MAXIMUM LOSS CALCULATION USING SCENARIO ANALYSIS, HEAVY TAILS AND IMPLIED VOLATILITY PATTERNS

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Key words: Risk Management, Scenario Analysis, Maximum Loss Calculation, Non-normality in Returns, Hyperbolic Distributions

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Abstract

The objective of this paper is to improve option risk monitoring by examining the information content of implied volatility and by introducing the calculation of a single-sum expected risk exposure similar to the Value-at-Risk. The figure is calculated in two steps. First, there is a need to estimate the value of a portfolio of options for a number of different market scenarios, while the second step is to summarize the information content of the estimated scenarios into a single-sum risk measure. This involves the use of probability theory and return distributions, which confronts the user with the problems of non-normality in the return distribution of the underlying asset. Here the hyperbolic distribution is used to describe one alternative for dealing with heavy tails. Results indicate that the information content of implied volatility is useful when predicting future large returns in the underlying asset. Further, the hyperbolic distribution provides a good fit to historical returns enabling a more accurate definition of statistical intervals and extreme events.

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1 Introduction

Risk management is a key concern to any participant operating in the field of financial derivatives. As portfolios are becoming increasingly complex, the importance of useful and practical horizontally integrated risk management methods has increased, whilst for instance new regulations have simultaneously motivated the need for vertically integrated risk management systems.

The sensitivity of the option premium to changes in the price of the underlying asset and the level of implied volatility are usually considered the most significant risks of vanilla option trading. Quantifying these exposures using an appropriate pricing model and differential calculus yields the Greeks commonly referred to as delta and vega, respectively. These measures are ideal in a horizontal aspect at the trading desk when closely monitoring and adjusting individual portfolios but might be more difficult to communicate vertically in an organization.

The Basle Committee on Banking Supervision (1995) has primarily recommended two methods for generating a unified set of risk measures on a daily basis, contingent on important qualitative and quantitative standards. These have rapidly become industry standards for measuring risk for both internal and external constituencies (see for example Alexander [2000]). The first approach is to calculate a Value-at-Risk (VaR) measure, which is represented as the lower percentile of an unrealized profit and loss distribution where the distribution is based on movements of appropriate market risk factors over a fixed horizon. The second approach is to quantify the maximum loss over a given set of scenarios with regard to these risk factors. Further, as noted by Saita & Sironi (1999), the Basle Committee on Banking Supervision endorsed the use of such internal models in its 1996 market risk-based capital ratios amendment proposal. The critical step in the increasing use of VaR models has foremost been their recognition by the international regulatory community and the fact that it conveniently aggregates the several components of market risk into a single summary measure.

Hence, the purpose of this study is to improve option risk monitoring mainly by introducing the calculation of a single-sum expected risk exposure, the Expected Maximum Loss (EML), similar to the VaR and applicable to any (continuously traded) portfolio of vanilla options. This is accomplished in two steps. First, there is a need to properly calculate the value of a portfolio of options for a number of different
market scenarios. This in turn would require a model that at least captures the
dynamics of the volatility smile, especially if the time span of the ex ante estimated
period is longer than a few days. It is rather obvious that this phenomenon referred to
as the volatility smile, meaning that implied volatilities across strike prices and
maturities exhibit patterns that are not compatible with standard Black & Scholes
theory, is the cause of the most common pricing dilemmas. Some of the most
significant contributors to this topic are Derman & Kani (1994), Dupire (1994),
Rubinstein (1994) and Derman (1999). As the rest of the thesis consists of four essays
dealing with this topic, this essay will instead focus on the next step of the maximum
loss calculation and also examine the information content of implied volatility in
order to predict market regimes and future large returns.

The second step in the calculation process of the EML is to summarize the
information content of the estimated scenarios into a single-sum risk measure. This
involves the use of probability theory and return distributions, which confronts the
user with the problems of non-normality in the returns. It is well known that the
returns of most financial assets exhibit heavy tails, suggesting that there is a presence
of leptokurtosis (positive excess kurtosis) in the conditional return distribution (see
for example Bates [1996] and Prause [1997]). Here a short review is given of the
increasingly popular Extreme Value Theory (EVT) while the hyperbolic distribution
is examined to provide an appropriate solution for dealing with these heavy tails. The
hyperbolic approach is also successfully applied in for example Eberlein et al. (1998)
and Ahn et al. (2000). By constructing a simple portfolio of vanilla options the EML
measure presented herein is tested on real market data to demonstrate how it applies
to real market surroundings.

The rest of the study is structured as follows. Basic scenario analysis and
Greek calculation are topics covered in Chapter 2 whilst a review of the patterns of
implied volatility is documented in Chapter 3. Further, the risk-exposure calculation
and heavy-tail modeling is viewed in Chapter 4 and the essay is concluded with a
summary in Chapter 5.
2 Managing the Risks of Option Trading

Risk management or monitoring means keeping track of the risk exposure. Concerning vanilla options the most important risk are of probably the delta and vega risks, that is the sensitivity of the value of the portfolio to changes in the underlying asset and volatility, respectively. The delta, $\Delta$, for a European index call is defined as

$$\Delta = \frac{\partial c}{\partial S} = N(d_1),$$

where

$$d_1 = \frac{\ln \frac{S}{X} + \left( r + \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{t}},$$

and $c$ is the price of a European call option, $S$, the value of the underlying asset, $N$, the cumulative normal distribution, $X$, the strike price of the option, $r$, the risk free interest rate, $t$, the time to maturity (in years) and $\sigma$, the volatility. Vega${}^1$, $\upsilon$, is

$$\upsilon = \frac{\partial c}{\partial \sigma} = S \sqrt{t} * N'(d_1),$$

where

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-d_1^2/2}.$$

Delta and vega both provide instantaneous information and changes in market conditions alter these significantly. Hence, these risk parameters must be recalculated when such occur. To get a picture of the monetary risks of a portfolio for different outcomes of the two market risk factors associated with these two estimates, several scenarios can be simulated and a new value of the portfolio calculated. This sort of stress testing is usually referred to as scenario analysis.
### 2.1 Scenario analysis

Scenario analysis is visualized in matrix form where the net change in the value of the portfolio for different market outcomes using a suitable pricing method and a specified time-period, is calculated. Here, both changes in the underlying asset as well as changes in the implied volatility can be included in the same analysis. Consider a simple portfolio consisting of a short strangle (short call and put with different strike prices, where \( X_{\text{put}} < X_{\text{call}} \), turn to Appendix A for closer detail) using DAX-index options. The change in the underlying asset is calculated horizontally with steps 2, 4, 6, 8 and 10 percent, and accordingly, the change in the implied volatility is calculated vertically with steps 10, 20, 30, and 40 percent, respectively. The maximum changes in the underlying asset and in the implied volatility for a specific time-period are hence assumed to stay within these limits.

![Table](image)

**Figure 1**

**The scenario analysis for the portfolio consisting of a short strangle (€1,000)**

A change in the underlying asset is viewed horizontally and a change in the implied volatility vertically.

The portfolio is simulated under the different scenarios and the respective values calculated. On the setup day of the portfolio (4.10.1999), scenario analysis is performed to cover the time-span of one day, demonstrating the ex ante risk exposure of the portfolio with regard to 5.10.1999. Given this, the value of the portfolio of options will increase by approximately €510,800 if the DAX-index rises six percent and volatility falls by twenty percent, according to Figure 1. The payoff is visualized graphically in Figure 2. It is rather obvious that the nightmare scenarios considering this portfolio are those represented by an increase in volatility and a decline in the DAX-index, as these conditions cause significant losses to the specified portfolio.
2.2 Non-linearities and additional Greeks

One aspect that becomes evident when studying the values of Figure 1 and the shape of Figure 2 is the non-linear payoff of options. For example the gearing of delta, defined as gamma

$$\Gamma = \frac{\partial^2 \Delta}{\partial S^2} = \frac{N'(d_1)}{S * \sigma \sqrt{t}},$$

(5)

is clearly visualized across the horizontal axis where the profit/loss of the portfolio changes in a non-linear fashion.

![Figure 2](image.png)

**Figure 2**
The payoff for the portfolio visualized graphically

This non-linearity highlights the importance of deriving at least one additional Greek parameter, namely the gearing of vega or vega-gamma, in addition to the common ones to enhance the horizontal aspect of risk monitoring. Thus, the second derivative of \( \psi \) is needed. From equation (3) we know \( \partial c / \partial \sigma \), which differentiated with respect to \( \sigma \) gives
\[
\frac{\partial^2 c}{\partial \sigma^2} = \frac{S \sqrt{t}}{\sqrt{2\pi}} \cdot e^{-d_1^2/2} \cdot \left( \frac{d_i}{\sigma} - \sqrt{t} \right). \tag{6}
\]

The strangle-portfolio used in this study has a delta exposure of 1,368 and a vega exposure of –56,839. Calculating the vega-gamma exposure yields –2,142, indicating that this portfolio is short vega-gamma, which means that the negative vega exposure is accentuated if the level of volatility increases, and vice versa.

The next ‘obvious’ question is; how does a change in the underlying asset affect the level of vega? In similar fashion the sensitivity of the vega exposure of a portfolio/option to changes in the level of the underlying asset, or the sensitivity of the delta exposure to changes in the level of volatility, is measured by delta-vega (vega-delta). Differentiating \( \frac{\partial c}{\partial S} \) with respect to \( S \), gives

\[
\frac{\partial^2 c}{\partial S \partial \sigma} = \frac{\partial^2 c}{\partial S \partial \sigma} = \frac{L}{\sqrt{2\pi}} \cdot e^{-d_1^2/2} \cdot \left( \sqrt{t} - \frac{d_i}{\sigma} \right). \tag{7}
\]

The same formula is of course perceived if differentiating \( \frac{\partial c}{\partial S} \) with respect to \( \sigma \). This gives that the delta-vega exposure of the strangle-portfolio equals –31, implying that the vega exposure decreases with an increase in the underlying asset and accordingly, that the delta exposure decreases with an increase in volatility, and vice versa.

### 2.3 Adding the theta-dimension

The measure for how much the value of an option changes over time is commonly denoted theta. Theta for a long position is always negative. The theta for a call is defined as

\[
\Theta = -\left( \frac{\partial c}{\partial t} \right) = \frac{S \sigma \cdot N'(d_1)}{2\sqrt{t}} + r \cdot X \cdot e^{-r \cdot t} \cdot N(d_2). \tag{8}
\]

To account for this ‘risk’ in scenario analysis the lapse of time over the specified period is to be accounted for when calculating the different scenarios of the
analysis. Simulating the portfolio back (forth) the theta value is added (subtracted) to (from) the value of every scenario. The theta risk of the created portfolio can be read from scenario (0, 0) in Figure 1, as the time decay is the only market factor affecting the portfolio for this scenario. The theta of the short strangle is hence €8,700.

3 The Patterns of Volatility

As noted by Alexander (2000), scenario based maximum loss calculation requires at least the definition of scenarios for implied volatilities and underlying asset prices. In the absence of an efficient model of how volatilities change with market price, these scenarios may be rather simplistic. The base scenario that the smile surface remains unchanged over all risk horizons is often augmented by only a few simple scenarios such as parallel shifts in all volatilities that are assumed to be independent of movements in the underlying prices. Similar examples can also be found in Malz (1999). Within this essay, the concept of implied volatility is to be perceived as the implied ATM volatility for the options closest to maturity (10-30 days).

3.1 Implied volatility and the underlying asset

It is well documented for example in Cox & Ross (1976), Black (1976), Christie (1982), Koutmos (1996), Derman (1999) and Alexander (2000) that volatility increases when the market turns downwards, and vice versa, supposedly due to the leverage effect. Plotting the returns on the DAX-index against changes in the level of the implied ATM volatility also supports this hypothesis (see Figure 3). This would imply that realistic scenarios for the ATM volatility and index prices are those where these move in the opposite direction. The problem is of course to measure the magnitude of this relationship, when it is likely to be different at least for different market conditions and regimes.
Figure 3
The relationship between the DAX-index and the ATM volatility
Plotting the relationship between the daily returns of the DAX-index and the changes in the level of implied ATM volatility over the period 3.1.1995 to 15.9.1999 visualizes the negative correlation.

The characteristics of volatility have motivated Derman (1999) to formulate three different market regimes by studying the market under different time periods;

1. *Range bounded*, where the index moves within a certain range and there are no significant changes in realized volatility
2. *Trending*, where the level of the market is changing but in a stable manner implying again only small changes in realized volatility
3. *Jumpy*, where the probability of jumps is particularly high yielding an increase in realized volatility,

It is further suggested in Alexander (2000), that the current market regime can be determined by regressing

$$\Delta \sigma_{ATM} = \varphi(t) + \beta(t) \Delta \delta + \varepsilon(t),$$  \hspace{1cm} (9)

where an insignificant beta-value indicates a stable and trending market and a small but significant beta-value indicates a range-bounded market. Accordingly, the beta-value for a jumpy market is expected to be about twice the value of the range-
bounded market. Of course, due to the expected negative correlation, these betas are expected to exhibit negative values.

**Figure 4**

*Plotting values of the beta measure and t-statistic*

Plotting the beta-value estimated using a one-month rolling window together with the t-statistic over the period 3.2.1995 to 15.9.1999, visualizes the different regimes of the DAX-index. The values of the beta are presented on the left scale and the values of the t-statistic on the right scale.

Studying Figure 4, it seems that the beta-values are generally insignificant throughout the ‘silent’ markets of 1995, 1997 and 1999 (the shaded areas). However, the negative relationship between the underlying asset and volatility is significant and accentuated during most of 1996 and the volatile periods of late 1997 and early autumn 1998 during the Asian and Russian crises, respectively. By constantly re-estimating this regression it is thus possible to detect a change in conditions as a significant change in the value of the beta indicates that the market is entering a new regime. Hence, by estimating different betas for the relationship for different regimes, it is possible to determine which estimate is the most suitable during the present market conditions.

**3.2 The predicting power of implied volatility**

The next step would be to examine whether the information content of past implied volatility could be utilized to predict future large moves in the underlying index. This
would help the risk/portfolio manager to position himself according to risk preferences and perform the necessary stress testing in anticipation of such an event. Here, the concept of Granger-causality is implemented to determine whether there exists such a relationship. This methodology is also applied in for example Malz (2000).

A time series, \( y \), is said to cause another time series, \( x \), if past values of \( y \) help predict the current value of \( x \). A standard test for causality is to set a lag length \( l \) and to carry out the Ordinary Least Squares (OLS) regression

\[
x_t = \sum_{i=l}^1 \alpha_i x_{t-i} + \sum_{i=l}^1 \beta_i y_{t-i} + u_t, \quad t = 1, \ldots, T, \tag{10}
\]

where the null hypothesis that the coefficients \( \beta_1, \beta_2, \ldots, \beta_l \) are all equal to zero implies that \( y \) fails to Granger cause \( x \). The test can be performed by running a second OLS regression

\[
x_t = \sum_{i=l}^1 \gamma_i x_{t-i} + v_t, \quad t = 1, \ldots, T. \tag{11}
\]

Under the null hypothesis, \( \beta_1 = \beta_2 = \cdots = \beta_l = 0 \), the test variable

\[
\lambda(x, y) = \frac{\sum_{t=1}^T \hat{v}_t^2 - \sum_{t=1}^T \hat{u}_t^2}{\sum_{t=1}^T \hat{u}_t^2} \frac{T - 2l - 1}{l}, \tag{12}
\]

where \( \hat{u}_t \) and \( \hat{v}_t \) are the residuals from the regressions, has an asymptotic \( F(k, T-2k-l) \) distribution as \( T \to \infty \). If \( \lambda(a, b) \) exceeds the critical value of the \( F \) distribution for the specified confidence level, the null hypothesis is rejected. Here the number of lags \( (l) \) is arbitrarily set equal to 5 to capture the information content of one trading-week.

Performing this analysis on the data yields a lambda of 0.7 and 2.86 when implied ATM volatility is used to Granger cause future returns and squared returns of the DAX-index, respectively. Here the latter is significant at conventional levels,
implying that implied volatility can be a useful predictor of future large moves in the underlying asset. These results are very much in line with those of Malz (2000). Thus, a simple indicator based on moves in the underlying asset is created in order to examine whether this enables the prediction of large moves in the underlying index.

On seven occasions over the monitored period, the daily return of the DAX-index is found to be larger than \pm 3\%. To see whether the implied volatility and the market have in deed anticipated to these large moves, the level of the change in the implied ATM volatility on the five preceding days are compared to a ten day average of the absolute value of the changes in the implied volatility. On average, the absolute difference between the daily change in the implied volatility and its ten-day mean is equal to 0.06, while the average of the difference on the five days preceding the large moves in the underlying asset equals 0.093. As the latter value is obviously higher than the former, it seems as if this kind of warning signal can be useful when predicting sudden large moves in the underlying asset. For other kinds of warning signals, see for example Malz (2000).

### 4 The Expected Maximum Loss

To enable a vertical communication of risk in an organization with regard to investors, regulators and other parties in favor of the ever-convenient single-sum measures like the VaR, the information in the scenario analysis can be used to calculate a similar one-number risk exposure as well. This measure, like the VaR, visualizes the expected maximum loss for a given time-period, with a certain probability and will be referred to henceforth as the $EML_\alpha$, the expected maximum loss of the portfolio, where $\alpha$ represents the desired level of probability.

#### 4.1 Calculating historical probabilities

To estimate the $EML$, the historical probabilities for the different scenarios of the scenario analysis are needed. Observations ranging from 3.1.1995 to 15.9.1999, giving a total of 1,144 observations, on the close-open returns of the DAX-index future and daily changes in the implied ATM volatilities are examined and added up into the monitored scenarios.
By dividing the measures of Figure 5 by the total number of observations, historical probabilities are perceived. The establishment of these probabilities enables the calculation of the expected mean return implied by the analysis. This is achieved by multiplying the probabilities with the respective net change for the scenario. Using the probabilities in Figure 6 and the scenario analysis performed for the DAX-index portfolio in Figure 1, generates a total sum of €-1,103. It is now possible to calculate the precision of this value by examining the standard deviation, which in this example equals €156,638.

If knowing the distribution of the underlying asset, it would be possible to specify the intervals in which the risk exposure is to lie with a certain probability. Assuming a normal distribution and a probability of 90%, the boundaries are defined as the expected risk exposure ± 1.65 standard deviations generating an interval of [-259,556 ; 257,350]. Put in another way, there is only a 5% chance of losing more than €259,556 overnight, that is the \( EML_{0.95} = -259,556 \).
4.2 Accounting for the tails

As it is well known that high-frequency data exhibit heavy tail implying that returns differ from the normal distribution, a different distribution is needed to account for this phenomenon. Checking the returns on the DAX-index over the tested period (3.1.1995 to 15.9.1999) yields a skewness and kurtosis of −0.774 and 6.538, respectively, and a Bowman-Shelton statistic of 710.75. This implies that any assumption of normality in returns is in fact false. See also Figure 7 below. Below are two different methods for accounting for the presence of non-normality in returns presented, where the latter is examined empirically.

4.2.1 Extreme Value Theory

A methodology that has gained recent popularity is the modeling of the extreme tails of return densities, a method commonly denoted Extreme Value Theory. As noted in Christoffersen et al. (1998), extreme quantiles and probabilities are of particular interest as the ability to assess them accurately translates into the ability to manage extreme financial risks effectively. These include such events as currency crises, stock market crashes and large bond defaults.

Traditional parametric methods strive to produce a good fit in regions where most of the data fall, usually at the expense of a good fit in the tails where by definition few observations fall. Opposed to this, EVT concentrates only on the extreme values of the data thereby fitting the tail and only the tail. According to EVT, the survival function (equal to 1 minus the cumulative density function, see Christoffersen et al. [1998]) is assumed to be a power law times a slowly varying function

\[ P(Y > y) = k(y)y^{-\theta}, \]  

(13)

where the tail index, \( \theta \), is the parameter to be estimated. Hill (1975) orders the observations beginning from the largest, the second largest, and so on, and forms an estimator based on the difference between the average of the \( m \) largest log returns and the \( m \)-th largest log return according to

\[ \hat{\theta} = \frac{\log_{10} k(m_n)}{\log_{10}(n^{-1} \sum_{i=1}^{m_n} \log_{10} Y_{(i)} - \log_{10} Y_{(m_n)})}, \]
\[ \theta = \left( \frac{1}{m} \sum_{i=1}^{m} \ln(y_i) - \ln(y_m) \right)^r. \]  

(14)

As pointed out in Christoffersen et al. (1998), it is only a matter of converting an estimate of \( \theta \) into estimates of the desired quantiles and probabilities. However, one setback with this methodology is the amount of data needed. It would seem that the relevant size of the data sample is better approximated by the number of non-overlapping hundred-year intervals in the set, than by the actual number of observations, when performing statistical inference on objects happening once every hundred years. For results on the German market using an EVT methodology, see Broussard & Booth (1998).

### 4.2.2 The hyperbolic distribution

The hyperbolic distribution was first introduced in Barndorff-Nielsen (1977) for modeling grain size distribution of wind blown sand. It is named after the fact that its log-density is a hyperbola as opposed to the normal distribution and its parabola. This provides the distribution with the possibility to model heavy tails. The density of the hyperbolic distribution is

\[
f_{a,b,\mu,\delta}(x) = \frac{\sqrt{a^2 - b^2}}{2a\delta K_1\left(\frac{\delta \sqrt{a^2 - b^2}}{\sqrt{x^2 - b^2}}\right)} e^{-a(\delta \sqrt{x^2 - \mu^2} + b(x - \mu))},
\]  

(15)

where \( K_1 \) denotes the modified Bessel function of the third kind with index 1. The distribution has four parameters, \( \mu \in \mathbb{R}, \delta > 0 \) and \( 0 \leq |b| < a \). Roughly, \( a \) and \( b \) determine the shape where \( b \) is models the skewness, while \( \delta \) and \( \mu \) are the scale and location parameters, respectively (see Eberlein et al. [1998]). Given a set of independent observations these parameters can be estimated by maximum likelihood methods as documented in Blaesild & Sorensen (1992).

The hyperbolic distribution is found in Eberlein & Keller (1995) and in Ahn et al. (2000) to provide a good fit to historical distributions for different underlying assets. Estimating the parameters for the distribution yields the following results, as
presented in Table 1. From Figure 7 it is also quite evident that the hyperbolic
distribution fits the DAX-index returns quite well and at least far better than the
normal distribution.

Table 1
The results of the parameter estimation
Estimating the parameters of the hyperbolic distribution yielded the
following results. The frequency distributions are presented below.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>δ</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>163.913</td>
<td>-15.497</td>
<td>0.004</td>
<td>0.002</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>&lt;1σ</th>
<th>&lt;2σ</th>
<th>&lt;3σ</th>
<th>&lt;4σ</th>
<th>&lt;5σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stand. Norm</td>
<td>0.683</td>
<td>0.955</td>
<td>0.997</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Hyperbolic</td>
<td>0.736</td>
<td>0.937</td>
<td>0.988</td>
<td>0.993</td>
<td>0.994</td>
</tr>
</tbody>
</table>

From Table 1, it is obvious that the hyperbolic distribution is quite suitable
considering extreme events as more than 5 standard deviations are needed cover
100% of the possible returns. It is also interesting to see that one standard deviation
accounts for 73.6% of the returns when using the hyperbolic distribution but only for
68.3% if using the normal distribution, due to the presence of kurtosis.

Figure 7
The distribution of the DAX-index returns
Plotting the returns of the DAX-index versus the normal distribution (left) and the estimated
hyperbolic distribution (right).
4.3 Testing the EML measure

Due to its similarities especially with the analytical VaR estimate and rather hands-on and intuitive approach to measuring risk, the EML is to be tested in practice in a closed surrounding on real market prices to clarify its use and efficiency.

4.3.1 Methodology

The created portfolio of DAX-index options presented above is tested against realized market outcomes over the period 4.10-31.11.1999. For each day, the EML measure is estimated to cover the time-span of one day, and compared to the realized market value of the portfolio the next day. This short time span of one day is chosen for the sake of simplicity to avoid pricing problems caused for example by changes in the volatility smile. The realized market values are calculated using daily settlement prices provided by the exchange (EUREX™) and the scenarios are calculated according to the BSM. The level of the underlying asset is given by the closing value of the DAX-index on the respective day while the three-month EURIBOR is used as proxy for the interest rate. The used implied ATM-volatility is iterated using the BSM framework from the settlement price of the call option closest to the ATM-level.

On the set-up day 4.10.1999 the initial value of the portfolio is calculated according to the settlement prices of the options on that day. Based on this value, scenario analysis is performed displaying the net change in the portfolio for different market scenarios, which together with the probabilities estimated in the preceding chapter enables the estimation of the EML figure. The confidence intervals are calculated using both the normal and the estimated hyperbolic distribution. These figures are then compared to the realized net change in the value of the portfolio, where after the portfolio value is again set equal to the market value and the procedure is repeated. The time decay \( t = t-1 \) is of course included in the calculation of the scenarios.

4.3.2 Empirical findings

In this example, the interval is set to cover 94 % of the possible returns, that is the desired level of probability, \( \alpha \), is equal to 3 %. Using the normal distribution 1.88
standard deviations are needed to cover this interval, while the estimated hyperbolic distribution requires 2.0 standard deviations. Testing the measures using the proposed methodology generated the following empirical results as visualized in Figure 8. Turn to Appendix B for further detail.

Figure 8
Testing the EML
The graph shows the upper and lower boundaries of the expected returns of the portfolio together with the realized market values.

Results show that the lower limit \(EML_{0.97}\) is violated once (25.11), where both the figure based on the normal distribution as well as the figure based on the hyperbolic distribution, prove too small. More interesting is, however, the situation on 19.10 where the upper limit is exceeded only when using the normal distribution but not when using the hyperbolic distribution.

Still, as the test period is in deed very short and performed in a closed surrounding, the test can hardly be considered sufficient. It is merely a demonstration of how the EML works, and further implementation requires additional testing for instance by performing numerous simulations to extensively test the accuracy of the method.
4.4 Extending the EML measure

Using the calculation process presented above to estimate the EML might yield maximum loss values that underestimate the ‘real’ risk exposure especially during volatile periods, as the probabilities presented in Figure 6 rule out possibilities for a number of market scenarios. For instance, as the largest negative and positive return of the DAX-index during the observed period is 7.9% and 4.6%, respectively, any net change in the portfolio value for market scenarios exceeding these limits is not included in the EML measure.

To solve this problem the probabilities could instead be extracted from the estimated hyperbolic distribution which would put more emphasis on extreme events as results above (Table 1) indicate that this distribution accounts for a small possibility of events occurring even outside a five standard deviation interval. To find the appropriate change in the level of implied ATM volatility for each respective simulated return of the DAX-index, the theories presented in Chapter 3 regarding the relationship between these two can be utilized. First, the present market regime is determined and the most accurate beta estimate decided, which is then used to decide upon the most likely level of the implied ATM volatility for a given scenario regarding the DAX-index. Hence, the net change in the value of the portfolio is calculated for this scenario and weighted into the EML measure.

This ‘analytical’ approach to calculating the measure is more dependent on the modeling of relationships, whereas the use of historical probabilities is more hands-on and perhaps a bit more restricted. It is a question of user preference and extensive testing to determine which is the most adequate of these two.

5 Summary and Concluding Remarks

The purpose of this study was to improve option risk monitoring by examining the information content of implied volatility and by introducing the calculation of a single-sum expected risk exposure, the Expected Maximum Loss. This figure is very much similar to the VaR and applicable to any (continuously traded) portfolio of vanilla options.
By estimating historical probabilities, an expected VaR-alike single-sum risk exposure can be calculated from the scenario analysis, which combined with probability theory enables the calculation of the expected maximum loss or \( EML \) as introduced in this study. This measure, like the VaR, is suitable for vertical communication across any organization, as it is easy to communicate and understand. In this study the hyperbolic distribution was suggested as a suitable alternative for accounting for the apparent deviations from normality in returns, which is a common feature on most markets today.

All in all, it is fair to say that the EML based on the hyperbolic distribution and scenarios calculated using an efficient model, is a good alternative for calculating a single-sum risk measure. However, implementation inevitable calls for further testing. Equally important is of course the ability to forecast sudden large movements in the underlying asset. Here, it was shown that the information content of the implied volatility could be a useful predictor of future large moves in the underlying asset. Furthermore, the patterns of implied volatility also indicated links to moves in the underlying asset, which was used in order to document different market regimes.

Endnotes:

1 Vega is of course not a Greek letter and \( \upsilon \) (upsilon) is thus used as a substitute.
2 Additional papers by for example Pindyck (1984) and Campbell & Hentschel (1992) explain this negative correlation with the existence of time-varying risk-premiums.
3 With small variations from market to market, it is suggested that daily returns have an exponential tail while monthly returns are normally distributed. See Cont et al. (1997) or Prause (1997) for a review of some of the existing literature within this field.
References


Basle Committee on Banking Supervision (1996): “Amendment to the Capital Adequacy Accord for the Extension to Market Risks”, *Basle: Consultative Proposal, April*


## Appendix A

### Table A1
The constituents of the portfolio (short strangle)
The ATM-level on the day of set-up is 5219 and the ATM-volatility approximately 27.4%. The interest rate used is 3.2% and the time to maturity 74 days.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Strike</th>
<th>Call/Put</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 1999</td>
<td>5000</td>
<td>Put</td>
<td>-1000</td>
</tr>
<tr>
<td>Dec 1999</td>
<td>6000</td>
<td>Call</td>
<td>-1000</td>
</tr>
</tbody>
</table>
Appendix B

Table B1
The empirical results

The estimated ex ante measures $EML_{a,97}$ and the confidence interval (6%) are compared ex post to the realized portfolio value on the next day (net change). The intervals are calculated both using the normal and the hyperbolic distribution.

<table>
<thead>
<tr>
<th>Date</th>
<th>DAX ATM vola</th>
<th>E(R)</th>
<th>$\sigma$</th>
<th>EMLnorm Upper</th>
<th>EMLhyp Upper</th>
<th>Market Net change</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.10.1999</td>
<td>5218.86</td>
<td>0.274</td>
<td>-130</td>
<td>155 115</td>
<td>-291 746</td>
<td>291 486</td>
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<tr>
<td>5.10.1999</td>
<td>5301.85</td>
<td>0.241</td>
<td>-100</td>
<td>118 940</td>
<td>-224 609</td>
<td>222 603</td>
</tr>
<tr>
<td>6.10.1999</td>
<td>5353.32</td>
<td>0.221</td>
<td>76</td>
<td>135 124</td>
<td>-253 957</td>
<td>254 109</td>
</tr>
<tr>
<td>7.10.1999</td>
<td>5419.31</td>
<td>0.206</td>
<td>1000</td>
<td>-118 940</td>
<td>-224 609</td>
<td>222 603</td>
</tr>
<tr>
<td>8.10.1999</td>
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<td>-1399</td>
<td>106 170</td>
<td>-201 538</td>
<td>197 660</td>
</tr>
<tr>
<td>9.10.1999</td>
<td>5414.5</td>
<td>0.226</td>
<td>14172</td>
<td>100 458</td>
<td>-174 689</td>
<td>203 033</td>
</tr>
<tr>
<td>10.10.1999</td>
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<td>0.227</td>
<td>-7468</td>
<td>159 539</td>
<td>-217 943</td>
<td>215 400</td>
</tr>
<tr>
<td>11.10.1999</td>
<td>5295.43</td>
<td>0.238</td>
<td>-1125</td>
<td>117 609</td>
<td>-220 538</td>
<td>218 288</td>
</tr>
<tr>
<td>12.10.1999</td>
<td>5220.29</td>
<td>0.249</td>
<td>1427</td>
<td>128 087</td>
<td>-239 377</td>
<td>242 231</td>
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<tr>
<td>13.10.1999</td>
<td>5184.23</td>
<td>0.265</td>
<td>1859</td>
<td>139 980</td>
<td>-266 303</td>
<td>265 021</td>
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<tr>
<td>14.10.1999</td>
<td>5156.28</td>
<td>0.274</td>
<td>22019</td>
<td>147 254</td>
<td>-254 819</td>
<td>298 857</td>
</tr>
<tr>
<td>Mean</td>
<td>5570.04</td>
<td>0.21</td>
<td>2111</td>
<td>101 723</td>
<td>-189 127</td>
<td>193 352</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>259.94</td>
<td>0.03</td>
<td>8997</td>
<td>31 414</td>
<td>58 042</td>
<td>61 391</td>
</tr>
</tbody>
</table>

| 15.10.1999 | 5296.91      | 0.236 | 3924     | 155 146      | -287 750     | 295 598          |
| 16.10.1999 | 5301.85      | 0.241 | -100     | 118 940      | -224 609     | 222 603          |
| 17.10.1999 | 5353.32      | 0.221 | 76       | 135 124      | -253 957     | 254 109          |
| 18.10.1999 | 5419.31      | 0.206 | 1000     | -118 940     | -224 609     | 222 603          |
| 19.10.1999 | 5419.26      | 0.205 | -1399    | 106 170      | -201 538     | 197 660          |
| 20.10.1999 | 5414.5       | 0.226 | 14172    | 100 458      | -174 689     | 203 033          |
| 21.10.1999 | 5358.46      | 0.227 | -7468    | 159 539      | -217 943     | 215 400          |
| 22.10.1999 | 5295.43      | 0.238 | -1125    | 117 609      | -220 538     | 218 288          |
| 23.10.1999 | 5220.29      | 0.249 | 1427     | 128 087      | -239 377     | 242 231          |
| 24.10.1999 | 5184.23      | 0.265 | 1859     | 139 980      | -266 303     | 265 021          |
| 25.10.1999 | 5156.28      | 0.274 | 22019    | 147 254      | -254 819     | 298 857          |
| 26.10.1999 | 5296.91      | 0.236 | 3924     | 155 146      | -287 750     | 295 598          |
| 27.10.1999 | 5301.85      | 0.241 | -100     | 118 940      | -224 609     | 222 603          |
| 28.10.1999 | 5353.32      | 0.221 | 76       | 135 124      | -253 957     | 254 109          |
| 29.10.1999 | 5419.31      | 0.206 | 1000     | -118 940     | -224 609     | 222 603          |
| Mean       | 5570.04      | 0.21  | 2111     | 101 723      | -189 127     | 193 352          |
| Std. dev.  | 259.94       | 0.03  | 8997     | 31 414       | 58 042       | 61 391           |

| 30.10.1999 | 5296.91      | 0.236 | 3924     | 155 146      | -287 750     | 295 598          |