000</td>
<td>0.460</td>
</tr>
<tr>
<td>Market</td>
<td>−0.038</td>
<td>0.970</td>
<td>−0.086</td>
<td>−0.300</td>
<td>0.764</td>
<td>0.636</td>
<td>99.544</td>
<td>0.000</td>
<td>0.050</td>
</tr>
<tr>
<td>Standardised abnormal returns</td>
<td></td>
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</tr>
<tr>
<td>Constant-mean-return</td>
<td>10.608</td>
<td>0.000</td>
<td>0.583</td>
<td>5.233</td>
<td>0.000</td>
<td>0.795</td>
<td>220.513</td>
<td>0.000</td>
<td>0.408</td>
</tr>
<tr>
<td>Market</td>
<td>0.612</td>
<td>0.540</td>
<td>0.069</td>
<td>0.280</td>
<td>0.780</td>
<td>0.698</td>
<td>131.553</td>
<td>0.000</td>
<td>0.050</td>
</tr>
</tbody>
</table>
Table 6: Tests for abnormal returns. The table shows the standard errors, $J$ statistics and $p$-values over the spatial autocorrelation-consistent standard errors, corrected $J$ statistics and $p$-values.

<table>
<thead>
<tr>
<th>Model</th>
<th>Cross sectional test</th>
<th>Time series test</th>
<th>Crude dep adj</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{u}$ se $J_0$  $p$-value se $J_1$  $p$-value se $J_2$  $p$-value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bear Stearns. Event day 17 March 2008</td>
<td>Abnormal returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant-mean-return</td>
<td>-1.456 0.156 -9.355 0.000 0.099 -14.677 0.000 1.028 -1.417 0.158</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>-0.278 0.157 -1.768 0.077 0.092 -3.032 0.002 0.505 -0.550 0.583</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.353 -0.788 0.431 0.217 -1.281 0.200 - - -</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Standardised abnormal returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant-mean-return</td>
<td>-0.525 0.054 -9.818 0.000 0.034 -15.711 0.000 0.381 -1.379 0.169</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>-0.045 0.062 -0.719 0.472 0.034 -1.350 0.177 0.220 -0.204 0.838</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>0.144 -0.313 0.754 0.098 -0.462 0.644 - - -</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lehman Brothers. Event day 15 September 2008</td>
<td>Abnormal returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant-mean-return</td>
<td>-3.724 0.186 -20.033 0.000 0.135 -27.641 0.000 1.370 -2.718 0.007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>0.318 0.176 1.807 0.071 0.129 2.474 0.013 0.765 0.416 0.678</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- - - - 0.335 0.951 0.342 - - -</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Standardised abnormal returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant-mean-return</td>
<td>-1.057 0.047 -22.469 0.000 0.033 -31.704 0.000 0.390 -2.711 0.007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Market</td>
<td>0.174 0.052 3.348 0.009 0.033 5.355 0.000 0.252 0.691 0.490</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- - - - 0.106 1.695 0.090 - - -</td>
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</tbody>
</table>
B. Lehman Brothers’ Bankruptcy

The second event day is 15 September 2008, which is the day when Lehman Brothers filed for bankruptcy following losses of billions of dollars incurred in the US mortgage market. In the wake of financial turmoil caused by the bankruptcy of Lehman Brothers, the S&P 500 lost 4.71%, marking the biggest point fall since the September 2001 terrorist attacks. The S&P 500 Financials fell by 10.55%. Lehman Brothers’ bankruptcy had much larger repercussions on the economy than Bear Stearns’ collapse.

The sample covers one year of daily returns from 28 September 2007 to 29 September 2008. The total number of firms is 998 and the number of firms in the sample is 903. Table 4 shows the SIC codes and number of firms in the different groups. The number of time series observations and observations in the estimation period are the same as for the first event day, i.e. 253 and 232.

There is evidence of cross sectional correlation in the abnormal returns and standardised abnormal returns on the event day in the constant-mean-return model, indicated by a significant Moran’s $I$ statistic, as shown in Table 5. The estimated value of the spatial autoregressive parameter is 0.630 and is statistically significant at the 1% level. The asymptotic level $\alpha'$ of a nominal 5% test is estimated to be 46%. The results for the market model are different. Given both an insignificant Moran’s $I$ statistic and an insignificant estimate of the spatial autoregressive parameter, we find no evidence of cross sectional correlation.

Similar results are obtained for the standardised abnormal returns. However, estimating the spatial autoregressive parameter $\rho$ in the spatial panel from the estimation period results in a statistically significant estimate of 0.636 in the market model.

Table 6 shows that the mean abnormal return is estimated to be $-3.724$ per cent in the constant-mean-return model, while the market model somewhat surprisingly records a positive abnormal return of $0.318$ per cent. The table reports the tests for abnormal returns. There is some evidence of an event-induced increase in the volatilities, as shown by larger standard errors in the cross sectional test compared to the time series test. Comparing the
spatial autocorrelation-consistent standard errors with the standard errors assuming cross
sectional independence reveals that the spatial autocorrelation-consistent standard errors
are larger by a factor greater than 2. The crude dependence adjustment standard errors are
much larger than the spatial autocorrelation-consistent standard errors. However, as is the
case with Bear Stearns’ collapse, if some large outliers are removed, the standard errors are
more in line with the standard errors in the other tests.

In the constant-mean-return model all tests reject the null hypothesis that the abnormal
returns are zero at the 1% level. There is little evidence of an event effect caused by Lehman
Brothers’ bankruptcy once the abnormal returns are corrected for the market’s return and
cross sectional correlation. Lehman Brothers’ bankruptcy is an event affecting the whole
economy rather than an event confined to the financial sector. The market factor is therefore
able to capture all the cross sectional correlations in the event day abnormal returns (but
not in the abnormal returns in the estimation period). Lehman Brothers’ bankruptcy had
repercussions far beyond the financial sector, but these are accounted for by the market’s
return and there is no statistically significant event effect after controlling for the market’s
return.

VI. Conclusions

In event studies the abnormal returns are assumed to be cross sectionally independent.
Suppose, for example, that we want to test if a major economic event like Bear Stearns’
collapse or Lehman Brothers’ bankruptcy had an effect on the financial sector. In such
cases, since the event day is common and the firms are from the same industry, the abnormal
returns may not be cross sectionally independent. Tests for abnormal returns may find a
spurious event effect if the cross sectional correlation in the abnormal returns is ignored.

In this article we propose a solution to the problem with cross sectional correlation in
the abnormal returns. We define the abnormal returns with respect to a benchmark factor
model. The abnormal returns are modelled by a spatial autoregressive model. Based on
the spatial autocorrelations we derive spatial autocorrelation-consistent standard errors and tests for abnormal returns which are robust against cross sectional correlation. The size and power of the proposed tests are studied by simulation. The tests assuming cross sectional independence are severely oversized if the abnormal returns are cross sectionally correlated. The tests corrected for cross sectional correlation attain the correct size in large samples. Our Monte Carlo results document a tendency for the cross sectional test (corrected for cross sectional correlation) to overreject the null hypothesis in small samples. The time series test corrected for cross sectional correlation is more effective in dealing with cross sectional correlation because it uses a more precise estimate of the spatial autoregressive parameter from the estimation period. The crude dependence adjustment is also robust against spatial autocorrelation in the abnormal returns. The major drawback of the time series test and crude dependence adjustment is that they are not robust against an event-induced increase in the volatilities and cross sectional correlations of the abnormal returns.

We apply our tests to US stock returns around Bear Stearns’ collapse and Lehman Brothers’ bankruptcy. The empirical results show that there is cross sectional correlation in the abnormal returns. The tests assuming cross sectional independence find a spurious event effect. There is little evidence of an event effect caused by Bear Stearns’ collapse and Lehman Brothers’ bankruptcy once the tests are corrected for cross sectional correlation. Our theoretical and empirical results manifest the importance of accounting for cross sectional correlation in event studies.

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Appendix

The $J_0$ statistic in (9) and $J_1$ statistic in (10) can be written as

$$J_i = \frac{\sqrt{N\bar{u}}}{\sqrt{\text{Var}(\sqrt{N\bar{u}})}}, \quad i = 0, 1.$$ 

The square of the denominator of (9) is given by

$$\text{Var}(\sqrt{N\bar{u}}) = \sigma^2 \left( 1 + \frac{1}{N} \sum_{i\neq j} \sum \rho_{ij} \right)$$

and the square of the denominator of (10) is given by

$$\text{Var}(\sqrt{N\bar{u}}) = \frac{1}{N} \sum_{t=1}^N \sigma_t^2 + \frac{1}{N} \sum_{i\neq j} \sum \sigma_{ij},$$

which are the expressions given in the main text.

The derivations of the limiting distributions of the $J_0$ and $J_1$ statistics are based on results of Pesaran and Tosetti (2007, 2011).

**Proof of Theorem 3.** We detail the proof for the $J_0$ statistic. We can write the square of the denominator as

$$\text{Var}(\sqrt{N\bar{u}}) = \sigma^2 \left( 1 + \frac{1}{N} \sum_{i\neq j} \sum \rho_{ij} \right) = \frac{1' \Sigma_u 1}{1'1} \leq \lambda_1(\Sigma_u),$$

where the last inequality follows by the Rayleigh–Ritz theorem (see e.g. Horn and Johnson (1985), p. 176).

It follows from Assumption 1 and $|\rho| < 1$ that the eigenvalues of $\Sigma_u$ are bounded. In particular, the largest eigenvalue of $\Sigma_u$ satisfies the inequality (Pesaran and Tosetti (2007),
\[ \lambda_1(\Sigma_u) < \|\Sigma_u\|_c \leq \sigma_{\text{max}}^2 \left( \frac{1}{1 - |\rho| \|W\|_c} \right) \left( \frac{1}{1 - |\rho| \|W\|_r} \right) < K, \]

which implies that \( u_t \) is a cross sectionally weakly dependent (CWD) process such that (17) tends to a finite limit \( \gamma_0 \).

The final part of the proof concerns the consistency of the estimator \( \hat{\sigma}^2 \) of \( \sigma^2 \). By Theorem 17 of Pesaran and Tosetti (2007), the second term on the right hand side of

\[ \hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^{N} (u_i - \bar{u})^2 = \frac{1}{N} \sum_{i=1}^{N} u_i^2 - \bar{u}^2 \]

tends in probability to 0. The first term tends to \( \sigma^2 \). These convergencies require the condition of finite fourth order moments \( E(u_i^4) < \infty \), a condition which is satisfied since the \( u_i \)s are normal.

The asymptotic normal distribution follows from the fact that since the \( u_i \)s are normal, the numerator \( \sqrt{N} \bar{u} \) is also normal, with mean 0 and variance given by \( \sigma^2 (1 + \gamma_0) \).

The proof for the \( J_1 \) statistic is entirely analogous and is therefore omitted.
Notes

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