Small Open Economy Study for Hong Kong

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Economics
Master’s Thesis
May 2013
In this paper we derive a dynamic stochastic general equilibrium (DSGE) model, following Gali and Monacelli (2005) for Hong Kong. The model features a small open economy with a currency board. We simulate the model and illustrate impulse response functions, comparing three different monetary rules: PEG, domestic inflation target (DIT) and a Taylor rule.

The model is estimated with conventional Bayesian approach, then we perform model comparison of PEG against other two rules, and PEG wins the overwhelming support of the data. Our results show substantial openness of Hong Kong, and firms reset prices roughly every three quarters. Cyclical variations of Hong Kong seem mostly come from productivity and cost push-up shock. Finally a DSGE-VAR model is estimated, results are similar to DSGE model, however, estimated weight parameter indicates that cross equation restrictions are too stylised to capture the essential dynamics of the data than a pure VAR model.
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Table 1: Notation of Parameters

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Table 2: Notation of Variables
Chapter 1

Introduction

Hong Kong enjoys the highest independence among all territories in China. Hong Kong government, who manages its own tax income, has independent monetary policy to mainland as well. Due to a different political system, Hong Kong maintained its own institutions after sovereignty handover in 1997. With these attractive features, Hong Kong plays a role of global financial centre and free port.

The Hong Kong Monetary Authority (HKMA, the de facto central bank of Hong Kong) was founded in 1993, by the consolidation of ‘Office of the Exchange Fund’ - the well-known currency board - and ‘Office of the Commissioner of Banking’. Before consolidation, the currency board had started functioning in 1983. HKMA fixes the exchange rate of HK$7.80 to one US dollar, through a so-call ‘Linked Exchange Rate System’, namely every issuance of HK$7.80 is backed up by one US dollar in HKMA’s vault.

Without independent monetary policy to US, the interest rate of Hong Kong moves in tandem with US interest rate, which is shown in first panel of figure 1.1. Note that HK interest rate follows US interest rate closely, the spread between two series is the risk premium, the spike in 1997 is due to Asian financial crisis. Most recently, HKMA maintains an annual interest rate of 0.5%. The second panel presents the HP filtered real GDP of HK. We can identify three recessions with negative growth rates after 90s, 1997 Asian
financial crisis, 2003 SARS (Severe Acute Respiratory Syndrome) and 2008 great recession. After 1997, Hong Kong had several years of disinflation up to 2004. More recently, the inflation seems to step out of ‘Great Moderation’ era.

In order to corroborate our model, we present relevant empirical evidences. The first issue is about New Keynesian Philips curve (NKPC), which is the corner stone of New Keynesian models. For the last two decades, it has been considerably controversial whether price-setting firms are forward-looking, backward-looking, or both. Fuhrer (1997)\[6\] argues that their empirical results show that pure forward-looking term is unimportant in ex-
plaining the pricing behaviour. However, Gali and Gertler (1999)\[9\] develops a structural model allowing for backward-looking behaviour in NKPC, concludes that lag term, though significant, is quantitatively unimportant. Genberg and Pauwels (2005)\[11\] estimates a hybrid NKPC modelling the pricing behaviour of firms in Hong Kong, their results claim that forward-looking is the dominant effect of firms behaviour in Hong Kong. Although pure forward-looking NKPC might fail to capture certain dynamics of data, we will not use hybrid NKPC in this paper.

The second issue is about exchange rate pass-through of Hong Kong. Parsley (2003)\[23\] finds that Hong Kong has remarkably rapid import pass-through, between 80 and 95% to nominal exchange rate in short run. The findings are in line with Zitzewitz (2000)\[26\], which shows that Hong Kong had much higher price flexibility than most of OECD countries. Hence we do not model import firms, for reference model with incomplete pass-through see Monacelli (2003)\[21\].

A great amount of researches has been devoted to the studies of Hong Kong’s market economy, from pure empirical studies to partial equilibrium modelling. In recent years, general equilibrium models have become a popular framework for the study of Hong Kong. Cheng and Ho (2009)\[3\] estimates a small-scale New Keynesian model and results indicate that wages and prices in HK are quite flexible relative to other developed economies. Funke et al. (2011a)\[8\] identifies the positive wealth effects from stock markets on consumption. And further Funke et al. (2011b)\[7\] extends the small open economy model with a housing sector. Lim and McNelis (2012) sets up a similar small-scale DSGE model to perform counterfactual analysis from fixed to flexible exchange rate regime, results implies that switching from PEG to flexible exchange rate does not significantly raise welfare for Hong Kong.

As a convention, our basic assumptions feature Hong Kong as a small open economy with trivial influence to world economy which is represented by US in this study. One of the most intriguing elements of Hong Kong
economy is the fixed exchange regime, and our goal is to analyse the model qualitatively, comparing different monetary policies in the small open economy with currency board. We are also interested to have some empirical views into the deep parameters such as price rigidities, elasticities of substitutions, and etc. Furthermore, we would like to ask how much data dynamics a stylised DSGE model can capture when competing with a VAR model.

The remainder of this paper is organised as follows. Chapter two presents the model and its log-linearised form, chapter three discusses calibration and impulse response functions, while chapter four describes priors, estimation techniques and results. The final chapter concludes.
Chapter 2

The Small Open Economy Model

In this chapter we present the model derived from Galí and Monacelli (2005). There are four sectors: households, firms, monetary authority and foreign economy. Agents are modelled by explicit preferences with intertemporal constraints. We follow the notation in Galí and Monacelli (2005) to denote goods and services originated in home country with subscript $H$, imported products related variables marked by a subscript $F$, foreign variables are denoted with superscript $\ast$. For instance, $C_{F,t}^\ast$ is the bundle of consumption goods produced in foreign economy (with subscript ‘F’) and consumed by foreign economy (indicated by superscript ‘$\ast$’).

2.1 The Households

The representative household seeks to maximise her lifetime utility

$$E_t \sum_{t=0}^{\infty} \beta^t \left( C_t^{1-\sigma} \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right)$$

(2.1)

where $C_t$ is consumption bundle and $N_t$ is the hours of labour supply to domestic firms; $\sigma$ is the inverse elasticity of intertemporal substitution and $\varphi$ is the inverse elasticity of labour supply to real wage.

Define $C_t$ as a composite consumption index by a constant elasticity of

\footnote{All notations are listed in table and }
substitution (CES) form,

\[ C_t = \left[ (1 - \alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{1}{\eta}} \right]^{\eta - 1} \tag{2.2} \]

where \( \eta \) is the elasticity of substitution of domestic goods to foreign goods; \( \alpha \) stands for import ratio, which represents the openness of the small economy. \( C_{H,t} \) and \( C_{F,t} \) are indices of domestic goods and foreign goods. Both are defined by CES aggregators, following Dixit-Stiglitz formulation,

\[ C_{H,t} = \left[ \int_0^1 C_{H,t}(j)^{\varepsilon - 1} \frac{dj}{\varepsilon - 1} \right]^{\frac{1}{\varepsilon - 1}}, \quad C_{F,t} = \left[ \int_0^1 C_{F,t}(j)^{\varepsilon - 1} \frac{dj}{\varepsilon - 1} \right]^{\frac{1}{\varepsilon - 1}} \]

where \( C_{H,t}(j) \) and \( C_{F,t}(j) \) are indexed goods from a continuous interval \( j \in [0,1] \). \( \varepsilon \) is the elasticity between various types of goods for domestic and foreign consumption.

Utility maximisation problem of (2.1) subjects to the intertemporal budget constraint

\[ \int_0^1 P_{H,t}(j) C_{H,t}(j) \, dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) \, dj \, di + E_t \left( \frac{D_{t+1}}{1 + r_t} \right) = D_t + W_t N_t \tag{2.3} \]

where \( P_{H,t}(j) \) is domestic price of commodity \( j \) for home economy; \( P_{i,t}(j) \) is the price of commodity \( j \) imported from country \( i \) and \( C_{i,t}(j) \) is the consumption of commodity \( j \) from country \( i \); \( D_{t+1} \) is financial wealth (including dividends from firms) held at the end of period \( t \); \( r_t \) is nominal interest rate and \( W_t \) is nominal wage rate.

The total consumption of domestically and foreign produced goods is defined

\[ P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \]

where domestic and foreign consumptions are, respectively

\[ P_{H,t} C_{H,t} = \int_0^1 P_{H,t}(j) C_{H,t}(j) \, dj \]

\[ P_{F,t} C_{F,t} = \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) \, dj \, di = \int_0^1 P_{F,t}(j) C_{F,t}(j) \, dj \]

\footnote{We assume both domestic and foreign firms produce a continuum of infinite goods in a monopolistically competitive manner.}
Following the same pattern of CES consumption aggregators, we define the price indices of domestically produced goods and foreign goods in Dixit-Stiglitz formulation,

\[ P_{H,t} = \left[ \int_0^1 P_{H,t}(j)^{1-\varepsilon} \, d\varepsilon \right]^{1/(1-\varepsilon)}, \quad P_{F,t} = \left[ \int_0^1 P_{F,t}(j)^{1-\varepsilon} \, d\varepsilon \right]^{1/(1-\varepsilon)} \]

where both indices are expressed in domestic currency.

The demand functions of optimal allocation of expenditures for domestic and foreign goods are derived,

\[ C_{H,t} = (1-\alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \quad C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \]  

(2.4)

and also consumer price index (CPI),

\[ P_t = \left[ (1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{1/(1-\eta)} \]  

(2.5)

Furthermore, the optimal allocation of consumption within each group, i.e. variety demand functions are given,

\[ C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}, \quad C_{F,t}(j) = \left( \frac{P_{F,t}(j)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \]  

(2.6)

In order to simplify the intertemporal budget constraint, we make use of identity \( P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \), the constraint \(2.3\) reduced to

\[ P_tC_t + E_t \left( \frac{D_{t+1}}{1+r_t} \right) = D_t + W_t N_t \]  

(2.7)

Household’s utility maximisation problem yields following F.O.C.s,

\[ C_t^\alpha \frac{W_t}{P_t} = N_t^{-\varphi} \]  

(2.8)

(2.8) is a standard intratemporal optimality condition between consumption and labour supply. And Euler equation \(2.9\) is

\[ \frac{1}{1+r_t} = \beta E_t \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \]  

(2.9)
Log-linearise (2.4), (2.8) and (2.9), yield

\[ c_{H,t} = -\eta (p_{H,t} - p_t) + c_t \]  
\[ c_{F,t} = -\eta (p_{F,t} - p_t) + c_t \]  
\[ w_t - p_t = \sigma c_t + \varphi n_t \]  
\[ E_t c_{t+1} = \frac{1}{\sigma} (r_t - E_t \pi_{t+1} - \rho) + c_t \]  

where small letters represent log variable, it approximates the percentage change, and \( E_t \pi_{t+1} = E_t p_{t+1} - p_t \) is the expected inflation rate. (2.13) tells that current consumption depends on expected consumption of next period, the higher expectation raises current consumption in order to smooth consumption. And also depends and real interest rate \( r_t - E_t \pi_{t+1} \), the higher the real interest rate, the lower the current consumptions, consumers tend to exploit interest gains, which reflects the intertemporal substitution.

### 2.2 The Firms

Domestic firms produce differentiated goods \( Y_t(j) \) with constant return to scale technology represented by

\[ Y_t(j) = A_t N_t(j) \]  

where technology process takes logarithm form \( a_t = \ln A_t \), which follows an AR(1) process

\[ a_t = \rho_a a_{t-1} + \varepsilon_t^a \]  

We define a Dixit-Stiglitz CES aggregator for aggregate output \( Y_t \)

\[ Y_t = \left[ \int_0^1 Y_t(j) \frac{\varepsilon^{-1}}{\varepsilon^t} \, dj \right]^\frac{1}{\varepsilon} \]  

With firm’s technology, total cost and marginal cost can be written as:

\[ TC_t = \frac{W_t Y_t}{P_{H,t} A_t}, \quad MC_t = \frac{W_t}{P_{H,t} A_t} \]  

where \( W_t/P_{H,t} \) is real domestic wage, \( Y_t/A_t = N_t \) is labour supply.
2.2.1 Price Setting

We introduce staggered price setting à la Calvo. Each period, each domestic firm reoptimises their price at a probability \(1 - \theta_H\), if it does not have ‘fortune’ to reoptimise, then sticks to the price of last period, namely

\[ P_{H,t}(j) = P_{H,t-1} \]

Let \(P'_{H,t}\) denote the optimised price, then we can define the aggregate domestic price level

\[ P_{H,t} = \left[(1 - \theta_H)P'_{H,t} + \theta_H P_{H,t-1}^{-\varepsilon}\right]^\frac{1}{1-\varepsilon} \tag{2.18} \]

The firms which have chances to reoptimise their price will seek to maximise the present discounted value of dividend stream

\[ \max E_t \sum_{k=0}^{\infty} \left( \prod_{k'=1}^{k} \frac{1}{1 + r_{t+k}} \right) \theta^k_H \left[ Y^d_{t+k}(P'_{H,t} - MC_{t+k}^n) \right] \]

\(MC_{t+k}^n\) is nominal marginal cost. We discount dividends by nominal interest rate \(r_{t+k}\) and probability \(\theta^k_H\) together, because once the \(P'_{H,t}\) is set, the probability remaining unchanged within \(k\) periods is \(\theta^k_H\). Also subject to sequence of demand constraints

\[ Y^d_{t+k} \leq \left( \frac{P'_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} (C_{H,t+k} + C_{H,t+k}^*) \]

where \(C_{H,t+k}^*\) is a bundle of goods produced domestically and consumed in foreign economy. We solve the problem to get decision rule

\[ \sum_{k=0}^{\infty} E_t \left\{ Y^d_{t+k}(j)(\theta_H)^k \left( C_{t+k}^* \frac{P_{H,t-1}}{P_{H,t}} \right) \left( \frac{P'_{H,t}}{P_{H,t-1}} + \frac{\varepsilon}{1 - \varepsilon} \Pi_{t-1,t+k}^H MC_{t+k} \right) \right\} = 0 \]

where \(MC_{t+k} = MC_{t+k}^n/P_{H,t+k}\) and \(\Pi_{t-1,t+k}^H = P_{H,t+k}/P_{H,t-1}\). Put differently, optimised price setting is

\[ \frac{P'_{H,t} - \varepsilon}{P_{H,t}} \frac{P_{H,t-1}^{1-\varepsilon}}{P_{H,t-1}^{1-\varepsilon}} = \frac{\sum_{k=0}^{\infty} Y^d_{t+k}(i)(\theta_H)^k \left[ C_{t+k}^* \frac{P_{H,t-1}}{P_{H,t}} \right] \left( \frac{P'_{H,t}}{P_{H,t-1}} + \frac{\varepsilon}{1 - \varepsilon} \Pi_{t-1,t+k}^H MC_{t+k} \right)}{E_t \left\{ \sum_{k=0}^{\infty} Y^d_{t+k}(i)(\theta_H)^k \left[ C_{t+k}^* \frac{P_{H,t-1}}{P_{H,t}} \right] \left( \frac{P'_{H,t}}{P_{H,t-1}} + \frac{\varepsilon}{1 - \varepsilon} \Pi_{t-1,t+k}^H MC_{t+k} \right) \right\}} \]

(2.19)
The equation shows that with an elasticity of substitution of $\varepsilon$, imperfectly competitive firms set prices as a markup over marginal cost, such that $\frac{\varepsilon}{\varepsilon - 1}$ multiplies discount expected marginal cost.

Log-linearise equation of optimised price-setting at zero inflation steady state, i.e. $\Pi_{t-1,t+k}^H = 1$, we have

$$p_{H,t} - p_{H,t-1} = E_t \sum_{k=0}^{\infty} (\beta \theta_H)^k \left( \pi_{H,t+k} + (1 - \beta \theta_H) mc_{t+k} \right)$$

(2.20)

Extract period 0 out of summation sign preparing for technical manipulation (2.20),

$$p'_{H,t} - p_{H,t-1} = \pi_{H,t} + (1 - \beta \theta_H) mc_t$$

$$+ E_t \sum_{k=0}^{\infty} (\beta \theta_H)^{k+1} \left( \pi_{H,t+k+1} + (1 - \beta \theta_H) mc_{t+k+1} \right)$$

Notice the expectation term is exactly $(\beta \theta_H)(p_{H,t+1} - p_{H,t})$

$$p'_{H,t} - p_{H,t-1} = \pi_{H,t} + (1 - \beta \theta_H) mc_t + \beta \theta_H (p'_{H,t+1} - p_{H,t})$$

(2.21)

And also we log-linearise CPI index (2.18)

$$\pi_{H,t} = (1 - \theta_H)(p'_{H,t} - p_{H,t-1})$$

(2.22)

Substitute (2.22) into (2.21) we have equation for inflation dynamics,

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda_H mc_t$$

(2.23)

where $\lambda_H = \frac{(1 - \beta \theta_H)(1 - \theta_H)}{\theta_H}$, since $0 < \theta_H < 1$ and $0 < \beta < 1$, $\lambda_H$ is always positive. Besides, $\lambda_H$ depends negatively on $\theta_H$ and $\beta$, lower the $\theta_H$ is, the higher sensitivity of inflation to marginal cost.

2.3 Inflation, Exchange Rate and Terms of Trade

2.3.1 Terms of Trade

Terms of trade (TOT) is defined as $S_t = \frac{p_{F,t}}{p_{H,t}}$, in log-linear form $s_t = p_{F,t} - p_{H,t}$, which represents the unit price of imported goods in terms of
home goods. Increase of TOT implies higher competitiveness for domestic
economy, which results either from a raise of imported goods prices $p_{F,t}$ or
a decline of domestic prices $p_{H,t}$. To see the relation between TOT and
aggregate price level, combine the log-linear domestic price index (2.5) with
TOT

$$p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t$$

(2.24)

After first difference, yields the relation of inflation and TOT

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t$$

(2.25)

where the inflation difference between foreign and domestic economy is

$$\Delta s_t = \pi_{F,t} - \pi_{H,t}$$

(2.26)

From equation (2.25), we see that the difference between foreign and do-
mestic inflation is proportional to the change in TOT. On the other hand,
$\Delta s_t = \frac{1}{\alpha}(\pi_t - \pi_{H,t})$ tells that the change of TOT is proportional to the dif-
ference of overall inflation and domestic inflation. The higher the openness
parameter $\alpha$, the smaller the change of TOT under shocks.

2.3.2 Law of One Price

Although import firms have some price-setting power and incentives to push
the prices above marginal cost, we assume the law of one price (LOP) holds
throughout our study, then import firms will not be modelled explicitly.
Here we follow a general case introduced by Monacelli (2003), ‘LOP gap’ is
the most general case of LOP.

Define $\mathcal{E}_t$ as nominal exchange rate in terms of domestic curreny per
unit of foreign currency, the law of one price gap (LOP) can be expressed
$\Psi_t = \frac{\mathcal{E}_t P^*_t}{P_{F,t}}$. In words, law of one price gap is the ratio of the price index of
world economy in terms of domestic currency $\mathcal{E}_t P^*_t$ to the domestic currency
price of imported goods $P_{F,t}$. If LOP holds, $P_{F,t} = \mathcal{E}_t P^*_t$. Define real
exchange rate as well

$$Q_t = \frac{\mathcal{E}_t P^*_t}{P_t}$$

(2.27)
which is the ratio of the foreign price level in terms of domestic currency $E_t P^*_t$ to the domestic price level $P_t$.

Log-linearise LOP gap and real exchange rate around symmetric steady-state:

\[ \psi_t = e_t + p_t^* - p_{F,t} \tag{2.28} \]
\[ q_t = e_t + p_t^* - p_t \tag{2.29} \]

Substitute domestic price index (2.24) into real exchange rate (2.29) and insert a zero term $-p_{F,t} + p_{F,t}$,

\[ q_t = (e_t + p_t^* - p_{F,t}) + p_{F,t} - p_{H,t} - \alpha p_{F,t} + \alpha p_{H,t} \]

Replace the first term inside brackets of equation above by (2.28),

\[ q_t = \psi_t + (1 - \alpha)(p_{F,t} - p_{H,t}) \]

Replace again with TOT (2.3.1)

\[ q_t = \psi_t + (1 - \alpha)s_t \tag{2.30} \]

We can see that the LOP gap

\[ \psi_t = q_t - (1 - \alpha)s_t \tag{2.31} \]

is positively proportionate to the real exchange rate and negatively to competitiveness of domestic economy.

Under our assumption of law of one price holds and complete pass-through, it follows

\[ q_t = (1 - \alpha)s_t \tag{2.32} \]

real exchange rate nevertheless still fluctuates over time, while nominal exchange rate is pegged, as long as prices fluctuate.

\footnote{It means simultaneous steady-state for both domestic and foreign economy.}
2.3.3 International Financial Market

We assume perfect capital mobility, there will be consequently uncovered interest parity (UIP) and international risk sharing. UIP is the key no-arbitrage condition in international financial markets, it takes the form

$$\mathcal{M}_t (1 + r_t) = E_t \left[ \frac{\mathcal{M}_t}{E_t} (1 + r_t^*) \mathcal{E}_{t+1} \right]$$

where $\mathcal{M}_t$ is the units of domestic currency that an investor holds, $r_t$ and $r_t^*$ are investment returns of domestic bonds and foreign bonds (nominal interest rates), respectively. Eliminate $\mathcal{M}_t$ from both sides

$$(1 + r_t) = E_t \left[ \frac{\mathcal{E}_{t+1}}{E_t} (1 + r_t^*) \right]$$

(2.33)

UIP assumes perfect substitutability between domestic and foreign bonds, thus the rates of return expressed in the same currency is supposed to be equal.

Log-linearise (2.33) around steady-state and take the first difference of terms of trade, combine them, yields uncovered interest parity (UIP) condition

$$E_t \Delta s_{t+1} = (r_t - E_t \pi_{t+1}) - (r_t^* - E_t \pi_{t-1}^*)$$

(2.34)

The foreign stochastic Euler equation resembles domestic Euler equation (2.9)

$$\frac{1}{1 + r_t^*} = \beta E_t \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{P_{t+1}^* \mathcal{E}_t}{P_{t+1} \mathcal{E}_{t+1}} \right)$$

(2.35)

Combine (2.35) and (2.9), and plug in log linear real exchange rate, it follows

$$C_t = \Xi C_t^* \mathcal{Q}_t^{\frac{1}{\sigma}}$$

(2.36)

where $\Xi$ is a constant, which will be dropped during log linearisation. Log linearise (2.36) and use (2.32), yields

$$c_t = c_t^* + \left( \frac{1 - \alpha}{\sigma} \right) s_t$$

(2.37)

This equation connects domestic and foreign consumption by TOT up to an constant. TOT increases, relative price of domestic good decreases, and domestic consumption will be boosted.
2.4 Equilibrium

In this section, we close the model by presenting the equilibrium conditions for both small open economy and the world economy.

2.4.1 The Supply Side

The same derivation procedures can be applied to the world economy for the inflation dynamics

$$\pi^*_t = \beta E_t \pi^*_{t+1} + \lambda mc^*_t$$ (2.38)

where

$$mc^*_t = -\nu^* + (w^*_t - p^*_t) - a^*_t = -\nu^* + (\sigma + \varphi)y^*_t - (1 + \varphi)a^*_t$$ (2.39)

where $-\nu^*$ is the technical term of employment subsidy. The second equation makes use of $w^*_t - p^*_t = \sigma c^*_t + \varphi n^*_t$ and $y^*_t = a^*_t + n^*_t$.

We define an exact relation between output $y_t$ and output gap $\tilde{y}_t$

$$\tilde{y}_t = y_t - \bar{y}_t$$ (2.40)

where $\bar{y}$ is the natural level of output, which is realised under full price flexibility.

For the domestic counterpart of marginal cost, we have

$$mc_t = -\nu + w_t - a_t - p_{H,t}$$

$$= -\nu + \sigma y^*_t + \varphi y_t + s_t - (1 - \varphi)a_t$$

$$= -\nu + \left(\varphi + \frac{\sigma}{\varphi}\right)y_t + \left(\sigma + \frac{\sigma}{\varphi}\right)y^*_t - (1 + \varphi)a_t$$ (2.41)

where we make use of log-linear CPI $2.24$ and international risk sharing $2.37$ for the second equation. The last equation is derived by substitute out $s_t$ by market clearing condition $2.50$.

Then $2.41$ becomes

$$\mu = -\nu + \left(\varphi + \frac{\sigma}{\varphi}\right)\tilde{y}_t + \left(\sigma + \frac{\sigma}{\varphi}\right)y^*_t - (1 + \varphi)a_t$$ (2.42)
Note that $mc_t$ stays on steady state $\mu$ under flexible pricing, then combine (2.42) and (2.50), yields

$$\hat{mc}_t = \left(\frac{\sigma}{\varpi} + \varphi\right)\tilde{y}_t \quad (2.43)$$

Plug (2.43) into inflation dynamics equation (2.23), yields New Keyesian Philips Curve (NKPC)

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa \tilde{y} \quad (2.44)$$

where $\kappa = \frac{(1-\theta_H)(1-\theta_H)}{\varpi \mu} \left(\frac{\sigma}{\varpi} + \varphi\right)$.

Impose equilibrium condition $mc_t = -\mu$, and use (2.40) to substitute out $y_t$, we arrive at the natural rate of output as a function of productivity and world output

$$\tilde{y}_t = \frac{1}{\sigma + \varpi \varphi} \left(\varpi (\nu - \mu) + (1 - \varpi) \sigma y^*_t + \varpi (1 + \varphi) a_t\right) \quad (2.45)$$

However, the effect of world output is ambiguous, depending on the effect of world output on domestic marginal cost.

2.4.2 The Demand Side

Market clearing condition for imported good $i$ in the small open economy can be defined as follows

$$Y_t(i) = C_{H,t(i)} + C^*_t(i) \quad (2.46)$$

Combining (2.4) and (2.6) yields

$$Y_t(i) = (1 - \alpha) \left(\frac{P_{H,t(i)}}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{P_{H,t(i)}}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{E_t P_t}\right)^{-\eta} C^*_t$$

$$= \left(\frac{P_{H,t(i)}}{P_{H,t}}\right)^{-\varepsilon} \frac{\alpha^*}{\alpha} Y^*_t \left[ (1 - \alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} Q^1_t + \alpha \left(\frac{P_{H,t}}{E_t P_t}\right)^{-\eta} \right] \quad (2.47)$$

Substitute (2.47) into aggregate output (2.16), and notice that apparently the integral equals to one

$$Y_t = \left\{ \int_0^1 \left[ \left(\frac{P_{H,t(i)}}{P_{H,t}}\right)^{-\varepsilon} \right]^{\varepsilon - 1} d\eta \right\}^{\frac{1}{\varepsilon - 1}} \times$$

$$\frac{\alpha^*}{\alpha} Y^*_t \left[ (1 - \alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} Q^1_t + \alpha \left(\frac{P_{H,t}}{E_t P_t}\right)^{-\eta} \right] \quad (2.48)$$
Our assumption of an infinitely small open economy renders $P_t^* = P_{F,t}^*$, i.e. world price equals to foreign currency price of foreign goods. It follows that $\left( \frac{P_{F,t}}{P_{H,t}} \right)^\eta \left( \frac{E_tP_t^*}{P_t^*} \right)^{-\eta} = \left( \frac{P_{H,t}}{P_t} \right)^{-\eta}$, with the definition of terms of trade and real exchange rate, we can get $S_t Q^\frac{1}{\sigma} Q^{-\eta} = \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} Q^\frac{1}{\sigma}$. (2.48) is rewritten as

$$Y_t = \frac{\alpha^s}{\alpha} Y_t^s S_t^\eta \left( (1 - \alpha) Q^\frac{1}{\sigma} - \eta \right)$$

(2.49)

Log linearise and ignore the constant terms

$$y_t = y_t^* + \frac{\varpi}{\sigma} s_t$$

(2.50)

where $\varpi = 1 + \alpha(2 - \alpha)(\sigma\eta - 1) > 0$. Use international risk sharing condition (2.37) to substitute out $s_t$, the domestic consumption can be represented by a convex combination of domestic and world output

$$c_t = \Phi y_t + (1 - \Phi)y_t^*$$

(2.51)

Combine (2.51), (2.50) and domestic Euler equation, derives following relations

$$y_t = E_t y_{t+1} - \frac{\varpi}{\sigma} (r_t - E_t\pi_{H,t+1} - \rho) + (\varpi - 1)E_t \Delta y_{t+1}^*$$

(2.52)

To derive the output gap version of IS curve, we combine (2.45) and (2.52)

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{\varpi}{\sigma} (r_t - E_t\pi_{H,t+1} - r_t^*)$$

(2.53)

where $r_t^*$ denotes natural rate of interest under flexible pricing

$$r_t^* = \rho - \frac{\sigma(1 + \varphi)(1 - \rho_a)}{\sigma + \varphi\varpi} a_t - \frac{\varphi\sigma(1 - \varpi)}{\sigma + \varphi\varpi} E_t \Delta y_{t+1}^*$$

(2.54)

In our model, openness of the economy forces natural rate of interest depends on expected world production growth as well as domestic productivity.
2.4.3 Monetary Policy

In order to compare the qualitative implications of impulse response function, we specify three different monetary policies: currency board with fixed exchange rate (PEG), domestic inflation target (DIT) and a Taylor rule. Currency board implies that Hong Kong has no independent monetary policy, thus $e_t = 0$, the goal of interest rate instrument is to keep nominal exchange rate fixed. The DIT monetary policy aims at full stabilisation of domestic prices, implying $\pi_{H,t} = 0$. Under DIT, firms’ monopolistic pricing power are neutralised, the effects of flexible prices can be reproduced in IRF, namely $y_t = \bar{y}_t$ and $r_t = r^n_t$. Domestic Taylor rules are specified as $r_t = \phi_r r_{t-1} + \phi_\pi \pi_t + \phi_y \bar{y}_t$, where $\phi_r$ is the degree of interest rate smoothing, $\phi_\pi$ and $\phi_y$ are responsive parameters of inflation and output gap. The parameterisation will be specified in next chapter.
Chapter 3

Model Simulation

In this chapter, we calibrate the model and present the impulse response functions. Five structural shocks are attached to the model:

- Technology shock
  \[ a_t = \rho_a a_{t-1} + \varepsilon_t^a + \rho_{corr} \varepsilon_t^* \]

- Cost push-up shock
  \[ \gamma_t^\pi = \rho_t \gamma_{t-1}^\pi + \varepsilon_t^\pi \]

- Exchange rate/DIT/Monetary shock
  \[ \gamma_t^e = \rho_t \gamma_{t-1}^e + \varepsilon_t^e \]

- Foreign demand shock
  \[ a_t^* = \rho_a a_{t-1}^* + \varepsilon_t^* \]

- Foreign output shock
  \[ \gamma_t^y^* = \rho_y \gamma_{t-1}^y^* + \varepsilon_t^y^* \]

The parameters are selected to keep the model on the unique path of equilibrium, see Table 3.1.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\eta$</th>
<th>$\varphi$</th>
<th>$\alpha$</th>
<th>$\theta_H$</th>
<th>$\phi_x$</th>
<th>$\phi_y$</th>
<th>$\phi_y^*$</th>
<th>$\rho_u$</th>
<th>$\rho_{corr}$</th>
<th>$\rho_e$</th>
<th>$\rho_a$</th>
<th>$\rho_y^*$</th>
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</thead>
<tbody>
<tr>
<td>0.995</td>
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<td>2</td>
<td>3</td>
<td>0.4</td>
<td>0.75</td>
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<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 3.1: Parameters for calibration

For 2% nominal annual interest rate, we set $\beta = 0.995^2$. For the inverse of elasticity of intertemporal substitution and elasticity of domestic to foreign goods, we set $\sigma = \eta = 2$. As an RBC literature convention, inverse elasticity of labour supply to real wage, $\varphi$, is set to 3. Following recent small open economy studies, such as Galí and Monacelli (2005), Liu (2006)^1^ and

---

^1^ See full linearised model in appendix.

^2^ According to steady state of Euler equation $\frac{\beta}{1+\gamma} = 1$
Funke et al. (2010), we set openness parameter $\alpha$ as 0.4. Calvo probability equals 0.75, implying an average of one year price adjustment period. $\phi_{\pi^*} = 1.5$ is consistent with Taylor (1993) for US economy (proxy for foreign economy). We superimpose three different monetary policies PEG, DIT and Taylor in the same figure, however we focus on explaining PEG, the other two we mention otherwise.

Our impulse responses reproduce some typical small open economy features under fixed exchange regime (PEG) as Gali and Monacelli (2005). In figure 3.1, the impulse response after a domestic productivity is given. The positive productivity shock reduce the real marginal cost except DIT, which in turn lowers the production price level as well as CPI. Natural rate of output is raised by the productivity shock, narrowing the output gap, hence
negative response of output gap. However under DIT, output gap and inflation level is fully stabilised. From the view of domestic economy, lower prices means higher competitiveness of domestic products, therefore terms of trade increases. Under a PEG the nominal exchange rate will be kept completely stable as the monetary authority has no independent monetary policy to respond with lower interest rate.

In figure 3.2 responses to a foreign demand shock is given. Under DIT, price level and domestic inflation are stabilised, in turn output gap is neutralised as well. Under PEG, foreign demand of domestic product rises, which stimulates the domestic production, thus the actual output deviates from natural rate of output, resulting a positive output gap, in medium term output gap closes when the actual output drops. The domestic consumption
Figure 3.3: IRF to an exchange rate, inflation target and Taylor monetary shock

and labour supply are accordingly pushed up, in turn higher wage will be paid to households, which implies higher real marginal cost. As a result, domestic inflation and price level go up. Terms of trade deteriorates which also brings down the real exchange rate, monetary authority reacts with lowering nominal interest to keep nominal exchange rate stable. The world inflation level is simply pushed up by the higher foreign productivity. Under Taylor rule, we notice an interesting propagation channel, nominal exchange rate and domestic price are permanently lower than former equilibrium.

The PEG responses are given in figure 3.3 as well as shocks from DIT and Taylor. Under PEG regime, a positive exchange rate shock equals a negative monetary shock. Because increase of nominal exchange rate is caused
by a positive interest differential, where foreign interest rate remain still. Loosening the monetary policy boosts actual output, hence a positive output gap. Firms are hiring more labours and paying more wages, the real marginal cost is pushed up, so are the price level and domestic inflation. Depreciation of domestic currency (nominal exchange rate rises) increase the export of domestic products, terms of trade shows the rising of competitiveness of domestic economy, which brings real exchange rate moving in tandem. Notice that Taylor rule has the exact opposite qualitative features of PEG and DIT.

The figure 3.4 presents the responses of a cost push-up shock. The first reaction of a cost push-up shock is the rise of producer prices and further for consumer price as well. The competitiveness of domestic products falls
as the falling of terms of trade shows, which in turn drags down the real exchange rate. The nominal interest rate has a tiny upwards movement (it will be seen if we scale the vertical axis to $0.2 \times 10^{-2}$). Tightening monetary policy lowers the actual output and output gap, price level also comes back to equilibrium in the medium run. Marginal cost is pushed down as lowering the output gap.
Chapter 4

Bayesian Estimation

In this paper, we estimate the model by conventional Bayesian approach, based on estimation results we perform model comparison by comparing numerical log marginal likelihood, \( \log p(y|M_i) \), where \( M_i \) denotes model \( i \). The fundamental philosophy of Bayesian is to be consciously aware that no model is true and uncertainty comes along whatever model we choose. With this idea in mind, we also provide results of DSGE-VAR estimation.

Bayesian approach considers the whole set of implications of the model and the estimation provides a full characterisation of the observed data via likelihood function. Prominent examples can be seen from Smets and Wouters (2003)\cite{24}, Lubik and Schorfheide (2005)\cite{20} and An and Schorfheide (2007)\cite{1}.

Another full-information method is the classical Maximum Likelihood Estimation (MLE), see examples in Kim (2000)\cite{15}, Ireland (2001)\cite{14} and Lindé (2005)\cite{17}. However, comparing with Bayesian approach, classical MLE might have two problems: firstly likelihood function might be flat in parameters subspace which imposes great difficulty in locating the optimised parameters, secondly likelihood function peaks in peculiar area, which contradicts with additional information that researchers have. In Bayesian inference, these two problems are controlled to the minimum degree, a prior serves as weight ‘reshaping’ the likelihood function, more curvatures ap-
pear in the area which researchers believe it is sensible. Thus the idea of Bayesian is plain and simple, $p(\theta|y,M) \propto p(y|\theta,M)p(\theta|M)$, namely the likelihood kernel is proportional to prior multiplies likelihood.

Although the Bayesian approach is straightforward, log posterior kernel, $K(\theta|M|y,M) \propto \log p(\theta|M) + \log p(y|\theta,M)$, does not take any closed form. We have several options on simulating the posterior kernel, such as importance sampling, Gibbs sampling and Metropolis-Hasting algorithm. The last two methods construct certain number of independent Markov chains ‘wandering’ around the posterior distribution area. Theoretically, if the Markov chain has infinite length to be ergodic\(^1\), all non-zero probability area will be proportionally covered. Gibbs sampler is rarely used in DSGE posterior sampling, because it is barely possible to write down the full conditional posterior distributions for each parameters. In this paper, we use random walk chain Metropolis-Hasting algorithm\(^2\) which is a most general class of sampling methods, and the ‘curse of dimensionality’ is avoided. For the convergence, we generate 500,000 draws for eight parallel chains, and the acceptance rate is fine-tuned around 0.4.

### 4.1 Estimation Results

The time series data are real GDP of Hong Kong, three month nominal interbank rate, GDP deflator of Hong Kong, real exchange rate of Hong Kong to US dollars, and real GDP of US, all ranging from 1985Q1 to 2011Q4. Data source is from IMF International Financial Statistics database. Real GDPs are HP filtered, GDP deflator is transformed into annual growth rate, and real exchange rate to Hong Kong is calculated according to (2.27).

Given the structure of Hong Kong economy, we choose the priors which

\(^1\) For details about MCMC, refer to Dejong and Dave (2011)\(^3\) and Givens and Hoeting (2012)\(^4\).

\(^2\) Independence chain Metropolis-Hasting is mainly used where jump distribution can be easily formed, not a common choice for DSGE posterior simulation. For details, see Koop (2004)\(^5\) and Koop and Korobilis (2010).
Table 4.1: Prior and posterior

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>95% Conf. Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution Mean</td>
<td>Posterior Distribution Mean</td>
<td>95% Conf. Int.</td>
<td></td>
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<td></td>
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<td>Deep parameters</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td>$\sigma$</td>
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<td>4</td>
<td>0.5</td>
<td>7.0482</td>
<td>0.1170</td>
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<td>$\rho_{\pi}$</td>
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<td>0.1672</td>
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<td>0.8416</td>
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<td>Shock processes</td>
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<td></td>
</tr>
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<td>$\sigma_a$</td>
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<td>$\infty$</td>
<td>0.5630</td>
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</table>

The persistence of shock processes are set roughly according to the autocovariance of the data. Moreover, all parameters bounded between $[0, 1)$ are weighted by Beta distribution. As a convention, all standard deviations take the inverse Gamma as prior distribution. Elasticity parameters follow normal distribution prior. In order to have data dominate prior to a certain extent, we loosen the prior standard deviation. Elasticity parameters are notoriously difficult to estimate, which are difficult to identify, thus we set their means far away from 1 and larger standard deviations than other parameters.

$\sigma$ turned out to be 7.0482, indicating a considerably low intertemporal substitution in HK. The estimated $\eta$ is around 1.1607, which indicates the consumption basket of Hong Kong is diversified evenly among foreign and

27
domestic goods. The estimated inverse elasticity of substitution for labour $\varphi$ implies 1% increase in real wage might result in a tiny increase of labour supply. Results also show that Hong Kong has a substantial degree of openness ($\alpha = 0.7182$), which is higher than results from Funke et. al (2010) and Lim et. al (2012). Calvo pricing probability is around 0.72, which not only implies that the prices are optimised nearly every three quarters, but also shows that - compared with Euro area - Hong Kong has much lower price rigidity, see Smets and Wouter (2003). We also notice a considerably high persistence of productivity shock $\rho_a$ of 90%, and modest persistence of foreign demand shock of 54%. Persistence of foreign supply shock and real exchange rate shock are both quite high, 0.8416 and 0.8959 accordingly.

Highest standard deviations come from productivity shock and cost push-up shock, it implies that cyclical variations are mostly driven by productivity and inflation fluctuations.

4.2 Shock Decomposition

In order to have some insights into the contribution of each shock at each period, we decompose the shocks, and present the historical shock decomposition of inflation and output gap in figure 4.1. We see that technology shock and foreign demand shock are two fundamental driven forces for CPI inflation, and extreme high inflation rate are mainly caused by mark-up shocks and foreign supply shocks. Output gap is mostly driven by foreign demand shock $\varepsilon_y$ before 90s, after that weight gradually falls upon foreign supply and domestic productivity shock.

4.3 Model Comparison

We specified three different monetary policies to compare. The first one is fixed exchange rate regime DSGE model, denoted by $\mathcal{M}_1$, the second model $\mathcal{M}_2$ undertakes domestic inflation target (DIT) monetary policy, the last one $\mathcal{M}_3$ assumes a Taylor rule $r_t = \phi_r r_{t-1} + \phi_\pi \pi_t + \phi_y y_t + \gamma_m m_t$, where $\gamma_m$
Figure 4.1: Historical decomposition of CPI inflation and output gap

<table>
<thead>
<tr>
<th>Specification</th>
<th>Prior</th>
<th>Log marginal likelihood</th>
<th>Bayes factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Laplace approx.</td>
<td>Modified harmonic mean</td>
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<tr>
<td>PEG</td>
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<td>-4.8362</td>
<td>-5.3762</td>
</tr>
<tr>
<td>DIT</td>
<td>0.3</td>
<td>-67.5836</td>
<td>-68.8236</td>
</tr>
<tr>
<td>Taylor rule</td>
<td>0.3</td>
<td>-283.9732</td>
<td>-284.3763</td>
</tr>
</tbody>
</table>

Table 4.2: Model comparison
stands for monetary shock. We compute both Laplace approximation and modified harmonic means for the log marginal likelihoods, and they are fair close to each other from our estimation results in table 4.2.

As we have expected, the inflation target and Taylor monetary rule are highly misspecified for Hong Kong. We receive overwhelming support for PEG in the results, the Bayes factor practically approaches to 1, meaning the PEG model fits data much better than the other two models.

4.4 DSGE-VAR comparison

Since we have said that all models come with uncertainty, then a natural question is to ask: how uncertain it is? Although we would never know the exact answer, setting a reference VAR model would usually be the standard procedure to characterise the joint distribution of endogenous variables. Bayesian approaches allow scientific honesty to assess such state of affairs. And in this section, we estimate the DSGE-VAR model proposed by Del Negro and Schorfheide (2004)\cite{22}, to see its applied example in Hodge et. al (2008)\cite{13} and Bährle and Menz (2008)\cite{2}.

DSGE-VAR approach belongs to a bigger class of Bayesian VARs\footnote{Another prominent BVAR model uses Minnesota prior, see Litterman (1986)\cite{18} and Doans et al. (1984)\cite{5} for details.}. In DSGE-VAR setting, the priors come from independently estimated DSGE. Furthermore, a weight $\lambda$ on the DSGE prior is the optimised argument of integral

$$
\lambda^* = \arg \max_{\lambda \in \Lambda} \int_{\Sigma_u, \Phi} p(Y|\theta, \Sigma_u, \Phi)p(\theta, \Sigma_u, \Phi|\lambda)d(\Sigma_u, \Phi, \theta)
$$

where $\theta$ is a vector of DSGE deep parameters, $\Phi$ is a matrix of VAR parameters, $\Sigma_u$ the VAR covariance matrix. The weight of a pure VAR relative to DSGE is measured by $1/(1+\lambda)$. Two extreme value of this weight spectrum are $\lambda = 0$, where pure VAR explains all data variations, and $\lambda = \infty$ where pure DSGE explains all dynamics. A common method is to try different values of $\lambda$ to evaluate posterior likelihoods, the $\lambda$ which achieves highest
likelihood will be the optimal value. In this paper, we set up a uniform
distribution for $\lambda$ ranging from 0 to 2, and perform Bayesian estimation.

With the same prior settings for the rest of the parameters, the DSGE-
VAR estimation results are given in figure 4.3. In general, we find that
DSGE-VAR has similar results as DSGE estimation, however the $\lambda$
is lower than 1, therefore we conclude that this stylised DSGE model explains around
31% variations of data. Lim and McNelis (2012) achieve $\lambda = 1.875$, namely
65% variations of data can be explained by their DSGE model. Our result
can be no surprise when the model is highly stylised with certain misspecification,
and cross equation restrictions are unable to capture the dynamics
of data. Notwithstanding, the result provides the preliminary step of our
further researches on DSGE modelling for Hong Kong.

<table>
<thead>
<tr>
<th>Posterior Distribution</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Conf. Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep parameters</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
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Table 4.3: DSGE-VAR results
Chapter 5

Conclusion and Reflection

In this paper, we reproduce Galí and Monacelli (2005)’s stylised small open economy DSGE model for Hong Kong, featuring fixed exchange rate regime. We compare qualitative characteristics of the model in IRF under different monetary policies, namely PEG, DIT and Taylor rule. And according to our Bayesian model comparison results, PEG overwhelmingly wins the support of the data.

Our results show that openness of Hong Kong is substantial, even higher than other recent researches such as Funke et al. (2010). Price rigidities are much lower than Euro zone, nearly 30% of firms reoptimise prices in each period. And largest standard deviations are from productivity shock and cost push-up shock, which indicates the cyclical variations mostly seem to be driven by productivity and inflation fluctuation.

Furthermore we also find that the model is too stylised to capture the essential dynamics of the data, given estimated results of the DSGE-VAR model. However, the bright-side is that we are assured that necessary modification needs to be added in future research.

The future research might include improving the model fit to capture as much dynamics as possible, for instance, financial sector shall be modelled explicitly. Otherwise most of weight of cyclical variations fall upon productivity and inflation shock. Finally, a more comprehensive analysis of
model’s properties shall be examined, for instance, different parameter sets could test sensitivity of model, exploring model’s forecasting ability.
Appendix A

Prior and Posterior Distribution

Figure A.1: Prior and posterior distribution
Appendix B

Linearised Model

\[
\begin{align*}
y_t^* &= y_{t+1}^* - \frac{r_t^* - \pi_t^*}{\sigma} + \gamma_t^* \\
m_c^* &= y_t^*(\sigma + \varphi) - (1 + \varphi) a_t^* \\
\pi_t^* &= \pi_{t+1}^* + \beta + m_c^* \lambda_H \\
r_t^* &= \phi_n \pi_{t+1}^* + \phi_a a_t^* \\
\bar{y}_t &= \bar{y}_{t+1} - \frac{\sigma}{\varphi}(r_t - \pi_{t+1}^*) \\
\pi_{H,t}^* &= \beta E_t \pi_{H,t+1}^* + \kappa \bar{y}_t + \gamma_t^* \\
r_n^* &= -\frac{\sigma}{1 - \rho_a} \left(1 - \frac{1 - \rho_a}{\sigma}\right)(1 - \rho_a) \\
m_c^* &= s_t + y_t^* \sigma + \varphi y_t - (1 + \varphi) a_t \\
e_t &= 0 \\
a_t^* &= \rho_n a_{t-1}^* + \varepsilon_t^a \\
a_t &= \rho_n a_{t-1} + \varepsilon_t^a + \rho_{\text{corr}} \varepsilon_t^a \\
\gamma_t^* &= \rho_n \gamma_{t-1}^* + \varepsilon_t^\gamma \\
\gamma_t^\pi &= \rho_n \gamma_{t-1}^{\pi} + \varepsilon_t^{\pi} \\
\gamma_t^r &= \rho_n \gamma_{t-1}^r + \varepsilon_t^r \\
\gamma_t^\gamma &= \rho_n \gamma_{t-1}^\gamma + \varepsilon_t^\gamma
\end{align*}
\]

(Foreign IS curve)  
(Foreign marginal cost)  
(Foreign NKPC)  
(Foreign Taylor rule)  
(IS curve)  
(NKPC)  
(Natural rate of interest)  
(Natural output)  
(Output gap)  
(Clear condition)  
(World inflation)  
(Real exchange rate)  
(TOT dynamics)  
(Domestic price level)  
(CPI level)  
(Risk sharing)  
(Real marginal cost)  
(PEG)  
(Foreign productivity shock)  
(Domestic productivity shock)  
(Real exchange rate shock)  
(Cost push-up shock)  
(Foreign demand shock)
Appendix C

Key equations derivation

C.1 Household’s Utility Maximisation Problem

We handle the derivation of Euler equation first, basic dynamic programming skills are required.

To define the value function

\[ V(D_t) = E_t \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) = U(C_t, N_t) + \beta E_t V(D_{t+1}) \]

If we choose \( C_t \) to be the control, then Bellman equation is formed

\[ V(D_t) = \max_{C_t} \{ U(C_t, N_t) + \beta E_t V(D_{t+1}) \} \]

In order to replace \( D_{t+1} \), we use budget constraint \((2.7)\)

\[ D_{t+1} = (1 + r_t)(D_t + W_t N_t - P_t C_t) \quad (A.13) \]

Bellman equation becomes

\[ V(D_t) = \max_{C_t} \{ U(C_t, N_t) + \beta E_t V[(1 + r_t)(D_t + W_t N_t - P_t C_t)] \} \]

F.O.C. with respect to control \( C_t \),

\[ \frac{\partial V(D_t)}{\partial C_t} = U_C(C_t, N_t) - \beta E_t [V'(D_{t+1})](1 + r_t)P_t = 0 \]

where \( U_C(C_t, N_t) \) is \( \partial U(C_t, N_t) / \partial C_t \). Then,

\[ U_C(C_t, N_t) = \beta E_t [V'(D_{t+1})](1 + r_t)P_t \quad (A.14) \]

In order to find \( V'(D_{t+1}) \) we need to use Benveniste-Scheinkman envelope theorem with respect to state \( D_t \)

\[ \frac{\partial V(D_t)}{\partial D_t} = U_C(C_t, N_t) \frac{\partial C^*_t}{\partial D_t} + \beta E_t [V'(D_{t+1})] \left[ (1 + r_t) - (1 + r_t)P_t \frac{\partial C_t}{\partial D_t} \right] = 0 \]

\(^1\) The notation here is bit confusing, since the only variable in the value function is \( D_t \), we still use \( \partial \) because we are using envelope theorem.
From (A.13) we know that $C_t$ is a function of $D_t$. Rearrange,

$$U_C(C_t, N_t) \frac{\partial C_t}{\partial D_t} + \beta E_t \big[V'(D_{t+1})\big](1 + r_t) \left(1 - P_t \frac{\partial C_t}{\partial D_t}\right) = 0$$

$$\beta E_t \big[V'(D_{t+1})\big](1 + r_t) \left(1 - P_t \frac{\partial C_t}{\partial D_t}\right) = 0$$

$$\beta E_t \big[V'(D_{t+1})\big](1 + r_t) \left(1 - P_t \frac{\partial C_t}{\partial D_t}\right) = 0$$

The second equation use the fact (A.14). Thus, we get

$$\frac{\partial V(D_t)}{\partial D_t} = \beta E_t \big[V'(D_{t+1})\big](1 + r_t) = 0 \quad \text{(A.15)}$$

Multiply (A.15) by $P_t$, we get (A.14), therefore

$$U_C(C_t, N_t) = \frac{\partial V(D_t)}{\partial D_t} P_t$$

Move one period forward,

$$U_C(C_{t+1}, N_{t+1}) = \frac{\partial V(D_{t+1})}{\partial D_{t+1}} P_{t+1} = V'(D_{t+1}) P_{t+1}$$

Resubstitute back to (A.14),

$$U_C(C_t, N_t) = \beta E_t \left[ \frac{1}{P_{t+1}} U_C(C_{t+1}, N_{t+1}) \right] (1 + r_t) P_t \quad \text{(C.1)}$$

Now specify the utility function form,

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - \sigma N_t^{1+\varphi}}{1 - \sigma}$$

Then we plug into (C.1), yields

$$\frac{1}{1 + r_t} = \beta E_t \left[ \frac{C_{t+1}}{C_t} \right]^{1-\sigma} \frac{P_t}{P_{t+1}} \quad \text{(C.2)}$$

Next we will derive intratemporal optimality condition by Lagrangian, which is a standard textbook UMP problem. The representative household maximise

$$\max_{C_t, N_t} \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi} \right]$$

s.t.

$$P_t C_t + E_t \left( \frac{D_{t+1}}{1 + r_t} \right) = D_t + W_t N_t$$

Form Lagrangian,

$$L(C_t, N_t, \lambda_t) = \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi} \right]$$

$$+ \lambda_t \left[ D_t + W_t N_t - P_t C_t - E_t \frac{D_{t+1}}{1 + r_t} \right]$$
F.O.C. w.r.t. $C_t$ and $N_t$,

$$\frac{\partial L}{\partial C_t} = C_t^{-\sigma} - \lambda_t P_t = 0$$
$$\frac{\partial L}{\partial N_t} = -N_t^\sigma + \lambda_t W_t = 0$$

Rewrite the second equation,

$$\lambda_t = \frac{N_t^\sigma}{W_t}$$

Then substitute into the first one,

$$C_t W_t = N_t^{-\sigma} \quad \text{(C.3)}$$

is what we have seen in (2.6).

### C.2 Optimal Price Setting

The firms maximise dividend stream

$$\max_{P_{t+1}^H} E_t \sum_{k=0}^{\infty} \frac{\theta_H^k}{1 + r_{t+k}} \left( Y_{t+k}^d (P_{t+1}^H - MC_{t+k}^n) \right)$$

s.t.

$$Y_{t+k}^d (i) = \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} (C_{t+1,k} + C_{t+1,k}^\sigma) = \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} Y_{t+k}$$

$Y_{t+k}^d (i)$ is demand constrain for good $i$. Substitute demand constraint into objective function,

$$E_t \sum_{k=0}^{\infty} \frac{\theta_H^k}{1 + r_{t+k}} \left( \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} Y_{t+k} (P_{t+1}^H - MC_{t+k}^n) \right)$$

Keep on manipulating,

$$E_t \sum_{k=0}^{\infty} \frac{\theta_H^k}{1 + r_{t+k}} \left[ \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} Y_{t+k} P_{t+1}^H - \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} Y_{t+k} MC_{t+k}^n \right]$$

$$= E_t \sum_{k=0}^{\infty} \frac{\theta_H^k}{1 + r_{t+k}} \left[ \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} Y_{t+k} P_{t+1}^H - \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} Y_{t+k} MC_{t+k}^n \right]$$

Then F.O.C. w.r.t. $P_{t+1}^H$,

$$E_t \sum_{k=0}^{\infty} \frac{\theta_H^k}{1 + r_{t+k}} \left[ (1 - \epsilon) \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} Y_{t+k} + \epsilon \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} \left( \frac{P_{t+1}^H}{P_{t+1,k}^H} \right)^{-\epsilon} Y_{t+k} MC_{t+k}^n \right]$$

$$= E_t \sum_{k=0}^{\infty} \frac{\theta_H^k}{1 + r_{t+k}} \left[ (1 - \epsilon) Y_{t+k}^d (i) + \epsilon Y_{t+k}^d MC_{t+k}^n \right]$$

$$= E_t \sum_{k=0}^{\infty} \frac{Y_{t+k}^d (i) \theta_H^k}{1 + r_{t+k}} \left[ (1 - \epsilon) + \epsilon MC_{t+k}^n \right]$$
The second equation make use of demand constraint for good $i$. And set above equation to zero, multiply both sides by $P'_{H,t}/(1 - \varepsilon)$, manipulation details are following

$$
E_t \sum_{k=0}^{\infty} Y_{t+k}^d(i) \frac{\theta_H^k}{1 + \varepsilon} \left[ P'_{H,t} + \frac{\varepsilon}{1 - \varepsilon} MC_{t+k}^\varepsilon \right] = 0
$$

$$
E_t \sum_{k=0}^{\infty} Y_{t+k}^d(i) \theta_H^k \frac{P_{t+k}^d}{P_{t+k}^c} \left[ \frac{C_{t+k}^d - P_{t+k}^c}{C_{t+k}^d - P_{t+k}^c} \right] \left[ P'_{H,t} + \frac{\varepsilon}{1 - \varepsilon} MC_{t+k}^\varepsilon \right] = 0
$$

$$
\frac{1}{P_{t+k}^c} \sum_{k=0}^{\infty} Y_{t+k}^d(i) \theta_H^k \beta^k \left[ C_{t+k}^d - P_{t+k}^c \right] \left[ P'_{H,t} + \frac{\varepsilon}{1 - \varepsilon} MC_{t+k}^\varepsilon \right] = 0
$$

The second equation make uses of Euler equation. The fifth equation above uses the fact $MC_{t+k}^\varepsilon = P_{H,t+k}MC_t$ and the seventh equation replace $P_{H,t+k}/P_{H,t-1}$ with $\Pi_{t,k}$. Rearrange you will get $P'_{H,t}$ in equation (2.10).

### C.3 Internation Risk Sharing

Stack domestic Euler equation over foreign Euler equation

$$
1 = \frac{\beta E_t Q_{t+1}^{-1} \left( \frac{C_{t+1}^d}{C_t^d} \right)^{-\sigma} \frac{P_{t+1}^c}{P_{t+1}^c} \left( \frac{P_{t+1}^d}{P_{t+1}^c} \right)^{\sigma - \varepsilon} \frac{E_{t+1} P_{t+1}^c}{E_{t+1} P_{t+1}^c}}{\beta E_t Q_{t+1}^{-1} \left( \frac{C_{t+1}^d}{C_t^d} \right)^{-\sigma} \frac{P_{t+1}^c}{P_{t+1}^c} \left( \frac{P_{t+1}^d}{P_{t+1}^c} \right)^{\sigma - \varepsilon} \frac{E_{t+1} P_{t+1}^c}{E_{t+1} P_{t+1}^c}}
$$

(C.4)

where $Q_{t,t+1} = \frac{1}{1 + \varepsilon}$. Canceling and collecting terms,

$$
C_{t}^{-\sigma} = E_t \left[ \left( \frac{C_{t+1}^d}{C_{t+1}^d} \right)^{-\sigma} \left( C_{t+1}^d \right)^{-\sigma - \varepsilon} \frac{E_{t+1} P_{t+1}^d}{E_{t+1} P_{t+1}^d} \right]
$$

(C.5)

Make use of the definition of real exchange rate,

$$
C_t = C_t^d Q_t^\frac{1}{\sigma} E_t \left[ \left( \frac{C_{t+1}^d}{C_{t+1}^d} \right) Q_{t+1}^{-\frac{1}{\sigma}} \right]
$$

(C.6)

If we set $\Xi = E_t \left[ \left( \frac{C_{t+1}^d}{C_{t+1}^d} \right) Q_{t+1}^{-\frac{1}{\sigma}} \right]$ which is a constant dependent on initial net assets positions. Under symmetric assumption, $\Xi = 1$, thus the log form of (C.6) is

$$
c_t = c_t^d + \frac{Q_t}{\sigma}
$$

(C.7)

which is the international risk sharing condition.
Bibliography


