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The Political Economy of Labour Market Regulation with R&D

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Abstract

Consider an economy where oligopolists employ skilled and unskilled labour in production and escape production costs by devoting skilled labour to R\&D. Employers and workers bargain over wages and lobby the local policy maker that determines union bargaining power. The main results are the following. When the elasticity of factor substitution exceeds the elasticity of product substitution, the labour markets are deregulated. When labour market policy is left at the local level in an otherwise integrated economy, the likelihood of labour market deregulation increases. This result explains the past development of declining union bargaining power in wage settlement.

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1. Introduction

Following Blanchard and Giavazzi (2003), the policy that supports labour unions (employers) in collective bargaining can be called labour market regulation (deregulation). Nickell et al. (2005, pp. 6-7) report that unionization has shown a downward trend for most OECD countries since the 1980s. Acemoglu et al. (2001) document that the US and UK experienced rapid labour market deregulation in the years 1975-2000. They explain this development by skill-biased technological change which increases the outside option of skilled workers, undermining the coalition among skilled and unskilled workers in support of unions. In this document, the same development is explained by a political process in which workers and employers lobby policy makers on labour market regulation.

Palokangas (2003) argues that distorting taxation would cause labour market regulation. He constructs a political equilibrium in which employers and workers bargain over wages and lobby the government for taxation and labour market regulation. He shows that if it is much easier to tax wages than profits, then the government protects union power by labour market regulation. In contrast, this document introduces in-house research and development (R&D) as an alternative cause of labour market regulation: firms invest in R&D to escape labour costs due to high wages.

A number of empirical documents argue that international trade, in particular the possibility of outsourcing, causes declining union bargaining power (cf. Abraham et al. 2009, Dumont at al. 2005, 2012, Boulhol et al. 2011). On the other hand, Brock and Dobbelaere (2006) find little evidence of international trade having an impact on the workers’ bargaining power. According to Potrafke (2010), other explanations than globalization are required to portray the development of labor market institutions. This document considers an economy which is otherwise integrated except that the labour markets are still regulated at the local level. Because a local policy maker controls only a small proportion of the labour markers of the integrated economy, it has only limited changes to exercise independent policy. This undermines the benefits from lobbying for labour market regulation.

The growth effects of union power depend decisively on the structure of the economy. Labour unions impose minimum wages that cause unemployment. If the same technology were used both in production and in R&D, then union power would decrease profits, undermining incentives to invest in productivity-enhancing R&D (cf. Peretto 2011). In that case, an increase in
union wages raises unemployment but lowers the productivity growth rate. There is, however, contrasting empirical evidence. Caballero (1993) and Hoon and Phelps (1997) find a positive dependence between unemployment and productivity growth. Vergeer and Kleinknecht (2010) show that the annual percentage growth of real wages has a positive effect on growth in value added per labour hour. They conclude that flexible (i.e., deregulated) labour markets can lead to a growth path that is associated with high employment, but slow productivity growth.

Following Palokangas (1996, 2000, 2004), this document establishes a positive dependence between unemployment and productivity growth by the assumption that production and R&D are subject to different technology. There are two categories of labour: key workers (called skilled labour, for convenience) and ordinary workers (called unskilled labour), so that production employs ordinary and R&D key workers more intensively. The minimum wages that are determined by collective bargaining are effectively binding for ordinary, but not necessarily for key workers. When those minimum wages increase, firms lay out unskilled workers, but transfer skilled workers from production to productivity-enhancing R&D to escape labour costs.

Without R&D, the workers and shareholders would obtain their highest income in full employment, in which case the political process would lead to labour market deregulation. With R&D, workers can have incentives to lobby for labour market regulation: they can accept unemployment for unskilled workers in exchange for higher prospective labour income. Labour market regulation increases wages for unskilled workers, decreasing output and transferring skilled workers from production to R&D. This promotes R&D, raising productivity and prospective income.

The remainder of this document is organized as follows. Section 2 characterizes the institutional structure of the economy. Sections 3, 4 and 5 construct the specific models of the households, final-good firms and intermediate-good industries, respectively. Section 6 establishes a common agency game where employers and workers lobby decision makers. Sections 7 and 8 construct the political equilibrium, on which the results are based.

2. The economy

Households supply land, skilled labour and unskilled labour inelastically. There are two sectors: the high-tech sector, in which oligopolists employ skilled and unskilled labour, producing intermediate goods and performing in-house
R&D to escape labour costs, and the traditional sector which produces its output from land.\textsuperscript{1} Investment in physical capital is ignored, because it would excessively complicate the analysis. Households consume the products of the oligopolists and the traditional sector, but it is equivalent to assume that competitive firms produce the consumption good from these products.

It is plausible to assert that skilled labour is more intensively used in R&D than in production. Following many growth models (e.g. Romer 1990), this assertion is translated into an extreme specification in which only skilled labour is devoted to R&D. This reduces the analysis of the dynamics of this system to a system of equations that can be explicitly solved by doing algebra. Presumably, the relaxation of the specification would not change the basic dynamics of the model. The earnings of unskilled labour are called wages and those of skilled labour salaries, for convenience.

In the economy, there is a large number (a “continuum”) of industries that are placed evenly in the limit $[0,1]$, and a number $n$ of equal but disjoint jurisdictions, each of which implements common labour market regulations:

\begin{equation}
[0,1] = \bigcup_{k=1}^{n} B_k, \quad B_k \cap B_\zeta = \emptyset \text{ for } k \neq \zeta, \quad \frac{1}{n} = \int_{i \in B_k} di,
\end{equation}

where $B_k$ is the set of industries belonging to jurisdiction $k$. Each industry $i \in [0,1]$ is controlled by one oligopolist (labeled $i$) that produces a different good (labeled $i$) and bargains over its wage with a labour union (labeled $i$) that represents its workers.

Public policy can be endogenized either by direct majority voting (cf. Saint-Paul 2002a, 2002b), or by lobbying. Majority voting is not applied in this document, because the main interest focuses on the relative bargaining power of employees and employers in the economy. Lobbying can be modeled either by the all-pay auction model, in which the lobbyist making the greater effort wins with certainty, or by the menu-auction model, in which the lobbyists announce their bids contingent on the politician’s actions. In the all-pay auction model, lobbying expenditures are incurred by all the lobbyists before the policy maker takes an action. This is the case e.g. when interest groups

\textsuperscript{1}The traditional sector is introduced into the model only to ensure that there is a stationary state equilibrium in the case where the labour markets are completely integrated, $n=1$ (cf. also footnote 7). If labour cannot freely move between the sectors, then the use of labour alongside land in the traditional sector does not change the results.
spend money to increase the probability of getting their favorite type of government elected (cf. Johal and Ulph 2002). In the menu-auction model, it is not possible for a lobbyist to spend money and effort on lobbying without getting what he lobbied for. Because the menu-auction model characterizes better the case in which the central planner’s decision variables (regulatory constraints, subsidies) are continuous – so that the interest groups obtain marginal improvements in their position by lobbying – it is chosen as a starting point in this document.

In jurisdiction \( k \in [0, n] \), there is a policy maker (labeled \( k \)) which determines relative union bargaining power, an employer lobby (labeled \( k \)) in which oligopolists \( i \in B_k \) are organized, and a labour lobby (labeled \( k \)) in which the workers of those oligopolists are organized. The lobbies influence the policy maker by their political contributions. If economic integration increases the size of the economy, but still leaves the regulation of the labour markets at the local level, then the relative size \( \frac{1}{n} \) of a single jurisdiction falls.

It is assumed that labour is industry specific, for simplicity.\(^2\) There is one unit of labour per industry. Of this, a fixed proportion \( \varphi \in (0, 1) \) is skilled and the remainder \( 1 - \varphi \) unskilled.\(^3\) It is assumed that the salaries for skilled labour are competitively determined, again for simplicity. The equilibrium condition for the market of skilled labour and the full-employment constraint for unskilled labour in industry \( i \) are then given by

\[
h_i + z_i = \varphi, \quad l_i \leq 1 - \varphi,
\]

where \( \varphi (1-\varphi) \) is the supply of skilled (unskilled) labour, \( h_i (z_i) \) skilled labour devoted to production (R&D) and \( l_i \) unskilled labour devoted to production in that industry. Although skilled labour is fully employed, it is assumed that unions and lobbies are common for both skilled and unskilled labour. It will be shown that skilled labour can indirectly benefit from union power.

In this document, the common agency model (c.f. Bernheim and Whinston 1986, Grossman and Helpman 1994, and Dixit et al. 1997) is used to establish the political equilibrium (cf. Fig. 1). The players in that model are households that consume, competitive firms that produce the final good,

\(^2\)If labour moved freely between industries, then it would be extremely difficult to obtain a stationary state equilibrium in a model where technological change is industry-specific.

\(^3\)The transformation of unskilled into skilled labour is a dynamic process where education plays a crucial role. The introduction of such dynamics into the model would unnecessarily complicate the analysis.
oligopolists that make intermediate goods, labour unions, labour and employer lobbies, and policy makers. It is assumed that there is the following sequence of decisions in the economy:

1. The employer and labour lobbies maximize the expected present value of their members’ income flow by their offers to the policy maker of their jurisdiction, relating their prospective political contributions to the latter’s policy on relative union bargaining power.
2. The policy maker sets relative union bargaining power in its jurisdiction to maximize the present value of the political contributions it receives from the employer and labour lobbies.
3. The oligopolists and labour unions bargain over the wages for unskilled labour, maximizing the expected present value of the flow of their profits and labour income, respectively.
4. The oligopolists employ unskilled labour for production and skilled labour for R&D to maximize the expected present value of the flow of their profits.
5. The salary for skilled labour is competitively determined.
6. The oligopolists employ skilled labour for production.
7. Competitive firms make the consumption good from the oligopolists’ outputs and the output of the traditional sector.
8. The households plan their consumption over time.

This game is solved in reverse order: stage 8 in section 3, stage 7 in section 4, stages 6, 5, 4 and 3 in section 5, and stages 2 and 1 in section 6.
3. Households

On the assumption that all households in the economy share the same preferences, they can be represented by a single household that chooses its flow of consumption $c$ to maximize its utility starting at time $T$,

$$\int_T^\infty (\log c) e^{-\rho(\theta-T)} d\theta,$$

where $\theta$ is time, $c$ consumption and $\rho > 0$ the constant rate of time preference. This utility maximization leads to the Euler equation

$$\dot{X}/X = r - \rho \quad \text{with} \quad X \equiv Pc,$$

where $P$ the consumption price index, $X$ consumption expenditure, $r$ the interest rate and $\dot{X} = dX/d\theta$. Because in the model there is no money that would pin down the nominal price level at any time, one can normalize the households’ total consumption expenditure $X$ at unity. This and (3) yield

$$Pc = X = 1, \quad P = 1/c, \quad r = \rho = \text{constant} > 0.$$  \hfill (4)

4. Final-goods producers

Because the supply of land is fixed, the output of the traditional sector, $\mu$, is a constant. The oligopolists $i \in [0, 1]$ produce high-tech goods. These are substitutes and form the composite product

$$\psi = \Psi = \left( \int_0^1 A_i y_i^{1-1/\epsilon} \, di \right)^{1/(\epsilon-1)}, \quad \epsilon > 1,$$

where $y_i$ is the output of oligopolist $i$, $\epsilon$ the constant elasticity of product substitution and $A_i$ the productivity of good $i$ in providing services to the households. Oligopolist $i$ can increase its productivity $A_i$ by investing in R&D. The composite high-tech good $\psi$ and the traditional good $\mu$ are substitutes: the consumption good is produced according to CES technology

$$c = \Phi(\psi, \mu) = \left[ \nu \psi^{1-1/\delta} + (1 - \nu) \mu^{1-1/\delta} \right]^{\delta/(\delta-1)} \quad 0 < \nu < 1, \quad \delta > 1,$$

where $\nu$ is a parameter and $\delta$ the constant elasticity of substitution.

Because all final-good producers are competitive, they can be represented by a single firm that maximizes its profit $Pc - \int_0^1 p_i y_i \, di$ by its inputs $y_i$,
\( i \in [0, 1] \), subject to technology (5) and (6), given the output price \( P \) and the input prices \( p_i, i \in [0, 1] \). Because the rent for land is equal to the marginal product for land, \( \partial \Phi / \partial \mu \), the expenditure share of rents is given by

\[
\frac{\mu \partial \Phi}{\Phi \partial \mu} \in (0, 1).
\]  

(7)

It is plausible to assume that the expenditure share of rents, (7), is smaller than \( (1 - 1/\epsilon)/(1 - 1/\delta) \).\(^4\) Given this and (4), the profit maximization yields the inverse demand curve of oligopolist \( i \) as follows (cf. A):

\[
p_i = b(\psi) A_i y_i^{-1/\epsilon} \quad \text{with} \quad \frac{1}{\epsilon} - 1 < \frac{\psi b'(\psi)}{b(\psi)} < 0 \quad \text{and} \quad \lim_{\nu \to 1} \frac{\psi b'(\psi)}{b(\psi)} = \frac{1}{\epsilon} - 1.
\]  

(8)

5. High-tech industries

Because the number of industries \( i \in [0, 1] \) is large, oligopolist \( i \) and union \( i \) take the interest rate \( r \) and the quantity of the composite high-tech good, \( \psi \), as given [cf. (5)]. Oligopolist \( i \) (union \( i \)) pays political contributions \( R_o^i \) (\( R_u^i \)) to the policy maker of its jurisdiction. Because \( R_o^i \) and \( R_u^i \) are determined by lobbying at the level of the jurisdiction, they are given for oligopolist \( i \) and union \( i \) as well.

5.1. Technological change

Following Grossman and Helpman (1991) and Aghion and Howitt (1998), technological change is specified as follows. Oligopolist \( i \) invests in R&D to improve its technology. During a short time interval \( d\theta \), it has an innovation \( dq_i = 1 \) with probability \( \Lambda_i d\theta \), and no innovation \( dq_i = 0 \) with probability \( 1 - \Lambda_i d\theta \). It is assumed that the arrival rate of innovations, \( \Lambda_i \), is in fixed proportion \( \lambda \) to skilled labour devoted to R&D, \( z_i \):

\[
\Lambda_i = \lambda z_i, \quad \lambda > 0, \quad z_i \geq 0.
\]  

(9)

\(^4\)This condition is satisfied already when the elasticity of substitution between two high-tech products, \( \epsilon \), is higher than that between the composite high-tech good and the traditional good, \( \delta \). In modern economies, the GNP share of agriculture, which approximates (7), is less than 20%. Given this, the condition holds true also when either \( \epsilon > \frac{4}{5} \) or \( \delta < (5/\epsilon - 4)^{-1} \).
The serial number of technology for oligopolist $i$ is denoted by $t_i$ and productivity corresponding to that technology by $A_i(t_i)$. It is assumed that an invention of a new technology raises $t_i$ by one and $A_i(t_i)$ by constant $a > 1$:

$$A_i(t_i + 1) = aA_i(t_i), \quad a > 1. \quad (10)$$

Because technology changes from $t_i$ to $t_i + 1$ with probability $\Lambda_i d\theta$, and does not change with probability $1 - \Lambda_i d\theta$ during interval $d\theta$, then, given (9) and (10), the expected average growth rate of productivity $A_i(t_i)$ is in fixed proportion $(\log a)$ to labour devoted to R&D, $z_i$:

$$\bar{g}_i = \Lambda_i E[\log A_i(t_i + 1) - \log A_i(t_i)] = (\log a)\Lambda_i = (\log a)\lambda z_i, \quad (11)$$

where $E$ is the expectation operator. The level of productivity $A_i(t_i)$ has the expected present value (cf. B, or Aghion and Howitt 1998, p. 61)

$$E \int_T^\infty A_i(t_i)e^{-r(\theta-T)}d\theta = \frac{A_T}{r + (1-a)\Lambda_i} = \frac{A_T}{r + (1-a)\lambda z_i}, \quad (12)$$

where $A_T$ is productivity at time $T$.

### 5.2. Production and profits

Oligopolist $i$ produces its output $y_i$ from unskilled labour $l_i$ and skilled labour $h_i$ according to the CES function

$$y_i = F(l_i, h_i), \quad F_l = \frac{\partial F}{\partial l_i} > 0, \quad F_h = \frac{\partial F}{\partial h_i} > 0, \quad F_{lh} = \frac{\partial^2 F}{\partial l_i \partial h_i} < 0,$$

$$F_{hh} = \frac{\partial^2 F}{\partial h_i^2} < 0, \quad F_{lh} = \frac{\partial^2 F}{\partial l_i \partial h_i} > 0, \quad \frac{F_l F_h}{F_{lh} F} = \gamma \in (0, 1) \cup (1, \infty), \quad (13)$$

where $\gamma$ is the constant elasticity of factor substitution. It pays the wage $W_i$ for its unskilled labour $l_i$ and the salary $S_i$ for its skilled labour $h_i + z_i$, of which $h_i$ is devoted to production and $z_i$ to R&D. Its profits are equal to sales revenue $p_i y_i$ minus wages $W_i l_i$, salaries $S_i(h_i + z_i)$, and its political contributions $R_o'$. Noting the inverse demand curve (8) and the production function (13), the profit of oligopolist $i$ can then be written as follows:

$$\Pi_i = p_i y_i - W_i l_i - S_i(h_i + z_i) - R_o' = b A_i g_i^{1-1/\epsilon} - W_i l_i - S_i(h_i + z_i) - R_o'$$

$$= b(\psi)A_i F(l_i, h_i)^{1-1/\epsilon} - W_i l_i - S_i(h_i + z_i) - R_o'. \quad (14)$$
5.3. Skilled labour

The oligopolist maximizes its profits by skilled labour devoted to production, \( h_i \), given the wage for skilled labour, \( S_i \), unskilled labour devoted to production, \( l_i \), skilled labour devoted to R&D, \( z_i \), productivity \( A_i \) and the quantity of the composite product, \( \psi \).\(^5\) The first-order condition of this maximization, \( \partial \Pi_i / \partial h_i = 0 \), leads to the inverse demand function of skilled labour [cf. (13)] as follows:

\[
S_i = \left(1 - \frac{1}{\epsilon}\right) \frac{F_h(l_i, h_i)}{F(l_i, h_i)^{1/\epsilon}} b(\psi) A_i. \tag{15}
\]

The salary \( S_i \) adjusts to balance the market for skilled labour, \( h_i + z_i = \varphi \) [cf. (2)]. Given this, (13) and (15), the equilibrium salary becomes a function of unskilled labour \( l_i \), input to R&D, \( z_i \), and the level of demand \( b(\psi) A_i \):

\[
S_i = s(l_i, z_i, \gamma, \epsilon) b(\psi) A_i, \quad s(l_i, z_i, \gamma, \epsilon) = \left(1 - \frac{1}{\epsilon}\right) \frac{F_h(l_i, \varphi - z_i)}{F(l_i, \varphi - z_i)^{1/\epsilon}},
\]

\[
\frac{\partial s}{\partial l_i} = \left(\frac{F_{lh}}{F_h} - \frac{F_l}{\epsilon F}\right) s = \left(\frac{1}{\gamma} - \frac{1}{\epsilon}\right) \frac{F_l}{\epsilon F} s > 0 \iff \frac{1}{\gamma} > \frac{1}{\epsilon} \iff \epsilon > \gamma,
\]

\[
\frac{\partial s}{\partial z_i} = \left(\frac{F_h}{\epsilon F} \frac{F_{hh}}{F_h} \right) s > 0. \tag{16}
\]

Results (16) can be interpreted as follows:

- When more skilled labour \( z_i \) is devoted to R&D, the demand for skilled labour increases. This raises the productivity-adjusted salary \( s = S_i / [b(\psi) A_i] \) for skilled labour, \( \partial s / \partial z_i > 0 \).

- The higher the price elasticity of demand for an oligopolist, \( \epsilon \), the stronger the output effect: an increase of unskilled labour in production raises both output and the input of skilled labour to production. The higher the elasticity of factor substitution, \( \gamma \), the stronger the substitution effect: an increase of unskilled labour in production substitutes for skilled labour at the given level of output \( y_i \). If the

\[\text{The oligopolist employs skilled labour } h_i \text{ for production at stage 6, while the wage for skilled labour, } S_i, \text{ unskilled labour devoted to production, } l_i, \text{ and skilled labour devoted to R&D, } z_i, \text{ are determined at earlier stages (cf. subsection 2).}\]
elasticity of product substitution, $\epsilon$, is greater than that of factor substitution, $\gamma$, then the output effect dominates over the substitution effect. In that case, an increase of unskilled labour in production raises the demand for skilled labour, increasing the productivity-adjusted salary $s$ for skilled labour, $\partial s/\partial l_i > 0$.

5.4. Output and the employment of unskilled labour

To obtain a stationary-state equilibrium where labour inputs ($l_i, z_i$) are constant for all serial numbers $t_i$ of technology over time, it is assumed that oligopolist $i$ and labour union $i$ bargain over the productivity-adjusted wage

$$w_i \equiv W_i/[b(\psi)A_i],$$

where $A_i$ and $b(\psi)$ are the levels of productivity due to past investment in R&D and the composite product $\psi$, correspondingly. Thus, oligopolist $i$ takes $w_i$ as given in its production plans. Noting (2), (16) and (17), the profit of oligopolist $i$, (14), becomes

$$\Pi_i = \pi_i A_i b(\psi) - R_o^i$$

with

$$\pi_i = \pi(l_i, z_i, w_i, \gamma, \epsilon) \equiv F(l_i, h_i)^{1-1/\epsilon} - w_i l_i - s(l_i, z_i, \gamma, \epsilon)(h_i + z_i)$$

$$= F(l_i, \varphi - z_i)^{1-1/\epsilon} - w_i l_i - \varphi s(l_i, z_i, \gamma, \epsilon).$$

(18)

Because the system has a stationary state solution where inputs ($l_i, z_i$) are constants, the optimum can be solved by choosing ($l_i, z_i$) from the class of constant controls. Oligopolist $i$ maximizes the expected discounted value of the flow of its profits (18) starting at time $\theta = T$, $E \int_T^\infty \Pi_i e^{-r(\theta - T)} d\theta$, by inputs ($l_i, z_i$), subject to the full-employment constraints (2) and technological change (cf. subsection 5.1), given the composite product $\psi$, the productivity-adjusted wage $w_i$ and political contributions $R_o^i$. In the stationary-state equilibrium, the productivity-adjusted profit (net of political contributions $R_o^i$), $\pi_i$, is constant for all serial numbers $t_i$ of technology [cf. (18)]. Given this, (12) and (18), the utility of oligopolist $i$ is

$$E \int_T^\infty \Pi_i e^{-r(\theta - T)} d\theta = b(\psi)\pi_i E \int_T^\infty A_i(t_i) e^{-r(\theta - T)} d\theta - R_o^i \int_T^\infty e^{-r(\theta - T)} d\theta$$

6 The use of stochastic dynamic programming (cf. Dixit and Pindyck 1994) leads to the same results. Detailed calculations of this will be provided to the reader on request.
\[
\pi_i b(\psi) A_{iT} \frac{R^i_t}{r} = b(\psi) A_{iT} \frac{\pi(l_i, z_i, w_i, \gamma, \epsilon)}{r} - \frac{R^i_t}{r}.
\]

Maximizing (19) by \((l_i, z_i)\) and noting (18), one obtains the value function of oligopolist \(i\) as follows:

\[
\mathcal{P}(w_i, c, \gamma, \epsilon, \lambda, R^o) = \max_{l_i, h_i, z_i} E \int_T^\infty \Pi_t e^{-r(t-T)} d\theta = b(\psi) A_{iT} \frac{\pi(l^*_i, z^*_i, w_i, \gamma, \epsilon)}{r} - \frac{R^o_t}{r} \quad \text{with} \quad \frac{\partial \mathcal{P}}{\partial w_i} = - \frac{l_i b(\psi) A_{iT}}{r + (1 - a) \lambda z^*_i} < 0,
\]

where the oligopolist’s optimal inputs \((l^*_i, z^*_i)\) are taken as given. Maximizing (19) by \((l_i, z_i)\) leads also to the oligopolist’s response functions (cf. C):

\[
l_i = \tilde{l}(w_i, \gamma, \epsilon, \lambda), \quad z_i = \tilde{z}(w_i, \gamma, \epsilon, \lambda), \quad \tilde{z}_w = \frac{\partial \tilde{z}}{\partial w_i} > 0 \iff \epsilon > \gamma,
\]

\[
\tilde{l}_w = \frac{\partial \tilde{l}}{\partial w_i} < 0, \quad y_i = \tilde{y}(w_i, \gamma, \epsilon, \lambda) = F(\tilde{l}, \tilde{h}), \quad \tilde{y}_w = \frac{\partial \tilde{y}}{\partial w_i} < 0 \quad \text{for} \quad \epsilon > \gamma.
\]

(21)

The results (21) can be interpreted as follows:

- An increase of the productivity-adjusted wage \(w_i\) for unskilled labour decreases the demand for unskilled labour, \(\tilde{l}_w < 0\).

- The higher the price elasticity of demand for an oligopolist, \(\epsilon\), the stronger the output effect: an increase in the productivity-adjusted wage \(w_i\) for unskilled labour lowers both output and the demand for skilled labour in production, \(\tilde{h}\). The higher the elasticity of factor substitution, \(\gamma\), the stronger the substitution effect: an increase in the productivity-adjusted wage \(w_i\) for unskilled labour raises the demand for skilled labour in production, \(\tilde{h}\). If the elasticity of product substitution, \(\epsilon\), is greater than that of factor substitution, \(\gamma\), then the output effect dominates over the substitution effect and an increase in the productivity-adjusted wage \(w_i\) lowers the demand for skilled labour in production. Because skilled labour is fully employed, this generates a transfer of skilled labour from production to R&D, \(\tilde{z}_w > 0\).
fall-back income is the discounted value of the flow of these contributions $-R_u$ but pays its political contributions also forestall production alone. Because oligopolist $i$ attempts to maximize its value function (20) and labour union $i$ its value function (23) by the productivity-adjusted wage $w_i$ in an alternating-offers game, given the quantity of the composite high-tech good, $\psi$, the interest rate $r$ and political contributions $(R^u, R^o)$. Both of them can also forestall production alone. Because oligopolist $i$ (union $i$) earns nothing but pays its political contributions $R^o$ ($R^u$) in the case of no production, its fall-back income is the discounted value of the flow of these contributions $-R^o/r$ ($-R^u/r$). The outcome of the alternating-offers game is obtained by maximizing the Generalized Nash Product (GNP) of the utilities of the parties, (20) and (23),

$$\Theta(w_i, c, \gamma, \epsilon, \lambda, R^u) = \alpha_i \log \left[ \mathcal{W}(w_i, c, \gamma, \epsilon, \lambda, R^u) - (-R^u/r) \right]$$

$$+ (1 - \alpha_i) \log \left[ P(w_i, c, \gamma, \epsilon, \lambda, R^u) - (-R^o/r) \right]$$

$$= \log A_{iT} + (1/\epsilon - 1)b(\psi) + \alpha_i \left\{ \log v(w_i, \gamma, \epsilon, \lambda) - \log[r + (1 - a)\lambda z] \right\}$$

$$+ (1 - \alpha_i) \left\{ \log \pi(l^*_i, z^*_i, w_i, \gamma, \epsilon) - \log[r + (1 - a)\lambda z^*_i] \right\}$$

\( (24) \)
by the productivity-adjusted wage $w_i$, where $\alpha_i \in [0, 1]$ is the relative bargaining power of union $i$. On the assumption that the equilibrium is unique, this maximization implies that the productivity-adjusted wage $w_i$ is an increasing function of relative union bargaining power $\alpha_i$ (cf. D):

$$w_i = w(\alpha_i, \gamma, \epsilon, \lambda), \quad \partial w_i / \partial \alpha_i > 0. \quad (25)$$

In this document, the skilled and unskilled workers are assumed to belong to the same labour union, for simplicity. In E, it is shown that skilled worker can have economic incentives to stay as a union member, although his/her salary is competitively determined.

6. Lobbies and policy makers

Employer lobby $k$ represents the oligopolists $i \in B_k$ and labour lobby $k$ the workers of these in jurisdiction $k$. It is assumed that relative union bargaining power $\alpha_i$ and political contributions ($R_{ku}^i, R_{ko}^i$) are uniform throughout the industries $i \in B_k$ of the same jurisdiction $k$:

$$\alpha_i = \beta_k, \quad R_{u}^i = R_{ku}^i \text{ and } R_{o}^i = R_{ko}^i \text{ for } i \in B_k. \quad (26)$$

Given (21) and (25), this unifies the productivity-adjusted wages throughout that jurisdiction

$$w_i = \varpi_k \text{ for } i \in B_k. \quad (27)$$

Define the average productivity in jurisdiction $k$ by [cf. (1)]

$$\tilde{A}_k = \int_{i \in B_k} A_{iT} di / \int_{i \in B_k} di = n \int_{i \in B_k} A_{iT} di, \quad (28)$$

the average productivity in the other jurisdictions $\zeta \neq k$ by the vector

$$\tilde{A}_{-k} \equiv \{\tilde{A}_\zeta | \zeta \neq k\}, \quad (29)$$

and the productivity-adjusted wages $\varpi_\zeta$ in jurisdictions $\zeta \neq k$ by the vector

$$\varpi_{-k} \equiv \{\varpi_\zeta | \zeta \neq k\}. \quad (30)$$
Given (5), (21), (27), (28), (29) and (30), the composite product $\psi$ is determined by the productivity-adjusted wages $(\bar{w}_k, \bar{w}_{-k})$ and the levels of productivity, $(\bar{A}_k, \bar{A}_{-k})$, as follows (cf. F):

$$
\psi = \tilde{\psi}(\bar{w}_k, \bar{w}_{-k}, \bar{A}_k, \bar{A}_{-k}, n, \gamma, \epsilon, \lambda),
\tilde{\psi}_w = \frac{\partial \psi}{\partial \bar{w}_k},
\tilde{\psi}_{\bar{w}_k} = \frac{\partial \psi}{\partial \bar{w}_k},
\tilde{\psi}_{\bar{w}_{-k}} = \frac{\partial \psi}{\partial \bar{w}_{-k}}.
$$

The utility functions of employer lobby $k$ and labour lobby $k$, $F_k$ and $U_k$, are obtained by plugging productivity-adjusted wages (27) and the composite high-tech good (31) into the utility functions of oligopolist $i$ and labour union $i$, (20) and (23):

$$
F_k(\bar{w}_k, \bar{w}_{-k}, n, \gamma, \epsilon, \lambda) = \mathcal{P}(\bar{w}_k, \tilde{\psi}, \gamma, \epsilon, \lambda, \lambda_k),
U_k(\bar{w}_k, \bar{w}_{-k}, n, \gamma, \epsilon, \lambda) = \mathcal{W}(\bar{w}_k, \tilde{\psi}, \gamma, \epsilon, \lambda, \lambda_k).
$$

The contribution schedule of the labour (employer) lobby $R_{ku}$ ($R_{ko}$) depends on the arguments $(\bar{w}_k, \bar{w}_{-k}, n, \gamma, \epsilon, \lambda)$ of its utility function (32) ((33)):

$$
R_{ku}(\bar{w}_k, \bar{w}_{-k}, n, \gamma, \epsilon, \lambda),
R_{ko}(\bar{w}_k, \bar{w}_{-k}, n, \gamma, \epsilon, \lambda).
$$

Policy maker $k$ collects the flow of the political contributions $R_{ko} + R_{ku}$ from all oligopolists and labour unions in jurisdiction $k \in [0, n]$, $\int_{i \in B_k} (R_{ko} + R_{ku})di$. It maximizes the present value of this flow of income [cf. (1), (27) and (34)]:

$$
G_k(\bar{w}_k, \bar{w}_{-k}, n, \gamma, \epsilon, \lambda) = E \int_0^{\infty} \left[ \int_{i \in B_k} (R^i_{ko} + R^i_{ku})di \right] e^{-r(\theta - T)}d\theta
= \frac{R_{ko} + R_{ku}}{rn} = \frac{1}{rn} \left[ R_{ko}(\bar{w}_k, \bar{w}_{-k}, n, \gamma, \epsilon, \lambda) + R_{ku}(\bar{w}_k, \bar{w}_{-k}, n, \gamma, \epsilon, \lambda) \right].
$$

7. Political Equilibrium

Plugging the functions (21) and the conditions (27) into the sector-specific full-employment constraints (2) yields the full-employment constraint for jurisdiction $k$:

$$
\bar{I}(\bar{w}_k, \gamma, \epsilon, \lambda) \leq 1 - \varphi.
$$
Employer lobby $k$ and labour lobby $k$ influence policy maker $k$ over union bargaining power $\beta_k$. These three agents take the productivity-adjusted wages elsewhere, $\overline{w}_{-k}$, as given and observe the full-employment constraint (36). Because there is a one-to-one correspondence from $\beta_k$ to $\overline{w}_k$ through (25), (26) and (27), it is equivalent to assume that the lobbies influence policy maker $k$ over the productivity-adjusted wage $\overline{w}_k$ subject to (36), given $\overline{w}_{-k}$.

In each jurisdiction $k$, there is a common agency game that yields the following equilibrium conditions (cf. G). First, the policy maker has no incentives to depart from the policy $\overline{w}_k$, i.e. the policy $\overline{w}_k$ maximizes its welfare (35):

$$\overline{w}_k = \arg \max_{\overline{w}_k \text{ s.t. } (36)} G_k(\overline{w}_k, \overline{w}_{-k}, n, \gamma, \epsilon, \lambda). \quad (37)$$

Second, the lobbies have no incentives to depart from the policy $\overline{w}_k$, i.e. the employer (labour) lobby cannot have a feasible strategy $R_{ko}$ ($R_{ku}$) that yields it higher utility (32) ((33)) than in equilibrium, given the policy maker’s expected policy:

$$\overline{w}_k = \arg \max_{\overline{w}_k \text{ s.t. } (36)} F_k(\overline{w}_k, \overline{w}_{-k}, n, \gamma, \epsilon, R_{ko}(\overline{w}_k, \overline{w}_{-k}, n, \gamma, \epsilon, \lambda)),$$

$$\overline{w}_k = \arg \max_{\overline{w}_k \text{ s.t. } (36)} U_k(\overline{w}_k, \overline{w}_{-k}, n, \gamma, \epsilon, R_{ku}(\overline{w}_k, \overline{w}_{-k}, n, \gamma, \epsilon, \lambda)). \quad (38)$$

Considering the economy in the vicinity of the point where average productivity $\tilde{A}_k$ is initially the same for all jurisdictions $k \in [0, n]$, and noting (21), (31), (32), (33), (35) and (38), one can transform the policy maker’s equilibrium conditions (37) into the following form (cf. H):

$$\tilde{l} < 1 - \varphi \iff \Delta = 0 \text{ with } \frac{\partial \Delta}{\partial \overline{w}_k} < 0, \quad \tilde{l} = 1 - \varphi \iff \Delta < 0, \quad (39)$$

where

$$\Delta = \left[ 1 - \frac{1}{\epsilon} + \frac{b'(\tilde{\psi}) \tilde{\psi}}{b(\tilde{\psi}) n} \frac{\overline{y}_w}{\overline{y}} + \frac{(a - 1) \lambda \overline{z}_w}{r + (1 - a) \lambda \overline{z}} \right]. \quad (40)$$

The result (39) and (40) can be explained as follows. The term

$$\frac{(a - 1) \lambda \overline{z}_w}{r + (1 - a) \lambda \overline{z}} \quad (41)$$
characterizes the *growth effect* of the productivity-adjusted wage $\bar{w}_k$. If the output effect dominates over the substitution effect, $\epsilon > \gamma$, then the growth effect (41) is positive [cf. (21)]. If vice versa $\epsilon < \gamma$, then (41) is negative.

If the productivity-adjusted wage $\bar{w}_k$ in jurisdiction $k$ increases, then the sales revenue $p_i y_i$ of any industry $i \in B_k$ falls [cf. (8), (21) and (31)]:

$$\frac{1}{p_i y_i} \frac{\partial (p_i y_i)}{\partial \bar{w}_k} = \frac{\partial \log(p_i y_i)}{\partial \bar{w}_k} = \frac{\partial \log[b(\bar{w}) \bar{y}^{1-1/\epsilon}]}{\partial \bar{w}_k} = \left(1 - \frac{1}{\epsilon}\right) \frac{\bar{y}_w}{\bar{y}} + \frac{b' \bar{\psi}_w}{b \bar{\psi}}$$

$$= \left[1 - \frac{1}{\epsilon} + \frac{b' (\bar{\psi} \bar{\psi})}{b(\bar{\psi}) \bar{\psi}}\right] \frac{\bar{y}_w}{\bar{y}} < 0. \quad (42)$$

If this *level effect* dominates over the growth effect (41), i.e. $\Delta < 0$, then the productivity-adjusted wage $\bar{w}_k$ is decreased, until the full-employment $\tilde{l}(\bar{w}_k, \gamma, \epsilon, \lambda) = 1 - \varphi$ is attained [cf. (36)], and the political process ends up with labour market deregulation. Otherwise, an increase in $\bar{w}_k$ raises the welfare of some lobby, which creates incentives for labour market regulation.

Because, by (8) and (42),

$$\frac{\partial}{\partial (\frac{1}{n})} \left[\frac{1}{p_i y_i} \frac{\partial (p_i y_i)}{\partial \bar{w}_k}\right] = \frac{\psi b'(\bar{\psi})}{b(\bar{\psi})} \frac{\bar{y}_w}{\bar{y}} > 0,$$

the fall in the sales revenue, (42), is the steeper, the more the industries $i \in B_k$ face competition from the industries $i \notin B_k$ outside the jurisdiction $k$, i.e. the smaller the relative proportion $\frac{1}{n}$ of that jurisdiction $k$.

8. Labour market integration

Without R&D, $\lambda \to 0$ [cf. (9)], there is no growth effect (41) and $\Delta < 0$ [cf. (40)]. From (39) it then follows that $\tilde{l} = 1 - \varphi$. In other words:

**Proposition 1.** The existence of R&D (i.e. $\lambda > 0$) enables an equilibrium with labour market regulation and unemployment $\tilde{l} < 1 - \varphi$.

7From (8) it follows that that if there is no traditional sector, $\nu \to 1$ [cf. (6)], then there is no level effect (42) and no equilibrium with full labour market integration, $n = 1$. That is why the traditional sector is introduced into the model.
Without R&D, all skilled labour is devoted to production. In that case, both lobbies attain their highest level of welfare in the presence of full employment, having no incentives to lobby for labour market regulation.

If \( \epsilon \leq \gamma \), then, from (21), (39) and (40), it follows that \( \tilde{z}_w \leq 0, \Delta < 0 \) and \( \tilde{l} = 1 - \varphi \). Thus, \( \tilde{l} < 1 - \varphi \) is possible only if \( \epsilon > \gamma \). In other words:

**Proposition 2.** Labour market regulation \( \tilde{l} < 1 - \varphi \) is possible only if the elasticity of product substitution, \( \epsilon \), exceeds that of factor substitution, \( \gamma \).

If the output effect dominates over the substitution effect, \( \epsilon > \gamma \), then the growth effect is positive and can outweigh the negative level effect. Otherwise, a decrease in the productivity-adjusted wage \( \varpi_k \) benefits the lobbies and the political process ends up with labour market deregulation.

From (8) and (40) it follows that

\[
\frac{\partial \Delta}{\partial \left( \frac{1}{n} \right)} = \frac{\tilde{\psi} b'(\tilde{\psi}) \tilde{y}_w}{b(\tilde{\psi}) y} > 0.
\]

Given (39), this implies \( \Delta < 0 \) and \( \tilde{l} = 1 - \varphi \) for low values of \( \frac{1}{n} \), and \( \Delta = 0 \) and \( \tilde{l} < 1 - \varphi \) for high values of \( \frac{1}{n} \). In other worlds:

**Proposition 3.** Assume that there exists a positive growth effect, \( \epsilon > \gamma \). In that case, the labour markets are deregulated \( (l = 1 - \varphi) \) for small, and regulated \( (l < 1 - \varphi) \) for high relative proportions \( \frac{1}{n} \) of a single jurisdiction.

If competition from outside the jurisdiction is weak (i.e. if \( \frac{1}{n} \) is close to one), then the growth effect (41) outweighs the level effect (42) and lobbying leads to labour market regulation. Otherwise, the labour markets are deregulated.

Differentiating the first-order condition \( \Delta = 0 \) [cf. (39)] and noting (39) and (43), one obtains

\[
\frac{d\varpi_k}{d\left( \frac{1}{n} \right)} = -\frac{\partial \Delta}{\partial \left( \frac{1}{n} \right)} / \frac{\partial \Delta}{\partial \varpi_k} > 0.
\]

From (11), (21) and (27) it then follows that

\[
\frac{\partial g_k}{\partial \varpi_k} = \frac{\partial g_k}{\partial w_i} = (\log a) \lambda \frac{\partial \tilde{z}}{\partial w_i} > 0 \iff \frac{\partial \tilde{z}}{\partial w_i} > 0 \iff \epsilon > \gamma.
\]
According to Proposition 2, inequality $\epsilon > \gamma$ holds true for $l < 1 - \varphi$. From this, (44) and (45) it follows that when $l < 1 - \varphi$, both $\varpi_k$ (for all $k$) and $g_i$ (for all $i$) increase with an increase in $\frac{1}{n}$. In other words:

**Proposition 4.** If the labour markets are initially regulated, $l < 1 - \varphi$, then an increase in the size of jurisdictions, $\frac{1}{n}$, raises both the productivity-adjusted wages ($\varpi_k$ for all $k$) and the productivity growth rates ($g_i$ for all $i$).

If the labour markets are initially regulated, then the growth effect is positive. The expansion of jurisdictions weakens the negative level effect, for there will be less competition from outside the jurisdiction. This strengthens the incentives to lobby for labour market regulation, promoting R&D and productivity growth.

9. Conclusions

In the economy under consideration, oligopolists employ unskilled and skilled labour, produce high-tech goods and perform research and development (R&D) to escape labour costs. Labour is unionized, but skilled labour is fully employed. There are many jurisdictions, each of them having a self-interested policy maker that can regulate (deregulate) the labour markets by supporting labour unions (employers). The workers’ and employers’ interest groups lobby the policy makers. The main results are the following.

Labour market regulation raises the wages for unskilled labour. This affects productivity growth through two channels. On the one hand, the oligopolists increase their output price and decrease their output (the output effect). With a lower level of output, there are less skilled labour in production. On the other hand, the oligopolists replace unskilled by skilled labour at the given level of output (the substitution effect). The higher the elasticity of substitution between goods, the higher the price elasticity of demand for an oligopolist and the stronger the output effect. The higher the elasticity of factor substitution, the stronger the substitution effect. If the elasticity of product substitution is higher than that of factor substitution, then the output effect dominates over the substitution effect: labour market regulation decreases skilled labour devoted to production. Because skilled labour is fully employed, more skilled labour is devoted to productivity-enhancing R&D. If this positive growth effect outweighs the negative effect of wage increases on income, then there are incentives to lobby for labour market regulation. Otherwise, the labour markets are deregulated.
If jurisdictions expand, they face less competition from elsewhere in the economy. This weakens the fall of income due to wage increases, strengthening the incentives to lobby for labour market regulation. If the labour markets are well integrated (i.e. if jurisdictions are large), then the growth effect outweighs the competition effect and there is labour market regulation. On the other hand, if the labour markets are incompletely integrated (i.e. if jurisdictions are small), then they are deregulated. In the presence of labour market regulation, the integration of the labour markets (i.e. the increase of the size of jurisdictions) strengthens the growth effect even further, increasing wages and speeding up productivity growth.

While a great deal of caution should be exercised when a highly stylized model is used to explain the relationship of productivity growth, collective bargaining and lobbying, the following judgement nevertheless seems to be justified. The observed tendency to labour market deregulation (Acemoglu et al. 2001, Dumont et al. 2012) can result from labour market policy being left at the local level. Once labour market policy is established and the workers’ and employers’ interest groups are organized at the level of the otherwise integrated economy, labour market regulation can reappear.

### A. Equation (8)

From (5) it follows that $\Psi^{1-1/\epsilon} = \int_0^1 A_i \zeta y_i^{1-1/\epsilon} d\zeta$. Differentiating this with respect to $y_i$ and noting (5) yield

$$\frac{\partial \Psi}{\partial y_i} = A_i y_i^{-1/\epsilon} \Psi^{1/\epsilon} = A_i y_i^{-1/\epsilon} \Psi^{1/\epsilon}. \quad (46)$$

Maximizing the profit $P_c - \int_0^1 p_i y_i di$ by $y_i$ subject to (5) and (6), noting (4) and (46), and holding $P$ and $p_i$ for $i \in [0, 1]$ constant, one obtains

$$p_i = P \frac{\partial \Phi}{\partial \psi} \frac{\partial \Psi}{\partial y_i} = \frac{1}{c} \frac{\partial \Phi}{\partial \psi} A_i y_i^{-1/\epsilon} \Psi^{1/\epsilon} = \frac{1}{c} \frac{\partial \Phi}{\partial \psi} A_i y_i^{-1/\epsilon} \Psi^{1/\epsilon} = b(\psi) A_i y_i^{-1/\epsilon}$$

with $b(\psi) = \nu (\Phi / \psi)^{1/\delta - 1} \Psi^{1/\epsilon - 1}$ and

$$\frac{\psi b'(\psi)}{b(\psi)} = \left( \frac{1}{\delta} - 1 \right) \left( \psi \frac{\partial \Phi}{\partial \psi} - \frac{1}{\epsilon} - 1 \right) + \left( 1 \frac{\mu}{\Phi} \frac{\partial \Phi}{\partial \mu} + \frac{1}{-1} - 1 > 1 - 1. \right.$$

From this and (6) it is easy to see that

$$\lim_{\nu \to 1} \frac{\psi b'(\psi)}{b(\psi)} = \lim_{\Phi \to \psi} \frac{\psi b'(\psi)}{b(\psi)} = \lim_{\nu \to 1} \frac{\psi b'(\psi)}{b(\psi)} = \frac{1}{\epsilon} - 1,$$
\[
\frac{\psi b'(\psi)}{b(\psi)} < 0 \quad \text{for the assumption} \quad \frac{\mu}{\Phi} \frac{\partial \Phi}{\partial \mu} < \left( 1 - \frac{1}{\epsilon} \right) / \left( 1 - \frac{1}{\delta} \right).
\]

B. Function (12)

Define the expected value
\[
\Omega(t_i) = E \int_T^\infty A_i(t_i)e^{-r(\theta-T)}d\theta. \tag{47}
\]

Given technological change (cf. subsection 5.1), the Bellman equation is (cf. Dixit and Pindyck 1994)
\[
r \Omega(t_i) = A_i(t_i) + \Lambda_i \left[ \Omega(t_i + 1) - \Omega(t_i) \right], \tag{48}
\]
where \(r \Omega(t_i)\) is the revenue from assets \(\Omega(t_i)\) at the market interest rate \(r\), \(A_i(t_i)\) current income from assets \(\Omega(t_i)\) and \(\Lambda_i \left[ \Omega(t_i + 1) - \Omega(t_i) \right]\) the expected increase of the value of assets \(\Omega(t_i)\). Let us try the solution
\[
\Omega(t_i) = \frac{A_i(t_i)}{\omega}, \tag{49}
\]
in which the discount factor \(\omega > 0\) is independent of \(t_i\). Inserting (49) into the Bellman equation (48) yields
\[
r = \frac{A_i(t_i)}{\Omega(t_i)} + \Lambda_i \left[ \frac{\Omega(t_i + 1) - \Omega(t_i)}{\Omega(t_i)} \right] = \omega + (a - 1)\Lambda_i. \tag{50}
\]
Solving for \(\omega\) from (50), inserting this into (49), and noting (47), one obtains
\[
E \int_T^\infty A_i(t_i)e^{-r(\theta-T)}d\theta = \Omega(t_i) = \frac{A_i T}{r + (1 - a)\Lambda_i},
\]
where \(A_i T\) is productivity at time \(T\).

C. Functions (21)

Noting (18) and (19), it holds true that
\[
(l_i, z_i) = \arg \max_{l_i, z_i} E \int_T^\infty \Pi_i e^{-r(\theta-T)}d\theta = \arg \max_{l_i, z_i} \pi_i \frac{r + (1 - a)\lambda z_i}{r + (1 - a)\lambda z_i}
\]
\[
= \arg \max_{l_i, z_i} \{ \log \pi_i - \log[r + (1 - a)\lambda z_i] \} = \arg \max_{l_i, z_i} \Xi(l_i, z_i, w_i, \gamma, \epsilon, \lambda)
\]
with \(\Xi(l_i, z_i, w_i, \gamma, \epsilon, \lambda) \equiv \log \pi(l_i, z_i, w_i, \gamma, \epsilon) - \log[r + (1 - a)\lambda z_i].\)
Given (18), this leads to the first-order conditions
\[
\frac{\partial \Xi}{\partial l_i} = \frac{1}{\pi} \frac{\partial \pi}{\partial l_i} = 0, \quad \frac{\partial \Xi}{\partial z_i} = \frac{1}{\pi} \frac{\partial \pi}{\partial z_i} + \frac{(a - 1)\lambda}{r + (1 - a)\lambda z_i} = 0. \tag{51}
\]

From (16) and (18) it follows that
\[
\frac{\partial \pi}{\partial l_i} = \left(1 - \frac{1}{\epsilon}\right) \frac{F_i}{F^{1/\epsilon}} - \varphi \frac{\partial s}{\partial l_i} - w_i = \left(1 - \frac{1}{\epsilon}\right) \frac{F_i}{F^{1/\epsilon}} - \left(\frac{1}{\gamma} - \frac{1}{\epsilon}\right) \varphi \frac{F_h}{\epsilon F} F^{1/\epsilon} - w_i = \left(1 - \frac{1}{\epsilon}\right) \frac{F_i}{F^{1/\epsilon}} \left[1 - \left(\frac{1}{\gamma} - \frac{1}{\epsilon}\right) \varphi \frac{F_h}{\epsilon F} (l_i, \varphi - z_i) \right] \frac{F_i(l_i, \varphi - z_i)}{F(l_i, \varphi - z_i)^{1/\epsilon}} - w_i = 0. \tag{52}
\]

Given this and (13), one obtains
\[
\left[1 - \left(\frac{1}{\gamma} - \frac{1}{\epsilon}\right) \varphi \frac{F_h}{\epsilon F}\right]^{-1} = \left(1 - \frac{1}{\epsilon}\right) \frac{F_i}{F^{1/\epsilon}} \frac{1}{w_i} \quad \text{and}
\]
\[
\frac{\partial^2 \pi}{\partial l_i \partial z_i} = -w_i \left\{ \left[1 - \left(\frac{1}{\gamma} - \frac{1}{\epsilon}\right) \varphi \frac{F_h}{\epsilon F}\right]^{-1} \left(\frac{1}{\epsilon} - \frac{1}{\gamma}\right) \frac{\varphi}{\epsilon F} \left(F_{hh} - \frac{F_h^2}{\epsilon F}\right) + \frac{F_{hh}}{\gamma F} - \frac{F_h}{\epsilon F}\right\} + \frac{F_h}{F} \left(1 - \frac{1}{\epsilon}\right) \frac{F_i}{F^{1/\epsilon} w_i} \left(\frac{1}{\epsilon} - \frac{1}{\gamma}\right) \varphi \frac{F_h}{\epsilon F} \left(\frac{F_{hh}}{F_h} - \frac{F_h^2}{\epsilon F}\right) + \frac{F_{hh}}{\gamma F} - \frac{F_h}{\epsilon F}\right\} = -w_i \left\{ \left(1 - \frac{1}{\epsilon}\right) \frac{F_i}{F^{1/\epsilon} w_i} \left(\frac{1}{\epsilon} - \frac{1}{\gamma}\right) \varphi \frac{F_h}{\epsilon F} \left(\frac{F_{hh}}{F_h} - \frac{F_h^2}{\epsilon F}\right) + \frac{F_{hh}}{\gamma F} - \frac{F_h}{\epsilon F}\right\} = \left(1 - \frac{1}{\epsilon}\right) \frac{F_i}{F^{1/\epsilon} w_i} \left[1 + \left(\frac{1}{\epsilon} - \frac{1}{\gamma}\right) \frac{F_i}{F^{1/\epsilon} w_i} \left(\frac{F_{hh}}{F_h} - \frac{F_h^2}{\epsilon F}\right) + \frac{F_{hh}}{\gamma F} - \frac{F_h}{\epsilon F}\right\} < 0
\]
\[
\iff \epsilon > \gamma. \tag{53}
\]

If the oligopolist’s equilibrium is unique, the function \( \pi \) must be strictly concave. This implies
\[
\frac{\partial^2 \Xi}{\partial z_i^2} < 0, \quad \mathcal{J} = \left| \begin{array}{ccc} \frac{\partial^2 \Xi}{\partial l_i \partial l_i} & \frac{\partial^2 \Xi}{\partial l_i \partial z_i} & \frac{\partial^2 \Xi}{\partial z_i \partial z_i} \\ \frac{\partial^2 \Xi}{\partial l_i \partial l_i} & \frac{\partial^2 \Xi}{\partial l_i \partial z_i} & \frac{\partial^2 \Xi}{\partial z_i \partial z_i} \end{array} \right| > 0. \tag{54}
\]

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Differentiating the equations (51) totally yields
\[
\begin{bmatrix}
\frac{\partial^2 \Xi}{\partial l_i^2} & \frac{\partial^2 \Xi}{\partial l_i \partial z_i} & \frac{\partial^2 \Xi}{\partial z_i^2}
\end{bmatrix}
\begin{bmatrix}
dl_i \\
dh_i
\end{bmatrix}
+ \begin{bmatrix}
-1 \\
0
\end{bmatrix} dw_i = 0.
\]

Noting (51), (53) and (54), one obtains the functions
\[
l_i = \tilde{l}(w_i, \gamma, \epsilon, \lambda), \quad z_i = \tilde{z}(w_i, \gamma, \epsilon, \lambda), \quad \frac{\partial \tilde{l}}{\partial w_i} = \frac{1}{J} \frac{\partial^2 \Xi}{\partial l_i \partial z_i^2} < 0,
\]
\[
\frac{\partial \tilde{z}}{\partial w_i} = \frac{1}{\mathcal{J}} \frac{\partial^2 \Xi}{\partial l_i \partial z_i} < 0 \quad \Leftrightarrow \quad \frac{\partial^2 \pi}{\partial l_i \partial z_i} < 0 \quad \Leftrightarrow \quad \epsilon > \gamma. \quad (55)
\]

D. Results (25)

It is equivalent to choose \(w_i\) to maximize
\[
\Theta / \alpha_i = [\log A_{iT} + (1/\epsilon - 1) \log c] / \alpha_i + \log v(w_i, \gamma, \epsilon, \lambda) - \log [r + (1 - a) \lambda \tilde{z}]
+ (1/\alpha_i - 1) \\{\log \pi(l_i^*, z_i^*, w_i, \gamma, \epsilon) - \log [r + (1 - a) \lambda z_i^*]\},
\]
where \((l_i^*, z_i^*)\) must be taken as constants. Noting (18) and (21), one obtains the first-order condition for this maximization as follows:
\[
\frac{1}{\alpha_i} \frac{\partial \Theta}{\partial w_i} = \frac{1}{v} \frac{\partial v}{\partial w_i} + \frac{(a - 1) \lambda \tilde{z}}{r + (1 - a) \lambda \tilde{z}} + \left( \frac{1}{\alpha_i} - 1 \right) \frac{1}{\pi} \frac{\partial \pi}{\partial w_i}
= \frac{1}{v} \frac{\partial v}{\partial w_i} + \frac{(a - 1) \lambda \tilde{z}}{r + (1 - a) \lambda \tilde{z}} - \left( \frac{1}{\alpha_i} - 1 \right) \frac{1}{\pi} l_i = 0.
\]

From this it follows that
\[
\frac{\partial}{\partial \alpha_i} \left( \frac{1}{\alpha_i} \frac{\partial \Theta}{\partial w_i} \right) = \frac{1}{\alpha_i^2} \frac{l_i}{\pi} > 0. \quad (56)
\]

On the assumption that the equilibrium is unique, the second-order condition
\[
\frac{1}{\alpha_i} \frac{\partial^2 \Theta}{\partial w_i^2} < 0
\]
holds true. Noting this and (56), one obtains
\[
w_i = w(\alpha_i, \gamma, \epsilon, \lambda), \quad \frac{\partial w_i}{\partial \alpha_i} = -\frac{\partial}{\partial \alpha} \left( \frac{1}{\alpha_i} \frac{\partial \Theta}{\partial w_i} \right) / \left( \frac{1}{\alpha} \frac{\partial^2 \Theta}{\partial w_i^2} \right) > 0.
\]
E. A skilled worker’s incentives to belong to the union

Because skilled labour in fully employed, the expected present value of the flow of salaries for a skilled worker is given by [cf. (12) and (16)]

\[
E \int_{T}^{\infty} S_i e^{-r(\theta-T)} d\theta = b(\psi)s(l_i, z_i, \gamma, \epsilon)E \int_{T}^{\infty} A_i(t_i)e^{-r(\theta-T)} d\theta
\]

\[
= b(\psi)A_i T s(l_i, z_i, \gamma, \epsilon) \frac{r}{r + (1 - a)\lambda z},
\]

(57)

The effect of relative union bargaining power \( \alpha_i \) on a skilled worker’s welfare (57) can be calculated by (16), (21) and (25) as follows:

\[
\frac{\partial}{\partial \alpha_i} \log E \int_{T}^{\infty} S_i e^{-r(\theta-T)} d\theta
\]

\[
= \left\{ \frac{1}{s_i} \frac{\partial s}{\partial l_i} \frac{\partial l_i}{\partial w_i} + \left[ \frac{1}{s_i} \frac{\partial s}{\partial z_i} + \frac{(a - 1)\lambda}{r + (1 - a)\lambda z} \frac{\partial z_i}{\partial w_i} \right] \frac{\partial w}{\partial \alpha_i} \right\} \text{ for } \epsilon > \gamma.
\]

If the output effect dominates over the substitution effect, \( \epsilon > \gamma \), then an increase of the productivity-adjusted wage \( w_i \) for unskilled labour has two opposite effects on a skilled worker’s welfare:

- It increases the demand for skilled labour in R&D, raising both the level and the expected growth of the salary \( S_i \).
- It decreases the demand for unskilled labour in production, lowering the salary \( S_i \).

If the former effect dominates over the latter, then an increase in relative union bargaining power \( \alpha_i \) benefits a skilled worker. This shows that a skilled worker can have incentives to belong to the same labour union together with unskilled workers, although his/her salary is competitively determined.

F. Function (31)

Noting (1), (5), (21), (28) and (27), total consumption is determined as:

\[
\psi = \tilde{\psi}(\varpi_k, \varpi_{-k}, \tilde{A}_k, \tilde{A}_{-k}, n, \gamma, \epsilon, \lambda) = \left[ \int_{0}^{n} \left( \int_{i \in B_k} A_i y_i^{1/\epsilon} di \right) dk \right]^{\epsilon/(\epsilon - 1)}
\]

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This implies

\[
\frac{\psi_{\overline{w}_k}}{\psi} = \frac{\partial \log \tilde{\psi}}{\partial \overline{w}_k} = \frac{\epsilon}{\epsilon - 1} \frac{\partial}{\partial \overline{w}_k} \log \int_0^n \tilde{y}(\overline{w}_\zeta, \gamma, \epsilon, \lambda) \frac{1}{\epsilon - 1} \tilde{A}_\zeta d\zeta
\]

\[
= \frac{\tilde{y}(\overline{w}_k, \gamma, \epsilon, \lambda)^{-1/\epsilon} \tilde{A}_k}{\int_0^n \tilde{y}(\overline{w}_\zeta, \gamma, \epsilon, \lambda)^{-1/\epsilon} \tilde{A}_\zeta d\zeta} \tilde{y}_w(\overline{w}_k, \gamma, \epsilon, \lambda) < 0 \, \text{ with}
\]

\[
\frac{\psi_{\overline{w}_k}}{\psi} \bigg|_{\overline{w}_\zeta = w} = \frac{1}{n} \frac{\tilde{y}_w}{\tilde{y}} < 0.
\]

**G. Equilibrium conditions (37) and (38)**

According to proposition 1 of Dixit et al. (1997), a subgame perfect Nash equilibrium for the game between the employer lobby, the labour lobby and the policy maker in jurisdiction \( k \) is a set of contribution schedules (34) and a policy \( \overline{w}_k \) s.t. the following conditions (i) – (iv) hold:

(i) The contributions of the union and employer lobbies, \( R_{ku} \) and \( R_{ko} \), are non-negative but no more than the contributor’s income. This is evident in the model.

(ii) The policy \( \overline{w}_k \) maximizes the policy maker’s welfare (35):

\[
\overline{w}_k = \operatorname{arg\ max}_{\overline{w}_k \text{ s.t. } (36)} G_k(\overline{w}_k, \overline{w}_{-k}, n, \gamma, \epsilon, \lambda).
\]

(iii) The employer (labour) lobby cannot have a feasible strategy

\[
R_{ko}(\overline{w}_k, \overline{w}_{-k}, n, \gamma, \epsilon, \lambda) \leq \left( R_{ku}(\overline{w}_k, \overline{w}_{-k}, n, \gamma, \epsilon, \lambda) \right)
\]
that yields it higher utility (32) ((33)) than in equilibrium, given the policy maker’s expected policy:

\[
\bar{\omega}_k = \arg \max_{\omega_k \text{ s.t. } (36)} F_k(\bar{\omega}_k, \omega_{-k}, n, \gamma, \epsilon, R_{ko}(\bar{\omega}_k, \omega_{-k}, n, \gamma, \epsilon, \lambda)),
\]

\[
\bar{\omega}_k = \arg \max_{\omega_k \text{ s.t. } (36)} U_k(\bar{\omega}_k, \omega_{-k}, n, \gamma, \epsilon, R_{ku}(\bar{\omega}_k, \omega_{-k}, n, \gamma, \epsilon, \lambda)).
\]

(iv) The employer (labour) lobby provides the policy maker at least with the level of utility than in the case where the lobby offers nothing \(R_{ko} = 0\) \(R_{ku} = 0\), and where the policy maker responds optimally given the other lobby’s contribution function (33) ((32)). This is assumed to hold true in the model, because otherwise there is no lobbying.

H. Results (39) and (40)

Because the equilibrium conditions (37) and (38) are symmetric throughout the jurisdictions \(k \in [0, n]\), the productivity adjusted wages will be uniform, \(\bar{\omega}_k = \bar{\omega}\) for \(k \in [0, 1]\), in general equilibrium. Thus, the properties of the partial equilibrium of a single jurisdiction \(k\) can be examined in the vicinity of general equilibrium \(\bar{\omega}_k = \bar{\omega}\) for \(k \in [0, 1]\). In addition to this, calculations are carried out in the vicinity of the point where average productivity \(\bar{\omega}_k\) is initially the same for all jurisdictions \(k \in [0, n]\), for simplicity.

Noting (18), (20), (21), (22), (23) and (27), the sum of the present values of oligopolist \(i\) and labour union \(i\), (32) and (33), is obtained as follows:

\[
F_k(\bar{\omega}_k, \omega_{-k}, n, \gamma, \epsilon, \lambda, R_{ko}) + U_k(\bar{\omega}_k, \omega_{-k}, n, \gamma, \epsilon, \lambda, R_{ku})
= \mathcal{P}(\bar{\omega}_k, \bar{\psi}, \gamma, \epsilon, \lambda, R_{o}^i) + \mathcal{W}(\bar{\omega}_k, \bar{\psi}, \gamma, \epsilon, \lambda, R_{i}^o) = \frac{A_{iT} b(\pi + v)}{r + (1 - a)\lambda\bar{z}} - \frac{R_{o}^i + R_{o}^u}{r},
\]

\[
A_{iT} b(\bar{\psi}) \left( F(l, \varphi - z) 1^{-1/\epsilon} - \bar{\omega}_k l + \varphi s + \bar{\omega}_k \tilde{l} + \varphi s \right) - \frac{R_{o}^i + R_{o}^u}{r}
\]

\[
= \frac{\bar{y}(\bar{\omega}_k, \gamma, \epsilon, \lambda) 1^{-1/\epsilon} b(\bar{\psi}) A_{iT}}{r + (1 - a)\lambda\bar{z}(\bar{\omega}_k, \gamma, \epsilon, \lambda)} - \frac{R_{o}^i + R_{o}^u}{r}, \text{ for which}
\]

\[
\frac{\partial (F_k + U_k)}{\partial \omega_k} = \frac{\partial}{\partial \omega_k} \left[ \frac{\bar{y}(\bar{\omega}_k, \gamma, \epsilon, \lambda) 1^{-1/\epsilon} b(\bar{\psi}) A_{iT}}{r + (1 - a)\lambda\bar{z}(\bar{\omega}_k, \gamma, \epsilon, \lambda)} \right]
\]

\[
= \frac{\bar{y} 1^{-1/\epsilon} b(\bar{\psi}) A_{iT}}{r + (1 - a)\lambda\bar{z}} \left[ 1 - \frac{1}{\epsilon} \frac{\bar{y} w}{\bar{y}} + \frac{b'(\bar{\psi})}{b(\bar{\psi})} \bar{\psi} w \right] + \frac{(a - 1)\lambda\bar{z}}{r + (1 - a)\lambda\bar{z}}
\]
\[ \frac{\vec{y}^{1-1/\epsilon} b(\vec{\psi}) A_{\epsilon} T}{r + (1 - a) \lambda \bar{z}} \Delta(\varpi, n, \gamma, \epsilon, \lambda) \text{ with } \]

\[ \Delta(\varpi, n, \gamma, \epsilon, \lambda) \equiv \left[ 1 - \frac{1}{\epsilon} + \frac{b(\vec{\psi})}{n} \right] \vec{y} w + (a - 1) \lambda \bar{z} \Delta(\pi, n, \gamma, \epsilon, \lambda). \] (58)

Noting (32) and (33), the equilibrium conditions (38) become

\[ 0 = \frac{dF_k}{d\varpi_k} = \partial F_k \partial \varpi_k + \partial R_{ko} \partial \varpi_k = \frac{1}{r} \partial R_{ko}, \]

\[ 0 = \frac{dU_k}{d\varpi_k} = \partial U_k \partial \varpi_k + \partial R_{ku} \partial \varpi_k = \frac{1}{r} \partial R_{ku}. \] (59)

These equations are equivalent to

\[ \frac{\partial R_{ku}}{\partial \varpi_k} = r \frac{\partial U_k}{\partial \varpi_k}, \quad \frac{\partial R_{ko}}{\partial \varpi_k} = r \frac{\partial F_k}{\partial \varpi_k}. \] (59)

Given (35), (58) and (59), one obtains

\[ \frac{\partial G_k}{\partial \varpi_k} \Bigg|_{\varpi = \varpi, \bar{A}_\epsilon = \bar{A}} = \frac{1}{r} \left( \frac{\partial R_{ku}}{\partial \varpi_k} + \frac{\partial R_{ko}}{\partial \varpi_k} \right) = \frac{\partial U_k}{\partial \varpi_k} + \frac{\partial F_k}{\partial \varpi_k}. \]

Conditions (37) are equivalent to the maximization of the Lagrangean

\[ L_k = G_k(\varpi, \varpi, n, \gamma, \epsilon, \lambda) + \xi_k [1 - \varphi - \bar{\ell}(\varpi, \gamma, \epsilon, \lambda)] \]

by the wage \( \varpi_k \), where the multiplier \( \xi_k \) is subject to the conditions

\[ \xi_k [1 - \varphi - \bar{\ell}(\varpi, \gamma, \epsilon, \lambda)] = 0, \quad \xi_k \geq 0. \] (61)

The first-order and second-order conditions for this maximization are

\[ \frac{\partial L_k}{\partial \varpi_k} \Bigg|_{\varpi = \varpi, \bar{A}_\epsilon = \bar{A}} = \frac{\partial G_k}{\partial \varpi_k} \Bigg|_{\varpi = \varpi, \bar{A}_\epsilon = \bar{A}} = -\xi_k \tilde{l}_w = \vec{y}^{1-1/\epsilon} b(\vec{\psi}) A_{\epsilon} T \Delta(\varpi, n, \gamma, \epsilon, \lambda). \]

\[ + r + (1 - a) \lambda \bar{z} = 0, \]

\[ 27 \]
\[
\frac{\partial \Delta}{\partial \omega_k} < 0 \iff \frac{\partial^2 G_k}{\partial \omega_k^2} < 0 \iff \xi_k = 0.
\]

Given (61), these are equivalent to
\[
\tilde{l} < 1 - \phi \iff \Delta = 0 \quad \text{with} \quad \frac{\partial \Delta}{\partial \omega_k} < 0, \\
\tilde{l} = 1 - \phi \iff \xi > 0 \iff \Delta = \frac{\xi_k}{l_w} + r + (1-a)\lambda\tilde{z} + \frac{\tilde{y}(\tilde{y}/e)^{1-1/\epsilon}}{A}\frac{\epsilon}{\epsilon-1} < 0.
\]

References:


