Research Reports
Publications of the Helsinki Center of Economic Research, No 2014:2
Dissertationes Oeconomicae

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PROPAGATION OF FINANCIAL SHOCKS: EMPIRICAL STUDIES ON FINANCIAL SPILLOVERS

ISBN 978-952-10-8724-0 (paperback)
ISBN 978-952-10-8725-7 (PDF)
ISSN 2323-9786 (print)
ISSN 2323-9794 (online)
Acknowledgments

So, it is finally here, the moment to write these acknowledgments. At the most stressful times of this PhD project, when I was either struggling with the exercises of the PhD courses or simply stuck with my research, I used to motivate myself by thinking that, if I only focused my thoughts and solved one problem at time, this day would inevitably come. Luckily those moments of frustration usually lasted only for a short while. By far the majority of the time I have felt myself privileged to be able to devote my full attention to PhD studies. However, finalizing this thesis would not have been possible to do alone, and I owe my thanks to a great many people. Apologies to all of you who I forget to mention here.

First of all, I want to thank my supervisor, Professor Markku Lanne who has been very supportive throughout the project and, probably what is still more important, with his practical advices, helped me to rediscover the red line of my research whenever I lost it. I am sure that a PhD student could not ask for a better supervisor. Second, I want to thank the pre-examiners of my thesis, Professors Tom Engsted and Charlotte Christiansen from the Aarhus University. They have given me good comments and suggestions on how I could improve my still unpublished papers. I am grateful for those comments.

Third, there are the many great colleagues–professors, post-docs, fellow PhD students and others–with whom I have had the pleasure to discuss on economics and variety of other topics during these four and a half years. I want to separately thank Tatu, Otto, Juha I., Harri T. and Gero for the peer support and many great lunch discussions. Thanks to those, often excited, exchanges of views my economic intuition has deepened enormously. Naturally, I am also very grateful to HECER/KAVA for the graduate school position and to the OP-Pohjola Research Foundation and Säästöpankki Säätiö for financial support. Although economists are said not to be very interested in money, you still need some.

Then comes my family. My mother has always been there for me and encouraged me to go on, to find and follow my own path. Also, my father and grandparents have never failed to show me that they believe in me, no matter what I choose to do. I must also mention the academic
example of Håkan and Mikki which probably put the idea in my head of, first, choosing the Faculty of Social Sciences and, later, to follow graduate studies.

Finally, I want to thank my wonderful wife Eeva who usually do not accept my economist’s jargon or half-thoughts as an answer but insists that I make my points clear. Your loving and tender support is the most valuable asset I can imagine.

Helsinki, March 2014.
Anssi Kohonen
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Chapter 1

Introduction

This thesis is a collection of empirical essays that try to understand the way how financial markets propagate shocks across borders, and from the financial sector to the real sector of an economy. The word *empirical* means that we develop statistical methods to model, measure, and test for such spillover effects. The part *collection of essays* highlights that we do not even pretend to cover the whole topic of the propagation of financial shocks. This literature is vast and includes both theoretical and empirical studies, exploring a multitude of potential channels of propagation. Instead, what we have in mind, is to consider a few particular examples of financial spillovers: contagion, volatility spillovers, and the real sector effects of uncertainty. Let us now present these three concepts, around which our three essays are organized.

Contagion

The widespread use of the term *contagion* as related to financial crises is a relatively new phenomenon; before the Asian financial crisis in 1997, references to contagion in economic and financial press were almost non-existent (Forbes (2012)). Although contagion is nowadays often cited in the media as an explanation to an escalation of financial turmoils, it is seldom that commentators exactly define what they mean by it. On a general level it seems clear that contagion should refer to a transmission of (negative) shocks, turmoil, or even panic, but a formal inspection of the subject is not self-evident. The trouble is that, in the interconnected world of today, one can expect the majority of shocks to one country to have an influence also elsewhere. Certainly not all of them should be counted as contagion.

In their survey on the contagion literature, Pericoli & Sbracia (2003) list five definitions of contagion that are generally used in the academic research. And, although the definitions resemble each other, they are not quite the same. Their fifth definition is basically the one...
that we follow in this thesis, according to it: "contagion occurs when the transmission channel (between markets or countries) changes after a shock in one market". So, for example, Figure 1.1 shows the long term interest rates of several euro countries in 1993–2013. Clearly, the introduction of the euro pushed the interest rates down and together, whereas the euro debt crisis has again pushed them apart. Then, if we reckoned the first ten years of the euro time (1999–2008) as a normal period, we would consider there being contagion, say, between Greece and Portugal if the transmission channels of shocks, or financial linkages, between the bond markets of these countries changed after the beginning of the Greek debt crisis in the late 2009/early 2010.

**Volatility spillovers**

Volatility, or standard deviation, of asset prices or stock market returns is a commonly used measure of uncertainty. So, behind the concept of **volatility spillovers**, there is an idea that uncertainty might be transmitted across countries during a crisis, or even that the transmission of uncertainty is the reason for an escalation of financial crises. As both concepts, contagion and volatility spillovers, refer to an idea that, due to an international propagation of financial
shocks, financial crises might escalate, it appears then that volatility spillovers is a closely related concept to contagion.\footnote{Actually, one definition of contagion that Pericoli & Sbracia (2003) provide is the spillover of the volatility of asset prices across countries.}

But, although they closely resemble each other, we prefer to keep the concepts separated. In particular, they help to highlight that spillover effects can occur either in the first moment of financial returns, or in the second moment (volatility). For instance, as Figure 1.2 shows, it seems like a plausible hypothesis that during the euro debt crisis there has been volatility spillovers between the national financial markets.

### Uncertainty and business cycle

Until now, the discussion on financial spillovers has concentrated on the transmission of shocks between the financial markets of different countries. If the effects of a financial shock were only financial, the Main Street should probably not need to be extremely worried about what happens...
in the Wall Street. However, there are plenty of reasons to assume that financial shocks have spillover effects also on the real sector of an economy. For example, as explained by Bernanke (1983), higher uncertainty might induce firms to postpone their investments, which could then affect also employment (Bloom (2009)). Indeed, it appears that, during the last almost hundred years in the US, higher volatility, which is our measure of uncertainty, at least coincide with recessions and declining industrial production (Figure 1.3). The spillover of financial shocks to the real sector is the third theme of the thesis.

The rest of this introductory chapter is organized as follows. The next section introduces, on a very general level, our main tools of analysis. After this, Section 1.2 summarizes the research essays of the thesis.

Figure 1.3: Monthly US stock market volatility, change in industrial production, and the US recessions in Jan/1920–July/2013

Note: Volatility corresponds to twelve months rolling standard deviation of the monthly stock market returns which is multiplied by two, and change in industrial production to the first differences of the logarithms of the monthly values of the industrial production index. Recessions months correspond to the contraction months as defined by the NBER. Source: NBER, St.Louis FED’s FRED database, own calculations.
1.1 Models for Analyzing Spillovers

The three basic models that underlie the analysis of this thesis are vector autoregressive model, structural vector autoregressive model, and autoregressive conditional heteroskedasticity model. In order to set ideas for the main chapters of this thesis, this section introduces the general principals of these models. The more detailed discussion of the models of the essays is left for the actual chapters.

1.1.1 Modeling Dynamic Interrelationships: VAR model

From the perspective of our analysis, the single most important statistical model is that of the vector autoregressive (VAR) model. Since the seminal paper by Sims (1980), the VAR model has been a standard tool in the toolbox of econometricians. Because there are many very good textbook representations of the VAR model (see for example Hamilton (1994), or Lütkepohl (2007)), we will only sketch the basic idea behind the model, and, for simplicity, focus on the first order VAR model. This will prepare us for the discussion on the structural VAR (SVAR) model and its identification which is an important theme in all of our essays.

Consider \( n \) random variables, call them \( y_{1,t}, y_{2,t}, \ldots, y_{n,t} \), that we observe on regular intervals (\( t \) denotes the time period); as an example, take monthly stock market returns in \( n \) countries. Then, the basic VAR model provides a simple framework to study the interrelationships between the variables. By collecting the variables into a \( (n \times 1) \) random vector \( y_t = [y_{1,t}, \ldots, y_{n,t}]' \), the first order VAR model can be written as

\[
y_t = Ay_{t-1} + u_t,
\]

where, for simplicity, we have assumed no intercepts, and where \( A \) is a \( (n \times n) \) coefficient matrix, and \( u_t \) is the \( (n \times 1) \) error vector which is assumed to have zero mean and the covariance matrix \( \Sigma_u = E(u_t u_t') = \Omega \). Also, it is assumed that \( u_t \) and \( u_{t-k} \) are uncorrelated to each other for all \( k \neq 0 \) and \( k \in \mathbb{Z} \), where \( \mathbb{Z} \) is the set of integers.

From the perspective of the effects of the \( n \) variables on each other, let us specify two classes of interrelationships: first, there are the contemporaneous linkages between the variables. This channel of interrelationship is measured with the off-diagonal elements of the matrix \( \Omega \), that is, with the covariances between the individual elements of \( u_t \). This point is explained in the next subsection where we discuss the structural VAR. Second, there are the dynamic interrelationships, that is, the effect of \( y_{k,t} \) on \( y_{l,t+m} \) for all \( k, l = 1, \ldots, n \) and \( m \geq 1 \). These effects are, of course,
dictated by the matrix $A$. And so, for example, the next-period effect of a realization $y_{1,t}$ on $y_{2,t+1}$ equals $a_{21} y_{1,t}$, where $a_{21}$ is the second row, first column element of $A$. The long run dynamic interrelationships are best captured with the impulse response (IR) functions.

In order to see the intuition behind the IR functions, consider equation (1.1). Clearly, we can equally well write the following:

$$y_{t-1} = Ay_{t-2} + u_{t-1}. $$

By now plugging $y_{t-1}$ back into equation (1.1), and then, again, by solving $y_{t-2}$ as a function of $y_{t-2}$ and $u_{t-2}$, and so forth, we can write down the moving average representation of the VAR model (1.1):

$$y_t = u_t + Au_{t-1} + A^2 u_{t-2} + \ldots + A^k u_{t-k} + \ldots, \quad (1.2)$$

where $k \geq 0$. Hence, the long term dynamic effect of, say, the first element of $u_{t-k}$, shock $u_{1,t-k}$, on $y_{2,t}$ depends on $[A^k]_{2,1}$, the second row, first column element of the matrix $A^k$. The full plot of the values $[A^k]_{2,1}$ as a function of $k$ is the IR function of the first element of $u_t$ on the second variable of $y_t$. The matrix $A$ can be consistently estimated by the method of the ordinary least squares (OLS). Once we have the estimate ($\hat{A}$), a simple way to compute the IR functions is, for instance, by simulating the responses of the (estimated) system in equation (1.1) to a shock of magnitude one on each element of $u_t$ separately at "period 1" while holding $y_0 = 0$.

1.1.2 Modeling Contemporaneous Effects: SVAR

One problem with the IR functions, as detailed above, is that, whenever there is no good reason to assume that the covariance matrix $\Omega$ would be diagonal, it is not logical to assume a non-zero realization solely of, say, the second element of $u_t$, and hence, to use its IR function to study the effects of $y_{2,t}$ (specific shock) on the other variables of the system. So, whenever the elements of $u_t$ are correlated to each other, as usually is the case with economic variables, we need to impose some (structural) model to detail with the contemporaneous linkages between the elements of $u_t$ and, hence, the elements of $y_t$. A structural vector autoregressive (SVAR) model is one approach to do this.

The SVAR framework was developed for the purposes of policy analysis, especially to study the effects of monetary policy, but it works fine also for our purposes as we want to understand the propagation of country or financial market specific shocks to other countries or the real sector. First, we need to assume $n$ structural shocks $\varepsilon_{1,t}, \ldots, \varepsilon_{n,t}$ that we collect into the $(n \times 1)$ vector

\footnote{For a very good discussion on the VAR and SVAR models from the perspective of the history of macroeconomic literature, see Sims (2011).}
The structural shocks are assumed to be mutually uncorrelated and to have unit variances, and so the covariance matrix of \( \varepsilon_t \) is the \((n \times n)\) identity matrix, that is, we assume that \( \Sigma_\varepsilon = I_n \). Second, assume that the reduced form error vector \( u_t \) of the VAR model (1.1) is the following linear function of the structural shocks \( \varepsilon_t \):

\[
u_t = B\varepsilon_t,
\]

(1.3)

where \( B \) is a \((n \times n)\) coefficient matrix.\(^3\)

Clearly, to take again our example of monthly stock market returns in \( n \) countries, assume for a moment that, by somehow, we knew that the structural shocks \( \varepsilon_{1,t}, \ldots, \varepsilon_{n,t} \) corresponded to country specific stock market shocks, respectively, then, the off-diagonal elements of the matrix \( B \) would tell us the contemporaneous effects between the stock markets. In this case, to answer the question on what are the contemporaneous spillover effects of a shock to, say, the stock market of the second country (return \( y_{2,t} \)), we should concentrate on the effects of the second structural shock \( \varepsilon_{2,t} \). Its contemporaneous effect on country 1 equals \( b_{1,2} \), the first row, second column elements of \( B \), the effect on country 3 equals \( b_{3,2} \), and so forth. In order to see the dynamic effects, use equation (1.3) to write the moving average representation (1.2) of our VAR model as

\[
y_t = B\varepsilon_t + AB\varepsilon_{t-1} + \ldots + A^kB\varepsilon_{t-k} + \ldots
\]

for \( k \geq 0 \). Hence, the second column of the matrix \( A^kB \) gives us the \( k \)-periods ahead effect of \( \varepsilon_{2,t} \). The problem is that, without any further assumptions about the SVAR model (1.3), we cannot estimate the matrix \( B \), the structural shocks are not identified, and, so, we can not associate the structural shocks with the \( n \) countries of our example.

### 1.1.3 Identification of SVAR

In order to see the problem with the identification of the SVAR model (1.3), let us start from the fact that we can always consistently estimate the \((n \times n)\) covariance matrix \( \Omega \) of the reduced form errors consistently with the standard estimations methods, such as the OLS, for example. Call this estimate \( \hat{\Omega} \). Because a covariance matrix is symmetric, \( \hat{\Omega} \) has only \( n(n+1)/2 \) distinct elements whereas the matrix \( B \) has \( n^2 \) elements. This means that, without any further

---

\(^3\) The specification of the SVAR model as in equation (1.3) corresponds to the B-model framework of the SVAR model. This is probably the most widely used specification of the SVAR model. Lütkepohl (2007, 358–67) talks of the B-model as well as other two frameworks. The difference between the frameworks come from whether we focus on imposing the contemporaneous linkages on the elements of \( u_t \) or directly on the elements of \( y_t \). Basically, this is just a question of taste and does not affect the analysis.
assumptions, the system of equations

\[ \hat{\Omega} = BB', \]  

(1.4)

which is implied by equation (1.3) and by our assumption that \( \Sigma_{\varepsilon} = I_n \), and where \( B' \) is the transpose on \( B \), is not well defined. So, given any data \( y_1, \ldots, y_T \), there is no way for us to solve for the elements of \( B \) based on (1.4).

Another way to look at the identification problem is to focus directly on equation (1.3). For simplicity, assume \( n = 2 \), then this equation becomes

\[
\begin{bmatrix}
u_1,t \\ u_2,t
\end{bmatrix} =
\begin{bmatrix}
b_{11} & b_{12} \\ b_{21} & b_{22}
\end{bmatrix}
\begin{bmatrix}
e_{1,t} \\ e_{2,t}
\end{bmatrix} =
\begin{bmatrix}
b_{11}\varepsilon_{1,t} + b_{12}\varepsilon_{2,t} \\ b_{22}\varepsilon_{1,t} + b_{22}\varepsilon_{2,t}
\end{bmatrix}. 
\]

(1.5)

And so, whenever \( b_{kl} \neq 0 \) for all \( k, l = 1, 2 \), both reduced form errors are linear combinations of both structural shocks. Hence, always having our stock market returns example in mind, we cannot associate the structural shocks with the countries. But, if we assumed, for instance, that \( b_{12} = 0 \), the equation would become

\[
\begin{bmatrix}
u_1,t \\ u_2,t
\end{bmatrix} =
\begin{bmatrix}
b_{11} & 0 \\ b_{21} & b_{22}
\end{bmatrix}
\begin{bmatrix}
e_{1,t} \\ e_{2,t}
\end{bmatrix} =
\begin{bmatrix}
b_{11}\varepsilon_{1,t} \\ b_{22}\varepsilon_{1,t} + b_{22}\varepsilon_{2,t}
\end{bmatrix}. 
\]

(1.6)

and so, at period \( t \), the second structural shock would affect only the stock market in country 2. This would allow us to identify the shock \( \varepsilon_{2,t} \) as country 2 specific shock. The shock \( \varepsilon_{1,t} \) would still be allowed to affect the stock markets in both countries at period \( t \). If, for instance, country 1 was a big economy whereas country 2 a small neighbor country, it would make sense to interpret the shock \( \varepsilon_{1,t} \) as the country 1 specific shock.

Assuming that the matrix \( B \) is a lower-triangular matrix as in our example (1.6), has probably been the most common method to identify a SVAR model. But it is also a controversial identification strategy as it assumes a specific recursive order of the contemporaneous effects between the variables of the system, and the estimations results can be sensitive on the selected order of the variables. Especially, if using a triangular \( B \) matrix is not well justified by economic theory, or by some other good reasons to justify a recursive ordering of the variables, one might have hard time in defending one’s identifying assumptions.\(^4\) In the third essay of this thesis, we will use a triangular \( B \) matrix to identify a financial shock. There, it is argued that, on the

\(^4\)Another famous strategy to identify a SVAR model was proposed by Blanchard & Quah (1989). They considered unemployment and changes in output in the US and identify (the structural) demand and supply shocks by assuming that only supply shocks can have permanent, long-run effect on the output. For a good discussion on the most common identification methods, both recursive and non-recursive, and their shortcomings, see Kilian (2011).
monthly frequency, due to the persistence in the industrial production, financial shocks should not have any contemporaneous effect on the change in industrial production.

From the perspective of our first two essays, the traditional SVAR identification methods are problematic as they require us to make \textit{a priori} restrictions on the matrix $B$. In those essays our desire is to study (possible changes in) the contemporaneous linkages between financial variables during the euro debt crisis. So, being forced to restrict them from the start is something that we would be willing to avoid. Hence, in those chapters we rely on some more recent ideas of identifying a SVAR model based on heteroskedasticity or non-normalities in data (such methods were introduced, for example, by Rigobon (2003b), Lanne & Lütkepohl (2008), and Lanne & Lütkepohl (2010)). And actually, these methods allow for us to test for potential restrictions on $B$.

In order to see how heteroskedasticity might help us to identify a SVAR model, we take a simple example from Lütkepohl (2012) (which is also a good survey on the literature). Assume $n = 2$ and, also, consider a sample of $T$ time periods, which we divide in two and assume that, for the first time interval ($t = 1, \ldots, T_1$), the covariance matrix of the reduced form errors equals $\Omega_1$, and that during the second sub sample ($t = T_1 + 1, \ldots, T$) it equals $\Omega_2$ (and, of course, that $\Omega_1 \neq \Omega_1$). There is a result in linear algebra according to which the two covariance matrices can be decomposed in the following way (see, for example, the appendix in Lanne & Lütkepohl (2010)):

$$
\Omega_1 = \tilde{B}\tilde{B}' \quad \text{and} \quad \Omega_2 = \tilde{B}\Psi\tilde{B}',
$$

where $\tilde{B}$ is a general $(2 \times 2)$ matrix, and $\Psi = \text{diag}(\psi_1, \psi_2)$ is a diagonal matrix. Furthermore, assume also that the covariance matrix ($\Sigma_\epsilon$) of the structural shocks changes in the following way:

$$
\Sigma_\epsilon = \begin{cases} 
I_2, & \text{when } t = 1, \ldots, T_1, \\
\Psi, & \text{when } t = T_1 + 1, \ldots, T,
\end{cases}
$$

where $I_2$ is the $(2 \times 2)$ identity matrix, and $\Psi$ as in equation (1.7). Especially, notice that, in equation (1.8), although we allow for heteroskedasticity in the distribution of the structural shocks, we maintain the assumption that the individual shocks are mutually uncorrelated.

Hence, now, a natural way to identify the structural shocks of the SVAR model in equation (1.3) is to assume that, for the first subsample, the matrix $B$ of equation (1.3) equals the matrix $\tilde{B}$ in equation (1.7), and that, during the second subsample, the matrix $B$ equals $\tilde{B}\Psi^{1/2}$. The benefit is that, unlike the system of equation (1.4), we now have the system of equation
(1.7) which is well defined as it gives the following six equations with six unknown parameters 
\(b_{11}, b_{12}, b_{21}, b_{22}, \psi_1, \psi_2\):

\[
\begin{align*}
\omega_{11,1} &= b_{11}^2 + b_{12}^2, \\
\omega_{12,1} &= b_{11}b_{21} + b_{12}b_{22}, \\
\omega_{22,1} &= b_{21}^2 + b_{22}^2, \\
\omega_{11,2} &= \psi_1 b_{11}^2 + \psi_2 b_{12}^2, \\
\omega_{12,2} &= \psi_1 b_{11}b_{21} + \psi_2 b_{12}b_{22}, \\
\omega_{22,2} &= \psi_1 b_{21}^2 + \psi_2 b_{22}^2,
\end{align*}
\]

where \(\omega_{i,j,k}\) refers to the row \(i\), column \(j\) element of the covariance matrix \(\Omega_k\) with \(i, j, k = 1, 2\).

Hence, given data, we can solve the parameters. (Remember that the elements of matrices \(\Omega_1\) and \(\Omega_2\) can always be consistently estimated, hence, we can treat them as known variables.)

### 1.1.4 Effects of Uncertainty: GARCH Model

As Figures 1.2 and 1.3 show, financial market volatility can sometimes increase for extended periods of time. Since Engle (1982) and Bollerslev (1986), a standard approach to model this phenomenon called *volatility clustering* is to consider a model of generalized autoregressive conditional heteroskedasticity (GARCH). In our third essay, we will use a standard GARCH(1,1) model to measure uncertainty, so let us briefly describe it here (for a detailed discussion, an interested reader should consult, for example, Lütkepohl (2007)). Assume a univariate random variable, call it \(u_t\), which has zero mean, conditional variance \(E(u_t^2|I_{t-1}) = \omega_t^2\), where \(I_{t-1}\) denotes the observations of \(u_t\) up to to time period \(t - 1\), and follows the subsequent GARCH(1,1) model:

\[
\begin{align*}
u_t &= \omega_t \varepsilon_t, \\
\omega_t^2 &= \alpha + \beta u_{t-1}^2 + \gamma \omega_{t-1}^2,
\end{align*}
\]

where the shock \(\varepsilon_t\) is assumed to be identically and independently distributed, and \(\alpha, \beta\) and \(\gamma\) are parameters.

The main point is to see that, according to equation (1.9), the conditional volatility of \(u_t\) depends on its first lag and the first lags of \(u_t\). So, from the perspective of our discussion, assume \(u_t\) corresponds to stock market return, a large realization of \(u_t\) today will increase the conditional
variance ($\omega_{t+1}$), and hence uncertainty, from the next period onwards. We will measure the real sector effects of uncertainty by considering the effect of $\omega_{t+1}$ on the growth rate of industrial production (in a multivariate setting).

1.2 Summary of the Essays

Let us now briefly summarize the essays and review their main empirical findings. The three essays consider volatility spillovers in stock markets, dynamic and contemporaneous interrelationships (or contagion) in the government bond markets, and the real sector effects of financial shocks, respectively.

1.2.1 Chapter 2: On Detection of Volatility Spillovers in Overlapping Stock Markets

The first essay considers volatility spillovers in stock markets. The starting point of the analysis is the model proposed by King & Wadhani (1990) which is a theoretical model to explain volatility spillovers in stock markets. According to the model, volatility spillovers are due to there being two types of investors, informed and uniformed. As the uniformed investors know that part of the changes in stock market prices reflect the private information of the informed investors, they are prone to react to price changes. This creates a potential channel for volatility to spill across national stock markets.

The model of King & Wadhani (1990) is not identified as such. The contribution of the essay is to interpret the model as a SVAR model and, then, use non-normalities of stock market data to identify the model. More precisely, we use the SVAR identification method proposed by Lanne & Lütkepohl (2010). The model can be estimated with the method of maximum likelihood, and volatility spillovers can be tested with the standard likelihood ratio test.

In the empirical application of the essay we consider stock markets of Greece, Italy, Germany, Ireland, and Spain in 2010–2011 and find evidence of volatility spillovers. Especially, the stock market volatilities of the large countries (Italy and Germany in particular) have large effects on all countries, both large and small. On the contrary, the stock market volatilities of the small countries (Ireland and Greece) mostly have effects on each other.
1.2.2 Chapter 3: Transmission of Government Default Risk in the Eurozone

The second essay extends the SVAR model of Favero & Giavazzi (2002) to analyze the reasons behind the rising ten year government bond spreads in the eurozone during the recent euro debt crisis. Also, we propose an alternative way to implement the "contagion" test of Favero and Giavazzi. The implementation is based on combining the ideas of Lanne & Lütkepohl (2008, 2010) on how to use heteroskedasticity and non-normalities of data to identify a SVAR model.

The SVAR model of the essay allows us to test for the stability of the contemporaneous and dynamic interrelationships between the spreads, as well as changing intercepts which we interpret as country specific risk factors. In the empirical application, we analyze the government bond spreads of Greece, Portugal, Ireland, Spain, and Italy over the German bond in the years 2001–2012. Although contagion seems to be an important factor in explaining the increasing spreads during the crisis, there are substantial differences between the countries. For Ireland, Italy and Spain also the idiosyncratic risk factors seem to play an important role. Also the Irish and Italian spreads become dynamically less interdependent with the spreads of the other countries. For Greece and Portugal contagion seems to be an important factor to explain the increases in their spreads.

1.2.3 Chapter 4: Uncertainty and Business Cycles

The third essay considers the real sector effects of uncertainty and, for this purpose, introduces a specification of the vector autoregressive model with autoregressive conditional heteroskedasticity in mean effects to model the joint dynamics of the monthly US stock market return and the change in industrial production in 1919–2013. The model is an extension of the multivariate GARCH model of Vrontos et al. (2003) and allows us to decompose the effect of a stock market crash on industrial production into two components, the effect of negative returns and the effect of higher volatility. The latter effect is our proxy for business cycle effects of uncertainty.

The empirical analysis finds uncertainty in the US to be significantly countercyclical. This result is robust for varied time periods. Also, the impulse response analysis shows that a monthly drop of ten percent in stock market prices is followed by a cumulative decline of three percent in the industrial production. Of this decline, around two thirds are explained by higher uncertainty.
References


Chapter 2

On Detection of Volatility Spillovers in Overlapping Stock Markets

Abstract

This paper applies a recently proposed structural vector autoregressive model identification method to an established, previously unidentified theoretical model of stock market volatility spillovers. The structural model is identified and can be estimated with the method of maximum likelihood. Volatility spillovers can then be tested with the standard likelihood ratio test. This way our test, unlike the majority of the existing volatility spillover tests, has its foundations firmly in the economic theory. Our test is developed for fully overlapping stock markets. The empirical application of the paper considers stock markets of the eurozone in the years 2010–2011. Evidence of volatility spillovers is found.

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1This chapter is based on an article with the same title, published in the *Journal of Empirical Finance*, 22, 140–158, 2013.
2.1 Introduction

Periods of financial distress are usually accompanied by simultaneous increases in volatility of the world’s financial markets. The literature on volatility spillovers claims that these international volatility clusters are due to volatility being transmitted across borders; a rise in volatility in one country increases volatility elsewhere. But what causes the rise in volatility in the origin country? This paper combines two themes in the literature: the economic theory of volatility and the statistical modeling of its spillovers.

Since the seminal papers by Engle et al. (1990) and Hamao et al. (1990), volatility spillovers have been extensively studied. Quite naturally, different specifications of the generalized autoregressive conditional heteroskedasticity (GARCH) model have been popular. The majority of the empirical studies find evidence of international interdependencies in volatilities. However, most of these models are purely statistical. The theoretical literature on the causes of the interdependencies is much more limited (Soriano & Climent, 2006). Furthermore, papers trying to estimate a theoretical model that would explain both volatility and its transmission are rare.

We will try to fill this void by proposing a way to estimate the classical theoretical model of King & Wadhwani (1990) (henceforth, the KW model). It is a rational expectations model with informed and uninformed investors. Stock market returns and their volatilities depend on arriving new information that only the informed investors observe. Volatility spillovers are, then, the result of the uninformed investors’ efforts to estimate the informed investors’ private information by solving a signal-extraction problem where price changes act as signals. We will concentrate on the simplest version of the KW model, that of fully overlapping stock markets. The author is not aware of any other paper that would have estimated the KW model for overlapping markets.

Unfortunately, King and Wadhwani were unable to identify their structural model. To do this, we will augment the model with an additional assumption about the variables’ distribution. Especially, we assume the daily stock market returns follow a mixed-normal distribution. Then, by interpreting the KW model as a structural vector autoregressive (SVAR) model, we can use a recent identification method by Lanne & Lütkepohl (2010). However, as emphasized by Lanne and Lütkepohl, their identification method "only" guarantees statistical identification of a SVAR model. This means that the identified structural shocks are guaranteed to be orthogonal, a generally accepted (minimum) requirement in the SVAR literature. Hence, the method does not guarantee that the identified structural shocks would have any economically meaningful

\footnote{For example, Soriano & Climent (2006) provide an extensive survey on the volatility spillover literature. Also, Hong (2001), and Savva (2009) briefly review this literature.}
In our context, this means that the Lanne-Lütkepohl method only provides partial identification of the KW model. To fully identify it, this paper suggests to use the Google search engine data as an external source of information. After this the model can be estimated with the method of maximum likelihood and volatility spillovers tested with the standard likelihood ratio test. In the empirical application of the paper, we estimate the KW model using the eurozone stock market data from 2010 to 2011 with five countries: Italy, Ireland, Spain, Greece, and Germany. We find evidence of volatility spillovers.

This paper is related to several different topics in the volatility spillover–and also contagion–literature. First, the main idea of the Lanne-Lütkepohl identification method is to identify a system of simultaneous equations by exploiting non-normalities in data; this relates our model to the papers that use some specific characteristic of the probability distribution of data as an additional source of information for structural model identification. For example, Rigobon (2003a) presents a heteroskedasticity-based identification method that has been successfully applied.

A common feature with the non-normality and heteroskedasticity based approaches is that, contrary to the more traditional identification methods, they try to avoid imposing any specific (usually zero) restrictions on the parameters of the instantaneous effects between the countries. Given that the objective of the volatility spillover models is usually to test the statistical significance of these parameters, it is of course a desirable feature of our approach if we can avoid restricting any of them ex ante. The Lanne-Lütkepohl and Rigobon methods differ, however, in that the latter assumes heteroskedasticity in the structural shocks whereas the former focuses on non-normalities (or heteroskedasticity) in the distribution of the reduced form errors.

Second, many papers study volatility spillovers with (latent) factor models. The KW model can be interpreted as an explanation for common factors because in the KW model news can be relevant to equity valuations in several countries. Such news could then be interpreted as unobserved common variables. However, unlike usually done in the latent factor models, the KW model does not model in conditional heteroskedasticity. Assuming news having a GARCH effect could be an extension of the model considered here.\footnote{Lin et al. (1994) estimate two variants of a model similar to the KW model, one with homoskedastic news process and one with heteroskedastic news. However, their model identification is based having stock markets that do not overlap, namely New York and Tokyo. Hence, their approach is not directly applicable to our context.}

\footnote{The distinction between statistical and "economically meaningful" identification is further discussed in Herbarts & Lütkepohl (2011), and Lütkepohl (2012).}

\footnote{See, for example, King et al. (1994) and Dungey & Martin (2007).}

\footnote{See, for example, Caporale et al. (2005b,a), Rigobon (2002), Rigobon & Sack (2003).}

\footnote{Of course, sometimes the market structure under consideration, or differences in trading hours allow for such zero restrictions, see for example Billio & Caporin (2010) for an application. Favero & Giavazzi (2002), in contrast, identify their model by restricting the dynamic—not contemporaneous—effects of the system.}

\footnote{In fact Sentana & Fiorentini (2001) show that variation in conditional covariance is important for the identification of dynamic factor models.}
Finally, one prevailing theme in the finance literature is transmission of information across countries. Wongswan (2006) shows that information is transmitted from the world’s major economies to smaller ones, and that there is a short-lived volatility effect in the target countries’ stock markets. Our paper shows that, in addition to information—and hence volatility—spreading from large countries to small countries, it gets transmitted also between small countries as well as between large ones.

The rest of the paper is organized as follows. Section 2.2 presents the KW model. Section 2.3 shows how to identify and estimate it, and to test for volatility spillovers. Section 2.4 provides the empirical example. Finally, section 2.5 concludes.

2.2 Theoretical Model of Volatility Spillovers

The KW model of volatility spillovers is a rational expectations model with two types of investors, informed and uninformed. Domestic investors are always the informed ones whereas foreign investors remain uninformed. The model is a variant of the Grossman & Stiglitz (1980) model on the impossibility of competitive equilibrium prices to fully reveal all information. The exposition of the KW model in sections 2.2.1 and 2.2.3 follows quite closely that in the original article.

2.2.1 Two Countries Case

Let us begin with only two countries, countries 1 and 2. There is one stock market in each country. All investors are risk neutral and there is no international (cross-border) stock trading. Also, both markets are assumed to be open around the clock, so the opening hours of the markets are fully overlapping.

There are two types of information: systematic and idiosyncratic, denoted by $\varepsilon$ and $\nu$, respectively. Systematic information is relevant globally to all stock markets whereas the idiosyncratic only to local stock prices. So, any news, denoted by $\eta$, can be of either type of information. However, the problem for foreign investors is that they never observe the type of information of a piece of domestic news. Only local investors observe (or correctly interpret) it. So, in the

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9 According to King and Wadhwani, if we allowed risk neutral investors with possibility of arbitrage between national stock markets, in the equilibrium all information would be revealed. Prohibiting international trade in stocks makes a non-fully-revealing equilibrium (equilibrium with information asymmetries) possible with risk-neutral investors. This modeling strategy simplifies the model’s structure without affecting the general conclusions. Alternatively, one could permit international stock trading and obtain the non-fully-revealing equilibrium by assuming risk-averse investors. For more discussion on alternative assumptions, see the original paper.

10 Given the modern information technology and international news agencies, the assumption that foreign investors are not able to observe (or interpret) information as well as domestic investors might seem too restrictive—surely news are widespread almost instantaneously. However, King and Wadhwani point out that there is a difference between news in the media and information as an assessment on consequences of the news to equity valuations. For example, the findings of Groß-Klußmann & Hautsch (2011) support this important distinction. Hence, as valuation assessment is costly, some investors might prefer to try to infer the new valuations from the
KW model, domestic investors are always the ones who are informed and foreign investors stay uninformed.

Hence, in our two countries case, both information types, \( \varepsilon \) and \( v \), come in two different forms depending on where they are observed, in country 1 or country 2. This said, when we denote by \( \eta^{(i)}_{t} \) news in country \( i \) at period \( t \), it can be decomposed in the following way:

\[
\eta^{(i)}_{t} = \varepsilon^{(i)}_{t} + v^{(i)}_{t} \quad \text{for} \ i = 1, 2, \tag{2.1}
\]

where the superscripts on the information variables \( \varepsilon \) and \( v \) emphasize the country where the information is observed. The four information variables–\( \varepsilon^{(1)}_{t}, \varepsilon^{(2)}_{t}, v^{(1)}_{t} \) and \( v^{(2)}_{t} \)–follow white noise processes and, so, are uncorrelated to each other.

Any change in a country’s stock market price index \( S \) during a time period is the result of new information released during that period. Because investors never directly observe systematic information in foreign countries, they need to estimate it. Let \( E_i \) denote the expectation operator of country \( i \) investors conditional on all information they observe at period \( t \). The stock market price indexes of our two countries will then follow these two equations:

\[
\Delta S^{(1)}_{t} = \varepsilon^{(1)}_{t} + \alpha_{12} E_1 \left( \varepsilon^{(2)}_{t} \right) + v^{(1)}_{t}, \tag{2.2}
\]

\[
\Delta S^{(2)}_{t} = \alpha_{21} E_2 \left( \varepsilon^{(1)}_{t} \right) + \varepsilon^{(2)}_{t} + v^{(2)}_{t}, \tag{2.3}
\]

where \( \Delta S^{(i)}_{t} \) denotes the percentage return in country \( i \)'s stock market in the period between time \( t - 1 \) and \( t \). This is measured by the change in the logarithm of the price index. The parameter \( \alpha_{ij} \) captures the importance of systemic information observed in country \( j \) on the equity prices in country \( i \).

Assume that the only information available to the (domestic) investors of a country about the systematic information observed elsewhere is the change in the foreign equity prices. So, for example, considering the signal extraction problem of country 1’s investors, the conditional expectation \( E_1(\varepsilon^{(2)}_{t}) \neq 0 \) whenever they observe a non-zero \( \Delta S^{(2)}_{t} \). However, \( \Delta S^{(2)}_{t} \) is a function of both the systematic information \( \varepsilon^{(2)}_{t} \) and the idiosyncratic information \( v^{(2)}_{t} \). Hence, there is noise in the signal. Also, country 1’s investors understand that, while they try to estimate \( \varepsilon^{(2)}_{t} \), the investors of country 2 undergo a similar type of reasoning in trying to estimate \( \varepsilon^{(1)}_{t} \) based on the price changes in country 1. Hence, investors in country 1 will adjust their expectations accordingly. Symmetric reasoning applies to the investors in country 2.
When we assume the distributions of the stochastic news processes and the values of the model parameters are common knowledge, the investors can solve their signal extraction problems. The minimum-variance estimators are

\[ E_i \left( \epsilon_i^{(j)} \right) = \lambda_j \left[ \Delta S_i^{(j)} - \alpha_j E_j \left( \epsilon_j^{(i)} \right) \right], \]

where

\[ \lambda_j = \frac{\sigma^2_{\epsilon^{(j)}}}{\sigma^2_{\epsilon^{(j)}} + \sigma^2_{\nu^{(j)}}} \]

for \( i, j = 1, 2 \) and \( i \neq j \). Here \( \sigma^2_x \) denotes the (known) variance of \( x \). By substituting these estimators into equations (2.2) and (2.3), and using the decomposition of country \( i \) news \( \eta_i^{(i)} \) in equation (2.1), we get

\[ \Delta S_t^{(1)} = (1 - \alpha_{12}\lambda_1\lambda_2) \eta_t^{(1)} + \alpha_{12}\lambda_2 \Delta S_t^{(2)}, \quad (2.4) \]

\[ \Delta S_t^{(2)} = (1 - \alpha_{12}\lambda_1\lambda_2) \eta_t^{(2)} + \alpha_{21}\lambda_1 \Delta S_t^{(1)}. \quad (2.5) \]

Because the parameters \( \alpha \) and \( \lambda \) are not separately identifiable, we combine them into a new parameter \( \beta \):

\[ \beta_{ij} = \alpha_{ij}\lambda_j \quad (2.6) \]

for \( i, j = 1, 2 \) and \( i \neq j \). Solving the system of equations (2.4)–(2.5) with respect to the price changes yields us the equilibrium laws of motions for the stock market returns as a function of the news variables \( \eta_t^{(1)} \) and \( \eta_t^{(2)} \):

\[ \Delta S_t^{(1)} = \eta_t^{(1)} + \beta_{12}\eta_t^{(2)}. \quad (2.7) \]

and

\[ \Delta S_t^{(2)} = \beta_{21}\eta_t^{(1)} + \eta_t^{(2)}. \quad (2.8) \]

The variances and covariances of the market returns are

\[ \text{Var} \left( \Delta S_t^{(1)} \right) = \sigma^2_{\eta^{(1)}} + (\beta_{12})^2 \sigma^2_{\eta^{(2)}}. \quad (2.9) \]

\[ \text{Var} \left( \Delta S_t^{(2)} \right) = \sigma^2_{\eta^{(2)}} + (\beta_{21})^2 \sigma^2_{\eta^{(1)}}. \quad (2.10) \]

\[ \text{Cov} \left( \Delta S_t^{(1)}, \Delta S_t^{(2)} \right) = \beta_{21}\sigma^2_{\eta^{(1)}} + \beta_{12}\sigma^2_{\eta^{(2)}}. \]
This system consists of four unknown parameters \((\sigma_{\eta(1)}, \sigma_{\eta(2)}, \beta_{12}, \beta_{21})\) and three equations, so the KW model is unidentified. In section 2.3, we will augment the model with a specific distributional assumption. This additional assumption, it is argued, provides us the necessary additional information for the identification.

### 2.2.2 Volatility Spillovers

From equations (2.9) and (2.10) we can solve, for example,

\[
Var\left(\Delta S_{t}^{(1)}\right) = (1 - \beta_{12}^2 \beta_{21}) \sigma_{\eta(1)}^2 + \beta_{12}^2 \sigma_{\Delta S_{t}^{(2)}}^2.
\]

The volatility of country 1’s market returns would be the square root of this. Clearly, whenever, \(\beta_{12} = 0 (\beta_{21} = 0)\), there is not any volatility spillovers from country 2 (1) to country 1 (2). Also, the greater is the absolute value of \(\beta_{ij}\), the greater will be the volatility spillover effect from country \(j\) to country \(i\).

This said, a testing of the volatility spillovers boils down to testing whether in our structural model

\[
\beta_{12} = \beta_{21} = 0 \text{ (no spillovers)}
\]

or

\[
\beta_{ij} \neq 0 \text{ for some } i \neq j \text{ (some spillovers).}
\]

Notice that, if for example we had \(\beta_{12} = 0\) together with \(\beta_{21} \neq 0\), we would have spillovers from country 1 to country 2 but not vice versa.

It seems worthwhile to shortly consider how, for example, a finding \(\beta_{12} = 0\) should be interpreted. Remember that \(\beta_{12} = \alpha_{12} \lambda_2\), so as long as we assume that both information variables observed in country 2 are genuine random variables (both \(\sigma_{\epsilon(2)}^2\) and \(\sigma_{v(2)}^2\) are non-zero real numbers which means \(\lambda_2 \neq 0\)), then, whenever \(\beta_{12} = 0\), we must have \(\alpha_{12} = 0\). So, there would not be any volatility spillovers from country 2 to country 1, for the simple reason that the systematic information observed in country 2 is not considered relevant for the equity valuations in country 1. The investors in country 1 would know this and, hence, they would not try to infer information component \(\epsilon_{(2)}\) from the price changes in country 2’s markets. There would still, however, exist the information asymmetry in the sense that the investors in country 1 would not be able to directly observe \(\epsilon_{(2)}\). Only this time, the information asymmetry would not matter.

Conversely, the greater is the relevance of \(\epsilon_{(2)}\) on the equity valuations in country 1, that is the greater is the absolute value of \(\alpha_{12}\), the greater will–\(ceteris paribus\)–the volatility spillover
effect be. Notice, however, that because we are unable to identify parameter $\alpha_{12}$ from $\lambda_2$, simply by observing a large (estimated) value of parameter $\beta_{12}$ does not tell us whether or not the systematic information in country 2 is very relevant for country 1’s markets. A large value of $\beta_{12}$ could also be the result of large $\lambda_2$ which would mean the variance of $\varepsilon^{(2)}$ is large compared to the variance of country 2 specific idiosyncratic information $v^{(2)}$.

### 2.2.3 General Model of Volatility Spillovers

As King and Wadhwani show in their paper, the KW model generalizes to a multiple country case in a straightforward manner. Assume $n \geq 2$. The countries’ stock market price changes are given by the equation below (comparable to equations (2.2) and (2.3))

$$\Delta S_t = \eta_t + Ae_t,$$  \hspace{1cm} (2.11)

where $\Delta S_t$ is a $n \times 1$ vector of the price changes at period $t$, $\eta_t$ is a $n \times 1$ vector of news at period $t$ with the typical element

$$\eta_t^{(i)} = \epsilon_t^{(i)} + v_t^{(i)}$$

being the news released in country $i$, $A$ is a $n \times n$ coefficient matrix with the typical element $\alpha_{ij}$, $i, j = 1, \ldots, n$, and $\alpha_{ii} = 0$ for all $i = 1, \ldots, n$ (all the main diagonal elements), and finally $e_t$ is a $n \times 1$ vector of the conditional expectations on the systemic informations $\varepsilon^{(i)}$, $i = 1, \ldots, n$, held by the (foreign) investors in the markets $j \neq i$ at period $t$.

The solution to the signal extraction problem is

$$e_t = \Lambda (\Delta S_t - Ae_t),$$  \hspace{1cm} (2.12)

where $\Lambda$ is a $n \times n$ diagonal matrix with parameter $\lambda_i$ as the $i$th element on its main diagonal. Then, by combining equations (2.11) and (2.12), and solving for $\Delta S_t$, one gets the laws of motion of the price changes in the $n$ market case as a function of the news in the $n$ countries:

$$\Delta S_t = (I_n + B) \eta_t$$  \hspace{1cm} (2.13)

where $B = AA$ is a $n \times n$ matrix, and its $ij$th element $\beta_{ij}$ is the response of the market $i$ prices to the news in market $j$. The matrix $I_n$ is a $n \times n$ identity matrix. Again, a test for volatility spillovers, for example, from country $j$ to country $j$ should test whether or not the element $\beta_{ij}$ equals zero. Notice that by construction the main diagonal elements $\beta_{ii}$ for all $i = 1, \ldots, n$ are
2.2.4 On Information Asymmetry

In the KW model it is assumed that only domestic investors are able to correctly interpret news on their country. Foreigners remain uninformed. Is there any empirical support for such information asymmetries?

Frankel & Schmukler (1996) analyze differences in the Mexican stock market valuations and the valuations of the Mexican closed-end country funds that were traded in the New York Stock Exchange (NYSE). They argue that around the Mexican devaluation in December 1994 the local investors were better informed than the international investors, and so they were more pessimistic and better prepared to react to negative local news.

More recently, Chen & Choi (2012) analyze the stock market valuations of the 56 Canadian companies listed both in the Toronto Stock Exchange (TSX) and the NYSE. They find evidence of the local (TSX) investors being better informed than the foreign (NYSE) investors. This information asymmetry, they argue, explains the small share price premiums detected in the NYSE prices over the TSX prices of these companies. Also, according to Chan et al. (2008) the information asymmetry between Chinese and foreign investors is able to explain the price differences between the locally owned (A-)shares and the foreign own (B-)shares of the Chinese companies.

2.2.5 The KW Model & the Theoretical Contagion Literature

In the KW model, because of asymmetric information, the idiosyncratic country shocks are transmitted across the borders. The authors also show that the correlations between the countries’ market returns are higher in the KW model than in a comparable full information model. Hence, the authors label the KW model as a "contagion" model. This lexicon is in-line with the theoretical contagion literature where many authors define contagion as transmission of an idiosyncratic shock–or crisis–from one country to other countries.\footnote{See, for example, Kaminsky & Reinhart (2000), Kodres & Pritsker (2002), Corsetti et al. (2005), Dungey et al. (2005), Pesaran & Pick (2007). For surveys on the contagion literature, see for example Pericoli & Sbracia (2003), Dungey et al. (2005), Dornbusch et al. (2000), Forbes (2012). The empirical contagion literature defines contagion slightly differently, see for example Forbes & Rigobon (2002).}

Similar to the KW model, Kodres & Pritsker (2002) analyze contagion in a set up with informed and uniformed investors. Then, in their model also, contagion is the consequence of the uninformed investors trying to infer the informed investors’ private information from the price changes. One interesting insight of the Kodres and Pritsker analysis is that the "magnitude"
of contagion depends on the relative share of the informed investors over the uninformed ones. Hence, for any given number of uninformed investors, increasing the number of informed investors, in the limit, makes contagion vanish. The intuition is that, as the relative share of the informed investors increases, the price system becomes more informative than before and, hence, better reflects the informed investors’ private information.

Both, King and Wadhwani as well as Kodres and Pritsker, consider a case where the numbers of informed and uninformed investors are exogenously given. However, because information is costly (individual time and effort, payments for professional analysts, etc.), some investors might actually prefer to stay ignored about the detailed macroeconomic conditions of a country. Calvo & Mendoza (2000) analyze investors’ incentives to pay information costs. In their model an investor chooses either to pay for information on a country’s fundamentals or to remain uninformed. In the latter case she would simply track a generic, global stock market index. Hence, the number of informed investors becomes a model endogenous variable. The authors show that, once there are exogenous information costs, or binding institutional or legislative constraints on short-selling, the more global the world financial markets become, the more tempting it is for a rational investor to stay uninformed. The intuition behind the result is that, for example, the constraints on the short-selling limit the informed investors’ opportunities to benefit from their private information. Hence, the expected value of country specific information decreases. This, then, decreases the incentives to pay for such information. Meanwhile, more global financial markets permit investors to more easily, by following a general market index, enjoy the benefits of asset diversification.

2.3 Estimation of the Structural Model

Consider the KW model with \( n \) countries in equation (2.13). By redefining

\[
\mathbf{u}_t = (\mathbf{I}_n + \mathbf{B}) \eta_t,
\]

we get the following simple identity:

\[
\Delta S_t = \mathbf{u}_t. \tag{2.14}
\]

Equation (2.14) can be interpreted as a zero order reduced form vector autoregressive (VAR) model. The vector \( \mathbf{u}_t \) is the reduced form error vector. Throughout the paper the stochastic process described by this VAR model is assumed to be stationary. When considering the fact that the data will consist of the changes in the stock market price indexes, this assumption seems reasonable.
Alternatively, we can redefine
\[ \tilde{B} = (I_n + B). \]

In this case, we can write equation (2.13) as
\[ \Delta S_t = \tilde{B} \eta_t. \] (2.15)

This, in its turn, can be interpreted as a zero order SVAR model. The \( n \times 1 \) random vector \( \eta_t \) (the KW model’s news vector) denotes the model’s structural shocks which are uncorrelated to each other. By combining equations (2.14) and (2.15) we then get the reduced form errors as a function of the structural shocks (news);
\[ u_t = \tilde{B} \eta_t. \] (2.16)

This representation corresponds to the so-called B-model framework of the SVAR model (see, for example, Lütkepohl (2007, 362–64)) where the \( n \)-dimensional reduced form error term \( (u_t) \) depends on the \( n \) structural shocks \( (\eta_t) \) via a \( n \times n \) coefficient matrix \( (\tilde{B}) \). The fundamental question of the SVAR literature is how to estimate the coefficient matrix and, hence, identify the structural shocks.

If we denote the covariance matrix of the reduced from errors as \( \Sigma_u \) and that of the structural shocks as \( \Sigma_\eta \) (which is by assumption diagonal), we get from equation (2.16) that
\[ \Sigma_u = \tilde{B} \Sigma_\eta \tilde{B}'. \] (2.17)

Typically, a SVAR model is normalized by assuming \( \Sigma_\eta = I_n \). This gives us the following system of \( n \) equations:
\[ \Sigma_u = \tilde{B} \tilde{B}'. \] (2.17)

where \( \Sigma_u \) can be estimated consistently with standard estimation methods. However, as the matrix \( \tilde{B} \) consists of \( n^2 \) unknown parameters and equation (2.17) provides us only \( n (n + 1)/2 \) independent equations (a covariance matrix is always symmetric), we need some extra information to be able to estimate the matrix \( \tilde{B} \).

One standard method to identify a SVAR model is to use economic theory or institutional knowledge to directly restrict (to zero) sufficiently many elements of \( \tilde{B} \). Other methods include, for example, restricting the signs of the impulse responses of the system, or restricting long-
run effects of the structural shocks on the observed variables.\textsuperscript{12} However, most of the standard identification methods are not suitable for our SVAR model. In our context, the non-diagonal elements of $\tilde{B}$ are the volatility spillover coefficients. Our very goal is to estimate them and test whether or not some (or all) of them equal to zero.

### 2.3.1 Identification Based on Non-normalities

Assume the reduced form errors, that is the stock market returns, follow a mixed-normal distribution:

$$u_t = \begin{cases} 
  u_{1t} \sim N(0, \Sigma_1) & \text{with probability } \gamma, \\
  u_{2t} \sim N(0, \Sigma_2) & \text{with probability } 1 - \gamma.
\end{cases} \tag{2.18}$$

Here $N(0, \Sigma)$ refers to a multivariate normal distribution with zero mean and a $n \times n$ covariance matrix $\Sigma$. The parameter $\gamma \in (0, 1)$ is the mixture probability. In order to $\gamma$ being identifiable, the covariance matrices $\Sigma_1$ and $\Sigma_2$ must be distinct. Parts of $\Sigma_1$ and $\Sigma_2$ may still be identical.

According to the distributional assumption (2.18), the random vector $u_t$ has zero as its mean and the covariance matrix $\Sigma_u = \gamma \Sigma_1 + (1 - \gamma) \Sigma_2$. Notice that the distributional assumption does not violate the assumptions of the KW model. In the KW model it is only assumed that the news (elements of $\eta_t$) are uncorrelated.\textsuperscript{13} Because the stock market price changes are the result of the news being multiplied by the matrix $\tilde{B}$, the price changes do not need to be uncorrelated. The KW model does not make any further assumptions on how the stock market price changes are distributed. We, however, assume that they follow a mixed-normal distribution. Also notice that the distribution (2.18) is non-normal. Because non-normality is a general feature of financial time series, the assumption seems reasonable also in this respect.

Lanne & Lütkepohl (2010) show that, given the mixed-normal distribution (2.18), there exists a diagonal matrix $\Psi = \text{diag}(\psi_1, \ldots, \psi_n)$ with $\psi_i > 0$ for all $i = 1, \ldots, n$, and a nonsingular $n \times n$ matrix $W$ such that $\Sigma_1 = WW'$ and $\Sigma_2 = W\Psi W'$.\textsuperscript{14} As long as all the elements $\psi_i > 0$ are distinct, the matrix $W$ is unique apart from changing all signs in a column. The covariance matrix of the reduced form error vector $u_t$ can then be written as

$$\Sigma_u = \gamma WW' + (1 - \gamma) W\Psi W' = W(\gamma I_n + (1 - \gamma) \Psi)W' \tag{2.19}$$

\textsuperscript{12}Kilian (2011) provides a good survey on the different SVAR model identification methods.

\textsuperscript{13}Notice that the KW model assumes that the agents know the distribution that the news follow. The agent’s problem has stayed the same even though we, as econometricians, assume that the observed market returns follow a mixed-normal distribution.

\textsuperscript{14}The decomposition of the matrices $\Sigma_1$ and $\Sigma_1$ is the result of them being symmetric and positive definite matrices. For details, see the appendix in Lanne & Lütkepohl (2010, 167).
A comparison of this with equation (2.17) lets us to choose

\[ \hat{B} = W (\gamma I_n + (1 - \gamma) \Psi)^{1/2}. \]  

(2.20)

The model given by the reduced form representation in equation (2.14), the distribution (2.18), and the decomposition of the covariance matrices \( \Sigma_1 \) and \( \Sigma_2 \) can be estimated by the method of maximum likelihood (ML). The distribution of \( \Delta S_t \) can be written as\(^{15}\)

\[
f(\Delta S_t) = \gamma \det(W)^{-1} \exp \left\{ -\frac{1}{2} \Delta S_t' (WW')^{-1} \Delta S_t \right\} \\
+ (1 - \gamma) \det(\Psi)^{-1/2} \det(W)^{-1} \exp \left\{ -\frac{1}{2} \Delta S_t' (W\Psi W')^{-1} \Delta S_t \right\}.
\]

Collecting all the parameters into the vector \( \Theta \), the log-likelihood function becomes

\[
l_T (\Theta) = \sum_{t=1}^{T} \log f(\Delta S_t).
\]

This can be maximized with the standard nonlinear optimization algorithms.

There is one severe limitation in a straightforward application of the identification method of Lanne and Lütkepohl to the KW model. The uniqueness of the coefficient matrix \( \hat{B} \) in equation (2.20) depends on the chosen, or assumed, order of the elements \( \{\psi_1, \ldots, \psi_n\} \) on the main diagonal of the matrix \( \Psi \). Hence, without any \textit{a priori} information on the correct these elements, we have in total \( n! \) possible B-matrices.

This can be seen in the following way. As it is formally shown in the appendix of this chapter (page 42), when equation (2.20) holds, we can equally well choose as our B-matrix the following matrix \( \hat{B} \):

\[
\hat{B} = (WP')(\gamma I_n + (1 - \gamma) P\Psi P')^{1/2} \\
= \hat{W} \hat{\Psi}^{1/2}.
\]

Above \( P \) is an arbitrary \( n \times n \) permutation matrix, \( \hat{W} = WP' \), and \( \hat{\Psi} = \gamma I_n + (1 - \gamma) P\Psi P' \). The matrix \( P\Psi P' \) is diagonal with a different permutation of the elements \( \{\psi_1, \ldots, \psi_n\} \) on its main diagonal than the matrix \( \Psi \). The matrix \( \hat{W} \) is simply a column-wise permutation of \( W \).

Clearly \( \hat{B} \) in equation (2.20) is not equivalent to the matrix \( \hat{B} \) unless \( P = I_n \). The permutation matrix \( P \) was arbitrary and there are \( n! \) possible permutations. This means that there are equally many matrices \( \hat{B} \) (the matrix \( \hat{B} \) in equation (2.20) included, corresponding to \( P = I_n \)). The

\(^{15}\)For details about deriving a conditional density for a VAR model with lagged values of the dependent variable, see Lanne & Lütkepohl (2010).
implication is that the B-matrix of the structural model is unique up to the permutation of the elements \( \{\psi_1, \ldots, \psi_n\} \); any permutation of these elements corresponds to only one permutation matrix \( P \).\(^{16}\)

Lanne et al. (2010) note the sensitivity of the estimated matrix \( \tilde{B} \) to different permutations of the main diagonal elements of \( \Psi \) (this is also discussed in Herwartz & Lütkepohl (2011), and Lütkepohl (2012)). Hence, they propose to use either the order of the elements \( \psi_i \) from the smallest to the largest, or from the largest to the smallest. However, nothing guarantees that either of these two permutations would identify the correct permutation of the KW model. Here, an alternative approach to identify the correct B-matrix is proposed.

### 2.3.2 Full Identification of the Model

Recall the identity in equation (2.16) between the reduced form error vector \( u_t \) (stock market price changes) and the structural shocks vector \( \eta_t \) (news). Given any permutation of the elements \( \{\psi_1, \ldots, \psi_n\} \), the Lanne and Lütkepohl identification method guarantees a locally unique matrix \( \tilde{B} \). Assume this matrix is also invertible. Then, by premultiplying the equation (2.16) with \( \tilde{B}^{-1} \), we get

\[
\eta_t = \tilde{B}^{-1} u_t.
\]

So, the news are now written as a function of the stock market returns. Then, we are able to calculate the covariance matrix of the news (\( \Sigma_\eta \)) as a function of (the estimated) matrix \( \tilde{B} \) and the covariance matrix of market volatilities \( \Sigma_u \):

\[
\Sigma_\eta = \tilde{B}^{-1} \Sigma_u (\tilde{B}'^{-1}). \tag{2.21}
\]

In the KW model the news covariance matrix \( \Sigma_\eta \) is a diagonal but not necessarily an identity matrix. So, the equation above is not a trivial identity but estimates the variances of the countries’ news variables \( \{\sigma^2_{\eta(i)}\} \) as a function of the reduced form errors’ covariance matrix and the estimated matrix \( \tilde{B} \) which in turn depends on the chosen order of the diagonal elements of \( \Psi \). In particular, we get that the ranking order for the variances \( \{\sigma^2_{\eta(1)}, \ldots, \sigma^2_{\eta(n)}\} \) depends on the specific matrix \( \Psi \) that we have used.

Hence, if we could find—from exogenous sources—some proximate variable for countries’ news, we would be able to get an alternative estimate for their variances. Especially, we are interested

\(^{16}\)This can be seen in the following way: assume we have some given permutation of the elements \( \{\psi_1, \ldots, \psi_n\} \) on the main diagonal of the matrix \( \Psi^* \) so that it is a result of the matrix multiplication \( P \Psi P' \), where \( \Psi = \text{diag}(\psi_1, \ldots, \psi_n) \) and \( P \) is an arbitrary permutation matrix. Because the matrices \( \Psi \) and \( P \) are nonsingular, \( \Psi^* = P \Psi P' \) is a bijection. So, whenever we fix the order of the main diagonal elements of \( \Psi^* \), we fix the permutation \( P \), and vice versa.
in the countries’ ranking based on these variances of the proximate news variables. If the countries’ order based on the ranking is unambiguous (unique) in such a way that no two or more countries share same ranking, we can use this ranking and equation (2.21) to identify the correct permutation of our model. We simply select among our $n!$ possible B-matrices the one that produces the same ranking of the countries based on their news’ variances as does our alternative news variable. In our empirical application, data from the Google trends is used to calculate a proximate news variable.

As a last note, an attentive reader might have noticed that in the beginning of this section 2.3 we assumed $\Sigma_\eta = I_n$ whereas here $\Sigma_\eta$ is diagonal but not necessarily an identity matrix. There is no contradiction here because these are only two alternative ways to normalize a SVAR model. First, when we use the Lanne-Lütkepohl method to identify the matrix $\tilde{B}$, we use the first normalization. Once we have identified the correct permutation of the B-matrix (notice that the countries’ order based on ranking does not depend on the chosen normalization), we need to swap to the normalization that was assumed in the KW model, namely allow the values of the diagonal elements of $\Sigma_\eta$ vary freely but restrict the main diagonal elements of the matrix $\tilde{B}$ to one. As shown in the appendix (page 44), this swap from the first normalization to the latter can be easily done: on each column $k = 1, \ldots, n$ of matrix $\tilde{B}$ provided by equation (2.20), divide all the elements $[\tilde{B}]_{ik}, i = 1, \ldots, n$, by the main diagonal element $[\tilde{B}]_{kk}$ of the column.

### 2.3.3 Testing the Volatility Spillovers

Once we have estimated the KW model’s volatility spillover parameters $\beta_{ij}$ for all $i \neq j$, we can test these spillovers across the countries. As suggested in section 2.2.2, we should test for the existence of volatility spillovers from country $j$ to country $i$ by comparing the unrestricted model where $\beta_{ij} \neq 0$ against the restricted model where $\beta_{ij} = 0$. As the model is estimated with the method of ML, the likelihood ratio (LR) test is a natural candidate for the test statistic. Given that the underlying stochastic process is assumed to be stationary, we can use the standard asymptotic distribution results of the ML estimation methodology.

Any restriction $\beta_{ij} = 0$ can be implemented by restricting to zero the corresponding element of the matrix $W$, that is by imposing the restriction $w_{ij} = 0$. This can easily be seen by considering equation (2.20), where the part $(\gamma I_n + (1 - \gamma) \Psi)^{1/2}$ is diagonal by construction with all the main diagonal elements $\gamma + (1 - \gamma) \psi_i$ being strictly positive. Because the coefficient of volatility spillover from country $j$ to country $i$ is

$$\beta_{ij} = [\tilde{B}]_{ij} = w_{ij} (\gamma + (1 - \gamma) \psi_i)^{1/2},$$
it is clear that

$$
\beta_{ij} = 0 \iff w_{ij} = 0.
$$

This same reasoning applies to any possible permutation of the estimated B-matrix

$$
B = W^{1/2}
$$

that were considered earlier. The matrix $\Psi$ is always diagonal with strictly positive main diagonal elements. Hence, we only need to consider restrictions on the elements of the matrix $W$.

### 2.4 Empirical Application: The Eurozone Stock Markets 2010–2011

As an example of how to estimate and test the volatility transmission parameters of the KW model, we consider the European stock markets. Since early 2010–or late 2009–the eurozone has been in the middle of a sovereign debt crisis. The countries we include into our sample are Germany, Greece, Ireland, Italy, and Spain. Figure 2.1 shows how the equity price indexes changed in these countries during our sample period, the years 2010–2011 (indexes have been rescaled; for details about the data, see the appendix (page 45)). During this period, the stock market prices decreased by almost 80 percent in Greece, in Spain and Italy by around 30 percent, and by less than ten percent in Germany and Ireland.

#### 2.4.1 Data: Daily Stock Market Returns

The data we will use represents the countries’ daily stock market returns. Consistent to the KW model, the daily returns are approximated by the first differences of the logarithmic transformations of the closing values of the price indexes:

$$
\Delta S^i_t = \log P^i_{C,t} - \log P^i_{C,t-1},
$$

where $P^i_{C,t}$ denotes the closing value of the price index in country $i$ at date $t$, and $i \in \{ITA, SPA, IRE, GRE, GER\}$ (for shortenings, see table 2.1).

During the sample period, there were 517 business days. So, this gives us 516 observations of the daily returns per country. However, for reasons of national banking holidays, each country has missing values (for details, see the appendix (page 45)). I have substituted any missing value with the previous available observation. Table 2.1 summarizes the data. Clearly, the empirical
distributions of the returns are not normal. This supports the idea of using a non-normal error
distribution in our VAR model (see equation (2.18)).

2.4.2 Estimation of the KW Model

In order to identify the sources of spillovers for each particular country, we need to fully identify
the KW model. To do this, section 2.3.2 suggests that we use some available (exogenous) news
variable, call it $\chi_{it}$ for country $i$. Then, for each country $i$, we can use the variance of this
outside-the-model news variable ($\sigma^2_{\chi_i}$) as a proxy for the variance of the KW model’s structural
shock ($\sigma^2_{\eta_i}$). Whenever the countries’ ranking order according to the variances of the alternative
news variables is unambiguous, in order to identify the KW model, we look after the permutation
of the model that sets the estimated structural shock variances into the same ordering.

As an example, assume that the variances of the alternative news variables are ordered in the following way: \( \sigma^2_{\chi_2} > \sigma^2_{\chi_3} > \sigma^2_{\chi_1} > \sigma^2_{\chi_4} > \sigma^2_{\chi_5} \). Then we try to find the permutation of the structural model that sets the countries’ (estimated) structural shock variances into the same order, namely into the order \( \sigma^2_{\eta_2} > \sigma^2_{\eta_3} > \sigma^2_{\eta_1} > \sigma^2_{\eta_4} > \sigma^2_{\eta_5} \). As it is shown in equation (2.21), the selection of the correct permutation of the structural shock boils down to selecting the correct B-matrix. There are five countries in the sample. This means there are 120 possible B-matrices.

**The unrestricted model: Estimated spillover effects**

The data that we use as a proxy for news is the changes in the global search volumes in Google on the economic conditions of our sample countries (figure 2.2). The data covers weekly observations on the search traffic about each country between 2010 and 2011. (More details on the Google Trends data are provided in the appendix, page 45.) For example, in one week of spring 2010, internet searches on the Greek economy increased around 200 percentages from the previous week. It would make sense to assume that such a heavy increase in the search traffic concerning the Greek economy reflects new information available to the markets about the country’s economic performance.¹⁷

Our identification will then be based in the countries’ descending ranking order according to the variances of the changes in the Google searches. Table 2.2 reports these variances and the countries’ subsequent ranking order. The changes in the Google traffic on Italy has the largest variance. Next comes Greece, then Spain, Ireland, and finally Germany. It must be said that the chances in Google searches are not perfect proxies for our structural shocks because the changes in search volumes are not uncorrelated to each other as they (ideally) should be. The sample correlation coefficients between the countries series range from 0.1 between Italy and Greece to 0.4 between Spain and Greece. As the next paragraph discusses this correlation between the weekly changes in the Google searches causes some problems for our model identification. Section 2.4.3 will discuss little further the strengths and weaknesses of the Google search data and considers an alternative variable.

There are only two permutations of the (estimated) KW model–among the possible 120 models–that create the same ranking of the countries according to the estimated variances of the countries’ structural shocks as in table 2.2. The reason that we are not able to identify one single permutation probably lies in the fact that the changes in Google searches are not uncorrelated to

¹⁷Of course, the peak coincides with the onset of the euro debt crisis and the first Greek bailout package, but even so, this does not contradict with what is said in the text.
Figure 2.2: Weekly percentage changes in Google search volume indexes and stock exchange trading volumes

Source: Google Trends, Bloomberg, Yahoo! Finance, own calculations.
Table 2.2: Variances of changes in Google search volumes

<table>
<thead>
<tr>
<th>Country</th>
<th>Variance</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italy</td>
<td>3833.0</td>
<td>1</td>
</tr>
<tr>
<td>Greece</td>
<td>1842.0</td>
<td>2</td>
</tr>
<tr>
<td>Spain</td>
<td>1183.0</td>
<td>3</td>
</tr>
<tr>
<td>Ireland</td>
<td>367.0</td>
<td>4</td>
</tr>
<tr>
<td>Germany</td>
<td>339.0</td>
<td>5</td>
</tr>
</tbody>
</table>

Source: Google Trends, own calculations.

Table 2.3: Parameter estimates of the KW model (estimated standard errors in parentheses)

<table>
<thead>
<tr>
<th>Elements of each vector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W[1,\cdot] \times 100$</td>
<td>0.81***</td>
<td>-0.11</td>
<td>0.06</td>
<td>0.10</td>
<td>-0.67</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.29)</td>
<td>(0.23)</td>
<td>(0.19)</td>
<td>(0.38)</td>
</tr>
<tr>
<td>$W[2,\cdot] \times 100$</td>
<td>0.62</td>
<td>0.34</td>
<td>0.02</td>
<td>0.31</td>
<td>-0.77**</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(0.30)</td>
<td>(0.57)</td>
<td>(0.18)</td>
<td>(0.30)</td>
</tr>
<tr>
<td>$W[3,\cdot] \times 100$</td>
<td>0.56***</td>
<td>0.31</td>
<td>0.29</td>
<td>-0.50</td>
<td>-0.41</td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.22)</td>
<td>(0.85)</td>
<td>(0.51)</td>
<td>(0.28)</td>
</tr>
<tr>
<td>$W[4,\cdot] \times 100$</td>
<td>0.64***</td>
<td>0.25</td>
<td>1.71</td>
<td>0.76</td>
<td>-0.24</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.49)</td>
<td>(1.28)</td>
<td>(2.97)</td>
<td>(0.37)</td>
</tr>
<tr>
<td>$W[5,\cdot] \times 100$</td>
<td>0.80***</td>
<td>0.22</td>
<td>-0.06</td>
<td>0.05</td>
<td>-0.19</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.19)</td>
<td>(0.16)</td>
<td>(0.16)</td>
<td>(0.37)</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>7.29***</td>
<td>3.86***</td>
<td>2.57***</td>
<td>2.40***</td>
<td>5.64***</td>
</tr>
<tr>
<td></td>
<td>(1.19)</td>
<td>(0.60)</td>
<td>(0.39)</td>
<td>(0.42)</td>
<td>(0.87)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.63***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Standard errors obtained from the inverse Hessian of the log-likelihood function. $W[i,\cdot]$ indicates $i$th row of matrix $W$. 

Each other. However, these two alternative models give quite different results, something that we can use to select the most plausible model between our two candidates. Especially, when it comes to the effects of the German news on the other countries’ stock markets, according to one of the two identified permutations a negative shock to German stock prices would increase the prices in some countries whereas according to the other permutation a negative shock to German stock prices always decreases prices in the other countries. Given that Germany is the core country of the eurozone and it is also its safety haven, it seems to me that this last permutation is the most plausible one. Table 2.3 reports the estimation results of this model.  

The first five rows in table 2.3 correspond to the five rows of the matrix $W$ (for reporting purposes, the parameter estimates and their estimated standard errors are multiplied by 100). The sixth row reports the estimated main diagonal elements of the matrix $\Psi$. Notice that the

18 All calculations were done with the programs in the GAUSS CMLMT library.
estimated matrices $W$ and $\Psi$ do not have any particular interpretation alone but they are used to calculate the (identified) $B$-matrix. The last row of table 2.3 reports the estimated mixture probability $\gamma$. Hence, with a probability of around 63.0 percent the reduced form error vector $u_t$ is from the multi-normal distribution with smaller variances (the estimated covariance matrices $\Sigma_1$ and $\Sigma_1$ are not reported here, but it is the $\Sigma_1$ that corresponds to the "regime" with less volatility).

It is easier to interpret the results of table 2.3 once we have calculated the corresponding $B$-matrix. By using the formula in equation (2.20), we get the following $B$-matrix$^{19}$:

$$\begin{bmatrix}
\Delta S_t^{ITA} \\
\Delta S_t^{SPA} \\
\Delta S_t^{IRE} \\
\Delta S_t^{GRE} \\
\Delta S_t^{GER}
\end{bmatrix} =
\begin{bmatrix}
1.00 & -0.32 & 0.22 & 0.13 & 3.58 \\
0.76 & 1.00 & 0.06 & 0.41 & 4.08 \\
0.69 & 0.89 & 1.00 & -0.66 & 2.18 \\
0.79 & 0.74 & 5.96 & 1.00 & 1.26 \\
0.98 & 0.63 & -0.20 & 0.06 & 1.00
\end{bmatrix}
\begin{bmatrix}
\eta_t^{ITA} \\
\eta_t^{SPA} \\
\eta_t^{IRE} \\
\eta_t^{GRE} \\
\eta_t^{GER}
\end{bmatrix}. \quad (2.22)$$

Equation above corresponds to the KW model’s structural equation (2.16) where the off-diagonal elements $\beta_{ij}$, $i \neq j$, are the spillover coefficients. First, observe that, as one would assume, news in Germany and Italy has the greatest impact on the other countries. Positive news in these two countries increases, ceteris paribus, stock market valuations also elsewhere. These two are large eurozone countries, so their stock market performance probably both represents and affects international investors’ overall confidence. Positive news in these countries increases this confidence and, so, support investments in equities across the whole currency zone. However, for example, during the sample period, the effect of the German news on Italy is over three times greater ($\beta_{15} = 3.58$) than the effect of the Italian news on Germany ($\beta_{51} = 0.98$). It is also interesting that, in absolute terms, the effect of German news on Italy and Spain were roughly twice as big as on the two smaller countries. This probably reflects the importance of Italy and Spain on the future course of the eurozone and, also, Germany’s role as both the most important member country and the largest creditor which means it might have the last word when the eurozone tries to navigate itself out of the crisis.

Second, the news in Ireland (Greece) affect valuations in Greece (Ireland). In absolute value, the effect of Irish news on the Greece stock market valuations gets the largest value ($\beta_{13} = 5.96$) over all of the estimated coefficients. According to this result the news concerning Ireland caused

$^{19}$Notice that the structural model has been renormalized to correspond the KW model. For details, see the end of section 2.3.2, or the appendix.
much more uncertainty (higher stock market volatility) in Greece than the opposite way. Also, the effects between these two countries are not symmetric: positive news in Greece decreases prices in Ireland whereas the positive news in Ireland increases prices also in Greece. This result might be a particularity of the sample period and/or reflect two things: first, being two small, peripheral countries of the eurozone, Greece and Ireland might traditionally be each others substitutes in international investors’ portfolios—if stocks in both countries were equally safe, investors would choose the country with lower prices. Hence, the news in these countries would also affect the other via the substitution effect. Second, however, during the sample period, Ireland was possibly considered as more safe a country for investments than Greece. If then there was good news in Greece, all other things equal, investing in Greece might have seem less risky and, also, at the same time more compelling given the low price levels compared to those in Ireland.

As a third observation from the estimated effects in equation (2.22), notice that, as one would assume, on average the effects of small countries’ news on the large countries’ stock markets are quite small whereas the news in the large countries have relatively large effects on the valuations in the small countries. This said, to summarize the results, although the estimated effects probably to some extend reflect peculiarities of the sample period, they feel quite intuitive. Overall, there seem to be relatively large volatility spillover effects across the countries, but the effects are necessarily not symmetric.

The restricted model: Testing spillovers

Table 2.3 suggests that we might be able to restrict to zero some of the elements of $W$. This will also constitute our volatility testing as $\beta_{ij} = 0$ iff $w_{ij} = 0$ for all $i \neq j$ (see section 2.3.3). The fact that we know we are now working with the identified KW model means that the within country effects of each country’s structural shocks (the main diagonal elements of the $W$ matrix) must be non-zero.

Our first test is to see if there were any volatility spillovers between the countries at all. This means to test whether the matrix $\mathbf{B}$ is diagonal or not. If it is, then there were not any spillovers during our sample period. The null-hypothesis of no spillover effects is rejected with the LR test. The test statistic comparing the restricted model to the unrestricted gets the value 1846.2 which is clearly greater than the critical values of the $\chi^2$-distribution at any reasonable significance level with 20 degrees of freedom (all the off-diagonal elements of the matrix $W$).

Next, we will proceed stepwise by first restricting to zero that off-diagonal element $w_{ij}$, $i \neq j$, which has the largest p-value and test with the LR-test whether or not the restriction is rejected
Table 2.4: Estimation results of the restricted KW model (estimated standard errors in parentheses)

<table>
<thead>
<tr>
<th>Elements of each vector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W[1,\cdot]\times 100$</td>
<td>0.80***</td>
<td>..</td>
<td>..</td>
<td>0.06</td>
<td>−0.69**</td>
</tr>
<tr>
<td></td>
<td>(0.27)</td>
<td></td>
<td></td>
<td>(0.11)</td>
<td>(0.35)</td>
</tr>
<tr>
<td>$W[2,\cdot]\times 100$</td>
<td>0.61**</td>
<td>0.46***</td>
<td>..</td>
<td>0.29***</td>
<td>−0.73***</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.06)</td>
<td></td>
<td>(0.10)</td>
<td>(0.27)</td>
</tr>
<tr>
<td>$W[3,\cdot]\times 100$</td>
<td>0.54***</td>
<td>0.32***</td>
<td>0.45**</td>
<td>−0.43***</td>
<td>−0.38</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.12)</td>
<td>(0.18)</td>
<td>(0.15)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>$W[4,\cdot]\times 100$</td>
<td>0.67***</td>
<td>..</td>
<td>1.47***</td>
<td>1.14**</td>
<td>−0.26</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td></td>
<td>(0.36)</td>
<td>(0.51)</td>
<td>(0.35)</td>
</tr>
<tr>
<td>$W[5,\cdot]\times 100$</td>
<td>0.78***</td>
<td>0.27**</td>
<td>−0.03</td>
<td>..</td>
<td>−0.17</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.11)</td>
<td>(0.08)</td>
<td></td>
<td>(0.34)</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>7.36***</td>
<td>3.85***</td>
<td>2.58***</td>
<td>2.40***</td>
<td>5.59***</td>
</tr>
<tr>
<td></td>
<td>(1.20)</td>
<td>(0.61)</td>
<td>(0.39)</td>
<td>(0.42)</td>
<td>(0.87)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.63***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Standard errors obtained from the inverse Hessian of the log-likelihood function. $W[i,\cdot]$ indicates $i$th row of matrix $W$. (***)/(*.*) means statistical significance at 5%/1% level. Estimated elements of $W$ are multiplied by 100 for reporting purposes. Log-likelihood function gets value 7964.00.

at the 5% significant level. If it is not, in the second step, we will restrict to zero the off-diagonal element of $W$ that now has the largest p-value. We should continue this process until no-more zero restrictions on the off-diagonal elements $w_{ij}$, $i \neq j$, are supported by the data at the 5% significant level. This process leads us to the restricted model of table 2.4. There are five restricted $w_{ij}$ elements. The LR-test statistic jointly testing these restrictions gets value 0.89 which is below 11.1, the critical value of the $\chi^2$-distribution with five degrees of freedom and at the 5% significance level.

The next element of the $W$ with the largest p-value is the element $w_{53}$. However, when we restrict this element to zero we loose our identification; the countries’ structural shock variances are not anymore ordered as in table 2.2. This means that we would again need to identify from the possible 120 permutations the one which gives the correct ranking of the countries and incorporates all the already imposed zero restrictions. Unfortunately this is not an easy task because in each permutation the indexes of the already restricted elements change, making the process quite intractable. Hence, I feel that the marginal benefit, compared to the costs, of extra restrictions would be quite minimal, especially as the most evident restrictions have already been imposed.

It is, again, easiest, to interpret the estimation results in table 2.4 once we have calculated
the corresponding (restricted model) B-matrix which is the following:

\[
\begin{bmatrix}
\Delta S_t^{ITA} \\
\Delta S_t^{ESP} \\
\Delta S_t^{IRE} \\
\Delta S_t^{GRE} \\
\Delta S_t^{GER}
\end{bmatrix} =
\begin{bmatrix}
1.00 & \cdots & 0.05 & 4.08 \\
0.76 & 1.00 & 0.25 & 4.34 \\
0.67 & 0.70 & 1.00 & -0.37 & 2.27 \\
0.83 & \cdots & 3.26 & 1.00 & 1.53 \\
0.97 & 0.60 & -0.07 & \cdots & 1.00
\end{bmatrix}
\begin{bmatrix}
\eta_t^{ITA} \\
\eta_t^{ESP} \\
\eta_t^{IRE} \\
\eta_t^{GRE} \\
\eta_t^{GER}
\end{bmatrix}.
\] (2.23)

So, during the sample period, there were not statistically significant spillover effects from Ireland to Italy and Spain; from Spain to Italy and Greece; and from Greece to Germany. Also, the absolute values of the coefficients of Irish news on Germany \((\beta_{53} = -0.07)\) and Greek news on Italy \((\beta_{14} = 0.05)\) are relatively small. When interpreting these results, it is good to recall from section 2.2.2 that any spillover coefficient \(\beta_{ij}\) is zero iff the investors do not consider the country \(j\) news being relevant for the market valuations in country \(i\). Hence, again not surprisingly, we could roughly say that, except for the effects from Greece to Spain, the news in our sample’s small countries (Greece and Ireland) were not considered very relevant for the stock market prices in the large countries (Germany, Italy, and Spain) whereas the stock market prices in the small countries were susceptible to news in the large countries, again the effect of Spanish news on Greece is an exception.

Given that Spain is a large member country of the eurozone, it is perhaps surprising that we find the importance of news concerning Spain to have relatively low importance. However, it should be remembered that our sample period consists of the beginning of the euro crisis. Especially in the beginning of the crisis much of the media’s attention focused on the public finances of Greece, Ireland, and Italy, and less so on the Spanish public debt. This seems natural when we note that in 2010–2011 the public debt in Spain was still at a reasonable level. Table 2.5 reports both the countries’ government debt levels (as percentage of GDP) and their yearly percentage changes for the years 2008–2010. Clearly the Greek and Italian governments were heavily indebted already before the euro crisis, and, in addition, the Greek public debt level increased by around 14–15 percent per annum in 2009–2011. Ireland started with low levels of public debt but, due to high yearly growth rates, the Irish public debt was already over 100 percent of GDP in 2011. Spain, in contrast, had still in 2011 a lower public debt level than Germany.
Table 2.5: Government debt 2008–2011: levels (percentage of GDP) and yearly percentage changes

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Greece</td>
<td>112.9</td>
<td>5.1</td>
<td>129.7</td>
<td>14.9</td>
<td>148.3</td>
<td>14.3</td>
<td>170.6</td>
<td>15.0</td>
</tr>
<tr>
<td>Italy</td>
<td>106.1</td>
<td>2.7</td>
<td>116.4</td>
<td>9.7</td>
<td>119.2</td>
<td>2.4</td>
<td>120.7</td>
<td>1.3</td>
</tr>
<tr>
<td>Germany</td>
<td>66.8</td>
<td>2.5</td>
<td>74.5</td>
<td>11.5</td>
<td>82.5</td>
<td>10.7</td>
<td>80.5</td>
<td>-2.4</td>
</tr>
<tr>
<td>Ireland</td>
<td>44.5</td>
<td>77.3</td>
<td>64.9</td>
<td>45.8</td>
<td>92.2</td>
<td>42.1</td>
<td>106.4</td>
<td>15.4</td>
</tr>
<tr>
<td>Spain</td>
<td>40.2</td>
<td>10.7</td>
<td>53.9</td>
<td>34.1</td>
<td>61.5</td>
<td>14.1</td>
<td>69.3</td>
<td>12.7</td>
</tr>
</tbody>
</table>

Source: Eurostat, own calculations.

2.4.3 Model Diagnostic and Discussion

Because our model assumes the mixed-normal distribution, conventional residuals are not available for model diagnostics. For a univariate model with a distribution like ours, Kalliovirta (2012) proposes to use quantile residuals. According to Kalliovirta, two transformations define quantile residuals: first, one transforms observations into approximately independent uniformly distributed random variables by using the estimated cumulative distribution function (CDF) of the model. Second, by using the inverse of the CDF of the standard normal distribution, one transforms the uniformly distributed variables into random variables that are approximately independent and follow the standard normal distribution. These results assume that the model is correctly specified. Hence, we can study the validity of the model by inspecting the properties of the quantile residuals.

Kalliovirta & Saikkonen (2010) propose a generalization of quantile residuals for multivariate models and, similarly to the univariate case, develop tests for normality, autocorrelation and conditional heteroskedasticity of the quantile residuals. Again, the same two transformations of the observations as in the univariate case define the multivariate quantile residuals. However, as we are dealing with a $n \times 1$ residual vector, in transforming the observations into uniformly distributed random variables, we need to use the CDFs of each county’s marginal distribution at period $t$, conditional on the countries’ order and the other countries’ observations at period $t$. Kalliovirta and Saikkonen define univariate joint quantile residuals $Q_t$ that, at each period $t$, incorporate into a single variable the $n \times 1$ vector of quantile residuals\textsuperscript{20}. If our model is correctly specified, the joint quantile residuals are independently distributed, homoskedastic, and follow the standard normal distribution. Hence, any great deviation from these characteristics should be considered as a case against the validity of our model.

Figure 2.3 plots the QQ-plot of the joint quantile residuals holds. Except for a few outliers

\textsuperscript{20}For further details on joint quantile residuals, see Kalliovirta & Saikkonen (2010).
at both tails, the normality assumption seems to hold quite well. When formally testing the normality of the quantile residual, as suggested by Kalliovirta and Saikkonen, we get the p-value of 0.70. Hence, normality seems to hold. As our data consists of daily observations, I consider lags from one to five in the autocorrelation and conditional heteroskedasticity testing. For the autocorrelation test, the p-values range from 0.01 (one lag) to 0.06 (four lags). Hence, there is some evidence of remaining autocorrelation in the quantile residuals. For the conditional heteroskedasticity test, the p-values range from 0.12 (five lags) to 0.65 (four lags). Hence, there does not seem to be much conditional heteroskedasticity in the quantile residuals. Overall it seems that the diagnostic tests support our model quite well.

Finally, as it was mentioned in section 2.4.2, the Google search data is not a perfect proxy for the KW model’s news variables because the changes in the countries’ search data are correlated to each other. However, compared to some other alternatives, the Google data fares quite well. For example, changes in the trading volumes in the countries’ stock markets is an alternative proxy for country specific news. The dashed lines in figure 2.2 depict the weekly changes in the average weekly trading volume for our sample countries. Overall the changes in the Google searches and the trading volume seem to be quite well related to each other. However, for Ireland the changes in trading volume are more volatile than the changes in the Google searches, and the opposite is true for Italy. This suggests that using the changes in trading volumes as proxies for our news variables might lead to different results than the ones reported in this paper.
The main reason for using the changes in the Google searches, instead of the changes in trading volumes, as a variable for identification is that the countries’ Google series are less correlated to each other than the changes in the trading volumes. The correlation coefficients of the latter range from 0.2 to 0.5 which is slightly higher than the range for the changes in the Google series. Moreover, it should be added that, for example, Choi & Varian (2012) show that using the Google Trends data improves the “nowcasting” performance of the standard forecasting models.

2.5 Conclusions and Discussion

The aim of this paper was to combine a theoretical explanation for volatility spillovers with a statistical model in order to develop a spillover test that would be well founded on economic theory. The selected approach was to augment the King & Wadhwani (1990) rational expectations model with an distributional assumption about the reduced form error distribution. This allowed us to identify the model and estimate it with the method of maximum likelihood. We were then able to test for the volatility spillovers with the likelihood ratio test. The whole model framework is designed for studying volatility spillovers in fully overlapping stock markets.

In the empirical application, the model was estimated using the eurozone stock market data in the years 2010–2011, the beginning of the sovereign debt crisis. Evidence of volatility spillovers was found. The large countries in the sample (Italy and Germany in particular) have large effects on all countries, both large and small. The small countries (Ireland and Greece) mostly have effects on each other. The results probably somewhat reflect the particularities of the sample period, but overall the results were quite intuitive. Generally speaking, the model diagnostics gave quite good support for our model.

Some possible extensions of the model are, first, to allow the structural shocks to have GARCH-effects. Second, contemplating only partially overlapping markets (with only a few common opening hours) might be fruitful as then one could set some zero restrictions on the instantaneous linkages between the markets.

Let us finally spend a few moments in discussing the possible implications of our empirical results. For investors, the main message of our empirical exercise is that there might be large volatility spillovers between countries and, also, that these spillover effects can be quite asymmetric. Obviously, this affects how an investor should allocate his assets and to her perception of risk. Whenever there is a core country whose volatility gets easily transmitted to other countries, it is of course quite hard for an investor to try to control and minimize her risk exposure to the
idiosyncratic shocks of this country by diversifying her assets.

The task of asset allocation is made even harder by the fact that, according to our theoretical model, volatility spillovers are due to information asymmetries between investors. In the theoretical model that this paper considered, the relative shares of informed and uninformed investors were assumed given. However, relating to the discussion in section 2.2.5, Kodres and Pritsker show that as the relative share of the uninformed investors grow, the spillover effects might become greater than before. In the case of the European debt crisis, it seems plausible to think that in the onset of the crisis with a lot of uncertainty about, for example, the public finances of Greece, Italy, Ireland, and later Spain, the number of bewildered investors grew and, hence, the informations asymmetries and spillover effects augmented. If at the same time some countries, for example, restrict or ban short-selling of stocks, this could reduce investors incentives to stay informed on the countries’ fundamentals, as is suggested by the analysis of Calvo and Mendoza that was also referred to in section 2.2.5. Hence, as the result of short-selling restrictions, the relative share of uninformed investors could increase further, making the spillover effects even greater during a crisis.

Appendix

Note on Lanne and Lütkepohl Identification Method

This appendix shows why the identification method of Lanne & Lütkepohl (2010) provides only a partial estimation of the coefficient matrix $\tilde{B}$. Assume the following $n \times n$ matrices: $\Psi = \text{diag}(\psi_1, \ldots, \psi_n)$ with $\psi_i > 0$, for all $i = 1, \ldots, n$; and $W$. Assume also a mixture probability $\gamma$ such that $0 < \gamma < 1$. Also, assume the $n \times n$ reduced form error vector’s covariance matrix $\Sigma_{\tilde{\eta}}$ can be written as

$$\Sigma_{\tilde{\eta}} = W (\gamma I_n + (1 - \gamma) \Psi) W' = W \bar{\Psi} W',$$  \hspace{1cm} (2.24)

where $I_n$ is the $n \times n$ identity matrix, and $\Psi = \gamma I_n + (1 - \gamma) \Psi$. Clearly $\Psi$ is also a diagonal matrix with its $i$th main diagonal element being $\bar{\psi}_i = \gamma + (1 - \gamma) \psi_i$. Hence, there is a bijective mapping between $\Psi$ and $\bar{\Psi}$. So, we can concentrate on the different permutations of $\bar{\Psi}$.

Now, take an arbitrary $n \times n$ permutation matrix $P$ such that $P \neq I_n$. Because permutation matrices are orthogonal matrices, it holds that $PP' = I_n$, where $P'$ denotes the transpose of $P$. 

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Hence, we can write

\[ \Sigma_{\tilde{\eta}} = (WP') (P\tilde{\Psi}P') (WP')' = \tilde{W}\tilde{\Psi}\tilde{W}', \]  

(2.25)

where we have redefined \( \tilde{W} = WP' \) and \( \tilde{\Psi} = P\tilde{\Psi}P' \).

It is straightforward to see that the matrix \( \tilde{\Psi} \) is diagonal with a different permutation of elements \( \{\psi_1, \ldots, \psi_n\} \) on its main diagonal than the matrix \( \bar{\Psi} \). First, write

\[ \hat{\Psi} = P(\bar{\psi}^{1/2}P^{1/2})P' = (P\bar{\psi}^{1/2})(P\bar{\psi}^{-1/2})' = (P\bar{\psi}^{1/2})(P\bar{\psi}^{1/2})', \]

which is possible because \( \bar{\psi}_i > 0 \) for all \( i \). Mark as \( e_k \) the \( 1 \times n \) vector whose \( k \)th element equals one and all the other elements equal zero. Then the \( j \)th column of \( \bar{\Psi}^{1/2} \) can be written as \( e_j'\bar{\psi}_j^{1/2} \).

Now, consider the permutation

\[ \Pi : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}, \Pi(k) = \pi_k \forall k \in \{1, \ldots, n\}, \]

that corresponds to the permutation matrix \( P \). Then the \( i \)th row of \( P \) is \( e_{\pi_i} \). And so, the \( ij \)th element of matrix \( P\bar{\psi}^{1/2} \) is

\[ \left[ P\bar{\psi}^{1/2} \right]_{ij} = e_{\pi_i} e_j'\bar{\psi}_{\pi_i}^{1/2} \]

that equals zero whenever \( \pi_i \neq j \) and \( \bar{\psi}_{\pi_i}^{1/2} \) when \( \pi_i = j \). The \( ij \)th element of matrix \( (P\bar{\psi}^{1/2})' \) equals to the \( ji \)th element of matrix \( P\bar{\psi}^{1/2} \). Hence,

\[ \left[ \Psi \right]_{ij} = \left[ (P\bar{\psi}^{1/2})(P\bar{\psi}^{-1/2})' \right]_{ij} = \sum_{k=1}^{n} \left( e_{\pi_i} e_k'\bar{\psi}_k^{1/2} \right) \left( e_{\pi_j} e_k'\bar{\psi}_k^{1/2} \right) = \sum_{k=1}^{n} \bar{\psi}_k e_{\pi_i} e_k' e_{\pi_j} e_k' \]

which equals zero whenever \( i \neq j \) and \( \bar{\psi}_{\pi_i} \) when \( i = j \). Hence, the matrix \( \hat{\Psi} \) is diagonal and the order of its main diagonal elements corresponds to the permutation \( \Pi \).

So, based on equations (2.24) and (2.25) we have now two equally possible choices for matrix \( \tilde{B} \) (see equation (2.20)):

\[ \tilde{B}_{(1)} = \hat{W}\bar{\psi}^{1/2} \] or \[ \tilde{B}_{(2)} = \hat{W}\psi^{1/2}. \]

These two alternative \( \tilde{B} \) matrices are not the same as long as \( P \neq I_n \). Because there are \( n! \)
different matrices $P$, there will also be $n!$ alternative matrices $\hat{B}$. Hence, unless we know exactly which of the permutation matrices $P$ to use, the structural model parameters will not be fully identified.

**Swapping between Alternative SVAR Normalizations**

The notation in this appendix is independent of the one used in the main text. Assume a $n \times 1$ reduced from error vector $u_t$ of a VAR model and a $n \times 1$ vector of the structural shocks $\varepsilon_t$ of a SVAR model. The reduced from error vector is assumed to be linear transformation of the structural shocks vector; $u_t = B\varepsilon_t$. The structural shocks are distributed as $\varepsilon_t \sim (0, \Sigma_\varepsilon)$ where $\Sigma_\varepsilon$ is a diagonal matrix. The reduced form errors are distributed as $u_t \sim (0, \Sigma_u)$ where $\Sigma_u$ is a general $n \times n$ covariance matrix. Consider then the two following options to normalize a SVAR model:

1. Assume $\Sigma_\varepsilon = I_n$ and let the elements of $B$ vary freely, or
2. Assume $\text{diag}(B) = 1_{n \times 1}$ and let the (diagonal) elements of $\Sigma_\varepsilon$ vary freely,

where $1_{n \times 1}$ refers to the $n \times 1$ vector with all elements equal to one.

The first normalization assumption is used in the paper by Lanne & Lütkepohl (2010) and the second in the original King and Wadhwani model. From this papers point of view, the relevant question is: How to switch from the normalization (1) to (2)?

First, assume normalization (1). For notational simplicity, let us suppress the time indexes. Then, we have the $n \times 1$ random vectors $\varepsilon$ and $u$, and the $n \times n$ matrices $\Sigma_\varepsilon$ and $B$, where $\Sigma_\varepsilon = I_n$. Following the identity $u = B\varepsilon$, we have

$$\Sigma_u = E(uu') = BE(\varepsilon\varepsilon')B' = B\Sigma_\varepsilon B' = BB', \quad (2.26)$$

where $E()$ is the expectations operator.

Secondly, assume normalization (2). Then, we have the $n \times 1$ random vectors $\tilde{\varepsilon}$ and $u$, where $\tilde{\varepsilon}$ is not necessarily equal to $\varepsilon$, the structural shocks of the previous paragraph. Also, we have the $n \times n$ matrices $\Sigma_{\tilde{\varepsilon}}$ and $\tilde{B}$, where $\text{diag}(\tilde{B}) = 1_{n \times 1}$. Again, following the identity $u = \tilde{B}\tilde{\varepsilon}$, we also have

$$\Sigma_u = E(uu') = \tilde{B}\Sigma_{\tilde{\varepsilon}} \tilde{B}' = \left(\tilde{B}\Sigma_{\tilde{\varepsilon}}^{\frac{1}{2}}\right) \left(\tilde{B}\Sigma_{\tilde{\varepsilon}}^{\frac{1}{2}}\right)', \quad (2.27)$$

Form equations (2.26) and (2.27) we get an identity

$$B = \tilde{B}\Sigma_{\tilde{\varepsilon}}^{\frac{1}{2}}. \quad (2.28)$$
For simplicity, let us limit our discussion to the two variable case \((n = 2)\). Thus, we have

\[
B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 1 & \tilde{b}_{12} \\ \tilde{b}_{21} & 1 \end{bmatrix}, \quad \text{and} \quad \Sigma^2 = \begin{bmatrix} \tilde{\sigma}_1 & 0 \\ 0 & \tilde{\sigma}_2 \end{bmatrix}.
\]

Equation (2.28) becomes

\[
\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} \tilde{\sigma}_1 & \tilde{\sigma}_2 \tilde{b}_{12} \\ \tilde{\sigma}_1 \tilde{b}_{21} & \tilde{\sigma}_2 \end{bmatrix},
\]

which gives us following four equations between the parameters. By solving for \(\tilde{b}_{12}\) and \(\tilde{b}_{21}\), we get

\[
\begin{cases}
\tilde{b}_{21} = b_{21}/b_{11} \\
\tilde{b}_{12} = b_{12}/b_{22}
\end{cases}
\]

Hence, we have derived that

\[
\tilde{B} = \begin{bmatrix} 1 & \frac{b_{12}}{b_{22}} \\ \frac{b_{21}}{b_{11}} & 1 \end{bmatrix}.
\]

So, once we have an estimate of \(B\) that is based on normalization (1), we can change to normalization (2) by dividing every column of \(B\) by the column’s main diagonal element. Clearly, the result generalizes to all \(n \geq 2\).

**Data Details**

**Stock market data**

The upper part of table 2.6 provides the details of the stock market price indexes that are used in this paper. All the stock market data is downloaded from Yahoo! Finance. In total the period under consideration covers 517 trading days. Due to banking holidays, none of the individual stock exchanges were open at every possible trading day. When there was a missing value, I selected the closing value of the previous (open) trading day.

**Google trends data**

The Google Trends data provides observations on how different topics (search terms) have been searched (in English) over time and provides weekly observations of Google’s search volume index. The search index reports the average amount of traffic (Google searches) on the chosen topic relative to the worldwide search traffic (in Google) during the week. Given that the data is only available for the searches in English, the generality of our results in the empirical application
of the paper could, of course, be questioned. However, because we are actually interested in the changes of the search volume data, as long as the data for the English tracks well the searches in other languages, this shortcoming should not affect too much our results. It is hard to imagine that trends in particular searches done in English would considerably differ (on average) from searches done in other languages.

The lower part of the table below reports the details of both the search topics I was interested to find data on and the actual Google Trends keywords I used to find the time series. So, for example, \((\text{italy gdp})\) means that the keyword in question is Italy and \((\text{its})\) GDP. The bar sign "\(|\)" between the keywords means that I wanted to find data on the Google searches including \(\text{at least one} \) of the keywords.

<table>
<thead>
<tr>
<th>Country</th>
<th>Stock market price indexes, daily closing values for the period Jan 4, 2010–Dec 30, 2011</th>
</tr>
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<tr>
<td>Index</td>
<td>Stocks trading included</td>
</tr>
<tr>
<td>Italy: FTSE MIB</td>
<td>40 most traded</td>
</tr>
<tr>
<td>Spain: IBEX 35</td>
<td>35 most traded stocks</td>
</tr>
<tr>
<td>Germany: DAX</td>
<td>30 most traded</td>
</tr>
<tr>
<td>Ireland: ISEQ Overall Index</td>
<td>All</td>
</tr>
<tr>
<td>Greece: FTSE/ASE 20</td>
<td>20 most traded</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Search topic</th>
<th>Actual keyword in Google Trends</th>
</tr>
</thead>
<tbody>
<tr>
<td>Italian economy OR debt OR stock market</td>
<td>((\text{italy gdp})</td>
</tr>
<tr>
<td>Spanish economy OR debt OR stock market</td>
<td>((\text{spain gdp})</td>
</tr>
<tr>
<td>German economy OR debt OR stock market</td>
<td>((\text{germany gdp})</td>
</tr>
<tr>
<td>Irish economy OR debt OR stock market</td>
<td>((\text{ireland gdp})</td>
</tr>
<tr>
<td>Greek economy OR debt OR stock market</td>
<td>((\text{greece gdp})</td>
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References


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Chapter 3

Transmission of Government Default Risk in the Eurozone

Abstract

The paper analyzes the reasons behind the rising ten year government bond spreads in the eurozone during the recent euro debt crisis. The structural vector autoregressive model that is developed allows us to test whether the upsurges in the spreads reflect breaks in the instantaneous shock propagation mechanisms between the spreads, changing dynamical effects, or changing country specific risk factors. Especially, we develop a new approach to test the stability of the instantaneous shock propagation mechanisms. Our results show that there are significant increases in the country specific risk factors of Ireland, Spain and Italy, while for Greece and Portugal contagion seems to be the main explanation.

1 An old version of this chapter has appeared as HECER Discussion Paper No. 359 (2012).
3.1 Introduction

The European debt crisis has been like a long-lasting drama, unfolding in many acts. Its beginning can be traced to October-November 2009 when the Greek government revealed the dire state of the country’s public finances (De Santis (forthcoming)). In the following years the crisis advanced rapidly; Greece, Portugal and Ireland were bailed out by the consortium of the European Union (EU), European Central Bank (ECB) and the International Monetary Fund, Spain was provided financial guarantees, and the EU and ECB needed quickly to adopt for new rules and institutions for financial crisis management. Only in 2013 the crisis started slowly to calm down, and, at the end of the year, Ireland became the first country to exit the bailout program.

One of the primary scenes of the crisis has been the government bond markets where the yields of the crisis countries rose quickly to levels last seen in the 1990’s when the countries still had their national currencies (figure 3.1). This paper introduces a non-linear structural vector autoregressive (SVAR) model to analyze the ten year government bond spreads of the crisis countries over Germany. The main motivation is to understand the sources of the increasing spreads during 2010–2012, the most acute years of the crisis.

Reckoning the German bond risk-free, then, for the eurozone member countries their ten year government bond spreads over the German bond basically measure the default risk of their government debt (Favero & Missale (2012)). There are then many possible reasons for the higher spreads during the crisis. Some have argued that the crisis simply reflects structural flaws in the architecture of the monetary union (De Grauwe & Ji (2013)), such as the slack enforcement of the fiscal discipline clause of the monetary union and the uncertainty over the joint bailout of an ailing government. Others see that the spreads rose during the crisis because the government default risk was actually “mispriced” prior to the crisis (Beirne & Fratzscher (2013)), or the feedback loop between the governments and national banking sectors changed over the course of the crisis (Mody & Sandri (2012)). But perhaps the most popular explanation has been contagion of government credit risk between the countries (for example Missio & Watzka (2011), Mink & de Haan (2013), Beetsma et al. (2013)). However, this explanation has recently been challenged; for example Caporin et al. (2013) do not find evidence of contagion in default risk between the eurozone countries during the crisis, also Beirne & Fratzscher (2013) reach a roughly similar conclusion.

\[\text{\footnotesize 2} \text{Pisani-Ferry (2013) discusses some of the flaws.}\]
\[\text{\footnotesize 3} \text{Intuitively, contagion means that a financial crisis in one country causes a crisis also in another country. However, as footnote 5 explains, the empirical contagion literature uses a more specific definition. There are several survey articles available on contagion: for example Dornbusch et al. (2000), Forbes & Rigobon (2001), Pericoli & Sbracia (2003), and Forbes (2012).}\]
\[\text{\footnotesize 4} \text{Beirne & Fratzscher (2013) refer transmission of credit risk between countries as regional contagion.}\]
Our model helps us to structure this discussion. In the model, there can be three types of crisis-contingent structural breaks in the stochastic process of the spreads which we can all test with statistical tests. First, the intercepts can change. Such a break can be interpreted as an exogenous change in the idiosyncratic country risk, due to, for example, a foreseeable national bailout of banking sector. Second, the autoregressive coefficients can change, especially the dynamic interdependencies between the countries. These dynamic interdependencies are called spillovers (Dungey & Martin (2007)). Changing spillovers can make spreads more volatile and could reflect bond markets becoming more speculative, for example due to lack of integrity in the decision making of the monetary union and uncertainty over its institutions. Third, the instantaneous (or contemporaneous) interdependencies between the countries can change due to a financial crisis. This means that a country specific shock would have a greater immediate impact on the spreads of the other countries during the crisis than before. This third structural break corresponds to contagion (Forbes & Rigobon (2002)).

\(^5\)Forbes & Rigobon (2002) underline the need to distinguish contagion from (mere) interdependence which refers to instantaneous linkages between financial variables in tranquil times. Contagion is then a crisis-contingent structural change in such linkages. The mainstream of the empirical research has adopted this definition of
contagion would be visible as a break in the correlations between the models residuals during a crisis.

We estimate the SVAR model with weekly data on ten year government bond spreads of the main crisis countries (Ireland, Greece, Portugal, Spain and Italy). As Greece joined the eurozone only in 2001, we consider the time period from January 2001 to December 2012. The analysis shows that for each country the intercept increases during the crisis but that only for Ireland, Spain and Italy the increase is statistically significant. There are statistically significant increases in the spillovers during the crisis only for Greece and Portugal. When it comes to breaks in the instantaneous linkages, there is evidence of contagion across all countries, but the contagious effects appear especially strong towards Greece and Portugal. So, all in all, although contagion is clearly the single most important factor in explaining the higher spreads during the crisis, for Ireland, Spain and Italy also the country specific risk factors appear to play their role. The importance of spillovers appears small.

The main contribution of the paper is to introduce a new way to test for the stability of the instantaneous linkages. Our approach is based on the idea of Favero & Giavazzi (2002) to test for contagion in the government bond spreads with a SVAR model. They allow structural breaks only in the instantaneous interdependencies and identify their model with restrictions on spillovers. Also, their framework requires analysis of news in order to identify source countries of exogenous shocks. In addition to that our model allows for several sources of breaks, it does not require any identifying restrictions on parameters, nor do we need to analyze news. Rather, the identification is based on the ideas of Lanne & Lütkepohl (2008, 2010) to use heteroskedasticity and non-normalities in the error distribution as the necessarily extra source of information for identification.6

Our approach to test for the stability of instantaneous linkages is closest to the test of parameter stability in a multivariate setting proposed by Rigobon (2003b). His test is based on the covariance matrices of market returns in normal and crisis times and computing the element-wise changes in these covariances. If the determinant of the matrix that includes these changes is zero, and assuming that we know that at least one country is not a source country of the crisis, the instantaneous interdependencies between the variables would have stayed stable during the crisis. However, Rigobon’s test is a joint test for the stability of all parameters of the model. Our

---

6Identification of a SVAR model based on properties of data, contrary to parameter restrictions, has received some attention in the recent literature. For example, Rigobon (2003a) considers a heteroskedasticity based identification plan that has been applied in the contagion and volatility spillover literature (Caporale et al. (2005b,a), Rigobon (2002), Rigobon & Sack (2003)). Rigobon’s approach assumes heteroskedasticity in the distribution of the structural shocks whereas our approach assumes that in the distribution of the reduced form residuals which is closer to the assumptions of Favero and Giavazzi.
framework allows to test separately for the stability of intercepts, spillovers and instantaneous interdependencies.

Also Dungey et al. (2005) and Pesaran & Pick (2007) consider testing of contagion in a multivariate setting. The former proposes to test contagion in a latent factor model context. But with more than three countries the model is unidentified without further restrictions. Our approach is basically applicable for any number of countries. The latter article discusses of several possible extensions of the Favero and Giavazzi model, including the idea of breaks in the intercepts and spillovers. In order to identify turmoil periods, Pesaran & Pick (2007) set predefined threshold values for the residuals. The actual identification is based on using country specific regressors as instruments for the endogenous variables. In our context this would mean using the past values of the spreads.

The rest of the paper is organized as follows. The next section presents the model and discusses the testable hypotheses. Section 3.3 estimates the model. Finally, Section 3.4 concludes.

### 3.2 Model for government bond spreads

This section first introduces the SVAR model. Then we talk about how to identify it and test for the stability of the instantaneous linkages between the variables. The final subsection shows how to estimate the model.

#### 3.2.1 The SVAR model

Denote the government bond yield of country \(i\) in period \(t\) as \(y_{it}\) and that of the German government bond as \(y^*_t\). The bond spread of country \(i\) over Germany in period \(t\) is defined as \(s_{it} = y_{it} - y^*_t\). Assume there are two distinct time periods: normal times (\(N\)) and the crisis period (\(C\)). Also, suppose that there are \(n\) countries and collect their spreads into the \((n \times 1)\) vector \(s_t\). Furthermore, assume that \(s_t\) follows the subsequent non-linear first order vector autoregressive (VAR) model:

\[
\begin{align*}
    s_t &= \begin{cases} 
        \mu_N + A_N s_{t-1} + u_t, & \text{when } t \in N, \\
        \mu_C + A_C s_{t-1} + u_t, & \text{when } t \in C,
    \end{cases} 
\end{align*}
\]

(3.1)

where \(\mu_N\) is the \((n \times 1)\) vector of the intercepts during the normal times, and \(\mu_C\) during the crisis period; similarly \(A_N\) and \(A_C\) are the \((n \times n)\) autoregressive coefficient matrices during the normal period and the crisis period, respectively; and \(u_t\) is the \((n \times 1)\) reduced form error vector.

---

7In our empirical analysis, the lag order of the VAR model is selected with the Bayesian information criterion.
which we will assume to follow the mixture-normal distribution with mean zero and covariance matrices $\Sigma^N_u$ and $\Sigma^C_u$ in the normal period and the crisis period, respectively.

Hence, in our model during a crisis period, there can be a structural break in the stochastic process of the government spread of a country for three reasons (some intuition for these breaks was discussed in the introduction). First, the intercept of a country may change. If, for example, $\mu_{i,C} > \mu_{i,N}$, there is an exogenous increase in the level of the government default risk of country $i$. Another possible reason for a break is a change in the dynamic interdependencies between the spreads. Denote by $a_{i,N}$ ($a_{i,C}$) the $i$th row of matrix $A_N$ ($A_C$) excluding the $i$th element $a_{ii,N}$ ($a_{ii,C}$). The vector $a_{i,N}$ ($a_{i,C}$) measures the joint dynamic effect of the other countries lagged values on $s_{it}$ during the normal (crisis) period while $a_{ii,N}$ ($a_{ii,C}$) measures the effect of its own lag. Changes in the autoregressive coefficients can make $s_{it}$ more volatile during the crisis period if, for example, all the elements of $a_{i,C}$ are in absolute value greater than the corresponding elements in $a_{i,N}$. Finally, there is the possibility of a break in the instantaneous interdependencies between the spreads.

As mentioned in the introduction, the empirical contagion literature is concerned about the stability of the instantaneous interdependencies (or conditional correlations) between financial variables. From the practical point of view this means that, for example, all predictability of stock market returns is first filtered away and the testing occurs in the residuals.\textsuperscript{8} The main motivation for this comes from the field of finance where the idea of the benefits of international diversification of assets is based on the exploitation of the estimated conditional correlations between the asset returns. If contagion causes these correlations to break during a crisis, the diversification may fail to provide the expected hedge at the moment such a hedge is usually the most needed (for a more detailed discussion, see Forbes & Rigobon (2001, 45)).

In model (3.1) breaks in the instantaneous linkages identifies as breaks in the interrelationships between the elements of $u_t$ during a crisis. In order to formalize this idea, assume that there are $n$ country specific structural shocks $\varepsilon_{1,t}, \ldots, \varepsilon_{n,t}$ which are assumed to be uncorrelated with each other. This is a standard assumption in the SVAR literature (Lütkepohl (2007)). Now, collect the structural shocks into the $(n \times 1)$ vector $\varepsilon_t$ and assume that $\varepsilon_t$ is linearly associated to the reduced form errors $u_t$ in the following way: in normal times $u_t = B_N \varepsilon_t$, while in a crisis $u_t = B_C \varepsilon_t$. As the matrices $B_N$ and $B_C$ depict the propagation of country specific shocks during normal times and a crisis, respectively, they determine the instantaneous interdependencies between the

\textsuperscript{8}For example, Forbes & Rigobon (2002) first filter our all autocorrelation from stock market returns and then focus on the residuals of the VAR model. Similarly, when discussing contagion, Bekker et al. (2005) concentrate on the residuals of their model once all predictability is filtered out from market returns. Also, for example, Pesaran & Pick (2007) and Chiang et al. (2007) concentrate on residuals of an autoregressive model.
spreads in both periods. Hence, our objective is to test whether or not $B_N = B_C$. In case they are, there is not evidence of contagion.

As it is explained in more detail in the next subsection, the testing of the stability of the B-matrices is based on allowing heteroskedasticity in the distribution of $u_t$. Given that crisis periods are usually more volatile than normal times, this is a natural assumption. The main point is the following: in the SVAR framework, it is usually assumed that the structural shocks have a unit variance. This means that we have $\Sigma_\varepsilon = I_n$, where $I_n$ is the ($n \times n$) identity matrix. It then follows that we can write the covariance matrices of the reduced form errors during the normal period and the crisis period as $\Sigma_u^N = B_N B_N'$ and $\Sigma_u^C = B_C B_C'$, respectively. So, in our model, the heteroskedasticity of the reduced form errors is a direct consequence of possible breaks in the instantaneous shock propagation mechanisms.

Model (3.1) is an extension of the SVAR model of Favero & Giavazzi (2002). First of all, they assume $A_N = A_C$ and $\mu_N = \mu_C$. Furthermore, following our notation, we can write their key assumptions as $B_N = F$ and $B_C = F(I_n + GD_t)$, where $F$ and $G$ are ($n \times n$) coefficient matrices, and $D_t$ is a crisis period indicator which equals a zero-matrix during normal times and a diagonal matrix with ones and zeros on its main diagonal during a crisis period (the number of ones depends on the number of crisis countries). Hence, when there is a crisis, and so $D_t \neq 0$, the instantaneous shock propagation mechanisms between the countries change from the normal times only if $G \neq 0$. Ideally then, in Favero and Giavazzi’s model, one should test the stability of the instantaneous linkages between the spreads by testing the null-hypothesis $G = 0$. Favero and Giavazzi collect the data on the dummy variable $D_t$ by determining turmoil periods based on outliers in the empirical distribution of the reduced form errors of a VAR model. Once they have these dates, they analyze news to see whether they can determine the source country of the turmoil. In order to identify their model, Favero and Giavazzi set specific restrictions on the spillovers. The next subsection considers an alternative approach.

### 3.2.2 Identifying the SVAR model

We will apply the idea presented by Lanne & Lütkepohl (2010) to identify model (3.1). They assume a specific type of non-normal distribution, mixture-normal distribution, and we will maintain their distributional assumption. However, inspired by their earlier idea (Lanne & Lütkepohl (2008)), we will allow for two types of periods, normal and crisis periods.

First note that any two positive definite ($n \times n$) covariance matrices $\Sigma_1$ and $\Sigma_2$ can be decomposed as $\Sigma_1 = WW'$ and $\Sigma_2 = W\Psi W'$, where $W$ is a nonsingular ($n \times n$) matrix and $\Psi = \text{diag}(\psi_1, \ldots, \psi_n)$ is a ($n \times n$) diagonal matrix with all the diagonal elements being strictly
positive (for details and further references, see the appendix in Lanne & Lütkepohl (2010)). As long as all the elements $\psi_i$ are distinct from each other—and in some predetermined order (see the discussion in the appendix, page 67), the matrix $W$ is unique (apart from changing the signs of the elements in every column).

Consider then a sample of $T$ time periods where the first $T_1$ periods represent normal times and the periods $T_1 + 1$ onwards the crisis period. Assume further that the reduced form errors of the model (3.1) are distributed as follows:

$$ u_t = \begin{cases} e_{1t} \sim N(0, W_N W_N') & \text{with probability } \gamma, \\ e_{2t} \sim N(0, W_N \Psi W_N') & \text{with probability } 1 - \gamma \end{cases} \quad (3.2) $$

for $t = 1, \ldots, T_1$, and

$$ u_t = \begin{cases} \tilde{e}_{1t} \sim N(0, W_C W_C') & \text{with probability } \gamma, \\ \tilde{e}_{2t} \sim N(0, W_C \Psi W_C') & \text{with probability } 1 - \gamma \end{cases} \quad (3.3) $$

for $t = T_1 + 1, \ldots, T$. So, we assume that both the mixture probability $\gamma$ and the diagonal matrix $\Psi$ stay the same during the normal and crisis periods but allow for crisis contingent heteroskedasticity by considering two different ($n \times n$) $W$-matrices, $W_N$ and $W_C$.

From the distributional assumptions, it follows that the covariance matrices of $u_t$ are

$$ \Sigma^N_u = W_N (\gamma I_n + (1 - \gamma) \Psi) W_N' \quad (3.4) $$

and

$$ \Sigma^C_u = W_C (\gamma I_n + (1 - \gamma) \Psi) W_C' \quad (3.5) $$

in the normal period and the crisis period, respectively. On the other, in the previous section it was argued that we also have

$$ \Sigma^N_u = B_N B_N' \quad (3.6) $$

and

$$ \Sigma^C_u = B_C B_C' \quad (3.7) $$

From equations (3.4)–(3.7) it follows that

$$ B_N = W_N (\gamma I_n + (1 - \gamma) \Psi)^{1/2} \quad (3.8) $$
and

\[ \mathbf{B}_C = \mathbf{W}_C (\gamma \mathbf{I}_n + (1 - \gamma) \Psi)^{1/2}. \]

(3.9)

Hence, we can test the stability of the instantaneous linkages between the spreads by testing the null hypothesis \( \mathbf{W}_N = \mathbf{W}_C \) under which \( \mathbf{B}_N = \mathbf{B}_C \). The following subsection proposes to conduct this test with the likelihood ratio (LR) test.

Notice that, given our assumptions that the parameters \( \gamma \) and \( \Psi \) stay constant throughout the whole sample, the equality \( \mathbf{W}_N = \mathbf{W}_C \) is the necessary and sufficient condition for the equality \( \mathbf{B}_N = \mathbf{B}_C \). It should also be noticed that the identification of the matrices \( \mathbf{B}_N \) and \( \mathbf{B}_C \) in equations (3.8) and (3.9) is conditional on the ordering of the diagonal elements of \( \Psi \) (again, see the discussion in the appendix, page 67). Hence, because there are \( n! \) possible permutations of the elements \( \{\psi_1, \ldots, \psi_n\} \), there are \( n! \) possibilities for the \( \mathbf{B}_N \) and \( \mathbf{B}_C \) matrices.

In this paper, we will assume that the diagonal elements of \( \Psi \) are in the descending order, i.e. \( \psi_1 > \psi_2 > \ldots > \psi_n \). This uniquely identifies the matrices \( \mathbf{B}_N \) and \( \mathbf{B}_C \), call them for a moment \( \bar{\mathbf{B}}_N \) and \( \bar{\mathbf{B}}_C \). Denote the corresponding vector of the structural shocks as \( \bar{\epsilon}_t \). As it is shown in the appendix (page 67), all the other \( n! - 1 \) possible \( \mathbf{B}_N \) and \( \mathbf{B}_C \) matrices are simply column-wise permutations of the matrices \( \mathbf{B}_N \) and \( \mathbf{B}_C \), respectively, and all the other \( n! - 1 \) vectors of the structural shocks are row-wise permutations of the elements in \( \bar{\epsilon}_t \). The implication is that, based on the identification of the matrices \( \bar{\mathbf{B}}_N \) and \( \bar{\mathbf{B}}_C \) in equations (3.8) and (3.9) (and assuming the main diagonal elements of \( \Psi \) are in the descending order), we are not able to associate the structural shocks with the countries. However, as long as we assume that our model in equations (3.1)–(3.3) is true, we know that we have identified the country specific structural shocks but we are simply unable to identify the ordering of them so that we could say which one of the shocks corresponds to which country.

However, as our only objective is to test for the stability of the instantaneous propagation mechanisms of the structural shocks between the spreads, this does not affect the generality of our test; stability would mean that for every possible permutation of the matrices \( \mathbf{B}_N \) and \( \mathbf{B}_C \), it must hold that \( \mathbf{B}_N = \mathbf{B}_C \). We simply choose one of these permutations to work with. For now on, we refer to \( \bar{\mathbf{B}}_N \) and \( \bar{\mathbf{B}}_C \) simply as \( \mathbf{B}_N \) and \( \mathbf{B}_C \) (see the discussion in the appendix, page 68).
3.2.3 Estimation of the model

The model described in equations (3.1)–(3.3) can be estimated with the method of maximum likelihood (ML). During the normal times the conditional density of \( s_t \) is

\[
    f^N(s_t|s_{t-1}) = \gamma \det(W_N)^{-1} \times \\
    \exp \left\{ -\frac{1}{2} (s_t - \mu_N - A_N s_{t-1})' (W_N W_N')^{-1} (s_t - \mu_N - A_N s_{t-1}) \right\} + \\
    (1 - \gamma) \det(W_N)^{-1} \det(\Psi)^{-1/2} \times \\
    \exp \left\{ -\frac{1}{2} (s_t - \mu_N - A_N s_{t-1})' (W_N \Psi W_N')^{-1} (s_t - \mu_N - A_N s_{t-1}) \right\},
\]

where we have ignored the constant terms of the Gaussian density function. Obviously, we get the conditional density \( f^C(s_t|s_{t-1}) \) during the crisis period in the same way by simply replacing the parameters \( W_N, \mu_N, \text{and} A_N \) with \( W_C, \mu_C, \text{and} A_C \), respectively. Then, by collecting all the parameters (\( \mu_N, \mu_C, A_N, A_C, W_N, W_C, \gamma, \text{and} \Psi \)) into the vector \( \theta \), the log-likelihood function of the model can be written as

\[
    l(\theta, s_T, \ldots, s_0) = \sum_{t=1}^{T_1} \log f^N(s_t|s_{t-1}) + \sum_{t=T_1+1}^{T} \log f^C(s_t|s_{t-1}),
\]

where \( s_0 \) is the initial value. This can be maximized numerically with the standard optimization algorithms.

As explained in the previous section, we can test for the stability of the instantaneous linkages by testing the null hypothesis \( W_N = W_C \) against the alternative hypothesis \( W_N \neq W_C \). A rejection of the null hypothesis would be an evidence against the stability of the instantaneous linkages during a crisis. Also, as explained in section 3.2.1 there are two other potential sources of changing government default risk during a crisis: changing intercept, and changing spillovers and own lagged effect. To test changes in the intercepts, we consider for each country separately the null hypothesis \( \mu_{i,N} = \mu_{i,C}, \ i = \{1, \ldots, n\} \). To test changing spillovers and own lagged effects we can test the null hypotheses \( a_{-i,N} = a_{-i,C} \) and \( a_{ii,N} = a_{ii,C} \) for each \( i = \{1, \ldots, n\}, \) respectively. All the null hypotheses can be tested with the standard LR test.

3.3 Eurozone government bond spreads

In this section, we analyze the ten year government bond spreads of Ireland, Greece, Portugal, Spain and Italy over the German bond during the years 2001–2012. The data consist of weekly
observations of the spreads which are calculated from the secondary market yields of the bonds. The data on yields are from the Eurostat database, and there are 626 observations for each country. The aim is to implement the tests on changing intercepts, changing spillovers, and changing instantaneous linkages.

In order to apply our testing approach we need to determine the normal and crisis periods. Although the euro crisis can be seen as a continuation of the subprime crisis that began in the U.S. financial markets already in 2007–2008 (Mody & Sandri (2012)), we follow De Santis (forthcoming) and set the beginning of the crisis to November 2009. This is when the newly elected Greek government announced the revised government budget deficit and debt figures which were on unexpectedly high levels. Hence, our normal period covers the weeks from January 2001 to October 2009 (461 weeks) and the crisis period is from November 2009 to December 2012 (165 weeks).

3.3.1 Estimation of the model

The lag order of the model in equations (3.1)–(3.3) is selected with the Bayesian information criterion. Comparing models where the lags range from one to three for the normal and crisis periods, order one is selected for both periods. There are 116 parameters in our model. Table 3.4.3 in the appendix (page 69) reports the estimation results. Let us first provide a few general remarks on the results before we conduct the actual testing.

First, the intercepts are substantially larger during the crisis period than during the normal period. Second, there appears to be less statistically significant spillovers between the spreads during the crisis periods than during the normal period. In both periods, for all of the countries, it is the own lagged effect of the spread that has by far the largest coefficient. Third, the number of statistically significant elements in the matrix $W_C$ is somewhat smaller than in the matrix $W_N$. As seen from equations (3.8) and (3.9), this suggests that, at least for some countries, the number of statistically significant instantaneous linkages decreased during the crisis period; suggesting the shock propagation mechanisms became “weaker” during the crisis. Fourth, the elements of the matrix $\Psi$ are clearly distinct from each other which is the prerequisite for using the identification scheme of Lanne & Lütkepohl (2010). Fifth, the estimated value of $\gamma$ tells us that both during the normal and crisis periods the error term is with the probability of 77 percent from the less volatile normal distribution.

In order to study the goodness of our model, we have applied the joint quantile residuals (JQRs) proposed by Kalliovirta & Saikkonen (2010). For a model with a mixture distribution, the estimation was done with the procedures in the CMLMT library of GAUSS.
quantile residuals provide a convenient diagnostic tool (Kalliovirta (2012)), and the JQRs are a concise way to analyze a multivariate model. When the model is correctly specified, JQRs follow the standard normal distribution and are free from autocorrelation in the first and second moments. Using the diagnostic tests provided by Kalliovirta & Saikkonen (2010) we find that the normality test gets the p-value 0.08. Hence, the null hypothesis of normality is not rejected at the 5% level of significance. The p-values of the autocorrelation tests for the lags from one to five range in between 0.44 (five lags) and 0.94 (one lag). So, there is not evidence of remaining autocorrelations in the JQRs. Similarly, The p-values of the conditional heteroskedasticity range in between 0.03 (one lag) and 0.32 (five lags). Hence, some evidence of remaining serial correlation in the second moments is found. Overall, the diagnostic tests give quite good support for our model.

3.3.2 Determinants of the government default risk

Let us now test the three possible explanations for structural breaks in the governments’ default risk during the crisis: changing intercept, changing autoregressive coefficients, and breaks in the instantaneous linkages between the spreads. These can all be tested with the standard LR test. This subsection presents only the test results while the next subsection interprets them.

First, in testing the stability of the intercepts, we test the null hypotheses $\mu_{i,N} = \mu_{i,C}$ for all $i \in \{Ire, Gre, Spa, Ita, Por\}$ with the obvious country abbreviations. At the 5% percent level of significance, the null hypothesis is rejected for Ireland, Spain, and Italy while for Greece and Portugal the null hypothesis is not rejected.

Second, as explained in Section 3.2.1, the testing of changing spillover effects can be conducted separately for each country by testing the null hypothesis $a_{-i,N} = a_{-i,C}$, where $a_{-i,N}$ ($a_{-i,C}$) denotes the $i$th row of matrix $A_N$ ($A_C$), excluding the $i$th element $a_{ii,N}$ ($a_{ii,C}$). It captures the dynamic cross-country effects of the lagged observations of the spreads of other countries on the spread of country $i$ during the normal (crisis) period. Also, it is of interest whether the effect of a country’s own lagged value changes or not during the crisis. This can be tested with testing the null hypothesis $a_{ii,N} = a_{ii,C}$. First, at the 5% level of significance, the null hypothesis $a_{-i,N} = a_{-i,C}$ is not rejected for Spain. For all the other countries, the null hypothesis is rejected. Second, again at the 5% significance level, the null hypothesis $a_{ii,N} = a_{ii,C}$ is not rejected for any country.

Finally, let us test the stability of the instantaneous linkages between the spreads. As explained earlier, this is obtained by testing the null hypothesis $W_N = W_C$ against the alternative hypothesis $W_N \neq W_C$. At the 1% level of significance, the null hypothesis is rejected. Hence,
the evidence speaks in favor of crisis-contingent changes in the instantaneous shock propagation mechanisms between the spreads.

### 3.3.3 Summary and interpretation of the findings

Table 3.1 summarizes our findings from the previous subsection. In order to facilitate the interpretation of the results, for the intercepts and the elements of matrices $A_N$ and $A_C$, the table shows for each rejected null hypothesis the estimated values of the parameters for the normal period and the crisis period. This way it is easier to see if there is an increase in the parameter values, which could help us to understand the increasing government bond spreads during the euro debt crisis.

However, as one of our null hypothesis ($a_{-i,N} = a_{-i,C}$) jointly tests the change in all off-diagonal elements of the $i$th row of the autoregressive coefficient matrix, the table introduces a measure for the joint impact of the lagged observations of other countries on the spread of country $i$. This measure is $\sum |a_{-i,N}| = \sum_{j \neq i} |a_{ij,N}|$ ($\sum |a_{-i,C}| = \sum_{j \neq i} |a_{ij,C}|$) for the normal (crisis) period and it simply sums together the absolute values of the off-diagonal elements on the $i$th row of matrix $A_N$ ($A_C$). If $\sum |a_{-i,N}| < \sum |a_{-i,C}|$, we can deduct that the combined dynamic effect of the spreads of the other countries on the spread of country $i$ increases during the crisis.

So, by first focusing on the intercepts and the dynamic effects, during the crisis, there was not statistically significant change in spillover effects of the other countries’ spreads on the Spanish spread. Also, the Irish and the Italian spreads became dynamically less interdependent with the spreads of the other countries. All the three countries see a substantial increase in their intercepts, our measure of the idiosyncratic risk factor. There was not a statistically significant change in the intercepts of the Greek and Portuguese spreads, but the spreads of these countries became dynamically more interdependent with the spreads of the other countries during the crisis. This could be one potential explanation for the Greek and Portuguese spreads becoming more volatile during the crisis.
### Table 3.1: Summary of the results

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Country i</th>
<th>Ireland</th>
<th>Greece</th>
<th>Spain</th>
<th>Italy</th>
<th>Portugal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0: \mu_{i,N} = \mu_{i,C}$</td>
<td>rejected</td>
<td>not rej.</td>
<td>rejected</td>
<td>rejected</td>
<td>not rej.</td>
<td></td>
</tr>
<tr>
<td>$\mu_{i,N} \text{ vs. } \mu_{i,C}$</td>
<td>-0.01 vs. 0.23</td>
<td>⋯</td>
<td>-0.01 vs. 0.10</td>
<td>0.00 vs. 0.90</td>
<td>⋯</td>
<td></td>
</tr>
<tr>
<td>$H_0: a_{-i,N} = a_{-i,C}$</td>
<td>rejected</td>
<td>rejected</td>
<td>not rej.</td>
<td>rejected</td>
<td>rejected</td>
<td></td>
</tr>
<tr>
<td>$\sum</td>
<td>a_{-i,N}</td>
<td>\text{ vs. } \sum</td>
<td>a_{-i,C}</td>
<td>$</td>
<td>0.40 vs. 0.26</td>
<td>0.13 vs. 0.42</td>
</tr>
<tr>
<td>$H_0: a_{ii,N} = a_{ii,C}$</td>
<td>not rej.</td>
<td>not rej.</td>
<td>not rej.</td>
<td>not rej.</td>
<td>not rej.</td>
<td></td>
</tr>
<tr>
<td>$W_N = W_C$</td>
<td>rejected</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note**
For details of the tests, see Section 3.3.2. $\sum |a_{-i,N}| = \sum_{j \neq i} |a_{ij,N}|$ ($\sum |a_{-i,C}| = \sum_{j \neq i} |a_{ij,C}|$) is the sum over the absolute values of the non-diagonal elements of matrix $A_N$ ($A_C$). This is our measure of how large is the combined effect of the lagged observations of other countries on country $i$ during the normal (crisis) period.
We also find that there are statistically significant changes in the instantaneous linkages between the countries. However, unfortunately, as explained at the end of Section 3.2.2, our identification method does not allow us to associate the structural shock with the countries because the identification of the matrices $B_N$ and $B_C$ in equations (3.8) and (3.9) depend on the selected permutation of the diagonal elements of the matrix $\Psi$ (we have assumed the descending order). In our case, as we have five countries, there are 120 permutations which means there are 120 different $B_N$ and $B_C$ matrices. On a more positive note, however, as the appendix shows (page 68), all permutations simply reshuffle the order of structural shocks $\{\varepsilon_{1,t}, \ldots, \varepsilon_{5,t}\}$. From the perspective of the matrices $B_N$ and $B_C$ this means, that only the order of the columns changes. So, as long as we believe that our five structural shocks correspond to five country specific shocks, we know that, although we are unable to say which shock corresponds to which country, in each permutation the effect of the shock on the spreads stays the same: for example, if two permutations are otherwise same except that the first and the fifth structural shocks change places, then in the matrices $B_N$ and $B_C$ only the first and fifth columns change places.

From the perspective of our analysis the discussion in the previous paragraph implies that, given our chose permutation of the matrix $\Psi$, we can measure the increases in the effect of each structural shock on each country during the crisis by computing how much the absolute value of each element of $B_N$ changes compared to the absolute value of the corresponding element of $B_C$. Table 3.2 reports these calculations. So, for example the effect of the first structural shock on Ireland decreases by 44 percent during the crisis, but at the same time its effect on Greece increases by 2508 percent. Then, for example, the first figure was computed by comparing the row 1, column 1 element of matrix $B_N$ ([$B_N$]$_{11}$) to [$B_C$]$_{11}$. For each column of the table, darker color indicates a larger increase in the effect of the corresponding structural shock. Although it is evident from Table 3.2 that the effects of the structural shocks on the countries increase in almost all cases, there are substantial differences in the averages. For Greece and Ireland the average increase of the effects of the structural shocks are around five and three times larger, respectively, than for Ireland, Spain, and Italy.

All in all, our results show that there are clear indications of contagion between all spreads during the crisis. However, the spreads of Ireland, Italy and Spain seem to become dynamically less interdependent to the spreads of the other countries. These three countries see also significant increases in their idiosyncratic risk factors. Hence, contagion probably explains a smaller share of the increases in their spreads during the crisis than for Greece and Portugal. Furthermore, while there probably was contagion of default risk from Greece and Portugal to the spreads of the other countries, at the same time, it seems that the Greek and Portuguese spreads became
much more susceptible to changes in the spreads of the other countries during the crisis.

### 3.4 Conclusions

This paper presents a non-linear SVAR model to analyze the reasons behind the rising spreads of Greece, Ireland, Spain, Italy and Portugal during the recent euro crisis. We identify the model by exploiting the heteroskedasticity and non-normalities of the reduced form errors of a VAR model. The model allows us to test for three possible reasons for a break in the stochastic process of the spreads during the crisis: changing intercept, changing spillovers between the spreads, and changing instantaneous interdependencies. Our test on the stability of the instantaneous interdependencies corresponds to the contagion tests in the previous literature. Changing intercept could be interpreted as an idiosyncratic change in the level of the default risk of a government.

Contagion seems to be an important factor in explaining the increasing spreads of all of the countries considered, but there are substantial differences between the countries. It is especially for Greece and Portugal to whom contagion of default risk from the other countries seems to be an important factor to explain their increasing spreads. Greece and Portugal are also the only countries who see an increase in the spillover effects of the spreads of the other countries on their spreads. Hence, it appears that, during the crisis, the Greek and Portuguese spreads became more responsive to the changes in the spreads of the other countries.

For Ireland, Italy and Spain, in addition to the changes in the instantaneous linkages between their spreads and those of the other countries, there are statistically significant increases in their the idiosyncratic risk factors. Furthermore, unlike with Greece and Portugal, the spreads of these three countries either become dynamically less interdependent with the spreads of the other countries or there is not a statistically significant change in the spillover effects.

The paper provides several possible research questions for future research. First, our model

Table 3.2: Percentage changes in the absolute values of the coefficients of the structural shocks during the crisis

<table>
<thead>
<tr>
<th>Effect on</th>
<th>Structural shocks</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_1$</td>
<td>$\varepsilon_2$</td>
</tr>
<tr>
<td>Ireland</td>
<td>-44 %</td>
<td>174 %</td>
</tr>
<tr>
<td>Greece</td>
<td>2508 %</td>
<td>524 %</td>
</tr>
<tr>
<td>Spain</td>
<td>-71 %</td>
<td>370 %</td>
</tr>
<tr>
<td>Italy</td>
<td>100 %</td>
<td>859 %</td>
</tr>
<tr>
<td>Portugal</td>
<td>49 %</td>
<td>1636 %</td>
</tr>
</tbody>
</table>

Note

For each structural shock (column), a darker color of a cell indicates a greater increase in the corresponding coefficient.
assumes that the structural shocks (but not the reduced form errors) are homoskedastic. This, of course, need not be the case, and an increase also in the variances of the structural shocks would be an interesting extension. Second, it would be interesting to further investigate the identification method that is applied in this paper so that one could associate the structural shocks with the countries. Here, we needed to content ourselves with testing the stability of their effects. Third, to allow for an endogenously determined crisis period would be a logical extension of the model framework.

Appendix

Mathematical appendix

This appendix shows why the matrices \( B_N \) and \( B_C \) in equations (3.8)–(3.9) become uniquely identified once we assume that the diagonal elements of the matrix \( \Psi \) are in some predetermined order. We also show why this assumption does not affect the generality of our test for the stability of the instantaneous linkages.

Uniqueness of the B-matrices

Consider first the covariance matrix of the reduced form errors during normal times, that is the matrix \( \Sigma_u^N \) in equation (3.4).

Take an arbitrary \((n \times n)\) permutation matrix \( P \). For permutation matrices it holds that \( P'P = I_n \). Hence, we can equally well write \( \Sigma_u^N \) in the following way:

\[
\Sigma_u^N = W_N(\gamma I_n + (1 - \gamma)\Psi)W_N'
= \gamma W_N W_N' + (1 - \gamma)W_N \Psi W_N'
= \gamma W_N P' P W_N' + (1 - \gamma)W_N P' \Psi P' P W_N'
= \gamma W_N P' (W_N P')' + (1 - \gamma)W_N P' \Psi P' (W_N P')'
= W_N P' (\gamma I_n + (1 - \gamma)\Psi P') (W_N P')'
= \hat{W}_N (\gamma I_n + (1 - \gamma)\hat{\Psi}) \hat{W}_N',
\]

where \( \hat{W}_N = W_N P' \) is a column-wise permutation of the matrix \( W_N \), and \( \hat{\Psi} = P \Psi P' \) is a column- and row-wise permutation of the diagonal matrix \( \Psi \). This means that, because all the off-diagonal elements of \( \Psi \) are zero, \( \hat{\Psi} \) is also a diagonal matrix with the diagonal elements of \( \Psi \) reordered.
So, instead of the matrix $B_N$ in equation (3.8), we could choose the following matrix as our normal times $B$-matrix:

$$\hat{B}_N = W_N (\gamma I_n + (1 - \gamma) \Psi)^{1/2}. \quad (3.4.10)$$

$\hat{B}_N$ is equal to matrix $B_N$ only when $P = I_n$. Because there are $n!$ different $(n \times n)$ permutation matrices $P$, there are equally many possibilities for our normal period $B$-matrix. By similar reasoning, we could, instead of choosing the matrix $B_C$ in equation (3.9), choose as our crisis period $B$-matrix the following matrix:

$$\hat{B}_C = W_C (\gamma I_n + (1 - \gamma) \Psi)^{1/2}, \quad (3.4.11)$$

where $\hat{W}_C = W_C P'$ is again a column-wise permutation of the matrix $W_C$.

Because the mapping $\Psi = P \Psi P'$ is a bijection and the elements $\psi_1, \ldots, \psi_n$ are assumed distinct, determining the order of the diagonal elements of $\Psi$ uniquely determines the permutation matrix $P$. Hence, the matrices $W_N$ and $W_C$ are also determined and, so, the $B$-matrices.

**Generality of our test for stability of instantaneous linkages**

Finally, let us show why the requirement to select one permutation $P$ does not affect the generality of our test for stability of instantaneous linkages. First, because the part

$$(\gamma I_n + (1 - \gamma) \Psi P')^{1/2} = (P(\gamma I_n + (1 - \gamma) \Psi) P')^{1/2}$$

in the equations (3.4.10)–(3.4.11) is diagonal, it equals to

$$P(\gamma I_n + (1 - \gamma) \Psi)^{1/2} P'.$$

Then it follows that, for example, equation (3.4.10) can be written as

$$\hat{B}_N = W_N P' P(\gamma I_n + (1 - \gamma) \Psi)^{1/2} P' = W_N(\gamma I_n + (1 - \gamma) \Psi)^{1/2} P' = B_N P'.$$

Hence, the matrix $\hat{B}_N$ is simply a column-wise permutation of the matrix $B_N$ in equation (3.8) which corresponds to the permutation $P = I_n$.

Denote as $\hat{\varepsilon}_t$ the vector of structural shock corresponding to matrix $\hat{B}_N$ during normal times.
Then, from the equality $u_t = B_N \varepsilon_t = \hat{B}_N \hat{\varepsilon}_t$, it follows that

$$\hat{\varepsilon}_t = (\hat{B}_N)^{-1} B_N \varepsilon_t = (B_N P')^{-1} B_N \varepsilon_t = P'^{-1} B_N \varepsilon_t = P \varepsilon_t,$$

where $P'^{-1} = P$ is a result from $P$ being orthogonal. So, $\hat{\varepsilon}_t$ is simply a row-wise permutation on the vector $\varepsilon_t$. The same reasoning holds for the crisis period.

Hence, our identification of the matrices $B_N$ and $B_C$ in equations (3.8) and (3.9), respectively, does not allow us to associate the corresponding structural shocks with the spreads of the countries, although assuming that the diagonal elements of $\Psi$ are in descending, uniquely identifies one permutation of the matrices matrices $B_N$ and $B_C$. Notwithstanding, following Favero & Giavazzi (2002), as long as we believe that our extension of their model in equations (3.1)–(3.3) correctly depicts the stochastic process of the spreads, we know that the structural shocks are country specific shocks to the spreads. So, all the $n!$ permutations of the vector of structural shocks simply reshuffle the individual shocks and, more importantly for our purposes, all the permutations of the matrices $B_N$ and $B_C$ measure the same effects. This means that by selecting one of the permutations to work with, we can always test the stability of the matrices $B_N$ and $B_C$ but we can not say which structural shock corresponds to which country.

**Parameter estimates**

Table 3.4.3: Estimation results (standard errors are in parentheses)

<table>
<thead>
<tr>
<th>Normal period</th>
<th>$S_{ire,t}$</th>
<th>$S_{gre,t}$</th>
<th>$S_{spa,t}$</th>
<th>$S_{ita,t}$</th>
<th>$S_{por,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intercepts</strong></td>
<td>$-0.009$</td>
<td>$0.003$</td>
<td>$-0.005$</td>
<td>$0.003$</td>
<td>$0.002$</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>$S_{ire,t-1}$</td>
<td>$0.917^{* * *}$</td>
<td>$0.000$</td>
<td>$-0.016$</td>
<td>$-0.035^{* * *}$</td>
<td>$-0.036^{* *}$</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.01)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$S_{gre,t-1}$</td>
<td>$0.171^{* * *}$</td>
<td>$0.984^{* * *}$</td>
<td>$0.054^{* * *}$</td>
<td>$0.095^{* * *}$</td>
<td>$0.124^{* * *}$</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.024)</td>
<td>(0.021)</td>
<td>(0.021)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>$S_{spa,t-1}$</td>
<td>$0.050$</td>
<td>$0.039$</td>
<td>$0.959^{* * *}$</td>
<td>$0.024$</td>
<td>$0.098^{* * *}$</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.022)</td>
<td>(0.023)</td>
<td>(0.021)</td>
<td>(0.035)</td>
</tr>
<tr>
<td>$S_{ita,t-1}$</td>
<td>$-0.085$</td>
<td>$0.032$</td>
<td>$-0.002$</td>
<td>$0.913^{* * *}$</td>
<td>$-0.028$</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.032)</td>
<td>(0.030)</td>
<td>(0.029)</td>
<td>(0.043)</td>
</tr>
<tr>
<td>$S_{por,t-1}$</td>
<td>$-0.097^{* * *}$</td>
<td>$-0.062^{* *}$</td>
<td>$-0.026$</td>
<td>$-0.05^{* *}$</td>
<td>$0.784^{* * *}$</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.023)</td>
<td>(0.024)</td>
<td>(0.022)</td>
<td>(0.036)</td>
</tr>
<tr>
<td>Crisis period</td>
<td>( \text{Intercepts} )</td>
<td>( \beta_{sre,t-1} )</td>
<td>( \beta_{gxe,t-1} )</td>
<td>( \beta_{spa,t-1} )</td>
<td>( \beta_{tre,t-1} )</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td></td>
<td>0.232*** 0.243  0.096  0.086** 0.028</td>
<td>(0.088) (0.148) (0.05) (0.042) (0.092)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.968*** 0.063  −0.014 −0.010 0.059**</td>
<td>(0.026) (0.044) (0.015) (0.013) (0.027)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.013  0.946*** 0.004  0.002 0.088***</td>
<td>(0.015) (0.027) (0.009) (0.008) (0.017)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.018 −0.180 0.952*** 0.007 −0.304***</td>
<td>(0.071) (0.132) (0.040) (0.036) (0.076)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>−0.200** 0.113  −0.035 0.909*** 0.123</td>
<td>(0.097) (0.192) (0.059) (0.053) (0.110)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.038  0.046  0.027  0.023 0.854***</td>
<td>(0.038) (0.066) (0.022) (0.019) (0.04)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Matrix \( W_N \) | 0.017*** −0.016*** −0.017*** 0.003 0.20***        | (0.002) (0.005) (0.005) (0.005) (0.003) |
|                 | 0.016*** 0.009*** −0.003 0.007*** 0.004**          | (0.001) (0.002) (0.003) (0.002) (0.002) |
|                 | 0.004*** −0.003 −0.004 0.020*** 0.009**            | (0.001) (0.002) (0.003) (0.002) (0.004) |
|                 | 0.005*** 0.006 −0.015*** 0.008*** 0.008***         | (0.001) (0.004) (0.002) (0.003) (0.002) |
|                 | 0.008*** 0.004 −0.006** 0.008 0.036***              | (0.001) (0.003) (0.003) (0.006) (0.002) |

| Matrix \( W_C \) | −0.010 0.012 0.004 0.056 0.323***                | (0.017) (0.029) (0.037) (0.084) (0.027) |
|                 | −0.417*** −0.057 −0.064 0.207*** 0.210***        | (0.037) (0.131) (0.109) (0.08) (0.081) |
|                 | 0.001 −0.007 0.049 0.161*** 0.059               | (0.015) (0.044) (0.034) (0.022) (0.049) |
|                 | −0.011 0.061 0.052 0.116*** 0.042             | (0.019) (0.042) (0.055) (0.023) (0.036) |
|                 | −0.011 0.078 −0.191** 0.188*** 0.178***       | (0.039) (0.14) (0.076) (0.07) (0.069) |

| Matrix \( \Psi \) | \( \psi_1 \) \( \psi_2 \) \( \psi_3 \) \( \psi_4 \) \( \psi_5 \) | 71.558*** 25.446*** 16.991*** 8.013*** 3.985*** |
|                 | \( \xi_1 \) \( \xi_2 \) \( \xi_3 \) \( \xi_4 \) \( \xi_5 \) | (10.142) (3.584) (2.359) (1.18) (0.6) |

| Mixture prob. \( \gamma \) | 0.770*** | (0.018) |
NOTE:
Standard errors obtained from the inverse Hessian of the log-likelihood function.

(***) / (***) indicates statistical significance at 5 % / 1 % significance level.
The log-likelihood function gets value 6789.10
Abbreviations: Ireland (ire), Greece (gre), Spain (spa), Italy (ita), and Portugal (por).

References


Chapter 4

Uncertainty and Business Cycles

Abstract

The paper develops a vector autoregressive model with autoregressive conditional heteroskedasticity in mean effects to decompose the effect of a stock market crash on industrial production into two components, the effect of negative returns and the effect of higher volatility. Our special attention is on the effect of volatility as it is our proxy for business cycle effects of uncertainty. We estimate the model with US data from 1919 to the mid 2013 and find uncertainty significantly countercyclical. Impulse response analysis shows that a monthly drop of ten percent in stock market prices is followed by a cumulative decline of three percent in the industrial production. Of this decline, around two thirds are explained by higher uncertainty.

\footnote{This chapter has appeared as \textit{HECER Discussion Paper No. 373 (2013)}.}
4.1 Introduction

Does uncertainty aggravate business cycles? The recent financial crisis has again brought up this question. For example, the bankruptcy of the Lehman Brothers in Autumn 2008 was followed by a jump in the US stock market volatility which in its turn coincided with declining industrial production (see Figure 4.1). There are many theoretical reasons why higher uncertainty could cause a negative business cycle. For example, higher uncertainty might lead firms to scale down and postpone their investments and hiring (Bernanke (1983), Bloom (2009)), or consumers to postpone purchases of durable goods (Romer (1990)). On the other hand, declining (and hence usually more volatile) stock market prices in a recession could be a consequence of investors expecting lower future dividends and capital gains.

Either way, one would assume that stock market returns and volatility predict business cycles. This paper brings together two (possible) real economic effects of a stock market crash: the first order effect of negative returns, and the second order effects via higher uncertainty which we measure by stock market volatility. The methodological contribution of this paper is to extend the multivariate general autoregressive conditional heteroskedastic (GARCH) model of Vrontos et al. (2003) to a vector autoregressive (VAR) model with GARCH-in-mean effects. This model provides an ideal framework to study both jointly and separately the importance of the two possible effects of a stock market crash.

The variables of our model are the monthly stock market return and the change in industrial production. In order to study the business cycle effects of negative stock market returns and higher uncertainty, we need to identify a structural shock that generates stock market surprises. By referring to the high autocorrelation in the monthly capacity utilization in the manufacturing industry, we argue that the production of an industrial company necessarily is quite persistent. This enables us to identify one of the structural shocks of our model as a stock market specific shock which is interpreted as financial news. Then, financial news can affect the stock market returns immediately but the industrial production only with a lag. Hence, our financial news variable can generate unexpected increases in the stock market volatility. The model can be estimated with the method of maximum likelihood (ML), and we can statistically test the significance of the first and second order effects of a stock market crash on the industrial production. Also, our model allows us to separately study the importance of these two effects by means of

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2 Actually, Romer (1990) simply extends the intuition of the "wait and see" hypothesis for investments of Bernanke (1983) to consumable durable goods.

3 Although there are many other possible measures of uncertainty, stock market volatility is probably the most common one. Also, it is highly correlated with the other measures (see, for example, Arnold & Vrugt (2008), Bloom (2009)).
Figure 4.1: Monthly US stock market returns, estimated stock market volatility, and change in the US industrial production in January 2007–December 2010

Note: Here volatility of the stock market returns is computed as the conditional standard deviation of the returns implied by the univariate GARCH(1,1) model estimated from our full sample period (January 1919–July 2013). Sources: http://www.econ.yale.edu/~shiller/data.htm (stock market data), St.Louis FED’s FRED database (industrial production), and own calculations.

In the empirical application, we estimate the model with US data covering the period from the beginning of 1919 to the mid 2013. According to the estimation results the stock market volatility (as well as the return) is a statistically significant predictor of the change in industrial production. Furthermore, as the theoretical contemplations would predict, financial volatility is countercyclical, meaning that higher volatility decreases the growth rate of industrial production. The impulse response analysis shows that a (monthly) negative stock market shock of ten percent is followed by a slump in the growth rate of industrial production that lasts for around two years, with the cumulative effect on the industrial production of roughly minus three percent. Approximately half of the duration of the business cycle is explained by the direct effects of negative stock market returns. The other half is due to the higher volatility, or uncertainty.

As the emphasis of the paper is to study the business cycle effects of uncertainty, it is closely related to the literature on the linkages between financial and macroeconomic volatility. The question of predicting financial volatility with macro variables (and their volatility), is of course
an old theme in the financial literature (Schwert (1989) is a classical reference, whereas Beltratti & Morana (2006), Diebold & Yilmaz (2008), and Engle et al. (2013) are more recent ones, only to mention a few). According to Diebold & Yilmaz (2008) the main finding of this research is that, perhaps unsurprisingly, stock market volatility is higher in recessions. This is also the main conclusion of Hamilton & Lin (1996) who, furthermore, notice that it is the higher stock market volatility that precedes a fall in the US industrial production by one month. However, in their model stock market volatility and industrial production follow the same latent process which determines the state of the economy. Hence, they do not consider the direct links between the two variables.

The literature that studies the macroeconomic effects of financial volatility is not very voluminous but growing, especially due to the recent financial crisis. From the perspective of this paper, the most relevant part of this literature consists of the papers that explicitly focus on the effects of uncertainty or volatility shocks. The main methodology of this line of research is to write down and calibrate a theoretical macroeconomic model where the uncertainty shock is modeled as a second order shock to the productivity process (see, for example, Bloom (2009), Bloom et al. (2012)). The main finding is that uncertainty shocks can create business cycles that last for about six to twelve months. Our result on the magnitude of the second order effect of a stock market crash is consistent with this.

There are only few truly empirical studies on the subject. Alexopoulos & Cohen (2009), Beetsma & Giuliodori (2012), and Denis & Kannan (2013) are some rare exceptions. As the model specifications of these papers come quite close to our model, we will discuss them (and problems in their identification) in detail later on. Also Bachmann et al. (2013), and Baker & Bloom (2013) use statistical methods to determine the business cycle effects of uncertainty. But the methods and data they use are quite different from ours. As a measure of uncertainty, the former uses dispersion of forecasts for economic conditions of manufacturing companies. The latter considers an event study framework where natural disasters, coups, and revolutions are used as exogenous sources of uncertainty shocks. Overall, these studies find that higher uncertainty has a statistically significant negative effect on economic growth. Our results reconfirm this and, also highlight the relative importance of the second order effects to the first order effects in explaining a recession.

The rest of the paper is organized as follows. In the next section, we introduce the model and discuss its estimation and identification of the structural shocks. In Section 4.3, we present the estimation results on the US data. Finally, Section 4.4 concludes.
4.2 The Empirical Framework

In this section, we first introduce the model for the joint dynamics of industrial production and the stock market return and discuss its estimation. Then, we discuss both the identification of the structural shocks and the econometric analysis of the identified model. Finally, we briefly compare the model to a number of similar ones in the literature.

4.2.1 The Model

Let us denote by $\Delta \text{ind}_t$ the monthly percentage change in industrial production and by $r_t$ the monthly stock market return. We collect the variables into the $(2 \times 1)$ vector $y_t = [\Delta \text{ind}_t, r_t]'$ and assume that $y_t$ follows a bivariate GARCH model with a non-zero conditional mean. Specifically, we assume the following multivariate specification of the (G)ARCH-in-mean model of Engle et al. (1987):

$$A(L)y_t = \mu + Ch_t + u_t,$$

(4.2.1)

where $A(L) = I_2 - A_1 L - \ldots - A_p L^p$ is a $(2 \times 2)$ matrix polynomial, $\mu$ is a $(2 \times 1)$ vector of the intercepts, $C$ is a $(2 \times 2)$ coefficient matrix, and $h_t = [h_{1,t}, h_{2,t}]'$ is a $(2 \times 1)$ vector of the conditional volatilities $h_{1,t}$ and $h_{2,t}$ of the structural shocks $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$, respectively. The structural shocks are discussed shortly. Finally, $u_t = [u_{\text{ind},t}, u_{r,t}]'$ is the $(2 \times 1)$ reduced from error vector. In the empirical application, the order of $A(L)$ is determined with the Bayesian information criterion (BIC).

To complete the model, we assume that the reduced form errors are a linear function of the two structural shocks with the following simple specification:

$$u_t = B \varepsilon_t,$$

(4.2.2)

where $\varepsilon_t = [\varepsilon_{1,t}, \varepsilon_{2,t}]$ is the $(2 \times 1)$ vector of the structural shocks, and $B$ is a $(2 \times 2)$ coefficient matrix. To identify the shocks, we assume that $B$ is the following lower triangular matrix:

$$B = \begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}.$$  

(4.2.3)

This assumption identifies the second structural shock ($\varepsilon_{2,t}$) as a stock market specific shock. The assumption that the diagonal elements are equal to one restricts the number of parameters and, hence, normalizes the model. The structural shocks are assumed to be mutually orthogonal.
and to follow univariate GARCH(1,1) processes with Gaussian conditional distributions:

\[ \varepsilon_{i,t} | I_{t-1} \sim N(0, h_{i,t}^2), \]

\[ h_{i,t}^2 = \alpha_i + \beta_i \cdot \varepsilon_{i,t-1}^2 + \gamma_i \cdot h_{i,t-1}^2 \]

for \( i = 1, 2 \). Here \( I_{t-1} \) denotes the information set up to time period \( t - 1 \) (the period \( t - 1 \) included), and \( \alpha_i, \beta_i, \) and \( \gamma_i, \) \( i = 1, 2, \) are parameters.

The multivariate GARCH model in equations (4.2.2)–(4.2.5) was proposed by Vrontos et al. (2003). The model is well defined under rather mild assumptions. To see this, collect first the conditional variances \( h_{1,t}^2 \) and \( h_{2,t}^2 \) into the \((2 \times 2)\) diagonal matrix \( H_t^2 = \text{diag}(h_{1,t}^2, h_{2,t}^2) \). Then, given specification (4.2.3), the conditional covariance matrix of the reduced form error vector \( \mathbf{u}_t \),

\[ \Sigma_u | I_{t-1} = B H_t^2 B', \]

is always positive definite as long as the conditional variances \( h_{1,t}^2 \) and \( h_{2,t}^2 \) are well defined. In order to guarantee this, we follow Vrontos et al. and assume that \( \alpha_i > 0, \beta_i \geq 0, \) and \( \gamma_i \geq 0 \) for \( i = 1, 2, \)

The model can be estimated with the method of ML. Assume a sample of \( T \) observations, and denote by \( \mathbf{Y}_{t-1} \) the vector of observations up to the time period \( t - 1 \) (\( y_{t-1} \) included). Then, given the initial values \( \{y_0, \ldots, y_{-p}, h_0, \varepsilon_0\} \), the conditional density function of the model (4.2.1)–(4.2.5) becomes

\[
f(\mathbf{y}_t | \mathbf{Y}_{t-1}) = \text{det}(B H_t)^{-1} \times \\
\exp \left\{-\frac{1}{2} (A(L)\mathbf{y}_t - \mu - Ch_t)'(B H_t^2 B')^{-1}(A(L)\mathbf{y}_t - \mu - Ch_t) \right\},
\]

where we have omitted the constant terms of the Gaussian distribution. After collecting all the parameters of the model into the vector \( \delta \), the log-likelihood function of the model can be written as

\[
l(\delta, \mathbf{Y}_T) = -T \times \ln(\text{det}(B)) - \sum_{t=1}^{T} \ln(\text{det}(H_t)) - \\
-\frac{1}{2} \sum_{t=1}^{T} (A(L)\mathbf{y}_t - \mu - Ch_t)'(B H_t^2 B')^{-1}(A(L)\mathbf{y}_t - \mu - Ch_t)
\]

which can be maximized numerically with standard optimization algorithms. In our empirical analysis, we take the first \( p \) observations of \( \mathbf{y} \) as the initial values for the dependent variables,
set \( h_0 \) equal to the sample standard deviations of the residuals \( \hat{u}_{\text{ind},t} \) and \( \hat{u}_{\text{r},t} \) of a standard \( p \)th order VAR model estimated from the full sample, and finally assume that \( \varepsilon_0 = 0 \).

### 4.2.2 Identification and Econometrics Analysis

Assumption (4.2.3) on matrix \( B \) serves two purposes. On the one hand, as mentioned above, it guarantees that the model is well defined under the stated assumptions (see Vrontos et al. (2003, 314–15)), on the other hand it identifies the second structural shock \( (\varepsilon_{2,t}) \) as the stock market specific shock, a shock which affects the stock market returns instantaneously, in period \( t \), but industrial production only from period \( t + 1 \) onwards.

The second shock \( \varepsilon_{2,t} \) can be interpreted as financial news. As we are especially interested in the effects of unexpected surges in uncertainty on the real side of an economy, we will restrict our attention to studying the business cycle effects of this shock. The first structural shock \( (\varepsilon_{1,t}) \) does not have any specific interpretation here. It is a shock that can affect both the real sector and the stock markets contemporaneously, and hence it could incorporate, for example, productivity shocks.

Why does it make sense to assume that financial news in period \( t \) instantly affects only the stock markets but not the industrial production? The monthly capacity utilization in the US manufacturing, mining, and electric and gas sectors (in 1972–2012) is a highly persistent variable (with the coefficient estimate of 0.99 in a first order autoregressive model). Hence, a high level of orders of an industrial company this month predicts a high level of orders also in the next month. This seems natural as probably many industrial products are (investment) goods whose production take more than a month. Assume there is negative financial news, such as the bankruptcy of the Lehman Brothers, in period \( t \). During this same period, given the high persistence in the industrial orders, the companies would still be busy in fulfilling their orders from the previous months, and so, the shock would not affect the current production. Of course, it could affect the number of new orders received in period \( t \) and, hence, the future production, but this is exactly the effect we are interested in.

In order to avoid any misunderstanding, let us briefly discuss the limitations of our identification scheme. Such dramatic news as the default of the Lehman Brothers and the subsequent stock market crash could of course be a consequence, not necessarily the cause of slowing economic activity; after all, stock market prices should reflect discounted future dividends and capital gains. However, such reversed causality between stock market prices and future economic activity is irrelevant for our purposes of quantifying the business cycle effects of uncertainty shocks. To this end, we only need to identify a stock market specific shock that can generate stock market
volatility surprises. With such a shock available we can separate the direct effect of the drop in the stock market prices from the volatility effect.

To take an example, consider Figure 4.1, which shows that after the default of the Lehman Brothers, the US stock market prices collapsed and the estimated volatility tripled. Whether the subsequent stock market crash was the cause or the consequence of the recession, the upsurge in the volatility suggests that at least many investors perceived a huge rise in the uncertainty over the actual and future state of the US economy. Our question is whether there is any evidence of this uncertainty prolonging the slump as the theoretical literature referred to in Introduction suggests. This means that higher volatility should be an important variable in explaining variation in the industrial production.

In order to study this question we can first test for the statistical significance of the parameter $c_{1,2}$ (the first row, second column element of the matrix $C$ in equation (4.2.1)). This measures the direct effect of the volatility of the structural shock $\varepsilon_{2,t}$ on industrial production. One would expect that $c_{1,2} \leq 0$, i.e. higher volatility tends to decrease industrial production. Furthermore, assume a negative realization of $\varepsilon_{2,t}$, say, at period $T_0$, and call it $\tilde{\varepsilon}_2$. According to our assumptions, $\tilde{\varepsilon}_2$ affects the stock market return already at period $T_0$ but the growth rate of industrial production only from period $T_0 + 1$ onwards. In our model, the effect of $\tilde{\varepsilon}_2$ on $\Delta ind_{T_0+1}$ comes from two channels; on the one hand via the lagged (negative) stock market return, and on the other hand via the conditional variance of $\varepsilon_{2,t}$ which, according to equation (4.2.5), increases at period $T_0 + 1$. The first channel corresponds to the first order effect of a stock market shock, and the second channel to the second order, or uncertainty, effect.

The dynamic effects of $\tilde{\varepsilon}_2$ and the importance of the two channels on the growth rate of industrial production can be studied with two different impulse response functions. First, as suggested by Elder (2003)\(^4\), we can calculate the total impulse response function by simply introducing $\tilde{\varepsilon}_2$ and numerically compute the responses of $r_t$ and $\Delta ind_t$ to the shock. This gives us the total effect of $\tilde{\varepsilon}_2$. (The details of our actual calculations are explained in the next section.) Then, in order to separate the effect of higher volatility on $\Delta ind_t$ from the total effect of $\tilde{\varepsilon}_2$, we can calculate the responses of the system to another shock which we refer as a "volatility jump". This jump corresponds to an increase only in $h_{2,t}$ at period $T_0 + 1$ which exactly matches the increase in it due to the shock $\tilde{\varepsilon}_2$. The two impulse response functions of $\Delta ind_t$ give us the total effect of the stock market shock and the effect of higher volatility on the growth rate of industrial production. The difference of the two (at each period) tells us the first order effect.

4.2.3 Related Literature

Hamilton & Lin (1996) model the joint dynamics of the changes in the US industrial production and the excess stock market return. The framework they consider is a bivariate Markov-switching VAR model with ARCH-effects. Their main finding is that both variables are more volatile in recessions and that increasing volatility in stock markets precedes declines in the industrial production by one month. This result supports our identifying assumption.

Alexopoulos & Cohen (2009), Denis & Kannan (2013), and Beetsma & Giuliodori (2012) study the business cycle effect of uncertainty (financial market volatility) with the VAR framework. Unlike us, the first two papers assume that volatility shocks (to financial markets) affect the real sector immediately but that the real sector specific shocks affect volatility only with a lag. On the basis of the discussion above, this seems an incorrect timing of events. At the very least, we should expect shocks to real sector to have an immediate effect on the financial markets.

Like us, Beetsma & Giuliodori (2012) assume that stock market volatility affects the real sector only with a lag, but strangely enough they include the quarterly return of the Dow Jones index and its volatility as separate variables in their VAR model and assume that the return can immediately affect the volatility but not vice versa. However, as their volatility variable is necessarily a function of the returns data, dealing these two variables as separate time series is questionable. In our framework, the effect of returns on the volatility is explicitly modeled, and the parameters of the model jointly estimated.

Nonetheless, Alexopoulos & Cohen (2009), Denis & Kannan (2013), and Beetsma & Giuliodori (2012) find that uncertainty (or volatility) shocks can predict recessions which, depending on the size of the shock, last one to two years. In the next section we find quite similar results with our model fitted to US data.

4.3 Uncertainty and Business Cycles in the US

In order to estimate the model, we consider monthly percentage changes in the US stock market prices (returns \( r_t \)) and industrial production (\( \Delta \text{ind}_t \)). The stock market prices are downloaded from Robert Shiller’s home page and here we use the nominal prices (the correlation between the nominal and real prices is 0.96). The industrial production data is from the online database (FRED) of the Federal Reserve Bank of St.Louis. As the industrial production data is available only from January 1919 onwards, our data on monthly changes covers the period from February

\[ \Delta \text{ind}_t = 100 \times \left( \ln \text{ind}_t - \ln \text{ind}_{t-1} \right), \]
\[ r_t = 100 \times \left( \ln P_t - \ln P_{t-1} \right), \]

where \( P_t \) is the monthly stock market price index.


The variables are computed in the following way: \( \Delta \text{ind}_t \) and \( r_t \) are the percentage changes in industrial production and stock market prices, respectively.
1919 to July 2013. This means that there are 1134 observations.

4.3.1 Estimation Results and Testing

Table 4.3.1 reports the estimation results for the model (4.2.1)–(4.2.5) where the lag length \( p \) is set to two by the BIC.\(^7\) In the equation of the change in industrial production, all the coefficients of the lagged returns and changes in industrial production are statistically significant at the 5% significance level. On the contrary, in explaining the stock market return, only the first lag of the return seems to have a statistically significant coefficient at the 5% significance level while the coefficient of the first lag of the change in industrial production is statistically significant only at the 10% significance level. All the parameters of the GARCH-processes are statistically significant at the 1% significance level.

As explained in Section 4.2.2 our main interest is in testing whether the coefficient of \( h_{2,t} \), the conditional volatility of the shock market specific structural shock, on the change in industrial production is statistically different from zero. Its estimated value in Table 4.3.1 equals -0.08 and, hence, is negative as expected. The value of the likelihood ratio test statistic for the null hypothesis of \( c_{1,2} = 0 \) against the alternative hypothesis of \( c_{1,2} \neq 0 \) gets value 9.40, which means that the null hypothesis is rejected at any reasonable significance level. Hence, we conclude that uncertainty is a statistically significant explanatory variable for the changes in industrial production. It is also countercyclical in the sense that when it rises, as it usually does in recessions, it decreases the growth rate of industrial production.

Neither of the conditional standard deviations seem to be statistically significant variables in explaining the stock market return. However, somewhat surprisingly, the coefficient of the conditional standard deviation of the first structural shock appears to be a statistically significant predictor of the change in industrial production. However, as the robustness checks in Section 4.3.3 below shows, this result seems to depend on the fact that our sample period includes the Great Depression of the 1930’s. As a final remark on Table 4.3.1, notice that the parameter \( b \) of the matrix \( B \) in equation (4.2.3) is statistically significant only at the 10% significance level. Hence, there is some weak evidence of the second structural shock being a real sector specific shock.

4.3.2 Impulse Response Analysis

In order to study the economic significance of uncertainty in explaining recessions, we generate a stock market crash by introducing a large negative realization of \( \varepsilon_{2,t} \). The magnitude of the shock

\(^7\)All the estimations were done with the procedures in the CMLMT library of GAUSS.
Table 4.3.1: Estimation results (standard errors in parentheses)

<table>
<thead>
<tr>
<th>Dependent variables</th>
<th>$\Delta \text{ind}_t$</th>
<th>$r_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intercepts</strong></td>
<td>0.239</td>
<td>0.560</td>
</tr>
<tr>
<td></td>
<td>(0.101)</td>
<td>(0.384)</td>
</tr>
<tr>
<td>$\Delta \text{ind}_{t-1}$</td>
<td>0.244</td>
<td>-0.135</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.075)</td>
</tr>
<tr>
<td>$r_{t-1}$</td>
<td>0.020</td>
<td>0.269</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>$\Delta \text{ind}_{t-2}$</td>
<td>0.151</td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>$r_{t-2}$</td>
<td>0.033</td>
<td>-0.061</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>$h_{1,t}$</td>
<td>0.241</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.124)</td>
</tr>
<tr>
<td>$h_{2,t}$</td>
<td>-0.080</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.113)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$h_{1,t-1}$</th>
<th>$h_{2,t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intercepts</strong></td>
<td>0.062</td>
<td>0.622</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.169)</td>
</tr>
<tr>
<td>$h_{1,t-1}^2$</td>
<td>0.646</td>
<td>⋮</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>⋮</td>
</tr>
<tr>
<td>$h_{2,t-1}^2$</td>
<td>⋮</td>
<td>0.832</td>
</tr>
<tr>
<td></td>
<td>⋮</td>
<td>(0.021)</td>
</tr>
<tr>
<td>$\varepsilon_{1,t-1}$</td>
<td>0.394</td>
<td>⋮</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>⋮</td>
</tr>
<tr>
<td>$\varepsilon_{2,t-1}$</td>
<td>⋮</td>
<td>0.140</td>
</tr>
<tr>
<td></td>
<td>⋮</td>
<td>(0.022)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$u_{\text{ind},t}$</th>
<th>$u_{r,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{1,t-1}$</td>
<td>⋮</td>
<td>0.117</td>
</tr>
<tr>
<td></td>
<td>⋮</td>
<td>(0.077)</td>
</tr>
</tbody>
</table>

Note: Standard errors in parenthesis are obtained from the inverse Hessian of the log-likelihood function.
is minus ten. This generates a drop of ten percentage points in the monthly stock market return which roughly corresponds to the average (-14%) of the monthly returns in September–October 2008, the period when the Lehman Brothers defaulted (Figure 4.1).

The shock happens in period 0. We assume that in period -1 the variables equal their long run levels (unconditional means) which are computed based on our parameter estimates. The long run levels (monthly percentage changes) of stock market return and the change in industrial production are 0.72 percent and 0.37 percent, respectively. When these are transformed into yearly percentage changes, we get 8.64 percent stock market return and 4.44 percent increase in the industrial production. These are reasonable figures, which lends support to our estimation results. Notice also that in our model there is no feedback in equations (4.2.4)–(4.2.5) from the variables of y_t back to the GARCH processes of the conditional variances of the structural shocks. This makes the calculation of the impulse responses straightforward.

Figure 4.2 reports the impulse responses of the stock market return and the change in industrial production to the stock market shock in period 0 with the 95% confidence intervals. The impulse responses are demeaned with the long run levels of r_t and Δind_t in order to highlight the effect of the shock. Hence, for example the period 0 value of -10 for r_t means that, due to the shock, the stock market return is ten percentage points lower than the return in the long run. Gradually, both variables converge back to their long run levels (level zero in the graphs).

For both variables the drop is significant. As the upper panel of Figure 4.2 shows, for the stock market return the highest impact of the shock is right after the shock (period 0), and the stock market accommodates to the shock quite quickly; after four to six months the stock market return has converged back to its long run level. In period 4, there is even a small boom in the stock markets as the volatility has started to decrease from its high levels right after the shock (the volatility time series is not reported) and uncertainty decreases. In the lower panel, as assumed in our model, the effect of the shock on the change in the industrial production in period 0 is nil. The highest impact of the shock on Δind_t comes three months after the shock, and the negative effect lasts much longer than for the stock market return. It is only after around two to two and a half years (!) that Δind_t has basically converged back to its long run level.

8The computation of the 95% confidence intervals was carried out in five steps: (Step 1) the ML estimates of the parameters of the model were used to simulate a data set of the same size as our actual data. The simulation of the data consisted of three phases: first, the initial values of ε1,t and ε2,t were drawn from normal distributions with the variances equal to the long run values of h1,t and h2,t, respectively. Second, the estimated univariate GARCH processes (4.2.4)–(4.2.5) were used to simulate the realizations of the structural shocks. Third, the simulated structural shocks (and the estimated equations (4.2.1)–(4.2.3)) were used to construct the new data set on r_t and Δind_t. In 'Step2', the model was re-estimated for these new (simulated) data. 'Step 3' consisted of using the new parameter estimates from the previous step to compute the impulse responses of r_t and Δind_t to a negative realization of ε2,t = −10 (the same magnitude as in the original case). In 'Step 4', the previous three steps were repeated 10000 times. Finally, in 'Step 5', for each lag separately, we ordered the 10000 impulse responses into ascending order and selected the elements that were the 500th and 9500th in order. We did the fifth step separately for both variables, r_t and Δind_t.
Figure 4.2: Response of $r_t$ and $\Delta ind_t$ to a negative stock market specific shock

Note: Negative stock market specific shock at period $t=0$. The panels show demeaned impulse responses (demeaned by the long-run levels of the variables). Hence, the levels $r_t = 0$ and $\Delta ind_t = 0$ correspond to the long run stock market return and the growth rate of industrial production, respectively. The bootstrapped confidence intervals are based on 10000 replications (for details, see footnote 8).
Figure 4.3 decomposes the total effect of the stock market crash into the first and second order effects (for details on the concepts, see the end of Section 4.2.2). The second order effect measures the share that uncertainty explains of the negative business cycle following the stock market crash. As seen from the figure, the first order effect lasts only around nine to ten months which is consistent with the quick recovery in the stock market return. After this, for around one more year, it is only the effect of higher volatility that still drags down the growth rate of industrial production. At the trough of the business cycle (period 3), uncertainty explains around one third of the minus 0.6 percentage points deviation of $\Delta \text{ind}_t$ from its long term level.

Figure 4.4 shows the cumulative effect of the stock market crash on the change in industrial production and decomposes it into the shares explained by the first and second order effects. The cumulative effects were computed by summing the demeaned impulse responses in the previous figure. As Figure 4.4 shows, the total cumulative effect of the stock market crash on $\Delta \text{ind}_t$ is around minus three percentage points. This means that around two years after the shock, the level of industrial production is three percent lower than without the shock (assuming the growth rate of industrial production at its long run level). As seen from the figure, the second order effect, or uncertainty, explains around two thirds of the total cumulative effect. Based on Figure 4.3, this result is intuitive as it is the second order effect that prolongs the business cycle with another year while the first order effect dies out quickly. Clearly, uncertainty is an important factor in propagating and prolonging business cycles.

### 4.3.3 Robustness Checks

Beetsma & Giuliodori (2012) argue that the responsiveness of the real sector of an economy to stock market volatility shocks changes in time. They find that, after the 1980's, the GDP growth has become less responsive to volatility shocks. This raises the question of how robust our findings are for different time intervals, especially as our sample period includes two severe economic crises, one at the beginning and the other at the end of the sample.

Table 4.3.2 shows the estimates of the coefficients $c_{1,1}$ and $c_{1,2}$ for a number of subsamples. Encouragingly, the estimate of the effect of stock market volatility on the industrial production (the coefficient $c_{1,2}$) is always negative with p-values below 0.05, but we also reconfirm the finding of Beetsma & Giuliodori (2012) that the absolute value of $c_{1,2}$ decreases towards the end of the sample period. Also, according to Table 4.3.2 the coefficient $c_{1,1}$ appears to become statistically insignificant towards the end of the sample. It seems that this coefficient gets its largest value in the period including the Great Depression and the Second World War. Overall, our main finding that uncertainty is countercyclical, seems robust.
Figure 4.3: Decomposition of the IRF of $\Delta ind_t$

Note: The first order effect refers to the direct effect of $\varepsilon_{2,t}$ on $\Delta ind_t$ via stock market returns, the second order effect refers to the effect of $\varepsilon_{2,t}$ on $\Delta ind_t$ via higher $h_{2,t}$ only, or uncertainty. For details, see the end of Section 4.2.2.

Table 4.3.2: Robustness of volatility coefficients (p-values in parentheses)

<table>
<thead>
<tr>
<th>Time period</th>
<th>$c_{1,1}$</th>
<th>$c_{1,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample period</td>
<td>0.24</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Feb/1919–Dec/1954</td>
<td>0.43</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Jan/1955–Dec/1989</td>
<td>0.07</td>
<td>-0.22</td>
</tr>
<tr>
<td></td>
<td>(0.73)</td>
<td>(0.03)</td>
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<tr>
<td>Jan/1955–Jul/2013</td>
<td>0.21</td>
<td>-0.09</td>
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<tr>
<td></td>
<td>(0.23)</td>
<td>(0.02)</td>
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Note: p-values are based on the standard errors as detailed in the note to Table 4.3.1, $c_{1,1}$ ($c_{1,2}$) is the effect of $h_{1,t}$ ($h_{2,t}$) on $\Delta ind_t$.
4.4 Conclusions

The aim of this paper was to study the business cycle effects of uncertainty. According to theory, one expects uncertainty to be countercyclical. To examine this, we proposed measuring uncertainty with stock market volatility and introduced a bivariate VAR-GARCH-in-mean model for the monthly stock market return and the change in industrial production. We identified stock market specific structural shock which can generate volatility surprises whose effects on industrial production we study. The framework enables us to test the statistical significance of uncertainty in explaining variations in the industrial production.

In analysis of US data from the beginning of 1919 to the mid 2013, we found that, in accordance with the theoretical models, uncertainty is countercyclical with statistically significant coefficient. The result was robust for varied time periods. The impulse response analysis shows that a ten percent monthly decrease in the stock market prices is followed by a slump in the growth rate of the industrial production that lasts for about two years and leaves the industrial production three percent lower than without the stock market crash. Roughly half of the duration of the business cycle and two thirds of the total cumulative effect of the stock market shock are explained by higher uncertainty.
References


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<td>1</td>
<td>Matthijs Lof</td>
<td>ESSAYS ON EXPECTATIONS AND THE ECONOMETRICS OF ASSET PRICING</td>
<td>10.4.2013</td>
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<td>100</td>
</tr>
<tr>
<td>2</td>
<td>Anssi Kohonen</td>
<td>PROPAGATION OF FINANCIAL SHOCKS: EMPIRICAL STUDIES ON FINANCIAL SPILLOVERS</td>
<td>25.3.2014</td>
<td>978-952-10-8724-0</td>
<td>97</td>
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