THE DAY OF THE WEEK EFFECT AND OPTION PRICING
- A STUDY OF THE GERMAN OPTION MARKET

DECEMBER 2000
Key words: Option pricing, Day of the week effect, Weekend effect, Time units, Implied Volatility

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Abstract

The use of different time units in option pricing may lead to inconsistent estimates of time decay and spurious jumps in implied volatilities. Different time units in the pricing model leads to different implied volatilities although the option price itself is the same. The chosen time unit should make it necessary to adjust the volatility parameter only when there are some fundamental reasons for it and not due to wrong specifications of the model. This paper examined the effects of option pricing using different time hypotheses and empirically investigated which time frame the option markets in Germany employ over weekdays. The paper specifically tries to get a picture of how the market prices options. The results seem to verify that the German market behaves in a fashion that deviates from the most traditional time units in option pricing, calendar and trading days. The study also showed that the implied volatility of Thursdays was somewhat higher and thus differed from the pattern of other days of the week. Using a GARCH model to further investigate the effect showed that although a traditional tests, like the analysis of variance, indicated a negative return for Thursday during the same period as the implied volatilities used, this was not supported using a GARCH model.

Keywords: Option pricing, Day of the week effect, Weekend effect, Time units, Implied volatility

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1 Introduction

In option pricing the model counts the time left in an option as a fraction of a year. The calculation of these fractions reveals a great deal about one’s view of the passing of time. Calculating the time remaining using calendar days implies complete indifference about what kind of time, volatile or non-volatile, is passing. In calendar time, a weekday is given the same weight as a Saturday or Sunday, as if every day had equal value. The argument for paying time value over the weekend is that floods, draughts and other pertinent news do happen during weekends, too.

Taking the view that weekends are relatively worthless, then by the close of business on Friday, option prices should have fallen to levels at which they are expected to open on at the beginning of the next week. In such a world, it makes sense to calculate the time remaining using trading days. The use of different time units in options pricing may lead to a phenomenon that most option traders have encountered, the spurious gap in implied volatilities that seem to appear between the close of trading on Friday afternoons and the opening of trading on Monday mornings. These gaps are the result of nothing more than treating all calendar days as if they were equally important to the value of the option, when in practice weekend days are more or less worthless.

Each passing day reduces the time value of an option; the standard name for this time decay is “theta”. In the language of options, an option’s theta measures the effect of a change in time remaining to expiration in the price of the option while holding all other things constant. Among these other things is the volatility of the underlying price. Therefore the choice of time unit in option pricing is important. The chosen time unit should make it necessary to adjust the volatility parameter only when there are some fundamental reasons for it and not due to wrong specifications of the model.

As option traders are far from indifferent about the kind of time that passes, the choice of which time unit in option pricing becomes an important factor, and neither calendar time nor trading time seem to account for the weekend properly. This can be seen from empirical studies of the problem. The day of the week or weekend effect is a well-documented phenomenon and a number of studies have empirically examined the issue of the effect, e.g. those of Fama (1965), French (1980), Gibbons and Hess (1981), Ball et al. (1982) and Lakonishok and Levi (1982). The results of variances for returns are inconsistent with both the calendar- and trading-time hypotheses, but they are much
closer to the variances linear in trading time than to variances linear in calendar time. Fama (1965) indicated that the variance from Friday close to Monday close was only 22% higher than between other trading days of the week and French (1980) found the variance to increase by 19% in a similar study. The expected mean and variance is three times as high from Friday close to Monday close as between subsequent trading days under the calendar time hypothesis. In a study of the FDAX on the German Dax-index Sundkvist and Vikström (2000) found the variance to increase by approximately 19% over the weekend, which is closer to trading time than to calendar time.

In an empirical study of the day of the week or weekend effect and the implications for option pricing, French (1984) suggests using the Black and Scholes formula on a composite-time basis. This model allows for two different time units for the interest rate and volatility respectively. One of the empirical findings of French’s study was that options seem to be priced under trading days with interest rate accrued during calendar days. Burghardt and Hanweck (1993) went one step further and argue that information releases increase volatility, and their survey rank non-farm payroll as the most important statistic. They also outline a technique for adjusting volatilities in the Black and Scholes model to account for these changing volatilities. Essentially their technique is based on trading days but also rescale time, according to whether information is released on that day or not. Days with information announcements are given more weight in time (perhaps equal to three ordinary days).

The objective of this study is to examine the effects of option pricing using different time hypotheses and empirically investigate which time frame the option markets in Germany may be using. The majority of papers dealing with option pricing problems naturally examine option prices. Often different pricing models are compared and tested against option prices in the market to find out which model that best “fits” or explains the market prices.

To compute model option prices, implied volatility needs to be estimated as the only input parameter not available directly from the market. In this study we use market-implied volatilities. The use of different time units in the pricing model leads to different implied volatilities although the option price itself is the same. Furthermore, we are not testing any specific model; instead we try to get a picture of how the market prices options. First implied volatilities from the market are examined, this to see if participants have taken the weekend effect into consideration. Then calculated implied volatilities are compared to the market values and in this way it is possible to find out
which time frame the actual option market uses. Finally, we examine the daily returns of the German Dax-index. In short, the paper examines the German market and uses recent data from Eurex, the world’s largest electronic market for options and futures.

The study showed that the option market accounted for the weekend effect and thus prices the options in a way that deviate from the most traditional time units in option pricing, calendar and trading days. The study also revealed that the implied volatility of Thursdays differed from the pattern of other days of the week and was somewhat higher. Using a GARCH model to further investigate the effect showed that although a traditional tests, like the analysis of variance, indicated a negative return for Thursday during the same period as the implied volatilities used, this was not supported using a GARCH model. Also no significantly higher variance for Thursday during the same period was observed. This paper continues in section two by presenting the methodology more thoroughly. Section three presents the data and the empirical findings. In section four we discuss the implied volatility ratios with respect to different time units. Section five presents findings using GARCH-models to catch the weekday effect. The paper ends with a summary and concluding remarks.

2 Methodology

The methodology used in the study was to first examine implied volatilities from the market, to see if market participants accounted for the weekend effect. The implied volatilities were collected from an option-pricing model using calendar days. Because the level of implied volatilities varies considerably over time we needed to modify them in order to investigate the weekend effect better. This was done by first calculating a mean implied volatility ($IV_{Week}$) for each week as in (1).

$$IV_{Week} = \frac{IV_{Mo} + IV_{Ta} + IV_{We} + IV_{Th} + IV_{Fr}}{5}.$$  

(1)

This weekly mean was then used to count an implied volatility ratio ($IVR$) for each day. This was done using (2)

$$IVR_{Mo} = \frac{IV_{Mo}}{IV_{Week}},$$  

(2)
and the same procedure goes for Tuesday, Wednesday and so on. When we got these implied volatility ratios for each day of the week and if the market seemed to have taken the weekend effect under consideration, then the next step was to see by how much the volatilities were modified. As mentioned above, the pricing of the weekend will appear in different ways in the implied volatility under the different time units employed.

The methodology used next was to simulate at-the-money option prices using one time unit (for example 252 days) and then to calculate the implied volatility and implied volatility ratios that corresponded to same prices using calendar days. The maturity of the options was between two and six weeks. We used the Black and Scholes (1973) model (see Appendix 1), and the composite-time model by French (1984) (see Appendix 1). The use of calendar days as base in this study is due to the fact that the data was collected from a calendar days-based model. When these implied volatilities and IVR:s were calculated we could compare them to implied volatilities that the market used. The comparison revealed which time frame the actual option market used.

Previous studies have indicated that market prices appear to be priced closer to a trading-time model than a calendar-time model. French (1984) demonstrates that the composite-time definition of the parameter of the Black and Scholes model provides better estimates of market option prices. The composite-time definition uses calendar days for interest rates and trading days for the volatility.

Burghardt and Hanweck (1993) also use trading days as base for their technique, and also rescale time, but in their study according to whether information is released on a specific day. This study is not restricted to the usual calendar or trading day hypotheses, but we also assume different time unit models that could perhaps be more in line with what empirical studies indicate.

Normally there are 365 calendar days during a year and of course a normal weekend is two days long, resulting in a time decay of three days from Friday to Monday. This is one of the most common ways of counting days in option pricing. On the other hand using trading days we do not give any time value to the weekend. Also we might assume an unobserved volatility over the weekend to be about half that of other trading days of the week and then count the weekend as a half-day. In the same way we can assume a weekend volatility equal to that of other trading days of the week
and count the weekend as one additional day. The time parameter of these different
time models can be written in a general way as,

\[ w_{\Theta d} = cd - W(2 - \Theta) - \frac{H}{2}(2 - \Theta) \]  \hspace{1cm} (3)

where \( 0 \leq \Theta \leq 2 \) \hspace{1cm} (4)

In the equation \( w_{\Theta d} \) is the different time models, \( cd \) is calendar days left, \( W \) is the
number of weekends before expiration, \( H \) is the number of trading holidays before
expiration and \( \Theta \) is the number of days the weekend is accounted for. Using the
composite-time basis model by French (1984) the time factor that corresponding to
volatility can be written as follows

\[ \tau_\Theta = \frac{w_{\Theta d}}{365 - 52(2 - \Theta) - \frac{9}{2}(2 - \Theta)} \]  \hspace{1cm} (5)

where \( 0 \leq \Theta \leq 2 \). \hspace{1cm} (6)

We use \( \tau_\Theta \) to denote the time for volatility. In a normal year there are about 9 holidays,
but this number might naturally vary from year to year\(^1\). Using trading days we then
expect there to be about 252 days during a year. With the weekend-half days we expect
there to be about 280.25 days and with the weekend-one days we expect there to be
308.5 days during a year.

In order to evaluate these different time units, simulated implied volatilities and
implied volatility ratios are presented in Table 1. In the table, different implied
volatilities were calculated with the composite-time model using one time unit and then
transformed into calendar days. For example on Monday, an option price with the
implied volatility of 45 percent with the weekend-one model gives an implied volatility
of 45.3 percent transformed into calendar days. The use of different time units in the
pricing model leads to different implied volatilities although the option price itself is
the same.

As can be seen from the table, if the market priced options according to
calendar days then our hypothesis was that the volatility should be the same for each

\(^1\) Also, in a leap year there are 366 calendar days.
day of the week. On the other hand if the market priced according to any of the other presented models then the hypothesis was that the volatility should be higher at the beginning of the week and lower at the end of the week. It should be noted that in Table 1 these ratios are averages of options with two to six weeks to maturity. For longer maturity options the differences between the time units are smaller and for shorter maturity options the differences are larger (see Appendix 2).

Table 1
Implied volatility ratios and implied volatilities with different time bases

<table>
<thead>
<tr>
<th></th>
<th>Calendar</th>
<th>Weekend-one</th>
<th>Weekend-half</th>
<th>Trading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>1.000</td>
<td>45.0 %</td>
<td>1.006</td>
<td>45.3 %</td>
</tr>
<tr>
<td>Tuesday</td>
<td>1.000</td>
<td>45.0 %</td>
<td>1.003</td>
<td>45.1 %</td>
</tr>
<tr>
<td>Wednesday</td>
<td>1.000</td>
<td>45.0 %</td>
<td>1.000</td>
<td>45.0 %</td>
</tr>
<tr>
<td>Thursday</td>
<td>1.000</td>
<td>45.0 %</td>
<td>0.997</td>
<td>44.9 %</td>
</tr>
<tr>
<td>Friday</td>
<td>1.000</td>
<td>45.0 %</td>
<td>0.993</td>
<td>44.7 %</td>
</tr>
</tbody>
</table>

The different implied volatilities were calculated with the composite-time model using one time unit and then transformed into calendar days. Underlying price 100, strike price 100, interest rate 3.5 % and volatility 45 %.

Traded options are usually American options on dividend-paying stocks and for these options the standard Black and Scholes model cannot be used. For call options it is common to use the Compound Option Model by Roll (1977), Geske (1979) and Whaley (1981) and for put options the Quadratic Approximation Method by MacMillan (1986) with a dividend correction by either Blomeyer (1986) or Barone-Adesi and Whaley (1988). For index options it is common to use the model by Black (1976a). Although the presentation was based on the Black and Scholes model the time unit problem should be the same also for these models.

### 3 Data

At-the-money implied volatilities were collected for four stocks and the index traded at the Eurex. These were DaimlerChrysler (Dcx), Deutsche Bank (Dbk), Siemens (Sie), Volkswagen (Vow) and Odax. All data was retrieved from the Estlander & Rönnlund Financial Products Ltd database. The at-the-money spread average implied volatilities were collected from an option-pricing model using calendar days and were collected

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2 The idea was to use also different levels of volatility, but the difference in the results using different volatility levels turned out to be minimal and therefore only one volatility level is used.
each day for the duration of January 1, 1995 through December 30, 1999. These implied volatilities were for options that had two to six weeks to expiration, which basically was the contract nearest expiration but switch to the second nearest two weeks before expiration to avoid any irregularities before expiration. The behaviour of shorter-maturity options is somewhat erratic and the implied volatility is more subject to supply and demand of the options.

The pricing model for stock call options was the Compound Option Model by Roll (1977), Geske (1979) and Whaley (1981) and for stock put options the Quadratic Approximation Method by MacMillan (1986) with a dividend correction by Blomeyer (1986). Index options used the Black (1976a) model. Descriptive measures for the data are presented in Table 2.

### Table 2
**Descriptive measures for the data**

<table>
<thead>
<tr>
<th></th>
<th>Volkswagen</th>
<th>Siemens</th>
<th>DeutscheBank</th>
<th>DaimlerChrysler</th>
<th>Odax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imp.vola</td>
<td>Min 15.0</td>
<td>9.8</td>
<td>11.0</td>
<td>12.0</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>Max 80.0</td>
<td>70.0</td>
<td>81.0</td>
<td>62.0</td>
<td>62.0</td>
</tr>
<tr>
<td></td>
<td>Mean 31.8</td>
<td>24.8</td>
<td>28.6</td>
<td>28.2</td>
<td>21.0</td>
</tr>
<tr>
<td>Imp.vola</td>
<td>St.dev. 11.9</td>
<td>11.5</td>
<td>13.0</td>
<td>10.1</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>Observations</td>
<td>1248</td>
<td>1250</td>
<td>1213</td>
<td>1207</td>
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<tr>
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<td>1233</td>
</tr>
</tbody>
</table>

The data was based on at-the-money implied volatilities from the time period 1/1/1995 – 12/30/1999 with a calendar time model.

As can be seen the volatility differed much in level and we needed to modify them in order to investigate the weekend effect better. This was done with the implied volatility ratio method presented in (1) and (2).

### 4 Examination of the implied volatility ratios

The IVR:s for each of the stocks and for the index and for each day of the week is presented in Table 3. Also the IVR for all the papers put together are presented. It was rather obvious that the option market accounted for the weekend effect and did not price the options according to the calendar day hypothesis. If the calendar-days model was correct then the IVR:s should be the same for each day of the week. As can be seen from the table the market seemed to price the volatility lower at the end of the week and to increase it at the beginning of the week. This was in line with expectations using
any of the other presented time hypotheses and also what previous studies have indicated. As can be seen, the ratios were not constant over time, but the pattern was clear.

Table 3
Implied volatility ratios for each day of the examined papers

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dcx</td>
<td>1.0086 (47)</td>
<td>1.0041 (50)</td>
<td>1.0019 (51)</td>
<td>0.9940 (50)</td>
<td>0.9920 (50)</td>
<td></td>
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<tr>
<td></td>
<td>0.9998 (41)</td>
<td>0.9993 (42)</td>
<td>0.9965 (41)</td>
<td>1.0042 (41)</td>
<td>1.0002 (41)</td>
<td></td>
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<td></td>
<td>1.0052 (50)</td>
<td>1.0064 (52)</td>
<td>0.9961 (50)</td>
<td>0.9990 (47)</td>
<td>0.9931 (50)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0132 (50)</td>
<td>1.0040 (52)</td>
<td>0.9998 (52)</td>
<td>0.9907 (50)</td>
<td>0.9917 (47)</td>
<td></td>
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<tr>
<td></td>
<td>1.0018 (50)</td>
<td>1.0003 (52)</td>
<td>1.0019 (52)</td>
<td>1.0071 (50)</td>
<td>0.9885 (49)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0059 (238)</td>
<td>1.0030 (248)</td>
<td>0.9994 (246)</td>
<td>0.9988 (238)</td>
<td>0.9929 (237)</td>
<td>3.12*</td>
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<tr>
<td>Dbk</td>
<td>1.0116 (47)</td>
<td>1.0044 (50)</td>
<td>0.9979 (51)</td>
<td>0.9980 (50)</td>
<td>0.9888 (50)</td>
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<td></td>
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<tr>
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<td>1.0085 (50)</td>
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<td>0.9924 (47)</td>
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<td>All</td>
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<tr>
<td></td>
<td>1.0054 (249)</td>
<td>1.0045 (259)</td>
<td>0.9961 (249)</td>
<td>0.9999 (234)</td>
<td>0.9946 (249)</td>
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</tr>
<tr>
<td></td>
<td>1.0116 (250)</td>
<td>1.0029 (260)</td>
<td>0.9964 (260)</td>
<td>0.9984 (250)</td>
<td>0.9901 (234)</td>
<td></td>
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<tr>
<td></td>
<td>1.0052 (250)</td>
<td>1.0034 (259)</td>
<td>1.0031 (260)</td>
<td>1.0047 (250)</td>
<td>0.9830 (245)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.0072 (1216)</td>
<td>1.0032 (1262)</td>
<td>0.9993 (1257)</td>
<td>0.9999 (1201)</td>
<td>0.9905 (1215)</td>
<td>8.44*</td>
</tr>
</tbody>
</table>

The data was based on at-the-money implied volatility from the time period 1/1/1995 – 12/30/1999 with a calendar time model. Number of observations is in parenthesis. The t-value comes from a two-sample t-test which examines if the implied volatility in calendar time was statistically lower on Friday compared to Monday. * indicates significance at 5% level.
To examine if the implied volatility in calendar time was statistically lower on Friday compared to Monday a two-sample t-test assuming equal variances was computed. First an F-test was utilised to test the two samples for equal variances. All the differences were significant at a 5% level and we can therefore say that the implied volatility was statically lower on Friday compared to Monday.

Figure 1
Market IVR against simulated IVR
Market IVR based on average IVR from the time period 1/1/1995 – 12/30/1999 with a calendar time model. Simulated IVR based on the composite-model using different time models transformed to calendar days.
To examine which time unit the market might use, option prices using one time unit were first simulated and then these prices were used to calculate the implied volatility that corresponded to same prices using calendar days. The simulated options had a time to maturity between two and six weeks, which was the same as the prices in the database. In Figure 1 the result of simulated prices are graphically compared to market prices. The simulated prices here use trading days, weekend-half days, weekend-one days and calendar days. The market prices are for all of the papers used in the study.

Additionally to the graphical examination a non-parametric method to compare the models for significant differences was computed. The Wilcoxon matched-pairs signed-rank test was used to compare the performance of one time unit relative to another. To implement this test, first each market IVR was matched to the corresponding model IVR. This means that for example on a Friday with five weeks to expiration the market IVR was matched to the corresponding model IVR for a Friday with five weeks to expiration according to the table in Appendix 2. Second, deviations of model prices from market prices were calculated for each time unit as

\[ IVRD = IVR_{Market} - IVR_{Model} \]  

where \( IVRD \) is the deviation of model price from market price and \( IVR_{Market} \) is the observed market implied volatility ratio and \( IVR_{Model} \) is the computed model implied volatility ratio. Finally, the difference between the absolute value of the paired deviations was computed for each observation as follows

\[ D = |IVR_{Calendar, i} - IVR_{\Theta, i}|, \quad i = 1, 2, \ldots, 1250 \]  

where \( IVR_{Calendar} \) is the implied volatility ratio deviation for the calendar time model and is \( IVR_{\Theta} \) the implied volatility ratio for any of the other presented time models. The difference \( D \) tests which of the models that performed better. If the calendar time model were closer to observed market prices, then \( D \) should be predominantly negative. The Wilcoxon test ranks the differences and computes the sum of the negative and positive ranks, it therefore also accounts for the magnitude of the differences. The sum of the negative ranks should exceed the sum of the positive ranks if the calendar time model performs better relative to the other time model. The \( D \) values were also
computed and tested comparing all other possible model combinations. Table 5 gives the results of the tests.

Table 4
Z-scores for Wilcoxon signed-rank tests comparing time models

<table>
<thead>
<tr>
<th></th>
<th>Dcx</th>
<th>Dbk</th>
<th>Sie</th>
<th>Vow</th>
<th>Odax</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calendar Trading</td>
<td>Calendar Trading</td>
<td>Calendar Trading</td>
<td>Calendar Trading</td>
<td>Calendar Trading</td>
<td>Calendar Trading</td>
</tr>
<tr>
<td>Weekend-half</td>
<td>3.53*</td>
<td>2.14*</td>
<td>3.03*</td>
<td>2.14*</td>
<td>3.07*</td>
<td>4.50*</td>
</tr>
<tr>
<td>Weekend-one</td>
<td>2.37*</td>
<td>1.72*</td>
<td>1.75*</td>
<td>1.84*</td>
<td>3.51*</td>
<td>2.19*</td>
</tr>
<tr>
<td></td>
<td>0.86</td>
<td>2.86*</td>
<td>1.16</td>
<td>2.14*</td>
<td>3.33*</td>
<td>1.84*</td>
</tr>
<tr>
<td></td>
<td>0.51</td>
<td>0.28</td>
<td>0.36</td>
<td>0.85</td>
<td>1.20</td>
<td>1.61</td>
</tr>
</tbody>
</table>

The model specified by the label to the left provided implied volatility ratios closer to the market volatility ratios than the model specified by the label at the top of the column. The implied volatility ratios were for the period 1/1/1995 – 12/30/1999. * indicates significance at 5% level.

This empirical test appears to verify that the German market behaves in a fashion that deviates from the most traditional time units in option pricing, calendar and trading days. For DaimlerChrysler and Siemens the weekend-one model seems to fit the market quite well, the only day that differs a little from the pattern is Thursday. For Deutsche Bank it appears like the weekend-half model could be the best, although there exist clear deviations and the choice is not that obvious. Also for Deutsche Bank the Thursday differs from the pattern. Volkswagen also seems to be priced according to the weekend-one model, but here the Tuesday seems to deviate. For Odax the Thursday effect is very extreme and Thursday even has an IVR that is considerably higher than for Wednesday. The best model for Odax is the weekend-half model.

To summarise the studies it can be said that the weekend-one model appears to be best for the stocks and the weekend-half the best for the index options Odax. When all the papers are put together it is quite hard to draw any conclusions, but perhaps the
weekend-one seems to be the best. This should not be any surprise because here the four stock options are put together with the single index.

One interesting feature in Figure 1 is the Thursday effects of almost all the papers. Is this an evidence of the fact documented by Donders and Vorst (1996) and Burghardt and Hanweck (1993) that scheduled news announcements increase volatility? Their results show that implied volatilities increase prior to a news release, reach a maximum on the announcement date, and return to the long run level thereafter. Burghardt and Hanweck (1993) rank the non-farm payroll and unemployment numbers in the U.S. typically on the first Friday of each month as the most important statistics.

Another possible explanation for the Thursday effect is the negative returns and a downward move on the volatility skew as a consequence. It is well known that at-the-money implied volatilities rise as the market comes down, at least for index options. The phenomenon that volatility increases as the underlying declines has been reported by e.g. Cox and Ross (1976), Black (1976b), Christie (1982) and Koutmos (1996). A volatility skew is due to the fact that the constancy of volatility in the Black and Scholes theory cannot easily be reconciled with the observed structure of implied volatilities for traded options, often lower strike implied volatilities are much higher than both at-the-money and higher strike implied volatilities. This indicates a bias in the marketplace for a higher probability of a significant collapse compared to a significant explosion to the upside. The Black and Scholes model assumes that stock prices are lognormally distributed, which implies in turn that stock log-prices are normally distributed. Hull (1993) and Natenberg (1994) point out that volatility skews are a consequence of empirical violations of the normality assumption. Volatility skews are very popular research objects and there exist a lot of papers dealing with the problem, e.g. those of Jarrow and Rudd (1982), Tompkins (1994), Corrado and Su (1997) and Peña (1999).

Another similar explanation for the Thursday effect might be the so-called “leverage effect”. According to the “leverage effect” negative stock returns automatically produce a higher debt to equity ratio and hence higher risk and volatility. The “leverage-effect” is a well-documented phenomena and a number of studies have empirically examined this issue, for example those of Black (1976b), Christie (1982), Nelson (1991), Poon and Taylor (1992) Koutmos (1992) and Cheung and Ng (1992).

To examine these theories, returns and variances for the Dax-index were examined. The choice of the Dax-index was justified by the extreme Thursday effect in
Odax during the examination of the implied volatility ratios. We also wanted to examine the overall German market and to be able to compare our study with other similar studies of the German market. Therefore the Dax-index was chosen as an approximate of the German market.

5 Examination of returns and variances for the Dax-index

Since French (1980) and Gibbons and Hess (1981) first documented unusual stock returns over weekends, there have been a number of additional studies of the weekend or day of the week effect. Researchers regard the pattern of negative Monday stock returns as an anomaly. Much of the empirical work on the weekend or day of the week effect rests on a foundation of simple econometric models with strong statistical assumptions. Connolly (1989), (1991) and Chang et al. (1993) questions these assumptions and they find that the sample size, the error term adjustment and alternative heteroskedasticity corrections weakens the significance of the day of the week effect. Connolly argues that the weekend anomaly is dependent on the estimation method and the sample period. Chang et al. (1993) argues that the day of the week effect has disappeared for the most recent periods. In studies of the German day of the week effect, Chang et al. (1993) and Dubois and Louvet (1996) obtain varying results depending on the estimation method used. With a classical approach they both find negative returns for Mondays (at least to 1985), but with alternative methods the results are not that clear and often insignificant.

To examine the Thursday effect in our study of implied volatilities, returns and variances for the German Dax-index were calculated for each day for the duration of January 1, 1990 through December 29, 1999. The returns were calculated as differences in log close to close prices, \( R_t = 100 \times (\ln(P_t) - \ln(P_{t-1})) \) where \( P_t \) denotes the price at time \( t \). Table 4 reports the mean returns and variances for each weekday of the German Dax-index as well as statistics testing for independence. Also a traditional analysis of variances to test if the returns and variances are equal across days is computed.

As previously mentioned the choice of the Dax-index was justified by the extreme Thursday effect in Odax during the examination of the implied volatility ratios and by the fact that we wanted to compare our study with other similar studies of the German market. We also briefly present the main results for the four stocks
DaimlerChrysler, Deutsche Bank, Siemens and Volkswagen. The main focus should be on the variance but the returns were also of interest.

Table 5
Average returns and variances of the German Dax-index

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>F</th>
<th>Q(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 - 99</td>
<td>$R_t$</td>
<td>0.128 (490)</td>
<td>0.055 (508)</td>
<td>0.033 (502)</td>
<td>-0.004 (485)</td>
<td>0.056 (494)</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>$R_t^2$</td>
<td>2.212 (490)</td>
<td>1.544 (508)</td>
<td>1.330 (502)</td>
<td>1.503 (485)</td>
<td>1.350 (494)</td>
<td>3.80*</td>
</tr>
<tr>
<td>90 - 94</td>
<td>$R_t$</td>
<td>-0.007 (244)</td>
<td>-0.065 (253)</td>
<td>-0.032 (249)</td>
<td>0.116 (245)</td>
<td>0.050 (250)</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>$R_t^2$</td>
<td>2.412 (244)</td>
<td>1.208 (253)</td>
<td>1.108 (249)</td>
<td>1.161 (245)</td>
<td>1.014 (250)</td>
<td>4.80*</td>
</tr>
<tr>
<td>95 - 99</td>
<td>$R_t$</td>
<td>0.262 (246)</td>
<td>0.174 (255)</td>
<td>0.097 (253)</td>
<td>-0.127 (240)</td>
<td>0.063 (244)</td>
<td>2.95*</td>
</tr>
<tr>
<td></td>
<td>$R_t^2$</td>
<td>2.033 (246)</td>
<td>1.877 (255)</td>
<td>1.548 (253)</td>
<td>1.853 (240)</td>
<td>1.693 (244)</td>
<td>0.48</td>
</tr>
</tbody>
</table>

The data was for the time period 1/1/1990 – 12/29/1999. Returns were based on close-to-close prices and are defined as $R_t = 100*(\ln(P_t)–\ln(P_{t-1}))$ and variances as $R_t^2$. Numbers of observations are in parenthesis. The F-value comes from an analysis of variance where the equality of daily mean returns and variances are tested. The H0-hypothesis is that the expected returns and variances are equal across days and the H1-hypothesis is that any day deviates from the others. Q(5) are the Ljung-Box statistics for 5 lags (distributed as $\chi^2$ with $n$ degrees of freedom). * indicates significance at 5 % level.

Looking at the means for the subperiods and the full 10 years indicated that the mean return was not constant through the week and that there existed quite large differences between the periods. The returns for Thursday during the first subperiod 1990 – 1994 was largely positive while the returns for the second subperiod 1995 – 1999 was largely negative. Examining at the full period also indicated negative returns for Thursdays. The annual mean returns did not show any clear pattern. Rather the returns seemed to follow a random pattern. Looking at the variances we can see that during the full period and the first subperiod the variance for Monday was considerably higher than for other days of the week. For the last subperiod there were not any large differences between the days. The F-values indicated that the returns for the last subperiod were not equal across days and that the variances were not equal for each day for the first subperiod and for the full period.
The variances indicated that Mondays had a variance that was 54.50% higher than for the other weekdays during 1990–1999, but was only 16.65% higher during the subperiod 1995–1999. This also supports the study of the implied volatilities where we found that a model in trading time was not pricing options optimally, neither was a model in calendar time.

As mentioned classical tests for the day of the week effect are not fully satisfactory because they rely on some less realistic assumptions. Among other things these models cannot account for autocorrelation and heteroskedasticity. As can be seen from Table 3 the Ljung-Box statistics for 5 lags, calculated for both the return and the variance, suggested the presence of significant linear and non-linear dependencies respectively. Non-linear dependencies are most likely due to autoregressive conditional heteroskedasticity i.e., volatility clustering. It was interesting to note that the Q statistic for the variance was in all cases several times greater than that calculated for the returns, especially for the second period. This was an indication that second moment dependencies were far more significant than first moment dependencies.

In this study a time varying heteroskedasticity model like the GARCH model by Bollerslev (1986) was used to test the day of the week effect. First a GARCH (1,1) model with dummies for each day and for both returns and variances was computed. This model was as follows

\[ R_t = \alpha_1 + \alpha_2 T_{u_t} + \alpha_3 W_{e_t} + \alpha_4 T_{h_t} + \alpha_5 F_{r_t} + \phi_1 R_{t-1} + \epsilon_t \]

(7)

\[ (\epsilon_t | \Phi_{t-1}) \sim N(0, h_t) \]

(8)

\[ h_t = \beta_1 + \beta_2 T_{u_t} + \beta_3 W_{e_t} + \beta_4 T_{h_t} + \beta_5 F_{r_t} + \delta_1 \epsilon_{t-1}^2 + \delta_2 h_{t-1} \]

(9)

where \( R_t \) was the return for the Dax-index and (7) described the conditional mean and (9) described the conditional variance. The coefficient \( \epsilon_t \) was the error term and \( \Phi_{t-1} \) was the available information at time \( t-i \) and the variable \( R_{t-1} \) was the lagged return for the Dax-index. The conditional variance follows a GARCH (1,1) process whereby volatility at time \( t \) depends on last period’s squared innovation \( \epsilon_{t-1}^2 \) and last period’s volatility \( h_{t-1} \). The dummy variables indicate the day of the week on which the return and variance was observed (\( T_{u_t} = \) Tuesday, \( W_{e_t} = \) Wednesday, etc.). If the return and
variance was the same for each day of the week, the estimates of $\alpha_2$ to $\alpha_5$ and $\beta_2$ to $\beta_5$ would be close to zero and insignificant.

Another GARCH (1,1) model with dummies for only Monday and Thursday was also computed. Earlier studies generally found differences for other days than Mondays to be zero. As we found the implied volatility of Thursdays to deviate from the pattern over weekdays, use of dummy variables only for Monday and Thursday was also of interest. This model was as follows

$$R_t = \alpha_0 + \alpha_1 M_0 + \alpha_4 T_0 + \phi_1 R_{t-1} + \epsilon_t$$

$$(\epsilon_t|\Phi_{t-1}) \sim N(\theta, h_t)$$

$$h_t = \beta_0 + \beta_1 M_0 + \beta_4 T_0 + \delta_1 \epsilon^2_{t-1} + \delta_2 h_{t-1}$$

where $\alpha_0$ and $\beta_0$ now describe the returns and variances for Tuesday, Wednesday and Friday. Other variables were as in (7), (8) and (9). To be able to capture any systematic increase in Monday and Thursday variances, dummy variables for these days were included in the variance equation (10)\footnote{Models with dummies for each day for the returns and with dummies for each day for the variances were also computed. These models said the same as the one presented and are not presented in the paper.}. Assuming normality, the log likelihood can be written as

$$L(\Theta) = -\frac{1}{2} \log h_t - \frac{1}{2} \epsilon^2_t h_t^{-1}$$

The log likelihood function is highly non-linear in $\Theta$ and therefore numerical maximisation techniques have to be used. We use the numerical algorithm of Berndt et al. (1974). The maximum likelihood parameter estimates for the GARCH models are presented in Table 5.

This GARCH approach tries to capture the excess day of the week return over the yesterday or lagged return. It also means that the model estimates the instantaneous day of the week effect and not the expected effect\footnote{The expected effect for the AR(1) or (7) and for Monday can be shown that it can be modeled as, $\mathbb{E}[\epsilon_{t+h_t}] = \frac{\alpha_0}{1-\phi} + \frac{\phi_1 \alpha_1}{1-\phi^2} + \frac{\phi_2 \alpha_2}{1-\phi^3} + \frac{\phi_3 \alpha_3}{1-\phi^4} + \frac{\phi_4 \alpha_4}{1-\phi^5}$. The other days of the week could also be modeled in a similar way.}. On the other hand the coefficient
for autocorrelation $\phi_t$ turns out to be insignificant for both models and all the periods so this distinction between instantaneous effect and expected effect ought not to make any difference.

The conditional heteroskedasticity is one of the most important properties describing the short-term dynamics of stock markets. The conditional variance is a function of past innovations and past conditional variances. The relevant coefficients $\delta_1$ and $\delta_2$ were statistically significant for both models and all the periods.

With the first model utilizing four dummies, the whole period and the first subperiod 1990 – 1994 showed similar patterns. None of the return parameters were significant ($\alpha_1$ to $\alpha_5$), thus no weekday effect in returns was observable when accounting for heteroskedasticity and autocorrelation. The parameters for the variances for Tuesday to Friday ($\beta_2$ to $\beta_5$) were significantly lower compared to Monday ($\beta_1$). For the last subperiod the parameter for the return for Monday was significantly higher ($\alpha_1$) compared to the other days of the week, but none of the variance parameters were significant.

With the second model and for the full period, the parameter for the variance for Monday $\beta_1$ was significantly higher, representing the excess weekend volatility that should be realized on Monday morning. Also in this second model none of the return dummies representing the weekday effect were significant, thus the negative Thursday returns are not significant in a GARCH-model. The subperiod 1990 – 1994 also indicated significantly higher variance for Monday and similar to the full period none of the return parameters were significant. For the last subperiod 1995 – 1999 the return was significantly positive for Monday $\alpha_1$ but none of the variance parameters were significant.

Diagnostics tests on the residuals showed that the GARCH (1,1) model worked satisfactorily. The Ljung-Box statistics for 5 lags showed no dependence in the standardised residuals $z_t$ and the standard squared residuals $z^2_t$. Also for all the periods the Kolmogrov-Smirnov statistic cannot reject the null hypothesis that the residuals were normally distributed.
Table 6
Estimates for the GARCH (1,1) models

\[ R_t = \alpha_0 + \alpha_1 T_{ut} + \alpha_2 W_{et} + \alpha_3 T_{ht} + \alpha_4 M_{rt} + \phi_1 R_{t-1} + \epsilon_t \]
\[ (\epsilon_t | \Phi_{t-1}) \sim N(0, h_t) \]
\[ h_t = \beta_0 + \beta_1 T_{ut} + \beta_2 W_{et} + \beta_3 T_{ht} + \beta_4 M_{rt} + \delta_1 \epsilon_{t-1}^2 + \delta_2 h_{t-1} \]

I

\[ R_t = \alpha_1 + \alpha_2 T_{ut} + \alpha_3 W_{et} + \alpha_4 T_{ht} + \alpha_5 M_{rt} + \phi_r R_{t-1} + \epsilon_t \]
\[ (\epsilon_t | \Phi_{t-1}) \sim N(0, h_t) \]
\[ h_t = \beta_1 + \beta_2 T_{ut} + \beta_3 W_{et} + \beta_4 T_{ht} + \beta_5 M_{rt} + \delta_1 \epsilon_{t-1}^2 + \delta_2 h_{t-1} \]

<table>
<thead>
<tr>
<th></th>
<th>I 90 – 99</th>
<th>I 90 – 94</th>
<th>I 95 – 99</th>
<th>II 90 – 99</th>
<th>II 90 – 94</th>
<th>II 95 – 99</th>
</tr>
</thead>
<tbody>
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<td>0.1039</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>0.0674</td>
<td>-0.0288</td>
<td>0.1533</td>
<td>0.0057</td>
<td>0.0101</td>
<td>0.0732</td>
</tr>
<tr>
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<td>(1.84)</td>
<td>(-0.02)</td>
<td>(2.17)*</td>
<td>(0.10)</td>
<td>(0.13)</td>
<td>(0.96)</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
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<td>0.0431</td>
<td>0.0242</td>
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<tr>
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<td>(0.30)</td>
<td>(0.43)</td>
<td>(0.26)</td>
<td></td>
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</tr>
<tr>
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<td>0.0033</td>
<td>-0.0506</td>
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<td>( (\alpha_3) )</td>
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<td>(0.03)</td>
<td>(-0.54)</td>
<td></td>
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<tr>
<td>( \alpha_4 )</td>
<td>0.0009</td>
<td>0.1102</td>
<td>-0.1197</td>
<td>0.0160</td>
<td>0.0927</td>
<td>-0.0893</td>
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<td>(0.25)</td>
<td>(0.96)</td>
<td>(-1.07)</td>
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<td>(0.82)</td>
<td>(-0.87)</td>
<td></td>
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<tr>
<td>( \phi_1 )</td>
<td>0.0320</td>
<td>0.0465</td>
<td>0.0129</td>
<td>0.0344</td>
<td>0.0316</td>
<td>0.0227</td>
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<tr>
<td>( (\phi_1) )</td>
<td>(1.40)</td>
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<td>(0.38)</td>
<td>(1.42)</td>
<td>(0.90)</td>
<td>(0.65)</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
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<td></td>
</tr>
<tr>
<td>( (\beta_0) )</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
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<td>( \beta_1 )</td>
<td>0.3214</td>
<td>0.4323</td>
<td>0.1051</td>
<td>0.1438</td>
<td>0.1937</td>
<td>0.0442</td>
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<td>( (\beta_1) )</td>
<td>(9.68)*</td>
<td>(8.37)*</td>
<td>(1.31)</td>
<td>(2.91)*</td>
<td>(2.43)*</td>
<td>(0.44)</td>
</tr>
<tr>
<td>( \beta_2 )</td>
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<td>-0.8531</td>
<td>-0.2097</td>
<td></td>
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</tr>
<tr>
<td>( (\beta_2) )</td>
<td>(-14.58)*</td>
<td>(-14.96)*</td>
<td>(-1.46)</td>
<td></td>
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<td>0.0734</td>
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<td>( (\beta_4) )</td>
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<td>(-5.63)*</td>
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<td>(0.89)</td>
<td>(0.60)</td>
<td>(0.31)</td>
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<td>-0.1879</td>
<td>0.0279</td>
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<td>( (\beta_5) )</td>
<td>(-2.49)*</td>
<td>(-1.97)*</td>
<td>(0.22)</td>
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<td>( \delta_1 )</td>
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<td>0.0678</td>
<td>0.0754</td>
<td>0.0606</td>
<td>0.0731</td>
</tr>
<tr>
<td>( (\delta_1) )</td>
<td>(8.78)*</td>
<td>(5.34)*</td>
<td>(6.03)*</td>
<td>(7.70)*</td>
<td>(4.74)*</td>
<td>(5.82)*</td>
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<tr>
<td>( \delta_2 )</td>
<td>0.9234</td>
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<tr>
<td>( (\delta_2) )</td>
<td>(111.25)*</td>
<td>(56.76)*</td>
<td>(83.83)*</td>
<td>(69.35)*</td>
<td>(42.43)*</td>
<td>(73.28)*</td>
</tr>
<tr>
<td>( Q(5) z_t )</td>
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<td>( Q(5) z_t^2 )</td>
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<tr>
<td>( D )</td>
<td>0.0168</td>
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<td>0.0191</td>
<td>0.0134</td>
<td>0.0126</td>
<td>0.0164</td>
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</table>

The data is for the time period 1/1/1990 – 12/29/1999. Returns are based on close-close prices and are defined as \( R_t = 100*(\ln(P_t)–\ln(P_{t-1}) \). The dummy variable indicate on which day of the week each return and variance is observed \((Tu = Tuesday, We = Wednesday etc.)\), the t-statistics are in parentheses. \( Q(5) z_t \) and \( Q(5) z_t^2 \) are the Ljung-Box statistic for 5 lags of the standardised residuals and the standard squared residuals (distributed as \( \chi^2 \) with \( n \) degrees of freedom), \( D \) is the Kolmogrov-Smirnov statistic testing for normality (5 % critical value 1.36/√T where \( T \) is number of observations). * indicates significance at the 5 % level.
The results for the subperiod 1995 – 1999 and for the four stocks DaimlerChrysler, Deutsche Bank, Siemens and Volkswagen using the second GARCH model, showed that the conditional heteroskedasticity coefficients were all statistically significant and also the autocorrelation coefficients were significant except for Siemens. A negative and significant return for Thursday was observed for DaimlerChrysler and rather surprisingly also a significantly lower variance on Thursday was observed. Also Siemens showed a significantly lower variance on Thursday.

In a similar study of the day of the week effects, in the spot return on U.S. dollars relative to British pounds, Copeland and Wang (1993) found a weak but regular pattern of low returns on dollars relative to pounds on Thursdays. The same study also shows an unmistakable cycle of high volatility on Mondays and Thursdays, but no explanation for the phenomena was provided.

The most important observation from this study was that it showed that although a traditional tests, like the analysis of variance, indicated a negative return for Thursday during the same period as the implied volatilities used, this was not supported using a GARCH (1,1) model. Also no significantly higher variance for Thursday during the second subperiod was observed. This test may indicate that the theory that scheduled news announcements increase volatility could be a possible explanation for the deviations of Thursdays, in the examination of the implied volatility ratios.

6 Summary and conclusions

The choice of time unit in option pricing is important. The chosen time unit should make it necessary to adjust the volatility parameter only when there are some fundamental reasons for it and not due to wrong specifications of the model. This paper investigated the effects of option pricing using different time hypotheses and empirically investigated which time frame the option markets in Germany were using.

The empirical test appears to verify that the German market behaves in a fashion that deviates from the most traditional time units in option pricing, calendar and trading days. The four stocks in the study seem to use a time frame that we call weekend-one days and the index options seems to use what we call weekend-half days.

More detailed results from this study are available upon request.
Weekend-half days assume an unobserved volatility over the weekend to be about half that of other trading days of the week and count the weekend as a half-day. On the other hand weekend-one assumed the weekend volatility to be one times that of other trading days of the week and counts the weekend as one additional day.

An interesting observation in the study was the Thursday effect that was present in almost all the examined underlying assets. The implied volatility of Thursdays differed from the pattern of other days of the week and was somewhat higher. Using a GARCH model to further investigate the effect showed that although a traditional tests, like the analysis of variance, indicated a negative return for Thursday during the same period as the implied volatilities used, this was not supported using a GARCH model. Also no significantly higher variance for Thursday during the same period was observed. This test supports that the theory that scheduled news announcements increase volatility could be a possible explanation for the deviations of Thursdays, in the examination of the implied volatility ratios.

Perhaps the most important observation is the fact that the choice of time units can influence the implied volatilities. There can exist spurious gaps in the implied volatilities that in fact are due to nothing more than the choice of time unit in the option model. Understanding volatility is one of the keys to successful option trading.
References


Appendix 1

The Black and Scholes (1973) formula gives the price of a European call option as

$$C = SN(d_1) - Xe^{-rt} N(d_2),\quad (A1)$$

$$d_1 = \frac{\ln(S/X) + (r + \sigma^2/2)t}{\sigma \sqrt{t}},\quad (A2)$$

$$d_2 = d_1 - \sigma \sqrt{t},\quad (A3)$$

where $C$ is the price of the call option with strike $X$ and underlying $S$, $r$ is the interest rate, $\sigma$ is the volatility and $t$ is time to maturity, $N$ is the cumulative univariate normal distribution and $\ln$ is the natural logarithm.

The French (1984) model adjust the Black and Scholes formula in (A2) and (A3) as demonstrated in (A4) and (A5) to have two different time basis

$$d_1 = \frac{\ln(S/X) + rt + \sigma^2 \tau / 2}{\sigma \sqrt{\tau}},\quad (A4)$$

$$d_2 = d_1 - \sigma \sqrt{\tau},\quad (A5)$$

where $\tau$ is time to expiration on a new time basis that has yet to be estimated. French used in his study 252 days per year corresponding to volatility only during trading time. French still used $t$ as calendar days in (A1) in order to count interest rate on a daily basis.
### Appendix 2

**Table A1**

<table>
<thead>
<tr>
<th></th>
<th>Calendar</th>
<th>Weekend-one</th>
<th>Weekend-half</th>
<th>Trading</th>
</tr>
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<td><strong>Monday</strong></td>
<td>1.000</td>
<td>1.004</td>
<td>1.007</td>
<td>1.010</td>
</tr>
<tr>
<td></td>
<td>45.0 %</td>
<td>45.2 %</td>
<td>45.3 %</td>
<td>45.4 %</td>
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<tr>
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<td>1.000</td>
<td>1.002</td>
<td>1.004</td>
<td>1.005</td>
</tr>
<tr>
<td></td>
<td>45.0 %</td>
<td>45.1 %</td>
<td>45.2 %</td>
<td>45.2 %</td>
</tr>
<tr>
<td><strong>Wednesday</strong></td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>45.0 %</td>
<td>45.0 %</td>
<td>45.0 %</td>
<td>45.0 %</td>
</tr>
<tr>
<td><strong>Thursday</strong></td>
<td>1.000</td>
<td>0.998</td>
<td>0.997</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
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<td>44.8 %</td>
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<tr>
<td><strong>Friday</strong></td>
<td>1.000</td>
<td>0.996</td>
<td>0.993</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>45.0 %</td>
<td>44.8 %</td>
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<tr>
<td><strong>Monday</strong></td>
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<td>1.005</td>
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</tr>
<tr>
<td></td>
<td>45.0 %</td>
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<tr>
<td><strong>Tuesday</strong></td>
<td>1.000</td>
<td>1.003</td>
<td>1.004</td>
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<tr>
<td><strong>Wednesday</strong></td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<tr>
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<td>45.0 %</td>
<td>45.0 %</td>
<td>45.0 %</td>
<td>45.0 %</td>
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<tr>
<td><strong>Thursday</strong></td>
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<td>0.996</td>
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<td>45.0 %</td>
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<td>44.7 %</td>
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<tr>
<td><strong>Friday</strong></td>
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<td><strong>Thursday</strong></td>
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<td>44.5 %</td>
<td>44.3 %</td>
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</table>

The different implied volatilities were calculated with the composite-time model using one time unit and then transformed into calendar days. Underlying price 100, strike price 100, interest rate 3.5 % and volatility 45 %. The maturity of the options was between two and six weeks and the table starts with the longer maturities.