Monetary Policy Implementation in the Interbank Market

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Key words: interbank market, overnight interest rate, monetary policy, central bank operational framework

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Chapter 1

Introduction

1.1 Introduction

The liquidity crisis that swept through the financial markets in 2007, triggered multi-billion losses and forced buyouts of some large banks. The resulting credit crunch is sometimes compared to the great recession in the early twentieth century. But the crisis also serves as a reminder of the significance of the interbank market and of proper central bank policy in this market.

This thesis deals with implementation of monetary policy in the interbank market and examines how central bank tools affect commercial banks’ decisions. In particular, I focus on the following questions:

- What is the relationship between the policy setup and interbank interest rate volatility?
- What can explain certain patterns in the Eurosystem, such as a weak relationship between market liquidity and the interest rate?
- What determines banks’ decisions on when to satisfy the reserve requirement?
- How did the liquidity crisis that began in 2007 affect interbank market behaviour?

In order to answer these questions, I first analyse the behaviour of the overnight interest rate and demand for reserves in the Eurosystem, the UK and the US. I then provide a theoretical framework that allows me to model individual bank behaviour and market equilibrium. Using Monte-Carlo simulations, I replicate the observed patterns in the interbank markets and find that:

- averaging of the reserve requirement helps reduce interest rate volatility
- since the banks need not borrow from the ECB in the middle of the maintenance period, their response to aggregate liquidity changes is slower
- the banks do not target average required reserves for the whole maintenance period, which can be attributed to the existence of market frictions
- the liquidity crisis in 2007 resulted in increased credit risks, trading frictions and expected liquidity shortage.

This chapter starts with a brief introduction of monetary policy implementation and its place within the monetary policy framework. In the second part, I focus on the interbank market and its role for the central banks. Finally I present a summary of the remaining chapters.

1.2 Monetary policy implementation

Monetary policy has historically been a target of intensive research, which has produced a number of studies that characterise efficient central bank monetary policies. Woodford (2003) and Walsh (2003) are excellent surveys of existing research and theoretical models of monetary policy. This thesis contributes to this literature and focuses on monetary policy implementation, which is introduced in this section.

In terms of general goals, most central banks focus on price stability although some countries (e.g. the United States) also require the central bank to promote maximum sustainable output and employment. Since the central bank cannot control inflation and the output level directly, it uses intermediate operational targets to attain its objectives. The procedures employed by the central bank to achieve the operational targets are referred to in the literature as monetary policy implementation.

According to Bindseil (2004, p.7), monetary policy implementation consists of three elements:

- selection of the operational target of monetary policy
- establishment of an operational framework that allows for control of the operational target
- use of the instruments on a daily basis to achieve the operational target.

The choice between operational targets can be reduced to the choice between monetary aggregates and interest rates. The ‘reserve position doctrine’, which focused on monetary aggregates, was used by some central banks until the 1970s but led to several problems (see Bindseil (2004, p.25) for details) that eventually resulted in a shift to interest rate targeting. The relationship between the interest rate and inflation and output level is captured by the New Keynesian Model, which is presented in Walsh (2003, p.244), but similar formulations are used by Woodford (2003, p.246), Svensson (2003), Clarida et al. (1999), Gali (2002) and many others.
The operational target that is most popular among central banks is the interbank overnight interest rate.\(^1\) It can be argued that longer maturities have stronger impacts on the economy and hence should be better operational targets. However, the relationship between short and long term maturities (the yield curve) is in practise stable; thus a change in short term rates is likely to result in a corresponding change in long rates.\(^2\) At the same time, targeting short rates allows one to avoid certain time series anomalies which would occur if the long term target were changed.

In order to steer the level of the interbank interest rate, the central bank must influence market liquidity and expectations of future market rates, which is described below. The choice of tools depends on the policy framework; for instance in Chapter 3, I show that a high reserve requirement renders the interest rate almost entirely dependent on expectations of future liquidity and the level of the interest rate. This requires certain policies that focus on controlling the liquidity toward the end of the maintenance period.

Market expectations depend to a large extent on central bank policy announcements. For instance, the announcement of a target interest rate serves as a natural anchor for the expected interest rate. However, the policy details can play an important part in determining market behaviour, which is illustrated by ECB policy before March 2004. In that period, the announcement of ECB targets (following Governing Council meetings) occurred in the middle of the maintenance period,\(^3\) which resulted in greater volatility of the interest rate on those days. Synchronising announcement days with settlement days has effectively removed that effect and contributed to greater stability of the interest rate.

Central bank control over market liquidity is achieved by a combination of reserve requirements and various types of liquidity supply operations.\(^4\) The reserve requirement is an obligation imposed on commercial banks, to maintain a certain value of the current account balance. A bank must use the standing facilities\(^5\) if its end-of-day balance is below or above the required level. Some countries adopt a so-called averaging provision, which means that a certain average current account balance must be held at the central bank account. This gives the banks more flexibility in managing their reserves. However, it also makes the interest rate less vulnerable to aggregate liquidity changes, which can have a substantial effect.

\(^1\)In the sample of fourteen banks analysed by Borio (1997), eleven choose the overnight interest rate and the remaining three followed the rest after his study was published.

\(^2\)In fact this issue has been a subject of heated discussion, briefly summarised by Bindsell (2004, p.38). The relationship between long and short term interest rates depends on the rational expectations assumption, which was initially rejected in the empirical data, which in turn undermined the foundation of short-term interest rate targeting. However Mankiw and Miron (1986), Rudebush (1995) and more recently Kuttner (2001), have shown that under certain assumptions the rational expectations do hold, so that there is a significant relationship between short and long term rates.

\(^3\)With an averaging reserve requirement, banks must maintain a certain average current account balance during a period of time, which in this thesis is referred to as the maintenance period.

\(^4\)Note the difference between reserve requirement ratio (which relates to the central bank deposit) and capital ratio requirement, which is specified in Basel II.

\(^5\)The standing facilities are defined by Bindsell (2004, p.103) as "monetary policy operations which eligible counterparts of the central bank can use at their discretion at any moment during business hours".

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on the efficacy of certain central bank tools, as documented in Chapter 3.

The existence of a reserve requirement creates a structural deficit of funds in the market, which must be covered by central bank liquidity supply operations. The central bank can perform either early or late liquidity supply operations. Early operations take the form of open market operations or liquidity tenders before or during the trading day. During open market operations the central bank provides the market with reserves in exchange for securities such as government bonds. The distribution of liquidity is typically achieved via an auction. Late operations include the standing facilities, which give banks access to lending at the rates that are significantly higher (lower for deposits) than the market rate.

Since the banks always have a choice as to use of standing facilities, the deposit and lending rates serve as a corridor for the overnight market interest rate. Most central banks adopt a symmetric corridor around the target.\(^6\)

By balancing the reserve requirement and the supply of liquidity, the central bank can control the amount of available liquidity in the interbank market. However, the policy details can influence how fast the interest rate reacts to the change in aggregate liquidity. For instance, an averaging provision gives banks more flexibility in their liquidity management, which could reduce the impact of current liquidity. Weekly frequency of open market operations might temporarily leave the market with a surplus or shortage of liquidity in the presence of unexpected liquidity shocks, which might increase interest rate volatility and move the rate away from the target level. These issues are covered in detail in Chapter 2, where I analyse the behaviour of the interbank market under different regimes for monetary policy implementation.

Even if the central bank can control market liquidity, there remains the question of how much reserves to supply, in order to bring the interest rate to the target. To determine the liquidity injection, the ECB and the Bank of England use a 'benchmark allotment', which leaves the market with exactly the right amount of liquidity to meet the reserve requirement. Random changes in the autonomous liquidity factors (discussed below) can, however, cause market liquidity to deviate from the level set by the central bank. Thus apart from determination of the liquidity injection, it is equally important to take into account the impact of potential imbalances. I tackle this issue in Chapter 2, where I find that reserve averaging allows the market to absorb short term liquidity imbalances. On the other hand, without averaging there may be high interest rate volatility whenever market liquidity deviates from the neutral level.\(^7\)

### 1.3 The interbank market

Every day commercial banks engage in multiple transactions that alter the balances of their current accounts at the central bank. Some of these transactions derive from activities of Bank's customers and cannot be directly controlled. For instance,

\(^6\)Välimäki (2003) discusses the case of an asymmetric standing facilities corridor and timing of the liquidity supply operations.

\(^7\)By neutral liquidity I refer to the current account balance that allows the bank to smoothly satisfy the reserve requirement
a customer might order a bank transfer to an Internet store with an account at a different bank.

The general direction of the flow of funds might be predictable and linked to bank size or profile,\(^8\) which allows the bank to arrange proper finance in advance. For instance, a small bank serving local households might use a temporary surge in liquidity on salary payment day to satisfy its reserve requirement.

Unexpected liquidity flows (referred to as \textit{liquidity shocks}) cause banks’ current accounts to deviate from planned levels. Since the market is closed (the aggregate liquidity is determined by the balance sheet of the central bank) a surplus of one bank corresponds to a shortage for its counterparties. Thus the role of the overnight interbank market is to redistribute liquidity imbalances resulting from random liquidity shocks.

If the liquidity shocks occur early (I use the term \textit{early liquidity shocks}), the banks still have a chance to offset them in the interbank trade. However, some transactions occur late in the day, when the interbank trade is no longer possible. It is also possible that some transaction will not be processed (for instance, due to a clerical error). These transactions cannot be offset by interbank trade and hence will cause the end-of-day current account balance to deviate from the expected value. In the thesis, I refer to such transactions as \textit{late liquidity shocks}.

As mentioned above, banks’ transactions in the interbank market do not affect aggregate liquidity. However, certain types of transactions such as changes in the currency, government accounts or foreign exchange reserves, affect aggregate liquidity, which can have a strong impact on the behaviour of the interest rate. These transactions are referred to as autonomous liquidity factors, which Bindseil (2004, p.46) defines as “items in the balance sheet of the central bank that do not reflect monetary policy operations or reserve holdings of banks with the central bank”. Thus aggregate interbank market liquidity depends only on autonomous liquidity factors and central bank liquidity interventions, which are independent of the commercial banks.

\section*{Modelling the interbank market}

An early model of the interbank market was constructed by Poole (1968). Here, an individual bank minimises the expected cost of finance, obtained from either the interbank market or the central bank. The commercial bank, which starts with a certain balance on current account at the central bank, must decide on the borrowing volume. After the trade is completed, the bank is subject to a late liquidity shock such as that described above. The sum of starting balance, interbank trade and the liquidity shock is the end-of-day balance. If the balance is positive, the bank can use the deposit facility; if negative, it must borrow from the lending facility.

Solving the model reveals that, with optimal borrowing, the marginal cost must be the same for market or central bank finance. Thus the interbank interest rate equals the expected cost of using the standing facilities. If, for example, there is

\footnote{In fact, ECB (2007) finds that small banks are on average suppliers of liquidity, reflecting their role as intermediary between household savings and company investments.}
an aggregate shortage of liquidity, the probability of using the lending facility is higher, thus raising the expected cost of central bank finance and corresponding market interest rate.

A recent discussion on the interbank market was initiated by Hamilton (1996), who analysed the behaviour of the US overnight market. He pointed out that if the funds on each day of the maintenance period are perfect substitutes in terms of satisfying the reserve requirement, then the interest rate must exhibit the following martingale property:

\[ i_t = E(i_{t+1}) \]  

(1.1)

Furthermore, if this is true, the interest rate will move independently of market liquidity, which is controlled by the central bank. Thus the martingale hypothesis is very important for the implementation of monetary policy.

Hamilton (1996) rejects the martingale hypothesis for US data, attributing the deviation to the existence of certain market frictions, such as credit lines. However, some econometric studies come to a different view. In particular, Thornton (2001) shows that Hamilton's (1996) findings hold only for selected time periods and that extending the analysis to capture more recent data affects the results. Würtz (2003) and Moschitz (2004) find no evidence against the martingale hypothesis for the Eurosystem, except for the last couple of days of the maintenance period.

In Chapter 3, I refer to this discussion and show that the martingale property of the interest rate depends on the relationship between liquidity shock variance and average account balance. Calibrating the parameters for the Eurosystem, I find that the martingale hypothesis is likely to hold for most of the maintenance period.

From a theoretical point of view, the Poole (1968) model has been extended to the averaging provision case by Välimäki (2003), Pérez-Quirós and Rodríguez-Mendizábal (2006), Gaspar et al. (2008) (for the Eurosystem framework) and Ho and Sanders (1985), Bartolini, Bertola, and Prati (2001), Bartolini et al. (2002), Bartolini and Prati (2006), Clouse and Dow Jr. (1999), Clouse and Dow Jr. (2002) (for the US). All these papers adopt a similar approach, treating Poole (1968) as a single period model and extending the analysis to a dynamic framework. Since just one maintenance period is considered, the bank's cost minimisation problem has a finite time horizon and a different solution for settlement and non-settlement days.

On the last day of the maintenance period, any deviation from the reserve requirement will result in the use of standing facilities; thus resembling the problem without averaging that was solved in Poole (1968).

For the days preceding the end of the maintenance period, the problem can be captured by the Bellman equation

\[ V_t = \min_{b_t} \{ \kappa(b_t, i_t) + E(c_t) + V_{t+1} \} , \]  

(1.2)

where \( V_t \) is the value function that the bank minimises at time \( t \), \( \kappa(b_t, i_t) \) is the cost of finance from the market, \( E(c_t) \) is the expected cost of using the standing facilities at the end of the day, \( b_t \) is the borrowing and \( i_t \) is the interbank market interest rate. A similarly formulated problem is often the starting point in the
articles cited above. This thesis is a part of the literature that extends (1.2) to capture more sophisticated features of the interbank market.

The solution to (1.2) indicates that at the optimal borrowing the marginal cost of finance from the interbank market, $i_t$, is equal to the expected cost of using the standing facilities at the end of the day\(^9\) plus the dynamic cost factor that measures the marginal value of the reserve requirement for the bank. The reserve requirement is beneficial, since it provides the bank with a buffer: preventing the use of the deposit facility until the account balance exceeds the entire reserve requirement. This, however, creates an incentive to postpone fulfilment of the reserve requirement until the end of the maintenance period, to maintain the buffer as long as possible. This behaviour can explain the rise in the average interest rate toward the end of the maintenance period, which is documented e.g. by Pérez-Quirós and Rodríguez-Mendizábal (2006).

Poole’s (1968) model provides an elegant framework that links the interest rate with market liquidity and the cost of the standing facilities. The paper is considered the seminal work on interbank market modelling. The key assumption of this model is that banks are concerned with the liquidity shock, which ultimately determines their borrowing decisions. If volatility of the shocks is very small compared with the average account balance, the banks perceive the probability of using the standing facilities as negligible, which gives them more flexibility in determining their borrowing amounts. Thus the value of the parameter that measures the standard deviation of the liquidity shock is crucial for the model, as is detailed in Chapter 3.

The papers that use Poole’s (1968) framework typically choose high volatility of the shocks. For instance Pérez-Quirós and Rodríguez-Mendizábal (2006) assume the standard deviation is equal to the current account balance. For the US market, Bartolini et al. (2001) assume the shock deviation of 0.5 million, and the average current account balance is 3 million. Using Eurosystem data, Cassola (2008) calibrates the shock standard deviation at less than half the average current account balance. For his estimate, the martingale hypothesis still holds and the probability of using the standing facilities is very low (below 1%). In Chapter 3 I introduce a method to estimate shock volatility by analysing settlement day bank balances and obtain a much lower estimate of the liquidity shock standard deviation when compared with the average current account. As mentioned earlier, this means that the overnight interest rate is likely to exhibit the martingale property for most of the maintenance period.

Poole also assumes that banks are risk neutral. Thus the interbank cost function takes the form of a simple product, $i_t \times b_t$. Casual observations indicate, however, that the interbank market is subject to certain market frictions and transaction costs, such as the cost of staff salaries or the cost of finding a counterparty. In addition, the banks might be reluctant to borrow excessive amounts, fearing that this might alert counterparties to potential liquidity problems. The trading costs have been considered to some extent in Bartolini et al. (2001), Clouse

\(^9\)During the maintenance period, the lending facility is only used with an overdraft (i.e. negative current account balance), while on the settlement day it is also used if the bank fails to satisfy the reserve requirement. The deposit facility in both cases is used when the bank already satisfies the entire reserve requirement and still has a surplus on its central bank current account.
and Dow Jr. (1999) and Hamilton (1996), but a different treatment of this issue is offered in Chapters 4 and 5, where I find that trading frictions can explain some of the patterns in commercial banks behaviour that are observed in the Eurosystem.

**Interbank lending risk**

The Poole (1968) model was very successful in explaining certain patterns in the interbank market, and his framework is the basis of the model presented in this thesis. However, the model assumes no credit risk related to interbank lending, i.e. there are no failures to meet debt obligations. Historically speaking, this assumption does not seem particularly strong. After all, there have been only a few cases of banks running into serious trouble, and the affected banks were usually bailed out by the authorities or bought out by competitors.

The classic paper on this topic is Diamond and Dybvig (1983), who were among the first to provide a comprehensive model of bank runs and stressed the importance of the central bank as lender of last resort in preventing bankruptcies. Another seminal contribution is by Rochet and Tirole (1996) who construct a framework for analysing the interbank market and systemic risks. Systemic risk, which they define as "propagation of an agent’s economic distress to other agents linked to that agent through financial transactions" is particularly important for the interbank market, where banks rely on their ability to raise finance from other banks. Rochet and Tirole (1996) point out that central bank deposit insurance might decrease the monitoring incentives of lending banks (which in turn monitor the borrowing banks). Without these incentives, the borrowing banks engage in risky lending, which will eventually lead to more bankruptcies (compare also Cooper and Ross (1998)). Other papers on the optimal central bank regulation that accommodate similar framework include Ringbom et al. (2004), Ayuso and Repullo (2000), and Freixas et al. (2000).

The papers cited in this paragraph employ a different approach to interbank lending compared from that of Poole (1968) and this thesis. In Rochet and Tirole (1996), interbank lending is mainly used to transfer funds from banks with liquidity surpluses (such as small banks serving local customers) to large investment banks. The investing bank must choose the initial investment value in period 0 such that it will not run out of cash if hit by a random liquidity shock in period 1. The interbank trade volume is thus determined by the investment return, which allows one to model issues related to monitoring, moral hazard and central bank deposit insurance.

In contrast, in this thesis (and in the literature based on Poole (1968)) liquidity shocks occur as a result of transactions between banks’ customers, which are not directly affected by the banks’ decisions. I focus only on that part of interbank lending that is aimed at offsetting such liquidity shocks. This type of lending typically has a very short (overnight) horizon. This allows me to determine the overnight lending and corresponding overnight interest rate.

Despite different focus, Rochet and Tirole (1996) raise the very important issue of systemic risk in interbank lending. This turned out to be particularly relevant in 2007, when global financial markets were severely wounded by a liquidity crisis. I analyse these issues and the impact of the liquidity crisis in Chapter 5, where I
introduce several modifications to the model used in Chapters 2-4. These modifications allow me to capture credit risk, the expected liquidity shortage and the increase in trading costs.

1.4 Summary of the chapters

This thesis focuses on the interbank market and its role in monetary policy implementation. The model I use is an extension of Pérez-Quirós and Rodríguez-Mendizábal (2006), who combined the standard Poole (1968) framework with averaging of the reserve requirement, which is currently applied by most central banks.

Each chapter of the thesis focuses on a different aspect of the interbank market. In Chapter 2, I analyse the role of the monetary policy framework. In Chapter 3, I calibrate the model to duplicate certain patterns in the Eurosystem and find that the liquidity shock, which is crucial for the model results, seems to be relatively small. This finding raises certain questions about determination of commercial banks demand, and I answer them in Chapter 4, where I analyse the impacts of trading costs and market frictions. Finally, the liquidity crisis of 2007 resulted in dramatic changes in banks’ behaviour, which I model in Chapter 5.

1.4.1 Interbank market with different implementations of monetary policy

Even though there seems to be a consensus among central banks to use the overnight interbank interest rate as an operational target, there are differences in monetary policy implementation. Some countries (e.g. ECB, Federal Reserve) adopt an averaging reserve requirement while others (Bank of England until 2006) used daily reserves. In addition, central banks supply liquidity at different frequencies, from several times a day (Bank of England) to once a week (ECB). The differences in policy frameworks lead to quite different behaviour of the interbank market and result in certain patterns in interest rate volatility and use of standing facilities.

In Chapter 2, I modify the classic Poole (1968) model to allow for different types of implementation of monetary policy. I focus on the averaging provision and the frequency and magnitude of liquidity supply operations. Using Monte-Carlo simulation, I successfully replicate the pattern observed in the Eurosystem, where the averaging of reserve requirement leads to lower volatility during most of the maintenance period but regular spikes on settlement days. Not having reserve averaging results in much higher interest rate volatility unless the central bank is able to perform very frequent operations. These results are consistent with observations of the interbank market in the UK.

I also devise an algorithm to analyse the role of the forecast errors of the aggregate liquidity shocks. Comparing the market behaviour under neutral (liquidity equal to remaining reserve requirement) and optimised (central bank can perfectly predict the aggregate shocks) liquidity supply allows me to conclude that the averaging provision seem to cope best with the problem of asymmetric information.

Finally, in Chapter 2 I analyse a new type of regime, which has been implemented only by the Bank of Brazil, where banks are divided into groups, each
with a different settlement day. I find that in such a regime the periodic spikes in interest rate volatility vanish, albeit at the cost of higher average volatility during the maintenance period as a whole. In addition the expectations of change in the interest rate target lead to significant disruptions in the market behaviour.

1.4.2 Liquidity shocks in the Eurosystem interbank market

In Chapter 3, I expand the model of Chapter 2 and focus on the relationship between liquidity shock variance and size of the reserve requirement. As mentioned above, this relationship is crucial for the martingale property of the overnight interest rate.

Calibrating the model for the Eurosystem, I find that the standard deviation of the shock is roughly 22% of square root of the average current account holdings, which means very low volatility, especially compared to the values used in previous research.

Using these parameters and a standard interbank model, I was able to reproduce and explain fairly well the dual pattern of EONIA behaviour. In the early stage of the maintenance period, when the rate is typically stable, it is the expectations that drive the rate, while the liquidity effect is low. Toward the end of the maintenance period, the interest rate is mostly determined by market liquidity.

These findings have important policy implications. Central bank operations performed before the end of the maintenance period are not likely to have direct impact on the behaviour of the interest rate. However, they help the central bank influence market expectations of the future liquidity position and level of the interest rate. Late liquidity supply operations, such as fine tuning operations, are key to determining the interest rate on the last day of the maintenance period.

1.4.3 What determines commercial banks demand for reserves in the interbank market

In Chapter 3, I show that volatility of the liquidity shock in the Eurosystem is very low. Thus assuming risk neutrality (as in original Poole (1968)), it is not possible to predict exactly the volume of commercial banks' demand for reserves. These predictions are important for the central bank, which must decide on the intervention value for open market operations. A solution to this problem is presented in Chapter 4.

I start by documenting the pattern in the Eurosystem, where commercial banks deviate from the required reserves balance at the start of the maintenance period and choose to adjust closer to the settlement day. I test this hypothesis statistically for each bank separately, which extends the panel data analysis of Cassola (2008) and allows analysing the heterogeneity in bank's preferences. I find that most banks exhibit statistically significant pattern but also that the large banks (which constitute the majority of the sample) are split fairly evenly between front and backloading.

\[10\] I refer to the periodic spikes in volatility around the settlement days. During the remaining of the maintenance period, the volatility of EONIA remains at a very low level.
I then argue that this behaviour can be attributed to the existence of certain trade related frictions and costs. Examples of these include potential extra expenses tied to large transactions or the asymmetry between costs of borrowing and profits from lending. The trade frictions encourage banks to initially reduce trading volumes, thus allowing the balances to deviate from required reserves. However, as the end of the maintenance period approaches, the banks become concerned about the use of standing facilities and their balances return to the required level.

The simulated behaviour that resembles the Eurosystem holds for a low level of the market frictions. When the frictions become more severe, the banks have stronger incentives to avoid significant trade volumes and adjust for liquidity shocks on a daily basis. However, a very high level of market frictions renders the standing facilities more attractive than the interbank finance trade, which results in a significant reduction of trade volumes, such as observed during the liquidity crisis analysed in Chapter 5.

1.4.4 Liquidity crisis in the interbank market

In the last chapter of the thesis I focus on developments in the financial markets that took place in 2007 and resulted in a major liquidity crisis. The systemic risk of the interbank market was already discussed in Rochet and Tirole (1996), where they pointed out the role of peer-monitoring (between banks) and central bank insurance schemes. In Chapter 5, I take a different approach, modifying the model based on Poole (1968) to allow for credit risk, trade frictions and possibilities of liquidity shortage.

I first document certain patterns that occurred in the overnight markets in the Eurosystem, US and UK and point out that the crisis resulted in liquidity hoarding and an increase in interest rate volatility and spread (compared with the central bank target). Based on a Monte-Carlo simulation I show that these developments might have been caused by an increase in trading cost, credit risk and expected liquidity shortage. In particular, it seems that it was the combination of all three elements of the crisis (rather than a single one) that led to the patterns observed in the market.

I also analyse the role of a liquidity injection, such as those performed by major central banks. I find that additional reserves helped to alleviate the problems of lower trade volumes but had no direct impact on the level of the interest rate. The market behaviour can be, however, substantially affected by the allotment method and in extreme case the intervention can substitute, rather than restore interbank trade.
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Chapter 2

Interbank market with different implementations of monetary policy

2.1 Introduction

Central banks rely on control of the interest rate to achieve their objectives. The most efficient way to do this is to target the interest rate in the overnight interbank market, where the central banks can use a wide array of direct tools. However, disagreements on the optimal setup are reflected in significant differences in the monetary policy frameworks of different countries.

The purpose of this chapter is to contribute to the ongoing discussion on the optimal setup for monetary policy. I construct a model of the interbank market that allows me to compute the market equilibrium and analyse the behaviour of commercial banks under different monetary policy frameworks. The main differences between the countries concern:

- the averaging provision
- determination of liquidity to supply in open market operations
- frequency of liquidity supply operations.

In this chapter I concentrate on monetary policy setups that differ in these respects.

The money market, despite its key role in central bank policy, has only recently started to attract more attention. An excellent and extensive introduction to the field is presented by Bindseil (2004). The seminal contribution is Poole (1968), who was the first to link market liquidity and interest rate level. More recently, Hamilton (1996) presented his own model and expressed formally one of the most important hypotheses related to the market: under an averaging provision, all money market finance should be perfect substitutes on all days of the reserve maintenance period. If that is true, however, the interest rate on each day
must equal the expected future rate. Analysing data on the US money market, Hamilton rejected this so-called martingale hypothesis. The deviation, he claimed, was caused by the existence of various trading costs and trading frictions, which took the form e.g. of credit lines. His paper sparked a new wave of interest in the interbank market, even though Hamilton’s results were later rejected by Thornton (2001).

The model derived in this chapter is based on that of Pérez-Quirós and Rodríguez-Mendizábal (2006), who adapted the early Poole (1968) model for the case of an averaging provision and German data. Similar models were also constructed by Välimäki (2003), Bartolini et al. (2001), Bartolini et al. (2002), Bartolini and Prati (2006) and Clouse and Dow Jr. (2002).

The researchers cited above adopted a similar approach to the modelling of the interbank market. They typically start with an introduction of a dynamic equilibrium model based on a single bank optimisation problem (which was solved by Poole (1968)). Then, they extend the analysis by adding new elements. For instance, Bartolini and Prati (2006) analyse imperfect commitment of the central bank to interest rate targeting; Clouse and Dow Jr. (2002) include the carryover provisions, etc. In this chapter, which should be considered a part of the above literature, I introduce endogenous central bank liquidity supply. This extension allows me to analyse the differences in monetary policy implementations.

The models cited above focus on country-specific features, in an attempt to match the empirical patterns as closely as possible. Papers that employ a framework broad enough to capture the monetary policy features of several countries include Bartolini and Prati (2006) and Borio (1997), which are also the closest to this chapter.

Bartolini and Prati (2006) investigate market volatility under a spectrum of different policies for liquidity supply, using a broadly similar method based on Poole (1968). They, however, focus on the impact of possible restrictions on the supply of liquidity, which may be caused by central bank reluctance to fully offset a liquidity shock. They claim that those restrictions are the source of interest rate volatility. In this chapter I focus on a different issue. I assume the central bank can choose any value of the intervention but also must decide on its frequency. I also discuss the merits of the policy that is actually used most often by central banks, where the intervention volume is determined by the change in the autonomous liquidity factors and use of the standing facilities.

Borio (1997) offers an excellent and very detailed overview of monetary policy frameworks of fourteen different countries. He focuses, however on many aspects of monetary policy that are not covered in this chapter, as, for instance, the choice of central bank operating target. He also has a short discussion on the reserve requirement and concludes, similarly to this chapter, that the averaging provision is likely to result in lower interest rate volatility. His findings are not, however, backed by a theoretical model, which prevents the sort of analysis that is performed in this chapter.

This chapter contributes to the existing research in several ways. First, as mentioned above, I expand the model of Pérez-Quirós and Rodríguez-Mendizábal (2006) and introduce endogenous central bank intervention. This modification allows me to analyse the behaviour of the money market under monetary policy
regimes that can differ in the frequency of open market operations and reserve requirement regime.

Second, this chapter introduces a Monte Carlo simulation that is used in this thesis. Compared with previous research (such as Pérez-Quirós and Rodríguez-Mendizábal (2006)), my simulation design allows me to analyse endogenous liquidity supply and overlapping maintenance period regime (this Chapter), examine the role of the assumptions and extend analysis into 20 day maintenance period (Chapter 3) or modify the cost function (Chapters 4 and 5).

Third, I present a method to evaluate the cost of aggregate liquidity forecast errors. I can accomplish that by simulating two scenarios:

- the central bank cannot predict the liquidity shock and relies on the past data to determine the allotment
- the central bank chooses the intervention value while being able to accurately estimate future market liquidity.

The first scenario can be considered as realistic, while the second as the “best case” scenario. By comparing, for instance, the interest rate volatility in two scenarios I can establish a measure of the information value for different monetary policy implementations. I find that the performance of the averaging provision regime is closest to the optimised scenario, which suggests that it copes best with the problem of asymmetric information.

Fourth, apart from the analysis of the regular reserve maintenance period, I also examine the possibility of banks having different settlement days. In principle, such a regime should remove the typical periodic increase in interest rate volatility at the end of a regular maintenance period. At the same time, however, significant problems might arise when the market expects changes in the level of the target rate. Analysis of such a regime, which is currently used only by the Bank of Brazil, has not been performed before. An exception is Cox and Leach (1964), who however do not use the rigorous approach provided by the framework based on Poole (1968).

This chapter is structured in the following way. Section 2.2 introduces the concept of monetary policy implementation and compares the frameworks used by the Federal Reserve, the ECB and the Bank of England. Section 2.3 presents the model used to compute the market equilibrium. The results of Monte-Carlo simulations and policy implications are presented in Section 2.4. Section 2.5 concludes.

2.2 Background

2.2.1 Implementation of monetary policy in the Eurosystem, the United Kingdom and the United States - instruments and procedures

In this section, I introduce the concept of monetary policy implementation. I also point out the differences between the policies adopted by several developed countries and the corresponding patterns of the interbank market behaviour.
Most developed countries adopted similar goals for monetary policy. The central bank's main task is to control the rate of inflation and, in some cases, economic growth. The process that allows the central bank to reach its goals is referred to as monetary policy implementation. Bindseil (2004) defines it as the combination of three elements:

1. Selection of an operational target of the monetary policy
2. Operational framework that allows the central bank to control the operational target
3. Daily use of the instruments to achieve the operational targets.

The direct operational target that is most popular among central banks is the interest rate in the overnight interbank market. In fact, in the sample of fourteen banks analysed by Borio (1997), eleven choose the overnight interest rate while the remaining three targeted longer maturities. The ECB targets EONIA (Euro Overnight Index Average), the Federal Reserve targets federal funds rate and the Bank of England targets SONIA (Sterling Overnight Index Average).

Apart from the current target level of the interest rate, the central banks can also communicate their stance to the money markets. The stance signals the direction of future changes in the target rate, which affects the level of long-term rates. Thus the central banks can effectively control the whole term structure of the interest rates.

Once the operational target is selected, central banks worry not only about the level but also about the volatility of the instrument. This might initially seem counter-intuitive. Assuming the policy is effective and the average interest rate remains close to the target, the agents should adapt their expectations and temporary and random deviations from the target should not influence their decisions. However, the overnight rate volatility can induce the volatility in the longer maturities, increase the term premium and affect the interpretation of the monetary policy stance (see eg. Cassola and Morana (2003)). Thus, the monetary policy implementation requires low volatility of the interest rate.

The main instruments that facilitate monetary policy implementation include: reserve requirement, standing facilities and open market operations. However, a comparison of the frameworks adopted in different countries reveals significant differences in how these instruments are used. In this chapter, I analyse monetary policy implementations in the Eurosystem, the Bank of England and the Federal Reserve, which are briefly presented below.

---

1By the year 2001 these three countries also shifted to overnight maturity as the operational target.
2At first, such a choice of the operational target might seem counter-intuitive; after all longer maturities are probably more relevant for economic agents' decisions and hence should be a better tool for the monetary policy transmission mechanism. However, as pointed out by Bindseil (2004, p.78), targeting the overnight rate allows the central bank to avoid certain anomalies in the yield curve and time series properties of the longer rates, which would occur if the long term target level of the interest rate were changed.
The European Central Bank

The ECB employs all basic monetary policy instruments: open market operations, standing facilities and minimum reserves. The Eurosystem monetary operations are presented in Table 2.1 (ECB (2006)).

The ECB’s open market operations include main refinancing operations, longer-term refinancing operations, fine-tuning operations and structural operations. The main liquidity supply operations are reverse transactions with weekly frequency and maturity. Executed on a tender basis, they have historically constituted the bulk of the financial sector refinancing. The long-term finance operations aim is to satisfy additional part of the structural deficit of funds that occurs as a result of the reserve requirement. Long-term operations do not serve any signalling role and the ECB normally acts as rate taker. The fine tuning operations are performed irregularly to manage current liquidity and interest rate. They are also used on the last day of the maintenance period, in order to adjust for changes in the autonomous liquidity factors that occurred during the week since last regular operations.

In the reverse transactions, the ECB buys (or sells) eligible assets under repurchase agreements or collateralized loans. The eligible assets must fulfil specific criteria, outlined in the Eurosystem documentation, however, a fairly broad scope of different asset types is accepted. Other transactions include outright transactions (direct purchase or sale of assets on the market), the ECB’s debt certificates, foreign exchange swaps (such as buying spot Euro and selling it back in forward transaction) and collection of fixed-term deposits.

The standing facilities are offered at pre-determined rates on a daily basis to all eligible counterparties. There is no limit to the amount of liquidity that can be obtained from lending facility, provided that sufficient collateral is presented. Similarly, there is no limit to the amount that can be deposited. The maturity is overnight. The facilities are not intended as a regular source of funding, which is reflected in the level of the deposit and lending rates. Typically the cost of borrowing from standing facilities is significantly above the market, while the deposit facility yields much less profit. The current spread between standing facilities and the target rate is 100 basis points.

The majority of the operations performed by the ECB aim to supply, rather than absorb the market liquidity. This is due to the structural deficit of funds, mainly created by the reserve requirement and autonomous liquidity factors (such as currency in circulation). The requirement is imposed on credit institutions in the euro area and forces the banks to maintain certain level of deposits in the central bank. The ECB has also adopted, so called, reserve averaging provision which means that the banks need to maintain an average value for a certain time period, referred to as the maintenance period (currently roughly equal to one month). This offers the banks an opportunity to compensate for temporary shortage (or surplus) of liquidity, which contributes to the stability of the money markets. The

---

3The autonomous liquidity factors are “all items in the balance sheet of the central bank that do not reflect monetary policy operations or the reserve holdings of banks with the central bank” (Bindseil (2004, p.46)). They consist mainly of banknotes, government deposits and net foreign assets.
<table>
<thead>
<tr>
<th>Monetary policy operations</th>
<th>Types of transaction</th>
<th>Maturity</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Provision of Liquidity</td>
<td>Absorption of Liquidity</td>
<td></td>
</tr>
<tr>
<td><strong>Open market operations</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main refinancing operations</td>
<td>Reverse transactions</td>
<td>-</td>
<td>One week</td>
</tr>
<tr>
<td>Longer-term refinancing operations</td>
<td>Reverse transactions</td>
<td>-</td>
<td>Three months</td>
</tr>
<tr>
<td>Fine tuning operations</td>
<td>Reverse transactions</td>
<td>Reverse transactions</td>
<td>Non-standardised</td>
</tr>
<tr>
<td></td>
<td>Forex swaps</td>
<td>Forex swaps</td>
<td>Collection of</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>deposits</td>
</tr>
<tr>
<td></td>
<td>Outright purchase</td>
<td>Outright sales</td>
<td>-</td>
</tr>
<tr>
<td><strong>Standing facilities</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal lending facility</td>
<td>Reverse transactions</td>
<td></td>
<td>Overnight</td>
</tr>
<tr>
<td>Deposit facility</td>
<td></td>
<td>Deposits</td>
<td>Overnight</td>
</tr>
</tbody>
</table>

Table 2.1: Eurosystem monetary policy operations. Source: ECB(2006)
required reserves are remunerated at the rate of the Eurosystem main refinancing operations.

The Eurosystem open market operations are executed in the form of liquidity tenders. The ECB has the option to either run fixed rate or variable rate liquidity tender. In the former, the tender rate is announced in advance and - if the total amount of the bids exceeds the planned intervention value - the allotted liquidity is reduced proportionally. This, however, means that the banks that are interested in securing the ECB finance have strong incentives to overbid. This mechanism is described in Välimäki (2003).

In the variable rate tender, the participants must specify both the bid size and the rate that they are willing to pay. The liquidity is allotted to the bidders that offer the highest rate (in the case of the liquidity providing operations). The minimum bid rate is determined during the Governing Council meeting and constitutes the ECB target interest rate. The variable rate tenders greatly neutralises the incentives for overbidding, creating, however, the problem of the spread between the minimum bid rate (the ECB target) and the marginal bid rate (all the bids above this rate are accepted). Välimäki (2006) attributes this spread to money market inefficiencies and banks’ risk aversion. Intuitively, the banks that want to secure the liquidity have strong incentives to always bid slightly above the minimum level, to increase chances of successful allotment at relatively little additional cost.

The Bank of England (before May 2006)

In May 2006 the Bank of England engaged into difficult and complicated reform of its monetary policy implementation framework (see for example BoE (2005)). As a result, its currently existing (as of 2008) instruments and procedures are very similar to the ones employed by the ECB. However, the earlier framework could serve as an example of a different approach to monetary policy implementation and offer an interesting basis for comparison. Thus, in the remaining of this section and this chapter, whenever I refer to the Bank of England monetary policy implementation, I refer to the framework used before May 2006 reforms. For more details on the Bank of England implementation of the monetary policy refer to BoE (2002), which is also the main source of information for this section.

The Bank of England’s control of the money market derived from its monopoly over the supply of central bank money (the Bank of England notes and the deposits at the Bank of England). The BoE satisfied the demand for liquidity by performing open market operations. The operations involved mostly outright purchase and repurchase transactions with two weeks maturity at the interest (repo) rate determined by the Monetary Policy Committee.

Unlike the ECB, the Bank of England had no reserve requirement until 2006. Most commercial banks’ accounts were held with settlement banks, which in turn had their own accounts with the Bank of England. The settlement banks were still required to maintain non-negative balance every day, effectively constituting one-day maintenance period with zero reserve requirement. In the absence of remunerated reserve requirement, the settlement banks accounts with the BoE were fairly low. For instance in February 2001, the current accounts were roughly 18% of the banknotes issue value, while in the Eurosystem that ratio was around
35%. Thus, the demand for liquidity was mainly determined by the value of banknotes in circulation.

The Bank of England used to run regular open market operations twice a day, at 9.45am and 2.30pm. These operations were conducted at the official repo rate set by the Monetary Policy Committee.

The counterparties could choose whether to obtain the liquidity in outright bill purchase or repurchase transaction. The BoE only accepted highest quality securities for outright purchase, while the requirements for the eligible collateral for the repurchase transaction were slightly more relaxed.

The daily allotment value depended on the value of maturing operations, changes in the banknotes level and other possible corrections. The allotment was typically split between two regular daily operations, allowing the BoE to revise the early estimate of liquidity demand and respond to changing market conditions. In the liquidity providing operations, fixed rate tenders were used. When the total value of bids exceeded planned allotment value, the liquidity was distributed pro rata. In liquidity absorbing operations, variable rate tenders were used.

If the market remained unbalanced after the 2.30pm operation, the BoE could enable an overnight lending and deposit facility at 3.30pm. The facilities were offered at 100 basis points above (for lending) or below (for deposit) the official target rate. If there was still a shortage of liquidity in the market, an additional overnight lending opportunity was enabled at 4.20pm at the rate 150 basis points above the official target.

**The Federal Reserve**

The implementation of the monetary policy in the USA resembles the system adopted in the Eurosystem. The Federal Reserve control of the money market is based on the reserve requirement (see Federal Reserve System (2008)), the open market operations (see Edwards (1997)) and the standing facilities. In addition, as a result of the liquidity crisis in 2007, the Federal Reserve introduced the following new tools: Term Auction Facility, Primary Dealer Credit Facility and Term Securities Lending Facility. Since the purpose of the chapter is the comparison between commonly used central bank instruments, those new tools are not analysed here.

The open market operations involve purchase and sales of U.S. Treasury securities and the Federal Reserve has established different procedures for temporary and permanent operations. Temporary operations that have an overnight horizon, are performed on a daily basis and serve as a fine tuning operations, to influence current market conditions. Temporary operations with longer maturities (up to 65 days) are used for seasonal adjustments and are executed on an irregular basis. Both types of temporary operations involve repurchase or reverse repurchase transactions. The permanent open market operations are executed irregularly and are implemented through outright purchase or sales of securities.

The Federal Reserve executes the open market operations between 11.30am and 11.45am. The transactions are completed with selected security dealers (primary dealers), in the form of variable rate tenders. The offers are ranked in descending order of rate and the highest bids are satisfied until the total value of offers equals the amount of reserves the desk wants to inject.
Similarly to other central banks, the Federal Reserve benefits from the structural deficit of funds, which facilitates the efficient allotment of desired liquidity. The deficit is created by the obligatory reserve requirement and autonomous liquidity factors (mostly banknotes in circulation). The reserve requirement covers two week maintenance period, during which the commercial banks are required to maintain an average level of the current account balance and/or vault cash. Unlike other central banks, the Federal Reserve allows for carryover of reserves. This means that a part of the current account balance excess over required reserves in one period, can contribute to the reserve requirement in the next maintenance period (the value is quite small though and cannot exceed 4% of reserve requirement).

In order to provide the banks with the emergency lending, The Federal Reserve uses the standing facilities. The lending is provided for overnight maturity in three types of credit facility, depending on the institution type. The primary credit is granted to the most reliable banks, the secondary credit is granted to the less reliable parties and the seasonal credit is addressed for the smallest institutions. The first two types of loans are granted at a fixed discount rate that exceeds the Federal Reserve target rate, while the seasonal credit rate depends on the market situation. Borrowing from the discount window requires the collateral, similar to open market operations.

**Determination of the allotment value**

Despite the differences in monetary policy instruments, the central banks analysed in this chapter have adopted a very similar approach to the determination of the market reserves.

In general, the demand for liquidity is determined by the size of the reserve requirement and changes in autonomous liquidity factors. The reserve requirement is a constant value, fixed for the duration of the maintenance period. On the other hand, the autonomous liquidity factors tend to fluctuate on a daily basis. Thus the short term liquidity supply is mostly determined by the changes in the autonomous liquidity factors. Below, I present the formula that the ECB is using to determine so-called benchmark allotment (ECB (2002), Moschitz (2004)):

\[
M_{\text{bench}} = \frac{1}{H - X} \left[ \begin{array}{c}
\text{Accumulated liq. imbalance} \\
\text{Future liq. needs} \\
\text{Liq. already provided}
\end{array} \right]
\]

where \(M_{\text{bench}}\) is the benchmark allotment, \(H\) is the number of days covered by the forecast of autonomous factors (usually 9), \(X\) is the number of days included in \(H\) for which open market operations have not been settled (usually takes 2 days), \(D\) the number of days until the start of maintenance period, \(RR\) the estimated daily average reserve requirement, \(ER\) the excess reserves, \(CA\) the average current account, \(AF\) the estimated average amount of autonomous factors, \(L\) the expected daily average amount of liquidity from long term operations, \(M^{\text{out}}\) the other outstanding refinancing operations and \(M^{\text{mat}}\) the maturing refinancing operations.
Table 2.2: Operational policy frameworks of the Eurosystem, the USA and the UK, 1 January 2006.

<table>
<thead>
<tr>
<th></th>
<th>EU</th>
<th>US</th>
<th>UK</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserve Requirement</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Maintenance Period (days)</td>
<td>28</td>
<td>14</td>
<td>1</td>
</tr>
<tr>
<td>Market operations frequency</td>
<td>1week</td>
<td>1day</td>
<td>&lt;1day</td>
</tr>
</tbody>
</table>

The value of the benchmark allotment is available to the public. Comparing it with the actual intervention value can be therefore used to interpret some of the ECB decisions.

The comparison between the Eurosystem, the Bank of England and the Federal Reserve

Comparison of the instruments used by the Eurosystem, the Bank of England and the Federal Reserve leads to some interesting insights and is presented in Table 2.2 as of 1 January 2006. The main difference between selected countries is the reserve requirement and open market operations regime. While the US and the Euro area central banks used the reserve requirement to smooth the temporary liquidity shortages, the Bank of England opted to run more frequent open market operations to reach the same goal. The ECB also adopted a maintenance period twice as long as in the US. This means that the reserve requirement buffer in the Eurosystem is considerably larger compared with the other central banks.

The last issue to be discussed in this subsection is whether the instrument choice has any impact on the behaviour of the operational target, which in all cases is the interbank market overnight interest rate. Figures 2.1, 2.2 and 2.3 present the spreads between EONIA, SONIA and federal funds rate and their respective central banks’ targets.

Inspection of the figures reveals that the highest volatility of the interest rate is experienced in the UK. The volatility in the Eurosystem and the US is significantly lower except on settlement days (last days of maintenance periods). Those observations have been also extensively documented in the literature by Hamilton (1996), Bartolini et al. (2002), Bartolini and Prati (2006) for the US markets and Pérez-Quirós and Rodríguez-Mendizábal (2006), Moschitz (2004), Würtz (2003) for the Eurosystem. In addition, all estimations mentioned indicate that there is a propagation of volatility for the days directly preceding the settlement day.

These observations indicate that the framework for implementation of monetary policy is important for the behaviour of the interest rate and the volatility of the interest rate in particular. In the following part of this chapter, I present a theoretical model that can be used to explain this relationship.

---

4The date of this information is important since the Bank of England engaged in a series of reforms resulting in bringing its framework very close to that of the Eurosystem by May 2006.

5To keep the figure clear, I removed the extreme observations for the Federal Funds Rate from the following days: 29.12.2000, 18-19.09.2001.
Figure 2.1: Euro Overnight Index Average spread
Source: The European Central Bank
Figure 2.2: Sterling Overnight Index Average spread
Source: British Bankers Association
Figure 2.3: Federal funds rate spread. Source: Federal Reserve Bank of New York
2.2.2 Interbank Market

Every day commercial banks engage in multiple transactions that alter the balances of their current accounts at the central bank. Part of these transactions derives from activities of bank customers, which might involve orders of money transfer or payments received from debtors. The bank has little direct impact on such transactions, although there certainly is a link between bank characteristics and expected liquidity flows. For instance, ECB (2007) finds that small banks are on average suppliers of liquidity, reflecting the fact that they mostly deal with households rather than large companies. However, the relationship between a bank’s profile and its behaviour in the interbank market is not analysed here.

Regardless of whether the transactions result in liquidity surplus or shortage, each bank needs to engage in interbank trade to bring its current account balance close to the reserve requirement. No overdrafts are allowed; thus if the bank’s balance turns negative, the bank needs to use the central bank lending facility, which is offered at a penalty rate (compared with the market). Correspondingly, a current account balance that exceeds the remaining part of the reserve requirement can be deposited at a rate significantly below the market rate.

In the case of a regime with an averaging provision, which is analysed in this thesis, the situation is slightly more complicated. The banks need to maintain an average value during the maintenance period, rather than a fixed value each day. This gives the banks more flexibility. In case their balances fall below the average they are not immediately forced to borrow from the central bank and can compensate for the shortfall during the remaining days of the maintenance period.

Even with the averaging provision, the banks have strong incentives to trade, in order to avoid the use of standing facilities. Thus, the role of the trade, and the interbank market more generally, is to redistribute liquidity to the banks that need it.

If the information were perfect (or freely available), the bank would know in advance all its transactions, which would allow it to plan how much to borrow (or lend). In reality, this is only partially true. Daily liquidity flows follow certain patterns that can be relatively well modelled. For instance, mortgage banks know the due dates for credit tranches and can anticipate liquidity surpluses on those days. This allows them to arrange long term contracts in advance. However, some part of the transactions cannot be anticipated, which forces banks to adjust their borrowing. In the remaining part of this chapter I refer to such a random transaction as an early liquidity shock and denote it $\varphi_t$.

Additionally, some transactions occur late in the day, when the interbank trade is no longer possible or might not be processed at all (for instance due to an error). Such transactions cannot be offset by interbank lending and hence will cause the end-of-day current account balance to deviate from the expected value. In the remaining part of this chapter, I refer to such a random transaction as a late liquidity shock and denote it $\varepsilon_t$.

Both early and late shocks are relevant for commercial banks’ borrowing decisions. The early shocks force the banks to engage in extra trade and might result from an aggregate change in liquidity if e.g. the shock derived from an unexpected transaction with the government. The late shocks, in the extreme case, can result
in a bank being forced to use costly standing facilities.

I make the following assumptions about shocks. I assume they are identically and independently distributed, following the normal distribution with mean zero and standard deviation $\sigma$. The late shock is normalised to have no impact on aggregate market liquidity, so that for $N$ banks in the market, $\sum_{i=1}^{N} \varepsilon_{ti} = 0$. To justify the independence assumption, note that the sources of the shocks are quite different: while the early shock results from unexpected trade flows during the day (such as large customer ordering unexpected payment request), the late shock is caused by processing errors or transactions that occur after the interbank trade is over. Thus the shocks in my model are not likely to be correlated.

2.3 The Model

A dynamic equilibrium model based on single bank optimisation problem has recently become a very popular approach in the interbank market literature. Similar types of models were developed in the series of papers written eg. by Bartolini et al. (2001), Bartolini et al. (2002), Bartolini and Prati (2006), Clouse and Dow Jr. (2002), Välimäki (2003) and Pérez-Quirós and Rodríguez-Mendizábal (2006).

All those papers extend early Poole (1968) into the averaging provision framework by introducing dynamic programming structure. Poole (1968) was a single period model and expanding it into multiple period finite horizon resulted in very similar Bellman’s formulation. Therefore, the basic structure of these cited papers might sometimes resemble my thesis.

The researchers cited above adopted the following approach. After the introduction of the dynamic framework (in the form of Bellman equation), new elements are added. For instance, Bartolini and Prati (2006) extend the model by adding imperfect commitment of the central bank to interest rate targeting; Clouse and Dow Jr. (2002) include the carryover provisions, Gaspar et al. (2008) add market segmentation etc.

A model with exogenous liquidity was solved by Pérez-Quirós and Rodríguez-Mendizábal (2006) and most of the algebra in sections 2.3.1 and 2.3.2 is presented in the mathematical appendix to their paper. Compared with them, the major innovation of this chapter is the introduction of endogenous liquidity in sections 2.3.4 and 2.3.7. This change allows analysing the impact of different monetary policy implementations on the behaviour of the interbank market.

The model is based on a risk neutral bank that minimises the cost of funding by adjusting its overnight borrowing value while complying with the averaging provision reserve requirement and no-overdraft condition. The timing of activities is the following:

1. A commercial bank starts day $t$ with the starting balance $m_t$ in its central bank account and remaining reserve requirement $d_t$

2. Central bank open market operations $\theta_t$ are executed and an early liquidity shock $\varphi_t$ occurs

3. The interbank money market opens and the bank borrows $b_t$ at the market clearing interest rate (negative $b_t$ reflects lending)
4. A late liquidity shock $\varepsilon_t$ occurs after no more trading is possible.

5. All the operations described above are added up to calculate the final end-of-day current account balance, which is used to determine the use of standing facilities and satisfy the reserve requirement.

$$m_t, d_t, \theta_t, \varphi_t, b_t, \varepsilon_t$$

After the final liquidity shock, the bank’s balance at the current account is equal to:

$$m_t + \theta_t + \varphi_t + b_t + \varepsilon_t,$$  \hspace{1cm} (2.2)

where $m_t$ denotes the starting current account balance, $\theta_t$ is the extra liquidity from open market operations, $\varphi_t$ is the early liquidity shock, $b_t$ denotes interbank borrowing and $\varepsilon_t$ is the late liquidity shock. Depending on the value of eq.(2.2), the bank then uses the standing facilities. The market borrowing and standing facilities have an overnight maturity, but the liquidity shocks and central bank borrowing are assumed permanent liquidity injections ($\theta$ can be also interpreted as the difference between new and maturing operations). Thus, the starting balance on the next day is equal to:

$$m_{t+1} = m_t + \theta_t + \varphi_t + \varepsilon_t.$$  \hspace{1cm} (2.3)

The variable $d_t$ denotes the remaining part of the reserve requirement, referred in this thesis as the deficiency. This term perhaps requires further explanation. In principle, the averaging provision, which I assume in this model, means that the commercial banks must satisfy an average value of its final current account balance over the maintenance period. Let this average value be $r$. Then the total accumulated value of the current account balance that the bank needs to hold during the $T$-day long maintenance period is $R = r \cdot T$.

On day 1, the starting deficiency is equal to $R$. The deficiency at the end of day $t$ depends on the starting value $d_t$ and the ending value of the current account balance, given by (2.2):

$$d_t = \begin{cases} 
    d_t & \text{if } m_t + \theta_t + \varphi_t + b_t + \varepsilon_t < 0 \\
    d_t - m_{t+1} - b_t & \text{if } 0 < m_t + \theta_t + \varphi_t + b_t + \varepsilon_t < d_t \\
    0 & \text{if } m_t + \theta_t + \varphi_t + b_t + \varepsilon_t > d_t.
\end{cases}$$  \hspace{1cm} (2.4)

The overnight standing facilities are used after the final value of the current account is determined. If the end-of-day balance exceeds the remaining deficiency, the entire reserve requirement is satisfied and any remaining surplus on the current account will be deposited in the central bank deposit facility. Correspondingly, should the current account balance drop below zero, the bank will be forced to use the lending facility.

The commercial bank can obtain funding from two sources: borrowing from other banks or from the central bank. The cost of market finance is the equilibrium rate $i_t$. In this chapter I assume that the bank is risk neutral and that there is no transaction cost related to interbank trading. The cost of reserves obtained from the central bank includes the cost of open market operations $\theta_t$ and the expected cost of using the standing facilities. In this thesis, I assume that the
liquidity is allotted proportionally among all market participants, at the rate equal to the central bank target. I do not analyse bidding behaviour, which is a common assumption in related literature (eg. Bartolini and Prati (2006)). Since banks have no effective control of open market operations cost, there is no point in including it in cost minimisation problem.

This assumption has significant implications for the analysis. In particular, it restricts the analysis of the impact of the intervention rate (rate at which the liquidity is allotted in central bank tenders) and potential differences between target rate and intervention rate. Such differences occur as a result of variable rate tenders, where banks specify both the bid volume and price. If the target rate is set as a minimum bid rate (as is the case in the Eurosystem), bank incentives will cause the intervention rate to exceed central bank target (see Välimäki (2006)).

Modelling variable rate tenders requires heterogeneous banks that have a past bidding history, which allows them to predict allotment rate and average bid rate. Such an analysis (done in Välimäki (2006)) is not possible within a single maintenance period model with homogeneous banks.

The spread between intervention and target rate can affect the market behaviour. In order to avoid arbitrage opportunities, the expected market rate must equal the expected intervention rate. Otherwise the banks would either suspend or bid excessively in central bank auctions, until no more (expected) profits can be made. In this case, higher intervention rate will cause the expected market rate to deviate from the central bank target. I postpone the analysis of the role of expectations until the following chapter.

Apart from deviations in the intervention rate, the central bank can also face problems with allotting the desired liquidity volume. In particular, if the commercial banks have incentives to postpone the satisfaction of reserve requirement, they might bid for less liquidity in the early stages of the maintenance period. This problem is typically addressed by maintaining a structural deficit of liquidity, which requires the commercial banks to roll over their borrowing. As a result, the deficit creates strong incentives to obtain the central bank finance reducing the problems with potential underbidding. Nonetheless, the central banks did run into problems with allotting sufficient liquidity in the past, which significantly affected the market behaviour. As I do not analyse bidding incentives, I assume that the central bank can always determine the market liquidity.

I make two seemingly contradictory assumptions: the redistribution effect is present in the aggregate liquidity shock but is missing in the central bank allotment. As the banks are identical, risk neutral and there is no trading frictions the distribution of the liquidity before the morning trade does not affect the banks behaviour.

The expected cost of using standing facilities depends on the relative probability that either deposit or lending facility will be used. The bank needs to use the lending facility whenever the late liquidity shock exceeds the current account balance, i.e. when

\[ -(m_t + \theta_t + b_t + \varphi_t) < \varepsilon_t. \]  

(2.5)

If the liquidity shock distribution function is \( F(\bullet) \), the probability of such a shock realisation is \( F(-m_t - \theta_t - b_t - \varphi_t) \) and the amount that must be borrowed to bring the current account to zero is \( (-m_t - \theta_t - b_t - \varphi_t - \varepsilon_t) \). Hence, the expected
The cost of using the lending facility is

\[ i^l (-m_t - \theta_t - b_t - \varphi_t - \varepsilon_t) F(-m_t - \theta_t - b_t - \varphi_t), \]  

(2.6)

where \( i^l \) is the lending facility rate. Whenever the liquidity shock realisation is so large that the current account balance ends the day above the remaining deficiency, the bank must use the deposit facility. Proceeding as above, I obtain the expected cost of using the deposit facility:

\[ i^d (m_t + \theta_t + b_t + \varphi_t + \varepsilon_t - d_t) (1 - F(-m_t - \theta_t - b_t - \varphi_t + d_t)). \]  

(2.7)

Rearranging and combining eq.(2.6) and (2.7) yields the expected cost of using the standing facilities:

\[
E(\varepsilon_t) = i^l \left[ \int_{-\infty}^{-m_t - \theta_t - b_t - \varphi_t} (-m_t - \theta_t - b_t - \varphi_t - \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \right]
- i^d \left[ \int_{-m_t - \theta_t - b_t - \varphi_t + d_t}^{\infty} (m_t + \theta_t + b_t + \varphi_t - d_t + \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \right]. 
\]  

(2.8)

A similar expression can be obtained in an analogous manner for the expected cost on the settlement day.

Since the reserve requirement can be satisfied on different days of the maintenance period, the bank’s problem of minimising its finance cost, \( V_t \), has a recursive structure, captured by the Bellman equation

\[ V_t = \min_{b_t} \{ i_t b_t + E(\varepsilon_t) + E(V_{t+1}) \}, \]  

(2.9)

where \( V \) is the value function evaluated at time \( t \). The problem, analysed for instance in Pérez-Quirós and Rodríguez-Mendizábal (2006) before, has different structures for settlement days versus days before the end of the maintenance period. Hence I will present them separately.

### 2.3.1 Last day of maintenance period

In this thesis I consider a single maintenance period and assume there is no connection between maintenance periods. Thus, the cost minimisation problem on settlement day is a single period problem, similar to the case with no averaging provision.

The bank arrives at day \( T \) with a certain balance on current account, \( m_T \), and remaining deficiency \( d_T \). If the current account at the end of the day exceeds the deficiency, the bank will deposit the surplus. If the balance falls short of the reserve requirement, the bank will need to borrow from the central bank. Thus, unless the end-of-day balance is zero, the bank will be forced to use one of the standing facilities.\(^6\)

---

\(^6\)In reality, the existence of certain operational costs related to use of standing facilities might discourage some banks from using the deposit facility if the value to be deposited is very small. This leads to “excess reserves”, which are carefully analysed by central banks due to the fact that they affect the aggregate market liquidity supply.
The bank’s problem thus reduces to determine its borrowing so as to minimise the following expression:

\[ V_T = \min_{b_T} \{ i_T b_T + E(c_T) \} , \] (2.10)

with the expected cost of using standing facilities given by the following expression:

\[
E(c_T) = i^l \left[ \int_{-\infty}^{-m_T-b_T-\varphi_T+d_T} (-m_T-\theta_T-b_T-\varphi_T-\varepsilon_T+d_T) f(\varepsilon_T) d\varepsilon_T \right] \\
- i^d \left[ \int_{-m_T-\theta_T-b_T-\varphi_T+d_T}^{\infty} (m_T+\theta_T+b_T+\varphi_T-d_T+\varepsilon_T) f(\varepsilon_T) d\varepsilon_T \right] . \] (2.11)

Equation (2.11) is very similar to eq. (2.8), the only difference being that now the bank needs to use the lending facility also when it fails to satisfy the reserve requirement, rather than only with an overdraft.

The model in this form was solved in a number of studies starting from Poole (1968), who shows (see appendix 2.6.1) that the first order condition takes the form

\[ i_T = i^l F(-m_T+b_T-\theta_T-\varphi_T+d_T) + i^d (1 - F(-m_T+b_T-\theta_T-\varphi_T+d_T)) . \] (2.12)

This result is often used in the interbank literature, for instance Pérez-Quirós and Rodríguez-Mendizábal (2006), Välimäki (2003) or Bartolini et al. (2001). The interpretation of this condition is the following. At the optimal borrowing, the bank will balance the cost of financing from the market \( i_T \) with the expected cost of using the standing facilities, which is determined by central bank rates \( i^l \) and \( i^d \) and the probability of running short of reserves, \( F(-m_T+b_T-\theta_T-\varphi_T+d_T) \).

### 2.3.2 Days before the end of the maintenance period

For the days before the end of the maintenance period, the problem takes a slightly different structure:

\[ V_t = \min_{b_t} \{ i_t b_t + E_t(c_t) + E_t(V_{t+1}) \} . \] (2.13)

Thus the bank needs to minimise the expected cost of the sum of current borrowing from the market, \( i_t b_t \), using the standing facilities at the end of the day, \( E_t(c_t) \) (given by equation 2.8), and cost of future funding, \( E_t(V_{t+1}) \).

Pérez-Quirós and Rodríguez-Mendizábal (2006) show that the first order condition for this problem is\(^7\)

\[
i_t = i^l \underbrace{F(-m_t-\varphi_t-\theta_t-b_t)}_{1.} + i^d \underbrace{[1 - F(d_t-m_t-\varphi_t-\theta_t-b_t)]}_{2.} \\
- \int_{-m_t-\varphi_t-\theta_t-b_t}^{d_t-m_t-\varphi_t-\theta_t-b_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t . \] (2.14)

\(^7\)Also shown in Appendix 2.6.2.
With optimal borrowing, the cost of obtaining an extra unit of finance from the market \((i_t)\) is equal to the sum of the probability-weighted average of the cost of financing from the lending facility (term 1), the proceeds from depositing the excess in the deposit facility (term 2) and the cost of financing on the following day. The last term, referred in the literature as a dynamic cost factor (Välimäki (2003)), depends on the derivative of the value function, given by \(^8\)

\[
\frac{\partial V_t}{\partial d_t} = -i^d [1 - F(d_t - m_t - \varphi_t - \theta_t - b_t)] + \int_{-\infty}^{d_t-m_t-\varphi_t-\theta_t-b_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f^t(\varepsilon_t) d\varepsilon_t.
\]  

(2.15)

The dynamic cost factor can be interpreted as the marginal cost of an extra unit of deficiency. The value of the cost depends on whether the bank will satisfy the entire remaining reserve requirement at the end-of-day (term 1 in eq. (2.15) or carry it over to the next day (term 2). Ultimately, on day \(T\), the value of the deficiency is equal to the expected market rate.

### 2.3.3 Market equilibrium on settlement day

I assume that market equilibrium is reached when the market clears at interest rate \(i_t\), i.e. when the sum of commercial bank’s borrowing and lending is zero: \(\sum b_{it} = 0\). It is possible to analytically calculate the market clearing interest rate for the last day by aggregating over all banks:

\[
i_T = i^d F(-\bar{m}_T - \bar{\theta}_T - \bar{\varphi}_T + \bar{d}_T) + i^d (1 - F(-\bar{m}_T - \bar{\theta}_T - \bar{\varphi}_T + \bar{d}_T),
\]  

(2.16)

where \(\bar{m}_T, \bar{\theta}_T, \bar{\varphi}_T, \bar{d}_T\) denote the aggregate values of \(m_T, \theta_T, \varphi_T\) and \(d_T\),\(^9\) and \(F(\bullet)\) is the late liquidity shock distribution function. Thus, on settlement day \(T\), the interest rate is directly determined by the aggregate market liquidity and the reserve requirement.

Suppose that the central bank aims for an interest rate in the middle of the corridor, \(i_T = \frac{i^d + i^l}{2}\). What is the optimal aggregate liquidity supply?

Rearranging condition (2.16) yields

\[
i_T = i^d + (i^l - i^d) F(-\bar{m}_T - \bar{\theta}_T - \bar{\varphi}_T + \bar{d}_T) \quad (2.17)
\]

or

\[
F(-\bar{m}_T - \bar{\theta}_T - \bar{\varphi}_T + \bar{d}_T) = \frac{i_T - i^d}{i^l - i^d}. \quad (2.18)
\]

For the interest rate to be in the middle of the corridor,

\[
\frac{i_T - i^d}{i^l - i^d} = \frac{i^l + i^d}{2} = 0.5
\]

\(^8\)Proof in Appendix 2.6.3.

\(^9\)For \(N\) market participants, \(m_T = \sum_i^N m_{iT}, \theta_T = \sum_i^N \theta_{iT}, \varphi_T = \sum_i^N \varphi_{iT}, d_T = \sum_i^N d_{iT}\).
thus reducing (2.18) to

\[ F(-\bar{m}_T - \bar{\theta}_T - \varphi_T + \bar{d}_T) = 0.5. \]  

(2.19)

Assuming that the shock is symmetrically distributed with zero mean yields

\[ -\bar{m}_T - \bar{\theta}_T - \varphi_T + \bar{d}_T = 0, \]  

(2.20)

which enables the central bank to compute the aggregate liquidity supply:

\[ \bar{\theta}_T = \bar{d}_T - \bar{m}_T. \]  

(2.21)

A central bank that aims to have the interest rate to end in the middle of the corridor must ensure that market liquidity is exactly equal to the remaining reserve requirement.

2.3.4 Market equilibrium on preceding days

For the days preceding the end of the maintenance period an analytical solution is not possible because the dynamic cost factor (term 3 in equation (2.14) takes different values for individual banks. Thus, to solve for the market equilibrium one needs to use numerical methods, which are presented in the following section.

The central bank, however, must still determine the value of the optimal liquidity. Consider first the policy where the central bank supplies \( \bar{\theta}_t^1 \), which is intended to supply the liquidity needed to satisfy the remaining reserve requirement:

\[ \bar{\theta}_t^1 = \frac{\bar{d}_t}{T - t + 1} - \bar{m}_t - E(\bar{\varphi}_t), \]  

(2.22)

where \( \bar{d}_t \) is the aggregate remaining deficiency, \( \bar{m}_t \) is the aggregate current account balance and \( E(\bar{\varphi}_t) \) is the expected aggregate early liquidity shock (the intervention takes place before the aggregate shock occurs). In this thesis, I refer to this policy as the neutral liquidity supply. Such a policy is followed to some extent by the ECB, which uses, so called, “benchmark liquidity” in deciding on the liquidity supply (the ECB executes periodic operations, which are discussed in detail below).

The advantage of this policy is its transparency; central bank interventions mostly reflect changes in autonomous liquidity factors and use of standing facilities. Thus, by supplying a surplus or a shortage versus the benchmark, the central bank can communicate its stance to the markets. However, there are several risks related to such a policy.

First, estimation of expected changes in autonomous liquidity factors, \( \bar{\varphi}_t \), is subject to errors. Even though the central banks have maintained high accuracy of the forecasts, there is a risk of leaving the market with an unintended surplus or shortage of liquidity.

Second, sometimes the aggregate liquidity does not convey all the information. Consider, for instance, the example included in Table 2.3, which presents two days of the maintenance period and a market consisting of two banks. On day \( T - 1 \),
Table 2.3: Aggregate vs. Individual Liquidity

<table>
<thead>
<tr>
<th>Case 1.</th>
<th>Day T-1</th>
<th>Day T</th>
</tr>
</thead>
<tbody>
<tr>
<td>bank A</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>bank B</td>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10</strong></td>
<td><strong>20</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2.</th>
<th>Day T-1</th>
<th>Day T</th>
</tr>
</thead>
<tbody>
<tr>
<td>bank A</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>bank B</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>10</strong></td>
<td><strong>20</strong></td>
</tr>
</tbody>
</table>

both the aggregate starting current account balance and aggregate deficiency are the same, and just sufficient to satisfy the reserve requirement. However, in case 1, the market is left short of funds on the following day $T$, whereas the market is perfectly balanced in case 2. The difference results from the uneven distribution of the deficiency among the banks, which leads to some part of the reserves being "wasted" i.e. not contributing to meeting the reserve requirement. Such a situation could for instance occur when the banks, for some reason, refrain from interbank trade, deciding instead to hoard liquidity, which is discussed in more detail in Chapter 5. A central bank that chooses the intervention values for day $t$ based solely on aggregate risks might therefore leave the market out of balance.

Above I discussed the policy that supplies the market with the liquidity that is equal to the reserve requirement. Such a policy is relatively straightforward and transparent to the markets. However, its efficiency might be influenced by the quality of the autonomous liquidity factors forecasts. It is therefore interesting to compare it to the benchmark where the central bank has all the possible information. In this chapter I refer to such an allotment as the optimised liquidity supply.

In order to find the value of the allotment that anchors the market rate on the target, I assume that the central bank has a perfect model of the interbank market, can accurately predict the change in aggregate liquidity and can compute the condition (2.14) for each commercial bank in the market. Thus, if the allotted liquidity is distributed evenly to all participants, $\theta_{it} = \frac{\theta_i}{N}$, the optimal liquidity $\bar{\theta}_i^2$ will satisfy

$$\bar{\theta}_i^2 = \arg \min_{\theta} \left( (i_t - i^*)^2 \right), \quad (2.23)$$

where $i^*$ is central bank target rate. Such a liquidity supply, if implemented on a daily basis, would keep the interest rate equal to the target level.

The concept of optimised liquidity supply is an important contribution of this chapter. How useful is the additional information is an important issue for the central banks that often cannot perfectly model the changes in autonomous liquidity
factors and have to deal with a forecast error. By comparing the results between "neutral" and "optimised" regime, the central bank can estimate the cost of these errors measured, for instance, by increased interest rate volatility.

In addition, by comparing the information value between different monetary policy implementations, it is possible to conclude which regime performs better in asymmetric information environment. Such an analysis can be used by the central banks to assess the cost and benefits of monetary policy implementations.

Further analysis that is based on the simulations is performed in Subsection 2.4.6.

2.3.5 Frequency and timing of central bank interventions

The frequency and timing of open market operations are important parameters of monetary policy implementation. Recall from Table 2.2, in the previous section, that different policies are possible and the choice might affect the market behaviour.

At first, daily liquidity injections seem to be best choice if the central bank is determined to keep the level of the interest rate exactly on target. This way the central bank can react to changing market conditions almost instantly. However, they can be technically difficult to execute in countries with large financial systems. In the USA, the problem is solved by only allowing a selected group of dealers to participate in the open market operations. Frequent operations also do not automatically guarantee smooth interest rates. For instance in the UK before 2006, the operations were executed twice a day, which did not prevent high interest rate volatility. This suggests that additional factors, such as timing of the operations, might play an important role.

Periodic operations require considerably less administrative burden, but they too come at a cost. First, the liquidity shock predictions become less accurate as the time horizon stretches. Second, even if the shock is perfectly predicted, its realisation is spread over several days. Thus a central bank that supplies liquidity equal to the simulated value of all expected shocks up to the next intervention, creates a temporary imbalance on the first days, and achieves equilibrium only on the last day. Similar to the daily operations, the timing might be an important factor in determination of the market behaviour. If the last operation of the maintenance period is executed one week before the end of the maintenance period, the market might be left with substantial liquidity shortage or surplus and the banks forced to use the standing facilities. An extra operation on the last day of the maintenance period will give the central bank a chance to adjust for any errors in the shock forecasts, thus contributing to the market stability. I discuss those issues in detail in the next section.

The timing of the operations is equally important as the frequency. Using the example presented in Table 2.3, if the central bank executed extra intervention on the second day, the market would be still balanced. I discuss the importance of the operations timing is Subsection 2.4.1 below.

The central banks that adopted periodic open market operations determine the allotment value using the approach similar to the one discussed above. For instance, the ECB’s formula for benchmark allotment looks very similar to eq.(2.22).
The major difference is that periodic operations require autonomous liquidity factors forecasts for the whole period until the next intervention (a week in the Eurosystem).

In order to evaluate the importance of additional information, in the above section I defined the concept of the "optimised liquidity supply". In case of periodic operations, the optimised liquidity supply $\tilde{\theta}_t^3$ satisfies

$$\tilde{\theta}_t^3 = \arg \min_{\theta_t} \left( \sum_{T} (i_t - i^*)^2 \right).$$

(2.24)

Further discussion and comparison between optimised and neutral liquidity supply are also included in the next section.

### 2.3.6 Different settlement dates

The majority of central banks that use the averaging requirement apply the same maintenance period for all participating banks. However, the periodic spikes at the ends of maintenance periods raise the question of whether the volatility can be avoided by splitting the banks into groups, each having a different settlement day. In fact, such a system is currently used by the Bank of Brazil, where the banks are divided into groups A and B, with a one week gap between settlement days. The maintenance period in Brazil is two weeks, so that the settlement day of one group occurs exactly in the middle of the maintenance period of the second group.

Referring to the model presented in this section, individual bank behaviour remains governed by equation (2.14). However, the market clearing rate cannot be computed directly, which once again forces the use of the numerical methods presented in the following section.

There are several interesting issues related to such a regime. Recall first that, if all banks have the same maintenance period, market liquidity on settlement day must equal the remaining deficiency for the interest rate to remain in the middle of the corridor. In the case of different settlement days, however, the banks at the end of their maintenance period can borrow from the ones in the middle of their maintenance period; thus the aggregate liquidity will not have such a strong effect on the level of the interest rate.

Second, a significant problem may arise whenever a change in the target rate is expected. In this case, banks have strong incentives to postpone (or speed up) the satisfaction of the reserve requirement, which causes the current rate to partially converge toward the expected level, even before policy change is announced. Unless no change in policy is expected, this feeds interest rate volatility.

Synchronisation between maintenance periods and policy announcements can substantially reduce this source of volatility. This is recognised e.g. in the Eurosystem, where the target rate changes have recently been synchronised with the maintenance period, in order to prevent speculative accumulation of liquidity during the period. However, in the case of overlapping maintenance periods, such a policy will not be possible, which is discussed in the section where the simulation results are presented.
2.3.7 Interest rate volatility

The final issue that I discuss in this section is interest rate volatility. As mentioned in Subsection 2.2.1, the central bank dislikes interest rate volatility, as it distorts information about the monetary policy stance. It is important then to understand how interest rate volatility is captured in my model.

In the next section I present results of a Monte Carlo simulation study of the interbank market. The volatility of the interest rate comes from different interest rate levels when different liquidity shocks realisations are drawn. Thus the standard deviation of the interest rate is a proxy for how strongly the interest rate is affected by the aggregate liquidity (combination of early shock $\varphi_t$ and central bank intervention $\theta_t$) and the distribution of liquidity among the market participants (late liquidity shock $\varepsilon_t$).

The relationship between aggregate liquidity and interest rate level is referred to in the literature as the liquidity effect, and it is an important element of monetary policy implementation. Recall that one of the tools at the central bank’s disposal, open market operations, is often used to affect the level of the interbank interest rate. Thus the absence of a liquidity effect forces the central bank to rely on the control of expectations. Further analysis of this issue is postponed until the next chapter of this thesis.

One would expect the highest volatility to be observed on the last days of the maintenance period. This is due to the fact that the banks have no further chance to correct for potential imbalances that occur on this day and any difference between current account balance and reserve requirement will result in the use of standing facilities.

The scenario where the central bank gauges the liquidity intervention so that the market rate will end up on target is likely to exhibit less volatility, in comparison with the scenario with neutral liquidity supply, where only the liquidity shock is offset. In the extreme case, where daily optimised operations are performed, there should be no volatility at all.

2.4 Simulation study

2.4.1 Methodology

This subsection presents an outline of the numerical method used to compute the market equilibrium. The general approach is to use equations (2.12) and (2.14) to model individual bank behaviour and search for the interest rate that clears the market.

The goal of the chapter is to analyse the impact of different monetary policy designs on the behaviour of the interbank market interest rate and the use of standing facilities. In particular, I model the following monetary policy aspects:

1. Reserve requirement regime:
   
   (a) averaging provision regime with the same maintenance period for all banks

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(b) zero reserve requirement regime
(c) averaging provision regime with different maintenance periods for different groups of banks

2. Frequency of open market operations
(a) daily liquidity supply operations
(b) single liquidity supply operation at the start of the maintenance period

3. Volume of open market operations
(a) liquidity supplied to provide market with reserves needed to satisfy the requirement (neutral liquidity supply)
(b) liquidity supplied in accord with a simple optimisation algorithm.

The choice of features is related to their relative resemblance to the actual policies of central banks analysed. For example, the ECB uses the averaging provision reserve requirement, periodic liquidity supply equal to the predicted change in autonomous liquidity factors (taking into account the remaining deficiency). On the other hand, the Bank of England used to have no reserve requirement with daily liquidity operations. In the intermediate case, the Federal Reserve has averaging provision (albeit with shorter, 2 week maintenance period) with daily operations. Finally, I would also like to analyse the scenario with overlapping maintenance periods, currently used only by the Bank of Brazil.

The results are presented in three subsections: the regimes with averaging reserve requirement, no reserve requirement and overlapping maintenance periods. For each regime, I analyse the following liquidity supply policies:

- I. daily liquidity supply operations, neutral liquidity supply
- II. single liquidity supply operation, neutral liquidity supply
- III. daily liquidity supply operations, liquidity based on the optimisation algorithm
- IV. single liquidity supply operation, liquidity based on the optimisation algorithm.

For cases with a single operation, I assume the intervention occurs at the start of the maintenance period. This is important assumption and I discuss its implications below.

\[\text{In practise, the ECB performs four or five Main Refinancing Operations in each maintenance period, with weekly maturity. Additionally, the ECB has recently introduced fine tuning operations, performed on a regular basis on the settlement day.}\]
Neutral liquidity supply  Under neutral liquidity supply (scenarios I. and II.), I assume the central bank does not have a perfect foresight of the incoming liquidity shock. In order to determine the allotment value an expected shock value is used (under symmetric distribution assumption it equal to zero). The central bank has, however, access to other information, such as the remaining reserve requirement or the use of standing facilities.

If interventions are performed on a daily basis, the neutral liquidity supply on day $t$ satisfies:

$$\bar{\theta}_t = \sum_i \left( \frac{d_{ti}}{T - t + 1} - m_{ti} \right),$$

(2.25)

where $d_{ti}$ is the deficiency, and $m_{ti}$ is the current account balance of bank $i$. Thus, the liquidity injection might still result in liquidity shortage/surplus, if the actual aggregate shock realisation $\bar{\phi}_t$ differs from zero. The only way to ensure that the market is balanced is to postpone the operations until late part of the day.

If open market operations are performed periodically, the allotment value depends on the timing of the operations. If the central bank can supply the liquidity after the final aggregate shock on the last day of the maintenance period, the liquidity injection is identical to the daily intervention case analysed above:

$$\bar{\theta}_T = \sum_i (d_{Ti} - m_{Ti}).$$

(2.26)

According to condition (2.21) the market would then clear at the interest rate exactly equal to the target, thus reducing the volatility to zero. Observe, that the market liquidity is then identical regardless whether periodical or daily operations are executed. I return to this issue in Chapter 3, where I analyse the interbank market when the central bank performs late fine tuning operations.

Since the late liquidity supply is likely to result in an outcome very similar to daily operations, in this chapter I assume that the intervention takes place at the start of the maintenance period. In fact, similar policy was actually adopted at some point in the Eurosystem, where the open market operations were executed one week before the end of the maintenance period (currently, the ECB runs fine tuning operation on the last day of the maintenance period).

With the neutral liquidity supply, such an operation would compensate for aggregate liquidity shocks and standing facilities used since the preceding intervention. In my simulation however, I only consider short period and single operation during whole maintenance period. This means that the open market operations occur before any shock realisation and thus their value is zero.

The periodic liquidity interventions executed in this manner lead to an aggregate liquidity surplus (or shortage) for most of the maintenance period. Late operations could help in bringing back the balance, but only on the last day. Thus, the scenarios with periodic operations can be used as an excellent benchmark in analysing the consequences of an aggregate liquidity imbalance for the behaviour of the interbank interest rate. Additional analysis of this issue is presented in the next chapter.
**Optimised liquidity supply** The procedure for computing the optimised liquidity injection in regimes III and IV is slightly more involved. I first chose an initial value of the intervention and computed the corresponding interest rate $i_t$ (or interest rates $i_{t1}$, $i_{t2}$, $i_{t3}$ for all three days for the case of a single intervention). I could then pick another intervention value and repeat the same steps until no improvement can be made and the interest rate remains equal (or close) to central bank target.

The results for each regime are presented in a form of a table. Each table contains the following information:

1. average aggregate use of central banks deposit and lending facilities at the end of the day
2. average spread between interest rate and central bank target, measured across simulation runs
3. volatility of the interest rate measured across simulation runs
4. average difference between current and starting market liquidity value (which can be interpreted as the average intervention value)

The banks use standing facilities whenever they run short of funds (when faced with negative liquidity shock) or when they satisfy the entire reserve requirement for the period. The interest rate volatility was already interpreted in Subsection 2.3.7, and it indicates the strength of the liquidity effect, i.e. how much the liquidity shocks (aggregate and individual) affect the level of the interest rate. The interpretation of specific policy setups is conducted below.

The average difference between current and starting market liquidity value reflects changes in aggregate liquidity, which result from the combined early liquidity shock $\varphi_t$ and central bank supply of liquidity. With neutral liquidity supply, the central bank intervention is mainly intended to offset aggregate shock and the use of standing facilities, thus the aggregate liquidity should barely change. However, under optimised liquidity supply, this difference will indicate the direction of central bank intervention necessary to set the rates exactly on the target.

In certain scenarios, such as no averaging of reserves or different settlement days and daily liquidity supply operations, the market behaved in the same way every day. For those scenarios, I reported the results in single row, and left the other days marked with N.A. (not-applicable).

### 2.4.2 Simulation design

The simulation design constitutes an important contribution of the thesis and this chapter in particular. Therefore, it is important for the reader to understand the innovations I introduced in the programming code.\(^\text{11}\)

I designed the simulation code in order to accomplish the following goals in my thesis:

\(^{11}\)The programming code used in simulations in all 5 chapters of this thesis is available upon request at author's email michal.kempa.thesis@gmail.com
• simulate the behaviour of the interbank market with different monetary policy implementations

• analyse the key parameters of the interbank market, such as interest rate level and volatility

• build a framework that allows me to change some key assumptions, such as interest rate expectations (Chapter 3), different cost function (chapters 4 and 5) and asymmetric liquidity expectations (Chapter 5)

• design an efficient code, which can be used to extend the analysis into multiple periods (chapters 3-5).

The literature on the interbank market includes a number of simulation studies, which, however, cannot be used to attain all of the listed objectives. This is because the simulation designs often focus on country specific details (e.g. carryover provision in Clouse and Dow Jr. (2002)), different aspects of the market (central bank's commitment to intervention in Bartolini and Prati (2006)) or prevent the calculations of certain market parameters (such as interest rate volatility in Pérez-Quirós and Rodríguez-Mendizábal (2006)). These problems are addressed in my simulation design, which makes it a useful contribution to the existing interbank market literature.

The work that is perhaps closest to mine is Pérez-Quirós and Rodríguez-Mendizábal (2006), to whom I am very grateful for providing me with the version of the code they used in their paper. The same authors, in their second paper, Gaspar et al. (2008), made some significant changes in their simulation design. However, since I had no chance to verify their updated program, I focus on pointing out the differences between my code and the version they used in their original paper.

The novelty of the simulation used in this thesis stems from the combination of three elements:

• application of the Monte Carlo method,

• using efficient programming algorithms and interbank market properties that significantly reduce the computational burden,

• using a flexible framework that allows to verify the assumptions.

Let me discuss each of these points separately.

Unlike Pérez-Quirós and Rodríguez-Mendizábal (2006), my program applies a Monte Carlo method, which offers numerous advantages that can be used in the modelling of the interbank market. The model used in this thesis (presented in Section 2.3.) has a finite horizon and there is no carryover of reserve requirement. Thus, a single iteration of the Monte Carlo simulation can be easily interpreted as an independent maintenance period.

Apart from straightforward interpretation, large number of iterations is necessary to compute additional statistics such as interest rate volatility. In addition,
I can analyse the banks in different stages of the maintenance period, which allows me to simulate a regime with overlapping maintenance periods that was not modelled in the literature before.

Finally the Monte Carlo method reduces the computation time by using previously saved intermediate results in each iteration. Thus, I only compute the value functions once per simulation. For comparison, Pérez-Quirós and Rodríguez-Mendizábal (2006) repeat their calculations for each guess of the interest rate.

Pérez-Quirós and Rodríguez-Mendizábal (2006) claim that the extension of the analysis beyond three day maintenance period is numerically unfeasible, while I manage to simulate the behaviour of 20 day maintenance period in Chapter 3. In fact, simulations for periods exceeding 3 days are fairly common in the literature. Hamilton (1996) already 14 years ago managed to simulate 7 day period. In 2002 Clouse and Dow Jr. (2002) and Bartolini et al. (2001) had no problems simulating 10 day maintenance periods. Hence extending the analysis beyond 3 days is certainly feasible.

Computing banks’ behaviour during 20 day maintenance periods was possible by focusing on efficient programming methods and taking advantages of certain overnight interbank market properties. The Monte Carlo method was mentioned above already. Another source of efficiency is avoiding repeating identical computations which were fairly common in Pérez-Quirós and Rodríguez-Mendizábal (2006). Finally, thanks to efficient algorithms such as bisection I can quickly solve for the market clearing rate or the optimised liquidity injection, while Pérez-Quirós and Rodríguez-Mendizábal (2006) relied on initial guess of the interest rate to reach the solution.

The second source of efficiency in my simulation is taking advantages of certain unique properties of the interbank market. For instance, I show in Appendix 3.8.3 that \( \frac{\partial b_t}{\partial m_t} = -1 \). This analytical result allows me reduce the grid of the state values by one dimension! Another property used is the link between interest rate expectations and the central bank target rate. In Subsection 3.2.4 I show that the interest rate expectations do seem to closely follow the central bank target (at least in the Eurosystem). Using the framework derived in this chapter, such behaviour is a logical consequence of the Central Bank ability to control liquidity supply (“optimised liquidity”). In next chapter I also show that a large reserve requirement buffer has very similar consequences and hence the assumption seems

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12 Pérez-Quirós and Rodríguez-Mendizábal (2006) designed their code in order to compute the expected level of the interest rate. They started with an initial guess of interest rates in 3 analysed periods. For the initial guess of the interest rates, they computed the corresponding value functions (using condition 2.8 in this chapter) and other state variables. Assuming certain liquidity shock distribution allowed them to compute expected borrowing values.

The part that was really time consuming in their calculations was the algorithm for finding the level of the market clearing rate. The way they proceed in their analysis was to compute the whole simulation for each interest rate +/- one basis point for each day of the 3 day maintenance period. So for instance, in order to compute the market clearing conditions for 3 day period they needed \(3^3 = 27\) iterations, for 4 day period they needed \(81\) etc. Only after all iterations were computed, they checked the market clearing condition for all days. If the condition was not satisfied, they moved the interest rate by 1 basis point and repeated the whole process again duplicating in effect 2/3 of their calculations. The fact that they had to compute the value functions each iteration and that they did not check the market clearing condition until computations for whole period were done, rendered the process extremely time consuming.
justified. I abandon it in Chapters 4 and 5, where market frictions and credit risk require additional algorithm to ensure rational expectations.

The last significant novelty in my simulation is the possibility to analyse the role of different assumptions. The design is sufficiently flexible to allow extending the analysis into overlapping regime (Chapter 2), 20 days maintenance period and assumption sensitivity analysis (Chapter 3), different functional forms of interbank funding cost (Chapter 4) and expected liquidity shortage (Chapter 5).

Since the contribution of the thesis depends heavily on the simulations, it is important that the reader clearly understands the algorithm used.

The general idea is as follows. I start from an individual bank’s optimisation problem, which in this chapter is given by equations (2.14) and (2.12). For an initial guess of the interest rate and a set of state variables I compute the individual borrowing that satisfies these conditions. If the aggregate borrowing is different from zero, I repeat these steps until I find that the interest rate that satisfies the market clearing condition.

The process described above is repeated for each day of the maintenance period, which constitutes one full Monte Carlo iteration. In Chapter 2, where I analyse the Central Bank intervention, one additional step is needed to find the “optimised” intervention value. In Chapters 4 and 5 I deal with credit risk and information asymmetry, which forces me to introduce another loop to ensure rational expectations with respect to the interest rate.

More technical details are included in Appendix 2.6.4. In this chapter, I only use 3 day maintenance period, similarly to Pérez-Quirós and Rodríguez-Mendizábal (2006). The reason for choosing such a short period is the time required to compute the optimised supply liquidity value. To find this value, I need to calculate the market equilibrium for the whole 3-day maintenance period for each guess of the allotment value. In the next chapter, where I focus on different aspects of the interbank market, I am able to extend the analysis into 20 periods.

The parameter values resemble the ones used in previous works of Pérez-Quirós and Rodríguez-Mendizábal (2006) and Bartolini et al. (2001). More specifically, I assume $N = 9$ identical banks on the market, which are divided into three groups in the overlapping maintenance period scenario. Each group has starting current account $m_1 = 100$ and a corresponding reserve requirement of $R = 3 * m_t = 300$.

Both early and late shocks are i.i.d, according to the normal distribution with mean zero and standard deviation $\sigma = 60$. The late shock, $\epsilon$, is normalised to always sum to zero. Note that the assumed value of the shock variance is fairly high, especially when compared with my own estimations in Chapter 3 made for the Eurosystem. A similar value was used in other simulations of Pérez-Quirós and Rodríguez-Mendizábal (2006) and Bartolini et al. (2001). In addition, in this chapter I only analyse three day maintenance period. Thus, in order to observe the liquidity effect, I need higher variation in the liquidity shock, when compared with 20 day maintenance period analysed in the next chapter. Finally, even though I estimate the shock variance as very low in the Eurosystem, I do not have the data to compare it with the variance in the UK and the USA. This is especially important for the analysis of the regime where there is no reserve requirement.

\footnote{Thus the combined standard deviation of both shocks is around 80, whereas Pérez-Quirós and Rodríguez-Mendizábal (2006) used a value of 100.}
### Table 2.4: Averaging provision reserve requirement

<table>
<thead>
<tr>
<th>Regime type:</th>
<th>Day</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The recursion to deposit facilities</td>
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<td>0.0</td>
<td>0.0</td>
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<td></td>
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<td>33.1</td>
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<td>3</td>
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<td>346.8</td>
<td>202.4</td>
<td>269.3</td>
</tr>
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<td>-8.9</td>
<td>9.8</td>
<td>12.6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-20.2</td>
<td>-23.3</td>
<td>24.1</td>
<td>19.4</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-223.4</td>
<td>-300.1</td>
<td>216.8</td>
<td>239.0</td>
</tr>
<tr>
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<td>-0.04</td>
<td>0.00</td>
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<td>0.01</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.01</td>
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<td>0.05</td>
<td>0.01</td>
<td>0.04</td>
</tr>
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<td>-99.8</td>
<td>-15.9</td>
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<tr>
<td></td>
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<td>-5.50</td>
<td>0.70</td>
<td>109.6</td>
<td>-15.9</td>
</tr>
</tbody>
</table>

For detailed discussion of the actual variance value refer to Section 3.4.

#### 2.4.3 Averaging provision reserve requirement

I first analyse the case where the commercial banks must satisfy a reserve requirement with an averaging provision. Within that framework, I also analyse different liquidity supply policies, such as daily and single operations or neutral versus optimised liquidity supply volumes. A framework similar to regime III was adopted in the Eurosystem before March 2004. The results are presented in Table 2.4.

Starting from the standing facilities, the first observation is that the banks use standing facilities only on the settlement day. This feature is strictly related to the averaging provision, which has two major effects. First, it forces the banks to maintain a positive account balance, which can be used to offset negative liquidity shocks, thus reducing the need to use the lending facility.\(^{14}\) Second, positive balances on central bank accounts are not deposited until they exceed the remaining

\(^{14}\)In the Eurosystem, the balances used to satisfy the reserve requirement are remunerated, which significantly reduces the cost of the reserve requirements for the commercial banks.
reserve requirement, which is typically very large at the start of the maintenance period. Thus the deposit facility will only be realistically utilised at the end of the maintenance period.

A similar pattern of use of standing facilities can be observed for the Eurosystem, where the average use of lending facilities in the period 1999-2005 was roughly five times higher on the last day of maintenance period than in other periods (for the borrowing facility the ratio was about nine).

Concerning the level of the interest rate, the results partially confirm the pattern observed already by Pérez-Quirós and Rodríguez-Mendizábal (2006), namely the tendency for the interest rate to increase toward the end of the maintenance period. This is due to the fact that banks cherish the buffer provided by the reserve requirement, which protects them from use of the deposit facility. Thus reserves are less desired at the start of the maintenance period, which corresponds to a lower level of the interest rate.

Under regime III, where the central bank performs daily operations with an optimised intervention volume, the interest rate always ends the day on target. Apart from perfect foresight of the aggregate shock, this result also required substantial deviations of the market liquidity from neutral level that corrected for the Bank's incentives described above.

Concerning the volatility of the interest rates on different days of the maintenance period, the first observation is that there is higher volatility toward the end of the maintenance period. A similar observation was made by Bartolini et al. (2001) and Gaspar et al. (2008), who used comparable models, and it reflects the actual pattern of the interest rate in the Eurosystem, as observed by Moschitz (2004) and Würtz (2003).

The increase in the interest rate volatility is caused by a property of the demand functions, as originally pointed out by Välimäki (2003). He observed that the demand curves of the commercial banks become less elastic toward the end of the maintenance period. Thus a deviation from neutral liquidity results in higher interest rate spread\(^{15}\) on the settlement day, when compared with the rest of the maintenance period.

Besides these general observations, note also that there are substantial differences in the interest rate volatility across different liquidity supply regimes. The general impression from Table 2.4 is that frequent liquidity injections and optimised allotment value result in lower volatility. However, the timing of the operations also plays an important role.

Concerning first the settlement day, according to condition (2.21), whenever market liquidity is equal to the remaining reserve requirement the interest rate ends on target.

Since the operations in regime I. are performed early and the central bank does not have perfect foresight, this condition is satisfied if the aggregate shock on that day is zero. Daily operations, however, still help to compensate for the imbalances that aroused during first two days of the maintenance period.

In regime II the operations are performed once, at the start of the maintenance period. Therefore, condition (2.21) requires the sum of the aggregate shocks and

\(^{15}\)By interest rate spread I refer to the spread between market rate and the central bank target.
the use of standing facilities during the whole period to be zero. Since this condition is more restrictive compared with regime I, the volatility of the interest rate is significantly higher in regime II.

Observe that the timing is crucial in this analysis. If the liquidity supply operations were executed late on the last day of the maintenance period (as they are in fact in the Eurosystem), the interest rate on that day would always end on the target level with zero volatility, regardless of the operations frequency. This is, in fact, what happens in regime III, where the central bank has perfect foresight and can adjust the liquidity on daily basis. In regime IV, the central bank has to worry also about the level of the interest rate on the days preceding the end of the settlement day; thus the market liquidity might differ from the neutral level.

Regimes III and IV exhibit significantly lower volatility of the interest rate, which can be attributed to the perfect foresight assumption. Thus, by investing resources into better forecasts, the central banks can improve their implementation of the monetary policy. However, just how important is the improvement depends on the general framework such as existence of the reserve requirement, timing of the operations etc. I return to these issues in Subsection 2.4.6.

Apart from imperfect forecast of liquidity shocks, the neutral liquidity supply policy is also limited by only considering aggregate balances. In Subsection 2.3.4 I pointed out the importance of individual liquidity. Note, however, that once again the timing of last operation can play very important role. Consider the example presented in Table 5.3, where different distributions of liquidity and deficiency forced some banks to use the standing facilities and resulted in different market outcomes. Having an extra intervention on the settlement day would address that problem by allowing the market to adjust the liquidity to balance the remaining reserve requirement.

To summarise this subsection, the most important result is that the neutral liquidity provision policy that does not assume perfect foresight might be improved by leaving the market with a liquidity shortage for most of the maintenance period and correcting that on the settlement day. This allows banks to longer maintain the buffer provided by the reserve requirement, thus allowing them to avoid using costly standing facilities.

Frequent operations in general decrease the interest volatility by reducing the market imbalances. However, similar effect can be reached by adjusting the timing of the operations. This was recognised by the ECB, which introduced fine tuning operations at the end of each maintenance period.

2.4.4 No averaging provision

In this subsection I present the behaviour of the market in the absence of the averaging provision reserve requirement. Such a system was for instance used by the Bank of England before May 2006.

In practise, the countries that do not use the averaging provision often require the banks to maintain a minimum current account balance. In this case, however, the reserve requirement role is reduced to a tax on banks, or a mechanism that could be used to control the expansion of the monetary base (depending on how the reserve requirement is calculated).
Table 2.5: No averaging provision
I. daily liquidity supply operations, neutral liquidity supply
II. single liquidity supply operations, neutral liquidity supply
III. daily liquidity supply operations, liquidity based on optimisation algorithm
IV. single liquidity supply operations, liquidity based on optimisation algorithm

In terms of control over the overnight interbank interest rate, the level of the reserve requirement in this case makes little difference. If a bank’s balance falls below the required level, it is forced to use the lending facility, as on the last day of the maintenance period under the averaging provision regime. Thus the behaviour of the market can be captured by equation (2.12), which implies that rather than the absolute value of liquidity, it is the difference between market liquidity and the reserve requirement that determines the level of the interest rate. For practical reasons, I have assumed a zero reserve requirement.

As in the subsection above, I analyse the behaviour of the market in four different scenarios. The results are presented in Table 2.5.

Use of the standing facilities follows similar values for all regimes and in fact is almost identical to the last day of the maintenance period in an averaging provision regime. This reflects the fact that in both cases the banks target an end-of-day balance of zero, which makes them vulnerable to both positive and negative liquidity shocks, which in turn force them to use the standing facilities.

In the regime where the central bank engages in daily intervention, the average interest rate stays on target, since the average market liquidity is equal to
zero. However, the interest rate volatility depends on the liquidity supply regime. With neutral liquidity supply, the central bank does not have the perfect foresight and the market balance is determined by the aggregate liquidity shock. This case results in high volatility that was, in fact, exhibited by SONIA in the UK before 2006. With optimised liquidity supply, the central bank can offset the aggregate shock and leave the market with the liquidity equal to the remaining reserve requirement. According to condition (2.21) the interest rate then always ends on target. Similar to the above section, the same results as in optimised liquidity could be obtained if the central bank executed late, instead of early operations. Thus the timing is very important also in the case where there is no reserve requirement.

In regimes with only periodic operations, interest rate volatility is substantially higher regardless of the liquidity supply policy. This is caused by the fact, that without reserve requirement even a small deviation from neutral liquidity has a strong effect on the level of the interest rate. Without possibility to offset daily changes in aggregate liquidity, the market balance then almost always deviates from the level given by condition (2.21). In this case, the change of timing of the operations would not lower the volatility (except for the day of the intervention), which means that the central banks that do not wish to adopt reserve requirement are essentially forced to commit to daily operations.

To conclude, the most important finding of this subsection is the importance of tight control of aggregate liquidity for the regime without averaging provision. Even a relatively small deviation from neutral liquidity is likely to result in a large deviation of the interest rate from the target. Such a deviation can result e.g. from an unexpected change in autonomous liquidity factors or from the central bank's inability to allot desired amounts of funds and in reality is difficult to avoid. The central bank can counter this by performing daily late, fine tuning operations, to correct for possible errors in the aggregate liquidity estimations. Such a policy however, requires a great deal of micro-management and has been only implemented in relatively small countries, such as Sweden or New Zealand.

### 2.4.5 Different settlement days

In the regime analysed in this subsection, the banks must meet an average value of the reserve requirement (as in Subsection 2.4.3 above), but the settlement day is different for three groups of banks. Without changing the main assumptions of this simulation, I assume that each group consist of three banks (1/3 of all banks in the market). The first group ends the maintenance period on day 1, the second on day 2, and the third on day 3. I also assume that the starting liquidity is sufcient to satisfy the reserve requirement.

---

16The fact that the average interest rate remains above the target in scenario IV is related to certain assumptions made about lending and that the banks target zero reserves. In particular, I assumed that the lending cannot exceed Bank's current account balances or reach a positive value if the balance is negative (in fact, the bank is required to borrow from the central bank then). This sometimes prevents the banks from making profits on lending at a market rate higher than the expected cost of using standing facilities, which results in small part of liquidity being locked and a slightly higher interest rate. In reality, however, where banks target positive balances for other reasons (such as a reserve requirement), this assumption is not likely to hold, and the average interest rate ends up on target.
Table 2.6: Different settlement days
I. daily liquidity supply operations, neutral liquidity supply
II. single liquidity supply operations, neutral liquidity supply
III. daily liquidity supply operations, liquidity based on optimisation algorithm
IV. single liquidity supply operations, liquidity based on optimisation algorithm

The results of the simulation are presented in Table 2.6. Note that unless the central bank engages in periodic liquidity supply operations, the situation on each day will be identical, with one of the groups completing, one starting, and one being in the middle of its maintenance period.

The average use of standing facilities remains at a level significantly lower compared with the previous scenarios. The reason for this is that there are only three banks ending the maintenance period on a given day, while in Subsection 2.4.3 there were nine. The three banks determine their borrowing according to condition (2.12), which implies that at a rate close to the central bank target, the banks end the trading day with a current account balance equal to the remaining reserve requirement. Thus the use of standing facilities only reflects the late liquidity shock hitting the banks in this group. In the meantime, the banks in the other groups still enjoy the averaging buffer, which shields them from the late liquidity shock realisations.

The average interest rate has a tendency to drop slightly below the target level. This is caused by the effect already covered in the previous scenarios: under the averaging provision, the banks have incentives to postpone meeting the reserve requirement until the last day of the main maintenance period. In the case of overlapping maintenance periods, two-thirds of the banks in the market are always in days
before settlement, which puts constant downward pressure on the level of the interest rate. This is also reflected in scenarios III and IV, where I find that the level of liquidity necessary to bring the market rate to the target is on average below the neutral level.

Comparing the performance of the market under daily and periodic supply operations allows me to study the impact of aggregate liquidity on the level of the interest rate. As one could expect, the daily operations are much better for reducing the volatility of the interest rate to a low level. However, the difference between daily and periodic operations is not as large, as for instance, in the averaging reserve requirement case. This suggests that the market is much more resilient to changes in the aggregate liquidity.

Comparing directly the overlapping with the traditional reserve requirement regime, (Tables 2.6 and 2.4 respectively) reveals that the interest rate volatility is slightly higher for the overlapping maintenance period regime on the days before settlement day and significantly lower on the last day of the maintenance period. Thus applying different maintenance periods for banks causes the interest rate volatility to be more evenly spread across the whole period.

Based on the results reported above, the overlapping maintenance period regime seems a feasible solution that offers less volatility to the market, even when the central bank has no perfect foresight. Yet, few countries have adopted such a system. The reason for this is the system instability when target rate change is expected (even if the change does not occur in reality). In the previously analysed regimes with averaging provision, the target interest rate and rate expectations were assumed constant. This is because any change in the target rate takes place after the last day of the maintenance period, which does not affect the bank’s behaviour. However, with overlapping maintenance periods such an assumption cannot be made, since there will be always some banks for which the change will occur in the middle of their maintenance period.

In order to illustrate this situation, I conducted additional simulations. Unlike in the previous cases, I now assume the banks expect a 50 basis points rise in the interest rate on the third day of the maintenance period. The results are presented in Table 2.7, for the case of neutral liquidity supply only, which is sufficient to illustrate the market behaviour.

The examination of the results reveals first that the expected change in the interest rate rise significantly affects the bank’s propensity to use the lending facility. This is due to the banks frontloading (satisfy the reserve requirement early), in order to avoid higher finance cost. The effect is spread over three days. On the first day, two days before policy change, it is only one group of banks that is directly affected, while two other groups will end their corresponding maintenance periods before policy change is in place. However, already this group frontloading is enough to increase the market clearing interest rate. On the day directly preceding the policy change, there are already two groups of the banks affected and the impact on the interest rate level is even stronger. Finally on the last day, when the policy actually comes into force, the banks that were frontloading before find themselves with actual surplus of liquidity, which results in the interest rate below the new target level.

To summarise the above observations, with the overlapping maintenance per-
iods, the expected change in the policy rate substantially affects the market behaviour and the level of the interest rate. This means, however, that any time the market expects change in the central bank target, the market rate will deviate toward the expected new level. From the central bank perspective, this is a serious disadvantage that negatively affects the interpretation of central bank stance and general implementation of the monetary policy.

2.4.6 Monetary policy implications

The above subsections present the behaviour of the interbank market under different policy setups. I start with a regular averaging provision, adopted by the majority of developed countries. I then move on to analyse a regime without averaging provision, which was used in the United Kingdom (before 2006) and small countries such as Sweden or New Zealand. Finally, I consider a regime where banks have different settlement days, which was implemented only in Brazil.

In this chapter I focus on the most important features of monetary policy implementation: determination of the allotment value, frequency and timing of liquidity supply operations. This sub-section contains a brief summary and comparison between the analysed regimes.

Using the spread and the volatility of the interest rate as metrics, the regimes where the amount of liquidity allotted is decided in accord with my simple optimisation algorithm perform better compared with the neutral liquidity supply. This is not a surprising result, given that the central bank has perfect foresight of

<table>
<thead>
<tr>
<th>Regime type:</th>
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<td>The recursion to CB deposit facilities</td>
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<td>2</td>
<td>0.36</td>
<td>0.33</td>
</tr>
<tr>
<td>3</td>
<td>0.46</td>
<td>0.40</td>
</tr>
<tr>
<td>Standard deviation of the interest rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>2</td>
<td>0.09</td>
<td>0.18</td>
</tr>
<tr>
<td>3</td>
<td>0.09</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 2.7: Different settlement days and change in rate expected rate
I. daily liquidity supply operations, neutral liquidity supply
II. single liquidity supply operation, neutral liquidity supply
liquidity shocks in the optimised case. However, as discussed in Subsection 2.3.4, comparing the results of optimised and neutral liquidity supply can be still used to estimate the cost of information.

The first observation is that, in most cases, the additional information plays little role in the average level of the interest rate as it remains close to the target for the whole maintenance period. The only exception is the regime with averaging provision and single operations. This is due to the effect described already by Pérez-Quirós and Rodríguez-Mendizábal (2006): the interest rate in averaging provision regime has a tendency to increase toward the end of the maintenance period. Thus, the central bank that wishes for the rate to remain on the target for all days, needs to keep the market with shortage of liquidity, and only compensate on the last day of the maintenance period.

The second observation is that additional information significantly reduces interest rate volatility. Comparing the optimised and neutral regime reveals that perfect forecast of liquidity shocks can reduce the market volatility by 100% (with no averaging provision), 20-70% (averaging provision) or 75% (different settlement days).

This effect is present in all scenarios, but perhaps the most dramatic change can be observed in the regime without averaging provision. In this case, even small deviation of the liquidity from the neutral level results in the interest rate ending far from the target. Thus, the central bank that wishes to reach its interest rate target needs to either perfectly predict the aggregate liquidity changes or perform the operations very late, when the aggregate shock value is already known. On the opposite side is the averaging provision regime with a single operation. In this case, the perfect foresight of the liquidity shock is not required to maintain low interest rate volatility, except for the settlement day. The problem of higher volatility on the last day can be (and indeed was in the Eurosystem) solved by introduction of late liquidity supply operations. Therefore it is possible to conclude that in an environment with volatile and unpredictable shocks, the averaging requirement regime copes better with asymmetric information problems.

Concerning the frequency of supply operations, it seems that daily operations result in much less volatility in the market than periodic operations. This result varies for different regimes, but the frequency seems to be most important for the case of no averaging provision. This is caused by the fact that such a regime is strongly affected by deviation of liquidity from neutral level, which in turn depends on the amount of days between interventions. The frequency is less important with averaging provision, which offers the banks a buffer against liquidity shocks for majority of the maintenance period. Note that the timing plays a significant role in the analysis of the impact of changing frequency and, as mentioned above, late operations can result in a less volatile market.

2.5 Conclusions

This chapter uses simulation studies to compare the behaviour of the interbank market under different monetary policy frameworks. My model is based on the po-
popular Poole (1968) framework with extensions that allow for averaging provision by Pérez-Quirós and Rodríguez-Mendizábal (2006). The model captures the central bank that chooses the allotment value and borrowing behaviour of an individual bank that minimises the cost of market and central bank finance.

I start with a brief introduction and comparison between monetary policy frameworks in the Eurosystem, the United States and the United Kingdom. I find that the Eurosystem and the United States exhibit much less volatility in the interest rate compared to the United Kingdom. Using my model and numerical simulation, I attribute these differences to use of an averaging provision.

In Subsection 2.4.3 I show that whenever only periodic operations are possible, the averaging provision results in the best market performance in terms of interest rate spread and volatility. The downside is the spike in volatility on settlement day, which however can be removed by executing late liquidity supply operations. This periodic effect can be spread over the days of the maintenance period if the commercial banks are divided into groups, each having a different settlement day. However, the benefits of such system are overshadowed by the instability that occurs each time a change in the target rate is expected.

A central bank that commits to frequent operations and is able to perfectly predict the liquidity shock, can ensure that the market is left with exactly neutral liquidity at the end of each day. In this case it is possible to obtain perfect control over the interest rate with zero volatility. Difficulties in obtaining such outcome can be, however, illustrated by the case of the United Kingdom, where the rate volatility was persistently high before 2006.

Finally, I also evaluate the role of the forecast errors of liquidity shock. I find that perfect forecasts can reduce the interest rate volatility by up to 100%. However, the benefits vary according to the regime used, are in case of averaging provision they remain quite limited for the majority of the maintenance period.

2.6 Appendix

2.6.1 Proof of results from Subsection 2.3.1

The expected cost function can be then rewritten as

\[ i_T \delta_T + E(c_T) \]  

(2.27)
where $i_T b_T$ denotes the cost of interbank borrowing and $E(c_T)$ the expected cost of using the standing facilities, given by

$$E(c_T) = i^T \int_{-\infty}^{\infty} (-m_T - b_T - \theta_T - \varphi_T + d_T - \varepsilon_T)f(\varepsilon_T)d\varepsilon_T$$

$$- i^d \int_{-\infty}^{\infty} (m_T + b_T + \theta_T + \varphi_T - d_T + \varepsilon_T)f(\varepsilon_T)d\varepsilon_T$$

$$= -i^T(m_T + b_T + \theta_T + \varphi_T - d_T)F(-m_T - b_T - \theta_T - \varphi_T + d_T)$$

$$- i^d(m_T + b_T + \theta_T + \varphi_T - d_T)(1 - F(-m_T - b_T - \theta_T - \varphi_T + d_T))$$

$$- i^T \int_{-\infty}^{\infty} \varepsilon_Tf(\varepsilon_T)d\varepsilon_T - i^d \int_{-\infty}^{\infty} \varepsilon_Tf(\varepsilon_T)d\varepsilon_T$$

(2.28)

The first order condition for the problem with respect to $b_t$ is

$$-i_T = -i^T F(-m_T - b_T - \theta_T - \varphi_T + d_T)$$

$$+ i^T(m_T + b_T + \theta_T + \varphi_T - d_T)f(-m_T - b_T - \theta_T - \varphi_T + d_T)$$

$$- i^d(1 - F(-m_T - b_T - \theta_T - \varphi_T + d_T))$$

$$- i^d(m_T + b_T + \theta_T + \varphi_T - d_T)f(-m_T - b_T - \theta_T - \varphi_T + d_T)$$

$$- i^d(m_T + b_T + \theta_T + \varphi_T - d_T)f(-m_T - b_T - \theta_T - \varphi_T + d_T)$$

$$+ i^d(m_T + b_T + \theta_T + \varphi_T - d_T)f(-m_T - b_T - \theta_T - \varphi_T + d_T),$$

(2.29)

where the last line follows from the Leibniz rule. Rearranging yields

$$i_T = i^T F(-m_T - b_T - \theta_T - \varphi_T + d_T) + i^d(1 - F(-m_T - b_T - \theta_T - \varphi_T + d_T))$$

(2.30)

### 2.6.2 Proof of results from Subsection 2.3.2

Similar proof is presented in Pérez-Quirós and Rodríguez-Mendizábal (2006). The value function for the problem takes the form

$$V_t = \min_{b_t} \{i_t b_t + E_t(c_t) + E_t(V_{t+1})\},$$

(2.31)

which can be written using the slightly modified expected cost expression (2.8) as

$$V_t = i_t b_t - i^T(m_t + \theta_t + \varphi_t + b_t)F(-m_t - \theta_t - \varphi_t - b_t)$$

$$- i^d(m_t + b_t + \theta_t + \varphi_t - d_t)(1 - F(-m_t - \theta_t - \varphi_t - b_t + d_t))$$

$$- i^T \int_{-\infty}^{\infty} \varepsilon_t f(\varepsilon_t)d\varepsilon_t - i^d \int_{-\infty}^{\infty} \varepsilon_t f(\varepsilon_t)d\varepsilon_t$$

$$+ E(V_{t+1}).$$

(2.32)
The first order condition with respect to $b_t$ is
\[ i_t = i^d F(-m_t - \theta_t - \varphi_t - b_t) + i^d (1 - F(-m_t - \theta_t - \varphi_t - b_t + d_t)) + E \frac{\partial V_{t+1}}{\partial b_t} . \tag{2.33} \]

To calculate the last element of the F.O.C., note that the borrowing maturity is one period. This means that it has no direct impact on the current account value and borrowing in the next period. The borrowing does however affect the deficiency value, which allows to write the last term as
\[ \frac{\partial V_{t+1}}{\partial b_t} = \frac{\partial V_{t+1}}{\partial d_{t+1}} \frac{\partial d_{t+1}}{\partial b_t} . \tag{2.34} \]

Realisation of the liquidity shock determines the outcome of (2.34); hence I analyse three separate cases:

1. The shock forces the bank to use a lending facility, whereby the deficiency remains unchanged.

2. The intermediate case, where the shock value only reduces the deficiency without forcing the bank to use any of the facilities.

3. The shock forces the bank to use a deposit facility, whereby the whole deficiency gets satisfied.

In the first case, the deficiency remains unchanged. In the third case, the bank satisfies the whole reserve requirement and the deficiency turns to zero regardless of the borrowing. Thus, for those cases where $\frac{\partial d_{t+1}}{\partial b_t} = 0$,
\[ \int_{-\infty}^{-m_t - \theta_t - \varphi_t - b_t} \frac{\partial V_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon_t = \int_{d_t - m_t - \theta_t - \varphi_t - b_t}^{\infty} \frac{\partial V_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon_t = 0. \tag{2.36} \]

In the intermediate case, whenever the deficiency is carried over one period, one more unit of borrowed funds decreases it by the same value, $\frac{\partial d_{t+1}}{\partial b_t} = 1$; hence
\[ \int_{-m_t - \theta_t - \varphi_t - b_t}^{d_t - m_t - \theta_t - \varphi_t - b_t} \frac{\partial V_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = \int_{-m_t - \theta_t - \varphi_t - b_t}^{d_t - m_t - \theta_t - \varphi_t - b_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} \frac{\partial d_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = \]
\[ - \int_{-m_t - \theta_t - \varphi_t - b_t}^{d_t - m_t - \theta_t - \varphi_t - b_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon. \tag{2.37} \]
Combining (4.48) and (4.51) yields the profit maximising conditions

\[ i_t = i^d F(-m_t - \varphi_t - \theta_t - b_t) + i^d [1 - F(d_t - m_t - \varphi_t - \theta_t - b_t)] \]

\[- \int_{-m_t - \varphi_t - \theta_t - b_t}^{d_t - m_t + \varphi_t - \theta_t + b_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t. \]  

(2.38)

2.6.3 Proof of dynamic cost factor equation from Subsection 2.3.2

The derivative of the value function with respect to deficiency can be solved using an envelope theorem (so that \( \frac{\partial V_t}{\partial b_t} = 0 \)):

\[ \frac{\partial V_t}{\partial d_t} = \frac{\partial}{\partial d_t} (E(c_t) + EV_{t+1}) \]

\[ \frac{\partial}{\partial d_t} \left\{ i^d (m_t + b_t + \theta_t + \varphi_t - d_t)(1 - F(-m_t - \theta_t - \varphi_t - b_t + d_t)) + 
\right. \]

\[ i^d \left[ \int_{-m_t - \theta_t - \varphi_t - b_t + d_t}^{\infty} \varepsilon_t f(\varepsilon_t) d\varepsilon_t \right] + EV_{t+1} \} . \]  

(2.39)

Since the deficiency has no impact on the lending facility use, I need only analyse the deposit facility probability:

\[ \frac{\partial V_t}{\partial d_t} = i^d \left\{ -(1 - F(-m_t - \theta_t - \varphi_t - b_t + d_t)) 
\right. \]

\[ - (m_t + \theta_t + \varphi_t + b_t - d_t) f(-m_t - \theta_t - \varphi_t - b_t + d_t) 
\]

\[ -(-m_t - \theta_t - \varphi_t - b_t + d_t) f(-m_t - \theta_t - \varphi_t - b_t - d_t) \]  

\[ + E \frac{\partial V_{t+1}}{\partial d_t} \]

\[ = -i^d [1 - F(-m_t - \theta_t - \varphi_t - b_t + d_t)] + E \frac{\partial V_{t+1}}{\partial d_t} . \]  

(2.40)

In a manner similar to that above, I analyse 3 cases:

1. The shock forces the bank to use a lending facility, whereby the deficiency remains unchanged.

\[ \frac{\partial d_{t+1}}{\partial d_t} = 1 \rightarrow \int_{-\infty}^{-m_t - \theta_t - \varphi_t - b_t} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon) d\varepsilon = \int_{-\infty}^{-m_t - \theta_t - \varphi_t - b_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon . \]  

(2.41)

2. The shock forces the bank to use a deposit facility, whereby all deficiency gets satisfied, so that

\[ \frac{\partial d_{t+1}}{\partial d_t} = 0 \rightarrow \int_{d_t - m_t - \theta_t - \varphi_t - b_t}^{\infty} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon) d\varepsilon = 0 . \]  

(2.42)
3. The intermediate case, where the shock value only reduces the deficiency without forcing the bank to use any of the facilities, is similar to case 1.

\[
\frac{\partial d_{t+1}}{\partial d_t} = 1 \rightarrow \int_{-\infty}^{d_t-m_t-\theta_t-\phi_t-b_t} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon)d\varepsilon = \int_{-\infty}^{d_t-m_t-\theta_t-\phi_t-b_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon)d\varepsilon,
\]

which yields:

\[
\frac{\partial V_t}{\partial d_t} = -i^d [1 - F(d_t - b_t - m_t - \theta_t - \phi_t)] + \int_{-\infty}^{d_t-b_t-m_t-\theta_t-\phi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon)d\varepsilon.
\]

2.6.4 Simulation method

Demand side

I start with the basic case with reserve requirement and 3-day maintenance period (\(T\)). In the first period each bank decides on the optimal borrowing level based on a) current interest rate b) its own liquidity and c) expectations of future rates. The last element is necessary to calculate the derivatives of the value function, which I assume equal to the central bank target rate. Using the traditional grid method, one can calculate the value functions for all possible states in period \(T\) (last day). By substituting these values into eq. (2.15) one can compute analogous derivatives for period \(T-1\) and finally use them to calculate the optimal borrowing for some initial guess as to the interest rate at \(T-2\). The same procedure is repeated for all \(N\) banks to come up with the aggregate borrowing. Using bisection algorithm I can then quickly find the interest rate that satisfies market clearing condition.

Once the interest rate and optimal borrowing for each bank participant in \(T-2\) are known, one can move on to the following periods \(T-1\) and \(T\). Each time shock realisations move the system to new states (i.e. current accounts and deficiency) for which the above procedure is repeated.

The behaviour of the interbank system without reserve requirements simply replicates the last stage of the procedure above, using different parameters of monetary policy (such as a narrow corridor).

Finally in the last regime, banks have different maintenance periods. To capture that, I set up three groups of banks, with \(n = 3\) banks in each, that share a common ending date of the maintenance period. Thus the total number of banks is the same in all setups.

Using the procedure described above, it is possible to compute the banks borrowing and the market clearing rate. Then resulting current account balance is used to compute the next day’s deficiencies (except of course for the group that starts a new maintenance period, in which case they are reset to the initial values). To ensure that these results are comparable with other setups (with only three periods) I reset the current account balances of all banks at every third iteration. Otherwise, the current account balance, which follows a random walk becomes more volatile with each round of the simulation.
Supply side

In case of a neutral liquidity supply, the intervention value is equal to the remaining reserve requirement divided by the number of remaining days of the maintenance period.

In case the optimisation algorithm is used, I compute the market equilibrium for initial guess and check (using bisection algorithm) whether an improvement can be made by increasing or decreasing the allotment value. The allotted value is distributed evenly to the market participants.

The parameters of the simulation are set so that the model is comparable with earlier research. Increasing the shock variance or decreasing the market liquidity results in higher volatility, but the results are generally robust.
References


Pérez-Quirós, G., Rodríguez-Mendizábal, H., 2006. The daily market for funds in Europe: What has changed with the emu. Journal of Money, Credit and Banking 38, 91–118.


Välimäki, T., 2006. Why the marginal mro rate exceeds the ecb policy rate? Discussion Papers 20, Bank of Finland.

Chapter 3

Liquidity shocks in the Eurosystem interbank market

3.1 Introduction

European Central Bank control over inflation in the Eurosystem is based on the power to affect interest rates in the economy. In practical terms the attention is focused on the average transaction rate in the overnight interbank market, which is later used as a benchmark for loans and deposits of commercial institutions. How the central bank tools can affect the interbank market is then the central issue for the effectiveness of the monetary policy implementation and the topic of this chapter.

Analysing the main features of the money market might suggest that the central bank has a near complete control. Commercial banks need funds to process and facilitate transactions performed by their customers every day. In addition, they need to satisfy the reserve requirement and are not allowed any overdraft on their account in the central bank. Available sources of liquidity, which include open market operations and standing facilities, are directly controlled by the central bank. Closer inspection, however, reveals areas where that control is not so tight: banks need to maintain an average value of funds on their account, meaning they might choose to keep less one day and compensate later on. This can have very important consequences (first pointed out by Hamilton (1996) as the so-called martingale hypothesis): if there were no additional barriers to trade, the current rate must be the same as the expected rate in the future in order to prevent arbitrage opportunities. In that case, however, the interest rate is determined by the market expectations rather than by liquidity controlled by the central bank.

The martingale hypothesis was initially rejected by Hamilton (1996), who attributed the deviation to the trading costs. More recent econometric papers by Moschitz (2004), Würtz (2003) (for Eurosystem) and Thornton (2001) (for the USA) fail to find evidence against this hypothesis for most of the days of the maintenance period. However, the same researchers find that the market liquidity becomes an important determinant of the interbank interest rate on the last days
of the maintenance period.

The theoretical contributions include models of Pérez-Quirós and Rodríguez-Mendizábal (2006), Välimäki (2003), Clouse and Dow Jr. (2002) and Bartolini et al. (2001) that focus on explaining the increase in the average level of the interest rate and interest rate volatility at the end of the maintenance period without using trade barriers. The models that have a similar structure to this thesis, all include so-called liquidity shock as defined initially by Poole (1968). This term captures all the liquidity flows that were not anticipated by a commercial bank when it was deciding on its borrowing value in the interbank market. One example might be a late transaction request received after the trading day is over.

From the banks’ perspective, such liquidity shocks introduce a costly randomness into the final end-of-day current account balance that is, in turn, used to satisfy the obligatory reserve requirement. Consider, for example, a situation where a positive liquidity shock causes the bank account to exceed the remaining part of reserve requirement for this maintenance period. Any excess funds must then be deposited at the penalty rate, which is significantly below the market rate. To avoid such an expensive operation, banks need to make sure that the current account before the shock realisation remains at the appropriate level. Pérez-Quirós and Rodríguez-Mendizábal (2006) show that this can result in additional incentives for, so-called, backloading. If all banks share similar incentives, the liquidity becomes more valuable at the end of the maintenance period, which might explain the increasing pattern in the interest rate behaviour that was observed in many interbank markets in the late 90s and discussed in the previous chapter.

Even though the liquidity shock is a central issue in most of the papers on the interbank market, its properties have not been properly analysed thus far. In fact, to the author’s knowledge, until recently there has been no attempt to estimate the statistical parameters based on actual market data. The only study that deals with similar issues is Cassola (2008), whose findings are very similar to this chapter.

This chapter contains several important additions that contribute to existing interbank market literature.

First, I present a calibration of the size of the liquidity shock volatility based on actual Eurosystem money market data. Regressing the deviations of the banks’ current account balances from the remaining reserve requirement, I find that the standard deviation of the liquidity shock is roughly 22% of square root of the average current account holdings. This estimate is significantly lower compared with the values assumed in the previous research.

Second, I show that the magnitude of the shock compared to the current account balance determines the results of the model widely used in the interbank related literature. In particular low liquidity shock variance might result in weak liquidity effect. I also simulate the impact of other assumptions on model behaviour.

Third, I show how the liquidity effect and martingale hypothesis can be related

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1 Backloading bank postpones satisfaction of the reserve requirement until the later part of the maintenance period by keeping the current account value below the average level required.

2 By liquidity effect I refer to the relationship between the equilibrium interest rate and the aggregate liquidity.
to the length of the maintenance period and that the expectations determine the interest rate level until the reserve requirement drops below $6 \times \sigma$, where $\sigma$ is the standard deviation of the liquidity shock.

The combination of these two observations can then be used to explain the behaviour of the Eurosystem interbank interest rate during the maintenance period.

The results of this chapter can have important consequences for the implementation of the monetary policy. Low liquidity shock variance implies that for the majority of the maintenance period, the interest rate is determined by market expectations. These expectations are, in turn, affected by the central bank controlled liquidity supply on the settlement day. These findings are in line with the econometric analyses performed by Würtz (2003) and Moschitz (2004).

In Section 3.2, I start with a general discussion on the interbank market by describing the components of the demand and supply of funds.

In Section 3.3, I briefly present a modified version of the standard Poole (1968) model based on Pérez-Quirós and Rodríguez-Mendizábal (2006) and the previous chapter and show the analytical solution limitations that justify the use of numerical methods.

In Section 3.4, I use the data obtained from the ECB to calibrate the liquidity shock volatility.

Section 3.5 contains the theoretical portion of the chapter. I show the impact of shock variance on the interbank market behaviour. Assuming the parameter values calibrated in Section 3.4, there exists a flat part in the demand schedule where borrowing cannot be uniquely determined under risk neutrality. The range of the flat schedule depends on the day of maintenance period and essentially disappears when the remaining part of the reserve requirement drops below $6 \times \sigma$ (where $\sigma$ is the standard deviation of the liquidity shock). This suggests that the interest rate will follow the martingale process for the majority of the maintenance period and might only deviate shortly before the settlement day (when the remaining required reserves drop low).

Finally, in Section 3.6, I simulate the behaviour of the market under several different scenarios using the calibrated parameters. I find confirmation for the theoretical results from the previous section; it seems that the interest rate follows the martingale hypothesis for all but the last days of the maintenance period, lending strong support to the results obtained by Moschitz (2004) and Würtz (2003). Section 3.7 concludes.

### 3.2 The interbank market in the Eurosystem

This section reviews some of the key features of the interbank market that were introduced in Section 2.2 in the previous chapter. Here, I focus on those aspects that are relevant for the analysis of the liquidity shock.

#### 3.2.1 Aggregate liquidity effect

The supply of aggregate liquidity remains at the central bank discretion. Once the aggregate liquidity is settled during the open market operations, its balance can only be altered by changes in the past and expected autonomous liquidity factors
(ALF). These include, for example, cash, government accounts and other elements of the base money that are not controlled directly by the central bank. The ability to predict these changes is typically one of the main concerns of the central bank, which uses the aggregate liquidity to affect the interbank rate. Even though sophisticated and accurate tools were developed to forecast such changes, policy design occasionally limits the ability to fully accommodate them. For instance, in the Eurosystem, the main refinancing operations are performed only once a week, while changes to ALF take place every day. Therefore, the market liquidity might and will deviate (at least for a short time) from the central bank’s target level.

The degree to which the aggregate change in the liquidity will influence the interest rate is signified by the *liquidity effect* and can be well assessed using econometric methods. A common approach involves regressing changes in the market rate on the aggregate imbalance measured by the net recourse to standing facility. Studies done by Würtz (2003) and Moschitz (2004) for the Eurosystem data show that the effect is only present in the last two days of the maintenance period. My own calculations, based on the publicly available data for the period March 2004 - April 2006, return an estimate for the liquidity effect on the last day of the maintenance period at 0.0754 (a EUR 1B imbalance results in a rate change of 7 basis points). Similar studies by Hamilton (1997) and Thornton (2001) for the US markets also find little evidence of the effect.

### 3.2.2 Individual liquidity shock

The demand for reserves by the commercial banks is determined first by the obligatory reserve requirement and, second, by their balance of transactions with other market participants. The reserve requirement is known in advance at the start of the maintenance period. The balance of transactions can typically be estimated quite accurately by the bank itself based on past time experiences. For example, a large mortgage bank can predict substantial inflows of liquidity on days when its customers are making loan repayments.

Even though the forecasts might be quite accurate, there is still some uncertainty that banks need to account for. It stems primarily from late transactions (finalised after the trading day is over) or errors in transaction processing. Historically, when the information technology was barely developed, the transactions between banks’ customers were processed in clearing sessions. Sometimes, the outcome of the session was announced after trade was concluded (or close to the end of the trading day). In this case, banks that needed liquidity to cover the balance of payments of their customers had no way of obtain the funds on the market and had to resort to the central bank facilities. Nowadays, with the RTGS systems in place, this problem might be less severe.

Alternatively the uncertainty can also occur as a result of forecast error. For instance, the commercial bank expects outflow of EUR 100 million and plans longer term finance accordingly. During the day, only EUR 80mln is actually realised. If the bank in the meantime secured EUR 100 million finance, its current account balance is EUR 20mln higher than expected.

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3Real Time Gross Settlement
From a commercial bank's perspective, the late transactions or forecast errors constitute a random deviation between the current account balance at the end of the day and the expected level. In this chapter I refer to them as *liquidity shocks*. In the remaining part of this section, I discuss the various statistical properties and theoretical assumptions required for liquidity shock modelling.

### 3.2.3 Modelling liquidity shock

The liquidity shock, denoted as $\varepsilon_t$ for the remainder of the chapter, is the key element of the model. If all banks could perfectly predict their current account balances, no buffer would be necessary for the purpose of averaging the reserve requirement.\(^4\) Some of the modelling assumptions need, however, a bit of explanation.

First I have chosen to focus on individual sources of uncertainty rather than aggregate ones. In practice, I assume that the central bank compensates for changes in the autonomous liquidity factors (which affect the total market liquidity) on a daily basis, even though, in reality, the operations take place once a week.

Second, I assume that the shock is identically and independently distributed. I assume a normal distribution with an expected value equal to zero and the standard deviation $\sigma$. A zero mean reflects the fact that the banks do not expect the shock to be biased toward neither a surplus nor a shortage of liquidity. There might be a pattern in the actual market that would link some bank characteristics with expected liquidity shocks. For instance, large banks tend to be net liquidity borrowers, which might suggest that the liquidity shock they face would be, on average, negative. However, the forecast errors can result in the current account balance being over or below the expected level, regardless of the average direction of funds flow (just as in the example above). In addition, I have no data on banks' borrowing, thus for the remainder of the chapter, I simply assume a symmetric distribution.

The value of $\sigma$ is calibrated in Subsection 3.4.1. I assume the standard deviation of the liquidity shock to be proportional to the size of the bank as measured by the average central bank account balance. Let me please discuss this assumption.

On one hand, it is quite sensible to assume that large banks with a sizable consumer base, involved in multiple complicated operations are subject to a larger absolute amount of errors in transaction processing and a larger number of late transactions. In addition, large banks are more likely to run into problems with market depths, especially if the trading groups they operate with are small and unable to satisfy their demand for liquidity (or alternatively absorb surpluses). These limitations might result in higher shocks. Finally, the magnitude of liquidity forecast errors will be probably much larger, when compared to a bank with only small number of customers. Therefore, it is very likely that the shock variance increases with the bank size.

---

\(^4\)Averaging reserve requirement means that the banks have to satisfy an average reserve requirement within the maintenance period, but are free to choose their daily values. That means, that random deviation from the target assigned by the central bank will not force the immediate use of the standing facilities provided that the bank can compensate for this in the following days.
On the other hand, however, large banks can probably afford more staff, whereby the transactions would be handled more carefully, and liquidity flows would be more accurately predicted with the ultimate result that fewer errors would be made. Therefore, the linear relationship between the bank size and shock variance is unlikely. In this chapter, I account for that and assume that the shock variance scales with the square root of the bank average current account.

3.2.4 Interest rate expectations

Interest rate expectations are an important aspect of the monetary policy implementation and also play a significant role in this thesis. For instance, in the simulation design I assume that the banks expect the future interest rate to remain at the central bank target level. In this section, I analyse this assumption in more detail.

I provide an extensive discussion of the role of expectations in Subsection 3.5.1, where I also point out their implications for central bank policy implementation. To summarise the discussion in that section, the central bank must control the market expectations that determine the level of the interbank interest rate. I also show that this can be accomplished by adjusting the market liquidity on the settlement day.

In this section, I test statistically how efficient is this control in practice in the Eurosystem.
In order to test whether the interest rate expectations follow central bank target, I compare 1 week Eonia swaps with the corresponding ECB target interest rates. The Eonia Swap index, calculated by European Banking Federation, is available since 20 June 2005 and I analyse the relatively stable period 20 June 2005 - 31.12.2006. Figure 3.1 presents the ECB target rate and 1 week Eonia Swap index for this period. The figure reveals that Eonia Swap seems to follow the target rate very closely, and the observed volatility was mostly related to the expected changes in the target rate.

To remove the effect of expected changes in the target rate, I exclude the observations taking place 7 days before Governing Council’s policy announcements. This way I avoided the problems with Eonia swaps capturing market expectations extending into next maintenance period.

I test the null hypothesis of unit root using the augmented Dickey-Fuller (ADF). I reject the null at 99% confidence level, which allows me to conclude that the

5The following table presents the results of testing using ADF (** denotes statistically significant at 99% confidence level)

<table>
<thead>
<tr>
<th>D-lag</th>
<th>t-adf</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-5.361**</td>
</tr>
<tr>
<td>1</td>
<td>-6.156**</td>
</tr>
<tr>
<td>0</td>
<td>-5.991**</td>
</tr>
</tbody>
</table>
spread between Eonia swaps and the ECB target is stationary. Thus, the interest rate expectations do follow the central bank target rate.

3.3 The model

This section presents an overview of the model of the interbank market that constitutes the basis of the simulation study in Section 3.6. The model is based on Poole (1968), followed by Hamilton (1996) and, more recently, Pérez-Quirós and Rodríguez-Mendizábal (2006), Välimäki (2003) and Bartolini et al. (2001). Compared with the previous chapter, here I concentrate on the late liquidity shock and do not consider the open market operations. This allows me to base the analysis on the model presented in Section 2.3. In this section, I only review the key equations that are used later in the chapter.

The model illustrates the behaviour of the individual bank in the interbank market with the averaging reserve requirement provision regime described in more detail in Chapter 3.

The timing of events in the model for each day of the maintenance period is as follows:

\[ m_t, d_t \rightarrow b_t \rightarrow \epsilon_t \rightarrow m_{t+1}, d_{t+1} \]

The bank starts the day with current account balance \( m_t \) and a deficiency \( d_t \) from the previous day. It can obtain more liquidity through overnight interbank borrowing represented by the term \( b_t \), which can also be interpreted as a simple demand for liquidity at market rate. The liquidity shock, \( \epsilon_t \), was explained in more detail in the previous section. These three factors contribute to the final end-of-day current account balance \( m_t + b_t + \epsilon_t \) (a negative \( b_t \) means that the bank is lending), which, depending on its value, is used to either satisfy the reserve requirement or deposited in the central bank. Overdraft is not allowed, therefore if \( m_t + b_t + \epsilon_t < 0 \) the bank needs to use lending facility.

The last term, deficiency \( d_t \), represents the remaining portion of the reserve requirement to be satisfied, as explained already in Chapter 2, Section 2.3. I assume that the starting value of the deficiency is the value of starting current account balance multiplied by the length of the maintenance period \( d_1 = m_1 \ast T \), thus each bank has (initially) sufficient reserves to satisfy the requirement.

The bank’s problem is to minimise the borrowing cost by choosing an appropriate value for \( b_t \) for each day of the maintenance period. The problem has a finite horizon and has a different structure for the settlement day and for the days preceding it.

3.3.1 Time T

The optimal borrowing problem for the last day of the maintenance period is indeed the same as for the case without an averaging provision reserve requirement regime and was already solved by Poole (1968). I provided the derivations in the previous
chapter, Section 2.3.1. The scenario analysed in this chapter does not include early aggregate liquidity shock nor central bank intervention. Thus, the optimal borrowing on the last day of the maintenance period satisfies slightly modified condition (2.12)

\[ i_T = i^d T F( -m_T - b_T + d_T ) + i^d (1 - F( -m_T - b_T + d_T )). \]  

(3.1)

At the optimal volume of borrowing, the probability weighted cost of using standing facilities is equal to the cost of obtaining funds in the market.

The equilibrium condition for the interest rate in the middle of the interest rate corridor is identical to equation (2.20), in Chapter 2, and, in the absence of early liquidity shock or central bank intervention, yields:

\[ \bar{m}_T = \bar{d}_T, \]  

(3.2)

where \( \bar{m}_T = \sum^N_i m_{T_i} \) is the aggregate market liquidity and \( \bar{d}_T = \sum^N_i d_{T_i} \) is the aggregate deficiency. This result is used by the central bankers that choose the value of the aggregate market liquidity in their operations executed on the last day of the maintenance period.

3.3.2 Periods \( t < T \)

Before the end of the maintenance period, the commercial banks’ borrowing also affects also the deficiency that needs to be satisfied in remaining days of the maintenance period. The problem becomes dynamic and in Chapter 2, Section 2.3.2 I showed that the optimal borrowing satisfies condition (2.14), which after abandoning aggregate shock and liquidity intervention becomes:

\[ i_t = i^d T F( -b_t - m_t ) + i^d T [1 - F( d_t - b_t - m_t )] - \int_{- \infty}^{d_t - b_t - m_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f( \varepsilon_t ) d\varepsilon_t. \]  

(3.3)

Very similar formulation is presented eg. in Pérez-Quirós and Rodríguez-Mendizábal (2006). The interest rate is the probability weighted average of three terms, each of them with a specific interpretation. Term 1 reflects the expected cost of the shock (negative) falling below the bank’s current account balance, thus forcing it to use the lending facility. Term 2, in a similar fashion, reflects the cost of the shock that exceeds the current deficiency and forces the bank to use the deposit facility. Finally, the last term gives the value required to satisfy only part of the reserve requirement and to carry over the deficiency to the next period.

The last term (sometimes called the dynamic cost factor in literature and in this chapter) depends on the derivative of the value function that I derived in the previous chapter:

\[ \frac{\partial V_t}{\partial d_t} = -i^d T [1 - F( d_t - b_t - m_t )] + \int_{- \infty}^{d_t - b_t - m_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f( \varepsilon_t ) d\varepsilon_t. \]  

(3.4)
The cost of an additional unit of deficiency depends on how close the bank is to satisfy total reserve requirement, in which case any positive account balance is deposited at the central bank interest rate $i^d$ (term 1).

A demand schedule based on equation (3.3) is illustrated in Figure 3.2. Note, that some parts of the figure are flat, which means that the demand for liquidity, at certain interest rate levels, is not uniquely determined. More details and exact interpretation of the shape will be included in the latter part of the chapter, in Subsection 3.5.1.

Figure 3.2: 3-day maintenance period
Liquidity(x-axis): demand for liquidity, when the bank’s current account equals the remaining deficiency/days until the end of the maintenance period.

3.3.3 Example: reserve maintenance period with 3 days
In this example, I am going to show how the model can be used to analytically calculate the individual bank demand for liquidity under the assumption that the expected rate is equal to the central bank’s target set in the middle of the standing
facilities corridor. Similar results, for the last day of the maintenance period, were obtained by Cassola (2008).

Due to the recursive structure of the problem, I start the solution from the last day of the reserve maintenance period.

**Day 3** On day 3, the solution is identical to Subsection 3.3.1. At the market equilibrium rate:

\[ i_3 = i^d F(-m_3 - b_3 + d_3) + i^d (1 - F(-m_3 - b_3 + d_3)), \]  

(3.5)

where \( i_3 \) is the interest rate, \( m_3 \) is the starting current account balance, \( b_3 \) is the interbank borrowing and \( d_3 \) starting deficiency on day 3. At the central bank target, the rate \( i_3 \) is in the middle of the corridor system, and bank’s borrowing is given by

\[ b_3 = d_3 - m_3. \]  

(3.6)

The derivative of the value function is \( \frac{\partial V_3}{\partial d_3} = -i_3 \) with the same interpretation as in Subsection 3.3.1.

**Day 2** On day 2, I can use the fact that the future value function derivative is fixed for all banks regardless of their current liquidity, hence for each bank:

\[ i_2 = i^d F(-b_2 - m_2) + i^d [1 - F(d_2 - b_2 - m_2)] \]

\[ + \int_{-b_2 - m_2}^{d_2 - b_2 - m_2} E(i_3) f(\varepsilon_2) d\varepsilon_2, \]  

(3.7)

so

\[ i_2 = i^d F(-b_2 - m_2) + i^d [1 - F(d_2 - b_2 - m_2)] \]

\[ + E(i_3) [F(d_2 - b_2 - m_2) - F(-b_2 - m_2)], \]  

(3.8)

which has a similar interpretation as in Subsection 3.3.2.

Assume for a moment that the expected interest rate on day 3, \( E(i_3) \), is equal to the central bank target in the middle of the corridor system. At the interest rate equal to the central bank target, bank’s borrowing is given by:

\[ b_2 = d_2 \frac{d_2}{2} - m_2. \]  

(3.9)

This is an intuitive result that indicates that half of the remaining reserve requirement should be satisfied in order for the interest rate to remain in the

---

6 The corridor is defined by standing facility rates. The central bank promises to accept any values of deposits at its lower boundary (\( i^d \)), and similarly grants unlimited loans at its upper boundary (\( i^l \)), thus rendering any transactions outside that range unfeasible. In the Eurosystem, the corridor is symmetric around a target rate.

7 As shown in Appendix 3.8.1
middle of the corridor system. The derivative of the value function takes the following form for each individual bank:

\[
\frac{\partial V_2}{\partial d_2} = -i^d \left[ 1 - F(d_2 - b_2 - m_2) \right] - E(i_3) F(d_2 - b_2 - m_2). \tag{3.10}
\]

Unless deficiency \(d_2\) is significantly larger than the current account \(m_2\) (so \(F(d_2 - b_2 - m_2) = 1\)), this expression is going to take a different value for each individual bank, thus rendering the aggregation process more complicated and analytically intractable.

**Day 1** The analytical solution for this period is not possible without further assumptions. To illustrate this fact, I use equation (3.3) once again. Substituting (3.10) for \(\frac{\partial V_2}{\partial d_2}\) yields the following relationship at the equilibrium rate:

\[
i_1 = i^d F(-b_1 - m_1) + i^d \left[ 1 - F(d_1 - b_1 - m_1) \right] - \int_{-b_1 - m_1}^{d_1 - b_1 - m_1} \left[ -i^d \left[ 1 - F(d_2 - m_2 - b_2) \right] - E(i_3) F(d_2 - m_2 - b_2) \right] f(\varepsilon_1) d\varepsilon_1,
\tag{3.11}
\]

which can be further simplified using (3.9):

\[
i_1 = i^d F(-b_1 - m_1) + i^d \left[ 1 - F(d_1 - b_1 - m_1) \right] + \int_{-b_1 - m_1}^{d_1 - b_1 - m_1} \left[ -i^d \left[ 1 - F(d_2/2) \right] - E(i_3) F(d_2/2) \right] f(\varepsilon_1) d\varepsilon_1. \tag{3.12}
\]

The first two terms look familiar and have the same interpretation as above. The value of the dynamic cost factor depends, however, on the deficiency \(d_2\), which, in turn, is determined by the liquidity shock realisation. This means that the analytical solution is no longer possible without distribution function assumptions. This motivates the use of the numerical tools presented in Section 3.6.

### 3.4 Calibration of the parameters for the Eurosystem

#### 3.4.1 Liquidity shock estimation

In this subsection, I show the procedure and results for the calibration of the liquidity shock properties. The key parameter is the standard deviation \(\sigma\) of the liquidity shock. If the shock volatility is low, the commercial banks using the market could plan the path of the reserve requirement well in advance to avoid any use of the facilities. A high volatility of the shock means their forecasts are prone to errors, and recourse to the standing facilities are likely and need to be taken into consideration. As discussed in Subsection 3.2.3, I assume the volatility of the shock is related to the bank size, hence, my focus is on the estimation of that relationship.
Table 3.1: Sample Banks

<table>
<thead>
<tr>
<th>Country</th>
<th>Number of banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>15</td>
</tr>
<tr>
<td>France</td>
<td>12</td>
</tr>
<tr>
<td>Spain</td>
<td>12</td>
</tr>
<tr>
<td>Italy</td>
<td>11</td>
</tr>
<tr>
<td>Belgium</td>
<td>6</td>
</tr>
<tr>
<td>Netherlands</td>
<td>7</td>
</tr>
<tr>
<td>Ireland</td>
<td>4</td>
</tr>
<tr>
<td>Finland</td>
<td>4</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>3</td>
</tr>
<tr>
<td>Greece</td>
<td>3</td>
</tr>
<tr>
<td>Portugal</td>
<td>4</td>
</tr>
<tr>
<td>Austria</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>86</strong></td>
</tr>
</tbody>
</table>

To accomplish that goal, I have gathered the following data for $N = 86$ selected banks covered by the reserve requirement in the Eurosystem, for each day during the period 24 January 2004 - 31 May 2005:

1. Current account value at the end of the day
2. The use of standing facilities
3. The reserve requirement applicable for the relevant maintenance period

The information was provided by national central banks, and includes a number of banks of different sizes from each country (see Table 3.1). In addition, I have obtained an extended sample (including the year 2003) for all the banks except German. The results with the extended period (but without Germany) were very similar to the ones presented below; hence, for the remainder of the chapter, I use a shorter but broader sample.

The results of the calibration are presented in Table 3.2. In the remaining part of the subsection, I will explain the calculation procedure and interpret the value of the parameters.

First of all, we recall that the liquidity shock occurs after trade is no longer possible and constitutes an error in the current account forecasts. Hence, if I knew the target current account value for a sample bank, I could compare it to the actual realisation and the difference attribute to the shock. This is difficult to perform during the maintenance period, when banks might choose different paths for the reserve requirement, depending on their individual expectations for the interest rate evolution or other motives such as risk aversion (see next chapter for discussion). However, on the last day of the maintenance period, any cost-minimising

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8The data were obtained during an internship at the ECB during the period April - June 2006. I am very grateful to the ECB for the access to the data and all help provided during this period.
bank avoiding costly standing facilities would simply target the current account balance that is equal to the remaining value of the reserve requirement. That gives me enough information to compute the liquidity shock for those days. The sample covers 15 maintenance periods, thus providing 15 realisations of liquidity shock for each individual bank.

The exact procedure to calculate the shock value is fairly straightforward. I have the data on each bank’s end of the day current account balance after the use of the standing facilities. This balance, according to the central bank regulations, for the last day of the maintenance period, must not be lower than the remaining deficiency.

I also have the data on the use of the standing facilities (included in the final current account balance), so I am able to compute the current account value directly after the liquidity shock realisation.

Linking this data to the theory presented in Section 3.3 results in the following relationship:

\[
(f_{\text{final current account}})_{it} - (\text{net recourse to standing facilities})_{it} = m_{it} + b_{it} + \varepsilon_{it}. \tag{3.13}
\]

From the available data, I can also compute how much of the reserve requirement each bank has satisfied earlier in the period. That corresponds to term \(d_{it}\) from Section 3.3. According to formula (3.6), a cost minimising bank would target the current account value equal to the remaining deficiency.

\[
m_{it} + b_{it} = (\text{deficiency})_{it} \tag{3.14}
\]

This can then be substituted into eq.(3.13) to obtain an estimate for the liquidity shock \(\varepsilon_{it}\):

\[
\varepsilon_{it} = (f_{\text{final current account}})_{it} - - (\text{net recourse to standing facilities})_{it} - (\text{deficiency})_{it}. \tag{3.15}
\]
Index \( it \) indicates the realisation for bank \( i \) on day \( t \). The other statistics from the Table 3.2 are straightforward. The top entry in the first column gives the average liquidity shock across all banks included and periods and is computed as:

\[
\frac{1}{NT} \sum_{i}^{N} \sum_{t}^{T} \epsilon_{it},
\]

(3.16)

where \( \epsilon_{it} \) is a single shock realisation for bank \( i \) at time \( t \). The standard deviation reported in the second column is computed as the average standard deviation across all banks included:

\[
\frac{1}{N} \sum_{i}^{N} SE_{i},
\]

(3.17)

where \( SE_{i} \) is the standard deviation of the liquidity shock for bank \( i \).

In order to compute the aggregate shock, I have added up the shocks of all the banks in the sample. In the last column, I have included the average current account value I used as an approximation of the bank size. Large banks, with a wide scale of operations, are likely to have a higher reserve requirement base and, hence, maintain a higher current account value.

Is the liquidity shock significant in the Eurosystem in light of the results from Table 3.2? Compared to the daily current account value, the shock does not seem to be substantial enough (the next section concentrates on that link a bit more) to constitute a serious threat of overdraft to the commercial bank. Unless the bank decides to satisfy the reserve requirement prematurely, the possibility of a positive shock large enough to force the use of the deposit facility seems even lower. This means, however, that the probability of using the standing facilities during the maintenance period is negligible, an issue further covered in the next section.

Any estimates of the liquidity shock for the Eurosystem should be approached with care due to the existence of previously mentioned excess reserves\(^9\). While the model predicts that any excess liquidity will be deposited, this is often not the case in reality. This has profound consequences for the efficient determination of the liquidity supply. It is not just sufficient to calculate the proportion of the reserve requirement that remains to be satisfied together with expected changes in the autonomous liquidity factors. The central bank must also estimate how much of the supplied liquidity will remain "unused" for any of these purposes, and the error in that estimate will result in the market imbalance. What makes the exercise tricky is that, unlike the autonomous liquidity factors, the value of the excess reserves might indeed be correlated with the allotment. As an example, consider a policy in which the central bank forces frontloading behaviour by supplying excess liquidity early in the maintenance period and removing it toward the end. One consequence of such a policy is the increased number of banks that will satisfy the reserve requirement before the end of the maintenance period, thereby implying an

---

\(^9\)Excess reserves are the funds stored in the ECB account which are not reported to the deposit facility after satisfying all the reserve requirement. The underlying reason for the existence of those reserves are the costs associated with the standing facilities, which might discourage some banks from their use.
increased probability of excess reserves. An increase in excess reserves works the same way as an increase in autonomous liquidity factors and has a negative effect on the available market liquidity thereby potentially diminishing the effectiveness of the liquidity supply. In practice, however, no such pattern is observed. Miscalculations of the excess reserve value add to the errors when estimating changes in autonomous liquidity factors and do not distort the aggregate deficiency in a significant manner.

3.4.2 Parameter calibration

In order to simulate the behaviour of the whole market, I use the numerical methods presented in the following sections. I will show that the results are sensitive to the assumed variance of the liquidity shock as compared to the average current account balance. Using the individual bank data, I can actually try to estimate that relation for the Eurosystem and compare it to the assumptions made in existing literature.

In Subsection 3.2.3, I discuss the relationship between shock volatility and bank size. To briefly summarise, large banks probably experience more late transaction requests or transaction errors simply due to the scale of their operations. However, since they might employ additional staff to deal with those problems more efficiently, the relationship between liquidity shock variance and bank size is unlikely to be linear.

Considering the available data, I should mention extreme heterogeneity within the sample. Out of the 86 banks from which I collected data, the smallest bank average current account value was equal to EUR 230 thousand, while the largest one EUR 4.2 billion. The liquidity shock variance is likely to depend on several variables other than just the size of the bank. Bank type, country of origin and even ownership structure might determine the degree of randomness the bank is facing. Unfortunately, I do not have such detailed information, hence, the estimates presented here must be taken with caution.

Given the available information, I can approximate the parameters of the liquidity shock using a basic regression. I assume non-linear, square root relationship between the bank size and shock standard deviation in order to capture the economies of scale. The estimated equation takes the form:

$$ SE_i = \alpha + \beta \sqrt{a_i} + \epsilon. \quad (3.18) $$

$SE_i$ is the average standard deviation of liquidity shocks for bank $i$ measured across 15 realisations for the last days of the maintenance period. $a_i$ is a simple average current account value for bank $i$ and $\epsilon$ is the error term. The results of the regression are presented in the Table 3.3.

Parameter $\beta$ is statistically significant with a sufficient level of confidence to justify its use in the simulation stage. It seems that the volatility the banks are facing is indeed quite small when comparing the values assumed, for example, in Pérez-Quirós and Rodríguez-Mendizábal (2006) (they assumed the liquidity shock standard deviation roughly equal to current account). In their second paper
<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>standard error</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-23.45</td>
<td>22.82</td>
<td>-1.03</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.27666</td>
<td>0.02609</td>
<td>10.6</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.57</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.3: Regression results

(Gaspar et al. (2008)) they have however lowered their estimate of shock variance to around 30% of the current account. The discussion of the impact of the parameter value on the model behaviour is included in Section 3.5.

In the simulation, I had to take into account grid limitations (I calculate the possible states for grid size equal to 3 times the shock standard deviation) and assumed the shock with a standard deviation of 10% of the current account value. Recalculating the simulation with even lower volatility does not change the results; using significantly higher values, however, yields results that are close to those obtained by other researchers. This only occurs at the shock standard deviation that is equal to roughly daily account value, which leaves me plenty of room for potential estimation errors.

3.5 The simulation study description and discussion

In this section, I apply the model discussed above to verify the behaviour of the interbank market given the estimated parameter values. The detailed description of the simulation method can be found in Chapter 2.

3.5.1 The shape of the demand schedule for risk neutral banks

Before I move on to simulations, it is useful to once again return to the demand schedule for an individual bank. Figure 3.2. presents the demand curves for the last 3 days of the maintenance period. To construct that figure, I arbitrarily assumed the current account balance and a daily reserve requirement equal to roughly 160 units. The liquidity shock has an assumed standard deviation of 50 units, which is much larger than my estimate but allows the pictures to remain clear.

Flat parts of the demand schedule, multiple borrowing values and the reserve requirement

First of all, note that for the days preceding the end of the maintenance period, there is a flat part of the demand schedule where different values of market liquidity result in the same interest rate. The size of the flat buffer is determined by the reserve requirement as documented in Figure 3.3, where I drew several demand curves with different value for the starting current account balances (corresponding to the reserve requirement). As the relative volatility increases, the flat part becomes shorter.
Figure 3.3: Different values of reserve requirement
Liquidity (x-axis): demand for liquidity, when the bank’s current account equals the remaining deficiency/days until the end of the maintenance period.

It is perhaps easier to understand the relationship between the average current account value and the shock volatility by combining the graphical and algebraic interpretation. In order to draw Figure 3.3, I have assumed a normal distribution of the liquidity shock and have limited the grid of possible shocks to three times the standard deviation $\sigma$.\footnote{The assumption is also used in the simulation study.} This is a fairly mild assumption, widely used in the related literature, that covers 95% of the distribution and it is necessary to use numerical tools. With this assumption, however, I can solve equation (3.3) for three special cases.

**Case 1** Large lending value ($b_t < -m_t - 3 \ast \sigma$)

The condition $b_t < -m_t - 3 \ast \sigma$ can be transformed into $-b_t - m_t > 3 \ast \sigma$. Thus, assuming the shock realisation will never exceed three times its standard deviation, the expressions $F(-b_t - m_t) \rightarrow 1$ which implies also $F(d_t - b_t - m_t) \rightarrow 1$. This means that the probability of a negative current account balance, at the end of the
day, is one, and the bank will be forced to use the central bank borrowing facility. Lending even more does not affect that probability (since it is already close to one), hence, the demand schedule is flat. Of course, the bank would only agree to lend so much at a market rate that is no lower than the lending facility rate. In fact, using \( F(-b_t - m_t) = 1 \) and \( F(d_t - b_t - m_t) = 1 \) in equation (3.3) reduces it to:

\[
i_t \approx i^d.
\]

**Case 2** Large borrowing value \((b_t > d_t - m_t + 3 \times \sigma)\)

For a sufficiently large borrowing value, \( d_t - b_t - m_t < -3 \times \sigma \), which implies \( F(d_t - b_t - m_t) \to 0 \) and \( F(-b_t - m_t) \to 0 \). The probability of the current account balance that exceeds all the remaining reserve requirement, at the end of the day, is one. Thus, by borrowing enough liquidity the bank is sure to satisfy its entire reserve requirement and is forced to deposit any remaining reserves in the central bank. The bank would only borrow such reserves at the rate that is no higher than the deposit facility rate. Substituting \( F(d_t - b_t - m_t) = 0 \) and \( F(-b_t - m_t) = 0 \) in eq.(3.3) leads to:

\[
i_t \approx i^d.
\]

**Case 3** Intermediate borrowing value \((-m_t + 3 \times \sigma < b_t < d_t - m_t - 3 \times \sigma)\)

At some intermediate borrowing value, within a specific range of values of \( m_t, \sigma \), \( d_t \) (so that there exists a \( b_t \) that can satisfy this condition),

\[
-b_t - m_t < -3 \times \sigma \quad \text{and} \quad d_t - b_t - m_t > 3 \times \sigma,
\]

assuming normality means that the distribution functions of shock realisations converge to \( F(-b_t - m_t) \to 0 \) and \( F(d_t - b_t - m_t) \to 1 \). The last expression can be also presented as \( 1 - F(d_t - b_t - m_t) \to 0 \). Now both the probability of a negative current account balance and the probability of satisfying all the remaining reserve requirement, are zero. It can be shown that in order for such a borrowing value to exist, the deficiency must be significantly larger than the standard deviation of the shock. A simple transformation of eq. (3.21) yields:

\[
d_t > 6 \times \sigma.
\]

Since \( d_t \) is diminishing throughout the main period, the condition is much more likely to hold in the earlier rather than in the later part of the main period. Additionally, as the starting deficiency is linked with the current account value and the length of the main period, the condition is also more likely to hold for markets with long main periods and high values of reserve requirement (compared with the liquidity shock volatility).

Using \( F(-b_t - m_t) = 0 \) and \( F(d_t - b_t - m_t) = 1 \) in the equilibrium condition (3.3) results in:

\[
i_t \approx - \int_{m_t - b_t}^{d_t - b_t - m_t} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon_t) d\varepsilon_t,
\]

\(85\)
so the interest rate is fully determined by the dynamic cost factor. For the analysed range of borrowing, this expression can be simplified as

\[ i_t \approx -\int \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t. \]  

(3.24)

I can compute the derivatives of the value function \( \frac{\partial V_t}{\partial d_t} \) for the last day of the maintenance period:

\[ \frac{\partial V_T}{\partial d_T} = -i_T, \]  

(3.25)

and using eq.(3.4) for the preceding days:

\[ \frac{\partial V_{T-1}}{\partial d_{T-1}} = -i^d [1 - F(d_{T-1} - b_{T-1} - m_{T-1})] \]

\[ + \int_{-\infty}^{d_{T-1} - b_{T-1} - m_{T-1}} \frac{\partial V_T}{\partial d_T} f(\varepsilon_{T-1}) d\varepsilon_{T-1}. \]  

(3.26)

These results are not new and were obtained e.g. by Pérez-Quirós and Rodríguez-Mendizábal (2006). However, my assumptions allow me to further simplify this expression. I use \( F(d_t - b_t - m_t) = 1 \), and the above condition (3.25) to obtain:

\[ \frac{\partial V_{T-1}}{\partial d_{T-1}} = i_T \]  

(3.27)

Provided the reserve requirement buffer given by \( (d_t - b_t - m_t) \) is sufficiently large, and the borrowing remains in a certain range, so that \( F(d_t - b_t - m_t) = 1 \), the property (3.27) holds for all days \( t \) preceding the end of the maintenance period. Recall that the derivative of the value function indicates how much an extra unit of deficiency changes the expected cost of borrowing and is referred in a literature as the dynamic cost factor. The interpretation of eq.(3.27) is that under certain assumptions, the dynamic cost factor remains constant, equal to the expected level of the interest rate for every day of the maintenance period.

This property combined with condition (3.24) can be presented as:

\[ i_t = E(i_T), \]  

(3.28)

which is an illustration of the celebrated martingale hypothesis, which states that the current rate is equal to the expected interest rate on the settlement day.

The results presented in this section constitute a very important contribution of this chapter. Based on my analysis I can solve the puzzle of dual pattern in Eonia behaviour, where the volatility remains low for most of the maintenance period and only spikes toward the settlement day.

With such a long maintenance period as adopted in the Eurosystem, the starting deficiency \( d_1 \) is very high when compared with the current account balance (assuming linear satisfaction of reserve requirement \( d_t = (T - t + 1) * m_t \)) and
liquidity shock variance. Even if one assumes very high shock standard deviation, e.g., \( \sigma = m_t \), conditions (3.22) and (3.28) hold for all except last five days of the maintenance period.\(^{11}\) Thus, the interest rate is determined primarily by the expectations. If the expectations are consistent with the central bank target (as documented in Subsection 3.2.4), the interest rate remains very close to the target rate, which is actually observed in the Eurosystem.

As the end of the maintenance period draws closer, the remaining deficiency becomes smaller compared to the shock volatility and it is the current liquidity and implied probabilities of using the standing facilities that determine the level of the interest rate. Thus, the rate volatility is higher, which leads to observed spikes on the settlement days.

This observation is confirmed in the empirical papers published in the ECB working series. Moschitz (2004) builds a model of Euro Overnight Index Average (EONIA) taking into account policy variables and operating policy (such as open market operations). He finds that the liquidity effect only holds for the four last days of the maintenance period, and that the main determinants of the interest rate for the rest of the maintenance period are the expected policy rate and expected liquidity on the last days of the maintenance period. Similar results are obtained by Würtz (2003), who reports that the liquidity only has an effect on the last two days of the maintenance period. The most significant determinant of the interest rate is the expected policy rate.

The martingale hypothesis has also implications for the determination of the market equilibrium within the framework of this model.

If the current interest rate is equal to the expected rate in the future, the optimal borrowing will be carried out within a range that will ensure that neither the deposit nor lending facilities are ever used. The range of possible borrowing decisions that could satisfy this condition is determined by the size of the remaining deficiency when compared to liquidity shock volatility. Early on in the maintenance period the deficiency is large, so the borrowing range will be also substantial.

This means that there might be multiple market clearing equilibria, each of them at the same level of interest rate at different individual borrowing values. In fact, using Figure 3.2, an equilibrium where all banks chose the point close to X1 is possible. But this implies that even in case of, say, an aggregate surplus of liquidity, the interest rate will remain at the expected level.

This observation explains the sluggish reaction of the whole market to aggregate liquidity changes. On one hand, the reserve requirement secures the market from unexpected aggregate shocks, such as a change in the autonomous factors which leads to a very low volatility of the interest rate. This pattern is well documented in Moschitz (2004) and Würtz (2003), but another illustration is presented in Figure 3.4, which shows the spread before the ECB target rate and EONIA. The only spikes on otherwise flat graph occur at the end of the maintenance period.

To summarise the findings of this section, the shape of the demand schedule might have significant policy implications. Provided that the expectations for future interest rates remain unchanged, the market will not react to small changes in liquidity, resulting, for example from errors in the estimations of the autonomous liquidity factors.

\(^{11}\)Substituting \( d_t = (T - t + 1) \ast m_t \) and \( \sigma = m_t \) into condition (3.22) yields \( t < T - 5 \).
Figure 3.4: Euro Overnight Index Average (EONIA) Spread
3.5.2 Central bank intervention and interest rate expectations

Based on the discussion above, it is clear that expectations play a key role in the determination of the interest rate level and therefore monetary policy implementation.

The situation where the interest rate is entirely dependent on the current expectations, means less effective tools that are based on the control of liquidity. This affects the open market operations and standing facilities used before the last day of the maintenance period. In the Monte Carlo simulations below I show that the interest rate is largely independent on the aggregate liquidity and the cost of standing facilities.

Weaker impact of liquidity-based tools before the end of the maintenance period does not imply less efficient monetary policy implementation. In this situation the central bank can still rely on its ability to affect the market expectations.

Such control can be accomplished by adjusting the liquidity in late, settlement day operations (such as fine tuning operations run by the ECB). If the market liquidity is sufficient to satisfy the reserve requirement, the interest rate should stabilise on the target level. According to condition (3.28) this should be reflected in the interest rate expectations.

The results of the analysis presented so far can be affected by several factors that might drive expected rate away from the central bank target:

1. Due to the fact that banks avoid the "lock-in" state (when all the deficiency is exhausted) the model predicts some incentives for backloading (postponing satisfying the reserve requirement until the end of the maintenance period). If the aggregate market liquidity does not change, the average rate at the start will be lower compared with the end of the maintenance period.

2. During the maintenance period, some banks might use standing facilities. In addition, the change in autonomous liquidity factors might differ from the forecasted level. This might result in market liquidity deviating from reserve requirement, which, unless corrected by the central bank, will affect the interest rate (and corresponding expectations) on the settlement day.

3. Variable rate tenders might end up with an intervention rate (defined before as rate at which the central bank allots liquidity to commercial banks) that exceeds the minimum bid rate (set at the target level). In order to avoid arbitrage opportunities the expected rate and intervention rate must be equal (otherwise the banks have incentives to either borrow infinite amounts from the central bank or not borrow at all). Thus the liquidity tender procedure might indirectly affect interest rate expectations.

Even though these factors can result in market expectations deviating from policy target, the central banks can minimise their impact:

1. The tendency to backload is related to the size of the reserve requirement buffer. If the reserve requirement is set at a sufficiently high level (related to liquidity shock variance as discussed above), these incentives will largely diminish, reducing the pressure on the interest rates.
2. The problem of aggregate liquidity imbalance on the settlement day can be fixed by executing late liquidity supply operations (such as fine tuning operations in the ECB).

3. The last problem is more difficult to fix as it involves evolution of banks expectations that can cover multiple maintenance periods (see Välimäki (2006) for extensive discussion). The resulting spread between intervention and policy rate tends to, however, remain relatively stable.

In the statistical analysis in Subsection 3.2.4 I show that the interest rate expectations closely follow the changes in central bank target rate and exhibit relatively little volatility. This indicates the ECB success in general control of market expectations. However, inspection of Figure 3.1 also reveals a certain fixed spread between the ECB target and Eonia swaps. This spread results from the tender procedures employed during the analysed period.

The discussion included in this section is reflected in the design of the simulation used in this thesis. In order to justify constant rate expectations at the central bank target level, I first assume that the central bank executes late fine tuning operations on the settlement day. This ensures that the market liquidity is equal to remaining reserve requirement. Second, unlike Chapter 2, I use the calibrated, much lower values of shock variance which reduce the incentives for backloading. Finally, I assume that liquidity is allotted proportionally among all banks, which helps avoiding problems with the spread between target and intervention rate.

3.5.3 Risk aversion

Before moving to the actual simulation study, let me elaborate on the issue of risk aversion.

In the model above I have assumed risk neutrality which implied the banks only care for the expected cost of funding. In reality however, one might question such an assumption and argue instead that banks are risk averse. That means that the banks not only care about the expected value of their funding cost but also about the cost variance.

In particular, one might suspect that since the last day typically experiences the highest variance of the interest rate, banks that expect a net outflow of liquidity would like, for example, to frontload in order to avoid using the market as a funding source.

To see how a bank might accomplish that within this framework, one can use the model to compute the last day profit variance. For the last day of the maintenance period, the cost equation (2.10) in Chapter 2 implies:

\[
\text{var}(\text{cost}) = \text{var} (i_T b_T + E(c_T)) = \text{var} \left( i_T b_T + (i^l - i^d) \left[ \int_0^\infty \varepsilon_T f(\varepsilon_T) d\varepsilon_T \right] \right). \tag{3.29}
\]

\footnote{As shown in Appendix 3.8.2.}
If the expected interest rate is in the middle of the standing facilities corridor, the bank chooses borrowing value equal to $d_T - m_T$ (see condition (3.6), at which the expected cost of using the standing facilities is zero. Thus, the second part of the expression in brackets vanishes, which allows for a further simplification of eq. (3.29):

$$\text{var}(iTb_T) = b_T^2 \text{var}(iT) = (d_T - m_T)^2 \text{var}(iT).$$

(3.30)

The variance of the cost depends on the interest rate variance and the value of interbank trade.\(^{13}\)

What can a risk averse bank do to minimise its risk exposure before the settlement day? I can safely assume that the variance of the interest rate is exogenous, and the starting value of current account $m_T$ on day $T$ is determined by past shock realisations. The only variable that can be controlled by the bank is the deficiency $d_T$ given by

$$d_T = d_{T-1} - m_{T-1} - \varepsilon_{T-1} - b_{T-1}.$$  

(3.31)

A bank that is willing to minimise the variance in the last period needs to adjust his borrowing activity so that

$$E(d_T) = E(m_T).$$

(3.32)

This condition will be satisfied at a borrowing value of

$$b_{T-1} = d_{T-1} - 2 \times m_{T-1},$$

(3.33)

Thus, the risk aversion might determine the borrowing value for the penultimate day of the maintenance period.

However, this conclusion cannot be extended for the preceding days. This is because the bank can reach the value $d_T = E(m_T)$ in multiple different ways, which include front, backloading as well as simply following average required value every day. This issue is discussed in much more detail in the next chapter.

### 3.6 Simulation results

The above sections contain the analytical analysis of the base model. The crucial equations become, however, analytically intractable for more than 3 periods while in reality, the maintenance periods stretch as far as 20 days (in the Eurosystem). Hence, to analyse the behaviour of the model in a more realistic scenario, I turn to numerical simulations.

The first scenario that I look into is the benchmark model where I assume homogeneous banks with average reserve requirement 100 units, a symmetric corridor system (with deposit and lending facility at 2% and 3% respectively), aggregate liquidity sufficient to satisfy the reserve requirement and a low liquidity shock variance (10 units), which resemble the Eurosystem. In later sections, I allow for

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\(^{13}\)To understand the last transition, recall from the previous sections that on last day of the maintenance period, the optimal borrowing is the simple difference between the bank's current account balance and remaining deficiency.
heterogeneity, an asymmetric corridor system, aggregate shock and different values for the idiosyncratic shock variance to see the impact of those assumptions on the market behaviour. A summary concludes this section.

The results of the simulations are presented in the form of tables. Each table includes the following information:

- the average market clearing rate for each day of the maintenance period (the central bank target is 2.5%),
- the interest rate volatility,
- the average use of the standing facilities.

3.6.1 Benchmark scenario

I start out by presenting the results for a benchmark scenario with the following assumptions:

1. The banks are homogeneous; each of them has the same starting current account balance and deficiency value as well as being faced by i.i.d. liquidity shock (for heterogeneity, see Subsection 3.6.2.).

2. The liquidity shock is idiosyncratic in that there is no aggregate shock to the market (as it is fully offset by the central bank). For the analysis of the aggregate shock, see Subsection 3.6.4.

3. The parameters of the simulation are calibrated to the Eurosystem values. The most important parameter is the shock volatility as compared to the bank's current account balance. Based on the discussion in the section above, I have used a shock standard deviation equal to roughly 1/10 of the bank current account size. For a different size of the shock variance, check Subsection 3.6.5.

4. With no aggregate liquidity outflow (or inflow), there is no practical need for extra central bank intervention on the last day of the reserve maintenance period apart from a countering of the use of the standing facilities during the maintenance period.

5. A symmetric corridor system with the target rate that lies between the standing facilities rate. For an asymmetric corridor, see Subsection 3.6.3.

The parameters of this scenario were chosen to resemble the Eurosystem. The reserve requirement is very high, which forces the commercial banks to maintain a high value for the current account (in the simulation, I use current a account size of 100, where the liquidity shock standard deviation is 10). That, however, means that the shocks are not likely to force the commercial bank to use the standing facilities during the maintenance period. That is, indeed, what I observed in the simulation results presented in Table 3.4, where the differences in the target interest rate level drop to zero for all days of the maintenance period. With the shock so small, hardly any bank risks making mistakes and losing their reserve requirement
<table>
<thead>
<tr>
<th>Day</th>
<th>Mean Rate</th>
<th>Rate Volatility</th>
<th>Deposit facility</th>
<th>Lending facility</th>
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</table>

Table 3.4: The benchmark scenario
The mean and standard deviation (volatility) of the interest rate; deposit/lending facility values refer to the average use across all simulation runs.

buffer too soon; this means that, on the last day of the reserve maintenance period, the market liquidity will be just sufficient to satisfy the reserve requirement hence no last day operations are necessary (or their volume is zero). This, according to equation (3.1), means that the rates remain exactly on target.

For earlier days, it seems that the interest rate closely follows the martingale hypothesis and remains determined mostly by the expected level on the settlement day, offering strong support for condition (3.28).

Despite having no volatility at all, the scenario can be used to gain some interesting insight. Probably the most important factor one concerns the importance of the idiosyncratic shock on the behaviour of the interbank market. Pérez-Quirós and Rodríguez-Mendizábal (2006) explain the deviation from the martingale hypothesis with simply an idiosyncratic shock that stimulates backloading\(^{14}\) to avoid a lock-in state (where the deficiency is satisfied before the end of the maintenance period). Here, I find that in case of the values of the liquidity shock variance that are close to those of the Eurosystem, this effect - while still possibly present - is too small to make any difference. This is related to the modelling strategy and the flat shape of the demand curves (see Subsection 3.5.1 for details), which results in market clearing rate equal to the expected rate for a wide range of borrowing decisions. Thus, the individual realisation of the idiosyncratic shock and distribution of liquidity among the banks seem to play no significant role in the evolution of the interest rate.

In the remaining part of the section, I will attempt to verify the robustness of those results by analysing some of the assumptions.

### 3.6.2 Heterogeneous banks

In the benchmark scenario from Subsection 3.6.1, I have assumed that all the banks are homogeneous and are facing shocks drawn from the same distribution. With the reserve requirement (or average current account volume) at a fairly high level in comparison with the volatility of the liquidity shock, the probability of being forced to use the standing facilities was very low. That meant that there was nearly always an equilibrium where the market cleared at the target rate (equal to the expectations for the last day’s rate).

In this subsection, I attempt to verify if the results still hold when I allow for:

\(^{14}\)Postponing satisfaction of the reserve requirement.
• Heterogeneous banks - more specifically, I use two groups of banks with different current account balances.

• Heterogeneous exposure to the liquidity shock - by forcing both small and large banks to draw their shocks from the same distribution.

This might be justified in the following way. In reality, the market experiences much more variety. There are both large and small banks subject to the reserve requirement and liquidity shock, as indicated in my sample. Recall that the average current account size was EUR 760M with a standard deviation EUR 960M, which indicates significant heterogeneity. In practise, I divided the banks into 2 groups. One group was given a current account balance and the corresponding reserve requirement of 100 (as in the benchmark case) and the other of 10 (equal to the assumed shock standard deviation).

In the benchmark scenario, I have assumed that the bank size and liquidity shock variance is proportional. The regression from Section 3.4 confirms that such a link indeed exists, but also that there is a lot left unexplained by simply looking at the current account size, and small banks might be, in fact, subject to larger shocks (relative to their size) than big banks. Having banks with different reserve requirement facing identical liquidity shock distribution allows me to analyse the impact of heterogeneity in liquidity shock.

The results from the simulation are presented in Table 3.5. The smaller banks are much more likely to use their standing facilities, especially the lending facility. The reason is that, with such a small reserve requirement, their average current account balance is way too small to sufficiently protect them against late negative shocks. After several periods, some of them also lose the reserve requirement buffer and (starting from period $t = 11$) they start using the deposit facility as well. Those in a lock-in state have no more incentives to maintain a positive balance on the current account (as the relative expected cost of lost profits is equal to the expected cost of the borrowing facility), thereby reducing their balances to zero and further increasing the frequency of the need to use the lending facility. These observations only concern small banks. Those banks with a large buffer are not affected by the use of the standing facilities until the last day of the maintenance period, when the aggregate liquidity shortage (or excess) is transmitted to the behaviour of the interest rate. In this scenario, however, I do allow for central bank intervention, which assures that the interest rate level on the last day of the reserve maintenance period stays on target (hence, no volatility on the last day).

The impact of heterogeneity on the interest rate is very small. For the better part of the maintenance period, even though there is a constant inflow of liquidity (from the borrowing facility used by small banks), it is still possible to find borrowing levels that would clear the market at the target rate. This only changes later on in the maintenance period, resulting in a tiny volatility of the interest rate, which at most reaches a value of 0.007. This is due to the fact that small banks have very little effect on the market behaviour as a whole.

The conclusion for this section is that the introduction of heterogeneity does have an impact on the use of the standing facilities, but it is too small to substantially affect the market rate.
<table>
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<tr>
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<th>Rate Volatility</th>
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Table 3.5: Heterogeneous banks

The mean and standard deviation (volatility) of the interest rate; deposit/lending facility values refer to the average use across all simulation runs.
3.6.3 Asymmetric corridor system and interest rate expectations

In the simulation design this so far, I have assumed that the standing facilities corridor is symmetric and the expected market rate follows the central bank target. The implications of these assumptions are discussed in this subsection.

The spread between the standing facility rates offered by the ECB is fixed at 200 basis points and the target rate falls precisely in the middle of the corridor. The symmetry of the corridor might be, however, distorted by additional factors that restrict the use of the facilities.

This can be confirmed by analysing the available data on the spread between the EONIA and ECB target and the liquidity conditions on the last day of the maintenance period. It can be shown that even though the model predicts that for the symmetric corridor, the neutral liquidity in the last day of the maintenance period should result in a rate exactly on the target, this is not actually the case. A simple regression of the spread on the net use of the standing facilities has a constant component equal to roughly 7 basis points. Thus, even though the market liquidity is sufficient to satisfy the reserve requirement, the average interest rate on the settlement day exceeds, on average, the ECB target.

One explanation for this puzzle might be related to the collateral required to use the lending facilities, which raises the cost of central bank borrowing. Also, the banks that use the facility incur some operating expenses. Since the penalties for overdraft far exceed the cost of lost profits from the deposit facility, banks that want to avoid using standing facilities prefer to maintain a surplus rather than risking a shortage. Finally, there might be several other factors which are not so easy to quantify that could distort the symmetry of standing facilities. An example of this would be window dressing at the end of the year.

Those restrictions introduce a certain asymmetry into the actual cost of the standing facilities as perceived by the commercial banks, and, in this subsection, I analyse the impact of this asymmetry on the model behaviour. In particular, I analyse a case where the cost of the lending facility is higher than the nominal value.

The actual modification of the model that reflects asymmetric facilities, is fairly straightforward. Since the incentives I mentioned above are present in all the days of the maintenance period, I increase the lending facility rate by 20 basis points throughout the entire simulation. A brief inspection of condition (3.1) for the last day of the maintenance period reveals that raising the lending rate $i^l$ means that the equilibrium interest rate at neutral liquidity (when the use of both standing facilities is equally likely) will be above the central target.

For the days preceding the settlement day, the interest rate still follows martingale process. If the expectations of the settlement day conditions accommodate higher lending facility cost, the market clearing equilibrium rate will adjust accordingly.

This theoretical predictions are confirmed in the Monte Carlo simulations. It turns out that under asymmetric corridor, the equilibrium interest rate is, on average, 10 basis above the target. Apart from this, the market behaves exactly the same way as in the benchmark scenario (which is why I did not report the
If the interest rate follows the expected level, the efficiency of the monetary policy rules that rely on liquidity control somewhat diminishes. Indeed, running the above simulation with benchmark levels of standing facilities (symmetric corridor) but with the expected rate exceeding the target by 10 basis points, results in exactly the same market behaviour as in the case of asymmetric corridor for all except last days of the maintenance period. This once therefore reinforces the importance of central bank ability to control market expectations that was discussed before.

3.6.4 Aggregate shock

In the scenarios described above, I have assumed that the liquidity shock is idiosyncratic and have found that its impact on the interbank market (for the parameter values close to the Eurosystem) is small. If the central bank's interventions to offset the changes in the autonomous liquidity factors are performed frequently, the only outflow (or inflow) of liquidity could result from the use of the standing facilities. In this subsection, I modify that assumption and allow the aggregate liquidity to change every day.

In terms of modelling, this can be accomplished by adding an additional early shock realisation to the model, similarly to chapter 2. The timing looks almost identical to the one presented before:

\[ m_t, d_t, \varphi_t, b_t, \varepsilon_t, m_{t+1}, d_{t+1}. \]

The additional shock \( \varphi_t \) occurs in the morning, before the market opens and \( b_t \) is calculated. I assumed this shock to have parameters similar to the idiosyncratic one (i.e. the mean zero and standard deviation \( \sigma = 10 \)), but its aggregate value might be different from zero. In order to assure that the market expectations of the interest rate are correct, I need the central bank intervention on the last day of the maintenance period.

The results are presented in Table 3.6. Note that the use of the lending facility does not change from the benchmark scenario. This is obvious: unless the banks lose their reserve requirement bonus, they will be inclined to maintain the current account balance sufficient to offset a negative liquidity shock. Recall that the late liquidity shock has a small variance compared to the current account balance value.

On the other hand, the deposit facility is used much earlier and more extensively, when compared with the benchmark scenario. This suggests that the aggregate liquidity shocks result in the reserve requirement buffer being exhausted early.

I will analyse the cases of aggregate liquidity shortage and surplus separately as they occur with the same probability, but have quite a different impact on the market behaviour.

First, assume that the additional shock caused an aggregate liquidity shortage on the market. The banks’ current account balances are lower, but, with a low shock volatility, even the remaining balances are sufficient to protect them from the borrowing facility. The satisfaction of the reserve requirement is behind schedule.
Table 3.6: Aggregate shock
The mean and standard deviation (volatility) of the interest rate; deposit/lending facilities values refer to the average use across all simulation runs.

(in comparison to the benchmark), but the banks assume that by the last day of the maintenance period the central bank will provide the liquidity necessary to satisfy the reserve requirement. That causes the temporary shortage of liquidity to have little impact on the use of the standing facilities and interest rate.

On the other hand, the scenario involving a liquidity surplus is more interesting. Even a minor liquidity surplus from, for example, day $t = 1$ that is not countered by the central bank intervention (which is assumed to happen only on the last day) contributes to the reserve requirement for each day of the remaining part of the maintenance period. This might cause some banks to end up in the lock-in state and explains the use of the deposit facilities.

The behaviour of the interest rate can be explained by a combination of those two effects. The market is not affected by the liquidity shortage, but the surplus will cause some banks to become locked up, thereby forcing them to use the deposit facility and depressing the interest rate.

This subsection offers some interesting findings in terms of the optimal policy setup. The results suggest that in order to keep the interest rate equal to the target level in the presence of aggregate shocks, it is actually better to maintain an aggregate liquidity shortage for the entire maintenance period and to supply the amount required to satisfy the reserve requirement on the last day. This would allow the banks to sustain the reserve requirement buffer while maintaining sufficient funds to offset negative late shocks. A reverse policy (i.e. liquidity surplus during the maintenance period and a draining of the excess on the last day) could result in several banks finding themselves in a lock-in state, which drives the rate below the target as they have no way to insure against positive shocks.
### 3.6.5 Large idiosyncratic shock

In the scenarios presented above, I have demonstrated that an idiosyncratic shock of a size similar to the one observed in the Eurosystem has little impact on the behaviour of the interest rate. In this subsection, I analyse what happens if the liquidity shock volatility is higher and the model is otherwise unchanged.

In terms of modelling assumptions, I set the standard deviation of the liquidity shock at $\sigma = 100$ (compared with $\sigma = 10$ in the benchmark scenario), which is equal to the starting current account balance. Similar parameter values were used in earlier research. I have assumed that the central bank intervention provides the market with neutral liquidity on the last day of the maintenance period (equal to remaining reserve requirement), thus also setting the expected interest rate at the target level. The results are presented in Table 3.7.

Note that the increase in the standard deviation has a substantial impact on the market behaviour. The lending facility is used much more extensively, which can be explained by a much higher volatility of the shock compared to the average current account balance. Contrary to the benchmark case, this buffer does seem to be too small to save the banks from the negative shocks. Increased volatility has, however, much less impact on the use of the deposit facility. This is due to the fact that the reserve requirement buffer (for 20 periods) is sufficient to secure

<table>
<thead>
<tr>
<th>Day</th>
<th>Mean Rate</th>
<th>Rate Volatility</th>
<th>Deposit facility</th>
<th>Lending facility</th>
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<tr>
<td>1</td>
<td>2.47</td>
<td>-</td>
<td>-</td>
<td>71.09</td>
</tr>
<tr>
<td>2</td>
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<td>2.47</td>
<td>0.00</td>
<td>-</td>
<td>72.11</td>
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<td>0.01</td>
<td>-</td>
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</tr>
<tr>
<td>20</td>
<td>2.50</td>
<td>-</td>
<td>380.88</td>
<td>380.88</td>
</tr>
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</table>

Table 3.7: Large idiosyncratic shock

The mean and standard deviation (volatility) of the interest rate; deposit/lending facilities values refer to the average use across all simulation runs.
the banks from even very large positive shocks. This explains the discrepancy in using the standing facilities observed in the simulation.

The constant use of the lending facility, without a corresponding increase in the deposit facility constitutes a steady inflow of liquidity in the system which, at some point, will drive the rates down. The banks still have the incentive for backloading (as in the Pérez-Quirós and Rodríguez-Mendizábal (2006) model), which is reflected by a starting rate that is lower than the central bank target level. The backloading is, however, smaller than the liquidity effect so the interest rate is decreasing. This was not observed by Pérez-Quirós and Rodríguez-Mendizábal (2006), as their model only covered 4 periods.

3.6.6 Simulation summary

In the subsection above, I have presented the results of the simulations using the parameters calibrated in Section 3.4. I have also verified whether changing different assumptions will change the results in a significant way.

The main conclusion is that the benchmark model provides a good approximation of the actual market behaviour. In all days of the maintenance period, other than the last one the interest rate remains equal to the expected level. I observe little variation in the interest level, and hardly any bank satisfies the reserve requirement early. Since some of the underlying assumptions could be perceived as strong, I have also run the simulation by allowing banks to exhibit heterogeneity, an asymmetric corridor, aggregate shock and a large shock volatility.

In the first scenario small banks are much more vulnerable to randomness and have to use the standing facilities more extensively. However, their influence is too small to significantly affect the whole market.

In the second scenario, I allow for an asymmetric corridor between the standing facilities. I find that the interest rate remains tied to the expected level on the last day of the maintenance period and that market behaviour depends on central bank's ability to affect those expectations.

In the third scenario, I allow for an additional early shock that affects the aggregate market liquidity. Perhaps the most interesting finding is that there is now an asymmetry between the use of the deposit and lending facilities. The average use of the lending facility is not affected, since the banks still have enough liquidity to secure safe end of the day balances (a shortage of liquidity is assumed to be covered by the central bank on the last day). If the market is, however, flooded with cash, more and more banks would find themselves in an irreversible lock-in state. In this case, the market rate would be pushed below the target. However, those results assume that the only central bank intervention takes place on the last day of the maintenance period.

Finally, I attempt to compare my model with the model of Pérez-Quirós and Rodríguez-Mendizábal (2006) by verifying what happens if I increase the liquidity shock variance. My model covers 20 periods (whereas they only look at a 4-day maintenance period) and the effect of the liquidity inflow from the lending facility exceeds the motivation to backload, which is why I actually observe a drop rather than an increase in the interest rate. The rate, however, starts at a level below the target, which is consistent with their finding.
3.7 Conclusions

To summarise, the main finding of this chapter is that the behaviour of the inter-bank interest rate is related to the relative size of the liquidity shock volatility and reserve requirement. I find that, for the Eurosystem, the volatility of the liquidity shock is significantly lower than the average current account balance, which has significant implications for the determination of the interest rate.

I also find the link between martingale hypothesis and the length of the maintenance period. I show that the interest rate will be primarily driven by the expectations until the remaining reserve requirement (deficiency) falls below $6 \times \sigma$, where $\sigma$ is the standard deviation of the liquidity shock. Given the lengthy maintenance period in the Eurosystem, this condition holds for all days except the ones directly preceding the settlement day.

In this chapter, I calibrated the size of the liquidity shock standard deviation relative to the average current account holdings, which seems to be a crucial relationship for the behaviour of the model. Based on the sample from 86 banks in the Eurosystem, I have found that the average shock standard deviation stands roughly at 22% of the square root of the current account on the last days of the maintenance period.

Such a low value of the liquidity shocks means that the flat part in the middle of the demand curve will be fairly large; thus the interest rate will not react even to substantial changes in aggregate market liquidity. This means, however, that in order to steer the interest rate, the central bank must focus on the control of the banks expectations. Since the Eonia swaps do seem to follow the ECB target, there is no reason to doubt the efficiency of such control.

In the numerical part, I simulate the behaviour of the market for 20 periods. I found that, for the calibrated parameters, I was able to duplicate the behaviour of the Eurosystem fairly well. The market rate remains on target and the use of the standing facilities is very low. The robustness of the assumptions has been verified by running the simulations with bank heterogeneity, an asymmetric corridor system, aggregate shock and a large idiosyncratic shock. For each of these factors, I was able to explain the difference in comparison with the benchmark model.

3.8 Appendix

3.8.1 Proof of the results from Subsection 3.3.3

Proof of equation (3.9): The equilibrium interest rate at time 2 is given by

$$i_2 = i^d F(-b_2 - m_2) + i^d [1 - F(d_2 - b_2 - m_2)] + E(i_3) [F(d_2 - b_2 - m_2) - F(-b_2 - m_2)].$$ (3.34)
To find the level of liquidity that corresponds to the middle of the corridor system, assume that \( i^2 = E(i_3) = (i^d + i^l)/2 \). That implies, \( i^l - i_2 = -(i^d - i_2) \). Now I can deduct \( i_2 \) from both sides of the equation:

\[
0 = (i^l - i_2)F(-b_2 - m_2) + (i^d - i_2)\left[1 - F(d_2 - b_2 - m_2)\right].
\] (3.35)

Assuming the shock distribution is symmetric around zero \( 1 - F(\bullet) = F(-\bullet) \) and divided by \((i^l - i_2)\)

\[
F(-b_2 - m_2) = F(-d_2 + b_2 + m_2),
\] (3.36)

and finally

\[
m_2 + b_2 = \frac{d_2}{2}.
\] (3.37)

3.8.2 Proof of the results from Section (3.5.3)

Proof of equation (3.30) The cost function is given by the following expression:

\[
var(cost) = var \{i_t b_t + E(c_t)\}.
\] (3.38)

Recall that the optimal borrowing in the case of a symmetric corridor system is equal to \( b_t = m_t - d_t \). But, in that case, the expected cost reduces to:

\[
E(c_T) = i^l \left[ \int_{-\infty}^{-m_T-b_T+d_T} (-m_T - b_T + d_T - \varepsilon_T)f(\varepsilon_T)d\varepsilon_T \right] - \\
i^d \left[ \int_{-m_T+b_T+d_T}^{\infty} (m_T - b_T - d_T + \varepsilon_T)f(\varepsilon_T)d\varepsilon_T \right] \\
- i^l \left[ \int_{-\infty}^{0} \varepsilon_T f(\varepsilon_T)d\varepsilon_T \right] - i^d \left[ \int_{0}^{\infty} \varepsilon_T f(\varepsilon_T)d\varepsilon_T \right]
\]

\[
= (i^l - i^d) \int_{0}^{\infty} \varepsilon_T f(\varepsilon_T)d\varepsilon_T,
\] (3.39)

since

\[
\int_{-\infty}^{0} \varepsilon_T f(\varepsilon_T)d\varepsilon_T = - \int_{0}^{\infty} \varepsilon_T f(\varepsilon_T)d\varepsilon_T,
\] (3.40)

so it is a constant non-random value. The cost variance then reduces to:

\[
var(-i_t b_t + E(c_t)) = var \left(i_t (d_t - m_t) + (i^l - i^d) \int_{0}^{\infty} \varepsilon_T f(\varepsilon_T)d\varepsilon_T \right) = \\
(d_t - m_t)^2 var(i_t).
\] (3.41)
3.8.3 Proof of the equation used in the simulation

In the simulation study, the following result greatly simplifies the complexity of the computations:

\[
\frac{\partial b_t}{\partial m_t} = -1 \quad (3.42)
\]

for all periods before the end of the maintenance period.

The key equations for the days prior to the end of the reserve maintenance period are:

\[
i_t = i^d F(-b_t - m_t) + i^d [1 - F(d_t - b_t - m_t)] - \int_{-\infty}^{d_t - b_t - m_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t \quad (3.43)
\]

\[
\frac{\partial V_t}{\partial d_t} = -i^d [1 - F(d_t - b_t - m_t)] + \int_{-\infty}^{d_t - b_t - m_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t. \quad (3.44)
\]

In all of these equations, the terms \( b_t \) and \( m_t \) appear only as a pair \((b_t + m_t)\). That means, however, that the derivative is

\[
\frac{\partial i_t}{\partial m_t} = \frac{\partial i_t}{\partial b_t}. \quad (3.45)
\]

Hence, the application of the implicit theorem yields:

\[
\frac{\partial b_t}{\partial m_t} = - \frac{\partial i_t}{\partial m_t} = -1 \quad (3.46)
\]
References


Pérez-Quirós, G., Rodríguez-Mendizábal, H., 2006. The daily market for funds in Europe: What has changed with the emu. Journal of Money, Credit and Banking 38, 91–118.


Välimäki, T., 2006. Why the marginal mro rate exceeds the ecb policy rate? Discussion Papers 20, Bank of Finland.

Chapter 4

What determines commercial banks demand for reserves in the interbank market

4.1 Introduction

Commercial banks use the interbank market to trade funds that are used in daily payments and to satisfy reserve requirement. A bank’s demand (or supply) for funds is based on its liquidity and expectations of the future interest rate. This market is crucial for the central bank, since the average transaction rate is the benchmark for the level of interest rates in the economy and is therefore regarded as the primary tool for implementing the monetary policy.

This chapter analyses commercial banks’ behaviour in the interbank market. Compared with the previous chapters of this thesis, I introduce a new element, trade related frictions, which can be used to determine the banks demand for funding in different days of the maintenance period. In a Monte Carlo simulation of the interbank market with trade frictions, I successfully reproduce a pattern observed in the Euro interbank market; banks initially deviate from the average reserve requirement and reverse their behaviour when the settlement day draws closer.

The interbank market has already been analysed quite extensively but much of the literature has focused on determination of the interest rate, rather than bank’s individual behaviour. The most influential articles include early Poole (1968), followed more recently by Hamilton (1996), Välimäki (2003), Pérez-Quirós and Rodriguez-Mendizábal (2006), Bartolini et al. (2001), Bartolini et al. (2002), Bartolini and Prati (2006) and Clouse and Dow Jr. (2002) all of which were revised extensively in the previous chapters.

Although the papers are reasonably successful in predicting the patterns in the interest rate, they sometimes cannot capture individual bank behaviour. This is due to the fact that under martingale hypothesis (which holds for the majority of the Eurosystem maintenance period), the borrowing cannot be uniquely
determined (see Chapter 3 for details). This is related to weak liquidity effect, documented for instance by Moschitz (2004) or Würtz (2003), who argue that for a certain parameter range, the unexpected changes in liquidity have no effect on the interest rate.

The fact that the amount of borrowing cannot be uniquely determined within the framework of the standard model is a shortcoming from the perspective of central bankers who need to decide on the allotment size for open market operations. If the banks are indifferent between keeping different reserves at the same interest rate, the market becomes partially immune to the liquidity conditions, and the interest rate is driven mainly by expectations. Although the central bank has tools to affect interest rate expectations (for instance by controlling liquidity on the last day of the maintenance period), finding demand determinants might be important for the effectiveness of operating policy.

The papers that focus on the determination of the interbank borrowing include Gaspar et al. (2008) and Cassola (2008), who are the closest to the analysis done in this chapter.

Gaspar et al. (2008) document the increasing volatility of the interest rate toward the end of the maintenance period and attribute this to trading frictions that are related to market segmentation. They model the interbank market as divided into groups of banks that only trade with each other. Within each group the banks remain risk neutral which allows to apply standard Pérez-Quirós and Rodríguez-Mendizábal (2006) model to the case of aggregate liquidity shortage or surplus. Therefore their model cannot be used to analyze the potential asymmetric information problem that leads to asymmetric cost of borrowing discussed in this chapter.

Cassola (2008) answers the question whether banks satisfy the reserve requirement in a linear manner by deriving a series of tests that compare the current account balance with the average reserve requirement. Statistical testing done using panel data analysis indicates that banks on average do satisfy the reserve requirement in a linear manner. In this chapter, I further extend his analysis by considering each bank separately, which allows me to analyze the heterogeneity in banks’ preferences.

This chapter offers important contribution to the existing literature on the interbank market.

First, I extend the results of panel data analysis of Cassola (2008) to verify the potential front or backloading patterns for individual banks. Such an approach allows me to avoid the restriction imposed by the ECB aggregate liquidity supply and analyze the heterogeneity in banks’ preferences. I find that quite a few banks do seem to exhibit a tendency to deviate from the average required reserves value and that they are split almost equally in their preferences.

Second, I attribute these deviations to the existence of certain market frictions or cost asymmetry. I show how the standard, in the literature, risk neutral cost function can be modified to capture these frictions.

Third, I verify the impact of trading frictions on the market behaviour using a Monte Carlo simulation. I find that depending on the value of parameter that captures the market frictions, I can duplicate the behaviour observed in the Euro-system.
Fourth, I find that high market frictions can sometimes lead to a situation where the banks turn to central bank finance instead of using the interbank market. The analysis of such situation is particularly relevant in case of liquidity crisis, which is discussed in the following Chapter.

The chapter is structured in the following way. Section 4.2 presents the data for the sample of Eurosystem banks and their actual reserve demand for a period of 16 months. I compare actual liquidity holdings to required reserves, and document an interesting pattern: banks tend to deviate from the required level at first and compensate for it in the latter part of the maintenance period. Statistical analysis in this section is done on an individual bank data level and I find that out of 86 analysed banks, 75 exhibit statistically significant deviation from reserve requirement and the large banks (that constitute majority of the sample) are split almost equally into front and backloading banks.

Section 4.3 very briefly reviews the model introduced in the previous chapter.

Section 4.4 presents the modifications to the original model, together with a Monte-Carlo simulation study. In looking at the reserve demand, the benchmark is of course the required reserve requirement, but apart from this I analyse other variables that can play significant role:

- existence of trading cost
- incentives to avoid excess trading
- increasing marginal borrowing cost
- asymmetry between borrowing cost and lending profit.

The trading cost (suggested initially by Hamilton (1996)) might be the result of obligatory collateral (in case of secured transactions) or matching problems (cost of finding parties willing to trade). Also Bartolini et al. (2001) incorporate the trading cost in their model, however, they assume it as a fixed term, occurring only when the bank decides to trade. Avoiding volatility is a natural assumption, which holds for most financial markets, including the interbank market. As for the increasing marginal cost, excess borrowing in the interbank market might send a negative signal to other participants, in suggesting that the bank faces liquidity problems, which could hurt its reputation. Finally, the cost of borrowing extra reserves might be higher than profit from corresponding lending of temporary surpluses, which creates a certain asymmetry.

The results of the simulation roughly follow the patterns documented in Section 4.2. I find that market related frictions and trading costs might indeed play a significant role in reserves demand and their inclusion yields results that resemble the actual behaviour of the Eurosystem.

## 4.2 Reserve demand of Eurosystem banks

This section contains a brief review of the interbank market that is necessary to understand the analysis of the Eurosystem. For more detailed analysis refer to Chapter 2, Section 2.2.
4.2.1 Interbank market

The interbank market is used by commercial banks to trade reserves that are held on account at the central bank\(^1\). Developed markets facilitate all sorts of trading contracts: secured and unsecured, long-term and short-term. In this chapter, I focus on short term, unsecured lending.

During the day, reserves held on account at the central bank are used to process liquidity flows and transfers between the banks, which result from transactions between customers. The final end-of-day balance is used to satisfy the obligatory reserve requirement.

The reserve requirement is typically linked to the bank’s balance sheet and is known at the start of the maintenance period. In the Eurosystem, an averaging provision applies, meaning that the average account balance over the maintenance period must be sufficiently high. Required reserves are also remunerated at the rate based on past liquidity tenders. Thus, the reserve requirement does not earn the central bank any profits.

Each day commercial banks process large numbers of transactions. Since their amounts are so significant, there are likely to be statistically significant patterns that can be anticipated in advance. For example, a mortgage bank can very well model its loan repayments and predict its net market position in advance. If the market functions properly, the banks are able to offset the predictable patterns by long term or pre-arranged contracts, which do not contribute to the variation in the daily interest rate. If all transactions could be perfectly foreseen and anticipated, the banks end-of-day balances would follow a predetermined path and the standing facilities would never be used. The market is subject to liquidity shocks that were discussed in detail for instance in Sections 3.2.2 and 3.2.3 before. In this chapter I consider both early and late liquidity shocks that occur respectively:

- during trading hours
- after trading hours.

The early, unanticipated transactions are typically due to large customer payment orders or errors in estimated liquidity flows. Since those shocks are symmetric, one bank’s shortage of liquidity is reflected in a corresponding surplus of some other bank, which creates a natural opportunity for trade. If there are no trade restrictions and the aggregate liquidity does not change, those banks meet and effectively neutralise any possible realisation of the shock, which is the reason I did not include it in the model presented in Chapter 3. These types of transactions are referred to as early liquidity shock, and following the notation from Chapter 2 I denote them \(\phi_t\).

The random transactions that occur after trading hours could push the current account below zero or above the remaining reserve requirement, which would force the bank to use the standing facilities. Their sources include mainly errors in transaction processing or certain market features that prevent banks from obtaining the required liquidity. For instance, the bank can place an “ask” order, which

\(^1\)An excellent description of various aspects of the interbank market and monetary policy implementation can be found in Bindseil (2004).
will not be accepted by any other bank until the end of the day. These types of transactions were modelled and discussed in detail in Chapter 3, Section 3.2, and I refer to them as late liquidity shock, denoted \( \varepsilon_t \).

In the theoretical model I assume that both early and late liquidity shocks are identically and independently distributed, with mean zero and standard deviation \( \sigma \). This assumption should be justified since the banks do in fact exhibit systematic patterns in the interbank trade (see for instance ECB (2007)). As in Section 3.2.3 I interpret the late liquidity shock as “forecast error” in bank’s estimations of daily liquidity flows. Thus, even though the banks might exhibit permanent surplus or shortage of liquidity, there is no reason to expect that the forecast errors will be biased. As I have no access to individual data, I cannot verify this hypothesis, and I am forced to assume homogeneous banks and symmetric shocks.

It is important to distinguish between a commercial bank’s behaviour in the interbank market and its regular activities (such as deposits, customer loans), and they should be treated separately, as they refer to different time scales. The overnight interbank market does not provide liquidity for new customer loans, which typically have longer maturities. Increasing customer deposits will eventually result in a higher reserve requirement in the next maintenance period, but the extra liquidity needed can be easily obtained in open market operations at nearly no additional cost (since required reserves are remunerated at the same rate as liquidity supply operations). This essentially means that in normal circumstances, there is no direct link between interbank and the other activities performed by commercial bank.

### 4.2.2 Eurozone banks

In this subsection I focus on the analysis of the commercial banks’ behaviour in the Eurozone. A similar analysis can be found in Cassola (2008), who adopted a slightly different approach using panel data analysis.

The interbank market is a closed system, where the aggregate value of commercial banks’ current account balances is determined by the central bank balance sheet. In the analysed period 24 January 2004 - 31 May 2005, the ECB was pursuing the policy of supplying the market with the liquidity value very close to the average reserve requirement (benchmark liquidity). Indeed, the average liquidity of the Eurosystem banks in the tested period was EUR 138.9 billion, while the average reserve requirement was EUR 138.2 billion. Thus, the deviation from the benchmark liquidity was on average only EUR 0.6 billion.

Such policy has important implications for banks’ behaviour. At the aggregate level it means that banks’ eg. frontloading (satisfy the reserve requirement early) will be offset by other banks’ backloading. This restriction must be taken into account if the individual bank’s preferences are to be analysed. A similar conclusion is reached by Cassola (2008) when he tests whether his panel of commercial banks is representative of the euro area. However, as his sample covers only 42% of market liquidity, it is unclear how much this restriction affects his testing.

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\[ ^2 \] I am very grateful to ECB and Nuno Cassola from the liquidity management division for permission to collect and use the data obtained during an internship of the April - June 2006.
The liquidity supply policy also affects the behaviour of the aggregate deficiency (the remaining reserve requirement). On the individual level, the deficiency is governed by eq.(2.4) from Chapter 2. Since the maintenance period in the Eurosystem is fairly long, banks rarely use the standing facilities or satisfy the requirement before the settlement day. Thus, the deficiency follows roughly a simple difference equation

\[ d_{t,i} \simeq d_{t-1,i} - b_{t-1,i} - m_{t,i}, \]  

where \( d_{t,i} \) is the deficiency, \( b_{t,i} \) borrowing from interbank market and \( m_{t,i} \) is the starting current account balance of bank \( i \) on day \( t \). Assuming market clearing condition, eq.(4.1) can be aggregated over \( N \) banks

\[ \sum_{i=1}^{N} d_{t,i} = \sum_{i=1}^{N} d_{t-1,i} - \sum_{i=1}^{N} m_{t,i}. \]  

The ECB liquidity supply policy implies that

\[ \sum_{i=1}^{N} m_{t,i} = \sum_{i=1}^{N} R_{i}, \]  

where \( R_{i} \) is the average reserve requirement for bank \( i \). Thus,

\[ \sum_{i=1}^{N} d_{t,i} = \sum_{i=1}^{N} d_{t-1,i} - \sum_{i=1}^{N} R_{i}. \]  

Since the deficiency on the first day of the maintenance period is equal to \( T \times R \), (4.4) can be further simplified to

\[ \frac{\sum_{i=1}^{N} d_{t,i}}{\sum_{i=1}^{N} R_{i}} = (T - t + 1), \]  

which means that, assuming no standing facilities are used before the end of the maintenance period, the ECB liquidity supply policy implies that the aggregate deficiency is satisfied in a linear manner. This restrictions must be taken into account when analysing banks’ behaviour.

Cassola (2008) estimates a very similar equation for the last two days of the maintenance period:

\[ \left( \frac{d_{i,t}}{R_{i,t}} \right) = \delta + \nu_{i,t}, \]  

where \( d_{i,t} \) is the deficiency, \( R_{i,t} \) the average reserve requirement and \( \nu_{i,t} \) liquidity shock of bank \( i \) on day \( t \). If the banks satisfy the reserve requirement in a linear manner, the parameter \( \delta \) should be 2 on the penultimate and 1 on the last day of the maintenance period. He solves the problem of the restriction imposed by the ECB liquidity supply policy by applying panel data analysis method and finds that condition (4.6) is satisfied for last two days of the maintenance period.

This section extends the results obtained by Cassola (2008), whose method does not allow to analyse individual banks’ preferences. By considering the banks
one-by-one, I can address the problem of substantial heterogeneity in banks’ preferences, which is presented in more detail below. In addition, my method is also not affected by the restriction of the aggregate liquidity supply discussed above.

The sample obtained from the ECB includes information on 86 commercial banks, their end-of-day current account balances and corresponding reserve requirement for the period 24 January 2004 - 31 May 2005. The banks were classified by the national central banks as large (48 banks), medium (22 banks) and small (16 banks).\footnote{Details on the sample selection can be also found in previous chapter.} Unfortunately the sample choice is biased toward large banks. For example, Germany was represented by 15 banks, 10 of which were large, 3 medium and 2 small. The benefit of this procedure is that the data cover a much larger part of market than is suggested by the number of banks. At roughly 50% of the market liquidity, the average sum of current accounts of 86 analysed banks was EUR 65.7 billion, while the sample reserve requirement was only EUR 0.4 billion lower. Thus, the condition (4.3) was almost exactly satisfied in the analysed sample and the ECB liquidity supply policy constituted a significant restriction.

An illustration of the impact of the aggregate liquidity restriction on commercial banks’ behaviour is presented in Figure 4.1. The average deviation of current account from reserve requirement on different days of the maintenance period was calculated across individual imbalances:

\[ \text{imbalance} = m_{t+1,i} + b_{t,i} - R_{t,i}. \]  

\[ m_{t+1,i} \] is the starting current account balance on the day \( t + 1 \), which means that the end-of-day balance on the previous day is \( m_{t+1,i} + b_{t,i} \). I adopted the notation from Chapter 2, eq. (2.3). Not surprisingly, given that the aggregate liquidity is equal to aggregate reserve requirement the average fluctuates around zero. Intuitively, the frontloaders and backloaders preferences must offset each, leaving the average around zero. Observe also that there is apparently no change in the average positive and negative deviation along the maintenance period. This is a consequence of averaging reserve requirement provision; the bank that holds positive imbalance for the first half of the maintenance period, must switch to negative for the second half in order to satisfy the requirement.

A more interesting picture emerges once I start to look at the behaviour of average accumulated imbalance for front and backloading banks, presented in Figure 4.2. For an individual bank, the accumulated imbalance is given by:

\[ \text{accumulated imbalance} = \sum_{j=1}^{t} (m_{j+1,i} + b_{j,i} - R_{t,i}). \]  

Inspection of Figure 4.2 shows that the average accumulated imbalance does not deviate significantly from zero (just as in the case of daily imbalance). Similarly to above, this is partially due to the averaging provision which requires the accumulated imbalance to drop to zero on the last day of the maintenance period. In addition, the restriction caused by the ECB liquidity policy means that the aggregate market liquidity stays close to the aggregate reserve requirement.

Dividing the banks according to the sign of expression (4.8) reveals, however, a certain pattern. It seems that the average positive deviation initially gradually
Figure 4.1: Daily average difference between current account and reserve requirement
Figure 4.2: Average accumulated imbalance in the sample
Figure 4.3: Average accumulated imbalance for a large bank
Figure 4.4: Average accumulated imbalance for a small bank
Figure 4.5: Average accumulated imbalance for a medium bank
increases, to start dropping toward the end of the maintenance period. The initial increase can be attributed to the random walk property of the liquidity shock. While for most banks the negative and positive shocks will at least partially offset each other, there will be unlucky group of banks receiving a series of shocks with the same sign. In the absence of interbank market trade, this will result in the accumulation of the deviation from neutral liquidity.

Such deviation is indeed observed in the early part of the maintenance period and only start to adjust toward the settlement day. The discussion of this behaviour is provided in the latter part of the chapter.

It is important to evaluate whether the deviation of the current account balance from reserve requirement is significant. The maximum average deviation from neutral liquidity (market reserves equal to reserve requirement) is roughly EUR 1.6 billion, and the average reserves over the sample were EUR 765 million. A bank with a shortage of that proportion would have to borrow more than twice its average current account value in order to immediately raise its reserves to the average required level, so it seems that the deviation might be actually significant. This observation is verified in the statistical analysis done below.

**Statistical analysis** Extending the analysis done in Cassola (2008), I perform the statistical analysis on an individual bank level and verify possible front and backloading patterns for each bank separately. A bank that is systematically front-loading will keep the initial current account balance above the required reserves. Eventually, in order to avoid using standing facilities, the bank will have to reduce the account balance below the reserve requirement but the accumulated imbalance will stay positive for the whole period. Correspondingly, the backloading bank will exhibit negative, on average, accumulated imbalance. If the bank attempts to satisfy the reserve requirement in a linear manner, the accumulated imbalance will not exhibit any statistically significant pattern and will fluctuate around zero.

In order to test it statistically, I employ the T-test and verify the null hypothesis

$$H_0 : \sum_{j=1}^{t} (m_{j+1,i} + b_{j,i} - R_{t,i}) = 0 \quad (4.9)$$

for each bank separately. Note that individual banks might have different days when their accumulated imbalance reaches maximum, thus testing $H_0$ for each day of the maintenance period separately might not return concise results. Instead, I test $H_0$ for all days of the maintenance period combined using full 494 daily observations for each bank. The summary of my testing is provided in Table 4.1.

<table>
<thead>
<tr>
<th></th>
<th>All banks</th>
<th>Large</th>
<th>Medium</th>
<th>Small</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frontloading</td>
<td>47</td>
<td>25</td>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>Backloading</td>
<td>28</td>
<td>18</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>No significant relationship</td>
<td>11</td>
<td>5</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>86</td>
<td>48</td>
<td>22</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 4.1: Individual testing results
Out of 86 banks in my sample I reject the null hypothesis $H_0$ at 95% confidence level for 75 banks. Of those, there is only slightly more frontloading than backloading banks, and this imbalance mostly reflects the tendency among medium and small banks.

Concerning medium banks, inspection of Figure 4.5 reveals that the average medium bank's imbalance remains close to zero. This indicates that among medium banks, the frontloading are, on average, smaller than the backloading.

Concerning small banks, also the average bank's accumulated imbalance tends to turn positive toward the settlement day (see Figure 4.4). Cassola (2008) reaches a similar conclusion for small banks and finds significant frontloading pattern for last two days of the maintenance period. Note however that small banks are significantly underrepresented in the analysed sample of banks.

Concerning the behaviour of large banks that dominate the sample, I find that they are almost equally split into front and backloaders. This confirms the panel analysis results obtained in Cassola (2008). As the average accumulated imbalance is close to zero (see Figure 4.3), the large banks seem to be also most restricted by the aggregate ECB liquidity supply policy.

The number of banks that front or backload can be used only as a rough indication of the market preferences and I return to this issue below. The most important, however, conclusion from this analysis is whether the commercial banks satisfy the reserve requirement in a linear manner. Using my simple test, I find that only 13% of the analysed banks keep their accumulated imbalance close to zero. The remaining banks seem to favour the policies that keep the current account statistically different from the average reserve requirement.

The analysis executed above does not reveal the reasons why the commercial banks allow their current accounts to deviate from the reserve requirement. Two scenarios can be realistically considered:

1. The banks preferences for back or frontloading stem from exogenous reasons not related to the interbank market structure. For instance, a mortgage bank that receives credit repayments at the end of the month might want to postpone satisfaction of the reserve requirement. Similar pattern might also arise as a result of active trading behaviour allowing banks to actively run surplus (or shortage) balance.

2. The banks postpone their adjustment to liquidity shock, which allows them to reduce their interbank trade volumes. One reason for such a behaviour might be related to certain trading frictions, which are analysed in the second part of this chapter.

Since I have no information on bank characteristics (beside the country of origin), I cannot exclude neither of the above scenarios. My model can be, however, used to test the second hypothesis.

As mentioned above, the number of banks that front or backload can be used only as a rough indication of the market preferences as it does not distinguish between the average size of the imbalance. For instance, the restriction resulting from the ECB liquidity supply does not exclude a hypothetical scenario where a single bank is massively frontloading, while the remaining banks in the market
Figure 4.6: Statistically significant accumulated imbalances
are backloading. An illustration of this is Figure 4.6, which, for the 75 banks that exhibit statistically significant pattern, presents their average current account balance (which can be used as an indication of their size), average deviation of the current account from the reserve requirement and the average imbalance/current account ratio.

First observation derived from the figure is that the bulk of front and backloading value can be attributed to several large banks. Comparison between top and middle graphs in Figure 4.6 reveals that the largest backloaders (top figure) are also the largest banks in the sample.

To analyse the heterogeneity among banks, one can also look at the imbalance/current account ratio (bottom graph in Figure 4.6). The analysis of this ratio reveals some heterogeneity among banks, which does not seem to be related to bank size. The average of the absolute imbalance/current account ratio is 1.13 with standard deviation (calculated across banks) 1.7. This suggests more random patterns in banks’ behaviour, which offers some support to scenario 2. mentioned above.

On the other hand, there is also an indication that some of the banks preferences might be more systemic. In Figure 4.6, there is one bank, for which the average accumulated deviation of the current account from the reserve requirement is EUR -7 billion; there are also three banks that consistently exceed the reserve requirement early in the maintenance period and the accumulated deviation is above EUR 3 billion. It turns out that those banks behaviour might be related. The backloading bank and two of the frontloading banks come from the Netherlands, which might suggest a relationship of the type described in the scenario 1. above. Since the names of the banks are considered confidential, I cannot, however, explore this hypothesis further.

The general conclusion from the analysis in this section seems to indicate that the banks typically choose reserve amounts that are only loosely related to the required reserves. Due to the fact that the interbank market is closed, some banks frontloading must cause their counterparties to backload, which is reversed toward the end of the maintenance period. The individual deviations from the neutral liquidity (equal to the average reserve requirement) can be considered substantial but it is unclear whether they result from sluggish response to liquidity shocks or genuine preference for front or backloading.

4.3 Standard model

In this section I introduce the standard model of the interbank market based on Poole (1968) and later extensions to the version with averaging provision by Välimäki (2003) and Pérez-Quirós and Rodríguez-Mendezábal (2006). The model has been extensively discussed in Chapter 2, Section 2.3 and readers familiar with that chapter should proceed directly to Section 4.4.

The starting point of the model is a commercial bank that minimises the cost of funding and needs to satisfy a reserve requirement. It can obtain the funds from open market operations, interbank market or standing facilities. Open market operations take the form of tenders (at least in the Eurosystem) that allot liquidity
pro rata. Since the funds obtained during the liquidity tenders can be traded away in the interbank market, to prevent arbitrage opportunities, the allotment and the market rate must be the close to each other.\textsuperscript{4} Hence, from the commercial bank perspective it should make no difference whether the funds are obtained from the central bank or the market (in practise the collateral requirements differ a bit). Therefore in the model I focus on two sources of funding: interbank market and standing facilities.

The timing of the model is the following. Each bank starts the day with a current account balance \( m_t \) and deficiency (remaining reserve requirement) \( d_t \).

\[
\begin{align*}
  m_t, d_t, \varphi_t, b_t, \varepsilon_t, m_{t+1}, d_{t+1}
\end{align*}
\]

During the trading day, the bank faces both expected and unexpected liquidity changes based on its customers’ payment decisions. Those transactions were discussed in detail in Subsection 4.2.1 above. Here I am interested in liquidity flows that have not been anticipated before. The early shocks, denoted \( \varphi_t \), can be offset by the bank during the interbank trade by choosing the value \( b_t \) (positive \( b_t \) means the bank is borrowing). After the trading day is over, the bank might face a late liquidity shock \( \varepsilon_t \), and the sum of all those terms is the end-of-day current account balance, which is used to satisfy the reserve requirement.

In the remaining part of the chapter, I assume both early and late shocks are identically and independently distributed with mean zero and constant standard deviation. In the simulation study in Subsection 4.4.2, I normalise the shocks to follow a normal distribution. Let me spend some time discussing this assumption. First of all, some banks are perhaps more likely to be hit by positive and others by negative shocks. There might be also a connection between early and later shock realisation, but this is likely to be determined by the individual bank’s characteristics, which are beyond the scope of this chapter. Finally, the variance of early shocks might in reality be higher than of later shocks; the scale of unexpected customer-driven transactions is much larger than random errors in processing. Having different variances of the shocks would not introduce any new mechanism but would greatly complicate the computations, which is why I decided in favour of identical distributions.

The final end-of-day balance on current account is then \( m_t + \varphi_t + b_t + \varepsilon_t \). If that expression turns negative, the bank must borrow from the standing facilities. If positive, it is used to satisfy the reserve requirement or is deposited at the central bank (if the entire reserve requirement has been satisfied).

In the standard model, a single period cost function \( K \) for risk neutral commercial bank takes the form

\[
K_t = i_t b_t + E(c_t).
\]  \hspace{1cm} (4.10)

At interest rate \( i_t \), the cost of borrowing from the interbank market is \( i_t b_t \). \( E(c_t) \) is the expected cost of standing facilities, which are used when the bank balance either falls below zero or exceeds the remaining part of the reserve requirement.

\textsuperscript{4}A model of the relationship between the open market operations and interbank interest rate has been constructed by Välimäki (2006).
Since the bank can choose when to satisfy the reserve requirement, the problem has a dynamic nature captured by the following Bellman's equation, where $V$ is the value function:

$$\min_{b_t} V_t = \{i_t b_t + E(c_t) + E(V_{t+1})\}. \quad (4.11)$$

There is no continuation value beyond the end of the maintenance period (I assume no carryover provision), so the problem on the last day is slightly different and takes the single period form

$$\min_{b_T} V_T = \{i_T b_T + E(c_T)\}. \quad (4.12)$$

First order conditions\(^5\) for equations (4.11) and (4.12) link the optimal borrowing and the interest rate:

$$i_T = i^d F(-m_T - b_T - \varphi_T + d_T) + i^d(1 - F(-m_T - b_T - \varphi_T + d_T)) \quad (4.13)$$

for the last day of the maintenance period and

$$i_t = i^d F(-b_t - m_t - \varphi_t) + i^d[1 - F(d_t - m_t - b_t - \varphi_t)]$$

$$- \int_{-b_t-\varphi_t}^{d_t-m_t-b_t-\varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t, \quad (4.14)$$

for the prior days. These conditions are standard in the literature and were derived in similar forms eg. by Pérez-Quirós and Rodríguez-Mendizábal (2006) or Bartolini et al. (2001). In eq.(4.14) $m_t$ denotes current account balance, $b_t$ interbank borrowing (negative means lending) $d_t$ is the remaining part of reserve requirement, $F(*)$ is the late shock ($\varepsilon_t$) distribution function and can be interpreted as the probability of the shock realisation falling below the value of the expression in brackets.

Those conditions have the following interpretation: at market rate $i_t$ the bank would choose the borrowing value $b_t$, so that the expected cost of interbank borrowing ($i_t$) is equal to the expected cost of using the standing facilities and the dynamic cost factor that captures the impact of an extra unit of deficiency on the future cost of funding.

The demand curves based on above equations were presented on Figure 3.2 in Chapter 3, where I used it to draw several interesting observations. The most important is perhaps that for the parameter range that resembles the values used in the Eurosystem, there exists a flat part in the demand schedule for the days just prior to the end of maintenance period. Its interpretation is the following: given that the interest rate is equal to the expected level, the bank will be indifferent between several borrowing values. This is due to the fact that for a large (compared to the shock variance) reserve requirement, the probability of using the standing facilities remains very small, hence parts (1) and (2) of equation (4.11) vanish. I

\(^5\)Proofs of those equations can be found in Chapter 2, Sections 2.6.1 and 2.6.2.
show in the previous chapter, Subsection 3.5.1, that the last part, the dynamic cost factor, is very close to the expected interest rate, reducing eq. (4.11) to

\[ i_t \approx E(i_{t+1}). \]

(4.15)

Intuitively, if the reserves are large enough compared to market volatility, small changes in borrowing do not affect the probability of using the standing facilities in the following days. Under risk neutrality, this means that the alternative cost of borrowing today, is the expected future interest rate. If that holds, however, the current liquidity does not affect the borrowing value and the demand for reserves cannot be uniquely determined.

In reality, that view might be questionable. After all, the banks appear to follow some specific policy, such as the one documented in Subsection 4.2.2, even though the martingale property of the interest rate holds. It might be front or backloading, and some banks might even constantly change their policies. This is however corrected well before the end of the maintenance period, and cannot be explained using the standard model. In order to address that shortcoming, I modify the original profit equation (4.10) to include some additional motives, while keeping the assumption that the interest rate follows the martingale property.

4.4 Model of the interbank market with trading frictions

The standard profit function used in previous research assumes that the bank is risk neutral, which implies that only the nominal cost of finance matters. That assumption was the key to the martingale hypothesis, which states that funds on different days of the maintenance period are perfect substitutes. In reality, however, the banks might consider additional factors that affect their borrowing decisions. In this section, I explore the implications of banks trying to avoid excess volatility of their costs, transaction costs and an asymmetry between cost of borrowing and profit from lending. In this chapter, I refer to these effects as the trading frictions.

Consider, for a moment, the setup used widely in the existing literature with a bank that needs to borrow a given amount of funds in order to satisfy the reserve requirement. The assumption of risk neutrality implies that the bank can choose to either borrow a small part of the requirement every day of the maintenance period or borrow all the reserves in a single, large transaction. If the expected rate is constant, there will be no difference in the cost of finance between those two cases and hence they would be chosen by the banks with comparable frequencies.

The analysis of patterns in interbank borrowing and in the average current account balances of the commercial banks (Subsection 4.2.2) indicates that this is not the case for the Eurosystem. In fact, even though the banks do deviate from the average required reserves, they reverse this deviation already in the middle of the maintenance period, and avoid trading large volumes. This means that a model of the interbank market that is to capture the patterns present in the Eurosystem must incorporate some trading frictions and abandon the risk neutrality.
assumption. In this section, I present possible motivation and the intuition for the existence of such frictions and a modification of the standard interbank model to properly reflect them.

There might be several arguments against the risk neutrality assumption, but perhaps the most important are the transaction cost and banks' aversion to excess variation in the cost of financing from the market.

The transaction cost includes the operational expenses of finding the counterparty and staff costs. These costs affect equally lenders and borrowers of the liquidity.\(^6\) Generally defined transaction costs were already included in the previous works of interbank market. For instance Hamilton (1996) and Clouse and Dow Jr. (1999) use fixed costs that occur whenever transaction is made. The cost in their models does not, however, depend on the borrowing value.

Different approach to trading frictions is applied by Gaspar et al. (2008). They focus on the problem of bank segmentation and argue that trade frictions result in banks divided into groups. While I agree with their observation that the Eurozone might be segmented, I employ a different method to analyse this issue. In Gaspar et al. (2008) the liquidity shocks affects all market participants but the interbank trade is allowed only within the group. Thus, despite the aggregate liquidity sufficient to satisfy the reserve requirement, each group faces either shortage or excess of liquidity, which is reflected in the level of the internal clearing interest rate. However, within each group banks behave such as in Pérez-Quirós and Rodríguez-Mendizábal (2006) model that assumes risk neutrality and no market frictions.

Banks aversion to excess variation means that a bank expecting a certain cost of borrowing to satisfy the reserve requirement will choose to split the cost over several periods rather than bear all the risks on a single day. In other words, the banks perceive the cost of a single large transactions to be larger than the combined cost of several smaller transactions, even if they occur at the same interest rate. Apart from simple risk aversion, such behaviour can also be motivated by the segmentation mentioned above (see Cocco et al. (2003) and Gaspar et al. (2008) for the discussion of Portugal and the Eurosystem). A large transaction could introduce significant instability to the group and might require finding counterparties outside of the trading group, which involves extra effort and possible additional expenses.

Second, even though the expected interest rate level is fixed, there is still some volatility in the actual interest rate realisation. Spreading the trade over several days is therefore more prudent and allows the bank to avoid being forced to trade in unfavourable market conditions.

A good example of diseconomies of scale is the case of Societe Generale in January 2008, when the bank discovered fraudulent transactions by one of its traders. In order to minimise the loses, the bank decided to unwind his transactions. Ho-

\(^6\)An additional component of the transaction cost is the bid-ask spread. The difference between the cost of immediate borrowing and lending means that the bank taking a round trip (a purchase and sale together) will incur a loss. However, the issue is slightly more complicated, in comparison with the operating expenses, as the cost of the spread for one bank constitutes profit to its counterparty and depends on the market power of trading banks. The model presented in this thesis is the perfect competition model, with one uniform market clearing rate. Hence, in the remaining part of this chapter I focus on the interpretation of the transaction cost as operating cost.
ever, the scale of the operation, USD 75 billion, was significant compared to the market liquidity at that time. As the result, in order to execute the transaction, the bank had to agree to conditions costing it USD 7 billion in losses.

The transaction costs described above affect equally borrowers and lenders, increasing the borrowing cost and reducing the lending profit. This means that the total sum of the borrowing costs for all banks is larger than the corresponding profits from lending, rather than equal as in the risk neutral case. This difference constitutes the overhead that the banking sector must pay in order to participate in the market. Its value is partially determined by the technology (such as staff salaries), but partially depends on how liquid and fragmented the market is. For instance, a crisis that destroys the mutual trust between banks is likely to discourage banks with surplus liquidity from lending, thus increasing the cost of finding a trade partner. Thus, the transaction cost reflects current market conditions and can change rapidly in the case of a crisis.

The transaction cost sources that were analysed so far, affected all banks in a symmetric way. However, the interbank trade is also subject to other effects, which might result in an asymmetry; the cost of borrowing a unit of funding might exceed the corresponding profit from lending the same amount.

Significant volume of the interbank trade is executed in an unsecured lending. The remaining trade, however, requires collateral to secure the transactions. Previous research on secured and unsecured lending did not deal directly with the interbank market due to the problems with obtaining reliable data set. These difficulties are solved in the empirical paper of Demiralp et al. (2006), who analyse the repurchase agreements in the brokered federal funds markets (that do not involve the Fed current accounts). Their analysis is, however, focused on the Fed funds rate, rather than individual bank’s incentives.

There is also quite an extensive literature that deals with the securitisation of central bank lending. The papers that analyse the role of collateral in liquidity tenders include for instance Ayuso and Repullo (2003) and Välimäki (2003). However, in this chapter I analyse the transaction cost that occurs as a result of transaction between banks, where the central bank lending is not included.

The asymmetry between cost of borrowing and profit from lending may derive also from sources other than transaction costs. For instance, the information cost means that the banks’ reputation is an important factor in determination of trade conditions. Consider for a moment a bank that attempts to borrow large amounts of reserves; it is possible that other banks will perceive this as an early sign of potential losses or liquidity problems damaging its reputation. This turned out particularly relevant during the market turnover in 2008 described in the following chapter. The fact that some banks actually used the lending facility offered at the rate exceeding the market rate suggests that Central Bank finance was favoured for reasons other than cost minimisation.

The asymmetry might be also related to market depth. In particular the situation where one bank finds itself in need of substantial funding might give its counter parties stronger bargaining position and thus result in worse borrowing conditions. Finding lenders is even more expensive when the bank is forced to search for funding among foreign banks.

Before I present the functional form, note that the asymmetry discussed here
is likely to appear only at excessive borrowing volumes. Even though majority of trade will be not affected, the potential penalty might still affect on their decisions. In the remaining of this chapter I refer to the above properties as trading frictions. In the next section I present a practical way to modify the cost function to capture the increasing transaction cost and the asymmetry between borrowing cost and lending profit.

4.4.1 Model

In this subsection I modify the standard model of the interbank market to capture the properties described above. I assume that a commercial bank has only two possible means to obtain funding. First, it can use the central bank standing facility, and I denote the expected cost of using them by $E(c_t)$. Second, the bank can obtain reserves $b_t$ from the interbank market at interest rate $i_t$. I denote the cost of interbank market finance by $\kappa(i_t, b_t)$. The total cost of funding $K_t$ is then:

$$K_t = \kappa(i_t, b_t) + E(c_t). \quad (4.16)$$

I discuss the two sources of finance separately.

**Financing from the market** In the introduction to the section above, I argued that the cost of market finance increases with the scale of transactions and that there exists an asymmetry between cost of borrowing and profit from lending. Those properties must be captured by the expression $\kappa(i_t, b_t)$.

In order to capture the increasing transaction cost, the function must satisfy

$$\frac{\partial \kappa(i_t, b_t)}{\partial b_t} < \frac{\partial \kappa(i_t, b_t + \epsilon)}{\partial (b_t + \epsilon)} \quad (4.17)$$

for all $b_t$ and positive $\epsilon$.

To capture the asymmetry of trading cost (borrowing more costly than lending), the cost function must satisfy

$$\kappa(i_t, b_t) > -\kappa(i_t, -b_t) \quad (4.18)$$

for any $b_t > 0$.

It is easy to see that both properties are satisfied for any non-decreasing, convex function such that

1. $$\frac{\partial \kappa(i_t, b_t)}{\partial b_t} > 0 \quad (4.19)$$

---

7As mentioned above, the model is built on the perfect competition assumption, which implies one market clearing price. This prevents direct modelling of the reputation, which requires variation in the price. However, the method applied in this chapter and the resulting convexity of the cost function, constitute a good approximation of the behaviour of the banks that worry about the impact of borrowing on their reputation.

8The exact expressions are given in Appendix 4.6.3, in equations (4.43) and (4.46).
2. \[
\frac{\partial^2 \kappa(i_t, b_t)}{\partial b_t^2} > 0
\] (4.20)

3. \[
\kappa(i_t, 0) = 0.
\] (4.21)

Those are not particularly restrictive conditions, and in fact they can be captured by a quadratic function of the following type:

\[
\kappa(i_t, b_t) = i_t b_t + \frac{\rho}{2} (i_t b_t)^2.
\] (4.22)

Equation (4.22) has the following economic interpretation: the total cost of borrowing from the interbank market \( \kappa \) is given by the nominal cost of borrowing \( i_t b_t \) and an extra term \( \frac{\rho}{2} (i_t b_t)^2 \), which is determined by the borrowing value and includes the transaction cost.

Seen (4.22) as Taylor expansion, it can be transformed into:

\[
\kappa(i_t, b_t) = \rho^{-1} \exp(\rho i_t b_t) - 1/\rho,
\] (4.23)

which significantly simplifies the algebra with no effect on the interpretation. It can be shown that this function satisfies all three properties discussed above: (4.21), (4.19) and (4.20). In addition, in the frictionless case, where \( \rho \to 0 \)

\[
\lim_{\rho \to 0} \rho^{-1} \exp(\rho i_t b_t) - \frac{1}{\rho} = i_t b_t;
\] (4.24)

the cost equation converges to the case discussed in the previous chapters of the thesis.

Since the constant term \(-1/\rho\) has no affect on bank decisions it disappears from the first order condition that determine bank’s borrowing.

There are several additional restrictions that are necessary (for example maximum lending cannot exceed current account balance), which are discussed below, in the simulation description. Also, for discussion of the role of parameter \( \rho \), which captures the significance of trading costs please refer to Section 4.4.2.

**Financing from the central bank** Commercial bank can also choose to finance through the standing facility. In this case, its expected cost for the days just before the end of maintenance period\(^{10}\) is given by

---

\( ^9\)This step can be reproduced by substituting term \( x \) by \((\rho i_t b_t)\) in standard Taylor series expansion of exponential function: \( e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=1}^{\infty} \frac{x^n}{n!} \) for \( -\infty < x < \infty \)

\( ^{10}\)The formula for the last days is presented in Appendix 4.6.2.
\[
E(c_t) = i^l \left[ \int_{-\infty}^{-m_t-b_t+\varphi_t} (-m_t - b_t - \varphi_t - \varepsilon_t)f(\varepsilon_t)d\varepsilon_t \right] \\
- i^d \left[ \int_{-m_T-b_T+\varphi_T+d_T}^{\infty} (m_T + b_T + \varphi_T - d_T + \varepsilon_T)f(\varepsilon_T)d\varepsilon_T \right].
\]

Whenever negative shock realisation exceeds the current account balance (term 1), the bank must use the lending facility, costing \( i^l \). If the current account balance is higher than the required reserves (term 2), the surplus is deposited at the rate \( i^d \). The expected cost \( E(c_T) \) is a decreasing convex function of market borrowing \( b_t \).

Taking into account the above considerations, the total cost of finance for the commercial bank is given by

\[
K = \rho^{-1} \exp(\rho i_t b_t) - 1/\rho + E(c_t),
\]

which consists of both terms described above. Notice an interesting property of such a formulation: the marginal cost of obtaining an extra euro from the market increases with the borrowing volume. On the other hand, the marginal cost of the central bank finance is fixed at the \( i^l \) rate. This means that in certain situations banks might prefer central bank finance to using the market. When that happens, depends specifically on model parameters such as the relative cost of the standing facilities (spread between lending and deposit facility), or the degree of market frictions, captured by \( \rho \).

Since the total cost is the sum of two convex functions, in solving for the optimal borrowing for the individual bank on the last day of reserve maintenance period, I can use the first order condition

\[
i_T \exp(\rho i_T b_T) = i^l F(-m_T - b_T - \varphi_T + d_T) + i^d (1 - F(-m_T - b_T - \varphi_T + d_T)).
\]

For the days before the end of the reserve maintenance period, the first order

\footnote{Formal proofs in Appendix 4.6.2. Intuitively, the more the bank borrows from the market, the more likely it will use the deposit facility, which reduces the cost of the central bank finance. To see the convexity, note that first order condition is just the inverted function (4.14), plotted on Figure 3.2 in Chapter 3, and is clearly increasing. Also note that the derivative becomes effectively flat at extremely low or high values of borrowing, when the probability of using the standing facility converges to unity. At this point, an extra unit of funds lent (borrowed) in the market results in exactly the same increase in the use of standing facilities, which are offered at fixed rates.}
condition takes the form

\[ i_t \exp(\rho_i b_t) = i^1 F(-b_t - m_t - \varphi_t) + i^d [1 - F(d_t - m_t - b_t - \varphi_t)] \]

1.

\[ + i^d [1 - F(d_t - m_t - b_t - \varphi_t)] \]

2.

\[ - \int_{-b_t - m_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t. \quad (4.28) \]

3.

The proofs are very similar to the standard model and are presented in Appendix 4.6.3. Note that compared to the standard model, the right-hand side of eq. (4.28) remains unchanged, since the expected cost of using standing facilities does not change. However, the cost of market finance is equal to the market rate modified by the additional term, \( \exp(\rho_i b_t) \), This term can be interpreted as a tax on revenues from lending or an extra cost of borrowing, which the bank incurs when it uses the interbank market.

The interpretation of equation (4.28) is the same as in the standard model: at the optimal borrowing, the marginal cost of obtaining one unit of finance from the market is equal to the expected cost of obtaining one unit of finance from the central bank.

Figure 4.7: Demand curves with market frictions
Figure 4.7 shows the demand curves for the modified model. In comparison with Figure 3.2 (presented in Chapter 3), the flat part in the middle of the graph is now steep. This suggests that the demand for funds can be uniquely determined within this model.

4.4.2 Simulation

To verify the behaviour of the whole market, I ran a simulation of a 10-day maintenance period with 10 homogeneous banks. I assume that early $\phi_t$ and late $\varepsilon_t$ shocks are identically and independently distributed, approximated by the normal distribution with mean zero and standard deviation 10% of the starting current account value. I assume any aggregate inflow and outflow of liquidity, is adjusted in the open market operations; hence the sum of the shocks is normalised to zero. Given the distribution assumption (mean equal to zero), this is not a restrictive assumption. On the last day of the maintenance period, additional central bank fine tuning operations correct for possible aggregate liquidity shortage (or surplus) from the use of standing facilities. To analyse the impact of this assumption, I also ran a simulation with an aggregate shortage of funds for the whole maintenance period, that was corrected (by central bank intervention) on the last day, which is discussed in more detail below.

I verified three parameter values $\rho_1 = 10^{-2}$, $\rho_2 = 10^{-3}$, $\rho_3 = 10^{-5}$, which were chosen quite arbitrarily, based on grid size and average current account balance. The value of the parameter might seem small, but recall that the whole product $(\rho_i b_t)$ is crucial, as it enters the first order conditions exponentially. The right side of the equation (4.28), is a number between $i_l$ and $i_d$, which is typically linked very closely to market rate expectations. The left side is the market rate $i_t$ multiplied by a factor: in order to keep those two terms equal, the factor must be close to 1, or the expression $(\rho_i b_t)$ close to zero. I return to this issue in more detail below.

The target rate of the central bank is 2.50%, which is in the middle of the standing facility rates (2.0% and 3.0%). Those rates were actually used by the ECB for most of the period 2001-2003. Banks are homogeneous, with average reserve requirement set at 100 units.

The empirical results by Moschitz (2004) and Würtz (2003) and analysis presented in Subsection 3.2.4 indicate that the policy rate is crucial in the determination of the expected and current interest rate. However high trading frictions might upset this relationship and require explicit modelling of the expectations in order to ensure rationality. To accomplish this I introduce the following change to the algorithm. Instead of having fixed interest rate expectations, I only use the Central Bank target as an initial guess. I then use the results as expected rates in the next iteration. It turns out that this simple algorithm converges in 3 iterations, which ensures that the expected rate is equal to the average obtained in Monte Carlo simulation.

The results of the simulation are presented in Table 4.2 at the end of the chapter. The first three columns present the results of the scenarios with different values of parameter $\rho$ (reflecting different level of trade friction). The fourth column presents the result of the scenario under a possible liquidity shortage. Numbers 1-10 stand for days of the maintenance period. Starting from the top row of the
table, information is given on the average (across simulation runs) interest rate, the rate volatility (across simulations), and the total market accumulated negative and positive deviation from the neutral liquidity.\textsuperscript{12} To compute the deviation from neutral liquidity for the first day, I calculated the simple difference between the current account balance (after interbank trade) and average required reserve requirement (the same procedure was used with the Eurosystem data in Subsection 4.2.2). For the following days, I accumulate the positive and negative differences from the first day of the maintenance period for all banks. Reported values are averages across simulations.

**Observation 1** At low levels of market frictions banks initially deviate from the average level of required reserves.

This result was obtained for low levels of market frictions, for parameter $\rho = 10^{-5}$ and lower, which is presented in the third column of Table 4.2 and the scenario marked “low” in Figure 4.8.

![figure](image-url)  
**Figure 4.8:** Deviation from reserve requirement

Inspection of the table and figure reveals that for the most of the period, the imbalances (deviation from neutral liquidity) initially increase, which means that the banks do not fully adjust for the liquidity shock. The banks change their behaviour around the middle of the maintenance period, so that the deviation from

\textsuperscript{12} Similarly to the previous chapters, I define the neutral liquidity as current account balance that allows to exactly satisfy the remaining reserve requirement.
neutral liquidity starts to decline. In my example, this happens on the eighth day. At this point, the banks realise that in order to satisfy the reserve requirement, they must either use the standing facilities or engage in substantial market trade, both of which are expensive. Since the asymmetry between cost of borrowing and profit from lending is fairly small at the low level of the market frictions, the demand of one group matches the supply of the other, and there is no pressure on the level of interest rates.

This result can also be obtained analytically, by combining the martingale hypothesis with the first order condition (4.28).

The martingale hypothesis and law of iterated expectations imply the following condition for the interest rate:

\[ i_t = E(i_T) = i^*, \]  
(4.29)

so that the current interest rate is equal to the expected rate on the last day of the maintenance period, which in low-frictions scenario is very close to the central bank target rate \( i^* \).

Considering the first order condition (4.28), during the maintenance period both the reserve buffer \( (d_t - b_t - m_t) \) and the current account balance \( (m_t + b_t) \), are very high compared to the shock variance. This means that the probability of the shock exceeding those values is very low. It can be shown that this in turn allows for a simplification of the first order condition: \(^{13}\)

\[ i_t \exp(\rho_i b_t) \approx i^d (1 - F(-m_T - b_T - \varphi_T + d_T)) + i^l F(-m_T - b_T - \varphi_T + d_T) \]  
(4.30)

The current cost of market finance (LHS of the equation) is equal to the expected cost of using the standing facilities on the last day of the maintenance period.

On the first day of the maintenance period, the liquidity shocks realisations are still unknown, but their expected value is zero. This implies that the probabilities of using the deposit and lending facilities on the last day of the maintenance period are equal, so that assuming a symmetric corridor the expected cost of central bank finance is equal to the target rate:

\[ i_1 \exp(\rho_i b_1) = i^*. \]  
(4.31)

In order for both conditions (4.29) and (4.31) to be satisfied, the following must hold:

\[ \exp(\rho_i b_1) \approx 1 \]  
(4.32)

or

\[ (\rho_i b_1) \approx 0, \]  
(4.33)

which means that the product \( \rho b_t \) should be close to zero. This implies that the borrowing value, \( b_t \), will be close to zero, which means that the interbank trade

\(^{13}\)Proof in Appendix 4.6.4.
will not fully compensate for the liquidity shock and the deviation from required liquidity will increase.

Toward the end of the maintenance period, the frontloading banks face a higher probability of depleting reserve requirement buffer and consequent use of deposit facility. This will drive the right hand side of equation (4.30) toward the deposit rate and below the central bank target rate:

\[ i_t \exp(\rho_i b_t) < i^*. \tag{4.34} \]

Assuming condition (4.29) holds, this implies

\[ \exp(\rho_i b_t) < 1, \tag{4.35} \]

and

\[ b_t < 0 \tag{4.36} \]

In other words, the frontloading banks increase their lending, which reduces their deviation from neutral liquidity. Similarly, backloading banks increase their borrowing, which leads to the pattern of deviation from neutral liquidity documented for the Eurosystem in Subsection 4.2.2.

The role of the parameter \( \rho \) is crucial here. When \( \rho \) is low, the restrictions (4.33) are fairly mild and the diseconomies of scale are not significant for determining the optimal borrowing. This means that the banks will allow the deviation from neutral liquidity to rise, hoping it will be corrected by future shock realisations. As the end of the maintenance period approaches, the probability of using standing facilities increases for both front and backloading banks, giving them an incentive to trade and neutralise the deviation from neutral liquidity. The cases for higher \( \rho \) values are discussed below.

Another, perhaps striking, feature of the scenario with a low level of market frictions is that the average interest rate remains equal to the target level and the interest rate volatility is low. This is a direct consequence of the assumption of a high reserve requirement, which allowed me to state condition (4.30) above. Recall that on the last day of the maintenance period another first order condition holds, given by equation (4.27). Combining these two equations leads to the following conclusion:

\[ i_t \exp(\rho_i b_t) \approx i_T \exp(\rho_i b_T). \tag{4.37} \]

The functional form assumed in this chapter implies that when the parameter \( \rho \) is very low, the volume of borrowing has little impact on this condition. For instance, at a borrowing volume of 30 units, which is three times the shock standard deviation, \( \exp(\rho_i b_t) \approx 1.00075 \). At twice that level of borrowing, \( \exp(\rho_i b_t) \approx 1.0015 \), which is less than 1% higher. Now, the borrowing value is affected mostly by realisations of the liquidity shocks, which are random in my model. If, however, the borrowing value has little effect on condition (4.37), neither does the liquidity shock. In other words, the realisation of a liquidity shock does not affect the level of the interest rate, which remains close to the expected level. Since I assumed that the banks expect the rate to stay at the central bank target, the average market clearing interest rate must remain equal to the target level, which is what I observe in this scenario. I discuss the other implications of condition (4.37) in the remaining part of this section.
**Observation 2** At high levels of market frictions, banks substantially reduce trade volumes over the whole maintenance period.

This result holds for high values of parameter $\rho$, such as the 0.01 used in this example. At this point, trading in the market becomes less favourable comparing to borrowing (or lending) from the central bank. Intuitively, the cost of using the standing facilities is capped, while the cost of market finance moves in line with trade volume. This means that there is some maximum borrowing/lending amount, which depends on the relative cost of market frictions. This value can be found using first order condition (4.28) repeated here:

$$i_t \exp(\rho i_t b_t) = \underbrace{i^d F(-b_t - m_t - \varphi_t)}_{1} + \underbrace{i^d [1 - F(d_t - m_t - b_t - \varphi_t)]}_{2} - \int_{-b_t - m_t - \varphi_t}^{d_t - m_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon_t) d\varepsilon_t$$

The RHS of the equation is a probability-weighted average of three terms, and its value is in between $i^d$ and $i^d$. It is now possible to find the values of $b_t$, at which the LHS of the equation is equal to those boundaries. At the maximum possible lending value, $b^d_t$, the bank is indifferent between lending the funds to other banks or depositing them at the central bank deposit facility rate:

$$i_t \exp(\rho i_t b^d_t) = i^d$$

or

$$b^d_t = (\rho i_t)^{-1} \ln(i^d / i_t)$$

For the values used in the simulation, $\rho = 0.01$, $i^d = 2$ and the interest rate in the middle of corridor, $i_t = 2.5$, the maximum volume the bank can lend is $b^d_t = -8.9$. Any unit of lending beyond $b^d_t$ yields a smaller profit than depositing it in the deposit facility. Similarly, it is possible to compute the value of maximum borrowing: $b^l_t = 7.2$. Any units of funds borrowed from the market beyond $b^l_t$ cost more than using the central bank lending facility. The $b^l_t$ and $b^d_t$ values are below the standard deviation of the liquidity shock ($\sigma = 10$), which means they will likely become binding. This means, however, that the shocks will not be offset during the whole maintenance period and the deviation from neutral liquidity will widen continuously, even on the last day of the maintenance period. In addition, without the balancing trade some banks will find themselves spending their reserve requirement buffers early, which forces them to use the deposit facility for any outstanding positive balances. With no trading to redistribute the liquidity shocks, the banks will be forced to use the central bank facilities on the last day of the maintenance period in order to comply with the reserve requirement.

A high value of $\rho$ also implies that the imbalance between borrowing and lending will be substantial. Comparing $b^l_t$ and $b^d_t$ reveals that the maximum borrowing is 20% lower than maximum lending, which impacts the behaviour of the interest rate. Intuitively, if the cost of market borrowing is higher than the profit from the corresponding market lending, the borrowing facility is much more attractive.
Hence, the lenders actually must compete for borrowers, which lowers the market clearing interest rate. This observation can be confirmed in the results of the scenario presented in the first column of Table 4.2.

The analysis presented in the paragraph above are important for the monetary policy implementation and constitute an important contribution of this chapter. Observe that in the scenario with very high trading frictions, it is possible that $b_t^l$ and $b_t^d$ can drop to such low levels that the market trade will effectively freeze. In this case the commercial banks will compensate for the liquidity shocks using central bank standing facilities, which is first costly, and second, it transfers all the lending risk onto the central bank. Such a situation also violates one of the goals of the central banks that are obliged to ensure efficient market functioning.

A situation where the frictions rise could occur as a result of a liquidity crisis is discussed in the following chapter.

**Observation 3** At the medium level of market frictions banks track the average level of required reserves.

Using an intermediate parameter value, $\rho = 0.001$, produced a very interesting outcome, reported in the second column of Table 4.2 and marked as medium on Figure 4.8. It turns out that at this value of $\rho$ the banks compensate for liquidity shocks, which means that account balances remain close to the average required reserves. This can be explained as follows. In comparison with the scenario with low trade frictions, adjusting on the last days of the maintenance period, which requires a single massive transaction, is very costly. In the medium range of $\rho$, the cost of one large transaction is higher than the cumulative cost of several smaller transactions, even though the total volumes traded in the latter case will probably be higher. At the same time, the costs of trade are still lower than the expected cost of using standing facilities, which helps to avoid the market freeze scenario under high trade frictions. Combined, this means that the banks find that the most profitable way is to remain in the safe range, where the costs of daily trade during whole period are lower than the cost of one large market transaction on the last day or using standing facilities.

The same results can be obtained by inspection of condition (4.37), presented above. Recall that for low values of market frictions, I pointed out that liquidity shocks have little effect on the level of the interest rate, which remains close to the central bank target. However, a higher level of market frictions, represented by a higher value of $\rho$, changes that conclusion. In fact, for a borrowing volume of 30 units, $\exp(\rho \int_0^T b_t^d) = 1.078$, and for borrowing of 60 units $\exp(\rho \int_0^T b_t^d) = 1.162$, i.e. the difference is around 10%. If the current interest rate is equal to the expected rate, so that $i_t \approx i_T$, condition (4.37) also implies $b_t \approx b_T$, which leads to the smoothing of borrowing observed in this scenario.

**Observation 4** No liquidity effect in the market.

Recall from the discussion in previous chapter that one of the most important issues for central bankers is the presence of the liquidity effect, which links the market reserves (controlled in open market operations) and the interest rate. In
the empirical works of Moschitz (2004) and Würtz (2003), the liquidity effects were significant only on the last days of the maintenance period, and the model derived in Chapter 3 confirmed their prediction.

In order to analyse the impact of market liquidity, I ran a simulation with an aggregate shortage of liquidity in the market (total value of the shortage was 150 units compared to market liquidity 1000 units). The shortage is corrected by the central bank on the last day of the maintenance period, so that the expected interest rate remains on target. In this scenario I use low values of market frictions, to resemble the actual behaviour of the Eurosystem. The results are reported in the last column of Table 4.2. It seems that the liquidity shortage had no effect on the level of the interest rate, and the banks that were previously deviating from the required level of reserves, now almost perfectly adjust for liquidity shocks on a daily basis. This is reflected in an essentially zero positive deviation from neutral liquidity and a negative deviation equal to the aggregate imbalance (which was set at 150 units value).

To first understand the behaviour of the interest rate, recall the results from the scenario 1, where a similarly low level of market frictions was assumed. In particular, condition (4.37) can explain why the interest rate does not depend on the liquidity shock realisation. The case of liquidity shortage does not affect this condition in any significant way. The central bank intervention anchors the interest rate expectations, and an average liquidity shortage of 15 units/bank is too low to put the bank at risk of using the standing facilities.

Intuitively, when market frictions are low, the banks are not willing to accept interest rates that significantly deviate from the expected level. The aggregate shortage does not seem to affect this, since even the banks with substantial shortages expect they will be able to obtain the reserves from the market with little additional cost.

Unlike interest rate, the borrowing patterns seem to be affected by the liquidity shortage. The banks that experienced positive shocks now trade the excess liquidity away immediately instead of accumulating it until the end of the maintenance period. The market symmetry implies that the banks faced with negative shocks are left with an imbalance equal only to the aggregate shortage. In contrast to scenario 1 it seems that the banks adjust for that deviation right away. This might be due to the fact that the shortage accumulates for each day it is present, so, for instance, on day 1 it might be 150 units, but on the second day the market would require an injection of 300 units, to bring it in line with neutral liquidity. This means that the effect of shortage on banks’ expected cost of funding is considerably stronger, which stimulates interbank trade.

The lack of liquidity effect means that the market response to the change in the aggregate liquidity is weak and the interest rate is primarily driven by the expectations. It stabilises the market in the presence of aggregate liquidity shocks, but could also weaken the effect of the open market operations shifting the focus to the control of the expectations. This can be obtained by adjusting the liquidity on the settlement day when the liquidity effect is strong. An example of such a policy is the ECB special liquidity injection that occurs in the form of fine tuning operations on the last day of the maintenance period.

In order for the settlement day intervention to be fully effective, the central
The ECB would have to stand ready to drain or supply reserves at the rate equal to policy rate. This addresses any potential problems related to market power, unexpected liquidity shocks or errors in the estimations of the aggregate autonomous liquidity factors.

However, as a side effect, it would also reduce the banks' incentives to trade in the interbank market. If the bank can obtain any desired level of reserves from the central bank, there is little reason to deal with other banks, which might incur additional cost. This creates a conflict with one of the key policy objectives, which (in the Eurosystem) require the central bank to “act in accordance with the principle of an open market economy with free competition, favouring an efficient allocation of resources” (ECB (2006)). The ECB solved that problem by designing the fine tuning operations similar to regular open market operation tenders. The liquidity draining operations are executed as fixed rate tenders, and liquidity supply as variable rate tenders. Thus, there is a degree of uncertainty that encourages active participation in the interbank trade. I return to this issue in the following chapter.

Observations 1-4 represent the key contribution of the chapter. There are, however, several other issues that should be addressed as well, and which are related to the key parameter $\rho$, which measures the impact of trading frictions on the behaviour of commercial banks.

First of all, different values of $\rho$ seem to induce different bank behaviour. For high values of $\rho$, the increasing cost of trade frictions becomes quite important, and banks are more reluctant engage in trade, turning to the standing facilities for finance. The exact numbers might be misleading, as they depend on the relative values of the parameters used (such as reserves volume), but, for instance, I find that at the parameter value $\rho = 0.01$, the banks refrain from trade until almost the last day of the maintenance period (scenario marked “high” on the Figure 4.8).

As the value of $\rho$ decreases, so does the cost of daily trade, and offsetting shocks on a daily basis becomes a viable option. At a very low level of $\rho$, the banks find that large transactions do not cost so much, and it is cheaper to wait for several liquidity shocks realisations before making final adjustments.

The second important issue is how close the market rate follows the central bank target. Inspection of Table 4.2 reveals that the central bank target is easily achieved for low and intermediate parameter values, and that the market deviates quite a bit at high values of $\rho$. At high $\rho$, the asymmetry between cost of borrowing and profits from lending become quite substantial, and the market clearing rate adjusts in order to compensate for that. Intuitively, in the absence of credit risk, the lending facility becomes an attractive alternative much faster than deposit facility at comparable deviations from the neutral liquidity.

Finally, one should compare the simulations results with the actual behaviour of the Eurosystem. The analysis presented above is subject to several assumptions, such as bank homogeneity. These assumptions mean that I cannot dismiss the possibility that the pattern observed in the market is caused by external factors and even by active trade. Taking into account this considerations, it seems that of the three scenarios presented, the one with the low level of market frictions seems to best capture the behaviour of the Eurosystem. First, it successfully replicates the pattern, where the initial deviation from the target rate increases then declines in
the latter part of the maintenance period. Second, the interest rate remains closely
tied to the central bank target, which was the actual behaviour of the Eonia in
the period studied.

4.4.3 Section summary

The model presented in this section is a standard model of interbank trade mo-
dified to include the asymmetry between borrowing and lending and potential
market frictions-related costs. I ran a Monte-Carlo simulation for different values
of parameter $\rho$ and found that I could duplicate some of the patterns observed in
the Eurosystem market for a relatively low level of the market frictions:

- The banks that initially deviate from neutral liquidity return to the required
  level later on, during the maintenance period.
- There is no liquidity effect in the market.
- The interest rate follows the central bank target.

The results of the simulation seem to indicate that the Eurosystem might be sub-
ject to some trade-related costs which encourage commercial banks to reduce their
trading volumes early in the maintenance period. This results in a deviation of
current accounts balances from average required reserves, which is gradually cor-
rected in the second part of the maintenance period. The interest rate follows the
central bank target, with relatively low volatility; however, the aggregate liquidity
does not seem to affect it in any significant way.

These findings could have potentially interesting implications for central bank
policy. Assuming that the risk of using standing facilities is small during the
maintenance period, banks trading decisions are affected more by the expected
interest rate level than by their current liquidity. This is perhaps not a new finding
and was indeed stated already by Välimäki (2003) and in Chapter 3, both of which
analyse the risk neutral case. This chapter extends those results to a more robust
case where banks face certain limitations and market frictions.

The Central bank could use this finding in determining the frequency and the
scale of liquidity supply operations. It seems that, even if the central bank supplies
liquidity over or below benchmark level, the market rate will remain unaffected.
Similarly, more frequent (and thus exact) operations would not affect the interest
rate. This gives the central bank room for potential errors but complicates its job
if interest rate expectations run out of control.

4.5 Conclusions

The chapter explains what determines the demand for reserves in the interbank
market. I point out certain patterns in the Eurosystem using sample information
from 86 banks. Contrary to popular belief, commercial banks do not strictly follow
the required reserves level every day of the maintenance period. In fact, most of
them choose to either front or backload (satisfy requirement early or late). Their
preferences do not seem to be affected by their size or country of origin.
Using a modification of the standard model of the interbank market developed by Poole (1968) and Pérez-Quirós and Rodríguez-Mendizábal (2006), I show how market-related frictions can explain that pattern. It seems that in the beginning of the maintenance period banks are slow to react to liquidity changes. With a large reserve requirement buffer a bank will prefer to postpone adjustments to required reserves, which reduces the costs of interbank trade. As the end of the maintenance period approaches, the threat of being forced to use the standing facilities becomes more credible, and banks are more willing to trade and revert to the neutral liquidity level. Instead of postponing that process until the very last day of the period, banks choose to spread it, in order to avoid large transactions, which are likely to incur extra expenses. This suggests that both daily operating expenses and increasing marginal transactions costs are important in the banks’ decision-making.

At the Eurosystem level of market frictions, the banks’ individual patterns in the reserves demand seem to have little impact on the level of the interest rate. Indeed I find that the interest rate remains on a level very close to the expected one with little volatility, and aggregate liquidity does not affect it significantly. This is related to the previous paragraph: the banks, under certain circumstances, do accept that their current accounts will deviate from the required reserves level.

Those findings have specific policy applications. The central bank’s actual control over the interest rate is exerted mainly through expectations, rather than market liquidity. Since the required reserves are high compared to the volatility banks face, it offers banks a sufficient buffer not only from random liquidity shocks but also from aggregate liquidity changes.
### 4.6 Appendix

#### 4.6.1 Tables

#### Average rate on day

<table>
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<tr>
<th></th>
<th>$\rho = 10^{-2}$</th>
<th>$\rho = 10^{-3}$</th>
<th>$\rho = 10^{-5}$</th>
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#### Rate volatility on day

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<td>0.000</td>
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<td>7</td>
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<td>0.011</td>
<td>0.000</td>
<td>-</td>
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<td>8</td>
<td>0.067</td>
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#### Accumulated positive deviation

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<tr>
<td>2</td>
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<td>56.8</td>
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<td>89.4</td>
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<td>10</td>
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<td>101.4</td>
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</table>

Table 4.2: Simulation results.
4.6.2 Proofs of results from Subsection 4.4.1.

To prove the convexity of the cost function, I need to show that

$$\frac{\partial^2 E(c_t)}{\partial b_t^2} > 0. \quad (4.40)$$

For example, for the last day of maintenance period $T$, I have calculated above in (4.44) that

$$\frac{\partial E(c_t)}{\partial b_t} = -i^l F(-m_T - b_T - \varphi_T + d_T) - i^d(1 - F(-m_T - b_T - \varphi_T + d_T)) =$$

$$= -i^d - (i^l - i^d) F(-m_T - b_T - \varphi_T + d_T) < 0. \quad (4.41)$$

Taking second order conditions yields

$$\frac{\partial}{\partial b_t} \left( \frac{\partial E(c_t)}{\partial b_t} \right) = \frac{\partial}{\partial b_t} \left[ i^d + (i^l - i^d) F(-m_T - b_T - \varphi_T + d_T) \right] =$$

$$= (i^l - i^d) f(-m_T - b_T - \varphi_T + d_T) > 0. \quad (4.42)$$

4.6.3 Proof of the first order conditions from Subsection 4.4.1.

The expected cost of using standing facilities on the last day of maintenance period is given by the expression

<table>
<thead>
<tr>
<th>Average rate on day</th>
<th>$\rho = 10^{-2}$</th>
<th>$\rho = 10^{-3}$</th>
<th>$\rho = 10^{-5}$</th>
<th>Liquidity shortage</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>-51.9</td>
<td>-41.6</td>
<td>-46.9</td>
<td>-157.0</td>
</tr>
<tr>
<td>2</td>
<td>-108.7</td>
<td>-56.8</td>
<td>-92.3</td>
<td>-307.4</td>
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<tr>
<td>3</td>
<td>-175.9</td>
<td>-70.6</td>
<td>-145.2</td>
<td>-457.6</td>
</tr>
<tr>
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<td>-254.3</td>
<td>-78.8</td>
<td>-204.9</td>
<td>-606.7</td>
</tr>
<tr>
<td>5</td>
<td>-342.2</td>
<td>-85.5</td>
<td>-268.6</td>
<td>-753.0</td>
</tr>
<tr>
<td>6</td>
<td>-438.0</td>
<td>-89.3</td>
<td>-334.8</td>
<td>-898.4</td>
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<tr>
<td>7</td>
<td>-542.6</td>
<td>-92.3</td>
<td>-399.2</td>
<td>-1,045.4</td>
</tr>
<tr>
<td>8</td>
<td>-655.1</td>
<td>-94.9</td>
<td>-458.1</td>
<td>-1,192.1</td>
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<tr>
<td>9</td>
<td>-777.0</td>
<td>-94.5</td>
<td>-447.8</td>
<td>-1,340.9</td>
</tr>
<tr>
<td>10</td>
<td>-730.0</td>
<td>-98.9</td>
<td>-40.3</td>
<td>-41.2</td>
</tr>
</tbody>
</table>

Table 4.3: Table 4.2 continued
\[
E(c_T) = i^d \left[ \int_{-\infty}^{d_T-m_T-b_T-\varphi_T} (d_T - m_T - b_T - \varphi_T - \varepsilon_T) f(\varepsilon_T) d\varepsilon_T \right] - i^d \left[ \int_{-\infty}^{\infty} (m_T + b_T + \varphi_T - d_T + \varepsilon_T) f(\varepsilon_T) d\varepsilon_T \right] = \\
=-i^d(m_T + b_T + \varphi_T - d_T) F(-m_T - b_T - \varphi_T + d_T) - i^d(1 - F(-m_T - b_T - \varphi_T + d_T)) \\
- i^d(\int_{-\infty}^{m_T-b_T-\varphi_T+d_T} \varepsilon_T f(\varepsilon_T) d\varepsilon_T - i^d \int_{-\infty}^{\infty} \varepsilon_T f(\varepsilon_T) d\varepsilon_T). \tag{4.43}
\]

Substituting this equation into the cost function (4.12),

\[
V_T = \rho^{-1} \exp(\rho i_T b_T) - 1/\rho + E(c_T),
\]

and solving the first order conditions with respect to \(b_T\) yields

\[
-i_T \exp(\rho i_T b_T) = -i^d F(-m_T - b_T - \varphi_T + d_T) + i^d (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T) - i^d(1 - F(-m_T - b_T - \varphi_T + d_T)) \\
- i^d(m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T) - i^d(m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T) + i^d(m_T + b_T - d_T) f(-m_T - b_T - \varphi_T + d_T), \tag{4.44}
\]

which can be simplified to

\[
i_T \exp(\rho i_T b_T) = i^d F(-m_T - b_T - \varphi_T + d_T) + i^d(1 - F(-m_T - b_T - \varphi_T + d_T)). \tag{4.45}
\]

\[\blacksquare\]

For days before the last one, the cost function is

\[
E(c_t) = i^d \left[ \int_{-\infty}^{m_t-b_t-\varphi_t} (-m_t - b_t - \varphi_t - \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \right] \\
- i^d \left[ \int_{-\infty}^{m_t-b_t+\varphi_t+d_t} (m_t + b_t + \varphi_t - d_t + \varepsilon_t) f(\varepsilon_t) d\varepsilon_t \right] = \\
- i^d(m_t + b_t + \varphi_t) F(-m_t - b_t - \varphi_t) - i^d(m_t + b_t + \varphi_t - d_t)(1 - F(-m_t - b_t - \varphi_t + d_t)) \\
- i^d \left[ \int_{-\infty}^{m_t-b_t-\varphi_t} \varepsilon_t f(\varepsilon_t) d\varepsilon_t \right] - i^d \left[ \int_{-\infty}^{\infty} \varepsilon_t f(\varepsilon_t) d\varepsilon_t \right]. \tag{4.46}
\]
The resulting value function is

\[ E_t(V_t) = \rho^{-1} \exp(\rho i_t b_t) - 1/\rho - i^d(m_t + b_t + \varphi_t)F(-m_t - b_t - \varphi_t) \]
\[ - i^d(m_t + b_t + \varphi_t - d_t)(1 - F(-m_t - b_t - \varphi_t + d_t)) \]
\[ - i^d \left[ \int_{-\infty}^{-m_t - b_t - \varphi_t} \varepsilon_t f(\varepsilon_t) d\varepsilon_t \right] - i^d \left[ \int_{-m_t - b_t - \varphi_t + d_t}^{\infty} \varepsilon_t f(\varepsilon_t) d\varepsilon_t \right] \]
\[ + EV_{t+1}. \] (4.47)

The first order conditions are

\[ i_t \exp(\rho i_t b_t) = i^d F(-m_t - b_t - \varphi_t) + i^d (1 - F(-m_t - b_t - \varphi_t + d_t)) \]
\[ + E \frac{\partial V_{t+1}}{\partial b_t}. \] (4.48)

To calculate the last element of the F.O.C. first note that the borrowing maturity is one period; hence it has no direct impact on current account balance or borrowing in the next period. It, however, does impact the deficiency value, which must be analysed for 3 separate cases:

1. The shock forces the bank to use the lending facility, meaning the deficiency remains unchanged.
2. The shock forces the bank to use the deposit facility, meaning the whole deficiency is satisfied.
3. The intermediate case, where the shock value lowers the deficiency without forcing the bank to use any of the facilities.

In the first two cases, the deficiency either remains unchanged (compared to previous period) or drops to zero, meaning the borrowing decision has no impact on its value.

\[ \int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = \int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} \frac{\partial d_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = 0 \] (4.49)

and similar

\[ \int_{-m_t - b_t - \varphi_t + d_t}^{\infty} \frac{\partial V_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = 0. \] (4.50)

In the last case, whenever the deficiency is carried over by one period, one more unit of borrowed funds decreases required reserves by one unit, hence

\[ \int_{-m_t - b_t - \varphi_t}^{-m_t - b_t - \varphi_t + d_t} \frac{\partial V_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = \int_{-m_t - b_t - \varphi_t}^{-m_t - b_t - \varphi_t + d_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} \frac{\partial d_{t+1}}{\partial b_t} f(\varepsilon) d\varepsilon = \]
\[ - \int_{-m_t - b_t - \varphi_t}^{-m_t - b_t - \varphi_t + d_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon. \] (4.51)

This leads to the profit maximising conditions.
\[ i_t \exp(\rho i_t b_t) = i^l F(-b_t - m_t - \varphi_t) + i^d [1 - F(d_t - b_t - m_t - \varphi_t)] - \int_{-m_t-b_t-\varphi_t}^{d_t-b_t-\varphi_t} \frac{\partial V_{t+1}}{\partial d_t+1} f(\varepsilon) d\varepsilon. \]  

(4.52)

\[ 4.6.4 \text{ Proofs of the property used in Subsection 4.4.2.} \]

To prove the property used in Subsection 4.4.2, start with the equilibrium interest rate:

\[ i_t \exp(\rho i_t b_t) = \underbrace{i^l F(-b_t - m_t - \varphi_t)}_{(1)} + \underbrace{i^d [1 - F(d_t - m_t - b_t - \varphi_t)]}_{(2)} - \int_{-m_t-b_t-\varphi_t}^{d_t-b_t-\varphi_t} \frac{\partial V_{t+1}}{\partial d_t+1} f(\varepsilon) d\varepsilon. \]  

(4.53)

Terms (1) and (2) are the respective expected costs of using the lending and deposit facility. If the reserve buffer is large, a positive shock will not exceed it, which would force the use of the deposit facility. If the current account balance is high, a negative shock will never force use of lending facility. Hence, terms (1) and (2) will converge to zero. This reduces the first order condition to

\[ i_t \exp(\rho i_t b_t) \approx \int \frac{\partial V_{t+1}}{\partial d_t+1} f(\varepsilon) d\varepsilon. \]  

(4.54)

The derivative of the value function with respect to deficiency can be solved using an envelope theorem \((\frac{\partial V}{\partial m_t} = 0)\), so the problem reduces to

\[ \frac{\partial V_t}{\partial d_t} = \frac{\partial}{\partial d_t} (E(c_t) + EV_{t+1}), \]  

(4.55)

where \(E(c_t)\) is the expected cost of using standing facilities. However, the probability of using the lending facility is not affected by the deficiency value, which allows for simplification by considering only the expected cost of using the deposit facility:

\[ \frac{\partial V_t}{\partial d_t} = \underbrace{\frac{\partial}{\partial d_t} \left\{ -i^d (m_t + b_t + \varphi_t - d_t)(1 - F(-m_t - b_t - \varphi_t + d_t)) \right\}}_{(4.56)} - i^d \left[ \int_{-m_t-b_t-\varphi_t+d_t}^{\infty} \varepsilon f(\varepsilon) d\varepsilon \right] + EV_{t+1}. \]  

(4.56)

Solving the partial derivative:
\[
\begin{align*}
\frac{\partial V_t}{\partial d_t} &= i^d \{(1 - F(-m_t - b_t - \varphi_t + d_t)) + (m_t + b_t + \varphi_t - d_t) f(-m_t - b_t - \varphi_t + d_t) \\
&\quad + (m_t - b_t - \varphi_t + d_t) f(-m_t - b_t - \varphi_t - d_t)\} + E \frac{\partial V_{t+1}}{\partial d_t} = \\
&\quad i^d [1 - F(-m_t - b_t - \varphi_t + d_t)] + E \frac{\partial V_{t+1}}{\partial d_t}.
\end{align*}
\]

The expected derivative of the value function can be presented as

\[
E \frac{\partial V_{t+1}}{\partial d_t} = \int \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon) d\varepsilon = \int \frac{\partial V_{t+1}}{\partial d_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon.
\]

Similarly to above, I analyse 3 cases:

1. The shock realisation forces the bank to use the lending facility, which means that the deficiency remains unchanged. One extra unit of deficiency today means one extra unit of deficiency tomorrow:

\[
\frac{\partial d_{t+1}}{\partial d_t} = 1.
\]

Hence the value function derivative

\[
\int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon) d\varepsilon = \int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon.
\]

2. The shock forces the bank to use the deposit facility, which means that the whole deficiency is satisfied. A deficiency today thus has no impact on the deficiency tomorrow and

\[
\int_{d_t - m_t - b_t - \varphi_t}^{\infty} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon) d\varepsilon = 0.
\]

3. The intermediate case, where the shock value only lowers the deficiency without forcing the bank to use any of the facilities is similar to case 1:

\[
\frac{\partial d_{t+1}}{\partial d_t} = 1 \rightarrow \int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon) d\varepsilon = \int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon.
\]

Combining those 3 cases yields

\[
\frac{\partial V_t}{\partial d_t} = i^d [1 - F(d_t - b_t - m_t - \varphi_t)] + \int_{-\infty}^{-m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon.
\]

If the current account balance is high compared to the size of the liquidity shock, \( F(d_t - b_t - m_t - \varphi_t) \) converges to one, hence the first part of that expression vanishes. This leaves

\[
\frac{\partial V_t}{\partial d_t} = \int \frac{\partial V_{t+1}}{\partial d_{t+1}} f(\varepsilon) d\varepsilon.
\]
which can be interpreted as
\[
\frac{\partial V_t}{\partial d_t} \approx E \frac{\partial V_{t+1}}{\partial d_{t+1}}. \tag{4.65}
\]

Using the law of iterated expectations leads to
\[
\frac{\partial V_t}{\partial d_t} \approx E \frac{\partial V_T}{\partial d_T}. \tag{4.66}
\]

So now it is sufficient to solve for the value function on the last day of the maintenance period:
\[
\frac{\partial V_T}{\partial d_T} = \frac{\partial c_T}{\partial d_T} = \frac{\partial}{\partial d_T} \left\{ -i^d (m_T + b_T + \varphi_T - d_T) (1 - F(-m_T - b_T - \varphi_T + d_T)) \\
- i^l (m_T + b_T + \varphi_T - d_T) F(-m_T - b_T - \varphi_T + d_T) \\
- i^d \int_{-m_T - b_T - \varphi_T + d_T}^{\infty} \varepsilon_T f(\varepsilon_T) d\varepsilon_T - i^l \int_{-\infty}^{-m_T - b_T - \varphi_T + d_T} \varepsilon_T f(\varepsilon_T) d\varepsilon_T \right\}, \tag{4.67}
\]

which can be solved as
\[
\frac{\partial V_T}{\partial d_T} = i^d [(1 - F(-m_T - b_T - \varphi_T + d_T)) + (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T)] \\
+ i^l [F(-m_T - b_T - \varphi_T + d_T) - (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T)] \\
- i^d (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T) \\
+ i^l (m_T + b_T + \varphi_T - d_T) f(-m_T - b_T - \varphi_T + d_T), \tag{4.68}
\]

which can be simplified to
\[
\frac{\partial V_T}{\partial d_T} = i^d (1 - F(-m_T - b_T - \varphi_T + d_T)) + i^l F(-m_T - b_T - \varphi_T + d_T), \tag{4.69}
\]

which is the expected cost of using the standing facilities on the last day of the maintenance period. Combining (4.54) and (4.69) yields
\[
i_t \exp(\rho_t b_t) \approx i^d (1 - F(-m_T - b_T - \varphi_T + d_T)) + i^l F(-m_T - b_T - \varphi_T + d_T). \tag{4.70}
\]

\[\blacksquare\]
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Välimäki, T., 2006. Why the marginal mro rate exceeds the ecb policy rate? Discussion Papers 20, Bank of Finland.

Chapter 5

Liquidity crisis in the interbank market

5.1 Introduction

In the summer of 2007, the problems related to a relatively small fraction of the US based mortgages, quickly spread to the majority of the world's financial markets and resulted in a major liquidity crisis. Caballero and Krishnamurthy in the Bank of France Stability Report (BoF (2008)) estimate the potential losses from subprime mortgages at USD 270 billion, which should be compared to trillions of dollars worth of assets traded every day in the financial markets. Yet, as they argue “the incidences of defaults have been the trigger for the current severe liquidity crisis that has ensnared markets from consumer credit to corporate credit”.

The overnight interbank markets, which are crucial for monetary policy implementation, are not directly connected with asset-backed securities (ABS) markets; yet they have been caught up in the turmoil. A severe liquidity crunch combined with excessive volatility of interest rates forced the central banks to take an active role in order to maintain their credibility and commitment to policy targets.

The focus of this chapter is on developments in the interbank markets in the latter part of 2007. I identify the key elements of the liquidity crisis and their effects on the interbank market rate and trade volumes. Additionally, I analyse the impact of central bank intervention, and find that liquidity surplus alone is not sufficient to bring the interest rate to the target level. To accomplish this, the central bank needs to be able to significantly influence market conditions such as credit risk.

The recent crisis has immediately attracted a great deal of attention. The first, preliminary analysis of events leading to the worldwide credit crunch was presented in a special report in The Economist (The Economist, 18 October 2007). A more structured approach, backed by market data, was presented in periodic reports published by the UK, French and European central banks (ECB (2007), BoF (2008) and BoE (2007)) and the Bank for International Settlements (BIS (2007), BIS (2008b)). A survey of central banks’ policy reactions to the liquidity crisis is
presented in Chailloux et al. (2008a) who also wrote second paper more focused on the collateral issue: Chailloux et al. (2008b). All these publications point to issues related to the pricing, risk assessment and management of asset-backed securities as sources of the turmoil in the financial markets.

The crisis in asset-backed securities eventually affected the core money markets. The average interbank market interest rate, which is the main tool of operational policy, soared, especially for longer maturities and unsecured lending. This was accompanied by a corresponding increase in interest rate volatility. The banks started to stockpile cash beyond the level necessary to satisfy the reserve requirements ("liquidity hoarding"), which prevented the efficient redistribution of liquidity. Regarding the channel through which the liquidity crisis affected the interbank markets, the central banks’ view is that “the squeeze in the interbank money market reflected the fact that participants in the market became fearful about counterparty credit risks and they also hoarded liquidity in case of unexpected need” (ECB (2007) p.30).

The classic references on the interbank risk is Diamond and Dybvig (1983) who were among the first to model bank runs and stress the importance of the central bank role as the lender of last resort. Rochet and Tirole (1996) and Freixas et al. (2000) analyse systemic risk in the interbank market that results from decentralised nature of interbank lending. Central bank insurance reduces the incentives for peer monitoring and might lead to the banks taking excessive risks. These papers assume the interbank lending is mainly used to transfer the funds from the banks with liquidity surpluses (such as small banks serving local customers) to large investment banks. In contrast, as discussed in previous chapters, in this thesis I focus on overnight lending, which is used to offset random liquidity shocks that occur as a result of transactions between banks’ customers.

Gaspar et al. (2008) analyse the impact segmentation, where banks only trade within a single trading group. To find the equilibrium rate for each group they adopt standard risk neutral model, which does not allow for credit risk or asymmetry between lenders and borrowers. In the crisis banks might have responded to greater uncertainty by restricting their activities to trusted partners (within trading group). In this chapter, however, I focus on the behaviour of individual banks, which (as in the case of Northern Rock) had strongest impact on the market.

The model used in this chapter is based on the interbank market model developed in the previous chapters of this thesis, based on Poole (1968), Pérez-Quirós and Rodríguez-Mendizábal (2006), Bartolini et al. (2001) and Välimäki (2003), all of which were reviewed extensively before.

The most important modifications include the introduction of the credit risk, increase in transaction cost and expected liquidity shortage. To include the market frictions, I employ the procedure developed in the previous chapter. Expected liquidity shortage can be captured by a modification of the liquidity shock expected distribution. Finally, to introduce credit risk, I assume that lending profits are reduced by (perceived) default probability.

I also analyse the efficiency of central bank intervention. In order to accomplish this, I introduce liquidity injection on the first day of the crisis. The liquidity is allotted proportionally, but the results should not change under variable rate tenders. The excess is eventually removed, so that market liquidity on the last day
of the maintenance period is equal to the remaining reserve requirement.

I find a link between trade frictions, expected liquidity shortage, credit risk and the observed market developments. In particular, it seems that all three elements of the crisis were necessary ingredients in the patterns observed in the market. I also find that an injection of liquidity substitutes (rather than restores) interbank market trade. Bringing the interest rate back to the target level requires central bank control over market conditions such as credit risk.

The chapter is structured as follows. Section 5.2 introduces the background of the crisis and documents patterns observed in the Eurosystem, UK and US interbank markets. I also briefly review the central banks' response and discuss their impact on the market behaviour. In Section 5.3, I identify the key elements of the liquidity crisis from 2007. I then proceed to modify the standard interbank model to include trade frictions, expected liquidity shortage and credit risk. In Section 5.4, I report and interpret the results of Monte Carlo simulation of the interbank market under the liquidity crisis. Section 5.5 concludes.

5.2 Background information

5.2.1 Asset market crisis

The financial crisis in 2007 was first analysed in the reports published by major central banks (ECB (2007), BoF (2008) and BoE (2007)) and the Bank for International Settlements (BIS (2007), BIS (2008b)). This section contains a brief discussion of the crisis that is based on those studies.

Period 2003-2005 exhibited unusual stability and historically low interest rates in the financial markets of the developed countries. The European central bank target rate was set at 2% on 6 June 2003, and remained unchanged until 6th of December 2005. A similar situation occurred in the USA, where the federal funds rate between December 2001 and June 2004 changed only from 1.75% to 1.25%. In addition to the stable monetary policy, the markets managed to deal effectively with several mini-crises that occurred during the period, further strengthening the perception of stability and safety.

Investors, facing historically low interest rates, pursued higher yields and turned to more risky assets and heavy leveraging (BoE (2007) p.6). Increasing market liquidity, however, led to a dangerous feedback: once a certain market section was targeted, increasing liquidity reduced the volatility and resulting yields, which forced investors to move to even more risky assets. At the same time the commercial banks' adoption of the "originate and distribute" model provided a fresh inflow of assets. The idea behind "originate and distribute" schemes was to securitise and sell the loans in the financial markets. This allowed the banks that originally granted the loans to transfer part of the risk to counterparties and obtain the liquidity for new lending (ECB (2007)p.12).

The Asset Backed Securities (ABS) or Asset Backed Commercial Papers (ABCP) markets were at the centre of the financial crisis in 2007 and 2008. Therefore it

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is important to understand their structure and their impact on the originating banks. This discussion in this paragraph is based on Financial Stability reports of BoE (2007) and ECB (2007).

According to ECB (2007, p.86), the purpose of ABCP programs is to “remove assets, which have a risk-weighted capital requirement, from bank’s balance sheet while retaining some economic interest through income generation from the management of the special purpose vehicle (SPV) which issues the securities”.

There is a number of different SPV structures that vary according to their funding mode and investment profile. The portfolio of traditional SPVs was well diversified and included various assets eg. repos, trade receivables, commercial loans, etc. The liquidity was obtained in short term commercial papers market and, in case of problems, they relied on liquidity provision from originating bank.

In the 2007 crisis the key role was played by one class of SPVs, so called, “structured investment vehicles” (SIV) and SIV-lites, which heavily invested in residential mortgage-backed securities (RMBS). SIVs had their own capital and their financing relied on issuance of capital notes and junior and senior medium term notes (MTNs), which typically had maturity over one year. At the end of 2007 the traditional types of SPVs had accumulated USD 1.5 trillion assets (globally), while SIVs held over USD 350 billion.

The problems with assessing the default risks of the vehicles resulted in their inability to obtain independent finance. In case of traditional SPVs, they used the credit lines provided by sponsoring bank and sometimes were taken back into bank’s balance sheet. The SIVs, which were less dependent on the originating bank, were forced to sell their high-rated assets in order to fund maturing liabilities. Their relied on longer term finance, so it is possible that their financing problems will surface in the near future.

Special purpose vehicles problems affected the originating banks in two ways. First, the banks had to provide their conduits with liquidity (or at least consider the risk of being forced to do it in the near future) to refinance past loans. Second, the banks were forced to obtain additional finance for the loans that were initially intended for securitisation.

The “originate and distribute” business model is not entirely new and in fact has been widely used for many years. It allows risk to be spread among a larger number of parties, thus contributing to the stability of the system. Additionally, the financial institutions can use new risk exposures to diversify their portfolios. However, those benefits are based on correct pricing, accurate risk assessment, proper management of underlying assets and the assumption that securitisation does not affect the incentives for ex ante screening and ex post monitoring (ECB (2007)p.12). As it turned out, none of these assumptions survived the liquidity crisis in the summer of 2007. Let me discuss them separately:

- Since the underlying assets often consisted of long term loans, such as mortgages, the asset-backed securities were historically considered low-liquidity commodities reserved for long term investors. However, the abundance of liquidity in the asset-backed securities markets, which occurred in the period 2003-2007, reduced both the volatility and the liquidity risks, and consequently the market price. This, however, led to the situation where the
market price did not fully take into account the underlying long-term nature of the traded assets (BoE (2007) p.42).

- Due to complex nature of the traded financial instruments, fund managers heavily relied on credit ratings. However, since many of the assets were “buy and hold” securities, the secondary market and history of ratings was often not available. When the rating downgrades spread even to highest rated papers, the investors lost their trust in the credit ratings, which resulted in great uncertainty concerning the valuation of the assets (BoE (2007) p.43).

- Large number of different participants in the securitisation chain leads to asymmetry and loss of information. Originators, which were rewarded based on the business volume, might have lower incentives for proper risk screening than the end-investors (BoE (2007) p.42). This was typically addressed by forcing the originators to share some part of the risk by keeping “equity tranches” in their balance sheets. However, the surge in demand allowed the originators to reduce their own risk exposures to levels that diminished the incentive to properly monitor the loans (ECB (2007)p.13).

- The essence of the “originate and distribute” model is a leverage that allows the originating bank to lend much more than its assets would normally allow. This implies that the originating bank depend on “fresh” finance to cover current liquidity needs (BoE (2007) p.44).

The developments cited above contributed to the creation of the asset bubble that burst in mid-July 2007. The original trigger was a revaluation of US based securities backed by “sub-prime mortgage loans”, which were severely hurt by the increase in US interest rates. Even though the initial revaluation affected a relatively small part of the assets traded, their effect turned out to be significant for many other financial markets. There were several reasons for that.

First, as pointed out by Tobias Adrian and Hyun Song Shin in the Bank of France Financial Stability Review (BoF (2008)), the rapid drop in securities prices forced highly leveraged banks and their conduits to reduce their risk exposures in other markets, in order to meet the regulatory criteria. For example, banks that incurred losses on BBB rated bonds, responded by reducing their exposure to BB bonds. Since there were so many banks affected by the initial crisis, their synchronised reaction brought crisis to the next market, which forced them to repeat the same steps yet again. This explains why the AAA securities were hit by the crisis that originally affected only low quality paper.

Second, as reported by the Bank of England (BoE (2007), p.19), “these losses in mortgage-backed securities seemed to trigger a wider loss of confidence in all structured credit products and rating agencies’ valuation models”. The investors responded by reducing their exposure not only to the directly affected paper, but to the broad class of asset-backed paper, which created a substantial liquidity problem in these markets.

Third, the loan originators relied on new issues to finance their operations. When the demand for new assets-backed paper vanished, they faced severe liquidity constraints. In some cases, such as the Northern Rock case in the UK, this resulted
in insolvency. In other cases, the banks were forced to liquidate other assets, which often involved prime rated paper. This contributed further to the developments described above and resulted in losses for institutions that were never involved in mortgage-backed paper.

5.2.2 Interbank markets

It was not immediately clear that interbank markets, especially overnight markets, would be affected by the developments described above. First, aggregate liquidity in the interbank market is controlled by central banks, and at least initially there were no changes in the supply policy. The demand for funds is in principle determined by the reserve requirement, which also offers substantial buffer to counter temporary liquidity shortages. Neither aggregate supply nor aggregate demand for funds seems to have been directly connected with the liquidity crisis. However, as the symptoms of the crisis became more apparent and widespread, it became clear that the interbank markets were not immune, which was reflected in several developments described below.

1. Liquidity hoarding

   - The term “liquidity hoarding” means that the banks prefer to store funds at central bank deposit facility, preferring zero or very low rates to the risk of the counterparty risk. Such behaviour reduces interbank market efficiency in the distribution of liquidity. The euro money market exhibited such a behaviour between August 2007 and May 2008, which is reflected in higher use of deposit facility displayed in Figure 5.1. In addition, the deposit facility was used not only on the settlement day, but also during most of the maintenance period.

2. Higher volatility of the interest rate

   - Figure 5.2 presents the spread between target rates and market rates for Eurosystem, US and UK markets. Even though the central bank interventions managed to halt the increase in rates, volatility remained at a considerably higher level than in the earlier part of the year.

3. Deviation of interest rate from central bank target

   - The upward pressure on the interest rate in the overnight market was promptly countered with the central banks intervention that supplied massive liquidity in the form of short term lending (see below). However, that pressure persisted in markets where the intervention was not present (at least initially), for example for longer maturities. This gap between the long-term and short-term markets generally widened and became much more volatile after the crisis began as documented in Figure 5.3.

4. Spread between secured and unsecured lending

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Figure 5.1: Deposit facility in the Eurosystem  
Source: European Central Bank
Figure 5.2: Spread between market and target rates for Eurosystem (EONIA), US (Federal Funds Rate) and UK (LIBOR)
Source: Fédération Bancaire Européenne, British Bankers Association and Federal Reserve.
Figure 5.3: Spread between EONIA and 1m Euribor
Source: Fédération Bancaire Européenne
Figure 5.4: Spread between 1m Euribor and 1m Eurepo
Source: Fédération Bancaire Européenne
The problems related to asset valuations, downgrades of credit ratings and funding shortages of off-balance sheet entities (SPVs structures discussed above) resulted in significant increase in credit risk. The evolution of credit risk can be analysed by comparing the borrowing rates between secured and unsecured lending markets. In Figure 5.4 I present the spread between 1m Euribor (unsecured lending) and 1m Eurepo (secured lending). The liquidity crisis is very clearly reflected in significant increase in the spread.

5.2.3 Central bank intervention

The central banks recognised the increasing pressure in the markets and, as interest rates drifted away from targets, decided to intervene with a temporary liquidity injection. Excellent survey of central banks’ actions can be found in BIS (2008a) and Chailloux et al. (2008a), and here I only briefly review the developments that are relevant for the analysis of this chapter.

Early central banks’ interventions were mostly executed in the form of extra liquidity tenders. Although the operational details differ between the ECB, Bank of England and Federal Reserve, the underlying principle remained the same: the commercial banks were allowed access to extra liquidity. There was another element of the policy which was perhaps often overlooked: the excess liquidity supplied in those extra operations was removed completely until the end of the maintenance period. This means that the market as a whole, on the last day of the maintenance period had aggregate liquidity roughly equal to the remaining part of the aggregate reserve requirement.

This is illustrated on Figure 5.5 which displays the accumulated (during a maintenance period) excess of liquidity over reserve requirement in the Eurosystem. Before the crisis in the early 2007, this deviation remained very close to zero. When the crisis started, the ECB initially allowed for surplus of liquidity when compared with average reserve requirement. However, by the last day of the august 2007 maintenance period market liquidity was brought back to the level sufficient to satisfy the reserve requirement. The following maintenance periods have seen the similar pattern: supply of abundant liquidity early and subsequent correction in late part of the maintenance period.

Apart from short term interventions, the central banks also adopted different measures aimed to calm the markets. In order to address the increasing term spread, additional tenders were executed to supply liquidity at longer maturities. Table 5.1 presents the analysis of the balance sheets of the Eurosystem, the Bank of England and the Federal Reserve.

Comparison between those three central banks reveals significant differences in adopted policies. In case of the ECB, increased longer term financing mostly substituted regular main refinancing operations, with the total amount of lending to the financial sector mostly unchanged. The initial increase in long term finance offered by the Bank of England was later partially reverted and, by August 2008, there has been little change in its balance sheet compared with August 2007. An extra 50% increase in long term operations mostly contributed to the expansion of its lending to financial sector. The Federal Reserve has perhaps experienced
Figure 5.5: Accumulated market liquidity excess compared with reserve requirement in the Eurosystem.

Source: ECB. Vertical lines indicate the start of maintenance periods.
most significant change in its balance sheet, with new long term lending facility reaching USD 150 billion and traditional repurchase agreements rising by 200%. This was accomplished by reducing the holdings of US treasury assets.

In addition to changes in the term structure of interventions, also the basis of collateral and the list of eligible parties was expanded.

This was perhaps most apparent in the USA, where the Federal Reserve launched a brand new market tool, Term Securities Lending Facility (TSLF). Banks could borrow from TSLF for 28 days (unlike traditional, overnight lending facility) and the base of collateral was significantly expanded to include federal agency debt, federal agency residential-mortgage-backed securities (MBS), and non-agency AAA/Aaa-rated private-label residential MBS (Federal Reserve (2008)). In addition the Federal Reserve narrowed the standing facilities corridor to 50 basis points on August 2007 and further to 25 basis points on March 2008. These facilities are available to much larger number of banks than traditional open market operations.

A very similar program, called Special Liquidity Scheme (SLS) was launched on April 21, 2008 by the Bank of England. Under SLS, the banks could swap high quality illiquid assets for Treasury bonds for a period of 1 year (although several restrictions apply and the risk stays with the original banks) (Bank of England (2008)).

In the Eurosystem, the revised (in 2007) framework already allowed a broad base of collateral, therefore it was not necessary to make further changes.

Changes in collateral framework allowed commercial banks to use less liquid assets to obtain central bank funding, while using government debt as a collateral in interbank funding. This phenomena was referred to as “Gresham’s law of collateral” and described in detail in Chailloux et al. (2008b).

The impact of the interventions, measured in volatility and the spread between market rate and central bank target, is difficult to gauge. However, comparison of performances of the UK and European markets yields some interesting insights. The crisis escalated first around 9 August 2007, which was also when the ECB performed its first intervention. The Bank of England, however, initially refrained from intervention and only followed the other central banks in September. The
comparison between EONIA and LIBOR only, for August and September 2007, is presented in Figure 5.6, which shows that compared with EONIA the LIBOR rose on 9 of August and remained at a much higher level until the early-September intervention. This can be interpreted in two ways. First, the early ECB intervention could send a signal to the market that the central bank is committed to the target even on the short term basis. This could affect market expectations, which are the key element in determination of the current rate. Second, the interventions could actually help to address the temporary problems with liquidity. Since the excess funds were later removed by the ECB, towards the end of the maintenance period the relative liquidity of both markets was similar (roughly equal to the required reserves); yet EONIA remained much closer to the target than did LIBOR.

As the scale of interventions increased, the central banks started to play even more important role as suppliers of liquidity. This was also reflected in changes in their liquidity tenders and banks’ bidding behaviour, which, for the case of Eurosystem, are reviewed in detail.

The ECB allots majority of the liquidity in variable rate tenders. Since the funds obtained from the central bank and other banks are substitutes, in the equilibrium banks should place the bids at the price equal their expected cost of
Figure 5.7: Accumulated market liquidity excess compared with reserve requirement in the Eurosystem.
Source: ECB. Vertical lines indicate the start of maintenance periods.
market finance (opportunity cost). The cost of unsecured lending is related to individual bank credit risk, and the cost of secured lending to the quality of the assets used as collateral.

If the credit risk is generally low for most of the banks, the cost they pay in unsecured borrowing is similar and close to the expected equilibrium market rate. Therefore banks place similar bids and obtain similar share of central bank finance. Of course even before the crisis the banks were associated with different risk levels, which was reflected in their credit ratings. However, as the size of losses and leveraging became apparent, the market segmentation increased. In particular, two groups were considered risky: the banks with large exposures to mortgage backed assets and heavily leveraged banks that relied on short term market to finance long term liabilities. These banks might have seen substantial increase in risk premiums paid in unsecured lending in the interbank market.

In addition, central banks expanded the base of eligible collateral (see above), which resulted in less liquid assets being used as collateral. Due to the crisis in the asset backed securities market the risk associated with these papers greatly increased, which also led to higher risk premiums (see Chailloux et al. (2008a) for discussion).

As the opportunity cost in both unsecured and secured lending increased, the banks had stronger incentives to bid more aggressively in the central bank liquidity tenders, which was reflected in rising spread between minimum and marginal bid rate in ECB’s main refinancing operations (grey area in Figure 5.7).

The more problems the banks had, the higher was their opportunity cost of market funding and the more incentives they had to secure the central bank finance. Thus the troubled banks bid even more aggressively, which was reflected in rising spread between average allotment and marginal bid rate (dark area in Figure 5.7). Since very often those were the same banks that faced liquidity shortages, they used the central bank funding as a substitute for the interbank market. Thus, the central banks, at least to some extent, became a substitute for a regular interbank market trade.

5.3 Modelling the liquidity crisis in the interbank market

In the model presented in Chapter 4, I analysed the behaviour of interbank markets in a period of a relative stability. My model predicted fairly well the following patterns actually observed in the markets in 2003-2005:

- The market rate remained very close to the target level,
- The banks initially deviated from required reserves level and corrected for it toward the end of the maintenance period.

In this section, I present an extension of this model that allows me to capture the behaviour of the market during a liquidity crisis. The literature on the interbank market was discussed extensively in the previous chapters of the thesis. Below I present the problem of the individual bank that minimises the cost of finance and
the optimal borrowing condition. To analyse the equilibrium, however, I need to use the numerical methods that are presented and applied in Section 5.4.

As mentioned before, the liquidity crisis changed many aspects of the financial markets. Here, I will concentrate on three crucial, in my opinion, changes that affected the banks and their borrowing volumes. These changes include an increase in the trade frictions, a perceived liquidity shortage and an increase in credit risk.

The key assumptions of the model from Chapter 4 remain unchanged: banks avoid excess trade volumes, minimise the cost of finance and are subject to two identically and independently distributed shocks, $\varphi_t$ and $\varepsilon_t$, with zero mean and standard deviation $\sigma$. The cost of funding, $K_t$, depends on the average market clearing interest rate, $i_t$, borrowing volume, $b_t$, and the expected cost of using the standing facilities, $c_t$, given by the function

$$K_t = \kappa(i_t, b_t) + E(c_t).$$

(5.1)

$K_t$ is total cost of finance on day $t$, $\kappa(i_t, b_t)$ is the cost of finance from the interbank market and $E(c_t)$ is the expected cost of using standing facilities. Let me briefly comment on the structure of the cost function and explain how it is affected by the liquidity crisis.

**Financing from the central bank**

Similarly to previous chapters in this thesis I only consider the central bank finance from the standing facilities. Since the open market operations are assumed fixed rate tenders with proportional allotment, from commercial bank's perspective their cost is exogenous. I discuss the implications of this assumption in the results section below.

A bank will resort to the central bank lending facility only in two situations: when faced with a liquidity shortage that exceeds its current account balance, or if it has failed to accumulate sufficient resources to satisfy the remaining reserve requirement by the end of the maintenance period. The deposit facility is available to banks that have already satisfied their reserve requirements in the current maintenance period.

The structure of the expected cost of finance from the central bank was not directly affected by the liquidity crisis. The cost of using standing facilities is fixed, decided and announced in advance for a given maintenance period. Even though some central banks eventually responded by lowering their target rates, the corridor system (difference between standing facility rate and target rate) remained unchanged at 100 basis points.\(^2\)

In the crisis in 2007 the central banks also executed liquidity supply operations at maturities much longer compared with regular operations. The goal of these interventions was to reduce the term spread (see Subsection 5.2.2) but also raises an interesting question: even though in this chapter I analyse the overnight market, should I consider the impact of different operations maturities?

To answer this question recall first that the framework used in this thesis deals with overnight interbank borrowing. It allows me to reduce the analysis to a single

\(^2\)An exception is the Federal Reserve, which reduced the corridor first to 50 then to 25 basis points.
period and consider the bank optimisation problem as finite horizon. Thus I cannot analyse interbank lending maturities longer than overnight within model based on Poole (1968). Regarding open market operations, I model a net value of liquidity supply i.e. the difference between all maturing operations and all new operations. In this respect extending the maturity of the open market operations does not matter, as long as it does not affect the aggregate liquidity on the settlement day.

Trading frictions

The only alternative to central bank finance is borrowing from the interbank market. These transactions are subject to certain frictions, which were covered in more detail in the previous chapter:

- Transaction costs,
- Diseconomies of scale,
- Asymmetry between profits from lending and cost of borrowing.

The transaction costs affect both unsecured and secured trading, and comprise mainly operational expenses, such as the cost of finding the counterparty that is willing to trade (in terms of unsecured trade) or the cost of collateral (used in secured trading). The diseconomies of scale are related to fragmentation of the market and to banks’ aversion to excess variation. The existence of transaction costs that increase with the size of a transaction implies convexity of the cost function, which creates a certain asymmetry between perceived lending profits and borrowing costs. Intuitively, if the transaction cost is included, the expected cost of borrowing 100 units is higher than the corresponding profit from lending those 100 units, and a round trip (borrowing and lending together) will generate a loss. This effect is further reinforced by the asymmetric information problem, which gives extra value to the banks’ reputation.

In the previous chapter I showed that any non-decreasing convex function will capture those properties of the cost of finance from the market. For simplicity, I assumed the exponential form

\[ \kappa(i_t, b_t) = \rho^{-1} \exp(\rho i_t b_t) - 1/\rho \]  

where \( \kappa(i, b) \) is the cost of market finance, \( i_t \) is the interest rate, \( b_t \) is the interbank borrowing volume and \( \rho \) is the parameter measuring the significance of trade frictions. The higher the value of \( \rho \), the sooner the additional costs start to play a role. In the previous chapter I found that during stable periods, such as that observed in the Eurosystem in 2003-2005, the value of the parameter \( \rho \) is likely to be low. This means that banks can perform even relatively larger transactions without incurring heavy additional costs, and the asymmetry between borrowing cost and lending profit is almost negligible.

The liquidity crisis of 2007 might had an impact on trade friction. The general perception of insecurity, which surfaced as news of heavy losses came out, has probably increased the incentive to smooth out the expected cost of funding, rather than adjust to the reserve requirement in a single transaction. Additionally,
a perceived higher risk made the lending banks more cautious, which means that finding a counterparty, especially for borrowers, became harder. Also, the losses announced by some of the largest banks shattered mutual trust and reputations; hence reliable information became scarce and expensive. Finally, the borrowers were in particular predicament, as large borrowing was perceived to indicate liquidity problems, which made the task of finding reserves even harder. All those factors combined suggest that the liquidity crisis substantially increased the trading frictions, which in my model are captured by the parameter $\rho$. The resulting higher $\rho$ increased the convexity of the cost function.

Credit risk

A large part of the interbank trade involves unsecured lending, which does not require costly collateral. This however means that the trading rates and availability of funds depend on the mutual trust and reputations of market participants. During stable periods, obtaining finance is relatively easy and cheap, hence the possibility that a bank will become insolvent and default on its loans can generally be ignored. However, the valuations of many assets dropped dramatically during the crisis in the summer of 2007, which forced the banks to disclose heavy losses and ultimately search for additional sources of finance. In addition, the actual size of future losses was still unknown due to the unclear and complicated nature of the assets traded. The uncertainty in the market was further fuelled by several defaults, such as the German bank IKB and UK-based Northern Rock (which were however bailed out by the authorities).

This combination of developments forced lenders to consider the credit risk, which reduced the expected profits from lending.

In terms of the model, credit risk can be introduced in the following way. If the probability of default is $\psi$, the expected cost of the interbank lending is given by

$$
\begin{cases}
(1 - \psi) i_t b_t - \psi b_t & \text{if } b_t < 0 \\
i_t b_t & \text{if } b_t > 0,
\end{cases}
$$

(5.3)

where a positive $b_t$ means the bank is borrowing, and negative $b_t$ means the bank is lending. This assumes limited liability for borrowers, which means all the risk is borne by the lenders. Combining (5.3) with the cost function presented above yields the cost expression

$$
\kappa(i_t, b_t) = \begin{cases}
-\frac{1}{\rho} \exp \left( \frac{\rho b_t ((1 - \psi) i_t - \psi) - 1}{\rho} \right) & \text{for } b_t \leq 0 \\
-\frac{1}{\rho} \exp \left( \frac{\rho i_t b_t - 1}{\rho} \right) & \text{for } b_t > 0.
\end{cases}
$$

(5.4)

Thus the general cost of finance for single period is

$$
K(i_t, b_t) = \begin{cases}
-\frac{1}{\rho} \exp \left( \frac{\rho b_t ((1 - \psi) i_t - \psi) - 1}{\rho} + E(c_t) \right) & \text{for } b_t \leq 0 \\
-\frac{1}{\rho} \exp \left( \frac{\rho i_t b_t + E(c_t) - 1}{\rho} \right) & \text{for } b_t > 0.
\end{cases}
$$

(5.5)

A bank that is minimising the cost of finance over the whole maintenance period needs to solve an inter-temporal problem to choose the value of borrowing for each
day of the maintenance period. Such a problem can be solved by minimising the value function $V(i_t, b_t)$

$$V(i_T, b_T) = \begin{cases} 
\rho^{-1} \exp [\rho b_T ((1 - \psi) i_T - \psi)] - 1/\rho + E(c_T) & \text{for } b_T \leq 0 \\
\rho^{-1} \exp (\rho i_T b_T) + E(c_T) - 1/\rho & \text{for } b_T > 0 
\end{cases} \quad (5.6)$$

on the last day of the maintenance period, $T$, and

$$V(i_t, b_t) = \begin{cases} 
\rho^{-1} \exp [\rho b_t ((1 - \psi) i_t - \psi)] - 1/\rho + E(c_t) + E_t(V_{t+1}) & \text{for } b_t \leq 0 \\
\rho^{-1} \exp (\rho i_t b_t) - 1/\rho + E(c_t) + E_t(V_{t+1}) & \text{for } b_t > 0 
\end{cases} \quad (5.7)$$

for the days before. The analytical solution to the equilibrium would require unrealistic assumptions about the length of the maintenance period or the distribution function for liquidity shocks. However, numerical methods can be used to determine the optimal value of borrowing, $b_t$, for each participant, given the level of the interest rate $i_t$. Assuming perfect competition and no market power, the equilibrium can be characterised by the interest rate $i_t$, which clears the market consisting of $N$ banks, i.e. $\sum_n^N b_{tn} = 0$.

Even though the equilibrium is solved numerically, the first order conditions for this problem can be useful in interpreting some of the results. Solving eq.(5.7) for the optimal $b_t^*$ yields

1. For $b_t^* \leq 0$ (lending)

$$((1 - \psi) i_t - \psi) \exp [\rho b_t^* ((1 - \psi) i_t - \psi)] = 
= i_t F(-m_t - b_t^* - \varphi_t) + i_t^d (1 - F(-m_t - b_t^* - \varphi_t + d_t)) + E \frac{\partial V_{t+1}}{\partial b_t^*}. \quad (5.8)$$

2. For $b_t^* > 0$ (borrowing)

$$i_t \exp [\rho b_t^* i_t] = 
= i_t F(-m_t - b_t^* - \varphi_t) + i_t^d (1 - F(-m_t - b_t^* - \varphi_t + d_t)) + E \frac{\partial V_{t+1}}{\partial b_t^*}. \quad (5.9)$$

The interpretation of the first order conditions is the following. The left hand sides of equations (5.8) and (5.9) are the marginal current costs of obtaining (or lending) an extra unit of finance from the market. This term depends on the present interest rate $i_t$, the probability of default, $\psi$, the degree of market frictions, $\rho$, and the trade volume. The right hand sides are the expected future costs, which include the expected cost of using the standing facilities at the end of the day and the marginal change in the future cost of funding for a bank obtaining one more unit from the market today, $E \frac{\partial V_{t+1}}{\partial b_t^*}$. At optimal borrowing $b_t$, the cost of funding from the market today must be equal to the expected cost tomorrow, otherwise the bank can make an extra profit by borrowing extra finance today or tomorrow. If the reserve requirement is sufficiently high, the probability that the liquidity shock will
exceed the current account balance (which forces the central bank lending) or the
remaining deficiency (which forces central bank deposits) is very low. This implies
that, at least for the days before the end of the maintenance period, the expected
cost of standing facilities can be dropped from the expressions above. This means
that at given parameter values, the interest rate $i_t$ will be mainly determined by
the expected future cost of funding, $E \frac{\partial V_{t+1}}{\partial b^*}$. If there is no crisis and the banks
are confident that they will be able to obtain finance from the market, the future
funding cost is equal to the expected market rate, which in this model is assumed
equal to the central bank target (as in Chapter 2 and Chapter 3). In the following
section I discuss how this assumption changes in a liquidity crisis.

Since I assumed there are no extra restrictions or costs related to the use of
standing facilities, the bank can always ignore the market and obtain (or deposite) all necessary liquidity from the central banks, at rates between $i^l$ and $i^d$ (the
standing facilities rates). This, however, imposes restrictions on the right hand
sides of the equations (5.8) and (5.9); and, at given values of parameters $\psi$, $\rho$ and
$i_t$, it is possible to determine the maximum and minimum volume of borrowing.
At maximum borrowing, the cost of obtaining one more unit of reserves from the
market exceeds the expected cost of obtaining it from the lending facility. Correspondingly at maximum lending, the profit from lending an extra unit is lower than
the profits from depositing it in the central bank deposit facility. The convexity
of the cost function means that there is a certain asymmetry between maximum
lending and maximum borrowing, as discussed in Chapter 4, Section 4.4, which
results in maximum lending exceeding maximum borrowing. Introducing credit
risk means, however, that the maximum lending decreases and, for the parameter
values used in the simulation, it is half the size of the maximum borrowing. This
implies that the lending banks will be much more likely to use the deposit facility,
which was in fact observed in the Eurosystem.

In this chapter I assume that the expected bankruptcy rate, $\psi$, is exogenous.
In reality, however, actual bankruptcies could depend on expected liquidity flows (discussed in the following section) or the degree of market frictions. For instance,
a liquidity crisis can increase transaction costs, which might cause losses for some
banks, which might in turn run into problems and eventually default. In this
chapter, however, I do not consider the possible implications of endogeneity of
credit risk.

**Expected liquidity shortage**

The aggregate liquidity is directly under central bank control, and a majority of
the central banks adopted a policy of supplying the market with the liquidity
necessary to satisfy the reserve requirement. This is based on the assumption that
banks with liquidity exceeding reserve requirements are willing to trade the excess
with banks facing corresponding shortage. Since the aggregate liquidity does not
change, a shortage for one bank must be reflected in a surplus of for other bank,
which creates an opportunity for trade. This property was used in the previous
chapters to motivate one of the assumptions of the model: the distribution of
liquidity shocks has zero mean. Intuitively, the banks had no reason to expect that
deviations from their anticipated liquidity flows will be biased in either direction.\(^3\)

The crisis that emerged in the summer of 2007 affected these expectations. Uncertainty concerning the condition of the structural investment vehicles, which relied on the assets-backed paper markets as the source of finance, forced the banks to consider possible support of their subsidiaries. Since the exact magnitude of the losses was initially unknown, the banks assumed the worst and acted as if they were facing an impending outflow of liquidity.

Such behaviour constitutes a deviation from the rational expectations hypothesis since it is impossible for all the banks to become short of liquidity when aggregate liquidity does not change.

In this chapter this is modelled by assuming that the expected liquidity shock realisations are negative on average. However, the distribution of actual liquidity shocks remains unchanged (in comparison to the case of no crisis), with the mean equal to zero.

### 5.4 Simulation

#### 5.4.1 Simulation description

Solving the model for the equilibrium interest rate analytically is complicated and requires strong distribution assumptions. Thus, as in the previous chapters of this thesis, I use numerical methods.

In order to analyse the impact of developments discussed in the previous section, I ran Monte Carlo simulations in several scenarios. First, I looked at the market behaviour and separately introduced greater trade frictions, credit risk and expected liquidity shortage. Next, I repeated the experiment with all such changes implemented simultaneously. Finally, in order to verify the impact of central bank intervention, I ran a scenario where an extra liquidity is supplied on the day of the crisis.

To obtain the results presented in the next subsection, I applied the same procedure used in the previous chapter of the thesis. The basic assumptions are that the market consists of 10 banks, each initial current account balance and reserve requirement set at 100 units. Hence, the aggregate market liquidity is 1000 units. The standard deviation of the liquidity shock is set at 10 units, which is 10% of the average current account balance. The standing facilities are set at 2% and 3% respectively for the deposit and the lending facilities. The central bank target is 2.5%. The most important modifications, compared to the model of the previous chapter, are

1. I assume the crisis starts on the sixth day of the 10-day maintenance period. This means that the first five days are identical for all scenarios, and correspond to the case where no crisis occurs. There are two main benefits

\(^3\)A liquidity shock was defined in the previous chapters as the random part of the liquidity flows, faced by each bank. The liquidity flows are mostly determined by the balance of transactions with other banks, which include transfers between bank customers and interbank lending. The random part in those flows comes from unexpected payment orders or errors in transaction processing, which results in the final end-of-day balance to differ from the expected one.
of this procedure, versus allowing the crisis to affect the whole maintenance period. First, I can analyse the behaviour of the banks that are affected by the crisis while front or backloading. On the sixth day, most banks will have some accumulated imbalance, compared with the reserve requirement, which they hope to offset by the end of the maintenance period. It is interesting to see how these plans are affected by the crisis. This leads to the second advantage of my procedure, which is to see the change in banks behaviour, when they need to adapt their expectations of the future cost of funding to new parameter values.

2. To verify the impact of an increase in trading frictions, I allow the parameter $\rho$ to change from $\rho = 0.0001$ to $\rho = 0.001$. The starting value was chosen to reflect a market with low trading costs, such as the Eurosystem.

3. To see how the credit risk alters market behaviour, I apply a probability of $\psi = 3\%$ that the loan will not be paid.\(^4\)

4. To gauge the effect of changes in expected liquidity, I assume the new expected liquidity shock mean is one standard deviation below zero.

5. To introduce central bank intervention, I assume that all banks receive equal liquidity injections of 10\% of assets on the first day of the crisis. This liquidity is then removed in the fine tuning operations, on the last day of the maintenance period.\(^5\)

6. Since the liquidity conditions on the last day of the maintenance period are unlikely to resemble those observed in “normal” times, similarly to Chapter 4 I adopt an algorithm that computes the equilibrium assuming rational expectations of the interest rate level (rather than fixed expectations used in Chapter 2 and Chapter 3).

The parameters were chosen to resemble the Eurosystem interbank market. The results do not qualitatively change for different set of values.

Similarly to previous chapters I use the market clearing condition to find the market equilibrium. The method used to introduce credit risk ensures that there is no credit rationing in the sense that was defined in Stiglitz and Weiss(1981) paper. While increasing the borrowing cost, the trade is unrestricted within the corridor of standing facilities.\(^6\)

\(^4\)The empirical ex post rate was much lower, and in fact even the banks that did default, were eventually bailed out by the authorities. However, $\psi$ captures the market expectations of default, which at the time of the crisis might have been considerably higher. I also ran the simulations at 5\% and 10\% and got similar results (although the magnitudes of change were more significant). A lower default rate was difficult to use because the numerical methods require certain approximations and the results were affected by the rounding procedures.

\(^5\)For comparison, the ECB on 9 August 2007 supplied the market with extra liquidity of EUR 90 billion, which constituted almost half the aggregate market liquidity on that day. The excess liquidity was removed within a week.

\(^6\)The following algorithm is used to find market equilibrium. I start out with the individual bank profit maximisation problem, which determines the optimal level of borrowing given the interest rate and current deficiency (how much reserve requirement remains to be satisfied). Depending on whether the sum of individual borrowing is positive or negative I adjust the
As mentioned in point 6 above, it is unlikely that the market expectations of the interest rate remain unaffected by the liquidity crisis. One of the key assumptions that justified this property in Chapter 3 was that the market liquidity will be equal to reserve requirement on the last day of the period, which is unlikely as demonstrated in results below. Therefore I introduce an algorithm that ensures the rational expectations. The exact method is similar to Chapter 4: I start with an initial guess of the interest rate and compute the market equilibrium for all 10 days. I then use these rates as expected interest rate in next iteration. Since the crisis occurs on 5th day, I effectively need to calculate two different sets of expectations - pre- and after-crisis. The algorithm converges in 3 iterations, which allows the model to satisfy rational expectations.

5.4.2 Simulation results

Preliminary results of the simulation are presented in Tables 5.2 and 5.3, where each column corresponds to one of the scenarios and each row represents a different day of the 10-day maintenance period. The scenarios presented are

1. The benchmark scenario, where no crisis occurs.
2. The increase in trading frictions, where other parameters are unchanged.
3. The expected liquidity shortage, where other parameters are unchanged.
4. The credit risk, where other parameters are unchanged.
5. The increase in trading frictions, expected liquidity shortage and credit risk combined.
6. Scenario #5, with the liquidity injection on the first day of the crisis.

Table 5.2 focuses on the behaviour of the interest rate and the use of standing facilities. The average interest rate is the average market clearing rate over 1000 simulation runs, and should be compared to the central bank target 2.5%. Volatility is measured by the standard deviation of the rate across simulations. Note that aggregate liquidity is always the same (except for the scenario with central bank intervention), and the liquidity shock only affects the way the liquidity is distributed among the banks. Low interest rate volatility indicates that the interest rate does not depend on the distribution of the liquidity among the banks. The average use of the deposit facility is the aggregate value for the whole market.

The deviation from neutral liquidity for individual banks is the difference between the current account balance at the end of the day and the required reserves. In Table 5.3, I report the sum of those deviations for all the frontloading banks. If the whole market is left with neutral liquidity, the deviations for the backloading banks are symmetric, at least until the central bank injects additional liquidity. 

interest rate and recalculate each bank’s optimisation problem until the aggregate borrowing reaches zero. As discussed in previous chapter, with market frictions the lending banks will have a limit of lending. However, this limit is reached at the lending facility rate, which also constitutes maximum rate for the borrowing banks. Hence the situation where the borrowing bank is willing to pay higher rate and the lending bank declines its offer is not possible.
The accumulated deviations track the differences between current account balance and reserve requirement for a bank since the start of the maintenance period. Again, the reported number is the sum of accumulated deviations for all the front-loading banks.

In the rest of this section I comment separately on each of the scenarios.

5.4.3 Interpretation of results

The first scenario (denoted scenario #1) serves as the benchmark case of no crisis. Here, the market follows the pattern explained in the previous chapters and the interbank rate remains at the expected level, equal to the central bank target, with very little volatility (Table 5.2). The trading costs were calibrated so that banks initially deviate from the average required level, and return to the required level in the second part of the maintenance period (Table 5.3), which was the case for the Eurosystem in 2003-2005.

Observation 1 Higher trading frictions (scenario #2) increase interest rate volatility and the probability of using standing facilities.

In the second scenario, I look what happens when the trading frictions become more severe. The market behaviour with different values of the parameter $\rho$ was covered in Subsection 4.4.2 of the previous chapter. However, the case where the parameters and expectations change in the middle of the maintenance period was not analysed.

With higher trading frictions, the interest rate shows a propensity to decrease toward the end of the maintenance period, which is consistent with the findings of the previous chapter. This is the result of asymmetry between borrowing cost and lending profit, which renders the central bank lending facility more attractive in comparison with the deposit facility.

The deviation from the neutral liquidity, reported in Table 5.3, is very similar to the benchmark scenario. However, the accumulated deviation increases substantially, compared to the case of no crisis.

Recall that the market in this scenario is always left with the same aggregate liquidity. Hence, any volatility of the interest rate is an indication that the interest rate level is sensitive to the distribution of liquidity among banks. Intuitively, in the absence of trading frictions, the banks are able to trade almost any amounts they wish neutralising any effect of the liquidity shock. High trading frictions mean that the distribution of liquidity matters.

This can be also interpreted by analysing the borrowing volumes. In the first part of the maintenance period, the banks allow for the accumulated deviation to increase, expecting to correct it by the end of the maintenance period. Depending on the liquidity shock realisations, some of those banks will arrive on the sixth day with small, and others with large, deviations from required reserves. The rise in trade frictions and corresponding increase in convexity of the cost function means that the extreme values of borrowing and lending become very expensive. Hence, the banks that enter the crisis with large imbalances choose to use the standing facilities rather than obtain the liquidity from the market, which results in an increase in the accumulated deviation from neutral liquidity. If it is the backloading
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Table 5.2: Simulation of a liquidity crisis.

Scenario #1: Benchmark scenario with no crisis
Scenario #2: Increase in trading cost
Scenario #3: Expected liquidity shortage
Scenario #4: Credit Risk
Scenario #5: Scenarios 2-4 combined
Scenario #6: Scenario 5 with central bank intervention.
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Table 5.3: Simulation of a liquidity crisis.

Scenario #1: Benchmark scenario with no crisis
Scenario #2: Increase in trading cost
Scenario #3: Expected liquidity shortage
Scenario #4: Credit Risk
Scenario #5: Scenarios 2-4 combined
Scenario #6: Scenario 5 with the central bank intervention.
banks that decide in favour of central bank facilities, the whole market will suffer from the surplus of liquidity, which will depress the interest rate. Correspondingly, the existence of frontloading banks with large surpluses will boost the rate above the central target. This means, however, that the interest rate is tied to liquidity shock realisations and hence is likely to be more volatile than in benchmark case. This is indeed documented in Table 5.2.

Observation 2 An expected liquidity shortage (scenario #3) increases the average interest rate but does not affect the borrowing volumes

In the third scenario I look at the consequences of an expected liquidity shortage. In the model, this is accomplished by assuming that the mean of expected liquidity shocks is one standard deviation below zero. However, the actual shock realisation still has mean zero, since there is no aggregate outflow of funds from the market.

It seems that, compared to the benchmark scenario (where no crisis occurs), the crisis impact is limited to the average level of the interest rate of transactions. Even though the interest rate is higher, the banks decisions on how much to borrow or lend do not seem to be affected. As a result, the daily deviation and accumulated deviation remain at the same level, as if no crisis occurred.

To interpret the results, I look at the first order conditions (equation (4.27)) derived in Subsection 4.4.1. For the last day of maintenance period $T$, the following relationship determines the market equilibrium:

\[ \frac{i_T \exp(\rho_T b_T)}{B_T} = \frac{i_l F(-m_T - b_T - \varphi_T + d_T)}{B_T} + \frac{i_d (1 - F(-m_T - b_T - \varphi_T + d_T))}{B_T}. \]  

This has the following interpretation: the equilibrium interest rate $i_T$, corrected for a possible friction cost $\rho_T$, is equal to the probability-weighted cost of using the standing facilities. In the case analysed here, the expected liquidity shortage means that the probability of using the lending facility is higher (and deposit facility lower), which increases the right hand side of equation (5.10) for all the banks. In terms of the market equilibrium, this means that the interest rate $i_T$, must also increase, which is in fact observed in Table 5.2, where the average interest rate computed in my simulation is 30 basis points above the central bank target.

For the days before the end of the maintenance period, the first order condition (4.28) derived in previous chapter, Subsection 4.4.1 take the form

\[ i_t \exp(\rho_t b_t) = \frac{i_l F(-b_t - m_t - \varphi_t)}{B_t} + \frac{i_d [1 - F(d_t - m_t - b_t - \varphi_t)]}{B_t} \]

\[ - \int_{-b_t - m_t - \varphi_t}^{d_t - m_t - b_t - \varphi_t} \frac{\partial V_{t+1}}{\partial d_t} f(\varepsilon_t) d\varepsilon_t. \]  

The expected liquidity shortage affects the probability of lending, given by the term $F(-m_t - b_t - \varphi_t)$, and the dynamic cost factor, given by term 3 in equation (5.11). Since the liquidity shock standard deviation is roughly 10% of the average current account value, the probability of using lending facilities before the end
of the maintenance period is very low, and expected shortage of one standard deviation does not change that, which allows me to focus on the dynamic cost factor.

The dynamic cost factor, which captures the expected cost of future finance, depends not only on the liquidity flows expected to happen on day $t$, but also on the accumulated liquidity flows that will occur every day until the end of the maintenance period. The larger the expected accumulated shortage, the larger the expected cost of funding and the higher the market clearing rate $i_t$. If the bank expects the shocks to be i.i.d. every day, the expected accumulated shortage will be largest on the day of the crisis and will decline in the following days.\(^7\) This corresponds to the pattern observed in Table 5.2, where, after the initial increase, the interest rate also falls.

The expected liquidity shortage has little effect on the banks’ borrowing and lending volumes, and the market deals efficiently with the redistribution of a liquidity shock.\(^8\) Since the distribution function of the realised liquidity shock is the same as in the benchmark case, similar borrowing decisions lead to similar ending values of banks’ current accounts, and corresponding similar use of the deposit and lending facilities (the last row in Table 5.2).

**Observation 3** *Credit risk (scenario #4) temporarily reduces the market’s ability to offset a liquidity shock*

The results of the simulation that takes into account credit risk indicate that this change initially has little effect on the level of the interest rate and interest rate volatility. On the last day of the maintenance period, the average interest rate is 2 points higher than the central bank target. This increase is a natural consequence of the fact that credit risk only affects lenders, albeit only moderately.\(^9\) In order to encourage lenders to trade, rather than use the standing facilities, the interest rate must increase. However, no such behaviour is observed for the days preceding the final day of the maintenance period. This can be explained by considering the borrowers and lenders separately.

For the borrowers, the expected future funding cost is not affected by credit risk; hence, their demand for funds is identical to the benchmark case. In addition, since the trading costs are relatively low, borrowing large volumes is not penalised, which means that the banks can choose to postpone the borrowing rather than accept higher interest rate. On the other side, the lenders’ profits are directly and negatively affected by the credit risk. This means that they will reduce lending volumes at an interest rate equal to the central bank target.

Because the borrowing banks refuse to accept higher interest rates, and the lending banks refuse to supply enough liquidity, the market trade volumes are

\(^7\) If the expected shock has a mean equal to $(-\sigma)$, which is one standard deviation below zero, the expected accumulated value of all shocks that will occur until the end of the maintenance period is $2 \times (T - t)(-\sigma)$ (since there are two shocks, $\varphi$ and $\varepsilon$).

\(^8\) I have verified that, given identical liquidity shock realisations, individual banks borrowing decisions are the same as in the benchmark scenario (with no crisis), although the funds are traded at the higher interest rate.

\(^9\) For comparison, with a default rate of 10% the interest rate on the last day was 20 basis points above the target.
significantly reduced, but the market clearing interest rate level remains close to the expected level.

Toward the end of the maintenance period, a large deviation from neutral liquidity means a higher probability of using costly standing facilities. This encourages banks with surplus reserves to increase their lending and reduces the market imbalances.

Observations 2 and 3 are crucial in understanding how the liquidity crisis affects the interbank market. The expected liquidity shortage increases the interest rate level but does not affect trading volumes. On the other hand, the credit risk leaves the interest rate unchanged but instead hampers market trade volumes in a significant way. How are these results related?

In the equilibrium, the banks balance the current cost of market finance (which is directly linked with the interest rate) with the expected future cost of finance. The expected liquidity shortage increases the future expected cost of funding for both lenders and borrowers. Hence, in order to avoid arbitrage, the current cost of market finance must increase as well, which is reflected in the higher level of the interest rate. On the other hand, the credit risk reduces the expected profits from lending but has no effect on banks with liquidity shortages. Since the borrowing banks would rather postpone satisfaction of the reserve requirement than pay a higher rate today, the market clearing rate remains at a level close to the central bank target, albeit at low trade volumes. This means that changes in the expected future cost of funding are key to understanding the change in market behaviour.

In the scenarios analysed so far, only the effect of a single crisis element was taken into account; hence, only a single parameter was changed while the rest remained at benchmark levels. However, in the liquidity crisis that emerged in the interbank markets in 2007, all the developments mentioned in Section 5.3 happened simultaneously. The next scenario covers the case where all parameters change on sixth day of the maintenance period.

Observation 4 The simultaneous increases in trading frictions, the expected liquidity shortage and the credit risk (scenario #5) increase the interest rate level, interest rate volatility and substantially affect the trading patterns.

In scenario #5, I incorporate the trading frictions, the expected liquidity shortage and the credit risk into a single model, in order to verify their combined impact on market behaviour. Despite the fact that the effect of each of these developments taken separately is fairly limited in size and scope, their combination substantially changes nearly every aspect of the market that is analysed in this chapter.

The crisis has perhaps the strongest impact on the interbank average trading rate, which increases on the sixth day by nearly 30 basis points and gradually decreases almost to the target on the last day of the maintenance period. The volatility of the interest rate also increases substantially, suggesting that the market behaviour is very sensitive to the allocation of liquidity. Such a high level of interest rate apparently discourages banks from trading, which is reflected in higher-than-benchmark deviation from neutral liquidity. Even though the trade

\footnote{Similarly to scenario #4, the deviation of the rate from the target depends on the credit risk; for instance, at a 10\% default probability, the interest rate ends the maintenance period 12 points above the target.}

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volumes increase slightly during the days following the crisis, the aggregate deviation from required reserves continues to rise. This is also illustrated in Table 5.4 below, which directly compares trading volumes for three selected scenarios. Since the imbalances persist also on the last day of the maintenance period, the banks need to use the standing facilities in order to meet the reserve requirement. This simulation results is shown in Table 5.2, where on the last day almost 20% of total market liquidity (set at 1,000) is deposited and borrowed from the central bank.

Such market behaviour is the result of simulating of all the elements of the crisis. In the scenarios discussed above, the trading behaviour was mainly affected by the higher trade frictions and credit risk. Not surprisingly, when both of these developments take place at the same time, the deviation of the current account from the required reserves rises substantially, which indicates that the market is less efficient in allocating the liquidity to the banks that need it. This result should not come as a surprise. Credit risk has a negative impact on lenders profits and reduces their incentives to lend. A higher interest rate, as discussed below, offsets the increase in the cost of future funding, which affects all the banks but does not compensate lenders for the increase in risk. In addition to lenders' reluctance, the increase in trading frictions renders the standing facilities generally more attractive in comparison with market trade and lowers the maximum and minimum borrowing values as discussed in Section 5.3. The banks that were affected by the crisis, while their current account balances were substantially higher or lower compared to the reserve requirement, will most likely turn to central bank finance. All those developments are reflected in the increase in the accumulated deviation from neutral liquidity.

The interest rate in Table 5.2, increases substantially already on the first day of the crisis, which requires some explanation. Brief inspection of the first order conditions (5.8) and (5.9), presented in Section 5.3, reveals that the interest rate is directly connected with the expected future cost. This implies that the interpretation of the higher interest rate must link the liquidity crisis with the expected cost of finance.

The effect of the expected liquidity shortage is fairly straightforward: it forces all banks to obtain extra liquidity either from the market or from the central bank, and hence increases the cost of finance. The credit risk, discussed in the scenario #4, introduces additional asymmetry between profits from lending and costs of borrowing, thus reducing the incentive for lending banks to participate in the market, which in turn leads to an increase in accumulated deviation from neutral liquidity. A large deviation of current account balance from reserve requirement increases the expected cost of funding for all banks. The banks with liquidity surpluses will find themselves using the deposit facility more often, which reduces their expected profits. Corresponding banks with liquidity shortages are even in a worse situation due to the increase in trading frictions, which increases the convexity of the cost function and penalises large borrowing values. To conclude, it seems that the combination of higher trading frictions, an expected liquidity shortage and credit risk has a strong positive effect on the expected future cost.

\[11\] This is due to the fact that a larger deviation from neutral liquidity increases the chances that banks will be forced to use the standing facilities, which are generally offered at the rates significantly less attractive compared with the market.
Table 5.4: Trading volumes in selected scenarios

<table>
<thead>
<tr>
<th>Day</th>
<th>Scenario #1</th>
<th>Scenario #5</th>
<th>Scenario #6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43.23</td>
<td>43.38</td>
<td>42.96</td>
</tr>
<tr>
<td>2</td>
<td>78.93</td>
<td>78.92</td>
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</tr>
<tr>
<td>3</td>
<td>102.47</td>
<td>101.8</td>
<td>103.38</td>
</tr>
<tr>
<td>4</td>
<td>124.41</td>
<td>124.88</td>
<td>125.84</td>
</tr>
<tr>
<td>5</td>
<td>145.34</td>
<td>146.9</td>
<td>145.89</td>
</tr>
<tr>
<td>6</td>
<td>165.33</td>
<td>105.15</td>
<td>117.91</td>
</tr>
<tr>
<td>7</td>
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<td>132.69</td>
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<td>8</td>
<td>199.71</td>
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<td>9</td>
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</tr>
<tr>
<td>10</td>
<td>251.48</td>
<td>219.9</td>
<td>201.60</td>
</tr>
</tbody>
</table>

Scenario #1: Benchmark scenario with no crisis
Scenario #5: Scenarios 2-4 combined
Scenario #6: Scenario 5 with central bank intervention.

which explains the higher interest rate on all the days affected by the liquidity crisis.

The effects mentioned above appear strongest on the first day of the crisis. For the following days, the expected accumulated liquidity shortage decreases, thus reducing pressure on the expected future cost and corresponding interest rate.

Observation 5 Central bank intervention with proportional allotment lowers the interest rate level but also fails to stimulate market trade

In the final scenario analysed in this chapter, I look at what happens to the crisis scenario described above, when the central bank supplies the market with extra liquidity. This intervention is then reversed on the last day of the maintenance period, so that the market reserves are equal to the remaining reserve requirement.

A quick inspection of the results of the simulation indicate partial success of the intervention in controlling the interest rate. The average level and volatility of the interest rate are lower than in the case with no intervention. This holds until the extra liquidity is removed, on the last day of the maintenance period, when the interest rate increases rapidly. In addition the deviation from neutral liquidity is much lower, but the trading volumes remain at lower level compared with the no-crisis scenario. Let me discuss these results in detail.

In scenario #5, I identified two major effects that take place during the crisis: reluctance of a lending bank to trade, and perceived higher cost of future funding, both of which seem to be affected by the liquidity injection.

Higher credit risk reduces the profits from interbank lending, which diminish the trade incentives for the banks with surplus liquidity. These banks prefer to deposit their reserves at the central bank, which has the side effect of creating an aggregate deficit of funds. Their incentives are reflected in average deviations

\[ \text{Recall the value recorded in Table 5.3 includes also the liquidity injection of 100 units. Deducting this value reveals that the average deviation is significantly lower than in scenario #5.} \]
from neutral liquidity (Table 5.3) but also directly in trading volumes. To present it more clearly, I report additional results of the simulation in Table 5.4, which contains the total market turnover for three selected scenarios.

I assume that the interventions allot the liquidity proportionally, regardless of the banks' actual liquidity needs. In this scenario, extra reserves have little effect on the banks with surplus liquidity that hoard liquidity anyway. Intuitively, the banks that are already at the stage where they are ready to accept the opportunity cost and deposit the excess liquidity at the central banks, are not likely to change their preferences when given even more liquidity. On the other hand, the banks with liquidity shortages greatly benefit from this intervention as it allows them to offset some negative liquidity shocks. Thus, the accumulated deviation from neutral liquidity decreases compared with scenario #5.

In reality the central banks allotted the liquidity in variable rate tenders, where the banks with shortages were likely to bid more aggressively. Incorporating this effect in my simulation would mean that the banks with surplus liquidity (over the reserve requirement) were essentially excluded from allotment, which would be fully divided between banks with shortage of liquidity. In this case, the banks have no more incentives to trade (as the liquidity shock is neutralised) and the interbank market role is greatly reduced. The discussion in Subsection 5.2.3 indicates that such behaviour, at least to some extent, actually took place in the Eurosystem.

The rise in the interest rate is related to the higher expected future cost of funding, which was affected by all three elements of the crisis. In this simulation I assumed that the injection of liquidity has no immediate effect on credit risk or trading frictions, as discussed in more detail below. Abundant liquidity can still help to offset the expected liquidity shortage. This effect is perhaps not so strong on the first day of the crisis: the injection increased each bank's reserves by 10 units, while the expected accumulated value of liquidity shocks until the end of the maintenance period was 100 units. For the following days, however, extra liquidity reduced the remaining reserve requirement, thus lowering the liquidity needs of all the banks in the market, which reduced their expected cost of finance. Eventually, the interest rate one day before the end of the maintenance period was close to the central bank target level.

Note that the average interest rate on the last day of the maintenance period is higher compared with the scenario with no injection. This is the result of the fact that the surplus erodes the reserve requirement buffer that the banks carry to the last day of the period. Thus, when the central bank adjusts aggregate liquidity to the outstanding reserve requirement in fine tuning operations, very little liquidity is left in the market, which raises the interest rate. In reality the central bank interventions were reversed gradually, which means that they did not affect the reserve requirement buffer to the extent presented in this model.

In 2007, the central bank's did manage to lower the overnight interest rate, which is not replicated in my simulation. This is mainly caused by the assumption that the central bank does not control the credit risk or degree of trading frictions. Simple allotment rule used in this thesis cannot capture new liquidity supply mechanisms introduced by the central banks. Expanding the available collateral allowed the banks to use secure treasury bonds in the interbank trade reducing the credit risk. Extra reserves could make it easier for banks to find
trading partners, thus alleviating the increase in trading frictions. The sole fact that the central banks implicitly committed to the stability of the financial system in number of statements must have also restored some level of confidence. The fact that simple liquidity injection failed to duplicate the pattern actually observed in the markets indicates that those factors were very important for the interbank market behaviour.

Another aspect of the crisis that is not replicated in the simulation is the increase in the volumes of overnight trading. This shift can be explained by banks' responding to higher risk by shortening their investment horizon. Hence, even though global money markets interest rates were higher and trade volumes lower (as predicted in the simulation), the overnight maturity market volumes increased.

5.5 Conclusions

The goal of the chapter is to define the liquidity crisis of 2007 and analyse its impact on the behaviour of the interbank market. I argue that the elements of the crisis that seem to have the strongest effect on the commercial banks are:

- Rising trading frictions, which occur as market participants question the stability of the system.
- Expected liquidity shortage, as a result of the liquidity problems of various off-balance sheet entities.
- Increase in credit risk.

Based on the framework developed in the previous chapter of the thesis, I construct a model and perform a Monte Carlo simulation of the interbank market. The analysis of the impact of the crisis on market behaviour reveals that:

- Larger trade frictions increase the volatility of the interest rate.
- An expected liquidity shortage leads to higher expected cost of funding and an increase in the interest rate,
- Credit risk reduces lenders’ profits and their incentive to participate in the interbank market, which results in much lower trading volumes.

The combination of the three elements that define the liquidity crisis seem to cause the behaviour that was actually observed in 2007: widespread liquidity hoarding of large lending banks and pressure on the market interest rate level and volatility. I also find that the crisis has a strong negative effect on trading volumes, which prevents the banks from accommodating liquidity shocks.

I also analyse the impact of the central bank liquidity injection on the behaviour of the interbank market. It seems that additional reserves substitute rather than help the market to recover. I find that proportional allotment is either deposited back with the central bank (for banks with liquidity surpluses) or serves as a mean to offset liquidity shock (for banks with liquidity shortages). Variable rate tenders are likely to result in a similar outcome.
I conclude that in order to successfully bring the interest rate to the target level, the central bank must control the expected credit risk and the degree of market frictions.
References


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