Alf-Erik Lerviks

SIMULATING AND FORECASTING THE DEMAND FOR NEW CONSUMER DURABLES

Forskningsrapporter från Svenska handelshögskolan

Swedish School of Economics and Business Administration
Research Reports

59

Helsingfors 2004
Simulating and forecasting the demand for new consumer durables

Key words: Consumer durables, Diffusion of innovations, Adoption behaviour, Interpersonal communication, Word of mouth, Diffusion models, Growth curve models, Simulation models, Forecasting, Calibration, Replacement demand

© Swedish School of Economics and Business Administration & Alf-Erik Lerviks

Alf-Erik Lerviks
Department of Marketing and Corporate Geography
Swedish School of Economics and Business Administration
P.O.Box 479
00101 Helsinki, Finland

Distributor:

Library
Swedish School of Economics and Business Administration
P.O.Box 479
00101 Helsinki, Finland

Telephone: +358-(0)9-4313 3376, +358-(0)9-4313 3265
Fax: +358-(0)9-4313 3425
E-mail: publ@hanken.fi
http://www.hanken.fi/

Yliopistopaino, Helsingfors 2004

ISBN 951-555-824-7 (printed)
ISBN 951-555-825-5 (PDF)
ISSN 0357-5764
PREFACE

When I started to work on this research report I did not know where the process or, rather, journey would take me. One thing was, however, clear from the very beginning. I simply felt that I had to collect, refine and adapt various ideas and experiences of mine gathered over the last 40 years concerning the modeling of diffusion processes for new consumer durables into a common framework, especially adjusted to long-term forecasting. It soon became clear that this journey turned out to be much more exciting and captivating than I could ever have expected at the beginning. As the journey went on, ideas and research findings from various times merged into each other giving birth to conceptualizations like the counteractive adoption model as well as further operationalizations of the concepts of this model into an overall simulation framework or demand simulator, as I have called the final model. The model turned out to work well in empirical validations, especially in a forecasting sense. However, before the journey could reach this point a serious obstacle had to be overcome. It is obvious that traditional statistical techniques are not very helpful in estimating model parameters on the basis of very few sales observations, which is the case in early stages of a diffusion process. To overcome this problem an optimization algorithm was developed by which the main parameters of the model could be calibrated on the basis of very few sales observations in iterative simulation runs. The validation outcomes using this algorithm have so far been promising. The journey has not yet ended, as some of my remarks in Chapter 6 imply. However, the journey has reached a point where I feel that the experiences achieved so far are worth reporting on.

Two colleagues have acquainted themselves with my journey by reading my report. They have given me valuable comments that have helped me to improve my text. I thank you both, Kristian Möller and Gunnar Rosenqvist, for that.

Espoo, January 30th, 2004
Alf-Erik Lerviks
CONTENTS

Preface ........................................................................................................................... i

List of figures ................................................................................................................ v

List of tables .................................................................................................................. vi

List of symbols used in DEMSIM ................................................................................ vii

Chapter 1 INTRODUCTION .................................................................................. 1

Sphere of interest ................................................................................................. 1
Purpose of the study ............................................................................................... 2

Chapter 2 THE DIFFUSION MODEL ..................................................................... 6

Earlier diffusion models for consumer durables ........................................... 6
Adopting and resisting new durables ............................................................... 11
The nature of diffusion processes ................................................................. 18
Interpersonal contact behaviour .................................................................... 20
Basic model structure ....................................................................................... 21
The promoting forces operationalized .......................................................... 22
Bringing the pieces together ........................................................................... 28

Chapter 3 THE REPLACEMENT MODEL ........................................................... 34

Research on consumer durable replacements .............................................. 34
A simple replacement mechanism ................................................................. 36
The replacement model summarized ............................................................. 38

Chapter 4 DEMSIM – A DEMAND SIMULATOR ................................................. 40

Structure of DEMSIM ....................................................................................... 40
Defining the market segments ....................................................................... 44
Model types and simulation purposes ............................................................. 45
Applying DEMSIM – Interacting and learning ............................................... 47
Validating DEMSIM ......................................................................................... 50
Calibration approaches ..................................................................................... 53

Chapter 5 DEMSIM IN ACTION .......................................................................... 59

Plan of the chapter ............................................................................................. 59
Owner shares, new demand and owner share corrections ................................ 60
Contact behaviours and advertising exposures .............................................. 65
Diffusion sensitivity to word-of-mouth parameters .......................... 68
Forecasting accuracy of the diffusion model ............................................. 76
Long-term demand scenarios ................................................................. 100

Chapter 6 CONCLUDING REMARKS ........................................... 106

Short summary .................................................................................. 106
Main findings .................................................................................. 107
Developing DEMSIM further .............................................................. 110
Applying DEMSIM ........................................................................... 110

REFERENCES .................................................................................. 112

APPENDICES
A Observed number of households in sample 1958-1968 (Periods 1-32) .... 118
B Observed number of owners in sample 1958-1968 (Periods 1-32) ........ 119
C Observed new demand in sample 1958-1968 (Periods 1-32) ............... 120
D Advertising volumes concerning television sets in daily newspapers and weekly magazines in 1958-1968 (Periods 1-32) ......................... 121
E Observed and simulated new demand in Periods 1-32 (Case 1, Simulation 3) ................................................................. 122
F Successively updated estimates of influence parameters with four decimals (Case 1) ................................................................. 123
G Simulation accuracy using revised influence parameter estimates (Case 1) ............................................................................. 124
List of figures

1. Phases preceding a possible adoption ....................................................... 12
2. The counteractive adoption model ............................................................ 16
3. Decreasing probability to discuss the durable in contacts with owners ...... 27
4. Survival curves .......................................................................................... 37
5. Basic structure of DEMSIM ................................................................. 41
6. Learning through repeated forecasting and validation processes .......... 49
7. Preparing for calibration ....................................................................... 57
11. Accumulated owner share corrections over time (Periods 1-32) .......... 64
12. Decreasing probability to discuss the durable for four \(d\)-values ............. 69
13. Observed and simulated owner shares (Illustration 1) ......................... 73
14. Observed and simulated owner shares (Illustration 2) ............................ 74
15. Observed and simulated owner shares (Illustration 3) ......................... 75
16. Observed and simulated owner shares (Illustration 4) ......................... 76
17. Observed and simulated new demand yearly (Case 1, Simulation 1) ...... 80
18. Observed and simulated new demand yearly (Case 1, Simulation 2) ...... 81
19. Observed and simulated new demand yearly (Case 1, Simulation 3) ...... 82
20. Observed and simulated owner shares (Case 1, Simulation 3) ............... 83
21. Simulated internal exposures over time (Case 1, Simulation 3) .......... 84
22. Simulated internal exposures according to origin (Case 1, Simulation 3) . 85
23. Observed and simulated new demand yearly (Case 2, Simulation 2) ...... 91
24. Observed and simulated owner shares (Case 2, Simulation 2) ............... 92
25. Observed and simulated new demand yearly (Case 3, Simulation 3) ...... 96
26. Observed and simulated owner shares (Case 3, Simulation 3) ............... 97
27. Simulated demand patterns (Scenario 1) ................................................ 103
28. Simulated demand patterns (Scenario 2) ................................................ 104
List of tables

1 The diffusion model summarized .............................................................. 32
2 The replacement model summarized ......................................................... 38
3 Exposure to personal contacts and mass media ....................................... 66
4 Contact probabilities within and between income segments ............... 67
5 Simulation accuracy for four $d$-values in twelve simulations .......... 71
6 Four validation cases ................................................................. 77
7 Successively updated estimates of influence parameters (Case 1) ....... 78
8 Simulation accuracy in eleven simulations (Case 1) ................................. 79
9 Some accuracy comparisons using iteration steps .001 and .0001 (Case 1) 87
10 Successively updated estimates of influence parameters (Case 2) ....... 89
11 Simulation accuracy in eleven simulations (Case 2) ................................. 90
12 Successively updated estimates of influence parameters (Case 3) ....... 94
13 Simulation accuracy in eleven simulations (Case 3) ................................. 95
14 Successively updated estimates of influence parameters (Case 4) ....... 98
15 Simulation accuracy in eleven simulations (Case 4) ................................. 99
16 Parameter values used in generating two scenarios ............................. 102
List of symbols used in DEMSIM

\( A(t) \)  
Amount of advertising (external signals) concerning a durable in mass media in time period \( t \)

\( a_m \)  
Coefficient of internal influence in segment \( m \)

\( AP_m(t) \)  
Probability to adopt a durable in segment \( m \) in time period \( t \)

\( b_m \)  
Coefficient of external influence in segment \( m \)

\( C_m \)  
Expected number of personal contacts per consumer in segment \( m \) in one time period

\( D_{mn}(t) \)  
Probability that a durable will be discussed in a personal contact between a non-owner in segment \( m \) and an owner in segment \( n \) in time period \( t \)

\( d \)  
Rate at which the probability to discuss a durable in a personal contact between a non-owner and an owner decreases when the time since the owner acquired the durable increases

\( \Delta Y_m(t) \)  
Change in owner share due to consumers adopting a durable in segment \( m \) in time period \( t \)

\( EM_m \)  
Expected exposure to mass media per consumer in segment \( m \) in one time period

\( EXTEXP_m(t) \)  
Expected exposure in segment \( m \) to advertising (external signals) concerning a durable in time period \( t \)

\( g_m \)  
Expected service-life of a durable in segment \( m \)

\( H_m(t) \)  
Number of consumers in segment \( m \) at the beginning of time period \( t \)

\( INTEXP_m(t) \)  
Expected exposure in segment \( m \) to information concerning a durable in personal contacts with owners in time period \( t \)

\( K \)  
Number of time periods the willingness to discuss a recently acquired durable prevails in personal contacts between a non-owner and the owner that recently acquired the durable

\( k \)  
Variable denoting number of time periods since a durable was acquired \((k = 1, 2, \ldots, K)\)

\( M \)  
Number of segments

\( m \)  
Subscript denoting specific segments \((m = 1, 2, \ldots, M)\)

\( \text{MAE}^{\text{dem}} \)  
Mean absolute error (deviation) between simulated and observed new demand in all segments and all simulated time periods \((\text{optimization criterion in calibrations})\)
MAE \textsuperscript{own} Mean absolute error (deviation) between simulated and observed owner shares in all segments and all simulated time periods (optimization criterion in calibrations)

MAE \textsubscript{m} \textsuperscript{dem} Mean absolute error (deviation) between simulated and observed new demand in segment \textsubscript{m} in all simulated time periods (accuracy measure)

MAE \textsubscript{m} \textsuperscript{own} Mean absolute error (deviation) between simulated and observed owner shares in segment \textsubscript{m} in all simulated time periods (accuracy measure)

ME \textsubscript{m} \textsuperscript{dem} Mean error (deviation) between simulated and observed new demand in segment \textsubscript{m} in all simulated time periods (accuracy measure)

ME \textsubscript{m} \textsuperscript{own} Mean error (deviation) between simulated and observed owner shares in segment \textsubscript{m} in all simulated time periods (accuracy measure)

MxE Maximal single error (deviation) in one simulation run (accuracy measure)

\textit{n} Subscript denoting specific segments (\textit{n} = 1,2,...,\textit{M})

\textit{obs} Superscript denoting that the variable in question is empirically observed

\textit{P(E1)} Probability that event 1 (\textit{E1}) will occur

\textit{P(E2 | E1)} Probability that event 2 (\textit{E2}) will occur, given that event 1 occurs simultaneously

\textit{P(E3 | E1 \cap E2)} Probability that event 3 (\textit{E3}) will occur, given that events 1 and 2 occur simultaneously

\textit{P(E1 \cap E2 \cap E3)} Probability that events 1, 2 and 3 will occur simultaneously

\textit{P_{mn}} Probability that, when a consumer in segment \textsubscript{m} is in contact with another consumer, this consumer belongs to segment \textsubscript{n}

PEAD \textsubscript{m} \textsuperscript{dem} Percentage error in simulated new demand accumulated over simulated time periods as compared to observed new demand accumulated over the same time periods in segment \textsubscript{m} (accuracy measure)

\textit{Q(t)} Total demand for a durable in time period \textit{t}

\textit{QN(t)} Total new demand for a durable in time period \textit{t}

\textit{QN_m(t)} New demand for a durable in segment \textsubscript{m} in time period \textit{t}

\textit{QR(t)} Total replacement demand for a durable in time period \textit{t}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$QR_m(t)$</td>
<td>Replacement demand for a durable in segment $m$ in time period $t$</td>
</tr>
<tr>
<td>$RP_m(x)$</td>
<td>Probability to replace a unit of a durable having been in use $x$ time periods in segment $m$, i.e. the failure rate</td>
</tr>
<tr>
<td>$s_m$</td>
<td>Parameter regulating the slope of the survival curve in segment $m$</td>
</tr>
<tr>
<td>$SR_m(x)$</td>
<td>Proportion of all units of a durable acquired in a specific time period that are still in use $x$ time periods later in segment $m$, i.e. the survival rate</td>
</tr>
<tr>
<td>$T$</td>
<td>Number of time periods simulated</td>
</tr>
<tr>
<td>$t$</td>
<td>Variable denoting specific time periods ($t = 1,2,...,T$)</td>
</tr>
<tr>
<td>$U_{m,t-x}(t)$</td>
<td>Number of consumers in segment $m$ owning a durable belonging to age category $x$ ($x = 1,2,...,X$) at the beginning of time period $t$, i.e. the age distribution of durables in use in segment $m$ at the beginning of time period $t$</td>
</tr>
<tr>
<td>$X$</td>
<td>Maximal life span of a durable expressed as number of time periods</td>
</tr>
<tr>
<td>$x$</td>
<td>Variable denoting age of a durable in use expressed as number of time periods ($x = 1,2,...,X$)</td>
</tr>
<tr>
<td>$Y(t)$</td>
<td>Owner share of a durable among all consumers at the beginning of time period $t$</td>
</tr>
<tr>
<td>$Y_m(t)$</td>
<td>Owner share of a durable in segment $m$ at the beginning of time period $t$</td>
</tr>
<tr>
<td>$YC_m(t)$</td>
<td>Owner share corrections in segment $m$ in time period $t$, i.e. changes in owner shares due to other reasons than consumers adopting a durable</td>
</tr>
</tbody>
</table>
Chapter 1

INTRODUCTION

Sphere of interest

In this study we are focusing on when consumers\(^1\) (individuals, families or households) on a given consumer market buy a new consumer durable for the first time (adopt the durable) and later on possibly buy the same durable again in order to replace old units of the durable. Furthermore we are concerned with durables mainly bought and owned in units of one. We are analysing the buying behaviour for such durables from the very beginning, i.e. from the time when the new durable is launched on the market and no consumers are owners of the durable. At that time the durable can be looked upon as an innovation unknown for most of the consumers. In such situations, supposing that the durable in due time will be accepted and adopted by many consumers, sales to non-owners, i.e. new sales, often follows a more or less distinct pattern. At the beginning new sales grows slowly, then accelerates, begins to slow down, reaches a peak, and after that starts to decrease. New sales decreases until all potential adopters have adopted. The market is then said to be saturated, a state when the diffusion of the innovation/durable has come to an end. The share of owners among all consumers at that time expresses the saturation level. However, total sales of the durable does not stop at this point. As a matter of fact, when new sales decreases replacement sales to owners usually increases. The magnitude of total sales at different times depends on, among other things, how quickly the durable penetrates the market, the level of saturation, the share of owners willing to replace the durable at different times, and the age of the durable when it is replaced (replacement time).

We are, in other words, interested in how total sales of a new consumer durable develops along with the diffusion process of that durable and further on when the market is saturated. Ultimately, we are looking for a tool that, right away after the introduction of a new durable on a market, is capable of producing more reliable long-term sales forecasts for the durable than traditional growth curve models usually do. Such a tool, or model, should obviously incorporate diffusion mechanisms generating new sales as well as replacement mechanisms generating replacement sales.

According to general diffusion theory, the crucial elements in analysing diffusions of innovations are (1) the innovation, (2) its communication from one individual to another (3) in a social system (4) over time (Rogers, 1962, p. 12). In our context we have a new durable (the innovation), communicated between consumers on a market

\(^1\) In the following we will use the word consumer as a synonym of the word consumption unit.
(a social system) by word-of-mouth and/or visual signs, all this taking place over time. The adoption behaviour is, in other words, supposed to be influenced by information and/or signs concerning the durable received in social interactions mainly with consumers on the market that already have adopted or otherwise are acquainted with the durable. This kind of, so to say, pure diffusion influence has frequently been called *internal influence*, mainly in connection with mathematical diffusion models (See e.g. Lekvall and Wahlbin, 1973, and Mahajan, Muller and Bass, 1990). In the present study we are strongly focusing on this concept. Actually, the concept will be operationalized far beyond what has been done in most existing diffusion or growth curve models.

Furthermore, it is obvious that adopting a new durable also can be influenced by different activities carried out by the marketer of the durable in order to speed up the adoption processes of the consumers. These kinds of influences originate, so to say, outside the social system of interactions just mentioned. Hence, they are often called *external influences*. This concept has been used together with the concept of internal influence in many mathematical diffusion models of which some will be discussed in the following. We also find the concept of external influences useful and incorporate it in our theoretical framework.

As it comes to replacement sales we can also distinguish between two main types of influences. We have, on one hand, influences due to how long the durable has been in use since it was acquired, i.e. the service-life (or age) of the durable. We call these influences *service-life influences*. On the other hand, external activities influencing new sales are probably also influencing replacement sales in a somewhat similar way. External influences are, in other words, crucial both for new sales and for replacement sales. Consequently our overall model will hold in it three main types of influences affecting sales, i.e. *internal influences*, *external influences* and *service-life influences*.

It should also be stated that we are dealing solely with *product class sales* in the present study. As a consequence, e.g. effects of promotional activities on brand switching behaviour within a product class fall outside our sphere of interest. However, if promotional activities also affect total product class sales of the durable they represent external influences in our context and should, in such cases, be accounted for. Naturally, a similar reasoning also holds for price effects.

**Purpose of the study**

In the following a diffusion/replacement model for new consumer durables will be presented and, as it comes to the diffusion part of the model, empirically validated as to its predictive ability. The main purpose of the diffusion/replacement model is to serve as a long-term forecasting tool for product class sales. Actually the model combines two mutually independent models, a diffusion model generating *new sales* among non-owners and a replacement model generating *replacement sales* among owners. The overall model can be characterized as a *sales or demand simulator*\(^2\) for

\(^2\) The *sales* and *demand* concepts will be used synonymously in the following. In doing so we implicitly assume that supply meets demand in each time period.
(new) consumer durables. This demand simulator will be called DEMSIM in the following.

From a strategic marketing point of view it is important to be able to make reliable long-term forecasts concerning future sales of a new durable as soon as possible after the launch of the durable on a market. Early answers to questions of the following type are crucial for a company producing and/or marketing a new durable.

- At what time will a possible sales peak appear and what will the magnitude of that peak be?
- At what time will the market get saturated and what will the share of owners then be?
- What will the approximate magnitude of replacement sales be when the market is saturated and how stable will this magnitude be over time?

DEMSIM is especially designed and developed to give answers to questions like these in early stages of a diffusion process. This purpose has determined

- how the model is specified,
- how the parameters of the model are calibrated and
- how the interface between user and model is designed.

Each of these three ‘hows’ will be commented on below.

**How the model is designed.** To begin with the basic driving force of any diffusion process, the internal influence, has to be much more thoroughly defined and operationalized than the case has been in most traditional growth curve models if we want to increase the predictive power as compared to these models. However, this cannot be done without some kind of explicit specification of the communication patterns among consumers. In our case we use a segmentation approach specifying communication patterns (habits) within and between a number of predefined consumer segments. In an explanatory diffusion model such communication patterns are crucial and should be considered explicitly. This is obvious since these patterns represent the channels through which, so to say, an innovation diffuses a market. Accordingly we use these communication patterns in our operationalization of the internal influence. In the demand simulator our specification of the internal influence constitutes an intervening construct affecting the speed and the shape of the diffusion process. Furthermore, the internal influence, as specified in our model, determines the saturation level of the diffusion process. This being the case, no prespecifications whatsoever of the saturation level are needed in applying the model.

**External influences** naturally also affect the adoption behaviour. However, external activities (e.g. advertising and pricing decisions) that influence the adoption behaviour are mostly of a strategic nature making them difficult to predict accurately, especially in long-term forecasting situations. This is, of course, due to the fact that these activities originate from strategic considerations of human beings acting in a continuously changing and usually heavily competitive marketing environment. Such activities will therefore always be more or less impossible to predict, especially in the long run. This does not, however, mean that it is difficult to model and measure the
effects of such activities ex post, i.e. when we know what the activities were. The problem lies, as just stated, in predicting the occurrence and nature of such activities in the future. It does not help much to have an accurate model of how external activities usually have affected the adoption behaviour in the past if we don’t have an accurate prediction of what these external activities will be in the future. Actually there are two kinds of forecasting problems embedded here. The first problem concerns whether the model of how the external activities have affected the adoption behaviour so far also will be valid in the future. The second problem concerns how well future external activities can be predicted. Since the second problem is more or less impossible to overcome, and since the first problem is conditional on the second problem in forecasts, there is, in a pure forecasting sense, no use in specifying the external influences too closely. However, they should not be forgotten either. It is important to specify them and analyse their impact on sales ex post continuously in order to deepen our understanding of the phenomenon that we are forecasting. This may lead to respecifications and further developments of our basic forecasting model as well as various adjustments to changing conditions in the market place.

Due to the uncertainty associated with predicting the external influences the accuracy of long-term forecasts based on all three types of influences (internal, service-life and external) are not necessarily better than the accuracy of forecasts based on only internal and service-life influences. This statement will be empirically demonstrated in Chapter 5. Forecasts based solely on internal influences and service-life influences represent a type of baseline sales forecasts.³ In our case we define baseline sales as expected sales under conditions of no external influences. The basic purpose of our demand simulator is then to produce long-term baseline forecasts of, on one hand, new sales generated through internal influences and, on the other, replacement sales generated through service life expectancies.

**How the parameters of the model are calibrated.** Since our model should be able to produce reliable sales forecasts as soon as possible after the launch of a new durable on a market the use of traditional statistical estimation techniques are more or less out of the question. Instead we have developed an optimization algorithm by which some of the parameters of the diffusion model can be calibrated in very early stages of a diffusion process. The algorithm is especially adapted to the structure of our diffusion model and developed as an integrated part of DEMSIM. However, parameter estimates based on very few sales observations are, of course, not very reliable. Therefore new estimates should be made each time sales data for a new time period has been collected in order to continuously improve the predictive power of the model. These approaches will be empirically demonstrated in Chapter 5.

---

³ The concept *baseline sales* has been defined as sales under nonpromoted conditions (Leeflang, Wittink, Wedel and Naert, 2000, p. 478). The concept has been frequently used in analysing promotional impacts on sales from scanner data within retailing. In this context promotions usually include price, display, advertisements and various feature conditions. See e.g. Cooper, Baron, Levy, Swisher and Gogos (1999), Bucklin and Gupta (1999) and Silva-Risso, Bucklin and Morrison (1999). However, the concept has been used in other areas as well. E.g. Gupta, Jain and Sawhney (1999) use the baseline sales concept in forecasting sales for new durables within the U.S. digital television industry. See also Pindyck and Rubinfeld (1998, p. 455) and Armstrong (1985, p. 42). Armstrong uses the concept in connection with scenario-building and talks about “baseline projections” in the sense “surprice-free projections”. Hair, Anderson, Tatham and Black, in turn, use the expression “baseline prediction” for predictions without independent variables (1998, p. 150).
How the interface between user and model is designed. Finally we have developed our demand simulator DEMSIM into an interactive validation and forecasting tool, which should be easy to use and understand. The basic idea is to facilitate ongoing interaction between the user and the model. By an extensive use of graphical presentations of simulated results the user can immediately analyse the results visually, adjust parameter values and/or respecify the model and then right away simulate the process all over again. A user setting is created that enables interactive trial and error search procedures that enhance learning. It could be said that applied forecasting is very much about continuous learning, i.e. continuously learning more about and understanding better the phenomenons we are forecasting.

Concluding this chapter we want to state that DEMSIM should be looked upon as a general frame for models set out to explain and forecast the diffusion of and demand for new consumer durables in a market. As such the frame incorporates those mechanisms that are supposed to be more or less similar regardless of what type of consumer durable we are dealing with. Using the frame in a specific application means that we have to further adapt the frame to conditions specific for the durable and market in question. This applies to various external influences as well as to the specification of interpersonal contact behaviours. In Chapter 5 a specific application based on our general DEMSIM frame is carried out.
Chapter 2

THE DIFFUSION MODEL

Earlier diffusion models for consumer durables

In the past 30 years or so a great number of studies on aggregate diffusion models for consumer durables have been reported on in journals and books (e.g. Bass, 1969, Bernhardt and Mackenzie, 1972, Bottomley and Fildes, 1998, Dodson and Muller, 1978, Heeler and Hustad, 1980, Lekvall and Wahlbin, 1973, Lilien, Kotler and Moorthy, 1992, Mahajan and Peterson, 1978, Meade and Islam, 1995, Midgley, 1977, Tanny and Derzko, 1988). These models have primarily been intended for forecasting purposes. Excellent reviews on this matter can be found in Mahajan and Muller (1979) and in Mahajan, Muller and Bass (1990).

The development in these 30 years has been greatly influenced by an article by Bass in Management Science in 1969 on a new product growth model for consumer durables (Bass, 1969). Actually a great deal of the research done since then within this area can be categorized as extensions and refinements of the Bass model. The article by Bass was especially important in the sense that it, in the area of new consumer durable forecasting models, combined the internal influences and the external influences in the same model. Actually the Bass model is a combination of two earlier growth models, the modified exponential growth model as applied by Fourt and Woodlock (1960), mainly representing external influences, and the logistic growth curve model as applied by Mansfield (1961), mainly representing internal influences (Mahajan and Muller, 1979). Bass (1969) was talking about innovation and imitation effects and distinguished between innovators and imitators in his theoretical framework.\(^4\) Later on, the connection between the theoretical framework and the mathematical formulations of the Bass model as well as the interpretation of the parameters have been widely discussed (see e.g. Mahajan and Muller, 1979, Tanny and Derzko, 1988, and Lilien, Kotler and Moorthy, 1992, pp. 469-470).

Independently of the Bass model Lekvall and Wahlbin (1973) presented a similar model specifying external influences and internal influences as the two main forces promoting diffusion processes. They defined external influence as “the direct influence on the innovative behavior of an individual which, for example, the marketer of a new product exerts through his various promotional activities”, while internal influence was defined as “the influence that the members of a social system exert on one another as a result of their social interaction” (Lekvall and Wahlbin, 1973).

\(^4\) Bass was apparently influenced by the hypothesis of a two-step information flow (cf. Katz and Lazarsfeld, 1955).
1973, p. 367). They also stressed that these two types of influences operate simultaneously on individual consumers at any point in time. This way of specifying the two basic diffusion-driving forces has been widely accepted by other researchers. The innovation and imitation effects of the Bass model were early compared to the external and internal influences as specified by Lekvall and Whalbin (Mahajan and Muller, 1979). Today it can be said that the parameters of the Bass model most often are interpreted as parameters reflecting external and internal influences.

Let us take a closer look at some aspects of the Bass model. The core assumption is that the probability to adopt at time $T$ is a linear function of the number of previous adopters, i.e.

\[
P(T) = p + \left(\frac{q}{m}\right)Y(T)
\]

where $p$ is referred to as the coefficient of innovation, $q$ as the coefficient of imitation, $m$ as the ultimate number of adopters and $Y(T)$ is equal to the accumulated number of adopters up to time $T$. The expression $\left(\frac{q}{m}\right)Y(T)$ reflects the pressures operating on imitators as the number of previous buyers increases (Bass, 1969, p. 216).

The number of adopters at time $T$ can then in discrete-time form be expressed as follows.

\[
Q(T) = p(m - Y(T)) + \left(\frac{q}{m}\right)Y(T)(m - Y(T))
\]

The expression $p(m - Y(T))$ stands for the innovation effect (the external influence) and the expression $\left(\frac{q}{m}\right)Y(T)(m - Y(T))$ for the imitation effect (the internal influence) (cf. Lilien, Kotler and Moorthy, 1992, p. 469). Note that these two influences are supposed to be additive in the model.

Various extensions of the Bass model have, among other things, concerned the nature and the dynamics of the market potential $m$ and the incorporation of marketing mix variables into the model. Such extensions are reviewed in Mahajan, Muller and Bass (1990). Especially the role of price has been dealt with extensively. Bottomley and Fildes (1998) distinguish between two schools of thought in this respect. The market potential school of thought assumes that the market potential, $m$, is dependent on the product’s price, i.e. the size of $m$ is supposed to increase when the price decreases (see e.g. Mahajan and Peterson, 1978, and Mahajan, Peterson, Jain and Malhotra, 1979). The other school assumes that a reduction in price accelerates the adoption decision of those who belong to the market potential but does not affect the size of the market potential $m$ (see e.g. Robinson and Lakhani, 1975). Later on Kamakura and Balasubramanian (1988) have proposed a family of twelve nested discrete-time diffusion models, three of which represent the first school of thought, three of which represent the second school of thought and three of which represent a combination of the two schools, i.e. allowing for simultaneous price effects on the probability to adopt as well as on the size of the market potential (see also Bottomley and Fildes, 1998).

---

5 This parameter has frequently also been called *market potential* or *saturation level.*
The Bass model and various extensions of it have frequently been empirically validated (see e.g. Bass, 1969, Mahajan and Peterson, 1978, Heeler and Hustad, 1980, Tigert and Farivar, 1981, Meade and Islam, 1995, Bottomley and Fildes, 1998). Without going into details it can be stated that the results of such validations have in many cases been more unsatisfactory than satisfactory. E.g. Heeler and Hustad (1980) show that, with short periods of actual sales data as input, the predictive ability of the Bass model usually is poor when it comes to predicting the time and magnitude of the sales peak as well as the size of the potential market. Their study covers the diffusion of more than fifteen household durables in many countries. A general finding from several empirical validation studies is, furthermore, that less complex, more straightforward, models usually perform better from an explanatory as well as from a predictive point of view than more complex models with more parameters to estimate. Bottomley and Fildes (1998) validated the twelve nested discrete-time diffusion models proposed by Kamakura and Balasubramanian (1988) using short series of observations for long-term forecasts on twelve consumer durable products, six from the UK and six from the USA. As was mentioned above, nine of the nested models include price, while the last three models are simple time-dependent models, one incorporating only external influences (the modified exponential growth model), one only internal influences (the logistic growth model) and the last one both external and internal influences (the Bass model). In most cases the three simple time-dependent models explained the diffusion processes better than those incorporating prices. In a few cases, however, price succeeded to increase the explanatory power. In these cases price influenced the probability to adopt rather than the market potential. This was the case for relatively expensive durables. Noteworthy is, however, that in no case did price influence the market potential significantly. In another study Meade and Islam (1995) validated seventeen growth curve models (including the Bass model) on twentyfive time series describing the diffusion of telephones in fifteen countries. As a general conclusion they found that, in the actual cases, more straightforward models offered better overall forecasting performances. In most cases logistic and Gompertz models performed best. In terms of model complexity, they concluded that the data generally is better represented by models with two or three parameters than by models with four parameters. As to the Bass model the estimates of the parameter $p$ were negative in all cases and, so to say, illogical (Meade and Islam, 1995, p. 209).

These were only some examples of validations. Of course there are also more successful validations (see e.g. Bass, 1969, and Mahajan and Peterson, 1978). Many successful validations are, however, based on long series of observations usually covering periods far beyond the sales peak. In such cases the forecasting ability of the models in early stages of a diffusion process is not properly validated. Instead, such validations mainly concern the theoretical correctness of the models and their explanatory power *ex post*. As to the validation of diffusion models for new durables in general Mahajan and Muller stated in their review article from 1979 that, until then, empirical research findings on the validity and reliability of such models were rare. In a later review article on the same topic from 1990 Mahajan, Muller and Bass conclude that, for most diffusion model studies, the analytical elegance surpasses the empirical validation of the derived results. Later on several empirical validations have been conducted, some reported on above. The results of these have, however, not been especially encouraging, at least when it comes to the forecasting ability of more
complex diffusion models for consumer durables incorporating marketing mix variables.

The fact that the incorporation of marketing mix variables into diffusion models sometimes tends to make the predictive as well as the explanatory power of these models poorer does not actually tell us anything about the real impact of such marketing mix variables on diffusion processes. The poor performance obviously stems from other reasons. In our opinion such reasons can be gathered under three headings.

- **General properties of mathematical growth curves**
- **Theoretical assumptions underlying diffusion models based on growth curves**
- **Problems in estimating several parameters from few observations**

**General properties of mathematical growth curves.** We are here concerned with mathematical growth curves that illustrate how a variable grows over time from a starting level (usually zero) to an asymptotic or saturation level, i.e. a level where the growth has come to an end. The growing or dependent variable is expressed basically as a mathematical function of time. If this function is expressed as a discrete-time function, the derivative of this function expresses the change in the dependent variable per unit of time. This derivative is usually formulated as a function of time and/or the dependent variable and/or some other variable(s). Equation (2) above of the Bass model is an example of such a function. The form of the derivative determines naturally the form of the final growth curve. Above we have touched upon growth curve forms ranging from exponentially decreasing forms to s-shaped forms with both stable and varying inflection points.

Although there are growth curve models that are very flexible as to the form of the growth process, they all have one characteristic in common that can be considered as a serious restriction as it comes to explaining and predicting diffusion processes. This characteristic is the saturation level. A mathematical growth curve model is more or less impossible to specify without specifying the saturation level in one way or another prior to applying the model empirically. This specification has been made in various ways in models referred to above. In the original Bass model (1969) the saturation level is specified as the parameter $m$, which is estimated simultaneously with the parameters $p$ and $q$. Bottomley and Fildes (1998) carry out validations in which the saturation level is specified as being dependent on price and Mahajan and Peterson (1978) specify the saturation level as dynamic, only to mention some examples. In the following we will more closely elaborate on the drawbacks of being forced to specify the saturation level in growth curve models, especially when these are used as diffusion models intended for explanatory and predictive purposes.

---

6 For a review of such growth curve models, see e.g. Lerviks (1967).

7 The inflection point of a s-shaped growth curve appears when the derivative of the curve is in its optimum. Two well-known growth curves with stable inflection points are the logistic curve and the Gompertz curve. The inflection point of the logistic curve always appears when the value of the growth variable is 0.5 of the saturation level and for the Gompertz curve 0.368 of the saturation level.
Theoretical assumptions underlying diffusion models based on growth curves. To begin with we return to the basic assumption of the Bass model put forward in Equation (1). According to this assumption the probability that a non-adopter adopts at time $T$ is a linear function of the number of previous adopters. Equation (1) states that this probability can be looked upon as a sum of two terms. The first term, $p$, can be said to represent external influences, which are supposed to be constant over time in the model. The second term, $(q/m)Y(T)$, represents internal influences, which are supposed to be dependent on the number of previous adopters, $Y(T)$. According to Bass this term reflects the pressures operating on imitators (non-adopters) as the number of previous adopters increases. In other words, we have a situation where the probability that a non-adopter will adopt continuously grows until all non-adopters belonging to the market potential (saturation level) have adopted. The last non-adopter to adopt does this with the highest probability of them all. For the non-adopters not belonging to the market potential, however, the probability to adopt is zero according to the model. This means that no kind of pressure from previous adopters are supposed to affect non-adopters not belonging to the market potential. In our opinion it is hard to understand where the continuously increasing pressure from previous adopters suddenly disappears when $Y(T)$ reaches $m$, i.e. when the market is said to be saturated. Why does not this pressure continue to affect those who at that moment do not belong to the market potential but maybe will do so in the future? It is, in other words, hard to see the behavioural rationale behind such an implicit assumption.

Although we have used the Bass model as an example, a similar reasoning holds for most diffusion models based on mathematical growth curves, simply because they all incorporate a specification of the saturation level in one way or another. Of course, as we have noted earlier, there are approaches that e.g. allow for a dynamic saturation level or let the saturation level be price-dependent. However, in such approaches, what do we really model? Do we model a diffusion process? Do we model the demand on a saturated market? Or do we model both of these? Without going further into these questions, we state that approaches forcing us to explicitly specify a saturation level in a diffusion model, no matter how we do it, usually brings about generalizations and assumptions that can be considered as unrealistic from a consumer behavioural point of view. Therefore we are, in the following, looking for approaches that avoid this drawback.

Problems in estimating several parameters from few observations. It goes without saying that the quality of the estimates are much better when we estimate few parameters from many observations than when we estimate many parameters from few observations. However, when using a diffusion model as a forecasting tool the forecasts are most urgently needed at the very beginning of the diffusion process when we still have very few, if any, sales observations. The more parameters our model contains, the more observations we need for estimation purposes and the later we can start forecasting with the model. This statistical fact is obviously one contributory cause to the poorer validation results achieved for growth models with more parameters to estimate than for growth models with fewer parameters to estimate, reported on above.
Other causes are, however, also possible. We should not automatically exclude the possibility that incorporating marketing mix variables into aggregated non-linear diffusion models can bring about specification errors that result in poorer predictive as well as explanatory power of the models. You could even go as far as saying that the poor validation results as to more complex growth curve diffusion models possibly indicate the existence of (serious) specification errors in these models. Since the assumptions behind these aggregate diffusion models are extremely simplified and even unrealistic, we believe that any attempt to explicitly incorporate marketing mix variables into them is, in most cases, doomed to fail in the sense that such model extensions would improve the explanatory and predictive power of the models. It should be remembered that the foundation of these models consists of rather simple basically time-dependent growth curves. To build sophistication and elegance upon such a foundation does not make the end construct better than the foundation itself. Therefore we believe that other approaches are needed in the search for more powerful forecasting models for new consumer durables.

**Adopting and resisting new durables**

From here on the development of our diffusion model starts. In this and the following sections we will present and discuss ideas, concepts and assumptions that, as the text proceeds, builds up into a theoretical framework, which finally will be specified into a mathematical diffusion/replacement model.

We start our development of the diffusion part of DEMSIM by looking at consumers on a non-aggregated level and more precisely the process that, possibly, leads to an adoption of a new consumer durable. This process has usually been called the adoption process. Mostly this process has been specified as a number of consecutive steps or stages ending up with an adoption (or a rejection) of the durable in question (see e.g. Rogers, 1962, Lekvall and Wahlbin, 1973, Midgley, 1977). The number and naming of the stages of the process differ somewhat between researchers, but the basic idea is the same, to model a kind of decision preparing process usually supposed to start when a consumer becomes aware of the new durable and ending when the consumer decides to adopt (or reject) the durable.\(^8\) One typical example of such an adoption process is Rogers’ paradigm specifying five stages: Awareness, Interest, Evaluation, Trial, and Adoption (Rogers, 1962, p. 306). A variety of different factors, both personally related as well as situational and informational factors, are supposed to affect the speed as well as the outcome of the adoption process or, put another way, the timing of the adoption/rejection decision.

It is important to note that adoption process frames of this type mainly try to describe and explain the behaviour of those consumers that show some kind of interest in the durable and at least in some way consider to adopt it. However, for most new durables there are always consumers that are unwilling even to consider, in any way, adoption of the durable after they have become aware of it. They are simply not interested. They, so to say, deliberately want to stand outside the actual adoption process. As a matter of fact, traditional diffusion research has shown little interest in these

---

\(^8\) The resemblance to ordinary buying behaviour process models is obvious.
consumers and the reasons for their unwillingness or lack of interest. Rogers stated in 1976 (Rogers, 1976, p. 295) that most diffusion research until then had held in it a pro-change bias according to which the innovations studied were supposed to be “good” and, hence, should be adopted by everyone. Still, twenty-three years later, Sheth and Sisodia (1999, p. 76) come to a similar conclusion. This matter has also been pointed out by, among others, Sheth (1981) and Ram (1987). The unwillingness to adopt has been called innovation resistance or more generally speaking resistance to change. Sheth points out that most humans prefer consistency and status quo instead of continuous search for new behaviours. Strong habits are much more prevailing among people than innovativeness. Furthermore, Sheth claims that innovation resistance also originates from perceived risks associated with adopting an innovation (Sheth, 1981, p. 275). Such risks can be financial, of a health character or something else. In this connection it seems plausible to state that innovations that are e.g. expensive in relation to the income of the consumer are resisted stronger than cheaper innovations. Also, innovations that bring about considerable adjustments in earlier habitual behaviours will be resisted stronger than if this is not the case. Furthermore, innovations that according to the consumers meet their needs better than earlier products or services do are probably not resisted as strongly as if this would not be the case. According to Ram innovation resistance is affected by three major groups of factors. The first group holds in it perceived characteristics of the innovation, the second psychological and demographic characteristics of the consumer and the third different types and characteristics of propagation mechanisms, i.e. channels through which information concerning the innovation reaches the consumer (Ram, 1987, p. 209).

**Figure 1. Phases preceding a possible adoption**

<table>
<thead>
<tr>
<th>Phase 1</th>
<th>Phase 2</th>
<th>Phase 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unawareness State</strong>&lt;br&gt;Consumer unaware of the existence of the new durable</td>
<td><strong>Inactive State</strong>&lt;br&gt;Consumer aware of but not showing active interest in the new durable</td>
<td><strong>Adoption Process</strong>&lt;br&gt;Consumer aware of and showing active interest in the new durable</td>
</tr>
</tbody>
</table>

In order to thoroughly understand diffusions, and especially non-diffusions, of new consumer durables the resistance of consumers and the reasons behind the resistance should be studied more closely. Since the resistance mainly is present before the consumer gets interested in the durable and enters the adoption process we should concentrate more on the times preceding the adoption process. Actually we can

---

9 Note that Lekvall and Wahlbin used the concept *Resistance to Innovation* in their important article from 1973 (Lekvall and Wahlbin, 1973, p. 371).
distinguish between three possible consecutive phases preceding an adoption. These are illustrated in Figure 1.

Note that the first two phases are called states, while the third is called a process. This indicates that a consumer’s relation to the durable is, so to say, active in phase 3 (a process concerning the durable is going on), while it is mainly passive in phases 1 and 2 (nothing happens regarding the durable except maybe a conscious or unconscious mental rejection in phase 2). How these three phases are realized differs strongly between consumers and between different durables. One consumer becomes, maybe, aware very quickly but is not at all interested in the durable, at least to begin with, and remains in the inactive state for the time being. This consumer has obviously a strong resistance to adopt the durable. Another, again, proceeds directly from unaware to aware, likes the durable at once, moves straight on into the adoption process and adopts shortly after that. This consumer has obviously a weak resistance to adopt the durable from the very beginning. However, most durables are never adopted by all consumers, meaning that there are always consumers that never enter the adoption process. They remain either as unaware or inactive consumers. In the case of such inactive consumers the resistance is obviously strong and also enduring.

In other words, consumers behave very differently in these respects. Based upon the three phases in Figure 1 at least four main patterns or types of behaviour can be distinguished. Consumers can

(1) stay in phase 1 and never move on,
(2) move from phase 1 to phase 2 and stop there,
(3) move from phase 1 to phase 2 and later on to phase 3, or
(4) move from phase 1 directly to phase 3.

Types (1) and (2) never adopt the durable, while types (3) and (4) possibly adopt the durable. So called innovators and early adopters obviously follow a type (4) behavioural pattern, while later adopters probably follow a type (3) pattern. The later the adoption takes place, the more probable is it that the consumer has been inactive at least for some time. It is reasonable to believe that there is a clear connection between type of behaviour, as specified above, and strength of resistance. A type (2) behaviour is obviously the result of a strong and enduring resistance. In behaviours of type (3) the resistance may have been strong at first but not enduring. The resistance may have been lowered by external and/or internal influences leading, at last, to an adoption. Type (4) behaviours can finally be characterized by a rather low resistance from the very beginning.

In our opinion the concept of resistance deserves much more attention in diffusion research. To place the focus mainly on the conscious adoption process is a very restricted approach. Approaches are needed that, besides the conscious adoption process, also take into account the phases preceding this process as described above, but also phases succeeding it. Only then can we increase our understanding of e.g. the timing of the adoption process, i.e. whether it starts at all and if it starts, when it starts and when it ends? Further crucial questions are e.g. if there will be any replacement processes, and if so, what their timing will be? To bring all these behaviours and/or non-behaviours together in a common framework we need a concept that can be tied
to these behaviours in a logically approvable way regardless of which behaviour we are talking about. Such a concept is *innovation resistance*. The discussion that follows will present this concept more closely and especially the way we look at and define it.

Obviously the strength of innovation resistance *varies between consumers* due to their psychological and demographic characteristics as well as due to the type of new durable in question. Those who adopt a new durable very early (the innovators) can be said to have, from the very beginning, a relatively weak resistance to adopt that specific durable, while those who adopt very late (the laggards) obviously have a much stronger resistance to adopt the durable, at the beginning as well as later on. It is worth noting that the reasons behind a strong resistance can vary considerably from case to case. In one case the main reason may be plain ignorance towards the type of durable in question, in another case maybe unfavourable affections towards the durable and what it stands for, in a third case financial obstacles to adopt the durable, and so on. Some reasons can be rather stable over time, others vary frequently.

It is also plausible to assume that the resistance of a single consumer *varies over time*. Usually, in the early stages of a new durable’s market penetration non-adopters are more and more frequently exposed to information concerning the durable in personal contacts with adopters, simply because the number of adopters on the market increases. The growing amount of such exposures probably affects the non-adopter’s resistance but not necessarily. Furthermore, the resistance can be affected by exposures to marketer and non-marketer controlled messages in various media. These two types of exposures, those in personal contacts and those in various media, or rather the effects of these exposures, are synonymous to internal influences and external influences mentioned above. In many cases it seems justified to assume that each of these influences has a weakening effect on the resistance. The possibility that internal and/or external influences strengthen the resistance should not, however, be automatically excluded. This could be the case if, for example, the exposures strongly irritate the consumer.

An interesting question is how weak a consumer’s resistance has to be until an adoption is possible. At zero resistance obviously no obstacles to adopt exist. But do consumers adopt even if a resistance is present? The answer to this question is of course dependent on how we define resistance. By innovation resistance we mean *all kinds of perceived uncertainties and hesitations connected with becoming and being a user of a new durable*. Defined in this way it is obvious that some kind of resistance is present when most new durables are adopted, among other things because the adopter at that moment still usually lacks experience of the durable. The fact that adoptions take place regardless of this resistance is obviously a consequence of considerable internal and external influences affecting those who have not adopted, especially at times when a new durable strongly penetrates a market.

The processes preceding the adoption can actually be characterized as a battle between two basic forces that counteract each other. We have, on one hand, the *resisting forces* originating within the consumer and, on the other, the *promoting forces* (internal and external influences) originating outside the consumer. It is plausible to assume, at least for successful new durables, that the promoting forces may have a weakening effect on the resistance, as we argued above. This effect may
furthermore be of a long-lasting character, meaning that when resistance is weakened it remains on the new weaker level. However, it is also possible that the promoting forces may strengthen the resistance for various reasons. If this happens often it is possible that the new durable turns out to be a failure. In spite of such possible effects, we believe, in general, that the resistance of most consumers is rather stable over time and not very easily affected by promoting forces. This, we believe, is due to strong habits and consistency-seeking behaviours, typical qualities of most consumers. The fact that many consumers adopt when a new durable strongly penetrates a market is, to our belief, mainly a consequence of heavy exposures to promoting forces, especially internal, leading to situations in which the promoting forces exceed the resisting forces and adoption takes place.

Adopting a new durable does not, however, mean that the resistance necessarily disappears after the adoption. In most cases it is reasonable to assume that the resistance lives on, in some cases even to such an extent that the adopter regrets that he adopted and, if possible, abandons the new durable. In such cases the promoting forces obviously exceeded the resisting forces only temporarily due to exceptionally strong internal and/or external influences for the time being. It is also possible that negative experiences in using the durable strengthens the post-adoption resistance. In other cases, again, the post-adoption resistance may be lower and not cause abandonment of the durable but, possibly, influence future replacement behaviour in a delaying or rejecting way. The experience of using the new durable may also be very positive causing the resistance to more or less disappear. It seems, generally speaking, plausible to assume that post-adoption experience of an innovation heavily influences post-adoption resistance. A positive experience obviously weakens the resistance and a negative experience strengthens the resistance.

In Figure 2 the discussion so far is brought together. Note that the promoting forces are supposed to influence the final adoption decision both indirectly through the innovation resistance construct and directly. The direct influence means that the promoting forces at every point in time act against the resisting forces leading to an adoption if and only if the promoting forces at some point in time are stronger than (exceed) the resisting forces, i.e. $A > B$. If this never happens, no adoption takes place. However, the promoting forces also affect the adoption behaviour indirectly by weakening (or strengthening) the innovation resistance. For durables that successfully penetrate new markets it is reasonable to assume that the promoting forces weaken the innovation resistance of most consumers over time continuously leading to greater willingness to adopt. If this is so, less and less promoting forces are needed to overcome the weaker and weaker resisting forces as time goes by. For durables that do not succeed as well on new markets it is, on the other hand, reasonable to assume that the promoting forces have not succeeded in weakening the resistance.

---

10 It is worth noting that resistance on a zero level can be reawakened due to e.g. negative experiences. Therefore it can be said that resistance never disappears. Instead, on a zero level, we prefer to talk about a latent resistance.

11 Note that the rejection-concept, used in many adoption process models, is not included in our model. Either a new durable is adopted (sooner or later) by a consumer or never adopted by that consumer. Instead of talking about rejecting a new durable, which sounds very final and conclusive to us, we prefer to talk about postponing the decision to adopt the durable up to some future point in time or forever.
innovation resistance of most consumers appreciably. The promoting forces may even have strengthened the innovation resistance of consumers that are, for some reason, annoyed by the promoting forces. In situations like this more and more direct promoting forces are needed to overcome the resisting forces that have not weakened but have stayed strong or maybe even got stronger. This is apparently the case when diffusions fail.

Traditional process oriented research on adoption behaviour does not distinguish between direct and indirect influences of the promoting forces. This distinction is, however, in our opinion important. It is clear that a new durable cannot diffuse a market successfully if the promoting forces do not succeed in clearly weakening the innovation resistance among consumers over time. This indirect influence on the adoption behaviour is the key to a “successful” diffusion process. Most consumers will not adopt if their counteractive force (the resistance) does not weaken over time. For this to happen, in turn, the consumers have to experience that adopting a new durable brings additional value into their lives. The weakening of the resistance, if it takes place, is often a slow process continuously influenced by the promoting forces, especially the internal influences. The direct influence, in turn, determines, so to say, the point in time when the adoption takes place, if it takes place at all. The direct influence is, however, not as crucial for the overall “success” of the diffusion process as the indirect influence is. The direct influence serves more as a regulator of the speed of the diffusion process.
The *counteractive adoption model* described above and summed up in Figure 2 portrays, so to say, processes inherent in and close to all consumers at all times. These processes, concretized as two counteracting forces, are always present, at some times more actively, e.g. in the various stages of a conscious adoption process, at other times again more passively, e.g. in the stages preceding an adoption process or in the stages succeeding an adoption. The counteractive adoption model holds in it the essential variables and constructs of the diffusion model that will be presented and validated later on. The model comprises one way of looking at circumstances and variables affecting the adoption behaviour of consumers on an individual level. The model constitutes the theoretical and conceptual foundation of the aggregate diffusion model that will be developed in the following.

We have argued that a consumer’s possible adoption of a new durable is the outcome of a “battle” between two basic forces counteracting each other, the resisting forces originating within the consumer and the promoting forces originating outside the consumer. If and when the promoting forces beat (grow stronger than) the resisting forces an adoption takes place. If this never happens, the consumer never adopts. The core element of this approach is the concept of innovation resistance. This concept is, as we see it, always inherent in every consumer. Even when a consumer is unaware of a new durable a general resistance to change can be said to exist. When the consumer gets aware of a new durable the general resistance develops into a resistance adjusted to the specific durable in question, or rather, the perceived characteristics of that durable. Furthermore, the resistance does not necessarily disappear after an adoption has taken place. Negative experiences of using the durable may even increase the resistance again. This, in turn, may influence future replacement behaviour.

The benefits of using the resistance concept is apparent at least in two respects. Firstly, by focusing on the resistance, i.e. the forces working against an adoption, along with the forces promoting an adoption, we get a deeper understanding of the adoption behaviour than if we solely look upon the promoting forces, which is the case in most traditional adoption process oriented research. Especially the timing of the adoption process can be better understood if we learn more about powers resisting an adoption before any real adoption process has started. Secondly, it seems plausible to assume that, in most cases, a consumer with a high resistance shortly after a durable has been launched also tends to have a relatively high resistance later on. If this is true we can state that consumers with a high resistance at the beginning has a lower probability to ever adopt the durable than consumers with a weaker resistance at the beginning. If this is the case, resistance measurements shortly after the launch of a durable can be very helpful in forecasting e.g. the time when a possible sales peak will appear or the level at which the market will be saturated. This will be demonstrated later on.

12 Note that by replacing *Innovation Resistance* by *Resistance* and *Adoption* by *Decision to Buy* in Figure 2 the model also can be interpreted as a general buying behaviour model.

13 Note that this concept can be said to portray the mental readiness of the consumer to adopt the durable. If the resistance is high the mental readiness to adopt is low and if the resistance is low the mental readiness is high.
The nature of diffusion processes

The diffusion process of a new consumer durable on a market starts when the first consumer becomes an owner of the durable (adopts it) and ends when the share of owners (adopters) among all consumers does not increase any more. In the period between these two points in time the new durable is, so to say, spreading among the consumers through word-of-mouth communication and/or visual signs. Especially the communication taking place in social interactions between adopters and non-adopters of the durable is supposed to be an important driver of the diffusion process. Diffusion processes for consumer durables are, however, also obviously influenced by activities (advertising, sales promotions, and so on) carried out by marketers of the durable. As we have stated earlier, diffusion processes for new successful consumer durables usually result in s-shaped growth patterns illustrating the growth of the share of owners (adopters) over time. When the growth of the owner share ends the market is said to be saturated. Usually there are consumers that never adopt, meaning that the saturation level is less than 1.

This description of an s-shaped growth curve ending up in a stable saturation level is, of course, a strong generalization. In reality all consumer populations change over time in various ways. Consumers get older, new consumers enter the market, some consumers die or move away, and so on. Furthermore, innumerable factors and circumstances, originating outside as well as inside consumers, continuously affect the adoption behaviour. This being the case, there is, of course, no such thing as a stable saturation level. The concept is, however, important in the sense that it, theoretically, implies when the actual diffusion process has come to an end.

We have distinguished between internal and external influences as the two key forces affecting the adoption behaviour of a consumer. We have specified internal influences as influences mainly originating through word-of-mouth communication taking place in social interactions between consumers or, more generally speaking, influences originating within a social system of consumers. The external influences, in turn, are specified as all those influences originating outside the social system of consumers usually aimed at speeding up the adoption processes of consumers within the social system. These two types of influences affect the adoption behaviour of consumers simultaneously over time. This is the case when a new durable penetrates a market but also after the market has been penetrated or, so to say, is saturated. For example, word-of-mouth communication concerning products takes place regardless of whether a product is new or old in the marketplace. Such communication does not disappear when a durable has penetrated a market and the share of owners is more or less stable. Internal influences, as well as external influences, on a saturated market affect newcomers on that market who have not yet adopted, but also possibly non-owners who so far have been inactive as to the durable. Finally internal and external influences on saturated markets also affect the replacement behaviour of owners, i.e. earlier adopters.

Let us for a moment return to the Bass model and Equation (1). In that equation it is assumed that the probability to adopt is affected by an ever-increasing pressure from owners or, in other words, an ever-increasing internal influence up to the point when the last potential non-owner adopts. After that point the pressure on non-owners
suddenly disappears. In our opinion a much more logic assumption would be that the overall influence stemming from word-of-mouth communication between owners and non-owners on the market is at its highest approximately at the same time as the rate of ownership growth is at its highest. The logic behind this assumption seems obvious. It is reasonable to assume that, when a non-owner meets an owner, the probability that the non-owner is exposed to word-of-mouth information concerning the durable in question at that particular meeting is higher the shorter the time since the owner became an owner is.\textsuperscript{14} Put another way, when a non-owner and an owner meet for the first time after the owner adopted it is much more probable that the owner tells about (and maybe even demonstrates) the recently bought durable than the next time they meet. It is reasonable to believe that such a willingness to talk about (and demonstrate) the durable decreases each time these two consumers meet again after the owner adopted. In the following we will call this assumption or phenomenon the decreasing word-of-mouth willingness. Combining this assumption with the fact that the total number of owners who recently have adopted is at its highest when the rate of ownership growth is at its highest, it can be concluded that exposure to word-of-mouth information concerning the durable in contacts with owners also is at its highest at approximately the same time. When the diffusion process gets closer and closer to the saturation level the exposure to word-of-mouth information concerning the durable in the defined sense also gets smaller and smaller, simply because the word-of-mouth willingness in most contacts at this time is low due to the fact that most owners have been owners of the durable for a considerable time.

We are, in other words, letting the internal influence act through word-of-mouth communication and visual signs appearing solely in contacts between non-owners and owners in the market place. We assume that these contacts are those that in particular are driving diffusion processes (Assumption 1). Furthermore we assume that non-owner exposure to information concerning a new durable in such contacts depends on the time since the owner became an owner (acquired the durable) or, more precisely, the longer this time is, the smaller is the probability that an exposure will take place (Assumption 2). Using these two assumptions when operationalizing the internal influence later on in this chapter gives us a specification that is narrower in scope than the usual definition of internal influence. According to the usual definition internal influence acts through all social interactions among consumers (cf. Lekvall and Wahlbin, 1973, p. 367).

Our way of specifying the internal influence makes any prespecification of a saturation level in our diffusion model unnecessary. The internal influence, as we specify it, ceases to exist as a diffusion driver at some point in time. When this happens the saturation level is reached. This can happen at any owner share level whatsoever. Where the saturated owner share level will lie depends on how successful the promoting forces have been in their “struggle” against the resisting forces of the consumers. If the forces resisting the durable have, in general, been strong it is likely that the share of owners on the whole market is relatively low when the market is saturated. If, on the other hand, the resistance has been weak the ultimate share of owners will probably be much higher.

\textsuperscript{14} The Bass model assumes that the influence exerted by owners on non-owners is independent of the time since the owner adopted, i.e. the probability of word-of-mouth communication in a contact is the same regardless of whether e.g. the owner adopted yesterday or five years ago.
It goes without saying that the forces driving the diffusion process cannot depend on the saturation level. Such a dependence is, however, implicitly inherent in many growth curve models, since the saturation level in these models functions as a “ceiling of growth”. Actually it is this “ceiling” that slows down the growth and not a decreasing internal influence. In other words, the causality goes, so to say, in the wrong direction. The saturation level should, naturally, be dependent on the internal influences and not the other way around.

So far we have stated the following. The core of our diffusion model is the conceptual framework for adopting new consumer durables on an individual level put forward in the previous section and summed up in the counteractive adoption model, cf. Figure 2. One of the concepts of this model is internal influence. In this section we have argued in favour of the existence of a decreasing word of mouth willingness in repeated contacts between a specific owner and a specific non-owner. Such an assumption, easy to agree with, makes any specification of saturation levels *ex ante* unnecessary, which will be demonstrated later on. This has, among other things, far-reaching consequences as it comes to forecasting the growth of new consumer durables in early stages of their market penetration processes. This assumption represents a first step towards an operationalization of internal influences in our model. However, to be able to complete our intentions we must, as a next step, specify interpersonal contact behaviours within our model setting. This will be dealt with in the following section.

**Interpersonal contact behaviour**

Every diffusion model contains, explicitly or implicitly, assumptions about interpersonal contact behaviour within a social system. How explicit and how detailed such assumptions are depends, among other things, on the aggregation level of the adoption behaviour modeled. Building an aggregate consumer behaviour model means that we are modeling the behaviour of aggregates of consumers. An aggregate consisting of all consumers on a market represents the highest aggregation level. The other extreme is a model in which each consumer is modeled separately. It is obvious that an aggregated diffusion model on the highest level cannot incorporate specifications allowing for differentiated interpersonal contact behaviours simply because all consumers are dealt with equally in such a model. If, however, each consumer is dealt with individually no restrictions as to specifying interpersonal contact behaviours, at least in principle, exist. The drawback of such a model is, however, that the model gets very complicated and very difficult to handle very quickly when behavioural details are added.

Traditional growth curve models represent highly aggregated models describing the growth of consumer durables by means of, usually, one single growth curve equation. In such models the underlying assumption is that interpersonal contact behaviour is random in the marketplace, i.e. the probability that a consumer will be in contact with a specific other consumer is independent of who this other consumer is. Actually the interpersonal contact behaviour among all consumers within an aggregate is supposed to be perfect in the sense that “everyone meets everyone all the time”. Perfect interpersonal communication is, in other words, supposed to prevail. To diverge from
this assumption is more or less impossible in such a model setting. In other words, we have to live with this assumption if we base our diffusion model on a single growth curve.

One way of allowing for differentiated contact behaviours in a diffusion model and, at the same time, keeping the overall model relatively simple is to use a segmentation approach (Lerviks, 1973, 1976a, 1976b, and 1984). This can be done by dividing the market into a number of mutually exclusive aggregates or segments of consumers according to, for example, age or income. The contact behaviours are then considered as behaviours of these aggregates of consumers. These behaviours are expressed in terms of probabilities or expected values. Using this approach contact behaviours within and between segments can be differentiated in the model by specifying contact probabilities within and between the predefined segments. We can, for example, within such a model frame let consumers from one specific segment be in contact with consumers from their own segment with a greater probability than with consumers from other segments. Still, however, the underlying assumption is that the contact behaviour of a consumer within a segment is random.

The above approach is used in our diffusion model. The fact that the segments in this approach are connected with each other through interpersonal contacts means that the diffusion processes generated within the model, one for each segment, are dependent on each other. In other words, the adoption behaviour in each segment affects and is affected by the adoption behaviours in all other segments through interpersonal communication taking place between the segments. This interdependence between the segments has some consequences as to the further development of our basic model structure.

**Basic model structure**

As to the model structure we obviously need a simulation approach to handle the interconnected diffusion processes of various segments. We have chosen an approach in which the adoption behaviours of the segments are simulated in a step-by-step (period-by-period) manner letting simulated behaviours in each step affect simulated behaviours in succeeding steps. This means that our approach is recursive. We are not allowing for simultaneous interdependencies between segments which would complicate the model considerably. Since our objective is to make the model manageable and usable as a forecasting tool we also use, besides the recursive model structure, a deterministic simulation approach instead of a stochastic one. In other words, our model can be characterized as a deterministic and recursive simulation model with a constant time unit.

The simulation model works, generally speaking, in the following way. The simulated probability to adopt in a segment in time period $t$ is multiplied by the share of non-owners at the beginning of time period $t$ in that segment giving the share of adopters (new owners) in time period $t$. Adding the share of adopters (new owners) in time period $t$ to the share of owners at the beginning of time period $t$ gives the share of owners at the beginning of time period $t+1$. These calculations are repeated for each segment. Then, in the next simulation step, the procedure is repeated for time period
$t+1$, now, however, using simulated adoption probabilities for time period $t+1$ and the shares of non-owners at the beginning of time period $t+1$.

A model of this type can be called a semi-aggregated behavioural model in which the criterion variable stands for the adoption behaviour of aggregates (segments) of consumers, in our case the probability to adopt a new durable in time period $t$. We use, in other words, an expected value approach. In such an approach we express the modeled behaviour as a probability function. Equation (1) in the Bass model is an example of such a probability function. In our case we have one probability function for each aggregate or segment. The criterion variable of these functions, the probability to adopt in a segment, can be interpreted as an average or expected value of all individual probabilities to adopt within that segment. One can say, according to Rapoport (1957), that we in the interest of achieving a more manageable model are dealing only with the expected events at each step of our simulation instead of dealing with the entire distribution of all possible events. This means, that the expected share of adopters in each simulated step is calculated assuming that the expected share actually also adopted in the previous step. Models of this type have also been called aggregate flow models (see e.g. Urban, 1970). As mentioned above, we also use the expected value approach when we operationalize the contact behaviours within and between our segments. This also holds for exposure to mass media in each segment as well as the word-of-mouth willingness in personal contacts between non-owners and owners.

**The promoting forces operationalized**

Our model development has now reached the point when we are starting to tie our basic framework (Figure 2) as well as assumptions and specifications made so far more closely together into a mathematical model. Since we are dealing with a diffusion model it is natural to start with the core diffusion driver for most diffusion processes, i.e. the internal influence. From now on we also define the market as a number of mutually exclusive and collectively exhaustive segments. Parameters and variables specified and modeled use the subscripts $m$ and/or $n$ to imply segments. The total number of segments is $M$, while $m, n = 1, 2, \ldots, M$.

**Internal influence.** At first it should be stated that internal influence holds in it at least two elements. We can say that a consumer, on the one hand, is exposed to something and, on the other, that this exposure has (or has not) an effect on the consumer. In our context the consumer is a non-owner of a durable and belongs to a segment $m$. This non-owner is exposed to a certain amount of word-of-mouth information and/or visual signs concerning the durable in contacts with owners in his own segment as well as owners in other segments in time period $t$. Let us in the following call this amount $\text{INTEXP}_m(t)$. The possible effect this exposure has on the adoption behaviour of the non-owner is expressed by the parameter $a_m$. Then we can define internal influence on non-owners in segment $m$ in time period $t$ in the following way.

\begin{equation}
\text{Internal influence}_m(t) = a_m \text{INTEXP}_m(t)
\end{equation}
The parameter \( a_m \) expresses, in other words, how strongly the internal exposure influences the adoption behaviour in segment \( m \). We are calling this parameter the \textit{coefficient of internal influence}. 

We have argued above that the core driving force behind most diffusion processes works through interpersonal contacts taking place between owners and non-owners in the market place. We have assumed that the willingness to discuss a durable in such contacts decreases when the time since the owner of the durable became an owner increases. We will now, starting from that discussion, move on into an explicit operationalization of the internal exposure variable, \( \text{INTEXP}_m(t) \).

A situation in which a non-owner of a durable is exposed to information concerning the durable in a personal contact with an owner from a specific segment can be said to take place when and only when the following three events, expressed by \( E_1, E_2 \) and \( E_3 \), occur simultaneously.

- \( E_1 = \) The consumer that the non-owner meets belongs to a specific segment.
- \( E_2 = \) The consumer that the non-owner meets is an owner of the durable.
- \( E_3 = \) The durable will be discussed in that meeting.

Note that we are assuming that a personal contact is taking place. In other words, given a personal contact, our three events represent conditions which have to be fulfilled simultaneously for an exposure of the defined type to take place in that contact.

The probability that these three events will occur simultaneously in a personal contact can be expressed as a product of three probabilities in the following way.

\[
P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2|E_1) \cdot P(E_3|E_1 \cap E_2)
\]

Of these probabilities \( P(E_1) \) is the probability that \( E_1 \) will occur, \( P(E_2|E_1) \) the probability that \( E_2 \) will occur, given that \( E_1 \) occurs simultaneously, and \( P(E_3|E_1 \cap E_2) \) the probability that \( E_3 \) will occur, given that \( E_1 \) and \( E_2 \) occur simultaneously. Each of these probabilities has a distinct meaning of its own. The approach, so to say, separates the contact network from the message communicated within that network. The first event represents the general contact network (contact probabilities within and between the segments), the second identifies the type of consumer contacted (an owner), and the third specifies whether a certain durable is discussed in a contact between a non-owner and an owner of that durable. Using this approach it is rather easy to express two of these probabilities (the second and third) solely as functions of lagged endogeneous variables of the model, i.e. owner shares in segments and changes in these owner shares from time to time. The first probability, in turn, enters the model exogenously.

We will denote the probabilities for each one of these three events to occur by \( P_{mn}, Y_n(t), \) and \( D_{mn}(t) \), which are equivalent to the expressions \( P(E_1), P(E_2|E_1), \) and \( P(E_3|E_1 \cap E_2) \) in Equation (4). The product of these three probabilities for a given segment \( m \) and a given segment \( n \) represents the probability that a non-owner in segment \( m \) is exposed to information concerning the durable in a contact with an
owner in segment $n$. By keeping $m$ fixed and letting $n$ vary ($n = 1, 2, \ldots, M$), we can calculate $M$ such probabilities. One of these represents the probability that the non-owner is exposed to information concerning the durable in a contact with an owner in the own segment while the others represent probabilities for such exposures in a contact with an owner in each of the other segments. The sum of these $M$ probabilities represents the overall probability that a non-owner in segment $m$ will be exposed to information concerning the durable in a personal contact with an owner on the market, cf. Equation (5). This overall probability cannot exceed 1 since none of the three probabilities is, according to our model specifications, allowed to exceed 1 and, furthermore, the sum of $P_{mn}$ over $n$ ($n = 1, 2, \ldots, M$) always is equal to one. This is so because $P_{mn}$ expresses the probabilities that a consumer contacted belongs to a specific segment $n$ ($n = 1, 2, \ldots, M$), given that a contact takes place.

Finally, by multiplying the overall probability that a non-owner in segment $m$ will be exposed to information concerning the durable in a personal contact with an owner on the market by the number of personal contacts a consumer in segment $m$ on average experiences per time unit, $C_m$, we get the expected number of contacts in which a non-owner in segment $m$ is exposed to information concerning the durable, i.e. the internal exposure variable. In other words, we have the following expression.

\begin{equation}
\text{INTEXP}_m(t) = C_m \sum_{n=1}^{M} P_{mn} Y_n(t) D_{mn}(t)
\end{equation}

In Equation (5) the general contact network is represented by $C_m$ and $P_{mn}$. As was pointed out above, $C_m$ stands for the number of contacts a consumer in segment $m$ on average experiences per time unit. $P_{mn}$, in turn, represents the first event above, $E_1$, expressing the probability that a consumer contacted belongs to segment $n$, i.e. the contact probabilities within and between all segments. The average or expected number of contacts a consumer has with consumers in various segments can then be calculated by multiplying $C_m$ by $P_{mn}$ for all possible values of $n$ ($n = 1, 2, \ldots, M$). The sum of these products is naturally equal to $C_m$. The two parameters of the contact network, $C_m$ and $P_{mn}$, enter the model exogenously. As such they have to be estimated separately. From Equation (5) it can be seen that we are using constant values for these two parameters. The reason for this is twofold. Firstly, we want to avoid too laborious empirical measurements of the contact behaviours in order to keep the model easy to apply. Secondly, we believe that the type of contact behaviours we are dealing with often can be regarded as rather stable over time. As long as applications do not cover extremely long time intervals, we believe that contact behaviours measured at some point in time not too far away from the actual time interval can serve as reasonable approximations to be used in applications.

The next probability concerns the second event above, $E_2$. Here we have to specify the probability that a consumer contacted in segment $n$ is an owner of the durable in question. Falling back on our basic assumption of random individual contact behaviours within segments it is reasonable to assume that the probability that a consumer contacted in segment $n$ in time period $t$ is an owner coincides approximately with the share of owners in segment $n$ at that time. The share of owners
in segment \( n \) at the beginning of time period \( t \) is denoted by \( Y_n(t) \) and serves in Equation (5) as an approximation of the probability that a consumer contacted in segment \( n \) is an owner of the durable. Under these assumptions the probability that a consumer contacted in time period \( t \) is an owner in segment \( n \) can be expressed as \( P_{mn}Y_n(t) \). The total number of such contacts in time period \( t \) can, in turn, be expressed as \( C_mP_{mn}Y_n(t) \).

Finally we have the probability that the third event above, \( E3 \), will occur in a contact with an owner. In other words, we are talking about the probability to discuss the durable in such a contact. We have denoted this probability \( D_{mn}(t) \) in Equation (5). It is no overstatement to say that a sound and trustworthy derivation of such a probability from empirical observations is more or less impossible to achieve. Obviously we have to lean on theoretical assumptions and reasoning in specifying this probability. Such reasoning led us above to conclude that it seems likely that the probability to discuss a durable in a contact between a non-owner and an owner of the durable is dependent on how recently the owner became an owner. If the owner has acquired the durable recently the probability that the durable will be discussed in a contact is supposed to be higher than if the owner has been an owner for some time. We have called this phenomenon or assumption the decreasing word-of-mouth willingness. This assumption is built into our specification of \( D_{mn}(t) \).

In specifying \( D_{mn}(t) \) we assume that the probability to discuss the durable in a contact between a non-owner and an owner decreases exponentially with increasing time since the owner acquired the durable. We believe, in other words, that the probability to discuss the durable decreases at first quickly and by time slower and slower. It seems obvious to assume that the probability to discuss a new durable is at its highest when two consumers meet for the first time after one of them has acquired the durable. Furthermore, we find it reasonable to assume that the probability to discuss the durable the next time these two specific consumers meet will lie on a lower level. Since, however, the durable still is perceived as “new” for some time, at least by the owner, the durable will probably be discussed now and then, however less and less as time goes by. These assumptions lead us to our choice of functional relationship, i.e. a decreasing exponential relationship, between, on the one hand, the probability to discuss the durable in a contact between a non-owner and an owner and, on the other, the time since the owner acquired the durable.

As can be seen from the above reasoning we are not trying to include all kinds of possible word-of-mouth exposures concerning a new durable into our model. Instead we are trying to capture those word-of-mouth exposures that we believe are crucial forces in driving diffusion processes among consumers. Such exposures are, to our belief, those that take place in personal contacts between non-owners and owners especially when the owners recently have become owners.

We can now specify the probability that a durable will be discussed in a personal contact between a non-owner in segment \( m \) and an owner in segment \( n \) in time period \( t \) in the following way.
\[ D_{mn}(t) = \sum_{k=1}^{K} \exp[-(k-1)d] \frac{\Delta Y_n(t-k)}{Y_n(t)} \]

Here \( \Delta Y_n(t-k) \) stands for the change in owner share in segment \( n \) in time period \( t-k \), i.e. \( k \) time periods ago. When this change is divided by the owner share in segment \( n \) at the beginning of time period \( t \), \( Y_n(t) \), we get the share of the owners in segment \( n \) that acquired the durable \( k \) time periods ago. This share is used as an approximation of the probability that the owner in a contact is one that became an owner \( k \) time periods ago. As can be seen from Equation (6) we have \( K \) such probabilities representing mutually exclusive and collectively exhaustive groups of owners, i.e. owners for \( 1, 2, ..., K \) time periods. The decreasing exponential function in Equation (6) serves, in turn, as an approximation of the probability that a new durable will be discussed in a contact with an owner that became an owner \( k \) time periods ago. By multiplying the probability that an owner in a contact is one that became an owner \( k \) time periods ago, \( \Delta Y_n(t-k)/Y_n(t) \), by the probability that the durable will be discussed when the owner has owned the durable \( k \) time periods, \( \exp[-(k-1)d] \), and summing up the products over \( k \) gives us the overall probability that the durable will be discussed in a contact between a non-owner in segment \( m \) and an owner in segment \( n \) in time period \( t \), \( D_{mn}(t) \). The probability \( D_{mn}(t) \) cannot exceed 1 since the expression \( \exp[-(k-1)d] \) is always equal to or smaller than 1, cf. Figure 3, and the sum of all \( \Delta Y_n(t-k) \) up to \( t \) cannot be greater than \( Y_n(t) \). Note that Equation (6) is slightly modified when \( t \leq K \). In such cases \( K \) in Equation (6) is replaced by \( t-1 \). This is the case in early stages of a diffusion process when the number of time periods since the durable was launched on the market is lower than or equal to \( K \). In the very first time period after launch, when \( t = 1 \), Equation (6) is reduced to the following form: \( D_{mn}(1) = Y_n(1) \).

We have, in other words, expressed the probability that a durable will be discussed in a personal contact between a non-owner in segment \( m \) and an owner in segment \( n \) in time period \( t \), \( D_{mn}(t) \), as a function of lagged endogenous variables of the model. This function holds in it two parameters, \( d \) and \( K \). Of these \( d \) regulates the rate at which the probability to discuss the durable decreases with increasing time since the durable was acquired. \( K \), in turn, expresses how many time periods the willingness to demonstrate and discuss a durable due to the fact that it was recently acquired by the owner prevails. In Figure 3 the exponentially decreasing probability to discuss the durable is demonstrated for three values of \( d \) and one value of \( K \) (\( K = 9 \)).

Let us for a moment look at the assumptions behind Equation (6) and compare these with the corresponding assumptions inherent in the Bass model. The main difference is that our model specifies a probability that a new durable will be discussed in a contact between a non-owner and an owner as a function of the time since the owner became an owner. In the Bass model (and most other growth curve models) such a probability is not explicitly specified but can be interpreted as being equal to 1 all the time. This is so because the share of owners, \( Y(T) \) in Equation (1) above, directly serves as an “exposure” (or pressure according to Bass) on non-owners (Bass, 1969). As we have pointed out earlier, such an ever-increasing pressure forces the model builder to incorporate some kind of saturation level in the model, a level that serves as a ceiling for growth, e.g. the parameter \( m \) in the Bass model. A more realistic
assumption is that the internal influence (pressure) decreases more and more when the diffusion process approaches a saturation level. This phenomenon in combination with the strength of the resistance of the consumers determines where the saturation level will lie.

**Figure 3. Decreasing probability to discuss the durable in contacts with owners**

![Graph showing decreasing probability to discuss the durable over time](image)

**External influences.** The other promoting force specified in our basic framework in Figure 2 is external influences. Like the internal influence the external influences also can be split into two elements. We can say that a non-owner is *exposed* to external signals concerning a new durable, for example advertising in mass media, and that such an exposure may have an *effect* on the adoption behaviour of the non-owner. We will call the amount of such external signals that a non-owner in segment $m$ is exposed to in time period $t$ external exposure and denote it by $EXTEXP_m(t)$. The external influence on non-owners in segment $m$ in time period $t$ can then, for each type of external signals, be specified in the following way.

\[
(7) \quad External\ influence_m(t) = b_m \cdot EXTEXP_m(t)
\]

The parameter $b_m$ expresses how strongly the external exposure of the specified type influences the probability to adopt in segment $m$. We are calling this parameter the *coefficient of external influence*.

A non-owner can externally be exposed to a lot of various types of signals concerning a new durable that can affect the adoption behaviour. Such signals can, for example, be advertising for the durable in different media, sales promotions of various types, price changes and special offerings, writings in newspapers and magazines about the durable, and so on. It is, however, not our intention in this study to explicitly specify several such possible signals. We will instead use exposure to advertising in mass media as an illustration in the following. The external exposure, $EXTEXP_m(t)$, in Equation (7) could represent some other type of signal as well. Then the specification
of the external exposure variable would probably be different from the one we will present for advertising in mass media. Within our model frame it is also possible to let several external exposure variables representing various signals affect the probability to adopt simultaneously. We would then have several coefficients of external influence, one for each type of external signal.

In our specification of $EXTEXP_m(t)$ we distinguish between, on one hand, the average or expected exposure to certain mass media in segment $m$, $EM_m$, and, on the other, the amount of advertising for the durable in these media in time period $t$, $A(t)$. Hence we get the following expression.

$$EXTEXP_m(t) = EM_m A(t)$$

In applying our model in Chapter 5 we assume that the expected exposure to mass media within each segment, $EM_m$, is rather stable over time. Hence we use empirical observations on $EM_m$ made at one point in time as approximations for the whole period of analysis. When we use our model in ex post analyses the advertising volume, $A(t)$, represents exogenously made observations on historical time series of advertising volumes in mass media. If we use our model as a forecasting tool the advertising variable represents future advertising volumes in mass media and has, in a first step, to be separately forecasted before the variable can be used as input in forecasting the diffusion process.

**Bringing the pieces together**

After having operationalized the promoting forces of our model we can start to bring the pieces dealt with so far together. This is done by specifying the probability to adopt a new durable as a function of the promoting forces. We specify the following general and simple relationship.

$$Adoption probability_m(t) = Internal influence_m(t) + External influence_m(t)$$

As can be seen we assume that the two types of influences are additive. We are, in other words, not allowing for interactions between the influences, which of course can be regarded as a considerable simplification. However, bearing the purpose of our study in mind we choose to proceed in this way. We specify the probability to adopt in segment $m$ in time period $t$ as a linear function of the internal exposure in segment $m$ in time period $t$ and the external exposure in segment $m$ in time period $t$, i.e.

$$AP_m(t) = a_m INTEXP_m(t) + b_m EXTEXP_m(t)$$

An alternative and theoretically more sound solution would have been, for example, to choose a logit transformation in Equation (10). This would obviously have been our solution if our intention had been to estimate the parameters $a_m$ and $b_m$ statistically on the basis of a sufficient number of observations on the criterion and the
predictor variables.\textsuperscript{15} Our primary purpose here is, however, to achieve estimates of $a_m$ and $b_m$ as soon as possible after the launch of a new durable on a market. To enable this we have developed an optimization algorithm that needs very few sales observations in order to produce the estimates. The quality of the estimates will, naturally, improve the more time periods our sales observations cover. The algorithm is still in its first development state. In developing the algorithm it felt natural to start from a rather simple functional form like the one in Equation (10), implicitly assuming that the probability to adopt is linearly dependent on internal and external exposures within reasonable intervals.

Although the resisting forces are not explicitly specified in Equation (10) they are, so to say, implicitly present. When we allow for solely internal exposures to affect the adoption probability in Equation (10) the estimates of the coefficient of internal influence, $a_m$, can be said to represent the net effect of the counteracting promoting and resisting forces. This can, of course, be stated about any single equation growth curve model allowing for solely internal influences to affect the adoption behaviour as well. In the present case we have, however, several estimates of the coefficient of internal influence, $a_m$, one for each segment. Furthermore we have expected values of internal exposures in each segment over time, $INEXP_m(t)$. The product of this coefficient and the internal exposure represents the internal influence. Under these conditions the coefficient $a_m$ can be interpreted as a measure of \textit{how much the adoption probability in segment $m$ is changed when the internal exposure is changed with one unit}. Since the measurement unit of the internal exposure is the same for each segment the estimated coefficients of internal influence, $a_m$, are directly comparable among themselves. We can, in other words, state that our estimated coefficients of internal influence measure \textit{how willing consumers in various segments are to adopt under equal social pressure}.

Our interpretation is then that the coefficient $a_m$ serves as an indicator of resistance. If, for example, the estimated value of this coefficient is smaller in one segment than in another, then the resistance has been greater in that segment than in the other segment. In other words, low estimates of $a_m$ correspond to high resistance and high estimates to low resistance. Since the internal exposure can vary considerably between segments due to, among other things, different contact behaviours it is possible that a new durable penetrates a segment with relatively high average resistance (a low $a_m$-value) quicker than a segment with lower resistance (a higher $a_m$-value) simply because the internal exposure was much higher in the first segment. Such a situation is, however, probably more an exception than a rule. The point is, however, that the speed by which a segment is penetrated is not necessarily a very reliable indicator of the level of resistance in that segment as compared to other segments. A much more reliable indicator is the coefficient of internal influence, $a_m$, since this indicator measures the effect \textit{per unit of internal exposure}. As to the resistance concept inherent in our basic framework we can conclude that our model produces estimates of $a_m$ that can be regarded as indicators of that resistance. However, we cannot measure the resistance as such, but we can, thanks to our

\textsuperscript{15} Such estimations using a logit transformation based on a similar type of model can be found in Lerviks (1984).
segmentation approach, get an understanding of how the resistance differs between segments. In this context it is, however, important to stress that the reasoning above only holds when we are applying our model as a pure diffusion model, i.e. the case when we only allow for internal exposures to affect the adoption probability. This is so because it can be argued that all influences on the adoption probability are, so to say, summed up in one parameter estimate, $a_m$, when we are applying the pure diffusion model. In this case the $a_m$-estimates are, in a way, comparable between segments. Such comparisons are, however, not meaningful any more when our applications simultaneously produce estimates of internal as well as external influences. This is obvious when we look at our empirical findings in Chapter 5, cf. Table 12 as compared to Table 7.

The flexibility of our diffusion model should also be stressed. By keeping $a_m = 0$ we are only allowing external influences to affect the probability to adopt through $b_m$ resulting in diffusion curves of an exponential type (cf. Fourt and Woodlock, 1960). Note that when $a_m = 0$ the diffusion curves in various segments are independent of each other. By, on the contrary, keeping $b_m = 0$ we are only allowing for internal influences to affect the probability to adopt through $a_m$ resulting in s-shaped diffusion curves in each segment (cf. Mansfield, 1961), each diffusion curve being dependent on the others. Finally, by allowing both internal and external influences to affect the probability to adopt we get a great variety of differently shaped diffusion curves due to the values of $a_m$ and $b_m$. Using an analytical growth curve model Lekvall and Wahlbin (1973) illustrate how various combinations of internal and external influences affect the shape of the growth curve. The shape varies from a logistic curve when only internal influences are allowed for to a modified exponential curve when only external influences are allowed for. In our model all such combinations of internal and external influences are possible. One further dimension that enters our model is that the influence parameters can differ between segments leading to different shapes of the growth curves in different segments.

An important quality of our diffusion model is that no saturation level whatsoever needs to be specified within the model. The owner share levels at which a durable has penetrated the various segments depend on how well the promoting forces have succeeded in overcoming the resisting forces of the consumers in these segments. How well our simulation model can predict these saturation levels depends, among other things, on how well we have succeeded in specifying the promoting forces, especially the internal influences, in our model. On this point our approach diverges strongly from approaches used in most traditional growth curve models. We are striving at identifying those internal exposures to information concerning a new durable that we believe are crucial as diffusion drivers, i.e. such word-of-mouth information concerning the durable that is generated in contacts between non-owners and owners that recently have become owners. Our basic assumption is that when such an internal influence ceases to exist the market will be saturated.

The fact that a diffusion takes place among consumers on a market is, according to our view, a result of an imbalance arising between forces promoting the adoption of a new durable and forces resisting such an adoption. For such a diffusion driving imbalance to appear the durable has to have such qualities that most, or at least many,
consumers sooner or later will acquire the durable. In such cases the promoting forces will overcome the resisting forces of consumers, first in a few cases, then in more and more cases, then again in fewer and fewer cases until the market is saturated. At this point the imbalance has, so to say, disappeared. Our two counteracting forces are then roughly in balance. We have reached a state of equilibrium, the saturation level. However, as long as the durable diffuses the market an imbalance between our two forces are prevailing. The stronger this imbalance is in favour of the promoting forces, the faster the durable will penetrate the market and the higher the final saturation level generally will lie. An imbalance in the other direction, i.e. situations in which the promoting forces do not overcome the resisting forces of most consumers, means that very few consumers will adopt the durable.

To end up this chapter we summarize our diffusion model in Table 1. The first four equations are equivalent to Equations (10), (5), (6), and (8) above. In using the model as a simulator the output from each simulated step is the probability to adopt the durable in each of a given number of predefined market segments. Due to the recursive structure of the model the adoption probabilities of each simulated step affect all subsequent adoption probabilities throughout the whole simulation. Technically this recursivity works through the changes in owner shares in previous time periods in each segment in combination with the interpersonal contact behaviour within and between segments as well as how quickly the word-of-mouth willingness decreases after an acquisition, cf. Equations (5) and (6). Note that \( K \) in Equation (6) is replaced by \( t-1 \) when \( t \leq K \). When \( t = 1 \) Equation (6) has the form \( D_{mn}(1) = Y_{n}(1) \).

Simulations carried out with our diffusion model start from the end of the time period in which the durable was launched on the market. The first simulated period is then the second period the durable is available on the market. To make a simulation including only internal influences we need initial values of the owner shares in each segment, \( Y_{m}(1) \), estimates of the parameters of the model, i.e. the contact parameters, \( P_{mn} \) and \( C_{m} \), the word-of-mouth parameters, \( d \) and \( K \), and the coefficient of internal influence, \( a_{m} \). If we also incorporate advertising in our simulation, as specified above, we further need a time series for advertising volumes covering the simulated time period, \( A(t) \), as well as estimates of the parameters \( E_{m} \) and \( b_{m} \).

The criterion variable of the diffusion model is the probability to adopt a new durable in a number of predefined market segments, \( AP_{m}(t) \). This probability is calculated (simulated) using Equations (6), (5), (8), and (10). When the adoption probabilities \( AP_{m}(t) \) are multiplied by the shares (fractions) of non-owners at the beginning of time period \( t, 1 - Y_{m}(t) \), we get the changes in owner shares in time period \( t, \Delta Y_{m}(t) \). Adding these changes to the owner shares at the beginning of time period \( t, Y_{m}(t) + \Delta Y_{m}(t) \), gives us the owner shares at the beginning of the next time period \( t + 1 \), i.e. \( Y_{m}(t+1) \). In the next simulation step these procedures are repeated, but now for time period \( t + 1 \). This process goes on until all time periods, \( T \), have been simulated. The simulated owner shares over time, \( Y_{m}(t) \), constitute the simulated diffusion or growth curves for various segments and the changes in these owner shares, \( \Delta Y_{m}(t) \), the simulated speed of growth. The speed of growth tells us at what time sales to non-owners will peak in each segment according to the simulation. If a simulation covers a
sufficient number of time periods, $T$, the growth curves also reveal simulated saturation levels.

**Table 1. The diffusion model summarized**

\begin{equation}
AP_m(t) = a_m \text{INTEXP}_m(t) + b_m \text{EXTEXP}_m(t)
\end{equation}

\begin{equation}
\text{INTEXP}_m(t) = C_m \sum_{n=1}^{M} P_{mn} Y_n(t) D_{mn}(t)
\end{equation}

\begin{equation}
D_{mn}(t) = \sum_{k=1}^{K} \exp[-(k-1)d] \Delta Y_n(t-k)/Y_n(t)
\end{equation}

\begin{equation}
\text{EXTEXP}_m(t) = EM_m A(t)
\end{equation}

where

$m,n = 1,2,...,M$

$k = 1,2,...,K$

$t = 1,2,...,T$

\[
\sum_{n=1}^{M} P_{mn} = 1
\]

\[
\sum_{k=1}^{K} \Delta Y_n(t-k)/Y_n(t) \leq 1
\]

\[
\Delta Y_n(t) = AP_n(t)[1 - Y_n(t)]
\]

\[
Y_n(t+1) = Y_n(t) + \Delta Y_n(t)
\]

and

$AP_m(t)$ = probability to adopt the durable in segment $m$ in time period $t$

$a_m$ = coefficient of internal influence in segment $m$

$\text{INTEXP}_m(t)$ = expected exposure in segment $m$ to information concerning the durable in personal contacts with owners in time period $t$

$b_m$ = coefficient of external influence in segment $m$
The diffusion model, as specified above, does not simulate sales volumes over time simply because the sizes of the segments have still not been incorporated in the model. The sizes of segments enter our model building process in Chapter 4 when we bring different parts together into an overall demand simulator.
Chapter 3

THE REPLACEMENT MODEL

Research on consumer durable replacements

A great deal of research on replacement demand for consumer durables has been done through the years. In spite of this there seems, however, to be a lack of practical forecasting models in this area reported on in the literature. For example, the book on marketing models by Lilien, Kotler and Moorthy (1992) touches this area only slightly although repeat-purchase models for non-durables are thoroughly dealt with. A contributory cause for this could be that the demand situation when it comes to durable replacements differs substantially from demand situations as described and modeled in traditional demand theory. As a consequence of this replacement demand modeling cannot, at least directly, be based on traditional demand theory. The usually long time span between acquiring a durable and replacing it also creates troublesome and laborious data collection procedures in order to enable validation of such models. Furthermore, the existence of second-hand markets for most consumer durables does add to the complexity of modeling accurate replacement processes.

A characteristic feature of durable replacement demand is an element of timing not present in traditional demand theory. This element concerns the time that elapses between acquisition and replacement, i.e. the age of the durable when replaced. One could say that every durable sooner or later reaches a state of irreparable failure. Then it has to be replaced if the consumer wants to go on using the durable. Most durables are, however, replaced (long) before such an irreparable failure state is reached. It can be said that, when a durable gets older, the consumer will be more and more strongly faced with the question of when the durable should be replaced. We have, in other words, a situation in which the timing of the replacement act in relation to the time of acquisition is the actual phenomenon that constitutes the essence of durable replacement demand. What we need to understand better as researchers is how the process leading to a replacement decision works for various durables and for various types of consumers, what forces or events do usually have an impact on this process, either slowing it down or speeding it up, and so on.

Earlier research on replacement demand can be said to fall into at least two main categories. We have, on the one hand, research mainly of a descriptive type estimating removal rates, survival rates and service-life expectancies of durables and, on the other, research more of an explanatory type striving to explain and understand why some consumers are replacing a durable sooner (or later) than others or, more
generally speaking, to understand all the forces and circumstances that regulate this behaviour.

Research of a descriptive type concerning life expectancy of industrial equipment and durables was practised already in the first half of the twentieth century (see e.g. Winfrey and Kurtz, 1931, de Wolff, 1938, Barfod, 1945, and Altman and Goor, 1946). The approaches used then stemmed from population statistics, more precisely from how life expectancies of people born in various years were calculated. The approach has been called the actuarial method. The core of this method is to calculate (estimate) independent removal rates for each age interval throughout the maximal life span of, for example, a durable. If we know the number of units acquired of the durable in time period $t$ and the number of these units replaced in each time period $t + x$ ($x = 1, 2, \ldots, X$), we can estimate the removal rates and then calculate the survival rates and finally the expected service-life of those units that were originally acquired in time period $t$ (Pennock and Jaeger, 1957). The main purpose of this approach is, in other words, to estimate the expected service-life of various durables and also, by estimating the expected service-life of the same durable at different times, to find out whether the service-life is changing over time. Numerous such studies have been made in most western countries. In the Nordic countries such research has, for example, been backed up by the Nordic Council of Ministers and covers each of the five countries (Dahl, 1977 and 1980).

As it comes to research trying to explain why replacement timing varies between consumers as well as over time a lot of studies can be found in the literature. We will here refer to only a few, the findings of which can be considered to have bearing on our model approach. It has, for example, been shown that durables are replaced sooner if the household has a higher income, if the head of the household is younger, and if the children still live in the household. Also attitudinal profiles have been found to have impact on the timing of replacements as well as information seeking and usage of mass media (Tippett, Magrabi and Gray, 1978, and Bayus, 1991 and 1993). Other studies have reported similar findings, especially when it comes to the age of the household head and the household income (see e.g. Box, 1981). It has, however, also been shown that reasons affecting replacement timing differ, sometimes even considerably, between various durables. Durable-specific characteristics are, in other words, also important in trying to understand replacement timing (see e.g. Box, 1981, and Bayus and Mehta, 1995). An increasing speed of product development, typical of our time, has furthermore brought about shorter and shorter lifecycles (earlier and earlier replacements) for many durables.

In analysing the impact of various factors on replacement timing different types of statistical models have been used. Since the criterion variable on the individual level usually can be considered as a binary variable (replace or not replace) various types of limited dependent variable techniques like probit and logit models have been utilized (see e.g. Cragg, 1971, and Deaton and Muellbauer, 1985). A more recent development is to use so called duration models or hazard models in which the impact of various factors on the length of time until a discrete event, for example a replacement, occurs is analysed (see e.g. Raymond, Beard and Gropper, 1993, Chandrashekaran and

16 $X$ stands for the maximal life span of the durable in question.
Sinha, 1995, and Leeflang, Wittink, Wedel and Naert, 2000). We do not, however, go further into these and other approaches here, as they fall outside the primary purpose of our study.

**A simple replacement mechanism**

The replacement model that will be described in the following is not intended as a free-standing model. Rather the model should be looked upon as an extension of the diffusion model presented above. It is, contrary to the diffusion model, a simple descriptive model in the sense that it does not try to explain variations in replacement timing between consumers or over time. It uses an actuarial approach, but the other way around, meaning that it starts from a given replacement distribution whose parameter-values, estimated in one way or another, are the inputs of the model. On the basis of these parameter-values survival rates are calculated and finally replacement probabilities for durables of various age. This is done separately for each segment in the model. We are, in other words, modeling the replacement behaviour of aggregates (segments) of consumers using a similar expected value approach as in the diffusion model dealt with in the previous chapter.

According to research on product replacements the probability to replace a durable should be increasing with growing age of the durable up to the point when this probability equals one (see e.g. Bayus and Mehta, 1995). Consequently, when choosing a specific replacement distribution this condition should be fulfilled. Among possible distributions we have the Weibull and the Gamma distributions, which for certain values of the parameters have increasing failure rates (Cox, 1962). Bayus (1991 and 1993) has achieved excellent fits between the Weibull distribution and aggregate durable replacement behaviour. Similar results concerning the Gamma distribution has been achieved by Oomens (1974).

As the core of our replacement model we use the **Weibull distribution** (see e.g. Stuart and Ord, 1987, pp. 181-182, and Dudewicz and Mishra, 1988, pp. 151-153). The Weibull distribution can take the form of a decreasing failure rate distribution as well as an increasing failure rate distribution depending on the value of one of the parameters of the distribution. The survival rate, i.e. the proportion of all units acquired in a specific time period that are still in use \( x \) time periods later, is expressed as follows.

\[
SR_m(x) = \exp\left[-\left(\frac{x}{g_m}\right)^{sm}ight]
\]

In our specification the survival rate, \( SR_m(x) \), is allowed to differ between segments by permitting the values of each of the two parameters of the expression to vary between segments. Of these parameters \( g_m \) stands for the expected service-life (mean age) of the durable and \( s_m \), in turn, determines the slope of the survival curve. For \( s_m \)-values greater than one the expression delivers increasing failure rates when \( x \) is growing, for \( s_m \)-values equal to one the failure rate is constant and for \( s_m \)-values

---

\( ^{17} \) This probability is sometimes also called *failure rate, hazard rate or force of mortality* (Dudewicz and Mishra, 1988, p. 151).
smaller than one the failure rates are decreasing (Dudewicz and Mishra, 1988, p. 152). As was pointed out earlier we are interested in $s_m$-values greater than one in what case the probability to replace a durable in use (the failure rate) is increasing with increasing age of the durable. In Figure 4 the survival curve is illustrated for some values of the two parameters.

**Figure 4. Survival curves**

![Survival curves graph](image)

Due to our segmentation approach we can in an application let the expected service-life of a durable, $g_m$, be, for example, lower in a high-income segment than in a low-income segment, hereby taking into account research findings on consumers’ durable replacement behaviour of the types referred to above.

The probability to replace a durable of age $x$ (the failure rate) is finally calculated from the survival rates in the following way.

(12) $RP_m(x) = (SR_m(x) - SR_m(x+1))/SR_m(x)$

This probability is increasing with increasing $x$ when the parameter $s_m$ in Equation (11) is greater than one.

Our replacement mechanism follows a similar expected value approach and uses the same segments as the diffusion model. Actually the replacement model is an extension of the diffusion model working side by side with the diffusion model in simulation runs. The overall demand simulator, more closely described in Chapter 4, updates information concerning the number of units in use in each segment including their age distribution after each simulated step. By multiplying the probability to replace units of age $x$ in segment $m$ by the number of units of age $x$ in segment $m$ the number of replaced units of age $x$ in segment $m$ is simulated for each time period.
The replacement model summarized

We end this chapter by summarizing the replacement model in Table 2. The two equations in the table are equivalent to Equations (12) and (11) above. Estimates of two parameters are needed when applying the model in simulation runs, i.e. the expected service-life of the durable in each segment, $g_m$, and the slope of the survival curve in each segment, $s_m$.

Table 2. The replacement model summarized

\[
(12) \quad RP_m(x) = \frac{[SR_m(x) - SR_m(x+1)]}{SR_m(x)}
\]
\[
(11) \quad SR_m(x) = \exp\left(-\frac{x}{g_m} s_m^m\right)
\]

where

\[
m = 1, 2, \ldots, M \\
x = 1, 2, \ldots, X \\
s_m > 1
\]

and

\[
RP_m(x) = \text{probability to replace a unit of the durable having been in use } x \text{ time periods in segment } m, \text{ i.e. the failure rate}
\]
\[
SR_m(x) = \text{proportion of all units acquired in a specific time period that are still in use } x \text{ time periods later in segment } m, \text{ i.e. the survival rate}
\]
\[
g_m = \text{expected service-life of the durable in segment } m
\]
\[
s_m = \text{parameter regulating the slope of the survival curve in segment } m
\]
\[
M = \text{number of segments}
\]
\[
X = \text{maximal life span of the durable expressed as number of time periods}
\]

It should be stressed that the replacement mechanism proposed here does not, in its present shaping, allow for any exogenous variables to affect the replacement behaviour. This means that only influences due to how long the durable has been in use since it was acquired affect replacement sales through the expected service-life of the durable specified by the parameter $g_m$. As can be seen this parameter is supposed
to be unchanged over time in our model setting. We have called these influences on replacement demand service-life influences above. Under these conditions the replacement model represents a kind of baseline replacement simulator, i.e. a simulator that forecasts replacement demand under conditions of no external influences. Furthermore the present model assumes that a consumer having adopted the durable once sticks to it after that and, when the time comes, replaces it.
Chapter 4

DEMSIM - A DEMAND SIMULATOR

Structure of DEMSIM

Our demand simulator DEMSIM consists of the two models presented above, i.e. the diffusion model generating the probability to adopt a new durable and the replacement model generating the probability to replace a unit in use of that durable. The first model simulates, in other words, the adoption behaviour of non-owners while the second model simulates the replacement behaviour of owners. These behaviours are simulated for each one of the predefined market segments. In the diffusion model, as explained above, the behaviour of each segment is affected by the behaviours of all segments in previous time periods through word-of-mouth influences in interpersonal contacts taking place within and between the segments creating an endogenous diffusion mechanism. Hence the diffusion model is explanatory in character while the replacement model, only allowing for service-life influences, can be characterized as descriptive.

So far we have only dealt with behavioural probabilities, owner shares and changes in owner shares as outcomes of our simulations. To simulate demand or sales volumes of a new durable we have to incorporate an exogenous variable representing the size of each market segment into our model. After this the different parts of DEMSIM can be drawn together. This is done in Figure 5.

Figure 5 is simplified in order to give an overall view of the model structure and of how the different parts relate to each other. The structure is identical for each segment. The upper part of Figure 5 contains the two models presented in Chapters 2 and 3. Only within the diffusion model interdependencies between segments are present. The accounting identities in the middle of Figure 5 transform the probabilities generated in the diffusion model and the replacement model into simulated output using the number of consumers in each segment (exogenously given) split into non-owners and various owner categories. The owner categories represent owners that acquired the durable 1 time period ago, 2 periods ago, and so on, up to X periods ago. The sizes of these categories constitute the age distribution of units in use in each segment. Before each new simulated step the number of non-owners and the age distribution of units in use are naturally corrected according to the changes that took place in the previous step. The boxes connected by solid arrows in Figure 5 represent the core of DEMSIM, i.e. the basic diffusion model without external influences affecting the adoption probability. The dotted arrows represent various extensions that can be added to the basic diffusion model in applications.
More precisely the accounting identities are the following ones. As for the diffusion model the number of adopters in segment $m$ in time period $t$ is calculated as

\begin{equation}
QN_m(t) = AP_m(t) [1 - Y_m(t)] H_m(t)
\end{equation}

$H_m(t)$ stands for the number of consumers in segment $m$ at the beginning of time period $t$, cf. “Sizes of Segments” in Figure 5. By multiplying the number of
consumers by the share of non-owners at the beginning of \( t \), \( 1 - Y_m(t) \), we get the number of non-owners at that time. Using the expected value approach discussed earlier we multiply the number of non-owners by the probability to adopt, \( AP_m(t) \), getting the expected number of adopters in segment \( m \) in time period \( t \). The number of adopters in all segments in time period \( t \) is then

\[
QN(t) = \sum_{m=1}^{M} QN_m(t)
\]

The total number of adopters, \( QN(t) \), equals the demand by non-owners and is called new demand in DEMSIM.

As for the replacement model the number of replacements made in segment \( m \) in time period \( t \) is calculated as follows.

\[
QR_m(t) = \sum_{x=1}^{X} RP_m(x) U_{m,t-x}(t)
\]

The variable \( U_{m,t-x}(t) \) stands for the number of owners in various owner categories in segment \( m \) at the beginning of time period \( t \), the first category representing owners that acquired the durable 1 time period ago \( (x=1) \), the second owners that acquired the durable 2 periods ago \( (x=2) \), and so on up to the last category, i.e. owners that acquired the durable \( X \) periods ago \( (x=X) \) or, when \( t \leq X \), \( t-1 \) periods ago \( (x=t-1) \). In other words, the variable stands for the age distribution of durables in use at the beginning of time period \( t \) in segment \( m \). Using the expected value approach, we get the number of replacements in each age category by multiplying the probability to replace units of age \( x \) by the number of such units. By summing up the replacements over all age categories we get the expected number of replacements in segment \( m \) in time period \( t \). The total number of replacements in time period \( t \) is then

\[
QR(t) = \sum_{m=1}^{M} QR_m(t)
\]

The total number of replacements, \( QR(t) \), equals the demand from owners and is called replacement demand.

Finally we get the total demand in time period \( t \) as

\[
Q(t) = QN(t) + QR(t)
\]

After each simulated step a new category is added to the age distribution of units in use. The total number of categories in the age distribution increases as long as the total number is less than \( X \). After that the oldest category of the age distribution disappears each time a new category enters the distribution simply because all units
of the durable in the oldest category have by then been replaced. The new age category entering the distribution in each simulated time period \( t \) represents all units of the durable acquired in the previous time period \( t-1 \), i.e.

\[
U_{m,t-1}(t) = QN_m(t-1) + QR_m(t-1)
\]  

After each simulated period the age distribution of units in use is naturally corrected due to the simulated replacements made in that period.

The “Owner Share Corrections” in Figure 5 represent *such changes in owner shares that are due to other reasons than adoptions*. The sizes of segments are changing all the time. A consumer may belong to one segment in time period \( t \) and to another segment in time period \( t+1 \). Some of the consumers moving between segments may have adopted our durable, others not. If then, for example, the proportion of owners among those who are moving into a given segment in time period \( t \) is higher than the proportion of owners among those who are moving out of that segment and the total number of consumers moving in is about the same as the total number of those moving out, then the owner share of that segment increases in time period \( t \), even if not a single consumer in that segment adopts the durable in period \( t \). Owner share changes of this type can be calculated *ex post* from observed owner shares and those observed changes in owner shares that are due to observed adoptions in the following way.

\[
YC_{m}^{obs}(t) = Y_{m}^{obs}(t+1) - Y_{m}^{obs}(t) - \Delta Y_{m}^{obs}(t)
\]

\( YC_{m}^{obs}(t) \) represents the observed owner share change in segment \( m \) in time period \( t \) that is due to other reasons than adoptions. We will call this variable the *correction factor*. This factor represents, so to say, the net effect of all other circumstances besides adoptions that affect the owner shares. It is clear that the simulated owner shares should be corrected with this factor in simulation runs. Otherwise we are ignoring the fact that changes in owner shares on real markets result, besides from new adoptions, also from a lot of other circumstances affecting the owner shares from time to time. We have, in other words, to take these other circumstances into consideration by eliminating them from our simulation runs. This is done in the following way.

\[
Y_{m}(t+1) = Y_{m}(t) + \Delta Y_{m}(t) - YC_{m}^{obs}(t)
\]

The correction factor represents an exogenous variable in our model. In validating our model *ex post* we adjust the owner shares with observed correction factor values, cf.

---

18 The superscript \( obs \) indicates that the variable in question *has to be observed* in the specific context where the superscript is used. Throughout this report a variable without superscript indicates, generally speaking, that the variable can be observed or simulated or separately forecasted.

19 The owner share corrections are also included in the diffusion model, as indicated in Figure 5, although they were not introduced and discussed in Chapter 2.
Equation (20). These are calculated according to Equation (19). In doing forecasts, in turn, we first have to make predictions on the correction factor values, $Y_{Cm}(t)$, which then are used as inputs in our forecasts. The size and direction (+ or -) of the correction factor depends heavily on how the market segments are defined. This will be empirically demonstrated in Chapter 5.

Besides owner shares the correction factors naturally also affect the age distribution of durables in use in each segment. This effect is incorporated in DEMSIM by correcting each durable age category with the same factor value as the value used in correcting the overall owner share. Then the sum of these corrections, counted as numbers of owners, equals the correction made on the overall number of owners.

**Defining the market segments**

As we have seen, the segmentation approach used in DEMSIM enables us to explicitly incorporate interpersonal contact behaviours in our model. In other words, by segmenting the market we have been able to, on a semi-aggregated level, specify such channels through which we believe that most innovations usually are spreading within societies. These channels are then used in specifying the internal diffusion influences within each predefined market segment using an expected value approach. This is, so to say, the core idea of our diffusion model. This idea would not have been possible to realize within a manageable model without using some kind of segmentation approach or, in other words, to allow for differentiated contact, exposure and adoption behaviours on a semi-aggregated level.

If the market segments are defined in such a way that each segment more or less is a replica of the others in those behavioural aspects specified in DEMSIM the advantages of using a segmentation approach are obviously small. If, however, the contact and exposure behaviours specified in DEMSIM differ clearly between segments, then the consequences of such differences on the adoption behaviour can be analysed within the model frame leading to a better understanding of the diffusion process in question. This in turn should lead to improved model specifications and more accurate forecasts.

Actually benefits of the type just mentioned have been advocated in the litterature through the years. A thorough discussion on these matters can be found in Armstrong (1985, pp. 249-270). The advantages of disaggregating economic data into meaningful homogeneous groups in order to better understand and forecast future economic behaviour has been stressed and empirically illustrated by e.g. Elton and Gruber (1971). Also Shlifer and Wolff (1979) as well as Zellner and Tobias (2000) make conclusions in similar directions. Zellner and Tobias (2000) also stress the fact that, with disaggregation, there are more observations to base parameter estimates on, a fact that we believe is especially important in dealing with the very first phases of a diffusion process when we don’t have any historical time series data at all to fall back on.

An ideal segmentation in our context would be one where the current behaviours within each segment are as homogeneous as possible while the same behaviours are
as heterogeneous as possible between the segments. By increasing the number of segments we can, at least up to a given point, get closer and closer to such a goal. From an application point of view too large a number of segments is, however, in most cases problematic. Increasing the number of segments too much will probably bring about serious measurement problems. One important aspect to consider here is, of course, the segmentation criteria that we use. We must be sure that the contact and exposure behaviours can be adequately described and that sales observations for each segment (easily) can be obtained for each time period. Therefore it can be suitable to use segments for which some kind of official data exists. Such segments are, for example, age and income segments.

Using age and income as segmentation criteria has some obvious advantages. Firstly, the probability to adopt a new durable, especially when the durable is very new on a market, differs in most cases between various age segments as well as different income segments. Also the contact and exposure behaviours can be supposed to differ between these segments. Secondly, age and income are relatively easy to measure. For most markets data already exist on e.g. the sizes of various age and income segments. Thirdly, the model calls for estimates of inter- and intrasegmental contact behaviours as well as exposure behaviours in segments. Such behaviours can be supposed to be rather stable over time when age and income are used as segmentation criteria. If this is so, estimates of these behaviours made at one point in time can be used as approximations of these behaviours at other points in time. Furthermore, the contact and exposure estimates can be used in other applications of DEMSIM as well if the segments are the same and the durable is of the same type. Fourthly, age and income represent segmentation criteria that are familiar to most companies and can usually be said to fit well into strategic marketing thinking.

**Model types and simulation purposes**

At least three basic types of DEMSIM applications can be distinguished. To begin with each type will be shortly presented and discussed below.

**The Pure Diffusion Model.** The core of DEMSIM is the diffusion model generating the probability to adopt a new durable in each one of a predefined set of market segments over time through an endogenous diffusion mechanism. By stating the sizes of these segments and excluding the external influences as well as the replacement model, DEMSIM simulates new demand generated through a pure endogenous diffusion mechanism. This means that we only allow for internal influences to affect the adoption behaviour. Such a DEMSIM application can be called a pure diffusion application representing the simplest way to apply the present model. In Figure 5 such an application is denoted by solid arrows.

**The Mixed Diffusion Model.** The dotted arrows in Figure 5 represent various options that can be added to the pure diffusion model. If we allow for external influences the pure diffusion model turns into a mixed diffusion model. In this state DEMSIM simulates new demand generated through a combined endogenous-exogenous diffusion mechanism allowing for both internal and external influences to affect the adoption behaviour simultaneously. This approach, theoretically sound as it is, has,
however, drawbacks as it comes to long-term forecasting. This stems from the fact that the external influences hold in them exogenous variables that have to be forecasted separately and then used as exogenous inputs in the DEMSIM forecasts. The DEMSIM forecasts are in other words conditional on the separately forecasted exogenous variables. These exogenous variables mostly represent phenomenons that heavily depend on changing conditions in the marketplace due to human will and strategic decision making. As such their predictability, especially in the long run, is very poor. However, as it comes to validating our model \textit{ex post}, i.e. on the basis of historical time series, this problem disappears. Then the mixed diffusion model usually outrules the pure diffusion model as a tool for validation.

Generally speaking it seems obvious that the more the external influences, as compared to the internal influences, affect the demand for a new durable, the harder it is to make reliable long-term demand forecasts for that durable. If, however, the internal influences dominantly affect the growth of demand for a durable it is easier to make reliable long-term demand forecasts, simply because internal influences follow a more law-like (or at least a more predictable) pattern over time than external influences generally do.

**The Diffusion-Replacement Model.** Another option in Figure 5 is to include the replacement model either in the pure diffusion model or in the mixed diffusion model. In these states DEMSIM simulates \textit{total demand} as the sum of new demand and replacement demand. This type of application is, due to the descriptive nature of the replacement model in its present shaping, mainly an explorative tool that e.g. can be used to generate various scenarios under different service-life expectancies for a durable.

**Simulation purposes.** Regardless of which of these model types we use our simulations can have different purposes. As to possible purposes we can distinguish, at least, the following ones. Either we are interested in \textit{ex post} analyses, i.e. analyses based on historical time series data (simulated and/or observed), or in \textit{ex ante} applications, i.e. forecasting. In the first case we want to analyse and find out, as soon as possible after the launch of a new durable, in what ways and to what an extent various exogenous variables (external influences) as well as the endogenous forces (internal influences) affect the behaviour observed. In doing so we are deepening our understanding of these behaviours and the forces affecting them. Such knowledge should improve our ability to make reliable forecasts \textit{ex ante}. Such \textit{ex post} analyses are usually called validation analyses. In our context we mean by validation more precisely comparisons between simulated and observed time series. Such comparisons can be made using various accuracy measures but also simple visual comparisons of simulated and observed time series are useful.

We will, furthermore, distinguish between two types of \textit{ex post} analyses in DEMSIM. In addition to the validation purpose just commented on we have also incorporated a calibration purpose into our demand simulator. This means that we, with the help of an optimization algorithm built into DEMSIM, are able to estimate (calibrate) some of the model parameters, i.e. the coefficient of internal influence, $a_m$, and the coefficient of external influence, $b_m$, in very early stages of a diffusion process. This is especially the case when we are applying the pure diffusion model having only one parameter,
\(a_m\), to estimate instead of two parameters, \(a_m\) and \(b_m\), as the case is when applying the mixed diffusion model. In Chapter 5 it is shown that rather good predictions are achieved using the pure diffusion model and estimates based on only 2 or 5 sales observations per segment, cf. Figures 17 and 18.

In conclusion we can state that our demand simulator has three distinct simulation purposes, i.e.

- a calibration purpose (ex post parameter estimation)
- a validation purpose (ex post analyses, simulated versus observed)
- a forecasting purpose (ex ante applications)

In other words, DEMSIM is designed to be used as a parameter estimation tool, as a validation tool and as a forecasting tool. One important goal in designing the computer model has been to enable continuous interactions between the user and the program including possibilities to fluently shift between the three simulation purposes. It should be noted that calibration and validation simulations in DEMSIM always have to start from the very beginning of the diffusion process, while forecasting can start from a later point in time. However, when starting a forecast from a later point in time the diffusion history up to the starting point first has to be simulated in validation runs. In doing this we are forming historical owner share changes as well as age distributions of durables in use needed to carry out our forecasts.

On the whole it is important to note that DEMSIM is intended to be used in such a way that the three simulation purposes continuously are backing up each other in ongoing applications. Not until then do we make the most out of the calibration-validation-forecasting design inherent in DEMSIM.

**Applying DEMSIM – Interacting and learning**

Building models to support marketing decision making has been practised at least since the middle of the last century. At the beginning this happened mainly within disciplines like operations research and management science.\(^\text{20}\) In a recent review article by Leeflang and Wittink (2000) the developments within marketing modeling in the last fifty years are thoroughly described and discussed as well as possible future developments commented on. Behind most of these developments a common goal can be traced. This goal has simply been to improve the implementability of marketing decision support models or, in other words, to learn more about how to build models that predict accurately and are easy to understand and use in practice. Pioneering work on these matters was provided by Little (1970). Since then we have experienced

\(^\text{20}\) Actually the roots can be traced back at least to the years preceding the second world war, a time when the term operations research was still unknown. E.g. from 1935 on, for more than ten years, a journal named Nordic Journal of Technological Economics (my translation from the Danish title *Nordisk Tidskrift for Teknisk Økonomi*) was published in Denmark. The articles of this journal dealt mainly with applied models intended to support management (and marketing) decision making within firms. Among the authors names like Ragnar Frisch, Frederik Zeuthen, and Börge Barfod can be mentioned.
developments like increased model flexibility achieved e.g. through modular model building, more frequent uses of subjective data and subjective calibration techniques, richer data bases enabled through improved tracking services and improved possibilities to store, maintain, and access data, and many others. All these developments clearly serve the same goal, to improve the implementability of marketing decision support models.

Model building developments similar to those in marketing science can be observed within the operations research and management science fields as well. However, within these fields, for example, various simulation approaches have perhaps been used more frequently than within marketing science. One such approach is visual interactive simulation or VIS (Bell and O’Keefe, 1995, and Chau and Bell, 1995). The basic idea behind this approach is to combine iconic computer-generated animation and/or graphic display (the visual component) with flexible user interaction possibilities (the interactive component). In such a setting the user can stop the execution of the model due to visual signals, recalibrate one or several parameters of the model or even respecify the model, and then resume model execution. In this way the user takes a more active role in using the model and experimenting with it leading to, hopefully, a deeper understanding of the functioning of the model as well as the phenomenons modeled. Ideas resembling these have been guiding our development of DEMSIM.

It has been pointed out that, when sales for a new product or service is forecasted for practical purposes, the forecasting model(s) are extremely likely to be re-estimated and alternative functional forms considered after each new observation has been recorded (Bewley, 1997). Bewley states further that practical diffusion modeling is an on-going process of learning more about both the phenomenon modeled and the forecasting model(s) used after each new observation has been collected. Such forecast updating processes are, according to Bewley, rarely described in academic work. We strongly agree on this point. Actually one of the basic ideas behind DEMSIM is that the model, in order for us to be able to improve its forecasting abilities, should be recalibrated and revalidated ex post after each new sales observation has been recorded. These processes may reveal that our forecasting model needs respecification. In this way an on-going process of learning takes place that continuously should improve the qualities of our model as a forecasting tool.21 The approach is illustrated in Figure 6.

As suggested each forecast should be validated as soon as new sales observations have been recorded. First a validation is carried out using estimates of the coefficients $a_m$ and $b_m$ from the previous forecast. Then these coefficients are calibrated again on the basis of earlier sales observations plus the new sales observations for the last period. Then a new validation is carried out using the new estimates. After that the results from the two validations are compared and we decide whether a respecification of our forecasting model is needed or not. If we respecify the model we return after that to the previous stage and repeat the calibration and validation steps to check

---

21 We are here talking about recommended activities in ongoing real time forecasting work. In validating forecasting models on the basis of historical time series data the same approach has been used under the name of successive updating (Armstrong, 1985, p. 343). Successive updating will be used in validating our diffusion model in Chapter 5.
whether the respecified model performs better \textit{ex post} than the old model did. Naturally several respecifications can be considered and validated. The model that performs best is then used in making a new forecast \textit{ex ante}. This procedure is repeated each time sales observations for a new time period have been recorded. In this way a continuous interaction between \textit{ex ante} forecasting and \textit{ex post} validating takes place constituting an \textit{on-going learning process}.

\textit{Figure 6. Learning through repeated forecasting and validation processes}

Launch of new durable
\rightarrow
Specification and Calibration
\rightarrow
Forecast
\rightarrow
Validation and Calibration
\rightarrow
Possible Respecification
\rightarrow
Forecast
\rightarrow
Validation and Calibration
\rightarrow
Possible Respecification
\rightarrow
Forecast
\rightarrow
And so on
As has been pointed out earlier DEMSIM represents a general frame or model for explaining and forecasting the diffusion of new consumer durables on a market. As such the frame incorporates those mechanisms that are supposed to be more or less similar regardless of what type of durable we are concerned with. Using the frame in a specific application means that we probably have to further adapt the model to conditions specific for the durable as well as the market in question. This applies especially to various external influences but it is also possible that e.g. the interpersonal contact behaviour needs to be respecified. The process of adapting the model to a specific durable follows, in a natural way, the learning process depicted in Figure 6. At first, when our knowledge obviously is more restricted than later on, we start with a rather simple model, e.g. a pure diffusion model. When our knowledge increases through the recurrent validation processes we probably will respecify our forecasting model from time to time. As it comes to the exogenous variables we must always bear in mind their predictability if we want to include them in our forecasting model. One way to proceed is to have two models running alongside each other, i.e. one model without exogenous variables that are more or less impossible to predict accurately, at least in the long run, and one model that includes such variables. The last model then serves as an *ex post* learning device in the process of continuously learning more about the phenomenon we are forecasting, hereby e.g. increasing our ability to understand and interpret our forecasts made by the first model.

As we have stated above, DEMSIM simulates demand in several mutually exclusive market segments simultaneously. Hence the sum of these demands represents total market demand. This means that, instead of having only one new sales observation to consider in each new validation round, which usually is the case when traditional growth curve models are used, we will have $M$ new sales observations to consider, one for each market segment plus the sum of these, i.e. total market sales. This gives us richer data to base our validations on than in cases where we are using single growth curve models. This improves our possibilities to better understand how our three basic diffusion constructs, internal influences, external influences and innovation resistance, affect the demand for a new durable in each of a number of predefined market segments leading to richer learning and better forecasts.

**Validating DEMSIM**

A vast literature going back to the middle of the last century exists on simulation models and the testing of them. This literature originated at first mainly from research areas like operations research and management science but also e.g. sociology and philosophy (cf. Naylor and Finger, 1967, Van Horn, 1971, and Mihram, 1972). Testing a model can mean many things of which validation is one. Validating a diffusion model has mainly come to mean testing the agreement between the behaviour of the simulation model and the behaviour of the real system simulated. Verification, in turn, insures that a simulation model behaves as the model builder intends. (Fishman and Kiviat, 1967) We will in the following concentrate on the validation of simulation models in general and on the validation of DEMSIM in particular.
Since the validation of simulation models mainly is concerned with the examination of how closely simulated values of certain variables track observed values of these variables, we need quantitative measures of how close this fit is and, furthermore, the nature of the errors. By errors we mean the differences between simulated and observed values. Such measures are often called *accuracy measures*. There is a considerable number of accuracy measures and a lot of various reminders of important aspects to observe when validating simulation models (cf. Armstrong, 1985, pp. 346-354, Pindyck and Rubinfeld, 1998, pp. 384-389, and Leeflang, Wittink, Wedel and Naert, 2000, pp. 500-508). However, any established standard of how a proper validation should be carried out does not exist, which is understandable. A validation design should always be based on the purposes of the simulation model. The validation tools and approaches should be chosen according to how well they can contribute in reaching the purposes of the model. Since such purposes can vary considerably between models the validation approaches should, as a rule, be specially designed in each case. Two specific simulation situations have, however, been pointed out in the literature mentioned above. That is whether the main purpose of the model is to analyse *ex post* how well the model fits historical data or to forecast *ex ante*. For the first type of situation a lot of sophisticated statistical procedures have been developed that are not necessarily very strongly related to forecasting accuracy and therefore not especially helpful in predictive validation, i.e. the validation of forecasts. Two examples of such measures are according to Armstrong and Brodie (1999) $R^2$ and the standard error of the estimate of the model. They state that predictive validation should lean more on *ex post* forecasts carried out in an *ex ante* manner, meaning that forecasts over historical time periods are made using only such information as input that would have been available at the starting point of the forecasts.

As we have seen the core of DEMSIM is a diffusion model generating new demand. Our validations of DEMSIM in Chapter 5 are solely concerned with this part of the model. In these validations we compare simulated outcomes with observed outcomes for *owner shares* and *new demand*. For each of these two variables we have one time series for each predefined market segment and one for the whole market. In our validations we are using mainly two accuracy measures, i.e. the *Mean Error* (ME) and the *Mean Absolute Error* (MAE). The first measure is calculated for owner shares in each segment as

\[
(21) \quad \text{ME}_{m}^{own} = \frac{1}{T} \sum_{t=2}^{T+1} \left[ Y_m(t) - Y_{m}^{obs}(t) \right] / T
\]

and for new demand as

\[
(22) \quad \text{ME}_{m}^{dem} = \frac{1}{T} \sum_{t=1}^{T} \left[ QN_m(t) - QN_{m}^{obs}(t) \right] / T
\]

where $Y_m(t)$ and $QN_m(t)$ represent simulated outcomes.
Equations (21) and (22), as well as Equations (23) and (24) below, apply to segments. If the subscript \( m \) is left out the equations apply to the whole market. The mean error is useful as an indicator of systematic errors, i.e. such errors that, for example, cause a simulated time series to systematically fall mainly on one side of the observed time series. This is not the case when \( \text{ME} = 0 \). Then we can state that the simulated values are, on average, equal to the observed values. However, this tells us nothing about the variance of the errors, since errors with opposite signs offset each other. Therefore the ME-measure should be used together with some other statistic measuring the magnitude of the errors. There are several such measures, of which we have chosen the mean absolute error. For owner shares this measure is calculated as

\[
\text{MAE}_{own}^{m} = \frac{\left\{ \sum_{t=2}^{T+1} |Y_m(t) - Y_{\text{obs}}^m(t)| \right\}}{T}
\]

and for new demand as

\[
\text{MAE}_{dem}^{m} = \frac{\left\{ \sum_{t=1}^{T} |QN_m(t) - QN_{\text{obs}}^m(t)| \right\}}{T}
\]

where \( Y_m(t) \) and \( QN_m(t) \) represent simulated outcomes.

Other approaches could, for example, have been to use a measure based on squared errors or one based on errors expressed as percentages, i.e. a relative measure. Both of these approaches are frequently used, but we prefer to use absolute errors. Measures based on squared errors have in predictive validation been criticized mainly for their sensitivity for outliers (cf. Chatfield, 1992, Fildes et al., 1998, and Armstrong and Brodie, 1999). The criticism has also concerned the scale dependence of such measures, which also holds for the mean error and the mean absolute error when these measures are applied to new demand volumes, cf. Equations (22) and (24). When they are applied to owner shares, cf. Equations (21) and (23), this is not the case since owner shares already represent a kind of relative measure. Furthermore it can be stated that relative accuracy measures, although they are dimensionless, have a serious drawback in our specific model setting. This follows from the fact that owner shares as well as new demand of a durable diffusing a market are not especially stable over time. And above all, at the very beginning of the diffusion process observations on these two variables often lie close to zero and sometimes are even equal to zero. This brings about calculation problems when zero-values on owner shares and new demand are involved and abnormal and unstable accuracy measures when (very) low values on owner shares and new demand are involved. Considerations like these led us to choose the mean absolute error as our main accuracy measure. Absolute errors are also advantageous in the sense that they are easy to understand and interpret. For owner shares they (multiplied by 100) express the error as a percentage (of all consumers). For new demand the errors simply stand for numbers of units.
We will also use one kind of relative measure in applying DEMSIM. This measure concerns simulated and observed new demand volumes. We call this measure the \textit{Percentage Error in Accumulated Demand} (PEAD). The measure is calculated as

\begin{equation}
\text{PEAD}^{\text{dem}}_m = \left\{ \left( \sum_{t=1}^{T} Q_N^{m}(t) \right) - \left( \sum_{t=1}^{T} Q_N^{\text{obs}}_m (t) \right) \right\} / \left( \sum_{t=1}^{T} Q_N^{\text{obs}}_m (t) \right) \cdot 100
\end{equation}

where \(Q_N^{m}(t)\) represents simulated outcomes.

This measure compares, in relative terms, simulated new demand, accumulated from the start of the diffusion process, to observed new demand, also accumulated from the start of the diffusion process. This approach excludes, at least to some extent, the drawbacks due to low sales volumes mentioned above.\(^{22}\)

One important validation criterion is further how well a model simulates so called \textit{turning points} in the data (cf. Armstrong, 1985, p. 353, and Pindyck and Rubinfeld, 1998, p. 386). This is particularly interesting in our case since DEMSIM simulates the diffusion of a new consumer durable or, in other words, a process that, as it comes to new demand, usually contains a clear turning point. This is the point in time when new demand ceases to grow and usually begins to decrease. Another typical feature of diffusion processes for durables is that, when new demand decreases more and more, the owner share approaches some kind of saturation level. From a forecasting point of view it is important to be able to, as soon as possible after the launch of a new durable, make reliable predictions of when the turning point in new demand will appear as well as when the saturation level will be reached and what the share of owners then will be.

\textit{Calibration approaches}

It has been argued that statistical parameter estimation for diffusion models is primarily of historical interest (Mahajan, Muller and Bass, 1990). The reasons for this are obvious. By the time we have a sufficient number of observations for reliable estimation, we have long ago passed the times when we most urgently would have needed the estimates for forecasting purposes. Actually the estimates would have been needed most urgently \textit{before} the new durable was launched on the market, i.e. at a point in time when we did not have any empirical observations at all to base our estimates on. Our present diffusion/simulation model has, among other purposes, been designed in order to handle these problems at earlier stages of the diffusion process than what is possible using traditional growth curve models. Hence the calibration approaches used in estimating the parameters of DEMSIM are specially adapted to this model. The approaches will be presented in the following.

\(^{22}\) In DEMSIM-validations the PEAD-measure is calculated after each simulated step.
The parameters of DEMSIM can be grouped into four distinct types. These are the following ones.

1. **Contact and exposure parameters** ($P_{mn}$, $C_m$, and $EM_m$)
2. **Word-of-mouth parameters** ($d$ and $K$)
3. **Internal and external influence parameters** ($a_m$ and $b_m$)
4. **Replacement parameters** ($g_m$ and $s_m$)

Each parameter type has its own calibration approach. These are explained below. The first three parameter types originate from the diffusion model, while the fourth type concerns the replacement model. It should be noted that the estimates of the internal and external influence parameters (type 3) are conditional on the estimates of the contact and exposure parameters (type 1) and the word-of-mouth parameters (type 2). This means that, in the present shaping of DEMSIM, the type 1 and type 2 parameters have to be estimated before the type 3 parameters can be estimated constituting a two-step estimation process. If we, for example, reestimate the word-of-mouth parameter $d$, we also have to reestimate the internal and external influence parameters $a_m$ and $b_m$.

**Contact and exposure parameters.** These parameters are concerned with the interpersonal contact behaviours within and between our segments as well as personal exposure behaviours to mass media. The personal contact network is operationalized through the parameters $P_{mn}$ and $C_m$ and personal exposure to mass media through the parameter $EM_m$. These parameters were closer discussed in connection with Equations (5) and (8) in Chapter 2. They are entering our model exogenously and have to be estimated separately. The nature of these behaviours can, in most cases, be supposed to be rather time-invariant. If this is the case, using estimates made at one point in time as approximations for other points in time can be justified. This is the approach we will use in Chapter 5. These parameters can be estimated independently of the actual diffusion process, i.e. even before the process starts, because the actual behaviours exist regardless of and unaffected by the diffusion process under study. The estimates should, however, be based on how information concerning the durable in question most probably is spread among consumers. Different durables may be spread through different types of contacts and networks. Therefore the estimates of the contact and exposure parameters have to be adjusted to the application in question including type of durable, type of consumer, and so on. The estimates we use in applying our diffusion model in Chapter 5 are based on data collected in a cross-sectional study especially designed for the present purpose.

**Word-of-mouth parameters.** The parameter $d$ regulates the rate at which the probability to discuss the durable is decreasing when the time since the durable was acquired is increasing in contacts between owners and non-owners. $K$, in turn, stands for the number of time units this specific probability exceeds zero. These two parameters were discussed in detail in connection with Equation (6) and Figure 3 in Chapter 2. It is obvious that the values of $d$ and $K$ are dependent on the length of the time unit used. With increasing length of the time unit follows an increasing $d$-value and a decreasing $K$-value, everything else unchanged. Furthermore, the parameter $d$ has a much greater impact on simulated outcomes than the parameter $K$. This is
especially so when we use relatively high values of $K$ in our simulations (cf. Figure 3 in Chapter 2).

The estimation of $d$ and $K$ for forecasting purposes can hardly be based on empirical market data concerning the specific behaviours in question. Such data would have to be collected alongside with the diffusion process and could not be used for estimation purposes in, at least, early stages of the diffusion process. Due to, among other things, the fact that the phenomenons studied are long-lasting such empirical data would probably suffer from considerable measurement errors. Consequently other calibration approaches have to be used.

As we have pointed out above we can, by choosing a high enough $K$-value, concentrate on solely estimating the parameter $d$. One possible approach after the durable has been launched on the market is to use a kind of search process in which we are testing ex post which of a given number of $d$-values produces the best fit between simulated and observed time series. In applying our model in Chapter 5 we will use this type of explorative approach in estimating the word-of-mouth parameter $d$. If we want to estimate $d$ before the new durable is launched we have to rely on judgemental estimation approaches.

**Internal and external influence parameters.** If we want to simulate the diffusion of a new durable on a market before the durable has been launched we naturally have to use judgemental or subjective estimates of the influence parameters. One way to arrive at such estimates is to, first, make judgements of how many units of the new durable that will be sold in each segment in the very first period(s) after the launch. These judgements are then used, as if they were observed sales figures, to estimate the influence parameters by means of the optimization algorithm presented below. This approach gives us a first feeling of the levels of the influence parameters based on our own judgement of how many units of the durable that will be sold in various segments right away after the launch.

The optimization algorithm by which we, after a new durable has been launched, estimate the internal and external influence parameters is a simple search algorithm. The algorithm lets you choose between two optimization criteria. The first applies to owner shares

\[ MAE^{own} = \frac{1}{M} \left( \sum_{m=1}^{M} MAE^{own}_m \right) 
\]

The second applies to new demand

\[ MAE^{dem} = \frac{1}{M} \left( \sum_{m=1}^{M} MAE^{dem}_m \right) 
\]
As we can see, MAE stands for the mean absolute error over all segments.\textsuperscript{23} MAE is, in other words, an overall measure of the absolute deviations between simulated and observed owner shares, Equation (26), or new demand, Equation (27). In estimating the influence parameters we first choose either of the two optimization criteria. Then we decide on the interval to be searched for each parameter as well as the search points (steps) applied in the search process. After these preparatory phases we can start the search engine, i.e. the estimation process. The search engine looks for the combination of parameter values, among all possible combinations within the predefined search frames, that optimizes the chosen criterion, i.e. minimizes the chosen MAE. If we, for example, have chosen MAE\textsubscript{OWN} as optimization criterion our search process ends up with parameter estimates that minimize the absolute deviations between observed and simulated owner shares in all segments according to Equation (26). More precisely, the search for the optimal combination of parameter values means that our diffusion model, given estimates of the contact and exposure parameters as well as the word-of-mouth parameters, simulates over and over again a diffusion process covering as many time periods as we have observations for. The only difference between these simulations is the combination of parameter values, $a_m$ and/or $b_m$, used in each simulation. In other words, the total number of simulations (iterations) equals the total number of all possible combinations of our predefined influence parameter values or levels, cf. Figure 7. The combination of parameter values that produces the best fit (smallest MAE) between the observed and simulated owner shares or sales volumes constitute our influence parameter estimates.\textsuperscript{24}

As has been stated earlier the influence parameters can, in the approach just described, be estimated objectively at very early stages of the diffusion process. Technically this can be done already when we have recorded sales volumes for only one time period, i.e. the very first time period after the launch of a new consumer durable. Then we have, in other words, one observation for each segment. Such a first calibration produces estimates that obviously are very unreliable, especially if we have observed zero-sales in some segments in this time period or if we are using the mixed diffusion model with more parameters to estimate than if we are using the pure diffusion model with fewer parameters to estimate. However, with more observations the reliability should increase and the estimates get more stabilized. When time passes and the number of sales observations reaches a level where traditional statistical estimation procedures can be used, we get a further tool to analyse the diffusion processes \textit{ex post}. Such analyses should, however, not compensate our optimization algorithm. Instead they should be looked upon as complementary means in an ongoing calibrating-validating-forecasting process.

---

\textsuperscript{23} An alternative would have been to weight the MAEs according to the sizes of the segments. In the present version of DEMSIM we, however, choose an approach with equal weights. We are, in other words, stressing the fits between observed and simulated outcomes in each segment equally.

\textsuperscript{24} It should be noted that the number of iterations needed to find an optimal solution is dependent on which diffusion model we use (the pure diffusion model or the mixed diffusion model) and especially on how we define the search points. Note that the number of iterations increases very rapidly when we (even slowly) increase the number of search points for each parameter to be estimated. Using a standard personal computer a considerable time may be needed to find an optimal solution.
The interface between the user and DEMSIM when preparing for calibration is shown in Figure 7. Note that the figure concerns an application with three segments.

**Figure 7. Preparing for calibration**

In calibrating with DEMSIM a very first step is to find the appropriate intervals for each parameter within which the optimal solution is going to be sought. This is crucial since the solution we will arrive at is, of course, optimal only within the boundaries we define, i.e. the starting points, the ending points and the search points (Step) in Figure 7. To find these intervals we can use, for example, a trial and error search approach. By visually examining the simulation outcomes using various combinations of parameter values we are building up a feeling for the intervals within which the optimal solution most likely will lie. What we then actually are doing is that we, step by step, are eliminating such parameter values that produce predictions that are clearly far from optimal. The approach seems to work well. It represents a type of visual interactive simulation (cf. Bell and O’Keefe, 1995, and Chau and Bell, 1995) enabled through the specific design of DEMSIM, i.e. instant graphic display of simulated and observed time series in combination with flexible parameter handling possibilities meaning that a trial can be carried out and the results visually examined almost at once.

It should be remembered that our calibration approach as to estimating the internal and external influence parameters is, so far, on a preliminary and explorative stage. This also holds for estimating the word-of-mouth parameter $d$. Further research and
testing is needed in order to develop these calibration approaches further. However, the results are promising so far. This will be demonstrated in Chapter 5.

**Replacement parameters.** The replacement model holds in it two parameters to be estimated. The first parameter represents the expected service-life of the durable in each segment, \( g_m \), and the second controls the slope of the survival curve, \( s_m \) (cf. Figure 4 in Chapter 3). Sophisticated procedures to estimate these two parameters would call for extensive data, something that we usually do not have when we want to make long-term demand forecasts for new durables. Therefore we have to be satisfied with subjective estimates based on, for example, experiences from markets where the same durable has been launched earlier. Also earlier diffusions of durables of a similar type can be helpful when the two parameters are subjectively estimated.
Chapter 5

DEMSIM IN ACTION

Plan of the chapter

In this chapter we are mainly concerned with validating the diffusion model inherent in DEMSIM. Our main emphasis is on the long-term forecasting qualities of the model. This means that our validation approaches are designed to give answers to questions like, for example, how soon after product launch the diffusion model can produce reliable long-term forecasts and how dependent these forecasts are on the values of the word-of-mouth parameters of the model, especially the parameter $d$. This could also be expressed in another way. What we utmost are testing is how well our operationalization of internal influences serves as a predictor of future adoption behaviour.

First the empirical market data used will be presented including observations on owner shares and new demand in segments and totally and on owner share corrections in segments. Estimates of contact and exposure parameters adjusted to the market and segments in question are also presented and discussed.

After that the diffusion model is validated in various ways. We start by analysing how different values of the word-of-mouth parameters affect the simulated outcomes or, in other words, how sensitive the simulated diffusion processes are especially to various values of the parameter $d$. These analyses give us the estimate of $d$ that will be used in the subsequent validations. After that follows a section in which we are especially concerned with how soon after product launch our diffusion mechanism is able to produce reliable long-term forecasts taking into account the type of model we are using as well as the optimization criterion used in estimating the internal and external influence parameters. In these sections an approach resembling successive updating (Armstrong, 1985, p. 343) is used. First calibrations are made on very few observations. Then the number of observations is increased in steps and new calibrations are made in each step. The results of each new calibration are used in repeated unconditional forecasts.\(^{25}\) In this way we analyse the model sensitivity for

\(^{25}\) By an unconditional forecast we mean a forecast in which all exogenous variables of our model are known with certainty for the entire forecast period. From this follows that an unconditional forecast always is an ex post forecast in our model context. A conditional forecast, on the other hand, is a forecast in which one or more of our exogenous variables are not known when the forecast is made. Supposing that we have one such exogenous variable we first have to forecast this variable separately and after that use this forecast as input in forecasting the diffusion process. The diffusion forecast is, in other words, conditional on the exogenous variable forecast (cf. Pindyck and Rubinfeld, 1998, p. 203).
various \( d \)-values as well as the overall predictive power of the model starting from various stages of the diffusion process. Together these validations should improve our knowledge of the suitability of DEMSIM as an \textit{ex ante} forecasting tool.

Chapter 5 is concluded with an illustration using the complete DEMSIM diffusion-replacement model. This illustration serves mainly as an example of how the whole model can be used. In the illustration two scenarios for future demand for a new durable are generated under conditions of different future service-life expectancies for the durable. Since we lack empirical observations in our database on the replacement behaviour of the consumers we have not been able to validate the replacement part of DEMSIM in the same way as the diffusion part.

\textbf{Owner shares, new demand and owner share corrections}

The validations carried out in this chapter are based on data collected in 1970-71 covering a local market in Finland.\textsuperscript{26} The consumers or consumption units were specified as \textit{households}, the durable as \textit{black and white television sets}, and the local market as the small town of Porvoo situated 50 kilometres east of Helsinki. The observed data covered the period 1958-1968. The time unit was set to \textit{four months}, beginning each January, May and September. The data covers 32 time periods, starting from the May-August period in 1958. In this period the very first television sets were bought in Porvoo shortly after television had been introduced on the Finnish market at the beginning of 1958. At the end of 1968, 51% of the households of our Porvoo-sample were television owners. Colour television did not enter the Finnish market until 1969.

Our time series observations are based on a sample drawn from a population consisting of all households living in Porvoo in the years 1958 and 1968, letting the same household appear only once in the population. Official household registers of Porvoo in 1958 and 1968, organized according to an apartment by apartment and street by street principle, served as a basis. By systematically sampling every fifth household with the starting point randomly drawn we could assure that our sample was evenly distributed over the whole Porvoo area while, at the same time, each household in our population had the same chance of being drawn. In this way we ended up with a sample consisting of 20% of the households living in Porvoo in 1958 and 1968. In all, our miniature market of Porvoo consisted of 1523 households. By drawing our sample at two points in time, the starting point and the ending point of

\textsuperscript{26} The data was originally collected in order to test a predecessor to the present model (Lerviks, 1973, 1976a, 1976b, 1984). In these tests ordinary least squares and weighted least squares were applied on linear probability functions. Later on logit analyses were also used (Lerviks, 1984). These tests mainly served \textit{ex post} validation purposes, especially the first ones. From a forecasting point of view these parameter estimation approaches have, however, serious restrictions in the sense that they cannot be applied at early stages of a diffusion process because of the low number of time series observations available then. One purpose of this report, among several others, is to present alternative approaches in this respect.
our observed time series, we wanted to secure at least an approximate representativity of the sample also at times in-between 1958 and 1968. Our approach gave us a sample with a varying number of households over time, a sample that included households which moved into Porvoo, which moved out of Porvoo, which were established and which ceased to exist in the years 1958-1968, in other words, a replica of the Porvoo household market in these years.

For each household in our sample a number of characteristics were collected from different official registers. These characteristics were, among others, the date of the acquisition of the first TV-licence, the number of household members at each point in time, and for each of these members, income after taxes each year, birth date and occupation. In this way information was gathered that enable us to carry out segmentations within the sample and to observe the time series needed in applying our model.

In the present application we are grouping our sample households into segments according to household income. Household income in one time period (4 months) is calculated as the sum of each household member’s yearly income after taxes divided by three. We are using three income segments: Low Income Households (Segment 1), Medium Income Households (Segment 2) and High Income Households (Segment 3). Income level cutpoints between the segments vary from time to time due to adjustments made for inflationary and nominal wage level changes. The observed number of households at the beginning of each time period is shown in Appendix A for each income segment as well as for the whole sample. Our sample portrays, just like the Porvoo market as a whole, two kinds of household movements over time. On
one hand we have households moving between segments at various times due to income changes, on the other we have households entering or leaving our sample at various times. The observed number of households in our three segments over time constitute the exogenous variable $H_{m}^{obs}(t)$, cf. Appendix A.

**Figure 9. Observed new demand 1958-1968 (Periods 1-32)**

Having decided on our segments we count (observe), for each segment, the number of households owning a TV set at the beginning of each time period, cf. Appendix B, and the number of households acquiring their first TV set in each time period, cf. Appendix C. This information was collected from official TV-licence data. The observed owner shares over time for each segment, $Y_{m}^{obs}(t)$, as well as for the whole sample, $Y^{obs}(t)$, are shown in Figure 8. As can, for example, be seen, the low income segment had, by the end of 1968 (Period 32), still not reached a 40% owner share level, while high income households exceeded this level already in 1962 (Period 14), i.e. 6 years earlier.

Figure 9, in turn, shows observed sales to non-owners in each time period, $QN_{m}^{obs}(t)$ and $QN^{obs}(t)$, i.e. new demand, also shown as numbers in Appendix C. Note that the sales figures follow a distinct seasonal pattern. As a rule less TV-sets were sold in the summer months (Periods 1,4,7,...) than in other months of the year, which is understandable. Since our segments contain rather few households the observed sales
figures are also rather small. This being the case, random fluctuations can be supposed to show up more heavily in our observed sales figures than the case obviously would have been if the sizes of our segments would have been bigger. In spite of these circumstances, i.e. the seasonal patterns and the rather small sizes of our segments, Figure 9 depicts trends and regularities that obviously would have been even more prominent with bigger segments.

We can see a demand peak for the whole sample in late 1963 with 41 units sold in Period 17. This period indicates the position in time of the inflection point of the whole sample growth curve, cf. Figure 8. Among low income households the pattern is not as clear. A first peak appears in early 1964 and a couple of even higher peaks in the years 1966 and 1967. One interpretation could be that the growth of TV-ownership among low income households had not yet reached its inflection point in 1968. Among medium income households the new demand pattern resembles closely the one of the whole sample with the new demand peak appearing in the same time period. This is also the case for high income households. However, in this segment we can see two earlier peaks in 1961, each situated almost on the same level as the single peak in 1963, which implies an earlier inflection point within this segment than within the whole sample.

**Figure 10. Observed new demand 1958-1968 on a yearly basis**

To illustrate these observations more clearly we have summed up the sales observations on a yearly basis according to calendar years in Figure 10, hereby eliminating the seasonal fluctuations in Figure 9. Number 1 on the time axis represents Year 1 (1958) and so on up to Year 11 (1968). Looking at our sales data on a yearly basis we can conclude that the new demand peak among all households as
well as among medium income households appeared in 1964 (Year 7). Among low income households the same peak appeared at least not until 1967 (Year 10), while among high income households the peak appeared as early as in 1961 (Year 4). The new demand peaks of the three segments lie, in other words, clearly apart from each other, i.e. in the years 1961, 1964, and 1967 (or later). One purpose of our validations in the following is to test how soon after launch our demand simulator is capable of producing accurate forecasts of the timing and the volume of these peaks.

Finally the observed owner share corrections for each time period and each segment were calculated using Equation (19). In Figure 11 these corrections are accumulated over time for each segment. In other words, for each time period in Figure 11 the corresponding curve value holds in it the sum of all observed owner share corrections up to the end of that time period.\(^{27}\)

\[\text{Figure 11. Accumulated owner share corrections over time (Periods 1-32)}\]

Figure 11 clearly shows that certain regularities are present in our observed owner share corrections over the years 1958-1968. For low income households the accumulated correction trend is slightly increasing revealing an average increase in owner share with 0.0011 per time unit due to other reasons than the adoption behaviour within that segment. What we can say about these other reasons is that the owner share among those households who entered and left the low income segment in the analysed period was, on average, slightly higher than among households staying in the segment. We can see an opposite trend in the two other segments. Here the

\(^{27}\) Actually the calculation of mean corrections started from the period when the first non-zero correction was observed. In other words, we did not let zero-corrections at the very beginning of the diffusion processes, when owner shares were zero or very low, affect the calculated mean corrections.
conclusion is that the owner share among households entering and leaving either of these segments was, on average, lower than among households staying in the segment. Especially among high income households the average decrease in owner share due to other reasons than the adoption behaviour was on average as high as −0.0084 per time unit. This gives us a roughly 0.26 decrease in the owner share over the whole analysed period, a decrease that was due to the fact that the owner share among households entering and leaving the segment was clearly higher than among households staying in the segment. This may explain why the growth curve of the high income segment in Figure 8 seems to have reached a saturation level, which is necessarily not true. One possible reason could be that, for example, the fraction of owners among households leaving the segment was higher than among households entering the segment and/or households staying in the segment.

Thanks to our detailed database we have been able to observe (measure) changes in owner shares due to other reasons than the adoption behaviour analysed. This is very helpful in ex post validations of the diffusion processes. By continuously correcting the owner shares in simulation runs, cf. Equation (20), we are eliminating the consequences of these other reasons on our owner shares, which, so to say, enables “cleaner” validations of our diffusion mechanism. Furthermore Figure 11 reveals that the owner share corrections within a segment are rather stable and similar over time. If this is also more generally the case, good forecasts of these corrections can be made in relatively early stages of the diffusion processes to be used as exogenous inputs in forecasting with DEMSIM, cf. Figure 5.

Contact behaviours and advertising exposures

Besides collecting time series observations of owner shares, new demand and owner share corrections in a number of predefined market segments we have to calibrate or estimate the contact and exposure behaviours of these segments. According to our model we need to know the probability that household members in segment \( m \) will be in contact with household members in segment \( n \), \( P_{mn} \). Furthermore we need to know the expected number of such contacts that a household in segment \( m \) is exposed to in one time period, \( C_m \). Finally, if we use external influence variables in our validations, like for example advertising in mass media, we also need to know the expected exposure of a household in segment \( m \) to the mass media in question, \( EM_m \).

In our case these estimates were based on data collected in a mail survey in 1971 (Lerviks, 1973). The respondents (households) were chosen by simple random sampling from the very same Porvoo household sample that we have described above. Naturally households that had moved away from Porvoo or ceased to exist after 1968 as well as those who had left our sample before 1968 had to be excluded. The final number of households in the survey was 396 or, in other words, about one third of all households in our 1968 Porvoo sample. In the case of our smallest segment, i.e. the high income households, about 50% of these were represented in the mail survey. As to each of the two bigger segments, i.e. the low income segment and the medium income segment, about 30% of the households were represented in the mail survey. The overall response rate was remarkably high, 84%, and for each income segment clearly over 80%.
The behaviour of a household actually represents a kind of aggregate behaviour built up of the behaviours of several household members. Consequently information from several household members were collected in the survey. It was, however, obvious that too young members could not be included in the survey. The solution was that each household member aged 15 or more answered an own questionnaire containing a small number of highly structured questions. The questionnaires were identical. The survey data used in our present simulations concern (1) each respondent’s approximate exposure to mass media per time unit, more precisely daily newspapers and weekly magazines, (2) the approximate number of times each respondent visits relatives, neighbours, and acquaintances per time unit, daily work contacts excluded, and (3) a couple of questions by means of which contact probabilities within and between our three income segments are estimated.28

Our two exposure variables, $C_m$ and $EM_m$, were estimated in two steps. Firstly, the individual member exposures within each household were summed up resulting in exposures per household. Secondly, the average of these household exposures were calculated for each income category giving us expected values for the exposure variables in each such category. The calculations are described and discussed in detail in Lerviks (1973, pp. 118-130 and pp. 139-144). Note that our estimates of $C_m$ only contain contacts appearing when household members visit other households and their members, not contacts appearing when household members are visited by other household members. According to our diffusion model contacts between a non-owner and an owner that recently has become an owner are crucial for the diffusion process. This is especially the case when the contact takes place in the owner’s home where the new durable, in our case a television set, usually is situated. Due to this reasoning we base our estimates solely on contacts taking place in visiting other households and their members. The estimated exposure variables are shown in Table 3.

<table>
<thead>
<tr>
<th>Household category</th>
<th>$C_m$</th>
<th>$EM_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low income</td>
<td>127</td>
<td>7.73</td>
</tr>
<tr>
<td>Medium income</td>
<td>174</td>
<td>10.67</td>
</tr>
<tr>
<td>High income</td>
<td>152</td>
<td>10.50</td>
</tr>
</tbody>
</table>

In estimating the probability that household members in segment $m$ will be in contact with household members in segment $n$, $P_{mn}$, the following approach was used. The question about how often each household member visits relatives, neighbours, and acquaintances was immediately followed by questions concerning the persons visited

28 As mentioned earlier the data used here was originally collected in order to validate an earlier version of the present model. This validation included ten segments specified according to income, age of household head, and language (Finnish or Swedish). (Lerviks, 1973).
in such contacts, questions that enabled us to decide to which segment the person(s) visited most frequently belonged. From the answers to these questions the desired contact probabilities on a household level were calculated using an approach in which the contact behaviours of single household members were weighted with their contact frequencies per time unit. The specific questions and calculations are described and discussed in detail in Lerviks (1973, pp.130-139). The estimated contact probabilities within and between our three income segments are shown in Table 4. Note that the table is calculated on the basis of contacts appearing when household members are visiting other households and their members. This is indicated by From\To in the table. As a consequence of this the rows sum up to one but not the columns.

Table 4. Contact probabilities within and between income segments

<table>
<thead>
<tr>
<th>From \ To</th>
<th>Low income</th>
<th>Medium income</th>
<th>High income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low income</td>
<td>0.73</td>
<td>0.21</td>
<td>0.06</td>
</tr>
<tr>
<td>Medium income</td>
<td>0.29</td>
<td>0.54</td>
<td>0.17</td>
</tr>
<tr>
<td>High income</td>
<td>0.23</td>
<td>0.27</td>
<td>0.50</td>
</tr>
</tbody>
</table>

The estimates in Tables 3 and 4 are, as was pointed out earlier, based on survey data collected in 1971. We will, however use these estimates in our simulations covering the years 1958-1968. We are, in other words, using these estimates as approximations for earlier time periods assuming that the contact and exposure behaviours in question are rather stable over time.

Finally, when applying the mixed diffusion model we also need observations on the amount of advertising concerning television sets in daily newspapers and weekly magazines in Porvoo for each of our 32 time periods. These data were obtained, also in 1971, from a market research company (Markkinatutkimus Oy) that for many years regularly had collected advertising data in daily newspapers and weekly magazines all over Finland. The advertising volumes were measured in column millimetres.

Seasonal fluctuations in the advertising volumes were removed from the time series because there was no reason to believe that the low sales figures in the summer periods resulted from the low advertising volumes in the corresponding periods. In removing the seasonal fluctuations a seasonal index was used. This index was calculated using the percentage moving average method (Spiegel, 1961, p. 287). The advertising time series used here, $A(t)$, is shown in Appendix D. The advertising time series is dealt with in detail in Lerviks (1973, pp. 154-158 and p. 252). Observed advertising exposures in our three segments are achieved by multiplying $EM_m$ with $A(t)$, cf. Equation (8).
**Diffusion sensitivity to word-of-mouth parameters**

So far we have presented our validation time series consisting of observed time series for the television adoption behaviour in three income segments covering 32 time periods or close to 11 years. We have also presented the first step of our calibrations, i.e. the estimation of the contact and exposure parameters that will be used in validating the diffusion model. What remains for us to do is to calibrate the word-of-mouth parameters, $d$ and $K$, as well as the internal and external influence parameters, $a_m$ and $b_m$. In calibrating these parameters it should be remembered that the influence parameters are conditional on the word-of-mouth parameters. This being the case we start our calibrations from the word-of-mouth parameters analysing how sensitive simulated diffusion processes are to changes in (different values of) these parameters and, at the same time, looking for parameter values producing good fits between simulated and observed time series. Naturally, these analyses have to include repeated calibrations of the influence parameters (a new calibration for each new word-of-mouth parameter value) in order for us to be able to simulate the diffusion processes according to the new conditions. The word-of-mouth parameter values giving the best fit then serve as estimates in further validations of the diffusion model in the next section. We are, in other words, using validation through simulations as a means of calibrating our word-of-mouth parameters.

Furthermore we use, both in this section and in the next one, an approach resembling successive updating as described by Armstrong (1985, p. 343). In this approach we, in a first step, estimate the influence parameters using only the very first time series observations (our calibration time series). This calibration is carried out using the optimization algorithm described in Chapter 4. Then we simulate the diffusion processes over all 32 time periods (our validation time series) using these parameter estimates. In these simulations we use the observed values of our exogenous variables for each simulated time period. In a second step we increase the length of the calibration time series, reestimate the influence parameters and simulate the diffusion processes all over again using the new parameter estimates, everything else being unchanged. We proceed in this way by increasing the length of the calibration time series in each new step up to the point when the calibration time series coincides with the validation time series. This means that each simulation covers the calibration time series plus the time periods left (if any) of the validation time series. The part of a simulation covering the time periods beyond the calibration time series can be called an unconditional ex post forecast, i.e. a forecast made over past time periods with the exogenous variables known with certainty while the influence parameters are calibrated on the basis of observations preceding the forecast period. In the following these parts of our simulations sometimes will be referred to as unconditional forecasts. By letting observed exogenous variables affect the simulated outcomes in this way we can analyse and evaluate various aspects and qualities of our diffusion mechanism as such better, especially its predictive qualities.

Our estimation of the word-of-mouth parameters $d$ and $K$ follows the approach put forward in Chapter 4 under the heading ‘Calibration approaches’. In a first step we choose a high enough $K$-value, i.e. $K = 12$. From Figure 12 we can see that the probability to discuss the durable when $K > 12$ is marginal for most $d$-values. In our case a $K$-value equal to 12 holds in it an assumption that the willingness to discuss the
durable in a contact between a non-owner and an owner due to the fact that the owner acquired the durable not too long ago exists up to four years after the acquisition. In all calibrations and validations in the following we will keep this assumption unchanged.

In a second step we estimate the internal and external influence parameters given different $d$-values. In these calibrations, one for each $d$-value, we use the mixed diffusion model and as optimization criterion the Mean Absolute Error for new demand, cf. Equation (27). Using the parameter estimates of each such calibration we simulate the diffusion processes over all 32 time periods using the observed exogenous variables. This gives us as many simulations as we have calibrations, i.e. one for each prespecified $d$-value. Finally simulated time series are compared with observed ones using accuracy measures discussed in Chapter 4. All this is done repeatedly for increasing lengths of the calibration time series, everything else being unchanged. In other words, we are successively updating our estimates of the influence parameters for each $d$-value. Our first calibration covers 5 time periods (1958-1959), the following 17 time periods (1958-1963), and the last one 32 time periods (1958-1968).

**Figure 12. Decreasing probability to discuss the durable for four $d$-values**

![Graph showing decreasing probability to discuss the durable for four $d$-values](image)

In preliminary analyses different values of $d$ were tested in the way just described. Very soon it became clear that $d$-values exceeding 0.5 consistently led to simulated outcomes that differed much more from the observed time series than simulated outcomes based on $d$-values lower than 0.5. A systematic test design was then put up in which four $d$-values were chosen, 0.2, 0.3, 0.4, and 0.5, to be used in calibrating the influence parameters on the basis of observed time series of various length, i.e. 5, 17, and 32 time periods. This gave us twelve simulation cases. Each of these cases produced eight simulated time series covering our 32 time periods, i.e. time series for
new demand and owner shares in each income segment as well as in the sample as a whole representing the corresponding time series in segments aggregated.

As stated above we use three calibration time series, the first covering only a few observations at the beginning of the analysed diffusion processes (1958-1959), the second covering observations up to a level when the diffusion processes are well on their way (1958-1963), and the third all our observations, i.e. up to a level when the diffusion processes are deteriorating (1958-1968). By this approach we want to find out whether $d$-values performing well using estimates based on the first calibration time series also perform well when we use estimates based on the second and on the third calibration time series. If this is so, i.e. that good-fitting $d$-values appear to be rather stable over time, early estimates of $d$ can be useful in long-term forecasting. Furthermore such findings would support the assumptions underlying the word-of-mouth behaviour and its operationalization in our diffusion model.

The decreasing probability to discuss the durable in a contact between a non-owner and an owner due to the time since the owner acquired the durable is illustrated in Figure 12 for each of the prespecified $d$-values used in our test design. We can see that this probability in, for example, one year (three periods) after the acquisition has decreased approximately with one third when $d = 0.2$ and with as much as two thirds when $d = 0.5$. In analysing the fit between simulated and observed time series we are especially interested in whether any of the analysed $d$-values performs better than the other ones and, furthermore, how stable these performances are when the length of the calibration time series is increased.

In evaluating how well simulated time series fit observed ones we need accuracy measures that are relative in the sense that a measurement for one segment has to be comparable with corresponding measurements for other segments or the whole sample. Such measures, among those discussed in Chapter 4, are the Mean Error (ME) for owner shares, cf. Equation (21), the Mean Absolute Error (MAE) for owner shares, cf. Equation (23), and the Percentage Error in Accumulated Demand (PEAD), cf. Equation (25). Of these the first two measure the fit between simulated and observed owner shares. The first one expresses the mean value of the errors (deviations) between simulated and observed owner shares letting errors with opposite signs offset each other. The second one, in turn, expresses the mean magnitude of these errors. The last one measures the fit between simulated and observed new demand accumulated over all simulated time periods in relative terms. We use these three accuracy measures simultaneously here since they clearly complement each other.

As mentioned above we use MAE for new demand as optimization criterion when we are testing the performance of different $d$-values. Preliminary simulation tests showed that calibrations based on new demand tend to predict the diffusion patterns better at an early stage of the diffusion processes than calibrations based on owner shares, a finding that will be further demonstrated in the next section. Due to this finding we chose MAE for new demand as optimization criterion. We also use exactly the same

---

29 The ME and MAE for new demand are dependent on the sizes of the segments and consequently not directly comparable between segments.
iteration steps in all our calibrations, i.e. 0.001 for internal influences and 0.00001 for external influences. We are, in other words, keeping the calibration precision unchanged. In this way we are not letting different levels of precision affect the simulated outcomes. Since our main purpose in this section is to find out which one of the prespecified $d$-values performs best in simulation runs, we will concentrate on comparing simulated time series with observed ones here and leave presentations of estimated influence parameters to the next section.

**Table 5. Simulation accuracy for four $d$-values in twelve simulations**

<table>
<thead>
<tr>
<th>Calibr. series $d$-value</th>
<th>Accuracy measurements</th>
<th>Low Income</th>
<th>Medium Income</th>
<th>High Income</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>5</td>
<td>.2</td>
<td>-.13</td>
<td>.13</td>
<td>-100.0</td>
<td>.01</td>
</tr>
<tr>
<td>5</td>
<td>.3</td>
<td>-.01</td>
<td>.02</td>
<td>-12.4</td>
<td>-.06</td>
</tr>
<tr>
<td>5</td>
<td>.4</td>
<td>-.13</td>
<td>.13</td>
<td>-100.0</td>
<td>-.17</td>
</tr>
<tr>
<td>5</td>
<td>.5</td>
<td>-.08</td>
<td>.09</td>
<td>-69.8</td>
<td>-.21</td>
</tr>
<tr>
<td>17</td>
<td>.2</td>
<td>-.04</td>
<td>.04</td>
<td>-43.9</td>
<td>.05</td>
</tr>
<tr>
<td>17</td>
<td>.3</td>
<td>.01</td>
<td>.01</td>
<td>-10.7</td>
<td>-.02</td>
</tr>
<tr>
<td>17</td>
<td>.4</td>
<td>-.01</td>
<td>.02</td>
<td>-37.4</td>
<td>-.06</td>
</tr>
<tr>
<td>17</td>
<td>.5</td>
<td>-.04</td>
<td>.04</td>
<td>-51.7</td>
<td>-.04</td>
</tr>
<tr>
<td>32</td>
<td>.2</td>
<td>.06</td>
<td>.06</td>
<td>38.8</td>
<td>.06</td>
</tr>
<tr>
<td>32</td>
<td>.3</td>
<td>.02</td>
<td>.02</td>
<td>-5.6</td>
<td>.00</td>
</tr>
<tr>
<td>32</td>
<td>.4</td>
<td>.05</td>
<td>.05</td>
<td>12.3</td>
<td>-.07</td>
</tr>
<tr>
<td>32</td>
<td>.5</td>
<td>.02</td>
<td>.03</td>
<td>-10.8</td>
<td>-.03</td>
</tr>
</tbody>
</table>

A = Mean Error (ME) for owner shares  
B = Mean Absolute Error (MAE) for owner shares  
C = Percentage Error in Accumulated Demand (PEAD)

Model: Mixed Diffusion Model  
Simulated time periods: 32  
Optimization criterion: Mean Absolute Error for new demand  
Iteration steps: .001 in calibrating $a_m$ and .00001 in calibrating $b_m$

In Table 5 each row represents one simulation run covering 32 time periods. The first column specifies the length of the calibration time series, i.e. the time series on the basis of which the influence parameters used in each simulation are estimated. If, for example, the length of the calibration time series is 5, the unconditional forecast...
covers the rest of the 32 time periods, i.e. 27 time periods. The second column shows, in turn, the \( d \)-value used in each specific calibration/simulation. For each simulation run twelve accuracy measurements are shown. Comparisons are made between measurements in each column for each calibration time series, each measurement representing different \( d \)-values. The measurement representing the best fit is underlined in the table. We can then easily see which \( d \)-values produce the best fits on various accuracy measures.

For the calibration time series consisting of 5 time periods (1958-1959) we can see that the \( d \)-value 0.3 performs better in the whole sample than the other \( d \)-values. PEAD shows that the simulated new demand among all households in the whole period 1958-1968 lies about 10% under the real one when \( d = 0.3 \). The corresponding new demand when \( d = 0.2 \) lies about 22% under and for the remaining two \( d \)-values almost 60% under. The \( d \)-value 0.3 also gives the best fit between simulated and observed owner shares with MAE = 0.05 meaning that the simulated owner share on average deviates from the observed one with 0.05 or with 5% of all households. The ME-value –0.04 tells us that the simulated owner share in most cases has been lower than the observed owner share. Furthermore the \( d \)-value 0.3 performs clearly best in the low income segment. The \( d \)-value 0.2, in turn, performs best in the medium income segment and also in the high income segment as to ME and MAE but not PEAD. Mainly due to the fact that the \( d \)-value 0.3 performs best in the whole sample we conclude that this \( d \)-value seems to perform slightly better than 0.2 when we only look at simulations using influence parameters estimated from very few time series observations of which several are zero-observations. Actually the observed amount of new demand in the first 5 time periods after launch was among low income households 2 television sets, among medium income households 12 television sets and among high income households 4 television sets, cf. Appendix C.

When we increase the length of our calibration time series to 17 time periods the results are about the same. However, now the \( d \)-value 0.2 performs slightly better than 0.3 in the whole sample, while 0.3 performs better among low income households and also among medium income households as to ME and MAE. On the whole we conclude that the \( d \)-values 0.2 and 0.3 seem to perform about equally well in simulations using influence parameters estimated from observed time series covering the years 1958-1963. Also here the \( d \)-values 0.2 and 0.3 clearly outrule 0.4 and 0.5.

Finally, looking at simulations in which the calibration time series coincides with the validation time series, we can see that the \( d \)-value 0.3 again performs clearly best in the whole sample and also best or close to best in the three segments. The \( d \)-value 0.4 now performs about as good as 0.2, while 0.5 performs worst also here. Note, however, that almost all \( d \)-values, except 0.2, perform better here than in the earlier simulations, obviously due to the fact that the calibration time series and the validation time series here are the same.

To further illustrate the simulations discussed so far we will show the simulated and observed growth curves for some of our 12 simulation cases in Table 5. We choose to show the simulations using the \( d \)-value that gives the best performance, i.e. 0.3, and the simulations using the \( d \)-value that gives the worst performance, i.e. 0.5. For each
of these $d$-values we show simulations using influence parameters calibrated on the basis of observations covering 5 time periods and 17 time periods.

*Figure 13. Observed and simulated owner shares (Illustration 1)*

In Figure 13 the observed and simulated owner shares corresponding to row two in Table 5 are shown, i.e. $d = 0.3$ with the influence parameters calibrated on the basis of observations covering the first 5 time periods. Figure 13 represents the output as seen on the screen when using DEMSIM. Besides the ME’s and MAE’s the output also includes the maximal single error in each simulation, denoted by MxE. The length of the calibration time series is for each segment indicated by a black arrow in the figure, as well as in all similar figures in the following.

Figure 14, in turn, shows the corresponding outcomes with the only difference that the influence parameters are calibrated on the basis of observations covering the first 17 time periods, i.e. row six in Table 5. The good fits in Figure 14 are further improved when the calibration time series is increased to 32 time periods and $d = 0.3$, cf. Table 5. This is, however, not surprising since the calibration time series in this case coincides with the simulation time series. From a forecasting point of view it is much more important to analyse whether the fit still is good when the simulation turns into an unconditional forecast, i.e. also covers time periods not included in the calibration time series.
Figures 15 and 16 are in all respects equivalent to Figures 13 and 14 except for the $d$-value used. We can see that the fits are rather good up to the point when the simulations reach the end of the calibration time series. This is naturally so because the influence parameters are estimated on the basis of sales observations up to that point. However, after that point the simulated owner shares, now representing unconditional forecasts, lie clearly and heavily under the observed owner shares. This is not at all as obvious when we use the $d$-value 0.3, cf. Figure 14. We can, in other words, say that the $d$-value 0.3 performs much better than 0.5 in unconditional forecasts. Obviously the $d$-value 0.3 is superior to 0.5 in fitting our diffusion model to the observed diffusion process. This can in our case be interpreted in the following way. Since increasing $d$-values bring about faster decreasing exposures to word-of-mouth information concerning our durable, cf. Figure 12, the value 0.5 obviously results in a simulated internal exposure that is too low. In other words, the willingness to discuss and demonstrate the durable after acquisitions lasts longer than the $d$-value 0.5 implies, cf. Figure 12. This fact is clearly demonstrated in Figures (15) and (16).

We have found that, of our four prespecified $d$-values, 0.3 performs best regardless of which of our three calibration time series we use in estimating the influence parameters. This means that the $d$-value 0.3 is the one among our four values that produces the best fit between simulated and observed owner shares and accumulated sales volumes, everything else being unchanged. Especially we can state that 0.3 predicts best in unconditional forecasts, cf. the cases when the calibrations are made on the basis of 5 and 17 time periods. Similarly we can state that 0.3 also gives the
best fit when the calibration time series coincides with the validation time series and no unconditional forecasts are involved. Close to 0.3 lies 0.2, which performs as well in one of the successive updating cases but not in the other two cases. The values 0.4 and 0.5 perform clearly worse than 0.3 and 0.2 in all calibration cases.

**Figure 15. Observed and simulated owner shares (Illustration 3)**

Finally we want to stress that 0.3 performs best already at the beginning of the diffusion processes, i.e. when the influence parameters are estimated on the basis of observations covering only the first 5 time periods after launch when very few units of our durable still have been sold, cf. Appendix C. This position of best-performing $d$-value was maintained by 0.3 through each of our successive updating cases. This indicates, at least in our specific case, stability over time in the behaviour expressed through the $d$-value. From a forecasting point of view this is important. If this is so a $d$-value estimated in early stages of a diffusion process can be very useful in long-term forecasting.

We are ending this section by stating the explicit meaning of the $d$-value 0.3 in our specific application, cf. Figure 12. In the first year after the acquisition (adoption) of our durable the probability that the durable will be discussed in a contact between the owner and a non-owner falls to a level slightly higher than 0.5 of the original level, two years after to a level slightly higher than 0.2, three years after to approximately a 0.1-level and four years after to approximately a 0.05-level.
All validations carried out in the next section will use as \(d\)-value 0.3 and as \(K\)-value 12. It should be pointed out that this \(d\)-value by no means can be regarded as optimal in our case. We can only state that the value 0.3 seems to perform better than the three other values in the tests we have performed. It is, for example, possible that a \(d\)-value between 0.3 and 0.2 would perform better than 0.3 or that 0.2 would perform better than 0.3 if we would run further tests using the mean absolute error for owner shares instead of the mean absolute error for new demand as optimization criterion.

**Forecasting accuracy of the diffusion model**

In this section we will, among other things, concentrate on the calibration of the internal and external influence parameters using the successive updating approach while keeping the \(d\)- and \(K\)-values unchanged. Our main purpose is to find out how soon after launch our diffusion mechanism in combination with our calibration approach are able to forecast and detect crucial phases of our observed diffusion processes. Like in the previous section we use, after each new updated calibration, unconditional *ex post* forecasts (simulations) in validating the diffusion model. We use, in other words, observed values for each exogenous variable in our forecasts in order to be able to draw conclusions as to the predictive power of the diffusion mechanism built into DEMSIM.
The successive updating approach in this section works in the following way. In a first step we calibrate the influence parameters on the basis of observations covering only 2 time periods (year 1958)\textsuperscript{30}, in the next step our calibration time series covers 5 time periods (years 1958-1959), then 8 time periods (years 1958-1960), and so on up to 32 time periods (years 1958-1968). Mainly due to the seasonal fluctuations in our data we lengthen our calibration time series with 3 time periods (1 year) in each new calibration step. After each such new calibration we repeat our simulation over the 32 time periods using the updated parameter estimates. Using this approach we can see how soon after television was launched in Porvoo that our diffusion mechanism is able to e.g. predict the time and level of the observed sales peaks (cf. Figure 9) and, of course, whether the parameter estimates are stabilizing when the calibration time series is lengthened and if so, how soon after launch this happens. These are important questions in evaluating the predictive power of our model.

The successive updating and validation approach, described above, is carried out for four cases. We are analysing the simulated outcomes of the pure diffusion model and the simulated outcomes of the mixed diffusion model as well as the simulated outcomes using, on the one hand, the mean absolute error for new demand, \(\text{MAE}^{\text{dem}}\), as optimization criterion and, on the other, the mean absolute error for owner shares, \(\text{MAE}^{\text{own}}\), as optimization criterion. When we combine the two model types with the two optimization criteria we get four cases as shown in Table 6. We will refer to these cases by their case number specified in the table.

\begin{table}
\centering
\caption{Four validation cases}
\begin{tabular}{lcc}
\hline
Optimization criterion & \(\text{MAE}^{\text{dem}}\) & \(\text{MAE}^{\text{own}}\) \\
\hline
Pure Diffusion Model & Case 1 & Case 2 \\
Mixed Diffusion Model & Case 3 & Case 4 \\
\hline
\end{tabular}
\end{table}

In the following we will, besides analysing each case separately as explained above, also make comparisons between the cases. We are interested in how well the two optimization criteria perform in a forecasting sense as compared to each other. The same holds for the two model types. We will start by analysing the pure diffusion model, first Case 1 and then Case 2. The optimization criterion in Case 1 is the \(\text{MAE}\) for new demand and in Case 2 \(\text{MAE}\) for owner shares. Since the main purpose of DEMSIM is to forecast \textit{demand} for new consumer durables it seems natural to start by using the optimization criterion based on deviations between observed and simulated \textit{demand} volumes, in our case \(\text{MAE}\) for new demand (Case 1). But we may also be interested in, for example, forecasting the level of the owner share when the market is saturated. Then it seems natural to use \(\text{MAE}\) for owner shares as

\textsuperscript{30} Note that black and white television entered the Finnish market in the late spring of 1958. This explains why our time series cover only two 4-month periods in 1958.
optimization criterion (Case 2). Our main research question on these matters is to find out whether we will end up with different calibration results and simulation outcomes due to which optimization criterion we use and, if this happens, which the main differences are. In other words, does it matter which optimization criterion we use?

Following the successive updating approach in the way just described we are making 11 calibrations\(^{31}\) in each of our four cases, each calibration followed by a new simulation always covering our 32 time periods. After each calibration we use, in other words, the recalibrated influence parameter values in the subsequent simulation, everything else being unchanged as compared to the earlier simulations.

**Table 7. Successively updated estimates of influence parameters (Case 1)**

<table>
<thead>
<tr>
<th>Calibr. no.</th>
<th>Calibr. series</th>
<th>Low Income (a_1)</th>
<th>Medium Income (a_2)</th>
<th>High Income (a_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>0.000</td>
<td>0.001</td>
<td>0.007</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.002</td>
<td>0.003</td>
<td>0.004</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>0.002</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>0.001</td>
<td>0.003</td>
<td>0.006</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>0.002</td>
<td>0.003</td>
<td>0.006</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>0.002</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>0.002</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>0.002</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>0.002</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>0.002</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>0.002</td>
<td>0.003</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Model: Pure Diffusion Model  
Word-of-mouth parameters: \(d = 0.3\) and \(K = 12\)  
Optimization criterion: MAE for new demand  
Iteration step: .001 in calibrating \(a_m\)

\(^{31}\) Note that estimates appearing in the same row in Table 7 depend on each other and are, as a consequence of this, calibrated simultaneously in simulation runs. Estimating the three parameter values in one row using our optimization algorithm involves usually thousands of simulation runs repeatedly covering the calibration time series in question. Hence a row of estimates in Table 7 is to be looked upon as a whole. A change in the estimated parameter value for one segment may very well change the estimated parameter values for the other segments. In other words, in Case 1 our optimization algorithm looks for those three estimates that together minimize the mean absolute deviance (MAE) between observed and simulated sales volumes in all segments.
In Cases 1 and 2 our calibrations concern solely the internal influence parameter. We then have three values of the coefficient of internal influence, one for each segment, to be estimated using our optimization algorithm. In Cases 3 and 4 we are dealing with the mixed diffusion model having six parameter values to be estimated in each calibration, i.e., three estimates of the coefficient of internal influence and three of the coefficient of external influence. For each case we show the results of each of the 11 calibrations in one table and the accuracy measurements for each of the 11 simulations in another. The iteration steps and the accuracy measures are identical to those in Table 5. Consequently we can compare our results here with those in Table 5.

Case 1 (Pure Diffusion Model - MAE for new demand). The successively updated estimates of the internal influence parameter are shown in Table 7 for Case 1. As can be seen the estimates do not differ much between the calibrations. Actually they are identical in 7 of our 11 calibrations. The most frequent estimates are underlined in the table. From calibration 6 on we end up with the same estimates although we add new sales data to the calibration time series in 5 steps (Calibrations 7-11) covering up to 5 years (15 time periods) of new sales data in the last step (Calibration 11).

Table 8. Simulation accuracy in eleven simulations (Case 1)

<table>
<thead>
<tr>
<th>Simul. no.</th>
<th>Calibr. series</th>
<th>Accuracy measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Low Income</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A  B  C</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-.13 .13 -100.0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>-.01 .02 -12.4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>.01 .01 -4.9</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>-.07 .07 -62.8</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>.03 .03 -2</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>.01 .01 -4.9</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>.01 .01 -4.9</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>.01 .01 -4.9</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>.01 .01 -4.9</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>.01 .01 -4.9</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>.01 .01 -4.9</td>
</tr>
</tbody>
</table>

A = Mean Error (ME) for owner shares
B = Mean Absolute Error (MAE) for owner shares
C = Percentage Error in Accumulated Demand (PEAD)

Model: Pure Diffusion Model
Simulated time periods: 32
Word-of-mouth parameters: $d = 0.3$ and $K = 12$
Optimization criterion: MAE for new demand
Iteration step: .001 in calibrating $a_m$
In Table 8 the accuracy of the simulations carried out using the estimates in Table 7 are shown. We have, in other words, as many simulations as we have calibrations. The lowest accuracies appear, not surprisingly, in the first simulation using influence parameters estimated from sales observations covering only the first two time periods. Within our low income segment no one adopted in the first two time periods, cf. Appendix C. This leads to a 0-estimate as to the influence parameter, cf. Table 7, and a PEAD-value of –100.0, cf. Table 8. This value tells us that the simulated demand accumulated over all 32 time periods equals 0, i.e. it lies 100% under the observed. However, the simulation for high income households is surprisingly accurate with the simulated cumulative demand lying only 4.9% over the observed one, taking into account that this outcome is based on calibrations involving very low sales volumes, actually a total of only 3 television sets sold in the whole sample in the first two time periods, cf. Appendix C.

**Figure 17. Observed and simulated new demand yearly (Case 1, Simulation 1)**

To visualize some of the outcomes of Simulation 1 we show the observed and simulated sales volumes in Figure 17. As we could see from Figure 9 our observed sales volumes reveal strong seasonal fluctuations. Seasonal fluctuations, being a consequence of the specific time unit used in applications (in our case 4 months), are naturally not built into our general diffusion model. Consequently the visualization serves its purpose better here if we show observed and simulated new sales on a
yearly basis like we already did for observed new sales in Figure 10. In Figure 17 the simulated outcomes are aggregated according to calendar years and compared to the observed new sales (new demand) as presented in Figure 10.

Although the overall accuracy of Simulation 1 is not too good it can be pointed out that the model seems to forecast the timing of the sales peaks among medium and high income households rather well already at this very early stage of the diffusion processes, cf. Figure 17. Since no units were sold among low income households in the first two time periods it is natural that the simulation outcomes show a zero-forecast for this segment.

**Figure 18. Observed and simulated new demand yearly (Case 1, Simulation 2)**

Moving on to Simulation 2 the fit between observed and simulated new demand is clearly improved, cf. Figure 18 and Table 8. The unconditional forecasts in Figure 18 are denoted by red arrows, in this case covering the years 1960-1968. The observed sales peak in the whole sample appeared in year 7 (1964). The forecasted sales peak covers approximately the years 1964-1966 on a somewhat lower level. The observed and forecasted sales for these three years together are, however, about the same. The model forecasts, in other words, both the timing and the magnitude of the sales peak in the whole sample rather well as early as 5 years before the sales peak appears. The forecasted timing is, however, lagging about one year. The same pattern holds for the
medium income segment. For the low income segment the timing of the sales peak is about right but the forecasted magnitude too small, while the forecasted sales peak for the high income segment is too low and appears three years too late.

*Figure 19. Observed and simulated new demand yearly (Case 1, Simulation 3)*

The corresponding outcomes of Simulation 3 are shown in Figure 19. In Appendix E we also show these outcomes as simulated, i.e. on a 4-month basis including the seasonal fluctuations in the observed sales data, cf. Figure 9. As we can see these outcomes are more detailed but not as easy to get an overview of as those in Figure 19. This follows from the fact that the observed sales volumes contain seasonal fluctuations while the simulated do not.

In Simulation 3 the fit between observed and simulated new demand is further improved. For the medium income segment the fit is very good. Among low income households the fit is also good except for, maybe, the last two years. Among high income households the fit is better than in the previous simulation but still the simulated sales peak is slightly too low and is still lagging behind. This discrepancy between observed and simulated sales volumes for high income households does not affect the whole sample forecast much in the present case because the high income segment is small as compared to the two other segments, cf. Appendix A. As we can see the fit in the whole sample is very good, especially concerning the timing and
In Simulation 3 the unconditional forecast starts from the beginning of 1960, i.e. 4 years before the sales peak appears. Taking this into account the pure diffusion model seems to perform rather well in our specific application.

**Figure 20. Observed and simulated owner shares (Case 1, Simulation 3)**

Simulations 6-11 are identical to Simulation 3, cf. Tables 7-8. The two simulations in-between, Simulations 4 and 5, do not perform as well as Simulation 3 but not too bad either, cf. Table 8. To further illustrate the best-fitting simulation we show the observed and simulated owner shares for Simulation 3 in Figure 20. Note that Figures 19 and 20 also apply to Simulations 6-11 except for the length of the calibration time series and the length of the unconditional forecast, cf. Table 8. From Table 8 we could see that the ME and MAE for owner shares in most simulations were higher for the high income segment than for the two other segments. This is also evident looking at Figure 20. From Table 8 we see that the ME for owner shares among high income households is 0.05 in Simulation 3 meaning that the simulated owner share curve for this segment on average lies 5 percentage units above the observed owner share.

---

32 Note, however, that the simulation takes place within and between the three segments. The curves concerning the whole sample are simply the sum of the corresponding curves in each segment. If bad fits in two segments offset each other leading to a good fit in the whole sample we should analyse the bad fits carefully and not be mislead by the good overall fit.
curve. The MAE-value for owner shares, 0.07, in turn tells us that the simulated owner share curve on average lies 7 percentage units from the observed owner share curve. The MAE-value for the other two segments lie in the range of 1-2 percentage units. There may be many possible explanations to this. One could be that some kind of specification error within the model exists, another that important external influences are lacking from the model, to mention only two possible explanations.

In the pure diffusion model the adoption probability function, cf. Equation (10), is reduced to the following form.

\[ AP_m(t) = a_m \text{INEXP}_m(t) \]

Since only internal exposures affect the adoption probabilities in Equation (28) and since these internal exposures are based on the same measurement unit, the internal influence estimates, \( a_m \), are comparable among themselves and can, in this special case, be interpreted as indicators of the resistance to adopt in our segments. In our application the estimates express, so to say, the change in the adoption probability when the internal exposure is changed by one unit. In Figure 21 the simulated internal exposures, \( \text{INEXP}_m(t) \), for Case 1, Simulation 3, are shown.

Figure 21. Simulated internal exposures over time (Case 1, Simulation 3)

Table 7 shows the estimates of the internal influence parameter in Case 1, Simulation 3, i.e. .002 for low income households, .003 for medium income households, and .005 for high income households. From Figure 21 we see that high and medium income households are exposed to information concerning our durable in personal contacts with owners at least twice as much as low income households up to approximately period 23. Our internal influence estimates measure, however, the willingness of our households to adopt the durable under equal internal exposure. The lower these estimates are, the stronger is the resistance. In other words, in our application the resisting forces seem to be 2.5 times stronger among low income households than among high income households, 1.5 times stronger among low income households than among medium income households, and 1.7 times stronger among medium households. 33 This is naturally not the case when we use the mixed diffusion model getting estimates of both the internal and external influence parameters.
income households than among high income households. Note that the very slow owner share growth in the low income segment as compared to the two other segments, cf. Figure 8, is not solely a consequence of a stronger resistance but also of a lower internal exposure to information concerning the durable in personal contacts with owners. For example, the resistance among low income households as compared to medium income households is stronger (1.5 times) but not at all as much stronger as could be expected if we only look at the owner share growths in Figure 8.

The simulated internal exposures in Figure 21 are further split according to their origin in Figure 22. The sum of the three graphs in each row of graphs represents the corresponding exposure curve in Figure 21.

**Figure 22. Simulated internal exposures according to origin (Case 1, Simulation 3)**

![Graphs showing simulated internal exposures according to origin](image)

We can see that households in general were most frequently exposed to information concerning the durable in contacts with owners from their own segment. Furthermore the relative importance of the own segment in this respect is highest among high income households and lowest among low income households. This is not surprising since the high income households are, so to say, early adopters and the low income households late adopters in our case, cf. Figure 8.

**Case 1 (Precision of estimates).** One aspect that naturally affects the parameter estimates and consequently also the simulation outcomes is the precision of the estimates. We are here using the iteration step .001 when we estimate the internal influence parameter of the diffusion model. We use, in other words, a precision with three decimal points. We have seen above that this precision in our application produces good-fitting simulation outcomes also in unconditional forecasts. To test whether and to what an extent these outcomes are dependent on the precision of the estimates we repeat all calibration and simulation runs using a precision with four
decimal points. In other words, we are applying .0001 as our iteration step. Some results of these runs are presented in Table 9 and Appendices F and G. Actually Appendix F is identical with Table 7 except for the iteration step used in the calibrations and Appendix G identical with Table 8 except for the influence parameter estimates used. In Table 9 the consequences of changing the iteration step from .001 to .0001 on the optimization criterion MAE for new demand is analysed.34

We can see that also in Appendix F the estimates stabilize when the length of the calibration time series is increased. This does not, however, happen as soon as in Table 7. As we can see the last two calibrations, Calibrations 10 and 11, are identical and Calibration 9 almost identical to these two. In Table 7 this is the case already from Calibration 6 on. This finding is natural. If we use four decimal points the estimates adjust more closely to the observations than if we use three decimal points. At the same time we naturally get more variations in our estimates. From Table 9 we can see the closer adjustment. Column A (iteration step .001) and Column B (iteration step .0001) represent the optimal MAE-value for new demand in each calibration. We can see that this value without exception is smaller in Column B than in Column A, i.e. estimates with four decimal points produce closer fits (a smaller MAE) than estimates with three decimal points, which goes without saying.35 Note, however, that the difference is generally small. Comparing, in turn, rows in each Column A and B we can see that the MAE-values are increasing with increasing length of the calibration time series. This is natural since an increasing length of the calibration time series brings about an increasing number of observations to adjust to in combination with increasing sales volumes and increasing seasonal fluctuations in these when the diffusion processes proceed in the segments. When the sales volumes per time unit get smaller the optimal MAE-values should stop growing or even get smaller. This also happens in the last calibration based on the whole validation time series.

Columns A and B in Table 9, just commented on, represent pure ex post validations. Columns C to F, in turn, represent outcomes of eleven simulations using the successively updated estimates shown in Appendix F, each simulation covering all 32 time periods in the validation time series. This means that all these simulations, except for the last one, end up with an unconditional forecast. Columns C and D measure the same thing as Columns A and B, i.e. our optimization criterion calculated by Equations (24) and (27). The only difference is that the calculations now cover the entire validation time series. They are, in other words, not optimal in Columns C and D except for the last simulation. In this simulation the calibration time series coincides with the validation time series, in what case the MAE-values are identical.

34 The appropriate number of decimal points to use when estimating the influence parameters of the model naturally depends on the volumes of the internal exposure variable, cf. Equation (5), and the external exposure variable, cf. Equation (8), in combination with the functional relationship expressed in Equation (10). The volumes of the two exposure variables, in turn, depend on the measurement units applied to the observations on the Number of personal contacts, Exposure to mass media and Advertising volumes inherent in Equations (5) and (8). There is, in other words, no general rule as to what the appropriate number of decimal points should be. This has to be decided from application to application in an explorative manner like we are doing here.

35 In Table 9 the smaller value in each row and pair of columns (A-B, C-D, and E-F) is underlined.
as can be seen from Table 9 (Column A equals C and B equals D on the last line).\textsuperscript{36} In Columns E and F the mean errors of the simulated new demand in the whole sample are shown, in Column E for iteration step .001 and in Column F for iteration step .0001. A zero mean error means that the simulated new demand cumulated over all time periods is equal to the observed new demand cumulated over all time periods. Deviations with opposite signs do, in other words, offset each other. We see that the mean error is less than one unit in a clear majority of our simulations.

\textbf{Table 9. Some accuracy comparisons using iteration steps .001 and .0001 (Case 1)}

<table>
<thead>
<tr>
<th>Calibr. time series</th>
<th>Calibrations</th>
<th>Simulations (.32 time periods)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>2</td>
<td>.200</td>
<td>.177</td>
</tr>
<tr>
<td>5</td>
<td>.715</td>
<td>.705</td>
</tr>
<tr>
<td>8</td>
<td>.906</td>
<td>.879</td>
</tr>
<tr>
<td>11</td>
<td>1.329</td>
<td>1.261</td>
</tr>
<tr>
<td>14</td>
<td>1.930</td>
<td>1.866</td>
</tr>
<tr>
<td>17</td>
<td>2.263</td>
<td>2.228</td>
</tr>
<tr>
<td>20</td>
<td>2.622</td>
<td>2.593</td>
</tr>
<tr>
<td>23</td>
<td>2.765</td>
<td>2.721</td>
</tr>
<tr>
<td>26</td>
<td>2.943</td>
<td>2.918</td>
</tr>
<tr>
<td>29</td>
<td>3.043</td>
<td>3.004</td>
</tr>
<tr>
<td>32</td>
<td>2.951</td>
<td>2.906</td>
</tr>
</tbody>
</table>

\textsuperscript{36} Note that this MAE-measure represents the optimization criterion, i.e. a mean calculated from absolute deviations between simulated and observed new demand in all segments. We can, in other words, state that the absolute deviation between simulated and observed new demand within segments in most of our simulations on average lies slightly under 3 units per segment. Note that this number is
We have seen that an increased precision when estimating the internal influence parameter leads to slightly lower optimal MAE-values in pure \textit{ex post} validations, cf. Columns A and B. From our simulations, that include unconditional forecasts, it is, however, not clear that the increased precision also improves the forecasting power of our model, at least in the present application. The MAE-values in Columns C and D are in most cases lower in Column D but in three cases they are lower in Column C. This is further stressed in Columns E and F where the lowest values are almost evenly distributed between the two columns.

Furthermore Appendix G reveals, when compared with Table 8, that an increased precision in estimating the internal influence parameter lowers the simulation accuracies measured as ME and MAE for owner shares, especially in the low and medium income segments and in the whole sample. Stating this it should be remembered that our optimization criterion in Case 1 is MAE for new demand and not for owner shares. Our empirical findings imply, in other words, that an increased precision in estimating the internal influence parameter using MAE for new demand as optimization criterion leads to slightly deteriorated fits between simulated and observed owner shares.

A general observation is that the simulation outcomes based on estimates including three decimal points seem to be more stable and, above all, so in earlier phases of the diffusion process than simulation outcomes based on estimates including four decimal points. This is not surprising. There are, in every market, always extraordinary exogenous events affecting new demand suddenly and temporarily, events that are more or less impossible to forecast in advance. Such events are naturally not incorporated in our diffusion model but their effects on new demand are present in the observed sales volumes. Actually the pure diffusion model only allows internal influence, as we have defined it, to affect new demand. Under these circumstances it is possible that a higher precision in estimating the internal influence parameter can lead to estimates that are more influenced by temporary exogenous events than when we use a lower precision in our calibrations. The bigger fluctuations in our simulation outcomes when we are using a higher precision could be an indicator of this. Since our main purpose is concerned with long-term forecasting we want to keep the estimates of the internal influence parameter as clean as possible from temporary exogenous influences. This is so because we assume that the internal influence, as defined in Chapter 2, has a long-term predictive power, while sudden exogenous events that temporarily affect new demand are more or less impossible to forecast in advance.

To conclude, we have not found anything indicating that the four decimal point precision as compared to the three decimal point precision should improve the forecasting ability of the pure diffusion model in our application. On the contrary, it seems that the three decimal point precision should be preferred in the present case due to the higher stability of the successively updated estimates brought about using this level of precision. Such a stability turning up in early stages of the diffusion process is very important from a long-term forecasting point of view. We can also

---

affected by seasonal fluctuations and the sizes of our segments. The corresponding figures for the whole sample mostly lie between 7 and 8 units.
state, as we expected, that the simulated timing of the sales peaks did not change when the level of precision was changed.\textsuperscript{37}

**Case 2 (Pure Diffusion Model – MAE for owner shares).** The successively updated estimates of the internal influence parameter are shown in Table 10 for Case 2.

**Table 10. Successively updated estimates of influence parameters (Case 2)**

<table>
<thead>
<tr>
<th>Calibr. no.</th>
<th>Calibr. series</th>
<th>Low Income $a_1$</th>
<th>Medium Income $a_2$</th>
<th>High Income $a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>.000</td>
<td>.000</td>
<td>.000</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>.002</td>
<td>.003</td>
<td>.001</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>.002</td>
<td>.003</td>
<td>.004</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>.002</td>
<td>.003</td>
<td>.005</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>.002</td>
<td>.003</td>
<td>.005</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>.002</td>
<td>.003</td>
<td>.005</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>.002</td>
<td>.003</td>
<td>.005</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>.002</td>
<td>.003</td>
<td>.005</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>.002</td>
<td>.003</td>
<td>.005</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>.002</td>
<td>.003</td>
<td>.005</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>.002</td>
<td>.003</td>
<td>.005</td>
</tr>
</tbody>
</table>

In Calibrations 4 to 11 the estimates are identical (underlined in the table) and equal to the most frequent estimates in Case 1, cf. Table 7. In other words, 8 of our 11 calibrations produce identical estimates in Case 2. The estimates based on observations covering 1958-1961 (Calibration 4) do not change any more in the subsequent 7 calibrations. In Case 1 the corresponding stability in the estimates was achieved in the calibration covering 1958-1963 (Calibration 6). In other words, the estimates reach stability two years earlier when we use owner shares as optimization criterion (Case 2) instead of new demand (Case 1).

\textsuperscript{37} Repeated calibration and simulation runs using a higher precision were carried out for all four cases in this study. For each case the results were very similar. In other words, the conclusions made above concerning Case 1 are also valid for Cases 2-4.
In Table 11 the accuracy of our 11 simulations in Case 2 are shown. Naturally identical estimates in Tables 10 and 7 produce identical accuracies in the corresponding simulations in Tables 11 and 8.

### Table 11. Simulation accuracy in eleven simulations (Case 2)

<table>
<thead>
<tr>
<th>Simul. no.</th>
<th>Calibr. series</th>
<th>Accuracy measurements</th>
<th>Low Income</th>
<th>Medium Income</th>
<th>High Income</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-13</td>
<td>-100.0</td>
<td>-31</td>
<td>37</td>
<td>-100.0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>-07</td>
<td>-58.7</td>
<td>-21</td>
<td>37</td>
<td>-90.3</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>-01</td>
<td>-12.4</td>
<td>-06</td>
<td>0.9</td>
<td>-2.3</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>0.1</td>
<td>-4.9</td>
<td>-0.1</td>
<td>0.5</td>
<td>10.5</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>0.1</td>
<td>-4.9</td>
<td>-0.1</td>
<td>0.5</td>
<td>10.5</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>0.1</td>
<td>-4.9</td>
<td>-0.1</td>
<td>0.5</td>
<td>10.5</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>0.1</td>
<td>-4.9</td>
<td>-0.1</td>
<td>0.5</td>
<td>10.5</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>0.1</td>
<td>-4.9</td>
<td>-0.1</td>
<td>0.5</td>
<td>10.5</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>0.1</td>
<td>-4.9</td>
<td>-0.1</td>
<td>0.5</td>
<td>10.5</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>0.1</td>
<td>-4.9</td>
<td>-0.1</td>
<td>0.5</td>
<td>10.5</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>0.1</td>
<td>-4.9</td>
<td>-0.1</td>
<td>0.5</td>
<td>10.5</td>
</tr>
</tbody>
</table>

A = Mean Error (ME) for owner shares  
B = Mean Absolute Error (MAE) for owner shares  
C = Percentage Error in Accumulated Demand (PEAD)

Model: Pure Diffusion Model  
Simulated time periods: 32  
Word-of-mouth parameters: $d = 0.3$ and $K = 12$  
Optimization criterion: MAE for owner shares  
Iteration step: .001 in calibrating $a_m$

We have seen that Case 2 produces stable estimates earlier in the diffusion process than Case 1. However, in the very first simulations (Simulations 1-3) Case 1, in turn, is superior to Case 2. Simulation 1 in Case 2 forecasts zero new demand and zero owner share in each segment throughout the whole simulation. The fits between simulated and observed owner shares and new demand in Simulation 2 in Case 2 are also far away from those of Simulation 2 in Case 1. This also holds for Simulation 3, but now Case 2 is closing up on Case 1.

Especially as it comes to the timing of the sales peaks the superiority of Case 1 in the first two simulations is obvious. In Case 1 we already in the first simulation, cf. Figure 17, got more or less clear hints of the timing of future sales peaks, hints that
were considerably improved already in Simulation 2, cf. Figure 18. In Case 2 the first simulation ended up in zero-forecasts, cf. Table 11. Some of the results of the second simulation in Case 2 are shown in Figure 23. We also show the observed and simulated owner shares for this simulation in Figure 24. The superiority of Case 1 as compared to Case 2 gets very clear when we compare Figure 23 with Figure 18.

**Figure 23. Observed and simulated new demand yearly (Case 2, Simulation 2)**

As can be seen the unconditional forecasts in Figure 23 do not account for the observed sales peaks at all, not to talk about the sales volumes. In Simulation 3 the situation is, however, improved. Actually the estimates of the internal influence parameter for Simulation 3 in Case 2, cf. Table 10, coincide with the corresponding estimates for Simulation 2 in Case 1, cf. Table 7. Consequently Figure 18 also applies to Simulation 3 in Case 2. In Simulations 4 and 5 Case 2 seems to perform better than Case 1. In the rest of the simulations the two cases perform as well. This performance has already been illustrated in Figures 19 and 20.

We have seen that the Case 2-simulations perform badly in the unconditional forecasts using parameter estimates based on early owner share observations, i.e. observations covering the first two years after product launch. When the estimates are based on early sales observations instead, cf. Case 1, the diffusion model performs much better in the unconditional forecasts. One reason to this is obviously that the owner shares in our application are very small and grow very slowly or, in some
periods, not at all in the first two years after launch. From Table 10 we can see that this causes zero-estimates in each segment in Simulation 1 and a very low estimate for high income households in Simulation 2, the estimates for the two other segments being equal to those in later simulations. The overall bad performance in Simulation 2 is, in other words, caused by the very low estimate of the internal influence parameter for high income households. This is not surprising when we look at Figure 24 and the observed owner share for high income households in the first 5 time periods. As we can see this owner share even decreases. This naturally affects the estimate for this segment since we use owner shares as optimization criterion. The reason for the decreasing owner share is that many new non-owners entered the high income segment in period 2, cf. Appendix A, causing our relative owner share measurement to sink although the number of owners stayed unchanged, cf. Appendix B. We can, in other words, state that the poor performance in Simulation 2 largely seems to depend on this specific condition of our data. This does not, however, explain the poor performance also in Simulation 1. Here the poor performance seems to depend on the fact that the observed owner shares at this early stage of the diffusion process still lack indications of a clear growth. Also note that observed new demand always precedes observed owner share with, so to say, one time period. Therefore the use of new demand as an optimization criterion, especially in early stages of a diffusion process, should reveal future diffusion patterns better than the use of owner shares. So far our empirical findings clearly support this statement.

**Figure 24. Observed and simulated owner shares (Case 2, Simulation 2)**

![Graph of observed and simulated owner shares](image-url)
The owner share criterion holds in it the other optimization criterion, new demand, in an accumulated form. Since owner shares are relative measures expressing the fraction of all households in a segment that own the durable at various points in time changes in the number of households in a segment also affect the owner shares. Therefore we can say that new demand is a non-aggregated and, so to say, cleaner measure of the phenomenon we are forecasting than owner share, which is an aggregated measure of the same phenomenon, which furthermore, due to its relative nature, also is affected by changes in the number of households in the segments. This can be a disadvantage in applications, especially if we use owner share as optimization criterion in early stages of a diffusion process. However, as soon as the the owner share curves start to grow exponentially, i.e. from 1960-1961 in our application, our successively updated calibrations in Case 2 produce even more stable estimates than the corresponding calibrations in Case 1. One cause of the higher stability is apparently the aggregated nature of the owner share measure. In cumulating new demand over time sudden and temporary demand increases and decreases due to various exogenous influences offset each other, at least to some extent, in the owner share measure leading to a higher stability in the estimates of the internal influence parameter.

Case 3 (Mixed Diffusion Model – MAE for new demand). Moving on to Case 3 we enlarge the pure diffusion model by incorporating external influences into the model. This is done by letting exposure to advertising concerning television sets in daily newspapers and weekly magazines also affect the diffusion processes, cf. Appendix D, Table 3, and Equation (8). The successively updated estimates of the internal and external influence parameters are shown in Table 12. Note that the only way Table 12 (Case 3) differs from Table 7 (Case 1) is that Table 12 also includes the external influence variable exposure to advertising. In estimating the external influence parameter we use a five decimal point precision. Under these conditions the calibrations show that advertising affects the probability to adopt in the high income segment but not in the two other segments. This is not the case in Calibrations 1 and 2 but from Calibration 3 on a positive effect of advertising on the adoption probability among high income households is stable and clear. In other words, in 9 of 11 calibrations we get this result. None of the 11 calibrations, in turn, reveal that advertising affects the adoption probability among low and/or medium income households. Note that the internal influence estimate for high income households drops when the external influence estimate enters the solutions. This follows automatically from Equation (10) stating that the adoption probability equals the sum of internal and external influences. All other estimates in Table 12, except those mentioned above, are identical to the corresponding estimates in Table 7.

It is not hard to understand that advertising affects the adoption probability among high income households but not among low and medium income households, bearing in mind that we are dealing with product class sales and highly aggregated advertising volumes. From Figure 8 we can see that television penetrated the high income segment much quicker than the medium and especially the low income segment. In the end of 1962 over 40% of the high income households owned a television set, 20%

---

38 This means that the solutions of Calibrations 3-11 in Table 12 bring about lower optimal MAE-values than the corresponding solutions in Table 7. The solutions of Calibrations 1-2 are, as can be seen, identical in the two tables meaning that their optimal MAE-values also are identical.
of the medium income households and under 4% of the low income households. Roughly speaking we can, in our case, call the high income households innovators and the medium and low income households followers. Then it is natural that the innovators (the high income households) to a greater extent are influenced by external sources than the followers (the medium and low income households), who are mainly influenced by earlier adopters.

Table 12. Successively updated estimates of influence parameters (Case 3)

<table>
<thead>
<tr>
<th>Calibr. no.</th>
<th>Calibr. series</th>
<th>Low Income</th>
<th>Medium Income</th>
<th>High Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$a_1$</td>
<td>$b_1$</td>
<td>$a_2$</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>.000</td>
<td>.00000</td>
<td>.001</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>.001</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>.003</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>.003</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>.003</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>.003</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>.003</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>.003</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>.004</td>
<td>.00000</td>
<td>.003</td>
</tr>
</tbody>
</table>

Model: Mixed Diffusion Model
Word-of-mouth parameters: $d = 0.3$ and $K = 12$
Optimization criterion: MAE for new demand
Iteration step: .001 in calibrating $a_{mt}$ and .00001 in calibrating $b_{mt}$

In Table 13 some accuracy measurements of the simulations carried out using the estimates in Table 12 are shown. Comparing Table 13 (Case 3) with Table 8 (Case 1) we can see that introducing the external influence into our model does not seem to improve the unconditional forecasts, on the contrary, although the optimal MAE-values in the ex post Calibrations 3-11 are improved. We can also see that Simulations 1 and 2 in Case 1 and Case 3 are identical, meaning that Figures 17 and 18 for Case 1 above also apply to Case 3. In both Case 1 and Case 3 Simulation 3 is repeated in Simulations 6-11. In other words, 7 out of 11 simulations are identical with identical influence parameter estimates in Case 1 as well as in Case 3. These simulations do, however, differ between the two cases since a positive external influence enters the scene among high income households in Case 3. In Figure 25 the observed and simulated new demand are shown on a yearly basis for Case 3, Simulation 3.
Let us compare Figure 25 with Figure 19 representing the corresponding simulation without advertising affecting high income households. We can see that the simulated new demand is consistently lower in Figure 25 than in Figure 19, something that we also could see from the PEAD-measurements in Tables 13 and 8. However, the estimated advertising effect on the adoption behaviour among high income households results in a better forecast of the timing of the sales peak among these households although the magnitude of the peak is too small. The clear systematic error in the high income segment in Figure 19 with the simulated sales peak lagging about two years after the observed one has, as it comes to the timing, been corrected in Figure 25 by letting advertising affect the adoption behaviour. This can also be seen clearly from Figure 26 showing the observed and simulated owner shares for Case 3. We can see that the observed and simulated owner share curves among high income households follow each other much closer in Figure 26 than in Figure 20, the only difference between the two figures being that advertising affects the adoption probability among high income households in Figure 26 but not in Figure 20. Also

### Table 13. Simulation accuracy in eleven simulations (Case 3)

| Simul. no. | Calibr. series | Accuracy measurements |  |  |  |  |  |  |  |  |
|------------|----------------|-----------------------|---|---|---|---|---|---|---|
|            |                | Low Income            | Medium Income | High Income | Whole sample |
|            |                | A         | B         | C         | A         | B         | C         | A         | B         | C         |
| 1          |                | -13       | .13       | -100.0    | -.27      | .27       | -81.8     | .11       | .11       | 4.9       | -.17      | .17       | -74.0     |
| 2          |                | -.01      | .02       | -12.4     | -.06      | .07       | -11.1     | -.08      | .09       | -2.3      | -.04      | .05       | -10.2     |
| 3          |                | .01       | .01       | -10.7     | -.02      | .03       | -12.4     | -.02      | .04       | -11.6     | -.01      | .01       | -11.8     |
| 4          |                | -.07      | .07       | -65.3     | -.03      | .04       | -17.4     | .05       | .05       | -1.5      | -.04      | .04       | -27.2     |
| 5          |                | .02       | .02       | -5.6      | .00       | .02       | -9.7      | .06       | .06       | .5        | .01       | .02       | -7.2      |
| 6          |                | .01       | .01       | -10.7     | -.02      | .03       | -12.4     | -.02      | .04       | -11.6     | -.01      | .01       | -11.8     |
| 7          |                | .01       | .01       | -10.7     | -.02      | .03       | -12.4     | -.02      | .04       | -11.6     | -.01      | .01       | -11.8     |
| 8          |                | .01       | .01       | -10.7     | -.02      | .03       | -12.4     | -.02      | .04       | -11.6     | -.01      | .01       | -11.8     |
| 9          |                | .01       | .01       | -10.7     | -.02      | .03       | -12.4     | -.02      | .04       | -11.6     | -.01      | .01       | -11.8     |
| 10         |                | .01       | .01       | -10.7     | -.02      | .03       | -12.4     | -.02      | .04       | -11.6     | -.01      | .01       | -11.8     |
| 11         |                | .01       | .01       | -10.7     | -.02      | .03       | -12.4     | -.02      | .04       | -11.6     | -.01      | .01       | -11.8     |

A = Mean Error (ME) for owner shares  
B = Mean Absolute Error (MAE) for owner shares  
C = Percentage Error in Accumulated Demand (PEAD)

Model: Mixed Diffusion Model  
Simulated time periods: 32  
Word-of-mouth parameters: $d = 0.3$ and $K = 12$  
Optimization criterion: MAE for new demand  
Iteration step: .001 in calibrating $a_m$ and .00001 in calibrating $b_m$
note that the unconditional forecasts cover as many as 8 years. The forecasts being unconditional means that we use the actually observed advertising time series, cf. Appendix D, in our forecasts. In doing so the forecasting part of the simulations, in the present case covering 8 years or 24 time periods, is a test of how well the internal influence mechanism of the diffusion model succeeds in forecasting sales volumes (Figure 25) and owner shares (Figure 26) 8 years ahead.

**Figure 25. Observed and simulated new demand yearly (Case 3, Simulation 3)**

In presenting Case 1 above we commented on possible explanations to the relatively high discrepancy between the observed and simulated owner shares in the high income segment as compared to the other two segments. The present Case 3 simulations reveal that one possible explanation of the high discrepancy is that advertising is not allowed to affect the adoption probability in Case 1 although advertising really affects this probability among high income households. This statement seems, on the basis of our simulation runs, plausible. The statement does not, naturally, exclude other possible explanations.

39 In earlier *ex post* validation tests on the present data using ordinary and weighted least squares as well as logit analyses it could be stated that advertising, regardless of estimation technique, affected the adoption probability significantly among high income households but not among low and medium income households, cf. Lerviks (1973 and 1984).
Case 4 (Mixed Diffusion Model – MAE for owner shares). The last validation case concerns the mixed diffusion model using MAE for owner shares as optimization criterion. Table 14 shows the results of the successively updated calibrations and Table 15 the simulation accuracies associated with each calibration. The calibration and simulation results are similar to those in the earlier cases. Comparing with Case 2 (the pure diffusion model with the same optimization criterion) we see that the first two simulations are as bad, cf. Tables 15 and 11 and Figures 23 and 24. Simulation 3 is better in both cases, cf. Tables 15 and 11, but not as good as the stable solutions achieved from Simulation 4 on, similarly in both cases. In other words, Case 4 is very similar to Case 2 as it comes to how soon the quality of the simulations improve when the length of the calibration time series is increased. The solutions under stable conditions are, however, not fully as accurate in Case 4 as in Case 2, cf. Tables 15 and 11. These findings are identical to those found when we compared Case 3 with Case 1.

Comparing, in turn, Case 4 with Case 3 (the mixed diffusion model with new demand as optimization criterion) we can see that stable solutions are achieved from Simulation 6 on in Case 3, while this happens from Simulation 4 on in Case 4, the stable solutions being identical between the two cases. The very first simulations are, however, much more accurate in Case 3 than in Case 4. These findings are identical to those found when we compared Case 2 with Case 1, i.e. Case 1 performs much better
in the very first simulations but Case 2 achieves stable solutions earlier in the diffusion processes than Case 1.

Table 14. Successively updated estimates of influence parameters (Case 4)

<table>
<thead>
<tr>
<th>Calibr. no.</th>
<th>Calibr. series</th>
<th>Low Income</th>
<th>Medium Income</th>
<th>High Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(a_1)</td>
<td>(b_1)</td>
<td>(a_2)</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>.000</td>
<td>.00000</td>
<td>.000</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>.002</td>
<td>.00000</td>
<td>.003</td>
</tr>
</tbody>
</table>

Model: Mixed Diffusion Model  
Word-of-mouth parameters: \(d = 0.3\) and \(K = 12\)  
Optimization criterion: MAE for owner shares  
Iteration step:.001 in calibrating \(a_m\) and .00001 in calibrating \(b_m\)

Some conclusions. This section has dealt with the forecasting accuracy of our diffusion model. Four validation cases explore the impact of two different optimization criteria as well as two model types on the simulated outcomes and the forecasting ability of the diffusion model based on successively updated estimates of the influence parameters of the model. The findings are, in our opinion, promising.

We can, in general, state that our specific validations show that the diffusion mechanism of DEMSIM is able to predict new demand and owner share growth in our segments rather well in relatively early stages of the diffusion processes, cf. Figures 18-20 and 26. Especially the timing of the sales peaks in various segments, but also the magnitudes of these, are predicted surprisingly well on the basis of sales observations covering 2 (1958), 5 (1958-1959), or 8 (1958-1960) time periods, cf. Figures 17-19 and especially Figure 25. Note that the observed sales peak for the whole sample appeared in 1964. Based on sales observations covering 1958-1960 our model predicts both the timing and the magnitude of the whole sample sales peak.
correctly, cf. Figure 19, i.e. 4 years before the sales peak appears. As another general conclusion we can state that our successively updated parameter estimates rather quickly approach stable values that remain unchanged in further successive updatings. This happens in Calibration 4 (1958-1961) in Cases 2 and 4 and in Calibration 6 (1958-1963) in Cases 1 and 3, i.e. in each case before the observed sales peak on the market appears. This is important from a long-term forecasting point of view since stable parameter estimates should improve long-term forecasting ability.

Table 15. Simulation accuracy in eleven simulations (Case 4)

<table>
<thead>
<tr>
<th>Simul. no.</th>
<th>Calibr. series</th>
<th>Accuracy measurements</th>
<th>Low Income</th>
<th>Medium Income</th>
<th>High Income</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-.13</td>
<td>.13</td>
<td>-100.0</td>
<td>-.31</td>
<td>.31</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>-.07</td>
<td>.07</td>
<td>-58.7</td>
<td>-.21</td>
<td>.21</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>-.00</td>
<td>.01</td>
<td>-16.5</td>
<td>-.05</td>
<td>.05</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>.01</td>
<td>.01</td>
<td>-10.7</td>
<td>-.02</td>
<td>.03</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>.01</td>
<td>.01</td>
<td>-10.7</td>
<td>-.02</td>
<td>.03</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>.01</td>
<td>.01</td>
<td>-10.7</td>
<td>-.02</td>
<td>.03</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>.01</td>
<td>.01</td>
<td>-10.7</td>
<td>-.02</td>
<td>.03</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>.01</td>
<td>.01</td>
<td>-10.7</td>
<td>-.02</td>
<td>.03</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>.01</td>
<td>.01</td>
<td>-10.7</td>
<td>-.02</td>
<td>.03</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>.01</td>
<td>.01</td>
<td>-10.7</td>
<td>-.02</td>
<td>.03</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>.01</td>
<td>.01</td>
<td>-10.7</td>
<td>-.02</td>
<td>.03</td>
</tr>
</tbody>
</table>

A = Mean Error (ME) for owner shares
B = Mean Absolute Error (MAE) for owner shares
C = Percentage Error in Accumulated Demand (PEAD)

Model: Mixed Diffusion Model
Simulated time periods: 32
Word-of-mouth parameters: \( d = 0.3 \) and \( K = 12 \)
Optimization criterion: MAE for owner shares
Iteration step: \( .001 \) in calibrating \( a_m \) and \( .00001 \) in calibrating \( b_m \)

Looking at our four validation cases we can report on two main findings. The first finding is that new demand as optimization criterion performs much better than owner shares at the very beginning of our diffusion processes, i.e. when we have very few sales observations in each segment to base our parameter estimates on. When the diffusion proceeds and we get more sales observations the use of owner shares as optimization criterion tends to result in stable estimates in earlier phases of the
diffusion processes than if we use new demand. These stable estimates are the same regardless of which optimization criterion we use. The two criteria seem, in other words, to complement each other and should obviously be used side by side in applications.

Our second finding is deeply rooted in our specific application and cannot in any way be considered as a general finding. As we have seen the pure diffusion model seems to perform better than the mixed diffusion model in our application. Although the inclusion of advertising as an external influence variable improves the model performance as it comes to the high income segment, the performances concerning the other two segments as well as the overall performance are slightly worse than the corresponding performances of the pure diffusion model, cf. Tables 15 and 13 against Tables 11 and 8.

We have also seen that increasing the number of decimal places of our estimates when calibrating our influence parameters leads to lower optimal MAE-values, which is natural. In spite of this the unconditional forecasts are, however, not necessarily performing better using estimates with more decimal places than estimates with fewer decimal places, cf. Table 9. It can, in other words, be stated that good fits between observed and simulated time series in calibrations ex post do not in any way guarantee good fits in forecasts based upon these estimates. What we have seen here is that a better fit (lower optimal MAE-value) achieved in calibrating the parameters of our diffusion model is not automatically leading to better unconditional forecasts, a finding that is not actually surprising.

One final point can be stressed. We have seen, although we have used observed advertising time series in our forecasts, that the outcomes using the mixed diffusion model are not better than the outcomes using the pure diffusion model. As we furthermore, in an ex ante forecasting situation, have to, in a first step, predict the advertising volumes and then, in a second step, use these predicted volumes as input in our final forecasts, we believe that such an approach, although theoretically sound, does not necessarily fit practical forecasting work very well.

**Long-term demand scenarios**

As was pointed out in Chapter 4, DEMSIM is designed to be used as a parameter estimation tool, as a validation tool and as a forecasting tool, each of these usage areas interacting with each other through the user in applying the model. In the validations carried out above we used successively updated calibrations, each one followed by a simulation including an unconditional ex post forecast. Together these processes constituted our design as to validating the diffusion part of DEMSIM. In other words, when using DEMSIM as a validation tool we interactively calibrated, simulated and carried out unconditional ex post forecasts to achieve our purposes. These validations did not, however, include the replacement model because our empirical data does not include observations on replacement sales. Neither have any ex ante forecasts using DEMSIM so far been demonstrated.
We will end this chapter with a short illustration of a couple of simulations using the complete DEMSIM, i.e. the diffusion model and the replacement model simultaneously. These simulations can be looked upon as totally fictitious *ex ante* forecasts. However, since we are not trying to forecast the most probable future demand in these simulations we prefer not to talk about the simulations as forecasts. The purpose of the illustration is to give a simplified example of how DEMSIM works and furthermore how DEMSIM can be used as an explorative tool in creating, for example, various future demand scenarios.

In our illustration we choose to look at how demand in various segments as well as the total market demand develops over time conditional to different service-life expectancies of the durable. To emphasize the effects of the different service-life expectancies we keep possible exogenous variables affecting demand apart from or unchanged in the simulations. In this way we develop future demand scenarios, one for each simulation, that are conditional to different service-life expectancies. These scenarios portray, so to say, *baseline demand under conditions of no exogenous variables influencing the demand*. Only internal influences and different service-life expectancies are allowed to affect the demand. In Table 16 the parameter values used in our simulations are summarized.

We use, in other words, the pure diffusion model instead of the mixed diffusion model. The illustration includes all parts of Figure 5 in Chapter 4 except for the external influences. The other exogenous variables in Figure 5 are dealt with as follows. The owner share corrections are kept at zero and the sizes of the segments unchanged in the simulations. The database used in the validations above contains 20% of all households in Porvoo in 1958 and in 1968. Not to make our segment sizes fully fictitious we choose to base our sizes on the situation in Porvoo in 1968. We multiply these figures by five giving us figures for the Porvoo household market as a whole, at least at one point in time. As to the parameters regulating the internal influences we use the estimates that produced the best fits between simulated and observed time series in our validations above. We are making two simulations, the only difference between them being the service-life expectancies. We are calling these two simulations Scenario 1 and Scenario 2. The simulations start from the point in time when our durable is launched on the market and cover 90 time periods, i.e. 30 years. Note the underlying assumption that all durables in use are replaced sooner or later.

Simulations covering 30 years with no external influences affecting the demand and with the segment sizes unchanged is, of course, completely unrealistic. Our approach is, however, justified since the purpose is to explore demand levels and patterns conditional to different service-life expectancies, everything else being unchanged. The simplified assumptions, summarized in Table 16, will give us smooth and regular demand curves, c.f. Figures 27 and 28 below. The main point is, however, that DEMSIM allows us to explore, by an experimental approach, how demand levels and patterns are dependent on changes in parameters explicitly specified in DEMSIM. Such experiments should increase our understanding of the forces that generate demand volumes over time.
Table 16. Parameter values used in generating two scenarios

Scenarios 1 and 2

Time unit: 4 months
Simulated time periods: 90 (30 years)

Sizes of segments, $H_m(t)$
- Low income households: 2900 (unchanged over time)
- Medium income households: 2740 (unchanged over time)
- High income households: 480 (unchanged over time)

Owner share corrections: None

External influences: None

Contact behaviours: $C_m$ and $P_{mn}$ as in Tables 3 and 4

Word-of-mouth parameters: $d = 0.3$ and $K = 12$

Coefficient of internal influence, $a_m$ (cf. Tables 7 and 10)
- Low income households: .002
- Medium income households: .003
- High income households: .005

Slope of the survival curve, $s_m$: 5 in each segment

Scenario 1

Expected service-life, $g_m$
- Low income households: 18 time periods (6 years)
- Medium income households: 12 time periods (4 years)
- High income households: 9 time periods (3 years)

Scenario 2

Expected service-life, $g_m$
- Low income households: 27 time periods (9 years)
- Medium income households: 21 time periods (7 years)
- High income households: 18 time periods (6 years)

In Scenario 1 we have chosen 6 years as the expected service-life among low income households, 4 years among medium income households, and 3 years among high income households. The choice of these service-life expectancies is based on an assumption that the larger the income of a household, the sooner the household will replace the durable. The overall expected service-life in Scenario 1 is 4.9 years. In Scenario 2 we extend the expected service-life in each segment with 3 years, i.e. to 9 years, 7 years, and 6 years. The overall expected service-life in Scenario 2 is 7.9 years. The simulated outcomes are shown in Figures 27 and 28.
Since the parameters of the diffusion part of DEMSIM are identical in the two scenarios the simulated new demand volumes are also identical, as can be seen from Figures 27 and 28. Differences appear, however, in the replacement demand due to different service-life expectancies and consequently also in total demand. The number of time periods between launch of the durable and the new demand peak is denoted by $A$.
A and a black arrow in the figures. Since the simulated new demand volumes are equal in the two scenarios the time periods expressed by A are also equal, i.e. 7 years, 6 years, and 5 years in the three income segments respectively. The service-life expectancies, $g_m$, in turn differ with three years between the two scenarios.

**Figure 28. Simulated demand patterns (Scenario 2)**

- **Low Income Households**
  - $g_m = 27$ periods (9 years)
  - $A = 21$ periods (7 years)

- **Medium Income Households**
  - $g_m = 12$ periods (4 years)
  - $A = 18$ periods (6 years)

- **High Income Households**
  - $g_m = 18$ periods (6 years)
  - $A = 15$ periods (5 years)

- **Total Demand**
  - **Replacement Demand**
  - **New Demand**

$A$ = Number of time periods between launch of the durable and New Demand peak (black arrow)
Our two scenarios reveal basic differences in the shape of the simulated total demand curves depending on whether the expected service-life is shorter or longer than A. In Scenario 1 the expected service-life is shorter than the corresponding A in each segment. In Scenario 2 we have the opposite situation, the expected service-life is longer than the corresponding A in each segment. In Scenario 1 the simulated total demand grows, in most cases, smoothly towards a stable level. In Scenario 2, on the contrary, the simulated total demand reaches a peak after which it oscillates towards a lower stable level. If we would further increase the expected service-life in Scenario 2 we would get total demand simulations that would oscillate even stronger than in Figure 28. However, the core message here is that the expected service-life does not as such determine possible oscillatory movements in total demand. Instead it is the expected service-life in relation to how quickly a new durable penetrates a market that determines whether oscillatory movements in total demand will occur or not.

From our two scenarios we can also see that the total demand in the long run for a durable with an expected service-life of 4.9 years is about 70% higher than for a durable with an expected service-life of 7.9 years, everything else being unchanged. Further simulations using various service-life expectancies would give us a more thorough picture of the dependence between the expected service-life and the total sales volume in the long run. It goes without saying that such information can be very valuable in the strategic planning of companies producing and/or marketing (new) consumer durables.
Chapter 6

CONCLUDING REMARKS

Short summary

This research report is concerned with forecasting long-term demand for new consumer durables. The forecasting tool developed takes the form of a deterministic simulation model, called DEMSIM. This model forecasts both long-term demand by non-owners of the durable (new demand) as well as long-term demand by owners of the durable (replacement demand).

The development of the simulation model starts from a counteractive adoption model, i.e. a conceptual frame illustrating the basic forces that are supposed to affect the adoption behaviour of individual consumers, cf. Figure 2. The basic counteracting forces in this model are the promoting forces and the resisting forces. The promoting forces are further divided into internal influences and external influences. These promoting forces are refined and further developed into a multi-segmental diffusion model expressing the adoption and some other behaviours of the consumers in each segment as aggregates or expected values which are allowed to differ between the segments. This diffusion model is combined with a replacement model that is built upon the same segmental structure as the diffusion model. The replacement model, in turn, expresses the replacement behaviour of the consumers in each segment as aggregates that are allowed to differ between the segments. Together these two models constitute the overall simulation model DEMSIM. In applying DEMSIM several model alternatives are possible. We can use the model as a pure diffusion model letting only internal influences affect the adoption behaviour. We can also use the model as a mixed diffusion model letting internal and external influences affect the adoption behaviour simultaneously. Finally we can add the replacement demand part of DEMSIM either to the pure diffusion model or to the mixed diffusion model, in which cases DEMSIM serves as a diffusion-replacement model simulating total market demand.

Since we are interested in long-term forecasting and since exogenous variables affecting demand (external influences), especially policy type variables, are more or less impossible to forecast in the long run, we are in this study concentrating our model building efforts on the endogenous mechanisms (especially internal influences) that are supposed to affect the demand for new consumer durables. We believe that the internal influences (as well as the expected service life of a durable) represent phenomena that usually are more regular and stable in their appearance over time than most external influences. From this follows that they also should be easier to
forecast accurately in the long run. These assumptions are supported by our findings in Chapter 5.

Besides the model structure of DEMSIM as such, a number of aspects concerning the implementation of the model have been considered and built into DEMSIM. These aspects are concerned with three general application purposes, i.e. calibrating the main parameters of the model, validating the model and forecasting with the model. DEMSIM is designed to make it possible for a user in applying the model to freely and fluently move around among these application purposes and, with the help of instant graphic displaying of simulated and observed time series as well as flexible parameter handling possibilities, explore and learn more and more about the phenomenon analysed and modeled.

The main parameters of the model, the coefficients of internal and external influence, are calibrated using a parameter estimation algorithm especially designed for DEMSIM. This optimization algorithm minimizes the mean absolute error between simulated and observed time series representing either owner shares or new demand, i.e. our two optimization criteria. Technically the algorithm is capable of estimating the coefficients of internal and external influence in very early stages of a diffusion process. Such early estimates are, naturally, not very reliable. However, in using the optimization algorithm in our validations, described in Chapter 5, the successively recalibrated coefficients were stabilizing surprisingly soon after the launch of our durable.

In the present study we are using a two-step parameter estimation approach. In a first step we are looking for the best-performing $d$-value among a given number of such values and, in a second step, we use this $d$-value in our final calibrations and validations of our diffusion model. The $d$-value expresses the rate at which the probability to discuss a new durable in a contact between a non-owner and an owner decreases when the time since the durable was acquired by the owner increases, cf. Figure 12. Our two-step approach means that our final findings are conditional on the $d$-value used in the second step. In preliminary unconditional forecasts (step 1) we found that the $d$-value 0.3 produced better fits between observed and forecasted time series than the use of some other $d$-values did. It is worth noting that the chosen $d$-value performed well in unconditional forecasts regardless of the length of the calibration time series. In this sense the chosen $d$-value outruled the other values. This implies that the phenomenon expressed through the $d$-value seems to be better caught by using the value 0.3 than by using any of the other values tested. Above all this holds for the fits in unconditional forecasts. The results also imply that the best-performing $d$-value seems to be rather stable over time, meaning that it performs well regardless of the length of the calibration time series used.

**Main findings**

In the final validations of the pure diffusion model as well as the mixed diffusion model in Chapter 5 we use successively updated estimates of the influence parameters in repeated simulations containing unconditional *ex post* forecasts. This approach allows us to test how the forecasting ability of the diffusion model develops when we
successively increase the length of the calibration time series, 1 year (3 time periods) at a time, everything else being unchanged. Since our validation time series covers the years 1958-1968 or close to 11 years, we are making 11 successive updates. On the basis of these validations the following general conclusions can be made.

• The diffusion model forecasts the overall new demand and owner share growth in segments well already in early stages of the diffusion processes, especially from the beginning of 1961 on. At this time our durable had been less than 3 years (8 time periods) on the market. See Figures 19 and 20 (the pure diffusion model) and Figures 25 and 26 (the mixed diffusion model).

• The diffusion model forecasts the timing and the magnitude of the new demand peaks in segments well already in early stages of the diffusion processes, especially from the beginning of 1961 on, i.e. about 4 years (12 time periods) before the new demand peak of the whole sample is reached. See Figure 19 (the pure diffusion model) and Figure 25 (the mixed diffusion model). Note that fairly good indications of the timing of the new demand peaks are present already in the unconditional forecasts starting 1959\(^{40}\) and especially 1960. See Figures 17 and 18 (the pure diffusion model).

• The successively updated estimates of the influence parameters approach stable values relatively early in the diffusion processes and remain stable after that. When we use owner shares as optimization criterion this happens in 1961, cf. Tables 10 and 14, and when we use new demand as optimization criterion in 1963\(^{41}\), cf. Tables 7 and 12. These results are the same regardless of which model we use, the pure diffusion model or the mixed diffusion model.

• New demand performs better as optimization criterion than owner shares at the beginning of the diffusion processes when the owner shares are still (very) low. Compare Figure 18 to Figure 23. In later phases of these processes owner shares perform as well as new demand, even so that stable parameter estimates are achieved sooner when we are using owner shares as optimization criterion than when we are using new demand. The stable estimates are the same regardless of which optimization criterion we use, owner shares or new demand.

• Using the mixed diffusion model does not, at least in our specific application, lead to better unconditional forecasts\(^ {42}\) than using the pure diffusion model, on the contrary. Compare Table 15 to Table 11 and Table 13 to Table 8.

• Applying the pure diffusion model produces estimates of the coefficient of internal influence that, so to say, reflect the innovation resistance in relative terms. Hence it was found that the resistance among low income households was 1.5

\(^{40}\) In the unconditional forecast starting 1959 the influence parameters are estimated from very low sales figures. In periods 1 and 2 (1958) no units were bought among low income households, 1 unit was bought among medium income households (period 2), and 2 units among high income households (1 in each period), cf. Appendix C. In other words, a total of 3 units were bought in our sample in 1958.

\(^{41}\) The stable values from 1963 on also appeared in one calibration before 1963, i.e. in 1960.

\(^{42}\) Note that we are using the observed advertising time series in our unconditional forecasts.
times stronger than among medium income households and 2.5 times stronger than among high income households over the analysed period. The resistance among medium income households was, in turn, 1.7 times stronger than among high income households.

- Although the use of a higher estimation precision when calibrating the internal influence parameter brings about lower optimal MAE-values, unconditional forecasts based on these estimates do not perform better than forecasts based on lower precision estimates, cf. Table 9.

The fact that the successively updated estimates of the influence parameters without exception stabilize before the new demand peak appears is an important finding, especially from a forecasting point of view. Achieving stable parameter estimates in early stages of a diffusion process is an important condition in order to be able to carry out reliable forecasts of the timing and magnitude of such demand peaks. The stable estimates also imply that a corresponding stability as to the level of resistance within segments seems to prevail throughout the analysed period. If this is the case, estimates of the coefficient of internal influence made in early stages of a diffusion process can be very useful in long-term diffusion forecasting.

As a whole we consider our findings very encouraging. Our diffusion mechanism seems to be very sensitive in locating the right timing of the turning point of new demand already in early stages of a diffusion process and also to be able to forecast the magnitude of this peak in good time before it occurs. This holds for each of our three income segments and consequently for the whole sample as well. We believe that our way of operationalizing the internal influences but also the early calibrations of the influence parameters enabled by the optimization algorithm designed for the present model contribute heavily to the promising outcomes of our validation simulations. In other words, DEMSIM incorporates a diffusion mechanism that seems to succeed in forecasting, at least in our specific case, the timing and magnitude of new demand peaks well already in early stages of a diffusion process.

Furthermore the way in which we have specified the internal influence within our diffusion model makes any explicit specifications of saturation levels unnecessary. This is so because internal influence within our model setting exists only when a non-owner is exposed to word-of-mouth information in personal contacts with owners that have acquired the durable not too long ago. When new demand has passed its peak and decreases more and more the number of contacts with owners that have acquired the durable recently also decreases more and more, meaning that the non-owners become less and less exposed to information concerning the durable in such contacts. When this exposure has reached a zero-level new demand is also zero meaning that the owner share has reached its saturation level.\(^43\) The owner share level at which this happens depends on how strong the resisting forces of the consumers have been during the diffusion process and how frequently non-owners have been exposed to

---

\(^{43}\) From Figure 21 we can see that the simulated internal exposures in each segment in the end of the analysed time period still are clearly greater than zero, meaning that the diffusion processes had not at that time yet reached their saturation levels. Therefore any conclusions on how well our model forecasts the saturation levels of our segments are impossible to make on the basis of the outcomes of our validations. Our validation time series are simply too short for that.
information concerning the durable in contacts with owners. In our model setting this means that the resisting forces affect the saturation level, so to say, through the estimates of the coefficient of internal influence. Generally speaking we could say that higher estimates bring about higher saturation levels and the other way around, however, not forgetting that the amount of exposures also affects the diffusion outcomes.

**Developing DEMSIM further**

We have stated above that the optimization algorithm designed in order to estimate the internal and external influence parameters of our diffusion model represents a first version. One disadvantage of the approach used above is the two-step estimation procedure ending up in influence parameter estimates that are conditional on the value of $d$ used in estimating these influence parameters. The weak point is how $d$ is estimated. Above we have objectively chosen the $d$-value that performs best out of four predefined $d$-values. However, these four $d$-values are subjectively chosen. Although the finally chosen $d$-value performs well in our unconditional forecasts, it is possible that there exist $d$-values, not tested, that perform better than the $d$-value we have used.

A natural way of avoiding this problem, at least to some extent\(^\text{44}\), would be to also include the estimation of the parameter $d$ in the optimization algorithm, meaning that we would estimate $d$ as well as the two influence parameters simultaneously. In doing so we could also let $d$ differ between segments in the same way as the internal and external influence parameters are now allowed to differ in calibrations. Note that we have been using the same $d$-value for each segment above. Extending our optimization algorithm in this way could possibly increase the validation power and, hopefully, also the predictive power of our model. However, the proposed extension has most likely also its drawbacks. Increasing the number of parameters to be estimated simultaneously would probably increase the unreliability of estimates based on very few observations. We can also assume that stable estimates in the sense specified above would not be achieved as soon after launch as the case has been in our validations. Therefore the extensions proposed here should be handled and built into DEMSIM as optional complements of the optimization algorithm.

**Applying DEMSIM**

In spite of the promising empirical findings reported on above we want to conclude this report by stating that it still remains to be seen how well our diffusion model performs when applied to bigger markets with far less detailed information available for calibration purposes than the case has been in our validations. The fact that the validation data used here is about 40 years old should not make our findings less important from a forecasting point of view than if we would have used more recent data. This is so because we believe that the basic forces regulating the diffusion of

\(^{44}\) Note that our optimization algorithm does not produce optimal solutions in a global sense. Instead the solutions are optimal among those search points that we explicitly specify for each new calibration that we do, cf. Figure 7.
new consumer durables have not changed considerably in the last 40 years. However, it should be remembered that the core of the present model is the mechanism generating the internal influence working through word-of-mouth communication taking place in personal contacts between non-owners and owners of the durable in question. This means that the model is especially suited for durables that can be supposed to be spread by the means of such communication. The more influential this type of interpersonal communication is for the spread of a specific durable, the better our model should be able to make reliable long-term demand forecasts for that durable.
REFERENCES


### Appendix A

**Observed number of households in sample 1958-1968 (Periods 1-32)**

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Low Income</th>
<th>Medium Income</th>
<th>High Income</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>475</td>
<td>372</td>
<td>63</td>
<td>910</td>
</tr>
<tr>
<td>2</td>
<td>452</td>
<td>371</td>
<td>62</td>
<td>885</td>
</tr>
<tr>
<td>3</td>
<td>439</td>
<td>355</td>
<td>77</td>
<td>871</td>
</tr>
<tr>
<td>4</td>
<td>430</td>
<td>352</td>
<td>77</td>
<td>859</td>
</tr>
<tr>
<td>5</td>
<td>420</td>
<td>356</td>
<td>78</td>
<td>854</td>
</tr>
<tr>
<td>6</td>
<td>425</td>
<td>345</td>
<td>80</td>
<td>850</td>
</tr>
<tr>
<td>7</td>
<td>418</td>
<td>346</td>
<td>81</td>
<td>845</td>
</tr>
<tr>
<td>8</td>
<td>414</td>
<td>348</td>
<td>83</td>
<td>845</td>
</tr>
<tr>
<td>9</td>
<td>385</td>
<td>374</td>
<td>86</td>
<td>845</td>
</tr>
<tr>
<td>10</td>
<td>385</td>
<td>379</td>
<td>87</td>
<td>851</td>
</tr>
<tr>
<td>11</td>
<td>380</td>
<td>383</td>
<td>88</td>
<td>851</td>
</tr>
<tr>
<td>12</td>
<td>367</td>
<td>396</td>
<td>90</td>
<td>853</td>
</tr>
<tr>
<td>13</td>
<td>369</td>
<td>402</td>
<td>91</td>
<td>862</td>
</tr>
<tr>
<td>14</td>
<td>363</td>
<td>404</td>
<td>90</td>
<td>857</td>
</tr>
<tr>
<td>15</td>
<td>388</td>
<td>391</td>
<td>90</td>
<td>869</td>
</tr>
<tr>
<td>16</td>
<td>384</td>
<td>403</td>
<td>90</td>
<td>877</td>
</tr>
<tr>
<td>17</td>
<td>391</td>
<td>412</td>
<td>90</td>
<td>893</td>
</tr>
<tr>
<td>18</td>
<td>388</td>
<td>417</td>
<td>94</td>
<td>899</td>
</tr>
<tr>
<td>19</td>
<td>386</td>
<td>425</td>
<td>96</td>
<td>907</td>
</tr>
<tr>
<td>20</td>
<td>392</td>
<td>432</td>
<td>96</td>
<td>920</td>
</tr>
<tr>
<td>21</td>
<td>417</td>
<td>425</td>
<td>88</td>
<td>930</td>
</tr>
<tr>
<td>22</td>
<td>421</td>
<td>437</td>
<td>89</td>
<td>947</td>
</tr>
<tr>
<td>23</td>
<td>426</td>
<td>450</td>
<td>90</td>
<td>966</td>
</tr>
<tr>
<td>24</td>
<td>430</td>
<td>457</td>
<td>101</td>
<td>988</td>
</tr>
<tr>
<td>25</td>
<td>443</td>
<td>469</td>
<td>105</td>
<td>1017</td>
</tr>
<tr>
<td>26</td>
<td>451</td>
<td>484</td>
<td>105</td>
<td>1040</td>
</tr>
<tr>
<td>27</td>
<td>473</td>
<td>491</td>
<td>98</td>
<td>1062</td>
</tr>
<tr>
<td>28</td>
<td>481</td>
<td>503</td>
<td>100</td>
<td>1084</td>
</tr>
<tr>
<td>29</td>
<td>493</td>
<td>519</td>
<td>101</td>
<td>1113</td>
</tr>
<tr>
<td>30</td>
<td>536</td>
<td>523</td>
<td>91</td>
<td>1150</td>
</tr>
<tr>
<td>31</td>
<td>565</td>
<td>535</td>
<td>94</td>
<td>1194</td>
</tr>
<tr>
<td>32</td>
<td>580</td>
<td>548</td>
<td>96</td>
<td>1224</td>
</tr>
</tbody>
</table>

*Numbers refer to the beginning of each time period*
### Appendix B

*Observed number of owners in sample 1958-1968 (Periods 1-32)*

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Low Income</th>
<th>Medium Income</th>
<th>High Income</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>11</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>13</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>13</td>
<td>6</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>17</td>
<td>10</td>
<td>31</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>21</td>
<td>11</td>
<td>37</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>31</td>
<td>16</td>
<td>52</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>40</td>
<td>23</td>
<td>71</td>
</tr>
<tr>
<td>11</td>
<td>9</td>
<td>46</td>
<td>26</td>
<td>81</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>65</td>
<td>30</td>
<td>105</td>
</tr>
<tr>
<td>13</td>
<td>15</td>
<td>81</td>
<td>34</td>
<td>130</td>
</tr>
<tr>
<td>14</td>
<td>14</td>
<td>79</td>
<td>38</td>
<td>131</td>
</tr>
<tr>
<td>15</td>
<td>25</td>
<td>105</td>
<td>37</td>
<td>167</td>
</tr>
<tr>
<td>16</td>
<td>28</td>
<td>118</td>
<td>39</td>
<td>185</td>
</tr>
<tr>
<td>17</td>
<td>29</td>
<td>128</td>
<td>41</td>
<td>198</td>
</tr>
<tr>
<td>18</td>
<td>38</td>
<td>154</td>
<td>48</td>
<td>240</td>
</tr>
<tr>
<td>19</td>
<td>51</td>
<td>175</td>
<td>50</td>
<td>276</td>
</tr>
<tr>
<td>20</td>
<td>52</td>
<td>184</td>
<td>50</td>
<td>286</td>
</tr>
<tr>
<td>21</td>
<td>74</td>
<td>202</td>
<td>51</td>
<td>327</td>
</tr>
<tr>
<td>22</td>
<td>86</td>
<td>227</td>
<td>51</td>
<td>364</td>
</tr>
<tr>
<td>23</td>
<td>91</td>
<td>237</td>
<td>52</td>
<td>380</td>
</tr>
<tr>
<td>24</td>
<td>93</td>
<td>253</td>
<td>59</td>
<td>405</td>
</tr>
<tr>
<td>25</td>
<td>102</td>
<td>277</td>
<td>67</td>
<td>446</td>
</tr>
<tr>
<td>26</td>
<td>107</td>
<td>282</td>
<td>67</td>
<td>456</td>
</tr>
<tr>
<td>27</td>
<td>127</td>
<td>296</td>
<td>62</td>
<td>485</td>
</tr>
<tr>
<td>28</td>
<td>133</td>
<td>308</td>
<td>63</td>
<td>504</td>
</tr>
<tr>
<td>29</td>
<td>146</td>
<td>317</td>
<td>65</td>
<td>528</td>
</tr>
<tr>
<td>30</td>
<td>181</td>
<td>339</td>
<td>54</td>
<td>574</td>
</tr>
<tr>
<td>31</td>
<td>196</td>
<td>352</td>
<td>58</td>
<td>606</td>
</tr>
<tr>
<td>32</td>
<td>204</td>
<td>363</td>
<td>59</td>
<td>626</td>
</tr>
</tbody>
</table>

Numbers refer to the beginning of each time period.
Appendix C

*Observed new demand in sample 1958-1968 (Periods 1-32)*

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Low Income</th>
<th>Medium Income</th>
<th>High Income</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>6</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>9</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>9</td>
<td>7</td>
<td>19</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>17</td>
<td>7</td>
<td>26</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>15</td>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>25</td>
<td>3</td>
<td>34</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>12</td>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>10</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>17</td>
<td>7</td>
<td>26</td>
<td>8</td>
<td>41</td>
</tr>
<tr>
<td>18</td>
<td>12</td>
<td>21</td>
<td>2</td>
<td>35</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>8</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>23</td>
<td>4</td>
<td>37</td>
</tr>
<tr>
<td>21</td>
<td>10</td>
<td>20</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>22</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>23</td>
<td>4</td>
<td>15</td>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>24</td>
<td>8</td>
<td>21</td>
<td>6</td>
<td>35</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>26</td>
<td>13</td>
<td>11</td>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td>27</td>
<td>7</td>
<td>8</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>28</td>
<td>9</td>
<td>4</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>29</td>
<td>13</td>
<td>23</td>
<td>1</td>
<td>37</td>
</tr>
<tr>
<td>30</td>
<td>7</td>
<td>6</td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>31</td>
<td>4</td>
<td>5</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>32</td>
<td>4</td>
<td>13</td>
<td>2</td>
<td>19</td>
</tr>
</tbody>
</table>
Appendix D

Advertising volumes concerning television sets in daily newspapers and weekly magazines in 1958-1968 (Periods 1-32)

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Advertising Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>106</td>
</tr>
<tr>
<td>3</td>
<td>104</td>
</tr>
<tr>
<td>4</td>
<td>56</td>
</tr>
<tr>
<td>5</td>
<td>233</td>
</tr>
<tr>
<td>6</td>
<td>255</td>
</tr>
<tr>
<td>7</td>
<td>370</td>
</tr>
<tr>
<td>8</td>
<td>348</td>
</tr>
<tr>
<td>9</td>
<td>453</td>
</tr>
<tr>
<td>10</td>
<td>370</td>
</tr>
<tr>
<td>11</td>
<td>402</td>
</tr>
<tr>
<td>12</td>
<td>401</td>
</tr>
<tr>
<td>13</td>
<td>444</td>
</tr>
<tr>
<td>14</td>
<td>449</td>
</tr>
<tr>
<td>15</td>
<td>309</td>
</tr>
<tr>
<td>16</td>
<td>388</td>
</tr>
<tr>
<td>17</td>
<td>326</td>
</tr>
<tr>
<td>18</td>
<td>358</td>
</tr>
<tr>
<td>19</td>
<td>363</td>
</tr>
<tr>
<td>20</td>
<td>387</td>
</tr>
<tr>
<td>21</td>
<td>437</td>
</tr>
<tr>
<td>22</td>
<td>415</td>
</tr>
<tr>
<td>23</td>
<td>305</td>
</tr>
<tr>
<td>24</td>
<td>241</td>
</tr>
<tr>
<td>25</td>
<td>189</td>
</tr>
<tr>
<td>26</td>
<td>188</td>
</tr>
<tr>
<td>27</td>
<td>141</td>
</tr>
<tr>
<td>28</td>
<td>303</td>
</tr>
<tr>
<td>29</td>
<td>157</td>
</tr>
<tr>
<td>30</td>
<td>208</td>
</tr>
<tr>
<td>31</td>
<td>130</td>
</tr>
<tr>
<td>32</td>
<td>163</td>
</tr>
</tbody>
</table>

Seasonal fluctuations have been removed from the time series using the percentage moving average method (Spiegel, 1961, p. 287)

Measurement unit: Column millimeters
Appendix E

Observed and simulated new demand in Periods 1-32 (Case 1, Simulation 3)

As can be seen the observed new demand (sales volumes) contain strong seasonal fluctuations while the simulated new demand does not, simply because seasonal fluctuations are not incorporated in the diffusion model. One could, in other words, say that the simulated new demand is a smoothed version of the observed new demand obtained by minimizing the mean absolute deviance between observed and simulated new demand over the calibration time series. The main reason for the relatively high ME-, MAE-, and MxE-values for new demand (shown above) is then, of course, the fact that the observed time series contain seasonal fluctuations and the simulated ones do not.
Appendix F

Successively updated estimates of influence parameters with four decimals (Case 1)

<table>
<thead>
<tr>
<th>Calibr. no.</th>
<th>Calibr. series</th>
<th>Low Income $a_1$</th>
<th>Medium Income $a_2$</th>
<th>High Income $a_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>.0000</td>
<td>.0016</td>
<td>.0070</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>.0021</td>
<td>.0029</td>
<td>.0044</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>.0016</td>
<td>.0025</td>
<td>.0064</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>.0013</td>
<td>.0029</td>
<td>.0063</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>.0019</td>
<td>.0035</td>
<td>.0046</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>.0016</td>
<td>.0031</td>
<td>.0049</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>.0020</td>
<td>.0035</td>
<td>.0045</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>.0015</td>
<td>.0035</td>
<td>.0047</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>.0020</td>
<td>.0034</td>
<td>.0046</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>.0021</td>
<td>.0034</td>
<td>.0046</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>.0021</td>
<td>.0034</td>
<td>.0046</td>
</tr>
</tbody>
</table>

Model: Pure Diffusion Model
Word-of-mouth parameters: $d = 0.3$ and $K = 12$
Optimization criterion: MAE for new demand
Iteration step: .0001 in calibrating $a_m$
**Appendix G**

*Simulation accuracy using revised influence parameter estimates (Case 1)*

<table>
<thead>
<tr>
<th>Simul. no.</th>
<th>Calibr. series</th>
<th>Accuracy measurements</th>
<th>Low Income</th>
<th>Medium Income</th>
<th>High Income</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>A B C</td>
<td>A B C</td>
<td>A B C</td>
<td>A B C</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>-.13 .13 -100.0</td>
<td>-.21 .21</td>
<td>-64.5</td>
<td>.14 .14</td>
<td>7.8</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>.00 .01 -4.7</td>
<td>-.06 .06</td>
<td>-11.1</td>
<td>-.03 .06</td>
<td>4.1</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>-.03 .03 -34.9</td>
<td>-.06 .06</td>
<td>-23.7</td>
<td>.15 .15</td>
<td>13.7</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>-.04 .04 -47.2</td>
<td>-.01 .02</td>
<td>-14.3</td>
<td>.15 .15</td>
<td>13.6</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>.01 .02 -5.6</td>
<td>.05 .05</td>
<td>2.0</td>
<td>.03 .06</td>
<td>8.4</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>-.03 .03 -30.3</td>
<td>-.02 .02</td>
<td>-8.1</td>
<td>.03 .06</td>
<td>8.3</td>
</tr>
<tr>
<td>7</td>
<td>20</td>
<td>.02 .02 1.5</td>
<td>.05 .05</td>
<td>2.6</td>
<td>.02 .06</td>
<td>7.9</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>-.03 .03 -32.0</td>
<td>.04 .04</td>
<td>-.5</td>
<td>.03 .06</td>
<td>7.6</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>.02 .02 .5</td>
<td>.03 .03</td>
<td>1.0</td>
<td>.03 .06</td>
<td>8.6</td>
</tr>
<tr>
<td>10</td>
<td>29</td>
<td>.03 .03 8.0</td>
<td>.04 .04</td>
<td>1.7</td>
<td>.03 .07</td>
<td>9.0</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>.03 .03 8.0</td>
<td>.04 .04</td>
<td>1.7</td>
<td>.03 .07</td>
<td>9.0</td>
</tr>
</tbody>
</table>

A = Mean Error (ME) for owner shares  
B = Mean Absolute Error (MAE) for owner shares  
C = Percentage Error in Accumulated Demand (PEAD)  

Model: Pure Diffusion Model  
Simulated time periods: 32  
Word-of-mouth parameters: $d = 0.3$ and $K = 12$  
Optimization criterion: MAE for new demand  
Iteration step: .0001 in calibrating $a_m$


34. CORPORATE SUCCESS FACTORS DURING TIMES OF CRISIS. Edited by Anders Kjellman, Stefan Långström and Tage Vest. Vasa 1996.


