Role of the $\Delta(1232)$ in Parity Nonconserving Nucleon-Nucleon Interactions

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ACADEMIC DISSERTATION
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Abstract

The basic goal of nuclear physics is the understanding of atomic nuclei and their interactions. The knowledge of the interactions in the simplest two-nucleon systems forms the foundation for understanding more complex nuclear structures. However, unlike the two-nucleon strong interaction, its weak analog is poorly constrained. The observable part of this diminutive force arises exclusively from the parity nonconserving property of weak interaction.

The primary focus of this thesis is on the effects of the lowest-lying nucleon excitation, the $\Delta$-resonance, in the parity nonconserving two-nucleon interactions. The special attention is paid to the $\Delta$-effect in the two-pion exchange which represents the medium-range component of the weak nuclear force. The interest to the parity nonconserving two-pion exchange as a significant contribution has resurfaced recently. Conventionally, this effect has been omitted from the analysis of the parity nonconserving observables. Some previous calculations, however, have considered it partially by including the nucleonic intermediate states only. The calculations presented in this thesis include also the nucleon-$\Delta$ intermediate states explicitly. In general, the thesis revises a number of known parity nonconserving observables by taking into account the $\Delta$ and two-pion exchange effects within realistic wavefunctions and various potential models.
Acknowledgements

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Tero Partanen
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This thesis is based on the following four publications [1–4]:


Author’s contribution

The first publication based on the calculation method suggested by Niskanen. The author carried out the calculations and wrote the first draft of the manuscript. The paper was jointly written with Niskanen. In other joint publications the author carried out the calculations and wrote the manuscripts which were further refined and polished with Niskanen. All authors participated in discussion and manuscript revision.
Chapter 1

Introduction

One cannot emphasize enough the importance of nuclear physics, which lies not only in understanding of the basic structure of matter, but also in numerous practical applications that affect everyday life. However, despite a great deal of effort, the current knowledge of nuclear interactions is still largely based on phenomenology. Even the underlying mechanisms of the simplest two-nucleon ($NN$) interactions are not clear and coherent. This is seen as the model-dependence between various nuclear potentials on the market. The study of the parity nonconserving (PNC) nuclear reactions may yield important insights into the more fundamental description of nuclear forces. Such reactions are also the only access to the strangeness conserving hadronic weak interaction, which is the least understood sector of the Standard Model. This thesis considers the structurally simplest of the reactions, namely, the PNC $NN$ interactions. The PNC nuclear reactions arise from a complex interplay between both strong and weak nuclear forces of which history stretches over more than a century.

The discovery of radioactivity by Becquerel at the end of the 19th century provided the first evidence of weak interaction and nuclear forces in general, marking the birth of nuclear physics. Up until then, the electromagnetism and gravity were the only known fundamental interactions. The existence of strong interaction became evident in 1918 as a result of Rutherford’s identification of the proton as an elementary particle present in all nuclei. Thus, the current perception that Nature operates through the four fundamental interactions became established already for about a century ago. Nowadays, apart from gravity, they are combined into one modern, symmetry-based theory of particle physics, the so-called Standard Model (SM), which describes well the
behaviour of the elementary particles. Despite its great success over a wide energy range, it has yet remained unresolved at the ends of the spectrum. The interest of this thesis lies in the low energy end where the fundamental theory of strong interactions, quantum chromodynamics (QCD), is nonperturbative and thus too difficult to apply directly. In this energy regime, which belongs to the realm of nuclear physics, quarks and gluons prove to be ineffective degrees of freedom. The nuclear systems are therefore preferably understood in terms of nucleons and mesons. The proposal that a virtual pion acts as a carrier of strong nuclear force by Yukawa in 1935 launched the development of the meson exchange model. Nowadays, QCD based effective field theory (EFT) has taken over much of the field. Unlike the phenomenological meson exchange model, EFT provides a systematic and model-independent approach to study hadronic reactions at low energies.

Symmetries play an outstanding role in physics not only as they simplify theories but as they also sort out different interactions by constraining them in characteristic ways. It is long known that the continuous space-time symmetries and consequent conservation laws are exact in Nature. This, however, is not absolute for the discrete space-time symmetries of space inversion (parity P) and time reversal (T), let alone less prosaic charge conjugation (C). The originator of this peculiarity is solely weak interaction which is the most rebellious of all interactions. It violates not only the flavour conservation and isospin but also all those discrete symmetries individually and in paired combinations. This thesis focuses merely on parity nonconservation (PNC), a discovery that dates back to the mid-1950’s. However, before going into that topic, it is in order to give the mathematical and physical description of parity as a spatial state.

The parity transformation $P$ is defined as a reflection of all spatial coordinates about the origin $\mathbf{r} \rightarrow -\mathbf{r}$. Even though its mathematical definition corresponds to the space inversion, in practice, however, it turns out to be a mirror reflection of the transformed system. This one-on-one correspondence follows from the rotation invariance as a consequence of the conservation of angular momentum. Because $P$ reverses the spatial coordinates, it also reverses the direction of the momentum $\mathbf{p}$, leaving consequently the orbital angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ and its analogue the spin $\mathbf{S}$ unchanged. Especially, the concept of handedness or helicity is intimately related to the space inversion. The helicity $h$ of a particle is the spin projection along the direction of motion and is thus given by the pseudoscalar operator $h = \mathbf{S} \cdot \mathbf{\hat{p}}$.  


A spin-$\frac{1}{2}$ particle exhibits a helicity flip under $\mathcal{P}$ as illustrated in figure 1.1. As for the $NN$ system, $\mathcal{P}$ corresponds to a swap of the nucleons which produces a phase $(-)^L$. Therefore, the parity of the $NN$ system is defined uniquely by the relative orbital angular momentum state of the nucleons, from which follows that the odd and even $L$ transitions are due to the PNC and PC nuclear forces, respectively. Moreover, each particle has also a definite intrinsic parity, so that the total parity of a system is in fact obtained by the product of the spatial parity and the intrinsic parities of the particles in the system. Let it further be noted that if parity were a conserved quantity, the right-handed and (its mirror image) left-handed configurations would be simply equivalent conventions. This, however, is not the case in reality.

Parity conservation was taken for granted for a long time until the mid-1950’s, when it came as a surprise that Nature indeed makes the difference between the world and its mirror-reflected counterpart. The observation that lead to this discovery was the finding of two strange mesons $\tau$ and $\theta$ in cosmic rays a few years earlier. The puzzling thing about these new particles was that they appeared to be identical in every respect except one, namely their decay modes via weak interaction indicated that they had different parities. At that time they were considered as two distinct particles because parity symmetry was expected to hold in Nature. In the mid-1950’s, it was realised that the parity symmetry had never actually been tested in weak interaction and, therefore, the PNC was suggested as the possible solution of this so-called $\tau$-$\theta$-puzzle [5]. The PNC was shortly afterwards observed experimentally in the asymmetry of the angular distribution of electrons in the $\beta$-decay of polarized $^{60}$Co nuclei [6]. As a result the $\tau$ and $\theta$ turned out to be the one and the same particle (known nowadays as the kaon) decaying...
in two different ways. The first attempt to see hadronic parity violation in nuclear reactions [7] was already carried out in the very same year as the breakthrough $^{60}$Co experiment. Not until a decade later the first positive signal was found in the radiative neutron capture on $^{181}$Ta nuclei [8]. The PNC is thereafter observed in numerous nuclear reactions. In the $NN$ sector, it is so far observed only in the $\bar{p}p$ elastic scattering.

Weak interaction is distinct in the leptonic, semileptonic, and strangeness nonconserving hadronic processes. However, it is not so clear-cut in the strangeness conserving hadronic sector because of its diminutive strength against that of incessantly present strong interaction. Nevertheless, the PNC weak interaction is unique in the sense that it sorts out different helicity states unlike any other interaction. For this particular reason, it can, in principle, be extracted under those overwhelming but always parity conserving (PC) strong and electromagnetic interactions. The knowledge of the strangeness conserving hadronic weak interaction relies completely on the PNC observables. The PNC $NN$ interactions provide a possible access to this least understood sector of the SM. Experimentally such subtle particle spin control based PNC measurements are feasible but highly demanding. On the theoretical side, the challenge lies largely in the poorly known coupling values which parametrize the strength of the minuscule PNC signal.

Even though a direct heavy $Z^0$ or $W^\pm$ boson exchange is highly improbable over the internuclear distances, it is feasible between the nucleon and virtual meson. Consequently, the PNC $NN$ interactions may be parametrized by weak meson-$NN$ coupling constants modelled in terms of quarks and intermediate bosons. Traditionally the PNC $NN$ calculations have relied largely on the single meson exchange picture, based on the DDH potential [9] in which the PNC $NN$ interactions are due to $\pi^\pm$, $\rho$, and $\omega$ exchanges, supplemented by an appropriate strong potential. Nowadays at very low energies, the calculations are preferably done in the framework of the model-independent effective field theories (EFT). However, all these models are parametrized by about half a dozen weak meson-$NN$ couplings (see e.g. [10, 11]), which are, even today, insufficiently known despite all the experimental and theoretical efforts.

The isospin symmetry plays also an important role in the PNC $NN$ interactions. It can be shown that such interactions are in general composed of isoscalar, isovector, and isotensor channels. Especially, the isovector part merits special interest for a couple of reasons. Firstly, it is by far the longest-range component of the PNC $NN$ interaction, since only it is mediated by
the pion exchange. This restriction imposed by CP-invariance is known as Barton’s theorem [12]. Secondly, in this particular channel, the charged current is Cabibbo suppressed and, thus, the PNC effects are expected to arise exclusively from the neutral current, see e.g. [13]. Since the flavour changing neutral current is suppressed by the GIM mechanism [14], the isovector PNC \( NN \) interaction provides a window of opportunity to observe exclusively the hadronic neutral current. Moreover, the PNC \( NN \) interactions may also shed some light on the poorly understood experimental fact that in the strangeness nonconserving weak decays the observed isospin-change \( \Delta I = 1/2 \) branching ratios are greatly favored over the \( \Delta I = 3/2 \) ones. The dynamical origin of this empirical so-called \( \Delta I = 1/2 \)-rule is not clear. It is also yet to be seen whether there is complementary behaviour in the strangeness conserving sector, too.

Although there are a number of calculations of the observables discussed in the thesis, the contributions of the two-pion exchange (TPE) and the \( \Delta (1232) \)-isobar excitation are customarily neglected in them. The PNC TPE was already studied long ago by several authors [15–19], but it was concluded to be small and, thus, omitted for further analysis. Nevertheless, the interest in this effect has resurfaced recently [1, 3, 4, 10, 20–24]. The emphasis of this thesis is largely on the investigation of the importance of these corrections in the PNC \( NN \) processes. The thesis is based on the four aforementioned peer-reviewed research articles I-IV referred to as [1–4], respectively. In the papers [1] and [3], the PNC longitudinal analyzing power \( \bar{A}_L \) in elastic \( \vec{p}p \) scattering was considered. A PNC TPE potential including the \( NN \) and \( N\Delta \) intermediate states was derived in terms of time-ordered perturbation theory and used in the evaluation of the observable \( \bar{A}_L \) in [1]. The latest calculation [3] of \( \bar{A}_L \) is an improved version of [1] which did not take into account the Coulomb repulsion between the protons nor the nonlocal contributions of the short-range PNC interaction. Besides a careful inclusion of the Coulomb interaction and the fully employed PNC potentials, the latter work [3] considered also the pionful EFT calculations as a new feature. Also the importance of the TPE was emphasized further by employing two different EFT PNC TPE potentials as a comparison. The asymmetries in PNC deuteron photodisintegration \( \vec{\gamma}d \rightarrow np \) and neutron capture \( \vec{n}p \rightarrow \gamma d \) were investigated in [2]. The main purpose of the paper was to study the effect of the \( \Delta \)-isobar within the coupled-channels technique by using the derived PNC one-meson exchange \( NN \leftrightarrow N\Delta \) transition potential. The special interest was in the reaction \( \vec{\gamma}d \rightarrow np \) at threshold because it is insensitive to the pion
exchanges and, thus, the $\Delta$ was expected to be somewhat highlighted against the background that arise only from the short-range contributions. In [4] the focus was on the PNC interactions between cold neutrons and parahydrogen molecules. When travelling through a parahydrogen target, cold neutrons may exhibit three different PNC phenomena: transversely polarized neutron spin rotation, unpolarized neutron longitudinal polarization, and photon-asymmetry of radiative polarized neutron capture. All three pion sensitive observables were calculated using the same potential models as in [3]. In this last paper, the PNC reaction $\vec{n}p \rightarrow \gamma d$ calculated in [2] was improved further by taking into account the one-pion exchange currents and the PNC TPE. The reactions presented in the thesis provide a versatile learning ground in the field of nuclear physics including the bound- and continuum-state problems along with the TPE and $\Delta$ complications. All the calculations are based on the wavefunction formalism in the configuration space and carried all the way out to the numerical predictions of the observables.

The remainder of the thesis is organized as follows. Chapter 2 outlines the basic formalism and ingredients that are used in the evaluation of the observables. Chapter 3 presents the observables and results. Finally, in Chapter 4 the main findings of the thesis are summarized.
Chapter 2

Formalism

This chapter gives the essential ingredients used in the evaluation of the PNC $NN$ observables. The calculations are based on the wavefunction formalism involving electromagnetic, strong, and weak interactions.

2.1 Preliminaries

This section specifies the general notations and definitions that will be used throughout the thesis. We start with the remark that the Condon-Shortley phase convention as well as the natural $\hbar = c = 1$ and Gaussian $4\pi\varepsilon_0 = 1$ units are adopted in the thesis.

The nucleon is both a spin-$\frac{1}{2}$ fermion and an isospin-$\frac{1}{2}$ particle that forms a proton-neutron doublet. The irreducible operators that operate on the spin and isospin spaces of nucleons are the Pauli spin matrices $\sigma$ and $\tau$, respectively. Due to the Pauli exclusion principle, the wavefunction of the $NN$ system must be antisymmetric (odd) under the interchange of the nucleons. The overall $NN$ wavefunction is composed of spatial, spin, and isospin wavefunctions. The even/odd symmetries of these partial wavefunctions are respectively characterized by the relative orbital angular momentum ($L$), the total spin ($S$), and the total isospin ($T$). The even values of $L$ and $S = T = 1$ (spin and isospin triplets) correspond to the even symmetries while the odd values of $L$ and $S = T = 0$ (spin and isospin singlets) correspond to the odd symmetries. As a consequence, the sum of the $L$, $S$, and $T$ must be an odd integer, i.e. $(-)^{L+S+T} = -1$, which serves as a constraint on the $NN$ wavefunctions.
In order to simplify the notations, the quantum numbers $LSJT$, where $J$ is the total angular momentum, are abbreviated to $Q$. Since the $LST$ may be changed by the nuclear forces they are denoted by $\xi$ for brevity, that is $Q = \xi J$. Because of the antisymmetry requirement of the wavefunction, the isospin $T$ may as well be considered uniquely defined by the $LS$, and so, on occasion, the $Q$ is also designated for convenience with the spectroscopic notation $Q = 2S + 1 L J$. The first six used values of $L = 0, 1, ..., 5$ are for historical reasons typically labelled $S, P, D, F, G, H$.

The nonrelativistic kinematics is adequate, since the used nucleon laboratory kinetic energies $T_{lab}$ are relatively low. In the two-body systems $(i = 1, 2)$ where $m_i$ is the mass, $r_i$ the position, and $k_i$ the laboratory momentum (wavevector) of the $i$-th particle, the center-of-mass (C.M.) $R$ and relative $r$ coordinates are given by

\[
R = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \quad \text{and} \quad r = r_1 - r_2 \tag{2.1}
\]

and correspondingly the total $K$ and relative $k$ momenta are given by

\[
K = k_1 + k_2 \quad \text{and} \quad k = \frac{m_2 k_1 - m_1 k_2}{m_1 + m_2}. \tag{2.2}
\]

The $K$ and $k$ are the conjugate momenta of the $R$ and $r$ coordinates respectively: $K \leftrightarrow -i \nabla_R$ and $k \leftrightarrow -i \nabla_r$ ($\nabla_r \equiv \nabla$). The reduced mass $\mu$ that appears frequently is defined by

\[
\mu = \frac{m_1 m_2}{m_1 + m_2}. \tag{2.3}
\]

In the experimental setups one particle (the target) is considered at rest $k_2 = 0$. The relation between the kinetic energies in the laboratory and C.M. systems is ($1 \to 2$ (rest))

\[
T_{lab}^1 = T^{C.M.} + T^{rel} = \frac{k_1^2}{2 \mu} \frac{m_1}{m_2} + \frac{k_2^2}{2 \mu} = \frac{k^2}{2 \mu} \frac{m_1}{m_2}, \tag{2.4}
\]

where $T^{C.M.}$ and $T^{rel}$ are respectively the kinetic energies of the C.M. and the relative motion in the C.M. system. The Fourier transform and momentum transfer $q$ are given by

\[
f(r) = \int \frac{d^3q}{(2\pi)^3} \tilde{f}(q)e^{iqr}, \quad q = k' - k, \tag{2.5}
\]
where \( \mathbf{k} \) denotes the initial and \( \mathbf{k}' \) the final momentum.

The observables are built up matrix elements of operators sandwiched between wavefunctions. The \( NN \)-operators \( \hat{O}(\mathbf{r}) \) are basically combined isospin-spin-space operators. The matrix elements for coupled angular momenta of a spherical tensor operator are defined as

\[
\langle L'S'J'M' | \hat{O}(\mathbf{r}) | LSJM \rangle = \int d\Omega_r \mathcal{Y}^{J'M'}_{LS}(\mathbf{r}) \hat{O}(\mathbf{r}) \mathcal{Y}^{JM}_{LS}(\mathbf{r}),
\]

(2.6)

where the integration is over the solid angle \((\theta, \phi)\) in the direction of \( \mathbf{r} \) and

\[
\mathcal{Y}^{JM}_{LS}(\mathbf{r}) = \sum_{M_L M_S} \langle LM_L S M_S | JM \rangle Y_{LM_L}(\mathbf{r}) | SM_S \rangle
\]

(2.7)

are the eigenfunctions of the coupled angular momentum composed of the Clebsch-Gordan coefficient, spherical harmonic, and coupled spin state. The formulae for the determination of the matrix elements of (2.6) can be found, e.g., in [25].

### 2.2 Potentials

The relevant potential for the PNC \( NN \) observables contains both the PC and PNC components \( \hat{V} = \hat{V}^{PC} + \hat{V}^{PNC} \), where \( \hat{V}^{PC} \) splits up further into the Coulomb \( \hat{V}_C \) and strong nuclear force \( \hat{V}_N^{PC} \) potentials. The nuclear potential is denoted by \( \hat{V}_N = \hat{V}_N^{PC} + \hat{V}^{PNC} \). All these potentials are formally scalar operators, that is, they are tensors of zeroth rank, and thus their matrix elements for coupled angular momenta have unambiguous geometrical parts that ensure the conservation of the total angular momentum \( J \) and its projection \( M \). The matrix elements \( \mathcal{V}_{\xi}(r) \) of the \( NN \) potential \( \hat{V} \) are defined by

\[
\langle L'S'J'M' | \langle T'M_T | \hat{V} | T M_T \rangle | LSJM \rangle = \mathcal{V}_{\xi}(r) \delta_{J'J} \delta_{M'M},
\]

(2.8)

with \( M_T = 1 \) for \( pp \) and \( M_T = 0 \) for \( np \) interaction.

#### 2.2.1 Nucleon-nucleon potentials

There are many realistic strong \( NN \) interaction potentials on the market to choose from. These potentials are semi-phenomenological and designed to reproduce the experimental \( NN \) interaction data accurately at energies below
the inelastic threshold. The more or less common and model-independent feature of them is that their long-range tails are represented by the one-pion exchange (OPE) potential. The shorter-range correlations, however, are constructed in various ways and cause some model-dependence. This trouble also reflects directly to the short-range effects of the PNC interaction. In this research, three phenomenological local soft-core potentials Reid93 [26], Argonne $A_{v_{18}}$ [27], and Reid68 [28] were used. The Reid93 and $A_{v_{18}}$ are modern charge dependent high-precision potentials. The $A_{v_{18}}$ applies Yukawa functions for the intermediate-range and Woods-Saxon ones for the short-range contributions, while the Reid68 and its update Reid93 use the multiple pion range Yukawa functions for these ranges. The Reid93 was employed in [2–4], $A_{v_{18}}$ also in [2], and Reid68 in [1].

The Coulomb potential represents the longest-range part of the $NN$ interaction. It contributes only between protons and is given by

$$V_C(r) = \frac{\alpha}{r},$$

where $\alpha = 1/137.036$ is the fine-structure constant.

Due to the antisymmetry requirement the PNC $NN$ interaction is compelled to change either the spin or isospin of the system and consequently the potential of the interaction is composed of various spin-isospin operators weighted by coupling constants. There are at least three considerable candidates for the PNC $NN$ potentials. The new model-independent QCD based effective field theory (EFT) approach offers two alternative choices for these potentials, namely the pionless and pionful [10, 11]. The pionless one consists only of short-range contact interaction whereas the pionful also includes the long- and medium-range interactions mediated respectively by the one- and two-pion exchanges. The third potential is the most conventional DDH meson-exchange model [9] which takes into account the long- and short-range effects in terms of the single $\pi^{\pm}, \rho$, and $\omega$ exchanges but not the two-pion exchange (TPE) contributions which are supposedly important in the medium-range. In spite of the different approaches and underlying ideas, the operators appearing in the potentials turn out to be essentially the same, except that the ranges of the forces vary. Another similarity with the potentials is that they are all parametrized in terms of about half a dozen ill-known couplings. In EFT these so-called low energy constants (LECs) are expected to be extracted from the experimental data of a series of prospective high-precision measurements. As a downside, EFT potentials cannot be
2.2 Potentials

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$h^{(0)}_\alpha$</th>
<th>$h^{(1)}_\alpha$</th>
<th>$h^{(2)}_\alpha$</th>
<th>$g_\alpha$</th>
<th>$\chi_\alpha$</th>
<th>$m_\alpha$</th>
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<td>-</td>
<td>4.6</td>
<td>-13.45</td>
<td>-</td>
<td>-139.6</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
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<td>-0.2</td>
<td>-9.5</td>
<td>2.79</td>
<td>3.71</td>
<td>770.0</td>
</tr>
<tr>
<td>$\omega$</td>
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<td>-1.1</td>
<td>-8.37</td>
<td>-0.12</td>
<td>782.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1: Standard set of weak $h^{(k)}_\alpha$ and strong $g_\alpha, \chi_\alpha$ ($\alpha = \pi, \rho, \omega$) couplings. The weak couplings are the DDH ”best” values in units of $10^{-7}$. The meson masses $m_\alpha$ are in units of MeV. The average nucleon mass is $M = 939$ MeV.

used in the evaluation of observables from first principles yet. Nevertheless, the LECs may be (loosely) interpreted numerically in terms of the meson exchange model parameters in a similar fashion as done e.g. in [10, 29]. Regardless of the explicit TPE potential which is not implemented in the DDH model, this procedure restores the pionful EFT ($\pi$EFT) back to the traditional meson-exchange picture. Therefore, basically, for over thirty years up until today, the theoretical predictions have largely rested on the DDH model and their recommended ”best” values for the weak couplings. The PNC one-pion exchange (OPE) potentials of the $\pi$EFT and DDH model coincide and are proportional to the weak $NN\pi$ coupling $h^{(1)}_\pi$. The strength of the PNC TPE depends also on the same coupling.

There are no single neutral spinless meson exchanges in the on-shell PNC $NN$ interaction due to PC invariance [12]. The DDH potential is based on the light meson $\pi^\pm, \rho$, and $\omega$-exchanges in which one vertex is PC and the other one PNC. The DDH potential arises from the most general effective interaction Hamiltonians which fulfill the appropriate conditions, yielding

$$
\hat{V}^{\text{DDH}}(\mathbf{r}) = \frac{g_\pi}{M} \frac{i h^{(1)}_\pi}{2 \sqrt{2}} \mathbf{\hat{\tau}}_\pi \cdot \mathbf{\hat{X}}^\pi_{[+]}(\mathbf{r}) - \frac{g_\omega}{M} \frac{h^{(1)}_\omega}{2} \mathbf{\hat{\tau}}_\omega \cdot \mathbf{\hat{X}}^\omega_{[+]}(\mathbf{r})
$$

$$
- \frac{g_\rho}{M} \frac{h^{(0)}_\rho}{2} \mathbf{\hat{\tau}}_\rho + \frac{h^{(1)}_\rho}{2} \mathbf{\hat{\tau}}_\rho + \frac{h^{(2)}_\rho}{2 \sqrt{6}} \mathbf{\hat{\tau}}_\rho \left( \mathbf{\hat{X}}^\rho_{[+]}(\mathbf{r}) + i(1 + \chi_\rho) \mathbf{\hat{X}}^\rho_{[-]}(\mathbf{r}) \right)
$$

$$
+ \frac{g_\rho}{M} \frac{h^{(1)}_\rho}{2} \mathbf{\hat{\tau}}_\rho \cdot \mathbf{\hat{X}}^\rho_{[+]}(\mathbf{r}) - \frac{g_\rho}{M} \frac{i h^{(1)}_\rho'}{2 \sqrt{6}} \mathbf{\hat{\tau}}_\rho \cdot \mathbf{\hat{X}}^\rho_{[-]}(\mathbf{r}),
$$

(2.10)
where $\hat{X}_\alpha^\sigma (r)$ are the spin-space structures given by

$$\hat{X}_\alpha^\sigma (r) = (\sigma_1 \otimes \sigma_2) \cdot [-i \nabla, Y_\alpha (r)]_\pm ,$$  \hspace{1cm} (2.11)

with the commutator $[\otimes]_-$ and anticommutator $[\otimes]_+$ terms ($\otimes = \pm, \times$) and Yukawa functions

$$Y_\alpha (r) = \frac{e^{-m_\alpha r}}{4\pi r},$$  \hspace{1cm} (2.12)

where $\alpha = \pi, \rho, \omega$. The shorthand notations for the isospin operators are $\hat{\tau}^\otimes_\pm = (\tau_1 \otimes \tau_2)_z$, $\hat{\tau}^\cdot_\pm = \tau_1 \cdot \tau_2$, and $\hat{\tau}^\otimes = 3\hat{\tau}_z \hat{\tau}_2 - \hat{\tau}_1 \cdot \tau_2$. The values of the parameters are given in Table 2.1. The term proportional to $h_\rho^{(1)}$ will be omitted, as is customary, due to its smallness [30]. Since the DDH potential (2.10) is no more singular than the centrifugal barrier ($\sim 1/r^2$), a regularization is not necessary. However, in the case of high energies the wavefunctions probe relatively short ranges, and hence the form factors of the type $(\Lambda_\alpha^2 - m_\alpha^2)/(q^2 + \Lambda_\alpha^2)$ are included in vertices and (2.12) takes the modified form

$$Y_\alpha (r) = \frac{e^{-m_\alpha r}}{4\pi r} - \frac{e^{-\Lambda_\alpha r}}{4\pi} \left( 1 + \frac{\Lambda_\alpha^2 - m_\alpha^2}{2\Lambda_\alpha} \right).$$  \hspace{1cm} (2.13)

The total PNC $NN$ potential in the pionless effective field theory ($\pi$EFT) is completely "short-ranged" $\hat{V}_{1,SR}$ with $m = m_\pi$ and in the $\pi$EFT it includes also the long- and medium-range (LR,MR) parts $\hat{V}_{1,LR} + \hat{V}_{1,MR} + \hat{V}_{1,SR}$, where $\hat{V}_{1,SR}$ is with $m \sim 1$ GeV [10]. The short-range (SR) EFT potential [10] is given by

$$\hat{V}_{1,SR}(r) = \frac{2m_\pi^2}{\Lambda_\chi^2} \left[ \left( C_1 + C_3 \tilde{\tau}^\cdot + \frac{1}{2} (C_2 + C_4) \tilde{\tau}^\cdot + C_5 \tilde{\tau}^\otimes \right) \hat{X}_{[-]+} (r) \right.$$

$$+ i \left( \tilde{C}_1 + \tilde{C}_3 \tilde{\tau}^\cdot + \frac{1}{2} (\tilde{C}_2 + \tilde{C}_4) \tilde{\tau}^\cdot + \tilde{C}_5 \tilde{\tau}^\otimes \right) \hat{X}_{[+]-} (r) \right.$$  \hspace{1cm} (2.14)

$$+ \frac{1}{2} (C_2 - C_4) \tilde{\tau}^\cdot \hat{X}_{[+]+} (r) + iC_6 \tilde{\tau}_x \hat{X}_{[+]-} (r) \right] ,$$

where the $C_i$ and $\tilde{C}_i$ are so-called low energy constants expected to be determined experimentally and $\Lambda_\chi = 4\pi f_\pi$ with the pion decay constant $f_\pi = 92.4$ MeV. It is shown in [31] that the potential (2.14) reduces to the form which has only five independent operators instead of ten. This reduced potential
2.2 Potentials

Figure 2.1: The EFT PNC TPE diagrams. The black dot denotes the PNC vertex, the filled bar the Δ-resonance, solid line the nucleon, and the dashed line the pion. The potential $\hat{V}_{K}^{\text{TPE}}$ [20] considers all diagrams and $\hat{V}_{D}^{\text{TPE}}$ [24] the first three diagrams.

\begin{equation}
\hat{V}_{1,\text{SR}}(r) = \frac{2m^2}{\Lambda^3} \left[ \left( C_1 + \frac{1}{2} (C_2 + C_4) \hat{\tau}_+ - C_5 \hat{\tau}_\otimes \right) \hat{X}_{[-]}(r) + i \tilde{C}_1 \hat{X}_{[\times]}(r) + i C_6 \hat{\tau}_+ \hat{X}_{[\times]}(r) \right].
\end{equation}

If the parameters of the $\pi$EFT potential are interpreted in terms of the DDH ones, then the $\pi$EFT potential takes the form $\hat{V}_{\pi}^{\text{EFT}} \rightarrow \hat{V}_{\pi}^{\text{DDH}} + \hat{V}_{1,\text{MR}}$ (i.e. $\hat{V}_{-1,\text{LR}} \rightarrow \hat{V}_{\pi}^{\text{DDH}}$ and $\hat{V}_{1,\text{SR}} \rightarrow \hat{V}_{\pi}^{\text{DDH}}$, see the discussion, e.g. in [10, 29]). The medium-range part $\hat{V}_{1,\text{MR}}$ arises from the TPE and is denoted as $\hat{V}_{\text{TPE}}$.

The EFT TPE potentials $\hat{V}_{\text{TPE}}^{\text{EFT}}$ for the PNC $NN$ interaction are taken from [20] and [24] and denoted, respectively, as $\hat{V}_{K}^{\text{TPE}}$ and $\hat{V}_{D}^{\text{TPE}}$. Note that, besides $\hat{V}_{K}^{\text{TPE}}$, there exists also another chiral perturbation theory derivation for the PNC TPE $N\Delta$ potential [21], which however is not utilized in this work. One might also note that $\hat{V}_{D}^{\text{TPE}}$ is the same as the one given in [33] and essentially the same as the one first derived in [10]. Both $\hat{V}_{K}^{\text{TPE}}$ and $\hat{V}_{D}^{\text{TPE}}$ are local potentials of the form

\begin{equation}
\hat{V}_{\text{TPE}}^{\text{EFT}}(r) = h_{\pi}^{(1)} \hat{\tau}_+ (\sigma_1 \times \sigma_2) \cdot \hat{r} W(r) + h_{\pi}^{(1)} \hat{\tau}_x (\sigma_1 + \sigma_2) \cdot \hat{r} U(r),
\end{equation}

in which the radial functions $W(r)$ and $U(r)$ differ in the following two aspects. The first difference is that $\hat{V}_{K}^{\text{TPE}}$ takes into account the $NN$ and $N\Delta$ intermediate states, while $\hat{V}_{D}^{\text{TPE}}$ considers only the $NN$ ones as depicted in Figure 2.1. The second is that the term proportional to the delta function $\delta(r)$ is included in $\hat{V}_{D}^{\text{TPE}}$ but omitted in $\hat{V}_{K}^{\text{TPE}}$. The radial functions for $\hat{V}_{D}^{\text{TPE}}$ are [24]

\begin{equation}
W_{NN}^{D}(r) = -\frac{4\sqrt{2}\pi g_A^3}{\Lambda^3} \frac{\partial}{\partial r} L(r),
\end{equation}
U_{NN}^{D}(r) = -\frac{\sqrt{2}\pi}{\Lambda_{\chi}^3} \frac{\partial}{\partial r} \left( g_{\Lambda} L(r) - g_{\Lambda}^3 [3L(r) - H(r)] \right), \quad (2.18)

where $g_{\Lambda} = 1.3$ and $L(r)$ and $H(r)$ are given in momentum space by

$$
\tilde{L}(q) = \frac{\sqrt{q^2 + 4m_{\pi}^2}}{q} \ln \left( \frac{\sqrt{q^2 + 4m_{\pi}^2} + q}{2m_{\pi}} \right)
= -\int_{2m_{\pi}}^{\infty} d\mu \sqrt{\mu^2 - 4m_{\pi}^2} \left( \frac{1}{\mu^2 + q^2} - \frac{1}{\mu^2 - 4m_{\pi}^2} \right), \quad (2.19)
$$

$$
\tilde{H}(q) = \frac{4m_{\pi}^2}{q^2 + 4m_{\pi}^2} \tilde{L}(q) = \int_{2m_{\pi}}^{\infty} d\mu \frac{4m_{\pi}^2}{\sqrt{\mu^2 - 4m_{\pi}^2} \mu^2 + q^2}. \quad (2.20)
$$

Configuration and momentum spaces are related by a Fourier transform. One might note that in the configuration space the latter term in the integral of (2.19) turns out to be essentially a Dirac delta function $\delta(r)$. The corresponding $NN$ parts of $\hat{V}_{K}^{TPE}$ are [20]

$$
W_{NN}^{K}(r) = -\frac{g_{\Lambda}^3}{8\sqrt{2}\pi f_{\pi}^3} \int_{2m_{\pi}}^{\infty} \frac{d\mu}{\mu^2} \sqrt{\mu^2 - 4m_{\pi}^2} \frac{\partial}{\partial r} I(r, \mu), \quad (2.21)
$$

$$
U_{NN}^{K}(r) = \frac{g_{\Lambda}}{32\sqrt{2}\pi f_{\pi}^3} \int_{2m_{\pi}}^{\infty} \frac{d\mu}{\mu^2} \sqrt{\mu^2 - 4m_{\pi}^2} \left[ g_{\Lambda}^3 \frac{3\mu^2 - 8m_{\pi}^2}{\mu^2 - 4m_{\pi}^2} - 1 \right] \frac{\partial}{\partial r} I(r, \mu), \quad (2.22)
$$

where $I(r, \mu)$ expressed in the unregularized form in momentum space is given by

$$
\tilde{I}(q, \mu) = \frac{q^2}{q^2 + \mu^2}. \quad (2.23)
$$

The $\delta$-function is omitted in the configuration representation of (2.23) to be equivalent with the analytic result of [20]. The $NN$ parts of $\hat{V}_{K}^{TPE}$ and $\hat{V}_{D}^{TPE}$ would coincide, if the constant term $(\mu^2 - 4m_{\pi}^2)^{-1}$ in the dispersion relation (2.19) were excluded. Nevertheless, following the regularization procedure described in [22] for $\hat{V}_{D}^{TPE}$, where the $\delta$-term is included, and the calculation of $\hat{V}_{K}^{TPE}$ [20], where the corresponding term is omitted by default, the $NN$ parts of the two similarly regularized TPE potentials differ from each other. Normally, the $\delta$-function is not felt by the wavefunctions due to the short-range repulsion. However, it contributes to the matrix elements when

$$
U_{NN}^{D}(r) = -\frac{\sqrt{2}\pi}{\Lambda_{\chi}^3} \frac{\partial}{\partial r} \left( g_{\Lambda} L(r) - g_{\Lambda}^3 [3L(r) - H(r)] \right), \quad (2.18)
$$

where $g_{\Lambda} = 1.3$ and $L(r)$ and $H(r)$ are given in momentum space by

$$
\tilde{L}(q) = \frac{\sqrt{q^2 + 4m_{\pi}^2}}{q} \ln \left( \frac{\sqrt{q^2 + 4m_{\pi}^2} + q}{2m_{\pi}} \right)
= -\int_{2m_{\pi}}^{\infty} d\mu \sqrt{\mu^2 - 4m_{\pi}^2} \left( \frac{1}{\mu^2 + q^2} - \frac{1}{\mu^2 - 4m_{\pi}^2} \right), \quad (2.19)
$$

$$
\tilde{H}(q) = \frac{4m_{\pi}^2}{q^2 + 4m_{\pi}^2} \tilde{L}(q) = \int_{2m_{\pi}}^{\infty} d\mu \frac{4m_{\pi}^2}{\sqrt{\mu^2 - 4m_{\pi}^2} \mu^2 + q^2}. \quad (2.20)
$$

Configuration and momentum spaces are related by a Fourier transform. One might note that in the configuration space the latter term in the integral of (2.19) turns out to be essentially a Dirac delta function $\delta(r)$. The corresponding $NN$ parts of $\hat{V}_{K}^{TPE}$ are [20]

$$
W_{NN}^{K}(r) = -\frac{g_{\Lambda}^3}{8\sqrt{2}\pi f_{\pi}^3} \int_{2m_{\pi}}^{\infty} \frac{d\mu}{\mu^2} \sqrt{\mu^2 - 4m_{\pi}^2} \frac{\partial}{\partial r} I(r, \mu), \quad (2.21)
$$

$$
U_{NN}^{K}(r) = \frac{g_{\Lambda}}{32\sqrt{2}\pi f_{\pi}^3} \int_{2m_{\pi}}^{\infty} \frac{d\mu}{\mu^2} \sqrt{\mu^2 - 4m_{\pi}^2} \left[ g_{\Lambda}^3 \frac{3\mu^2 - 8m_{\pi}^2}{\mu^2 - 4m_{\pi}^2} - 1 \right] \frac{\partial}{\partial r} I(r, \mu), \quad (2.22)
$$

where $I(r, \mu)$ expressed in the unregularized form in momentum space is given by

$$
\tilde{I}(q, \mu) = \frac{q^2}{q^2 + \mu^2}. \quad (2.23)
$$

The $\delta$-function is omitted in the configuration representation of (2.23) to be equivalent with the analytic result of [20]. The $NN$ parts of $\hat{V}_{K}^{TPE}$ and $\hat{V}_{D}^{TPE}$ would coincide, if the constant term $(\mu^2 - 4m_{\pi}^2)^{-1}$ in the dispersion relation (2.19) were excluded. Nevertheless, following the regularization procedure described in [22] for $\hat{V}_{D}^{TPE}$, where the $\delta$-term is included, and the calculation of $\hat{V}_{K}^{TPE}$ [20], where the corresponding term is omitted by default, the $NN$ parts of the two similarly regularized TPE potentials differ from each other. Normally, the $\delta$-function is not felt by the wavefunctions due to the short-range repulsion. However, it contributes to the matrix elements when
expanded to a finite range by the form factors. As illustrated in Figure 2.1, $V_{\text{K}}^{\text{TPE}}$ includes also the effects of the $N\Delta$ intermediate states given by [20]

$$W_{N\Delta}(r) = \frac{g_A^3}{32\sqrt{2}\pi f^3} \int_{2m_\pi}^{\infty} \frac{d\mu}{\mu^2} \left[ 2\sqrt{\mu^2 - 4m^2_\pi} + \frac{\pi}{4\Delta}(4m^2_\pi - \mu^2) \right] \frac{1}{\Delta} \left( 4m^2_\pi - 4\Delta^2 - \mu^2 \right) \arctan \frac{\sqrt{\mu^2 - 4m^2_\pi}}{2\Delta} \frac{\partial}{\partial r} \mathcal{I}(r, \mu), \quad (2.24)$$

$$U_{N\Delta}(r) = -\frac{g_A^3}{32\sqrt{2}\pi f^3} \int_{2m_\pi}^{\infty} \frac{d\mu}{\mu^2} \left[ \sqrt{\mu^2 - 4m^2_\pi} - \frac{1}{\Delta}(\mu^2 + 2\Delta^2 - 2m^2_\pi) \arctan \frac{\sqrt{\mu^2 - 4m^2_\pi}}{2\Delta} \right] \frac{\partial}{\partial r} \mathcal{I}(r, \mu), \quad (2.25)$$

where $\Delta = 293$ MeV and $\mathcal{I}(r, \mu)$ is given by (2.23). In the cases when the TPE potentials are regularized two different form factors with different cut-off masses are applied in order to see the sensitivity of the regularization. The TPE potentials are modified by the monopole $\Lambda^2(q^2 + \Lambda^2)^{-1}$ and dipole $\Lambda^4(q^2 + \Lambda^2)^{-2}$ form factors and used with three different but typical pion cut-off masses ranging from soft to hard $\Lambda = 0.8, 1.0, \text{ and } 1.2$ GeV. The same type of a monopole form factor was also used in $V_{D}^{\text{TPE}}$ in [22] to calculate the $\gamma$-asymmetry in $\vec{n}p \rightarrow \vec{\gamma}d$ at threshold. The regularization and the TPE potentials used here are consistently the same as applied in the PNC $pp$ elastic scattering [3]. The results of the medium-range TPE are expected to be somewhat sensitive to the form factors, which tend to have an increasingly suppressing effect on them when the cut-off mass is decreased and the rank of the form factor is increased.

In the spirit of [34], the time-ordered perturbation theory (TOPT) was also applied in [1] to derive the PNC TPE potential for the $pp$ interaction. The resulting potential is denoted here as $V_{\text{TO}}^{\text{TPE}}$. Like $V_{\text{K}}^{\text{TPE}}$, also $V_{\text{TO}}^{\text{TPE}}$ takes into account the $NN$ and $N\Delta$ intermediate states, as illustrated in Figure 2.2. The applied nonrelativistic PC $\pi NN$, PNC $\pi NN$, and PC $\pi N\Delta$ interaction vertices are respectively of the form

$$\frac{f_\pi}{m_\pi} \mathbf{\sigma} \cdot \nabla \mathbf{T} \cdot \mathbf{\pi}, \quad \frac{h^{(1)}_\pi}{\sqrt{2}} (\mathbf{\tau} \times \mathbf{\pi})_z, \quad \frac{f^{*}_\pi}{m_\pi} \mathbf{S} \cdot \nabla \mathbf{T} \cdot \mathbf{\pi} + \text{H.c.}. \quad (2.26)$$

The $\mathbf{S}$ and $\mathbf{T}$ represent the $N\Delta$ spin and isospin transition operators with the normalization $S^i_i S^i_j = (2\delta_{ij} - i\epsilon_{ijk}\sigma_k)/3$ [35] and $f^{*}_\pi = \sqrt{\frac{72}{25}} f_\pi$ (f_\pi = ...
Figure 2.2: The TOPT PNC TPE diagrams for the \( pp \) interaction. The constituents of the diagrams are explained in Figure 2.1. The rest of the diagrams are obtained by shifting the black dot to the other \( \pi NN \) vertices and exchanging \( N \leftrightarrow \Delta \). The direction of time is upwards.

\( m_\pi g_\pi /2M \) is the strong \( \pi N \Delta \) coupling following from the static constituent quark model \([35]\). A totally symmetric kinematics where the initial, final, and intermediate state baryons were assumed to be static was used in the derivation of \( \hat{V}_{\text{TPE}} \). The potential \( \hat{V}_{\text{TPE}} \) is constructed from the sum of the diagrams of Figure 2.2, in which each diagram corresponds to a time-ordered product of the vertices (2.26) combined with an energy denominator. The potential was regularized by including a monopole form factor with a pion cut-off mass of 1.0 GeV at each vertex. Because of the complicated momentum space form of the potential, its functional part was fitted to a convenient function \( \hat{W}(q) \) which was further Fourier transformed into an analytic function in the configuration space. The resulting regularized potential \([1]\) reads

\[
\hat{V}_{\text{TPE}}(r) = h_\pi^{(1)} \hat{\tau}_+(\sigma_1 \times \sigma_2) \cdot \hat{r} W(r),
\]  

(2.27)

where \( W(r) \) splits up into crossed (C) and box (B) diagram contributions as follows

\[
W(r) = W_{\text{NN}}^C(r) + W_{\text{N}\Delta}^C(r) + W_{\text{N}\Delta}^B(r).
\]

The radial function is given by

\[
W(r) = \frac{AB^2}{8\pi} \left( \frac{C^2}{C^2 - B^2} \right)^2 \times \left\{ e^{-Cr} \left[ \frac{C^2 - B^2}{2} + \frac{1}{r}(C + \frac{1}{r}) \right] - e^{-Br} \left[ B + \frac{1}{r} \right] \right\},
\]  

(2.28)
Table 2.2: Fit parameters for (2.28) for the partial contributions.

<table>
<thead>
<tr>
<th></th>
<th>A (fm$^3$)</th>
<th>B (fm$^{-1}$)</th>
<th>C (fm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N\Delta^H$</td>
<td>0.148934</td>
<td>3.5987</td>
<td>10.8463</td>
</tr>
<tr>
<td>$N\Delta^C$</td>
<td>-0.0170383</td>
<td>3.18247</td>
<td>8.28774</td>
</tr>
<tr>
<td>$NN^C$</td>
<td>0.226128</td>
<td>2.77668</td>
<td>7.59613</td>
</tr>
</tbody>
</table>

where the values for the parameters $A$, $B$, and $C$ are given in Table 2.2.

2.2.2 Nucleon-Delta transition potentials

In [2] the PNC $\Delta$ contributions were considered by employing a PNC one-meson exchange $NN \leftrightarrow N\Delta$ transition potential based on the vertices and couplings of [36]. The $\pi$ and $\rho$-mediated strong transition potential of the standard form [35] is

$$
\hat{V}_{N\Delta}(r) = \frac{g_\pi g_\pi^*}{4M^2} T_1 \cdot \tau_2 (S_1 \cdot \nabla)(\sigma_2 \cdot \nabla) Y_\pi(r) + \frac{g_\rho g_\rho^*}{4M^2} (1 + \chi_\rho)^2 T_1 \cdot \tau_2 (S_1 \times \nabla)(\sigma_2 \times \nabla) Y_\rho(r) + (1 \leftrightarrow 2) + H.c.
$$

(2.29)

and the $\pi$, $\omega$, and $\rho$-mediated PNC transition potential derived from the vertex interaction Hamiltonians of [36] is

$$
\hat{V}_{N\Delta}^{PNC}(r) = i \frac{g_\pi h_\pi^{(1)}}{2\sqrt{2}M} \hat{T}_1 (T_1 \times \tau_2)_z S_1 \cdot [-i\nabla, Y_\pi(r)]
$$

$$
- \frac{g_\omega h_\omega^{(1)}}{2M} \hat{T}_1 \left[ S_1 \cdot \{-i\nabla, Y_\omega(r)\} + i(1 + \chi_\omega)(S_1 \times \sigma_2) \cdot [-i\nabla, Y_\omega(r)] \right]
$$

$$
- \frac{1}{2M} \left\{ g_\rho \left[ h_\rho^{(0)} + \frac{h_\rho^{(1)}}{3} \right] T_1 \cdot \tau_2 + h_\rho^{(1)} \sqrt{\frac{2}{3}} [T_1 \otimes \tau_2]_0 \right\} S_1 \cdot \{-i\nabla, Y_\rho(r)\}
$$

$$
+ \left\{ g_\rho^* h_\rho^{(0)} + g_\rho \left( h_\rho^{(0)} + \frac{h_\rho^{(1)}}{3} \right) \right\} T_1 \cdot \tau_2 + \left( \frac{g_\rho^* h_\rho^{(2)}}{2} + \sqrt{\frac{2}{3}} g_\rho h_\rho^{(1)} \right) [T_1 \otimes \tau_2]_0
$$

$$
+ g_\rho^* h_\rho^{(1)} \hat{T}_1 ) i(1 + \chi_\rho)(S_1 \times \sigma_2) \cdot [-i\nabla, Y_\rho(r)] \right\} + (1 \leftrightarrow 2) + H.c.,
$$

(2.30)

where $Y_\alpha(r) \ (\alpha = \pi, \omega, \rho)$ represent Yukawa functions and $g_\alpha^* = \sqrt{72/25} g_\alpha \ (\alpha = \pi, \rho)$ are the strong meson-$N\Delta$ couplings [35]. The used set of weak
Table 2.3: The weak $\alpha NN$ couplings $h^{(i)}_\alpha$ and $\alpha N\Delta$ couplings $h^{* (i)}_\alpha$ ($\alpha = \pi, \rho, \omega$) are the FCDH "best values" [36] in units of $10^{-7}$.

couplings values, which takes into account the $\Delta$-sector, are from [36] (DDH generalization). The FCDH [36] weak coupling values are given in Table 2.3. The other parameter values are the same as in Table 2.1. It may be noted that in the case of pion-exchange the $\Delta$ is generated only at the strong vertex, since there is no $\pi N\Delta$ coupling related to a PNC vertex [36] or it is small [37, 38] (we take it as zero). Note also that the strong $N\Delta$ transition potential (2.29) is more singular than $1/r^2$ and thus necessarily requires regularization. Therefore, in order to be thoroughly consistent, the modified Yukawa functions (2.13) are always used in the presence of the $\Delta$.

2.3 Quantum scattering

In order to calculate the observables, one needs to first determine the wavefunctions of the form $\langle r | k, \zeta \rangle$, in which $\zeta$ denotes the quantum numbers in general. Even though scattering particles are essentially moving wave packets, the stationary (time-independent) wavefunction description of them is nevertheless sufficient in scattering problems. Useful reference books for this section are, e.g., [39] and [40].

The properly normalized antisymmetric relative $NN$ wavefunctions of the free nucleons in terms of the plane waves read

$$
\langle r | k, SMST \rangle_0 = \frac{1}{\sqrt{2}} \left[ e^{ik \cdot r} - (-)^{S+T} e^{-ik \cdot r} \right] |SM_S\rangle |TM_T\rangle,
$$

where $r$ and $k$ are respectively the relative position and momentum of the two nucleons and the subscript 0 refers to the free particle system. The continuum $NN$ wavefunctions are gained by expanding the plane waves of (2.31) in spherical harmonics, coupling the spin and orbital angular momenta, and taking into account the effects of the $NN$ potentials. In order to specify the
individual polarization of the nucleons, the $pp$ wavefunctions are expanded into the form

$$\langle r | k, m_1 m_2 \rangle^{(\pm)} = \sum_{SM_S} \left( \frac{1}{2} m_1 \frac{1}{2} m_2 \right) |SM_S\rangle \langle r | k, SM_S \rangle^{(\pm)}, \quad (2.32)$$

$$\langle r | k, SM_S \rangle^{(\pm)} = \frac{4\pi \sqrt{2}}{kr} \sum_{LM_L S M_M_L} i^L \langle LM_L SM_S | JM \rangle \sum_{\xi} U^{(\pm)}_{\xi|\xi}(k, r) \langle r | k, SM_S \rangle^{(\pm)} \langle r | k, SM_S | T \rangle^0 \rangle, \quad (2.33)$$

where $U^{(\pm)}_{\xi|\xi}(k, r)$ are the complex-valued radial wavefunctions, $\mathcal{Y}_{L}^{JM}(\hat{r})$ are defined by (2.7), and the superscripts $(\pm)$ refer to the incoming ($-$) and outgoing ($+$) wave boundary conditions. The unprimed subscript $\xi (= LST)$ symbolizes the quantum numbers of the main (source) wavefunction while the primed one $\xi' (= L'ST$ for DWBA $pp$) the quantum numbers of the possible radial wavefunctions generated from it. The phase factor $e^{\pm i\sigma_L}$ is due to the presence of the Coulomb potential. The pure Coulomb wavefunctions are otherwise the same, except that $\langle r | k, \zeta \rangle^{(\pm)} \rightarrow \langle r | k, \zeta \rangle^{(0)}$ with $U^{(\pm)}_{\xi|\xi}(k, r) \rightarrow F_L(\eta, kr) \delta_{\xi\xi}$, where $F_L(\eta, kr)$ with $\eta(k) = \alpha \mu/k$ is the regular spherical Coulomb function. Analogously to the $pp$ case, the $np$ wavefunctions are

$$\langle r | k, m_n m_p \rangle^{(\pm)} = \sum_{SMST} \frac{(-)^{T+1}}{\sqrt{2}} \left( \frac{1}{2} m_n \frac{1}{2} m_p \right) |SM_S\rangle \langle r | k, SM_ST \rangle^{(\pm)}, \quad (2.34)$$

$$\langle r | k, SM_ST \rangle^{(\pm)} = \frac{4\pi \sqrt{2}}{kr} \sum_{LM_L S M_M_L} i^L \langle LM_L SM_S | JM \rangle \sum_{\xi} U^{(\pm)}_{\xi|\xi}(k, r) \langle r | k, SM_S \rangle^{(\pm)} \langle r | k, SM_S | T \rangle^0 \rangle. \quad (2.35)$$

The wavefunctions for free particles are recovered by $\langle r | k, \zeta \rangle^{(\pm)} \rightarrow \langle r | k, \zeta \rangle_0$ with $U^{(\pm)}_{\xi|\xi}(k, r) \rightarrow kr j_L(kr) \delta_{\xi\xi}$, where $kr j_L(kr)$ is the Riccati-Bessel function.

The dynamics of the relative motion of the $NN$-system is given by the stationary Schrödinger equation

$$\hat{H}|k, \zeta\rangle = E|k, \zeta\rangle, \quad (2.36)$$
where the Hamiltonian operator is defined as $\hat{H} = \hat{H}_0 + \hat{V}$, with the potential operator $\hat{V}$ and the kinetic energy operator $\hat{H}_0 = -\left(1/2\mu\right)\nabla^2$. The radial wavefunctions are obtained as exact solutions of the Schrödinger equation. The operation of the Hamiltonian for the relative motion with the energy eigenvalue $E = k^2/2\mu$, leads to a coupled set of Schrödinger equations of the form

$$\left(\frac{\partial^2}{\partial r^2} - \frac{L(L+1)}{r^2} + k^2\right)U^{J(+)}_{\xi}(k, r) = 2\mu \sum_{\xi'} \mathcal{V}_{\xi'\xi}(r) U^{J(+)}_{\xi}(k, r),$$

(2.37)

where $\mu$ is the reduced mass of the particles and $\mathcal{V}_{\xi'\xi}(r)$ are given by (2.8).

The radial wavefunctions are subjected to the regular boundary condition $(F_L(\eta, kr), krj_L(kr))$ at the origin having the behaviour $U^{J(+)}_{\xi}(k, r) \rightarrow r^{L+1}$. The asymptotic boundary conditions for the $pp$ wavefunctions at $r \rightarrow \infty$ can be expressed through the Lippmann-Schwinger equation

$$|k, SM_S^{(+)}\rangle = |k, SM_S^{(+)}_C\rangle + \frac{1}{E - \hat{H}_0 - V_C + i0_+} \hat{V}_N |k, SM_S^{(+)}\rangle,$$

(2.38)

which, by means of the completeness relation of the incoming Coulomb states, relates the radial wavefunctions to the T-matrix as follows

$$U^{J(+)}_{\xi}(k, r) = F_L(\eta, kr)\delta_{\xi\xi} + \frac{4\mu}{\pi} \int_0^\infty dk' \frac{F_L'(\eta', k'r)\mathcal{T}_{\xi\xi}^{J}(k', k)}{k'^2 - k^2 + i0_+},$$

(2.39)

where $\mathcal{T}_{\xi\xi}^{J}(k', k)$ are the T-matrix elements. The application of the distorted-wave Born approximation (DWBA) simplifies the PNC scattering calculations considerably. The DWBA is appropriate in the scattering problems where the interaction potential can be split into large and small components, as the case is with the PNC $NN$ scattering $\hat{V}_N^{PC} \gg \hat{V}_N^{PNC}$. The insertion of $\hat{V}_N = \hat{V}_N^{PC} + \hat{V}_N^{PNC}$ into the Lippmann-Schwinger equation for the T-matrix yields to the following relation

$$\mathcal{T}_{\xi\xi}(k', k) = (\sqrt{C}) (k', \zeta'|\hat{V}_N|k, \zeta)^{(+)}$$

$$\approx \sqrt{C} (k', \zeta'|\hat{V}_N^{PC}|k, \zeta)^{(+)} + (\sqrt{C}) (k', \zeta'|\hat{V}_N^{PNC}|k, \zeta)^{(+)}.$$  

(2.40)

The approximation arises from the fact that the distortions caused by $\hat{V}_N^{PNC}$ are insignificant and neglected. The insertion of $\hat{V}_N^{PNC}$ into the Schrödinger equation is thus unnecessary and the potentials in (2.40) are simply sandwiched between the Coulomb-distorted strong interaction wavefunctions.
The S- and T-matrix elements are related by

\[ S_J^{\xi}(k) = \delta_{\xi\xi'} - iT_J^{\xi}(k), \tag{2.41} \]

with redefined T-matrix elements \( T_J^{\xi}(k) = (4\mu/k)T_J^{\xi}(k) \). The asymptotic behaviour of the outgoing wavefunctions at large distances is obtained from (2.39)

\[ U_J^{(+)}(\xi') r \to \infty \to \frac{1}{2} \left( H_L^{(-)}(\eta, kr) \delta_{\xi\xi'} + S_J^{\xi}(k)H_L^{(+)}(\eta, kr) \right), \tag{2.42} \]

with \( H_L^{(\pm)}(\eta, kr) = F_L(\eta, kr) \pm iG_L(\eta, kr) \), where \( G_L(\eta, kr) \) is the irregular spherical Coulomb function. The asymptotics at large distances are

\[ H_L^{(\pm)}(\eta, kr) \to \pm i \exp \left[ \pm i \left( kr - \frac{L\pi}{2} - \eta \ln(2kr) + \sigma_L(\eta) \right) \right], \tag{2.43} \]

where \( \sigma_L(\eta) = \arg \Gamma(L + 1 + i\eta) \) is the Coulomb phase shift.

The T-matrix elements of (2.41) in the explicit form are as follows

\[ T_J^{\xi}(k) = \frac{4\mu}{k} \sum_{\xi''} \int_0^{\infty} dr F_L(\eta, kr) U_J^{(+)}(\xi', \xi''; k, r) \] \[ \tag{2.44} \]

for the Coulomb-distorted strong interaction. The tiny PNC T-matrix elements are off-diagonal \((\xi \neq \xi')\) and given in terms of the DWBA as

\[ \tilde{T}_J^{\xi}(k) = \frac{4\mu}{k} \sum_{\xi'' \xi'''} \int_0^{\infty} dr U_J^{(+)}(\xi', \xi''; k, r) \tilde{U}_J^{(+)}(\xi'''; \xi''; r) \] \[ \tag{2.45} \]

where the radial wavefunctions, which differ by \( \Delta L = 1 \), are merely Coulomb and strong interaction distorted.

The equations for the np interaction are readily obtained by turning off the Coulomb interaction, i.e. \( V_C = 0 \); \( \eta = 0 \), from which follows that \( F_L(0, kr) \) and \( G_L(0, kr) \) coincide respectively with \( krj_L(kr) \) and the spherical Riccati-Neumann function \( kln_L(kr) \). Thus \( H_L^{(\pm)}(0, kr) \) are the spherical Riccati-Hankel functions \( krh_L^{(\pm)}(kr) \), with \( h_L^{(\pm)}(kr) = j_L(kr) \pm in_L(kr) \) for the outgoing (+) and incoming (−) waves. In (2.40) the Coulomb wavefunctions \( \langle r | k, \zeta \rangle_C \) reduces to the form \( \langle r | k, \zeta \rangle_0 \).

The presence of the potential causes irregularity to the continuum wavefunctions which appears at large distances as a negative phase shift for repulsive and positive phase shift for attractive interaction. In the nuclear pp
scattering the Coulomb repulsion is inevitably present and thus the phase shift is \( \delta_L' = \sigma_L + \delta_L \), where \( \delta_L \) is the Coulomb-distorted nuclear phase shift. In the case of two coupled channels, the S-matrix may be written explicitly in terms of the partial wave nuclear phase shifts \( \delta_L' \) and mixing parameters \( \epsilon_J \). The S-matrix elements (2.41) given in the so-called Stapp parametrization [41] read

\[
\hat{S}(k) = \begin{pmatrix}
    e^{2i\delta_L'(k)} \cos[2\epsilon_J(k)] & i e^{i[\delta_L'(k)+\delta_{L'}(k)]} \sin[2\epsilon_J(k)] \\
    i e^{i[\delta_L'(k)+\delta_{L'}(k)]} \sin[2\epsilon_J(k)] & e^{2i\delta_{L'}(k)} \cos[2\epsilon_J(k)]
\end{pmatrix}.
\]  

(2.46)

The unitarity of the S-matrix \( \sum_{\xi''} S_{\xi''\xi}^* S_{\xi''\xi} = \delta_{\xi\xi} \) must be preserved due to the conservation of probability flux. In addition, because of the time-reversal invariance, the reciprocity condition \( S_{\xi\xi} = S_{\xi'\xi'} \) must also be satisfied. Besides the preserved unitary and symmetry properties of the S-matrix, (2.46) is used as a check of the correctness of the PC interaction wavefunctions. The nuclear phase shifts and mixing parameters are tabulated at different energies in [26–28].

The use of the optical theorem somewhat simplifies the calculations of the total scattering cross-sections. According to the optical theorem, the total scattering cross-section and the imaginary part of the forward-scattering amplitude are related by

\[
\sigma(k) = \frac{4\pi}{k} \text{Im} f(k, \theta = 0).
\]

(2.47)

The cross-section (2.47) arises completely from the elastic scattering at energies below the inelastic pion-production threshold \( \sim 350 \text{ MeV} \). When the \( z \)-axis is along the direction of propagation of the nucleons \( \hat{k} \parallel \hat{z} \), the following simplifications in (2.33) and (2.35) take place \( Y_{LM_\ell}(\hat{k}||\hat{z}) = \delta_{M_\ell,0} \sqrt{2L+1} \frac{4\pi}{4\pi} \) and thus \( M = M_S \).

When the energies of the \( pp \) scattering are of the order of MeV’s, the Coulomb force plays no significant role as a scatterer in general. The Coulomb effects can be neglected in the low-energy (\( \lesssim 50 \text{ MeV} \)) scattering experiments which measure directly the scattered particles. The higher energy transmission experiments, however, measure the particles passing through the target and are thus complicated by the fact that the Coulomb field is singular in the forward direction \( \theta = 0 \). Therefore, for completeness, the long-range Coulomb effects are also carefully taken into account. The determination of
the \( pp \) scattering cross-section for a transmission experiment is based on the optical theorem and follows closely [42] (see also [43, 44]).

The \( pp \) scattering amplitude, given by

\[
f(k, \theta) = f^C(k, \theta) + f^N(k, \theta),
\]

(2.48)

consists of the Coulomb scattering amplitude (superscripted by \( C \)) and the Coulomb-nuclear scattering amplitude (superscripted by \( N \)) including strong and weak interactions calculated in the presence of the Coulomb potential. The forward Coulomb-nuclear scattering amplitude is connected to the diagonal T-matrix elements (2.40) through

\[
f^N_\zeta(k, 0) = -\frac{\mu}{2\pi} T^\zeta_\zeta(k \hat{z}, k \hat{z}),
\]

(2.49)

with \( \zeta = m_1 m_2 \). An awkward feature of the Coulomb scattering amplitude (e.g. [39])

\[
f^C(k, \theta) = -\frac{\eta}{2k \sin^2 \frac{\theta}{2}} e^{i[2\sigma_0 - \eta \ln \sin^2 \frac{\theta}{2}]}
\]

(2.50)

is that it is undefined at \( \theta = 0 \). Symmetrized and properly normalized \( f^C(k, \theta) \) may be written as

\[
f^C_{m_1 m_2}(k, \theta) = \frac{1}{\sqrt{2}} \sum_{S M_S} \langle \frac{1}{2} m_1 \frac{1}{2} m_2 | S M_S \rangle \langle \frac{1}{2} m_1' \frac{1}{2} m_2' | S M_S \rangle
\]

\[
\times \left[ f^C(k, \theta) + (-)^S f^C(k, \pi - \theta) \right].
\]

(2.51)

In order to avoid the singularity at \( \theta = 0 \) in the Coulomb cross-section, only the directions in the forward hemisphere from a cut-off angle \( \theta_c > 0 \) are considered. The spin averaged Coulomb cross-section becomes

\[
\sigma^C_{m_1}(k) = \pi \sum_{m_2} \int_{\theta_c}^{\pi} d\theta \sin \theta |f^C_{m_1 m_2}(k, \theta)|^2
\]

\[
= \frac{\pi \eta^2}{2k^2} \left( \frac{1}{\sin^2 \frac{\theta_c}{2}} - \frac{1}{\cos^2 \frac{\theta_c}{2}} + \frac{1}{\eta} \sin[2\eta \ln \tan \frac{\theta_c}{2}] \right) \delta_{m_1 m_1}.
\]

(2.52)

In the calculation of the corresponding nuclear cross-section, the cut-off \( \theta_c \) is considered to be such a small angle that \( f^N(k, \theta_c) \approx f^N(k, 0) \). Even though the Coulomb cross-section is singular in the forward direction, the optical
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Theorem (2.47) is nevertheless applied to the \( pp \) scattering amplitude (2.48) from which it follows that \( \sigma = \sigma^N + \sigma^C \). Taking the differential cross-section near the propagation axis of the beam as \( d\sigma^N_{m_1} = d\sigma_{m_1} - d\sigma^C_{m_1} \) and using the result

\[
\int_{\epsilon \to 0}^{\theta_c} d\theta \sin \theta f^{C*}(k, \theta) = \frac{1}{ik} \left( 1 - e^{2i[\eta \ln \sin \frac{\theta_c}{2} - \sigma_0]} \right),
\]

(2.53)
derived in [42], the spin averaged nuclear cross-section becomes

\[
\sigma^N_{m_1}(k) = \sigma^N_{m_1}(k) - 2\pi \int_{0}^{\theta_c} d\theta \sin \theta \frac{d\sigma^N_{m_1}}{d\Omega}(k, \theta)
= \frac{\pi}{k} \sum_{m_2} \text{Im} \left( f^N_{m_1, m_2} (k, 0)e^{2i[\eta \ln \sin \frac{\theta_c}{2} - \sigma_0]} \right),
\]

(2.54)
where \( \sigma^N_{m_1}(k) \) is the total cross-section obtained by an application of the optical theorem.

2.4 Bound states

The two nucleons are most likely bound in the lowest energy state where the repulsive centrifugal barrier is zero, that is \( L = 0 \). The spin singlet channel is only due to central interaction, while the spin triplet channel, in addition, contains also noncentral interaction. The large negative values of the singlet \( np, nn, \) and \( pp \) scattering lengths imply that the \( ^1S_0 \) state is nearly, but not quite, a bound state. In the spin triplet channel, however, the tensor potential provides a sufficient amount of attraction between the proton and neutron in order to bind them together to form the deuteron. The deuteron is thus the simplest stable nuclear system that has only one rather loosely bound tensor coupled \( ^3S_1 - ^3D_1 \) state.

As a consequence of the PNC interactions, the deuteron contains also a number of tiny parity admixed \( P \)-wave components. The PNC \( NN \) potential produces \( ^1P_1 \) and \( ^3P_1 \) partial wavefunctions. The \( ^1P_1 \) wave arises from the spin-changing part of the interaction and thus does not include pion exchange. The \( ^3P_1 \) one, on the other hand, is practically completely pion dominated and also by far the largest PNC component. The PNC \( N\Delta \) transition potential produces additional short-range components, of which the \( ^3P_1 \) and \( ^5P_1 \) are relevant in PNC deuteron photoreactions. The essential deuteron wavefunction, which is a superposition of the PC and PNC
components, may be written as

\[ \langle \mathbf{r} | M_d \rangle_D = \sum_{\xi_d} \frac{\mathcal{D}_{\xi_d}(r)}{r} \mathcal{Y}_{L_d S_d}^{1 M_d}(\mathbf{r}) | T_d 0 \rangle, \tag{2.55} \]

with the normalization \( \int_0^\infty dr \sum_{\xi_d} | \mathcal{D}_{\xi_d}(r) |^2 = 1 \) and negative energy eigenvalue of \( E_d = -2.2246 \text{ MeV} \), i.e. the binding energy of the deuteron. The corresponding Schrödinger equation is

\[ \left( \frac{\partial^2}{\partial r^2} - \frac{L_d(L_d + 1)}{r^2} + 2 \mu E_d \right) \mathcal{D}_{\xi_d}(r) = 2 \mu \sum_{\xi_d'} \mathcal{Y}_{\xi_d \xi_d'}^{-1}(r) \mathcal{D}_{\xi_d'}(r), \tag{2.56} \]

which gives a coupled set of equations that determines (2.55). Near the origin, the \( \mathcal{D}_{\xi_d}(r) \) behave as the Riccati-Bessel functions, whereas for large \( r \), as the outgoing Riccati-Hankel functions with the imaginary-valued argument \( i r \sqrt{2 \mu |E_d|} \).
3.1 Longitudinal analyzing power $A_L(\vec{p}\vec{p} \rightarrow pp)$

The PNC $\vec{p}\vec{p}$ elastic scattering has received a great deal of experimental and theoretical attention in the $NN$ sector. So far, eight experiments at various energies have been performed, see e.g. [45] for a summary of them. In the experimental setup of these experiments, the transmitted proton beam is longitudinally polarized while the protons in the target are unpolarized. The idea of them is to measure the tiny difference between the polarized scattering and its mirror reflection achieved by simply reversing the polarization. The measured quantity is the longitudinal analyzing power, defined as

$$\bar{A}_L = \frac{\sigma_{\frac{1}{2}} - \sigma_{-\frac{1}{2}}}{\sigma_{\frac{1}{2}} + \sigma_{-\frac{1}{2}}}$$

(3.1)

where $\sigma_{m_1}$ are the total cross-sections of the transmitted protons with the spins parallel ($m_1 = \frac{1}{2}$) and antiparallel ($m_1 = -\frac{1}{2}$) along the direction of propagation.

Conventional theoretical analyses of the PNC $\vec{p}\vec{p}$ experiments are based on the DDH model, which neglects the effects of the PNC TPE. In that picture, $\bar{A}_L$ is assumed to arise only from the $\rho$- and $\omega$-exchanges, since the single pion-exchange does not appear according to Barton’s theorem [12]. However, not only due to the fact that the strong and weak (DDH) pion couplings are sizable, but also that the pions are nearly six times lighter than heavy mesons, it seems reasonable to assume that the longest-range and possibly the leading effects might nonetheless be due to pion exchanges
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(in this particular case, induced by the two charged pions). Even though the TPE is far more complicated than the single meson exchange, it should not be ignored in this particular case due to its considerable strength and range. Recent calculations of $\bar{A}_L$ show the importance of the TPE \cite{1, 3, 24}. This, of course, is grounded in an assumption that the weak $\pi NN$-coupling $h^{(1)}_\pi$ is of the size suggested by DDH.

The total PNC potential is assumed to have the form $\hat{V}^{\text{PNC}} = \hat{V}^{\text{DDH}} + \hat{V}^{\text{TPE}}$. The DDH potential and the employed PNC TPE potentials $\hat{V}^{\text{TPE}}_D$, $\hat{V}^{\text{TPE}}_K$, and $\hat{V}^{\text{TPE}}_{\text{TO}}$ are given in Subsection 2.2.1. The potential $\hat{V}^{\text{TPE}}_{\text{TO}}$ is slightly scaled to correspond to the strong $\pi NN$ coupling value of $g_\pi = 13.45$, which is then in line with the typical choice of parameters in the PNC calculations given in Table 2.1. Calculating out the isospin matrix element, $\hat{V}^{\text{PNC}}$, taken between the initial and final $pp$ states, is

$$\hat{V}^{\text{PNC}}_{pp} (r) = - \sum_{\alpha=\rho,\omega} \frac{g_\alpha h^{pp}_\alpha}{M} \left( i(1 + \chi_\alpha) (\sigma_1 \times \sigma_2) \cdot [-i \nabla, Y_\alpha (r)] + (\sigma_1 - \sigma_2) \cdot \{-i \nabla, Y_\alpha (r)\} \right) + 2 h^{(1)}_\pi (\sigma_1 \times \sigma_2) \cdot \hat{r} W (r), \quad (3.2)$$

with $h^{pp}_\rho = h^{(0)}_\rho + h^{(1)}_\rho / \sqrt{6}$ and $h^{pp}_\omega = h^{(0)}_\omega + h^{(1)}_\omega$, which have the numerical values of $-15.48$ and $-3.00$ in units of $10^{-7}$ respectively. Since the energies can be relatively high, $Y_\alpha (r)$ are the modified Yukawa functions (2.13) with the cut-off masses $\Lambda_\rho = 1.3$ GeV and $\Lambda_\omega = 1.5$ GeV. The strong distortions are given by $\hat{V}^{\text{PC}}_N$, which is taken as the Reid93 potential.

In the determination of the total scattering cross-section by means of the optical theorem (2.47), the singularity of (2.48) in the forward direction is simply removed by the subtraction of $f^C (k, 0)$, leaving only $f^N (k, 0)$ to contribute. The forward, $\theta = 0$, $\bar{p}p$ Coulomb-nuclear scattering amplitude in the DWBA is given by (2.49). By applying the optical theorem with (2.49) to (3.1) and carrying out the spin sum over $m_2$, the longitudinal scattering asymmetry becomes

$$\bar{A}_L (k) = \frac{\text{Im} \sum_{SS'} (-) \langle k \bar{z}; S'S'0| \hat{V}^{\text{PNC}} |k \bar{z}; S0\rangle^{(+)} }{\text{Im} \sum_{SS'} (-) \langle k \bar{z}; SS| \hat{V}^{\text{PC}}_N |k \bar{z}; SS\rangle^{(+)}}, \quad (3.3)$$

where the nuclear potentials are sandwiched between the Coulomb-distorted strong interaction wavefunctions (2.33). Basically, the PNC $pp$ effects are exclusive properties of nuclear interactions distorted by the Coulomb field.
3.1 Longitudinal analyzing power $A_L(\bar{p}p \rightarrow pp)$

A

within the range of the nuclear forces. To obtain a clean PNC signal, the external long-range Coulomb effects can be cut out. Therefore, the expression for the scattering analyzing power of (3.3) is used by default without the long-range Coulomb effects, i.e. the Coulomb phases $e^{i\sigma_L}$ in the wavefunctions (2.33) are omitted. These negligible effects are nevertheless considered and shown separately.

For the measurement of the PNC $\bar{p}p$ longitudinal analyzing power $\bar{A}_L$, there exist three experimental precision data points: Bonn at 13.6 MeV ($-0.93 \pm 0.21) \times 10^{-7}$ [46], PSI at 45 MeV ($-1.50 \pm 0.22) \times 10^{-7}$ [47], and TRIUMF at 221.3 MeV ($0.84 \pm 0.29) \times 10^{-7}$ [48]. Figure 3.1 depicts the contributions of the different parity admixed partial waves up to $J = 4$ to the asymmetry (3.3) by using (3.2) with $\hat{V}_{TPE}$. The potential $\hat{V}_{TPE}$ is used simply as a reference PNC TPE potential. The partial wave contributions provide important characteristic features of $\bar{A}_L$ that are independent of the weak couplings. The transitions with $J = 4$ (or higher) are indistinguishable from the 0-axis and, thus, the lowest three admixtures up to $J = 2$ are in fact

Figure 3.1: The partial-wave contributions of the total scattering asymmetry (3.3) by employing (3.2) with $\hat{V}_{TPE}$.
sufficient within the used energy range. Below energies of about 50 MeV, only the lowest $J = 0$ one is significant. Especially, the Bonn and PSI experiments are low energy scattering experiments, where the contribution to $\bar{A}_L$ arises only from the lowest $^1S_0 - ^3P_0$ transition. One particularly interesting feature of the asymmetry, as was first pointed out in [49] and utilized in the TRIUMF E497 experiment, is that the $^1S_0 - ^3P_0$ contribution vanishes at a specific energy due to the equal, but opposite phase shifts of the $^1S_0$ and $^3P_0$ partial waves. The energy of the experiment was chosen so that the contribution should arise just from the $^3P_2 - ^1D_2$ and $^1D_2 - ^3F_2$ transitions as a result of the serendipitous disappearance of the lowest transition due to strong interaction interference, which is seen at 224.7 MeV in Figure 3.1. Furthermore, for $J = 2$, the local and nonlocal contributions of the $\omega$ exchange mostly cancel out because of the small isoscalar anomalous moment $\chi_\omega$. This is seen in Figure 3.2 which shows separately the $\rho$, $\omega$, and TPE contributions to the asymmetry. In contrast for the $\rho$ exchange, the local contributions dominate over the nonlocal ones because of its large isovector anomalous moment $\chi_\rho$. Assuming then that the $J = 2$ mixing arises from the $\rho$ exchange, the central
goal of the TRIUMF experiment was to determine the weak $ppp$-coupling $h_{\rho}^{pp} = h_{\rho}^{(0)} + h_{\rho}^{(1)} + h_{\rho}^{(2)}/\sqrt{6}$, whereas the lower-energy experiments Bonn and PSI determined the $h_{\rho}^{pp} + h_{\omega}^{pp}$, where $h_{\omega}^{pp} = h_{\omega}^{(0)} + h_{\omega}^{(1)}$. In these experiments the reasoning was built on the DDH potential. However, already the work [50] including the effect of intermediate $N\Delta(1232)$ states via $\rho$ exchanges in the coupled channels model showed that the simplest and most straightforward interpretation of the TRIUMF experiment might not be sufficient. The $\Delta$ effect was significant enough to suggest that the extracted coupling could rather be an effective one involving $\rho$ exchange both in $NN$ and $NN \leftrightarrow N\Delta$ transitions. In the later work [1] on PNC $\vec{p}\vec{p}$ elastic scattering, we looked at the effects of the $N\Delta$-channels in the coupled-channels formalism as well as the influence of the two-body irreducible TPE. The effects were again found significant and cast doubt on the aforementioned $ppp$-coupling and whether its value is straightforwardly proportional to the TRIUMF data point. It may be noted that the TPE in $\vec{p}\vec{p}$ scattering has previously been investigated also in [33,51], of which the former considers it as a part of the short-ranged $\rho$ exchange and the latter discusses it in terms of the EFT with $V_{D}^{TPE}$.

The effects of the PNC TPE potentials $\hat{V}_{D}^{TPE}$ and $\hat{V}_{K}^{TPE}$ are examined next. Because the PNC potentials are in general treated perturbatively in the DWBA, the regularization from the point of view of solving Schrödinger equation is not vital. Nevertheless, since the effect of the singularity comes forth increasingly with the increasing energy, it is worthwhile to study the short-range effects of the form factors, anyway. One might note, however, that the chiral perturbation theory based $\hat{V}_{D}^{TPE}$ and $\hat{V}_{K}^{TPE}$ potentials would serve their purpose best as unregularized due to their model independent nature. As regularization, the monopole $\Lambda^2(q^2+\Lambda^2)^{-1}(F)$ and dipole $\Lambda^4(q^2+\Lambda^2)^{-2}(FF)$ form factors are incorporated using two different cut-off masses $\Lambda = 1.0$ GeV and $\Lambda = 1.2$ GeV. A monopole form factor of the same type is also used for $\hat{V}_{D}^{TPE}$ in [22]. The effects of $\hat{V}_{D}^{TPE}$ and $\hat{V}_{TPE}$ on the asymmetry are shown in Figures 3.3 and 3.4, respectively. The asymmetry given by $\hat{V}_{D}^{TPE}$ is very sensitive to the applied regularizations and even exhibits a different sign when the dipole form factor is used. This behaviour arises from the fact that this potential includes also the $\delta$-function term (omitted in $\hat{V}_{K}^{TPE}$), which normally does not contribute, as the wavefunctions are zero at the origin. With the form factors, however, this term gets finite range resembling a meson exchange with the opposite sign. It may further be noted that the "w/o" curve in Figure 3.3 is identical with that of the $NN$ part of $\hat{V}_{K}^{TPE}$.
Figure 3.3: The TPE by $\hat{V}_D^{\text{TPE}}$ with the monopole (F) and dipole (FF) form factors using the cut-off masses $\Lambda = 1.0$ GeV and 1.2 GeV and also without (w/o) the form factors. The $NN$ part of $\hat{V}_\text{TO}^{\text{TPE}}$ is also shown as a comparison.

Figure 3.4: The same as Figure 3.3 but the TPE is by $\hat{V}_K^{\text{TPE}}$. As a comparison, the asymmetry is also plotted using $\hat{V}_\text{TO}^{\text{TPE}}$ in which the coupling values are scaled to correspond to those of $\hat{V}_K^{\text{TPE}}$. 
3.1 Longitudinal analyzing power $A_L(\vec{p}\vec{p}\rightarrow pp)$

Figure 3.4 shows that for the dipole form factor with $\Lambda = 1.0$ GeV, the effect of $\hat{V}_K^{\text{TPE}}$ up to about 150 MeV is more or less indistinguishable from the time-ordered one $\hat{V}_T^{\text{TPE}}$. In other cases, with harder form factors, the asymmetry is larger as expected.

While the lower energy experiments measure directly the scattered particles, the TRIUMF and higher energy experiments measure the transmitted beam after passing through the target. In transmission experiments, a complication arises due to the fact that the Coulomb interaction is singular in the forward direction. Therefore, we consider the Coulomb distortions near the propagation direction of the transmitted beam, as done, e.g. in [43,44]. The application of (2.54) and (2.52) to (3.1) gives the longitudinal transmission asymmetry

$$\vec{A}_L^0(k) = \frac{\text{Im} \left[ \sum_{SS'} (-)(k\hat{z}; S'0|\hat{V}^{\text{PNC}}|k\hat{z}; S0)^{(+)} e^{2i[\eta \ln \sin \frac{\theta_c}{2} - \sigma_0]} \right]}{\text{Im} \left[ \sum_{SM_S} (-)(k\hat{z}; SM_S|\hat{V}^{\text{PC}}|k\hat{z}; SM_S)^{(+)} e^{2i[\eta \ln \sin \frac{\theta_c}{2} - \sigma_0]} \right] - \frac{4k}{M} \sigma C_{\theta_c}(k)}.$$ (3.4)

The long-range Coulomb effects to the asymmetries are illustrated in Figure 3.5 along with the cut-off angle $\theta_c$ dependence of the transmission asymmetry (3.4). The calculated scattering and transmission asymmetries at the energies of about 150 MeV and above become nearly indistinguishable by the angles $\theta_c \geq 2^\circ$. The same result was also obtained in [43,44]. Especially noteworthy is that at the energy of the only transmission experiment, TRIUMF, the asymmetry remains practically unaffected by changes of the $\theta_c$ once it is larger than 1°.

Lastly, as an additional result, a hybrid $\pi$EFT calculation of $\vec{A}_L$ is presented. In the $\pi$EFT case, there is no TPE and thus the total PNC potential is given by (2.14) or alternatively by its reduced form (2.15). For the $pp$ interaction, the $\pi$EFT potential reads

$$\hat{V}^{\text{PNC}}_{pp}(r) = \frac{2m^2_p}{\Lambda_\chi^3} \left[ C(\sigma_1 - \sigma_2) \cdot \{-i \nabla, Y_\pi(r)\} + \tilde{C}(\sigma_1 \times \sigma_2) \cdot [-i \nabla, Y_\pi(r)] \right],$$ (3.5)

where $Y_\pi(r)$ are unmodified Yukawa functions. The LECs for (2.14) are $C = \sum_1^4 C_i - 2C_5$ and $\tilde{C} = \sum_1^4 \tilde{C}_i - 2\tilde{C}_5$ and for (2.15) $C = C_1 + C_2 + C_4 - 2C_5$ and $\tilde{C} = \tilde{C}_1$. One might note that (3.5) is formally similar to the DDH part
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Figure 3.5: The scattering asymmetry of (3.3) with \( (A_L(C)) \) and without \( (A_L) \) long-range Coulomb effects and the different cut-off angle \( \theta_c \) transmission asymmetries of (3.4) are illustrated.

of (3.2). The asymmetry given at the energies (which somewhat exceed the comfort zone of the \( \pi \text{EFT} \)) of the Bonn and PSI experiments is

\[
\bar{A}_L^{\pi \text{EFT}}(13.6 \text{ MeV}) = -\left(3.6231C + 2.0560\tilde{C}\right) \times 10^{-3}, 
\text{(3.6)}
\]

\[
\bar{A}_L^{\pi \text{EFT}}(45 \text{ MeV}) = -\left(5.7973C + 2.2103\tilde{C}\right) \times 10^{-3}. 
\text{(3.7)}
\]

These results are consistent with those of [33].

3.2 Asymmetry \( A_\gamma(\gamma d \rightarrow np) \)

The PNC observables in the photoreactions arise from an interference between the strong and weak interactions giving rise to simultaneous presence of the photomagnetic and photoelectric effects. The asymmetry \( A_\gamma \) is associated with PNC deuteron photodisintegration by circularly polarized photons
3.2 Asymmetry $A_\gamma(\vec{\gamma}d \rightarrow np)$

on unpolarized deuterons $\vec{\gamma}d \rightarrow np$. It is to be noted that at threshold $A_\gamma$ equals to the photon polarization observable of the time-reversed reaction $np \rightarrow \vec{\gamma}d$. There are various theoretical works on $A_\gamma$ [33, 52–58] and polarization [33, 59–66]. The results are more or less in agreement with these works. The calculations are typically carried out exploiting the DDH model, aside from some of the recent works, which apply the modern state-of-art EFT techniques. As for the experiments, up to date, for the reaction $\vec{\gamma}d \rightarrow np$, there exist only two experimental data points: $(7.7 \pm 5.3) \times 10^{-6}$ and $(2.7 \pm 2.8) \times 10^{-6}$ at the photon energies of 3.2 and 4.1 MeV respectively [67]. The latest photon polarization measurement for the inverse reaction $np \rightarrow \vec{\gamma}d$ gives the value of $(1.8 \pm 1.8) \times 10^{-7}$ [68]. The data for these reactions are consistent with zero with rather a poor precision.

The threshold behaviour of $A_\gamma$ can be shown to be insensitive to the $\pi$-meson exchange, which represents the long-range isovector part of the PNC interaction. Consequently, in terms of the DDH, the threshold region is essentially dominated by the exchanges of heavy mesons $\rho$ and $\omega$. One may therefore expect that the relatively long-ranged $\Delta$-excitation could occur more clearly highlighted than what it would, if it appeared in a background where the pion is more intensely present. In paper [2], we wanted to estimate the size of this $\Delta$ effect, which was found to be significant in PNC elastic $\vec{p}\vec{p}$ scattering at higher energies [1, 50]. As in these works our calculation was carried out within the framework of the coupled channels meson-exchange model and hence, according to the common practice, we utilized the DDH potential as the starting point for the PNC $\pi$, $\omega$, and $\rho$-exchanges extending to use the weak couplings from the analysis of [36] consistent with the presence of the $\Delta$. Until [2], the $\Delta$ effect had only been checked in the form of exchange currents in [52, 69].

The asymmetry observable for the PNC reaction $\vec{\gamma}d \rightarrow np$ may be expressed as

$$A_\gamma = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}. \quad (3.8)$$

The deuteron photodisintegration helicity cross-section is obtained by Fermi’s golden rule and spin averaging, yielding

$$\sigma_\lambda = \frac{\mu_k}{48\pi^2} \sum_{MM_d} \int d\Omega_k \left| \langle k; M | \hat{H}_{\lambda}^{\text{em}} | M_d \rangle \right|^2, \quad (3.9)$$

where the deuteron and final state scattering wavefunctions are given by (2.55) and (2.35), respectively. The electromagnetic perturbing Hamiltonian
Figure 3.6: The leading threshold contributions of the M1 and E1 transitions to $A_\gamma$. The dashed line represents the heavy-meson ($\rho, \omega$) exchange and the wavy line the incoming photon. The black dot denotes the PNC vertex and $D$ the bound $^3S_1 - ^3D_1$ state.

$\hat{H}_{\text{e.m.}} = \hat{H}_{\text{E1}} + \hat{H}_{\text{M1}}$ which takes care of the disintegration (and formation) of the deuteron is considered in the dipole approximation $e^{ikr} \approx 1$. Thus, the normalized photon field is $A = \sqrt{2\pi/\omega_\gamma} \hat{\epsilon}$. The impulse approximation Hamiltonian of the $pn$-system for absorption of a circularly polarized photon reads

$$\hat{H}_{\text{e.m.}}^\lambda = -\sqrt{\frac{\alpha\pi\omega_\gamma}{2}} \hat{\epsilon}_\lambda \cdot \left\{ \frac{i}{\sqrt{3}} \left[ L + \mu_s(\sigma_1 + \sigma_2) + \mu_v(\sigma_1 - \sigma_2) \right] \right\}, \quad (3.10)$$

where $\hat{\epsilon}_\lambda$ ($\lambda = \pm 1$) is the circular polarization vector of the photon, $\omega_\gamma$ the center-of-mass energy of the photon, $\mu_s = 0.88$ and $\mu_v = 4.71$ the isoscalar and isovector magnetic moments of nucleons, and $\alpha$ the fine-structure constant. The $D \to ^1S_0$ transitions which dominate at threshold are illustrated in Figure 3.6. An additional set of $D \to ^3P_J$ ($J = 0, 1, 2$) transitions is considered above threshold. The magnetic dipole effect is dominant at threshold where the asymmetry (3.8) reduces to a simple form, which is explicitly given by

$$A_\gamma(k \to 0) \approx 2\text{Re} \left[ \frac{i}{\sqrt{3}} \int dr r U^{(+)}_{1S_0}(k, r) \left( D_{3S_1}(r) - \sqrt{2} D_{3D_1}(r) \right) - i \int dr r U^{(+)}_{1S_0}(k, r) \hat{D}_{1P_1}(r) \right]. \quad (3.11)$$

The low-energy limit (3.11) coincides with the photon polarization in the time-reversed reaction $np \to \bar{\gamma}d$ for thermal neutrons.

It is important to note that (3.11) arises from the spin changing PNC interaction, see Figure 3.6, and thus does not include the PNC one- and
3.2 Asymmetry $A_\gamma(\vec{\gamma}d \rightarrow np)$

Figure 3.7: The leading threshold $\Delta$ corrections to $A_\gamma$. The dashed line is strong interaction, the dotted line weak interaction, and the filled bar the $\Delta$-resonance.

two-pion exchanges. Therefore, the short-range contributions from heavier vector meson exchanges (and possibly $\Delta$) are exclusively highlighted. As for the direct delta transition, the $\gamma N\Delta$-vertex can occur in the presence of the M1 and E2 transitions. Only the dominant M1 multipole is considered, neglecting the small E2 effect. The electromagnetic $N \leftrightarrow \Delta$ Hamiltonian is

$$\hat{H}_{\text{M1}\Delta}^\lambda = -\frac{\lambda \mu^*}{M} \sqrt{\frac{\alpha \pi \omega}{2}} \varepsilon_\lambda \cdot \vec{S} \hat{T}_z + \text{H.c.}, \quad (3.12)$$

where the value of the transition magnetic moment is given by the quark model as $\mu^* = f^* \mu_v / 2f = 3\sqrt{2} \mu_v / 5$ [70]. The nucleon-Delta spin and isospin transition operators are denoted as $\vec{S}$ and $\hat{T}_z$ [35].

As a consequence of coupled-channel dynamics, in addition to the direct M1 $\Delta$-channel depicted in Figure 3.7, the $\Delta$ may also include possible higher order corrections, which are naturally taken into account as correlation effects. For instance, at the threshold, the leading $\Delta$ contributions originate from the once-iterated meson exchange diagrams, presented in Figure 3.7 as well.

The $NN$ part of the asymmetry is evaluated by employing the two phase-equivalent but different modern strong interaction potentials Reid93 [26] and Argonne $Av_{18}$ [27] and the DDH potential (2.10) for the PNC interaction with two sets of weak parameter values DDH and FCDH given in Tables 2.1 and 2.3, respectively. The $N\Delta$ corrections arise from the PC (2.29) and PNC (2.30) $N\Delta$ potentials through the coupled-channels treatment. Utilizing different model complexes, Table 3.1 summarizes some numerical results for $A_\gamma$ in the threshold limit $\omega_{\gamma\text{lab}} \rightarrow 2.22592$ MeV. It is clearly seen that the $\rho$ effect has an overpowering dominance over the $\omega$ one. The result $A_\gamma = 2.56 \times 10^{-8}$ gained with the standard set of couplings of Table 2.1 and the $Av_{18}$ potential agrees with most of the existing calculations and perfectly with those of us-
Table 3.1: The threshold effects of heavy mesons, regularization, and $\Delta$ to $A_\gamma$. The asymmetry $A_\gamma = A_\gamma^e + A_\gamma^\Delta$ is calculated using the unmodified Yukawa functions. The effect of regularization is shown by $A_\gamma^{mod}$ and $A_\gamma^{\Delta}$ represents $\Delta$-effect on top of it. The asymmetries are in units of $10^{-8}$.

<table>
<thead>
<tr>
<th>Model</th>
<th>$A_\gamma^e$</th>
<th>$A_\gamma^\Delta$</th>
<th>$A_\gamma$</th>
<th>$A_\gamma^{mod}$</th>
<th>$A_\gamma^{\Delta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{v18}$ &amp; DDH</td>
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<td>-0.10</td>
<td>2.56</td>
<td>1.05</td>
<td>0.96</td>
</tr>
<tr>
<td>Reid93 &amp; DDH</td>
<td>3.26</td>
<td>0.00</td>
<td>3.26</td>
<td>1.14</td>
<td>1.04</td>
</tr>
<tr>
<td>$A_{v18}$ &amp; FCDH</td>
<td>1.58</td>
<td>-0.24</td>
<td>1.34</td>
<td>0.62</td>
<td>0.65</td>
</tr>
<tr>
<td>Reid93 &amp; FCDH</td>
<td>1.76</td>
<td>0.00</td>
<td>1.76</td>
<td>0.70</td>
<td>0.72</td>
</tr>
</tbody>
</table>

The asymmetries are bigger with Reid93 which is explained by the distinctly bigger PNC $^1P_1$ component in the deuteron obtained by Reid93 potential. This is mainly because the $^1P_1$ partial wave potential is entirely repulsive for $A_{v18}$, but for Reid93, it becomes attractive at short ranges. However, with form factors the short-range differences are minimized and the results of the two potentials get closer to each other. The regularization by (2.13) causes a dramatic decrease to less than half of the unmodified value of the asymmetry with the cut-off masses of $\Lambda_\alpha = 1200$ MeV that were used. To extract the pure $\Delta$ effect, the FCDH values are used for the needed weak $\alpha N \Delta$-couplings in the case of $A_\gamma^\Delta$ with DDH, even though they would not necessarily be entirely consistent with the other DDH couplings. The $N\Delta$ effect in $A_\gamma$ was found to be small being only a few percent decrease for DDH and practically null for FCDH. In order to see the effects of the $P$-wave continuum states, the asymmetry was also considered at the photon laboratory energies from the threshold up to 10 MeV. As the photon energy increases, the asymmetry decreases steeply towards the zero axis. The asymmetry is already settled down near to zero at the energies of about 1 MeV above the threshold. It goes through zero somewhere between 3 and 4 MeV continuing its gentle monotonic decline hardly deviating from the zero axis. A similar result was also found in [56, 57]. Therefore, very soon above threshold $A_\gamma$ becomes probably too small to be experimentally informative.
3.3 Cold neutron-parahydrogen interactions

As slow neutrons collide with hydrogen molecules, they either scatter elastically or get captured by the protons resulting in deuterons and photons. This section presents three different pion sensitive PNC observables arising from the cold neutron interaction with parahydrogen. Two are due to the PNC elastic scattering enabling the spin rotation $\frac{d\phi}{dz}$ and polarization $\frac{dP}{dz}$ of the neutrons and the third one is the $\gamma$-asymmetry $A_\gamma (\vec{n}p \to \gamma d)$ in the radiative PNC capture of polarized neutrons. In these reactions the target must be parahydrogen in order to preserve the neutron polarization.

Unfortunately, there exist no measurements of the neutron spin rotation $\frac{d\phi}{dz}$ or polarization $\frac{dP}{dz}$ in parahydrogen. Nevertheless, there has been some experimental interest in measuring $\frac{d\phi}{dz}$ in a liquid parahydrogen target at the Neutron Spallation Source (SNS) [71]. The neutron spin rotation has recently been measured on more complicated $^4$He target [72]. The neutron spin rotation and polarization in the PNC $np$ scattering were first discussed in [73,74] and the wavefunction based calculations have later been performed in [33,52,75,76]. These works, however, simply consider the neutron scattering by free protons instead of parahydrogen molecules as the target particles should be in reality. The $\gamma$-asymmetry $A_\gamma$ was measured previously in the 70's and 80's, resulting in the insufficiently accurate values of $(0.6 \pm 2.1) \times 10^{-7}$ [77] and $-(1.5 \pm 4.8) \times 10^{-8}$ [78] consistent with zero. However, the ongoing NPDGamma experiment at the SNS [79] aims to measure $A_\gamma$ with high accuracy up to a level of 20% of the typical theoretical prediction $-5 \times 10^{-8}$, which employs the DDH "best value" for $h_{\pi}^{(1)}$. The radiative PNC reaction $\vec{n}p \to d\gamma$ is discussed in multiple papers [22–24,33,52,59–62,69,80–88] of which most also present numerical predictions. In neutron absorption, despite the neutron polarization concern in practice, it is irrelevant whether the proton is bound in a hydrogen molecule or not. This, however, is not the case with the scattering at very low energies and therefore the scattering by molecules must be considered.

Hydrogen exists in nature in molecular form, composed of two protons bound by two electrons. The hydrogen molecule ($H_2$) appears in two forms called parahydrogen ($H_2^p$) and orthohydrogen ($H_2^o$) with proton spins aligned antiparallel and parallel respectively. The de Broglie wavelength of a neutron at energies of a few meV is much greater than the internuclear separation $R_0 = 0.75$ Å between the protons in the hydrogen molecule. Consequently the neutrons, if not captured by protons, scatter coherently off the two pro-
tons in the molecules. Spinless parahydrogen molecules cannot depolarize polarized neutrons, when they scatter elastically. The protonic wavefunction of the hydrogen molecule must be antisymmetric from which it follows that in the ground states of the para- and orthohydrogen molecules, the rotational energies are respectively zero and $I^{-1} = 14.7$ meV, where $I = \mu R_0^2$ is the moment of inertia and $\mu$ the reduced mass of two protons. This energy determines the upper limit of the neutron center of mass (C.M.) energy in order not to get depolarized by the conversion of the para- to orthohydrogen molecule. Contrary to absorption, the slow neutron scattering cross-sections of parahydrogen, free protons, and orthohydrogen differ from each other by about half an order of magnitude $\sigma_{nH^p} < \sigma_{np} < \sigma_{nH^o}$. Since the neutron spin rotation and polarization are due to $nH^o$ scattering, one must carefully take into account the relative motion between the neutron and system of the chemically bound protons. The neutron-electron interaction is negligibly small compared to the $NN$ weak interaction [89].

### 3.3.1 Neutron spin rotation $\frac{d}{dz}\phi$ and polarization $\frac{d}{dz}P$

When a low-energy neutron comes across the molecules in the medium, it interacts collectively with a number of them. As a result, the scattered waves originating from the molecules, interfere with the passing neutron and change its momentum. Applying the Lippmann-Schwinger equation for multiple point-like scatterers each located at $r_j$, the wave of a slow neutron after travelling through the target in the z-direction then becomes

$$e^{iq'z} \approx e^{i\varphi} + \sum_j \frac{e^{iq|r-r_j|}}{|r-r_j|} e^{iqz_j}, \quad (3.13)$$

where $\hat{f}(q, \theta = 0)$ denotes the forward $nH^p$ scattering amplitude and $q$ is the relative momentum of the neutron and molecule. The sum of the spherical waves from the scatterers in (3.13) may be written in the form of an integral over a smooth distribution of scattering centers in a cylindrically shaped target of infinite radius. For the neutrons travelling along the axis of the target, it then follows that the right hand side of (3.13) becomes $e^{i(qz-\varphi)}$ [90], with

$$\varphi(q, z) = -\frac{2\pi N_{H_2}z}{q} \hat{f}(q, 0), \quad (3.14)$$

where $N_{H_2}$ is the molecule density of the medium. The neutron thus gains the $\varphi = (q-q')z$ amount of phase when propagating through a medium of length.
3.3 Cold neutron-parahydrogen interactions

z. Equation (3.14) is related to the index of refraction \( n = q'/q = 1 - \varphi/qz \) in neutron optics [90, 91].

By the initial choice of the transversely polarized spin in the positive x-direction \( \langle \sigma_x \rangle = +1 \), the neutron spin wavefunction \( |x\rangle = (|+\rangle + |-\rangle)/\sqrt{2} \) contains an equal amount of \( \pm \) helicities in the direction of its propagation along the z-axis. The PNC interaction favors one helicity state slightly more than the other and thus depending on this state, the neutrons scatter a bit differently. The neutron wavefunction accumulates the \( \varphi_m \) amount of phase (where \( m \) is the spin polarization of the incident neutron) labeled individually for each two states when passing through the target. It follows straightforwardly from the expectation value of the spin

\[
\langle \varphi x | \sigma | \varphi x \rangle = e^{-i \theta_x} \left[ \cos(\text{Re} \theta_-), -\sin(\text{Re} \theta_-), -\sinh(\text{Im} \theta_-) \right],
\]

with \( \theta_x = \varphi_{+\frac{1}{2}} \pm \varphi_{-\frac{1}{2}} \), that the neutron spin rotates in the xy-plane around the z-axis if the real value of the phase difference between the helicity states of (3.14) \( \phi \equiv \text{Re} \theta_- \) is non-zero. Similarly, the neutrons gain some amount of polarization in the z-direction \( P \equiv \sinh(\text{Im} \theta_-) \approx \text{Im} \theta_- \).

Low-energy \( nH_2^p \) scattering, characterised by the scattering length \( b \) (defined at zero-energy by \( \text{Re} \frac{k-q}{q} = -b \)) can be given by the contact interaction potential

\[
V_{nH_2^p}(r_n, r_{H_2^p}) = \frac{2\pi}{\mu} \bar{\delta}(r_n - r_{H_2^p}),
\]

where \( \bar{\mu} \approx 2M/3 \) is the \( nH_2 \) reduced mass. However, since there is no \( nH_2 \) potential from which \( b \) could be determined exactly, the \( nH_2^p \) scattering can be crudely simplified to the coherent \( np \) problem. The coherent \( np \) scattering potential is given by [92] (see also [93])

\[
\hat{V}_{np}^{\text{coh}}(r_n, r_{p1}, r_{p2}) = \frac{2\pi}{\mu} \sum_{i=1}^{2} \frac{1}{4} [a_s + 3a_t + (a_t - a_s)\sigma_n \cdot \sigma_{pi}] \delta(r_n - r_{pi}),
\]

where \( \mu \approx M/2 \) is the \( np \) reduced mass and \( a_s \) and \( a_t \) are, respectively, the \( np \) scattering lengths for the singlet \(^1S_0\) and triplet \(^3S_1\) channels.\(^1\) At low neutron energies, the two protons in the molecule cannot be distinguished and thus the molecule appears as a point-like particle, \( i.e. \ r_{p1} \approx r_{p2} \equiv r_{H_2^p} \). As a consequence of this, the molecular spin changing \( (H_2^p \leftrightarrow H_2^o) \)

\(^1\)The values \( a_s = -23.72 \text{ fm} \) and \( a_t = 5.43 \text{ fm} \) follow from the T-matrix as \( -\frac{1}{2\pi} T_{\xi\xi}(k) \).
term vanishes, which is crucial because the neutron polarization must be maintained. Furthermore, the remaining spin operator $\sigma_n \cdot (\sigma_{p1} + \sigma_{p2})$ gives zero in the case of the $nH_2^p$ scattering. The coherent scattering potential (3.17) thus reduces to the form

$$V_{np}^{\text{coh}}(\mathbf{r}_n, \mathbf{r}_{H_2^p}) = \frac{2\pi}{\mu} a_{\text{coh}}^p \delta(\mathbf{r}_n - \mathbf{r}_{H_2^p}),$$  (3.18)

where $a_{\text{coh}}^p = (a_s + 3a_t)/2$ [93] is the parahydrogen coherent scattering length, which in terms of the $np$ scattering amplitude is $\text{Re} f/2 \frac{k-n}{q} - a_{\text{coh}}^p$. The $nH_2^p$ and $np$ scattering amplitudes and momenta are related as $f \approx (\tilde{\mu}/2\mu) f$ and $q = (\tilde{\mu}/\mu) k$, where $f$ is the forward $np$ scattering amplitude in the DWBA and $k$ the relative momentum of the neutron and proton. With the coherent scattering approximation, the common factor $\theta_-$ of the neutron spin rotation $\phi = \text{Re} \theta_-$ and polarization $P = \text{Im} \theta_-$ becomes

$$\theta_-(k, z) = -\frac{N_{H_2} M z}{2k} \sum_{m_n} m_n^{(-)} \langle k\hat{z}; m_n | \hat{V}_{\text{PNC}} | k\hat{z}; m_n \rangle^{(+)},$$  (3.19)

where the matrix elements are Hermitian. The $S \leftrightarrow P$ transitions are sufficient in low energy PNC scattering and also equally important in both directions.

The neutron spin rotation is split into one-meson exchange (OME) and two-pion exchange (TPE) components as $\frac{d\phi}{dz} = \frac{d\phi}{dz}^{\text{OME}} + \frac{d\phi}{dz}^{\text{TPE}}$. The liquid parahydrogen particle density value of $N_{H_2} = 0.021$ molecules/$\text{Å}^3$ is used in the numerical results. In terms of the DDH model, the rotation may be written as

$$\frac{d}{dz} \phi_{\text{DDH}}^{\text{OME}} = \left(0.617 h_\pi^{(1)} - 0.132 h_\omega^{(0)} - 0.012 h_\omega^{(1)} - 0.145 h_p^{(0)} + 0.004 h_p^{(1)} + 0.130 h_p^{(2)}\right) \frac{\text{rad}}{m}. $$  (3.20)

This has the value of $3.52 \times 10^{-7} \text{ rad}/m$ when the DDH "best" values are inserted. With this model and values, the one-pion exchange (OPE) has a dominance of about 80%. The result of (3.20) is about half the size compared to those of [33, 52, 76]. It is otherwise consistent in detail with the one in [76] and also in line with the predictions of [33, 52] which all employ the Argonne $v_{18}$ potential ([52] also the Nijmegen and Bonn potentials). An older result using
3.3 Cold neutron-parahydrogen interactions

The TPE contributions to the neutron spin rotation $\frac{d}{dz}\phi_{TPE}$ in units of $h\pi^{(1)}_{\text{rad}}/m$. The K and D stand for the $\hat{V}_{TPE}^K$ and $\hat{V}_{TPE}^D$ while the F and FF stand respectively for the modification by $\Lambda^2(q^2+\Lambda^2)^{-1}$ and $\Lambda^4(q^2+\Lambda^2)^{-2}$, where the cut-off masses $\Lambda$ are in units of GeV.

<table>
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<th>$\Lambda$</th>
<th>K(FF)</th>
<th>D(FF)</th>
<th>K(F)</th>
<th>D(F)</th>
</tr>
</thead>
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</tr>
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<td>-0.103</td>
<td>-0.017</td>
<td>-0.152</td>
<td>-0.055</td>
</tr>
</tbody>
</table>

Table 3.2: The TPE contributions to the neutron spin rotation $\frac{d}{dz}\phi_{TPE}$ in units of $h\pi^{(1)}_{\text{rad}}/m$. The K and D stand for the $\hat{V}_{TPE}^K$ and $\hat{V}_{TPE}^D$ while the F and FF stand respectively for the modification by $\Lambda^2(q^2+\Lambda^2)^{-1}$ and $\Lambda^4(q^2+\Lambda^2)^{-2}$, where the cut-off masses $\Lambda$ are in units of GeV.

the Paris potential reported in [75] is of the same order with the above results but of the opposite sign. Also in the result of [75] some of the signs between the partial contributions are in disagreement with the mutually consistent results of [76] and (3.20).

Because $\hat{V}_{D}^{TPE}$ and the $NN$ part of $\hat{V}_{K}^{TPE}$ are identical in the unregularized form, they contribute the same amount to the rotation $\frac{d}{dz}\phi_{TPE}^{NN} = -0.101 h\pi^{(1)}_{\text{rad}}/m$ reducing the total effect by about 13%. By taking into account also the $\Delta$ effects of $\hat{V}_{K}^{TPE}$, the contribution doubles to $\frac{d}{dz}\phi_{K}^{TPE} = -0.209 h\pi^{(1)}_{\text{rad}}/m$ cutting down the total effect by about 27%. This is because even though the $NN$ part of the TPE potential dominates at long-ranges, the $N\Delta$ one dominates at short-ranges where the wavefunctions are substantial, as illustrated in Figure 3.8. The TPE is a medium-range effect and therefore sensitive to form factors. The form factor modified TPE contributions to $\frac{d}{dz}\phi_{TPE}$ are given in Table 3.2. As seen in Table 3.2, the results of $\hat{V}_{D}^{TPE}$ are more sensitive to the applied regularizations than the ones of $\hat{V}_{K}^{TPE}$ and even change sign when the dipole form factors (FF) are used. This can be traced to the fact that $\hat{V}_{D}^{TPE}$ includes the $\delta$-function term whereas $\hat{V}_{K}^{TPE}$ does not. The $\delta$-function does not normally contribute because the wavefunctions vanish at the origin. Therefore, the swelling of the $\delta$-term to a finite range by the form factors may produce an unphysical effect. This behaviour of $\hat{V}_{D}^{TPE}$ is also seen in Section 3.1, where exactly the same models as used here are applied to $\hat{A}_L(\vec{pp} \rightarrow \vec{pp})$.

Similarly to (3.20), the DDHOE contribution to the polarization as a constant form is $k_{\text{Lab}} T_{\text{Lab}}^{\text{DDHOE}} = 6.80 \times 10^{-18}$ fm$^{-1}$m$^{-1}$meV$^{-1}$ in which the OPE contribution has more than 70% dominance. For example, with $T_{\text{Lab}} = 10$ meV neutrons the polarization is only $\frac{d}{dz} P_{\text{DDHOE}}^{\text{OME}} = 6.19 \times 10^{-12} \frac{1}{m}$. The
polarization $\frac{d}{dz} P$ is thus much smaller than $\frac{d}{dz} \phi$ and therefore not particularly interesting because it is experimentally much less achievable. The TPE effects on $\frac{d}{dz} P$ were also calculated in [4] and showed similar features as in the case of $\frac{d}{dz} \phi$.

In addition, analogously with (3.20), $\frac{d}{dz} \phi$ can also be expressed in terms of the $\pi$EFT potential (2.14), resulting in

$$\frac{d}{dz} \phi^{\pi EFT} = \left( 1.15C_1 + 0.25C_2 + 1.75C_3 - 0.25C_4 + 5.21C_5 
+ 0.30C_6 + 0.96\tilde{C}_1 + 0.83\tilde{C}_3 + 3.73\tilde{C}_5 \right) \times 10^{-2} \text{rad/m}. \quad (3.21)$$

The result of the reduced $\pi$EFT potential (2.15) is readily obtained from (3.21) by setting $C_2$, $C_3$, $C_4$, $\tilde{C}_3$, and $\tilde{C}_5$ equal to zero.

3.3.2 Photon asymmetry $A_\gamma(\vec{n}p \rightarrow \gamma d)$

The nonzero $\gamma$-asymmetry $A_\gamma$ arises from the interference between the M1 transition and the PNC interaction generated E1 transitions. The $\gamma$-asymmetry is defined, in terms of differential cross-sections, as

$$A_\gamma = \frac{d\sigma_{+\frac{1}{2}} - d\sigma_{-\frac{1}{2}}}{d\sigma_{+\frac{1}{2}} + d\sigma_{-\frac{1}{2}}}. \quad (3.22)$$

The thermal neutron capture cross-section on molecular hydrogen does not depend on the interference or binding effects of the protons in the molecule [94]. It is therefore sufficient to simply calculate the neutron capture cross-section on free protons. The M1 $^1S_0 \rightarrow ^3S_1 - ^3D_1$ transition dominates the $np \rightarrow \gamma d$ reaction at threshold. By far the largest contribution (about 90%) of this reaction arises from the impulse approximation which connects the $S$-states. However, the one-pion exchange currents can also reach the $D$-state of the deuteron and play an important role in explaining the experimental value of the cross-section for thermal neutrons as was shown in [95].

In terms of Lagrangian densities, the relevant photoproduction vertices for the $\gamma NN\pi$ and $\gamma \pi\pi$ interactions are

$$\mathcal{L}_{\gamma NN\pi} = -e \frac{f_\pi}{m_\pi} \bar{N} \gamma_5 \gamma^\mu (\tau \times \pi)_z N A_\mu, \quad \mathcal{L}_{\gamma \pi\pi} = -e(\partial^\mu \pi \times \pi)_z A_\mu, \quad (3.23)$$

where $A_\mu = (\phi, -A)$. Since the energy of the resulting photon at threshold of the reaction is only about 2 MeV, its wavelength is much larger than the
Figure 3.8: The two top-most panels show the continuum S- and P-wavefunctions $U^{J(+)}_{\xi}(k,r)$ near zero-energy ($T_{\text{Lab}} = 10$ meV) and the bottom panel shows the PNC TPE potentials $U^K_{NN}(r)$ (2.22) and $U^K_{N\Delta}(r)$ (2.25). The dash-dotted line in the top panel represents the side wavefunction generated from the triplet S-wave.
Figure 3.9: The diagrams for the magnetic dipole moments considered in the calculation of $\sigma(np \rightarrow \gamma d)$. The wavy line is a photon, the solid line is a nucleon, the dashed line is a pion, and the filled bar is a $\Delta$-isobar. The $D$ denotes the bound $^3S_1 - ^3D_1$ state of the deuteron.

deuteron size, and thus the electric $E$ and magnetic $B$ fields can be taken as constants. The scalar and vector potentials of the uniform (static) fields $E$ and $B$ are $\phi(r) = -E \cdot r$ and $A(r) = \frac{1}{2} B \times r$, respectively. The magnetic and electric potentials are given accordingly by $\hat{V}_m = -\hat{\mu}_m \cdot B$ and $\hat{V}_e = -\hat{\mu}_e \cdot E$. The $\Delta$-excitation also contributes significantly to the exchange currents. The lowest multipoles that can excite the nucleon to the $\Delta$-resonance are the $M1$ and $E2$ transitions, of which the latter is assumed small and neglected as unimportant. The nonrelativistic interaction Lagrangian for the $\gamma N \Delta$ vertex is then

$$L_{\gamma N \Delta} = \frac{f_{\gamma N \Delta}}{m_\pi} \mathbf{S} \cdot \mathbf{B} \mathbf{T}_z + \text{H.c.}$$

In addition to (3.23) and (3.24), the other necessary non-photonic vertices for the currents are given in (2.26). The one-body operators can be extracted from (3.10).

A diagrammatic illustration for the one- and two-body magnetic dipole moment operators is given in Figure 3.9. The corresponding magnetic moment operator is given by $\hat{\mu}_m(r) = e\hat{m}(r)/2M$, with

$$\hat{m}(r) = \frac{\mu_v}{4} \hat{\tau}_- \mathbf{\sigma}_- - M \left( \frac{f_\pi}{m_\pi} \right)^2 \hat{\tau}_x \left\{ \hat{r} \mathbf{\sigma} \cdot (1 + m_\pi r) - \mathbf{\sigma} \cdot m_\pi r \right\} Y_\pi(r)$$

$$- \frac{\mu_\Delta f_\pi f_\pi}{9(M_\Delta - M)} (\hat{\tau}_- - i\hat{\tau}_x) \left\{ 2\mathbf{\sigma} \cdot \hat{r} \mathbf{\sigma} - i(\mathbf{\sigma}_1 \times \hat{r})(\mathbf{\sigma}_2 \cdot \hat{r}) \right\} + i(\mathbf{\sigma}_2 \times \hat{r})(\mathbf{\sigma}_1 \cdot \hat{r}) \left\{ 1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2} \right\} Y_\pi(r),$$

where $\hat{\tau}_\odot = (\mathbf{\tau}_1 \odot \mathbf{\tau}_2)_z$ and $\mathbf{\sigma}_\odot = \mathbf{\sigma}_1 \odot \mathbf{\sigma}_2$ ($\odot = -, \times$).

The E1 transitions connect the initial $^3S_1 - ^3D_1$ and final deuteron states through the parity admixed continuum and bound $^3P_1$ states, while the PNC
Figure 3.10: The E1 contributions to the $\gamma$-asymmetry $A_\gamma$. The black dot denotes the parity nonconserving vertex. In addition the $D$ denotes the bound and $^3C_1$ the continuum $^3S_1 - ^3D_1$ state.

E1 transitions connect them directly. Figure 3.10 gives the diagrams for electric dipole moment contributions. The resulting E1 operator is given by

$$\hat{\mu}_e(r) = \frac{e}{4} \hat{\tau} \cdot r + \frac{e}{4M m_\pi \sqrt{2}} \hat{\tau} \left[ i \sigma_+ Y_\pi(r) - r \sigma_+ \cdot \left\{ -i \nabla, Y_\pi(r) \right\} \right],$$

(3.26)

with $\hat{\tau} = \tau_1 \cdot \tau_2 - \tau_1z \tau_2z$ and $\sigma_+ = \sigma_1 + \sigma_2$. One might note that the one-body impulse operator (the first term) changes parity, while the two-body current operator (the latter term) conserves it. Reference [49] investigates the PNC E1 transitions and gives an additional PC $\gamma N N \pi$ vertex leading to the spin-changing PNC E1 operator. However, this vertex has a vanishing contribution to $A_\gamma$ because the term proportional to $B$ is negligibly small and the M1-E1 interference disappears as a consequence of the initial $^1S_0$ states in both amplitudes. In addition, [49] and [10] present PNC $\gamma N N \pi$ vertices, which also result in spin-changing PNC E1 operators that do not contribute.

The experimental value of the radiative thermal neutron capture cross-section of proton is $\sigma(np \to \gamma d) = 334.2 \pm 0.5$ mb [96]. Theoretically it may be given by

$$\sigma_{\text{cap}} = \frac{\alpha \pi \omega_{\gamma}^2}{6k^3 M} \sum_{L,s} \int_0^\infty dr D_{L_s}(r)(3L_0^L)\left| \hat{m}(r) \right|^2 U_{01}(r, k)^2,$$

(3.27)

where $\hat{m}(r)$ is the magnetic moment operator (3.25). The choice of coupling values is $g_\pi = 13.45$ ($f_\pi = m_\pi g_\pi/2M$), $f_\pi^* = \sqrt{72/25} f_\pi$, and $\mu_\Delta = f_\pi^* \mu_\pi/2f_\pi$ ($f_{\gamma N \Delta}/m_\pi = e \mu_\Delta/2M$), where $g_\pi$ is the $\pi NN$ coupling, $f_\pi^*$ the quark model result for the $\pi N \Delta$ coupling [97], $\mu_\Delta$ the transition magnetic moment, and $\mu_\pi = 4.71$ the isovector magnetic moment of the nucleon. Evaluation of (3.27) for thermal neutrons ($T_{\text{Lab}}^n = 25$ meV) gives $\sigma_{\text{cap}} = 334.42$ mb in
an excellent agreement with the experimental cross-section. In this result, the plain impulse approximation, \textit{i.e.} the first term of (3.25), produces the largest part of the cross-section giving \( \sigma_{\text{cap}}^{\text{imp}} = 303.81 \text{ mb} \). The missing \( \sim 10\% \) enhancement results from the OPE current corrections, as proposed in [95].

In terms of the reduced matrix elements of the electric (3.26) and magnetic dipole (3.25) transition operators, the \( \gamma \)-asymmetry \( \mathcal{A}_\gamma \) reads

\[
\mathcal{A}_\gamma(k) = \sqrt{2} \text{Re} \left[ \frac{i}{\sum_{LL} \int_0^\infty dr D_{Ls}(r) (3L_4(3L_1)U_{LL_s}^{(+)}(r,k) \langle 3L_1 | \hat{\mu}_m(r) | 1S_0 \rangle U_{S_0}^{(+)}(r,k) \right],
\]

(3.28)

where the multiplication factor \( \cos \theta \), in which \( \theta \) is the angle between the neutron spin and photon direction, is left out.

By using the DDH model and their "best" coupling values, the OME contribution to the \( \gamma \)-asymmetry (3.28) is \( \mathcal{A}_{\gamma\text{DDH}}^{\text{OME}} = -5.387 \times 10^{-8} \). Even though included, the effects of the PNC E1 current (\( \sim -0.2\% \)) and heavy meson \( \rho \) and \( \omega \)-exchanges (less than 1\%) are negligibly small. In terms of the weak pion coupling, the result may be written as \( \mathcal{A}_{\gamma\text{DDH}}^{\text{OME}} \approx -0.117 \hbar(1)\pi \), which is consistent with the previous predictions, \textit{e.g.} [22–24,33,52,60,69,88]. The results, in which the TPE contributions are added on top of \( \mathcal{A}_{\gamma\text{DDH}}^{\text{OME}} \), are shown in Table 3.3 and are rather self-explaning. The result of the model D(F) with \( \Lambda = 1.0 \text{ GeV} \) is consistent with [22] and shows a \( \sim 6\% \) smaller \( \gamma \)-asymmetry than without the TPE. Just like in scattering cases, the TPE contribution of the D(FF) model with \( \Lambda = 0.8 \) and 1.0 GeV differs by the sign from the other models and, therefore, increases the total asymmetry. Otherwise the TPE effect is destructive as in the case of \( \frac{d}{dx} \phi_{\text{TPE}} \). In paper [2], the PNC \( \Delta \) contributions to \( \mathcal{A}_\gamma \) were treated in a different manner by employing a PNC one-meson exchange \( NN \leftrightarrow \Delta N \) transition potential analogously to \( \mathcal{A}_\gamma(\gamma d \rightarrow np) \) in Section 3.2. However, in this approach the leading \( \Delta \) contributions originate from the once-iterated meson exchanges and result in a negligible net \( \Delta \) contribution to \( \mathcal{A}_\gamma \).
Chapter 4

Summary

The objective of the thesis was to investigate the effect of the $\Delta$ in the various PNC reactions outlined in the previous chapter. The PNC observables were revised within realistic wavefunctions and various potential models including the TPE and the $\Delta$. Two new significant corrections were found. Firstly, the $N\Delta$ intermediate state contribution to the TPE was considered for the first time at the observable level and was found doubling the TPE effect. Secondly, the PNC $np$ scattering observables were appropriately dealt with by considering a real parahydrogen target. As a result, the neutron spin rotation and polarization become a factor of 4 smaller than in the unrealistic case of free target protons.

In Section 3.1, the PNC longitudinal analyzing power $A_L(p\bar{p} \rightarrow pp)$ was calculated by taking into account the electromagnetic and TPE effects in various models. The Coulomb interaction plays virtually no role in the scattering or transmission asymmetries. It was found that $\hat{V}_{TPE}$ potential along with the DDH model gives approximately equally sizable effects for the TPE and heavy meson exchange throughout the energy scale. A nearly consistent result comes also from $\hat{V}_{K}^{TPE}$ with the dipole form factor and $\Lambda = 1.0$ GeV cut-off. However, it was found that $\hat{V}_{D}^{TPE}$ would have a very strong, perhaps superficial, dependence on the form factor. Further, in line with the results given by $\hat{V}_{TO}^{TPE}$ and $\hat{V}_{K}^{TPE}$ the inclusion of the $N\Delta$ intermediate states about doubles the asymmetry.

It is shown in [1,50] that besides the TPE, a noteworthy PNC contribution may also arise from the $\Delta$-resonance within the coupled-channels technique by employing a PNC one-meson exchange $NN \leftrightarrow \Delta N$ transition potential based on the vertices and couplings of [36]. In [2], this approach is also
applied to the asymmetries $A_\gamma(\vec{\gamma}d \rightarrow np)$ and $A_\gamma(\vec{n}p \rightarrow \gamma d)$ in Sections 3.2 and 3.3. However, in these cases the leading $\Delta$ contributions appeared only as second-order corrections from the wavefunction distortions and no significant $\Delta$-effects were found. Because of the large uncertainties related to the meson-$N\Delta$ couplings especially in the weak sector, the $\Delta$ was otherwise taken into account only to the extent it appears in the PNC TPE potentials and exchange currents.

Three PNC observables for cold neutron interaction with parahydrogen were calculated in Section 3.3. All the observables, the neutron spin rotation $\frac{d}{dz}\phi$, polarization $\frac{d}{dz}P$, and $\gamma$-asymmetry $A_\gamma$, were found to be dominated by the pion exchange. The calculation employed the same EFT TPE models as in $A_L$ and showed the same characteristic features. It was shown that even at low energies the $N\Delta$ intermediate state plays an important role in the PNC TPE by roughly doubling the effect. The OME contribution to $\frac{d}{dz}\phi$ was concluded to be a factor of two smaller than the most recent predictions and that the TPE decreased it up to $\sim 30\%$ further. The polarization $\frac{d}{dz}P$ was considered rather uninteresting due to its small size. In $A_\gamma$, the OPE currents gave the expected increase for the M1 transitions but were insignificant for PNC E1 ones. The asymmetry was found to arise almost completely from pion exchanges, i.e. OPE weakened by TPE up to $\sim 20\%$ or so. The $\Delta$ appears not only as a significant factor in the PNC TPE potential but it is also important in the PC sector where the $\Delta$ exchange current correction to the $\sigma_{\text{cap}}(np \rightarrow \gamma d)$ is nearly a $4\%$ and, therefore, the $\Delta$ is an important supplement to account for the experimental cross-section.

All in all, in spite of the inescapable model dependence of the PNC observables, the TPE causes most likely an important effect to them and should not be ignored. The TPE correction within the models discussed in the thesis decreases the overall pion-exchange effect in the PNC neutron scattering and absorption reactions by several percent at least. Especially in the case of the PNC $pp$ elastic scattering, which does not include the OPE, the TPE can be more or less the dominant contribution unless the true value of $h_\pi^{(1)}$ is significantly smaller than that given by DDH. The PNC OPE and TPE effects are unique in the sense that they depend only on one weak coupling, namely $h_\pi^{(1)}$. Besides the DDH, there are various calculations [36, 98–104] for $h_\pi^{(1)}$ (ranging between 0 and $3.4 \times 10^{-7}$) indicating a smaller value than the DDH ”best” recommendation. However, past experiments in a complicated nuclear environment seem to be in contradiction concerning $h_\pi^{(1)}$, the
$^{18}$F experiments [105–108] suggest a relatively small value for it, while the $^{133}$Cs experiment [109] large. Be that as it may, ultimately, the ongoing NPDGamma experiment [79], which aims to measure the $\gamma$-asymmetry $A_\gamma$ with high accuracy, or other future experiments will probably decide the reliability of this value.
Bibliography


