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MODELING NONLINEARITIES AND ASYMMETRIES IN ASSET PRICING

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Modeling Nonlinearities and Asymmetries in Asset Pricing

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April 2008

Sofie Kulp-Tåg

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Part I

RESEARCH AREA AND CENTRAL FINDINGS

1 Introduction

Modeling and predicting future returns and volatility is a central part of research carried out in the area of asset pricing. This interest has its origins in the fact that if it is possible to predict tomorrow's prices on specific assets, or indices, with some certainty, profits could be possible. Information available today could be used for forecasting the price tomorrow, and with this information, trading and investment decisions could be done easily. This however is not as simple as it sounds. It is not easy to forecast tomorrow's prices, but what could possibly be done, for example, is to predict the sign of tomorrow's returns. This, according to Franses and van Dijk (2004), would be enough in many cases for making the investment decision.

The Efficient Market Hypothesis (EMH) by Fama (1970) is a pillar in financial time series data modeling. Fama's definition of efficient markets is a market where prices always fully reflect all available information. This directly implies that it is not possible to outperform the market by using available information.

The first thing to consider when investigating stock returns is the expected return or the mean return. Of equal importance is the risk, and the degree of risk associated with the certain level of return. The investment decisions are made in relation to the risk and investors should be compensated for taking non-diversifiable risk through higher expected returns on investments. The risk is often referred to as the volatility or the variance. The variance is essential especially in options pricing since, for example, in the pricing models by Black and Scholes (1973), an estimate of the variance of the asset is needed. The price of an option is, according to the Black and Scholes (1973) and Merton (1973) models, a function of the return standard deviation and other variables that can be easily observed or estimated. The volatility used in this pricing model is called implied volatility. The importance of understanding the volatility of financial time series data grows as new and more complicated instruments are developed, and for example trading options is trading volatility.

Modern portfolio theory assumes that investors are rational and large returns are preferred to small returns and low risk preferred to high risk. An investor is interested in both the first and the second moment, or expected return and risk. The risk is more complicated than the return. To construct optimal portfolios, in line with the theory by Markowitz (1952), forecasts of the expected returns and covariances between the assets are needed. The risk in returns is measured through the corresponding variance, and investors should hold a mean-variance efficient portfolio with the highest possible return with a given level of variance.

Mandelbrot (1963) recognized some predictability in the variance, and following this, one family of models that accounts for these predictable patterns has been developed; the Autoregressive Conditional Heteroscedasticity (ARCH) model by Engle (1982) and the Generalized ARCH (GARCH) model by Bollerslev (1986). This family has turned out to be very successful.

Volatility does not behave in a constant, stable way. Instead large variations are commonly seen which is one reason for difficulties in the predictions. Sometimes the volatility is more moderate, when other periods contain high volatility (Franses and van Dijk, 2004). This is a reason for the development and interest in models attributed to trends and patterns in the time series. Shocks often disturb the trends and these effects might follow the data for a while; the shocks might even persist for years. The autocorrelation among the errors is also a thing that might cause problems and might lead to invalid estimates and a violation of the criteria for linear regression models.

Not only the size, but also the sign of a shock influences the response in volatility. Generally, negative returns increase the volatility more than positive returns of the same size. Asymmetric effects are however not only important in the variance; returns can also behave in an asymmetric way.

Both linear and nonlinear models will be introduced in this dissertation. The advantages of nonlinear models are that they can take into account asymmetries seen in the data. Asymmetric patterns usually mean that large negative returns appear more often than positive returns of the same magnitude. This goes hand in hand with the fact that negative returns are associated with higher risk than in the case where positive returns of the same magnitude are observed. Black (1976) observed that price changes followed by bad news tend to be larger in magnitude than positive news and negative returns generally tend to be associated with higher volatility. The problem of volatility clustering presented by Mandelbrot (1963) can again be mentioned; large changes in price are followed by large changes and small changes in the price are followed by small changes. Another problem in return series is that the distribution of the returns not only tends to be skewed but also tends to contain excess kurtosis; fat tails (Mandelbrot (1963) and Fama (1965)). Asymmetries should be taken into account in the modeling process of the financial series. Since the forecasts (out-of-sample; when e.g. predicting tomorrow's price based on the information today), depend heavily on the correctness of the model applied, the importance of using the correct model should be obvious.

This dissertation has the following structure: In the first part, concepts of nonlinearities, asymmetries and risk will be presented and relevant literature will be discussed. The summary, description of the methodology, contribution and results of the essays follow. Part two of the dissertation presents the four essays.

2 Concepts in Modeling Asset Prices

Asymmetries and irregular patterns in financial data are typical. First, linear models assume that the distributions of the returns are normal or close to normal. However, when investigating returns series, it is observed that this is not the case; instead, large returns are more common than acceptable for a normal distribution (Franses and van Dijk, 2004). Second, the outstanding returns tend to behave in clusters, meaning they follow upon each other. This might be an indicator of persistence. Third, large negative returns are far more common than positive returns of the same magnitude. The fourth and last characteristic is that periods characterized by high volatility, generally follow periods with large negative returns (Franses and van Dijk, 2004). Clustering effects or persistence in volatility means that large shocks or innovations are followed by large shocks. This is the case for both positive and negative innovations. Periods characterized by high volatility tend to be followed by periods of high volatility. Consequently, periods of low volatility follow periods of low volatility. Mandelbrot (1963) and Fama (1965) were the first to present these patterns. If the patterns hold, it is possible to some extent predict tomorrow's volatility based on the volatility today. The sign of the shock also affects the volatility. Following a negative shock, higher volatility can be expected than following positive shocks. Black (1976) studied this and referred to it as the leverage effect.

When introducing and discussing high-frequency financial data, the hypothesis of Random Walk (RW) is a concept that is of importance and should be addressed. Bachelier argued in his 1900 "Theorie de la Speculation" article that stock prices behave in a random fashion. This means that the market price includes all information available, and the best prediction of tomorrow's price is the price today. From this RW hypothesis, the theories by Fama (1970) and Fama (1991) were developed. When using linear setups for modeling time series data as for example stock market indices, it is assumed that the series are normally distributed, or that the logarithmic series[1], are normally distributed. The return r_t should behave as a random variable with variance σ^2 and mean equal to μ or; $r_t \sim N\left(\mu, \sigma^2\right)$. Prices should follow a martingale, and tomorrow's price should be possible to predict from the price today and the information available today;

$$E\left[lnP_{t+1} \mid \Omega_t\right] = lnP_t + \mu. \tag{1}$$

$E\left[\cdot\right]$ gives the expectation of the price (logarithmic), and the information available today is Ω. Notable models that build upon RW are Historical Average, Moving Average, Exponential Smoothing and Exponentially Weighted Moving Average. ARCH type models are seen as a more sophisticated group of models for volatility forecasting (Poon and Granger, 2003).

Skewness measures the asymmetry in the return series in the distribution. Symmetric distributions have skewness of zero. This does not generally hold for return series. Instead, negative skewness is seen, indicating that the left tail of the distribution contains more weight, or is fatter than the right tail relative to the normal distribution.

The normal value for kurtosis (fourth moment) is three, but for return series the value tends to be larger. The statement of normally distributed series therefore fails since the high kurtosis is an indication of fat tails, and shocks or large observations occur more frequently than would be in line with the normal distribution. The first to study excess kurtosis in financial time series was Mandelbrot (1963). He compared the distribution of price changes with the normal distribution, and observed that price changes were thick-tailed and peaked. High kurtosis is thus an indication of non-normality and thus the normal distribution is not flexible enough for modeling the irregular patterns in financial time series data. This was also pointed out by Fama (1965). To capture the excess kurtosis seen in daily returns series, other distributions have been proposed. The most popular alternatives are the Generalized Error Distribution (GED) by Nelson (1991) and the Student's t-distribution suggested by Bollerslev (1986).

It seems likely that linear models that assume normally distributed characteristics of the return series not can be fully employed. Nonlinear models could be an alternative for taking the general characteristics of the return series into account, and this is the main focus in the papers in this dissertation. A natural way to start is by first employing linear models, and from the linear models to build nonlinear models which in a satisfying way can handle the characteristics of the series. It is stressed that nonlinear models do not always outperform linear models, but in many cases they do, especially when applied to out-of-sample forecasts. This issue will be considered in the four essays of this dissertation.

Non-normality is commonly seen in the indices included in the empirical investigations in this dissertation. Directly from descriptive statistics excess kurtosis is observed. In essay 1, asymmetric patterns in mean and variance are studied, and support is given to a distribution that can handle these irregularities and to asymmetric models for mean and variance. In essay 2 it is directly investigated if more flexible models give better risk estimates. In essay 3, the importance of the proper distribution for producing correct risk estimates is highlighted. Finally, in essay 4, the importance of introducing not only information variables or impulses, but also introducing flexible asymmetric models for mean and variance is studied with the purpose of estimating the volatility

[1]Logarithmic returns are commonly used when modeling financial time series data.

of the returns correctly.

2.1 Estimating the Mean

As the background for linear and nonlinear models, some basic methods for price and returns are discussed. In a basic time series, the price can be explained as consisting of two parts; one predictable and one unpredictable. The predictable part consists of the information found in the information set Ω_{t-1} and the unpredictable or unknown part ν_t that satisfies $E\left[\nu_t \mid \Omega_{t-1}\right] = 0$. The price expected today then becomes the sum of these two parts or,

$$r_t = E\left[r_t \mid \Omega_{t-1}\right] + \nu_t. \tag{2}$$

The predictable part is often considered as being a linear combination of the lagged values of the returns as,

$$E\left[r_t \mid \Omega_{t-1}\right] = \phi_1 r_{t-1}. \tag{3}$$

The ϕ:s are unknown parameters. The simple model is an $AR(p)$ model, or Autoregressive model of order p. The autoregressive model naturally accounts for autocorrelations. Theoretically, the first-order autocorrelation for a first-order autoregressive model equals,

$$\gamma_1/\gamma_0 = \phi_1. \tag{4}$$

The coefficients in the autoregressive model of order p can be estimated simply by employing an Ordinary Least Squares (OLS) method. Usually it is assumed that the error term (ε) follows the normal distribution with a mean of zero and variance σ^2. It should be normal, independent and identically distributed; $\varepsilon_t \sim IID\left(0, \sigma^2\right)$. However, the problem is that the error term does not necessarily follow a normal distribution and thus is not $NIID$. This generates incorrect estimates when using a linear model when a nonlinear model should be more appropriate. For the purpose of testing this problem, statistical tests have been developed. For example, a $\chi^2(2)$ test for normality is employed. The normality test includes both a skewness and a kurtosis component, and can identify outlying observations that results in a non-homoscedastic (heteroscedastic) error term series. This may be an indication that the return series that is being modeled could be better estimated with nonlinear models.

2.2 Estimating Volatility

In making predictions of financial time series, the risk is essential. The investor wants to know the uncertainty of the investment. The famous Capital Asset Pricing Model (CAPM) by Sharpe (1964) and Lintner (1965) includes a direct relationship between the expected return and the risk of the asset through the inclusion of the beta parameter. The risk in the CAPM is given by the covariance between the return of the specific asset and a benchmark series and here the use of beta as a risk measure can be supported. Investors are, according to CAPM, homogenous in their expectations on risk and return and they have mean-variance efficient portfolios. Also, when looking at options pricing, the risk measured through volatility is an important factor for pricing the option of the underlying asset.

(G)ARCH type models estimate a conditional mean and a conditional variance equation jointly.

Other techniques that can be mentioned are: the random walk model, where the best forecast of the volatility tomorrow is the volatility today; historical mean models, where the best forecast of tomorrow's volatility is the average of past observed volatilities; moving average models are models where the estimate of tomorrow's volatility is based on an unweighted mean on a specified estimation period, and finally, last technique to mention is, exponential smoothing models, where more weight is on new observations in that they take into account the dynamic ordering (Angelidis and Degiannakis, 2007).

One typical characteristic of returns of financial assets is that they are exposed to changing volatility. Heteroscedasticity refers to the changing volatility over time. In the ARCH models, it is the conditional variance that changes with time, not the variance itself. The conditional variance depends on the data available, and the risk of future observations can be quantified.

A characteristic presented by Black (1976) is the pattern of large volatility following negative returns. This can be related to the leverage effect since if assets are more leveraged, they are also more risky and less valuable to the owner. This is therefore not the only explanation for asymmetric patterns. This will be discussed in the next section.

Consider the series $\{\varepsilon_t\}$. The ARCH(q) for this series can be generated by the conditional distribution of ε_t with information Ω_{t-1} available. The series $\{\varepsilon_t\}$, with the available information Ω_{t-1} is ARCH(q) as,

$$\varepsilon_t \mid \Omega_{t-1} \sim N\left(0, \sigma_t^2\right) \tag{5}$$

[2] and

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2. \tag{6}$$

The first of the two equations above states that the conditional distribution of ε_t with the information Ω_{t-1} should be normal. The second equation, the conditional variance, specifies how σ_t^2 is defined by the information Ω_{t-1} . The information Ω_{t-1} contains all available information, including values of the series itself, and information that can be calculated from the series. Further, Ω_{t-1} might even include information on related time series; other information that is available and can be of value for the prediction. The following observation ε_t should also be normally distributed with zero conditional mean ($E\left[\varepsilon_t \mid \Omega_{t-1}\right] = 0$) and with a conditional variance $\left(var\left[\varepsilon_t \mid \Omega_{t-1}\right] = \sigma_t^2\right)$. From this it follows that even if $\{\varepsilon_t\}$ cannot be forecasted, $\left\{\varepsilon_t^2\right\}$ can be forecasted. Thus, the forecast of ε_t^2 ($E\left[\varepsilon_t \mid \Omega_{t-1}\right] = var\left[\varepsilon_t \mid \Omega_{t-1}\right]$) equals the ARCH(q) specified in the second of the above equations.

The problem of using the simplest forms of (G)ARCH models is that the effect that a shock has on the volatility only depends on the size of the shock, but ignores the sign of it. Another problem is that the risk and return are not directly related, as in the CAPM described earlier. Alternatives of the GARCH models have been developed as a result. An Integrated GARCH (IGARCH) was suggested by Engle and Bollerslev (1986) to account for the problem of the parameters in the estimations summing up to unity in the GARCH and the problem that many lags are often needed to model volatility with a simple ARCH. Another popular specification accounting for

[2] The conditional variance is given as,

$$var\left[r_t \mid r_{t-1}\right] = var\left[e_t\right] = \sigma_{e_t}^2.$$

ARCH models allow the conditional variance to depend on the available data.

the asymmetric dependence is the Exponential GARCH (EGARCH) by Nelson (1991). Nonlinear models might be beneficial because of the characteristics in time series data. The first two essays, "Short-Horizon Asymmetric Mean-Reversion and Overreactions: Evidence from the Nordic Stock Markets" and "An Empirical Comparison of Linear and Nonlinear Modeling of Volatility in Stock Returns", are closely related to this and empirically investigate how the best estimations are made. Essay four, "The Relationship between Returns, Return Volatility and Information - An Asymmetric Approach", also focuses on building the best estimation model by including different impulses.

Essay 3 "An Empirical Investigation of Value-at-Risk in Long and Short Trading Positions" deals with measuring the risk from the perspective of a trader holding possibly both long and short trading positions.

For financial institutions especially, good risk measures are of high importance. Financial risk highly influences the behavior at financial markets, since it generates unpredictable changes in the risk factors. Risk generally is grouped in five areas (Angelidis and Degiannakis, 2007): market, liquidity, business, credit and finally operational risk. Market risk is the risk that arises from unpredictable changes in risk factors. Liquidity risk is a result of the lack of possibility to liquidate an asset without resulting in price changes. Business risk is the risk associated to the specific industry or area in which the firm is active. Credit risk is the risk that is associated with problems in fulfilling the obligations. Operational risk is the risk associated with problems in internal systems, catastrophes or errors caused by humans.

2.3 Linear Volatility Models

The basic ARCH model by Engle (1982) can handle volatility clustering since the conditional variance of the error term is an increasing function of the squared shock for time $(t-1)$. This means that if the shock in the previous period is large, the expected shock for this period is also large (meaning volatility appears in clusters). Large (or small) returns tend to be followed by large (or small) returns.

Another characteristic of the basic ARCH model is that it can handle the excess kurtosis, or fat-tailness. However, the ARCH(1) model cannot take care of the simultaneous autocorrelations that occurs in the return series. This extended persistence results in a desire for including additional lags of the squared shocks in the conditional variance.

Bollerslev (1986) suggested an alternative to the lagged ARCH model, adding instead of squared past shocks, one lag of the conditional variance. The GARCH model with lags p and q (GARCH(p,q)) is given as,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2. \tag{7}$$

The parameters must satisfy the conditions; $\omega > 0$, $\alpha_i > 0$ and $\beta_i \geq 0$. The persistence in the volatility in GARCH(1,1) is measured as $(\alpha_1 + \beta_1)$. If this sum is close to one, the persistence is high, and the autocorrelations decrease marginally. Usually, the basic model includes only one lag of the shock and one of the variance meaning the GARCH(1,1) gives fair forecasts.

In the GARCH it is assumed that the conditional distribution is normal but the unconditional distribution can still have thicker tails than the normal distribution, and thus be non-normal (Premaratne and Bera, 2001). The normal or Student's t version for ARCH do not completely account for the excess kurtosis that can be seen in the data (Premaratne and Bera, 2001); they are not heavily tailed enough. The Generalized Error Distribution (GED) in combination with the EGARCH was employed by Nelson (1991). The GED includes the normal as a special case, but further comprises both distributions with thicker and thinner tails than the normal. The reason for adapting these distributions is that the log-likelihood functions for the conditional Student-t and GED density produces estimates that are not affected by extreme observations excessively. Extreme observations occur with a low probability, and can be for example a stock market crash or an extreme boom. More reliable statistical conclusions can also be drawn since more robust standard errors of the parameters are generated. It is thus possible to test the reliability of models assuming normality (Bali and Demirtas, 2007).

The log-likelihood function for the normal distribution is;

$$LogL_{normal} = -\frac{n}{2}ln(2\pi) - \frac{n}{2}\sigma^2_{t|t-1} - \frac{1}{2}\Sigma^n_{t=1}(\frac{r_t - \omega - \alpha r_{t-1}}{\sigma_{t|t-1}})^2. \tag{8}$$

The log-likelihood function for the Student's t distribution is;

$$LogL_{Student-t} = nln\Gamma(\frac{v+1}{2}) - nln\Gamma(\frac{v}{2}) - \frac{n}{2}ln((v-2)\sigma^2_{t|t-1}) - (\frac{v+1}{2})\Sigma^n_{t=1}ln(1 + \frac{\varepsilon^2_t}{(v-2)\sigma^2_{t|t-1}}). \tag{9}$$

The log-likelihood function for the GED is;

$$LogL_{GED} = ln(v/2) + 0.5ln\Gamma(3/v) - 1.5ln\Gamma(1/v) - 0.5\Sigma^n_{t=1}ln\sigma^2_{t|t-1}$$

$$-exp((v/2)(ln\Gamma(3/v) - ln\Gamma(1/v))) \times \Sigma^n_{t=1} \mid \frac{r_t - \mu_{t|t-1}}{\sigma_{t|t-1}} \mid^v. \tag{10}$$

2.4 Nonlinear Volatility Models

There are some problems associated with linear (G)ARCH models. In an ARCH(q) model for example, q must in some cases be quite large to handle the dependence of the conditional variance. Another problem related to this is that q cannot be determined in advance. Further, restrictions of non-negativity on the parameters might not hold. The other linear model considered, GARCH(p,q) is somehow more flexible in terms of taking into account some of the characteristics not captured by the ARCH(q) model. Regarding non-negativity restrictions, the GARCH(p,q) is more flexible than the ARCH(q) (Angelidis and Degiannakis, 2007).

(G)ARCH type models are thus incapable of capturing stylized facts such as kurtosis. Kurtosis can often be observed in the standardized residuals. Previous research (French et al. (1987), Chou (1988) etc.) argues that the null hypothesis of a unit root in variance cannot always be rejected and when large shocks are controlled for in the data, GARCH effects disappear (see e.g. Frances and Ghijsels (1999)).

The features of a nonlinear model are foremost that they allow a drop in price to be of greater impact on the volatility than a price rise of the same size. Nelson (1991)'s EGARCH has been employed in many studies, and is used in the essays in this dissertation as well. The advantage of the EGARCH is that no estimation constraints need to be made to avoid a negative variance. This is because the logarithm of the variance is formulated. The EGARCH in this sense captures negative shocks resulting in higher volatility than would follow a positive shock. The GJR-GARCH model by Glosten et al. (1993) is also a popular nonlinear asymmetric model, as is the Threshold GARCH (TGARCH) by Rabemananjara and Zakoian (1993).

The essays in this dissertation focus on the possible superiority of using non-linear models for modeling and predicting volatility, and further considers appropriate models for the mean.

3 Volatility and Asymmetry

Volatility and expected return go hand in hand since if volatility is priced, it should be that increasing volatility results in higher required return. However, there are two conflicting hypotheses on this. The leverage hypothesis states that it is the shocks in returns that raise volatility. The time-varying risk premium theory on the other hand states that the shocks in returns in fact occur as a result of changes in volatility.

The relationship between return volatility and autocorrelation of the returns has been observed in research as well. There is strong evidence of this relationship being negative, e.g. Brock and LeBaron (1996). This indicates that the volatility is higher when negative autocorrelation is seen. Research on the relation between volatility and expected return is furthermore conflicting. Among others French et al. (1987) report a positive relationship, whereas for example Glosten et al. (1993) and Nelson (1991) reports a negative relationship. For a proficient summary on empirical research on asymmetric volatility, see Bekaert and Wu (2000).

Asymmetry in variance and mean is often ignored (Bekaert and Wu (2000)). A number of models (described in the previous section) that incorporate asymmetry components have been developed by, for example, Nelson (1991) (EGARCH), and Glosten et al. (1993) (GJR-GARCH). Research indicates that the importance and advantage of asymmetric models for volatility is statistically significant, but this may not hold economically.

Volatility persistence measures how fast financial markets or the time series forget large shocks in volatility. Autocorrelations tend to be significant for several lags. This is related to stationarity. In stationary (G)ARCH models, memory tends to decay rapidly at an exponential speed; in integrated (G)ARCH, no decay at all can be observed.

The volatility in financial time series data behaves asymmetrically in that negative shocks in the price result in increased volatility, whereas a positive shock of the same magnitude does not increase the volatility as much. Asymmetry in the return series can be observed as patterns where returns are reverting to a mean level more quickly after bad news or a negative drop in price, whereas following good news, a positive shock persists. Asymmetric patterns in the volatility of returns data were pointed out by Black (1976). Other researchers that have also proved this are e.g. French et al. (1987), Nelson (1991) and Glosten et al. (1993); just to mention a few.

The asymmetry concept has also been introduced as a characteristic in the mean process by Hagerud (1996), Lundbergh and Teräsvirta (1998), Koutmos (1998) and several papers by Nam (2003), Nam et al. (2002), Nam (2001). They point out that the asymmetric characteristics can be seen in that stock prices do not reflect the predicted risk, especially when it comes to bad news. If asymmetry is a characteristic observed both in mean and variance, nonlinear setups should be applied both to the mean equation and the variance equation. An interesting question that could be posed when introducing nonlinear models is whether the stock market requires higher premiums for negative shocks. This question was raised by Glosten et al. (1993) and by Nam (2001). Surprisingly, in both of these papers the relationship between the risk premium and volatility prediction is negative following a negative shock. The stock market does not require higher premium for bearing the higher risk that follows the negative return shock. Actually, reduced risk premium requirements can be observed. Glosten et al. (1993) explains the negative correlation between risk and expected return as a result of investors being very, or even exceptionally optimistic. This is not in line with the time-varying rational expectation hypothesis, which states that there is a positive relationship between volatility and the risk premium (Nam et al., 2001).

Finally, how the performance of the return and risk models should be evaluated is of great importance. Akgiray (1989) was the first to employ GARCH for volatility forecasting purposes. Many studies have followed since.

4 Summary, Joint Results and Contribution of the Essays

This section will present individual summaries, a description of methodology, the contribution and briefly discuss the results of the four separate essays in the dissertation.

4.1 Essay 1: Short-Horizon Asymmetric Mean-Reversion and Overreactions: Evidence from the Nordic Markets

In the existing literature evidence suggests that the conditional variance is asymmetric. Most of the literature does take into account the variance following shocks, and relates this to what is called the leverage effect. Large changes in the negative direction lead to increased variance, when positive reactions even reduce the variance. This asymmetric behavior in the variance is captured by different extensions of the traditional ARCH model by Engle (1982).

Also, if returns behave asymmetrically, it could be possible to use contrarian-type strategies where “loser-stocks” outperform “winner-stocks”, and studies indicate that stock returns generally revert more quickly following negative returns than following positive returns. (See for example Sentana and Wadhwani (1992)).

The purpose of the essay is to examine the asymmetric mean-reverting behavior of both mean and variance on the Nordic stock markets, including indices from Finland, Sweden, Norway and Denmark, in order to investigate asymmetric patterns not only in the conditional variance, but also in the conditional mean. From this contrarian trading strategies are to be developed.

Linear autoregressive models restrict the serial correlation coefficient in that the coefficient must remain constant. Such linear models cannot handle the asymmetric reverting patterns in the return dynamics. Therefore, an Asymmetric Nonlinear Autoregressive (ANAR) model is introduced. In the model, the serial correlation coefficient is allowed to react differently to positive and negative shocks that occur. The model includes an asymmetry parameter that measures what degree of asymmetry the series shows. To this basic model, an ”in mean” parameter is added, since an EGARCH-M model by Nelson (1991) is used for the variance equation. The ANAR model is further extended, to investigate three more quotations or changes in the same direction, and further one model is created to investigate possible contrarian strategies.

For the one period model, the serial correlation parameter for positive returns is positive and significant for all the included indices. This means that positive returns are generally persistent. The parameter measuring the reverting pattern is negative and significant. These results indicate that the returns are asymmetric in mean in that positive returns are followed by more positive returns and negative returns revert to positive returns faster than positive revert to negative. For the two- three- and four-period models, the results are not that convincing as for the one-period model, but the parameter for the mean equation for positive returns is significant in three out of four cases. The volatility is asymmetric and rises following negative returns. The results for the other two models are similar to the two-period model.

The most significant results are generated from the one-period model. However, significant asymmetry in the variance is observed for all the four markets under investigation, indicating that negative reactions increased volatility more than positive reactions. In the model where a requirement on the standard deviation is included in the dummy variable, the purpose was to find a model for using contrarian type strategies. According to the expectations, the result should be most significant for this model. Some significant results were generated, but not as significant as any of the other presented models.

To summarize, asymmetry could be observed, not only in the conditional variance, but also in the conditional mean. Negative returns generally turned out to be mean-reverting with a greater magnitude than positive. Contrarian strategies could be of some use following these results. These negative feedback strategies result in positive return autocorrelations, and a more lively trading based on these strategies is expected when the prices are exposed to larger movements.

4.2 Essay 2: An Empirical Comparison of Linear and Nonlinear Volatility Models for Nordic Stock Returns

Essay 2 "An Empirical Comparison of Linear and Nonlinear Volatility Models for Nordic Stock Returns" evaluates linear and nonlinear models for the variance in the perspective of forecasting performance, skewness and kurtosis.

The interest in finding a volatility model that can describe the data series most correctly lies in the ability to make the best possible predictions of future risk. The ARCH model by Engle (1982), and the GARCH model by Bollerslev (1986) have gained lots of support, and this paper builds upon these models for the conditional variance. Research has indicated that financial time series data and the corresponding risk behave in a nonlinear manner, giving support to volatility models that can handle these effects. EGARCH is one version of the traditional (G)ARCH models. Asymmetric or nonlinear patterns in the variance are closely related to the stability of the markets. In unstable markets, shocks tend to have a more prominent effect, meaning more influence of the shock.

The performance of different linear and nonlinear models with a closer look at how good the models are at absorbing skewness and kurtosis are the objectives of the paper. This essay partly works as a support to essay 1. The contribution of the paper concerns the way skewness and kurtosis are seen as indicators of the performance of the models and further in the application to small stock market indices.

For identifying nonlinear or asymmetric patterns, sign- and size-bias tests by Engle and Ng (1993) are employed. The variance is modeled with the linear GARCH by Bollerslev (1986), and the nonlinear models Quadratic GARCH (QGARCH) by Sentana (1995), the EGARCH by Nelson (1991), the GJR-GARCH by Glosten et al. (1993), the TGARCH by Zakoian (1994) and finally the Volatility Switching GARCH (VS-GARCH) by Fornari and Mele (1997).

To evaluate the models, linear and nonlinear ARCH effects are investigated with Engle (1982)'s Lagrange Multiplier (LM) test, and a modified LM test by Lundbergh and Teräsvirta (1998). Parameter stability is tested with the parameter constancy test introduced by Franses and van Dijk (2004).

The results indicate that nonlinear models are not necessarily better at predicting future risk. However, in many cases nonlinear models generate slightly better predictions. Further, the results show that the Nordic stock market indices are subject to asymmetric patterns to a certain degree. However, the asymmetric patterns in variance indicate that negative shocks are more prominent than positive shocks. In terms of absorbing typical patterns in time series data, skewness and kurtosis, nonlinear models seem to outperform linear ones.

4.3 Essay 3: An Empirical Investigation of Value-at-Risk in Long and Short Trading Positions

Essay 3 "An Empirical Investigation of Value-at-Risk in Long and Short Trading Positions" uses the Value-at-Risk (VaR) approach for measuring risk in both the left tail and the right tail of the distribution.

This essay evaluates VaR measures estimated under different approaches for modeling the variance. Asymmetry and volatility clustering, and also distributions that are not symmetric have been modeled. The essay focuses on the left and also the right tail of the distribution, consequently risk in both tails.

VaR was first introduced by Jorion (1996) as a result of the demand for a risk management tool for the downside risk. VaR is used to estimate the predicted financial loss that can be expected with some specific probability. Results from previous studies on VaR and the VaR estimated of the down side risk indicate that the correctness highly depends on the models used, and the most suitable model varies among different indices and time horizons.

This essay, in addition to looking at the left tail of the distribution, also investigates the right tail. This setup makes it possible to use VaR for e.g. traders holding both long and short trading positions. The contribution of the essay lies in the question of identifying how a trader holding both long and short positions can benefit from using VaR as risk measure. In addition, the paper applies different distributions and models for the volatility estimation (symmetric and asymmetric) for VaR calculations. What distinguishes this paper from previous papers on this topic (see e.g. Giot and Laurent (2003a,b)and So and Yu (2006)) is that this paper focuses on the difference between VaRs for long and short positions. Earlier papers have been directed at finding models appropriate for both sides.

Asymmetry is considered in two ways: First, asymmetry is related to the relationship between the conditional variance and the lagged squared error term; this by (G)ARCH type modeling and extensions. Second, asymmetry is considered in the distribution applied in the modeling of the variance; this is accomplished by introducing asymmetric distributions.

For estimating the variance, Bollerslevs traditional GARCH model is used as starting point or benchmark. Two asymmetric extensions are applied; the EGARCH by Nelson (1991) and the Asymmetric Power ARCH (AP-ARCH) by Ding et al. (1993). Three distributions are combined with these models: the Normal (Gaussian) distribution, the Student's t-distribution (symmetric) and the GED (asymmetric). To evaluate the combinations, Kupiec (1995)'s test of unconditional coverage rate and Giot and Laurent (2003b)'s failure rate is calculated for the different confidence levels and indices. The degree of current conditional coverage for the VaR:s are calculated with Christoffersen (1998)'s Likelihood Ratio test.

The results from the empirical investigation show that more flexible models do not necessarily generate better VaR forecasts. Different models/ methods for the variance estimation are needed for different confidence levels of the VaR and also for the different indices. The left respectively the right tail of the distribution requires different estimation models. The results from the empirical investigation on the indices Nikkei, FTSE 100, DAX and S&P 500 suggest that more complicated models do not necessarily produce better out-of-sample VaR forecasts. Consequently a trader holding long or short positions, or both, can measure the variance for the VaR in a simple straightforward way. The most preferable model and distribution is the Normal distribution and the basic GARCH model performs very well. However, asymmetric variance models in some cases generate more accurate predictions.

VaR estimates generally overestimate risk, and generates too safe predictions. Consequently the number of exceptions differ substantially from the theoretical values.

4.4 Essay 4: The Relationship between Returns, Return Volatility and Information - An Asymmetric Approach

In Essay 4 "The Relationship between Returns, Return Volatility and Information - An Asymmetric Approach" the risk-return-information relation is investigated. Both the contemporaneous and the dynamic relationship are of interest. The asymmetry concept is introduced both in the conditional mean and variance.

The essay builds on the ideas of Lamoureux and Lastrapes (1990) to use ARCH models for investigating the risk-return-volume relationship and the study by Glosten et al. (1993) in introducing volume and interest rates as impulse variables for explaining ARCH effects, and reducing volatility persistence.

The impulse variables are introduced into the variance equation and these impulses aim to serve as factors that can have effects on the correctness of the volatility estimations. The introduced factors are, first trading volume, and second interest rates. Trading volume as impulse has been investigated by among others Glosten et al. (1993). The factor has proven to be a good information variable to include into the variance equation. It is mostly positive and significant, indicating better estimations for the volatility can be made with the help of including the variable. The second impulse introduced, interest rates, are not as significant. Negative values are received of the factor, however not consistently significant.

Asymmetry is introduced first in variance through the EGARCH model and second in the mean equation employing a piecewise regression model. Asymmetric patterns in both conditional mean and variance are strong enough to be accounted for. Including the correction that can be seen in using asymmetric mean and variance equations produces better estimates of the volatility. The variables introduced and the addition of the asymmetry components contributed to a decrease in ARCH effects only to a limited extent. A strong reduction in volatility persistence could neither be seen. Alternative variables for the volume were therefore tested. A "high-low" parameter and an "absolute return" variable were introduced and contributed to a decreased persistence and an increase in Log-Likelihood value, meaning better estimates and possibilities to create better forecasts.

5 Concluding Remarks

The importance of good models for describing both mean and variance has been discussed above and this will also be the focus of the essays in this dissertation. The reason why these models are of high importance lies in the ability to make the best possible estimations and predictions of future returns and for predicting risk.

Financial time series tend to behave in a manner that is not directly drawn from a normal distribution. Asymmetries and nonlinearities are usually seen and these characteristics need to be taken into account. To make forecasts and predictions of future return and risk is rather complicated. The existing models for predicting risk are of help to a certain degree, but the complexity in financial time series data makes it difficult. The introduction of nonlinearities and asymmetries for the purpose of better models and forecasts regarding both mean and variance is supported by the essays in this dissertation.

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List of essays:

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Part II

THE ESSAYS

Short-Horizon Asymmetric Mean-Reversion and Overreactions: Evidence from the Nordic Stock Markets

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Abstract

This paper examines the asymmetric behavior of conditional mean and variance. Short-horizon mean-reversion behavior in mean is modeled with an asymmetric nonlinear autoregressive model, and the variance is modeled with an Exponential GARCH in Mean model. The results of the empirical investigation of the Nordic stock markets indicate that negative returns revert faster to positive returns when positive returns persist longer. Asymmetry in both mean and variance is observed on all included markets and behaves fairly similarly. Volatility rises following negative returns more than following positive returns which is an indication of overreactions. Negative returns increase the variance and positive returns decrease the variance.

Key words: asymmetric mean-reversion, overreactions, nonlinearity, Exponential GARCH in Mean, Nordic stock markets

JEL classification: G12

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1 Introduction

Already in 1914, Bachelier presented a theory of the behavior of changes in security prices, known as the hypothesis of random walk (RW). Fama developed these thoughts in 1965, and brought forward results supporting the RW hypothesis (RWH) in that price changes had no memory (Lendasse et al., 2000).

In the existing literature on financial time series, evidence suggests that the conditional variance is asymmetric. Most of the literature does take into account the variance following shocks. The asymmetry, also known as the leverage effect, has contributed to a number of extended versions of the autoregressive conditional heteroscedasticity (ARCH) model introduced by Engle (1982). A definition of asymmetric behavior in variance in financial time series is that large changes in a negative direction, lead to increased variance, whereas when the change is positive, the increase is smaller, or even decreased variance can be observed. Financial time series react in a more volatile manner following negative returns than following positive returns. This asymmetric behavior in the variance can be captured by versions of ARCH type models. However, the mean might also behave asymmetrically, in which case asymmetric models are also needed for the estimation of the mean. Some researchers have studied the asymmetric relationship in both mean and variance; among others Koutmos (1998) and Nam et al. (2001).

Studies have documented positive and negative autocorrelations over different intervals. Negative autocorrelation indicates a mean-reverting behavior in stock prices. According to Kim et al. (1991) changes in the stock price tend to be followed by predictable changes in the other direction during the following period. This would in turn be an indication that the RWH does not completely hold. Usually, tests of mean-reversion are done with variance ratio tests. With variance ratios, the autocorrelation is indirectly the interesting coefficient to look at, so is the case in this study.

De Bondt and Thaler (1987, 1985) first presented the overreaction hypothesis. According to them, investors react irrationally to different kinds of news. Patterns in the returns, containing both positive and negative jumps could be observed in the study. If these patterns can be observed, it could also be profitable to use contrarian type strategies. Following the articles by DeBondt and Thaler, many articles concerning the hypothesis have been published.

The concept of contrarian-type strategies is often referred to patterns where prior loser-stocks outperform prior winner-stocks, and studies have indicated that stock returns generally revert more quickly following negative returns, than following positive returns. The logic is that if stock returns revert faster following negative returns, positive returns following negative returns can be expected. Consequently, the contrarian strategy exploits the inherent reverting process. Asymmetry in the mean-reverting process is thus of high interest. Many studies have investigated the asymmetry, referred to as the leverage effect, and investigated the effect returns in either direction have on the volatility (see for example Sentana and Wadhwani (1992)). Accordingly, this study will investigate the asymmetry in the response in volatility of the variance process with a nonlinear asymmetric model for the mean equation. Further, the overreaction hypothesis is closely related to mean-reversion, and the behavior might contribute to contrarian profitability. This possibility will also be investigated in this paper.

The purpose of the paper is to study the asymmetric mean-reverting behavior of both mean and variance on the Nordic stock markets, including Finland (HEX), Sweden (OMX), Norway (OSEBX) and Denmark (KFX) with a version of an Asymmetric Nonlinear Autoregressive (ANAR) - Generalized ARCH (GARCH); an ANAR - Exponential GARCH -in Mean (EGARCH-M) model.

The study will survey the asymmetric patterns possibly present in the returns and variances thereof. The study contributes to the existing literature in several ways. First, the tests are carried out on data from the Nordic stock markets. No literature capturing the asymmetric mean-reverting behavior on these markets has been found. Second, daily data is used when most of the studies in this area are done on weekly or monthly observations, and generally not on smaller stock markets. Further, only one study using an ANAR model has been found. The asymmetry in the variance is further measured with the EGARCH-M model.

The empirical results generated in this study show that negative returns have stronger mean-reversion patterns than positive returns or shocks of the same magnitude. Negative returns also tend to result in higher volatility, meaning negative returns are producing higher risk than positive returns of the same magnitude. Asymmetric patterns should naturally be considered both in the mean and the variance when modeling financial time series data.

2 Summary of the Literature

A number of ARCH and GARCH type models have been extended in order to take into account the asymmetry in the variance. The EGARCH model by Nelson (1991) is perhaps the most known asymmetric model for the variance. Other commonly used models are the Threshold ARCH (TARCH) and GARCH (TGARCH) by Zakoian (1990) and Rabemananjara and Zakoian (1993), and the GJR-GARCH by Glosten et al. (1993). The Smooth Transition GARCH (ST-GARCH) by González-Rivera (1998), and the Volatility Switching GARCH (VS-GARCH) by Fornari and Mele (1997) can also be mentioned as good models for handling the asymmetries in the variance.

To avoid misspecification in the conditional variance, the mean should also be correctly modeled. Usually, models do not take into account nonlinearity. Many models for time series ignore the behavior of the first moment (mean), and only concentrate on the second moment (variance). Models for taking into account asymmetry and nonlinearity in both the mean and the variance have also been presented; some of these studies are discussed below.

Li and Li (1996) use a Double-Threshold ARCH (DTARCH) model. Both mean and variance have threshold structures which impose many possibilities, and the model is rather flexible since many other models are included in the model as special cases. The results from empirical work employing the model indicate that asymmetries are observed in both the mean and the variance. These asymmetries are significant to the extent that they should be accounted for when modeling financial data.

Hagerud (1996) discovers two Smooth Transition models; one logistic (LSTAR-GARCH) and one exponential (ESTAR-GARCH). Nonlinearity is considered in the models. The models in a way complement each other. Both asymmetry in the sign and the size of the error term is taken into account; in the logistic model, the signs of the residuals are considered, and in the exponential the absolute size of the residuals are considered. Results from estimations made with the logistic model indicate that large negative residuals contribute to increased variance more than for positive residuals. From the results of the exponential model, it can be observed that large residuals are too heavily weighted in the basic ARCH models.

Lundbergh and Teräsvirta (1998) use a Smooth Transition Autoregressive (STAR) model for modeling the mean, and a ST-GARCH model, which is a modification of the GJR-GARCH, for the variance. The model is good for characterizing high-frequency time-series and the STAR-STGARCH model also captures the nonlinear behavior in both the conditional mean and the conditional variance. The results of the modeling indicate that it is important to use models that capture asymmetries and nonlinearity in the first moment (the mean) as well as in the second moment (the variance).

González-Rivera (1998) further develops the ST-GARCH model. The asymmetry in the variance response is modeled by the smooth-transition mechanism. High- and low-volatility regimes are treated and a large advantage of the model is that the threshold model is also nested.

Koutmos (1998) uses an Asymmetric Autoregressive Threshold GARCH (asAR-TGARCH) model. The model takes into account asymmetries in the mean and the variance. The model has many similarities with the ANAR model used in this study. The results of the empirical investigation indicates that both the mean and the variance behaves asymmetrically. Further, as has been proven by other studies, negative returns reverts, more quickly than positive returns.

Nam (2003); Nam et al. (2002, 2001) use an Asymmetric Nonlinear Smooth Transition GARCH (ANST-GARCH) model. The model is constructed for measuring the mean-reversion and taking into account possible asymmetry in both mean and variance. As for other models, the asymmetry in the variance is referred to as the leverage effect, but the asymmetry in the mean is referred to as the reverting property of return dynamics. The results from the investigation confirms that negative returns generally reverted faster than positive returns. The dimension of the reversion seemed as well to be larger for negative returns.

Brännäs and DeGooijer (2004) combine an asymmetric moving average (asMA) model for the mean equation with an asymmetric Quadratic GARCH (as-QGARCH) model for the variance equation. This model (asMA-asQGARCH) allows the response to shocks to behave asymmetrically. The results indicate that both conditional mean and conditional variance behave asymmetrically to past information. For changes in the positive direction, the conditional heteroscedasticity reacted negatively, and to negative changes the reaction is positive; the volatility increases.

3 Asymmetry in the Serial Correlation Coefficient

Linear autoregressive models restrict the serial correlation coefficient in that the coefficient must remain constant. In linear models with a constant serial correlation coefficient, the reaction of stock returns has to be the same for both positive and negative shocks (Nam, 2001). Such linear models cannot handle the asymmetric reverting patterns in the return dynamics. In research it has been observed results against the statement that returns react the same way to both positive and negative shocks or returns. Asymmetry also in mean can be seen as an important issue when modeling financial time series data. To incorporate this asymmetry, Nam (2001) presented an Asymmetric Nonlinear Autoregressive (ANAR) model. In the model, the serial correlation coefficient is allowed to react in a different manner to positive and negative shocks that occur. This model will be presented and employed in this paper.

To support the understanding of the effect of the serial correlation coefficient, a model of price behavior presented by Amihud and Mendelson (1987) will be laid out. The model includes the logarithmic price, p at time t and $t-1$, the logarithm of the real value (intrinsic value) v, an adjustment coefficient ϕ, and a noise term ε. The return is calculated as,

$$p_t - p_{t-1} = \phi[v_t - p_{t-1}] + \varepsilon_t. \tag{1}$$

The adjustment coefficient gives the reversion of the price towards the real value. If the coefficient

(ϕ) is one, the price is completely adjusted. The model then becomes,

$$p_t = v_t + \varepsilon_t. \tag{2}$$

If the coefficient (ϕ) is zero, there is no reaction to value changes. This case is thus somewhat unrealistic. If the coefficient is between zero and one, a partial price adjustment can be observed, and if the coefficient is larger than one, it can be an indication of overreactions.

To explore this, and allow the stock returns to behave asymmetrically, the nonlinear autoregressive model can be used[1];

$$r_t = \mu + \phi^- r_{t-1} + \varepsilon_{t-1},\ if\, r_{t-1} < 0, \tag{3}$$

and,

$$r_t = \mu + \phi^+ r_{t-1} + \varepsilon_t,\ if\, r_{t-1} \geq 0. \tag{4}$$

In the equations above, $|\phi^-| < 1$, and $|\phi^+| < 1$ for the stationary condition of r_t to hold. The serial correlation of the returns is captured by ϕ^- for negative returns ($r_{t-1} < 0$) and by ϕ^+ for positive returns ($r_{t-1} > 0$), and measures the reverting tendencies. The hypothesis states that the serial correlation following a negative return is less than following a positive return. This asymmetry property can be illustrated as; $\phi^+ > \phi^-$. This hypothesis introduced in the studies by Nam (2003); Nam et al. (2002, 2001), further states that the reverting behavior of shocks or returns with negative signs, in fact reverts faster to positive returns, than do positive returns or shocks. The reverting magnitude also seems to be greater following negative returns.

As pointed out, ϕ^+ and ϕ^- measure the reverting tendency and also the reverting magnitude for the returns. Considering the reverting behavior, the coefficients for both positive and negative returns can be positive. If $\phi^+ > \phi^-$, and both these coefficients are positive, this means that a negative return shock on average is less persistent than a return shock of the same size, but positive. Also if ϕ^- is negative, the reverting tendency would be stronger for negative returns than for positive. When observing the reverting magnitude, it can be stated that when the condition $\phi^+ > \phi^-$ holds, the reverting magnitude of negative returns or shocks is larger than for the positive returns.

The condition that will be tested in this article is indirectly: $\phi^+ > \phi^-$. If this condition is true, then negative returns or shocks generally revert more quickly than do positive returns or shocks, and contrarian strategies may be profitable. If on the other hand, $\phi^+ < \phi^-$, positive returns revert more quickly. Of course, it may also be that $\phi^+ = \phi^-$. This would imply that there is no asymmetry in the reverting behavior, in which case the serial correlation is constant.

4 Modeling the Asymmetric Mean-Reversion Behavior

As argued in the previous section, an important point in the modeling process is to consider the asymmetry in both the conditional mean equation and conditional variance equation to generate correct estimates of the return reversals of the mean and response in volatility. It is of importance to look at both the reverting behavior and the possibilities to take advantage of the asymmetries with contrarian type strategies.

[1] Following the setup by Nam et al. (2002).

As noted in the introduction, there are arguments both for and against the overreaction hypothesis. According to some, the hypothesis does not involve complete predictability. Instead, there are theories saying that what goes up, must come down, and vice versa. For the overreaction hypothesis to hold however, price changes must be negatively correlated for some holding period. If there are overreactions present on the market, it can be profitable to use a contrarian portfolio strategy. This kind of strategy exploits the negative serial correlation, or dependence, and one should purchase securities that have performed poorly in the past and sell (possibly short) securities that have performed well. According to the theory, current losers are likely to become future winners and current winners likely to become future losers. If the reactions are large enough, this selling winners and buying losers strategy would earn positive expected profits.

Positive earnings using contrarian portfolio strategies however, must not be because of overreactions, as the earnings may also come from positive cross- autocovariances across securities. If there is positive cross- autocovariance, contrarian portfolio strategies may contribute to positive earnings. According to Lo and MacKinlay (1990), over half of the expected profits are due to cross effects, and not to negative autocorrelation in individual security returns. According to Jegadeesh and Titman (1995) however, short-horizon contrarian strategies can make substantial profits. The reason for these profitable strategies mainly lies in the fact that prices tend to overreact to information, and thus the strategies work. Lo and MacKinlay (1990) did point out another possible reason for the reactions. According to them, some stocks react more quickly to information than others; when the returns of some stocks lead the returns of others.

The univariate first-order ANAR model (ANAR (1)) by Nam (2001) will be used as a starting point in the empirical investigation of possible asymmetric patterns on the Nordic stock markets. This model is developed for modeling the mean equation, and captures the asymmetry in mean. The ANAR (1) model has the following setup,

$$r_t = \mu + \left[\phi + \rho D_1 \left(r_{t-1} < 0\right)\right] r_{t-1} + \varepsilon_t. \tag{5}$$

D_1 in equation 5 is an indicator factor (or dummy) that will be 1 if $r_{t-1} < 0$ and 0 otherwise. The serial correlation coefficient (given by ϕ^+ and ϕ^- in Section 3), is given by ϕ following positive returns ($r_{t-1} \geq 0$), and $\phi + \rho$ following negative returns, ($r_{t-1} < 0$). For the measurement of the asymmetry, the coefficient ρ can now be observed, and in fact if $\rho < 0$, since $\phi^+ > \phi^-$ in Section 3 above, is the same as $\phi > \phi + \rho$ in the ANAR (1) model above, which gives the same as $\rho < 0$ for the asymmetry according to the hypothesis. The reason for using this model lies in its flexibility in allowing the returns autocorrelation coefficient to vary with the sign of the return time $t-1$ with that serial correlation calculated as $\phi + \rho$ when $r_{t-1} < 0$ and ϕ when $r_{t-1} \geq 0$.

To the basic model in equation 5, an "in mean" coefficient; δ, and the variance, σ_t^2 is included in the mean equation. The ANAR model above is now modified to,

$$r_t = \mu + \left[\phi + \rho D_1 \left(r_{t-1} < 0\right)\right] r_{t-1} + \delta\sigma_t^2 + \varepsilon_t. \tag{6}$$

The model is similar to the basic ANAR model in equation 5, except for the introduction of the "in-mean" parameter $\left(\delta\sigma_t^2\right)$ from the variance equation. In this model, only one time period is included. The asymmetric reverting pattern will be investigated for three more quotations or changes in the same direction (negative). For two periods of decline (in this case two days following each other), the ANAR (1) model receives the following specification (equation 7),

$$r_t = \mu + [\phi + \rho D_2 (r_{t-1} < 0, r_{t-2} < 0)]\, r_{t-1} + \delta\sigma_t^2 + \varepsilon_t. \tag{7}$$

For three and four periods (days) of decline, the models are specified as (equation 8 and 9),

$$r_t = \mu + [\phi + \rho D_3 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0)]\, r_{t-1} + \delta\sigma_t^2 + \varepsilon_t, \tag{8}$$

and,

$$r_t = \mu + [\phi + \rho D_4 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0, r_{t-4} < 0)]\, r_{t-1} + \delta\sigma_t^2 + \varepsilon_t. \tag{9}$$

D_2 to D_4 in the models above take the value of one if the returns for the given days are all negative, and zero otherwise. However, the case when the returns are negative three and four days at a stretch can be rather rare.

To build the framework for the contrarian investment strategy, negative price shocks and reactions to such will be investigated. An advantage of the ANAR (1) model used is that it is flexible and it is easy to include requirements in the dummy variable. The shocks or overreactions are in this case measured as a change in the daily standard deviation (of the returns) with two units. The model will be obtained as a modification of equation 6;

$$r_t = \mu + [\phi + \rho D_1 (r_{t-1} < -2\sigma)]\, r_{t-1} + \delta\sigma_t^2 + \varepsilon_t, \tag{10}$$

for one period. Still, the interesting parameter is the autocorrelation coefficient ρ.

ARCH and GARCH models are very popular for modeling the volatility in financial time series. The basic ARCH models are symmetric, in which case the size of shocks matters, but not the sign (Nelson, 1991).

In the basic ARCH and GARCH models, the mean equation is generally modeled by an autoregressive process (AR(p)) and the series is regressed on past values. In the ARCH the variance equation is regressed on lagged values of the error term (squared) from the AR(p) process (the mean equation), and a constant. When considering the GARCH model, the variance equation is a function of the squared error term from the mean equation, and also on its own values. However, as already mentioned, the basic models fail to capture asymmetry and nonlinearity. With this in mind, the modeling of the variance equation is done with asymmetric models.

In many studies it is observed that volatility reacts asymmetrically to the sign of the shocks. Financial time series contain nonlinearity, in which case traditional linear ARCH and GARCH models are not capable of modeling the data.

The well-known and commonly used GARCH model by Bollerslev (1986) makes the variance a linear function of past conditional variances with squared residuals. The variance depends on the size, but not on the sign of the error term;

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \left(\alpha_i \varepsilon_{t-i}^2\right) + \sum_{j=1}^{p} \left(\beta_j \sigma_{t-j}^2\right). \tag{11}$$

The simple GARCH model is useful for modeling financial time series, since it can handle such things as thick tales and volatility clustering. The leverage effect, here referred to as asymmetry (in variance), occurs since changes in stock prices tend to be negatively correlated with volatility.

Simple ARCH and GARCH models require a constant variance, and do not take into consideration the leverage effect. Different models that handle the asymmetry have been presented. The Exponential GARCH by Nelson (1991) and the Threshold GARCH by Rabemananjara and Zakoian (1993) are commonly used models that can incorporate asymmetries in the volatility. Other popular models are the GJR-model by Glosten et al. (1993) and the Quadratic GARCH by Sentana (1995).

The Exponential GARCH model by Nelson (1991), in which the variance is an asymmetric function of past errors can be defined as,

$$log\left(\sigma_t^2\right) = \omega + \sum_{j=1}^{p}\left[\beta_j log\left(\sigma_{t-j}^2\right)\right] + \sum_{i=1}^{q}\alpha_i\left[\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) + \gamma_i\left(\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| - E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right|\right)\right]. \quad (12)$$

In the exponential model by Nelson (1991), the variance can behave asymmetrically since the logarithm of the variance is used. The asymmetry and leverage effect is captured by the parameters α and γ . The direct parameter for the asymmetry is α. If the parameter α is negative, ($\alpha < 0$), then the volatility is asymmetric in the way that negative shocks generate higher volatility than positive. If the parameter α equals zero, ($\alpha = 0$) then the volatility is symmetric. Alternatively asymmetries are seen by the leverage effect parameter γ. When the parameter γ is positive ($\gamma > 0$), leverage effect is present, which means that returns and future volatility will be negatively correlated. This means that when returns are negative, the volatility will rise. If on the other hand, the parameter γ equals zero, ($\gamma = 0$), no leverage effect is seen, in which case there is no negative correlation between return and volatility.

The EGARCH is extended so that the conditional mean is a function of the conditional variance. The model (ARCH-M) was first presented by Engle et al. (1987), and includes the conditional variance as a function in the conditional mean[2]. This gives the EGARCH-M model for the variance. With an EGARCH-M model, the problem with time-varying volatility effects affecting the asymmetric pattern can be reduced. This application has widely been used when modeling stock returns, and with that also index returns.

The model can, as discussed, handle the asymmetry in the variance, and positive and negative shocks do not have the same reaction pattern. The asymmetry is thus captured in both the variance and the mean equation.

For the stationary condition to hold, the sum of the β:s must be smaller than one; $(\sum_{j=1}^{p}\beta_j) < 1$. For the estimation, the Generalized Error Distribution (GED) will be used. This distribution is preferred, since it can handle both thick and thin tails; financial time series data is usually characterized by fat tails. Further, the GED contains the normal distribution as a special case, as well as other distributions; both thin and thick tailed (Nelson, 1991).

The density function for the GED distribution is,

$$f\left(z\right) = \frac{\nu exp\left[-\left(\frac{1}{2}\right)\left|z/\lambda\right|^{\nu}\right]}{\lambda 2^{(1+1/\nu)}\Gamma\left(1/\nu\right)}, \; -\infty < z < \infty, \; 0 < \nu \leq \infty, \quad (13)$$

where the gamma function is expresses as $\Gamma\left(.\right)$, and

$$\lambda = \left[2^{(-2/\nu)}\Gamma\left(1/\nu\right)/\Gamma\left(3/\nu\right)\right]^{1/2}. \quad (14)$$

ν is the parameter that measures the thickness of the tail. If the parameter has a value of two,

[2]The "in mean" coefficient already included in the mean equations; equation 6-10.

($\nu = 2$), then z is normally distributed, and if the parameter is smaller than two, ($\nu < 2$), then the tails of the distribution are thicker than for the standard normal. If $\nu > 2$, the tails of the distribution are thinner than for the normal (Nelson, 1991).

The number of lags included in the variance equation will be selected with the Schwarz criterion. This method for the selection was chosen because it has been shown to be a better selection criterion than for example the Akaike information criterion, since noise can be better handled by the Schwarz criterion.

5 Data and Summary Statistics

The data included in this paper consist of index returns from the Finnish, Swedish, Norwegian and Danish stock markets.

The Nordic, or Scandinavian markets shows similar characteristics. This is natural, as there is a very strong coordination and cooperation economically within the Nordic countries and the tax systems and exchange rate policy in the region are similar (Booth et al., 1997). Not only is extensive trading conducted within the counties; the Nordic counties also tend to have the same external trading partners, which consequently supports the similarities in the stock markets. However, as can be observed in the data presented below, the Nordic markets are not completely co integrated when it comes to reactions and patterns.

The sample period is January 1996 to December 2004, and includes nine years of daily observations, or 2250 daily observations for the Finnish, the Norwegian and the Swedish indices, and 2245 for the Danish index. In the modeling, log returns of the indices are used.

Table 1: Summary Statistics for Daily Market Index Returns

	HEX	OSEBX	OMX	KFX
Mean	0.0006	0.0004	0.0003	0.0004
Median	0.0014	0.0010	0.0007	0.0006
Maximum	0.1456	0.0618	0.1102	0.0497
Minimum	-0.1742	-0.0718	-0.0853	-0.0626
Std. Dev.	0.0224	0.0121	0.0164	0.0116
Skewness	-0.4441	-0.5380	0.1025	-0.2761
Kurtosis	8.7169	6.7461	5.5839	4.9866
Jarque-Bera	3138	1427	630	398
Probability	<0.0001	<0.0001	<0.0001	<0.0001

The summary statistics are for daily logarithmic returns on the HEX index (Finland), the OSEBX index (Norway), the OMX index (Sweden) and the KFX index (Denmark). The sample covers the period from January 2, 1996 to December 31, 2004. The Jarque-Bera normality test shows departure from normality for all indices.

From Table 1, the descriptive statistics for the Nordic markets can be seen. The Finnish, Norwegian and Danish series are skewed to the left, whereas the return series for the Swedish market is slightly skewed to the right. The Finnish and the Norwegian series especially are highly leptokurtic, but also the Swedish and the Danish markets show excess kurtosis, indicating fat tails. This supports the use of the GED and supports asymmetries toward the negative side. The Jarque-Bera (JB) statistics are highly significant for all the four series, and so the normality can be rejected. The Finnish index seems to show most extreme summary statistics with highest mean, standard deviation and most extreme minimum and maximum values. On the other hand, the Danish index seems to show most stable summary statistics.

To try to forecast the results of the examination with the different versions of the ANAR-EGARCH-M model, Table 2 presents the number of observations: positive and negative as well as the number of positive and negative returns following each other.

Table 2: Summary Statistics for Negative and Positive Returns

	HEX	OSEBX	OMX	KFX
Total observations	2250	2250	2250	2245
Negative observations	1043	1012	1076	1069
Positive observations	1207	1238	1174	1176
Two negative observations	523	485	550	540
Two positive observations	686	709	647	646
Three negative observations	260	245	290	261
Three positive observations	392	401	342	370
Four negative observations	133	129	148	126
Four positive observations	216	216	176	215

The statistics show the daily logarithmic returns on the HEX index, the OSEBX index, the OMX index and the KFX index. The total number of observations and the number of negative and positive observations following each other are also shown.

For all four indices it is observed that the number of positive returns are higher, and that for positive returns, more observations can be seen at a row than for negative returns. This means that positive returns are more often followed by other positive returns, than is the case for negative returns[3]. The prediction that can be made from this is that positive returns are more persistent whereas negative returns tend to revert to positive returns faster. This already gives the indication that returns are asymmetric, and positive returns are more persistent when negative returns reverts quickly. For the Norwegian market, the difference between the number of positive and negative observations is most prominent. The smallest difference between positive and negative returns can be seen for the Swedish market. This might be an indication that most significant asymmetries in mean-reversion are to be observed on the Norwegian market, whereas less significant results are expected for the Swedish market. If this is the case, the Swedish market can be seen as more effective than the other Nordic markets.

To test the unit-root hypothesis of the series, an Augmented Dickey-Fuller Unit Root test is carried out. The hypothesis is rejected for the four series[4]. The most preferable number of lags for all indices are, according to the Schwarz criterion, one for the ARCH effect and one for the GARCH effect; EGARCH-M(1,1).

6 Empirical Results

The models are estimated with the maximum likelihood function, and the optimization algorithm used is the Marquardt algorithm. The mean equations (equation 6-10) will be estimated with the same variance equation; EGARCH-M, and the results of every model will be presented separately below.

As described in Section 4, the interesting parameters to look at from the estimations are the values of ϕ and ρ, and the sum of these. The serial correlation for prior negative reactions is given by $\phi+\rho$, and for prior positive reactions the serial correlation is measured by ϕ. If $\phi > 0$, this implies that positive prior returns are relatively persistent. The most interesting and telling parameter ρ

[3]This after correcting for the fact that there are not as many single negative observations as positive observations.
[4]The results of the unit-root tests are available upon request.

alone measures the asymmetric reverting pattern, and if $\rho < 0$ and $\phi > 0$, the serial correlation for negative returns is smaller than for positive. Further, $\phi + \rho < \phi$ implies that positive returns show persistence while negative returns show strong reverting tendencies. If this is the case, negative returns revert faster to positive returns than positive returns revert to negative. This in turn, leads to profitability for contrarian trading strategies in that prior "loser-stocks" will outperform prior "winner-stocks", and the asymmetry can be exploited.

Results from the estimation of equation 6 and 7 (for the mean equation), and EGARCH-M model (for variance equation), are presented in Table 3. For the one period model; panel A, the serial correlation parameter for positive returns ϕ is positive and significant for all the included indices at least at a significance level of 5 %. This means that positive returns generally are persistent. The parameter for measuring the reverting pattern ρ is negative and significant at a significance level of 10 % for the Finnish and Norwegian indices, and negative and significant at a level of 1 % for the Swedish and Danish indices. From this it follows that the returns are asymmetric in mean and positive returns are generally followed by positive returns and negative returns revert to positive returns faster than positive revert to negative. This means that the autocorrelation for prior negative returns is smaller than that for positive returns ($\phi + \rho < \phi$), and negative returns therefore show tendencies for quicker reversion. The asymmetric patterns in the mean process are stronger for the Swedish and Danish markets than for the Finnish and Norwegian markets.

The results from the two-period model can be seen in panel B and the results are not as convincing as for one period. The parameter for the mean equation for positive returns is still positive and significant for all indices except the Swedish, indicating persistence in positive returns, and the parameter for negative returns is smaller than that of positive, ($\phi + \rho < \phi$). For the Finnish market, the parameter for the asymmetry is negative and significant at 1 %, which indicates that for that span, the returns are very asymmetric; while positive returns persist, negative returns revert. The Danish index also shows a significant and negative coefficient at a level of 10 %. For the Norwegian and Swedish markets, the asymmetric parameter is negative but not significant, which means that no asymmetry can be seen in mean.

The GARCH-M parameter (δ) is negative for both models and all included indices, but significant only for the Finnish market, at 5 %. The leverage effect parameter for the variance equation (γ) is positive and significant at a level of 1 % for the four indices and for both models. This means that there is negative correlation between volatility and return, indicating asymmetric volatility, and that the volatility rises following negative returns. The direct asymmetry parameter α is negative for all indices and both models. However, the parameter for the Finnish market is not significant. The parameters for the Norwegian, Swedish and Danish markets are negative and significant, indicating strong asymmetry in variance, and negative shocks or returns increase the variance, when positive returns actually reduce variance. The parameter for the tail-thickness, ν is significantly smaller than two for all the four indices and both models. The distribution is thus characterized by thick tails.

Table 3: Results of the Estimation of the ANAR-EGARCH-M (1; 1, 1) for the One and Two Period Models

Parameter / Index	HEX	OSEBX	OMX	KFX
Panel A				
One period				
μ	0.0015	0.0010	0.0007	0.0006
	(3.5063)***	(2.8330)***	(1.6344)*	(1.8711)*
ϕ	0.0960	0.1123	0.0902	0.1216
	(2.4731)**	(2.7859)***	(2.7096)***	(4.1388)***
ρ	-0.1083	-0.1287	-0.1325	-0.1334
	(-1.6997)*	(-1.6953)*	(-3.0129)***	(-3.1194)***
δ	-2.8469	-5.6267	-0.8932	-0.2207
	(-2.1085)**	(-1.6330)	(-0.4484)	(-0.0729)
ω	-0.1332	-0.5546	-0.3498	-0.3725
	(-7.3023)***	(-6.4940)***	(-5.7553)***	(-5.4513)***
β	0.9956	0.9589	0.9760	0.9757
	(529.50)***	(112.59)***	(160.99)***	(149.01)***
γ	0.1278	0.2321	0.1838	0.1925
	(9.2723)***	(9.0093)***	(7.5711)***	(8.0532)***
α	-0.0126	-0.0699	-0.0876	-0.0435
	(-1.1982)	(-4.3991)***	(-7.2897)***	(-3.2351)***
ν	1.3674	1.5118	1.8369	1.6330
	(36.4818)***	(24.8563)***	(24.8889)***	(26.9596)***
Panel B				
Two periods				
μ	0.0015	0.0011	0.0008	0.0007
	(3.7697)***	(3.2385)***	(1.7802)*	(1.9152)*
ϕ	0.0979	0.0633	0.0285	0.0853
	(3.9939)***	(2.3964)**	(1.0654)	(3.2233)***
ρ	-0.2189	-0.0222	-0.0472	-0.1057
	(-4.2650)***	(-0.3716)	(-0.8048)	(-1.7950)*
δ	-3.0321	-2.6914	-1.6011	-2.1649
	(-2.5018)**	(-0.8674)	(-0.7512)	(-0.6749)
ω	-0.1268	-0.5621	-0.3004	-0.3608
	(-7.0717)***	(-6.5114)***	(-5.5849)***	(-5.4680)***
β	0.9958	0.9583	0.9807	0.9768
	(550.63)***	(111.40)***	(185.81)***	(154.62)***
γ	0.1225	0.2352	0.1713	0.1896
	(8.8637)***	(9.0537)***	(7.4837)***	(8.1186)***
α	-0.0123	-0.0703	-0.0726	-0.0408
	(-1.2070)	(-4.3787)***	(-6.5066)***	(-3.1253)***
ν	1.3660	1.5115	1.8518	1.6520
	(35.8617)***	(24.7220)***	(25.1579)***	(26.9551)***

Table 3 presents the results of the estimation of the conditional mean equation and the conditional variance equation for the HEX index, the OSEBX index, the OMX index and the KFX index. The sample covers the period from January 2, 1996 to December 31, 2004.

Conditional Mean Equation, one period model: $r_t = \mu + [\phi + \rho D_1 (r_{t-1} < 0)] r_{t-1} + \delta\sigma_t^2 + \varepsilon_t$.

Conditional Mean Equation, two period model: $r_t = \mu + [\phi + \rho D_2 (r_{t-1} < 0, r_{t-2} < 0)] r_{t-1} + \delta\sigma_t^2 + \varepsilon_t$.

Conditional Variance Equation: $log\left(\sigma_t^2\right) = \omega + \sum_{j=1}^{p} \left[\beta_j log\left(\sigma_{t-j}^2\right)\right] + \sum_{i=1}^{q} \alpha_i \left[\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) + \gamma_i \left(\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| - E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right|\right)\right]$.

The z-statistics are reported with significance levels 10% (*), 5% (**) and 1% (***) in parenthesis.

In Table 4, the results from equation 8 and 9 are presented. Panel A gives the results for the three period model, and the results are similar to those of two days. For all markets except the Swedish, the correlation parameter for positive returns is positive and significant, but the asymmetry parameter is significant only for the Finnish market, indicating that the returns are asymmetric. The parameter is negative but insignificant for the Swedish and the Danish markets, and even positive for the Norwegian market. This is an indication of no asymmetry in the mean reversion behavior. Similar results in the mean-reversion behavior can also be observed for four periods; in Panel B, where the asymmetry coefficient for the mean is negative and significant for the Finnish and the Swedish markets, and negative but insignificant for the Norwegian and the Danish markets.

The GARCH-M parameter, δ is significant only for the Finnish market when three periods are included, but insignificant in all other cases. The leverage effect parameter for the variance equation for the three and four period models is positive and significant at a level of 1 % for all four indices indicating negative correlation and rising volatility following negative returns. The asymmetry parameter α is negative and significant at a level of 1 % for the Norwegian, Swedish and Danish markets, but insignificant for the Finnish market. However, the variance of all indices is asymmetric. The tail-thickness parameter is again significantly smaller than two, and thus the distribution is characterized by thick tails.

Table 4: Results of the Estimation of the ANAR-EGARCH-M (1; 1, 1) for the Three and Four Period Models

Parameter / Index	HEX	OSEBX	OMX	KFX
Panel A				
Three periods				
μ	0.0016	0.0011	0.0008	0.0006
	(4.1532)***	(3.2704)***	(1.8251)*	(1.8876)*
ϕ	0.0603	0.0576	0.0290	0.0645
	(2.7801)***	(2.4224)**	(1.1961)	(2.7328)***
ρ	-0.1194	0.0082	-0.0744	-0.0600
	(-1.9944)**	(0.1186)	(-1.1187)	(-0.8950)
δ	-1.9158	-2.3632	-1.6797	-0.7600
	(-1.6540)*	(-0.7784)	(-0.8070)	(-0.2454)
ω	-0.1332	-0.5656	-0.3013	-0.3616
	(-7.2022)***	(-6.5455)***	(-5.6095)***	(-5.4548)***
β	0.9956	0.9580	0.9806	0.9766
	(525.96)***	(111.18)***	(186.55)***	(153.84)***
γ	0.1279	0.2365	0.1717	0.1892
	(9.1675)***	(9.0905)***	(7.4989)***	(8.1152)***
α	-0.0129	-0.0709	-0.0727	-0.0415
	(-1.2419)	(-4.3923)***	(-6.5017)***	(-3.1858)***
ν	1.3684	1.5111	1.8554	1.6552
	(36.4430)***	(24.7272)***	(24.9248)***	(27.0208)***
Panel B				
Four periods				
μ	0.0017	0.0011	0.0008	0.0007
	(4.2599)***	(3.2299)***	(1.8841)*	(1.9219)*
ϕ	0.0555	0.0604	0.0304	0.0625
	(2.6848)***	(2.6060)***	(1.2993)	(2.7690)***
ρ	-0.1331	-0.0169	-0.1373	-0.0942
	(-1.6856)*	(-0.2027)	(-1.7141)*	(-1.1732)
δ	-1.5455	-2.4688	-1.8137	-0.8391
	(-1.3372)	(-0.8238)	(-0.8837)	(-0.2735)
ω	-0.1254	-0.5647	-0.3015	-0.3479
	(-7.3228)***	(-6.5381)***	(-5.5976)***	(-5.4491)***
β	0.9959	0.9581	0.9807	0.9776
	(560.37)***	(111.19)***	(186.52)***	(160.01)***
γ	0.1214	0.2360	0.1726	0.1829
	(9.3090)***	(9.0868)***	(7.4995)***	(8.0927)***
α	-0.0129	-0.0709	-0.0728	-0.0402
	(-1.2773)	(-4.4007)***	(-6.5183)***	(-3.1461)***
ν	1.3821	1.5111	1.8606	1.6580
	(36.3511)***	(24.6893)***	(24.7578)***	(26.8489)***

Table 4 presents the results of the estimation of the conditional mean equation and the conditional variance equation for the HEX index, the OSEBX index, the OMX index and the KFX index. The sample covers the period from January 2, 1996 to December 31, 2004.
Conditional Mean Equation, three period model: $r_t = \mu + [\phi + \rho D_3 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0)] r_{t-1} + \delta\sigma_t^2 + \varepsilon_t$.
Conditional Mean Equation, four period model: $r_t = \mu + [\phi + \rho D_3 (r_{t-1} < 0, r_{t-2} < 0, r_{t-3} < 0, r_{t-4} < 0)] r_{t-1} + \delta\sigma_t^2 + \varepsilon_t$.
Conditional Variance Equation: $log\left(\sigma_t^2\right) = \omega + \sum_{j=1}^{p} \left[\beta_j log\left(\sigma_{t-j}^2\right)\right] + \sum_{i=1}^{q} \alpha_i \left[\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) + \gamma_i \left(\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| - E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right|\right)\right]$.
The z-statistics are reported with significance levels 10% (*), 5% (**) and 1% (***) in parenthesis.

Of the four models above, the most significant results for asymmetry in mean are observed for the one period model. Out of all the included indices, significant results were obtained for this model (Table 3, Panel A). Significant asymmetry in the variance can be seen for all the four markets under investigation. This indicates that negative reactions increases volatility more than positive reactions do.

The results from equation 10 can be seen in Table 5. A restriction, or requirement on the standard deviation is included here in the dummy variable. For all indices except the Swedish, the correlation coefficients for positive returns are positive at the significance level of 1 %, meaning persistence in positive returns. The parameter for measuring the asymmetry in mean is insignificant in all cases, and even positive for the Norwegian and Swedish markets. The slightly negative parameters for the Finnish and Danish markets means that negative returns reverts faster to positive when the shock is greater, and the slightly positive parameter for the Norwegian and Swedish markets means that returns revert slower to positive following larger negative shocks. The results are however not significant.

Table 5: Results of the Estimation of the ANAR-EGARCH-M (1; 1, 1) with a Change in Stddev with 2

Parameter / Index	HEX	OSEBX	OMX	KFX
Observations	304	296	296	300
μ	0.0016	0.0011	0.0008	0.0006
	(4.0764)***	(3.2476)***	(1.8529)*	(1.8668)*
ϕ	0.0694	0.0592	0.0071	0.0635
	(3.1314)***	(2.6043)***	(0.3051)	(2.7758)***
ρ	-0.0690	0.0374	0.0441	-0.0258
	(-1.2470)	(0.2268)	(0.6194)	(-0.3464)
δ	-2.1605	-2.4611	-0.6112	-0.6322
	(-1.8672)*	(-0.8373)	(-0.3019)	(-0.2083)
ω	-0.1338	-0.5654	-0.2996	-0.3650
	(-7.2457)***	(-6.5322)***	(-5.6722)***	(-5.4434)***
β	0.9955	0.9580	0.9807	0.9764
	(521.47)***	(110.99)***	(188.87)***	(151.86)***
γ	0.1284	0.2362	0.1705	0.1907
	(9.2427)***	(9.0852)***	(7.5433)***	(8.1444)***
α	-0.0147	-0.0710	-0.07226	-0.0420
	(-1.4004)	(-4.4041)***	(-6.4245)***	(-3.1809)***
ν	1.3736	1.5118	1.8448	1.6585
	(36.4160)***	(24.6810)***	(25.0292)***	(27.0103)***

Table 5 presents the results of the estimation of the conditional mean equation and the conditional variance equation for the HEX index, the OSEBX index, the OMX index and the KFX index. The sample covers the period from January 2, 1996 to December 31, 2004.
Conditional Mean Equation: $r_t = \mu + [\phi + \rho D_1 (r_{t-1} < -2\sigma)] r_{t-1} + \delta\sigma_t^2 + \varepsilon_t$.
Conditional Variance Equation: $log\left(\sigma_t^2\right) = \omega + \sum_{j=1}^{p}\left[\beta_j log\left(\sigma_{t-j}^2\right)\right] + \sum_{i=1}^{q}\alpha_i\left[\left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) + \gamma_i\left(\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right| - E\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right|\right)\right]$.
The z-statistics are reported with significance levels 10% (*), 5% (**) and 1% (***) in parenthesis. Observations refer to the number of observations where the change in the standard deviation is two units.

The parameter δ is negative and significant (at a level of 10 %), for the Finnish market and negative but insignificant for the Norwegian, Swedish and Danish markets. The leverage effect parameter for the variance is the same as for the other estimated models, highly significant for all the indices, indicating asymmetric volatility. The asymmetry is stronger for the Norwegian, the Swedish and the Danish markets since the asymmetry parameter is significant. The distribution is characterized by thick tails, as can be expected for financial time series data.

According to expectations, the results should be most significant for the model where the change in standard deviation was included as a restriction. From the presented results it can be noted that this was not the case. Instead, the results were most significant for the one- and two-period models. Contrarian strategies would therefore be most profitable for these models, where current losers are likely to become future winners, and investments should be made at that stage. Since these losers revert to zero or positive earnings relatively fast, earnings could be possible.

7 Diagnostic Tests

To test the robustness of the models used in the investigation, diagnostic tests are conducted. The robustness is tested for both the mean and the variance equation. Diagnostic tests are done with the Ljung-Box Q statistics. With this test, the serial correlation in the normalized residuals is tested (for the mean), and also if the time-varying volatility is well handled with the model for the variance. The Ljung-Box Q-statistics will be calculated up to 30 lags. The Q-statistics are presented separately for the different indices.

From Table 6 to Table 9, the Q-statistics for the four markets are presented. The Q-statistics for Standardized Residuals measure the serial correlation in the first moment (mean), and the Q-statistics for the Squared Standardized Residuals measure the serial correlation for the second moment (variance). The results are insignificant for every model, for all the four indices and all lags. No serial correlation can be observed. This indicates that the estimated models are good, and do not have to be corrected in any way.

Table 6: Diagnostic Tests for the Finnish Market

	Standardized Residuals					Standardized Residuals Squared				
Mod Lag	1	2	3	4	5	1	2	3	4	5
5	1.7236	2.0947	1.6555	1.5129	1.4594	4.4575	3.1236	3.9330	5.5424	4.3402
10	2.9833	3.2583	2.9434	2.7853	2.7498	6.5369	5.0973	6.0480	7.7336	6.5631
15	6.6576	7.4783	6.7786	6.4352	6.4737	8.5911	6.9379	8.0680	9.7921	8.6538
20	7.7222	8.0652	7.5987	7.3079	7.4165	9.8995	8.4419	9.4995	11.425	9.9826
25	10.064	10.573	10.069	9.7148	9.7824	13.597	11.933	12.954	15.065	13.526
30	13.267	13.535	13.197	13.432	12.906	16.820	15.179	16.130	18.411	16.636

Ljung-Box Q-statistics reported for standardized residuals and standard residuals squared. Mod 1-5 refers to the ANAR-EGARCH-M models estimated earlier. Model 1 is the one period model, model 2-4, the two to four period models, and model 5 is the model that included a change in standard deviation in the dummy.

The Q-statistics for standardized residuals and standardized residuals squared are all insignificant.

Table 7: Diagnostic Tests for the Norwegian Market

	Standardized Residuals					Standardized Residuals Squared				
Mod	1	2	3	4	5	1	2	3	4	5
Lag										
5	6.1273	6.4443	5.8667	6.1454	5.8717	4.0890	4.0552	4.1216	4.0756	4.1428
10	9.0220	9.7116	9.1630	9.4237	9.1819	16.849	17.057	17.114	17.017	17.141
15	13.990	15.039	14.511	14.758	14.481	22.471	22.958	22.997	22.888	22.985
20	20.366	21.509	21.030	21.263	21.021	25.526	26.269	26.394	26.267	26.366
25	24.253	25.679	25.208	25.443	25.191	29.498	30.493	30.563	30.450	30.493
30	27.490	29.099	28.677	28.898	28.629	31.711	32.585	32.675	32.544	32.616

Ljung-Box Q-statistics reported for standardized residuals and standard residuals squared. Mod 1-5 refers to the ANAR-EGARCH-M models estimated earlier. Model 1 is the one period model, model 2-4, the two to four period models, and model 5 is the model that included a change in standard deviation in the dummy.

The Q-statistics for standardized residuals and standardized residuals squared are all insignificant.

Table 8: Diagnostic Tests for the Swedish Market

	Standardized Residuals					Standardized Residuals Squared				
Mod	1	2	3	4	5	1	2	3	4	5
Lag										
5	2.7900	3.1098	2.5606	2.3364	3.2266	0.7855	0.5283	0.4947	0.4796	0.4738
10	8.3419	8.3277	7.7847	7.7719	8.2915	2.5338	2.2550	2.1866	2.2918	2.2789
15	21.059	20.560	19.816	19.501	20.922	5.9689	5.7679	5.6421	5.8828	5.8150
20	24.009	23.504	22.737	22.498	23.765	13.878	10.745	10.821	11.079	10.622
25	36.512	35.802	34.864	34.892	36.369	15.274	12.682	12.846	13.235	12.541
30	37.620	36.855	35.927	35.929	37.472	16.249	14.460	14.684	15.196	14.226

Ljung-Box Q-statistics reported for standardized residuals and standard residuals squared. Mod 1-5 refers to the ANAR-EGARCH-M models estimated earlier. Model 1 is the one period model, model 2-4, the two to four period models, and model 5 is the model that included a change in standard deviation in the dummy.

The Q-statistics for standardized residuals and standardized residuals squared are all insignificant.

Table 9: Diagnostic Tests for the Danish Market

	Standardized Residuals					Standardized Residuals Squared				
Mod	1	2	3	4	5	1	2	3	4	5
Lag										
5	2.7059	2.3343	3.1775	3.3976	3.1477	2.3653	2.0912	1.9978	2.1668	2.0005
10	4.8525	4.0909	5.0106	5.1698	4.9793	3.4070	3.1878	3.0536	3.4974	3.1113
15	12.706	12.134	13.090	13.168	12.956	5.2583	4.4208	4.4122	4.7564	4.4516
20	17.716	16.864	17.810	17.762	17.728	20.697	17.926	18.238	18.706	18.582
25	20.622	19.774	20.861	20.738	20.837	24.665	22.380	22.455	22.865	22.770
30	24.024	23.187	24.293	24.124	24.261	26.997	25.068	24.982	25.341	25.187

Ljung-Box Q-statistics reported for standardized residuals and standard residuals squared. Mod 1-5 refers to the ANAR-EGARCH-M models estimated earlier. Model 1 is the one period model, model 2-4, the two to four period models, and model 5 is the model that included a change in standard deviation in the dummy.

The Q-statistics for standardized residuals and standardized residuals squared are all insignificant.

8 Conclusions

This study suggests that there is asymmetry, not only in the variance, but also in the conditional mean. Negative returns are mean-reverting with a greater magnitude than positive. This means that positive returns are more persistent. The results agree with other studies carried out, among others Li and Li (1996); Hagerud (1996) and Koutmos (1998).

The presented hypotheses are accepted, in that the serial correlation coefficients for positive returns are positive and significant, and the coefficients for negative returns in almost all cases are smaller than the coefficient for positive. The asymmetry parameters for the mean varies. For the Finnish market, the coefficient was always negative, and asymmetry in mean-reversion an irreproachable fact. The asymmetry coefficient (for the mean) is negative and significant only in one of the estimated models for the Norwegian market. Although the asymmetric pattern in the mean-equation cannot be seen for the Norwegian market for other models than the first one, this does not exclude the fact that the returns are asymmetric. For the Swedish and the Danish markets, the parameter is significantly negative in two of the estimated models, indicating some degree of asymmetry in mean.

The leverage parameter turned out significant for all the estimated models and all four indices, indicating negative correlation between return and future variance. This means that negative returns generate higher volatility, and that asymmetry is in the variance. The direct asymmetry parameter shows up significant and negative on all the markets except the Finnish. The asymmetric pattern in variance is stronger for the Norwegian, the Swedish and the Danish market, but still asymmetric behavior in variance is present on the Finnish market as well.

The results indicate that it might be profitable to use contrarian type strategies and overreactions, and take advantage of the fact that the returns also behave asymmetrically. However, to measure the shocks with a change in standard deviation with two units would not be the most profitable way.

One reason for asymmetric patterns in mean and variance often considered is that participants on the market react differently to positive and negative changes in prices. The participants react faster to downward movements, or bad news, than to good news.

The results from this study show that asymmetry both in mean and variance should be accounted for when modeling financial time series data. The results further indicate that negative returns generate higher volatility, which means that negative reactions are associated with higher risk. In this way investors are more sensitive to bad news. However, negative returns revert quicker to positive or zero returns, indicating that the behavior following bad news is characterized by an "overreaction behavior", which is followed by a fast correction towards zero. The conclusion of this is that the behavior on the market reflects the positive correlation between risk and return.

For further research it could be of interest to look at the autocorrelations, and trends in the autocorrelations and try to find a method for forecasting the autocorrelations. This would give a possibility to forecast if contrarian type strategies or trend following strategies are profitable.

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An Empirical Comparison of Linear and Nonlinear Volatility Models for Nordic Stock Returns

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Abstract

This paper examines how volatility in financial markets is modeled. The empirical examination conducted investigates how good the models for the volatility, both linear and nonlinear, are in absorbing skewness and kurtosis. The Nordic stock markets, including Finland, Sweden, Norway and Denmark are studied. Different linear and nonlinear models are applied, and the results indicates that a linear model can almost always be used for modeling the series, even though nonlinear models perform slightly better in some cases. The results indicate that the Nordic markets are exposed to asymmetric patterns only to a certain degree. Negative shocks generally have a more prominent effect on the markets, but these effects are not really strong. In terms of absorbing skewness and kurtosis, nonlinear models outperform linear ones.

Key words: conditional variance, linear, nonlinear, skewness, kurtosis, parameter stability

JEL classification: G12

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1 Introduction

Volatility has been a popular topic for research in finance during the last decades. The reason for this interest naturally lies in the possibility to make as correct estimations of the future risk as possible. A number of Autoregressive Conditional Heteroscedasticity (ARCH) models have been presented. The different versions of ARCH and Generalized ARCH (GARCH) models have gained support in performing better than other volatility measures, such as the Exponential Weighted Moving Average (EWMA) and Historical volatility (HIS) models, (see e.g. Akgiray (1989)). Some of the published studies during the last years, have suggested that financial time series, and the volatility in them, cannot be perfectly modeled with linear models, such as the traditional ARCH model by Engle (1982). Although persistence generally tends to be very high, the models may not necessarily characterize the real behavior in return series. As a result, nonlinear models, for example one of the most popular of them, the Exponential GARCH (EGARCH) by Nelson (1991), that can account for asymmetry in variance, have gained support for giving better volatility predictions. The use of nonlinear models is supported in research by Koutmos (1998), Bali (2000) and Franses and van Dijk (2004) among others. The nonlinear models might give better predictions, since the variance is not modeled as a function of the lagged errors (squared) and lagged variance (Taylor, 2004).

Return series behave in an unpredictable manner, whereas the volatilities of the series are more predictable. This can be connected to the hypothesis of efficient markets (EMH). The volatility is considered more predictable even though the volatility is argued to behave asymmetrically. This happens in certain ways. Therefore, predictions of future volatility can be made on current information about the return series or mean. Financial time series are characterized by volatility clustering, excess kurtosis and instead of being normally distributed, fat tails are often observed. Engle's traditional ARCH model takes some of these characteristics into account, but not all. This is why nonlinear models are introduced to improve the predictions of the volatility.

The use of linear, symmetric or nonlinear asymmetric models also depends on the stability of the return series or the market. In a market where the returns or the mean behave in a stable way, the volatility can also be expected to be stable and less asymmetric. This means that shocks that hit the market are not that common, and when they appear, they do not influence the market tremendously. On the other hand, if the market is unstable, shocks hit the market hard, contributing to high asymmetric volatility. This gives a rule of what model, linear or nonlinear, should be used. If the market is stable, linear models should work for the modeling of the volatility, whereas when the market is unstable nonlinear models are preferred. In reality it is not always clear what model does the best work, and naturally it is hard to determine if the series that are being modeled are stable or not.

This paper investigates linear and nonlinear models for estimating volatility in perspective to skewness and kurtosis. The purpose is to provide evidence for or against linear and nonlinear models for the estimation of the volatility with a closer look at the third and fourth moment; skewness and kurtosis. The performance of different nonlinear volatility models relative to traditional linear ones, how good they are at absorbing skewness and kurtosis will be investigated. The estimations are made on the Nordic stock markets; the Finnish, the Swedish, the Norwegian and the Danish. It can be expected that these relatively small stock markets generate different results in comparison to other larger stock markets investigated in earlier studies. The Nordic markets are thus highly developed, suggesting similar results as for other stock markets. Similar results can however be expected among the Nordic markets, since they are pretty close to each other when comparing size, trading activity etc.

The paper contributes to the current research in that no previous studies have investigated, and compared linear and nonlinear models for the volatility with all the models compared to each other in this study, and from that investigated how the models perform in terms of absorbing skewness and kurtosis. Second linearity and nonlinearity on smaller stock markets, such as the Nordic markets, have not been studied.

Different methods for testing linearity and nonlinearity in series have been presented. The Lagrange Multiplier (LM) test by Engle (1982) is the most known and extensively used test for GARCH effects. Engle and Ng (1993) presented sign- and size-bias tests for detecting asymmetric effects or patterns, and these tests have been widely used as well. These basic tests and various extensions of them will be developed and used in this study.

In a paper by Lundbergh and Teräsvirta (1998), GARCH models are evaluated. Parametric tests are employed, and suggestions for improvements of the models are presented. The null hypothesis of no ARCH effects in the standardized errors are tested for by a misspecification test, a test for symmetry against a Smooth Transition GARCH (ST-GARCH) model, and further, a parameter constancy test is employed. Tests suggested are the Lagrange Multiplier (LM) tests. The results of the investigation by Lundbergh and Teräsvirta (1998) indicate that misspecification in GARCH parameters can be detected to a satisfying extent.

Hagerud (1997) presents two modified LM tests of the basic GARCH models against asymmetric models, in this case the Generalized Quadratic ARCH (G-QARCH) and the logistic ST-GARCH model. The conclusions state that although the tests carried out suggest whether asymmetric, nonlinear models should be used or not, it cannot exactly be decided which nonlinear model should be used.

Parallel to different extensions of GARCH models, smoothing methods, as the one mentioned above, and other alternatives have been developed. The advantage of these models is that they allow the features in the variance model to incorporate sign- and size-effects of past returns or shocks (Taylor, 2004). The results from the empirical investigation conducted in this paper are in line with the above mentioned studies. The volatility of the stock returns on the Nordic markets can be modeled with linear models to a satisfying extent. However, nonlinear models that can take into account asymmetries sometimes generates better predictions. The fact that linear models produce satisfying estimates is an indication of stable markets that are not exposed to numerous and big shocks.

The paper proceeds as follows: Section 2 discusses the different models (linear and nonlinear models for the variance), and further explains the testing procedure and how the different features[1] in the models can be detected. Data and summary statistics are presented in Section 3, where specifications of the mean equations are also presented. Results of the models and estimations are presented in Section 4. Section 5 concludes the paper.

2 Linear vs Nonlinear Models and Testing Procedure

The testing procedure starts by computing descriptive statistics for the market indices and tests for unit roots in the return series to check for stationarity. Estimations are made to determine the mean equation and preliminary tests for the identification of possible linear and nonlinear

[1] With features it is meant e.g. remaining ARCH effects, stability of parameters etc.

ARCH effects are conducted with the sign- and size-bias tests by Engle and Ng (1993). The LM-test by Engle (1982) is used for detecting ARCH effects in the different models. The parameters in each model will be evaluated so that they satisfy stationarity conditions and other conditions restricted by different volatility models. As one main point, the models (linear and nonlinear) will be estimated and compared to each other in terms of how they absorb skewness and kurtosis. Specification and diagnostic tests will be performed to check for remaining ARCH effects and higher order ARCH/GARCH effects to get an indication of how good the different volatility models work for the series under study.

Traditional ARCH models cannot handle some of the important facts seen in the volatility. Among others, Poon and Granger (2003) points out that the basic ARCH model cannot capture kurtosis in a satisfying way. The standardized residuals in ARCH estimations tend to include large kurtosis, and with that tail thickness. Further, when large shocks are controlled for, the ARCH effect tends to be reduced or completely disappears.

The GARCH model by Bollerslev (1986) is probably the most commonly used model for modeling volatility. The GARCH model is a linear model where the conditional variance is modeled as an Autoregressive (AR) process of past returns (squared) and variances. The model has many attractive features; it accounts for volatility clustering, captures heteroscedasticity (or time-dependent conditional variance) in the mean equations, as well as leptokurtosis in the empirical distribution. However, many studies (see e.g. Nelson (1991), Glosten et al. (1993)) have proved that important facts such as the leverage effect, negative skewness and fat tails, cannot be captured by the traditional GARCH model, meaning asymmetry and nonlinearity in the conditional variance are not taken into account by the model. The different models, both the linear and the nonlinear and their applications will be described in more detail below.

The sign- and size-bias tests proposed by Engle and Ng (1993) are diagnostic tests for the models under study. The tests examine whether the residuals (squared, standardized) are independent and identically distributed (iid).

The sign-bias test is constructed to test the different impact on volatility that positive and negative innovations (or changes or shocks) contribute to. The negative size-bias test on the one hand tests the impact on the volatility that large and small negative innovations have, whereas the positive size-bias test tests the impact on the volatility that large and small positive innovations have.

In the sign-bias test employed, (presented by Engle and Ng (1993),) the squared residual is directly a function of the residual itself;

$$\varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^- + \xi_t. \tag{1}$$

The dummy variable S_{t-1}^- takes the value of one if $\varepsilon_{t-1} < 0$ and the value of zero otherwise. The term ξ_t is a white noise parameter. With the test it is possible to analyze the conditional variance since the squared residual is used. It can in this case be investigated whether the variance depends on the sign of the lagged residuals or not. The coefficient ϕ_1 measures if the sign of the residual matters, and in case the coefficient is statistically significant, it does matter for the conditional variance.

From the sign-bias test the negative size-bias test can be generated. The negative size-bias test is written as,

$$\varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^- \varepsilon_{t-1} + \xi_t. \tag{2}$$

As an extension, the negative size-bias test examines not only if the sign, but also if the size of a negative shock or reaction have an impact on the conditional variance or the squared residual expressed as the dependent variable.

The positive size-bias test is similar to the negative size-bias test, but the term $S_{t-1}^- \varepsilon_{t-1}$, is replaced by the term $S_{t-1}^+ \varepsilon_{t-1}$ where $S_{t-1}^+ = 1 - S_{t-1}^-$. This gives the test,

$$\varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^+ \varepsilon_{t-1} + \xi_t. \tag{3}$$

The three tests (the sign-, negative size-, and positive size-bias tests) can also be expressed jointly with the following model that includes all the bias tests described above;

$$\varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^- + \phi_2 S_{t-1}^- \varepsilon_{t-1} + \phi_3 S_{t-1}^+ \varepsilon_{t-1} + \xi_t. \tag{4}$$

In the following, the equations for the mean and the variance are presented and specified. The mean equation will be specified as a simple autoregressive model of order p (AR(p)). The equation used when modeling the return series starts from an AR model of order one (AR(1)),

$$r_t = \mu + \theta r_{t-1} + \varepsilon_t. \tag{5}$$

The number of lags included in the mean equation will be decided by the Schwarz criterion. The use of the Schwarz criterion before the Akaike information criterion has been suggested by for example Nelson (1991), with the argument that the Schwarz criterion can select the lag-length more precisely than other similar criterions. For testing for asymmetry in the residuals in the mean equation, sign- and size-bias tests described above are employed. Further, linear and nonlinear ARCH effects in the residual of the variance equation will be tested for with specification and parameter stability tests.

Linear and nonlinear models and their performance will be compared to each other. First, a simple GARCH model is estimated for the linear setup. The traditional GARCH (q,p) model by Bollerslev (1986), that depends on both the squared residuals of the mean equation and on its own past values, can be expressed as,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_i \sigma_{t-j}^2. \tag{6}$$

This paper will use a setup where both the ARCH parameter and GARCH parameter equals one; $q = p = 1$. This setup is used since the GARCH (1,1) model seems to give fairly good predictions, and according to Bollerslev et al (1992), the simple GARCH (1,1) model dominates other GARCH models when comparing the models with the Akaike or Schwarz criterions in many, not to say most of the cases. In the GARCH model, the parameters ω, α_i and β_i should be non-negative and with at least one $\alpha_i > 0$ (in this case however, only one α parameter and one β parameter is included).

Next, extensions of the traditional GARCH model allowing for nonlinear behavior in the conditional mean and conditional variance are presented.

The Quadratic ARCH (QARCH) model introduced by Sentana (1995) is an asymmetric model for the volatility. The generalized setup of the model, or the Q-GARCH(1,1) model can be expressed as,

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \lambda\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2, \tag{7}$$

The quadratic parameter in this model enables it to be used as a second-order Taylor approximation to interpret the unknown conditional variance function of the model. The parameters ω, α, λ and β are constants, and for the condition of covariance stationarity to hold for the model, the sum of α and β should be smaller than one ($\alpha + \beta < 1$). The individual parameters α and β should be larger or equal to zero, and $\lambda < 4\alpha\omega$ for the positivity requirement in the variance to hold.

The EGARCH model proposed by Nelson (1991) has become very popular for modeling asymmetries. In the EGARCH(1,1), the natural logarithm of the conditional variance is modeled and it is calculated as,

$$ln\left(\sigma_t^2\right) = \omega + \alpha\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \lambda\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right| + \beta ln\left(\sigma_{t-1}^2\right). \tag{8}$$

The parameters ω, α, λ and β are again constant parameters, and for the stationarity condition to hold, β should be smaller than one ($\beta < 1$).

The GJR-GARCH model by Glosten et al. (1993), is often seen as the simplest model for modeling asymmetries in the conditional variance. The GJR model has similarities with the EGARCH, and contains the standard GARCH model but further a term that can handle the asymmetry in the variance as follows,

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \lambda S_{t-1}^-\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \tag{9}$$

S_{t-1}^- is a dummy variable that takes the value of one if $\varepsilon_{t-1} < 0$ and zero otherwise.

The parameters ω, α, λ and β are constant parameters and ω, α, β and $(\alpha + \lambda)$ should be non-negative for the positive conditional variance to hold. The stationarity condition requires that $\frac{(\alpha+\lambda)}{2} + \beta < 1$. One advantage of the GJR-GARCH model mentioned by Poon and Granger (2003) is that the volatility persistence is allowed to change quickly as the return changes from positive to negative, and vice versa.

The Threshold GARCH (TGARCH) model by Zakoian (1994) is similar to the GJR-GARCH model. The only difference is that in the TGARCH, the conditional standard deviation is modeled instead of the conditional variance. The TGARCH (1,1) model is expressed as,

$$\sigma_t = \omega + \alpha\left|\varepsilon_{t-1}\right| + \lambda\varepsilon_{t-1}S_{t-1} + \beta\sigma_{t-1}. \tag{10}$$

The restrictions states that the included parameters should be non-negative, and further, that the sum of α and β should be smaller than one ($\alpha + \beta < 1$).

The Volatility Switching (VS) model was introduced by Fornari and Mele (1997). The VS-GARCH can take advantage of the mean-reversion behavior in the conditional variance, and is calculated as,

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \lambda S_{t-1}\nu_{t-1} + \beta_1\sigma_{t-1}^2. \tag{11}$$

The parameter S_{t-1} gets a value of one if $\varepsilon_{t-1} > 0$, and zero if $\varepsilon_{t-1} = 0$, and a value of minus one if $\varepsilon_{t-1} < 0$. The persistence and the asymmetry in the variance is measured through the parameter $\nu_{t-1} = \frac{\varepsilon_t^2}{\sigma_{t-1}^2}$. Regime switching models have attracted interest lately since financial markets react differently to large and small shocks, and traditional ARCH models cannot handle such facts (Poon and Granger, 2003).

The models in the paper are estimated with the maximum likelihood (ML) function, and the Berndt-Hall-Hall-Hausmann (BHHH) algorithm is used as an optimization algorithm. Further, the Generalized Error Distribution (GED) will be used for all estimations made.

The BHHH algorithm iteration process is described as,

$$\theta_{i+1} = \theta_i + \lambda_i H_t^{-1} g_i. \tag{12}$$

The parameter λ measures the length of a variable step that is chosen for maximizing the likelihood in the right direction. The parameter H is the Hessian matrix, and the iteration process is fulfilled when the parameter g (the gradient vector) is zero (Alexander, 2004)

The GED has the following density function;

$$f(z) = \frac{\nu exp\left[-\left(\frac{1}{2}\right)|z/\lambda|^{\nu}\right]}{\lambda 2^{(1+1/\nu)}\Gamma(1/\nu)}, -\infty < z < \infty, 0 < \nu < \infty, \tag{13}$$

where the gamma function is expressed as $\Gamma(.)$, and

$$\lambda = \left[2^{(-2/\nu)}\Gamma(1/\nu)/\Gamma(3/\nu)\right]^{1/2}. \tag{14}$$

The parameter ν measures the tail thickness, and the parameter should be close to two for z to be normally distributed. If the parameter is larger than two, the tails are thinner than the tails of the normal, and if the parameter is smaller than two, the tails are thicker than is the case for the normal distribution (Nelson, 1991).

The results of the sign- and size-bias tests ran offer a first indication about the series and if there are asymmetric effects or not. Linear and nonlinear ARCH effects in the series will then be tested with the LM-test by Engle (1982). The LM test controls for the presence of ARCH effects in the residuals of the series. LM is calculated as,

$$\hat{\varepsilon}_t^2 = \omega + \alpha_1 \hat{\varepsilon}_{t-1}^2 + ... + \alpha_q \hat{\varepsilon}_{t-q}^2 + u_t. \tag{15}$$

The residuals in the equation above are obtained from the mean of the time series, or return series. From the estimated parameters, the LM test is computed as the sample size (n) times the R^2 value from the above equation $\left(nR^2\right)$ and the statistics follows an asymptotic $\chi^2(q)$ distribution.

These effects are important since the magnitude of the present residuals can be related to previous residuals. The rejection of the null hypothesis; $\alpha_1 = \alpha_2 = ... = \alpha_q = 0$ states heteroscedasticity in the residuals, which supports the use of linear ARCH models for modeling the variance of the return series.

The effects in the GARCH, QGARCH, EGARCH, GJR-GARCH, TGARCH and VS-GARCH are tested for with the LM test. For the GARCH, the null hypothesis is; $\alpha_0 = ... = \alpha_q = \beta_0 = ... = \beta_q = 0$. For the QGARCH, EGARCH, GJR-GARCH, TGARCH and VS-GARCH the corresponding null hypotheses are; $\alpha_0 = ... = \alpha_q = \lambda_0 = ... = \lambda_q = \beta_0 = ... = \beta_q = 0$.

In diagnosing the linear and nonlinear models, the GARCH (1,1) model will be used as a benchmark and starting point when comparing the models. Nonlinear models are compared to this benchmark, and as one of the main point in this article, what effect linear versus nonlinear models have on the skewness and kurtosis will be tested.

Excess kurtosis can generally be observed in return series. This is the case for the return series in this paper as well. Further, negative skewness can commonly be observed, and is mostly the case in this data sample as well.

To test to what degree nonlinear models are better in taking into account skewness and kurtosis and how good the forecasts of the volatility are, the different linear and nonlinear models will be estimated. The mean equation in the estimations will be the one described earlier, or the AR(p) model. As described, the number of lags included will be selected with help the of the Schwarz criterion.

A simple GARCH model for the variance equation in combination with the AR(p) model for the mean equation will be used as a starting point for measuring the different models. When changing the variance equation to nonlinear alternatives, the mean equation will remain an AR(p) process with the same number of lags in all estimations.

To further control how well the benchmark model (GARCH (1,1)) performs in relation to the other models, or to check how good linear models are in relation to nonlinear models, tests for remaining ARCH effects and tests for higher order ARCH and GARCH effects are made. These tests follow the setup by Lundbergh and Teräsvirta (1998), and an earlier version by Bollerslev (1986). If the null hypothesis is rejected, the residuals still contain heteroscedasticity.

Remaining ARCH effects means that the standardized residuals contain conditional heteroscedasticity, which is important to locate. A commonly used method for testing for remaining ARCH effects is the LM-test. The method was proposed by Lundbergh and Teräsvirta (1998) and can be expressed by the equation;

$$\hat{z}_t^2 = \phi_0 + \phi_1 \hat{z}_{t-1}^2 + ... + \phi_m \hat{z}_{t-m}^2 + \lambda' \hat{x}_t + u_t. \tag{16}$$

Remaining ARCH effects are to be detected in $\hat{z}_t$. The LM statistics are calculated the same way as for the basic LM test; nR^2, that is the number of observations times the R^2 value from the equation above.

The vector $\hat{x}_t$ is built from the partial derivatives of the conditional variance (σ^2), in relation to the parameters in the basic GARCH model. This gives the null hypothesis;

$$\hat{x_t} \equiv (\hat{\sigma^2})_t^{-1} \partial(\hat{\sigma^2})_t / \partial\theta. \tag{17}$$

In the case of the simple GARCH(1,1) model, this becomes;

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \tag{18}$$

which gives;

$$\frac{\partial\left(\sigma^2\right)}{\partial\theta'} = (1, \varepsilon_{t-1}^2, \left(\sigma^2\right)_{t-1}) + \beta_1 (\frac{\partial\left(\sigma^2\right)_{t-1}}{\partial\theta'}). \tag{19}$$

With these statistics, the correlation in the standardized residuals squared can be tested for.

A test for higher order GARCH effects can also be generated as an alternative to the equations above. To perform the test for higher order GARCH effects, the $z : s$ are replaced by parameters from the GARCH model, or values of the parameter $\hat{\varepsilon}_{t-q-1}^2, ..., \hat{\varepsilon}_{t-q-r}^2$ or the parameter $(\hat{\sigma^2})_{t-p-1}, ..., (\hat{\sigma^2})_{t-p-s}$. Again, the LM statistics can be calculated as nR^2.

Generally when modeling financial time series and their volatility, a linear model is used as the basic starting point in the modeling. Nonlinear models are then considered only if the diagnostics indicate that nonlinear models could be more appropriate. This statement should be remembered here, and therefore the traditional linear GARCH model is used as a benchmark and as a starting point in the evaluation of the models. Nonlinear models will be considered as well, and the estimates are compared to each other.

If it can be stated that some kind of misspecification is observed in the linear model, the linear (in this case the GARCH) model is compared to nonlinear versions. This can again be carried out with small modifications of the sign-bias, negative size-bias and positive size-bias test by Engle and Ng (1993).

The standardized residuals (squared) are used as dependent variables and the partial derivatives as regressors. This gives the regression,

$$\hat{z}_t^2 = \phi_0 + \phi_1 \hat{\omega}_{t-1} + \lambda' \hat{x}_t + \xi_t. \tag{20}$$

The parameter ω_{t-1} is equal to one if the asymmetry measures the sign-, negative size-bias test or positive size-bias test; S_{t-1}^-, $S_{t-1}^- \varepsilon_{t-1}$ or $S_{t-1}^+ \varepsilon_{t-1}$. The vector $\hat{x}_t$ can be defined as in the equation above. For the joint test that was also estimated when describing simple sign- and size-effects, a similar regression can be done here;

$$\hat{z}_t^2 = \phi_0 + \phi_1 S_{t-1}^- + \phi_2 S_{t-1}^- \hat{\varepsilon}_{t-1} + \phi_3 S_{t-1}^+ \hat{\varepsilon}_{t-1} + \lambda' \hat{x}_t + \xi_t. \tag{21}$$

The rejection of the null hypothesis however, does not tell much about which nonlinear model will be the best one. Instead, the different models (nonlinear) must, and will be estimated and compared to each other.

To test the stability or the constancy of the parameters over time, the parameter constancy test is estimated. Franses and van Dijk (2004) introduced a test where the GARCH (1,1) parameter is tested against the alternative;

$$\sigma_t^2 = \alpha_1 + \alpha_2 \varepsilon_{t-i}^2 + \beta_1 \sigma_{t-i}^2 + \left(\alpha_3 + \alpha_4 \varepsilon_{t-i}^2 + \beta_2 \sigma_{t-i}^2\right) F(t). \tag{22}$$

There are three different hypotheses that are tested in the equation above. The first tests the stability of the intercept; $H_0 : \alpha_1 = \alpha_3$. In the second test, the stability of the ARCH (and GARCH) parameters are investigated; $H_0 : \alpha_2 = \alpha_4$ and $\beta_1 = \beta_2$. The third test investigates the stability of the intercept and the other parameters; $H_0 : \alpha_1 = \alpha_3$ and $\alpha_2 = \alpha_4$, and finally $\beta_1 = \beta_2$.

3 Data and Summary Statistics

Descriptive statistics can be seen in Table 1 below. The data used in the examination consists of daily index returns from the Finnish, the Swedish, the Norwegian and the Danish markets. The observations cover a period from January 1996 to December 2004, and include over 2000 daily observations. The logarithmic returns of the indices are used in the calculations. The logarithmic return is calculated as,

$$r_t = ln(P_t) - ln(P_{t-1}). \tag{23}$$

From these descriptive statistics it can be observed that minimum returns are larger than the maximum return. This indicates that negative shocks hit the market harder than positive ones. The descriptive statistics for the Swedish market are to some extent different to those of the other

Table 1: Summary Statistics for Daily Market Index Returns; Period 1.1.1996 -31.12.2004

	HEX	OSE	OMX	KFX
Observations	2250	2250	2250	2245
Mean	0.0006	0.0004	0.0003	0.0004
Median	0.0014	0.0010	0.0007	0.0006
Maximum	0.1456	0.0618	0.1102	0.0497
Minimum	-0.1742	-0.0718	-0.0853	-0.0626
Std. Dev.	0.0224	0.0121	0.0164	0.0116
Skewness	-0.4441	-0.5380	0.1025	-0.2761
Kurtosis	8.7169	6.7461	5.5839	4.9866
Jarque-Bera	3138	1427	630	398
Probability	$<$0.0001	$<$0.0001	$<$0.0001	$<$0.0001

The summary statistics are for daily logarithmic returns on the HEX index (Finland), the OSE index (Norway), the OMX index (Sweden) and the KFX index (Denmark). The sample covers a period from January 2, 1996 to December 31, 2004. Jarque-Bera normality test shows departure from normality for all indices.

markets. When the Finnish, Norwegian and Danish markets show negative skewness, the Swedish shows slightly positive skewness. The Finnish index shows higher standard deviation in the return series. This phenomenon is compensated for by higher mean return for the HEX index. The fact that the Finnish index is the most volatile and most extreme should be pointed out. The extreme patterns in the Finnish index might have disappeard or at least slowed down after the introduction of the Euro currency. The common currency should most likely result in more stable prices. All series included in the sample show excess kurtosis, supporting the use of a distribution function accepting excess kurtosis, for example the GED described in Section 2.

A unit root test is conducted; the Augmented Dickey-Fuller Unit Root test. The results show that the hypothesis is rejected for all indices included.

To decide the number of lags to include in the mean equation of the included markets, the Schwarz criterion is calculated up to ten lags for the indices. The most suitable number is collected according to the criterion.

Table 2: Number of Lags to Include According to the Schwarz Criterion

Index	HEX	OSE	OMX	KFX
Lags	AR(1)	AR(1)	AR(1)	AR(1)

The table presents the most preferable number of lags to include in the mean specification according to the Schwarz criterion. One lag selected for all indices under study.

Surprisingly, all included indices suggest one lag as the most preferable number of lags (Table 2). The observed value of the Schwarz criterion does not change much when changing the number of lags, but clearly generates the lowest values for one lag.

4 Results

To get a preliminary indication of the use of linear versus nonlinear ARCH models and how they perform, sign- and size-bias tests are conducted. Table 3 presents the results of the estimations of a sign-bias test, a negative size-bias test and a positive size-bias test.

The results of the estimation of the sign-bias test for the four indices are significant. This is an indication of asymmetry and already here, nonlinear models gain some support. The estimation of

Table 3: Sign- and Size Bias Tests

Index	HEX	OSE	OMX	KFX
SB	0.00017	0.00005	0.00008	0.00003
p-value	0.0038***	0.0003***	0.0008***	0.0120**
NSB	-0.01641	-0.01200	-0.00933	-0.00777
p-value	<0.0001***	<0.0001***	<0.0001***	<0.0001***
PSB	0.00165	0.00094	0.00299	0.00193
p-value	0.4626	0.3784	0.0148**	0.0220**

The table reports results of the asymmetry tests sign-bias test (SB), negative size-bias test (NSB) and positive size-bias test (PSB). Coefficients and p-values for sign-bias test, negative size-bias test and positive size-bias test for the HEX index, the OSE index, the OMX index and the KFX index are reported. Lag length selected using the Schwarz criterion.
Sign-bias test: $\varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^- + \xi_t$.
Negative size-bias test: $\varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^- \varepsilon_{t-1} + \xi_t$.
Positive size-bias test:$\varepsilon_t^2 = \phi_0 + \phi_1 S_{t-1}^+ \varepsilon_{t-1} + \xi_t$.
***, ** and * indicates a significance level of 1%, 5 % and 10 % respectively.

the negative size-bias test also generates very significant results, indicating that negative asymmetry can be seen. The results of the positive size-bias test on the other hand, generates significant estimates for the Swedish and Danish markets, which leads to accepting positive asymmetry for these markets. This is in line with the summary statistics presented in Table 1, where OMX (Swedish) show positive skewness and KFX (Danish) show negative skewness, but not as negative as HEX and OSE. For the Finnish and Norwegian markets, the hypothesis of positive asymmetries cannot be accepted.

According to the test above, nonlinear patterns can be expected since the return series shows asymmetric patterns. A further test for investigating this should therefore be conducted.

Table 4: Lagrange-Multiplier Test of ARCH Effects

Index	GARCH			QGARCH			EGARCH		
Lags	1	5	10	1	5	10	1	5	10
HEX	0.4237	0.8472	0.9301	0.2823	0.7406	0.8879	0.1173	0.5361	0.7968
OSE	0.8503	0.7283	0.1525	0.7978	0.6675	0.1134	0.9924	0.5730	0.0768
OMX	0.3774	0.7806	0.8974	0.3485	0.9605	0.9868	0.6818	0.9942	0.9960
KFX	0.8922	0.8584	0.9705	0.9846	0.8314	0.9648	0.8034	0.8605	0.9815

The table reports p-values estimated with the Lagrange Multiplier (LM) test for GARCH, QGARCH and EGARCH. For every index and model, 1, 5 and 10 lags are used in the test.

Table 5: Lagrange-Multiplier Test of ARCH Effects

Index	GJR-G			TGARCH			VS-GARCH		
Lags	1	5	10	1	5	10	1	5	10
HEX	0.4305	0.8244	0.9209	0.4149	0.8429	0.9284	0.9251	0.8920	0.9915
OSE	0.7722	0.6922	0.1122	0.5933	0.6518	0.0746	0.1621	0.5447	0.0085
OMX	0.2786	0.9302	0.9751	0.4092	0.9624	0.9925	0.2182	0.6283	0.1260
KFX	0.7336	0.7919	0.9621	0.9999	0.8181	0.9646	0.0381	0.2176	0.2142

The table reports p-values estimated with the Lagrange Multiplier (LM) test for GJR-GARCH, TGARCH and VS-GARCH. For every index and model, 1, 5 and 10 lags are used in the test.

The LM test searches for ARCH effects. The test investigates whether heteroscedasticity or homoscedasticity can be seen in the series (in this case the index returns). The LM-test is performed for all four included markets, and for different model specifications. First, a simple GARCH model is tested, and further the QGARCH, the EGARCH, the GJR-GARCH, the TGARCH and the

VS-GARCH are estimated. No nonlinear and asymmetric ARCH effects can directly be observed from the estimation of the LM test (Table 4 and Table 5). Thus nonlinear models may not be better than linear ones.

From the results above, it is hard to make conclusions about what kind of model should be used for the volatility; can linear models give close enough predictions, or can better estimates be generated with nonlinear models? For further analysis of the performance of different volatility models, the estimation results of the models in question are presented in Tables 6 to 8.

From the estimations, it can be observed that the parameters in the variance equations that have restrictions to be non-negative (ω, α and β) all show significant values at a significance level of at least at 5 %, mostly at a level of 1 %. This satisfies the restriction and the parameters are correctly signed. The other restriction for the stationarity condition to hold; that the sum of the parameters α and β should be smaller than unity ($\alpha + \beta < 1$) , seem to hold for most of the estimated models. The exception is the Finnish index HEX, where $\alpha + \beta$ equals one. This means than an Integrated GARCH can be observed.

The estimates of the benchmark model AR(1)-GARCH(1,1) and the non-linear AR(1)-QGARCH(1,1) can be seen in Table 6 below.

Table 6: Estimation Parameters of AR(1)-GARCH(1,1) and AR(1)-QGARCH(1,1)

Index	HEX	OSE	OMX	KFX	HEX	OSE	OMX	KFX
μ	0.0014	0.0010	0.0010	0.0007	0.0014	0.0009	0.0006	0.0006
t-stat	4.7720	5.3020	3.7786	3.5740	4.6767	4.2904	2.1876	2.8890
θ	0.0442	0.0553	0.0177	0.0677	0.0483	0.0603	0.0229	0.0650
t-stat	2.1640	2.4452	0.7896	3.0945	2.3809	2.6536	1.0167	2.9801
ω	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
t-stat	2.0956	3.9554	2.7409	2.9559	1.9458	4.7430	4.0931	3.2899
α	0.0578	0.1308	0.0926	0.1014	0.0521	0.1260	0.0918	0.0977
t-stat	7.8977	8.0420	7.1029	7.2795	7.9576	7.3136	6.6450	7.1099
β	0.9422	0.8449	0.8996	0.8872	0.9478	0.8337	0.8891	0.8883
t-stat	150.2184	47.4907	64.0435	61.4020	167.2121	43.1448	57.4327	61.6237
λ					<0.0001	-0.0008	-0.0011	-0.0004
t-stat					-0.0941	-3.8186	-5.9930	-2.6684
Skewness	-0.5652	-0.3196	-0.1060	-0.2690	-0.5445	-0.2471	-0.0631	-0.2224
Kurtosis	7.3515	4.0845	3.5939	3.9624	7.1444	3.9751	3.4645	3.8829
SB	0.0002	<0.0001	<0.0001	<0.0001	0.0001	<0.0001	<0.0001	<0.0001
p-value	0.0088	0.0030	0.0015	0.0312	0.0588	0.0022	0.0009	0.0305
NSB	-0.0163	-0.0114	-0.0091	-0.0076	-0.0056	-0.0115	-0.0093	-0.0077
p-value	<0.0001	<0.0001	<0.0001	<0.0001	0.0036	<0.0001	<0.0001	<0.0001
PSB	0.0021	0.0010	0.0033	0.0022	0.0005	0.0010	0.0032	0.0021
p-value	0.3641	0.3911	0.0082	0.0120	0.3597	0.3851	0.0102	0.0132

The table presents estimates of the benchmark AR(1)-GARCH(1,1) model and AR(1)-QGARCH(1,1) for the HEX index, the OSE index, the OMX index and the KFX index.
AR(1) mean equation: $r_t = \mu + \theta r_{t-1} + \varepsilon_{t\mu}$.
GARCH(1,1) variance equation: $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$.
QGARCH(1,1) variance equation: $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \lambda\varepsilon_{t-1} + \beta_1\sigma_{t-1}^2$.

The results presented indicates how models are capturing not only the first and second moments, but also the third and fourth moments; skewness and kurtosis. Inspecting the variance parameters it can be observed that for both models, the parameters are highly significant; meaning the models estimated suits the data well. For the QGARCH, negative values of the parameter λ can be seen for some series. This indicates that negative shocks or reactions have a larger impact on the conditional variance than the effect of positive shocks. To further inspect the third and fourth moments, it can easily be observed that the nonlinear model (the QGARCH) performs better in absorbing both skewness and kurtosis. This holds for all included indices. Skewness lies closer to zero and kurtosis closer to three.

The parameters estimated for the sign- and size-bias tests can be seen in the presented tables as well. Compared to the tests presented earlier, the GARCH and QGARCH shows not as significant asymmetries as when the variance was not modeled with an ARCH type model. However, the asymmetries (sign- and negative size-effect) are still significant.

Table 7: Estimation Parameters of AR(1)-EGARCH(1,1) and AR(1)-GJR-GARCH(1,1)

Index	HEX	OSE	OMX	KFX	HEX	OSE	OMX	KFX
μ	0.0014	0.0009	0.0006	0.0006	0.0015	0.0012	0.0010	0.0008
t-stat	4.6806	4.3655	2.2208	3.1051	5.0006	5.8633	3.7910	3.8608
θ	0.0440	0.0604	0.0189	0.0563	0.0480	0.0615	0.0270	0.0717
t-stat	2.2101	2.7228	0.8544	2.6099	2.3339	2.6850	1.1743	3.2226
ω	-0.1405	-0.5839	-0.3167	-0.3717	<0.0001	<0.0001	<0.0001	<0.0001
t-stat	-7.4546	-6.6901	-5.9485	-5.5235	2.2209	4.3574	2.9960	3.2562
α	0.1314	0.2389	0.1735	0.1921	0.0444	0.0792	0.0667	0.0743
t-stat	9.1903	9.0464	7.5593	8.1325	4.4033	4.4840	4.8210	5.0189
β	0.9950	0.9562	0.9790	0.9758	0.9435	0.8299	0.8939	0.8815
t-stat	510.30	109.28	191.76	151.65	153.69	42.979	58.892	58.566
λ	-0.0120	-0.0706	-0.0741	-0.0421	0.0263	0.1383	0.0605	0.0604
t-stat	-1.1247	-4.3204	-6.7378	-3.1770	1.6108	4.2973	3.4881	2.7828
Skewness	-0.6132	-0.2171	-0.0607	-0.2312	-0.5552	-0.1934	-0.0594	-0.2137
Kurtosis	7.5968	3.9569	3.3890	3.8421	7.1812	3.9819	3.5003	3.9631
SB	0.0002	<0.0001	<0.0001	<0.0001	0.0001	<0.0001	<0.0001	<0.0001
p-value	0.0079	0.0022	0.0010	0.0277	0.0424	0.0238	0.0014	0.0264
NSB	-0.0163	-0.0115	-0.0093	-0.0077	-0.0055	-0.0114	-0.0091	-0.0076
p-value	<0.0001	<0.0001	<0.0001	<0.0001	0.0042	<0.0001	<0.0001	<0.0001
PSB	0.0021	0.0010	0.0032	0.0021	-0.0067	0.0010	0.0033	0.0022
p-value	0.3669	0.3847	0.0110	0.0158	0.0002	0.3813	0.0092	0.0112

The table presents estimates of the AR(1)-EGARCH(1,1) model and AR(1)-GJR-GARCH(1,1) for the HEX index, the OSE index, the OMX index and the KFX index.
AR(1) mean equation: $r_t = \mu + \theta r_{t-1} + \varepsilon_t$.
EGARCH(1,1) variance equation: $ln\left(\sigma_t^2\right) = \omega + \alpha \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \lambda \left| \frac{\varepsilon}{\sigma} - \sqrt{\frac{2}{\pi}} \right| + \beta ln\left(\sigma_{t-1}^2\right)$.
GJR-GARCH(1,1) variance equation: $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \lambda S_{t-1}^{-}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$.

The same patterns that appeared for the first two models can be observed for the EGARCH and the GJR-GARCH model (Table 7). The parameters estimated from the variance equation are statistically significant. The parameter α is larger than the parameter λ for all series except the OSE (Norwegian index). When this holds ($\alpha > \lambda$), negative shocks or reactions contribute to a larger effect on the conditional variance than is the case for positive reactions of the same size. In the case where the nonlinear model is an EGARCH model, the traditional, linear GARCH model performs better in two out of four cases (in two out of four markets), and the EGARCH performs better in terms of skewness and kurtosis in the other two cases. The estimations of the GJR-GARCH model performs better than the linear one in terms of skewness and kurtosis for all markets. However, the QGARCH still performs better than the GJR-GARCH.

The sign- and size-effects for the EGARCH and GJR-GARCH model still indicate asymmetries in a sign-effect and a negative size-effect.

Table 8: Estimation Parameters of AR(1)-TGARCH(1,1) and AR(1)-VS-GARCH(1,1,1)

Index	HEX	OSE	OMX	KFX	HEX	OSE	OMX	KFX
μ	0.0014	0.0009	0.0006	0.0006	0.0017	0.0013	0.0014	0.0011
t-stat	4.7208	4.4822	2.3644	3.0488	6.2287	6.8624	5.5771	5.7572
θ	0.0441	0.0630	0.0235	0.0672	0.0469	0.0521	0.0413	0.0771
t-stat	2.1582	2.7710	1.0544	3.0697	2.1759	2.1666	1.8473	3.3239
ω	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
t-stat	2.0755	4.6445	3.7831	3.2078	4.3861	4.7240	-0.8511	1.9415
α	0.0582	0.0790	0.0332	0.0742	0.0674	0.1600	0.1135	0.1158
t-stat	5.1069	3.8399	2.9082	4.6733	7.1484	6.8228	5.8711	8.0199
β	0.9423	0.8373	0.8975	0.8879	0.9175	0.6574	0.8162	0.7775
t-stat	148.24	45.162	63.078	61.2974	107.93	21.4850	36.5339	42.4929
λ	-0.0009	0.0918	0.1094	0.0471	<0.0001	<0.0001	<0.0001	<0.0001
t-stat	-0.0659	3.5369	6.1726	2.2353	6.8819	6.6045	6.7336	8.2560
Skewness	-0.5640	-0.2567	-0.0742	-0.2449	-0.4096	-0.2223	0.0623	0.0311
Kurtosis	7.3474	3.9763	3.4667	3.9648	7.5065	4.0188	3.2385	3.4356
SB	0.0002	<0.0001	<0.0001	<0.0001	-0.0001	>0.0000	>0.0000	>0.0000
p-value	0.0088	0.0025	0.0011	0.0320	0.0324	0.0003	0.0093	0.1291
NSB	-0.0163	-0.0115	-0.0093	-0.0077	>0.0000	-0.0022	0.0004	0.0002
p-value	<0.0001	<0.0001	<0.0001	<0.0001	0.9067	0.0230	0.0186	0.0001
PSB	0.0021	0.0010	0.0032	0.0021	0.0007	0.0022	<0.0001	<0.0001
p-value	0.3643	0.3760	0.0093	0.0134	0.3452	0.0232	0.3485	<0.0001

The table presents estimates of the AR(1)-TGARCH(1,1) model and AR(1)-VS-GARCH(1,1) for the HEX index, the OSE index, the OMX index and the KFX index.
AR(1) mean equation: $r_t = \mu + \theta r_{t-1} + \varepsilon_t$.
TGARCH(1,1) variance equation: $\sigma_t = \omega + \alpha\,|\varepsilon_{t-1}| + \lambda\varepsilon_{t-1}S_{t-1} + \beta\sigma_{t-1}$.
VS-GARCH(1,1) variance equation: $\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \lambda S_{t-1}\nu_{t-1} + \beta_1\sigma_{t-1}^2$.

When evaluating the estimated parameters in the TGARCH model (Table 8), it can be observed that the parameters α and λ are both positive in almost all cases (the only exception being HEX, where the parameter λ is slightly negative). The nonlinear TGARCH model performed better than the linear GARCH model in all cases except for one, where the GARCH was better in absorbing kurtosis. Asymmetry is observed for the TGARCH model as well, referring to the sign-bias effect and the negative size-bias effect.

The results for the VS-GARCH model produces similar results as for the TGARCH model. The parameter α is for all markets positive and in that larger than the parameter λ which indicates that small positive shocks have a larger effect on the conditional volatility than small negative shocks, whereas when the shocks are larger the impact on volatility is the opposite. This means that large positive shocks contribute to a smaller increase in volatility than is the case when the large shock is negative.

When investigating the skewness and kurtosis parameters for the VS-GARCH, the model performs well in absorbing the features. In comparison to the benchmark model, (the basic GARCH), the VS-GARCH outperforms, and in comparison to other models the VS-GARCH performs well in both absorbing skewness and kurtosis.

As for the TGARCH model, the VS-GARCH model generates significant results for the sign-bias and the negative size-bias test. Tests for detecting higher order ARCH and GARCH effects are conducted, and reported below in Table 9 (ARCH) and in Table 10 (GARCH).

Table 9: Higher Order ARCH

Index/Lags	HEX	HEX	OSE	OSE	OMX	OMX	KFX	KFX
ARCH(1)1	0.876	0.175	0.086	0.286	0.560	0.143	0.966	0.024
5	0.863	0.002	0.001	0.000	0.693	0.000	0.867	0.000
10	0.714	0.000	0.003	0.000	0.268	0.000	0.936	0.000
30	0.216	0.000	0.002	0.000	0.004	0.000	0.371	0.000
ARCH(2)1	0.976	0.276	0.131	0.249	0.931	0.426	0.825	0.322
5	0.916	0.024	0.080	0.000	0.555	0.000	0.785	0.000
10	0.920	0.002	0.082	0.000	0.333	0.000	0.827	0.000
30	0.564	0.000	0.031	0.000	0.019	0.000	0.408	0.000
ARCH(3)1	0.567	0.733	0.088	0.327	0.560	0.492	0.923	0.165
5	0.833	0.387	0.073	0.000	0.628	0.000	0.756	0.005
10	0.939	0.658	0.099	0.000	0.424	0.000	0.845	0.000
30	0.907	0.049	0.098	0.000	0.070	0.000	0.419	0.000
ARCH(4)1	0.721	0.602	0.159	0.819	0.665	0.999	0.680	0.300
5	0.961	0.451	0.163	0.394	0.606	0.000	0.924	0.040
10	0.993	0.834	0.182	0.103	0.446	0.000	0.957	0.000
30	0.988	0.661	0.084	0.001	0.061	0.000	0.647	0.000
ARCH(5)1	0.746	0.627	0.137	0.852	0.594	0.659	0.899	0.295
5	0.939	0.508	0.140	0.341	0.676	0.002	0.947	0.123
10	0.986	0.869	0.182	0.194	0.495	0.000	0.965	0.004
30	0.996	0.944	0.109	0.010	0.099	0.000	0.689	0.000

The table presents higher order ARCH effects for the HEX index, the OSE index, the OMX index and the KFX index. ARCH(1)-(5) is the number of ARCH parameters included, and 5, 10 and 30 the number of lags included in the test.

Table 10: Higher Order GARCH

Index/Lags	HEX	HEX	OSE	OSE	OMX	OMX	KFX	KFX
GARCH(1)1	0.617	0.423	0.093	0.850	0.498	0.377	0.394	0.892
5	0.854	0.843	0.067	0.735	0.702	0.791	0.796	0.855
10	0.977	0.927	0.144	0.147	0.717	0.901	0.930	0.969
30	0.994	0.987	0.138	0.323	0.265	0.898	0.822	0.762
GARCH(2)1	0.748	0.291	0.098	0.673	0.471	0.554	0.389	0.998
5	0.833	0.744	0.079	0.771	0.678	0.971	0.793	0.852
10	0.973	0.845	0.162	0.156	0.699	0.983	0.929	0.968
30	0.993	0.985	0.145	0.350	0.266	0.930	0.824	0.762
GARCH(3)1	0.614	0.427	0.096	0.327	0.536	0.922	0.429	0.613
5	0.830	0.834	0.067	0.826	0.691	0.972	0.796	0.894
10	0.976	0.891	0.138	0.362	0.707	0.969	0.931	0.990
30	0.995	0.976	0.131	0.474	0.242	0.951	0.852	0.872
GARCH(4)1	0.720	0.959	0.091	0.560	0.418	0.558	0.399	0.664
5	0.843	0.992	0.069	0.990	0.594	0.958	0.779	0.988
10	0.972	0.989	0.154	0.367	0.670	0.964	0.919	0.985
30	0.990	0.998	0.123	0.512	0.260	0.922	0.802	0.783
GARCH(5)1	0.678	0.528	0.119	0.696	0.416	0.605	0.528	0.525
5	0.813	0.987	0.073	0.954	0.608	0.992	0.840	0.878
10	0.955	0.986	0.121	0.682	0.658	0.977	0.906	0.987
30	0.992	0.985	0.112	0.669	0.295	0.946	0.812	0.812

The table presents higher order GARCH effects for the HEX index, the OSE index, the OMX index and the KFX index. GARCH(1)-(5) is the number of ARCH and GARCH parameters included, and 5, 10 and 30 the number of lags included in the test.

For the Finnish index HEX, the Swedish index OMX and the Danish index KFX, no remaining

ARCH or GARCH effects are observed with the exception of the OMX index, when the number of lags reaches 30. Higher order ARCH or GARCH models would not perform better, with the exception when the number of lags reach 30 for the Swedish index. For OSE, remaining ARCH and GARCH effects can be observed, indicating the use of higher order models.

Diagnostic tests are conducted as a check for remaining ARCH and GARCH effects. From Table 11 to Table 13 it can be observed that no remaining ARCH/GARCH effects are present. The diagnostics shows that no clear misspecification in the models are observed.

Table 11: Diagnostic Tests GARCH and QGARCH

Index/Lags	HEX	OSE	OMX	KFX	HEX	OSE	OMX	KFX
1	0.617	0.093	0.498	0.394	0.802	0.218	0.583	0.284
5	0.854	0.067	0.702	0.796	0.870	0.161	0.691	0.705
10	0.977	0.144	0.717	0.930	0.980	0.290	0.639	0.898
30	0.994	0.138	0.265	0.822	0.993	0.315	0.160	0.824

The table presents Ljung-Box Q-statistics for standardized residuals and standardized residuals squared for AR(1)-GARCH(1,1) and AR(1)-QGARCH(1,1).

Table 12: Diagnostic Tests EGARCH and GJR-GARCH

Index/Lags	HEX	OSE	OMX	KFX	HEX	OSE	OMX	KFX
1	0.643	0.314	0.502	0.236	0.880	0.279	0.796	0.444
5	0.917	0.176	0.712	0.666	0.872	0.207	0.700	0.840
10	0.985	0.309	0.642	0.884	0.980	0.383	0.630	0.952
30	0.995	0.354	0.151	0.758	0.994	0.304	0.249	0.859

The table presents Ljung-Box Q-statistics for standardized residuals and standardized residuals squared for AR(1)-EGARCH(1,1) and AR(1)-GJR-GARCH(1,1).

Table 13: Diagnostic Tests TGARCH and VS-GARCH

Index/Lags	HEX	OSE	OMX	KFX	HEX	OSE	OMX	KFX
1	0.615	0.248	0.543	0.348	0.791	0.497	0.930	0.816
5	0.853	0.148	0.791	0.752	0.826	0.248	0.488	0.899
10	0.977	0.267	0.734	0.911	0.977	0.344	0.384	0.958
30	0.994	0.285	0.219	0.829	0.996	0.177	0.127	0.686

The table presents Ljung-Box Q-statistics for standardized residuals and standardized residuals squared for AR(1)-TGARCH(1,1) and AR(1)-VS-GARCH(1,1).

Table 14: Parameter Stability Test GARCH

Test / Index	HEX	OSE	OMX	KFX
$H_0 : \alpha_1 = \alpha_3$	0.0433	0.0001	0.0033	0.0015
$H_0 : \alpha_2 = \alpha_4,\ \beta_1 = \beta_2$	0.3767	0.3114	0.0002	0.0090
$H_0 : \alpha_1 = \alpha_3,\ \alpha_2 = \alpha_4,\ \beta_1 = \beta_2$	0.1123	0.8042	<0.0001	0.0014
Skewness	-0.5431	-0.3175	-0.0325	-0.1918
Kurtosis	7.0861	4.0807	3.4840	3.8931
Jarque-Bera	1674	147	22	88
Probability	<0.0001	<0.0001	<0.0001	<0.0001

The table presents estimates (p-values) of the parameter stability tests for the GARCH model.
Parameter stability test: $\sigma_t^2 = \alpha_1 + \alpha_2\varepsilon_{t-i}^2 + \beta_1\sigma_{t-i}^2 + \left(\alpha_3 + \alpha_4\varepsilon_{t-i}^2 + \beta_2\sigma_{t-i}^2\right) F\left(t\right).$

From the three parameter stability tests above, it can be observed that there are stability problems in the estimated coefficients (Table 14). The parameters or coefficients are significantly time varying, indicating that the problem of non-normality remains. On the other hand, when comparing skewness and kurtosis in Table 14 with the parameters from Table 1, it can be noticed that skewness is better absorbed when using the GARCH specification in all cases except for the HEX index and kurtosis is better absorbed in the later specification. Further, according to the Jarque-Bera statistics, non-normality can be reduced with the GARCH approach.

5 Conclusions

In this paper several linear and nonlinear models for the conditional variance have been presented and compared. Apart from concentrating on the variance process, focus has also been on the third and fourth moment; skewness and kurtosis. In short, it has been investigated how well linear versus nonlinear models perform in terms of absorbing skewness and kurtosis and further which model best suits the time series under study. The investigation has also concentrated on finding the best model for modeling the volatility.

As expected, the markets under study react in a similar manner, with some exceptions for the Swedish OMX index. For the variance process, the estimated results are not that convincing and it can not clearly be stated that nonlinear models perform better than linear models. However, nonlinear models seem to give slightly better results but linear models cannot clearly be ruled out. When concentrating on skewness and kurtosis clearer results are generated. When looking at these measures, the nonlinear models tend to perform better, which indicates that although the nonlinear models must not always outperform the linear ones in terms of volatility forecasting, they are superior when investigating conditional skewness and kurtosis.

One direction to take when continuing this line of research is to concentrate on periods with high volatility and periods of low volatility. Since volatility clustering is a well known and proven fact, it could be possible to model periods characterized by high volatility with one model and periods with low volatility with another. However, the challenge is to combine these possible models, and make the out-of-sample forecasts.

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An Empirical Investigation of Value-at-Risk in Long and Short Trading Positions

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Abstract

This paper uses the Value-at-Risk approach to define the risk in both long and short trading positions. The investigation is carried out on major market indices (Japanese, UK, German and US). The performance of models that takes into account skewness and fat-tails are compared to symmetric models in relation to both the specific model for estimating the variance, and the distribution of the variance estimate used as input in the VaR estimation. The results show that more flexible models do not necessarily perform better in predicting the VaR forecast; the reason for this is most probably the complexity of the models. A general result is that different methods for estimating the variance are needed for different confidence levels of the VaR and for the different indices. Also, different models are to be used for the respective left and right tails of the distribution.

Key words: Value-at-Risk, asymmetry, Exponential GARCH, Asymmetric Power ARCH, long-, short- trading.

JEL classification: G11, G15, G20

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1 Introduction

How to best model the risk is a major concern for financial institutions. The concept of Value-at-Risk (VaR) has appeared as a result of this demand and serves as a tool for risk management. The demand for risk management tools has increased considerably following recent crashes and losses made by financial institutions and the need for good measures for modeling the downside risk has grown (Kuester et al., 2006). Further, the globalization of markets, developments in technology and expansion of derivatives traded demand effective risk management tools (Angelidis and Degiannakis, 2007). The key issue is to have a good measure for the loss probabilities or conditional shortfall probabilities. The use of VaR can be introduced as a measure for the riskiness of investments (Mittnik and Paolella, 2000). Jorion (2000), one of the most cited researchers in the literature on VaR, has defined VaR exposure as "the worst expected loss over a great horizon within a given confidence level".

During recent years the Value-at-Risk approach has gained a lot of attention in the risk management research and literature on risk management. The reason for the interest also is that the measure is an "easy-to-work-with" approach and according to Giot and Laurent (2003a) an "easy-to-understand" definition of risk. The measure furthermore tends to give fairly good estimates for risk, mostly considering the down-side risk. The VaR approach is used to estimate the predicted financial loss that can be expected with some probability. The given confidence levels are of importance, since they give the loss limits for the VaR measures. If the confidence level is higher, the portfolio also gets higher VaR (van den Goorbergh and Vlaar, 1999).

To be more correct, the VaR does not generate the maximum possible loss, which is often used in its definition; instead the VaR measures the worst result that is expected for the specified period of time (van den Goorbergh and Vlaar, 1999). The VaR of a portfolio gives the maximum loss given the worst case occurs when holding the portfolio unchanged over a certain time-period.

Previous research in the area concentrates on the left tail of the error distribution. Some recent papers on the topic that are worth mentioning are articles by van den Goorbergh and Vlaar (1999), Mittnik and Paolella (2000), Guermat and Harris (2002), Huang and Lin (2004) and Angelidis et al. (2004). A common result in the aforementioned papers is that various Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized versions (GARCH) and the Student's t-distribution, model VaR most effectively. For an introduction to VaR, see Jorion (2000), for a background and overview of VaR, see Duffie and Pan (1997), and for a presentation and comparison of alternative models and strategies, see Kuester et al. (2006).

There are very few papers published that takes into account the right tail, and articles by Giot and Laurent (2003a), Giot and Laurent (2003b) and So and Yu (2006) should give an exhaustive picture of the research carried out on this. The paper by Giot and Laurent (2003a) concentrates on VaR for long and short trading positions and risk management in the commodity markets. The skewed Asymmetric Power ARCH (APARCH) model turned out to give good and accurate VaR forecasts for the assets. The other paper by Giot and Laurent (2003b) concentrates on international stock market indices and some U.S. stocks. VaR is modeled with univariate models (for the indices) and further with multivariate models (for the stocks). In modeling the indices, which is also the focus in this study, Giot and Laurent (2003b) find that an asymmetric model for the variance equation is appropriate, but the distribution applied need not necessarily be asymmetric. The paper by So and Yu (2006) uses indices from the major markets in Asia, and the US indices S&P500 and NASDAQ, the UK index FTSE100 and the AOI index of Australia. The results support the use of stationary and Fractionally Integrated GARCH (FI-GARCH) models. The concept of taking into account fat-tailed distributions also seemed to be of importance.

The overall result generated from previous studies is that the correctness of the VaR estimate highly depends on the models used, and the most suitable model varies highly among different indices and time horizons.

As can be seen from previous research the focus when using the VaR approach has been on the down-side risk. This paper on the other hand, also takes a look at the right hand tail of the distribution in order to investigate possible positive returns that can result in losses for traders holding short positions. This approach is of interest, not only for banks who the approach was originally aimed at, but for traders holding both short and long positions. Further, it can be possible to use VaR to measure the risk-return profile of for example traders or other participants in the market activity.

The interest in both upside and downside risk should be of interest especially for hedge funds, since they usually include both long and short trading positions (Kritzman and Chow, 2002).

VaR measures try to estimate the exact quintile of the left tail of the distribution. Taking this from the perspective of a trader, the trader is focused on the long positions he or she holds, and wants to measure what happens if the price of the asset decreases. However, the trader most likely holds short positions as well, and in that situation hopes for the price to decrease since that will generate a profit, whereas an increase in the price could generate losses. This means that the short daily VaR that will be estimated is the VaR level for traders with short positions. These traders are exposed to losses when the price of an asset increases in value, and thus big positive returns are interesting. The long daily VaR on the other hand, estimates the VaR level for traders holding long positions, and these traders are exposed to losses following decreased prices. The focus is to model big negative returns. This highlights the interest in measuring both the left and right hand side of the distribution, and can be seen as the first object of this study. The basic idea of the paper is to identify how a trader holding both short and long trading positions can benefit from using VaR as a risk measure. Additionally, the paper uses different distributions and models for estimating the volatility (symmetric and asymmetric ones) for VaR calculations.

The VaR approach is closely related to the distribution of returns of a security, a portfolio or an index. To assume that the returns follow the normal distribution would make the VaR calculation very simple. However, this simplification would result in VaR estimates that are too low or underestimations of the likelihood of extreme returns (Guermat and Harris, 2002) . Financial time series are known for their excess kurtosis. This effect in the conditional distribution follows the phenomenon of volatility clustering. Kurtosis is of major interest in the VaR framework, since the moment concentrates on the tails of the distribution, and the VaR is a measure that especially specifies the tails, or more precisely the left-tail (Guermat and Harris, 2002). The introduction of ARCH-type models in this setup, and further the use of non-normal distributions for estimating the volatility should allow clustering effects, or excess kurtosis, and hopefully improvements in the estimated VaR measures can thus be made. The solution for handling excess kurtosis and non-normality for giving the best possible VaR estimates is the second object of this study.

The purpose of this article is to examine the performance of estimated VaR measures when introducing different ways of estimating the variance both regarding the model and the distribution used as input value in the calculation of the VaR. More importantly, the paper aims at investigating the use of the VaR measure on both the left and the right tails of the distribution. The paper investigates VaR from a trader's perspective and thus concentrates not only on the left hand tail of the distribution. As argued above, a trader holding both long and short positions is influenced by the behavior in both the left and the right tail of the ditribution (the left tail measuring the down-side risk and the right tail measuring the up-side risk). Thus, both tails are investigated. (Giot and Laurent have introduced this, and at this point no other studies looking at both left and right hand tails have been found (Giot and Laurent, 2003a,b)). What makes this paper different to a great extent from the papers by Giot and Laurent (2003a,b); So and Yu (2006), is that this paper concentrates on the difference between VaR:s for long and short positions. Previous papers have concentrated more on finding models that are appropriate for both sides. This paper tries to identify the best suitable models for calculating VaR for both the left and the right tails separately.

It should be stressed at this point the two ways asymmetry is discussed in this article. First, asymmetry is related to the relationship between the conditional variance and the lagged error term (squared). This asymmetry is accounted for by various ARCH/GARCH extensions. This implies that the estimated VaR:s automatically translate into conditional VaR models. Second, asymmetry is also referred to the distribution applied in the modeling of the variance. This is taken into account by introducing distributions that are asymmetric. However, it has been argued by for example So and Yu (2006) that the distribution has a larger impact than the volatility model for the VaR forecast.

Based on previous research, it can be expected that the results differ for different models, indices and different confidence levels. According to the expectations, the skewed Exponential GARCH (EGARCH) and APARCH generate the best estimates; however, performance varying with the index and confidence levels selected. Distributions that take into account the kurtosis are expected to generate better one-step-ahead VaR forecasts as well. If this is really the case, traders as well as financial institutions should try to find the best fitting model, which generates the highest possible performance of the VaR for the specific asset or index.

The results of the empirical investigation conducted on the indices Nikkei, FTSE 100, DAX and S&P 500, indicates that more complicated models do not necessarily produce better out-of-sample VaR forecasts. Instead, a trader holding long or short positions, or both, can measure the variance for the VaR calculation in a straightforward way. The Normal distribution and basic GARCH model for estimating the variance performs well, however, asymmetric models for the conditional variance may in some cases give more accurate predictions. The results tend to vary to some extent between the different indices and different confidence levels.

The remainder of this paper is organized as follows. Section 2 introduces the VaR approach. In Section 3 the models for the empirical investigation are presented. Section 4 continues partly on the same subject, and covers issues regarding distributions and ways to test the estimated VaR:s. In Section 5 the data is presented, and Section 6 continues with results from the empirical investigation conducted. Section 7 summarizes the results, and concludes the paper.

2 The Value-at-Risk Approach

Value-at-Risk (VaR) is a measure used to calculate the worst outcome of a portfolio that is likely to occur within a given horizon, assuming a certain level of confidence. The application of this measure is of interest especially for financial institutions who want to calculate capital requirements in relation to the financial risk that they hold.

The VaR approach is usually linked to the Basel Accord, and the recommendations given by the Basel Committee on Banking Supervision to create a more stable and consistent banking system internationally. The framework of creating this stability is built upon three pillars; minimum capital requirements, supervisory review and finally market discipline B.I.S. (2004). The VaR approach can be seen as closely related to the first pillar. However, from the objectives given in this study, the VaR approach is used as a tool for a trader, not for a bank or a financial institution, and naturally the regulations in the Basel Accord are not of much importance.

Risk Metrics have introduced the Exponentially Weighted Moving Average (EWMA) approach for the VaR calculation. This is the easiest way to estimate the conditional standard deviation or the variance. The EWMA is a special case of a GARCH(1,1) model, which will be discussed later on. The measure calculates the variance for time period $t+1$ as the average of the current variance and the current actual return (squared) in the following way,

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)\,\varepsilon_t^2. \tag{1}$$

The parameter λ is the decay factor, and this factor is most usually set to be 0.94 (this value has performed well in prior estimations). The above equation equals

$$\sigma_{t+1}^2 = (1-\lambda)\,\Sigma_{i=0}^{\infty}\lambda^i\varepsilon_{t-i}^2. \tag{2}$$

This description gives that EWMA is an infinite weighted average of past returns (squared), where all information on past shocks and squared returns is incorporated with weights that are exponentially declining (Guermat and Harris, 2002).

VaR measures the market risk of a portfolio, security or an index and gives an estimate of the level of loss that the security, or portfolio or index could incur given a certain confidence interval over a certain time horizon. This is analytically written,

$$Pr\left[P_{t+h} - P_t < VaR_P\,(h)\right] = \alpha. \tag{3}$$

P is the value of the asset at time t and $t+h$, where h measures the time horizon. The parameter α is the degree of confidence. The level of confidence, calculated as $(1-\alpha)$ is usually chosen as 95 % or 99 %. This gives the value of α (degree of confidence) as 5 % or 1 % (Billio and Pelizzon, 2000). VaR can also be expressed in terms of returns as,

$$Pr\left[R_{P_{t+h}} < -VaR_R\,(h)\right] = \alpha. \tag{4}$$

R is here calculated as $ln(P_{t+h}/P_t)$. Taking this to the perspective of only one period, the above formula is (Guermat and Harris, 2002),

$$Pr\left[R_{t+1} < -VaR_{t+1}\right] = \alpha. \tag{5}$$

In other words, the VaR measure is a specific quintile of an assets potential loss (or distribution of losses) over a certain specified time horizon.

If the change in the portfolio value (R) is drawn from a distribution with finite first two moments, the VaR measure (of the portfolio) is written as;

$$VaR_{t+1} = \left|\delta\,(\alpha)\right|\sigma. \tag{6}$$

The parameter σ is the standard deviation of the distribution of the value change R and $\delta\,(\alpha)$ denotes the quintile of the distribution (Guermat and Harris, 2002).

Usually it is assumed that the returns are normally distributed, in which case the parameter $\delta(\alpha)$ gives the appropriate quintile from the perspective of a standard normal distribution. However, as the stylized characteristics of financial time series data, and returns overall, the series are probably not normally distributed. The presence of leptokurtosis or excess kurtosis when applying the normal distribution generates VaR estimates that are not correct. The true VaR is underestimated or overestimated as a result of skewness and kurtosis not having been taken into account by the Normal distribution (Angelidis et al., 2004). To take this problem into consideration, a model that can handle time-varying variances should be used, and should give more appropriate results. Thus, the equation above becomes,

$$VaR_{t+1} = \left|\delta(\alpha)\right| \sigma_{t+1}. \tag{7}$$

This gives instead of the unconditional distribution the conditional distribution (conditional on t). Relating to a statement by Brooks and Persand (2003), models that do not incorporate asymmetries in the specification of the volatility or in the unconditional return distribution, tend to underestimate the true VaR:s.

The ARCH model, and extended versions of this model (most often the GARCH), is one trusted way of estimating the variance. Variances estimated through ARCH type models have the advantage that unconditional symmetric and leptokurtic distributions can be observed and corrected for (Billio and Pelizzon, 2000). The advantage of incorporating conditional variance is that the distribution is allowed to change over time in which case typical facts (e.g. volatility clustering) seen in returns can be taken into account. When applying the conditional variance, the VaR estimates are automatically made conditional as well (Huang and Lin, 2004). ARCH/GARCH type models for the variance have generally contributed to more correct estimates, this being the reason for applying them in this paper.

As mentioned, EWMA is a special case of the GARCH model first introduced by Engle (1982) (ARCH) and Bollerslev (1986)(GARCH);

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \tag{8}$$

If ω equals zero, and $\alpha = 1 - \beta$ the GARCH(1,1) above reduces to an integrated GARCH (IGARCH), and at the same time to an EWMA model.

The problem of estimating the variance with the EWMA approach is that the measure assumes that the tails of the returns (or more precisely the tail thickness of the returns), is constant during the estimation period. This may however not always be true due to clustering. The GARCH model partly covers thick tailed returns and volatility clustering. A problem on the other hand is that the variance only depends on the size of the error term and not on the sign (Angelidis et al., 2004). The well known leverage effect is therefore ignored in the basic GARCH model. Returns under some periods may be close to normal, whereas other periods may be characterized by excess kurtosis. To correct for this, alternative distributions and models that can handle these clustering effects are introduced. The issue here, is the way the variance used for calculating the VaR should be estimated. This issue will be discussed below.

3 Model Specifications

Equations describing both mean and variance will be specified. These equations will work as a starting point and extensions to them are done using various distributions. These are described in the next section.

Before moving to the modeling of the variance, the mean equation must be specified. A simple first-order autoregressive AR(1) process will be used for that purpose. The use of the AR(1) process can be motivated by the fact that daily returns used in this study are known to be exposed to some degree of serial correlation. The AR(p) takes care of this problem properly. The AR(1) is given as,

$$r_t = \mu + \theta r_{t-1} + \varepsilon_t. \tag{9}$$

r_t[1] and r_{t-1} denotes the return at time t and time $t-1$ and μ is the mean parameter.

To move on to one of the central points in this paper, the focus is now on the variance equation. The focus here will not be on presenting different ARCH type models. Instead only the models that are of importance in this study are covered. (For a study comparing symmetric and asymmetric volatility models, see e.g. Balaban (2004)). In the models presented, the concept of asymmetry is introduced in both the variance equation and in the applied distribution. The symmetric model in the area of the variance equation is the basic GARCH model, and regarding the distribution, the Normal distribution and the Student's t-distribution are symmetric. Moving on to the asymmetric models for the variance equation, the EGARCH and the APARCH models are introduced. The APARCH model has proven to generate good results in other VaR studies (e.g. Giot and Laurent (2003b)), and the EGARCH model is seen as a good model for estimating volatility when asymmetries are present. Regarding the asymmetric distributions, the Generalized Error Distribution (GED) is investigated.

The GARCH approach for estimating the variance for the VaR measure will be estimated as a benchmark, and will be compared to the more sophisticated models for estimates of the variance. When introducing the setup using ARCH or GARCH type models for estimating the variance, conditional volatility is automatically introduced. This means that when using these measures, the estimated VaR measures are conditional (Huang and Lin, 2004).

The basic GARCH(1,1) model together with the AR(1) model for the mean equation will be considered as a "basic model", and the variance measure used as input in the VaR calculation. The AR(1)-GARCH(1,1) model will be estimated with first, the Normal distribution, second, the traditional Student's t-distribution and finally the GED. These distributions will be discussed in detail in the next section.

The data most likely have non-normal error distributions, and thus higher moments as skewness and kurtosis must be taken into account to get the variance to describe the data as correctly as possible. Therefore, the basic GARCH is expanded to account for asymmetric effects in the variance, this by introducing two asymmetric extensions of GARCH; the Exponential GARCH (EGARCH) model by Nelson (1991) and the Asymmetric Power ARCH (APARCH) by Ding et al. (1993). The APARCH introduced by Ding et al. (1993) is widely used in the VaR literature (see for example Mittnik and Paolella (2000); Huang and Lin (2004); Giot and Laurent (2003a,b); Angelidis and Degiannakis (2004)) and the model has the advantage of handling asymmetric effects from return shocks on future variance. According to statements by Huang and Lin (2004), the model

[1] r_t is the logarithmic return ($lnP_t - lnP_{t-1}$)

should give better in-sample fits compared to other models for the variance. The APARCH is expressed as,

$$\sigma_t^{\delta} = \omega + \alpha \left(|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1} \right)^{\delta} + \beta \sigma_{t-1}^{\delta}. \tag{10}$$

The parameters should satisfy the following conditions; $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\delta \geq 0$ and $|\gamma| < 1$[2]. As the model is asymmetric, good news ($\varepsilon_{t-i} > 0$) and bad news ($\varepsilon_{t-i} < 0$) have different impacts on the volatility, and the volatility depends both on the sign and the size of the error term.

The EGARCH(1,1) model by Nelson (1991) models the natural logarithm of the conditional variance, and is expressed as,

$$ln\left(\sigma_t^2\right) = \omega + \alpha \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \lambda \left| \frac{\varepsilon}{\sigma} - \sqrt{\frac{2}{\pi}} \right| + \beta ln\left(\sigma_{t-1}^2\right). \tag{11}$$

In this model two new parameters are introduced; the asymmetry is seen in the parameter α (asymmetry parameter), and in the leverage effect parameter λ. This model in combination with the autoregressive model for the mean gives an AR(1)-EGARCH(1,1) model, and for the APARCH, AR(1)-APARCH(1,1). This will be estimated with the same distributions as for the AR(1)-GARCH(1,1) model; the Normal distribution, the Student's t-distribution and the GED.

4 Return Distributions, Interpretation and Evaluation Measures for VaR

The VaR approach is closely related to the distribution of returns, since the estimate is the expected maximal loss under a certain level of confidence. Generally, it is assumed that the distribution is Normal or Student-t distributed (Guermat and Harris, 2002). Return distributions are typically not normal, instead they tend to be leptokurtic (excess kurtosis), meaning they are peaked and have fat tails. These patterns may be a result of low volatility and high volatility clusters (Billio and Pelizzon, 2000). According to van den Goorbergh and Vlaar (1999), the most important characteristic that is needed to be taken into account in the VaR modeling is volatility clustering. GARCH-type models should correct for this characteristic.

Depending on the type of data that is being modeled, the density function or distribution should be carefully chosen, so that it accounts for the characteristics in the data. The suitability of the distribution affects both the mean and variance process, and so both the upside and downside part of returns and naturally the risk. The requirements or properties that are desired are that the distribution should have a suitable shape, in which case it should be flexible in the sense that skewness and kurtosis are absorbed. This is important when modeling financial time series since financial time series are characterized by these moments. Of course, the distribution should not be too complicated and should be possible to calculate. From this it is seen that the process of choosing the distribution is important, as is the process of choosing the model for the estimation.

Three parameters are often brought forward when investigating the distribution function. The parameters are first, the location parameter closely related to the mean process, second, the scale parameter related to the variance and third, the shape parameter related to skewness and kurtosis (Alexander, 2004). The location parameter or the mean is the midpoint of the symmetric distribution. The parameter can change, and naturally changes over time, whereas the density function

[2]Note that if $\delta = 2$ and $\gamma = 0$ the APARCH reduces to the basic GARCH (Angelidis and Degiannakis, 2004). The asymmetric component is in that the parameter δ , and it is not constrained to be equal to two as in the symmetric versions.

shifts, but stays constant in terms of the shape. The scale parameter or the variance-related parameter measures the variance and variability of the distribution function. Again, the parameter can, and will change, resulting in an expanded distribution function, but still stays constant in terms of the shape. Finally, the shape parameter that is not influenced by the location and scale parameters determines the skewness and kurtosis parameters, which naturally implies a change in the shape parameter which is also reflected in changes in the density function (Alexander, 2004).

The first four moments are thus interesting and important both when selecting the most suitable model, and when selecting the best fitting distribution. The parameters are of importance for the stability of the outcome of the modeling. How these parameters and the change in them are taken into account will be discussed later on.

Three different basic distributions will be used for the models described in the previous section; the Normal (Gaussian) distribution, the Student's t-distribution (symmetric) or GED (asymmetric).

The basic case and starting point in this investigation is the Normal (Gaussian) density/ probability function. The density is;

$$f(x) = (2\pi)^{-1/2}\, exp^{-x^2/2}, \tag{12}$$

where $-\infty < x < \infty$.

The log-likelihood function of the normal distribution is given as,

$$L_T = -\frac{1}{2}\Sigma_{t=1}^{T}\left[ln\,(2\pi) + ln\left(\sigma_t^2\right) + z_t^2\right], \tag{13}$$

where T denotes the number of observations.

In the Normal distribution only two of the parameters described above in the density function are accounted for; the location and the scale parameter (not the shape parameter). This means that the first two moments, mean and variance are considered, and in the estimation process, the density function always has the same shape, but the midpoint and the variability can change. The weakness in this distribution is of course that it cannot handle the shape, and in that not skewness and kurtosis, which can be seen as important measures when modeling financial time series data, especially when dealing with short-term data. The other distributions that will be used and discussed below also includes a shape parameter in the density.

The density for the Student's t-distribution is;

$$g\,(z \mid \eta) = \frac{\Gamma\left(\frac{\eta+1}{2}\right)}{(\eta\pi)^{\frac{1}{2}}\,\Gamma\left(\frac{\eta}{2}\right)}\left(1+\left(\frac{z^2}{\eta}\right)\right)^{\frac{-(\eta+1)}{2}}, \tag{14}$$

where $-\infty < z < \infty$, and $\eta > 0$.

In this distribution, the mean parameter μ should satisfy $\mu = 0$, the variance parameter should be $\sigma^2 = \frac{\eta}{\eta-2}$, the skewness is $s = 0$, and the kurtosis is $k = \frac{6}{\eta-4}$.

The log-likelihood function for the Student's t-distribution is given as follows;

$$L_T = ln\left[\Gamma\left(\frac{\eta+1}{2}\right)\right] - ln\left[\Gamma\left(\frac{\eta}{2}\right)\right] - 0.5ln\left[\pi\,(\eta-2)\right]$$

$$-0.5\Sigma_{t=1}^{T}\left[ln\sigma_t^2 + (1+\eta)\,ln\left(1+\frac{z_t^2}{\eta-2}\right)\right]. \tag{15}$$

In the function above, η is the degree of freedom and should be $2 < \eta \leq \infty$ and $\Gamma(.)$ is the gamma function.

The above described Student's t-distribution is less restrictive in that it allows for variation in the location and the scale parameter, the same as for the Normal distribution, and it further allows for variation in the tail thickness. (The tail thickness parameter is included in the density function as η.) However, in this basic Student's t-distribution, the skewness is still not accounted for (but the kurtosis is through the tail thickness parameter). A more flexible extension of the distribution is needed to completely account for the shape parameter, meaning it also accounts or allows for skewness.

The third type of distribution is the GED. The GED has the density;

$$f(z) = \frac{\nu exp\left[-\left(\frac{1}{2}\right)\left|\frac{z}{\lambda}\right|^{v}\right]}{\lambda 2^{\left(1+\frac{1}{v}\right)}\Gamma\left(\frac{1}{v}\right)}, \tag{16}$$

where $-\infty < z < \infty$, and $0 < v \leq \infty$, and the gamma function is expressed as $\Gamma(.)$ and $\lambda = \left[2^{-\frac{2}{v}}\Gamma\left(\frac{3}{v}\right)\right]^{\frac{1}{2}}$. The parameter ν measures the thickness of the tail. If ν has a value of two ($\nu = 2$), then z is normally distributed. If ν is smaller than two ($\nu < 2$), tails are thicker than in the case of normal distribution. If the parameter is larger than two, ($\nu > 2$), the tails are thinner than is the case for the normal distribution.

The log-likelihood of the GED distribution is,

$$L_T = \sum_{t=1}^{T}\left[ln\left(\frac{\eta}{\lambda}\right) - \frac{1}{2}\left|\frac{z_t}{\lambda}\right|^{\eta} - \left(1+\eta^{-1}\right)ln\left(2\right) - ln\Gamma\left(\frac{1}{\eta}\right) - \frac{1}{2}ln\left(\sigma_t^2\right)\right]. \tag{17}$$

Since the VaR for both the left and the right side of the tail will be investigated, as both long and short positions are of interest, methods for measuring the results of the VaR calculations are described in detail below. Of interest is how well the VaR (for both left and right side tails) does in fact work and how the large returns of either sign is taken into account by the measure.

The usual way to look at the VaR measure is to look at the left tail of the distribution, and the losses that are observed in the case of decreased prices. When investigating the right tail of the distribution, the interest lies in the losses that are observed in the case of increased prices. To describe this in an easy way, investigating the left tail refers to negative returns, and how these can be best modeled, whereas investigating the right tail refers to positive returns, and how these can be best modeled to gain profits or avoid losses (Giot and Laurent, 2003a,b).

First looking at the most simple case, the variance is calculated with the Normal (Gaussian) setup, and this is used for the VaR estimation. Interpreting the left tail or the long position, the VaR can be calculated as,

$$\mu_t + z_\alpha \sigma_t. \tag{18}$$

For the right tail or the short position VaR is,

$$\mu_t + z_{1-\alpha}\sigma_t. \tag{19}$$

z_α naturally is the left quintile, and $z_{1-\alpha}$ the right quintile.

The interpretations of the VaR measures are the same for the Student's t-distribution. Again the VaR for the left and right tails is calculated as,

$$\mu_t + st_{\alpha,\upsilon}\sigma_t \tag{20}$$

and

$$\mu_t + st_{1-\alpha,\upsilon}\sigma_t. \tag{21}$$

$st_{\alpha,\upsilon}$ measures the left quintile and $st_{1-\alpha,\upsilon}$ the right quintile. υ is the degree of freedom parameter.

The VaR measures for the GED for the left and right tails are calculated as,

$$\mu_t + GED_{\alpha,\upsilon}\sigma_t \tag{22}$$

$$\mu_t + GED_{1-\alpha,\upsilon}\sigma_t. \tag{23}$$

The evaluation sample in previous studies ranges between 250 days, 500 days, 1000 days, and up to at least 2000 days. In this study however, 5 years of daily observations are selected as the evaluation sample, whereas the estimation sample is a period of 10 years. The data sample (described in detail in the next section) is divided into two sub-periods. The first part of the data consists of the first ten years of the data sample and is used for estimating the parameters that are not known in the different GARCH models. From these estimates, the out-of-sample VaR performance for the remaining five years of data can be derived using a rolling window. All in all, the three different models for calculating the variance that have been described will be applied (GARCH, APARCH and EGARCH), and the models will be estimated with the three presented distributions (Normal (Gaussian), Student's t distribution and GED).

For the estimation, four different commonly used values for measuring "critical values" are selected; the selected values of α are 0.5 %, 1 %, 2.5 % and 5 %.

For the indices included in this study, the sample coverage rate $\hat{\alpha}$ will be computed as the proportion of losses exceeding the VaR estimate. The sample coverage rate should be close to the value of α, and the absolute value of the difference between these $\left(\left|\alpha - \hat{\alpha}\right|\right)$ will be calculated and reported for both short and long trading positions.

The forecasts for the risk made with the VaR should give as true estimates of the risk as possible. This means that the true VaR should neither be overestimated nor underestimated. To evaluate how the different models and alternative distributions work for estimating the VaR, an evaluation procedure is carried out. This evaluation cannot be made only straightforward, since a comparison of the actual and forecasted VaR cannot be applied, as would be the case when evaluating various volatility models.

The basic requirement on VaR models is that the VaR forecasts should satisfy correct conditional coverage (Guermat and Harris, 2002). This means that on average, the VaR forecast should generate coverage rates that satisfy one minus the confidence level or $(1-\alpha)$ for long positions, and α for short positions. Conditional coverage is discussed later in this section.

Kupiec (1995) developed a test statistic for testing null hypothesis of correct unconditional coverage. The t-statistic is given as,

$$t_U = \frac{\hat{\alpha} - \alpha}{\sqrt{\hat{\alpha}\left(1-\hat{\alpha}\right)/N}}, \tag{24}$$

where α denotes the confidence level, $\hat{\alpha}$ is the exceptions proportion, and the parameter N is the number of VaR forecasts made (in this paper 1305 forecasts).

Failure rate is another similar measure. The failure rate f can be explained as the absolute number of times the returns exceeds the outcome of the forecast (Giot and Laurent, 2003b). This directly imposes that the failure rate should be close to the selected critical value (in this paper 0.5 %, 1 %, 2.5 % and 5 %). Since the interest lies in investigating the failure rate for both short and long positions, further development should be done on this subject. For long trading positions, the failure rate f_{long} equals the amount of negative returns (in percentage) that are below the estimated one step ahead VaR forecast for long positions. For short trading positions, naturally, the failure rate f_{short} equals the amount of positive returns (in percentage) that are above the estimated one step ahead VaR forecast for short positions. These estimates described above makes it possible to test the null hypothesis $H_0 : f = \alpha$ against the alternative $H_a : f \neq \alpha$.

As pointed out earlier, the estimated VaR should give correct conditional coverage. To evaluate the degree of correct conditional coverage for the VaR model, Likelihood Ratio (LR) test statistics derived by Christoffersen (1998) will be applied. The back-testing process measures whether a model is appropriate or not. The out-of-sample VaR:s are calculated and the number of times the result of the index (or generally portfolios) is worse than the VaR is counted. This rate should give an indication how well the estimated model can predict the market risk. If the number that is estimated for a certain model differs from what is expected (according to the confidence level), capital losses can be expected if that model is used. For a review of back testing methods, see e.g. Campbell (2005).

The Basel II approach includes some specific regulations, and when back-testing the estimated VaR measures, this will be taken into account. To do this back-testing required by the Basel approach, the number of notations (or days) for each level of confidence, which performs worse than the expected VaR estimate are observed. This means that the number of exceptions (denoted by N) will be divided by the total size of the evaluation sample denoted by T (in this case the five year period, including 1305 days). The rate that is obtained from this is the failure rate. The failure rate ($f = N/T$), is calculated for each index and model and is compared to the confidence level for both short and long positions.

To evaluate this, a likelihood ratio test by Kupiec (1995) will be employed. The following hypotheses are tested;

$$H_0 : N/T = \alpha \tag{25}$$

with the alternative hypothesis;

$$H_a : N/T \neq \alpha. \tag{26}$$

The parameter N denotes the number of times the estimates performs worse than the true VaR. In long positions this means that the loss of the index is worse than the true VaR, and for short positions this means that the gain on the index is better than the true VaR (this of course generates a loss when holding a long position, but the change in the index is positive). The LR needed for testing the hypothesis is,

$$LR_{UC} = 2\left[ln\left(\left(\frac{N}{T}\right)^N\left(1-\frac{N}{T}\right)^{T-N}\right) - ln\left(\alpha^N(1-\alpha)^{T-N}\right)\right] \sim \chi_1^2. \tag{27}$$

The test above (LR_{UC}) is the test for the unconditional coverage (UC). This test is a test for investigating whether the number of exceptions are correct. The test rejects a model when it has too many or few failures. As can be seen by the equation, the test is asymptotically χ^2distributed with one degree of freedom. Kupiec (1995) pointed out that the power of the test for the UC generally is poor. The estimate of this will be used separately, but also in relation to the correct conditional coverage (CC) rate.

The test for the correct CC introduced by Christoffersen (1998) is applied in this study as well. The CC test jointly tests if the total number of failures are in line with the expectations, but further if the failures are independent. The hypotheses specified are;

$$H_0 : N/\widehat{T} = \alpha, \ and \ \pi_{01} = \pi_{11} = \alpha, \tag{28}$$

with the alternative hypothesis;

$$H_a : N/\widehat{T} \neq \alpha, \ and \ \pi_{01} \neq \pi_{11} \neq \alpha. \tag{29}$$

The LR for the CC is calculated as;

$$LR_{CC} = -2ln\left((1-\alpha)^{\hat{T}-N}\alpha^N\right) + 2ln\left((1-\pi_{01})^{n_{00}}\pi_{01}^{n_{01}}(1-\pi_{11})^{n_{10}}\pi_{11}^{n_{11}}\right) \sim \chi_2^2. \tag{30}$$

n_{ij} denotes the number of observations taking the value of i that is followed by the observation taking the value of j . Further, $\pi_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$ are corresponding probabilities. When the UC test examines the observed failure rate in relation to the expected failure rate, the CC test further examines the issue of independence. This means that the CC test observes clustered failures.

5 Data and Summary Statistics

In the empirical part index-data from four major markets is used; the Japanese Nikkei index, the European British FTSE 100 index and German DAX index, and finally the U.S. index S&P 500. The data-set includes 15 years of daily observations (closing prices) 1.1.1991 to 31.12.2005. This gives a set of 3913 observations for each of the included indices. In previous studies different time-perspectives have been used. The selection of this time-period of 15 years can be motivated by the suitable time-span for estimation and testing that this set of daily returns offers. Returns r are calculated as the logarithmic difference between the price P time t and time $t-1$ as,

$$r_t = ln(P_t) - ln(P_{t-1}). \tag{31}$$

Summary statistics for the whole sample period, and two sub-periods can be seen in Table 1.

The return over the whole sample period for Nikkei is slightly negative, but for the other series, mean-returns are positive, on average about 0,03 %. The series for the whole sample period seems to be stable, showing maximum daily returns of 6 - 7 % , and minimum daily returns from -6 % to a decrease of almost - 10 % for the German DAX index. The standard deviation reaches 1.4 percent for Nikkei and DAX, and is close to one for FTSE and S&P 500. This is an indication that the U.K. and the U.S. markets are more stable than the other markets. All markets except the Japanese are slightly negatively skewed, whereas Nikkei is slightly positively skewed. All indices show excess kurtosis, the German DAX index again being the most prominent. Observed from the summary statistics in Table 1, normality is strongly rejected, based on the values of the Jarque-Bera (J-B) statistics.

Table 1: Summary Statistics

	Nikkei 1	Nikkei 2	Nikkei 3	FTSE 1	FTSE 2	FTSE 3
Obs.	3913	2608	1305	3913	2608	1305
Mean	-0.010	-0.021	0.012	0.025	0.041	<0.001
Max	7.661	7.661	7.223	5.903	5.440	5.903
Min	-7.234	-7.234	-6.865	-5.559	-4.140	-5.559
Stddev	1.401	1.409	1.386	1.017	0.928	1.173
Skew	0.076	0.144	-0.066	-0.115	0.005	-0.198
Kurt	5.477	5.760	4.884	6.424	5.047	6.886
J-B	1004	837	194	1920	455	830
Prob.	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
	DAX 1	DAX 2	DAX 3	S&P 1	S&P 2	S&P 3
Obs.	3913	2608	1305	3913	2608	1305
Mean	0.035	0.059	-0.013	0.034	0.054	<0.001
Max	7.553	7.289	7.553	5.573	4.989	5.573
Min	-9.871	-9.871	-8.875	-7.113	-7.113	-5.047
Stddev	1.410	1.225	1.720	0.997	0.926	1.125
Skew	-0.285	-0.503	-0.066	-0.097	-0.303	0.169
Kurt	7.361	8.115	5.800	7.229	8.518	5.603
J-B	3154	2953	427	2922	3348	375
Prob.	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001

1 denotes the whole sample; time period 1.1.1991-31.12.2005. 2 denotes the first sample period 1.1.1991-31.12.2000, 3 the second sample period 1.1.2001-31.12.2005. Numbers are given in percentage form.

The sample period is divided into two sub-periods. The first period, or estimation period is the first ten years of the sample; 1.1.1991 to 31.12.2000. This period includes 2608 daily observations for each of the four indices. The second sub-period that will be used as a testing period, stretching from 1.1.2001 to 31.12.2005, consists of 1305 daily observations for each of the four indices. These subperiods of ten years as an estimation period and five years as a test period should be fair in this kind of estimation. Both sub-periods show almost similar patterns as the whole datasample. However, the standard deviation is generally higher during the five last years; negative skewness can be more commonly seen but kurtosis is lower during the second sub-period.

More often than not financial returns do not follow the pattern of a normal distribution; the statement of being independently and identically distributed (iid) can in this case not be assumed. Facts that on the other hand can be applied to financial returns data contains first; volatility clustering, second, excess kurtosis and fat-tails and third, a mild skewness (Alexander, 2004). From the summary statistics the excess kurtosis can be directly observed, and also some degree of skewness. To extend this analysis, QQ-plots are reported. From the figures (Figure 1 to Figure 4 at the end of the paper), it can clearly be observed that the distribution of the returns of the indices are all fat-tailed. Furthermore, the tails are not symmetric. Generally, more extreme events can be seen on the down-side of the distribution.

Table 2 presents values on parameters for the persistence of shocks in the data. For both the whole sample period and the first sub-period, the parameters turn out to be very close to one, showing a high degree of persistence in shocks. This is also true over different models for the variance and for different applied distributions.

Table 2: Parameter Estimates for GARCH

Model+distr.	Nikkei1	Nikkei2	FTSE1	FTSE2	DAX1	DAX2	S&P1	S&P2
GARCH-n	0.982	0.974	0.991	0.993	0.983	0.973	0.996	0.996
GARCH-t	0.988	0.990	0.994	0.996	0.995	0.991	0.998	0.999
GARCH-GED	0.989	0.983	0.991	0.993	0.992	0.987	0.997	0.998
APARCH-n	0.923	0.929	0.943	0.958	0.926	0.886	0.946	0.941
APARCH-t	0.933	0.938	0.942	0.956	0.932	0.930	0.949	0.948
APARCH-GED	0.929	0.934	0.942	0.956	0.931	0.929	0.948	0.947
EGARCH-n	0.964	0.959	0.988	0.980	0.948	0.896	0.985	0.982
EGARCH-t	0.982	0.985	0.989	0.991	0.990	0.989	0.990	0.991
EGARCH-GED	0.981	0.982	0.989	0.991	0.988	0.985	0.989	0.989

1 denotes the whole sample; time period 1.1.1991-31.12.2005. 2 denotes the first sample period 1.1.1991-31.12.2000. The given estimates give the parameter of persistence; (in GARCH $\alpha + \beta$, in APARCH and EGARCH β). n denotes the Normal distribution, t the Student's t-distribution and GED the Generalised error distribution.

Parameter estimates of the basic AR(1)-GARCH(1,1) model under different distributions can be found in Table 3.

Table 3: Estimates AR-GARCH

	Nikkei1	FTSE1	DAX1	S&P1	Nikkei2	FTSE2	DAX2	S&P2
N-distr.								
μ	0.0002	0.0004***	0.0005***	0.0005***	<0.0001	0.0005***	0.0005**	0.0005***
θ	-0.0173	0.0180	0.0071	0.0022	-0.0317	0.0618***	0.0299	0.0310
ω	<0.0001***	<0.0001***	<0.0001***	<0.0001***	<0.0001***	<0.0001***	<0.0001***	<0.0001***
α	0.0719***	0.0717***	0.0777***	0.0522***	0.0720***	0.0457***	0.0711***	0.0491***
β	0.9104***	0.9190****	0.9050***	0.9436***	0.9018***	0.9471***	0.9017***	0.9474***
LL	11391	12971	11777	13019	7571	8709	8031	8825
t-distr.								
μ	0.0001	0.0005***	0.0007***	0.0005***	-0.0001	0.0005***	0.0007***	0.0006***
θ	-0.0325*	0.0127	-0.0093	-0.0126	-0.0545***	0.0475**	0.0057	0.0071
ω	<0.0001***	<0.0001***	<0.0001***	<0.0001**	<0.0001***	<0.0001**	<0.0001***	<0.0001*
α	0.0621***	0.0698***	0.0772***	0.0454***	0.0653***	0.0512***	0.0721***	0.0396***
β	0.9261***	0.9241***	0.9183***	0.9524***	0.9244***	0.9452***	0.9191***	0.9594***
LL	11481	13002	11902	13106	7657	8727	8151	8914
GED								
μ	0.0001	0.0004***	0.0006***	0.0004***	-0.0001	0.0004***	0.0006***	0.0004***
θ	-0.0223	0.0116	-0.0146	-0.0187	-0.0390**	0.0477**	0.0014	-0.0017
ω	<0.0001***	<0.0001***	<0.0001***	<0.0001**	<0.0001***	<0.0001**	<0.0001***	<0.0001*
α	0.0652***	0.0694***	0.0791***	0.0468***	0.0668***	0.0461***	0.0739***	0.0411***
β	0.9241***	0.9219***	0.9128***	0.9505***	0.9167***	0.9471***	0.9127***	0.9568***
LL	11509	13001	11880	13117	7665	8728	8129	8917

Mean equation: $r_t = \mu + \theta r_{t-1} + \varepsilon_t$.
Variance equation: $\sigma_{t+1}^2 = \omega + \alpha_1 \varepsilon_t^2 + \beta_1 \sigma_t^2$.
*, **, *** gives degree of significance levels of 10 %, 5 % and 1 % respectively.

When going from the Normal distribution to more complex distributions, as with the Student's t-distribution and the GED, the likelihood value increases, indicating better estimates for the variance. The estimates for the Student's t-distribution and the GED distribution generated results that are close to each other.

To get a better forecast on the advantage of using more developed distributions in the context of estimating the volatility for the series corresponding parameters will be presented for one of the asymmetric volatility models as well, namely the APARCH model, again with an application to different distributions.

Table 4: Estimates AR-APARCH

	Nikkei1	FTSE1	DAX1	S&P1	Nikkei2	FTSE2	DAX2	S&P2
N-distr.								
μ	<0.0001	0.0002*	0.0003*	0.0003**	-0.0003	0.0003**	0.0004**	0.0003**
θ	-0.0168	0.0218	0.0024	0.0103	-0.0322	0.0570***	0.0313	0.0401*
ω	<0.0001*	<0.0001	<0.0001	<0.0001*	<0.0001*	<0.0001	<0.0001	<0.0001*
α	0.0703***	0.0543***	0.0613***	0.0509***	0.0631***	0.0405***	0.0840***	0.0608***
γ	0.4818***	0.6149***	0.4051***	0.8256***	0.6328***	0.5099***	0.1767***	0.7232***
β	0.9229***	0.9431***	0.9264***	0.9456***	0.9290***	0.9581***	0.8859***	0.9413***
δ	1.2621***	1.2079***	1.5279***	1.1751***	1.1624***	1.2777***	1.7186***	1.0709***
LL	11432	13008	11807	13074	7606	8724	8037	8858
t-distr.								
μ	<0.0001	0.0002**	0.0005***	0.0004***	-0.0004	0.0003**	0.0006***	0.0005***
θ	-0.3252**	0.0201	-0.0133	-0.0051	-0.0475**	0.0534***	0.0025	0.0156
ω	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
α	0.0645***	0.0547***	0.0741***	0.0466***	0.0607***	0.0416***	0.0772***	0.0542***
γ	0.4874***	0.6206***	0.4280***	0.8263***	0.6270***	0.5053***	0.2543***	0.6521***
β	0.9329***	0.9419***	0.9316***	0.9490***	0.9383***	0.9560***	0.9299***	0.9478***
δ	1.3204***	1.2277***	1.0763***	1.2960***	1.2190***	1.3267***	1.0678***	1.2657***
LL	11519	13021	11932	13144	7682	8737	8165	8930
GED								
μ	<0.0001	0.0002**	0.0005***	0.0003**	-0.0002	0.0003**	0.0006***	0.0003**
θ	-0.0211	0.0129	-0.0171	-0.0116	-0.0362**	0.0435**	-0.0025	0.0029
ω	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
α	0.0672***	0.0542***	0.0709***	0.0486***	0.0616***	0.0418***	0.0738***	0.0559***
γ	0.4944***	0.6482***	0.4339***	0.8101***	0.6235***	0.5311***	0.2627***	0.6726***
β	0.9291***	0.9425***	0.9313***	0.9484***	0.9342***	0.9562***	0.9286***	0.9475***
δ	1.2663***	1.2341***	1.1238***	1.2400***	1.1991***	1.3147***	1.1272***	1.1590***
LL	11536	13034	11907	13153	7687	8742	8141	8935

Mean equation: $r_t = \mu + \theta r_{t-1} + \varepsilon_t$.
Variance equation: $\sigma_t^\delta = \omega + \alpha\left(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1}\right)^\delta + \beta\sigma_{t-1}^\delta$.
*, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

The estimates of the AP-ARCH are seen in Table 4. Again, strong persistence or memory effects are observed; the values obtained for the parameter β by far exceeds 0.9, with the exception of the German DAX index for the first sub-period. For all indices, and both for the full period and the sub-period the estimates of the parameter α is positive, indicating presence of a leverage effect in the case of negative returns (in the specification of the conditional variance). The parameter δ significantly differs from 2 in all cases, indicating that the use of a skewed distribution could give better predictions. In terms of Log-likelihood (LL) values, the Student's t-distribution and GED produces better predictions. Overall however, both the estimates of the parameters and the LL values are close to each other for the different distributions.

According to previous studies, how the different models perform tends to be very sensitive to the data (Angelidis and Degiannakis, 2007). For different sets of data, different models might be more preferable. It is expected that different models generate the best forecasts for different indices and different levels of confidence.

6 Results

First, to investigate the correctness of the VaR forecasts, absolute values of real and predicted VaR:s are presented in Table 5 for long positions and in Table 6 for short positions. Results from back-testing are presented in Table 7 to Table 10.

As described in Section 4, the sample coverage rate should be close to the value of α, and the absolute value of the difference between these $\left(\left|\alpha - \hat{\alpha}\right|\right)$ will be calculated and reported for both short and long trading positions.

The sample coverage rate $\hat{\alpha}$ reported in parentheses in Table 5 should be close to the given value of α and the absolute difference of these $\left(\left|\alpha - \hat{\alpha}\right|\right)$ should thus be close to zero. These two measures are used as a first indication of which models generate the best foercasts.

Table 5: Absolute value of the real and predicted VaR long positions

Panel A	α= 0.5 %	Nikkei	FTSE	DAX	S&P
GARCH-n		0.19 (0.69)	0.50 (1.00)	0.34 (0.84)	0.12 (0.38)
GARCH-t		0.12 (0.38)	0.27 (0.23)	0.27 (0.23)	0.35 (0.15)
GARCH-GED		0.19 (0.69)	0.50 (1.00)	0.42 (0.92)	0.12 (0.38)
APARCH-n		0.19 (0.69)	0.19 (0.69)	0.19 (0.69)	0.04 (0.46)
APARCH-t		0.27 (0.23)	0.04 (0.54)	0.27 (0.23)	0.42 (0.08)
APARCH-GED		0.19 (0.69)	0.11 (0.61)	0.19 (0.69)	0.04 (0.46)
EGARCH-n		0.19 (0.69)	0.19 (0.69)	0.34 (0.84)	0.04 (0.54)
EGARCH-t		0.27 (0.23)	0.19 (0.31)	0.19 (0.31)	0.04 (0.46)
EGARCH-GED		0.19 (0.69)	0.19 (0.69)	0.19 (0.69)	0.04 (0.46)
Panel B	α= 1 %	Nikkei	FTSE	DAX	S&P
GARCH-n		0.38 (1.38)	0.92 (1.92)	0.38 (1.38)	0.46 (1.46)
GARCH-t		0.54 (0.46)	0.54 (0.46)	0.69 (0.31)	0.77 (0.23)
GARCH-GED		0.07 (1.07)	0.84 (1.84)	0.23 (1.23)	0.30 (1.30)
APARCH-n		0.15 (1.15)	0.38 (1.38)	0.38 (1.38)	0.23 (0.77)
APARCH-t		0.62 (0.38)	0.23 (1.23)	0.62 (0.38)	0.77 (0.23)
APARCH-GED		0.15 (1.15)	0.30 (1.30)	0.15 (1.15)	0.31 (0.69)
EGARCH-n		0.07 (1.07)	0.38 (1.38)	0.69 (1.69)	0.23 (0.77)
EGARCH-t		0.62 (0.38)	0.54 (0.46)	0.69 (0.31)	0.23 (0.77)
EGARCH-GED		0.15 (1.15)	0.30 (1.30)	0.15 (1.15)	0.23 (0.77)
Panel C	α= 2.5 %	Nikkei	FTSE	DAX	S&P
GARCH-n		0.18 (2.68)	1.10 (3.60)	0.18 (2.68)	0.41 (2.91)
GARCH-t		0.74 (1.76)	0.81 (1.69)	1.04 (1.46)	1.73 (0.77)
GARCH-GED		0.11 (2.61)	1.02 (3.52)	1.41 (3.91)	0.34 (2.84)
APARCH-n		0.34 (2.84)	0.95 (3.45)	0.41 (2.91)	0.28 (2.22)
APARCH-t		1.50 (1.00)	0.64 (3.14)	1.27 (1.23)	2.04 (0.46)
APARCH-GED		0.26 (2.76)	0.95 (3.45)	0.18 (2.68)	0.28 (2.22)
EGARCH-n		0.57 (3.07)	1.02 (3.52)	1.33 (3.83)	0.35 (2.15)
EGARCH-t		1.66 (0.84)	0.43 (2.07)	1.27 (1.23)	0.35 (2.15)
EGARCH-GED		0.34 (2.84)	1.18 (3.68)	0.18 (2.68)	0.28 (2.22)
Panel D	α= 5 %	Nikkei	FTSE	DAX	S&P
GARCH-n		0.44 (5.44)	1.28 (6.28)	1.74 (6.74)	0.21 (5.21)
GARCH-t		0.63 (4.37)	1.02 (3.98)	0.33 (4.67)	2.16 (2.84)
GARCH-GED		0.10 (4.90)	1.36 (6.36)	1.90 (6.90)	0.36 (5.36)
APARCH-n		0.02 (4.98)	0.59 (5.59)	1.21 (6.21)	0.33 (4.67)
APARCH-t		2.16 (2.84)	0.44 (5.44)	1.32 (3.68)	2.32 (2.68)
APARCH-GED		0.06 (5.06)	0.67 (5.67)	1.21 (6.21)	0.48 (4.52)
EGARCH-n		0.25 (4.75)	0.67 (5.67)	1.90 (6.90)	0.06 (5.06)
EGARCH-t		2.09 (2.91)	0.02 (4.98)	1.02 (3.98)	0.25 (4.75)
EGARCH-GED		0.17 (4.83)	0.67 (5.67)	1.05 (6.05)	0.25 (4.75)

The first value represents the absolute difference of the confidence level and the sample coverage rate $|\alpha - \hat{\alpha}|$, and the value in parentheses is the sample coverage rate $\hat{\alpha}$.

For a confidence level of 0.5 % in the case of long positions, the APARCH normal version seems to give the best overall estimate. However, to pick one best model for each index, the GARCH t-version would be preferable for Nikkei and APARCH t-version for FTSE. APARCH normal-version and EGARCH GED-versions perform about the same for DAX. For S&P500 APARCH normal and GED, and further all the EGARCH versions generates about equally good estimates.

For a confidence level of 1 %, the best estimates are generally the same as for a confidence level of 0.5 %. For Nikkei however, the GARCH-GED and EGARCH-n now gives the best predictions, whereas the other indices suits with about the same models as for the lower confidence level.

For a confidence level of 2.5 %, the support of the asymmetric GED distribution grows. The variance models seems to be of less importance.

When moving on to the confidence level 5 %, the best fitting model differs. For Nikkei the APARCH-n gives the best prediction; for FTSE the EGARCH-t version, for DAX the GARCH-t and finally for S&P the GARCH-normal version gives the most correct predictions.

From the out of sample results reported in the tables it can be observed that more complex models do not necessarily generate more accurate VaR prediction. The normal version of the distribution used in the modeling of the variance delivers in almost all cases better predictions than the Student's t-distribution. The t-version tends to overestimate the risk and produce VaR results that are too "safe" in the out-of-sample investigation.

For lower confidence levels, the EGARCH model, both normal- and student's t-distribution generate the most correct estimates of VaR for Nikkei and for S&P 500. For FTSE 100 and DAX on the other hand, the APARCH model gives the best predictions for the lowest confidence levels. For higher α values however, the normal version and the GED version of the APARCH model, and the EGARCH model for the variance generate the best performing out-of-sample VaR results. The normal version of the GARCH model also gives best predictions in some cases. As a general observation, it can be stated that the versions where the volatility was estimated with a Student's t-distribution, generated good out-of-sample VaR estimates, that tended to be too safe, compared to the chosen level of confidence. The results vary in which case it is very hard to give an absolute recommendation as to what model gives the best VaR prediction for long positions. Overall the APARCH model estimated under the Normal distribution and partly also under the GED distribution generates the best and most stable correctness in the predictions. If not the best model in all cases, the APARCH performed in almost all cases as the second best model, indicating that it is stable over different selections of the confidence level.

For short positions, similar results as for long positions can be observed. The results are presented in Table 6.

Table 6: Absolute value of the real and predicted VaR short positions

Panel A	α= 0.5 %	Nikkei	FTSE	DAX	S&P
GARCH-n		0.12 (0.38)	0.35 (0.15)	0.04 (0.46)	0.04 (0.46)
GARCH-t		0.27 (0.23)	0.50 (0.00)	0.50 (0.00)	0.32 (0.15)
GARCH-GED		0.04 (0.46)	0.35 (0.15)	0.19 (0.31)	0.04 (0.46)
APARCH-n		0.04 (0.46)	0.35 (0.15)	0.27 (0.23)	0.19 (0.31)
APARCH-t		0.35 (0.15)	0.35 (0.15)	0.50 (0.00)	0.50 (0.00)
APARCH-GED		0.04 (0.46)	0.35 (0.15)	0.27 (0.23)	0.27 (0.23)
EGARCH-n		0.19 (0.69)	0.35 (0.15)	0.19 (0.31)	0.27 (0.23)
EGARCH-t		0.35 (0.15)	0.42 (0.08)	0.50 (0.00)	0.27 (0.23)
EGARCH-GED		0.04 (0.54)	0.35 (0.15)	0.27 (0.23)	0.27 (0.23)
Panel B	α= 1 %	Nikkei	FTSE	DAX	S&P
GARCH-n		0.39 (0.61)	0.54 (0.46)	0.39 (0.61)	0.08 (0.92)
GARCH-t		0.69 (0.31)	0.85 (0.15)	0.77 (0.23)	0.85 (0.15)
GARCH-GED		0.31 (0.69)	0.54 (0.46)	0.46 (0.54)	0.08 (0.92)
APARCH-n		0.23 (0.77)	0.69 (0.31)	0.54 (0.46)	0.39 (0.61)
APARCH-t		0.85 (0.15)	0.77 (0.23)	0.85 (0.15)	1.00 (0.00)
APARCH-GED		0.16 (0.84)	0.69 (0.31)	0.62 (0.38)	0.39 (0.61)
EGARCH-n		0.07 (1.07)	0.69 (0.31)	0.08 (0.92)	0.46 (0.54)
EGARCH-t		0.77 (0.23)	0.85 (0.15)	0.85 (0.15)	0.46 (0.54)
EGARCH-GED		0.00 (1.00)	0.69 (0.31)	0.62 (0.38)	0.46 (0.54)
Panel C	α= 2.5 %	Nikkei	FTSE	DAX	S&P
GARCH-n		0.51 (1.99)	0.97 (1.53)	0.74 (1.76)	0.11 (2.61)
GARCH-t		1.58 (0.92)	2.19 (0.31)	1.96 (0.54)	1.96 (0.54)
GARCH-GED		0.51 (1.99)	0.97 (1.53)	1.04 (1.46)	0.11 (2.61)
APARCH-n		0.03 (2.53)	1.43 (1.07)	0.81 (1.69)	0.43 (2.07)
APARCH-t		1.89 (0.61)	1.66 (0.84)	2.12 (0.38)	2.19 (0.31)
APARCH-GED		0.12 (2.38)	1.50 (1.00)	1.20 (1.30)	0.43 (2.07)
EGARCH-n		0.03 (2.53)	1.43 (1.07)	0.58 (1.92)	0.43 (2.07)
EGARCH-t		1.81 (0.69)	1.96 (0.54)	2.12 (0.38)	0.66 (1.84)
EGARCH-GED		0.11 (2.61)	1.50 (1.00)	1.04 (1.46)	0.51 (1.99)
Panel D	α= 5 %	Nikkei	FTSE	DAX	S&P
GARCH-n		0.02 (4.98)	1.63 (3.37)	0.86 (4.14)	0.79 (4.21)
GARCH-t		1.17 (3.83)	3.31 (1.69)	2.47 (2.53)	2.55 (2.45)
GARCH-GED		0.06 (5.06)	1.63 (3.37)	1.02 (3.98)	0.71 (4.29)
APARCH-n		0.13 (5.13)	1.63 (3.37)	0.79 (4.21)	0.63 (4.37)
APARCH-t		2.16 (2.84)	2.24 (2.76)	2.78 (2.22)	2.93 (2.07)
APARCH-GED		0.21 (5.21)	1.70 (3.30)	1.09 (3.91)	0.79 (4.21)
EGARCH-n		0.44 (5.44)	1.63 (3.37)	0.13 (5.13)	0.63 (4.37)
EGARCH-t		1.93 (3.07)	3.16 (1.84)	2.70 (2.30)	0.71 (4.29)
EGARCH-GED		0.36 (5.36)	1.63 (3.37)	0.86 (4.14)	0.63 (4.37)

The first value represents the absolute difference of the confidence level and the sample coverage rate $|\alpha - \hat{\alpha}|$, and the value in parentheses is the sample coverage rate $\hat{\alpha}$.

For a confidence level of 0.5 %, the overall result is that the GED versions produce good predictions. For Nikkei, this holds with all volatility models as well. Very inaccurate predictions are seen for FTSE. For DAX the simplest model, GARCH normal produces the best predictions. This is the case for S&P as well.

For a confidence level of 1 %, the GARCH model for the variance estimated with the normal distribution or with the GED distribution produces good forecasts, with Nikkei being an exception. For Nikkei, the EGARCH-GED version gives the best predictions.

Similar patterns can also be seen for confidence levels of 2.5 %. GARCH-GED and normal gives the best predictions. However, for Nikkei, the APARCH-normal or EGARCH-normal works most accurately. The EGARCH-normal version also seem to be sufficient for DAX.

As was the case for long positions when confidence levels reached 5 %, the results seems to be very irregular for short positions as well. For all indices, the EGARCH-normal and EGARCH-GED produce good predictions of the VaR.

In general the APARCH and EGARCH models generates the best out-of-sample results, but the simple GARCH model also generates good approximations in some cases. The EGARCH model with a Normal distribution for the variance tends to give the most correct estimates; the index FTSE being an exception here. For the British index, the GARCH - Normal generates the best VaR forecast.

The results are somehow varying, but not necessarily surprising. The Normal distribution should, according to the results, be preferred over the Student's t-distribution, a result that is not supported by the fact of non-normality in the data, and the fact of fat-tails present in the data. This result is very interesting in the view of a trader holding long- or short-positions, or both, since the results indicates that more complicated models do not generate better one-step-ahead forecasts. For measuring the risk in the positions held, both long and short, and at different levels of confidence, the simple GARCH model with a Normal distribution generated fair results.

Results from the Back-testing are presented from Table 7 to Table 10. If the observed frequency is less than the confidence level, then the model overestimates the risk of the current position; the z-values tend to be too large. If the observed frequency is higher than the confidence level, then the model underestimates the risk of the position; the z-values tend to be too small. For a confidence level of 0.5 %, the number of times that the VaR prediction is exceeded should be 7 in the sample of 1305 observations. Clearly it can be observed that this number, N, is exceeded for the normal versions of the different volatility models, whereas N is never exceeded when using the Student-t version. Similar patterns can be seen for the higher confidence levels. The number of expected losses is exceeded in almost all cases for the normal versions of the volatility models, and the number of predicted exceptions with reference to the Student t-version of the distribution is below the number according to the confidence level. Surprisingly, the number that is given by the confidence level is almost never exceeded for the Student's t-distribution (three exceptions however).

For short positions, similar results can again be seen. N is never exceeded for the Student's t-distribution, and for the Normal distribution, the rate is generally not exceeded either. For the Student's t-distribution, the $N : s$ are very low, and also zero in some cases for lower confidence levels. Again, it can be pointed out that the risk is overestimated.

Table 7: Back test long positions; failure rate f_{long}

Panel A α= 0.5 %	N=7	Nikkei	FTSE	DAX	S&P
GARCH-n		9	13	11	5
GARCH-t		5	3	3	2
GARCH-GED		9	13	12	5
APARCH-n		9	9	9	6
APARCH-t		3	7	3	1
APARCH-GED		9	8	9	6
EGARCH-n		9	9	11	7
EGARCH-t		3	4	4	6
EGARCH-GED		9	9	9	6
Panel B α= 1 %	N=13	Nikkei	FTSE	DAX	S&P
GARCH-n		18	25	18	19
GARCH-t		6	6	4	3
GARCH-GED		14	24	16	17
APARCH-n		15	18	18	10
APARCH-t		5	16	5	3
APARCH-GED		15	17	15	9
EGARCH-n		15	18	22	10
EGARCH-t		5	6	4	10
EGARCH-GED		15	17	15	10
Panel C 2.5 %	N=33	Nikkei	FTSE	DAX	S&P
GARCH-n		35	47	35	38
GARCH-t		23	22	19	10
GARCH-GED		34	46	51	37
APARCH-n		37	45	38	29
APARCH-t		13	41	16	6
APARCH-GED		36	45	35	29
EGARCH-n		40	46	50	28
EGARCH-t		11	27	16	28
EGARCH-GED		37	48	35	29
Panel D α= 5 %	N=65	Nikkei	FTSE	DAX	S&P
GARCH-n		71	82	88	68
GARCH-t		57	52	61	37
GARCH-GED		64	83	90	70
APARCH-n		65	73	81	61
APARCH-t		37	71	48	35
APARCH-GED		66	74	81	59
EGARCH-n		62	74	90	66
EGARCH-t		38	65	52	62
EGARCH-GED		63	74	79	62

N is the expected number of times the VaR is exceeded, the numbers in the table are the observed.

Table 8: Back test short positions; failure rate f_{short}

Panel A α= 0.5 %	N=7	Nikkei	FTSE	DAX	S&P
GARCH-n		5	2	6	6
GARCH-t		3	0	0	2
GARCH-GED		6	2	4	6
APARCH-n		6	2	3	4
APARCH-t		2	2	0	0
APARCH-GED		6	2	3	3
EGARCH-n		9	2	4	3
EGARCH-t		2	1	0	3
EGARCH-GED		7	2	3	3
Panel B α= 1 %	**N=13**	**Nikkei**	**FTSE**	**DAX**	**S&P**
GARCH-n		8	6	8	12
GARCH-t		4	2	3	2
GARCH-GED		9	6	7	12
APARCH-n		10	4	6	8
APARCH-t		2	3	2	0
APARCH-GED		11	4	5	8
EGARCH-n		14	4	12	7
EGARCH-t		3	2	2	7
EGARCH-GED		13	4	5	7
Panel C α= 2.5 %	**N=33**	**Nikkei**	**FTSE**	**DAX**	**S&P**
GARCH-n		26	20	23	34
GARCH-t		12	4	7	7
GARCH-GED		26	20	19	34
APARCH-n		33	14	22	27
APARCH-t		8	11	5	4
APARCH-GED		31	13	17	27
EGARCH-n		33	14	25	27
EGARCH-t		9	7	5	24
EGARCH-GED		34	13	19	26
Panel D α= 5 %	**N=65**	**Nikkei**	**FTSE**	**DAX**	**S&P**
GARCH-n		65	44	54	55
GARCH-t		50	22	33	32
GARCH-GED		66	44	52	56
APARCH-n		67	44	55	57
APARCH-t		37	36	29	27
APARCH-GED		68	43	51	55
EGARCH-n		71	44	67	57
EGARCH-t		40	24	30	56
EGARCH-GED		70	44	54	57

N is the expected number of times the VaR is exceeded, the numbers in the table are the observed.

P-values from the likelihood ratio test of unconditional coverage and conditional coverage are presented in Table 9 (long positions) and Table 10 (short positions).

Table 9: LR unconditional and LR conditional long

Panel A α= 0.5 %	Nikkei	FTSE	DAX	S&P
GARCH-n	(0.358)0.656	(0.025)0.082	(0.111)0.279	(0.532)0.823
GARCH-t	(0.532)0.823	(0.122)0.302	(0.122)0.302	(0.037)0.114
GARCH-GED	(0.358)0.656	(0.025)0.082	(0.055)0.158	(0.532)0.823
APARCH-n	(0.358)0.656	(0.3584)0.656	(0.358)0.656	(0.835)0.978
APARCH-t	(0.122)0.302	(0.854)0.983	(0.122)0.302	(0.007)0.026
APARCH-GED	(0.358)0.656	(0.576)0.855	(0.358)0.656	(0.835)0.978
EGARCH-n	(0.358)0.656	(0.358)0.656	(0.111)0.279	(0.854)0.983
EGARCH-t	(0.122)0.302	(0.286)0.565	(0.286)0.564	(0.835)0.978
EGARCH-GED	(0.358)0.656	(0.358)0.656	(0.358)0.656	(0.835)0.978
Panel B α= 1 %	Nikkei	FTSE	DAX	S&P
GARCH-n	(0.193)0.428	(0.003)0.013	(0.193)0.428	(0.121)0.301
GARCH-t	(0.028)0.009	(0.028)0.009	(0.003)0.013	(0.001)0.003
GARCH-GED	(0.794)0.966	(0.006)0.024	(0.428)0.730	(0.294)0.576
APARCH-n	(0.596)0.869	(0.193)0.428	(0.193)0.428	(0.376)0.676
APARCH-t	(0.010)0.038	(0.193)0.428	(0.010)0.038	(0.001)0.507
APARCH-GED	(0.596)0.869	(0.294)0.576	(0.596)0.869	(0.233)0.491
EGARCH-n	(0.596)0.869	(0.193)0.428	(0.023)0.076	(0.376)0.676
EGARCH-t	(0.010)0.038	(0.028)0.090	(0.003)0.013	(0.376)0.676
EGARCH-GED	(0.596)0.869	(0.294)0.576	(0.596)0.869	(0.376)0.676
Panel C 2.5 %	Nikkei	FTSE	DAX	S&P
GARCH-n	(0.677)0.917	(0.017)0.057	(0.677)0.917	(0.353)0.649
GARCH-t	(0.072)0.198	(0.045)0.135	(0.009)0.033	(<0.001)<0.001
GARCH-GED	(0.809)0.971	(0.025)0.082	(0.003)0.011	(0.447)0.749
APARCH-n	(0.447)0.749	(0.038)0.116	(0.353)0.649	(0.513)0.807
APARCH-t	(<0.001)0.004	(0.153)0.360	(0.001)0.005	(<0.001)<0.001
APARCH-GED	(0.556)0.841	(0.038)0.116	(0.677)0.917	(0.513)0.807
EGARCH-n	(0.206)0.450	(0.025)0.082	(0.003)0.012	(0.401)0.703
EGARCH-t	(<0.001)<0.001	(0.304)0.590	(0.001)0.005	(0.401)0.703
EGARCH-GED	(0.447)0.749	(0.011)0.039	(0.677)0.917	(0.508)0.807
Panel D α= 5 %	Nikkei	FTSE	DAX	S&P
GARCH-n	(0.469)0.771	(0.040)0.122	(0.006)0.023	(0.729)0.942
GARCH-t	(0.263)0.564	(0.081)0.219	(0.585)0.862	(<0.001)0.001
GARCH-GED	(0.873)0.987	(0.030)0.095	(0.003)0.012	(0.551)0.837
APARCH-n	(0.439)0.999	(0.334)0.623	(0.053)0.155	(0.585)0.862
APARCH-t	(<0.001)0.001	(0.471)0.771	(0.022)0.072	(<0.001)0.001
APARCH-GED	(0.924)0.996	(0.276)0.553	(0.053)0.155	(0.420)0.722
EGARCH-n	(0.460)0.917	(0.276)0.553	(0.003)0.012	(0.924)0.996
EGARCH-t	(0.001)0.001	(0.975)0.999	(0.081)0.219	(0.677)0.917
EGARCH-GED	(0.774)0.960	(0.276)0.553	(0.090)0.239	(0.677)0.917

The first number is the p-value of the likelihood ratio for unconditional coverage (LR_{UC}) and the second number is the p-value of the likelihood ratio for the conditional coverage (LR_{CC}).

Table 10: LR unconditional, LR independence and LR conditional short

Panel A α= 0.5 %	Nikkei	FTSE	DAX	S&P
GARCH-n	(0.532)0.823	(0.037)0.114	(0.835)0.978	(0.835)0.978
GARCH-t	(0.122)0.302	(<0.001)<0.001	(<0.001)<0.001	(0.037)0.114
GARCH-GED	(0.835)0.978	(0.037)0.114	(0.286)0.565	(0.835)0.978
APARCH-n	(0.835)0.978	(0.037)0.114	(0.122)0.302	(0.286)0.565
APARCH-t	(0.037)0.114	(0.037)0.114	(<0.001)<0.001	(<0.001)<0.001
APARCH-GED	(0.835)0.978	(0.037)0.114	(0.122)0.302	(0.122)0.302
EGARCH-n	(0.358)0.656	(0.037)0.114	(0.286)0.565	(0.122)0.302
EGARCH-t	(0.037)0.114	(0.007)0.026	(<0.001)<0.001	(0.122)0.302
EGARCH-GED	(0.854)0.983	(0.007)0.114	(0.122)0.302	(0.122)0.302
Panel B α= 1 %	Nikkei	FTSE	DAX	S&P
GARCH-n	(0.130)0.318	(0.028)0.090	(0.130)0.318	(0.767)0.957
GARCH-t	(0.003)0.013	(0.001)0.001	(0.001)0.003	(0.001)0.001
GARCH-GED	(0.233)0.491	(0.028)0.090	(0.065)0.182	(0.767)0.957
APARCH-n	(0.376)0.676	(0.003)0.013	(0.028)0.090	(0.130)0.318
APARCH-t	(0.001)0.001	(0.001)0.003	(0.001)0.001	(<0.001)<0.001
APARCH-GED	(0.558)0.842	(0.003)0.013	(0.010)0.038	(0.130)0.318
EGARCH-n	(0.794)0.966	(0.003)0.013	(0.767)0.957	(0.065)0.182
EGARCH-t	(0.001)0.003	(0.001)0.001	(0.001)0.001	(0.065)0.182
EGARCH-GED	(0.989)0.999	(0.003)0.013	(0.010)0.038	(0.065)0.182
Panel C 2.5 %	Nikkei	FTSE	DAX	S&P
GARCH-n	(0.224)0.477	(0.016)0.055	(0.072)0.198	(0.809)0.971
GARCH-t	(<0.001)<0.001	(<0.001)<0.001	(<0.001)<0.001	(<0.001)<0.001
GARCH-GED	(0.224)0.477	(0.016)0.055	(0.009)0.033	(0.809)0.971
APARCH-n	(0.947)0.998	(0.001)0.001	(0.045)0.135	(0.304)0.590
APARCH-t	(<0.001)<0.001	(<0.001)<0.001	(<0.001)<0.001	(<0.001)<0.001
APARCH-GED	(0.771)0.959	(<0.001)0.001	(0.002)0.001	(0.304)0.590
EGARCH-n	(0.947)0.998	(0.001)0.001	(0.159)0.371	(0.304)0.590
EGARCH-t	(<0.001)<0.001	(<0.001)<0.0001	(<0.001)<0.001	(0.109)0.276
EGARCH-GED	(0.800)0.971	(<0.001)0.001	(0.009)0.033	(0.224)0.477
Panel D α= 5 %	Nikkei	FTSE	DAX	S&P
GARCH-n	(0.975)0.999	(0.004)0.017	(0.141)0.339	(0.181)0.409
GARCH-t	(0.044)0.131	(<0.001)<0.001	(<0.001)<0.001	(<0.001)<0.001
GARCH-GED	(0.924)0.996	(0.004)0.017	(0.081)0.219	(0.229)0.485
APARCH-n	(0.825)0.976	(0.004)0.017	(0.181)0.409	(0.285)0.564
APARCH-t	(<0.001)0.001	(<0.001)0.001	(<0.001)<0.001	(<0.001)<0.001
APARCH-GED	(0.729)0.942	(0.003)0.011	(0.060)0.171	(0.181)0.409
EGARCH-n	(0.471)0.771	(0.004)0.017	(0.825)0.976	(0.285)0.564
EGARCH-t	(0.001)0.003	(<0.001)<0.001	(<0.001)<0.001	(0.285)0.564
EGARCH-GED	(0.551)0.837	(0.004)0.017	(0.141)0.339	(0.285)0.564

The first number is the p-value of the likelihood ratio for unconditional coverage (LR_{UC}) and the second number is the p-value of the likelihood ratio for the conditional coverage (LR_{CC}).

The results of the unconditional and the conditional coverage rates are in line with the results presented in Table 7 and Table 8. The presented p-values are high, in most cases they exceed 10 % by far. This means that the VaR forecasts are satisfying.

7 Summary and Conclusions

This paper analyses and compares VaR measures estimated using different approaches for modeling the variance. Models incorporating asymmetry and volatility clustering, and also distributions that are not symmetric (symmetric assumes iid), have been modeled. The focus of the study is on investigating not only the left but also the right tail of the distribution and in that the risk in both tails.

The out-of-sample VaR for four major indices; Nikkei, FTSE, DAX and S&P500 were calculated. The results of the different estimations and models for the variance were compared to each other, and most importantly, to the basic GARCH-normal estimation for the variance-parameter used in the VaR estimation.

The results indicated that a trader holding long and short positions, or one of these, can usually manage to use linear models well for estimating the variance. The conditional variance is mostly better modeled with an asymmetric version, but the use of symmetric or asymmetric distributions does not make that much of a difference. It seems that the asymmetry in the volatility should be accounted for by adapting asymmetric volatility models, but that a linear distribution in the modeling process is at work.

Overall, the VaR estimates obtained generally overestimates risk and generate measures that are too safe, and the number of exceptions differ substantially from the theoretical values. This is not fully in agreement with some studies (for example Bams et al. (2002)) that argues that more complicated models for the distribution overestimate the VaR whereas more simple models underestimates the VaR.

From a trader's perspective, it would be of importance to handle the right and the left tails of the distribution separately. Different models should be used for the tails, and furthermore, it is relevant to consider the choice of model and distribution in relation to the level of confidence.

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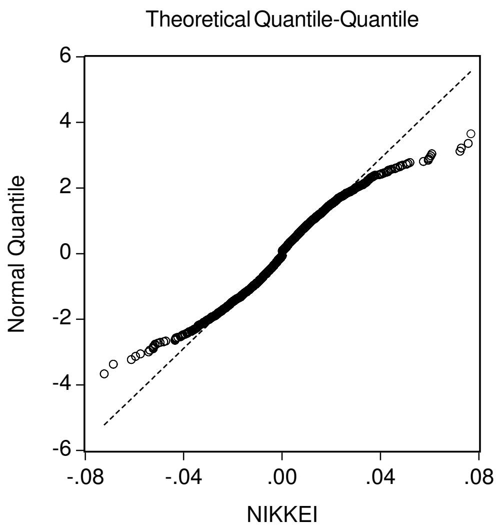

Figure 1: Nikkei QQ

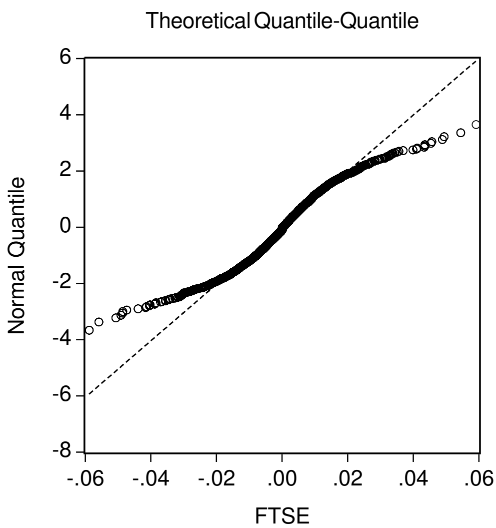

Figure 2: FTSE QQ

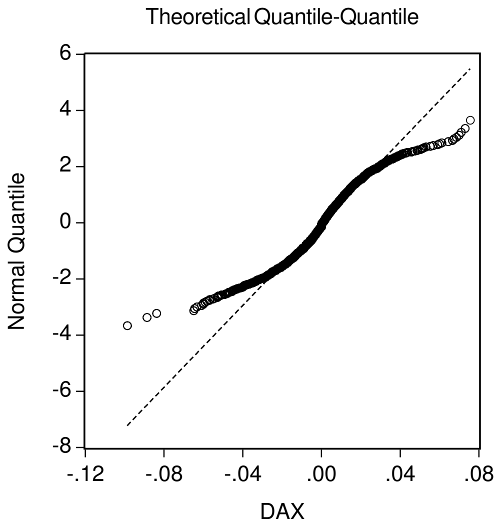

Figure 3: DAX QQ

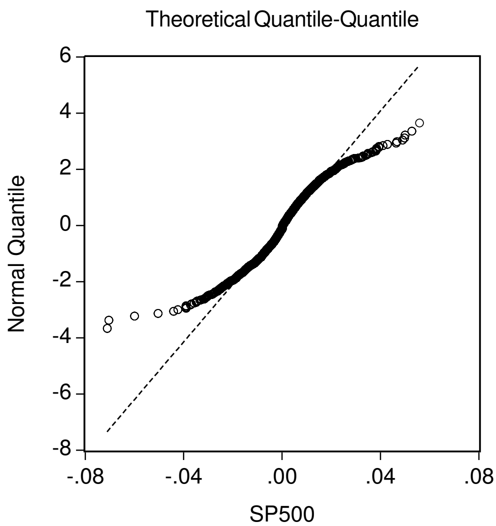

Figure 4: S&P500 QQ

The Relationship between Returns, Return Volatility and Information - An Asymmetric Approach

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Abstract

This paper investigates the relation between return, volatility with volume and interest rates as impulse variables. An impulse is a variable that might have an effect on the volatility. The concept of asymmetry in returns and volatility in this study contributes to the current literature. The risk-return-information relationship is investigated on a major market; the S&P 500 index on daily data. The results indicate that including volume as impulse affects the variance of the returns (indirect relationship). Interest rate is not a significant information parameter for modeling the volatility. Asymmetry in mean is modeled with a piecewise regression to take into account the asymmetric autocorrelation in the mean. Asymmetry in mean seems to be of some importance in modeling conditional mean and variance. By introducing alternative measures for the volume, especially in the variance equation, reduced persistence in conditional heteroscedasticity is observed.

Key words: risk, return, volume, asymmetric returns, asymmetric volatility, interest rates, piecewise regression, ARCH effects, persistence

JEL classification: G12, G14, G15

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1 Introduction

The behavior in stock market prices and the volatility of prices or of returns is affected by many different factors. The family of Autoregressive Conditional Heteroscedasticity (ARCH) models have been employed with various extensions in the purpose of measuring volatility persistence following new information since they were introduced by Engle (1982). This study will focus on the effect of introducing impulse variables [1] in the variance equation and introduce a way of modeling asymmetry in mean.

Does trading volume, or the change in trading volume have an effect on the outcome in returns, or are there other variables that are of greater importance? Further, what happens with the corresponding volatility? These questions will be empirically investigated in this paper. A related question that is of interest is the asymmetry issue. Asymmetry may be observed in both the conditional mean and the conditional variance, and can be regarded as important in relation to the volume.

This study focuses on the relationship between these elements; return, risk (volatility) and the information content of volume and interest rates, and further in relation to asymmetric behavior in returns and variances. The paper brings the literature one step forward in that, except for investigating the direct relationship between the elements, it further incorporates the concept of asymmetries. The asymmetry concept has previously been investigated for mean-variance setups, but not for the case when impulses have been taken into consideration. As an additional step, alternative variables to replace the direct volume variable are introduced.

The volume as a predictor or variable for modeling volatility of returns has been investigated in previous research, and has gained support for working as an information variable. Some studies that have concentrated on the relationship between stock returns and trading volume are Granger and Morgenstern (1963) and Karpoff (1987). Volume should not be argued as the only and best variable; other variables might be of importance. The effect different variables have on the heteroscedasticity is of large interest both in financial research and industry since the benefit of good forecasts are of importance economically.

The conditional variance is introduced first, and specifications for the conditional heteroscedasticity are applied. Change in volume and interest rates are introduced as impulses in the variance equation. Second, the concept of asymmetry in variance is introduced and the asymmetry in mean is the subject of study. The paper introduces a new way of modeling possible asymmetric autocorrelations in returns. In this case, the piecewise regression technique can be seen as the main contribution in the paper. What makes the asymmetry issue important is that positive and negative returns generally reverts differently to a mean level, and regarding the volatility asymmetric patterns can often be observed to follow increasing and decreasing prices. This means that there can be a benefit in modeling positive and negative returns separately. Third, alternative parameters replacing the volume parameter are introduced.

The empirical results generated in this study strongly support volume as an information contributing factor. The interest rate as impulse does not seem to contribute much in this context. The results support the importance of modeling positive and negative returns separately; done by introducing the piecewise regression technique. The study also supports an asymmetric modeling technique when it comes to the conditional variance. To summarize this, the modeling of conditional volatility can be improved by introducing asymmetric setups in conditional mean and

[1] An impulse variable is defined as information that can have an effect on the price and change it.

variance, and further by introducing volume in the variance. Introducing the alternative parameters for the volume results in reduced persistence in the autoregressive conditional heteroscedasticity.

The paper has the following setup: In Section 2, the traditional concept of risk-return relationship is described and some previous literature is discussed. In Section 3 asymmetries are introduced - both for the mean (return) and for the variance equation. Section 4 covers the methodology for the empirical part of the paper. Section 5 describes the data, and discusses summary statistics. Section 6 presents the results of the empirical part. Section 7 presents causality tests. In Section 8 some alternative proxies for the volume or trading activity are introduced and investigated. Section 9 concludes and summarizes the paper.

2 Risk-Return-Volume - Other Factors?

In recent years, research in returns and volatility has focused on ARCH effects, and support for ARCH type models is observed. The importance of taking these effects into consideration lies in the fact that heteroscedasticity needs to be modeled in order to generate reliable and correct volatility estimates. There is no straightforward explanation for the autoregressive patterns in the conditional variance. However, the most trusted theoretical explanation for the effects is the mixture of distribution hypothesis (MDH) by Clark (1973). ARCH effects are related to the information that arrives on the market, and these effects are a measure of the persistence in the information. Engle (1982) argues that the introduction of ARCH specifications for the variance estimation accounts for information variables not included. Consequently, ARCH effects could be reduced by introducing parameters including important information; referred to as impulses in this paper.

"It takes volume to make prices move" is a Wall Street expression that has been cited in almost every research paper on volume issues. The argument that there is a positive relationship between returns and volume sounds highly logical, but how do prices move in reality? Some studies have indicated a negative relationship between volatility and volume. Consequently, for prices to move, positive trading volume is a condition that must be satisfied, and naturally the volume parameter can be seen as an important information generator. Volume has an impact on the price, and of course on the volatility as well. However, volume cannot be the single predictor; other variables may be of equal importance. The importance of these variables, or the relationship between risk, return and the information lies in the possibility to model prices and volatility from the current information.

By investigating the relation between prices and information, the understanding and predictability of the structure of the market should be improved. The volume of assets traded on a specific day fluctuates. Possible reasons for the fluctuations are the amount of information that can be found on the company, possible options contracts that will soon expire and further how long the trading day is; full or half day. New information or impulses on the market seems to be the most influential element that contributes to the fluctuations in volume. New information can also be a question of regular annual reports, press releases, or information given by a third party.

According to previous studies, both risk and volume have a positive relationship to the return. The use of volume as a proxy for the information flow was first presented by Granger and Morgenstern (1963) and by Morgan (1976) with the motivation that volume is associated with risk. Generally, the behavior or the level of stock prices reflects the change in the beliefs of the investor that occurs because of new information and the change results in fluctuating returns. Trading volume on the other hand, is the sum of the real reaction of the investors. Volume should therefore be a more stable variable than the return. The risk-return relationship can in this case be described from the view of volume as an impulse; volume is said to have a large impact on risk (as described above, and proven in previous studies). The volume therefore could be of importance and should also be included in the equation for the risk (the variance equation).

The paper by Clark (1973) is one of the first papers that investigates trading volume and its role in the outcomes of returns. Clark presented the MDH that states that returns are a result of a mixture of distributions, and returns and volume are related to each other through their dependence on an information flow variable. Clark states that prices are correlated with the information flow, and prices tend to change more the more information is given to the market given a certain time interval. Clarke stresses the use of volume as a good information variable. Further, according to MDH, a strong relationship exists between trading volume and return volatility, and a moment pattern can be observed in the return volatility following information. Among others Epps and Epps (1976), Tauchen and Pitts (1983), Lamoureux and Lastrapes (1990), and Andersen (1996) have followed the theoretical setup presented by Clark (1973). Other papers on this topic that can be mentioned are Blume and O'Hara (1994) and Suominen (2001). These authors conclude that the volume variable contributes unique information to traders and other market participants. This information could not be seen directly from the prices, and therefore it is seen as unique.

Another key paper on the topic of return volatility and trading volume relationship is by Karpoff (1987). Karpoff proposes a simple model of the price-volume relationship that behaves in a consistent way. Karpoff suggests reasons why it is important to investigate the price-volume relation in equity markets. The relationship provides information on the structure of financial markets, and the relationship is important in the case of event study analysis because the inclusion of the price-volume relationship in the study increases the power of the tests. The price-volume relation also affects the empirical distribution of prices and should be taken into account. Karpoff argues that the price-volume relationship has strong implications in futures markets, and thus in research directed towards futures markets. Expected hypotheses by Karpoff (1987) are; 1) Small trading volume usually goes hand in hand with a fall in price. 2) Large trading volume is usually accompanied by a rise in price. 3) A large increase in trading volume usually indicates either a large price rise or a large price fall. The conclusion is that first, no correlation between volume and price exists; second, there is a correlation between the change in price and volume and third, the volume is higher when there is an increase in prices compared to when the prices are decreasing. Karpoffs' conclusions are that a positive relationship exists between volume and price changes. This relationship can be related to the information flow.

Following Karpoff (1987), Bessembinder and Seguin (1993), Brock and LeBaron (1996), Avouyi-Dovi and Jondeau (2000), and partly Lee and Rui (2002) have adopted the same ideas, and proposed evidence of a strong volatility-volume relationship. According to these authors, volume and volatility are related to each other in how they change. In contrast to the results mentioned above, Darrat and Zhong (2003) were not able to report a volatility-volume relationship directly. Instead, they could only report lead and lag relations, but no evidence of a correlation or a relationship between volume and volatility. Nguyen and Daigler (2005) present evidence that the effect of the return is more important in determining the volatility than is the effect of volume and neither volume nor volatility can be of use when predicting prices. Guo (2006) reports positive autocorrelation in stock market volatility and a significant correlation between realized variance and future returns.

The methodology in many published papers on the risk-return relationship is based on a paper by Lee and Rui (2002). Lee and Rui investigate the dynamic relationship between trading volume and stock returns on a daily basis. This imposes test of causality, and the results indicates that trading volume does not Granger cause the returns of stocks and second, a positive feedback relationship exists between volume and return.

Mestel et al. (2003) investigates the relationship between stock returns, return volatility and trading volume. The results indicate weak support for both the contemporary and the dynamic relationship between returns and volume on the Austrian market. In a similar paper Gurgul et al. (2005) find no support for a relationship between stock return and trading volume for the Polish market. A significant contemporaneous relationship between return volatility and trading volume can on the other hand be observed. Trading volume can thus be used as an information flow indicator, which supports the MDH.

Other factors to reduce ARCH effects and of help in modeling the variance have been of interest in research. For example Glosten et al. (1993) argue that an interest rate variable in the variance equation reduces the persistence, and interest rates are of help in modeling and predicting volatility. There has however not been any proof of interest rates producing better predictions; adding interest rates into the mean equation does not give better models, but benefits in the volatility modeling seems to be present.

There are several reasons why interest rates are of interest in the return-, risk- modeling of stock returns, or as here, index returns. For example, the relationship in a time when the interest rates are rising versus when they are falling (for example: September 2001 - falling interest rates hand in hand with a weak equity market) could be investigated. Interest rates further are directly linked to the overall economic situation since the central banks use the rates to govern development. This is closely related to the volume, since if the interest rates are lowered, it is highly likely that trading volume rises. Relating this to a state of economic recession, the activity is generally more modest. Consequently, the relationship between the information variables volume and interest rates should be inverse.

For example Schwert (1989), argues that increased volatility is accompanied by decreasing markets and low interest rates. Especially short-term interest rates are highly affected by the rate of inflation, or the expected inflation rate. This could be a motivation for using the interest rate as a control variable for the variance. Glosten et al. (1993) state that with the help of nominal interest rates, it could be possible to forecast periods of relatively large excess returns and significantly less volatility. This alone supports the use of interest rates as an information variable for the variance.

Lamoureux and Lastrapes (1990) introduced conditional heteroscedasticity (GARCH-type) models in their investigation of the price-volume relationship. Lamoureux and Lastrapes (1990) suggested a model where the volume is taken into account for the conditional variance. These authors emphasize the volume parameter for investigating trends.

Gallo and Pacini (2000) investigate the volatility persistence issue for GARCH type models. The results indicate that the volume used as an explanatory variable decreases persistence in the volatility when employing GARCH-type models for the conditional variance. How the risk-return-volatility relationship is affected by asymmetric modeling in the mean equation is hard to predict, but a higher volume might be correlated with positive returns, and lower volume with negative returns.

Salman (2002) applies a GARCH-in-mean model for the risk-return relationship, and concludes that both risk and return are integrated with information given to the market participants. This highlights the importance of volume as an impulse variable. Most studies have reported positive correlation and a positive relationship between price and volume. This relationship has been investigated with various methods; employing traditional regression analysis, GARCH applications and cointegration analysis. Some variations of course have been observed in the results, but most commonly a positive relation has been observed.

3 Asymmetries in Mean-Variance

Studies have shown that not only the variance behaves asymmetrically, but that this also holds for the return series or the mean. There is a difference in how fast a certain level of change in either direction reverts back to a mean level or to a zero return state, and empirical investigations indicate that positive returns persist for a longer period of time, meaning the trend continues to show positive returns. On the other hand, when the return at a specific time is negative, the negative patterns revert faster to the mean level. This means that financial time series reacts in a more volatile way following negative changes than following positive changes. The asymmetric relationship in both mean and variance have been investigated by among others Koutmos (1998) and Nam et al. (2001). The results indicate that asymmetries should be taken into account.

Regarding the asymmetric patterns in the conditional variance, research results are convincing. The variance behaves in an asymmetric way following shocks. Nelson's Exponential GARCH (EGARCH) model that takes into account asymmetric patterns in variance has gained a lot of support and will be employed in this study. This model is presented and discussed in the methodology part (Section 4).

As stated, asymmetry not only in the variance, but also in the mean should be taken into consideration and should be seen as an important factor in the modeling process. How the modeling of these asymmetric patterns in the return series can improve the modeling of return and volatility therefore is of highest interest. According to predictions, the introduction of the asymmetric model in the mean equation should decrease the volatility persistence, and should give significant results in the mean equation. This is done by modeling positive and negative returns separately. With further introducing asymmetric patterns in the variance equation, the persistence could further be reduced, indicating even better models.

For modeling asymmetry in the mean equation, a piecewise regression technique will be employed. The model includes an interaction variable that measures the asymmetry in the autocorrelation of the returns. The advantage of this rather flexible model is that both the intercept and the slope can vary with the data.

4 Methodology

As proposed, models will be specified for different setups of the risk-return relationship, and impulses for volume, interest rates and asymmetry will be introduced. The equations will be modeled including and excluding the impulse and combined versions including multiple variables.

ARCH specifications will be employed for taking into consideration the changing variance in the return series. The impulse variable will be included in the GARCH specification by Engle et al. (1987). By doing this, an Augmented GARCH in mean model is generated. The impulse variable is included in the variance equation, which means that the effect of the variable is investigated indirectly through the variance equation[2]. The factor included in the unrestricted model is a proxy for the information that flows into the market, but otherwise is unobservable.

Return R is calculated as the logarithmic change in the price (P) time t and $t-1$ as,

$$R_t = log\left[\frac{P_t}{P_{t-1}}\right]. \tag{1}$$

The volume is calculated the same way, and thus;

$$V_t = log\left[\frac{TV_t}{TV_{t-1}}\right], \tag{2}$$

where TV is the trading volume. The reason for using this concept in that changes and not directly the volume is that the change in volume (more or less trading activity compared to the previous day) gives more contribution in the volume factor as an impulse.

The interest rate parameter is calculated as the change in the rate between time t and one step back in time (time $t-1$, here one day) and is calculated as;

$$Int_t = log\left[\frac{TN_t}{TN_{t-1}}\right]. \tag{3}$$

The notation TN stands for 10 year treasury note, and so Int is the change in the rate.

For the basic case of the mean equation, the autoregressive model of order one (AR(1)) is used. The basic equation for the mean is thus[3];

$$R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t,\ \varepsilon_t \mid (\varepsilon_{t-1}, \varepsilon_{t-2}, ...) \sim N(0, \sigma_t^2). \tag{4}$$

Next the conditional variance is included. The GARCH model by Bollerslev (1986) and a generalization of the basic ARCH model by Engle (1982), is applied. To investigate the effect of volume and interest rate an additional variable is included in the variance equation as;

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 I_t. \tag{5}$$

The variable I is the impulse, and will be replaced by V for the change in volume, and Int for the interest rate.

Since the volatility is argued to behave asymmetrically, the EGARCH will be employed. The impulses are added to the EGARCH model by Nelson (1991);

$$log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2(I_t). \tag{6}$$

The persistence of the volatility for the GARCH(1,1) model is measured as $\alpha + \beta$ and for the EGARCH(1,1) model as β . In studies (see e.g. Li and Ding, 2003) it is argued that the "in-mean" version of the GARCH model generates good predictions, and this parameter ($\delta\sigma_t$) that investigates the mean/ variance relation will therefore be included in the mean equations. [4]

[2]Models where volume and interest rate were included in the mean equation (both with and without variance equation) were also estimated. No significant results could be observed, therefore they are not reported here.

[3]Various OLS regressions were conducted, where the information parameters (and lagged values of them) were regressed on the return, and return regressed on the information variables. The results did not indicate a strong direct relationship in the conditional mean, and the results are therefore not reported here. The causal relationship will anyway be investigated in Section 7 of the paper.

[4]It can be argued that the impulse one step back in time has a greater impact on the risk of today than today's information. To test this, different models were estimated to include lagged values of the impulse parameters. Weak or no significant results were obtained, consequently the results are not reported here.

Next, the asymmetry in mean equation is introduced. The asymmetry is already taken into account in the above covered variance equation, but will now be introduced for the mean equation. To measure the asymmetry and interaction in the mean equation, a dummy variable is first constructed to determine whether R_{t-1} is negative or not. If the return time $t-1$ is negative, the dummy (lagged) gets a value of one. If the return time $t-1$ is positive or zero, then the dummy gets a value of zero. Subsequently, if $R_{t-1} < 0$, the dummy is one and if $R_{t-1} \geq 0$, the dummy is zero. Further, an interaction term between the dummy variable and the return variable itself is constructed as the lagged dummy times the return time $t-1$. This gives the equation;

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 \left(D_{t-1} \left(R_{t-1} < 0\right)\right) + \phi_3 \left(D_{t-1} \left(R_{t-1} < 0\right) R_{t-1}\right). \tag{7}$$

In the case where positive or zero returns are observed ($R_{t-1} \geq 0$) the equation becomes,

$$R_t = \mu + \phi_1 R_{t-1}, \tag{8}$$

and when negative returns are observed ($R_{t-1} < 0$);

$$R_t = (\mu + \phi_2) + (\phi_1 + \phi_3) R_{t-1}. \tag{9}$$

This construction should fit the data in a better way, since not only the intercepts, but also the slopes are allowed to vary with the data. The constructed model is named piecewise regression. The advantage of this non-parametric type of model or technique is its flexibility in building the regressions.

The model for the mean equation above will in what follows be expressed as,

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t. \tag{10}$$

The results from the eight model specifications are presented in Section 6. First, the basic AR-GARCH model is estimated as such and with the asymmetric EGARCH for the variance equation. Then the in-mean parameter is included, volume in the variance equation, interest rates in the variance equation, and both. Models 5 to 8 use the same specifications for the variance but introduces an asymmetric model for the mean equation. This piecewise regression allows both intercepts and slopes to vary, which can be of importance in time series data.

If the asymmetry parameters are significant, their introduction in the mean equation might help in modeling the return. The combination of the asymmetry and the impulses for better estimates is thus investigated. This concept is applied both to the mean and the variance.

The equations are estimated using the maximum likelihood (ML) estimation. As indicators for selecting the best performing model specification, the Akaike Information Criteria (AIC), Schwarz Criteria (SC) and Log Likelihood (LL) measures are employed.

To compare the models, an out of sample forecast is utilized. The purpose of the forecast is to find the model specification that generates the smallest Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) values. RMSE is calculated as:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} e_i^2}. \tag{11}$$

The Mean Absolute Error (MAE) can be calculated as;

$$MAE = \frac{1}{n}\sum_{i=1}^{n} |e_i| \,. \tag{12}$$

The Mean Absolute Percentage Error (MAPE) is calculated as;

$$MAPE = \frac{1}{n}\sum_{i=1}^{n} \frac{|e_i|}{y_i} 100. \tag{13}$$

Further, the Theil Inequality Coefficient (TIC) will be calculated. The coefficient lies between zero and one, with zero representing a perfect fit. The coefficient is calculated with the formula;

$$TIC = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2}\sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^{-2}}}. \tag{14}$$

Mean Squared (Forecast) Error (MS(F)E) serves as a basis for the bias, variance and covariance proportions, and is calculated as;

$$MSE = \frac{1}{n}\sum_{i=1}^{n} (e_i)^2 \,. \tag{15}$$

Naturally, it holds that $RMS = \sqrt{MSE}$. The bias proportion measures how far the mean of the forecast is from the mean of the series. Variance proportion measures how far the variance of the forecast is from the actual variance. Finally, the covariance proportion is a measure of the unsystematic forecasting error remaining. These measures sum up to one. The smaller the proportions of the bias and the variance are, the better and thus most of the bias would be concentrated on the covariance proportion.

5 Data and Summary Statistics

The data sample consists of U.S. index S&P 500 and the corresponding trading volume. A 10-year U.S. treasury note is employed as the interest rate. The dataset includes seven years of daily observations (January 2000 to December 2006). Seven years of daily observations should be enough to deliver correct results. Summary statistics are presented in Table 1. The return of the index S&P 500, the change in volume of the index, and change in the interest rate are presented. From the summary statistics it can be seen that the mean return of the index is slightly negative. The change in volume has a positive sign, whereas the interest rate has a negative mean. The volume parameter is, as expected exposed to higher fluctuations (see for example maximum and minimum).

Correlations between the return and the information parameters (volume and interest rate and their lagged values) were calculated, but are not reported here. No strong correlation coefficients could thus be observed.

Table 1: Summary Statistics

	S&P500 return	S&P500 volume	Int. rate
Observations	1754	1754	1754
Mean	-0.0001	0.0335	-0.0105
Median	0.0382	-0.0000	0.0000
Max	5.5744	118.07	6.1538
Min	-6.0045	134.88	-4.4482
Std.dev	1.1296	1.8663	1.2906
Skewness	0.1034	-0.1231	0.4448
Kurtosis	5.6231	13.041	4.7434
Jarque-Bera	507.15	7389.55	279.98
Prob.	0.0000	0.0000	0.0000

Logarithmic returns and changes presented in %. S&P500 volume refers to the logarithmic change in the trading volume.

6 Results

When investigating the results, the focus is on the question whether the impulses are of importance in modeling returns and volatilities. Does the volume data only describe the trading activity, or does the volume data (the change in trading volume) offer unique information that can be taken advantage of in the modeling of returns and/ or volatilities? Furthermore, can the interest rate as a macro variable improve estimates and reduce persistence? Of further interest is whether the introduction of the asymmetry in mean and variance improve the predictions. Impulses that arrive on the market have an impact on the heteroscedasticity in returns. When introducing the above described factors in the modeling, the persistence in ARCH effects is affected, and will thus be investigated. If the impulses reduce the persistence and/ or improve the LL, AIC and SC values, the variable in question can be seen as a variable of importance for modeling variance. Investigating the mean, and possibilities in modeling the returns, significant coefficients or parameters from the piecewise regression can be of help in presenting the best estimation model.

Results from the eight specifications are presented from Table 2 to Table 9. First, the basic AR-GARCH model is estimated as such and with the asymmetric EGARCH for the variance equation. Then the in-mean parameter is included, volume in the variance equation, interest rates in the variance equation, and both. Models five to eight use the same specifications for the variance but introduce an asymmetric term for the mean equation. This piecewise regression allows both intercepts and slopes to vary, which can be of importance in time series data. The results from the regressions including the piecewise regression are presented from Table 6 to Table 9.

Models 1-8 refer to the earlier presented models, and a to d represent; a: simple model, b: in-mean parameter included, c: asymmetry in variance, d: in-mean parameter and asymmetry parameter in variance.

The estimates that are of interest are the values of the model-evaluation parameters, or Log-Likelihood (LL), AIC, SC and overall significant parameters. The LL value should be maximized, meaning higher LL value is an indication of a better model. The AIC and SC measures compares the in-sample fit, measured by the residual variance, in relation to the amount of estimated parameters. These values should consequently be minimized; smaller values indicating better models. The persistence of the volatility is also of interest. The volatility persistence can be measured as the sum of $\alpha + \beta$ for GARCH models, and by β for the EGARCH specifications. In the models where asymmetry is introduced in the mean equation, the parameters in the piecewise regression will be

Table 2: Estimation Results from Basic Mean Equation with Symmetric and Asymmetric Variance Equation (AR-GARCH-EGARCH-M)

Parameter	Model 1a	Model 1b	Model 1c	Model 1d
δ		0.0145		-0.0298
		(0.2169)		(-0.4782)
μ	0.0004	0.0003	0.0000	0.0003
	(2.2174)**	(0.5643)	(0.3324)	(0.5700)
ϕ_1	-0.0464	-0.0465	-0.0375	-0.0382
	(-1.8176)*	(-1.8137)*	(-1.4784)	(-1.4967)
α_0	0.0000	0.0000	-0.1641	-0.1501
	(1.7854)*	(1.7881)	(-5.0110)***	(-3.7413)***
α_1	0.0647	0.0650	0.0691	0.0681
	(5.6749)***	(5.6471)***	(4.1598)***	(4.1538)***
γ			-0.1064	-0.1065
			(-9.1689)***	(-9.2202)***
β_1	0.9313	0.9309	0.9885	0.9899
	(78.874)***	(78.067)***	(365.46)***	(272.67)***
LL	5667	5667	5705	5706
AIC	-6.4440	-6.4429	-6.4867	-6.4858
SC	-6.4253	-6.4211	-6.4650	-6.4609

Model 1a: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$.
Model 1b: $R_t = \mu + \phi_1 R_{t-1} + \delta\sigma + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$.
Model 1c: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi} \right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$.
Model 1d: $R_t = \mu + \phi_1 R_{t-1} + \delta\sigma + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi} \right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$.
Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

discussed as well.

From Model 1 (Table 2), an LL value of 5567 is observed. The persistence is very high, exceeding 0.99. When introducing the in-mean parameter, or the relationship mean/ variance, the estimates are about the same. The relationship itself is not significant. When including the asymmetry in the variance (EGARCH specification), the LL raises to 5706. The volatility persistence decreases slightly. In the estimation, the asymmetry parameter is also significant, and this in combination with the increased likelihood and reduced persistence gives support for using an asymmetric specification for the variance equation.

In model 2 (Table 3), the volume (β_2) is introduced in the variance equation. When it is introduced into the basic model, it is positive and significant at a level of 1 %, indicating the volume might have an effect on the variance. The LL value also increases, but no significant decrease in volatility persistence can be observed. The same results hold for the b specification. However, the in-mean parameter itself does not obtain significant values in any of the models estimated, indicating there is no mean/ variance relationship, and the parameter itself is not of importance in the correctness of the estimation.

In model 3 (Table 4), the interest rate is introduced as an information variable to the variance equation. The parameter itself (β_2) is negative and significant at a level of 10 %, meaning it is of importance. The LL value rises compared to the simple AR-GARCH model, but is not as high as when the volume parameter was included in the variance equation. Still, the persistence is close to 1. Again, the results give support to the use of asymmetric models for the variance equation since that specification gives higher LL, and a significant asymmetry parameter. There is no significant

Table 3: Estimation Results from Basic Mean Equation with Symmetric and Asymmetric Variance Equation With Volume (AR-GARCH-EGARCH-M-VOLUME)

Parameter	Model 2a	Model 2b	Model 2c	Model 2d
δ		0.1548		-0.0283
		(1.1295)		(-0.5442)
μ	-0.0000	-0.0017	0.0000	0.0002
	(-0.0970)	(-0.9284)	(0.2132)	(0.6040)
ϕ_1	-0.0203	-0.0182	-0.0574	-0.0583
	(-0.5027)	(-0.3943)	(-2.1691)**	(-2.1940)**
α_0	0.0000	0.0000	-0.1155	-0.1055
	(6.0846)***	(3.7033)***	(-4.9567)***	(-3.9612)***
α_1	0.1799	0.1687	0.0515	0.0508
	(5.2856)***	(4.0951)***	(3.7756)***	(3.7584)***
γ			-0.0866	-0.0867
			(-8.8773)***	(-8.8798)***
β_1	0.7987	0.7981	0.9923	0.9933
	(27.398)***	(17.836)***	(517.91)***	(432.60)***
β_2	0.0000	0.0000	1.6874	1.6897
	(4.1649)***	(8.6447)***	(13.517)***	(13.458)***
LL	5565	5550	5751	5751
AIC	-6.3270	-6.2519	-6.5373	-6.5363
SC	-6.3052	-6.2269	-6.5123	-6.5083

Model 2a: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Vol_t$.
Model 2b: $R_t = \mu + \phi_1 R_{t-1} + \delta\sigma + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Vol_t$.
Model 2c: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi} \right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Vol_t$.
Model 2d: $R_t = \mu + \phi_1 R_{t-1} + \delta\sigma + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi} \right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Vol_t$.
Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

mean/ variance relationship.

In model 4 (Table 5), both volume (β_2) and interest rate (β_3) are introduced in the AR-GARCH model. The volume parameter is positive and significant at a level of 1 % for all model specifications (a-d). The interest rate parameter is negative but insignificant. However, it is close to being significant in specification c and d (the EGARCH specification). The highest LL value for model 4 is obtained for the EGARCH specifications, and again, no significant reduction in the volatility persistence can be seen when introducing both volume, interest rate and asymmetry to the variance equation.

When comparing model 1 to 4, the highest LL value is received when including the volume as an information variable to the asymmetric volatility specification. This model also gives the lowest AIC and SC values. There is not much of a variation in the variables measuring volatility persistence, thus no conclusion will be drawn based on those.

Next, the piecewise regression is introduced in the mean equation. Models 5 to 8 all include this specification with an interaction variable in the mean equation, which accounts for asymmetric effects in the mean. This mean equation is estimated with the same variance equations as in models 1 to 4; both symmetric and asymmetric.

In interpreting the results from the piecewise models, the mean equation will also be discussed in more detail. The results of model 5 are presented in Table 6. The coefficient ϕ_1 generated from the lagged dummy variable (with value 0 or 1 depending on the sign of the return time $t-1$)

Table 4: Estimation Results from Basic Mean Equation with Symmetric and Asymmetric Variance Equation With Interest Rate (AR-GARCH-EGARCH-M-INTEREST)

Parameter	Model 3a	Model 3b	Model 3c	Model3d
δ		0.0154		-0.0290
		(0.2300)		(-0.4545)
μ	0.0004	0.0003	0.0000	0.0003
	(2.1566)**	(0.5292)	(0.3583)	(0.5487)
ϕ_1	-0.0459	-0.0460	-0.0380	-0.0382
	(-1.7922)*	(-1.7899)*	(-1.4995)	(-1.4994)
α_0	0.0000	0.0000	-0.1514	-0.1404
	(1.9462)*	(1.9503)*	(-5.1455)***	(-3.7471)***
α_1	0.0644	0.0646	0.0619	0.0618
	(5.5513)***	(5.5234)***	(3.9060)***	(3.9109)***
γ			-0.1014	-0.1018
			(-9.0535)***	(-9.0764)***
β_1	0.9306	0.9303	0.9892	0.9904
	(77.656)***	(76.982)***	(409.34)***	(289.57)***
β_2	-0.0001	-0.0001	-1.6655	-1.6530
	(-1.8090)*	(-1.8013)*	(-2.1231)**	(-2.1077)**
LL	5653	5654	5692	5692
AIC	-6.4421	-6.4410	-6.4852	-6.4842
SC	-6.4202	-6.4160	-6.4602	-6.4561

Model 3a: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Int_t$.
Model 3b: $R_t = \mu + \phi_1 R_{t-1} + \delta\sigma + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Int_t$.
Model 3c: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Int_t$.
Model 3d: $R_t = \mu + \phi_1 R_{t-1} + \delta\sigma + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Int_t$.
Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

shows possible asymmetric patterns in the data, and the coefficient ϕ_2 gives an indication how this asymmetry is connected to the return. Model 5, with GARCH and EGARCH specifications without additional impulses for the variance gives positive and significant estimates of the ϕ_2 parameter. This means that there are asymmetric patterns in the autocorrelation of the returns, and the asymmetry is positively related to the return. When introducing asymmetry also to the variance (EGARCH), the interaction parameter in the mean equation is no longer significant (however, close to significant). The LL value increases slightly compared to the first model, but no reduction in the persistence can be observed.

In model 6 (Table 7) the volume is introduced in the variance equation, and the parameter is positive and significant. The interaction parameter in the mean equation does not give significant values for specifications a and b, but when introducing asymmetry in the variance, significant parameters at a level of 10 % are obtained. The autocorrelation is negative for this specification. The value of LL also increases, indicating the model that accounts for asymmetry in both mean and variance, and further uses volume as an information variable should give fairly good volatility estimates.

Model 7 (Table 8) introduces the interest rate as the impulse in the variance equation in combination with the piecewise model for the mean equation. The interest rate parameter itself is negative and significant at least at a level of 10 % in all cases. The interaction parameter is close to being significant, but never reaches a significance level of 10 %. Compared to the model where

Table 5: Estimation Results from Basic Mean Equation with Symmetric and Asymmetric Variance Equations With Volume and Interest Rate (AR-GARCH-EGARCH-M-VOLUME-INTEREST)

Parameter	Model 4a	Model 4b	Model 4c	Model 4d
δ		0.1152		-0.0194
		(0.9832)		(-0.3590)
μ	-0.0000	-0.0011	0.0000	0.0002
	(-0.0011)	(-0.7185)	(0.1208)	(0.3894)
ϕ_1	-0.0282	-0.0212	-0.0571	-0.0574
	(-0.7561)	(-0.4776)	(-2.1485)**	(-2.1482)**
α_0	0.0000	0.0000	-0.1160	-0.1103
	(3.4863)***	(3.4545)***	(-5.2290)***	(-4.2347)***
α_1	0.1112	0.1690	0.0508	0.0506
	(4.4289)***	(4.1981)***	(3.8210)***	(3.8064)***
γ			-0.0852	-0.0855
			(-8.7550)***	(-8.7088)***
β_1	0.8509	0.8060	0.9922	0.9928
	(27.011)***	(18.537)***	(532.98)***	(432.54)***
β_2	0.0000	0.0000	1.6901	1.6905
	(6.1794)***	(9.8777)***	(13.383)***	(13.327)***
β_3	-0.0000	-0.0002	-1.1077	-1.1134
	(-0.4532)	(-0.9944)	(-1.5968)	(-1.6038)
LL	5555	5513	5737	5737
AIC	-6.3290	-6.2795	-6.5355	-6.5344
SC	-6.3040	-6.2514	-6.5074	-6.5032

Model 4a: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Vol_t + \beta_3 Int_t$.
Model 4b: $R_t = \mu + \phi_1 R_{t-1} + \delta\sigma + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Vol_t + \beta_3 Int_t$.
Model 4c: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Vol_t + \beta_3 Int_t$.
Model 4d: $R_t = \mu + \phi_1 R_{t-1} + \delta\sigma + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Vol_t + \beta_3 Int_t$.
Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

the mean equation was modeled as an AR model, this model improves the LL value to a small degree.

Model 8 (Table 9) includes the piecewise regression that accounts for the asymmetry in the mean equation, and for the variance equation both volume and interest rate are included, with estimations both with symmetric and asymmetric variance equations. In specifications a and b (modeled with simple GARCH), the interaction variable is insignificant. The volume parameter is positive and significant at a level of 1 %. The interest rate parameter does not generate significant estimates. When also introducing asymmetry in the variance, the autocorrelation parameter is negative and significant at a level of 10 %. The interaction parameter in the piecewise regression model is very close to being significant (positive); again, it never exceeds the level of 10 %. The asymmetry parameter in the variance equation is significant, so is the volume (positive and significant at a level of 1 %). Interest rate is negative and very close to significant at a level of 10 %. No reduction can be seen in the volatility persistence, but the LL values are relatively high, slightly higher than was the case without an asymmetric specification for the mean equation.

To summarize the results from the estimated models, there are asymmetric patterns in the mean process that can be accounted for by using a regression model that allows the intercept to vary. Also when it comes to variance, asymmetry should be accounted for. The results also showed

Table 6: Estimation Results from Piecewise Mean Equation with Symmetric and Asymmetric Variance Equations (Piecewise-GARCH-EGARCH-M)

Parameter	Model 5a	Model 5b	Model 5c	Model 5d
δ		0.0025		-0.0416
		(0.0371)		(-0.6583)
μ	0.0001	0.0001	-0.0001	0.0002
	(0.4984)	(0.2048)	(-0.5107)	(0.3165)
ϕ_1	-0.0510	-0.0506	-0.0446	-0.0458
	(-1.2385)	(-1.2375)	(-1.1431)	(-1.1723)
ϕ_2	0.0007	0.0007	0.0004	0.0005
	(1.7793)*	(1.7635)*	(1.1608)	(1.2502)
ϕ_3	0.0041	0.0041	0.0091	0.0090
	(0.0786)	(0.0789)	(0.1761)	(0.1744)
α_0	0.0000	0.0000	-0.1622	-0.1443
	(1.7460)*	(1.7414)*	(-4.9597)***	(-3.6687)***
α_1	0.0638	0.0638	0.0680	0.0670
	(5.6533)***	(5.6299)***	(4.1174)***	(4.1084)***
γ			-0.1033	-0.1035
			(-9.0271)***	(-9.0905)***
β_1	0.9323	0.9323	0.9886	0.9904
	(79.931)***	(79.496)***	(365.30)***	(279.27)***
LL	5669	5669	5706	5707
AIC	-6.4435	-6.4424	-6.4853	-6.4844
SC	-6.4186	-6.4144	-6.4572	-6.4533

Model 5a: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$ and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$.
Model 5b: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \delta\sigma + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$.
Model 5c: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$ and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$.
Model 5d: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \delta\sigma + \varepsilon_t$ and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$.
Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

that volume is an important factor in the variance equation when modeling the volatility. Interest rate also has an effect, however not as strong as the volume. The overall estimates of the variance can be improved by using the models and variables, and combinations of them introduced in this study.

To further evaluate the models above, an out of sample forecast was conducted. The forecasts were carried out for the base case model and for the models that gave the best estimates according to AIC, SC and LL. The results of the out of sample forecasts can be seen in Table 10.

From the out-of-sample forecasts of some selected models, the best performing predictions seems to be derived with the same models as were suggested by the AIC, SC and LL statistics. The first two measures are the same for all models in this sample. However, when it comes to the TIC[5] (equation 14), variations can be seen. The two models that take into account the asymmetry in the mean equation (and variance equation) generated the smallest values, which is preferred. The best forecast judging from the proportion measures is when the bias proportions (BP) [6] and variance proportions (VP) [7] are as small as possible, leaving the bias concentrated on

[5]The Theil inequality coefficient measures the RMS error in relative terms, and thus measures the deviation between actual and simulated values.
[6]The bias proportion measures how much the average of the simulated series differs from the historical value.
[7]The variance proportions measures the models' capacity in simulating the historical variability.

Table 7: Estimation Results from Piecewise Mean Equation with Symmetric and Asymmetric Variance Equation with Volume (Piecewise-GARCH-EGARCH-M-VOLUME)

Parameter	Model 6a	Model 6b	Model 6c	Model 6d
δ		0.1564		-0.0434
		(1.1978)		(-0.8245)
μ	-0.0002	-0.0018	-0.0002	0.0000
	(-0.3618)	(-1.0609)	(-1.0264)	(0.0905)
ϕ_1	-0.0136	-0.0051	-0.0679	-0.0684
	(-0.2014)	(-0.0681)	(-1.7301)*	(-1.7240)*
ϕ_2	0.0003	0.0003	0.0006	0.0007
	(0.4685)	(0.3170)	(1.7537)*	(1.8305)*
ϕ_3	-0.0140	-0.0225	0.0136	0.0116
	(-0.1631)	(-0.2341)	(0.2599)	(0.2196)
α_0	0.0000	0.0000	-0.1143	-0.0997
	(3.6171)***	(3.4939)***	(-4.9628)***	(-3.8827)***
α_1	0.1816	0.1688	0.0584	0.0498
	(4.9815)***	(4.9706)***	(3.7560)***	(3.7353)***
γ			-0.0833	-0.0833
			(-8.7237)***	(-8.7715)***
β_1	0.7976	0.7981	0.9924	0.9938
	(21.060)***	(19.854)***	(522.42)***	(449.50)***
β_2	0.0000	0.0000	1.7102	1.7146
	(166.70)***	(6.7084)***	(13.217)***	(13.207)***
LL	5554	5501	5753	5753
AIC	-6.3125	-6.2505	-6.5368	-6.5360
SC	-6.2844	-6.2193	-6.5057	-6.5018

Model 6a: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Vol_t$.
Model 6b: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \delta\sigma + \varepsilon_t$ and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Vol_t$.
Model 6c: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Vol_t$.
Model 6d: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \delta\sigma + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Vol_t$.
Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

the covariance proportions (CP)[8]. The variance proportion is very high in all models. The highest CP are obtained for the two last models which also seemed to be most preferable according to the other measures.

7 Causality Test

Above, the contemporaneous relationship between return volatility and impulses has been investigated, and further the effect of asymmetric mean and variance has been studied. Also of interest, however, is the causal or dynamic relationship between the information variables and the return. The dynamic relationship shows if information on the trading volume or interest rates can be of use in improving the forecasts on returns.

Granger (1969) introduced the Granger causality test. The test derives answers to what parameters drives the other. If an event x occurs before an event y , then x is said to cause y.

To test the causality between returns and volume change, and returns and interest rate, a

[8]The covariance proportion measures the models' ability to replicate the remaining errors.

Table 8: Estimation Results from Piecewise Mean Equation with Symmetric and Asymmetric Variance Equation with Interest Rate (Piecewise-GARCH-EGARCH-M-INTEREST)

Parameter	Model 7a	Model 7b	Model 7c	Model 7d
δ		0.0051		-0.0407
		(0.0752)		(-0.6296)
μ	0.0002	0.0001	-0.0001	0.0002
	(0.5626)	(0.2009)	(-0.4314)	(0.3373)
ϕ_1	-0.0501	-0.0502	-0.0472	-0.0484
	(-1.2248)	(-1.2242)	(-1.2072)	(-1.2361)
ϕ_2	0.0006	0.0006	0.0004	0.0004
	(1.5931)	(1.5745)'	(1.0490)	(1.1392)
ϕ_3	0.0045	0.0045	0.0140	0.0137
	(0.0859)	(0.0869)	(0.2708)	(0.2640)
α_0	0.0000	0.0000	-0.1512	-0.1348
	(1.9217)*	(1.9218)*	(-5.1236)***	(-3.6681)***
α_1	0.0637	0.0638	0.0613	0.0608
	(5.5320)***	(5.5077)***	(3.8745)***	(3.8782)***
γ			-0.0990	-0.0991
			(-8.9224)***	(-8.9569)***
β_1	0.9314	0.9313	0.9892	0.9909
	(78.336)***	(77.917)***	(407.48)***	(296.06)***
β_2	-0.0001	-0.0001	-1.6879	-1.6746
	(-1.7998)*	(-1.7972)*	(-2.1556)**	(-2.1456)**
LL	5655	5655	5693	5693
AIC	-6.4413	-6.4401	-6.4836	-6.4827
SC	-6.4132	-6.4089	-6.4524	-6.4483

Model 7a: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Int_t$.
Model 7b: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \delta\sigma + \varepsilon_t$.and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Int_t$.
Model 7c: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Int_t$.
Model 7d: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \delta\sigma + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Int_t$.
Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

bivariate autoregressive model is used. The model is expressed as;

$$x_t = \alpha + \sum_{0i=1}^{m} \alpha_i x_{t-i} + \sum_{i=1}^{n} \beta_i y_{t-i} + \varepsilon_t. \tag{16}$$

$$y_t = \gamma + \sum_{0i=1}^{m} \gamma_i x_{t-i} + \sum_{i=1}^{n} \delta_i y_{t-i} + \eta_t. \tag{17}$$

Assuming x_t stands for the return, and y_t denotes the impulse (volume or interest rate). If the coefficient β turns out to be significant, including lagged values of the parameter y (volume or interest rate), gives a better forecast of the return. If γ turns out to be significant, including lagged values of the return gives better volume and interest rate forecasts. The latter however is not of much interest here.

The results of the Granger causality test can be seen in Table 11 below. The causality runs from return to volume, but not the other way around. This is consistent with the mixture model by Clark (1973), that states no causality running from volume to returns, and with the finding by

Table 9: Estimation Results from Piecewise Mean Equation with Symmetric and Asymmetric Variance Equation with Volume and Interest Rate (Piecewise-GARCH-EGARCH-M-VOLUME-INTEREST)

Parameter	Model 8a	Model 8b	Model 8c	Model 8d
δ		0.1177		-0.0333
		(0.9850)		(-0.6085)
μ	-0.0000	-0.0013	-0.0002	-0.0000
	(-0.0490)	(-0.8191)	(-0.9107)	(-0.0082)
ϕ_1	-0.0029	-0.0097	-0.0695	-0.0695
	(-0.0403)	(-0.1321)	(-1.7732)*	(-1.7561)*
ϕ_2	0.0003	0.0003	0.0005	0.0006
	(0.3832)	(0.3698)	(1.4826)	(1.5377)
ϕ_3	-0.0227	-0.0196	0.0177	0.0153
	(-0.2444)	(-0.2092)	(0.3378)	(0.2902)
α_0	0.0000	0.0000	-0.1152	-0.1052
	(3.3180)***	(8.8790)***	(-5.2245)***	(-4.1460)***
α_1	0.1581	0.1666	0.0503	0.0499
	(4.0299)***	(6.8514)***	(3.8063)***	(3.7847)***
γ			-0.0825	-0.0827
			(-8.6231)***	(-8.6094)***
β_1	0.8135	0.8068	0.9926	0.9932
	(18.535)***	(86.781)***	(535.40)***	(443.96)***
β_2	0.0000	0.0000	1.7063	1.7086
	(59.895)***	(4.1084)***	(13.1150)***	(13.102)***
β_3	-0.0002	-0.0002	-1.0967	-1.1005
	(-1.1319)	(-0.6413)	(-1.5833)	(-1.5894)
LL	5505	5506	5739	5739
AIC	-6.2691	-6.2693	-6.5346	-6.5336
SC	-6.2379	-6.2350	-6.5002	-6.4962

Model 8a: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Vol_t + \beta_3 Int_t$.
Model 8b: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \delta\sigma + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 Vol_t + \beta_3 Int_t$.
Model 8c: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Vol_t + \beta_3 Int_t$.
Model 8d: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \delta\sigma + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \beta_1 log(\sigma_{t-1}^2) + \alpha_1 \mid \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} - \sqrt{2/\pi}\right) \mid + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_2 Vol_t + \beta_3 Int_t$.
Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

Lee and Rui (2002). They also show evidence for returns Granger causing volume. When testing the causality between return and interest rate, it can be found that the causality runs from interest rates to return.

8 An Alternative for Trading Activity

Since the results of the above investigations did not turn out as that convincing, alternatives for describing the heteroscedasticity and for reducing the persistence in the volatility will be tested. Price movements intradaily have an effect on the trading activity or the trading volume. With the purpose of investigating this, two new variables are introduced.

From the empirical investigations conducted in section 6, it was argued that the lagged volume parameter did not contribute to reduced volatility persistence. As pointed out by Gallo and Pacini (2000), the contemporaneous volume is autocorrelated and is too closely related to the returns,

Table 10: Out of Sample Forecast

Measure	Model 2a	Model 4a	Model 5d	Model 6c	Model 6d
RMSE	0.0113	0.0113	0.0113	0.0113	0.0113
MAE	0.0081	0.0081	0.0081	0.0081	0.0081
MAPE	101.22	99.920	100.64	117.37	119.82
TIC	0.9967	0.9977	0.9943	0.9710	0.9670
BP	0.0000	0.0000	0.0000	0.0000	0.0000
VP	0.9966	0.9953	0.9905	0.9442	0.9404
CP	0.0033	0.0047	0.0095	0.0558	0.0596

Model 2a: AR-GARCH+volume, Model 4a: AR-GARCH+volume+interest rate, Model 5d: AR-EGARCH-M+volume, Model 6c: Piecewise-EGARCH+volume, Model 6d: Piecewise-EGARCH-M+volume.

Table 11: Granger Causality Test

Panel A	F-statistic	Probability	Outcome
Return $\rightarrow^{G.C}$ Volume	8.420	0.000	1
Volume $\rightarrow^{G.C}$ Return	0.547	0.579	0
Panel B			
Return $\rightarrow^{G.C}$ Interest	0.492	0.611	0
Interest $\rightarrow^{G.C}$ Return	31.71	0.000	1

Panel A gives the causality between return and volume and Panel B the causality between return and interest rate.

and consequently to the squared returns. Since the volume parameter for time t was significant but the lagged volume was not, other measures replacing this volume variable should be considered.

The first variable to introduce is the difference between the highest and the lowest price intradaily (determined from returns). Since a higher trading volume also makes a reasonable contribution to larger difference between these two, this variable denoted HL , could be an impulse substituting the volume parameter. The variable is calculated as,

$$HL_t = log\left[\frac{R, high_t}{R, low_t}\right]. \tag{18}$$

This variable can be seen as high-low volatility. The lagged value of the parameter is to be included in the regressions. If this parameter is significant, the difference between intraday high- and low-values can be used as a predictor, and the variable contributes to a reduction in persistence.

If the first variable describes the volatility of daily extreme outcomes, the second variable is a measure of the volatility overnight. The variable ABS is the absolute value of the change in the price from one day to the other, or directly the lagged absolute value of the logarithmic returns.

$$ABS_t = \left|log\left[\frac{P_t}{P_{t-1}}\right]\right|. \tag{19}$$

This parameter is connected to the volume in the way that the price change and volume are related. The Granger causality test indicated that return caused volume, and therefore the above specified parameter could be a parameter to replace the volume. If the parameter turns out as significant in the regressions, shocks or surprises during the night that are reflected in the returns can be used as an information parameter, and consequently the persistence can be reduced.

When the mean equation is linear, as in Table 12, the introduced parameters are significant. The high-low parameter is positive and significant at a level of 1 % (Models A and B). The overnight or absolute parameter is negative and significant at a level of 10 % (GARCH) and 1 % (EGARCH)

(Models C and D). When both parameters are introduced in the variance equation (Models E and F), both parameters are significant at a level of 1 %. The persistence decreases compared to the models where parameters as the direct volume and interest rates were included, which supports the use of these parameters (HL and ABS) for reducing the persistence. The parameters introduced in this section therefore can be said to contribute to absorbed persistence. From the estimation results it can also be observed that the asymmetry component in the variance equation (when EGARCH is applied) is negative and significant at a level of 1 % in all models. This indicates high asymmetry in the conditional variance that should be taken into consideration.

When comparing the values of the measures LL, AIC and SC of the models specified in this section with the values in section 6, a slight overall improvement can be observed. Since these parameters also performs better in reducing persistence, the substitute parameters for the volume or trading activity that have been introduced in this section are the information parameters that are preferred.

Table 12: Estimation Results from Basic Mean Equation with Symmetric and Asymmetric Variance Equation with HL and ABS

Parameter	A	B	C	D	E	F
μ	0.0002	-0.0002	0.0005	-0.0001	0.0003	0.0000
	(0.9110)	(-1.1326)	(2.7845)***	(-0.5110)	(1.5408)	(0.3379)
ϕ_1	-0.0432	-0.0467	-0.0470	-0.0292	-0.0458	-0.0423
	(-1.6891)*	(-2.2342)***	-(1.8685)*	(-1.1003)	(-1.8472)*	(-1.7325)*
α_0	-0.0000	-2.2159	0.0000	-0.8319	-0.0000	-0.3598
	(-3.5913)***	(-11.505)***	(2.1001)	(-4.8160)***	(-1.4033)	(-3.3098)***
α_1	0.0181	-0.0937	0.0936	-0.0032	0.0749	0.0553
	(1.2500)	(-2.7551)***	(4.6074)***	(-0.0858)	(3.0650)***	(1.9299)*
γ		-0.0127		-0.1400		-0.1062
		(-0.9075)		(-9.5917)***		(-8.0760)***
β_1	0.9056	0.7828	0.9297	0.9217	0.8846	0.9713
	(45.221)***	(38.665)***	(79.7143)***	(52.269)***	(41.004)***	(92.386)***
β_2	0.0010	17.679			0.0020	13.896
	(3.5147)***	(10.2969)***			(4.7362)***	(5.4318)***
β_3			-0.0006	-12.7963	-0.0025	-18.226
			(-1.7131)*	(-3.2516)***	(-4.3871)***	(-4.6883)***
LL	5677	5622	5668	5694	5691	5732
AIC	-6.4547	-6.3908	-6.4445	-6.4719	-6.4693	-6.5150
SC	-6.4329	-6.3658	-6.4227	-6.4469	-6.4444	-6.4870

Model A-B: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 HL_{t-1}$. (Model A GARCH, Model B EGARCH)

Model C-D: $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$ and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_3 ABS_{t-1}$. (Model C GARCH, Model D EGARCH)

Model E-F : $R_t = \mu + \phi_1 R_{t-1} + \varepsilon_t$. and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 HL_{t-1} + \beta_3 ABS_{t-1}$. (Model E GARCH, Model F EGARCH)

Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

Table 13: Estimation Results from Piecewise Mean Equation with Symmetric and Asymmetric Variance Equation with HL and ABS

Parameter	A	B	C	D	E	F
μ	-0.0001	-0.0004	0.0002	-0.0001	0.0000	-0.0002
	(-0.4219)	(-1.6817)*	(0.8954)	(-0.5657)	(0.1224)	(-0.8697)
ϕ_1	-0.0465	-0.0509	-0.0511	-0.0401	-0.0516	-0.0499
	(-1.119)	(-1.6155)	(-1.2797)	(-1.0359)	(-1.3231)	(-1.3379)
ϕ_2	0.0007	0.0007	0.0007	0.0005	0.0006	0.0005
	(1.7284)*	(1.8204)*	(1.8004)*	(1.2176)	(1.4846)	(1.2947)
ϕ_3	0.0016	-0.0480	0.0034	0.0055	0.0069	0.0052
	(0.0309)	(-1.0940)	(0.0656)	(0.1049)	(0.1363)	(0.1026)
α_0	0.0000	-3.3374	0.0000	-0.2379	-0.0000	-0.6628
	(-3.5511)***	(-52.553)***	(2.1015)**	(-2.8009)***	(-1.4286)	(-3.9983)***
α_1	0.0177	-0.2882	0.0925	0.0502	0.0726	0.0363
	(1.2120)	(-7.1721)***	(4.6122)***	(2.1358)**	(3.0313)***	(1.0200)
γ		-0.1653		-0.1048		-0.1188
		(-7.3337)***		(-9.1457)***		(-7.9126)***
β_1	0.9056	0.6793	0.9312	0.9806	0.8866	0.9425
	(45.239)***	(86.800)***	(81.127)***	(114.518)***	(41.776)***	(59.321)***
β_2	0.0010	40.266			0.0019	17.965
	(3.4822)***	(27.549)***			(4.7149)***	(5.2560)***
β_3			-0.0006	1.8600	-0.0025	-19.098
			(-1.7330)*	(0.9394)	(-4.3835)***	(-4.0125)***
LL	5679	5701	5670	5707	5692	5733
AIC	-6.4542	-6.4785	-6.4441	-6.4846	-6.4684	-6.5139
SC	-6.4261	-6.4473	-6.4161	-6.4535	-6.4373	-6.4796

Model A-B: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$ and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 HL_{t-1}$. (Model A GARCH, Model B EGARCH)
Model C-D: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$ and $\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_3 ABS_{t-1}$. (Model C GARCH, Model D EGARCH)
Model E-F: $R_t = \mu + \phi_1 R_{t-1} + \phi_2 D_{t-1} + \phi_3 D_{t-1} R_{t-1} + \varepsilon_t$. and $log(\sigma_t^2) = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 HL_{t-1} + \beta_3 ABS_{t-1}$. (Model E GARCH, Model F EGARCH)
Numbers in parentheses give the test statistics. *, **, *** gives the degree of significance levels of 10 %, 5 % and 1 % respectively.

Table 13 presents regression results for models including asymmetry also in the mean equation and the "new" impulse parameters in the variance. Again, when comparing the results generated with these variables with the results in the previous section, reduced persistence can be observed. The HL parameter is positive and significant at a level of 1 % both in the model where it serves as the only information parameter, and also when combined with the ABS parameter. The ABS parameter is negative, however not highly significant in the model where it serves as the only impulse. When it is combined with the HL variable in the variance equation it is on the other hand significant at a level of 1 %. In the three first models (A, B, C) the asymmetry parameter for the mean equation is positive and significant at a level of 10 %. Since this parameter is significant, asymmetry in mean is sometimes that strong that it should be accounted for in the estimations. Better predictions can be made when allowing the mean to behave in an asymmetric manner.

Comparing the results presented in previous sections to the results presented here, a reduction can be observed in the persistence. This is especially true in model B where high-low is included as an information variable. The asymmetry parameter for the variance is negative and significant in all the above estimated equations. This is an indication of asymmetry in the variance.

On average, higher LL values and lower AIC and SC values are obtained for the estimations in this section. Also, decreased persistence can be observed, meaning the variables are able to capture persistence in the heteroscedasticity. Both impulses seem to be good parameters to include in the variance equation for making volatility estimates.

9 Summary and Conclusions

This paper has extended the ideas of Lamoureux and Lastrapes (1990) by adopting ARCH modeling with the purpose of investigating the return-risk-volume relationship and the study by Glosten et al. (1993) in investigating parameters for explaining ARCH effects and reducing the persistence in volatility. The concept of asymmetric, nonlinear behavior in the mean into the modeling setup is introduced.

Conditional information variables or impulses are introduced in the variance equation. These variables, the change in volume, the change in interest rates and further taking into account the asymmetric patterns, that are introduced both in mean and variance are of some importance in the modeling. ARCH effects are only in some cases decreased. Reduced persistence in volatility can hardly be seen. The relationships between the risk and impulses are of a more significant importance. The introduced information variables cannot capture part of the heteroscedasticity that is not captured by the ARCH process itself. However, the parameters reduce some nonlinearity in the series. The heteroscedasticity in stock returns can be explained in a more correct way in adding except the ARCH process also other impulses and the concept of asymmetry. Both volume and interest rates, and the combination of these and further taking into account the asymmetry generates higher LL values. The impulses provide important information for the modeling. What is interesting is that both conditions regarding the trading behavior and general behavior in the economic development can be used for the predictions. The same models seems to be preferred from the out of sample forecast as well, which gives an indication of robustness for the models.

When introducing alternative parameters for the volume, one parameter that is the change between the highest and the lowest notation during the day, and a second parameter that is the absolute value of the returns, a strong reduction in the persistence and higher LL values could be seen.

To summarize, the high-low parameter turned out to be of most importance when serving as an impulse or control variable. Also, the empirical study supports the concept of asymmetry in conditional mean and conditional variance. When the intercept in the mean equation is allowed to vary, better estimates can be received. Causality tests further indicates causality running from return to volume and from interest rate to return.

For further development on this research, it could be of interest to include other possible parameters, both for predicting returns and risk; meaning parameters both for mean equation and variance equation. For the variance, other complementary factors to the ARCH/ GARCH modeling setup are of interest, as are parameters that would work as substitute for this.

One problem with volume-augmented GARCH models is thus pointed out by Fleming et al. (2006). Adding volume to the GARCH models implies that volume is treated as exogenous variable, which is against most trading rules. If the volume parameter on the other hand is endogenous, problems arises in the estimation of the maximum likelihood making it hard to trust the significance of the results. Not all of the variation in the persistent component of volatility touches the variation in the component of volume. This implies that ARCH effects are not reduced by including the volume as information parameter for the volatility. To develop this further, it would make sense to run a simultaneous test including volume, returns and interest rates.

ARCH effects and the behavior in variance and heteroscedasticity seem to be a complicated equation that cannot be explained completely with a few single impulses. However, variables that can give a contribution in the modeling and predictability of both the risk and/ or the return should be incorporated in the modeling. More research is thus needed for better model combinations and explaining information variables, to get better estimates of prices and corresponding volatilities.

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