Efficient Pricing and Insurance Coverage in Pharmaceutical Industry when the Ability to Pay Matters

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Abstract

A non-trivial fraction of people cannot afford to buy pharmaceutical products at market prices. The paper derives the socially optimal pricing and coverage of pharmaceutical products when the producer price has to cover the R&D sunk cost of the firm, when the different abilities to pay by people are acknowledged and the health insurance is tax-financed. In our model, there is one producer having market power in pricing. We first characterize the Ramsey pricing rule in the absence of insurance coverage. Subsequently, conditions for a welfare increasing departure from Ramsey pricing are stated in terms of price regulation and insurance coverage both in the case where the regulator has access to lump sum taxes and where the social cost of public funds is positive. The resulting outcome is generally second best. While most previous papers have abstracted from different abilities to pay for the pharmaceutical products, our paper extends the analysis to the case of socially optimal third-degree price discrimination. We consider equilibrium where people with low ability to pay have access to the full coverage while those with high ability to pay have partial coverage.

JEL Classification: L1, L5

Keywords: pharmaceutical products, price regulation, public health insurance, third-degree price discrimination

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1 Introduction

People's ability to pay for the pharmaceuticals varies. As pharmaceutical products are created through expensive R&D programs committing the pharmaceutical firms to rather high and risky expenditures, those expenses should subsequently be covered by the product prices. Such prices, however, may turn out to be too high to be socially acceptable. The policy stance as related to the medical industry and expressed in official documents typically states that "the purpose of the medical policy is to provide to citizens a high-quality and cost-efficient health program at reasonable prices....". Moreover, "price regulation aims at providing people’s access to pharmaceutical products at reasonable prices...").¹

Previous work on the optimal price regulation of pharmaceutical products and health insurance has produced a number of contributions. The basic idea is cast in terms of optimal product taxation in one person or many person economy with Ramsey’s (1927) idea of equal percentage reductions in (compensated) demands for all commodities (Diamond, 1975). Based on such foundations, Besley (1988) explored the trade-off between risk sharing and incentives to consume increased medical care inherent in reimbursement insurance. Therefore, the price elasticity of demand appears to play a key role (Ringel et al.). Earlier, Feldstein (1973) had expressed concerns of the welfare cost of excess health insurance induced by the adverse incentives to increase the consumption of health care. Determination of the pharmaceutical prices was considered as a strategic game between the regulator and the pharmaceutical firms by Wright (2002) in a two-firm model with product quality differences. The economic case for patents and the potential for differential pricing was considered by Danzon and Towse (2003) in a multi-country framework. A multi-country case was also introduced by Marinoso et al. (2011) who studied the price negotiations of a pharmaceutical firm with two countries.

Two articles closely related to our work are Barros and Martinez-Giralt (2008) and Gaynor et al. (2000). Barros and Martinez-Giralt (2008) addressed the interaction of pricing and insurance coverage in the pharmaceutical market. Their paper assessed the normative allocation of R&D costs across the different markets served by a pharmaceutical firm. They showed that higher insurance coverage calls for higher prices not only because of lower demand elasticity but also due to a larger moral hazard effect on the consumption of the pharmaceuticals. The equilibrium pricing rule appeared to differ from the standard Ramsey pricing rule: for equal demand elasticities, and given the distortion cost of funds, a country with a higher coverage rate will have a higher price as well. The paper by Gaynor et al. (2000) comes close to our approach, but their context is different from ours. Their focus was in the excessive consumption of the pharmaceutical products caused by the insurance, that is the moral hazard. While they worked with the case of a private insurance market for health care, our focus is instead on the public insurance.

In a related area, Grassi and Ma (2011, 2012) studied the provision of public supply of health care services but with non-price rationing when the income levels of people are different. They showed, among other things, that if rationing is based on wealth information (as is the case in the USA), the optimal policy must implement a price reduction in the private market. If also the cost is observed, the optimal rationing turned out to be based on cost-effectiveness (as in most

¹Those quotations come from the official statements of Ministry of Social Affairs and Health in our country. Other countries with public health care have adopted similar policy lines.
European countries and Canada).

Besides efficiency considerations, policy-makers also emphasize equitable access to services due to the fact that in many countries, if not in most, low-income people are not able to buy the medication they need. At least 14 countries in the European Union apply or have applied some kind of third-degree price discrimination practices in their reimbursement systems (ISPOR 2013; WHO 2013). Such countries include, among others, Germany, Austria, Spain, Great Britain, France, and Italy among others. The most commonly used criteria have been income, social status and age. Consequently, our analysis focuses on the different abilities to pay for the product by individuals with different incomes.

To fix the ideas of our paper, we consider the market for a pharmaceutical product with one firm having innovated a new product. The firm is the sole producer of the product, say through a patent protection. The cost of innovation is sunk at the time the product is sold in the market, and it makes the average cost for the firm decreasing. We allow the consumer population to be heterogeneous in terms of ability to pay (income) and analyze questions related to access to pharmaceutical care. Our objective is to derive and analyze the optimal pricing and insurance coverage by first ignoring equity issues and, secondly, by directly dealing with the fact that a non-trivial fraction of patients cannot afford to buy pharmaceutical products even at regulated and subsidized market prices.

In the Finnish reimbursement system, for example, an equal access to reimbursed pharmaceuticals is pursued by applying a maximum out-of-pocket payment ceiling (670€ per calendar year per patient) and means-tested subsistence subsidy to low income citizens. However, the recent Finnish studies reveal that these practices do not guarantee that poor or socially disabled citizens get full access to the pharmaceuticals they need. In a recent survey, the payment ceiling is considered too high and people cannot necessarily purchase the prescribed pharmaceuticals (Kela, 2013). Strikingly, 11 per cent of the Finnish population has abstained from buying medication because of the prices they find too high. Among those entitled to the sickness benefit or the basic unemployment compensation, the share is 24-36 per cent.

Our approach incorporates the R&D cost of the product into the optimal pricing. As the equality between the marginal cost of production and the marginal revenue does not represent a feasible starting point for price regulation, we first characterize the Ramsey pricing rule in the absence of insurance coverage. Subsequently, conditions for a welfare increasing departure from Ramsey pricing in terms of price regulation and optimal coverage are derived taking the social cost of taxation into account. The resulting outcome is second best in general. The results provide insight as to why both the price regulation and the social insurance are desirable. Arising from the decreasing average cost of the firm and its pricing power, a social cost-benefit analysis is needed to capture both the social welfare gains from price regulation and public health insurance as well as

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2 There is another dimension of inequity in the health policy. In Finland, for example, the access to the health services is uneven in that the working population has an easy access to the occupational health care while the non-working population is serviced in the public health centers subject of a prolonged queuing (Doorslaer 2013).

3 These issues are related to the discussion on the value-based pricing in health economics.

4 The sunk cost of R&D is very large making the average cost decreasing. Thus, the marginal cost pricing is not feasible. However, the fixed cost of production is low and thereby the firm does not make up a natural monopoly. Under the Ramsey-pricing, the firm is allowed the access to pricing where its costs are covered but it is subject of the zero-profit condition.
the social costs in terms of the cost of public funds. We notice that the public insurance approach in our model solves automatically the potential adverse selection problem by the patients.

While most previous papers have abstracted from patients with low ability to pay for the pharmaceutical products, we extend our analysis to the case of the socially optimal third-degree price discrimination. We suggest how the health of low income people can be valued in the social cost-benefit analysis. The approach leads to studying an equilibrium where patients with low ability to pay have access to the full coverage while patients with high ability to pay have partial coverage. The results are informative as to how the cost-effectiveness analysis should be used in price regulation and in the creation of the insurance coverage. It turns out, for example, that in case of a high-quality drug, the optimal coverage for high-ability individuals approaches zero while the low-ability individuals have the full coverage.

Our paper has the following structure. We introduce the model in Section 2. Section 3 formulates the basic problem to be analyzed in different forms throughout the paper. Section 4 derives the Ramsey prices and Section 5 analyzes the optimal prices together with the endogenously determined insurance coverage. Such policy is called price-insurance policy. Section 6 examines the insurance mechanisms ensuring pharmaceutical consumption for all consumers independently of their ability to pay. Section 7 discusses the findings and concludes. A technical appendix follows.

2 Model

We consider a market for a new pharmaceutical product. There is a single monopoly producer holding a patent to sell the product. The size of the consumer population is normalized to one. The fraction \( \gamma \) of the consumers is ill and in need of the pharmaceutical treatment. We assume throughout the article that \( \gamma = 1 \), and that all consumers can be treated as patients. If a patient consumes the pharmaceutical, the product helps to recover health and ability to work.

2.1 Ability to pay, consumer surplus and demand

Each patient consumes regular commodities and at most one unit of the medication. It is the key ingredient of our approach that patients are heterogeneous in their ability to pay for the pharmaceutical. We introduce a randomly distributed income variable \( W \), which is assumed to follow the \( U[0,1] \) distribution. The small letter \( w \) denotes the realization of the income variable. The income variable measures the disposable income and hence includes patients’ tax payments to the government. The state of a patient’s health is either high or low with corresponding utilities \( u^H(s) > u^L \), depending whether or not the consumer consumes the pharmaceutical. The parameter \( s \) measures the quality of the pharmaceutical product. The better is the quality of the product, the better off is a healthy consumer but at a decreasing rate, ie. \( \partial u^H(s)/\partial s > 0 \), \( \partial^2 u^H(s)/\partial s^2 < 0.5 \).

We first show how willingness to pay for the pharmaceutical product, denoted \( \theta \), is determined by patient’s ability to pay and the quality of the pharmaceutical using the approach developed in Grassi and Ma (2011, 2012). We first introduce the following price structure: the variables \( P \) and \( p^f \) denote the price of the (composite of the) consumption goods and the consumer price of

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4In the current paper, the quality \( s \) is exogenous. We, however, acknowledge the possibility that the price regulation may have implications for the quality of the pharmaceutical.
the pharmaceutical product, respectively. Let \( x \geq 0 \) denote the amount of the consumption good consumed. The budget constraint of a patient with income \( w \) can be written as 
\[
\text{w} = \text{px} + \text{ip}^c,
\]
where the binary variable \( i = 1, 0 \) depending whether or not the patient buys the pharmaceutical.

For simplicity, we adopt the normalization \( P = 1 \) in the following analysis. Assuming a separable utility function and introducing a minor generalization to Grassi and Ma (2011, 2012), the patient obtains indirect utility
\[
u(w - \mu) + u^H(s)
\]
by consuming the pharmaceutical product. The utility function \( u(x) \) is assumed to be strictly increasing and concave in \( x \). If the pharmaceutical product is not consumed, the patient’s indirect utility is
\[
u(w) + u^L.
\]

The willingness to pay for the pharmaceutical product \( \theta \) for the patient with income \( w \) is now determined by the indifference condition
\[
u(w - \theta) + u^H(s) = u(w) + u^L.
\]

Stated more intuitively, the health benefit due to the consumption of the pharmaceutical equals the utility sacrifice in terms of foregone consumption of the regular good, that is
\[
u^H(s) - u^L = u(w) - u(w - \theta).
\]

The above condition defines implicitly the willingness to pay as a function of income and the quality of the pharmaceutical. The willingness to pay is increasing in income and the quality of the pharmaceutical, because the implicit differentiation of the condition (4) yields
\[
\frac{\partial \theta}{\partial w} = \frac{\left[ u'(w) - u'(w - \theta) \right]}{u'(w - \theta)} > 0,
\]
\[
\frac{\partial \theta}{\partial s} = \frac{\partial u^H(s)/\partial s}{u'(w - \theta)} > 0.
\]

Given the price \( p^c \), patients with willingness to pay satisfying \( \theta \geq p^c \) buy the pharmaceutical product while those with \( \theta < p^c \) abstain from buying. The former group creates the demand for the pharmaceutical product. For the latter group, the sacrifice is too high in terms of the foregone consumption of the regular goods. We can thus define the low-income patients in our model as those with ability to pay so low that that they abstain from buying the pharmaceutical. With the consumer price \( p^c \), the consumer surplus for buying patients is given by
\[
CS(1) = \theta - p^c.
\]

Those patients with positive consumer surplus have \( u^H(s) \) as their utility from health. For the marginal patient who is indifferent between buying and not buying the pharmaceutical, the consumer surplus is zero. Those patients, who abstain from consuming the pharmaceutical, have \( u^L \) as their utility from health.

In the above analysis, the willingness to pay \( \theta(w, s) \) has been regarded as an endogenous variable determined by income and the quality of the pharmaceutical. A natural parametrization applied in the following analysis is given by
\[
\theta(w, s) = ws.
\]
It can be shown that this parametrisation (6) is consistent with the general approach presented above. Setting \( u_L = 0 \), adopting the logarithmic specification \( u(x) = \ln(x) \) for the utility of the consumption goods and \( u_H(s) = -\ln(1 - s) \) for the valuation of quality levels \( s < 1 \), the condition (4) can be rewritten as

\[
\ln(w - \theta) - \ln(1 - s) = \ln(w)
\] (7)

The parametrisation (6) can be obtained by solving the above condition with respect to \( \theta \).\(^6\)

In what follows, we assume conditions which allow us to derive the parametrisation (6). Then, a patient with income \( w \) obtains a surplus

\[
CS(1) = ws - p^F,
\] (8)

if she consumes pharmaceutical and zero surplus \( CS(0) = 0 \) otherwise. In what follows, the variable \( p \) denotes the producer price of the pharmaceutical product and \( r \) the insurance coverage. The consumer price is then given as \( p^F = (1 - r) p \). For the indifferent (marginal) patient with income \( w_m \), the equality

\[
w_m s - (1 - r)p = 0
\] (9)

holds true. The equation (9) can be solved with respect to \( w_m \) to obtain

\[
w_m = \frac{(1 - r)p}{s}.
\] (10)

Assuming \( w_m < 1 \), those patients with incomes lower than \( w_m \) do not buy the pharmaceutical while the patients with incomes higher than \( w_m \) buy. If the price of the pharmaceutical is sufficiently high, all patients prefer not to consume the pharmaceutical. This occurs when the condition \( p(1 - r) > s \) holds true.

The demand for the pharmaceutical product is the amount of buying patients:

\[
q(p, r) = 1 - w_m = 1 - \frac{(1 - r)p}{s}.
\] (11)

Thus, the inverse market demand function is given as

\[
p = \frac{s}{1 - r} (1 - q).
\] (12)

For the intuition, this demand function is consistent with the idea that the product is a normal good with positive income elasticity. The consumers are ordered on the declining (linear) demand function in regard to their ability to pay. We also notice that an increase in the insurance coverage moves the demand function right.

\(^6\)We thank Albert Ma for pointing out the solution and utility specification to us.
2.2 Producer

The profit of the pharmaceutical firm is defined as follows

$$\pi = (p - c)q(p, r) - F,$$  \hspace{1cm} (13)

where $c \geq 0$ is the marginal cost of production and $F > 0$ is a fixed (sunk) cost from R&D activities prior to the launch of the pharmaceutical. In order to have an active market, we assume throughout the article that the quality of the pharmaceutical exceeds the marginal cost of production:

**Assumption 1** $0 < c < s$.

If we did not make Assumption 1, there would be no patients whose willingness to pay for the pharmaceutical exceeds the marginal cost of producing the pharmaceutical, and hence there would be no room for market exchange.

In addition, to ensure a positive monopoly profit, we assume the condition

**Assumption 2** $0 < F < \frac{(s - c)^2}{4s}$

to hold true. Assumption 2 ensures that there are prices on the demand curve, which exceed the average cost of production. Assumption 2 ensures that there are regulatory policies satisfying the firm’s participation constraint

$$\pi \geq 0.$$

2.3 Regulator

The regulator is benevolent and searches for the producer price and insurance coverage which maximize the social welfare. It is defined as the sum of the consumer surplus and the firm’s profit net of the cost of financing the health insurance

$$W = CS + \pi - (1 + \lambda)T.$$  \hspace{1cm} (14)

Here $T$ is the tax revenue collected to finance the health insurance. We assume that each euro raised through taxes to finance the pharmaceutical consumption costs $(1 + \lambda)$ for the society where $\lambda \geq 0$ measures the social cost of public funds. The regulator maximizes social welfare subject to the budget constraint

$$T \geq r p q(p, r).$$  \hspace{1cm} (15)

The right-hand side of equation (15) measures the insurance expenditure due to the consumption of the pharmaceuticals.

2.4 Timing

We will examine a strategic game between the regulator and the producer of the pharmaceutical. The sequence of moves is as follows. The regulator first chooses the producer price $p$ and insurance coverage $r$, after which the firm either accepts or rejects the proposal. If the firm accepts the proposal, patients decide whether or not to acquire the pharmaceutical.\(^7\)

\(^7\)The regulator acts as a Stackelberg leader relative to the producer and the consumers.
3 Setting up the problem

Since all patients with income higher than $[p(1 - r)]/s$ consume the pharmaceutical, their number can be computed as follows

$$q(p, r) = \int_{\frac{p(1 - r)}{s}}^{1} dw = 1 - \frac{(1 - r)p}{s}. \quad (16)$$

The consumer surplus arising from the consumption of pharmaceuticals is then given as

$$CS(p, r) = \int_{\frac{p(1 - r)}{s}}^{1} (ws - (1 - r)p) dw. \quad (17)$$

On the other hand, the firm’s profit can be defined as follows:

$$\pi(p, r) = (p - c) \int_{\frac{p(1 - r)}{s}}^{1} dw - F = (p - c)q(p, r) - F. \quad (18)$$

It is worth observing that one part of the firm’s revenue is paid by the patients and the other part reimbursed by the health insurance. Hence, we can decompose the profit into two parts,

$$\pi(p, r) = (1 - r)p \int_{\frac{p(1 - r)}{s}}^{1} dw + rp \int_{\frac{p(1 - r)}{s}}^{1} dw - c \int_{\frac{p(1 - r)}{s}}^{1} dw - F$$

$$= (1 - r)pq(p, r) + rpq(p, r) - cq(p, r) - F. \quad (20)$$

In equation (20), $(1 - r)pq(p, r)$ is the revenue that the firm earns from the patients’ out-of-pocket payments and $rpq(p, r)$ is the payment that the firm collects from the insurer. In the case of the social insurance, the latter payment is first collected as a tax from the citizens (including patients) and then transferred to the firm as a subsidy.

The aggregate insurance expenditure thereby amounts to

$$IE(p, r) = rp \int_{\frac{p(1 - r)}{s}}^{1} dw = rpq(p, r). \quad (21)$$

Note that $IE(p, 0) = 0$. Since the value of the social welfare function decreases strictly as tax revenue $T$ increases, the regulator is not willing to collect more tax revenue than the amount of the aggregate insurance expenditure. This implies that the budget constraint (15) must be binding

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8In the later section, we examine the policy under which also those with low ability to pay are eligible of the coverage.
at any solution of the regulator’s problem. The social welfare function can then be restated as follows:

\[ W = CS(p, r) + \pi(p, r) - (1 + \lambda)IE(p, r). \tag{22} \]

The regulator chooses the price-insurance policy \((p, r)\) which maximizes the value of the social welfare (22) subject to the profit constraint

\[ \pi(p, r) \geq 0 \tag{23} \]

and feasibility constraints

\[ p \geq 0 \tag{24} \]

and

\[ 0 \leq r \leq 1. \tag{25} \]

Throughout the article, we will analyze a relaxed problem in which social welfare is maximized without the feasibility constraints. After solving the problem we then check whether the obtained solution satisfies the constraints (24) and (25). This approach has become a standard analytical tool in the principal-agent literature (see e.g. Martimort and Laffont, 2002).

As an efficient benchmark for the regulator’s problem, we use the first-best price and quantity which maximize the social welfare not influenced by insurance coverage:

\[ W_f = CS(p, 0) + \pi(p, 0). \tag{26} \]

In the first-best solution, the price equals the marginal cost, \(p_f = c\). The amount of pharmaceuticals consumed at the first-best solution is \(q(c, 0) = 1 - (c/s)\). The corresponding value of the social welfare is

\[ W_f = CS(p_f, 0) + \pi(p_f, 0) = \frac{(s - c)^2}{2s} - F. \tag{27} \]

It is also known that the regulator can not implement the marginal-cost pricing because the firm will reject such proposals. If marginal-cost pricing were applied the firm would earn the profit \(-F\), and the pricing scheme would not satisfy the firm’s profit constraint. Our main objective in this article is to examine and assess the potential solutions to this problem with and without an insurance coverage.

### 4 Ramsey prices

We first consider the situation in which the society decides not to subsidize the patients’ pharmaceutical expenditures selecting \(r = 0\). Then, the consumption of the pharmaceutical has no effect on public expenditures and there is no need to fund social insurance through tax collection. Note, however, that the problem of covering the firm’s fixed R&D cost still remains.

The problem of the regulator can be defined as finding the price of the pharmaceutical which maximizes the social welfare
subject to the profit constraint
\[ \pi(p, 0) \geq 0. \]  

This is the problem of finding the Ramsey prices. With \( L \) denoting the value of the Lagrangean function, the necessary condition of the regulator’s problem can be defined as follows

\[
\frac{\partial L}{\partial p} = \frac{\partial CS(p, 0)}{\partial p} + (1 + \mu) \frac{\partial \pi(p, 0)}{\partial p} = 0,
\]

where \( \mu \) is the positive-valued Lagrange multiplier of the profit constraint. In addition to the condition (30), the profit constraint and the complementary slackness conditions require that \(-\pi(p, 0) \leq 0, \mu \geq 0 \) and \(-\mu \pi(p, 0) = 0 \).

It is now straightforward to establish that the profit of the firm must be zero in the solution of the regulator’s problem. Suppose this is not the case and \( \hat{p} \) solves the regulator’s problem but \(-\pi(\hat{p}, 0) < 0 \). Then \( \hat{\mu} = 0 \), and the first-order condition (30) yields \( \hat{p} = c \). Marginal-cost pricing can not solve the regulator’s problem, because the firm’s profit is strictly negative. This contradiction implies that the firm must earn normal (zero) profit in the solution of the regulator’s problem.

The first-order condition (30) can be rearranged as follows:

\[
\frac{p - c}{p} = \frac{\mu}{1 + \mu} \frac{s - p}{p}.
\]

This condition together with the zero-profit condition

\[
F - (p - c) \left(1 - \frac{p}{s}\right) = 0
\]

yields\(^9\) the Ramsey price

\[
p_R = \frac{1}{2} \left[ s + c - \sqrt{(s - c)^2 - 4sF} \right] < \frac{1}{2} (s + c) \equiv p_M.
\]

where \((1/2)(s+c) = \text{arg max}_p (p-c)(1/s)(s-p) - F\) is the monopoly price. It is worth pointing out that the Assumption 2 ensures that the Ramsey price is well-defined. The value of the Lagrange multiplier at the regulator’s solution is

\[
\mu_R = \frac{p_R - c}{s - 2p_R + c} > 0.
\]

\(^9\)The system of equations (32) and (31) has two solutions \( r_1 = (p_1, \mu_1) \) and \( r_2 = (p_2, \mu_2) \). The first (second, respectively) solution corresponds to the lower (higher) root of the zero profit condition. The value of social welfare is strictly decreasing at all price levels exceeding marginal cost. Since the prices in the feasible set (ie. prices which satisfy the profit constraint) all exceed the marginal cost, the lower root \( r_1 \) is the solution of the regulator’s problem.
Intuitively, the Ramsey price is sufficiently high so as to make the firm break even but it is lower than the monopoly price. We also notice that the Ramsey price is related not only to the marginal or the fixed R&D costs but also to the demand elasticity through the quality parameter $s$. The absolute value of the price elasticity of demand in our model is

$$|\kappa_p| \equiv \left| \frac{\partial q(p,0)}{\partial p} \frac{p}{q(p,0)} \right| = \frac{p}{s - p}$$

(35)

The above equation for demand elasticity suggests that the higher quality of the drug implies a less elastic demand which then should lead to a higher Ramsey price according to the arguments usually presented about Ramsey prices. However, in the context of our model, an increase in the quality of the pharmaceutical shifts the whole demand curve to the right and the Ramsey price needs to be reduced in order for the zero profit condition to be satisfied in a new equilibrium with a higher quality. Therefore, an increase in the quality of the pharmaceutical reduces Ramsey price, which can also be verified by differentiating the expression (33) with respect to quality parameter.

The firm earns zero profit at the Ramsey solution and, hence, the maximum social welfare equals the equilibrium value of the consumer surplus. The maximum social welfare is given as

$$W_R = CS(p_R,0) = \frac{1}{2s} (s - pr)^2 = \frac{1}{8s} \left(s - c + \sqrt{(s - c)^2 - 4sF} \right)^2.$$  

(36)

We notice from the Ramsey price that it even if it eliminates excess profits, it forcefully limits the number of people who are able to buy the pharmaceutical product when in need. In the next section, we turn to consider a tax-financed social insurance program and its possibilities to enhance social welfare.

## 5 Second-best efficient pricing and insurance coverage

We then introduce health insurance and ask whether adding a distortionary instrument to the regulator’s strategy set improves welfare. It is worth observing at this point that our analysis contains the cases of costless taxation, $\lambda = 0$, and costly taxation, that is $\lambda > 0$. Whether taxation is costless or costly for society has crucial implications for the efficiency of the optimal price-insurance policy. We first derive the optimal solution and then assess the welfare properties of it.

As above, we let $L$ to denote the value of the Lagrangean function. The interior solution of the relaxed problem must satisfy the following first-order conditions:

$$\frac{\partial L}{\partial p} = \frac{\partial CS(p,r)}{\partial p} + (1 + \mu) \frac{\partial \pi(p,r)}{\partial p} - (1 + \lambda) \frac{\partial IE(p,r)}{\partial p} = 0$$

(37)

and

$$\frac{\partial L}{\partial r} = \frac{\partial CS(p,r)}{\partial r} + (1 + \mu) \frac{\partial \pi(p,r)}{\partial r} - (1 + \lambda) \frac{\partial IE(p,r)}{\partial r} = 0.$$  

(38)

In addition, the solution of the relaxed problem must satisfy the feasibility and complementary slackness conditions $-\pi(p,r) \leq 0$, $\mu \geq 0$ and $-\pi(p,r)\mu = 0$. 

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The solution to the regulator’s problem is presented in Proposition 1. A general feature of the solution is that the value of the Lagrangian multiplier must equal the social cost of public funds and $\bar{\mu} = \lambda \geq 0$. To explain intuitively this result, we notice that the multiplier $\mu$ measures the marginal social benefit of relaxing the profit constraint of the pharmaceutical firm. It is part of the optimal solution that this benefit equals the marginal social cost of generating public funds, $\lambda$. The following proposition characterizes the optimal price-insurance policy.

**Proposition 1.** If $\lambda \geq 0$ and

$$\frac{(s - c)^2 \lambda (1 + \lambda)}{s(1 + 2\lambda)^2} < F < \frac{(s - c)^2}{4s}$$

the optimal price-insurance policy is

$$\bar{p} = c + \frac{sF(1 + 2\lambda)}{(s - c)(1 + \lambda)}$$

and

$$\bar{r} = \frac{sF(1 + 2\lambda)^2 - (s - c)^2 \lambda (1 + \lambda)}{(1 + 2\lambda)[sF + c(s - c) + \lambda(2sF + c(s - c))]}.$$  

**Proof.** See Appendix.

In the optimal policy, the producer price $\bar{p}$ exceeds the marginal cost $c$ in order to cover the fixed R&D costs. The optimal price-insurance policy $(\bar{p}, \bar{r})$ is designed in a way that it gives the normal profit for the firm. It is also worth noticing that the optimal price is increasing (and concave) in the social cost of public funds $\lambda$.

The condition (39) guarantees that $\bar{r} > 0$ and the optimal policy is an interior solution. If the condition was not satisfied, the necessary conditions of the problem would support the Ramsey solution $\bar{r} = 0$ and $\bar{p} = p_R$ analyzed in Section 4\footnote{This analysis is available from the authors.}. Furthermore, it can be shown that the effect of the fixed cost on the optimal insurance coverage is given as follows:

$$\frac{\partial \bar{r}}{\partial F} = \frac{(s - c)s(1 + \lambda)[s\lambda + c(1 + \lambda)]}{[(s - c)c(1 + \lambda) + Fs(1 + 2\lambda)]^2} > 0.$$  

Intuitively, these two observations suggest that the regulator is more likely to use health insurance and to increase the insurance coverage, the higher is the fixed cost $F$. The active use of insurance reimbursement allows the regulator to reduce out-of-pocket payments and enhance social welfare. If health insurance was not available, an increase in the fixed cost would increase the price of the pharmaceutical and reduce consumer surplus and social welfare. In other words, the introduction of health insurance improves the regulator’s possibilities to meet its policy objectives.

We then evaluate the consumer surplus, the insurance expenditure and the social welfare at the optimal price-insurance policy displayed in Proposition 1. The price that patients pay out of their own pockets is.
where the equality holds true when taxation is costless and $\lambda = 0$. When taxation is distortionary and $\lambda > 0$, the consumer price is strictly above the marginal cost of producing pharmaceuticals. From these observations it naturally follows that the demand for the pharmaceuticals is in general below the first-best level and

$$q(\hat{p}, \hat{r}) = \frac{(s - c)(1 + \lambda)}{s(1 + 2\lambda)} \leq \frac{s - c}{s} = q(c, 0). \quad (43)$$

When evaluated at the optimal price-insurance pair, the consumer surplus is

$$CS(\hat{p}, \hat{r}) = \frac{(s - c)^2}{2s} \left( \frac{1 + \lambda}{1 + 2\lambda} \right)^2 \leq \frac{(s - c)^2}{2s} = CS(c, 0). \quad (44)$$

The consumer surplus is also lower than the consumer surplus, which patients would enjoy by paying the efficient price and receiving no insurance coverage.

The insurance expenditure or the reimbursement at the optimal solution is given as

$$IE(\hat{p}, \hat{r}) = -\frac{(s - c)^2 \lambda (1 + \lambda)}{s(1 + 2\lambda)^2} + F. \quad (45)$$

The insurance expenditure is positive in the interior solution and, furthermore, we notice that, in the case of distortionary taxation, the expenditure is less than the fixed cost. On the other hand, if $\lambda = 0$, the insurance reimbursement equals the fixed cost at the optimal solution. The intuition behind this relationship between the optimal insurance reimbursement and the social cost of public funds is as follows: the higher is $\lambda$, the less (more, respectively) willing is the regulator to use taxation (patients’ out-of-pocket payments, respectively) as a means to finance the fixed cost of producing pharmaceuticals.

Recalling that the firm earns zero profit at the solution, the maximum value of the social welfare simplifies to

$$\hat{W} = \frac{(s - c)^2}{2s} \left( \frac{1 + \lambda}{1 + 2\lambda} \right)^2 - F (1 + \lambda). \quad (46)$$

The consumer surplus is below the first-best consumer surplus at the optimal price-insurance policy, but the insurance expenditure is less than the fixed cost (see Eq. 45). Hence, it is not immediately clear that the maximum welfare (46) is less than the efficient social welfare (26). However, Proposition 2 below demonstrates that, because of the positive social cost of public funds, first-best welfare exceeds the welfare at the optimal price-insurance policy (46).

It is also interesting to compare the maximum social welfare from the optimal price-insurance policy with costly taxation (46) with the maximum social welfare obtained from the Ramsey solution (36). This allows us to evaluate the welfare consequences of the health insurance in a setting where the benchmark case with no health insurance is not the first-best allocation. Compared with the Ramsey solution, the introduction of the insurance coverage separates the
producer and consumer prices from each other, induces patients to consume more pharmaceuticals and allows the regulator to design policies which satisfy the firm’s profit constraint. In comparison to the Ramsey solution, the optimal price-insurance policy \((\bar{p}, \bar{r})\) increases the consumer surplus by reducing the consumer price below the Ramsey price (33). The obvious cost of introducing the social health insurance is that it increases public expenditures. Social insurance improves welfare if the gain in consumer surplus exceeds the corresponding increase in public expenditures. Proposition 2 proves that this indeed is the case.

Proposition 2. The welfare ranking between the first-best solution, the Ramsey solution and the optimal price-insurance policy is the following:

\[
\bar{W}_f \geq \bar{W} \geq W_R.
\]

Proof. See Appendix.

Intuitively, the Ramsey solution produces smaller welfare than the optimal price-insurance, because a great many people are not able to acquire the drug at Ramsey prices. The optimal policy \((\bar{p}, \bar{r})\), however, does not reach the efficient solution because of the social cost of public funds.

6 Rawlsian approach

6.1 Third-degree price discrimination and quality of drugs

The previous analysis on the optimal price-insurance policy demonstrates how the introduction of health insurance can improve the efficiency of the pharmaceutical market in comparison with the situation where no health insurance is available. From the equity point of view, however, the policy has a serious limitation. The out-of-pocket payment exceeds the marginal cost, and the patients in the cohort of the lowest incomes cannot afford to buy the pharmaceutical even though in the presence of health insurance. The number of such patients is \(1 - q(\bar{p}, \bar{r}) > 0\). Health is, however, not like any other product, and equity considerations suggest that patients with limited ability to pay should also have access to the pharmaceutical treatment.

Our objective in this section is to examine whether the welfare criterion could be adjusted to cope with equity. We introduce a Rawlsian principle of equity into the health insurance in the form of the third-degree price discrimination. This amounts to analyzing a model where people with high ability to pay and people with low ability to pay are entitled to different coverage rates, say \(r_1 \leq r_2\), where subscripts 1 and 2 refer to high-ability to pay (high-income) and low-ability to pay (low-income) patients, respectively. In particular, in what follows we will focus on a price-insurance mechanism \((p, r_1, r_2)\) with the feature \(r_1 \leq r_2 = 1\). Under this mechanism, the regulator chooses the same price for all patients, selects the value of insurance coverage for high-income patients, and allocates the pharmaceutical to the patients with a low ability to pay for free. It is worth observing that the number of patients within these two groups is determined endogenously on the basis of the policy parameters \((p, r_1, 1)\).\(^{11}\)

\(^{11}\)There are good reasons for considering the full insurance coverage for low-income patients. If the insurance...
In order to define the welfare of the low-ability patients, we study two complementary criteria. The first criterion is a subjective one and based on the consumer surplus of low-income patients. The second criterion is socially determined and is based on the value of life and the ability of low-income patients to work. We will analyze the first criterion in this section and delay the analysis of the second criterion until the next subsection.

With full insurance coverage, the consumer surplus of the low-income patients is determined in a manner which is analogous to that of the high-income patients. Given the price-insurance mechanism \((p, r_1, 1)\), the consumer surplus of the low-income patients is

\[
\int_0^{\mu(1-r_1)} wsdw.
\]

The aggregate consumer surplus is thus given as follows:

\[
CS(p, r_1, 1) = \int_0^{\mu(1-r_1)} wsdw + \int _{\mu(1-r_1)}^1 (ws - (1-r_1)p)dw. \tag{48}
\]

All patients purchase the pharmaceutical under this mechanism, because the aggregate demand is

\[
q(p, r_1, 1) = \frac{p(1-r_1)}{s} + 1 - \frac{p(1-r_1)}{s} = 1.
\]

This implies that the firm’s profit

\[
\pi(p, r_1, 1) = p - c - F; \tag{49}
\]

is now independent of the insurance coverage. The total insurance expenditure consists of the insurance reimbursements paid to the high- and low-income patients

\[
IE(p, r_1, 1) = \frac{p^2(1-r_1)}{s} + r_1 p \left(1 - \frac{p(1-r_1)}{s}\right). \tag{50}
\]

The following analysis concentrates on the case of costly taxation only, which is also the approach used in the literature (Barros and Martinez-Xiralt, 2008). The optimal equity-adjusted price-insurance mechanism based on the subjective welfare criterion is characterized in the following proposition.

**Proposition 3.** If \(\lambda > 0\) and \(c + F > s/2\), the equity-adjusted optimal price-insurance policy with the subjective criterion is

\[
\hat{\rho} = c + F \tag{51}
\]

and coverage were not full, there would be a patient at the lowest end of the income distribution who would not be able to acquire the pharmaceutical. We note that many European countries have developed instruments to provide free medication for low-income individuals.
\[ \hat{r}_1 = 1 - \frac{s}{2(c + F)}. \] (52)

**Proof.** See Appendix.

In the optimal price-insurance policy described in Proposition 3, the high-income patients are entitled to partial coverage while the low-income patients obtain the medication with full coverage. The quality of the drug affects the optimal insurance coverage in an interesting way. In particular, the higher is the quality of the pharmaceutical, the lower is the optimal insurance coverage for high-income patients.

These results may appear counterintuitive, but the logic goes as follows. If the quality of the pharmaceutical is high, the wealthy patients are willing to pay more for the pharmaceutical product, and our results suggest that the regulator takes this into account by adjusting the insurance coverage accordingly. This feature is not unique to the Rawlsian mechanism described above, but as the analysis of the optimal price-insurance mechanism in Section 5 shows, the out-of-pocket payment of buying patients (see Eq. 42) is also increasing in the quality of the pharmaceutical.

The welfare properties of the optimal solution make some of these points explicit. The out-of-pocket payment for high-income patients is

\[ \hat{p}(1 - \hat{r}_1) = \frac{s}{2} \] (53)

and these patients’ demand for the pharmaceutical is

\[ q(\hat{p}, \hat{r}_1, 1) = \left(1 - \frac{\hat{p}(1 - \hat{r}_1)}{s}\right) = \frac{1}{2}. \] (54)

Another half of the market consists of the consumption of low-income patients. Equal division of the market shows up also in the consumer surplus that the two groups obtain from the consumption of the pharmaceutical. The aggregate consumer surplus that patients obtain from the optimal policy \((\hat{p}, \hat{r}_1, 1)\) is

\[ CS(\hat{p}, \hat{r}_1, 1) = \frac{s}{4}, \] (55)

which is split equally between the high- and low-income patients. Hence, and somewhat strikingly, although the patients with low ability to pay obtain the pharmaceutical for free, their surplus at the optimal solution is no higher than the surplus of the patients with high ability to pay.

When evaluated at the optimal solution \((\hat{p}, \hat{r}_1, 1)\), the aggregate insurance expenditure amounts to

\[ IE(\hat{p}, \hat{r}_1, 1) = c + F - \frac{s}{4}. \] (56)

The insurance expenditure due to the consumption of the high-income patients is

\[ \hat{r}_1 \hat{p} \left(1 - \frac{\hat{p}(1 - \hat{r}_1)}{s}\right) = \frac{1}{2} (c + F) - \frac{s}{4}. \] (57)
It is worth pointing out that the aggregate insurance expenditure and the insurance expenditure of the high-income patients are strictly positive under the assumption $s < 2(c + F)$, which ensures the interior solution (see Proposition 3). Finally, the insurance expenditure due to the consumption of low-income patients is

$$\frac{\hat{p}^2(1 - \hat{r}_1)}{s} = \frac{1}{2}(c + F),$$

which is strictly positive. As expected, the low-income group of patients is more expensive for the regulator in terms of their insurance expenditure. The higher is the quality of the pharmaceutical $s$, the lower is the aggregate insurance reimbursement paid for the high-income patients. Such a relationship arises because the optimal insurance coverage of the high-income patients decreases as the quality of the pharmaceutical increases and a larger share of the total price $\hat{p}$ is paid directly by the high-income patients.

The social welfare in the Rawlsian solution with the subjective criterion is given as

$$\tilde{W} = s - (1 + \lambda)(c + F - s).$$

We next compare the welfare obtained from the above policy paying explicit attention for equity with the welfare obtained from the optimal price-insurance policy with no concern for the access of low-income patients (Section 5). We address the question whether adjusting the optimal policy for the patients’ ability to pay improves the social welfare. The following proposition shows that this is possible and occurs when the quality of the pharmaceutical is sufficiently high.

**Proposition 4.** Suppose that $\lambda > 0$. Then the equity-adjusted price-insurance policy $(\hat{p}, \hat{r}_1, 1)$ with a subjective welfare criterion produces higher welfare than the optimal price-insurance policy $(\tilde{p}, \tilde{r})$ with no equity concern and $\tilde{W} \geq \tilde{W}$, if

$$s \geq 2c(1 + \lambda) \left( 1 + \sqrt{1 + \frac{1}{2\lambda}} \right).$$

**Proof.** See Appendix.

Stated verbally, under the Rawlsian criterion, the social welfare exceeds the social welfare under an optimal price-insurance policy with a uniform coverage rate. This result is, however, conditional on the quality of the drug. When the quality is high enough, the high-income people are willing to pay for the consumption of the pharmaceutical by themselves which limits the social cost of public funds needed to deliver the drug to all patients.

### 6.2 A social criterion

While the consumer surplus, based on the subjective valuation of those with sufficiently high ability to pay, might be an appropriate measure of welfare for those with high ability to pay, a social criterion might be more reasonable when valuing the welfare of the low-income patients. Taking care of everyone may be regarded as a social norm and a value as such. In order to study the implications of such a view, we denote the social value of a low-income patient by $v > 0$. A
possible interpretation is that by providing the medication, the society can recover the ability to work of these people. Then $v$ can be taken to measure the social value of low-income patients in terms of the value-added they produce in the labor market reflected in their market wage\footnote{We take that $\nu$ is the average productivity. Introducing the whole distribution is possible but would only add an unnecessary complexity in the model.}. Such an approach points to an interpretation that the society can regard the expenditure on the state of health of the patients with low ability to pay as an investment.\footnote{The approach is not appropriate in the case of the high-income people as those make the investment in their health by themselves in the model.}

As there are $1 - q(p, r_1, 1)$ such patients, the Rawlsian view with social criterion introduces a term $v [1 - q(p, r_1, 1)]$ into the social welfare\footnote{If the medication is absolutely necessary for the survival of the patient, $v$ can alternatively be regarded as the value of human life. The issue of the value of life has been long discussed in economics. Health care programs are but one of the many public policy initiatives that have mortality reductions as their primary goal. The proper social cost-benefit analysis requires an estimate of the value the society places on a life saved. Evaluating the economic value of a statistical life is now part of the generally accepted economic methodology and a large literature has developed to estimate its (Mrozek and Taylor (2001), Viscusi (2003), Brannon (2004,2005). Economists often estimate the value of life thus by looking at the risks that people are voluntarily willing to take, or how much they must be paid for taking them, see also Mankiw (2012).}. The appropriate policy target now is the maximization of the sum of the welfare of the self-paying customers and non-paying customers where the non-paying customers obtain the pharmaceutical product with full coverage. The coverage for the paying customers and the price of the pharmaceutical remain the optimizing variables of the regulator. We assume that both the consumer surplus of the high-income patients and the social value of low-income patients $v [1 - q(p, r_1, 1)]$ are measured in monetary units. The aggregate value of pharmaceutical consumption is then defined as follows:

$$CS(p, r_1, 1) = v \int_0^{p(1-r_1)} dw + \int_{p(1-r_1)}^1 (ws - (1 - r_1)p) dw.$$  \hfill (61)

The other ingredients of the model are similar to those analyzed in Section 6.1, and the following analysis also proceeds along the lines presented in that section. The following proposition displays the Rawlsian optimal price-insurance policy with a social criterion.

**Proposition 5.** If $\lambda > 0$ and $c + F > (v + s\lambda)/(1+2\lambda)$, the equity-adjusted optimal price-insurance policy with a social criterion is given by

$$\hat{p}^v = c + F$$  \hfill (62)

and

$$\hat{r}_1^v = 1 - \frac{v + s\lambda}{(1 + 2\lambda)(c + F)}.$$

**Proof.** See Appendix.
The current analysis is complementary to the previous analysis with the consumer surplus as a welfare criterion for low-income patients (see Section 6.1). When the social valuation of recovered health is introduced to provide an alternative measure of welfare of patients with low ability to pay, the result is rather similar to the one in the previous section but with some adjustments. The close similarity of the results under the two alternative formulations of the social welfare point to an emerging and rather robust view. The conflict of interest between the high-income and low-income patients depends on by how much the society values the recovered health of the low-income patients. In particular, if the social preferences justify the regulator to impute a high value of recovered health to patients with low ability to pay, the optimal coverage of those with high ability to pay is reduced, i.e. \( \partial r / \partial v < 0 \).

As above, we then carry out the welfare analysis. The out-of-pocket payment of the high-income patients is given as

\[
\hat{p}^*(1 - \hat{r}_1^*) = \frac{v + s\lambda}{1 + 2\lambda},
\]

which is strictly increasing in the social value of recovered health \( v \) of low-income patients. The high-income patients’ demand for the pharmaceutical is

\[
q(\hat{p}^v, \hat{r}_1^v, 1) = \left(1 - \frac{\hat{p}_v(1 - \hat{r}_1^v)}{s}\right) = \frac{s(1 + \lambda) - v}{s(1 + 2\lambda)},
\]

which, as expected, is decreasing in the value of the recovered health of low-income patients. The demand of the low-income patients is given as

\[
1 - q(\hat{p}^v, \hat{r}_1^v, 1) = \frac{v + s\lambda}{s(1 + 2\lambda)}.
\]

The surplus that the high-income patients receive from the consumption of the pharmaceutical is

\[
\int_{p=0}^{1} (ws - (1 - r_1) p) \, dw = \frac{(s(1 + \lambda) - v)^2}{2s(1 + 2\lambda)^2},
\]

and the social value of the consumption of low-income patients is

\[
v[1 - q(\hat{p}^v, \hat{r}_1^v, 1)] = \frac{v(v + s\lambda)}{s(1 + 2\lambda)}.
\]

By summing up the consumer surplus of the high-income patients and the social value of the pharmaceutical consumption of the low-income patients, we obtain the aggregate value of the pharmaceutical consumption

\[
CS(\hat{p}^v, \hat{r}_1^v, 1) = \frac{s^2(1 + \lambda)^2 - 2sv(1 - 2\lambda^2) + (3 + 4\lambda)v^2}{2s(1 + 2\lambda)^2}.
\]

It is worth pointing out that the consumer surplus of the high-income patients is monotonically decreasing in the social value of pharmaceutical consumption of the low-income patients, when
the social value of the pharmaceutical consumption of low-income patients is not too high and
\( v \leq s(1 + \lambda) \).

The aggregate insurance expenditure is given as
\[
IE(\hat{p}_v, \hat{r}_1, 1) = -\frac{s^2 \lambda (1 + \lambda) + s(1 + 2\lambda)^2 (c + F) + v(v - s)}{s(1 + 2\lambda)^2},
\]
which can be split into the insurance expenditure due to the consumption of high-income patients
\[
\hat{r}_1 \hat{p}_v \left( 1 - \frac{\hat{p}_v(1 - \hat{r}_1)}{s} \right) = \max\{0, \frac{[(c + F)(1 + 2\lambda) - (v + s\lambda)](s(1 + \lambda) - v)}{s(1 + 2\lambda)^2}\}
\]
and the insurance expenditure due to the consumption of low-income patients
\[
\frac{(\hat{p}_v)^2(1 - \hat{r}_1)}{s} = \frac{(c + F)(v + s\lambda)}{s(1 + 2\lambda)} > 0.
\]
The insurance expenditure due to the consumption of the high-income patients (71) is positive under the assumption \((c + F)(1 + 2\lambda) > v + s\lambda\) ensuring interior solution (see Proposition 5), if the social value of recovered health of low-income patients is sufficiently small and \( v \leq s(1 + \lambda) \). Finally, the social welfare is given as follows
\[
\hat{W}_v = \frac{s^2 (1 + \lambda)^2 - 2s(1 + 3\lambda + 2\lambda^2)(c + F) + v(v + 2s\lambda)}{2s(1 + 2\lambda)},
\]

The following proposition derives the conditions for the social welfare \( \hat{W}_v \) to exceed the social welfare in a situation with a uniform insurance coverage and no equity concern for low-income patients (see Section 5).

**Proposition 6.** Suppose that \( \lambda > 0 \). Then the equity-adjusted price-insurance policy \((\hat{p}_v, \hat{r}_1, 1)\) with a social welfare criterion produces higher welfare than the optimal price-insurance policy \((\tilde{p}, \tilde{r})\) with no equity concern, that is \( \hat{W}_v \geq \check{W} \), if \( v \geq c(1 + \lambda) \), namely, if the value of the recovered health of low-income people exceeds the social marginal cost of producing the medication.

**Proof.** See Appendix.

7 Final remarks
People’s ability to pay for urgently needed pharmaceutical products varies according to their income. Those products are created through expensive R&D programs where pharmaceutical firms commit themselves to rather high and risky expenditures. Those expenses should subsequently be covered by the product prices of successful new products. Such prices may indeed turn out to be too high to be socially acceptable. The policy stance with respect to the medical products and the insurance coverage is typically based on the purpose of providing to the citizens cost-efficient pharmaceuticals of high quality at reasonable prices.
Previous work on the optimal price regulation of pharmaceutical products and the health insurance has produced a number of contributions. Our paper has extended the previous work in three ways. First, we considered a market where the ability to pay differs in the patient population. Second, we made the insurance coverage endogenous and analyzed the optimal price regulation together with the insurance coverage. Third, we examined how the optimal coverage is related to the social cost of public funds. While most of the earlier papers have abstracted from different abilities to pay for the pharmaceutical products, we extended our analysis to the case of the socially optimal third-degree price discrimination. We derived an equilibrium where people with low ability to pay have access to the full coverage while those with high ability to pay have partial coverage. This turned out to be well motivated: the ability to pay of the high-income people is turned into willingness to pay while this cannot hold for the low-income people. Our results are informative in guiding decision-makers regulating the prices and the reimbursement of new pharmaceuticals.

References


Appendix

Proof of Proposition 1. The regulator’s problem is to find the price-insurance pair \((p,r)\) which maximizes the social welfare

\[
W = CS(p, r) + \pi(p, r) - (1 + \lambda)IE(p, r)
\]

subject to the profit constraint

\[-\pi(p, r) \leq 0\]  

and the feasibility constraints

\[-p \leq 0\]  
\[-r \leq 0\]  
\[r \leq 1.\]

The above problem is called the original problem, OP. In what follows, we analyze the solutions of the original problem without the feasibility constraints. Such a problem will be called the relaxed problem, RP. This approach to finding the solutions to the regulator’s problem through the relaxed problem rests on the intuition that, if the solutions of the relaxed problems also satisfy the feasibility constraints, then they must solve the original problem.

Let \((\bar{p}, \bar{r})\) denote the price-insurance policy solving the relaxed problem and \(\mu\) the Lagrange multiplier of the profit constraint. The Lagrangian function of the relaxed problem can be written as

\[
L = CS(p, r) + (1 + \mu)\pi(p, r) - (1 + \lambda)IE(p, r).
\]

The solution of the relaxed problem must satisfy the first-order conditions:

\[
\frac{\partial L}{\partial p} = -(1 - r) \left[ 1 - \frac{p(1 - r)}{s} \right] + (1 + \mu) \left[ 1 - \frac{2p(1 - r)}{s} + \frac{(1 - r)c}{s} \right]
\]

\[-(1 + \lambda) r \left[ 1 - \frac{2p(1 - r)}{s} \right] = 0\]  

\[
\frac{\partial L}{\partial r} = p \left[ 1 - \frac{p(1 - r)}{s} \right] + (1 + \mu)(p - c) \frac{p}{s}
\]

\[-(1 + \lambda) p \left[ 1 - \frac{p(1 - r)}{s} + \frac{pr}{s} \right] = 0\]

Moreover, the solution must satisfy the profit constraint and the complementary slackness conditions \(-\pi(p, r) \leq 0, \mu \geq 0\) and

\[
\mu \left( F - (p - c) \left( 1 - \frac{p(1 - r)}{s} \right) \right) = 0.
\]
Lemma 1. If \((\bar{p}, \bar{r}, \bar{\mu})\) solves the relaxed problem, then \(\bar{\mu} = \lambda \geq 0\).

Proof. Contrary to the claim suppose that \(\mu \neq \lambda \geq 0\) in the solution of the relaxed problem. Then the first-order conditions (80) and (81) have two solutions. The first solution is \(\bar{\mu} = 0\) and 
\[\bar{r} = \frac{s\mu + c(1 + \mu)}{(s + c)(1 + \mu)}\]
and the second solution is 
\[\bar{\mu} = \frac{s(1 + \lambda) - c(1 + \mu)}{(s + c)(1 + \lambda) - c(1 + \mu)}\]. When evaluated at these two solutions, the profit of the firm is 
\[-\pi(\bar{p}, \bar{r}) = c + F\] and 
\[-\pi(\bar{p}, \bar{r}) = F\], respectively. Therefore, the solutions of the first-order conditions (80) and (81) never satisfy the profit constraint. This implies that, if \(\mu \neq \lambda\), there is no price-insurance pair which would satisfy the necessary conditions of the relaxed problem. For solutions to exist, we must have \(\mu = \lambda \geq 0\). ||

Lemma 2. If \((\bar{p}, \bar{r}, \bar{\mu})\) solves the relaxed problem, then any pair \((\bar{p}, \bar{r})\) satisfying
\[p = \frac{s\lambda + c(1 + \lambda)}{(1 - r)(1 + 2\lambda)}\] (83)
satisfies both first-order conditions (80) and (81).

Proof. Suppose that \((\bar{p}, \bar{r}, \bar{\mu})\) solves the relaxed problem. Then, the first-order condition (80) holds true, if
\[p = \frac{s\bar{\mu} + c(1 + \bar{\mu}) - \bar{r}[s\lambda + c(1 + \bar{\mu})]}{(1 - r)(1 + 2\bar{\mu} - r(1 + 2\lambda))}\] (84)
and the first-order condition (81) is satisfied, if
\[p = \frac{s\lambda + c(1 + \bar{\mu})}{1 + \bar{\mu} + \lambda - \bar{r}(1 + 2\lambda)}\] or \(p = 0\). (85)

The solution \(p = 0\) can be ruled out, because it does not satisfy the profit constraint. By Lemma 1, the solution of the relaxed problem must satisfy \(\bar{\mu} = \lambda\). Evaluating the right-hand sides of the equations (84) and (85) at \(\bar{\mu} = \lambda\) yields the equation (83). ||

The rest of the proof proceeds by analyzing the cases \(\lambda = 0\) and \(\lambda > 0\) separately.

Part 1. Let us assume that \(\lambda = 0\). Then, by Lemma 1 we must have \(\bar{\mu} = \lambda = 0\). By Lemma 2, any price-insurance pair \((\bar{p}, \bar{r})\) satisfying
\[p = \frac{c}{1 - r}\] (86)
satisfies both first-order conditions (80) and (81). Any such pair is also the solution of the relaxed problem, if the profit constraint is satisfied. The profit constraint 
\[-\pi(p, r) \leq 0\] is satisfied, if
\[\frac{sF}{sF + c(s - c)} \leq r < 1\]. (87)

Hence any triple \((\bar{p}, \bar{r}, \bar{\mu})\), which satisfies (86) and (87) and \(\bar{\mu} = 0\), is a solution candidate of the relaxed problem. At any such solution for which the constraint (87) is not binding, the Hessian matrix of the second derivatives is indefinite, because under the condition (86)
\[ |H_2| = \frac{-1}{s^2} [c - p(1 - r)] [c - 3p(1 - r)] = 0. \]  

This implies that no interior solution where the profit constraint is not binding and the insurance coverage satisfies

\[ \frac{sF}{sF + c(s - c)} < r \]  

(89)
can be a local maximum.

Let us then evaluate the second-order conditions of the zero-profit solution, which satisfies

\[ \frac{sF}{sF + c(s - c)} = r. \]  

(90)

When the insurance coverage is (90), the optimal price is

\[ p = c + \frac{sF}{s - c}. \]  

(91)

When evaluated at the candidate solution, the determinant of the bordered Hessian matrix is given as

\[ |\ddot{H}| = \frac{(c(s - c) + sF)^2}{s^3} > 0. \]  

(92)

Hence, the zero-profit solution is a local maximum of the relaxed problem. It is also clear that the solution satisfies the feasibility constraints (76), (77) and (78).

**Part 2.** Let us then assume that \( \lambda > 0 \). Then by Lemma 1 we must have \( \bar{\mu} = \lambda > 0 \). By the complementary slackness conditions the zero profit condition \( \pi(p, r) = 0 \) must then hold true at the solution of the problem. Solving the first-order conditions (80) and (81) together with the zero profit condition yields the price and insurance policy and the value of the Lagrange multiplier:

\[ \bar{p} = c + \frac{sF}{s - c} \left( \frac{1 + 2\lambda}{1 + \lambda} \right) \]  

(93)

\[ \bar{r} = \frac{sF(1 + 2\lambda)^2 - (s - c)^2\lambda(1 + \lambda)}{(1 + 2\lambda)[sF(1 + 2\lambda) + c(s - c)(1 + \lambda)]} \]  

(94)

\[ \bar{\mu} = \lambda. \]  

(95)

When evaluated at the point \((\bar{p}, \bar{r}, \bar{\mu})\), the determinant of the bordered Hessian matrix is

\[ |\ddot{H}| = \frac{(c(s - c)(1 + \lambda) + sF(1 + 2\lambda))^2}{s^3(1 + 2\lambda)} > 0, \]  

(96)

which proves that the optimal policy is a local maximum.

Let us then check that the solution of the relaxed problem satisfies the feasibility conditions. It is straightforward to establish that the optimal insurance policy satisfies the condition \( \bar{r} < 1 \).
Proof of Proposition 2. We first prove that $\bar{W} \geq W_R$. Define the welfare difference

$$DW(F) \equiv \bar{W} - W_R = \frac{(s - c)^2 (1 + \lambda)^2}{2s} - F(1 + \lambda) - \frac{1}{8s} \left( s - c + \sqrt{(s - c)^2 - 4sF} \right)^2. \tag{97}$$

The first partial derivative of the welfare difference with respect to the fixed cost $F$ is given as

$$\frac{dDW}{dF} = -(1 + \lambda) + \frac{s - c + \sqrt{(s - c)^2 - 4sF}}{2\sqrt{(s - c)^2 - 4sF}}, \tag{99}$$

and the second partial derivative is

$$\frac{d^2DW}{(dF)^2} = \frac{s(s - c)}{\left( \sqrt{(s - c)^2 - 4sF} \right)^3} > 0. \tag{100}$$

Hence, the welfare difference is a strictly convex function of the fixed cost $F$. The strict convexity of the function $DW(F)$ implies that the unconstrained minimum of the welfare difference with respect to the fixed cost (if one exists) must be unique. Solving the first-order condition $dDW/dF = 0$ with respect to $F$ yields the minimum point

$$F_1 = \frac{(s - c)^2}{s} \frac{\lambda(1 + \lambda)}{1 + 2\lambda} \geq 0, \tag{101}$$

which corresponds to the infimum of the interval of the fixed cost in the interior solution. This implies that $DW(F) > DW(F_1)$ for all values of the fixed cost, which satisfy the condition (39). When evaluated at the minimum point the value of the welfare difference is zero:

$$DW(F_1) = \frac{(s - c)^2 (1 + \lambda)^2}{2s} - F_1(1 + \lambda) - \frac{1}{8s} \left( s - c + \sqrt{(s - c)^2 - 4sF_1} \right)^2$$

$$= \frac{(s - c)^2}{2s} \left( \frac{1 + \lambda}{1 + 2\lambda} \right)^2 (1 + 2\lambda) - \frac{(s - c)^2}{2s} \left( \frac{1 + \lambda}{1 + 2\lambda} \right)^2 2\lambda - \frac{(s - c)^2}{2s} \left( \frac{1 + \lambda}{1 + 2\lambda} \right)^2$$

$$= 0$$

Hence $DW(F) > DW(F_1) = 0$ and $\bar{W} > W_R$ for all interior solutions.

Secondly, we have $\bar{W}_f \geq \bar{W}$ when

$$\frac{(s - c)^2}{2s} - F \geq \frac{(s - c)^2 (1 + \lambda)^2}{2s} (1 + 2\lambda) - F(1 + \lambda), \tag{102}$$

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which implies that
\[
\frac{(s - c)^2}{2s} \frac{\lambda}{1 + 2\lambda} \leq F. \tag{103}
\]
But now
\[
\frac{(s - c)^2}{2s} \frac{\lambda}{1 + 2\lambda} = \frac{(s - c)^2}{2s} \frac{\lambda(1 + 2\lambda)}{(1 + 2\lambda)^2} < \frac{(s - c)^2}{2s} \frac{2\lambda(1 + \lambda)}{(1 + 2\lambda)^2}, \tag{104}
\]
where the last expression corresponds to the infimum of the set of fixed costs inducing the optimal price-insurance solution to be interior. Hence, the condition (103) is satisfied as a strict inequality in the interior solution (with conditions 39), if \( \lambda > 0 \). If \( \lambda = 0 \) and the solution is interior, then the optimal price-insurance pair induces the efficient consumption and production of pharmaceuticals and we have \( \bar{W} = \bar{W}_f \).

**Proof of Proposition 3.** Suppose that \( \lambda > 0 \). As above in the proof of Proposition 1, we will concentrate on solving the relaxed problem. The Lagrangian function of the relaxed problem is given as follows
\[
L = CS(p, r_1, 1) + (1 + \mu) \pi(p, r_1, 1) - (1 + \lambda)IE(p, r_1, 1), \tag{105}
\]
where the consumer surplus, the firm’s profit and the insurance expenditure are defined in (48), (49) and (50). The consumer surplus and the insurance expenditure simplify to
\[
CS(p, r_1, 1) = \frac{s}{2} - p(1 - r_1) + \frac{p^2(1 - r_1)^2}{s} \tag{106}
\]
and
\[
IE(p, r_1, 1) = r_1p + \frac{p^2(1 - r_1)^2}{s}. \tag{107}
\]

The solution of the relaxed problem must satisfy the first-order conditions:
\[
\frac{\partial L}{\partial p} = \mu - \lambda \left[ r_1 + \frac{2p(1 - r_1)^2}{s} \right] = 0 \tag{108}
\]
\[
\frac{\partial L}{\partial r_1} = -\lambda \left[ 1 - \frac{2p(1 - r_1)}{s} \right] = 0 \tag{109}
\]
Moreover, the solution must satisfy the profit constraint and complementary slackness conditions
\[-\pi(p, r_1, 1) \leq 0, \mu \geq 0 \text{ and } \]
\[
\mu(F + c - p) = 0. \tag{110}
\]
We can ignore the solutions in which \( p = 0 \), because such solutions do not satisfy the profit constraint.

**Lemma 1** \( \mu = \lambda \).
Proof. Suppose first that \( \lambda > 0 \). Then from the first-order condition (109) we obtain

\[
\frac{2p(1-r_1)}{s} = 1.
\]

(111)

Substituting this condition into the (108), we have \( \hat{\mu} = \lambda > 0 \). If \( \lambda = 0 \), then by the first-order condition (108) we have \( \mu = 0 \). Therefore \( \mu = \lambda \).

By Lemma 1 and assumption \( \lambda > 0 \) we have \( \hat{\mu} = \lambda > 0 \). The optimal price is \( \hat{p} = c + F \) by the condition (110). Substituting the optimal price into the condition (111) yields

\[
\hat{r}_1 = 1 - \frac{s}{2(c + F)}.
\]

(112)

The optimal insurance coverage is strictly positive, if \( c + F > \frac{s}{2} \). When evaluated at the solution of the problem, the determinant of the bordered Hessian matrix is

\[
|\hat{H}| = \frac{2(c + F)^2 \lambda}{s} > 0,
\]

(113)

which demonstrates that the solution is a local maximum. The solution satisfies the feasibility constraints \( p \geq 0 \) and \( 0 \leq r_1 \leq 1 \) and, therefore, solves the original problem.

Proof of Proposition 4. Now \( \hat{W} \leq \hat{W} \), if

\[
\frac{(s - c)^2 (1 + \lambda)^2}{2s} - F (1 + \lambda) \leq \frac{s}{4} (2 + \lambda) - (1 + \lambda)(c + F)
\]

(114)

which simplifies to the inequality

\[
\frac{s^2}{4} \frac{\lambda}{2} - 2sc\lambda(1 + \lambda) - c^2(1 + \lambda)^2 \geq 0.
\]

(115)

The above inequality holds true, if \( s \geq 2c(1 + \lambda)|1 + \sqrt{1 + 1/(2\lambda)}| \) or if \( s \leq 2c(1 + \lambda)|1 - \sqrt{1 + 1/(2\lambda)}| \). Since the lower root requires a strictly negative-valued quality \( s \) violating Assumption 1, we have \( \hat{W} \leq \hat{W} \), if \( s \geq 2c(1 + \lambda)|1 + \sqrt{1 + 1/(2\lambda)}| \).

Proof of Proposition 5. Suppose that \( \lambda > 0 \). The Lagrangian function of the relaxed problem is (105) except that now the consumer surplus is given as

\[
CS(p, r_1, 1) = \frac{vp(1-r_1)}{s} + \int_{\frac{p}{s} - r_1}^{1} (ws - (1-r)p) dw.
\]

(116)

The solution of the relaxed problem must satisfy the first-order conditions:

\[
\frac{\partial L}{\partial p} = \frac{v(1-r_1)}{s} - (1-r_1) \left[ 1 - \frac{p(1-r_1)}{s} \right] + (1+\mu) - (1+\lambda) \left[ r_1 + \frac{2p(1-r_1)^2}{s} \right] = 0
\]

(117)
and

\[ \frac{\partial L}{\partial r} = \frac{-vp}{s} + p \left[ 1 - \frac{p(1 - r_1)}{s} \right] - (1 + \lambda) p \left[ 1 - \frac{2p(1 - r_1)}{s} \right] = 0. \]  

(118)

The solution of the relaxed problem must also satisfy the profit constraint and the complementary slackness conditions \(-\pi(p, r_1, 1) \leq 0, \mu \geq 0\) and

\[ \mu (F + c - p) = 0. \]  

(119)

**Lemma 1.** The solution of the relaxed problem satisfies \(\hat{\mu} = \lambda \geq 0\).

**Proof.** Contrary to the claim suppose that \(\mu \neq \lambda \geq 0\) in the solution of the relaxed problem. Then the first-order conditions (117) and (118) hold true simultaneously only when \(\hat{p} = 0\) and \(\hat{r}_1 = [s\mu + v]/[s\lambda + v]\). At this point the firm’s profit is \(-\pi(\hat{p}, \hat{r}_1) = c + F > 0\) and therefore, we have no solution, which would satisfy the necessary conditions of the problem, if \(\mu \neq \lambda\). For a solution to exist, we must have \(\mu = \lambda \geq 0\). \|

**Lemma 2** If \((\hat{p}, \hat{r}_1, \hat{\mu})\) solves the relaxed problem, then any pair \((\hat{p}, \hat{r}_1)\) satisfying

\[ p = \frac{s\lambda + v}{(1 - r_1)(1 + 2\lambda)} \]  

(120)

satisfies both first-order conditions conditions (117) and (118).

**Proof.** Suppose that \((\hat{p}, \hat{r}_1, \hat{\mu})\) solves the relaxed problem. Then the first-order condition (80) holds true, if

\[ p = \frac{s(\hat{\mu} - r_1 \lambda) + v(1 - r_1)}{(1 - r_1)^2 (1 + 2\lambda)} \]  

(121)

and the first-order condition (81) is satisfied if

\[ p = \frac{s\lambda + v}{(1 - r_1)(1 + 2\lambda)} \text{ or } p = 0. \]  

(122)

The case \(p = 0\) can be ruled out because that solution never satisfies the profit constraint. By Lemma 1, the solution of the relaxed problem must satisfy \(\hat{\mu} = \lambda\). Evaluating the right-hand side of the equation (121) at \(\hat{\mu} = \lambda\) yields the equation (120). \|

Let us then assume that \(\lambda > 0\). Then \(\hat{\mu} = \lambda > 0\), and the optimal price is \(\hat{p}^v = c + F\) by the zero-profit condition. This solution together with the condition (120) yields the optimal insurance:

\[ \hat{r}_1^v = 1 - \frac{s\lambda + v}{(1 + 2\lambda)(c + F)} \]  

(123)

The optimal insurance coverage is strictly positive, if \(c + F > (s\lambda + v)/(1 + 2\lambda)\). At the solution, the determinant of the bordered Hessian matrix is

\[ |\hat{H}| = \frac{(c + F)^2(1 + 2\lambda)}{s} > 0, \]  

(124)
which shows that the solution is a local maximum. The solution also satisfies the feasibility constraints.||

**Proof of Proposition 6.** Now $\tilde{W} \leq \tilde{W}^v$ if

$$\frac{(s-c)^2 (1+\lambda)^2}{2s} - F(1 + \lambda) \leq \frac{s^2(1+\lambda)^2 - 2s(1 + 3\lambda + 2\lambda^2)(c + F) + v(v + 2s\lambda)}{2s(1 + 2\lambda)}$$

(125)

which, after some straightforward computation, simplifies to the inequality

$$v^2 + 2\lambda sv - c(1 + \lambda)[c(1 + \lambda) + 2s\lambda] \geq 0.$$  

(126)

The above inequality holds true if $v \geq c(1 + \lambda)$ or if $v \leq -(c(1 + \lambda) + 2s\lambda)$. Since negative values of the parameter $v$ are not feasible, we have $\tilde{W} \leq \tilde{W}^v$, if $v \geq c(1 + \lambda)$. ||