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Non-renewable Resources, Endogenous Growth, and Environmental Policy

Master’s Thesis

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Abstract:
I investigate the impact of pollution generated from the utilization of exhaustible resources in the endogenous growth model. The endogenous model is an appropriate treatment of technological change in analyzing a long-term growth with a constraint of polluting non-renewable resources. Under this model, I consider the relationship between environmental policies and technological change. The environmental policies are found to induce technological change which can avoid the depletion of exhaustible resources and the deterioration of the environment. I extensively discuss two approaches focusing on endogenous technological change. Although both approaches are formulated with the constraint of resources and focus on the negative externality, they lead to strikingly different results. An approach by Schou [2000] concludes that an environmental policy is unnecessary since the market behaves optimally. The other approach by Grimaud and Rouge [2005] concludes that the market shows a distortion at the equilibrium and an environmental policy instrument be introduced to correct it. In this approach, it is shown that the growth rate of tax impacts price, quantities, and growth rates.

Finally, I examine the current state of environmental policies that aim to mitigate, and adapt to, climate change, by presenting examples of environmental policy tools.

Keywords: Endogenous growth, Non-renewable resource, Pollution, Technological change, Environmental policy, Tax
To my father
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List of Abbreviations

In the alphabetical order,

- BC  British Columbia
- EPA  Environmental Protection Agency
- ETC  Endogenous Technological Change
- ETS  Emission Trading System
- EU  European Union
- EUR  Euro (Currency)
- GDP  Gross Domestic Production
- GHG  Greenhouse Gas
- GMM  Generalized Method of Moments
- LBD  Learning-by-doing
- MAC  Marginal Abatement Cost
- OECD  Organisation for Economic Co-operation and Development
- R&D  Research and Development
- TC  Technological Change
- UNFCCC  United Nations Framework Convention on Climate Change
List of Symbols

\(A_t\) Stock of knowledge at time \(t\)
\(C_t\) Consumption at time \(t\)
\(K_t\) Capital at time \(t\)
\(L_t\) Labour at time \(t\)
\(P_t\) The flow of pollution at time \(t\)
\(R_t\) The flow of non-renewable resource at time \(t\)
\(r_t\) The rate of interest at time \(t\)
\(S_t\) The stock of non-renewable resource at time \(t\)
\(Y_t\) Production of the basic good at time \(t\)
\(V_t\) The value of innovation at time \(t\)
\(w_t\) Wage at time \(t\)
\(\tau_t\) The tax rate at time \(t\)
\(L_{Y,t}\) The amount of labor devoted to production at time \(t\)
\(L_{RD,t}\) The amount of R&D devoted to production at time \(t\)
\(p_{R,t}\) The price of resource at time \(t\)
\(v_{Y,t}\) The willingness to pay the good sector
\(v_{RD,t}\) The willingness to pay the R&D sector
\(\pi_{Y,t}\) The profit of a firm at time \(t\)
\(F(C_t, P_t)\) or \(F(C_t)\) Aggregate production function
\(U(\cdot)\) Instantaneous utility function
\(U_X\) The partial derivative of the utility function \(U\) with respect to \(X\)
\(\phi(X)\) The flow of pollution generated by \(X\)
\(g_{z_t} = \frac{\dot{z}_t}{z_t}\) The growth rate of \(z_t\)
\(X^*\) The value of \(X\) at the steady-state optimum
\(X^e\) The value of \(X\) at the steady-state equilibrium
1 Introduction

In this paper, I analyze the possibility of positive long term growth with resource scarcity and the need for an environmental instrument. In the twenty first century, it is inevitable for humans to solve two problems closely intertwined and both fundamental to economic prosperity. First, growth precipitates the depletion of exhaustible energy resources. This has led some individuals particularly concerned with the possibility that zero growth was a only sustainable long-run objective. Second, growth induces a deterioration of the environment such as climate change by global greenhouse gas (GHG) emission. As a consequence, how to reduce the pollution caused by the consumption of resources and how to achieve long-run growth rate are among the most pressing policy challenges facing the world today [Acemoglu et al., 2009].

A pledging action to achieve national GHG emission reduction targets under the UNFCCC at Copenhagen and Cancun was an important first step by countries in finding a global solution. The countries which signed the convention have agreed to work collectively to achieve the ultimate objective: stabilization of GHG-concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference of the climate system (Article 2, UNFCC). In 2005, the Kyoto Protocol entered into force and created a legal obligation to limit or reduce their GHG emission between 2008 and 2020 to within agreed emission levels [OECD, 2011a].

Despite some progress in global cooperation, in order to tackle climate change the economies worldwide have to go through transformations in terms of energy production, consumption, transportation, and agricultural patterns. The transition to a low-carbon, climate resilient economy will require significant investment in mitigation and adaptation and a shift of investment from fossil fuels and conventional technologies to newer, cleaner technologies and less carbon-intensive infrastructure [OECD, 2011a]. This adaptation appears in growth models as technological change. This is why technological change is an important question in modeling.

In a same vein, recent research has focused on the interaction between long-run growth and environmental problems. The complex and salient questions remain-
ing in climate change policy modeling is the appropriate treatment of technological change [Gillingham et al., 2008]. The approach to modeling technological change is widely considered to be one of the most important determinants of climate policy analysis in terms of estimating the costs of regulation. There are two main approaches of analysis; the exogenous growth model\textsuperscript{1} and the endogenous growth model\textsuperscript{2}. Many authors in literatures have presented solutions with the endogenous growth model rather than with the exogenous model. I am going to analyze differences between the two models, first, to choose the appropriate approach instead of taking the endogenous model as granted to diagnose the possibility of long run growth. Then, the role of endogenous innovation in mitigating the resource scarcity will be discussed.

In an endogenous growth model, the following question is still raised: does endogenous growth overcome environmental problems in the form of pollution, allowing economic growth to be sustained at a positive rate indefinitely? If the unregulated market economy achieves the optimal extraction of non-renewable resources and growth rates, any intervention may not be necessary. Otherwise, an environmental instrument should be imposed to make the economy behave optimally.

Policies to foster technological change can be categorized into two approaches: command-and-control approach and a market-based one. For example, pollution charges, subsidies, and tradable permits are well known market-based instruments. It can encourage firms or individuals to make efforts to reduce pollution on the basis of their own interests. On the other hand, command-and-control regulations force firms to take allocated shares of pollution-control burden mostly by setting a uniform standard for the firms. In models with external polluting effects, an emission tax will be introduced as an environmental policy.

The paper is organized as follows: first, I present that the approach of the endogenous technological change is more plausible to explain the long-term growth and cope with the problem of the resource scarcity than the exogenous technological change is. In Sections 4 and 5, I introduce two interesting papers with the endogenous growth model. Both of them introduce the constraint of the resources and

\textsuperscript{1}This model takes the rate of technological change as being determined exogenously, by noneconomic forces.

\textsuperscript{2}In this model, technology is an endogenous variable, determined within the economic system.
focus on the negative externality; pollution. The pollution affects two targets: the
production process and the household’s utility. Schou [2000] analyzes the effects
on the first target while Grimaud and Rouge [2005] consider the other, leading to a
significantly different conclusion.

In the framework of Schou [2000], the environmental policy is unnecessary since
the market does not behave sub-optimally. On the other hand, Grimaud and Rouge
[2005] conclude that there is a difference between the optimum and equilibrium.
This is interpreted as non-optimality. To bring equilibrium conditions closer to
optimality, the environmental instruments should be levied. In this context, it is
shown that Hotelling rule is not a pure efficiency condition anymore.

In Section 6 an environmental policy is aimed to correct the distortion at the equi-
librium. It is found that a decreasing tax levied on the resource extraction yields
the optimum, not the tax level. Finally, Section 7 is devoted to comparing two
models of Schou [2000] and Grimaud and Rouge [2005]. Furthermore, I examine
some of the empirical research to disclose the effect of an environmental policy
and the policy instruments currently adopted in many industrialized countries to
tackle climate change.
2 Literature Review

When the standard analysis of economic growth considers the fact that the non-renewable natural resources are depleted over time, it raises a few questions: with resource scarcity, can we induce stable growth in the long run, what is the optimal growth path, and what is the appropriate environmental policy instrument to achieve the optimal condition to correct the distortion introduced at the equilibrium?

There are two main types of models which have contributed to answer these questions. One is the Ramsey growth model for which Stiglitz [1974] or Dasgupta and Heal [1979] can be quoted. Another strand is a group of endogenous growth models. In a series of papers, Romer [1991], Schou [2000], Barbier [1999], and Grimaud and Rouge [2005] have attempted to show that long-term positive growth is possible with technological progress.

In the first type of models, the key optimal point is developed by the Hotelling rule. Hotelling [1931] derives the rule of which the price of non-renewable resource grows at the rate of interest in the equilibrium. The logic behind the Hotelling rule is straightforward. A stock of natural resources is regarded as an asset in the owner’s portfolio. If the growth rate of the resource price rises less than the interest rate does, he will extract and sell the stock of natural resources to invest them to the capital asset. On the other hand, if the growth rate of the resource price is greater than the interest rate, the owner will leave the resources. Hence, the Hotelling rule is verified at the equilibrium, where the equilibrium and optimal paths are identical.

Dasgupta and Heal [1979] point out that the elasticity of substitution between the physical capital and the resource is an important factor. If this elasticity is below unity, the output will tend towards zero in the end as the resource is exhausted. If, on the other hand, it is greater than one, a consumption level, strictly greater than zero, can be sustained infinitely. In the where case the elasticity is one, the feasibility of long-run growth depends on the relative input shares of capital and the resource.

The key feature of the endogenous growth model is that people must be given incentives to improve technology. There is good reason to believe that technological
change depends on economic decisions, for it comes from industrial innovations made by profit seeking firms and depends on the funding of science, the accumulation of human capital, and other such economic activities.

For example, Barbier [1999] shows that technological progress offsets the problems caused by natural resource scarcity with the endogenous growth model. The endogenous growth model presents a truly more optimistic view of the contribution that an economic policy can make for long-run growth, because the rate of technological progress is determined by forces that are internal to the economic system.

In spite of the assumption that endogenous technological change overcomes the resource scarcity, there is one more key feature to be considered in the use of non-renewable resources. The combustion of oil, coal and natural gas emits greenhouse gas and causes the climate change. These negative byproducts raise a problem of economic distortion. While the environmental degradation impacts the household’s utility as well as humans’ welfare negatively as a whole, profit seeking firms do not consider social problems when making a decision. An environmental policy, therefore, is aimed at correcting the distortion introduced at the equilibrium.

Groth and Schou [2007] demonstrate that policies toward the returns to resource conservation influence growth. It shows that a tax on resource use, such as a carbon tax, matters when it is time-varying. In their framework, when it keeps declining over time, it favors conservation and growth. They conclude that resource taxes are decisive for long-run growth whereas traditional capital taxes and subsidies only influence the level.

In [Golosov et al., 2011], the three main factors; damages, discounting, and depreciation, influence the social cost and optimal tax. Golosov et al. [2011] characterize competitive market outcomes with and without taxes. In the model, the optimal per-unit tax on emissions is set to equal to the marginal externality cost of emissions. They assume that there are two sources of energy; oil and coal, and that these are perfect substitutes. In their framework, oil is more efficient than coal, extracted at zero cost, and is finite supply. Coal is extracted using labor (at constant marginal cost) and is in infinite supply. According to their conclusion, the no-tax market economy would empty oil supplies in three decades, while oil would be
used for five decades with the optimal taxation.

Acemoglu et al. [2009] analyze endogenous and directed technical change in a growth model with environmental constraints and limited resources. In the framework of Acemoglu et al. [2009], research can be biased towards improving the technology of machines. They characterize dynamic tax policies that achieve sustainable growth or maximize inter-temporal welfare, as functions of the degree of substitutability between clean and dirty inputs, environmental and resource stocks, and cross-country technological spillovers.

Their framework highlights the central roles played by the market size and the price effects\(^3\) on the direction of technical change. When the two inputs are strong substitutes, redirecting technical change using a temporary policy intervention can be sufficient to avoid a disaster. An important implication of the analysis in this literature is that optimal environmental regulation should always use both a carbon tax to control current emissions and research subsidies. They argue that the optimal policy relies less on a carbon tax and more on direct encouragement to the development of clean technologies.

There has been a long discussion on different environmental instruments. The market based instruments are generally regarded as more efficient in economic terms than command and control measures. In a series of papers, Parry [1997], Fischer et al. [2003], Montero [2002], and more recently Requate and Unold [2003] have attempted to rank environmental policy instruments with respect to their incentive to spur the adoption of advanced, low-polluting technologies.

Parry [1997] analyzes the welfare effect in the market for environmental R&D induced by alternative environmental policy instruments. The induced welfare gain is greater under the emission tax than the tradable emission permit. However, he finds that the empirical significance of this discrepancy crucially depends on the potential size of innovation. In addition, he shows that the efficiency discrepancies between the emission tax and the fixed performance standard are somewhat larger, although again they are very sensitive to the potential size of innovation.

Fischer et al. [2003] find that an unambiguous ranking of policy instruments is

\(^3\)The market size effect encourages innovation towards the larger input sector, while the price effect directs innovation towards the sector with higher price.
not possible. Rather, the ranking of policy instruments depends on the innovator’s ability to appropriate spillover benefits of new technologies to other firms, the costs of innovation, environmental benefit functions and the number of firms producing pollutants. Montero [2002] compares instruments under noncompetitive circumstances and finds that the result is less clear than under perfect competition. Standards and taxes yield higher incentives for R&D when the market is characterized by Cournot competition, but the opposite holds when the market is characterized by Bertrand competition.

Finally, Requate and Unold [2003] demonstrate that the comparison of environmental policies leads to quite different results if the number of firms which adopt new technology is determined endogenously through equilibrium considerations. In the case where the regulator makes long-term commitments to policy levels and does not anticipate the arrival of new technology, taxes provide stronger incentives than permits. Auctioned and free permits offer identical incentives, while standards may give stronger incentive than permits. On the other hand, when the regulator anticipates new technologies, the paper shows that the regulator can induce the first-best outcome with taxes and permits if he moves after firms have invested, whereas this does not hold if he moves first.
3 Exogenous or Endogenous Technological Change?

One of the most complicated and critical questions in environmental policy modeling is the proper treatment of technological change. The endogenous technology change implies that it incorporates a feedback mechanism by which policy encourages clean production. This feedback occurs through energy prices, R&D activities or accumulated production experience (learning-by-doing). In contrast, the exogenous technological change is in line with that the technological progress is independent of economic forces or policy. Then, the starting point is to take the differences of these approaches into account and find the appropriate perspective for environmental policy modeling [Gillingham et al., 2008].

3.1 Exogenous Technological Change

The neoclassical model\(^4\) takes the rate of technological change as being determined exogenously by non-economic forces. Solow [1956] and Swan [1956] first built this model. They show that economic policy can boost an economy’s growth rate by inducing households to save more.

However, the model leads to the pessimistic long-run result of which an increase in growth can not last indefinitely. This is because the marginal productivity is diminishing, given the state of technology. Then, how can the persistent long-run growth that has been observed be explained? Technological change gives an answer implying that it continually offsets the effect of diminishing returns.

The aggregate production function can be written as \(F(K, AL)\) where \(A\) is an exogenous productivity parameter that reflects the current state of technological progress and this parameter grows at the constant exponential rate \(g\). It assumes that the exogenous value of \(g\) reflects progress in science. It allows the stock of capital to grow indefinitely because the effect of diminishing returns is now offset by the continual rise in productivity.

However, the main limitation of the neoclassical model is not to account for the rate of technological progress which can specify the scientific discovery and diffusion

\(^4\)The Cass-Koopmans-Ramsey model is derived in Appendix A.
in the industry. Since the rate of technological progress is exogenously given, one can not analyze how environmental degradation can affect technological change and how technological change can be directed by policy to be more environmental friendly.

### 3.2 Endogenous Technological Change

Industrial innovation is induced by profit seeking firms and depends on the funding of science and the accumulation of human capital. The environmental issues caused by the utilization of the exhaustible resources can make effects on technological progress in an attempt to meet the needs of consumers and a society. In this sense, the endogenous innovation might alleviate resource scarcity and sustain growth in the long run.

However, a few questions still remain: how does the endogenous growth overcome increasing resource scarcity which constrains innovation? Can the stable growth be induced with this constraint, and which environmental policy should be implemented to avoid environmental disasters in the long run? First of all, I introduce endogenous approaches for measuring technological change and discuss the critical aspects of the process of technological change. Furthermore, I diagnose if endogenous technological change can overcome resource scarcity.

The innovation is carried out primarily in private firms. A successful innovation becomes available to other firms and individuals in related fields, of which the process is called diffusion. With the induced innovation approach, firms invest to the new technology actively to produce profitable new products and processes. This decision is made by the firms’ efforts to maximize their value, and it affects the overall rate of technology change. For instance, it occurs through energy prices, R&D activities or accumulated production experience of lowering costs.

Considering that the induced innovation rests on a profit-motivated investment activity, it can be assumed that the level and direction of the innovation respond to the change in relative prices. According to the assumption, the environmental policy which induces more expensive price of inputs can have impacts on technological change toward, for example, carbon-saving technology change.
Furthermore, the amount of induced innovation can be a criterion to evaluate different policy instruments. From this perspective, the induced innovation literature focuses on the potential for an environmental policy to give incentives adopting advanced technology through innovation. Then, how is the technological change made endogenous? What can we learn from these approaches? I aim to incorporate endogenous technological change to specify models for an environmental policy by providing the existing endogenous specification. Moreover, my review is restricted to select papers that illustrate key concepts and provide insight into the theoretical basis for the methodology of modeling endogenous technological change.

Although it is not easy to categorize neatly, the most commonly used approaches of endogenous technological change are direct price-induced, R&D-induced and learning-induced approaches. Direct price-induced technological change means that changes in relative prices can bring forth a new technology to minimize the cost of production. R&D-induced technological change implies that R&D investment affects the rate and direction of technological change. Finally, learning-induced technological change implies that the unit cost of a technology becomes a decreasing function as the experience with the technology is accumulated.

3.2.1 Direct Price-Induced Technological Change

The theory of induced innovation has been strengthened by a number of empirical studies. According to the empirical evidence, the price-induced form of technological change partly can explain why higher energy prices are inclined to extract faster improvements in energy efficiency. In particular, as the price of energy increases, price-induced technological change will result in greater energy efficiency in modeling the climate of policy.

For example, Newell et al. [1999] find that historically the increase in the price of energy accounts for one-quarter to one-half of the observed improvements in energy efficiency for consumer durables from 1958 to 1993. Popp [2002] finds that patenting in energy-related fields increases in response to the increased energy prices. In addition, Popp [2005] finds the interaction between technological change and environmental policy from the empirical literature. According to the result, innovation responds to incentives, exogenous technological change does not capture
the nature of technological change, and the social return to environmental research is high.

However, the reduced-form approach largely has been passed over for the R&D or learning-induced technological change methodologies. I now turn to those approaches in more detail.

3.2.2 R&D-Induced Technological Change

One of the most common approaches to modeling endogenous technological change is R&D-induced technological change. A number of literatures have focused on this approach. The endogenous-growth literature (see, e.g., Romer [1991], Aghion and Howitt [1998] and Acemoglu [1998]) shows that it is important to include a stock of knowledge capital in modeling economic growth. The stock of knowledge can, in a broad range, include information, skills, and experience in production.

There are three key points in this approach. One is whether R&D-induced technological change is relevant to an innovation market imperfection because of spillovers. Another is whether R&D-induced technological change on the purpose of reducing carbon emission crowds out R&D in other industries. Finally, the other one is whether there exists a substitutability or complementarity between the generation of output and the generation of new knowledge [Gillingham et al., 2008].

For the last issue the elasticity of supply of additional R&D plays an important role in R&D activity. If the supply of R&D is relatively inelastic, more effort on carbon saving R&D will result in crowding out R&D activity in other sectors. This behavior of firms has been shown in several models. Nordhaus [2002] as well as Goulder and Schneider [1999] present that environmental policy instruments, such as a subsidy or tax, will decrease R&D and aggregate economic output in the case where there is a relatively inelastic supply of R&D.

Another critical point is the spillover effect. Specifically, it is the degree to which firms fail to achieve the full benefits from investment in R&D. It means that social returns to R&D are relatively high. If a firm is able to obtain most of benefits from the R&D investment, the incentive to undertake the R&D will increase, and the social returns of the R&D will converge with the private returns of a firm. On the
other hand, if a firm makes less profits from the R&D, it will be less inclined to undertake R&D, and high social returns will take place due to spillover effects. There exists a tension between spillovers and crowding out; the former one points out that when endogenous technological change is included, more cost saving can be achieved, whereas the latter one dampens that effect.

### 3.2.3 Learning-Induced Technological Change

This approach is often described as learning-by-doing (LBD). LBD can be represented by the learning curves showing how much a unit cost decreases as a function of experience. The curve intuitively explains the process of learning which influences production. If more experience with one technology is accumulated, its production cost will be lower, and the technology becomes more competitive, resulting in even more accumulated experience.

However, in modeling LBD it is difficult to characterize the underlying mechanisms even though it is intuitively understandable. In spite of its disadvantages, the manageability of learning curves has led to the use of the model based on learning-induced technological change. A common result of including endogenous technological change is that the lower level of carbon tax is needed in models with LBD to achieve carbon mitigation target than in those without LBD.

This result is also intuitive. According to LBD models, R&D expenditure for abatement is not required, and any additional capacity of technologies reducing carbon emissions will decrease the costs of that technology as time goes by, leading to more emissions reduction per unit cost.

There are limitations of each approach to choose the best which corresponds to the purpose and structure of the model and to make assumptions. However, my focus is to show that it is more plausible to build a growth model including endogenous technological change rather than one without it. When technological change is not considered, it tends to overestimate the level of imposing environmental policy instruments [Popp, 2004].
4 Optimal Growth without Environmental Policy

Stiglitz [1974] and Romer [1991] have developed an endogenous growth model incorporating exhaustible resource depletion. The model demonstrates that innovation is endogenously determined by economic forces and endogenous growth overcomes resource scarcity. It implies that sustainable growth can be achieved in the long run even with the condition of natural resource scarcity.

In this section, the baseline framework proposed by Schou [2000] is introduced. Schou [2000] examines how the utilization of the exhaustible resources causing pollution affects the growth in economy. The pollution is considered to affect the production process negatively. For example, the climate change caused by greenhouse gas emission may influence production in agricultural sector. In this model, the source of endogenous growth is the continuous and proportional increase in human capital.

Next, I discuss how flow pollution problems will diminish over time as the economy moves along the balanced growth path. Furthermore, it is analyzed why the result is counter-intuitive that an unregulated market economy will behave optimally with a negative externality.

4.1 The Model

I consider the following standard specifications. At time $t$, production $Y_t$ depends on physical capital $K_t$, the share $u_t$ of its time that the labor force $L_t$ spends in production, the average human capital level $h_t$, the use of the exhaustible resource $R_t$, and pollution $P_t$ caused by the resource extraction. Physical capital $K_t$, labor $L_t$ and the non-renewable resources $R_t$ in the production function exhibit constant returns to scale:

$$Y_t = A_t K_t^\alpha (u_t h_t L_t)^\beta R_t^\gamma P_t^{-\delta} h_t^\theta$$  \hspace{1cm} (1)

with

$$A, \alpha, \beta, \gamma, \delta, \theta \geq 0 \text{ and } \alpha + \beta + \gamma = 1,$$
where $\theta$ is a possible positive externality from the average human capital level on the productivity, and $\delta$ is a negative pollution externality.

It is assumed that the elasticity of the pollution function with respect to resource use is constant. The pollution function follows

$$P_t = D R_t^\lambda,$$

where $D > 0$ and $\lambda > 0$.

Moreover, I impose the assumption that the positive effects of the resource utilization to production outweighs its negative effects, $\gamma > \delta \lambda$. The production function implies that the flow of pollution affects productivity in the economy negatively.

The production is consumed partly by the representative household and invested partly in capital goods with the initial capital $K_0$:

$$\dot{K}_t = Y_t - C_t L_t,$$

where $C_t$ is per capita consumption. Human capital is accumulated linearly depending on time spent on education with the initial human capital $h_0$:

$$\dot{h}_t = \kappa (1 - u_t) h_t,$$

where $\kappa > 0$. The resource is extracted from an initial finite stock $S_0$, and the standard resource stock law of motion is given by

$$\dot{S}_t = -R_t.$$

### 4.1.1 The Optimal Solution

The utility of the representative household only depends on consumption, and it is not affected by pollution in this model. The utility function is iso-elastic:

When $\epsilon > 0$, $\rho > 0$ and $\epsilon \neq 1$,

$$\max \int_0^\infty \frac{C_t^{1-\epsilon} - 1}{1-\epsilon} e^{-\rho t} L_t dt,$$

and when $\epsilon = 1$ and $\rho > 0$,

$$\max \int_0^\infty \log C_t e^{-\rho t} L_t dt,$$
where $\rho$ and $\epsilon$ are the constant rate of time preference and the inverse of the inter-temporal elasticity of substitution, respectively.

In this case, individuals have the same elasticity of substitution $1/\epsilon$ between present and future consumption no matter the level of consumption. This is the key parameter defining the household’s desire to smooth consumption over time, and in this class of utility that desire is independent of the level of consumption.

The social planner will maximize the utility of the representative households by maximizing the following Hamiltonian:

$$
\frac{C_t^{1-\epsilon}}{1-\epsilon}L_t + \mu_t \left[ A_t K_t^\alpha (u_t h_t L_t)^\beta R_t^\gamma P_t^{-\delta} h_t^\theta - C_t L_t \right] - \nu_t R_t + \eta_t (1 - u_t) \kappa h_t.
$$

Taking first-order conditions for an interior optimal solution with respect to the three control variables ($c_t$, $u_t$ and $R_t$) and the three state variables ($K_t$, $S_t$ and $h_t$) are given as follows:

$$
C_t^{\epsilon} = \mu_t, \quad (6)
$$

$$
\nu_t = \mu_t (\gamma - \delta \lambda) \frac{Y_t}{R_t}, \quad (7)
$$

$$
\frac{\mu_t \beta Y_t}{u_t} = \eta_t \kappa h_t, \quad (8)
$$

$$
\frac{\mu_t}{\mu_t} = \rho - \alpha \frac{Y_t}{K_t}, \quad (9)
$$

$$
\frac{\nu_t}{\nu_t} = \rho, \quad (10)
$$

$$
\frac{\eta_t}{\eta_t} = \rho - \frac{(\beta + \theta) \mu_t Y_t}{\eta_t h_t} - (1 - u_t) \kappa, \quad (11)
$$

where $\mu_t$, $\nu_t$ and $\eta_t$ are the shadow values of physical capital, the natural resource and human capital respectively. In order to maximize the utility, the social planner will choose variables $C_t$, $u_t$ and $R_t$ to satisfy the conditions from (6) to (11) and transversality conditions$^5$:

$$
\lim_{t \to \infty} \mu_t K_t e^{-\rho t} = 0, \quad (12)
$$

$$
\lim_{t \to \infty} \nu_t S_t e^{-\rho t} = 0, \quad (13)
$$

$$
\lim_{t \to \infty} \eta_t h_t e^{-\rho t} = 0, \quad (14)
$$

$^5$It states that either capital stock and the natural resource must be zero or it must be valueless. That is, you must not plan to die leaving anything valuable unconsumed.
where $K_t \geq 0$, $S_t \geq 0$, and $h_t \geq 0$. Along the balanced growth path, $Y_t$, $K_t$ and $C_t$ must have the same growth rate which is denoted by $g$. From (4), it is known that $u_t$ must be constant ($= u$) if $h_t$ grows with a constant rate. By differentiating logarithmically with respect to time in (6)–(8) in the steady-state, the following identities are obtained:

$$-\epsilon g = g\mu, \quad (15)$$
$$g\nu = g\mu + g - gR, \quad (16)$$
$$g\mu + g = g\eta + g\rho. \quad (17)$$

When the growth rates are analyzed in the steady-state, the subscript $t$ of the corresponding variable is omitted.

Differentiating with respect to time in the production function (1) in the steady-state and considering (2) yields

$$g = \frac{\beta + \theta}{\beta + \gamma} g\mu + \frac{\gamma - \delta \lambda}{\beta + \gamma} gR. \quad (18)$$

Consequently, the steady-state growth rate for production of basic goods is

$$g = \frac{(\beta + \theta)\kappa - (\beta + \gamma - \delta \lambda)\rho}{\delta \lambda + \epsilon(\beta + \gamma - \delta \lambda)}. \quad (19)$$

The growth rate of human capital follows

$$g_h = \frac{(\beta + \theta)(\beta + \epsilon \gamma + (1 - \epsilon)\delta \lambda)\kappa - \beta(\beta + \gamma)\rho}{(\beta + \gamma)((\delta \lambda + \epsilon(\beta + \gamma - \delta \lambda))}. \quad (20)$$

The growth rate of human capital is derived based on the assumption that labor spends time in both on education activities and the production. It implies that the growth rate of human capital is always positive, but less than the highest possible value $\kappa$, when the share of time spent in production $u$ is zero. Therefore, (20) should satisfy the condition:

$$0 < (\beta + \theta)(\beta + \gamma + (1 - \epsilon)\delta \lambda)\kappa - \beta(\beta + \gamma)\rho$$
$$< (\beta + \theta)(\delta \lambda + \epsilon(\beta + \gamma - \delta \lambda))\kappa, \quad (21)$$

since

$$0 < g_h < \kappa.$$
This implies that it is required to have appropriate values of parameters $\epsilon$ and $\rho$. $\epsilon \geq 1$ is a sufficient condition to hold the right inequality sign. According to the condition, the growth rate of resource extraction will be negative:

$$g_R = \frac{(1 - \epsilon)(\beta + \theta)\kappa - (\beta + \gamma)\rho}{\delta \lambda + \epsilon(\beta + \gamma - \delta \lambda)}$$  \hspace{1cm} (22)

For the derivations of (19), (20) and (22), see Appendix B.

Note that $g$ is always smaller than $g_h$ when there is no positive externality from the average human capital level on the productivity ($\theta = 0$). In addition, $g_h$ is constrained to be positive given (21), but the sign of $g$ is not determined. It means that if the representative household values more current consumption relative to future consumption and the elasticity of the exhaustible resource in production is relatively large, the long-run growth rate of consumption $g$ will be negative. In this case, although the stock of knowledge increases as people keep educating themselves, the positive impact on the production will be offset by the drain in the resource stock.

This is in line with Stiglitz [1974]. The presence of a resource in fixed supply that is indispensable in production does not necessarily imply that production will eventually go towards zero, when there is technological potential that can offset the negative influence of the resource. However, the optimality of declining consumption in the long run remains a possibility, depending on the relative size of the parameters representing discounting of the future on the one hand, and technology on the other hand.

### 4.1.2 Comparative Optimal Growth

The effect of the resource scarcity on the growth rate of consumption $g$ can be seen by taking derivative with respect to $\gamma$:

$$\frac{\partial g}{\partial \gamma} = \frac{-(\epsilon \kappa (\beta + \theta) + \delta \lambda \rho)}{(\delta \lambda + \epsilon(\beta + \gamma - \delta \lambda))^2} < 0.$$  \hspace{1cm} (23)

Here, the level of $\gamma$ indicates the importance of the resource in production. It is found that the resource should be saved to avoid exhausting it completely in finite time as the less input of the resource will lead to a negative effect to growth. The
negative effect is larger, the more important is the natural resource in the production technology.

When it comes to $\delta$, the effect of an increase in $\delta$ on $g$ will be

$$\frac{\partial g}{\partial \delta} = \frac{\lambda((\beta - 1) + (\beta + \gamma)\rho)}{(\epsilon + \beta + \gamma - \delta \lambda)^2} > 0. \tag{24}$$

Notice that the more important the negative impact of pollution on production, the larger long-run growth rate will be. Also, the larger the size of $\gamma$, the smaller the growth rate of production. $\delta$ affects the production in an opposite way, reducing the positive contribution of the resource to production. If $\delta$ rises, the non-renewable resource will be used less overtime. It will lead to not only the positive productivity effect from utilizing the resource but also less negative effect of pollution to the economy.

A higher value of $\delta$ means that the detrimental impacts on the growth rate of the consumption, caused by the need to keep diminishing resource extraction, will decrease. As a sequence, the growth rate of $Y_t$ will increase. This result opens the possibility for the long-run sustainable growth in the finite resource stock.

The analysis, therefore, shows that the environmental problem by pollution will diminish over time, and the larger is the negative impact of pollution (a rise in $\delta$) on the production, the higher the growth rate of $Y_t$ will be.

At last, the influence of $\delta$ on the growth rate of human capital and resource extraction will be following:

$$\frac{\partial g_R}{\partial \delta} = \frac{\beta + \theta}{\beta} \frac{\partial g_h}{\partial \delta} = \frac{-g_R(1 - \epsilon)\lambda}{\delta \lambda + \epsilon(\beta + \gamma - \delta \lambda)}. \tag{25}$$

The effect of parameter $\delta$ is depicted in Table 3. If $\epsilon$ is large, the economy will invest less in education and extract the resource more to make current generations better off. On the other hand, if $\epsilon$ is small, more patient agents invest more in education and concern preservation of the resource for future.

### 4.2 The Market Case

In the market economy, each firm does not consider pollution. A single firm has a negligible impact on the environment. It also does not take into account the positive
Table 3: Property of the growth rate of human capital and resource

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon &lt; 1$</th>
<th>$\epsilon = 1$</th>
<th>$\epsilon &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial g_R}{\partial s}$</td>
<td>$&gt; 0$</td>
<td>$= 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$\frac{\partial \gamma}{\partial s}$</td>
<td>$= 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The resource owners maximize their profits by setting that the price of the resource is equal to its marginal product. On the competitive natural resource market, the maximization of the profit yields the standard equilibrium “Hotelling rule”:

$$r_t p_t = \dot{p}_t,$$

(26)

where $r_t$ is the interest rate, and $p_t$ is the price of one unit of the resource. The resource owners must be indifferent between selling the resource and keeping it in the ground. It implies that the rate of return to selling the resource and investing the revenues at the rate $r_t$ is equal to the price rise if the resource is kept in the ground. In the market equilibrium, the initial resource price will adjust instantaneously to the unique value which ensures that the resource is exhausted exactly as time goes to infinity [Dasgupta and Heal, 1979].

In the market, the price of the resource is higher than its social value since only the positive effect on production is taken into consideration, not the externality from pollution. To see if it affects the extraction rate of the resource, (7) is modified as follows:

$$\nu_t = \mu \gamma \frac{Y_t}{R_t}.$$  

(27)

The first-order condition of the Hamiltonian to maximize the utility of the representative household is the same as before (with $\theta = 0$). Using (27), the same growth rates in production (19), human capital (20), and resource (22) are derived. It can be seen by that differentiating (7) and (27) with respect to time leads to the
same result (16). Therefore, without the consideration of the externality the inter-temporal distortion in the resource extraction will not take place. It implies that the market achieves the socially optimal condition.

The consequence is based on the fact that the total resource is finite and the assumption that the net contribution of a unit of the resource to production is always positive, considering the externality ($\gamma > \delta \lambda$). As a result, all the resource stock will be exhausted in both cases of the social planner and the market economy. According to this, the allocation of the whole resource extraction over time periods is the only thing that could possibly be suboptimal in the market.

By introducing an environmental instrument in the model, we can see if the market brings the optimum to the economy without an impact on the inter-temporal allocation of the resources. Here, a tax is levied on the firm with the amount that corresponds to the negative impact of pollution. A traditional emission tax on each pollutant is, hence, equal to

$$\tau_t = \delta \lambda \frac{Y_t}{R_t}. \quad (28)$$

It is assumed that the tax is less than the direct marginal product of the resource $Y_R = \frac{\partial Y_t}{\partial R_t}$ whereas it increases with the same rate. If tax is imposed, the firms would not pay the marginal product $Y_R$ for a unit of the resource, but only

$$p_t = Y_R - \tau_t. \quad (29)$$

However, the change of the price for a resource does not affect the amount the resource owners sell. They can expect that the tax will increase over time with the proportion of $Y_R$. Following the Hotelling rule, the individual resource owner will set the initial price in which the resource will be depleted when time goes to infinity. It implies that the initial price of the resource will decrease with the amount of the tax. It will lead to the same demand as before. Therefore, the levied tax plays a role as a constant profit tax for the resource owners.

In other words, tax has no impact on the inter-temporal allocation of resources, and no distortion takes place, not changing the behavior of the resource owners. It can be concluded that a green tax does not affect the environmental condition and the investment decisions. The market economy, therefore, behaves optimally without a regulation.
5 Welfare Analysis with Environmental Policy

In this section, I introduce the endogenous growth model constructed in Grimaud and Rouge [2005]. Grimaud and Rouge [2005] focus on the presence of non-renewable resources whose use in the production process generates a flow of pollution that has negative impacts on the household’s utility.

First of all, the general solutions are presented. At the optimum, it is found that the Hotelling rule is no longer a pure efficiency condition by introducing an externality caused by the utilization of polluting non-renewable resources. Then, it is shown that the market equilibrium features a socially suboptimal resource extraction rate. The non-optimal condition in the equilibrium implies that an environmental instrument is necessary in order to correct the economic distortion and to make the equilibrium condition closer to the optimality.

5.1 The Model

At each time, the production of basic good $Y_t$ depends on the amount of labor devoted to production $L_{Y,t}$, the stock of knowledge $A_t$ and the flow of non-renewable resources $R_t$:

$$ Y_t = F(L_{Y,t}, A_t, R_t). $$

I denote the marginal productivities of $L_{Y,t}$, $A_t$ and $R_t$ by $F_L$, $F_A$ and $F_R$, respectively. At each time, the stock of knowledge evolves as follows:

$$ \dot{A}_t = q(A_t, L_{RD,t}), $$

where $L_{RD,t}$ is the amount of labor devoted to R&D. I denote by $q_A$, $q_L$ the marginal productivities. The technologies $F(\cdot, \cdot, \cdot)$ and $q(\cdot, \cdot)$ have constant returns to scale in private inputs.

It is assumed that population is constant and normalized to one:

$$ 1 = L_{Y,t} + L_{RD,t}. $$
In the resource sector, there is an initial finite stock $S_0$, and at each time the amount $R_t$ is extracted according to the standard resource stock law of motion:

$$\dot{S}_t = -R_t.$$  \hspace{1cm} (33)

Produced goods are entirely consumed by the representative household at each time $t$:

$$Y_t = C_t.$$  \hspace{1cm} (34)

The instantaneous utility function of the household is affected by both consumption $C_t$ and the flow of pollution $P_t$. It is based on an assumption that pollution reduces the household’s utility. Therefore, I set up the inter-temporal utility function as

$$U_0 = \int_0^\infty U(C_t, P_t)e^{-\rho t}dt.$$  \hspace{1cm} (35)

The partial derivatives of the utility function with respect to consumption $C_t$ and pollution $P_t$ are denoted by $U_C$ and $U_P$: $U_C > 0$, and $U_P < 0$.

The use of the non-renewable resource generates pollution in the production process. In other words,

$$P_t = \phi(R_t),$$  \hspace{1cm} (36)

where the derivative of $\phi$ is positive ($\phi' > 0$).

In this model, pollution does not have a negative effect on production like in the model of Schou [2000]. Instead, the negative effect is felt by the households. For example, if climate change is thought to cause natural disasters and high oil prices, consumers will increasingly desire to reduce greenhouse gas emissions.

### 5.2 Welfare Analysis

The social planner maximizes the inter-temporal utility function (35) subject to constraints in (30)-(34) and (36). In order to find the solution to this problem, the current-value Hamiltonian is defined as

$$H : U(F(L_Y,t, A_t, R_t), \phi(R_t)) + \mu_t q(A_t, L_{RD,t}) - \nu_t R_t,$$  \hspace{1cm} (37)
where \( \mu_t \) and \( \nu_t \) are the co-state variables. By eliminating the co-state variables, the first order conditions reduce to the two following characteristic conditions:

\[
\rho - \frac{\dot{U}_C}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_{AQL}}{F_L} + q_A, 
\]  

(38)

and

\[
\frac{\dot{F}_R}{F_R} - \frac{U_P \phi'}{U_C F_R} \left( \rho - \frac{U_P \phi'}{U_P \phi'} \right) = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_{AQL}}{F_L} + q_A, 
\]  

(39)

where \( X_{\bar{y}} \) and \( X_{\bar{y}y} \) are shorthand notations for \( \frac{\partial X}{\partial \bar{y}} \) and \( \frac{\partial^2 X}{\partial \bar{y} \partial y} \), respectively.

Both the conditions are derived in Appendix C.

Equations (38) and (39) are modifications of the Ramsey-Keynes model and Hotelling rule\(^6\), respectively. Contrary to the standard neoclassical growth model, there is no physical capital, but a stock \( A_t \) of knowledge. Moreover, the Hotelling rule is not strictly a pure efficiency condition anymore when considering pollution. The term \( \left( \rho - \frac{U_P \phi'}{U_P \phi'} \right) \) in (39) explains that extracting one unit of resource not only allows to produce more, but also increases pollution that reduces utility.

5.3 Specification and Steady-State Analysis

In order to study the effect of an environmental instrument later, the standard specification needs to be considered. Production and R&D technologies have constant returns to scale in private inputs. The production function is written as:

\[
Y_t = L_\alpha R_t^{1-\alpha} A_t^\nu, 
\]  

(40)

where \( 0 < \alpha < 1 \) and \( \nu > 0 \).

The stock of knowledge follows, with a constant \( \sigma > 0 \),

\[
\dot{A}_t = \sigma L_{RD,t} A_t. 
\]  

(41)

It is assumed that the level of emission is a linear function of extraction flows:

\[
\phi(R_t) = \gamma R_t, 
\]  

(42)

\(^6\)The percentage change in net-price per unit of time should be equal to discount rate \( r \) in order to maximize the present value of the resource capital over the extraction period in an efficient exploitation of a non-renewable resource \( \left( \frac{\dot{p}}{p} = r \right) \). [Hotelling, 1931].
where $\gamma > 0$.

The utility of the representative household depends on consumption and pollution. The household, hence, has the separable instantaneous utility function:

$$U(C_t, P_t) = \frac{C_t^{1-\epsilon}}{1-\epsilon} - \frac{P_t^{1+\omega}}{1+\omega},$$

(43)

where $\epsilon > 0$ and $\omega > 0$.

When $g_{z_t}$ is denoted as the growth rate, $\frac{dz_t}{z_t}$, of any variable $z_t$, from (40) the growth rate of production $g_{Y_t}$ is

$$g_{Y_t} = \alpha g_{L_Y,t} + (1 - \alpha)g_{R_t} + \nu \sigma (1 - L_{Y,t}).$$

(44)

For its derivation, see Appendix D.

Using (40)–(44), the following proposition is defined.

**Proposition 1.** At the steady-state optimum, the values of the quantities and growth rates are given by

1. $L_0^{R_D} = \frac{(\sigma \nu - \rho \alpha)(\epsilon (1 - \alpha) + \alpha + \omega)}{\sigma \nu (\epsilon + \omega (1 - \alpha + \epsilon \alpha))},$

2. $L_0^{Y} = 1 - L_0^{R_D},$

3. $g_0^{A} = \sigma L_0^{R_D},$

4. $g_0^{R} = g_0^{P} = g_0^{S} = \frac{(\sigma \nu - \rho \alpha)(1 - \epsilon)}{\epsilon + \omega (1 - \alpha + \epsilon \alpha)},$

5. $g_0^{C} = g_0^{Y} = \frac{(\sigma \nu - \rho \alpha)(1 + \omega)}{\epsilon + \omega (1 - \alpha + \epsilon \alpha)}.$

The superscript $o$ is used for the optimum.

**Proof.** See Appendix E.

The initial quantities $Y_0^o$ and $R_0^o$ in this model are

$$Y_0^o = C_0^o = (L_Y^o)^{\alpha} (-S_0 g_R^o)^{1-\alpha} (A_0)^{\nu},$$

(45)

$$R_0^o = \frac{P_0^o}{\gamma} = -S_0 g_R^o,$$

(46)

given $S_0$ and $A_0$. (46) is readily obtained by dividing (33) by $S_t$. 24
The interior optimum exists when the following conditions hold:

\[ L_{RD}^o < 1, \quad (47) \]
\[ g_R^o < 0, \quad (48) \]

while \( L_{RD}^o \) is positive. These conditions are fulfilled if

\[ \epsilon > 1 \quad (49) \]

and

\[ \rho < \frac{\delta \nu}{\alpha}. \quad (50) \]

Figure 1 illustrates the existence of interior optimum. The derivation of the conditions and their consequences are shown in Appendix F.

In this model, the parameter \( \sigma \) indicates the effectiveness of the R&D sector in (41). It means that the higher \( \sigma \) is, the larger the labor in research \( L_{RD}^o \) and the growth rate of knowledge \( g_{RD}^o \) are. On the other hand, the elasticity \( \epsilon \) of marginal utility affects them in an opposite way. That is, as \( \epsilon \) increase, more utility is derived from uniform consumption paths. It leads to the social planner to allocate less labor to R&D and more to production. As a consequence, both the consumption growth rate \( g_C^o \) and the growth rate of resource extraction \( g_R^o \) will be lowered.

In the standard literature of the endogenous growth model, an increase in \( \rho \) means that current utility is more valued relative to utility in the future. Therefore, in the steady-state, the growth rate of output will decrease, as more output today means less tomorrow. According to this preference to consume more today, a social planner will increase both the utilization of resource \( R^o \) and the amount of labor \( L_{Y}^o \) into the production process. It also leads to a decrease in \( L_{RD}^o \) and the growth rate of knowledge in (31), \( \dot{A}_t = q(A_t, L_{RD,t}) \).
Table 4: The properties of the optimal path. The entries that are different from the equilibrium path are marked in boldface.

<table>
<thead>
<tr>
<th>ξ = δ</th>
<th>ξ = ε</th>
<th>ξ = ρ</th>
<th>ξ = ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>∂L/∂RD</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>∂L/∂g_A</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
</tr>
<tr>
<td>∂g_R/∂ξ</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>∂g_Y/∂ξ</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>

In the framework of Grimaud and Rouge [2005], there is a fundamental difference from the standard growth model in terms of the impact on $g^o_R$ by change in $\rho$. It is based on that the utility function depends on not only an amount of consumption but also pollution level. In case where current time is more valued (increase in $\rho$), it also means that the current state of environment is valued more. That is, $P^o_0$ should decrease. It means that the first generation reduces the pollution to increase welfare. As a social planner decreases the use of resource today and increases it tomorrow, both $g^g_{P}$ and $g^g_{R}$ increase contrary to the model without pollution.

In addition, the parameter $\omega$ in (43) reflects the household’s concern for a cleaner environment. Thus, an increase in $\omega$ implies less pollution today, but more tomorrow. It results in an increase of the growth rate of resource extraction. Then, to make up for the fall in production with less resource use, the planner will increase $L^o_Y$ today and decrease $L^o_{RD}$. Furthermore, from (40) we get $g^o_Y = (1 - \alpha)g^o_R + v g^o_A$ at the steady state. The combination of both effects on $g^o_R$ and $g^o_A$ by an increase in $\omega$ results in a domination of the $g^o_R$ which leads to a rise in $g^o_Y$. The results are summarized in Table 4.

5.4 Decentralized Equilibrium

5.4.1 Behavior of Agents and Equilibrium Conditions

I include private agents (the firms, households and government) into the model in order to study the market equilibrium. It is assumed that firms do not take into consideration the negative externality by the use of the non-renewable resources. In order to correct the market failure, an environmental policy, a carbon tax, is
implemented.

Consumption good sector: At time $t$, the firm’s profit is:

$$\pi_{Y,t} = F(L_{Y,t}, A_t, R_t) - w_t L_{Y,t} - p_{R,t}(1 + \sigma_t) R_t,$$

where $w_t$, $p_{R,t}$ and $\sigma_t$ are the wage, the resource price, and the unit tax on the resource use at time $t$, respectively. I will use $\tau_t = 1 + \sigma_t$ for mathematical convenience.

By differentiating $\pi_{Y,t}$ with respect to $L_{Y,t}$ and $R_t$, the first order conditions are obtained:

$$F_L = w_t, \quad (52)$$

$$F_R = \tau_t p_{R,t}. \quad (53)$$

It is assumed that knowledge is not embodied in intermediate goods and directly financed. Moreover, I suppose that the government finances R&D activities entirely.

At each time the value of innovation is:

$$V_t = \int_0^{\infty} v_s e^{-\int_s^t r_u du} ds, \quad (54)$$

where $v_s$ is the sum of the willingness to pay of the good sector $v_{Y,s}$ and the R&D sector $v_{RD,s}$ at time $s$, and $r_u$ is a discount rate at time $u$. The profit on innovations is set up at time $t$ as follows:

$$\pi_{RD,t} = q(A_t, L_{RD,t}) V_t - w_t L_{RD,t}. \quad (55)$$

A necessary first-order condition for maximizing this profit function with respect to $L_{RD,t}$ is

$$q_L V_t = w_t. \quad (56)$$

From the firm’s profit (51) and the profit on innovation (55), we get

$$v_{Y,t} = \frac{\partial \pi_{Y,t}}{\partial A_t} = F_A,$$

$$v_{RD,t} = \frac{\partial \pi_{RD,t}}{\partial A_t} = q_A V_t. \quad (57)$$
Hence,

\[ v_t = F_A + q_A v_t. \]  (58)

**Representative household**: The representative household owns resource-extracting firms. The inter-temporal utility function in (35) subject to

\[ \dot{B}_t = w_t + r_t B_t + p_{R,t} R_t - T_t - C_t \]

is maximized, where \( B_t \) denotes the stock of bonds at \( t \), and \( T_t \) is a lump-sum tax imposed by the government. It can be done by maximizing

\[ H = U(C_t, P_t) + \lambda \{ w_t + r_t B_t + p_{R,t} R_t - T_t - C_t \}, \]  (59)

where \( \lambda \) is the co-state variable. This maximization leads to the following condition:

\[ \rho - \frac{U_{CC} \hat{C} + U_{CP} \hat{P}}{U_C} = r_t. \]  (60)

**Resource sector**: The profit function in the competitive resource market is

\[ \int_0^\infty p_{R,s} R_s e^{-\int_s^t r_u du} ds \] subject to \( \dot{S}_t = -R_t \). The maximization problem yields the standard equilibrium, the Hotelling rule:

\[ \frac{\hat{p}_{R,t}}{p_{R,t}} = r_t. \]  (61)

**Government**: The government budget constraint is, for each time \( t \),

\[ T_t + \sigma_t p_{R,t} R_t = (v_{Y,t} + v_{RD,t}) A_t. \]  (62)

**Remark 1**. The characteristic equilibrium conditions under a market are

\[ \rho - \frac{U_{CC} \hat{C} + U_{CP} \hat{P}}{U_C} = \frac{\hat{F}_L}{F_L} - \frac{\hat{q}_L}{q_L} + \frac{F_{AqL}}{F_L} + q_A, \]  (63)

\[ \frac{\hat{F}_R}{F_R} - \frac{\hat{\tau}_t}{\tau_t} = \frac{\hat{F}_L}{F_L} - \frac{\hat{q}_L}{q_L} + \frac{F_{AqL}}{F_L} + q_A. \]  (64)

**Proof.** See Appendix G. □

28
Now, these two conditions can be compared to the optimal ones, (38) and (39). Note the difference between (39) and (64). In the absence of the environmental policy, the optimal Hotelling rule cannot be derived. This result comes from the negative externality cost of pollution which is not included in the resource price in this model. However, if the household’s utility does not depend on pollution \((U_P = 0)\), these two different characteristic conditions (39) and (64) become identical, and the first best optimum is achieved.

### 5.4.2 Specification

Using (40)–(43), the conditions in (38) and (64), respectively, become

\[
\rho + (\epsilon - 1)g_{Y_t} = -g_{L_{Y,t}} + \frac{\sigma\nu}{\alpha} L_{Y,t} \quad \text{and} \quad -g_{R_t} - g_{\tau_t} = -g_{L_{Y,t}} + \frac{\sigma\nu}{\alpha} L_{Y,t}.
\]

Together with (44), it leads to the growth rate of tax:

\[
g_{\tau_t} = \frac{g_{Y_t} - g_{R_t} - \nu\sigma}{\alpha}.
\]

\(g_{Y_t}\) and \(g_{R_t}\) are constant with respect to \(t\) if \(g_{\tau_t}\) is constant. Then, based on the assumption that \(g_{\tau_t}\) is constant (= \(g_{\tau}\)), the steady-state equilibrium is characterized by Proposition 2.

**Proposition 2.** At the steady-state equilibrium, the quantities and rates of growth take the following values:

1. \(L^e_{RD} = \frac{\sigma\nu(\epsilon(1 - \alpha) + \alpha) - \alpha\rho + \alpha(\epsilon - 1)(1 - \alpha)g_{\tau}}{\epsilon\sigma\nu}\),
2. \(L^e_{Y} = \frac{\sigma\nu(\epsilon - 1) + \alpha\rho - \alpha(\epsilon - 1)(1 - \alpha)g_{\tau}}{\epsilon\sigma\nu}\),
3. \(g^e_A = \sigma L^e_{RD}\),
4. \(g^e_R = g^e_P = g^e_S = \frac{\sigma\nu(1 - \epsilon) - \rho}{\epsilon} - \frac{\alpha\epsilon + 1 - \alpha}{\epsilon} g_{\tau}\),
5. \(g^e_C = g^e_Y = \frac{\sigma\nu - \rho}{\epsilon} - \frac{1 - \alpha}{\epsilon} g_{\tau}\).

The superscript \(e\) denotes the equilibrium.
Similarly to the case of steady-state optimum, the initial quantities are

\[
Y^e_0 = C^e_0 = (L^e_Y)^\alpha (-S^e_0 g^e_R)^{1-\alpha} (A^e_0)^\nu, \tag{66}
\]

\[
R^e_0 = \frac{P^e_0}{\gamma} = -S^e_0 g^e_R, \tag{67}
\]

where at each time \(x^e_t = x^e_0 e^{g^e x t}\) for any variable \(x\).

It is assumed that there is no environmental policy \((\tau_l = 1)\). As done for the optimal path, the set of parameter values is found in which \(0 < L^l_{RD} < 1\) and \(g^l_R < 0\) such that the equilibrium path exists. To do so, 1. and 4. in Proposition 2 are used. The set is illustrated in Figure 2.

I now observe the impact of different parameters’ variation on \(L^e_{RD}, g^e_A, g^e_R\) and \(g^e_Y\). The properties of the equilibrium path are mostly similar to the properties of the optimal one. There are only two differences. First, an increase in \(\rho\) decreases the growth rate of the use of the non-renewable resources \(g^e_R\). If \(\rho\) is higher, the representative household wants to consume more in current time. Correspondingly, firms will produce more today using more resources and labor. Secondly, \(\omega\) does not affect the equilibrium variables. \(\omega\) corresponds to the household’s concern for a clean environment. Since the resource market does not take pollution into account, pollution is not priced without an environmental policy, and, thus, \(\omega\) has no effects on market. The properties at the equilibrium are listed in Table 5.
Table 5: The properties of the equilibrium path. The entries that are different from the optimal path are marked in boldface.

<table>
<thead>
<tr>
<th></th>
<th>( \xi = \delta )</th>
<th>( \xi = \epsilon )</th>
<th>( \xi = \rho )</th>
<th>( \xi = \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{\partial r_{RE}}{\partial c} )</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>= 0</td>
</tr>
<tr>
<td>( \frac{\partial r_{A}}{\partial c} )</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>= 0</td>
</tr>
<tr>
<td>( \frac{\partial r_{R}}{\partial c} )</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>= 0</td>
</tr>
<tr>
<td>( \frac{\partial r_{Y}}{\partial c} )</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td>= 0</td>
</tr>
</tbody>
</table>

6 Impact of the Environmental Policy

6.1 General Analysis

Next, it is considered how the optimum path can be implemented. By comparing (39) and (64), it is clear that the optimum path can be achieved if the growth rate of the tax is levied by

\[
g_{\tau,t}^o = -\frac{U_P \phi' F R}{U_C \phi' - \rho}.
\]

(68)

The term \( -\frac{U_P \phi'}{U_C F R} \) can be seen as the relative disutility of resource extraction which is, therefore, positive. That is, \( g_{\tau,t}^o \) has the same sign as the term \( \left( \frac{U_P \phi'}{U_C \phi'} - \rho \right) \). As before, it is assumed that \( \phi' \) is constant (\( \phi(R_t) = \gamma R_t \)), and \( U_P = 0 \) meaning that the disutility of pollution is constant. Then, \( g_{\tau,t}^o \) is negative. In this case, tax is levied more today than tomorrow, which implies that the household postpones the use of resource. Moreover, a rise in \( \rho \) causes \( g_{\tau,t}^o \) to decrease, which means that the more people prefer the present, the more present pollution will be regulated by the tax.

6.2 Optimum versus Equilibrium

I found that both the optimal path and the equilibrium path are jointly defined if \( \epsilon > 1 \) and \( \rho < \sigma \nu / \alpha \). Under these conditions, the following proposition can be derived by comparing \( g_{R}^e \) and \( g_{R}^o \), and \( g_{Y}^e \) and \( g_{Y}^o \).
Proposition 3.

\[ g_R^e < g_R^e \]  \hspace{1cm} (69)
\[ g_Y^e < g_Y^e \]  \hspace{1cm} (70)

In this model, if there is no pollution, the decentralized equilibrium is a first best optimum. If there is pollution, but no tax, the equilibrium does not change since firms do not consider pollution. That is, the equilibrium is equivalent to the first best optimum without pollution.

The paths taken at the equilibrium and optimum are illustrated in Figure 3. First, the resource growth, or negative extraction, rate at time \( t \) at the equilibrium and optimum are, respectively,

\[ R^e_t = R^e_0 e^{g^e_R t}, \]
\[ R^o_t = R^o_0 e^{g^o_R t}. \]

They are both exponentially decreasing functions with respect to \( t \), since \( g^e_R \) and \( g^o_R \) are negative due to the transversality conditions. Also, it should be reminded that they are monotonically decreasing and approach zero as \( t \to \infty \).

Given an initial amount of stock \( S_0 \), the initial values of the two functions are

\[ R^e_0 = -S_0 g^e_R > R^o_0 = -S_0 g^o_R \]
due to the inequality in Proposition 3 ((A) in Figure 3). Hence, the equilibrium path stays higher than the optimal path until the two curves cross each other at a unique, finite moment \( t^* \) ((B) in Figure 3):

\[ t^* = \frac{\log R^e_0 - \log R^o_0}{g^e_R - g^o_R} > 0, \]

since

\[ \log R^e_0 - \log R^o_0 > 0, \]

and

\[ g^o_R - g^e_R > 0. \]
Figure 3: Optimum vs. Equilibrium without any public intervention. (t: time; \( R_t \): resource extraction) Reprinted from Grimaud and Rouge [2005].

At any time \( t > t^* \), however, the equilibrium path always stays lower than the optimal path, essentially approaching 0 faster ((C) in Figure 3).

Figure 3 shows the path of exhaustible resources at the optimum and decentralized equilibrium, respectively. As people value their environmental condition more heavily, the resource is depleted more slowly: \( g_{R}^{o} \) increases. Conversely, when they do not care about pollution, \( g_{R}^{o} \) will decrease. Likewise, at the decentralized equilibrium, the resource is extracted faster since the externality is not priced.

### 6.3 Impact of the Environmental Policy

**Proposition 4.**

- A change in the tax level has no impact on the equilibrium, and
- A change in the tax’s growth rate affects prices, quantities and growth rates.

When considering a market for which the demand is \( D = D(p\tau) \), and the supply \( S \) is fixed, we have \( D(p\tau) = S \). \( p \) is price, and \( \tau = 1 + \sigma \), where \( \sigma \) is a unit tax. It
leads to $p\tau = D^{-1}(S)$ which is constant. Hence, an increase in $\tau$ does not affect the quantities but only results in a decrease in $p$.

On the other hand, a change in $g_\tau$ affects the entire path. Using 1.-5. in Proposition 2, we can see how $g_\tau$ impact the variations of the parameters in Table 6.

<table>
<thead>
<tr>
<th>$\xi = L_{RD}$</th>
<th>$\xi = g_\tau^e$</th>
<th>$\xi = g_R^e$</th>
<th>$\xi = g_Y^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\partial \xi}{\partial g_\tau} &gt; 0$ if $\epsilon &gt; 1$</td>
<td>$&gt; 0$ if $\epsilon &gt; 1$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

Table 6: Effect of the environmental policy

If $g_\tau$ decreases, it leads to an increase in $g_R^e$, which lowers the initial value $R_0^e$. Then, the slope of the equilibrium path becomes less steep, which indicates that the resource is extracted is at a lower path.

From the production function (40), $Y_t = L_{Y,t}^\alpha R_{t}^{1-\alpha} A_t^\nu$, and the two first-order conditions (52)–(53), $F_L = w_t$ and $F_R = \tau_0 p R_{t}$, the equilibrium prices are given by

$$w_t = \alpha (L_{Y}^e)^{-\frac{1}{\alpha}} (R_{t}^e)^{1-\alpha} (A_t^e)^\nu,$$

$$p_{R,t} = (1 - \alpha)(L_Y^e)^{\alpha}(R_t^e)^{-\alpha}(A_t^e)^\nu(\tau_0 e^{g_{\tau,t}})^{-1}.$$ (71) (72)

The price ratio $\frac{\tau_0 p_{R,t}}{w_t}$ is a decreasing function of $g_\tau$. Thus, the lower $g_\tau$ is, the higher $\tau_0 p_{R,t}$ is, compared to the wage. It implies that if a tax is levied more today, the price firms should pay more for the use of resource today. It causes firms to use the resource less today and more tomorrow.

This effect of the tax can be illustrated by considering the equilibrium path of $R_t^e$ (See Figure 4). The decrease in the growth rate of tax $\Delta g_\tau < 0$ implies the increase in $g_R^e$ ($\Delta g_R^e > 0$), since $\frac{\partial g_R^e}{\partial g_\tau} < 0$ in Table 6. The initial value $\tilde{R}_0^e$ of the new resource growth rate is, then,

$$\tilde{R}_0^e = -S_0(g_R^e + \Delta g_R^e) < R_0^e,$$

which means that the initial value decreases toward the optimal path ((A) in Figure 4).

As mentioned previously, the resource growth rate is a monotonically decreasing function. A finite moment $t^*$ at which the old $R_t^e$ and the new $\tilde{R}_t^e$ cross each other
Figure 4: Impact of the environmental policy in a decrease in $g_r$. Reprinted from Grimaud and Rouge [2005].

is

$$t^* = \frac{\log R_0^e - \log \tilde{R}_0^e}{\Delta g_R^e} > 0,$$

since

$$\log R_0^e > \log \tilde{R}_0^e$$

and

$$\Delta g_R^e > 0.$$

The existence of a unique, finite $t^*$ implies that the new path will approach zero more slowly than the original equilibrium path ((B) in Figure 4), hence, resulting in a slower extraction of the resource. Therefore, it may be said that an environmental instrument brings the equilibrium condition closer to the optimal condition.
7 Comparison and Empirical Analysis of Innovation

Schou [2000] introduces an endogenous growth model with a constraint of non-renewable resources causing pollution problems. He examines how the results of the growth model are affected by the inclusion of pollution and resource scarcity. In the framework of Schou [2000], the externality has negative effects on the production process.

The results are different from typical findings in several ways. In this model, the pollution declines over time, because there is a need to save the non-renewable resources. Moreover, it turns out that the negative effect of the pollution on the production process does not prevent the economy from reaching the optimal growth path. It is even shown that the more important is the negative impact of pollution on the production, the higher the optimal long-run growth will be.

In contrast, Grimaud and Rouge [2005] take into consideration the effect generated by the pollution on the household’s utility. In this context, it was shown that there are differences between the equilibrium and optimal condition. It analyze that resources are extracted too rapidly in the steady-state equilibrium compared to the optimal rate.

Subsequently, it argues that the environmental policy is a necessary instrument in order to correct the equilibrium condition and make it closer to optimality. Another contribution is the result that the level of tax does not matter, but the distortion can only be corrected by choosing an optimal growth rate of the tax.

There exist two common assumptions in Grimaud and Rouge [2005] and Schou [2000]. One is that pollution is considered as a flow variable, not a stock. Another is that firms do not take into account the negative externality since their contribution to the aggregate pollution level is negligible.

The main difference between the two papers concerns the assumed effect of pollution. In Grimaud and Rouge [2005], pollution is considered to affect the household’s utility. According to the analysis of Grimaud and Rouge [2005], if pollution affects the household’s utility, a disparity takes place between the socially optimal growth path and the market economy. It is the so-called economic distortion.
Therefore, an environmental policy is aimed at correcting the distortion introduced at the equilibrium.

On the other hand, Schou [2000] focuses on the negative impact of pollution on the production process. In the framework of Schou [2000], a market economy without any regulation leads to the optimum in the presence of a pollution externality. He, therefore, concludes that environmental policy is unnecessary, considering the fact that the resource stock is finite, and the externality has no impact on the optimal inter-temporal allocation of the resource. It can be summarized that the market achieves the optimum as firms respond to pollution by themselves, by using less resources in the production process.

The approach of Grimaud and Rouge [2005] to the endogenous growth theory appears to be more useful than the simpler approach of Schou [2000] in addressing the question of sustainable development. For it reflects the current trend, in which consumers increasingly concern about the environmental condition, and the institutions of international cooperation to implement a sustainable growth policy are organized.

According to the assumption of Grimaud and Rouge [2005], the utility function depending on both consumption and pollution reflects social phenomena. It is important to consider the fact that consumers become increasingly aware of environmental problems. For instance, it can be intuitively understood that climate change, which causes the natural disaster, and high oil price encourage consumers to reduce the utilization of polluting non-renewable resources. Another example is the finding reported by OECD [2011b] that the suitable knowledge and information about the environmental problems have profound effect on the behavior of consumers. OECD [2011b] presents the following remark on how the public policies affect the household behavior, or utility:

*As consumers account for 60% of final consumption in the OECD area, their purchasing decisions have a major impact on the extent to which markets can work to promote green products. However, their decisions to buy green depends on the financial cost of green options and the infrastructure to support such choices; the quality and reliability of information on the products, and the knowledge consumers*
have of environmental issues. Industry, government, and civil society can play an important role in creating the enabling environment for consumers to make greener purchasing choices.

Recent OECD work on environmental policy and household behavior is exploring the factors driving household’s environment-related decisions in order to inform policy design and implementation. A survey of over 10,000 households across 10 OECD countries (Australia, Canada, Czech Republic, France, Italy, Korea, Mexico, the Netherlands, Norway, and Sweden) confirms the impact of economic incentives on household behavior and the important complementary role played by information-based measures such as energy-efficiency labeling of appliances and housing.

The findings confirm the importance of providing the right economic incentive to spur behavioral changes, in particular in energy and water savings. The evidence also indicates that pricing consumption by volume is partially useful; the mere act of metering and introducing a price on the use of natural resources has an effect on people’s decision making. The survey indicates that softer instruments, such as information to consumers and public education, can play a substantial complementary role. Eco labels are particularly useful, as long as they are clear and comprehensible, and that they identify both public and private benefits. These soft instruments need to be given close attention in developing more comprehensive strategies for influencing consumer and household environmental behavior.

In addition, the following empirical result presents evidence that it is not sufficient to passively count on technological change to solve environmental problems. There are extensive studies focusing on the effect of policy instruments on the innovation of energy-efficiency technologies. In particular, recent empirical papers have presented evidences showing that energy price affects the choice of types of innovations.

Newell et al. [1999] study the specific case of the air-conditioning industry. They examine how much home appliances had responded to the change of energy price with respect to energy efficiency between 1958 and 1993 using a model of induced
Table 7: Induced-Innovation Regression Results. Reprinted from Popp [2002].

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Unweighted Stock of Patents</th>
<th>Weighted Stock of Patents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-9.016 (-12.362)</td>
<td>-7.311 (-46.625)</td>
</tr>
<tr>
<td>Energy prices</td>
<td>0.028 (2.146)</td>
<td>0.060 (2.852)</td>
</tr>
<tr>
<td>Lagged knowledge stock</td>
<td>0.719 (25.612)</td>
<td>0.838 (72.323)</td>
</tr>
<tr>
<td>Government R&amp;D</td>
<td>0.006 (0.968)</td>
<td>-0.009 (-1.741)</td>
</tr>
<tr>
<td>Truncation error</td>
<td>1.924 (2.445)</td>
<td>-1.203 (-5.054)</td>
</tr>
<tr>
<td>Lambda</td>
<td>0.933 (18.905)</td>
<td>0.829 (13.662)</td>
</tr>
<tr>
<td>Long-run energy elasticity</td>
<td>0.421</td>
<td>0.354</td>
</tr>
<tr>
<td>Long-run government R&amp;D elasticity</td>
<td>0.085</td>
<td>-0.052</td>
</tr>
<tr>
<td>Median lag</td>
<td>13.81</td>
<td>4.86</td>
</tr>
<tr>
<td>Mean lag</td>
<td>9.92</td>
<td>3.71</td>
</tr>
<tr>
<td>GMM criterion</td>
<td>86.560</td>
<td>93.421</td>
</tr>
<tr>
<td>Number of technology groups</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

Notes: This table shows the induced-innovation regression results. Lagged party of the president and lagged government R&D are used as instruments for government R&D. A time trend and lagged values of other exogenous variables are used as instruments for the knowledge stocks. t-statistics appear in parentheses below estimates. Data are from 1971-91.

Innovation. In their framework, it is found that the changes in energy price induced both the commercialization of new models and the elimination of old models.

In detail, they find that the technological change in air conditioners was biased against the energy efficiency in the 1960s when the energy price was falling. However, this phenomenon was reversed after the two energy shocks in 1970s. In addition, it is found that the energy efficiency in 1993 would have been approximately one-quarter to one-half lower in air conditioners if the energy price had remained at their 1973 levels.

Popp [2002] takes into consideration the effect of energy price on energy-saving innovations using U.S. patent data from 1970 to 1994. For that purpose, the set of subclassifications is sorted into eleven distinct technology group. It includes six groups pertaining to the energy supply, such as solar energy, and five groups related
to energy demand, such as methods of reusing industrial waste heat.

The dependent variables of the innovation regression are the ratios of the number of successful non-government U.S. patent applications for technology field $i$ in year $t$ over the total number of successful non-government U.S. patent applications in the same year. In addition, the following variables are used in the model as independent variables: (1) the price of energy in that year; (2) the stock of knowledge that had been accumulated by the previous year; and (3) technology-specific variables, government R&D expenditures, and a dummy for the lagged political party of the president.

The regression results are presented in Table 7. The first column shows the result using an unweighted count of past patents as the knowledge stock. The second column presents the result using a weighted stock of patents to represent knowledge. The regression results show that both price and the stock of knowledge available to inventors play an important role in inducing new energy innovations. The coefficients of the lagged knowledge stock are positive and significant in each regression.

The most significant result is the strong, positive impact energy price has on new innovations. This finding suggests that environmental tax and regulations not only reduce pollution by shifting a behavior away from polluting activities but also encourage the development of new technologies that make pollution control less costly in the long run. The results also make it clear that it is not enough to simply rely on technological change as a panacea for environmental problems. There must be some mechanisms in place that encourage new innovation. To sum up, innovation is necessary, but it is not sufficient for unlimited sustainable growth when environmental pollution and non-renewable resources are taken into account.

The analysis of Grimaud and Rouge [2005] uncovers some conditions under which growth can possibly be sustained with the constraints imposed by the finiteness of resource, but it has not addressed the critical questions of what policies might implement the optimal sustainable growth paths that have been found. In the next chapter, environmental policy instruments will be compared with respect to their effect on technological change and investigated in terms of which policy is more appropriate to induce the innovation.
8 Environmental Policy Instruments and Their Current Status

Many industrialized countries have already implemented environmental policy instruments to reduce climate change. The objectives of these actions are to achieve the Kyoto commitments and long-term emission reduction targets. These targets are aimed to spur investment to lower carbon outcome and build a climate resilient economy. Table 8 shows the examples of policy tools for climate change mitigation at the national level.

Credible and long-term price on carbon emission across the economy through market-based instruments, such as emission trading schemes or carbon taxes, is necessary to drive investment in low-carbon technologies. It penalizes carbon-intensive technologies and processes, creates markets for low-carbon technologies and stimulates action in the energy, industry, transport, and agriculture sectors. Putting a price tag on carbon can also help trigger green innovations and enhance energy efficiency [OECD, 2010].

8.1 Emission Trading Scheme

Under emission trading systems (ETS) a government sets a limit, or a cap, on the amount of a pollutant that can be emitted. The limit is allocated or sold to firms in the form of emission permits which represent the right to emit or discharge a specific volume of a specified pollutant. Firms are required to hold the number of permits equivalent to their emission. The total number of permits cannot exceed the cap, limiting total emissions. Firms that need to increase their emission must buy permits from those who require fewer permits. In effect, a buyer is paying charge for polluting, while a seller is being rewarded for having reduced emissions [OECD, 2011a].

ETS is becoming increasingly important in the climate policy portfolio. In the last ten years, almost all Annex I parties have either established or strengthened the existing trading schemes and are in some ways participating in either national or international carbon markets [UNFCCC, 2011, Hood, 2010]. Nevertheless, there
### Table 8: Examples of policy tools for climate change mitigation. Reprinted from de Serres et al. [2010].

<table>
<thead>
<tr>
<th>Category</th>
<th>Examples</th>
</tr>
</thead>
</table>
| Price-based instruments       | Taxes on CO₂ emissions.  
                                 | Taxes on inputs or outputs of process (energy or vehicles).  
                                 | Removal of environmentally harmful subsidies (e.g., for fossil fuels).  
                                 | Subsidies for emissions-reducing activities.  
                                 | Emissions trading systems (cap-and-trade or Baseline-and-credit). |
| Command and Control Regulations | Technology standards.  
                                 | Performance standards.  
                                 | Prohibition or mandating of certain products or practices.  
                                 | Reporting requirements.  
                                 | Requirements for operating certification.  
                                 | Land-use planning, zoning. |
| Technology policies support    | A robust intellectual property rights system.  
                                 | Public and private R&D funding.  
                                 | Public procurement of low-carbon products and services.  
                                 | Green certificates (e.g., renewable portfolio standard).  
                                 | Feed-in tariffs for electricity from renewable.  
                                 | Public investment in infrastructure for new low-carbon technologies.  
                                 | Policies to remove financial barriers to green technology  
                                 | Capacity building for the workforce, infrastructure development. |
| Information and voluntary approaches | Rating and labeling programs.  
                                 | Public information campaigns.  
                                 | Education and training.  
                                 | Product certification and labeling.  
                                 | Award schemes. |
are several issues that need to be considered in order to increase the environmental effectiveness and economic efficiency or permit trading [OECD, 2008a]. The initial allocation of the emission allowance, and ways of limiting the transaction costs are the remaining issues of this system.

The world’s largest emission trading system is the EU-ETS. It has led to building an international carbon market. Another good example is the New Zealand ETS. It covers all six Kyoto Protocol Gases from energy, transport, industry, waste, synthetic gases, and forestry.

8.2 Carbon Tax

Carbon tax is a cost effective way to reduce emission. The tax provides incentives for polluters and resource users to change their behavior today. It also provides long-term incentives to innovate [OECD, 2008b].

Carbon tax is usually applied to fuel and electricity so that their price reflects their CO₂-emission factors. Carbon tax is currently used in 10 OECD countries, with Denmark, Finland, the Netherlands, Norway, Sweden, and the United Kingdom leading these efforts since the early 1990s [OECD, 2009]. Sweden was one of the first countries to introduce a carbon tax in 1991, with a general level of the tax increasing over the years to reach EUR 111 per ton in 2010. A positive side effect of the Climate Change Levy in the U.K., which taxes industrial and commercial GHG-emitting power production, has been to stimulate innovation [OECD, 2010]. Those companies that pay a lower than normal tax rate under the negotiated Climate Change Agreements have registered fewer patents for inventions to tackle...
climate change than those that pay the full levy. In Canada, British Colombia (BC) has had a carbon tax in place since 2008. The carbon tax is a critical component of Climate Action Plan in BC in reducing GHG emissions by 33% by 2020 [OECD, 2011a].

8.3 Command-and-Control Instrument

Regulations can be appropriate when a market cannot provide price signals to individuals or organizations that reflect the costs of polluting behavior. In the transport sector, for instance, CO$_2$-emission standards are increasingly mandatory and have been widely implemented in many countries.

Regulations are used to reduce the emission of gases that are subject to the Montreal Protocol on Substances that Deplete the Ozone Layer. For example, Australia has the Ozone Protection and Synthetic Greenhouse Gases Management regulation; the European Union has directives on fluorinated gases, mobile air conditioning, and integrated pollution prevention and control; and the United States has Significant New Alternatives Program. Their regulations have also long contributed to landfill methane emission reductions as well as industrial N$_2$O and HFC reduction. For instance, industrial N$_2$O emission in France was cut by 90% in the 1990s through those schemes [OECD, 2011a].
9 Conclusion

This paper aimed to investigate the incentives provided by the environmental policy instruments to encourage the adoption of an advanced abatement technology. My point of departure was to analyze the impact of pollution generated from exhaustible resources in the endogenous growth model.

First, the following problem was considered: what is the proper treatment of technological change? I presented the limitation of the neoclassical model which takes the rate of technological change as being determined exogenously by non-economic forces. Without technological change an economy can grow for a while, but eventually the growth will stop with a diminishing marginal product of capital.

With this limitation, I focused on showing that it is more plausible to build a growth model including endogenous technological change. Different processes were presented which induce technological progress and industrial innovations firms make: price-induced technological change, R&D, and learning-by-doing.

Therefore, I introduced the two endogenous growth models incorporating exhaustible resource depletion proposed by Schou [2000] and Grimaud and Rouge [2005]. An economy achieves the optimum without an environmental policy in the framework of Schou [2000]. On the other hand, Grimaud and Rouge [2005] showed that there is a difference between the equilibrium and optimum.

Considering the perspective of Grimaud and Rouge [2005], this non-optimality justifies the implementation of an environmental policy so that the environmental policy corrects the distortion in the model caused by polluting emission. A unit tax on polluting resources was chosen to deal with the problem. Applying it to the model, it was shown that the growth rate of tax matters, not its level in order to correct the distortion in the economy.

In Section 7, these two approaches, by Schou [2000] and Grimaud and Rouge [2005], were compared. They shared two common assumptions that pollution is a flow, not a stock and that the contribution of a single firm on the aggregate pollution level is negligible. The major difference between the two models came from their approaches to the effect of the pollution. Schou [2000] concerned the impact on the production process whereas Grimaud and Rouge [2005] focused on the
effect on the household’s utility. By comparing Schou [2000] and Grimaud and Rouge [2005], it was shown that the approach of Grimaud and Rouge [2005] is more plausible in the current condition of society and the changing behavior of households.

In the last section, the empirical studies were presented to show that an environmental regulation is an effective tool to reduce pollution and induce technological change. Moreover, some of existing environmental policy instruments adopted in developed countries were introduced.
10 Bibliography


A Exogenous Growth Rate

This section presents the Ramsey model [Ramsey, 1928], as elaborated by Cass [1964] and Koopmans [1963]. Technological progress is added to the Cass-Koopmans-Ramsey model to make growth sustainable in the long run.

Consider a representative who lives infinitely. The utility function is

$$ W = \int_0^{\infty} e^{-\rho t} U(C_t) dt, $$  \hspace{1cm} (73)

where \( \{C_t\} \), \( U(\cdot) \) and \( \rho \) are the time path of consumption, an instantaneous utility function and a positive rate of time preference, respectively.

The special isoelastic case is assumed:

$$ U_t = \frac{C_t^{1-\epsilon} - 1}{1 - \epsilon}, $$  \hspace{1cm} (74)

where \( 0 \leq \epsilon < 1 \). In this case, individuals have the same elasticity of substitution between present and future consumption. It means that the household desire to have smooth consumption over time.

The aggregate production function can be written as \( F(K_t, A_t L_t) \) where \( A_t \) is an exogenous productivity parameter that reflects the state of technological progress at time \( t \), and it grows at the constant exponential rate \( g \).

Suppose the aggregate production function in the Cobb-Douglas form

$$ F(K_t, A_t L_t) = (A_t L_t)^{1-\alpha} K_t^\alpha. $$  \hspace{1cm} (75)

This form shows that technological progress is equivalent to an increase in the effective supply of labor \( A_t L_t \) which grows at the rate of growth of population plus the growth rate of productivity \( n + g \). To simplify the analyses we assume a constant labor force \( L = 1 \). Then the aggregate production function becomes \( F(K_t, A_t) \).

We will maximize \( W \) subject to the constraint that consumption plus investment equals the net national product:

$$ \dot{K_t} = F(K_t, A_t) - \delta K_t - C_t. $$  \hspace{1cm} (76)
As the utility of the representative households should be maximized, this problem is traditionally written in terms of the Hamiltonian:

\[ H : U(C_t) + \lambda \left[ F(K_t, A_t) - \delta K_t - C_t \right], \]

The necessary condition for maximizing this with respect to \( C_t \) is, then,

\[ \frac{\partial H}{\partial C_t} = 0. \] (77)

It shows that the marginal utility of consumption equals the shadow value of investment is:

\[ U'(C_t) = \lambda. \] (78)

The state variable \( K_t \) has the condition

\[ \dot{\lambda} = \rho \lambda - \frac{\partial H}{\partial K_t}, \] (79)

or equivalently

\[ \rho \lambda = \lambda (F_K(K_t, A_t) - \delta) + \dot{\lambda}, \] (80)

where \( F_K = \frac{\partial F}{\partial K} \).

Finally, the transversality condition\(^{10}\) must hold:

\[ \lim_{t \to \infty} e^{-\rho t} K_t \lambda = 0. \] (81)

Substituting for \( \lambda = U'(C_t) \) in (80), we can get

\[ \frac{U''(C_t)}{U'(C_t)} \dot{C}_t = \rho - [F_K(K_t, A_t) - \delta]. \] (82)

Suppose we define the marginal rate of return \( r(K_t) \) as the extra net output that the household could get from a marginal unit of capital:

\[ r(K_t) = \frac{\partial}{\partial K_t} [F(K_t, A_t) - K_t \delta] = F_K(K_t, A_t) - \delta. \] (83)

According to the utility function, the preceding equation (80) becomes

\[ \frac{\dot{C}_t}{C_t} = \frac{F_K(K_t, A_t) - \delta - \rho}{\epsilon}. \] (84)

\(^{10}\)It states that inputs used for the production, either the capital stock or natural resource must be zero or it must be valueless not leaving anything valuable unconsumed before you die.
We have $F(K, A) = AF(K/A, 1)$. Differentiating both sides with respect to $K$, we get $F_K(K, A) = F_K(K/A, 1)$ which shows that the marginal product of capital is a function of the ratio $K/A$. According to this, the assumption that $F$ exhibits constant returns implies that the marginal product $F_K$ depends on the ratio $K/A$. Therefore, $K_t$ and $A_t$ can both grow at the exogenous rate $g$ presented.
B Growth Rates under Endogenous Model

Let us start by restating the production function (85):

$$Y_t = A_t K_t^\alpha (u_t h_t L_t^\beta) R_t^\gamma P_t^{-\delta} h_t^\theta$$  \hspace{1cm} (85)

with

$$A, \alpha, \beta, \gamma, \delta, \theta \geq 0.$$  

We get

$$\begin{align*}
(\beta + \gamma)g &= (\beta + \theta)gh_t + (\gamma - \delta \lambda)gR_t \\
\iff g &= \frac{\beta + \theta}{\beta + \gamma}gh_t + \frac{\gamma - \delta \lambda}{\beta + \gamma}gR_t \\
\iff (\beta + \gamma)g &= (\beta + \theta)gh_t + (\gamma - \delta \lambda)gR_t
\end{align*}$$  \hspace{1cm} (86)

by differentiating the production function with respect to time $t$. It should be noted that $P_t$ was replaced by $DR_t^\lambda$ according to (2) which states

$$P_t = DR_t^\lambda.$$  

With the identity $-\epsilon g = g_{\mu_t}$ in (15), the growth rate of the shadow value of natural resource $\nu_t$ given in (16) becomes

$$\begin{align*}
g_{\nu_t} &= \rho = -\epsilon g + g - gR_t \\
&= (1 - \epsilon)g - gR_t,
\end{align*}$$  \hspace{1cm} (87)

where $\rho$ is a shorthand notation defined in (10). This is equivalent to

$$gR_t = (1 - \epsilon)g - \rho.$$  \hspace{1cm} (88)

From (8) it is easy to see that

$$\eta_t h_t = \frac{\mu_t \beta Y_t}{u_t B_t}.$$  

Plugging this into (11), we get

$$g_{\eta_t} = \frac{\dot{\eta}_t}{\eta_t} = \rho - \frac{\beta + \theta}{\beta} u_t B_t - (1 - u_t) B_t$$  \hspace{1cm} (89)
which is the growth rate of the shadow value of the human capital.

The following identities follow from (4):

\[ g_{ht} = \frac{\dot{h}_t}{ht} = (1 - u_t)B_t, \]  
\[ u_t = 1 - \frac{g_{ht}}{B_t}, \]  
\[ g_{ht} = (1 - \epsilon) \left( \frac{\beta + \theta}{\beta + \theta} \right) u_t B_t - g_{ht}, \]

where \( g_{ht} \) is the growth rate of the human capital \( h_t \). Using the first identity (90), (89) becomes

\[ \dot{\eta}_t = \frac{\dot{\eta}_t}{\eta_t} = \rho - \frac{\beta + \theta}{\beta} u_t B_t - g_{ht}. \]

Equating the above formula of \( \dot{\eta}_t \) with

\[ \dot{\eta}_t = g_{\mu t} + g - g_{ht}, \]

which comes from (17), we get

\[ (1 - \epsilon)g = \rho - \frac{\beta + \theta}{\beta} (B_t - g_{ht}), \]

where we used the second identity (91). Equivalently, we may write

\[ g_{ht} = B_t + \frac{\beta (1 - \epsilon)}{\beta + \theta} g - \frac{\beta \rho}{\beta + \theta}. \]  
\[ g_{ht} = \frac{(\beta + \theta)B_t - (\beta + \gamma - \delta \lambda)\rho}{\delta \lambda + \epsilon(\beta + \gamma - \delta \lambda)} \]

which is the growth rate of the production presented earlier in (19). Similarly, we obtain the following growth rates of the human capital and natural resource by simply replacing \( g \) in (88) and (93) with (19):

\[ g_h = \frac{(\beta + \theta)(\beta + \epsilon \gamma + (1 - \epsilon)\delta \lambda)B_t - \beta(\beta + \gamma)\rho}{(\beta + \gamma)(\delta \lambda + \epsilon(\beta + \gamma - \delta \lambda))}, \]

\[ g_R = \frac{(1 - \epsilon)(\beta + \theta)B_t - (\beta + \gamma)\rho}{\delta \lambda + \epsilon(\beta + \gamma - \delta \lambda)}. \]

As usual, the time index \( t \) of the variables \( h \) and \( R \) have been omitted as this derivation assumes the steady-state.
C Non-specified Optimality Conditions

Here the derivation of the characteristic conditions (38) and (39) from the Hamiltonian in (37) is presented.

Given the Hamiltonian $H$, the first-order conditions $\frac{\partial H}{\partial L} = 0$ and $\frac{\partial H}{\partial R_t} = 0$ yield

$$- U_C F_L \mu_t q_L = 0 \iff \mu_t q_L = U_C F_L \iff \mu_t = \frac{U_C F_L}{q_L} \tag{94}$$

$$U_C F_R + U_P \phi' - \nu_t = 0 \iff \nu_t = U_C F_R + U_P \phi'. \tag{95}$$

Furthermore, $\frac{\partial H}{\partial A_t} = \rho \mu_t - \dot{\mu}_t$ and $\frac{\partial H}{\partial s_t} = \rho \nu_t - \dot{\nu}_t$ result in

$$U_C F_A + \mu_t q_A = \rho \mu_t - \dot{\mu}_t \iff \frac{\dot{\mu}_t}{\mu_t} = \rho - \frac{F_A q_L}{F_L} - q_A \tag{96}$$

$$0 = \rho \nu_t - \dot{\nu}_t \iff \frac{\dot{\nu}_t}{\nu_t} = \rho \tag{97}$$

Differentiating a logarithm of (94) with respect to time, we get

$$\frac{\dot{\mu}_t}{\mu_t} = \frac{\dot{U}_C}{U_C} + \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} \tag{98}$$

Before proceeding, it should be reminded that $\dot{U}_C = U_{CC} \dot{C} + U_{CP} \dot{P}$ and $\dot{U}_P = U_{PC} \dot{C} + U_{PP} \dot{P}$. Together with (96), it follows that

$$\rho - \frac{F_A q_L}{F_L} - q_A = \frac{\dot{U}_C}{U_C} + \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} \iff \rho - \frac{\dot{U}_C}{U_C} = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_A q_L}{F_L} + q_A$$

which is the first characteristic condition given in (38).

The derivation of a logarithm of (95) with respect to time is

$$\frac{\dot{\nu}_t}{\nu_t} = \frac{\frac{d}{dt} (U_C F_R + U_P \phi' \dot{\phi})}{U_C F_R + U_P \phi' \dot{\phi}}$$

$$= \frac{1}{U_C F_R + U_P \phi' \dot{\phi}} \left( U_C F_R \left( \frac{\dot{U}_C}{U_C} + \frac{\dot{F}_R}{F_R} \right) + U_P \phi' \left( \frac{\dot{U}_P}{U_P} + \frac{\dot{\phi}}{\phi} \right) \right) \tag{99}$$

With (97), one obtains

$$\rho \left( U_C F_R + U_P \phi' \dot{\phi} \right) = U_C F_R \left( \frac{\dot{U}_C}{U_C} + \frac{\dot{F}_R}{F_R} \right) + U_P \phi' \left( \frac{\dot{U}_P}{U_P} + \frac{\dot{\phi}}{\phi} \right)$$

$$\iff \rho - \frac{\dot{U}_C}{U_C} = \frac{\dot{F}_R}{F_R} + \frac{U_P \phi' \dot{\phi}}{U_C F_R} \left( \frac{\dot{U}_P}{U_P} + \frac{\dot{\phi}}{\phi} - \rho \right) \tag{100}$$
With the first characteristic condition (38), (100) becomes

\[
\frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{F_Aq_L}{F_L} + q_A = \frac{\dot{F}_R}{F_R} + \frac{U_P\phi'}{U_CF_R} \left( \frac{U_P\phi'}{U_P\phi'} - \rho \right),
\]

where \( U_P\phi' = \dot{U}_P\phi' + U_P\dot{\phi}' \) was used. It is easy to see that this is equivalent to (39) with a simple re-arrangement.
D Growth Rate of Production

Given the production function (40), the growth rate $g_{Y_t} = \frac{\dot{Y}_t}{Y_t}$ can be computed by the logarithmic differentiation:

$$g_{Y_t} = \frac{\dot{Y}_t}{Y_t} = \frac{d}{dt} (\alpha \log L_{Y,t} + (1 - \alpha) \log R_t + \nu \log A_t)$$

$$= \alpha \frac{\dot{L}_{Y,t}}{L_{Y,t}} + (1 - \alpha) \frac{\dot{R}_t}{R_t} + \nu \frac{\dot{A}_t}{A_t}.$$ 

By replacing $\frac{\dot{A}_t}{A_t}$ with $\sigma L_{RD_t}$ according to (41), one gets

$$g_{Y_t} = \alpha g_{L_{Y,t}} + (1 - \alpha) g_{R_t} + \nu \sigma (1 - L_{Y,t}),$$

where $g_{L_{Y,t}}$ and $g_{R_t}$ are the growth rate of the labor in the production and the resource input, respectively.
E Proof of Proposition 1

**Proposition.** At the steady-state optimum, the values of the quantities and growth rates are given by

1. \( L_{RD}^o = \frac{(\sigma \nu - \rho \alpha)(\epsilon(1 - \alpha) + \alpha + \omega)}{\sigma \nu (\epsilon + \omega(1 - \alpha + \epsilon \alpha))}, \)
2. \( L_Y^o = 1 - L_{RD}^o, \)
3. \( g_A^o = \sigma L_{RD}^o, \)
4. \( g_R^o = g_P^o = g_S^o = \frac{(\sigma \nu - \rho \alpha)(1 - \epsilon)}{\epsilon + \omega(1 - \alpha + \epsilon \alpha)}, \)
5. \( g_C^o = g_Y^o = \frac{(\sigma \nu - \rho \alpha)(1 + \omega)}{\epsilon + \omega(1 - \alpha + \epsilon \alpha)}. \)

**Proof.** It should be noticed that the following equations hold under (40)–(43) together with the characteristic conditions (38) and (39):

\[
\rho + (\epsilon - 1)g_Y^o = -gL_{Y,t}^o + \frac{\sigma \nu}{\alpha}L_{Y,t}^o, \tag{101}
\]

\[
-g_{R,t}^o + \frac{(\gamma R_t)^{1+\omega}}{(1 - \alpha)Y_t^{1-\epsilon}}(\rho - \omega g_R^o) = -gL_Y^o + \frac{\sigma \nu}{\alpha}L_{Y,t}^o, \tag{102}
\]

where, as before, \( g_{zt}^o \) denotes the growth rate of any variable \( z_t. \)

When all the growth rates are constant, solving for \( L_{Y,t}^o \) and logarithmically differentiating (101) with respect to time imply that \( g_Y^o = g_R^o = 0 \) and that \( L_{Y,t}^o = L_Y^o \)
and \( L_{RD,t}^o = L_{RD}^o, \) considering (101)–(102) and (44). Hence, at the steady state, 2. and 3. immediately follow from (32) and (41).

Furthermore, as there is no population growth (i.e., \( L_{Y,t}^o + L_{RD,t}^o = 1, \) (32)) at the steady state, the following equations follow from (101), (102) and (44):

\[
\rho + (\epsilon - 1)g_Y^o = \frac{\sigma \nu}{\alpha}(1 - L_{RD}^o) \tag{103}
\]

\[
-g_R^o + \frac{(\gamma R_t)^{1+\omega}}{(1 - \alpha)Y_t^{1-\epsilon}}(\rho - \omega g_R^o) = \frac{\sigma \nu}{\alpha}(1 - L_{RD}^o), \tag{104}
\]

\[
g_Y^o = (1 - \alpha)g_R^o + \sigma \nu L_{RD}^o, \tag{105}
\]

where the last equation can be rearranged into

\[
-\frac{g_Y^o}{\alpha} + \frac{1 - \alpha}{\alpha}g_R^o + \frac{\sigma \nu}{\alpha} = \frac{\sigma \nu}{\alpha}(1 - L_{RD}^o), \tag{106}
\]

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and from (103) and (106)

$$g^o_R = -\frac{\sigma \nu - \rho \alpha}{1 - \alpha} + \frac{1 - (1 - \epsilon)\alpha}{1 - \alpha} g^o_Y$$  \hspace{1cm} (107)

follows.

Adding $g^o_R$ to the both sides of (104) and taking its logarithmic derivative with respect to time gives

$$g^o_Y = 1 + \omega \frac{1}{1 - \epsilon} g^o_R.$$  \hspace{1cm} (108)

Plugging (108) into (107) results in

$$g^o_R = \frac{(\sigma \nu - \rho \alpha)(1 - \epsilon)}{\epsilon + \omega (1 - \alpha + \epsilon \alpha)}.$$  \hspace{1cm} (109)

After rearranging (105) into $L^o_{RD} = \frac{g^o_Y - (1 - \alpha)g^o_R}{\sigma \nu}$ and plugging (108),

$$L^o_{RD} = \frac{\epsilon (1 - \alpha) + \omega + \alpha}{\sigma \nu (1 - \epsilon)} g^o_R$$

which leads to 1. after plugging in (109).

Since the variables $R_t$, $P_t$ and $\dot{S}_t$ have the same growth rate according to (33) and (36), 4. clearly follows from (109). Then, 5. holds trivially by plugging 4. into (108).

\[\square\]
F Existence of the Steady-state Optimum

The first transversality condition is

\[ \lim_{t \to \infty} \mu_t A^\alpha_t e^{-\rho t} = 0 \]

which is equivalent to

\[ \lim_{t \to \infty} \mu_0 e^{\mu_1} A^\alpha_0 e^{\gamma^\alpha t} e^{-\rho t} = 0 \]

\[ \lim_{t \to \infty} \mu_0 A^\alpha_0 (g_{\mu t} + g_A - \rho) t = 0. \]

This transversality condition is satisfied when

\[ g_{\mu t} + g_A - \rho < 0. \]  \hspace{1cm} (110)

From (96), we know that the growth rate of the co-state variable \( \mu_t \) is

\[ g_{\mu t} = \frac{\dot{\mu}_t}{\mu_t} = \rho - \frac{F^A q_L}{F_L} - q_A \]

which can be re-written as

\[ g_{\mu t} = \rho - \frac{\delta \nu}{\alpha} + \delta L_{RD}^\alpha \frac{\nu - \alpha}{\alpha}. \]  \hspace{1cm} (111)

By plugging (111) into (110), we get

\[ \frac{\delta \nu}{\alpha} (1 - L_{RD}^\alpha) < 0 \]

which holds if and only if \( L_{RD}^\alpha \) is lower than one as in (47).

Similarly, the second transversality condition

\[ \lim_{t \to \infty} \nu_t e^{\nu_1} S^\alpha_0 e^{\gamma^S t} e^{-\rho t} = 0 \]

holds when

\[ g_{\nu t} + g_S^\alpha - \rho < 0. \]  \hspace{1cm} (112)

By plugging (97) into (112) we see that

\[ \rho + g_S^\alpha - \rho = g_S^\alpha < 0 \]
which is equivalent to the condition (48).

Due to the condition that $L^0_{RD}$ is positive, we can see from 1. of Proposition 1 that

$$\delta \nu - \rho \alpha > 0,$$

which leads to $\rho < \frac{\delta \nu}{\alpha}$. Furthermore, from 4. of Proposition 1 with the condition (48), it is clear that

$$(\delta \nu - \rho \alpha)(1 - \epsilon) < 0$$

which is equivalent to

$$\epsilon > 1$$

since $\delta \nu - \rho \alpha > 0$. 
G Proof of Remark 1

By combining the derivatives of (54) and (56) with respect to time $t$ we get

$$\dot{w}_t = r_t - \frac{v_t}{V_t} + \frac{\dot{q}_L}{q_L}$$

$$\iff r_t = \frac{\dot{w}_t}{w_t} + \frac{v_t}{V_t} - \frac{\dot{q}_L}{q_L}. \quad (113)$$

From (58),

$$\frac{v_t}{V_t} = \frac{F_A}{V_t} + q_A$$

$$= \frac{q_L F_A}{F_L} + q_A,$$

where the identity in (56) has been used. Plugging this into (113),

$$r_t = \frac{\dot{F}_L}{F_L} - \frac{\dot{q}_L}{q_L} + \frac{q_L F_A}{F_L} + q_A,$$

where we used $F_L = w_t$ from (52).

Replacing $r_t$ in (60) with the above equation leads to the characteristic equilibrium condition (63).

According to the Hotelling rule in (61) together with (53), we know that

$$r_t = \frac{\dot{p}_{R_t}}{p_{R_t}} = \frac{\dot{F}_R}{F_R} - \frac{\dot{\tau}_t}{\tau_t}.$$

By replacing $r_t$ in the first condition with this, we get the second condition (64).