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Dynamically and spatially efficient phosphorus policies in crop production

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Eutrophication of surface waters accelerated by nutrient runoff from agriculture is a growing concern in developed countries. Controlling the loss of phosphorus is complicated, among other things, due to its intertemporal nature. Phosphorus fertilisation affects crop yields and phosphorus losses mainly via the potentially plant available of soil phosphorus reserves whose development is very slow. Hence, both privately and socially optimal choices of phosphorus are results of a dynamic decision making process.

On the other hand, phosphorus loss is comprised of various phosphorus forms, differing in their contribution to eutrophication and in their sensitivity towards various phosphorus control measures. These forms can be roughly divided into particulate phosphorus and dissolved reactive phosphorus. The former can be controlled mainly by controlling the soil erosion, the latter by controlling the potentially plant available soil phosphorus reserves.

In this study, we solve analytically the privately and socially optimal steady state fertilisation levels and vegetative filter strip allocations, and design and analyse alternative instruments to sustain these allocations. We conduct an empirical application for an agricultural area of 37 parcels of one hectare, varying in their slopes and shapes. We find that the first-best taxes can be equivalently base on fertilisation or on soil phosphorus, but basing them directly on soil phosphorus might reduce the information burden of the regulator. We also find that the vegetative filter strip allocations are strongly differentiated, and that the second-best vegetative filter strip subsidies can be relatively easily be adjusted to sustain almost the first-best allocations.

Keywords: Dynamic programming, steady state, phosphorus fertilization, potentially plant available soil phosphorus reserves, dissolved reactive phosphorus, particulate phosphorus, vegetative filter strips, first- and second-best instruments

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Introduction

1.1 Background

Phosphorus is one of the nutrients needed in crop production. It has a special, intertemporal character in edaphic and economic sense. It accumulates into soil by sorbing into certain soil constituents. As the total amount of accumulated phosphorus increases, the amount of potentially plant available phosphorus in soil solution may gradually increase, too. This contributes to crop yields more than short-term fertilization levels. This complicated process is steered mainly by a balance of annual phosphorus input and output.

Phosphorus fertilization is largely an investment for future. Therefore, the valuation of future profits – reflected by the discount rates – plays a crucial role in farmer’s choice of phosphorus use. These intertemporal considerations may well be of implicit nature. That is, even though the annual choice of phosphorus would not be considered as a critical economic choice that particular year, in the long run the phosphorus applications are adjusted according to economic considerations based on the soil phosphorus status and dynamics.

The speciality of phosphorus runoff as an externality is that it is interestingly related to its edaphic and economic characteristics. It is roughly comprised of two forms: the loss of particulate phosphorus (PP-loss) and the loss of dissolved reactive phosphorus (DRP-loss). The main determinant of PP-loss is erosion actuated by runoff. The runoff detaches and transfers the soil particles – entailing phosphorus that is sorbed in these particles – into watercourses. During the runoff process and eventually in the receiving waters, part of the particulate phosphorus may change in a bioavailable form, and thereby contribute to eutrophication.

The erosion susceptibility varies strongly within an agricultural region. It is influenced, for instance, by the soil type and the stability of the soil structure. One of the most transparent determinants is the slope of the field: the steeper the field, the higher the risk for erosion.

The loss of dissolved reactive phosphorus (DRP-loss) is instead more clearly determined by the soil's potentially mobile phosphorus reserves. Hence, this form of phosphorus loss is linked to the same variable as the soil fertility in terms of phosphorus. The higher the level of plant available soil phosphorus reserves, the more beneficial it is to crop yield; and the higher the potential DRP-loss is from that field. The DRP-loss is directly in a bioavailable form and it can therefore contribute directly to primary production in the receiving waters, and hence to environmental damage.

The privately and socially optimal levels of phosphorus use and measures to mitigate phosphorus loss presumably diverge. In choosing the optimal phosphorus fertilization levels, the farmer balances current and future periods' profits. The socially optimal level acknowledges, in addition to this, the trade-off between higher private profits and higher environmental damage. Analyzing these two optima and the alternative ways to make them coincide, is the core of this study.

1.2 The research problem

In the present study we bring the economic dynamic decision making and the dynamic soil phosphorus processes under a single framework. We examine the implications of this twofold dynamics and the heterogeneity of farmland on optimal phosphorus use and on instrument design. We want to answer the following research questions: for a given soil type, what are the privately and socially optimal steady state levels of annual phosphorus fertilization, and the associated potentially plant available soil phosphorus reserves? How are the erosion control measures optimally allocated when the cultivated area is heterogeneous in terms of slopes and shapes of the field parcels? We also analyse the instruments to induce these allocations.

To answer these questions we construct an analytical dynamic programming model to solve the privately and socially optimal steady state choices of fertilization and erosion control measures. Then, we conduct an empirical application for a crop production area of 37 hectares. The instruments are analysed both within the analytical model and the empirical application.

We have made two important choices while postulating the framework. Firstly, we acknowledge the simplest possible form of heterogeneity: the shapes and the slopes of the field parcels may vary, otherwise they are identical. There are myriad sources of heterogeneity that affect the runoff from agriculture. By choosing two important but simple ones enables us to produce generalizable results. Secondly, we scrutinize the problem under a stationary, dynamic programming framework. Focusing on steady states helps us explore properly the underlining intertemporal characteristics: the edaphic dynamics and economic, dynamic decision making.

1.3 Previous literature

Agriculture is a multi-production system where agricultural products and certain environmental externalities are produced simultaneously. If an externality is positive, the socially optimal level of production is higher than the privately optimal and vice versa. Agri-environmental externalities and efficient policy design are analyzed from different angles in many studies. A good starting point is Griffin and Bromley (1982), who analyze the agricultural runoff as a non-point externality. Even though our model is not by nature a non-point pollution model, their framework is an essential justification for this kind of analysis. The work by Braden and Segerson (1993) analyses the issues further and provides us with requirements for environmental instruments. Schnitkey and Miranda (1993) serves as an early – and one of the few – contributions of analytically modeled agri-environmental phosphorus dynamics. Xepapadeas (1992) studies the dynamic features of controlling an accumulating pollutant. Goetz and Keusch (2005) conduct a dynamic phosphorus policy analysis and Lankoski et al (2006) provide us with a study combining analytically various nutrient runoff control measures.

Griffin and Bromley (1982) develop a theoretical framework for agricultural runoff as a non-point externality of whose origins and contributions to environmental damage cannot be defined without excessive costs. The fact that the individual non-point emissions are costly to monitor hinders the direct application of the instruments applicable for point source emissions. Therefore, Griffin and Bromley (1982) examine efficient ways to control for the factors influencing the emissions: input use, protective measures taken, etc.

The key requirement for their framework is that the production of externalities can be expressed with a continuously differentiable non-point production function. This function defines the amount of the externality produced with (partly) the same inputs that contribute to goods production. Hence, if we are able to define functions that determine the production of both the agricultural products and externalities, we can determine the socially optimal levels of input use. Furthermore, the instruments influencing the private decision maker may be conditioned on the use of these inputs. Griffin and Bromley (1982) show that under certainty we can always find efficient instruments that induce the socially optimal behavior, if their design is based on correctly defined production, and non-point production functions.

In this study, we define private profits from crop production and environmental damage from phosphorus loss from the same set of inputs to explore the nature of phosphorus as an agricultural input, and as a cause of externalities. Even though we examine a well defined, fully observable agents and target area, the justification of the current study lies in the fact that agricultural runoff is a non-point externality, as defined by Griffin and Bromley (1982).

Our analysis focuses on two types of phosphorus loss, on soil phosphorus dynamics, on its effect on crop yield and environmental damage, and on the implications of heterogeneity. Braden and Segerson (1993) outline the required features for instruments to control non-point source pollution: i) correlation with water quality ii) enforceability iii) temporal and spatial targetability.

In the present study the division of phosphorus types on grounds of their bioavailability is a reflection of their first requirement. We take cognizance of the enforceability by analyzing different tax bases and, for instance, their information requirements. The temporal aspects are strongly present: we acknowledge both the edaphic and economic dynamics. Our analysis also acknowledges the spatial aspects in allocating the erosion control measures.

The additional phosphorus in crop production can be given in the form of manure or in the form of mineral fertilizer. The amount of manure applied is often based on its nitrogen content. Therefore, phosphorus can easily be given in excessive amounts (see, e.g., Feinerman et al 2004). The excessive cumulated amounts of phosphorus can also be explained by the spatial characteristics of manure application and dynamic character of soil phosphorus. Schnitkey and Miranda (1993) conduct a steady state analysis on controlling phosphorus runoff from livestock producing farms with crop production. Their model solves for privately optimal manure application radius around the livestock facility. Inside this radius the farmer applies only manure, outside only commercial fertilizer. Their model provides an analytical explanation for the excess soil phosphorus levels in the vicinity of livestock facilities. They also compare the (stationary) welfare effects of two alternative phosphorus control policies.

Schnitkey and Miranda (1993) emphasize the special dynamic and spatial agronomial character of phosphorus. In their model, phosphorus dynamics is steered by phosphorus application rates, carryover rates (in the present study: transition function) and phosphorus uptake by crops. The decision on phosphorus application rates differs from those of, say, nitrogen in the sense that the accumulated soil phosphorus contributes to crop yield more than does the fertilizer application; in their model particularly for the lower values of soil phosphorus. The spatial aspects in their model are incorporated in the form of hauling costs which explain the abundantly high soil phosphorus levels in the vicinity of the livestock facility.

They do not, however, model endogenously the environmental effects. The focus of the analysis is on the economic effects of two alternative policies on private farmers, not on the policies' effects on phosphorus runoff. They, for instance, assume uniform erosion susceptibility across the farm. They also do not differentiate between DRP- and PP-losses, and hence do not consider erosion control measures and soil phosphorus level restrictions separately. The do not consider the interesting interlinkages between economic, dynamic determinants of phosphorus use and spatial, dynamic elements of phosphorus runoff. Indeed, the main difference in the analysis of the present study and Schnitkey and Miranda (1993) is that we incorporate the spatial and dynamic aspects of phosphorus runoff in a framework that solves both privately and socially optimal phosphorus use and the level of chosen phosphorus control measures. Also, within this study we focus only on the application of mineral fertilizers.

Instead of conducting a stationary (steady state) analysis one can analyse the paths and their convergence towards the desired states. Xepapadeas (1992) provides an example of this. He creates a framework for designing dynamically efficient instruments for accumulating pollutants, alike phosphorus. The instruments are conditioned on deviations of private paths from the socially optimal ones when moving towards optimal steady states.

The accumulation of the pollutant is indeed an important issue in designing phosphorus policies, but the dynamics of soil phosphorus as a source of both benefits (private profits) and losses (eutrophication) does not readily fit the framework of Xepapadeas (1992).[1] Firstly, we do not know the socially optimal paths of DRP- and PP-loss reductions. Secondly, and more importantly, we do not even know the socially optimal steady states. Therefore, a stationary analysis defining the socially optimal steady states in different circumstances provides us the necessary starting point for analysing further the dynamically efficient phosphorus policies.

Goetz and Keusch (2005) conduct a dynamic phosphorus policy analysis, but they consider soil erosion, and hence particulate phosphorus as the only source of phosphorus loss. Particulate phosphorus in eroded material, however, has a relatively low bioavailability (Uusitalo 2004). Therefore, its role in eutrophication is not proportional to its share of the total phosphorus loss. More importantly, PP-loss is not a distinctively dynamic phenomenon. The stochasticity in PP-loss due to rainfall or heterogeneity of farmland with respect to erosion risk would seem to be more relevant features. The essential dynamics in phosphorus control lie in the interlinkages of soil phosphorus processes, crop yield and DRP-loss. Hence, that is where the dynamic analysis should focus on.

[1] For more on phosphorus accumulation and optimal policies, see for instance Mäler et al (2003).

Lankoski et al (2006) analyzed no-till technology and complementary policy instruments to control for both nitrogen and phosphorus losses. Subsidizing the construction of vegetative filter strips (VFS) was part of their policy options. Their framework is capable of solving the static social and private optima for chosen parameter values. The static character, however, makes their research more appropriate for analyzing nitrogen policies. In spite of the static approach, the welfare effects of constructing VFS are modeled in a fairly similar fashion as in the present study. The VFS subsidies in their study, however, are uniform and conditioned on VFS acreage only. In the present study, the analyzed subsidies may vary according to farmland heterogeneity.

The current study focuses in the intensive margin effects, i.e. the farmers can adapt to prevailing policies only by changing the use of inputs, not by increasing or decreasing the acreage under cultivation or by changing the cultivated crops. Unquestionably, the extensive margin effects are relevant in analysing the environmental effects of any agricultural policy. For instance, Just and Antle (1990) model endogenously the farmers' possibility to leave the land idle. In addition of affecting the input use decision, policies may turn parcels of given characteristics unprofitable to cultivate, and vice versa. Policies that affect the farmers' decision making via economic instruments tend to have both intensive and extensive margin effects.

Lichtenberg (2002) provides an exhaustive analysis on agri-environmental policy assessment and design. Among other things, he examines efficient policy design under heterogeneity. It is well known that uniform regulations can not induce socially optimal allocations when the targets of regulation are heterogeneous. The severity of the efficiency loss is linked to the degree and type of heterogeneity. If, for instance, the environmental damage is influenced by the use of fertilizer identically from each parcel, the uniform fertilizer tax can provide the socially optimal allocation with minimum costs. If, on the other hand, the environmental damage would be caused by erosion of whose magnitude would differ significantly across parcels, an efficient instrument should be differentiated across the parcels. Also, Lichtenberg (2002) points out that due to the characteristics of crop production, implementing the social optimum with differentiated instruments would be very hard. On the other hand, due to the easier observability of the key determinants, differentiating the instruments of erosion control would in principle be possible. In the present study, we analyze the use of both uniform and differentiated instruments.

The rest of the study is organized as follows. Chapter 2 presents the natural science preliminaries. Chapter 3 presents the analytical private optimization model. The analytical model for the social optimization problem is presented in chapter 4. The empirical application is presented in chapter 5. Chapter 6 presents the results of both the private and the social optimization problems. Chapter 7 discusses the instrument design. Chapter 8 concludes.

2 Natural science preliminaries – the role of phosphorus

Before modelling the private and social economic problems we review the role of phosphorus in agricultural production; and the elementary natural science aspects of phosphorus loss from cultivated land. We do this to understand the motives for the use of phosphorus fertilizers, and to evaluate its consequences on crop production and phosphorus loss in the long run.

We start the review by discussing the role of (soil) phosphorus in crop production and the plant uptake of phosphorus. Then we discuss the various forms of phosphorus and the role of potentially plant available soil phosphorus reserves in determining the DRP-loss, and the determinants of PP-loss. Finally we review the role of vegetative filter strips (VFS) in abating the PP-loss.

2.1 Phosphorus and crop production

The amount of total phosphorus in soil does not give much information on soil fertility in terms of bioavailable phosphorus. A crucial factor is, how strongly phosphorus is sorbed to soil constituents (Kaila 1963a, 1963b). The more strong the sorption, the more difficult it is for plants to use soil phosphorus and vice versa.

In Finland, the soil phosphorus reserves are generally poorly available for plants. The poor availability is largely due to strong sorption of phosphate anions in soils. This is typical especially of clay soils high in aluminium and iron oxides that are the main sorption components for phosphorus. The soils are also typically acidic which enhances the sorption reactions.

Long term surpluses in phosphorus application gradually increase total phosphorus of soil and also the plant available fractions of it. For instance, in Finnish cultivated fields, phosphorus has been accumulating into soil during the periods of traditional farming ere 1900's. Heavy increase in fertilizer use after 1930's raised the total phosphorus content of cultivated soils overall, and also the plant available fractions of it (Saarela 2002). These fractions are essential for crop production. They provide the soil solution (the water in soil) with dissolved phosphorus forms which can be biologically uptaken by crops.

In order to estimate the level of potentially plant available soil phosphorus reserves, approximation methods called Soil Test Phosphorus (STP) have been developed. Farmers can base their fertilization decisions on STP measured from their fields. There are various types of STP methods differing in the chemical properties of the extractants. The method used in Finland is based on the extraction with acid ammonium acetate. The method was introduced in the early 1950s (Vuorinen and Mäkitie 1955).

Studies have differing stances concerning the long-term development of the STP measure by Vuorinen and Mäkitie (1955), in response to various phosphorus balances. The opposite poles can be seen, for instance, in the papers by Saarela et al (2004) and Ekholm et al (2005). Saarela et al (2004) found that STP shows only marginal changes in the course of time if zero phosphorus balance is applied. They found a distinct decrease in STP levels with zero phosphorus balance only with soils with very high initial STP. Ekholm et al (2005), in contrast, concluded that even soils with relatively low initial STP a constant phosphorus surplus is needed to maintain the STP level. For instance with initial STP of only 2 mg l^{-1}, a surplus of 7.5 kg $ha^{-1}a^{-1}$ is required to maintain the STP level unchanged.[2]

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[2] Henceforth, we will omit the unit (mg l^{-1}) and use only the abbreviation STP.

The common feature in the aforementioned studies is that only part of the potentially plant available soil phosphorus stays in this form for the next period – the difference is how they asses the magnitude of this change. Already Kaila (1963b) discussed the tendency of phosphorus bounds to become stronger in the course of time. The phosphorus sorption capacity is a result of a complexity of phosphate retention mechanisms. The natively bound phosphates are in contact with soil constituents for a long time period, and are thus exposed to a sequence of reactions. The phosphates sorbed during short laboratory treatments are likely to form less strong bounds. These differences will also affect the policy implications of the present study.

The plant availability of phosphorus can increase, and be increased, even when the total phosphorus amount is kept constant. This can happen, for instance, as a results of an increase in soil temperature, aeration or biological activity in rhizosphere (the soil region on and around plant root) (see e.g., Yli-Halla et al 2002). The pH of soil also largely determines the availability of phosphorus by changing the charge properties of phosphorus binding oxide surfaces. The more acid the soil, the more efficient the fixation of phosphorus into soil constituents.

Altogether, soil phosphorus dynamics is a very complex issue. We restrict ourselves to a very narrow part of the phenomenon: development of potentially plant available soil phosphorus reserves, approximated by the STP measure. For a fresh, thorough description of soil phosphorus dynamics, see for instance Yli-Halla et al (2005) who simulate the soil phosphorus process extensively.

As said, phosphorus application has only a minor immediate effect on crop growth. The influence comes from the long-run steering of soil phosphorus fertility. However, there seems to be a direct response as well. Saarela et al (1995) have estimated this response in their long-term field trials. According to their data, the response was statistically insignificant for the soils with very high soil phosphorus values. On soils with lower phosphorus status, the response could be estimated. They also found that phosphorus fertilization affected the quality of crop yield by increasing its mineral content.

The mineral phosphorus fertilizer is usually given together with other fertilizers, typically nitrogen (N) and potassium (K). There are alternative ratios of NPK-fertilizers. In the short run, the farmers would theoretically choose the fertilization levels and NPK ratios based on the nitrogen response of the crop yield. In the long run, also the level of phosphorus is chosen in a dynamically optimizing fashion.

Phosphorus uptake

Plant uptake is a central issue in the emergence of nutrient loads from agriculture; only the inputs not uptaken by crops can contribute to environmental damage. Earlier we saw that with phosphorus input and output exactly equal, the plant available soil phosphorus reserves become gradually depleted. Therefore at steady state, by definition, we will have a positive phosphorus input, positive output due to crop uptake and a positive amount of phosphorus loss.

The plant available soil phosphorus reserves are steered by annual phosphorus balance, i.e. the difference in phosphorus applied to and removed by the crops. The phosphorus output thus depends on the crop yield and on the phosphorus concentration of the yield. Saarela et al (1995) suggest in their long-term phosphorus trials, that phosphorus concentration of barley (and other crops) varies with STP levels but only slightly or not at all with phosphorus fertilization and crop yield levels.

The phosphorus uptake of other parts of the plant is also important. The roots and straw contain phosphorus, according to Saarela et al (1995) approximately 19% and 23% of the total uptake of crops, respectively. The phosphorus in the plant residues, however, is in organic form. Its contribution to soil phosphorus dynamics is not a straightforward issue. Adding undecomposed organic matter may first even decrease the level of plant available phosphorus reserves in soil due to microbiological processes during decomposition. Later, phosphorus assimilated by micro organisms is released again, which in turn increases the reserves of plant available phosphorus (see, e.g. Kaila 1949). Altogether, the dynamics of organic phosphorus is too intricate an issue to be included in our research. Also the phosphorus balances used in Saarela et al (1995) include only the phosphorus uptaken by the crop yield.

Summa summarum, the essential features for present research are: i) The main determinant of crop growth and/or soil fertility in terms of phosphorus is the plant available reserve of soil phosphorus; ii) The main role of phosphorus fertilizer applied any particular year is to control these reserves in the long run. These two features form the base of our dynamic, economic problem.

2.2 Phosphorus in runoff and drainage flow

From the viewpoint of runoff, phosphorus is divided into different categories according to their chemical and biological properties. Some pools are readily available for plant uptake, some have to be first transformed into biologically available forms. There are multiple ways to define these forms. For an extensive treatment, see for instance Ekholm (1998).

From the viewpoint of phosphorus in source areas (fields) it is most illustrative for us to divide the phosphorus roughly into water soluble and insoluble forms. The soluble forms may eventually find themselves in runoff waters in the form of dissolved phosphorus, which we will be calling as dissolved reactive phosphorus (DRP). They can also be sorbed back into soil phosphorus, and hence into insoluble forms. The insoluble forms can be transported from fields by detachment of soil particles, i.e. erosion. This form of phosphorus in runoff water is called the particulate phosphorus (PP).

In the receiving watercourses the readily bioavailable DRP contributes directly to phosphorus concentration of the water. Eventually, it will be assimilated as biotic phosphorus (e.g., algae uptake) or sorbed into lake sediment. The PP received by the watercourses is either deposited directly into sediments or part of it may be changed by desorption processes into bioavailable form, after which it faces the same potential processes as DRP.

The bioavailability of PP is affected by many factors, one of them the size of the particles the phosphorus is attached. Therefore, some authors further divide PP into bioavailable and unbioavailable fractions according to the particle sizes (see e.g., Uusitalo et al 2001). Alike almost all physical phenomena, the desorption (PP à DRP) and (ad)sorption (DRP à PP) processes are continuous, and the measurements of the fractions are thus conditional on definitions. Within this study, we use the simple division of PP and DRP.

DRP-loss

The DRP-loss is largely determined by reserves of plant available phosphorus in topsoil (McDowell & Sharpley 2001; Vadas et al 2005; Ekholm et al 2005;). Because the STP measures discussed in the previous section are developed to approximate these reserves, it would seem logical that the STP could be used to predict DRP-losses. This indeed seems to be the case.

Several studies have estimated the relationship between STP and DRP-loss. (see, e.g., Sharpley 1995; Pote et al 1996; Uusitalo & Jansson 2002). Schroeder et al (2004) provide a brief list of studies examining the relationship between easily soluble topsoil phosphorus, measured by STP, and the DRP-loss in runoff. Altogether, the distinct correlation between STP and DRP-loss seems to be a well established results in soil and environmental sciences.[3]

It must be noted, however, that the STP methods differ from each other, and that the above mentioned studies do not explain the causal relations between STP and DRP-loss. STP is not the sole determinant of DRP-loss. For instance the soil to solution ratio plays an important role in the adsorption-desorption processes. The phosphorus concentration of the solution affects the process; diluting the solution accelerates the desorption process and vice versa (see e.g., Sharpley et al 1981; and Yli-Halla et al 1995 for a laboratory experiment for Finnish conditions). Also, the amount and the saturation rate of oxides in soil affects the buffering capacity. Perhaps a more extensive variable than STP would be needed for estimating the DRP-loss. For instance Börling et al (2004) estimate potential phosphorus release using additional variables to only STP.

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[3] However, Djodjic at al (2004) find no statistically significant correlation between STP and observed DRP-loss. They analysed various soil types and assessed the significance of the STP as a overall predictor of DRP-loss.

Altogether, STP is a fairly good and readily available measure to use in estimating DRP-loss. For instance in Finland the STP data has been collected from the vast majority of agricultural fields on a regular basis.

PP-loss

Erosion is the most important determinant of the PP-loss, which we earlier defined to consist of water-insoluble phosphorus, detached by runoff waters. That is, the key determinants of PP-loss are the same as the determinants of erosion. The soil type, the stability of soil structure, cultivation technologies and practices, crop choice etc. affect the magnitude of erosion. In Finland, also the season has a great significance since most of the erosion, and hence PP-loss, occurs in the cold season when it tends to rain and the fields are often ploughed and bare. Particularly, most of the runoff occurs from the snowmelt in springtime (Kniesel and Turtola 2000).

An important factor affecting the magnitude of erosion is the slope of the field (see, e.g., Tattari et al 2001; Rankinen et al 2001). It does not vary on a given parcels and is therefore perhaps easiest to observe. That is, it is possible to determine whether a parcel is steep or gentle, and even to give a precise grade for the slope.

Typically, the nutrient runoff is estimated with various simulation models which mimic the complex natural conditions and are capable of predicting the nutrient losses under various conditions. One of such models is the ICECREAM model which describes the phosphorus cycle in the soil and the phosphorus losses from soil to water (Tattari et al 2001). This, alike many other simulation models, capture the whole process of soil phosphorus dynamics. It is open for a wide range of variables: crop choices, various soil characteristics etc. Such simulation models are very useful for an interdisciplinary study such as ours. In our case, the ICECREAM model will be used to provide us with simulated results on erosion from given parcels. With the help of these we estimate the explicit functions for PP-loss used in the empirical section.

In addition to intensity of erosion, the absolute amount of PP-loss depends on the amount of total phosphorus in soil and hence in the eroded particles. The contribution of this PP-loss on eutrophication in the receiving water body, however, is a very complex issue. A central feature affecting this contribution is the change in the environment for the sorption-desorption processes that takes place as the soil particles are detached from the soil and carried into the watercourses. As noted earlier, diluting the solution accelerates the desorption process. That is, as the eroded particles enter the receiving water, its milder phosphorus concentration increases desorption and hence part of the PP is transformed into bioavailable form (see, e.g., Ekholm 1998, Uusitalo et al 2003).

Another important process is related to the release of phosphorus from anoxic sediments (see Mäler et al (2003) for an inventive economic analysis of the issue). For the present study, the key element in the bioavailability processes of PP is that the correlation between the level of plant available soil phosphorus and the bioavailability of PP-loss does not seem to be very strong. In particular, the phosphorus released from anoxic sediments will not be much affected by a decrease of STP (Uusitalo et al 2003). Since we will focus our analyse on the bioavailable phosphorus forms we will use this result when constructing the analytical model for assessing the dynamic efficiency of phosphorus reductions from agricultural fields.

The routes of phosphorus from soil to waters

The routes of runoff into the receiving water body can also be defined in many ways. The simplest way is suitable for our purposes; it is also the most commonly used. It is the division between the surface runoff that takes place in the immediate field surface, and the subsurface flow that takes place mainly in the constructed drainage system.

Considering the detachment process described earlier, it is obvious that both forms of phosphorus find themselves in both routes of runoff waters. Particularly in Finland the PP-concentrations in drainage flow have been high. For instance Uusitalo et al (2001) found approximately similar concentrations of phosphorus fractions in surface runoff and drainage flow in Finnish clayey soils.

The dominant form of phosphorus in surface runoff and drainage flow in Finland is PP. For instance, according to reports of Turtola and Jaakkola (1995) and Turtola and Kemppainen (1998), the fraction of DRP from total phosphorus loss varies between

22% and 33% depending on crops and farming practices, the rest consisting of PP. Uusitalo et al (2003) report that the share of PP from the runoff phosphorus varied between 73% and 94%. Ekholm and Krogerus (2003) estimate that the fraction of bioavailable phosphorus of the field runoff is eventually 17±5%. Topography affects the PP-loss significantly, as the susceptibility for erosion is determined largely by the slope of the field (see, e.g., Rankinen et al 2001).

It seems that in Finland both the drainage flow and surface runoff are important channels for both forms of phosphorus loss. Particularly the substantial PP-loss occurring in drainage is typical of Finnish soils. Climate conditions and the fact that agriculture is concentrated on clayey soils, can partly explain this. The high amounts of eroded particles (and hence PP) in drainage flow can be explained by macroscopic cracks that are common in dry clayey soils containing expanding clay minerals. (Kniesel & Turtola 2000; Uusitalo et al 2001). The cracks emerge during the dry periods when the soil shrinks as a result of interlattice dehydration of clay minerals. A rainfall after such a period detaches particles from soil, which are flushed away by the runoff waters. Part of this runoff, and the eroded particles with it, flow into the cracks, i.e. macropores, and find their way into the watercourse via the drainage flow. Gradually the rain makes the soil swell again, closing the macropores, after which the PP-loss takes place mostly in surface runoff.

Both routes seem important transporters of DRP-loss as well. According to the four-year experiment at four different sites by Uusitalo et al (2003), the DRP-losses were slightly grater in the drainage system than in the surface runoff. In their field trials for 1991-1999, Uusi-Kämppä and Kilpinen (2000) found that the amounts of DRP (in their study, dissolved orthophosphate) measured from the surface runoff and in the subsurface flow (measured from the depth of 0.2 meters) were close to each other. For a grass filter strip the average sum of altogether 20 sequential measurements (8 years) of dissolved orthophosphate was in the surface runoff about 1.17kg ha^{-1} and in the subsurface flow about 1.39 kg ha^{-1}. According to this, the DRP-loss in the surface runoff would be 16% less than in the subsurface flow. In fields with no filter strips, the amounts were 1.12 kg ha^{-1} in the surface runoff and 0.95 kg ha^{-1} in the subsurface flow. That is, the DRP-loss in the subsurface would be about 19% less than in surface runoff, i.e. the other way around as with a grass filter strip. According the these results it seems that the total amounts of DRP lost in the two alternative routes are approximately similar, or at least we can not identify the direction or the magnitude of the difference. It is also interesting to note that according to Uusi-Kämppä and Kilpinen (2000) the DRP-loss seems to be higher with a grass filter strip than without a strip. This difference was even larger with strips with other vegetation than grass.

Also theoretically there seems to be no reason to assume that the DRP concentrations would, on average, be significantly greater in one route or the other. This follows from the way the DRP-loss occurs. The loss takes mainly place as phosphorus is desorbed from stagnant surface soil into the runoff waters. Only part of DRP-loss is due to desorption of phosphorus from the eroded particles into surface runoff. For instance Yli-Halla et al (1995) concluded that eroded soil in runoff contributed only 16-38% of DRP in surface runoff; the rest is desorbed from stagnant surface soil. This process is affected by soil hydrology. Finally, the magnitude of rainfall is the most important determinant of the absolute amounts of DRP- and PP-losses in both routes to watercourses.

2.3 Phosphorus abatement by vegetative filter strips

There are ways to reduce the erosion risk or to remove nutrients from runoff waters. The use of catch crops or no-till technology reduce the soil's susceptibility for erosion. For nutrient abatement one can construct wetlands, lime filter ditches or vegetative filter strips (VFS).

In Finland, the most commonly used measure is probably the construction of VFSs. The complex filtering process of the VFS has been analysed extensively in a number of hydrological studies (see, e.g. Munoz-Carpena et al 1999; and Dosskey 2002 for a review on VFS studies). The models simulating the VFS processes account for a very high amount of variables: the characteristics of runoff (e.g., velocity, concentration of nutrients and their edaphic and hydrological determinants), slope of the field preceding the strip and slope within the strip, type of vegetation in the strip etc. Some models account also for stochasticities in rainfall and soil permeability (see e.g., Munoz-Carpena et al 1999).

Some issues related to phosphorus filtration by VFS are particularly relevant in view of our study. Firstly, even though PP would be reduced from runoff, the DRP concentration can even increase (see, e.g. Uusi-Kämppä and Kilpinen 2000; Dosskey 2002). That is, according to some studies, it might be optimistic to assume that VFS does not *increase* DRP-loss. However, Uusi-Kämppä and Kilpinen (2000) noticed that the abatement of VFSs may increase over time. This is probably due to the soil becoming gradually poorer due to negative phosphorus balances on the VFS soil; and the vegetation of VFS getting firmer. The former increases the probability of phosphorus adsorption from runoff solution into VFS soil and the latter reduces the runoff velocity and enhances both the biotic uptake as well as adsorption (Whithers and Jarvis 1998; Uusi-Kämppä and Kilpinen 2000). Therefore, it might be possible that in the long run, VFS would abate the DRP-loss as well.

According to Dosskey (2002), however, studies analyzing the actual impact, i.e. evaluating stream nutrient flows before and after constructing the VFS, have not been reported. That is, it has been shown in several studies that existing buffer strips are important in maintaining the water quality, but the responses of water quality to the construction of a new strip has not been reported. The reason for this is obviously that it is technically difficult and time-consuming to construct experiments which satisfy this criterion. One would first have to gather runoff data from a long enough period without VFS, then construct the VFS(s) and again gather data from a long enough period to enable the comparisons between the runoff values before and after the construction of the VFSs.

What this could mean in practice? For instance in the study of Uusi-Kämppä and Kilpinen (2000) the DRP-losses are on the average higher from the 4 parcels which have a VFS, compared to the two parcels without VFSs. The explanation is either that the DRP-losses from the associated parcels are initially different, or that the DRP-losses are higher due to the construction of VFSs. Both explanations are theoretically feasible, but hardly verifiable on the basis of the empirical data. This, in essence, is the issue raised by Dosskey (2002). Due to the nature of agriculture, the researchers will always more or less confronted with it.

Hence, there are results from VFSs both increasing and decreasing DRP-loss. Also theoretically there are processes within the VFS that affect in both directions. It is thus not justified to claim that VFSs would abate DRP. However, practically all studies find that they do filter eroded materials from runoff. Therefore, they do reduce PP-loss from surface runoff. They do not, however, affect the PP-loss taking place in the drainage system.

On the grounds of all aforementioned reasons, it is justified to partition the efforts to reduce the phosphorus loss into PP-loss abatement (constructing VFS) and DRP-loss abatement (lowering STP). Most certainly there are interlinkages between these two but the main lines seem to be so separate that an efficient policy analysis can, and for the sake of simplicity even should, assume for separability of the issues.

2.4 Summary on phosphorus review

In this section we have shortly presented the relevant features of phosphorus with respect to crop production, phosphorus loss and abatement. Along the way we have made assumptions that will affect the results of the study.

We have assumed that in terms of phosphorus, the crop yield is influenced both by phosphorus fertilization and by the plant available soil phosphorus reserves, approximated by the STP. STP is dynamically and soil type specifically steered by the long term phosphorus balance, i.e. the difference of phosphorus input (fertilizers) and output (plant uptake). The uptake can be defined with the help of the crop yield and the STP, which correlates with the phosphorus concentration of the seeds.

In the present study, the phosphorus loss is assumed to comprise of two forms: the loss of particulate phosphorus (PP-loss) and the loss of dissolved reactive phosphorus (DRP-loss). The latter is in a readily bioavailable form in soil solution, the former has to be transformed into bioavailable form before it can contribute to primary production (crop growth, algae growth, etc.). The phosphorus loss occurs via two routes: surface runoff and drainage flow. Both phosphorus forms can be found in both routes.

With respect to these, we have made two important assumptions. Firstly, we have assumed that the bioavailable fraction of the PP-loss is not affected by the changes in STP. Because we optimize only with respect to the bioavailable phosphorus loss, this means that the PP-loss is not affected by the dynamic variable STP. Secondly, we have assumed that the magnitude of the DRP-loss is affected only by the level of STP. The VFSs reduce only PP-loss, whose level is determined largely by the slope of the field parcel.

3 Privately optimal phosphorus application

To model the privately choice of phosphorus fertilization, we consider a farmer who maximizes profits from growing a certain crop on a parcel of land. She receives revenues from selling the crop yield and incurs costs from using fixed and variable inputs.

We focus only on farmer's choice of phosphorus. Hence, we assume that the choice of other nutrients as well as other fixed and variable inputs, such as fuel, machinery, etc is exogenous in the model. Keeping the model as simple as possible helps us focus on the dynamic and spatial characteristics of optimal phosphorus use.

3.1 The framework

The farmer's choice variable is the annual phosphorus fertilization which determines the crop yield together with plant available soil phosphorus reserves. The crop yield in turn determines the phosphorus uptake of crops, and all these together determine the next period's level of plant available soil phosphorus reserves. Hence, the current period's crop yield depends not only on the current input use but also on the decisions made in the past. The farmer maximizes the sum of the discounted profits at this and all the subsequent periods. To make the structure of the private welfare consistent with the social welfare we will define in the next section, we also include an option to set land aside from production. However, since the land set aside yields no benefits for the farmer, its privately optimal choice will be zero.

We define the sum of discounted profits as private welfare (PW). For a unit acreage of land it is given by:[4]

$$\begin{aligned} &PW = \sum_{t=0}^{\infty} \beta^t \left[\left[py(s_t, x_t) - wx_t - C \right] (1 - A) - f(A) \right] \\ &s.t.\ \ s_{t+1} = \Gamma(s_t, x_t), \end{aligned} \qquad (1)$$

where:

PW	private welfare
β	the discount factor
p	the price of output (crop)
w	the price of phosphorus fertilizer
y	the crop yield function
s	the level of plant available soil phosphorus reserves
x	the use of phosphorus fertilizer
C	other (fixed and variable) costs of production
A	the acreage set aside from production
f	the costs of setting land aside from production (>0)
Γ	the transition function

Equation 1 defines the farmer's welfare as the sum of discounted profits from this period to infinity. The transition function defines the plant available soil phosphorus reserves in the following period as a function of current period's reserves and phosphorus fertilization. Throughout the study, we will use the discount factor (β), derived from the discount rate (ι): $\beta = \frac{1}{1+\iota}$. It gives the weight of a unit of tomorrows

[4] The feasible set of (1) is nonempty and the objective function is well defined for every point in the feasible set. That is, (1) is well defined.

profits on today's decision making. We assume that the discount rate applied by the farmer is the real interest rate.

We will also assume that the input and output prices remain constant during the dynamic optimization process, i.e. we focus on stationary analysis. For further purposes, let us denote the private profit as: $\pi = [\, py(s_t, x_t) - wx_t - C](1\text{-}A) - f(A)$.

The crop yield function, or the phosphorus response function, is given by $y(s_t,x_t)$. The level of production is a function of plant available soil phosphorus reserves (s) and the phosphorus fertilization (x). We make the usual assumptions about its first and second partial derivatives: $y_s > 0$; $y_x > 0$; $y_{ss} < 0$ and $y_{xx} < 0$. We thus assume that the crop yield is increasing and concave in plant available phosphorus reserves and in phosphorus fertilizer use. For the cross derivatives we assume: $y_{sx} < 0$, i.e. the effect of a unit increase in fertilizer use has a smaller effect on yield on higher levels of plant available phosphorus reserves. This is a usual assumption. It is also in accordance with the long term phosphorus trials in Finland, reported by Saarela et al (1995).

The analytical form of the transition function (Γ) is the following:

$$\Gamma(s,x) = \chi(s) + v(P^{bal}(s,x)), \qquad (2)$$

where:

χ function defining the reserves surviving to the next period.
P^{bal} phosphorus balance; the difference between input and output of phosphorus
v function defining the effect of the phosphorus balance on the reserves

There are three different functions affecting the development of the plant available soil phosphorus reserves. Firstly, the level of plant available soil phosphorus (s) has a direct effect, defined by the function χ. Even though the chemical bounds of soil phosphorus tend to tighten in the course of time, most of s survives to the following period. All other things equal, however, the higher the s in the current period, the higher the absolute difference between s_t and s_{t+1}. That is, $\chi_s > 0$ and $\chi_{ss} < 0$.

The second and the third functions are interlinked. The phosphorus balance is steered by the phosphorus fertilization and the crop uptake of phosphorus. This in turn is determined by the crop yield function and the phosphorus concentration of the crops. The crop yield is a function of *s* and *x*. The phosphorus concentration is assumed to be affected only by *s*. The partial derivatives of P^{bal} w.r.t. *s* are: $P_s^{bal} < 0$; $P_{ss}^{bal} > 0$. The plant available soil phosphorus contributes to the phosphorus output by increasing the crop yield and the phosphorus concentration of the crops. Both effects are by the assumptions on the crop yield function smaller as the *s* gets higher, i.e. the function is concave in *s*. The fertilization is the sole input, but affects also the output by increasing the crop yield. From the earlier assumptions we can derive that: $P_x^{bal} > 0$; $P_{xx}^{bal} < 0$. For the cross derivatives we assume: $P_{sx}^{bal} < 0$, i.e., for higher values of *s*, the effect of fertilization on phosphorus balance gets smaller.

The third function is determining how the phosphorus balance affects the development of *s*. The effect is presumably not a constant, and most definitely it is not equal to unity. We may assume that the underlining function is concave. Hence: $v_P^{bal} > 0$ and $v_{P^{bal}P^{bal}} < 0$. Altogether, the partial derivatives of Γ are thus assumed to be: $\Gamma_s > 0$; $\Gamma_{ss} < 0$; $\Gamma_x > 0$; $\Gamma_{xx} < 0$; and $\Gamma_{sx} < 0$.

Because this dynamic transition process is at the core of the present study, it is reasonable to examine it thoroughly. Before we proceed, let us therefore look at the optimal choice of current period's fertilization in a two period setting. Let us fix the input and output prices to unity and all other costs to zero. The profit maximization problem (1) becomes:

$$\max_{x_0} \hat{\pi} = y_0(s_0, x_0) - x_0 + \beta\left[y_1(s_1(s_0, x_0), x_1) - x_1\right], \tag{3}$$

where s_0 denotes the initial level of plant available soil phosphorus reserves and x_i denotes the level of phosphorus fertilizer use in period *i*. The optimal choice of x_0 will be characterized by:

$$\frac{d\hat{\pi}}{dx_0} = \frac{\partial y_0}{\partial x_0} - 1 + \beta \frac{\partial y_1}{\partial s_1}\frac{\partial s_1}{\partial x_0} = 0\,. \tag{4}$$

Plugging in the transition function yields:

$$\frac{\partial y_0}{\partial x_0} = 1 - \beta \frac{\partial y_1}{\partial s_1} \Gamma(s_0, x_0) = 0 \tag{5}$$

At the optimum, the marginal effect of x_0 on crop yield (and hence on profits) must equal its marginal costs (unity) minus its discounted marginal effect on second period's profits. Using the assumptions $y_x > 0$; $y_{xx} < 0$; $y_s > 0$ and $\Gamma_x > 0$ we see that the second term in the right hand side is positive, and that therefore it increases the optimal use of fertilizers in the first period.

Hence, x_0 has a direct effect on the first period's profits and an indirect effect on the second period's profits. The effect of the latter on the optimal choice is determined by three factors: the factual transition process captured by the function (Γ), the marginal effect of the STP on crop yield (y_s) and the farmer's evaluation of the next period's profits, captured by the discount factor (β). The higher the effect of this period's fertilization on following periods STP (s), the higher the optimal fertilization in current period; the higher the marginal effect y_s, the higher the fertilization; and the higher the farmer's appreciation of future profits, the higher the fertilization. In the introduction, we referred to these factors as intertemporal edaphic and economic characteristics. The former is captured by Γ and y_s and the latter by β. This, in essence, is how these concepts enter our framework. In the following, we conduct the analysis for infinite time horizon and examine the optimal steady state decisions.

3.2 The optimal steady state use of phosphorus

Dynamic programming is a solution concept often used for continuous state, discrete time optimization problems with no closed form solutions. In this study, the (continuous) state variable is the plant available soil phosphorus level (*s*) and time is measured in discrete units of one year. Dynamic programming is based on Bellman's Principle of Optimality (Bellman 1957). Heuristically, the principle states that if a path determined by decisions in time from T_0 to T_n is optimal, it must also be optimal from any point of time, T_{n-k}, $0 < k < n$ to T_n. As long as this holds, we can determine optimal decision by working with backward induction.

Dynamic programming can be conducted either in finite or infinite time horizons. With infinite time horizon stationary problems the optimal actions are typically time-independent. As the planning horizon is extended to infinity, we end up in a steady state where the only relevant decision is made between this and the next period. After this steady state is achieved, the solution for the optimization problem is identical at each period. In our framework, this would mean that at the steady state the farmer always chooses the same level of fertilizer use.

To derive the optimality conditions analytically, let us first define a function that is the value of maximized equation (1) from any period onwards:

$$V(s_t) = \max_{(x_\tau, A_t)_{\tau=t}^{\tau=\infty}} \Pi = \sum_{\tau=t}^{\infty} \beta^{\tau-t} \pi(s_\tau, x_\tau, A_t)\,. \tag{6}$$

The function defined in (6) is called the value function. Starting from the state variable at any period (s_t), the value function $V(s_t)$ gives the sum of discounted private profits (π) from *t* to infinity, as the fertilizer use (*x*) and the acreage of land set aside from production (*A*) are chosen optimally at each time period. The value of choosing optimally at period $t-1$ can thus be thought of as comprising of the value of an optimal choice in period $t-1$ plus the value function in period *t*. Formally:

$$V(s_{t-1}) = \max_{x_{t-1}, A_{t-1}} \left[\pi(s_{t-1}, x_{t-1}, A_{t-1}) + \beta V(\Gamma(s_{t-1}, x_{t-1})) \right], \tag{7}$$

where the value function at t is defined with the help of transition function: $V(s_t) = V(\Gamma(s_{t-1}, x_{t-1}))$. Note that under the assumptions made earlier, the land set aside from production does not affect s. Again, the value function at $t-1$ is the value of maximized objective function from that point on.

The optimal stationary choice of x and A for the farmer's infinite time horizon problem is by definition characterized by the fact that the choice and state variables remain unaltered between the periods. Hence, we can drop the time-indicating subscripts from (7) and come up with an equation often called as the steady state Bellman equation (8):

$$V(s) = \max_{x,A}\left[\pi(s,x,A) + \beta V(\Gamma(s,x))\right], \tag{8}$$

where the equation in brackets could be equally well written as $[\pi(s,x,A) + \beta V(s)]$, since at the steady state $s_t = s_{t+1}$.

With fairly simple calculus (see Appendix 1 for a closer derivation) and assuming that $\Gamma_A = 0$, we obtain the optimal conditions from (8):

$$V_s = \frac{\pi_s(s^*, x^*)}{1 - \beta\Gamma_s(s^*, x^*)} \tag{9a}$$

$$\pi_x(s^*, x^*) + \beta V_s \Gamma_x(s^*, x^*) = 0 \tag{9b}$$

$$\pi_A(s^*, x^*, A^*)(1 - \beta\Gamma_s(s^*, x^*)) = 0 \Leftrightarrow \pi_A(s^*, x^*, A^*) = 0 \tag{9c}$$

$$s^* = \Gamma(s^*, x^*) \tag{9d}$$

and the complementary condition: $x, A \geq 0$. By plugging (9a) into (9b), we obtain the standard Euler equation and the stationary condition:

$$\pi_x(s^*, x^*, A^*)(1 - \beta\Gamma_s(s^*, x^*)) + \beta\pi_s(s^*, x^*, A^*)\Gamma_x(s^*, x^*) = 0 \tag{10a}$$

$$\pi_A(s^*, x^*, A^*) = 0 \tag{10b}$$

$$s^* = \Gamma(s^*, x^*), \tag{10c}$$

where:

π_i the partial derivative of the private profit w.r.t. i

β the discount factor

Γ_i the partial derivative of the transition function w.r.t. i

V_s the shadow price of the state variable.

The conditions (9a-9b) and (10a-10c) are thus identical. The reason for presenting both forms is that the intuition of the Euler equation (10a) is easier to discuss when divided into (9a) and (9b).

The shadow price (V_s) in (9a) tells the change in the value of the optimization problem (1) as the state variable is changed marginally. At the optimum, it is defined by (9a). The condition in (9b) balances the effects of current period's choice of x on immediate and future profits. It states that the optimal action must have equal absolute values for marginal effects on immediate profits (π_x) and for the change in the state variable they cause in the next period (Γ_x), evaluated with the shadow price of the state variable V_s and discounted with β. Because β and $V_s > 0$, we see that Γ_x and π_x must have opposite signs at the optimum. Referring to our assumptions on Γ; and to the discussion on the two-period example, we know that at the optimum $\pi_x < 0$ and $\Gamma_x > 0$.

The partial derivative Γ_s (< 1) defines the rate of change in tomorrow's state variable as we marginally change the today's state variable. As noted in the previous section, $\Gamma_{ss} < 0$. Hence, increasing s increases also the depletion process. Now, if we increase either the valuation of future profits (β), or the rate at which the plant available phosphorus reserves are inherited to next period (Γ_s), the shadow price becomes higher. That is, in choosing the level of fertilization, we are willing to accept it to have a larger negative effect on today's profits (π_x to become more negative). Practically, this means that we will apply more fertilizers if the plant available soil phosphorus reserves are more stable.

Increasing the fertilization increases the plant available soil phosphorus reserves ($\Gamma_x > 0$). The immediate effect of fertilization on crop yield is increasing and concave

($\pi_{xx} < 0$) and also $\pi_{ss} < 0$. According to our assumption: $\Gamma_{ss} < 0$, i.e., the higher the s, the smaller proportion survives to next period. That is, increasing the level of plant available soil phosphorus reserves *lowers the shadow price* at all levels of s.

The second line, (10b), has become a static condition: the land set aside is chosen so as to make the marginal costs and benefits equal. In the private optimization problem the land set aside causes only costs. Hence, $\pi_A < 0$ for all A. Therefore, we know that in the private optimum, $A = 0$ (i.e., the complementary condition holds).

The stationary condition of (10c) guarantees that the plant available soil phosphorus reserves remain unchanged at the optimum. The complementary condition excludes the possibility to increase the acreage under cultivation (i.e., to choose $A < 0$), and to apply negative amounts of fertiliser. The complementary condition always holds; if equations (10a) or (10b) provide negative values for the choice variables, the optimal choice is forced to be zero by the complementary condition.

The conditions (10a)-(10c) capture the notion of the economic steady state. We see that it is not merely a choice of a state which satisfies the condition (10c), and provides the highest stationary profits. Instead, the choice is affected by the valuation of future profits, reflected by the discount factor. The lower the discount factor, the lower profits the farmer is willing to accept in the future as a trade-off from higher profits today. The profits today can be increased by lowering the phosphorus fertilization. This, on the other hand, reduces the future level of plant available soil phosphorus, and hence future profits. Hence, the farmer does not choose an optimal steady state STP value but an optimal steady state fertilization, acknowledging the phosphorus dynamics and her valuation of future profits.

3.3 Comparative statics

Let us now conducting comparative statics on conditions (10a)-(10c). At the private optimum, the choice of VFS acreage is always $A = 0$ and the only decision variable is the level of phosphorus fertilization. We will also use a simplified, linearized version of the transition function: $\Gamma = \chi s + \nu(x - y\delta)$, where δ is the phosphorus concentration of the crop yield, assumed for a moment to be constant, and $y\delta$ is the phosphorus removed by the crops, i.e. the phosphorus output. The partial derivatives of this version w.r.t. s and x remain unchanged due to nonlinearities in crop yield function. By using this version we can detect the effects of the transition function on the optimal solution.

We will vary the following exogenous parameters: the price of output (p), the price of fertilizer (w), the discount factor (β), the fraction of s surviving to the following period (χ), the fraction of the phosphorus balance affecting s in the following period (ν) and the phosphorus concentration of the crops (δ). Table 1 presents the results for phosphorus fertilizer use and plant available soil phosphorus reserves. We present the detailed calculations in Appendix 2a.

Table 1. Comparative statics of the private optimum.

	p	w	β	χ	ν	δ
Phosphorus fertilizer use (x)	+	–	+	**?**	+	+
Plant available soil phosphorus (s)	+	–	+	+	+	–

The results are fairly intuitive. Increasing the price of output (p) increases both the steady state use of phosphorus fertilizer and the level of plant available soil phosphorus reserves. It increases the immediate marginal profits (π_x) as well as marginal profits w.r.t. the level of the plant available soil phosphorus (π_s). From the latter it follows that it increases the shadow price of s (V_s). Looking at condition (9b) we notice that this increase must be outweighed by an increase of the absolute value of π_x. We know that π_x is negative at the optimum and $\pi_{xx} < 0$. Hence, the absolute value is increased by increasing the use of fertilizers. Using similar reasoning, an increase in the input price (w) has an opposite effect.

Increasing the discount factor, i.e. increasing the valuation of future profits increases the steady state fertilizer use. We can also detect this effect directly from (9a and 9b): increasing β increases the shadow price (V_s) and the second term of (9b). It must be balanced with an increase in absolute value of π_x, i.e., an increase in the use of fertilizers.

For the three first variables it is obvious that the direction of change is identical for the optimal phosphorus fertilization and for the plant available soil phosphorus reserves. However, the remaining variables affect the transition function. Therefore, the optimal choice and the optimal state variables do not necessarily move in same directions.

The fraction of plant available soil phosphorus reserves surviving to the following period is captured by (χ). It is intuitive that increasing it increases the steady state level of the plant available soil phosphorus reserves. However, both increase and decrease in optimal stationary fertilizer use are intuitively plausible. Consistent with this, the sign obtained from analytical calculations remains ambiguous (see Appendix 2a). There are two main effects that work in opposite directions. Firstly, the higher the steady state reserves, the more likely it is that an increase in (χ) is associated with an increase in optimal fertilization. The reason for this is probably that at these higher levels a larger phosphorus balance is required to maintain the reserves at this level. Higher reserves mean proportionally higher phosphorus output which must be outweighed with a higher fertilization.

Secondly, the higher the marginal effect of plant available soil phosphorus reserves on (Γ_s) and the larger the (negative) static welfare effect ($py_x - w$), the more it favors the opposite effect, i.e., that an increase in (χ) decreases the optimal fertilization. Increasing fertilization decreases its marginal effect on crop yield and hence increases the negative static welfare effect. This effect is fortified be the overall marginal effect of this period's reserve to the following period's reserves (Γ_s). The higher this effect, the more it enables us to focus on the negative static welfare effect. The combination of these two effects determines whether the total effect will be increasing or decreasing. The magnitude of these effects depends on the curvatures of the transition and crop yield functions; and on the location of the prevailing steady state value on these functions.

The effect of an increase in the phosphorus balance coefficient (v), on the other hand, is positive for both the decision as well as for the stock variable. This is reasonable: at the steady state we have, by definition, positive phosphorus balances. If the effect of this balance on next period's reserves increases, both stationary fertilization and phosphorus reserves increase. The former effect is presumably due to the fertilizer being the sole input determining the phosphorus balance.

Finally, an increase in the parameter defining the phosphorus concentration of the crops (δ) increases the use of fertilizers. However, it decreases the steady state level of plant available soil phosphorus reserves. The reason is fairly obvious: increasing the concentration increases the phosphorus output. The only way to balance this effect is to increase the input, i.e., increase the use of fertilizers. This increase will not be sufficient to increase the level of the phosphorus reserves. Due to the concavity of, for instance, the crop yield function, it seems obvious that we end up on a lower steady state level of *s*.

4 Socially optimal use and abatement of phosphorus

The social planner acknowledges the externalities of crop production. Therefore, her optimization problem differs from the farmer's one. In the following section, we first outline and discuss the elements of the social welfare. Then, we collect these elements into a dynamic programming problem. We derive and discuss the optimal stationary choices and conduct the comparative statics.

4.1 The framework

The social welfare consists of the sum of producers' and consumers' surpluses. Because we assumed fixed prices, the producers' surplus is captured by private profits and the consumers' surplus by the environmental damage. Define the social welfare as:

$$SW = \sum_{t=0}^{\infty} \beta_s^t \left[\pi_t - D_t\right] \tag{11}$$

where SW denotes the discounted sum of private profits, including the costs of environmental protection, (π) net of the monetary value for the environmental damage

(D). Whereas the discount factor for the private farmer was derived directly from the real interest rate, the social planner's choice of appropriate discount rate (β_s) is a more complicated issue. Without discussing this in more detail, we merely state that the social discount rate may be equal or lower than the private discount rate.[5] That is, the social planner should evaluate future welfare at least as much as the private decision maker does.

The phosphorus loss is affected by the level of plant available soil phosphorus reserves (determining the DRP-loss) and the erosion (determining the PP-loss). The magnitude of the DRP-loss can be influenced only by controlling the level of bioavailable soil phosphorus, whereas the PP-loss can be abated by constructing vegetative filter strips (VFS) on filed edges adjacent to water.[6]

Next, we discuss in detail the elements of (11). We start with functions related to the effects that the construction of VFS has on private profits. In the private optimization problem, the farmer had to choose simply the set aside acreage (which was zero). In this section, we specify the choice to be the width of a VFS. To justify this, we shortly present the linkages between the shapes of the parcel, and widths and acreages of VFS.

The acreage and cost function of a vegetative filter strip

The costs for VFS consist of establishment, operation and maintenance costs, and of the opportunity cost of land set aside from production. To define the opportunity cost of land, we first formulate how the VFS acreage is determined by the shape of the field and the width of the strip. We assume that our field parcels are rectangular and we normalize their size to unity. Figure (1) illustrates the idea.

[5] For a thorough discussion, see for instance Hanley and Spash (1993).
[6] We choose an explicit abatement measure already for the analytical model. The cost structures of any 'end of pipe' measures for agricultural nutrient losses are fairly similar: they comprise of opportunity cost and establishment, operation and maintenance costs. Also, throughout the study we use the word 'abatement', even though we do not know its duration: we merely compare the phosphorus losses of two steady states.

Figure 1. Vegetative filter strips.

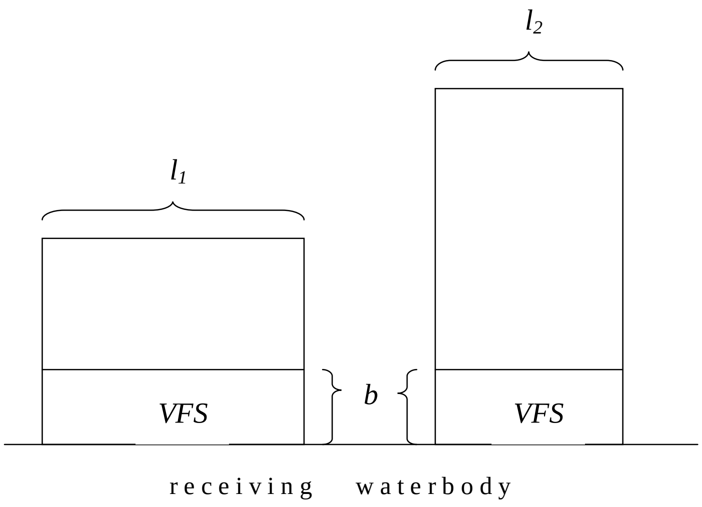


Figure (1) presents two unity-sized parcels, both having an equally wide VFS (b) constructed on them. The length of the field edge adjacent to water is denoted by l_i. The acreage of VFS is given by:

$$A_i = l_i * b_i. \qquad (12)$$

Depending on the shape of the field, the parameters l and b may attain values on the whole real axis, as long as it holds that $A \leq 1$. For a square formed field the acreage is given simply by b.

We have included the shape variation in our model because the width (b) presumably will have at least as important an effect in erosion control as the VFS acreage (A). The costs, however, will depend only on A. This makes the unit abatement costs depend on the shape of the field and the socially optimal allocation of VFS differ across the parcels.

The establishment, operation and maintenance costs of VFS are given by:

$$f = Y + XA(b,l); \quad A > 0 \qquad (13)$$

where:

Y the fixed cost of constructing a VFS

A the VFS acreage

X cost per acreage

The cost function presented in (13) is linear for $A > 0$ and discontinuous at $A = 0$ (since $f(0) = 0$). As soon as the farmer constructs a VFS, she faces the fixed cost Y and the cost per VFS acreage X. For $A > 0$, the partial derivatives are $f_b > 0$; $f_{bb} = 0$; $f_l > 0$; $f_{ll} = 0$.

The phosphorus loss and abatement functions

We define the phosphorus loss to be additively separable in PP- and DRP-losses. The DRP-loss is a function of plant available soil phosphorus reserves and the PP-loss is a function of the field shape, slope and the VFS width:

$$L = \xi L^{PP}(l,\gamma,b) + L^{DRP}(s) \qquad (14)$$

where :

ξ $\in [0,1]$: commensurates PP- to DRP-loss in terms of its bioavailability

L^{PP} the PP-loss function

l the length of the field edge adjacent to water (see figure 1)

γ the slope of the field

b the width of the VFS (see figure 1)

L^{DRP} the DRP-loss function

According to equation (14), the total amount of bioavailable phosphorus loss is attained by adding up the DRP-loss and the PP-loss, where the latter is weighed according to its bioavailability, captured by (ξ). The intensity of DRP-loss is determined only by the plant available phosphorus reserves. We assume: $L_s^{DRP} > 0$ and $L_{ss}^{DRP} > 0$. Hence, the DRP-loss is increasing and convex in the level of plant available soil phosphorus reserves.

The PP-loss function consists of the part determining runoff as a process launched by erosion; the part determining the abatement of VFS; and the constant part capturing the PP-loss in the drainage:[7]

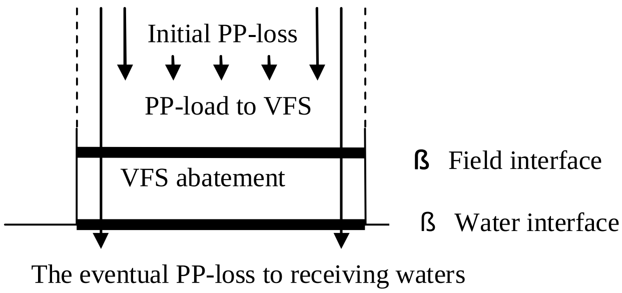
$$L^{PP} = l * r(\gamma,l,b) * z(b,r(\cdot)) + L^{drain}, \qquad (15)$$

where:

$r(\gamma,l,b)$ PP-load entering a point on the VFS
$z(\cdot)$ $\in [0,1]$ the VFS abatement; $z = 1$ stands for zero abatement
L^{drain} PP-loss in the drainage

The phosphorus abatement and the load entering the VFS are interdependent in equation (15). The eventual phosphorus loss is determined by the phosphorus load entering the VFS (r) and the abatement taken place within the VFS (z). On the other hand, the level of abatement (z) is affected by the amount of phosphorus to be abated (r). Let us illuminate the idea with a schematic presentation of the phosphorus loss process:

Figure 2. Phosphorus loss process.



The erosion (and the initial PP-loss) takes place in the field, before the VFS. The field area and the VFS are divided by an interface (Field interface). As we cross the Field interface, no more erosion takes place. The amount of PP entering the VFS is called

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[7] The PP-loss in drainage is assumed constant because it can not be affected with the choice variables in our model. It could be affected by, for instance, increasing the stability of the soil structure. See section 2.2 for discussion.

the PP-load (r). The VFS, located between the Field and Water interfaces, filters eroded materials and thereby reduces PP from the runoff waters. The eventual PP-loss is determined by these processes. All points along the interfaces are assumed to receive (release) an equal amount of PP-load (-loss) due to runoff (after the VFS abatement). Hence, to obtain the PP-loss from the whole parcel, we multiply by the length of the field edge adjacent to water (l).

The form of the function capturing the PP-load entering a single point along the field interface is:

$$r = \frac{r_0(\gamma)(1-A)}{l}, \qquad (16)$$

where:

r the PP-load entering a point on a field interface of a VFS

r_0 the initial PP-loss from the parcel of given slope

The starting point of (16) is the function defining the initial PP-loss from a field parcel (r_0). Each parcel is equally large and we assume that only the slope of the parcel affects the absolute amount of initial PP-loss. The PP-loss is increasing and convex in slope, hence $r_{0\gamma} > 0$ and $r_{0\gamma\gamma} > 0$ (see for instance Rankinen et al 2001).We assume that no erosion (and hence no PP-loss except the constant PP-loss in drainage) occurs within the VFS. To obtain the PP-loss actually entering the VFS, we have to multiply the initial PP-loss by $(1 - A)$, where A is the VFS acreage.

The shape does not affect the absolute amount of PP-loss from a field parcel. Instead, it affects the intensity of PP-load entering a point in the field interface. It is obtained by dividing the PP-loss with the length of the field edge adjacent to water. If the parcel is narrow ($l < 1$) the PP-load entering a point in the field interface is higher than if the parcel is wide ($l > 1$). The PP-load is thus decreasing and convex in the length of the field edge adjacent to water, i.e.: $r_l < 0$ and $r_{ll} > 0$ (see Appendix 3 for closer derivation).

The abatement effect of VFS is twofold: first, it sets land aside from production; the term (1-A) in (16). Therefore, we have: $r_b < 0$ and $r_{bb} = 0$. Second, it filters the eroded materials from runoff and reduces the eventual PP-loss; this is the function $z(b,r(\cdot))$ in (15).

We assume that the VFS abatement function $z(\cdot)$ is increasing and concave in b, i.e. the marginal abatement effect of widening the strip is positive but decreasing. We also assume that $z(\cdot)$ is decreasing in runoff intensity. According to this assumption, doubling the PP-load entering the VFS would reduce the percentage abatement of the strip. On the other hand, we assume that the absolute abatement increases as the runoff intensity increases. That is, doubling the PP-load entering the VFS would increase the absolute amount of particulate phosphorus filtered by the VFS. Acknowledging these and the construction of our abatement function (no reduction: $z = 1$; entire runoff reduced: $z = 0$) the partial and cross derivatives become: $z_b < 0$; $z_{bb} > 0$; $z_r > 0$; $z_{rr} < 0$; and $z_{br} < 0$.

Combining the partial derivatives of the sub functions of the PP-loss function (15) we find the following. PP-loss is decreasing and convex in VFS width (b) and increasing and convex in the slope of the parcel (γ). That is: $L_b^{PP} < 0$; $L_{bb}^{PP} > 0$; $L_\gamma^{PP} > 0$; and $L_{\gamma\gamma}^{PP} > 0$. The partial derivative with respect to l needs some simple calculations (presented in Appendix 3) and an assumption that the VFS width $b > 0$: $L_l^{PP} < 0$; $L_{ll}^{PP} > 0$. That is, the PP-loss is decreasing and convex in the length of the field edge adjacent to water (l). This feature is balanced by the fact that the costs of the VFS are increasing in l. Only in the empirical model we are able to assess which feature dominates, i.e. whether the strips are more efficient on narrow or broad parcels. Also the signs of cross derivatives L_{bl}^{PP}; $L_{b\gamma}^{PP}$; $L_{l\gamma}^{PP} >$ remain indefinite in the analytical model.

The damage function

The damage function ultimately separates the privately and socially optimal choices. It has basically two components: the factual environmental effect of the loss and the economic valuation of this effect. The eutrophicating effect of the phosphorus loss depends strongly on the characteristics of the receiving water body. For instance, in a lake whose algae growth is sensitive to nitrogen, the damage from phosphorus loss might be insignificant. Also, the stock character of phosphorus affects the environmental effect. If, for instance, a phosphorus sensitive lake already suffers from serious eutrophication, the effect of a unit increase in external phosphorus load might be lower than if the lake would be close to a threshold of moving from mesotrophic to eutrophic state. That is, the environmental characteristics make the eutrophication function nonlinear, strongly spatial and in the case of irreversible and unknown changes, even unidentified.

The environmental effect must be evaluated before it can affect decision making. This can be done, for instance, implicitly within the policy making process or it can be estimated with various techniques. In either case, there are substantial uncertainties involved in the evaluation process. The social costs include the monetary value of environmental damage caused by the phosphorus loss. In our deterministic study, we assume that the damage is entirely determined by the bioavailable phosphorus loss defined by (14). Hence, we do not account for the phosphorus stock accumulated in the receiving water body nor its effect on environmental quality. The damage function is of the form:

$$D = \eta(L), \tag{17}$$

where:

D the monetary equivalent for the environmental damage
L the phosphorus loss defined by (14)
η the damage function

We take the conventional approach and assume that the damage (17) is increasing and convex in L, i.e. $D_L > 0$; $D_{LL} > 0$.

4.2 Optimal steady state use and abatement of phosphorus

The social planner has two choice variables: the level of phosphorus fertilizer use and the width of the VFS. The maximization problem of the social planner is given by:

$$\begin{aligned} &\max_{x_t, b_t} \sum_{t=0}^{\infty} \beta_s^t \big[[py(s_t, x_t) - wx_t - C](1 - A(b_t, l)) - f(A(\cdot)) - \eta(L(s_t, b_t, l, \gamma)) \big] \\ &s.t. \;\; s_{t+1} = \Gamma(s_t, x_t), \end{aligned} \tag{18}$$

and the complementary conditions $x,b \geq 0$.

The social planner must choose a VFS width (b) and the level of phosphorus fertilization (x) for every period from this period to infinity, so as to maximize the present value of the sum of social welfare derived from crop production. The social welfare is determined by { $[py - wx - C](1 - A) - f - \eta(L)$}. The producers' surplus is captured by the private profit derived from the crop production: $\pi = [py - wx - C](1 - A) - f$. This includes the costs of constructing VFSs: the opportunity cost of setting land aside from production ($[py - wx - C]*A$) and the establishment costs (f). The consumers' surplus is captured by the environmental damage.

Deriving the optimal stationary conditions for (18) follows the same procedure as in the private problem, except that now we have specified the other choice variable to be the width of VFS. The optimality conditions can be derived from the steady state Bellman equation 19:

$$V(s) = \max_{x,b} \left[SW(s, x, b) + \beta_s V(\Gamma(s, x), b) \right], \tag{19}$$

The value function (V) in (19) is the sum of discounted social welfare from this period to infinity, as (x) and (b) are chosen optimally. At the steady state, the choices and the state remain unchanged between the periods (see section 3.2 for a more detailed presentation). The optimality conditions are derived similarly as in section 3.2 (see also Appendix 1), yielding two Euler equations and the stationary condition, combined with the complementary condition:

$$SW_x(s^*,x^*,b^*)(1-\beta_s\Gamma_s(s^*,x^*))+\beta_s SW_s(s^*,x^*,b^*)\Gamma_x(s^*,x^*)=0 \quad (20a)$$

$$SW_b(s^*,x^*,b^*)(1-\beta_s\Gamma_s(s^*,x^*))=0 \Leftrightarrow SW_b(s^*,x^*,b^*)=0 \quad (20b)$$

$$s^* = \Gamma(s^*,x^*), \quad (20c)$$

with $x,b \geq 0$.

The subscripts in (20a-20c) continue to refer to partial derivatives. Equation (20a) is the Euler equation characterizing the optimal stationary use of phosphorus fertilizer. The Euler equation characterizing the optimal stationary choice of VFS width b is given by (20b). Note that since b is not included in the transition function, its partial derivative (Γ_b) is zero. Therefore, we can drop out the second term ($\beta_s SW_s(s^*,x^*,b^*)\Gamma_b(s^*,x^*)$). Furthermore, assuming that $\beta_s \neq 0$ and acknowledging that $0 < \Gamma_s < 1$ (see section 2.4), the optimal VFS width is found when $SW_b(\cdot) = 0$. This is identical with static optimum: the welfare effect of constructing a marginal VFS has to be zero at optimum. That is, the marginal damage has to equal marginal benefit. The Euler equation has reduced to a static optimality condition.

Let us collect and compare the conditions for private and social optima in a table form:

Table 2. Comparing the steady state optimality conditions.

Private (π)	**Social (SW)**
$\pi_x(1-\beta_p\Gamma_s)+\beta_p\pi_s\Gamma_x = 0$	$SW_x(1-\beta_s\Gamma_s)+\beta_s SW_s\Gamma_x = 0$
$\pi_A = 0$	$SW_b = 0$
$s = \Gamma$	$s = \Gamma$

Technically, the derivation of the optimality conditions was similar to that of the private problem. The interpretation of the conditions, however, differ at some points. At the social optimum, the plant available phosphorus reserves contribute both to private profits and to environmental damage, and have thus opposite effects on social welfare. The Euler condition (20a) states that the marginal effect of the choice variable on social welfare today must be outweighed by the discounted marginal effects of the next period's state variable, evaluated with the shadow price (at the steady state). The respective condition in the private optimum (10a) refers only to marginal effects on private profit. The inclusion of environmental damage thus affects the level of socially optimal phosphorus application. Nevertheless, the direct marginal effect of phosphorus fertilization on social welfare and on private profits are identical: phosphorus contributes to environmental damage only via the plant available soil phosphorus reserves. Therefore, using the same reasoning as with the private optimum, the sign of SW_x must be negative at the social optimum.

Another feature where the social optimum differs from the private one stems from different discount factors. The discount factor β_s chosen by the social planner is equal to or higher than the privately applied discount factor β_p. Other things unchanged, a higher discount factor would mean that the absolute value of the first terms in both sides of the uppermost row in table 2 (or in equations 20a and 10a) would become smaller and the second terms would become larger. Acknowledging the signs of the partial derivatives of SW_x and SW_s (and π_x and π_s), this would mean that s and x would be higher at the social optimum. In other words, if the environmental damage would be zero, the socially optimal level of phosphorus fertilization would be either equal to or higher than the privately optimal level. The fact that the social planner may use a different discount rate in her decision making process might have an opposite effect on optimal solution than her acknowledging of the environmental damage's negative effect on social welfare.

Also, the optimal choice of VFS is not automatically zero alike in the private problem. On one hand VFSs increase social welfare by decreasing the PP-loss and hence the environmental damage. On the other hand, they reduce private profits. Because the optimality conditions were reduced to static conditions, the optimal VFS width is found at $SW_b = 0$, or at $b = 0$. At a social optimum with positive VFS width, the marginal costs from increasing the VFS width equals its marginal benefits; $\pi_b = D_b$. The complementary condition $b = 0$ holds if the marginal cost of first VFS width unit is higher than its marginal benefit; and if the marginal costs are increasing more rapidly than the marginal benefits.

Finally, the heterogeneity of the model is incorporated in the differing slopes and shapes. These do not affect private profits. On the other hand, the social optimum is affected by phosphorus loss and the costs of phosphorus control. Both these are affected by the model heterogeneity. Hence, the privately optimal steady state allocations will be identical, whereas the socially optimal allocations will be dissimilar across the parcels.

4.3 Comparative statics

By conducting analytical comparative statics we can not determine the direction of changes in the socially optimal level of choice variables as the other parameters are varied. Referring to Appendix 2a and 2b, the necessary requirement for determining any direction of changes is the unambiguity of the sign of the denominator when applying the Cramer's rule (as in equation A2.11). Determining this for the analytical social optimum would require several assumptions. Actually, it would require precise definition of the explicit functional forms. In other words, the exercise could be conducted only numerically.

However, we can limit the examination on the changes in the static optimality condition (20b). The phosphorus loss and abatement are new functions in the analytical social optimum and it might be useful to learn about their characteristics. Later, we will examine second-best instruments which treat DRP- and PP-losses as separate phenomena. Therefore, understanding the characteristics of the analytical PP-loss function may prove to be useful.

Due to the interlinkages in conditions (20a)- (20c), analyzing separately the direction of changes in condition (20b) will not necessarily provide identical results than would be obtained be analyzing the conditions as a whole. The cross effects between changes in optimal s and x are here ignored at the outset, as we focus the analysis only on (20b).

We thus examine how the optimal VFS width changes at the optimum as we vary the following exogenous parameters: the price of output (p), the price of fertilizer (w), the slope of the parcel (γ), the bioavailability coefficient (ζ) and the length of the field edge adjacent to water (l). Table 3 presents the comparative statics on the optimality condition $SW_b = 0$, assuming that $b > 0$ (the computations are presented in Appendix 2b):

Table 3. Comparative statics of the simplified optimal VFS choice.

	p	w	γ	ζ	l
VFS width (*b*)	–	+	+	+	–

An increase in the price of phosphorus fertilizer increases the VFS width. This is due to lowering of the opportunity cost of land, while maintaining the damage occurring from PP-loss: now it is worthwhile to reduce the environmental damage by VFSs more than prior to the change in *w*. The output price variation works in the opposite direction: it increases the opportunity cost of land and reduces the optimal use of VFS. Both these effects are intuitive: the more profitable the use of land, the more costly it is to set it aside from production.

Increasing the slope (γ) increases runoff and PP-loss. For a steeper slope the optimal VFS width is increased. Increasing the bioavailability (ζ) has similar effects. It increases the amount of bioavailable phosphorus on a given PP-loss level. Again, it will be optimal to increase the abatement of PP-loss by widening the VFS.

An increase in the field edge adjacent to water (l), which defines the shape of the field, decreases the optimal VFS width. That is, for a broader field, the socially optimal VFS width is smaller. We anticipated this result already in the beginning of the section as we justified the inclusion of the shape parameter into our model.

4.4 Instrument design

If the farmers would be taking the socially optimal actions, there would be no need to change this behaviour. If the socially and privately optimal steady states diverge the social planner may impose instruments (taxes, subsidies, best management practices, etc.) to make the privately optimal behaviour mimic the socially optimal one.

The social planner may use efficiently either input- or emission- (or emission proxy-) based instruments in controlling non-point sources under certainty. The emission-based control is conditioned on a deterministic non-point production function which relates each input combination with an emission estimate, acknowledging the farm characteristics. Typically, the instrument is a Pigouvian tax targeted at this emission estimate. The input based instruments are conditioned on input use. Under perfect information and certainty, the farmers will adopt the desired practices under both instrument bases (Griffin & Bromley 1982).

In the present study, the inputs are phosphorus fertilization and the VFS width. Phosphorus fertilization, however, also determines unambiguously the steady state STP level. Therefore, also the STP can be considered as an input in a stationary framework.[8] Since we know how these affect phosphorus loss, we can determine optimal instruments for each parcel. Because the parcels are heterogeneous, the instruments will be differentiated. In a deterministic world with zero transaction costs, we can always design an instrument scheme which induces the socially optimal outcomes with least possible costs. Such instruments are referred to as first-best instruments.

In reality, however, there are several constraints on designing and implementing the first-best instruments on non-point pollution control. Therefore, the regulator has to use second-best instruments. In general, the notion of second-best refers to welfare maximizing conditions when the Pareto-optimal (first-best) state of the world is no longer attainable due to at least one additional constraint to the original constrained maximization problem (Lipsey and Lancaster 1956). Consequently, the notion of second-best instruments refers to instruments that induce such second-best allocations. In our case, the additional constraint that distorts us to the second-best world might be, for instance, the lack of information on the parcels' shapes.

[8] In agri-environmental literature the plant available soil phosphorus reserves would probably be defined as an emission proxy (see e.g., Griffin and Bromley 1982).

In controlling agricultural nutrient runoff, there are three general features impeding the use of first-best instruments: the heterogeneity of farmland, environmental effects and production technology across production units; asymmetric information between the farmers and the social planner; and the stochastic nature of nutrient transport and fate (see e.g., Horan and Shortle 2001).

Within this study, the heterogeneity manifests itself in the slopes and shapes of field parcels; and it affects the phosphorus loss from these parcels. We also analyse the effects of dynamic characteristics of optimal phosphorus use on instrument design.

A first-best tax-subsidy scheme

We will examine two potential tax-subsidy schemes. In both of these, the subsidy is based on VFS width. The tax is based either on the use of phosphorus fertilizers (x) or on plant available soil phosphorus reserves (s). Before going into the specific schemes, let us define their general features. Under both schemes, the profit becomes $\pi^{\tau} = \pi - \tau(x) + \tau^{s}(b)$, where $\tau(x)$ refers to a tax on either of the bases and $\tau^{s}(b)$ refers to a VFS subsidy.

To adjust a stationary tax-subsidy scheme optimally, we must choose the size of the instruments so that the social optimum (20a) – (20c) and the private optimum (10a) – (10c), including the instruments, become identical. That is, the instruments must make the optimal choices $x*$ and $b*$, as well as the optimal state $s*$ resulting from the optimality conditions below coincide:

$$SW_{x*}(1-\beta_s\Gamma_{s*})+\beta_s SW_{s*}\Gamma_{x*}=0 \qquad \pi^{\tau}_{x*}(1-\beta_p\Gamma_{s*})+\beta_p\pi_{s*}\Gamma_{x*}=0 \quad (21a)$$

$$SW_{b*}=0 \qquad \pi^{\tau}_{b*}=0 \quad (21b)$$

$$s^{*}-\Gamma=0 \qquad s^{*}-\Gamma=0 \quad (21c)$$

The leftward equations in (21a) – (21c) are the conditions for the social optimum and the rightward equations the conditions for the private optimum, including the instruments. Alike with the optimality conditions themselves, we can not find the closed form solutions for the instrument schemes. Instead, we leave the interconnections of the taxes and subsidies implicit and solve the explicit forms only in the numeric examples in chapter 7.

The social optimum defines uniquely the steady state VFS width (b^*), optimal phosphorus fertilization (x^*), and the STP level (s^*) for each parcel. By definition, the first-best instruments are set to induce the private farmer to choose these allocations. Together with the additive nature (no cross derivatives) of the tax-subsidy scheme this means that by leaving the (partial derivatives of) damage function implicit in the equations, we can define the first-best instruments simply by making the pairs of equations in (21a) and (21b) identical. Intuitively this means that given the first-best tax induces the socially optimal s^*, the only way to make the farmers optimization process result in a socially optimal VFS width is to make the pair of equations in (21b) identical. The other way around; assuming the first-best subsidy induces b^*, the only way to make the private steady state optimum to be s^* is to make the pair of equations in (21a) identical. The two equations in (21c) are identical by the definition of the first-best.

Both the tax and the subsidy will be differentiated across dissimilar parcels. We can make certain assessments about the magnitude of the heterogeneity of the first-best instruments already at this point. First of all, the rates of taxes and subsidies will be the marginal payments *at the optimum*. They will thus not tell us anything specific, for instance, about the gross payment on subsidies for a given VFS width. Therefore, we may assume two things about the heterogeneity of the first-best instruments. Firstly, due to the minor anticipated heterogeneity in optimal levels of plant available soil phosphorus reserves, the first-best taxes will probably be close to uniform. Secondly, based on this and the static nature of the optimal VFS width, we should expect to find similar first-best marginal subsidies for *parcels of same shape*. This is because the marginal costs of a marginal increase in VFS width will be close to identical across identically shaped parcels.

Note also that the parametric values of the underlining subsidy functions will be different for each (dissimilar) parcel. Otherwise, due to decreasing average private costs of constructing a VFS (constant opportunity costs and decreasing average costs per unit of VFS width), the first best allocations could not be achieved. On the other hand, the scope of our analysis is only on maintaining the steady state, not achieving it.

A first-best tax-subsidy scheme based on phosphorus fertilization

To adjust the fertilizer-based tax optimally, we must thus choose the size of the fertilizer tax so that the pair of equations in (21a) are identical, i.e.:

$$SW_x(1-\beta_s\Gamma_s)+\beta_s SW_s\Gamma_x=\pi_x^\tau(1-\beta_p\Gamma_s)+\beta_p\pi_s\Gamma_x \qquad (22)$$

where:

π_x^τ the marginal profit w.r.t phosphorus fertilization

β_p the private discount factor

β_s the social discount factor ($\geq \beta_p$).

The marginal profit is thus: $\pi_x^\tau = \pi_x - \tau_x$, where τ_x is the marginal tax on fertilization. The following equalities continue to hold:

$$SW_x = \pi_x \qquad (23)$$

$$SW_s = \pi_s - D_s \qquad (24)$$

Hence, we can write the equation 22 as:

$$\pi_x^\tau=\frac{SW_x(1-\beta_s\Gamma_s)}{(1-\beta_p\Gamma_s)}+\frac{\Gamma_x(\beta_s SW_s-\beta_p\pi_s)}{(1-\beta_p\Gamma_s)} \qquad (25a)$$

$$\tau_x=-\pi_x\left[\frac{(1-\beta_s\Gamma_s)}{(1-\beta_p\Gamma_s)}-1\right]-\frac{\Gamma_x\left[\pi_s(\beta_s-\beta_p)-\beta_s D_s\right]}{(1-\beta_p\Gamma_s)} \qquad (25b)$$

The marginal tax from phosphorus fertilization is defined by (25b). It depends on the transition function (Γ), the profit function (π), the private and social discount factors ($\beta_{p,s}$), and on the damage function (D). Consider first the case where the socially and privately applied discount factors are identical. In this case (25b) simplifies to:

$$\tau_{x|\beta_s=\beta_p} = \frac{\Gamma_x \beta}{(1-\beta\Gamma_s)} D_s \tag{26}$$

The optimal tax based on fertilizer use is a product of two terms. The marginal damage w.r.t soil phosphorus (D_s) is multiplied with the propensity of the fertilizer use to affect the plant available soil phosphorus reserves, as a result of the farmer's optimization process. We see directly certain features from (26). First of all, the (marginal) tax rate is zero if the marginal environmental damage w.r.t. s is zero. Without externalities there is no need to alter the privately optimal choices. We also see directly that the tax is increasing in all its variables (Γ_x, Γ_s, β and D_s). That is, the more the phosphorus fertilization contributes to s in the following period, the higher the tax; and the higher proportion of s survives to the next period, the higher the tax. Increase in the discount factor increases the privately optimal s. This increases phosphorus loss and thereby the environmental damage, i.e. the tax must be increased. And trivially, the higher the environmental damage per unit of phosphorus loss, the higher the tax.

Now, let us examine (25b) under the assumption that the socially applied discount factor is higher than the privately applied, i.e. $\beta_s > \beta_p$. In this case, the signs of the different terms in the marginal tax function become:

$$\tau_x = -\pi_x \underbrace{\overbrace{\left[\frac{(1-\beta_s\Gamma_s)}{(1-\beta_p\Gamma_s)} - 1\right]}}_{+} - \pi_s \underbrace{\overbrace{\frac{\Gamma_x(\beta_s-\beta_p)}{(1-\beta_p\Gamma_s)}}}_{+} + \underbrace{\frac{\Gamma_x\beta_s}{(1-\beta_p\Gamma_s)}}_{+} D_s \tag{27}$$

We have shown earlier that $\pi_x < 0$ at the optimum, hence the sign for the first term. Denoting the terms after the partial derivatives of the profits as M and N, respectively, (27) can be expressed as:

$$\tau_x = \frac{\Gamma_x\beta_s}{(1-\beta_p\Gamma_s)} D_s - (\pi_x M + \pi_s N) \tag{28}$$

Now, the first term in (28) increases the tax rate whereas the second term decreases it. Increasing the discrepancy between the discount factors lowers *M*, i.e. makes it a larger negative number; and it increases *N*. Since $\pi_{x^*} < 0$ and $\pi_{s^*} > 0$, this means that increasing the discrepancy lowers the marginal tax. If the second term outweighs the first one, the tax in fact becomes a subsidy for phosphorus fertilization. That is, in case the environmental damage is low and the discrepancy between the privately and the socially applied discount factors is high, it may be that the privately optimal level of phosphorus fertilization is lower than the socially optimal level. Hence, it must be subsidized to upraise it to the socially optimal level.

This is a somewhat surprising result. We can think that the discrepancy in the discount rates is akin to an externality: it results in socially suboptimal outcomes. If this effect outweighs the other externality, namely the phosphorus loss, the instrument implemented by the social planner must be set to increase the input use.

The marginal VFS subsidy must be set to make the equations (20b) and (9c) equal:

$$SW_b = \pi_b^{\tau} = \pi_b + \tau_b^{s}. \qquad (29a)$$

$$-A_b(py - wx - C) - f_b - D_b = -A_b(py - wx - C) - f_b + \tau_b^{s} \qquad (29b)$$

$$\tau_b^{s} = -D_b \qquad (29c)$$

That is, the marginal subsidy on VFS is equal to the negative of its marginal effect on environmental damage. The complementary condition applies for the subsidy as well: if optimal $b = 0$, $\tau_b^s = 0$. By taking a closer look at D_b we obtain:

$$-D_b = -\frac{\partial D}{\partial L}\frac{\partial L}{\partial b} = -l\xi\frac{\partial D}{\partial L}\left[-r(\cdot)\frac{\partial A}{\partial b}z(\cdot) + \frac{\partial z}{\partial b}\left[r(\cdot)(1 - A(\cdot))\right]\right] = \tau_b^s \qquad (30)$$

Equation (30) tells us that $\tau_b^s > 0$, i.e. it is indeed a subsidy. Its magnitude is affected by various factors, of which some are heterogeneous in our model. The field edge adjacent to water (*l*) defines implicitly the shape of the field, and it affects the subsidy directly as well as via the PP-loss and abatement functions $r(\cdot)$ and $z(\cdot)$. Also, the slope of the parcel (γ) affects *D*. Earlier, we noted that we can not determine the sign of either L_{bl} or $L_{b\gamma}$ from the analytical model. The effects of the partial derivative of damage w.r.t PP-loss, and the affect of the bioavailability coefficient (ζ) on the subsidy are straightforward: increasing them increases the subsidy.

A first-best tax-subsidy scheme based on plant available soil phosphorus reserves

Consider next an alternative scheme, where the tax base is the level of plant available soil phosphorus reserves. In this case the profit becomes $\pi^{\tau} = \pi - \tau(s) + \tau^{S}(b)$, where $\tau(s)$ refers to soil phosphorus tax and $\tau^{S}(b)$ refers to VFS subsidy. That is, the subsidy remains unchanged. Again, to adjust the soil phosphorus -based tax optimally, the conditions (20a) and (10a) must be made equal:

$$SW_x(1-\beta_s\Gamma_s)+\beta_s SW_s\Gamma_x = \pi_x(1-\beta_p\Gamma_s)+\beta_p\pi_s^{\tau}\Gamma_x\,, \qquad (31)$$

where π_s^{τ} becomes: $\pi_s^{\tau} = \pi_s - \tau_s$. Acknowledging for equalities (23) and (24), we obtain after some simple manipulation:

$$\tau_s = \frac{\beta_s}{\beta_p}D_s - \pi_x\frac{\Gamma_s(\beta_p-\beta_s)}{\beta_p\Gamma_x} - \pi_s\frac{\Gamma_x(\beta_s-\beta_p)}{\beta_p\Gamma_x} \qquad (32)$$

Again, we see that the discrepancy in discount factors decreases the level of the tax. Setting the discount factors equal simplifies (32) to $\tau_{s|\beta_s=\beta_p} = D_s$. This is a classical result in environmental economics: at the social optimum, the marginal tax on input must equal its marginal damage.

Let us collect the two first-best taxes in a table format. In table 4, the middle column presents the taxes when the social discount factor is higher than the private discount factor. The rightmost column presents the taxes when the discount factors are equal.

Table 4. Comparing the first-best taxes.

Tax base	$\beta_s > \beta_p$	$\beta_s = \beta_p$
x	$\tau_x = \frac{\Gamma_x \beta_s}{(1-\beta_p \Gamma_s)} D_s - \pi_x \left[\frac{(1-\beta_s \Gamma_s)}{(1-\beta_p \Gamma_s)} - 1 \right] - \pi_s \frac{\Gamma_x (\beta_s - \beta_p)}{(1-\beta_p \Gamma_s)}$	$\tau_x = \frac{\Gamma_x \beta}{(1-\beta \Gamma_s)} D_s$
s	$\tau_s = \frac{\beta_s}{\beta_p} D_s - \pi_x \frac{\Gamma_s (\beta_p - \beta_s)}{\beta_p \Gamma_x} - \pi_s \frac{\Gamma_x (\beta_s - \beta_p)}{\beta_p \Gamma_x}$	$\tau_s = D_s$

Under both tax bases, the tax functions are fairly complicated if the private and social discount factors diverge. They consist of the marginal damage part (D_s) which is weighed with the ratio of the discount factors or with the fertilizer's propensity to affect the plant available soil phosphorus under the farmer’s optimal choice. The marginal damage part increases the level of the taxes. The two latter parts decrease the taxes. The discrepancy in the discount factor affects the optimal tax via the optimization process and via the marginal profit functions. A comprehensive interpretation of the mechanisms of these two latter terms, however, remains blurry.

However, if the discount factors are equal, the tax based on plant available soil phosphorus is fairly simple; as is the tax on fertilizer use. For the latter, both the transition function and the discount factor enters the tax function. For the former, the marginal tax on *s* is simply its marginal damage. Even though this is not important in the deterministic world, it is possible that the differences affect the efficiency of the instrument in the presence of imperfect or asymmetric information because the differences have implications on information and monitoring requirements. We will discuss these in more detail in section 8.2.

Second-best instruments

The first-best instruments of the previous section are differentiated. For dissimilar parcels, and for each socially optimal allocation, the instruments are dissimilar. In practice, we know that the costs of implementing and targeting such instruments are high. It might be that the social planner finds it feasible to monitor only certain but not all characteristics when determining the level of the instruments.

This is the motivation for second-best instruments. They are easier and/or less costly to use. The trade-off is that the first-best optimum can no longer be achieved. In analyzing the efficiency effects of using second-best instruments, we compare these gains and losses.

Within our model, the clear-cut simplification for the taxes is the change from differentiated to uniform taxes. Presumably, the differences will not be large, as the soil type is homogenous. However, there will be some variation because the optimal VFS allocations will affect the optimal fertilization levels, too.

For the VFS subsidy we have more potential variation: we can use uniform command and control instrument, or we can omit certain features from parcel characteristics that affect the PP-loss. For instance, we may analyze neglecting the effect of the shape of the parcel on the subsidy. In this case, the subsidy would be determined only by the width of the VFS and the slope of the parcel. This will necessarily distort us from the first-best. The interesting question is the size of this effect.

Our model poses two limitations on analyzing the efficiency effects. Firstly, under the stationary analysis will can not assess the control costs meaningfully: the path effects are neglected. Secondly, we can not say anything explicit about the gains of the second-best instruments without information about the transaction, and other costs that we avoid by using the second-best instruments. We present the instruments based on the empirical model in chapter 7.

5 Empirical application

We begin this section with an overview of the target area. We then present the functions entering the private and social optimization problems. Crop yield function, transition function and the parameter values for input and output prices enter both the social and private optimization problems. Besides these, phosphorus loss (and abatement) function, damage function and VFS cost function enter the social optimization problem.

For the sake of clarity, we present the functions related to private and social problems separately. We begin with the functions determining the private profit. Then we present those included only in the social welfare. The explicit transition function(s) are presented after these. The numeric values for the model parameters are presented with the associated functions. At the end of this section we collect all functions and parameters in a table format. Throughout the section we will use the Soil Test Phosphorus (STP) as an approximation for the potentially plant available phosphorus reserves.

5.1 Application target

Our target area is very stripped-down. It comprises of separate, equally sized parcels of varying characteristics in terms of shapes and slopes. In our model, those are the characteristics affecting the relative VFS acreage and the susceptibility to erosion. The soil type of all parcels is clay (see Appendix 4 for a precise definition). We assume that the parcels are allocated entirely on barley so that the only decision for the social planner (and for the farmer) is to choose the annual phosphorus fertilization level and the VFS width.

The total acreage of the area is 37 ha. The size is chosen to enable introducing the heterogeneity of the target area in a convenient way. The shares of shapes and slopes in the target area mimic the actual relative shares of Finnish field area. These are adopted from Puustinen et al (1994) who collected information on agricultural characteristics, particularly those related to environmental effects of the agriculture. They provided more detailed data than we use here; we have combined some of their classifications. The relative shares we are using approximate the average Finnish shares quite well.

The first source of heterogeneity is the slope of the parcel. We assume that the slope is uniform across the parcel. Also Puustinen et al (1994) classify more than 60% of all parcels as having a uniform slope. Following their work, the percentage distribution of parcels with respect to their slopes is given table 5.

Table 5. Division of slopes in the target area.

Slope	**<0.5**	**0.5-0.9**	**1-2.9**	**3-6.9**	**>7**
Percentage share	16%	19%	33%	24%	8%

The shapes of the parcels are defined in the same manner. According to Puustinen et al (1994) the share of rectangular parcels is more than 57% of all parcels. The researchers did not differentiate shapes in terms of parcels position towards water, i.e. shape 1:3 is equal to 3:1. We divide the rectangular parcels in two categories: square parcels (1:1) and narrow parcels with field edge ratio equal to 1:3. The latter category includes the parcels whose shape is defined to be 1:2, 1:4 or 1:8. It follows that 68% of the parcels are classified narrow.

Combining the information on shapes and slopes and choosing sensible discrete values for each parcel type provides us with the matrix representing the characteristics of our target area:

Table 6. Target area heterogeneity in terms of slope and shape.

slope	**0.3**		**0.7**		**2**		**5**		**7**		
shape	*1:1*	*1:3*	*1:1*	*1:3*	*1:1*	*1:3*	*1:1*	*1:3*	*1:1*	*1:3*	**Total**
acreage	***2***	***4***	***2***	***5***	***4***	***8***	***3***	***6***	***1***	***2***	***37***

Table 6 tells us, for instance, that the target area has 8 hectares of narrow parcels with slope 2˚. We see that there are altogether 10 different kinds of parcels with respect to their slope-shape combinations. According to our modelling assumptions, these differences will affect the optimal allocations of VFSs.

5.2 The private profit function

The farmer maximizes the sum of discounted profits, as presented in (1). The profit consists of the sales revenue from the crop yield minus the costs of production: $\pi = py(s_t, x_t) - wx_t - C$. Hence, we have to define the crop yield function (y), input price (w), output price (p), the other costs of production (C) and the discount factor.

Our crop yield function captures both the effect of accumulated phosphorus as well as the direct effect of phosphorus fertilizer use in a given period. All other variables affecting crop yield are assumed fixed. Following Myyrä and Pietola (2004), the crop yield is determined by two separate functions, y_s and y_x where the former captures the effect of STP and the latter the direct effect of phosphorus fertilization:

$$y = y_s(s) + y_x(s,x) \tag{33a}$$

$$y = 3367(1 - 0{,}74e^{-0{,}37s}) + \frac{1{,}05x}{s}(12{,}4 - 0{,}132x) + (24 - 0{,}367s)\sqrt{x} + 6{,}97\,, \tag{33b}$$

where:

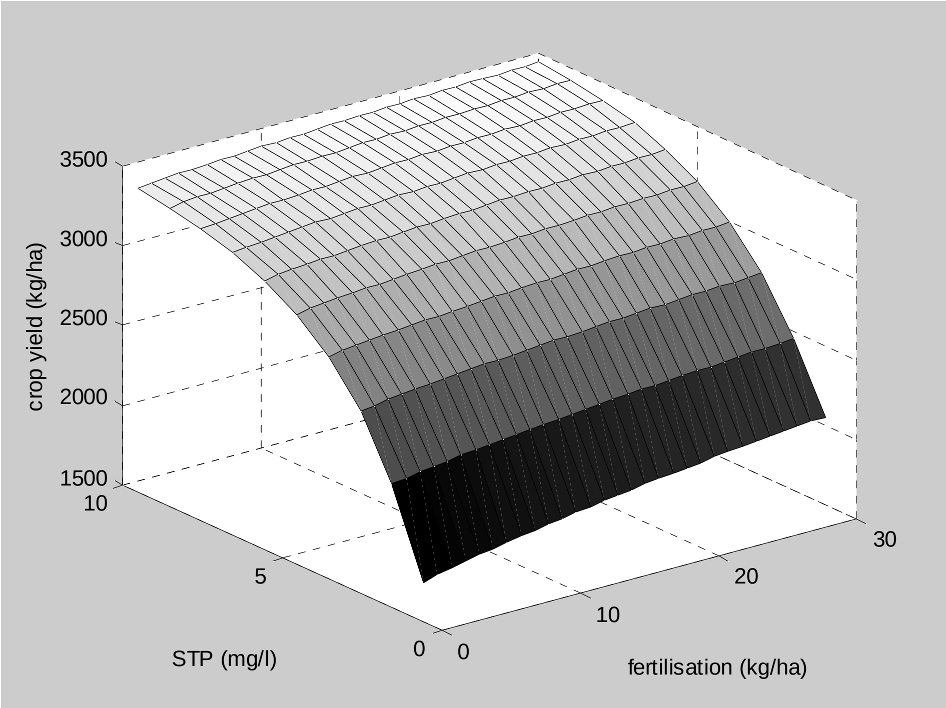
s STP

x phosphorus fertilization

In equation (33b) the first part ($3367(1 - 0.74e^{-0.37s})$), is the so-called Mitscherlich function. It depends only on the STP level (s). It is increasing and concave, i.e., increasing the STP level has a positive effect on crop yield but this effect becomes weaker as STP gets higher. The explicit form is adopted from Myyrä et al (2003). The remaining terms of (33b) comprise the function y_x. It captures the effect of the short-term phosphorus fertilization. This response depends both on the level of fertilization (x) and the level of STP (s). The effect is stronger for soils of low STP. The explicit form for this part is adopted from Saarela et al (1995).

The construction of (33b) is perhaps unconventional. An additive function is not as compact as possible. However, the specification has the desired properties as we can see from the following picture. Figure (3) depicts the crop yields for various STP-fertilization combinations. The vertical axis depicts the crop yield for barley (kg ha^{-1}). The left-hand side horizontal axis depicts the STP levels (mg l^{-1}). The right-hand side horizontal axis depicts the phosphorus fertilization levels (kg ha^{-1}) of the associated period.

Figure 3. Crop yields for fertilization-STP combinations.



We see from figure (3), that the effect of STP on crop yield is more significant than that of the associated period's phosphorus fertilization. Increase in the crop yield associated with a change in STP is larger than the increase associated with higher fertilization. Furthermore, the concavity of the crop yield function w.r.t. STP is more distinct. The effect of fertilization on the higher STP levels is weaker than it is for the lower STP levels. That is, for a high STP value the immediate effect of fertilization on crop yield is smaller than for a low STP value.

We consider a single price for barley (*p*). We use the average Finnish producer price between 2000 and 2005 which was 0.11€ kg^{-1} (Suomen maatalous ja maaseutuelinkeinot 2006).

The choice of the price for phosphorus fertilizer (*w*) is more ambiguous. Phosphorus fertilizer is mostly given in a mixture with other nutrients. Therefore, we have to make assumptions on the incidence of the total fertilizer price on for instance phosphorus, nitrogen and potassium. There is no unambiguous way to do this. One possibility is to scale the amount of nitrogen in the fertilizer mix on a given level and then to regress the (scaled) fertilizer prices against the amounts of phosphorus in the (scaled) fertilizer mix. Such a price estimate has been derived in Myyrä et al (2003) for 2002 prices. We will use their estimate for our default price for phosphorus fertilizer: 1.22 € kg^{-1}.

The constant term (C) includes the fixed and variable costs from tractors, machinery and labor, the costs of nitrogen application as well as seed costs for barley. It also includes a term approximating the social benefits derived from the existence of crop production.[9] Such term is often added into the analysis of agricultural production. For instance Lankoski et al (2007) fix its level to the LFA support (150€). We follow this choice. The other costs are presented in detail in Appendix 5. In our application, the constant cost term becomes: C = 248.6 € ha^{-1}. Mainly due to the inclusion of the LFA support, this term is fairly uncertain. It will not affect the optimal choice of phosphorus directly. Instead, it will have an influence on optimal choice of VFS. Therefore, we will vary the constant cost term and examine its effect on the socially optimal allocations in section 6.2.

For the discount factor we use a default value of 0.95. It is associated with a rate of discount of about 5.3%. For the moment, we assume that the discount factors applied by the farmer and the social planner are identical.

5.3 Phosphorus loss, abatement and damage functions

The social welfare consists of the private profits minus the costs of environmental protection and the monetary value for environmental damage: $SW = \pi(1 - A_t)) - f(A_t) - D(L(s_t,b_t,l,\gamma))$. The costs of controlling the PP-loss comprise of the opportunity cost of land set aside on VFS (πA_t) and its initial, operation and maintenance costs $f(A_t)$. The costs of controlling the DRP-loss comprise of changes in private profits due to changes in socially and privately optimal STP levels.

The VFS acreage function (A) is sufficiently determined in its analytical form (12). Therefore, we need to define here the phosphorus loss (and abatement) functions and to choose the necessary parameter values.

Phosphorus loss and abatement functions

Phosphorus loss is comprised of PP- and DRP-losses, whose analytical forms are determined in equations 14, 15 and 16. The losses are weighted according to their bioavailability.

We start with the function defining the PP-load entering the VFS (16). There, the key element is the function (r_0) which determines the potential PP-loss from a parcel of size one and a given slope. Since the PP-loss is related to erosion, we first have to find an estimate for erosion in terms of the slope.

Taking the average rainfall, soil type, crop choice and other variables affecting the magnitude of erosion as given, the only relevant variable is the slope of the parcel. We use simulation data generated by the ICECREAM model to estimate the relationship between the erosion and slope.[10] The simulation model mimics actual weather conditions and generates erosion data for parcels with varying slopes for a chosen time period. It uses the soil type of our target area (see Appendix 4 for a definition).

We regressed the erosion values against the squares of the slopes (0%,1%,3% and 7%) of the observations from a period of ten years (altogether 40 simulated observations). The soil type in the simulations remained the same, as well as the cultivation practices. Regressing the erosion against the slope squared we obtained a least squares estimate for erosion:

$$te = 174.12\gamma^2, \qquad (34)$$

where:

te the amount of annually eroded material, kg ha^{-1}.

γ slope

---

[9] In Finland, the crop production costs per unit are more than three times as high as the price of output per unit (Suomen maatalous ja maaseutuelinkeinot 2006).

[10] We are thankful for Jussi Lankoski and Markku Ollikainen for the kind permission to use the simulation data produced for the project: “Multifunctional policies in Agriculture”.

The coefficient is statistically significant with t-value 16.49 (standard error 10.56). The regression is conducted without the intercept, whose coefficient proved not to be statistically significant. The R^2 for the regression is 0.87. According to equation 34 the total eroded material is linearly dependent on the square of the slope, with a fairly high coefficient. Even though the simulations include the effect of stochastic rainfall, the slope remains to be statistically significant. Combining (34) and the acreage under cultivation yields the explicit function estimating the eroded material entering the VFS (see figure 2):

$$r = 174.12\gamma^2(1 - A). \tag{35}$$

Equation 35 states that there is no erosion taking place from the VFS itself. Note that in the empirical model, it is sufficient to define the erosion entering the VFS, not just a single point in it, as in the theoretical model (see appendix 6 for discussion).

Next, we define the explicit form of the VFS abatement function $z(b,r(\cdot))$ in (15). In the empirical model, the abatement will be conditioned on VFS width (b) only. We assume that the VFS filters only eroded material from runoff and hence affects only PP-loss, not DRP-loss (see section 2.6 and appendix 6).

In estimating the abatement function, we use parts of the same simulation data as earlier, and data derived from Uusi-Kämppä and Kilpinen (2000). With the help of these data we can calculate various abatement level for VFSs of various widths (see Appendix 6 for a closer description):

$$re = 25.57\ln(b+1). \tag{36}$$

Equation 36 gives us the percentage reduction of all eroded particles. If we can determine the PP content of soil we can use this to estimate the reduction in particulate phosphorus as well. Hence, an important modelling decision is whether the PP content should be made sensitive on fertilization decisions and STP levels.

As discussed in section 2.1 and 2.3, the links between total phosphorus content of soil and STP are not direct, even within a given soil type. Almost certainly, eroded soils of differing STPs will have differences in the desorption processes and effects on phosphorus concentrations in the receiving waters. For instance, Uusitalo et al (2003) report that the amount of bioavailable PP did vary with the reserves of plant available soil phosphorus (in their study assessed with the Olsen method). However, the variation was distinctively milder than that DRP-loss. Modelling the PP-content endogenously would increase the complexity of the model and the uncertainty with the bioavailability of PP-loss would not be remedied by this inclusion.

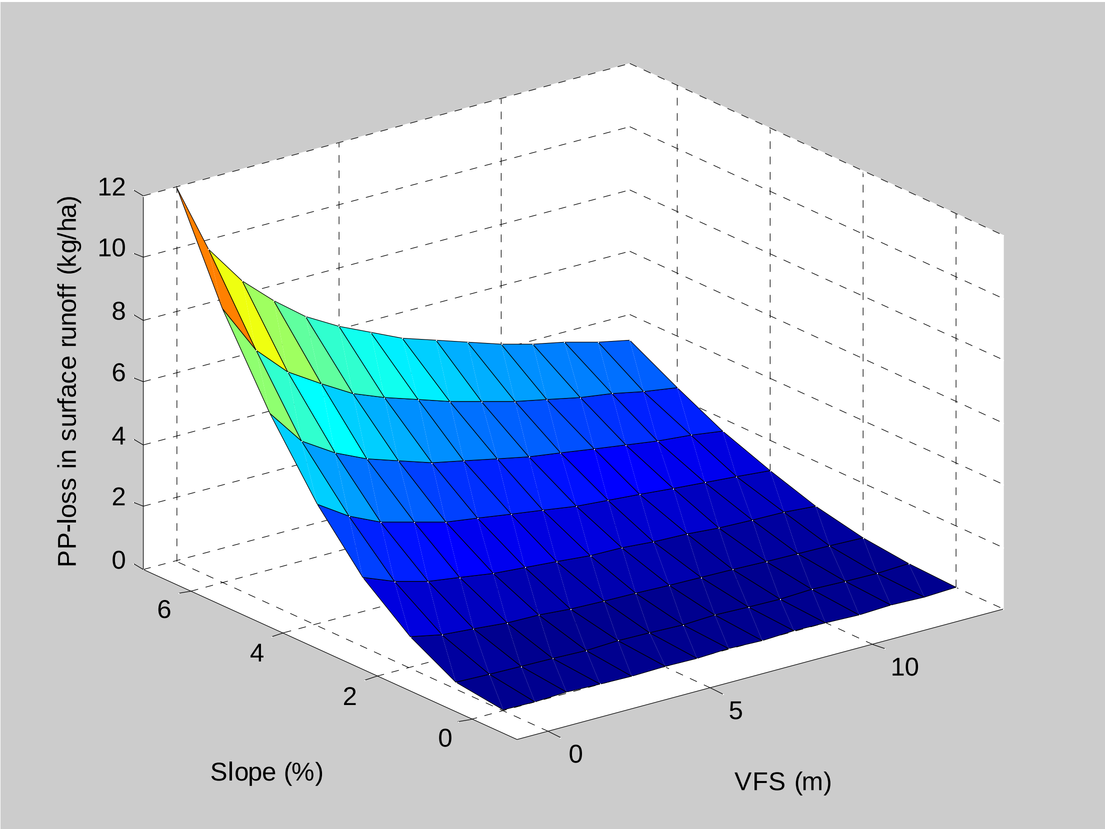
Hence, we keep the analysis as simple as possible and choose a constant value for the total phosphorus content of the soil. During the eight year field experiment of Uusi-Kämppä and Kilpinen (2000), the ratio of particulate phosphorus in eroded materials was on average 0.0014 on the parcels with no VFS, with a fairly modest variation (estimate for standard deviation: 0.0001). That is, one kilogram of eroded soil contained 0.0014 kg of particulate phosphorus.[11] We make the assumption that this ratio holds for all STP values. This assumption is also in accordance with Ekholm et al (2005) who also used a value 0.0014. With this assumption the same reduction percentage derived in (36) for eroded material holds for PP-loss reduction as well. To define the abatement fraction z (see 15), we have to convert the percentage reduction into a fraction that remains unabated: $z = 1 - re/100$. By this step we have completed the explicit definition of PP-loss from the field surface. The function defining the PP-loss from field surface becomes:

$$L^{PP} = 0.244\gamma^2(1-A)\left[1-\frac{25.57\ln(b+1)}{100}\right]. \qquad (37)$$

The first term in equation 37 thus defines the PP-load entering the VFS, determined by the slope of the parcel. The second term determines the fraction of the PP-load that is *not* filtered by the VFS. Referring to derivation of the function 37, it is justified to view the reduction percentage function 36 as the upper limit of VFS abatement. That is, based on our data we believe that filtering of particulate phosphorus can not exceed the percentage value denoted in (36) for a given VFS width. However, we can not say whether the eventual equation 37 over- or underestimates the underlining 'true' PP-loss for given slopes and VFS widths.

Figure 4 illustrates the development of PP-loss in surface runoff as the slope of a square parcel of one hectare and its VFS width are varied. The vertical axis denotes the PP-loss (kg ha^{-1}). The right hand side horizontal axis denotes the width of the VFS (m) and the other horizontal axis denotes the slope of the parcel (%).

Figure 4. PP-loss in surface runoff.



With slope equal to zero, no PP-loss occurs in the surface runoff. Trivially, the VFS has no effect on it. The steeper the parcel gets, the stronger the surface runoff, and hence the PP-loss. The percentage abatement is affected only by the VFS width. For the 15, 8 and 1 meter wide VFS the percentages are 71%, 56% and 17.5%, respectively. Note that the PP-loss depicted in figure 4 is not equivalent to DRP-loss

[11] The ratio on parcels with grass strip – which were initially on a lower STP level – was 0.00095, indicating that the STP does effect the PP-concentration as well.

in terms of its bioavailability. That is, it is not scaled with the bioavailability coefficient yet.

The PP-loss taking place in the drainage is modeled in a very straightforward manner. We use the observed results of eroded material in the drainage flow, reported by Kniesel and Turtola (2000). According to their results, the average amount of eroded material in the drainage was 567 kg ha^{-1} annually[12]. Again, we use the same constant value for the P content in soil as earlier: 0.0014 kg of total phosphorus in one kilogram of eroded material. Therefore, the PP-loss in drainage from a parcel of one hectare becomes:

$$L^{drain} = 0.79. \qquad (38)$$

The unit of L^{drain} is kg ha^{-1} of particulate phosphorus. We thus assume that constructing VFS does not affect the PP-loss taking place in the drainage. In fact, our model does not include any measures to reduce the PP-loss in the drainage. According to our model, the PP-loss in surface runoff is greater than in the drainage for parcels steeper than about 1.8% and smaller for parcels gentler than that. The phosphorus loss there is influenced by changes in the STP, but since it affects only the DRP-loss, the PP-loss remains unaffected.

The PP-loss is made equivalent with the DRP-loss using the bioavailability coefficient. The bioavailability of PP-loss varies according to its source. Ekholm (1998) estimated that the availability of PP in the agricultural surface soil varied between 17% to 24%. However, the STP method considered in the present study (Vuorinen and Mäkitie 1955) partly reflects these labile fractions of PP. In rivers, the availability varied between 0% and 13%, with mean of only 4% (Ekholm 1998). The bioavailability measures also vary depending on the estimation methods. Uusitalo et al (2003) found that 34%-54% of the PP-loss was redox-sensitive, i.e. this share of the PP-loss may be transformed in a bioavailable form in the sediment processes. These processes, on the other hand, are strongly dependent on the characteristics of the

[12] In their results, the variation between the periods is significant: in 1987- May 1991 the average PP-loss in drainage was 258 kg ha^{-1}, and in July 1991-1993 1122 kg ha^{-1}.

receiving water bodies. Ekholm and Krogerus (2003) and Uusitalo (2004) analyze the potential bioavailability of the PP-loss. Based on these studies, Ekholm et al (2005) use a constant bioavailability coefficient of 0.16. We will use this as our default value, i.e. $\zeta = 0.16$.

Estimating DRP-loss on the grounds of STP values is discussed in 2.3. In our model, we use the laboratory results of Uusitalo and Jansson (2002) to capture the DRP-loss:

$$DRP_{concentration} = 0{,}021 * STP - 0{,}015. \qquad (39)$$

The loss is given in mg/l, i.e. as a concentration. We assume that the DRP-concentrations are identical in drainage flow and surface runoff. To come up with the absolute value for DRP-loss we have to multiply the concentration with average annual runoff volume to get the DRP-loss we are interested at. We use the value of 270 mm, adopted from Ekholm et al (2005). Hence, the DRP-loss in kilograms becomes: DRP = 0.0567*STP – 0.0405. The entire phosphorus loss function is summarized at the end of this section.

Damage function

As discussed in section 4.1, the explicit choice of the damage function is a complex task. Our model links the damage to the annual phosphorus loss. This ignores the stock effects which are important in the case of phosphorus. Also, we are not able to include the spatial differences in the damage function. For some receiving water bodies, the phosphorus loss is extremely harmful, while others remain almost intact. And even if the environmental effect could be fully captured, the economic valuation of the water quality remains an open issue.

In choosing the explicit functional form, we have two objectives. Firstly, we want it to be as simple as possible. The aim of this study is primarily to analyse the dynamic decision making and optimal stationary instruments. Effectively, by choosing a damage function, we choose the socially optimal level of phosphorus abatement. On grounds of these, we choose to use a linear damage function.

Secondly, for consistency we want the marginal damage to be in the vicinity of those used in the few related studies. Vehkasalo (1999) uses an measure based on averted expenditures. He estimates that the average cost for phosphorus removal from the sewage is about 25 € kg^{-1}. On the other hand, in Finland's program for the protection of the Baltic sea, the average costs for phosphorus removal from agriculture are estimated to be 40 € kg^{-1} when using the suggested measures (Suomen Itämeren suojeluohjelma 2002). This estimated can be viewed as a simple revealed preference estimate: the government is prepared to accept costs of this magnitude from phosphorus abatement. Both measures are average measures and are based on fairly rough calculations. However, we choose our marginal damage value on grounds of these:

$$D = 30L, \tag{40}$$

where the unit of marginal damage is € kg^{-1} and L is the phosphorus loss in kilograms, as defined in (14).

The vegetative filter strip cost function (f)

The VFS cost function comprises of the investment and management costs. There are some tasks to be done to establish a VFS and the VFS should be harvested regularly. The time and machinery required for VFS management depend on its shape, slope and acreage, and on the chosen management practice. The smaller and narrower the VFS, the higher the management costs per acreage.

There is not much empirical data on machinery and labor requirements for VFS. Therefore, we have to make a few assumptions to come up with our eventual explicit function f. We do, however, have a fairly good understanding on the most important cost element of the VFS: the opportunity cost of land which is already included in (18).

We assume that the investment costs occur once in 15 years and comprise of 4 hours of machinery, 5 hours of labor and seed costs of 55 €. The values are adopted from Koikkalainen et al (1999). For the hourly cost of machinery and labor we use the lowest market price for tractor work, including the driver. In Finland, this was 21 € in

2004 (Pentti and Laaksonen 2005).[13] For the one hour of labor we use a value of 8 €. The initial costs for a hectare of VFS thus becomes 147 €.

The VFSs are harvested by mowing and baling. Palva and Peltonen (2006) provide us with some estimates on time and machinery requirements for such management scenario on two 15 meters wide VFSs. Naturally, the costs per acreage would be very different for narrower strips. However, constructing VFS in the present scale is a fairly new phenomenon in agriculture. Therefore, the actual costs of VFS management are yet by large part unknown. Technology and efficient practices will evolve in the course of time.

Based on Palva and Peltonen (2006), we estimate the time requirement for a hectare of VFS to be 3.5 hours with the tractor plus an hour for preparation. The tractor work includes mowing, baling and transportation. Again, we use the market price for the hourly tractor work: 21 € Combining their findings with the initial costs yields the following VFS cost function for VFSs with $A > 0$:

$$f = 10 + 81.5A \qquad (41)$$

Of course, there are many limitations in the explicit VFS cost function (41). For instance, the operation and maintenance costs would be affected by the shape of the field. This is because it is sensible to assume that for a given VFS acreage, the operation and maintenance costs would be higher for a narrow VFS than for a wide one. Also, the maintenance standards and requirements would practically be different for VFSs of differing widths. However, the direction and the shape of the cost function is correct: the costs are increasing in VFS acreage, but the average cost per acreage is decreasing (due to the constant term).

[13] We assume that the lowest price offer is the best approximation for the true costs of VFS management. The average price was 25.2€.

5.4 The transition function

To fully capture the soil phosphorus dynamics is obviously too mighty a task for an interdisciplinary study. Even within the discipline of soil sciences the long term development of the STP level is a controversial issue. In the following we derive and present the transition function used most of the time, based on Saarela et al (2004). At the end of this section, we present an alternative transition function by Ekholm et al (2005). As discussed in section 2.1, the studies above provide substantially different results on this issue.

The transition function we will use most of the time is derived from Saarela et al (2004). The core elements of the function are the phosphorus fertilizer application and the crop yield function (33b). We present the function piecewise, starting from the definition of phosphorus surplus and the empirically estimated equation for STP development under zero surplus, proceeding to the actual transition equation. To clarify notation we neglect time subscripts when possible. Equation 42 depicts the definition of phosphorus surplus:

$$P_{surplus} = x - \delta(s)y(s,x), \tag{42}$$

where $\delta(s)$ is the relative phosphorus content of the crop yield (y). Equation 42 states that phosphorus fertilization (x) has threefold effects on phosphorus surplus. Firstly, it is the sole phosphorus input. Secondly, it has a dynamic effect on STP (s), which determines the crop yield together with the phosphorus fertilization. Thirdly, the STP influences also the relative phosphorus content.

If we would consider the entire soil phosphorus dynamics, there would be more phosphorus inputs and outputs than those presented in (42). Equation 42 acknowledges only the phosphorus uptake by seeds. The phosphorus uptake of other parts of the plant is also important (see section 2.1 for a discussion).

The transition functions in the present study have been derived by acknowledging only the phosphorus in seeds. For consistency we must follow the same logic in estimating the phosphorus surplus. We thus neglect all other potential plant available phosphorus inputs and outputs. The surplus is thus defined as the difference in phosphorus fertilizer use as input and phosphorus removed with harvest as output.

The phosphorus concentration of crop is estimated by regressing the concentrations against the (logarithm of) STP values. The data (20 observations) is obtained from Saarela et al (1995). The least squares regression provides the following function:

$$\delta = 3.49 + 0.216\ln(s), \tag{43}$$

where s refers to STP value and the concentration is given in per mille. The relative phosphorus content is increasing and concave in STP. For instance with STP level 7 it is 3.91‰, and with STP of 12 it is 4.03‰. The standard errors are 0.04 for the constant term and 0.016 for the slope coefficient. That is, the coefficients are statistically highly significant (t-values 85.4 and 13.4, respectively). The R^2 for the regression is 0.91.

Applying a constant zero phosphorus balance alters the STP value. This phenomenon was discussed in sections 2.1 and 2.3. To assess how the STP evolves with zero phosphorus, we use an equation estimated in Saarela et al (2004):

$$s^0_{t+12.5} = -0.0031 s_t^2 + 0.887 s_t + 0.12, \tag{44}$$

where $s^0_{t+12.5}$ denotes the STP after 12.5 periods with constant zero phosphorus balance. Equation 44 represents the relationship of initial and final STP values after 9-15 years of zero phosphorus balance; thus applied phosphorus fertilizer outweighing exactly the phosphorus uptake of crop seeds. The equation 44 is increasing and concave in current period's STP. Above values of about unity it states that at zero balance the final STP is lower than initial STP at all initial levels. The peculiar result of increasing STP with zero balance at current STP values less than about one is due to extrapolation: the range of the STP values for which the estimate was made did not cover the very lowest values of STP. Essential for us is that in our range of analysis, we need to have a strictly positive P balance in order to increase the STP.

The function estimating the annual change in STP is:

$$s_{t+1} = s_t + \frac{[\Delta s(surplus)] + s^0_{t+12} - s_t}{12{,}5} \qquad (45)$$

where:

$$\Delta s(surplus) = (0{,}0013 + 0{,}001325 * s_t - 0{,}0000161 s_t^{\,2}) * P_{surplus} \quad . \qquad (46)$$

Equation (45) provides the desired equation to estimate the next period's STP as a function of the current STP and the phosphorus surplus. It consists of (44) and equation (46) from Saarela et al (2004).

We have to take into account the decrease of STP at zero balance which has to be offset by (46). The decrease is computed by subtracting the STP value at zero balance from the initial STP. For example with initial STP at 5 mg dm^{-3} this decrease is 0.12 in 12.5 years. Thus the increase (decrease) in STP due to phosphorus balance is outweighed (strengthened) by the natural change in STP. Dividing this change with 12.5 years we get the average yearly change. Adding this to s_t yields s_{t+1}. This transition function is graphically illustrated together with the alternative transition function in figure 5.

The alternative version of the transition function is provided by Ekholm et al (2005), who define the relationship between soil phosphorus in any particular period t and initial STP s_0 to be:

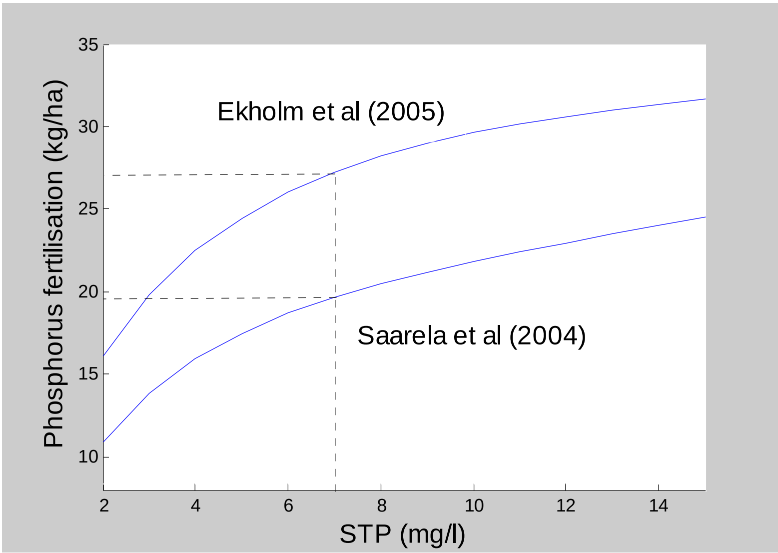
$$s_t = s_0 + (0.00084 * s_0 + 0.0032) * P_{surplus} * t - 0.0184 * s_0 * t, \qquad (47)$$

where $P_{surplus}$ is defined as in (42).

To view the properties – and differences – of (45) and (47) we depict the associated null isoclines. These capture the STP-phosphorus fertilization combinations which maintain the STP at an unchanged level. Figure 1 presents these isoclines for the functions derived from Ekholm et al (2005) and Saarela et al (2004).

In figure 5 the horizontal axis denotes the STP level. The vertical axis denotes the annually applied, constant amount of phosphorus fertilizer. The isoclines depict the fertilizer-STP combinations for which the STP remains unaltered. These levels correspond to possible steady state fertilizer-STP combinations.

Figure 5. Null isoclines for STP.



The vertical dashed line in figure 5 picks up one of these combinations. It crosses the horizontal axis at 7, and hence shows the level of annual fertilization required to maintain the STP level at 7. According to the transition function derived from Saarela et al (2004), maintaining this STP value would require applying about 20 kg of phosphorus fertilizer annually. According to Ekholm et al (2005), on the other hand, maintaining the STP value of 7 would require about 27 kg of annual phosphorus fertilization. It is obvious that these differences have a substantial effect on our results.

Apparently, the transition function presented in Ekholm et al (2005) takes a stronger stance towards the so-called hysteresis of soil phosphorus. The hysteresis states that the potentially plant available soil phosphorus is gradually transformed into more stable forms. The chemical bonds are assumed to become stronger in the course of time. Hence, even without crop uptake, a part of STP is ‘depleted’ by soil phosphorus processes. Therefore, the isocline derived from Ekholm et al (2005) is on a higher level for all values of STP. That is, according to Saarela et al (2004) a lower level of annual phosphorus fertilization is needed to maintain the STP at an unchanged level.

We also see from figure 5 that both isoclines are increasing and concave. Increasing the STP level has twofold consequences. It increases the crop yield and it increases the level of phosphorus surplus needed to maintain the STP level. The crop yield determined by (33b) is increasing and concave in STP (see figure 3). This together with the change in phosphorus concentration of the crops (42 and 43) increases the phosphorus uptake. The concavity of the isoclines suggests that as the STP increases, this uptake by crops is increasing only moderately in STP. A relatively larger part of the phosphorus fertilization may contribute to the higher phosphorus surplus required by a higher STP. This is the interpretation for the concavity of the isoclines.

Finally, we collect our explicit functional forms in a table. The upper part presents the social welfare function and its sub functions; the lower part the transition functions and their sub functions.

Table 7. The explicit functions and parameters of the empirical application.

The social welfare function: $[py(s,x) - wx - C](1-A) - f(A) - \eta L(s,b,l,\gamma)$	
Crop yield (kg ha^{-1})	$y = 3367(1-0.74e^{-0,37s}) + \frac{1.05x}{s}(12.4-0.132x) + (24-0.367s)\sqrt{x} + 6.97$
Input price for phosphorus (€ kg^{-1})	w = 1.22 € / kg
Output price (€ kg^{-1})	$p = 0.11$
VFS costs (€ ha^{-1})	$f = 10 + 81.5A$
Phosphorus loss (kg ha^{-1})	$L = \zeta L^{PP}(l,\gamma,b) + L^{DRP}(s)$
PP-loss (kg ha^{-1})	$L^{PP} = \xi\left\{0.244\gamma^2(1-A)\left[1-\frac{25.57\ln(b+1)}{100}\right] + L^{drain}\right\}$
Bioavailability coeff.	$\zeta = 0.16$
PP-loss, drainage (kg ha^{-1})	$L^{drain} = 0.79$
DRP-loss (kg ha^{-1})	$L^{DRP} = 0.0567s - 0.0405$
Damage (€ kg^{-1})	$D = 30L$
Fixed costs (€ ha^{-1})	$C = 248.6$
The transition function	
Phosphorus surplus (kg)	$P_{surplus} = x - \delta(s)y(s,x)$
Phosphorus concentr. (‰)	$\delta = 3.49 + 0.216\ln(s)$
Saarela et al (2004)	$s_{t+1} = s_t + \frac{\left[(0.0013+0.001325s_t-0.0000161s_t^2)P_{surplus}\right]+s_{t+12}^0-s_t}{12.5}$
Ekholm et al (2005)	$s_t = s_0 + (0.00084s_0 + 0.0032)\,tP_{surplus} - 0.0184\,ts_0$

6 Optimal phosphorus use and vegetative filter strip allocation

In this chapter we present both the privately and socially optimal choices of phosphorus fertilization and VFS-width for each parcel type in the target area. We begin with the private optimum. Then we examine how changes in input and output prices and the discount factor affect the optimal solutions. Also, we examine the effect of the transition function. After this, we present the social optimum in a similar fashion. Unless otherwise mentioned, the results will be based on the transition function derived from Saarela et al (2004)

6.1 The private optimum

Table 9 presents the privately optimal steady state phosphorus fertilization and the STP levels, and the associated steady state outcomes: the crop yield, the profits and the phosphorus loss from each parcel type. The phosphorus loss is a sum of DRP-loss and the bioavailable fraction of the PP-loss. Each entry in the table is a value of the variable given in the leftmost column for the associated parcel type.

Table 8. The private optimum.

Slope	**0.3**		**0.7**		**2**		**5**		**7**	
Shape	*1:1*	*1:3*	*1:1*	*1:3*	*1:1*	*1:3*	*1:1*	*1:3*	*1:1*	*1:3*
Fertilization (kg ha^{-1})	20.4	20.4	20.4	20.4	20.4	20.4	20.4	20.4	20.4	20.4
STP	7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9
Crop yield (kg ha^{-1})	3363	3363	3363	3363	3363	3363	3363	3363	3363	3363
Profits (€ ha^{-1})	96.4	96.4	96.4	96.4	96.4	96.4	96.4	96.4	96.4	96.4
Phosphorus loss (kg ha^{-1})	0.54	0.54	0.55	0.55	0.69	0.69	1.51	1.51	2.45	2.45

The annual steady state phosphorus fertilization is 20.4 kg ha^{-1} for all parcels. This is associated with a steady state STP value of 7.9. The annual crop yield associated with these is 3363 kg ha^{-1} and the annual profit equal to 73.5 € ha^{-1}. The information value of the steady state profit within this study is limited due to its stationary nature. However, we report it in some cases, as well as the stationary crop yields.

The heterogeneity in our model (slope and shape) does not affect crop yields or costs in our model. Therefore, all parcels have identical privately optimal solutions, only the associated phosphorus losses are different. These differences, on the other hand, are substantial. The three most gentle parcels have phosphorus losses fairly close to each other. However, the phosphorus loss from the parcel with slope 2% is already about 28% higher than from the most gentle parcel. For the steepest parcel, the loss is more than four times higher than from the gentlest one. The differences stem from dissimilar erosion intensities between parcels with different slopes. The phosphorus loss from the whole target area is 36.4 kg.

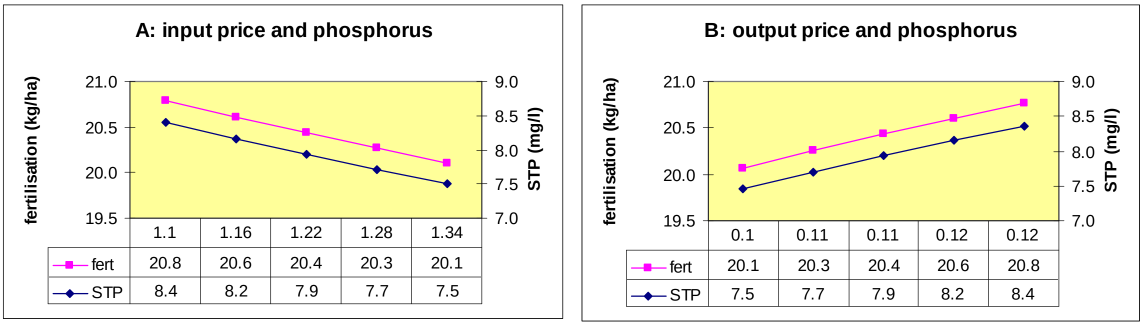
In chapters 3 and 4, we conducted already the analytical comparative statics. Now, we will do a similar exercise numerically to analyze the magnitudes of the effects of parameter variation on optimal allocations (fertilization and STP). If the effects on outcomes (crop yields and profits) are particularly interesting we will discuss these as well.

Variation in prices

We calculate the optima using 5 different input prices: the default value, a 5% decrease, a 10% decrease, a 5% increase and a 10% increase from the default input price. Similarly, we have analyzed the effects of identical percentage variations in the output price. We report the results for both price variations side by side.

Figure 6 depicts the optimal steady state phosphorus fertilization and the associated STP levels for different input (panel A) and output (panel B) prices. The rows under the graphs depict the associated numeric values. The leftward vertical axes depict the values of fertilization (kg ha^{-1}), whereas the STP values are given in the rightward vertical axes. The scales between the axes are dissimilar. They are, however, identical between the panels A and B. In the horizontal axis in panel A we have the values of input prices for which the steady states are calculated: 1.1 € kg^{-1}, 1.16 € kg^{-1}, 1.22 € kg^{-1} (the default value), 1.28 € kg^{-1} and 1.34 € kg^{-1}. In panel B, the values for output prices are given in the horizontal axis: 0.1 € kg^{-1}, 0.105 € kg^{-1}, 0.11 € kg^{-1} (the default value), 0.116 € kg^{-1} and 0.12 € kg^{-1}.

Figure 6. Price variations and phosphorus.



In conformity with the analytical model, the graphs show that increasing the input price reduces the steady state fertilization and lowers the steady state STP. Increasing the output price has an opposite effect.

The effects of input and output price variations on optimal steady state allocations are approximately of the same order of magnitude. Varying the default input price from -10% to +10% lowers the steady state fertilization (20.4 kg ha^{-1}) by 3.3% and the STP value (7.9) by 10.8%. The increase associated with a similar variation in output price is 3.5% for fertilization and 12.1% for the STP.

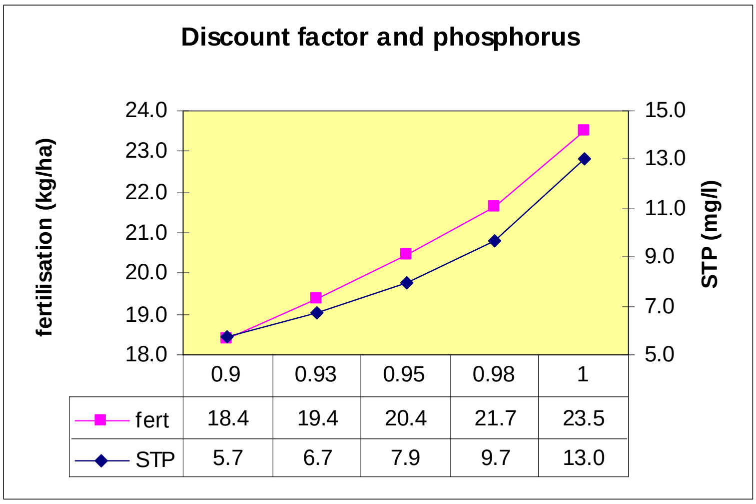
Since the price variations affect the optimal use of phosphorus, they will affect the crop yields and profits as well. The steady state crop yield associated with the highest input price (1.34 €) was 0.7% lower than initially. For the lowest price (1.10€) it was 0.6% higher. The percentage changes associated with the output price variation were 0.7% for the lowest price (0.10 €) and 0.6% for the highest (0.12 €).

A change in the input price changes the private profits only slightly: a 10% increase in the input price decreases the steady state profit by 6.0%. A similar change in the output price, however, increases the profits by about 52.8%. The reason for this is that the fertilizer costs are only one part of the overall costs which are left intact as the fertilizer price is varied. On the other hand, the sales revenues are the only source of returns. Therefore, varying the output price changes the profits substantially.

Discount factor

In figure 7 we display the optimal steady state fertilization and STP levels for 5 different discount factors: 0.9, 0.925, 0.9, 0.975 and 1 (corresponding discount rates: 11.1%, 8.1%, 5.3%, 2.6% and 0%, respectively). Discount factor equal to unity means that the farmer does not discount the future profits at all. The leftward vertical axis depicts the level of annual phosphorus fertilization (kg ha^{-1}). The rightward vertical axis depicts the STP level associated with this optimal steady state fertilization. The horizontal axis denotes the values of the discount factor. The numeric values for the examined variables at the respective steady state optimum are under the horizontal axis.

Figure 7. The discount factor and phosphorus.



With a discount factor 0.9 the optimal steady state phosphorus fertilization is 18.4 kg ha^{-1} which corresponds to an optimal steady state STP level of 5.7. With a default discount factor 0.95 the optimal fertilization is over 20 kilograms per hectare and the associated steady state STP level is 7.9. With a discount factor 1 the farmer considers only the edaphic dynamics and chooses an annual level of fertilization (23.5 kg) which corresponds to a significantly higher STP value: 13.0.

The results underline the importance of the role of the economic, dynamic decision making on the optimal choices. We discussed this already in the two-period example in section 3.1. Phosphorus fertilization can be considered mainly as an investment for the future which – like all investment decisions – is affected by the discount factor used in the decision making process.

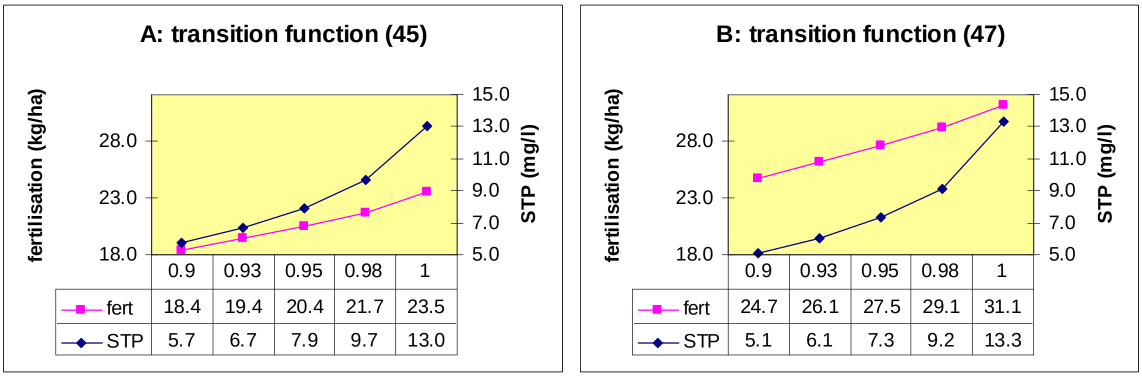
Again, the profits and crop yields are equally affected by the above variations, according to the new optima in phosphorus use. As we increase the discount factor, for instance, from 0.95 to 0.975, the steady state crop yield increases by 1.8%. Similarly, the steady state profit per hectare increase by about 7.0%. That is, even though there are no changes in price ratios or in production technologies, the differences in the discount factor of the decision maker affect the steady state outcomes. Lower discount rate induces the farmer to make decisions that increase her steady state profits. This effect, be it surprisingly significant, is nevertheless logical. Lower discount rate makes investments on soil more favourable. Investing in soil phosphorus status increases the steady state profits.

The effect of the transition function

Finally, let us examine how the above presented results would change if we were to use the alternative function to describe the development of the STP. Particularly, we want to study how the effect of the discount factor variation on optimal steady state allocations and outcomes will change when using the alternative transition function 47. To keep a given STP level unaltered, the transition function (47) requires higher annual fertilization than the transition function (45) (see also figure 5).

We first present side by side the steady state fertilization and STP levels sustained by the different transition functions. Panel A uses the (default) transition function (45) derived from Saarela et al (2004), and presents the privately optimal allocations for various discount factors. In panel B, we have a similar analysis using the transition function (47) by Ekholm et al (2005). In both panels the vertical axes on left denote the annual phosphorus fertilization (kg ha^{-1}). The vertical axes on right depict the STP levels at the various steady states. The horizontal axes depict the discount factors used while deriving the optimal steady states and the boxes under the horizontal axes depict the numeric values of the examined variables at the corresponding steady states.

Figure 8. The discount factor and phosphorus for different transition functions.



A: transition function (45)

	0.9	0.93	0.95	0.98	1
fert	18.4	19.4	20.4	21.7	23.5
STP	5.7	6.7	7.9	9.7	13.0

B: transition function (47)

	0.9	0.93	0.95	0.98	1
fert	24.7	26.1	27.5	29.1	31.1
STP	5.1	6.1	7.3	9.2	13.3

The comparison is interesting. The fertilization levels are approximately 32% - 35% higher if we use the transition (47) than if we use (45). Surprisingly however, the STP levels are fairly close to each other in both cases. The steady state STP using (47) is about 10% *lower* at the lowest level of discount factor. With discount factor 1, the STP is about 2% *higher* than when using (45). At the default discount rate (discount factor 0.95), the steady state STP associated with (47) is about 8% lower.

Based on this comparison we might conclude that the optimal amounts of phosphorus fertilization are very sensitive towards the scientific modeling choices of edaphic processes. However, the optimal STP levels are not as strongly affected. They seem to be more sensitive on the economic behavior of the decision maker, in particular on the appreciation of future profits, captured by the discount factor. This has implications towards instrument design. This will be discussed in more detail in chapter 7.

Acknowledging the above results, the comparison between the steady state returns is fairly anticipated. The crop yields are very close to each other. Because of the differences in optimal phosphorus fertilization levels, the profits differ more than crop yields. With the discount factor 0.95, the profits per hectare when using the transition function (47) are about 13.2% lower than when using the transition function (45).

6.2 The social optimum

The socially optimal allocations are the steady state solutions for the optimization problem (18). A parcel specific optimal allocation comprises of the steady state levels of annual phosphorus fertilization and STP, and the VFS width. We first present the social optimum and discuss its properties. Then, we vary some of the parameter values and examine the changes in the social optimum on a selection of parcels.

We present the results for the whole target area in table 10. In addition to the private optimum presented in table 9, we have new variables to report. Table paasiviki4 reports both the optimal VFS width (m) for a parcel, as well as the respective VFS acreage (ha). The crop yield is reported per parcel, i.e., the land allocated for the VFS is reduced from the crop yield per hectare. Profits denote private profits including the VFS costs. Phosphorus abatement is the percentage reduction from the privately optimal level of phosphorus loss from the corresponding parcel.

Table 9. The social optimum.

Slope	**0.3**		**0.7**		**2**		**5**		**7**	
Shape	***1:1***	***1:3***	***1:1***	***1:3***	***1:1***	***1:3***	***1:1***	***1:3***	***1:1***	***1:3***
Fertilization (kg ha^{-1})	19.69	19.69	19.69	19.69	19.69	19.69	19.67	19.67	19.64	19.64
STP	7.02	7.02	7.02	7.02	7.02	7.02	6.99	6.99	6.96	6.96
VFS width (m)	0	0	0	0	0	0.2	3.6	6.8	8.1	14.0
VFS acreage (ha)	0	0	0	0	0	0.001	0.04	0.04	0.08	0.08
Crop yield (kg parcel^{-1})	3313	3313	3313	3313	3313	3308	3190	3180	3040	3040
Profits (€ parcel^{-1})	91.8	91.8	91.8	91.8	91.8	81.5	75.3	74.8	67.3	67.3
Phosphorus loss (kg ha^{-1})	0.49	0.49	0.50	0.50	0.64	0.63	1.05	0.93	1.24	1.02
Phosphorus abat. (%)	9.6	9.6	9.3	9.3	7.5	8.7	30.3	38.7	49.1	58.3

We see from table 10 that the socially optimal STP level is throughout the target area approximately 7. For half of the parcel types, the optimal VFS width is zero, i.e., the socially optimal steady states do not include constructing any VFS on these parcels. For parcels with slopes 5% and 7% there will be a VFS in the optimum, as for the narrow parcel with slope 2%. The optimal width of the strip is influenced by the shape of the parcel. For the square parcel of 5% there will be a strip whose width is 3.6 meters. For the narrow parcel (1:3) the optimal width is 6.8 meters. For the steepest parcel (7%) the associated widths are 8.1 and 14.0 meters. As discussed in section 4.1, the reason for this is that for the narrower parcels the acreage set aside from production is smaller per unit of VFS width. Hence, the cost of a unit of VFS will be lower. The VFS acreages presented in table 10 underline this feature; the differences in optimal VFS acreages are minor.

The moderateness of the variation in optimal STP levels is notable. The maximum difference in the optimal levels for various parcel types is about 0.06 (0.9%). This is interesting because at the same time the differences in optimal VFS width are substantial. That is, even though it would be possible to substitute VFSs for lower STP values and vice versa, it is not optimal to do so.

To discuss why the wide VFSs are not compensated more intensively with higher STP values consider, say, the optimal allocation associated with the square parcel of slope 7% (STP level 6.96 and VFS width 8.1 meters). We might think that increasing the width of VFS would enable us to increase the STP level and thereby private profits while keeping the phosphorus loss constant. However, increasing the STP level a little would call not only a wider the VFS but also a more expensive VFS *per acreage*. That is, an increase in STP would increase the opportunity cost of land and increase incentives to reduce VFS, not increase.

Hence, increasing the STP could be offset with a wider VFS, in terms of phosphorus loss. However, a higher STP would make a acreage unit of VFS more expensive. Therefore, we observe only slight changes in the STP levels between the solutions including and not including the VFSs.

This is one of the key results of the present study. Firstly, constructing VFSs concentrates optimally on steeper parcels. Secondly, the differences in optimal STP levels are insignificant between the parcels. Later, we will see that these results hold fairly well also as we vary the marginal damage and thereby the abatement requirements.

The crop yields in table 10 are lowered from the private optimum due to the lower STP level and phosphorus fertilization. Also, the land allocated on VFSs reduces the crop yield per parcel. The private profits per parcel are diminished for the same reasons: at the social optimum, the STP level is at a lower level than would be privately optimal. Constructing the VFSs further reduces the profits. The discrete reduction in profits is due to the fixed cost of constructing the VFS (10€).

The phosphorus losses reported in table 10 are lower than at the private optimum for each parcel. Both the absolute and relative abatement is the highest for the steepest parcel. The percentage abatement is higher for the gentlest parcels than for the parcel of slope 2%.This is due to very low initial losses occurring from the gentle parcels. We also note that the shape of the parcel affects the socially optimal phosphorus loss, and abatement. The initial losses from parcels with identical slopes are identical. However, the absolute and percentage abatement from the narrower parcel is higher than from the square parcel.

The target area comprises of the parcel types presented in tables 9 and 10. If we apply the results for the whole target area (as defined in section 5.1) we see that the annual phosphorus loss associated with the private optimum is 36.4 kg. Under the social optimum it is 26.1 kg which corresponds to an 28% abatement. The total crop yield under the private optimum is 124 440 kg. The social optimum is associated with a crop yield of 120 550 kg, a reduction of about 3.1%. Altogether 0.60 ha is allocated on VFS, i.e. about 1.6% of the total acreage.

Analyzing the variations

Next, we examine the sensitivity of the social optimum on certain parameter values. Recall that we were not able to say much about this using the analytical comparative statics. We will vary the following parameter values: the fixed costs of production (C), the marginal damage (η), the bioavailability of the PP-loss (ζ) and the discount factor (β). We will also compute the social optimum using the alternative transition function.

We will illustrate the results in a similar fashion for the rest of this section. Each examination is condensed into a figure which will be a collection of one, two or four panels, depending on the focus. For each panel we will present two socially optimal steady state allocations or outcomes, derived using different values for the variable of interest. We will also report the numeric values of the variables under scrutiny.

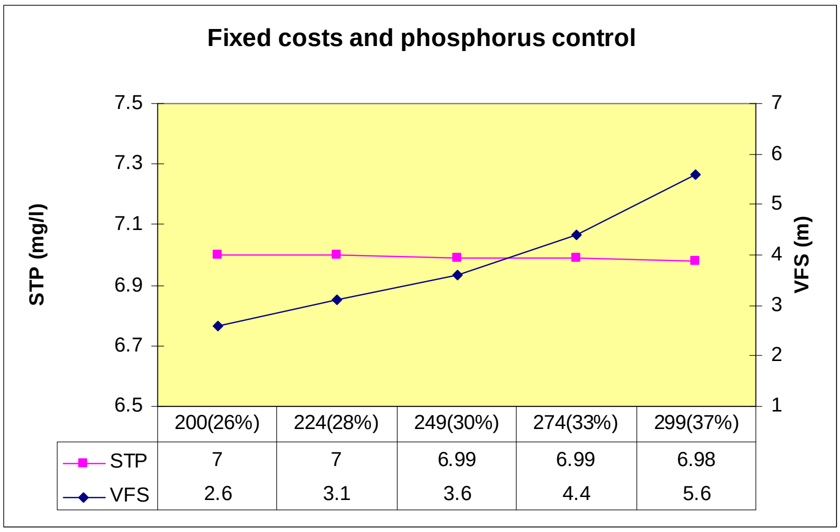
For the sake of clarity, we will use only few selected parcels for which we conduct the analysis. We will examine some – or all – of the following parcel types (as slope-shape combinations): (2%, 1:1); (2%, 1:3); (5%, 1:1) and (5%, 1:3). Hence, either the field edge adjacent to water is 58 meters and the orthogonal field edge is 173 meters long, or all edges are 100 meters. The slope is either 2% or 5%. The results could be easily generalized for the whole target area.

Fixed costs of production

The optimal allocation on VFSs is affected by the opportunity cost which is determined by the profits. In our profit function, the other costs of production (*C*) included for instance the LFA subsidy. Clearly, the magnitude of such a subsidy is by large a result of a political process, and it is not necessarily corresponding to the social value of active crop production. Therefore, it is important to examine how the optimal solutions change as we vary *C*.

In figure 9, we examine this for a square parcel with slope 5%. The optimal steady state STP levels are presented in the leftward vertical axis. The optimal VFS width are presented in the rightward vertical axis. The horizontal axis denotes the numeric value of other costs of production and the abatement percentages associated with the optima are given in parentheses. The boxes under the horizontal axis present the numeric values for the STP levels and VFS widths.

Figure 9. Fixed costs and phosphorus control.



The basic message of figure 9 is that increasing the other costs of production, and thereby decreasing the profits, increases the optimal use of VFS. For instance, increasing the fixed costs (i.e. lowering the profits) by 10% increases the optimal VFS width by 0.8 meters (22%) and the abatement by 3 percentage units. The variation in optimal STP levels is insignificant.

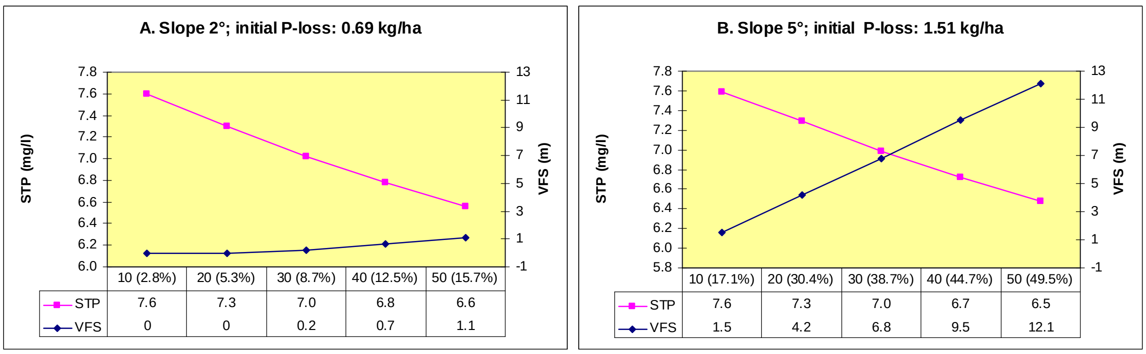
For the square parcel of 2% the optimum included a VFS of 0.1 meters under the highest value of fixed costs. For parcels gentler than 2% the optimal solution still did not include any VFS under these variations. The main reason for this is that the surface runoff from the gentler parcels is very modest and hence the potential gains in phosphorus abatement remain low, even if the opportunity costs would decrease substantially. For a gentle parcel, most of the PP-loss takes place in drainage, unaffected by the VFS.

Marginal damage

The higher the damage occurring from a given level of phosphorus loss, the more we will abate in the social optimum, compared to the phosphorus loss associated with the private optimum. We compare the socially optimal steady states of the parcels A: (2%, 1:3) and B: (5%, 1:3). We will use marginal damage values of 10, 20, 30, 40 and 50 euros. The initial (i.e. the privately optimal) annual phosphorus losses from the parcels under scrutiny are 0.69 kg ha^{-1} (parcel A) and 1.51 kg ha^{-1} (parcel B).

We present the results in figure 10. Panel A corresponds to optimal solutions for parcel A and panel B for parcel B. The leftmost vertical axes correspond to socially optimal level of STP. The rightmost vertical axes depict the optimal width of the VFS. The horizontal axes depict the marginal damage values, and the percentage abatement in parentheses; the first value in panel A, 10(2.8%) hence refers to marginal damage of 10 € which is associated at this parcel with optimal abatement of 2.8%. The numeric values for the examined variables are given in the boxes below the horizontal axes.

Figure 10. Optimal phosphorus control for various marginal damages.



The results reported in 10 are illuminating. For the lowest marginal damage value (10 €), it is not optimal to construct VFS on the gentler parcel. On the steeper parcel there will optimally by a 1.5 meter wide VFS. The steady state STP level is lowered from its privately optimal level (7.9) to 7.6 for both parcels. For parcel A, the associated phosphorus loss is 0.02 kg ha^{-1} lower than at the initial level., this means a reduction of about 2.8%; for parcel B it is 0.26 kg ha^{-1} (17.1%) lower.

As we increase the marginal damage to 20 €, the socially optimal steady state STP levels are further lowered for both parcels. The decrease is practically identical between the two parcels: the optimal STP levels are 7.3. The social optimum still includes a VFS only for the steeper parcel. Its width has increased to 4.2 meters. The phosphorus loss from the flatter parcel is reduced by 5.3% and from the steeper parcel by 30.4%.

A similar trend continues when further increasing the marginal damage and thus giving the environmental quality a higher relative value as a part of social welfare. The higher marginal damages are associated with lower steady state STP levels. For the steeper parcel, the VFS width increases fairly sharply. And for both parcels, the phosphorus loss becomes lower.

For the highest marginal damage value (50 €), the steady state STP level is about 6.6 for the gentler parcel and 6.5 for the steeper one. That is, it is approximately 17% lower for the flatter and 18% lower for the steeper parcel than at the private optimum. The VFSs and abatement percentages for the parcels are 1.1 (15.7%) and 12.1 (49.5%).

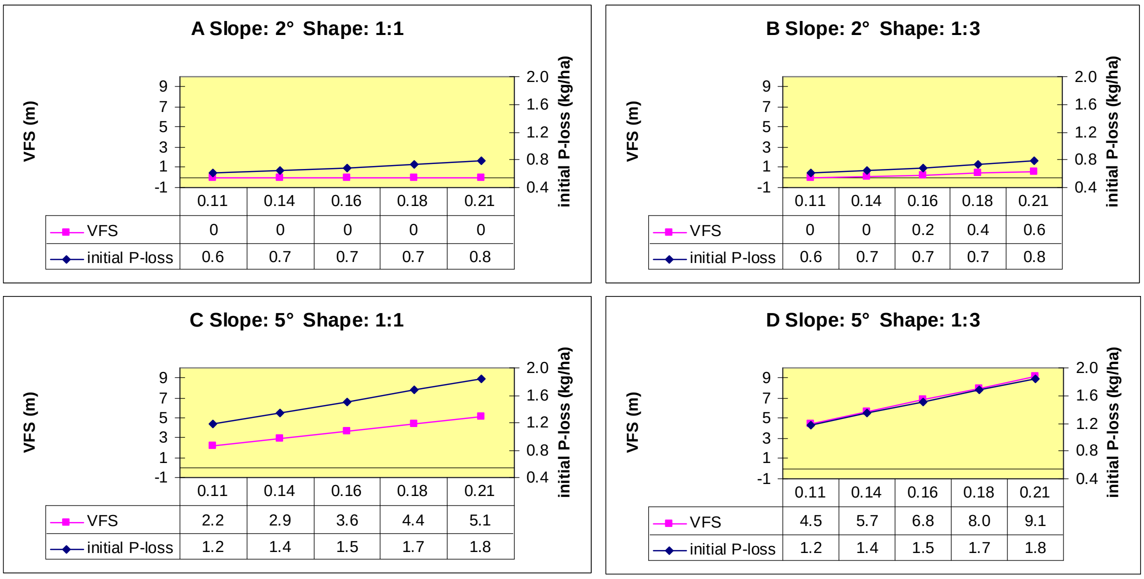
Bioavailability

Next, let us examine how the assumption about the bioavailability of PP-loss affects the socially optimal steady state solutions. We conduct the analysis for our four different parcels and five bioavailability coefficients. The parcels have the following slopes and shapes: (2%,1:1); (2%,1:3); (5%,1:1); and (5%,1:3). The examined bioavailability coefficients are 0.112, 0.136, 0.16 (the default value), 0.184 and 0.208.

As discussed earlier, the bioavailability of PP-loss is a controversial issue. Also, most of the real-world measures to control for the phosphorus loss are targeted on PP-loss reduction (the use of VFS, catch crops, no-till technology, etc). Therefore, it is useful to assess the results sensitiveness towards the bioavailability assumptions.

We report these results in the four panels of figure 11. The leftward vertical axis in each panel depicts the VFS width in meters. The rightward vertical axis in each panel depicts the initial phosphorus loss (i.e., the phosphorus loss associated with the private optimum). We report this to illustrate that defining the bioavailability coefficient influences the initial phosphorus loss which in our model is the sum of DRP-loss and the bioavailable fraction of the PP-loss. The horizontal axes depict the PP-loss bioavailability coefficients, for which the socially optimal steady states were calculated. The boxes under the horizontal axes depict the numeric values for the VFS widths and phosphorus losses for the respective bioavailability coefficients.

Figure 11. Bioavailability of PP-loss and optimal phosphorus control.



The slope of the square parcel in (A) and the narrow parcel in (B) is 2%. As we see, the shape does not affect the absolute amount of initial phosphorus loss from a parcel: the numeric values are identical between panels A and B. However, it does affect the socially optimal construction of VFSs. For the square parcel it is not optimal to construct any VFS, even for the highest bioavailability coefficient. For the parcel in panel B, the social optimum includes VFSs widths (0.2, 0.4 and 0.6 meters) for the three highest coefficients.

For the flatter parcels (2%) the effects of altering the bioavailability of PP-loss are anticipated but very modest. We can say that the VFSs get wider as the bioavailability increases. However, the use of VFS at the optima is very low, even at the highest coefficient values.

The square parcel in (C) the narrow one in (D) have slopes of 5%. The effects of varying the bioavailability coefficient are similar as in the upper panels, only that they are more distinct. The initial phosphorus losses are again identical.

There are differences in optimal VFS widths between the parcels C and D at the optimum. For the default value (0.16), the square parcel has a VFS of 3.6 meter width whereas the VFS constructed on the narrower parcel in panel D is 6.8 meters wide. The absolute difference grows further as we increase the bioavailability of the PP-loss. At the highest coefficient value the VFSs for the square parcel (panel C) is 5.1 meters and for the narrower parcel (panel D) 9.1 meters. However, the percentage difference grows smaller. This is due to decreasing marginal abatement of the VFS.

The optimal STP levels associated with the socially optimal steady states is not reported in figure 11. This is because the values are very close to each other for all levels of PP-loss bioavailability and for all parcels. The lowest values are encountered with the steepest, narrow parcel (STP equal to 6.98 with the highest coefficient) and the highest with more than half of the socially optimal steady states (STP equal to 7.02).

The insignificance of this variation is interesting. None of the variables affecting the DRP-loss directly are altered. Hence, the small variation is indeed the effect of ‘replacing’ some of the phosphorus abatement of the VFSs with a decrease in STP. Earlier, we gave some explanations for the stability of optimal STP, regardless of the VFS width. Already a very slight decrease in STP lowers the private profits – and hence the opportunity cost of land – in the extent that constructing VFS becomes less expensive. But at the same time, constructing the VFS gets less necessary as the total phosphorus loss is decreased due to lower STP level.

The discount factor and the transition function

Finally, we will analyse the effect of the discount rate on socially optimal allocations. For the sake of brevity, we conduct the analyses using both transition functions. For the time being, we assume that the discount rates are identical.

Finally, we examine the effect of the transition function on the social optimum. We saw earlier that the choice of the transition function had a substantial influence on optimal phosphorus fertilization levels but only minor influences on optimal steady state STP levels. Because the fertilization does not affect phosphorus loss, we anticipate that this feature is preserved also in the social optimization problem.

Figure 12 presents the socially optimal steady state STP levels and VFS widths using five different discount factors (0.9, 0.925, 0.95, 0.975, 1) and two transition functions. The parcel is square and its slope is 5%. Panel A presents the results for optima derived using the transition function (45) derived from Saarela et al (2004) and panel B using the transition function (47) of Ekholm et al (2005). The horizontal axes denote the discount factor used; phosphorus abatement is given in parentheses. The 2x5 tables underneath the horizontal axes present the numeric values for the examined variables. The leftward vertical axes depict socially optimal steady state STP levels and the rightward vertical axes the VFS widths.

Figure 12. The optimum for two transition functions and five discount factors.

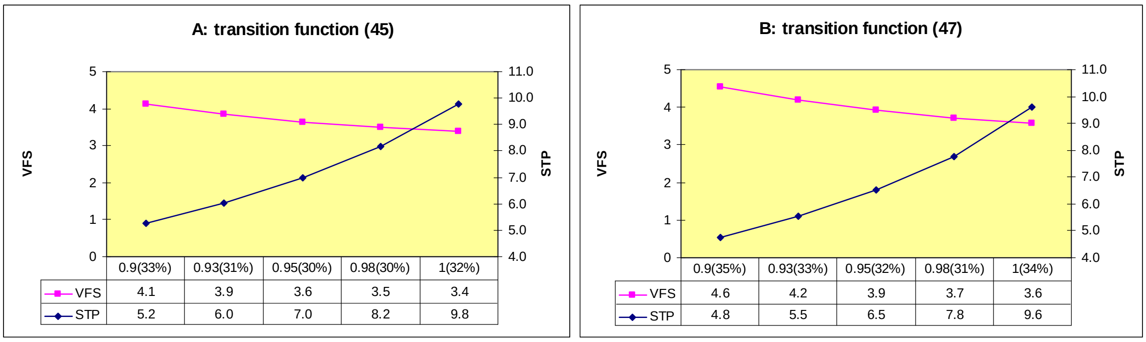


Figure 12 illustrates three distinctive features about the optimal use and abatement of phosphorus. Firstly, the optimal STP levels are higher for higher discount factors (lower discount rates) and they are practically identical between the two parcels. We encountered this already within the private optimization problem: given input and output price relations, the optimal steady state STP level is affected by the discount factor. The shape of the parcel has only a slight influence on the socially optimal STP (for the privately optimal it has a zero effect).

Secondly, the optimal VFS width decreases slightly as discount factor increases. The reason is that a higher discount factor is associated with a higher steady state STP. This is associated with higher private profits, i.e., with higher opportunity costs of land. This is interesting because higher STP level and a narrower VFS are associated with higher phosphorus losses. In our model, a higher discount factor is associated with higher phosphorus losses in the social optimum. The reason is that the damage is determined only by the flow of phosphorus instead of the stock.[14] However, the abatement percentage may either increase or decrease as we alter the discount factor, but we can not find any particular trend in it. This is because the initial (i.e. privately optimal) phosphorus loss increases along with the discount factor as well. The damage and DRP-loss functions, on the other hand, are linear. Therefore, the percentage reductions fluctuate around the default values.

Thirdly, the differences in optimal STP levels and VFS widths between the two transition functions are minor. In both panels, the optimal VFS width associated with the lowest discount factor is about 3 meters. If we use a higher discount factor, the optimal STP level (both socially and privately optimal) is higher which increases the opportunity cost of land. Therefore, the optimal VFS width decreases. With the highest discount factor (equal to 1, i.e. discount rate equal to 0%) the optimal VFS widths have decreased approximately to 2 meters.

The optimal STP levels are very close to each other for the two transition functions of panels A and B. The socially optimal steady state STP levels associated with the lowest discount factor (0.9) are 5.3 and 4.8. For the highest discount factors the optimal levels are 9.8 and 9.7. Hence, even though the differences in required annual phosphorus fertilization are substantial between the two transition functions (see figure 5) the optimal steady state STP levels in both social and private optimization problems are very close to each other.

[14] For certain cumulative damage function specifications, a higher discount factor might have had an opposite effect.

7 Instrument design

The final step in our study is to examine empirically the instruments presented in section 4.4. Due to the stationary nature of the study we can not assess the efficiency of various second-best instruments by comparing their welfare effects to those of the first-best instruments. For this reason, we will not report the stationary profits associated with outcomes induced by different instruments. However, the PP-loss and abatement processes do not include dynamic features. Therefore, we can assess the welfare effects of various VFS subsidies with reasonable accuracy. In doing this, we take advantage of the results which indicate that the optimal choices of STP levels and VFS widths are close to independent phenomena.

We start by examining phosphorus taxes. We then focus on VFS instruments, mainly on various second-best VFS subsidies.

7.1 Taxes on phosphorus

We saw in the previous section that for a given exogenous parameter values, the optimal steady state levels for STP are almost identical, regardless of the slope or the shape of the parcel. This is due to homogeneous soil type. Hence, the first-best taxes on either plant available soil phosphorus or on fertilization are close to uniform. This indicates that the (stationary) welfare effects of tax differentiation would be mild and the comparison between first-best and second-best taxes is not very illuminating.[15] Therefore, we focus our analysis on two features discussed shortly in the theoretical part. Firstly, we analyze the implications of a difference between the discount rates applied by the farmer and the social planner. Secondly, we analyze the implications of using alternative tax bases when the transition function used in the optimization process is incorrect.

[15] However, we will report the uniform tax rates while examining the tax bases under wrongly chosen transition functions.

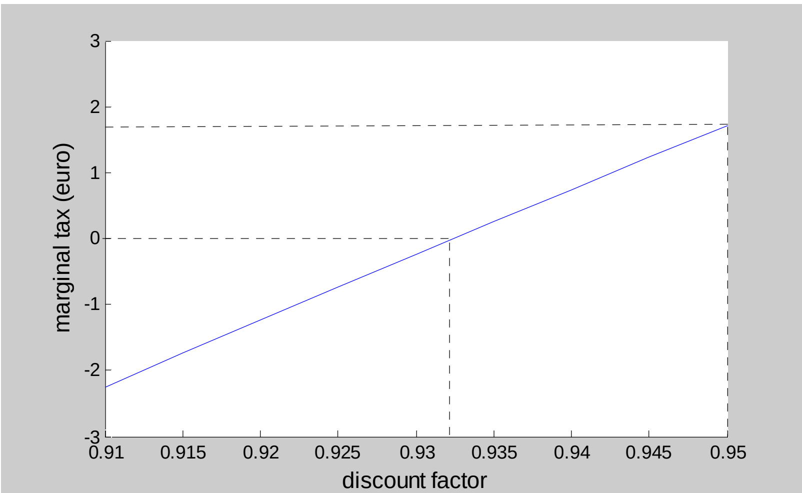
Discrepancy in discount factors

The possibility that the discount factors applied by the farmer and by the social planner are different has an implication on optimal tax level, as showed in section 4.4. The marginal tax based on STP under dissimilar discount factors is given in equation (32). Assuming that the social planner applies a discount factor $\hat{\beta}_s = 0.95$ we obtain:

$$\tau_s = \frac{0.95}{\beta_p} D_s - \frac{\pi_x \Gamma_s (\beta_p - 0.95) + \pi_s \Gamma_x (0.95 - \beta_p)}{\beta_p \Gamma_x} \qquad (48)$$

Figure 13 depicts the optimal steady state marginal taxes for a parcel (2% and 1:3) as the discount factor of the farmer varies between 0.9 and 0.95[16]. The discount factor of the social planner is fixed to 0.95. The vertical axis denotes the optimal steady state tax on STP. The horizontal axis denotes the discount factors applied by the farmer. The dotted lines depict two points on the graph: the discount factor for which the marginal tax rate is zero; and the marginal tax rate as the discount factors of the farmer and the social planner coincide.

Figure 13. Marginal tax under differing discount factors.



If the discount rates are identical, the optimal marginal tax rate based on STP for this parcel is 1.70 €. As the optimal STP level is 7.02, the marginal tax paid per hectare is

[16] In practice, the privately applied discount rate is set by the capital markets, and we can think that the discount rate applied by the social planner may vary. However, allowing the privately applied discount rate vary makes the presentation more illustrative.

11.94 €. If the privately applied discount factor is lower than 0.95, the optimal tax is lower than this. If it is equal to 0.932 (i.e., discount rate of 7.26%), the optimal marginal tax rate is zero. At this point, the socially and privately optimal choices of long term phosphorus fertilization converge. The private decision maker does not acknowledge the environmental damage, but her lower valuation of future profits coincidentally makes the optimal STP levels identical. In this case, there is no need to alter the private choice of steady state phosphorus fertilization.

If the discount factor is lower than 0.932, the privately optimal steady state STP is lower than the socially optimal. To achieve the social optimum, the farmer must be subsidized to increase the use of fertilizers.[17]

In practice, it would not be necessary for the social planner to observe the discount rates the farmers are applying. Observing the STP levels is enough, as long as the farmers' planning horizons are infinite.

Transition function and a uniform tax under alternative tax bases

We showed in section 4.4 that the tax can be based either on fertilization or on STP level directly. Now, we examine the difference of the tax designs when the transition function is chosen wrongly. For this, assume that the development of the STP follows the transition function (47) by Ekholm et al (2005) and that the farmers acknowledge this. However, the social planner wrongly assumes that the STP development is captured by the function (45) and poses a tax on either STP or phosphorus fertilizer use.

Assume that the applied discount factors are identical and the social planner is applying a uniform tax for the whole target area. It is set at the level to guarantee the socially optimal level of phosphorus loss together with first-best allocation of VFSs.[18]

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[17] This rather peculiar situation might be realistic, for instance, in the case of tenant farmer with a contract for a fixed time period. However, this case fits poorly our infinite time horizon framework.

[18] For instance, a uniform marginal tax rate based on STP would be 1.73 € if (45) would be the correct transition function. This would induce all parcels to maintain a steady state STP level of 7.01. This is 0.72% higher than the lowest first-best STP level (6.96) and 0.14% lower than the highest first-best STP level (7.02). That is, the difference in the STP-levels induced by a first-best tax and a uniform tax are minor. Therefore, simplifying the presentation by using a uniform tax is justified.

Table 10 presents the comparisons. The tax base is either phosphorus fertilization (x) or STP level (s). The correctly set uniform second-best marginal tax rates are taxes that together with the first-best allocations of VFSs are associated with the first-best total phosphorus loss from the whole target area. The incorrectly set taxes are computed using the wrong transition function (45). They *would* implement similar outcomes if the transition function would be correct. Both the correct and the incorrect tax rates induce the farmers to maintain STP levels that differ from each other. Consequently, the total phosphorus losses and the crop yields from the target area differ. All these are presented in the rows of table 10.

Table 10. Comparing tax bases when the transition function is wrongly chosen.

Tax base	**x**	**s**
Correctly set uniform marginal tax rate	0.24 € kg^{-1}	1.73 € STP^{-1}
Incorrectly set uniform marginal tax rate	0.27 € kg^{-1}	1.73 € STP^{-1}
STP induced by the correctly set rate	6.52	6.52
STP induced by the incorrectly set rate	6.41	6.52
Percentage diff. in the induced STP levels	1.7%	0.014%
Diff. in the associated phosphorus loss	0.23 kg (0.8%)	0.0019 kg (0.008%)
Diff. in the associated crop yield	330 kg (0.3%)	2.6 kg (0.002%)

If (47) would be the actual transition function, the correctly set uniform marginal tax rates under both bases induce an STP level of 6.53 (0.067% lower than the highest first-best STP level). A correctly set tax rate based on fertilizer use would be equal to 0.24 € kg^{-1}. If the social planner would implement a tax computed using a wrongly chosen transition function (45), its level would be 0.27 € kg^{-1}. This is 15.6% higher than the tax set using the correct transition function. The uniform STP level induced by this incorrect tax is 6.41 which is 1.7% lower than under the correctly set tax. The phosphorus loss associated with this is 0.23 kg (0.8%) lower than under the correctly set tax. The total crop yield from the target area is 324 kg (0.27%) lower than under the correctly set uniform tax.

The correctly set marginal tax rate based on the STP level is 1.732 €. The level of a wrongly set marginal tax would be 1.729 €. Implementing this would induce the farmers to maintain an STP level of 6.53 which is 0.014% higher than under the correctly set uniform tax. This is associated with a total phosphorus loss that would be 0.0019 kg (0.008%) higher than under the correctly set tax. Similarly, the differences in crop yields would be very close to each other.

The comparison suggest that STP is more robust a tax base than the phosphorus fertilizer use, when the social planner is uncertain about the explicit form of the transition function. Actually, we saw signs of this already in table 4 where we compared the analytical forms of alternative taxes. Also, we saw there that the information requirements for implementing an STP based tax are smaller than the requirements for a fertilizer use based tax. These results are in favor of basing the taxes directly on STP level. We discuss the policy implications of this observation in the last sections of this study in more detail.

7.2 Vegetative filter strip instruments

As opposed to the optimal STP levels, the optimal VFS allocations are strongly differentiated across the parcels. Therefore, it is interesting to analyze the welfare effects of distortions towards a more uniform solution. Luckily, in the case of VFSs we are also able to examine these welfare effects with a reasonable accuracy.[19] This can be done by sticking to the first-best STP levels throughout the analysis and solving for various second-best VFS allocations and instruments. Because the choice of the VFS is in effect a static decision (see equation 20b), the steady state profits associated with various first-best and second-best allocations do have information value. Hence, we examine the welfare effects of VFS differentiation under various second-best subsidies by keeping the STP at the optimal level.

[19] As discussed in section 4.4, the optimal choices of steady state STP and VFS width are interlinked. Hence the phrase ‘reasonable accuracy’. Comparing steady state profits of two different STP level does not give correct information on welfare effects. Sticking to single STP level makes the change in profits stem from VFS allocations only. However, the STP levels would actually react slightly on changes in VFS allocations, too.

Also, we depart slightly from the theoretical treatment of section 4.4 by presenting exemplifying VFS subsidies instead of only marginal subsidies at the optimum. This is more interesting in terms of policy implications. It also makes the first examined instrument – command and control regime for a uniform VFS width – consistent with other second-best VFS instruments. Of course, the marginal values of the subsidies are directly obtainable from the examples provided.

The first-best VFS allocation and the first-best subsidies

The first-best VFS allocations across the parcels were presented in table 10, section 6.2. The optimal VFS allocation included 0.6 hectares of VFSs (out of total field acreage 37 ha). The strips are located on parcels with slopes 2%, 5% and 7%. The annual costs of this allocation is 479.0 €. The costs of various second best instruments are compared against this value.

Equation 29c defined the first-best marginal VFS subsidies to be: $\tau_b^s = -D_b$. It is clear that the marginal subsidies will be identical for identically shaped parcels: the marginal benefits (i.e. the negative of marginal damage) from incremental VFS widths must be identical across all parcels. For parcels gentler than 2% the optimum contained no VFSs nor for the square parcel with slope 2%. Hence, the first-best subsidies are equal to zero for these parcels. The marginal first-best subsidies are presented in table 12:

Table 11. First-best nonzero marginal VFS subsidies.

Slope (%)	**2**	**5**		**7**	
Shape	***1:3***	***1:1***	***1:3***	***1:1***	***1:3***
Marginal subsidy (€ b^{-1})	1.00	1.73	1.00	1.73	1.00

The empirical results are consistent with the theoretical ones: the marginal subsidies are equal for equally shaped parcels and they are higher for square parcels than for narrow parcels (shape 1:3). The latter result may seem confusing because we know from table 9 that the optimally allocated VFSs are wider for the narrow parcels than for the square ones. This is nevertheless sensible: at the optimum, the marginal payments per VFS acreage, i.e. the marginal costs of VFS abatement, are identical; not the marginal payments per VFS width.

The subsidies for identically shaped parcels are identical up to the third integer, regardless of the slope of the parcel. The slight differences stem from the fact that the opportunity costs differ slightly across the parcels, as seen in the STP levels reported in table 9. The differences are there indeed to make the marginal costs and marginal benefits of VFS construction identical across all parcels at the optimum, as required by the static optimality.

Second-best instruments

We will examine five different second-best instruments that induce VFS allocations associated with the first-best level of phosphorus loss from the whole target area (together with the first-best level of STP). Four of the instruments are subsidies that are conditioned on various parameters and one is a command and control instrument. Finally, we will also consider a scheme external for our framework: setting aside the steepest parcels.

We present first collectively the features and outcomes of the four second-best subsidies and the command and control instrument in a table form. After this we examine graphically the VFS allocations on individual parcels under various instruments.

The information on the second-best instruments is collected in table 13. The leftmost column gives the subsidy type. The first subsidy is conditioned on the VFS acreage, the second on VFS width, the third on width and slope; and the fourth on width and slope squared. The second column presents the explicit form of the subsidy scheme, i.e. not just the level of marginal subsidies. The third column denotes the total acreage allocated on VFS under the respective subsidy. The costs are the annual costs, including the opportunity cost of land as well as the other costs of VFS. The last column denotes the percentage difference (increase) of the costs in comparison to the costs associated with the first-best VFS allocation. The subsidy payments are not analyzed nor reported.

Table 12. Second-best instruments.

Type	Form	A (ha)	Costs (€)	Increase (%)
Uniform C & C	5.7 meters	1.49	804	67.8
Subsidy (acreage)	$\tau_{Ab} = 0.42*(A)^{0.33}$	1.52	808	68.6
Subsidy (width)	$\tau_{wb} = 11.33*(b)^{0.33}$	1.45	797	66.3
Subsidy (width, slope)	$\tau_{w\gamma b} = 2.23*\gamma(w)^{0.33}$	0.65	527	10.1
Subsidy (width, slope^2)	$\tau_{w\gamma\gamma b} = 0.42*\gamma^2(w)^{0.33}$	0.61	481	0.4

Given the first-best level of STP, a uniform VFS of 5.7 meters on all parcels guarantees the first-best level of phosphorus loss. The annual costs associated with this would be 804 €, i.e. 68% more than the costs of the first-best. The distinct increase in costs has three sources. The initial costs are 170 € higher because the VFSs are constructed on 17 parcels where no VFSs existed under the first-best. The operation and maintenance costs increase by 72.5 €. The rest comprises of the opportunity cost of land set aside from production.

If we condition the subsidy on the VFS acreage, the total VFS acreage will be 1.52 hectares and the annual costs about 808€, an increase of about 69% compared to the first-best. The efficiency difference between this and the uniform VFS is fairly small. It is interesting that it is in the favor of the uniform instrument. For the subsidy conditioned on the VFS width, the total acreage allocated on VFSs is 1.45 ha. The associated annual costs are 797 €; an increase of about 66% from the first-best.

The results in table 12 do not indicate significant differences between the three first instruments. However, there are some. For the two first subsidies the induced allocations are identical for parcels with differing slopes, but dissimilar for parcels with differing shapes. The farmers choose to construct a wider VFS on a narrower parcel alike in the first-best allocation. In the case of subsidy on VFS width, the VFS will be 3.2 meters wide on square parcels and 7.4 meters wide on the narrow parcels. The reason for this variation is that on a narrow parcel the costs of constructing the first meter are smaller due to a smaller acreage set aside from production. The subsidies are identical. Hence, there will be more VFSs on narrow parcels. At the optimum, the marginal payments are *not* identical for the two parcels because the subsidy is not linear. And because the subsidy is concave in width, the marginal costs of the last width-unit are smaller on the narrow than on the square parcel.

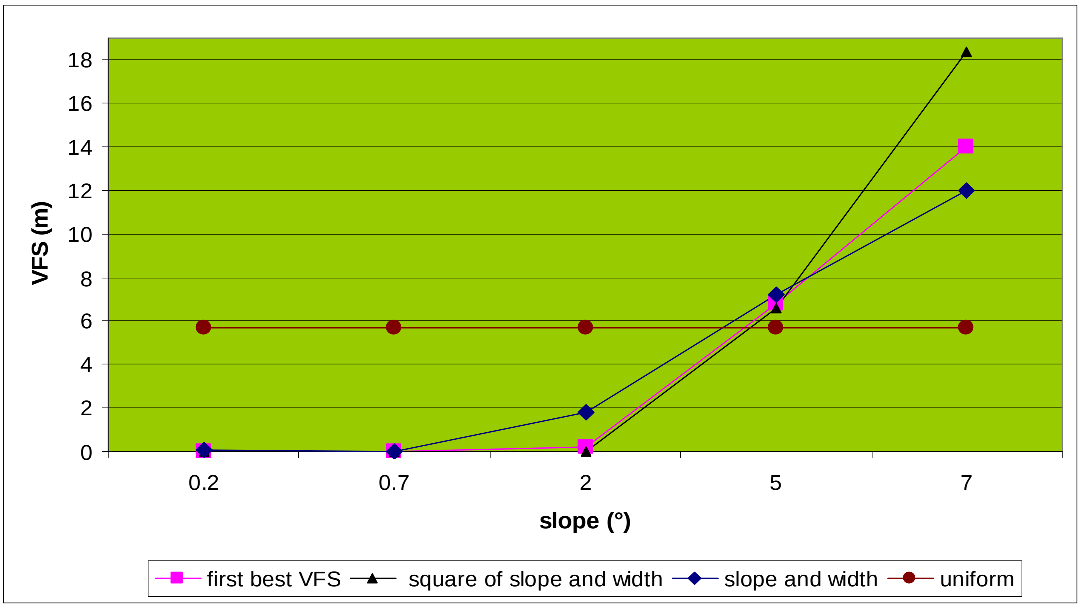
In case of the acreage subsidy, there will be 2.2 meters wide VFSs on the square parcels and 8.7 meters wide VFSs on the narrow parcels. In this case, the variation stems from the fact that when facing an acreage based subsidy, all farmers optimally choose an evenly *large* VFS. This means that the VFSs will be wider for narrower strips. That is, even though the outcomes of these two subsidies are almost identical, the reasons behind the variation are not identical. It is an interesting coincidence that the width subsidy provides efficiency gains when compared to the uniform instrument because it moves the allocation in the direction of the first-best, in terms of shape variation. On the other hand, the efficiency losses from the acreage subsidy result from going to the right direction, but too far. That is, the acreage subsidy differentiates too much in terms of shape variation. This is by no means a general result. With slightly different VFS abatement function, both subsidies are more efficient than the uniform instrument.

Altogether, allocating more efficiently only in terms of various parcel shapes does not provide us with significant efficiency improvements. Such an improvement is achieved when the subsidy is conditioned on the VFS width *and* the slope of the parcel. In this case, the induced total VFS acreage is only 0.65 hectares, as opposed to the first-best level of 0.60. Consequently, the costs are 527 € (about 10%) higher than under the first-best allocation. The difference in costs stems mainly from the fact that under the second-best subsidy, VFSs – if only very narrow – are constructed on some parcel types where none exists under the first-best allocation. The initial costs occurring from these parcels are 60€. If there would be no VFSs on these parcels, the costs would be only 2% higher. Of course, the phosphorus loss would be slightly higher than under first-best if these VFSs would be omitted.

Finally, if the subsidy acknowledges the width and the slope squared, we almost obtain the first-best allocation. The total acreage is 0.61 and the costs less than 1% higher. This, of course, is obvious: we have constructed the PP-loss function to be largely determined by the square of the slope. This is a general feature: the more accurately the social planner knows the PP-loss process, the more closely the allocations induced by the VFS subsidies can be made to mimic the first-best allocation.

Let us then examine graphically the allocations induced by the uniform instrument and two subsidies. Figure 15 collects these allocations for a narrow parcel. The vertical axis depicts the VFS width. The horizontal axis depicts the slopes of the various parcels. The lines illustrate the development of the VFS width as the slope increases.

Figure 14. VFS allocations induced by second-best instruments.



The benchmark case is the first-best VFS allocation, depicted by the pink line. It includes no VFSs for the gentlest parcels and a rapid increase in the VFS width as the parcels get steeper. The black line depicts the second-best allocation closest to the first-best. It is induced by the subsidy conditioned on the square of the parcel slope and the width of the VFS. The allocations induced by this subsidy follow the first-best allocations almost perfectly. The major difference is that the allocations are slightly biased towards the narrow parcels. This can be particularly seen with the steepest parcel where the VFS is wider than under the first-best for the narrow parcel and smaller for the square parcel (square parcel not presented in figure 15).

The subsidy conditioned on the slope and the width deviates already distinctively from the first-best. There are VFSs on parcels where there should be none and the VFS for the steepest parcel is too small. However, the subsidy is clearly sensitive for the slope of the parcel in the right direction: the VFSs get wider as the parcels get steeper.

We already discussed that the two other subsidies (conditioned on acreage or width) induce uniform allocation on a parcel of a given slope but varied with the shape. Therefore, we depict only the command and control allocation in 15. It simply induces a 5.7 meters wide VFS on all parcels. Altogether, figure 15 illustrates nicely that the closer the determinants of the second-best subsidies are to the determinants of the underlining natural phenomenon, the more efficient the subsidies get.

Setting aside the steepest parcels

Finally, let us consider the implications of taking the steepest parcels entirely away from production. We can not use our model for this calculation as the option was not included in it. Instead, we apply the results of the model and calculate the effects by hand. We do this to compare the abatement and stationary costs of two alternatives: setting aside the steepest parcels (7%, altogether 3 hectares) and allowing the remaining parcels to operate at a privately optimal level, and achieving a similar abatement without the option to set aside land from production.

We consider the steady state opportunity costs from lost production, which in our case is the annual net profit under the privately optimal allocation: 96.4 € ha^{-1}. Setting the steepest parcels aside from production would reduce the phosphorus loss from the target area in the steady state to 29.0 kg.[20] The phosphorus abatement would be 20.2%. A similar abatement percentage would be associated with a marginal damage of 17.2 €.

[20] The 3 hectares taken away from production are assumed to contribute to 0.07 kg ha^{-1} of phosphorus loss annually in the steady state. This corresponds to an estimate for background leaching in Finland (see for instance Uusi-Kämppä (1989)). Again, our stationary analysis does not take stance towards the duration and costs of the transition period.

Under the set aside scheme, all parcels are operating at the privately optimal levels of production with annual profits of 96.4€, making the annual total profits in the area 3278€. Under the first-best solution of our model that would guarantee the same phosphorus loss level, the profit per hectare would vary between 75.9 € and 93.8€, the total profit being 3296 € for the whole area. This is 0.6% more than under the set aside scheme. That is, by setting the steepest parcels permanently out of production and letting the remaining parcels to operate on the privately optimal level the sum of stationary private profits is approximately identical as with the socially optimal allocation associated with the same phosphorus loss, without the set aside option.

Taking into account that our model is using a very optimistic VFS abatement function, this result is interesting. If the VFS abatement would be lower, the set aside scheme would become more favorable. On the other hand, it will take a long time before the phosphorus losses from fields set aside could be as low as the overall background leaching. Also, we acknowledged only the opportunity costs and none of the potential costs taking place during the transition period. Altogether, this part of the analysis should perhaps be viewed as an exercise external for the rest of the research.

8 Discussion

One of the central elements of phosphorus loss is its time-dependent and spatial nature. The loss of dissolved reactive phosphorus (DRP-loss) is largely determined by the phosphorus accumulated in soil; in particular its potentially plant available reserves which accumulate and deplete slowly. The loss of particulate phosphorus (PP-loss) is mainly determined by erosion. The susceptibility for erosion varies strongly within agricultural areas.

Efficient phosphorus policies should acknowledge both these features. In this study, we developed a dynamic framework to analyse the privately and socially optimal steady state phosphorus fertilization levels and vegetative filter strip (VFS) construction. We also analysed instruments to induce the farmers undertake the socially optimal actions. Finally, we conducted an empirical application.

Phosphorus loss was modelled as a sum of PP- and DRP-losses. While PP-loss was dependent on the slope and the shape of the field, DRP-loss was determined solely by the level of potentially plant available soil phosphorus reserves. These reserves were estimated with a measure called soil test phosphorus (STP). DRP-loss had a direct effect on environmental damage whereas only a fraction of PP-loss contributed to it. The DRP-loss could be abated by lowering the steady state STP level and the PP-loss by constructing VFSs. Lowering the STP from private optimum lowered the profits. The VFS costs originated from initial and operational costs and allocating land away from production.

The optimal allocations were the steady state solutions for a dynamic optimisation problem, where either the farmer or the social planner had to choose the fertilization and VFS width levels to maximize her private discounted profits or social returns, respectively.

We examined two alternative first-best tax-subsidy schemes: a tax conditioned either on fertilizer use or on STP levels directly. The subsidies on VFS construction were identical.

In the empirical application, we defined the explicit functional forms and the parameter values for the model. The target was an agricultural area of 37 hectares allocated on barley. There were altogether 10 different kinds of parcels, differing in their shape-slope combination. We solved numerically the optimal steady state levels of phosphorus fertilization and VFS width for each parcel type in the area. We then examined how the optimal tax rates were affected by differences in the discount rates applied by the farmer and the social planner. We also analyzed the effect of the transition function (mis)specification on the efficiency of the taxes. The analysis on VFS instruments focused on the efficiency of second-best instruments.

8.1 Main findings of the study

The main theoretical results of the study were derived from the analytical examination of alternative phosphorus taxes. The main empirical results quantified the optimal steady state use and abatement of phosphorus on our target area.

First-best phosphorus taxes

Analytical examination of the instruments showed that a first-best tax can be based either on phosphorus fertilization or on the STP level directly. However, due to the underlining dynamics of the optimization problem, there were differences between these two. If the discount rates applied by the farmer and the social planner were identical, the optimal tax on STP was equal to its marginal damage. If the tax was based on fertilization, it also had to acknowledge the fertilizer's marginal contribution to the level of STP, as well as the dynamically optimal choice of the farmer. This made the tax based on STP distinctively simpler. This suggests that by using the STP level as a tax base, the information burden of the social planner might be reduced.

If the social planner would choose to strive for an exogenously set target on DRP-loss – which might easily be the case – she would still need to understand the interlinkages of the STP level of a given soil type and the level of DRP-loss. However, she would no longer need the knowledge of the dynamic links between the STP level and the phosphorus fertilization.

Achieving the socially optimal level of phosphorus use and abatement, on the other hand, would pose similar information requirements for setting the first-best tax, regardless of the tax base. However, the monitoring efforts of the social planner would be different. If the tax would be based on STP, there would be no need to monitor the fertilizer use. If the tax rate was based on fertilization, there would in principle be no need to monitor the STP level. Intuitively, however, it would seem logical that under both tax bases the social planner would find it necessary to monitor the development of the STP level as well. It would simply seem too risky to continuously rely only on the observed use of the control variable – fertilization – if it would possible to monitor the stock variable – the STP level – too. On the other hand, in the case of phosphorus, the stock variable (almost) alone matters for the environmental damage. Hence, monitoring fertilization would not be necessary if the tax would be based on the STP level.

Another interesting outcome of the theoretical part was the effect that a discrepancy in the discount rates had on the first-best taxes. If the discount rates of the social planner and the farmer differ strongly enough, the optimal tax rate might be zero, or even negative. That is, it might be socially optimal to subsidize the farmers' use of phosphorus if the discount rates applied be the farmers were high enough, compared to the rate applied by the social planner. It must be noted that this result is dependent on the damage function specification. If we would be using a dynamic damage function, the effect might be on a different direction. However, the discrepancy in the discount rates would still affect the first-best taxes in a similar manner, only the social welfare function would be differently determined.
In the empirical section we then showed that if the discount rate applied by the social planner was, for instance, 5.3% and the one applied by the farmer was 7.3%, the optimal tax based on the STP was zero. For higher discount rates the tax became a subsidy. There was a positive tax rate between the privately applied discount rate values of 5.3% and 7.3%.

We also conducted an analysis on tax bases using incorrectly chosen transition functions. As a result we saw distinctively higher deviations from the socially optimal level, when the tax was based on phosphorus fertilization. This also suggests that taxing directly the STP level might be more favourable.

Dynamically optimal steady state phosphorus fertilization

According to our empirical results, the socially optimal steady state phosphorus fertilization levels were only about 0.7 kg ha^{-1} lower than the privately optimal ones. This fairly moderate reduction decreased the steady state STP level from 7.9 to 7. Together with allocating VFSs on the two steepest parcels (5% and 7%) and on the square parcel of 2%, this reduced the loss of bioavailable phosphorus (DRP-loss and a fraction of PP-loss) by about 28% from the target area.[21] Using an alternative, strongly different transition function, the optimal STP levels were 7.3 for the farmer and 6.5 for the social planner. That is, the scale of decrease in STP levels remained almost identical. The optimal stationary phosphorus use varied only moderately within the target area.

The discount rate of the decision makers was perhaps the most interesting determinant of the optimal phosphorus use. Lowering the discount rate increased the steady state STP levels substantially. For instance, applying a discount rate of 2% instead of 5.3% increased the socially optimal steady state STP level from 7.0 to 8.2. The similar increase in the privately optimal STP level was from 7.9 to 9.7.

Consequently, in case the social planner applied a lower discount rate than the private farmer, the model showed that the privately optimal use of phosphorus could be above, below or exactly at the socially optimal level. This underlined the importance of first determining the socially optimal level and only then planning the necessary actions, instead of directly restricting the use of phosphorus from the privately optimal level.

Strongly differentiated optimal VFS allocation

The VFSs filter particulate phosphorus from surface runoff. Their efficiency is strongly related to the magnitude of erosion. According to our results, it was not efficient to construct VFSs on the gentlest parcels but the VFSs allocated on the steepest parcels were fairly wide. This result was quite robust on all parameter variations, including the varying of the bioavailability coefficient of the PP-loss. Even if 21 % of the PP-loss would have eventually been transformed in a bioavailable form, half of the parcels did not include any VFS at the optimum. The narrow parcel with slope 2% had a VFS of 0.6 meters. However, the square parcel with similar slope had none, alike all the parcel gentler than this. The intuition behind this is simple: if there

[21] The model did not analyze how long it takes to achieve this. Also, the model suggests that one could achieve a 28% abatement starting from the privately optimal allocation, not from the prevailing state of the Finnish crop production. Hence, actually inducing these allocations would not reduce – even theoretically – 28% of the currently occurring phosphorus loss; the Finnish agri-environmental program has already induced many of the allocations suggested here.

is only little surface runoff, the benefits of VFSs are minor. The costs, however, remain unchanged.

The results did not take into account the other potential benefits of VFSs. Including their effect on nitrogen removal or on providing biodiversity benefits might change the results. However, the general result is robust: when there is little surface runoff, there is little to be gained with VFSs in terms of nutrient abatement. Because the surface runoff is closely related to the slope of the parcel, the VFSs should be allocated on the steepest parcels.

Almost independent control of DRP- and PP-losses

There was only minor variation in the optimal STP levels between the parcels, regardless of the optimal VFS width. The optimal steady state STP level was 6.96 for the parcel with optimal VFS width of 14.0 meters (7%, 1:3), whereas for parcels with no VFS it was 7.02. In practice, a variation of this magnitude has no relevance.

The reason behind this was the inability of the VFSs to abate the DRP-loss. Therefore, the optimal control of the DRP- and PP-losses became almost independent problems. The key determinants of the socially optimal VFS width were the slope and the shape of the parcel, and the opportunity cost of land. For optimal phosphorus use, these were the input and output prices and the discount factor. The only way these were interlinked was that the optimal phosphorus use determines the opportunity cost of land. We saw that VFSs' effect on optimal fertilization remained negligible.

Second-best instruments

The use of a uniform tax rate had only minor effects on stationary outcomes. This was due to the negligible differences in the optimal phosphorus use between the parcels.

In incentivizing the construction of VFSs, on the other hand, it was essential to induce differentiated VFS allocations between dissimilar parcels. With our second best subsidies, the differentiation was achieved by making the subsidies sensitive towards the slopes and the shapes of the parcels. The closer the marginal payment function became the actual marginal damage of the empirical model, the more efficient the scheme became. The parcels where no VFS was needed were offered zero subsidies.

8.2 Policy implications

The immediate implication of the study is that the social planner should define the optimal phosphorus use under a dynamic framework. The instruments to alter the privately optimal behaviour must be designed and adjusted after this. It might be that the privately optimal levels are above, below or at the social optimum.

In Finland, the agri-environmental regulation uses subsidies instead of taxes in controlling the phosphorus applications. The farmers participating in the agri-environmental program must fulfil its compulsory conditions as well as certain voluntarily chosen supplementary measures. Participants receive a subsidy which is determined, for instance, by the type of the supplementary measures. Hence, the program offers a subsidy in exchange for a package of measures. Altogether 98% of the field acreage is included in the Finnish agri-environmental program (Suomen maatalous ja maaseutuelinkeinot 2006). That is, the program seems to be an effective device for posing the restrictions. At least it is considered beneficial by the farmers.[22]

The restrictions on phosphorus use posed by the current agri-environmental program are differentiated. The differentiation is done according to soil fertility analysis which classifies the soil into seven categories. The categories correspond with different ranges of STP levels.[23] For each category, the agri-environmental program poses fertilizer constraints. These define the category of the soil in the steady state. If the true transition function is (45), the farmers are allowed to apply fertilizers according to the private optimum. The private optimum (STP 7.9, fertilization 20.5 kg ha^{-1}) falls within the category 3 (STP range 4-8) where the maximum allowable fertilization level is 28 kg ha^{-1}. Also the private optimum derived using (47) falls within the allowable range (27.5 kg ha^{-1}). According to our results, the program does not restrict the use of phosphorus fertilizers in crop production.

[22] The high attendance might also indicate that the program is too generous. With certainty, the program has some adverse external margin effects. Whether these outweigh the environmental benefits attained in the intensive margin can not be assessed within our framework.
[23] STP is the sole determinant of the soil fertility class only for a fixed soil type and pH value.

However, the suggestion for a new Finnish agri-environmental program for 2007-2013 has more binding restrictions. According to compulsory actions of the program, the maximum quantity for the range of our (private and social) optima will be 22 kg ha^{-1}.[24] Again, the phosphorus use at the private optimum would not be restricted by the program if the transition process would be captured by (45). However, if it would follow (47) the steady state level of STP would fall into the second lowest category which would be below both the private and the social optimum. In the second lowest category, the maximum allowable application is 26 kg ha^{-1} which would correspond to a steady state STP level 6.0 which again would tighten the restriction into 22 kg ha^{-1}.

The restrictions suggested in the new agri-environmental program correspond to steady state STP levels and allowable phosphorus applications which are not very far from our results. The program's main purpose is to reduce the phosphorus fertilization in the fields with the highest STP levels. If the restrictions of both the current and the suggested new program are followed, this will eventually take place.

Technically, it would seem more sensible to use continuous values with certain confidence intervals instead of categories, when defining the restrictions. The ranges are presumably used because the readily available information is given in the form of fertility class instead of a continuous STP value.

Another implication is that the use of the VFSs should be strongly differentiated. Currently, the compulsory requirements for VFS construction are related to characteristics of the receiving water body instead of those of the field. According to our study, however, this is both ineffective and expensive. Focusing the measures to control for the PP-loss in the field surface should be distinctively targeted at the steepest parcels. Due to the inexpensive and readily available information on the parcel slopes, this should be easy to do. However, the inclusion of nitrogen removal provided by the VFSs, as well as their other potential welfare effects would affect their efficiency.

[24] The suggestion provides also subsidies for more stringent reductions.

Finally, it might be beneficial to focus the phosphorus use control directly on STP. Regular soil tests are already taken from most of the fields due to our previous and ongoing agri-environmental programs. In these programs, the allowable fertilization ranges are conditioned on the STP levels. Hence, the information requirements would be reduced in the direct control. Furthermore, the uncertainty about the transition processes would not affect the outcomes so much. We have seen that the steady state outcomes are affected by the true transition function. If, for instance, the social planner would find it justified to strive for an STP level of 7, the instruments could be conditioned directly on this. The farmers could then choose freely the levels of annual phosphorus fertilization. As the DRP-loss is related to the STP levels and not on fertilization, this might improve the efficiency of the regulation.

8.3 Limitations of the study and suggestions for further studies

The two major limitations of the study are confining the analysis on the steady state, and focusing on homogeneous soil. Particularly the analysis on instruments is evidently incomplete before the paths towards the steady states are properly examined. However, conducting first the stationary analysis for a homogeneous soil is a necessary starting point.

Then, allowing only for intensive margin effects restrains the analysis. We saw that setting aside the land from production might be welfare improving. Scrutinizing this issue, however, would require a framework with endogenously modelled options to switch the crops, set land aside from production or increasing the area under cultivation.

Analyzing only one of the two major nutrients causing eutrophication is naturally limiting. The interlinkages of phosphorus and nitrogen abatement measures would enhance the scope of the analysis. However, there is a large number of water bodies for which the reduction of external phosphorus load remains to be crucial. If the nutrient control policies could be differentiated basin-wise – as suggested for instance by the EU Water Framework Directive – we would be needing analysis focusing only on phosphorus control, only on nitrogen control, and on the control of both of the nutrients.

At the empirical side, the controversial nature of the STP measure poses some limitations on the applicability of the results. The STP measure developed for Finnish conditions (Vuorinen and Mäkitie 1955) is not a perfect estimate of soil fertility in terms of phosphorus. Also, basing the DRP-loss estimates only on STP level is problematic if the soil types vary. However, it seems evident that keeping other things fixed, both the DRP-loss and crop yields are positively correlated with the level of STP.

Also, the use of the STP level as a tax base has at least two difficulties. Firstly, the tests for estimating the STP levels produce results with a fairly large range. Peltovuori (1999) found that commercial laboratories analyzing identical soil samples reported STP with a high variation. However, the uncertainties due to the use of this measure are already present in the Finnish agri-environmental program. Hence, posing the control directly on STP would not alter the situation in this sense.

Secondly, obtaining reliable estimates on average STP is difficult because the results would be different in different parts of the same field. If there would be sufficiently many randomly selected samples this would not be a problem. However, when facing a tax conditioned on STP, there would be an incentive for the farmer to supply the sample(s) from the parts of the field known to be poor in phosphorus. That is, the issue of moral hazard should be taken into consideration if planning actual policies. Then again, the policy makers are confronted with this issue already under the current regulation.

Some of the explicit functions of our empirical model could be more accurate. The PP-loss and abatement are estimated using data generated by a simulation model. Although the model can mimic the stochasticity of rainfall, we know that the estimates for VFS abatement are very uncertain. For instance, our abatement function suggests that the percentage abatement of the VFS is insensitive for the variation in the incoming load. This can not hold for the narrowest VFSs and/or for the most intense rainfall periods which are the most important individual events contributing to total PP-loss. This is a serious drawback, and unfortunately very difficult to overcome. Also, the possible increase of DRP-loss is a serious disadvantage for the use of VFS, at least in Finnish conditions (see for instance, Uusi-Kämppä and Kilpinen (2000)). Finally, the phosphorus response function we have used is not ideal. However, as our steady state analysis did not focus on the quantitative welfare effects, only the derivatives of the phosphorus response function affected the main results; except for the effects of the absolute crop yield levels on phosphorus balance. Altogether, the limitations of the empirical model can be seen as separate of those of the theoretical model. Consequently, the robustness of the theoretical results is not affected by the limitations of the empirical part.

For further studies, an obvious extensions would be the analysis of other soil types, and a dynamic analysis evaluating the paths as well as the steady states. Also, introducing stochasticity would bring about important views, especially on the control of PP-loss. It would also be important to analyse extensive margin effects, both in terms of crop choices and increasing/decreasing the total acreage of farmland.

Expanding the analysis for areas of animal husbandry and the use of manure as a fertilizer would be important. It would be interesting to analyze whether a direct control of the STP levels seemed to be efficient there as well. However, the initial STP values in the areas of intense animal husbandry are so high that including the paths into the analysis would be absolutely necessary.

Perhaps the most interesting branch for further research is the analysis of information requirements under different tax bases. Monitoring the STP values is associated with asymmetric information and incentives for false reporting. It would be a useful exercise to model endogenously the required monitoring probabilities, penalty schemes, tax rates etc. that would induce the farmers to report their STP values truthfully.

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Appendices

Appendix 1. Deriving the steady state conditions with one choice variable.

To derive the first order conditions from the Bellman equation (8) we first derive the optimality condition for the value function: $\pi_x + \beta V_\Gamma \Gamma_x = 0$. Acknowledging that $\Gamma(s^*,x^*) = s^*$ at the steady state, we can write: $\pi_x + \beta V_s \Gamma_x = 0$. Since we choose x optimally, we can apply the Envelope theorem and differentiate the Bellman equation w.r.t. s, yielding: $V_s = \pi_s + \beta V_s \Gamma_s$. Solving the optimality condition for V_s and plugging it in yields the equation (10a) which together with the stationary condition and the complementary conditions constitute the optimality conditions.

Appendix 2a. Comparative statics on the private optimum.

For clarity, denote the optimality conditions (10a) and (10c) as F and G, respectively:

$$F \equiv \pi_x(s^*, x^*, A^*)(1 - \beta\Gamma_s(s^*, x^*)) + \beta\pi_s(s^*, x^*, A^*)\Gamma_x(s^*, x^*) = 0$$

$$G \equiv s^* - \Gamma(s^*, x^*) = 0$$, where:

$\pi_x = py_x - w$ A2.1

$\pi_s = py_s$ A2.2

$\Gamma_x = v(1 - \delta y_x)$ A2.3

$\Gamma_s = \chi - v\delta y_s$ A2.4

$\Gamma = \chi s + v(x - y\delta)$ A2.5

Take total differentials of F and G w.r.t the choice variables and our parameters of interest: p, w, β, χ, v and δ. First, dF:

$$D(py_x - py_x\beta\chi - w + w\beta\chi - w\beta v\delta y_s + \beta py_s v) =$$
$$(py_{xx} - py_{xx}\beta\chi - w\beta v\delta y_{sx} + \beta py_{sx}v)\mathrm{d}x + (py_{xs} - py_{xs}\beta\chi - w\beta v\delta y_{ss} + \beta py_{ss}v)\mathrm{d}s +$$
$$[y_x(1 - \beta\chi) + \beta y_s v]\mathrm{d}p + (-1 + \beta\chi - \beta v\delta y_s)\mathrm{d}w + (-py_x\chi + w\chi - wv\delta y_s + py_s v)\mathrm{d}\beta +$$
$$(-py_x\beta + w\beta)\mathrm{d}\chi + (-w\beta\delta y_s + \beta py_s)\mathrm{d}v + (-w\beta v y_s)\mathrm{d}\delta \equiv$$

$$F_x\mathrm{d}x + F_s\mathrm{d}s + F_p\mathrm{d}p + F_w\mathrm{d}w + F_\beta\mathrm{d}\beta + F_\chi\mathrm{d}\chi + F_v\mathrm{d}v + F_\delta\mathrm{d}\delta \qquad \text{A2.6}$$

Then, dG:

$$\mathrm{D}(\mathrm{s} - \chi s - v(x - y\delta)) =$$

$$[v(y_x\delta - 1)]\mathrm{d}x + (1 - \chi + vy_s\delta)\mathrm{d}s + (-s)\mathrm{d}\chi + (x - y\delta)\mathrm{d}v + (yv)\mathrm{d}\delta \equiv$$

$$G_x\mathrm{d}x + G_s\mathrm{d}s + G_\chi\mathrm{d}\chi + G_v\mathrm{d}v + G_\delta\mathrm{d}\delta \qquad \text{A2.7}$$

The signs of F_i and G_i are:

F_x	F_s	F_p	F_w	F_β	F_χ	F_v	F_δ	G_x	G_s	G_χ	G_v	G_δ
(–)	(–)	(+)	(–)	(+)	(+)	(+)	(–)	(–)	(+)	(–)	(–)	(+)

Defining these signs requires the following to hold, together with the assumptions on the crop yield function and the posed limits on parameter values:

- d$p > w\delta \Leftrightarrow yp > wy\delta$; that is: the revenues have to exceed the value of phosphorus uptake, evaluated with the price of phosphorus fertilizer. If the condition did not hold, crop production would not be meaningful: even if all other costs would be zero and there would be a perfect one-to-one relation in the applied and uptaken phosphorus, the farmer would be making losses.
- $w > py_x$; see the two-period example
- $y_x\delta < 1$; marginal effect of phosphorus input on phosphorus output less than unity.
- $x - y\delta > 1$; the phosphorus balance is positive at the steady state.

Holding all exogenous variables except for *i* constant, the total differential becomes:

$$F_x\mathrm{d}x + F_s\mathrm{d}s + F_i\mathrm{d}i = 0 \qquad \text{A2.8}$$

$$G_x\mathrm{d}x + G_s\mathrm{d}s + G_i\mathrm{d}i = 0 \qquad \text{A2.9}$$

Rearrange to obtain:

$$\begin{cases} -\frac{F_x dx}{di} - \frac{F_s ds}{di} = F_i \\ -\frac{G_x dx}{di} - \frac{G_s ds}{di} = G_i \end{cases} \Leftrightarrow \begin{bmatrix} F_x & F_s \\ G_x & G_s \end{bmatrix} \begin{bmatrix} \frac{dx}{di} \\ \frac{ds}{di} \end{bmatrix} = \begin{bmatrix} -F_i \\ -G_k \end{bmatrix} \qquad \text{A2.10}$$

Apply the Cramer's rule for the signs of d*x*/d*i* and d*s*/d*i*:

$$\frac{dx}{di} = \frac{\begin{vmatrix} -F_i & F_s \\ -G_k & G_s \end{vmatrix}}{\begin{vmatrix} F_x & F_s \\ G_x & G_s \end{vmatrix}} \quad \text{and} \quad \frac{ds}{di} = \frac{\begin{vmatrix} F_x & -F_i \\ G_x & -G_i \end{vmatrix}}{\begin{vmatrix} F_x & F_s \\ G_x & G_s \end{vmatrix}} \qquad \text{A2.11}$$

The sign of the denominator is (–). Plugging the respective partial derivatives into (A2.11) yields directly the signs of $\frac{dx}{dp}$, $\frac{dx}{dw}$ and $\frac{dx}{d\beta}$, as well the signs of $\frac{ds}{dp}, \frac{ds}{dw}, \frac{ds}{d\beta}, \frac{ds}{d\chi}, \frac{ds}{dv}$ and $\frac{ds}{d\delta}$.

The sign of the numerator in $\frac{dx}{d\chi}$ is determined by: $-(F_\chi G_s) + (G_\chi F_s) =$

$py_x\beta - \chi py_x\beta + py_x\beta vy_s\delta - w\beta + w\beta\chi - w\beta\, vy_s\delta - spy_{sx} + spy_{sx}\beta\chi + sw\beta v\delta y_{ss} - s\beta py_{ss}v =$ $\beta[py_x - \chi py_x + py_xvy_s\delta - w + w\chi - w\, vy_s\delta + spy_{sx}\chi + swv\delta y_{ss} - spy_{ss}v] - spy_{sx} =$ $\beta[(py_x - w)(1 - \chi + vy_s\delta) + svy_{ss}(w\delta_s - p)] + spy_{sx}(\beta\chi - 1)$. Hence, the sign remains ambiguous but we can characterize it using the expression above.

The sign of the numerator in $\frac{dx}{dv}$ is determined by: $-(F_vG_s) + (G_vF_s) =$

$-\beta y_sp + \beta y_sw\delta + \beta y_sp\chi - \beta y_sw\delta\chi - \beta y_spvy_s\delta - \beta y_sw\delta\chi vy_s\delta + (x - y\delta)(py_{sx} - py_{sx}\beta\chi -$ $w\beta v\delta y_{ss} + \beta py_{ss}v) = (w\delta - p)[\beta y_s(1 - \chi + vy_s\delta) + (x - y\delta)(-\beta vy_{ss} - y_{sx}(1 - \beta\chi))]$. Now, both terms in brackets are positive; the sign of the numerator is (–) and hence the overall sign is (+).

The sign of the numerator in $\frac{dx}{d\delta}$ is determined by: $-(F_\delta G_s) + (G_\delta F_s) =$

$-w\beta v y_s + w\beta v y_s \chi - w\beta v y_s v y_s \delta + \ yvpy_{sx} - yvpy_{xs}\beta\chi - yvw\beta v\delta y_{ss} + yv\beta p y_{ss} v =$

$w[\beta v y_s(-1 + \chi - v y_s \delta)] - yvw\beta v \delta y_{ss} + \ yp[v y_{sx}(1 - \beta\chi) + v\beta y_{ss} v].$

Now, because $w\delta < p$, it follows that $yvw\beta v\delta y_{ss} < ypv\beta y_{ss}v$. Therefore, the sign of the numerator is (–) and hence the overall sign is (+)

Appendix 2b. Comparative statics of the social optimum.

The optimality conditions are:

$$SW_x(1-\beta_s\Gamma_s) + \beta_s SW_s \Gamma_x = 0 \qquad \text{A2.12}$$

$$SW_b(1-\beta_s\Gamma_s) = 0 \Leftrightarrow SW_b = 0 \qquad \text{A2.13}$$

$$s^* = \Gamma, \qquad \text{A2.14}$$

Taking the total differential on the above conditions shows that we are not able to determine the sign of the determinant of the denominator in Cramer's rule. Hence, all signs would remain ambiguous in the analytical comparative statics.[25]

Comparative statics of the VFS optimality condition

Assume that the VFS construction is the only choice variable. This illuminates our PP-loss and abatement functions and provides some background for the oncoming analysis on second-best instruments. We consider changes in input and output prices (w,p), in the slope and width of the parcel (γ,l) and the bioavailability coefficient (ζ).

Assume that the DRP-loss as well as the PP-loss in drainage are zero. Also, assume a linear damage function with a slope coefficient $\eta = 1$. The optimality condition, *if constructing VFSs were the only choice variable* is given by:

$$SW_b = - A_b[py - wx] - f_b - \eta L_b \ = \ -lpy + lwx - X - L_b = 0, \qquad \text{A2.15}$$

[25] Already the sign of the partial derivative of (A2.12) w.r.t x is ambiguous without heavy assumptions. Note also that in a dynamic programming problem we do not have the 'usual' second order conditions, i.e. negative (semi)definiteness of the Jacobian matrix, which would aid in determining the signs. The existence and uniqueness of the steady state together with the first-order conditions and the Envelope theorem solve, by definition, for the maximum.

Taking a total differential on (A2.15) we obtain: $-L_{bb}\mathrm{d}b + (-py + wx - L_{bl})\mathrm{d}l - L_{b\gamma}\mathrm{d}\gamma - L_{b\zeta}\mathrm{d}\zeta - ly\mathrm{d}p + l\mathrm{d}w - (lpy_s + L_{bs})\mathrm{d}s + lw\mathrm{d}x = 0$.

The marginal changes are:

$$\frac{db}{dp} = \frac{-ly}{L_{bb}} = \frac{-ly}{\zeta\left(\underbrace{-z_{b}(l(r_0+1))}_{+} + \underbrace{z_{bb} r_0(1-bl)}_{+}\right)} = (-)$$

$$\frac{db}{dw} = \frac{l}{L_{bb}} = \frac{l}{(+)} = (+)$$

$$\frac{db}{d\xi} = \frac{-L_{b\zeta}}{L_{bb}} = -\frac{-lr_0 z + z_b r_0(1-bl) - lr_{0\zeta} z - lr_0 z_\zeta + z_{b\zeta} r_0 (1-bl) + r_{0\zeta} z_b (1-bl)}{(+)} = (+)$$

$$\frac{db}{dl} = \frac{-py + w - \zeta r_0\left[(-z) + (-z_l l) + z_{bl}(1-bl) + (-bz_b)\right]}{L_{bb}} = \frac{(-/+)}{(+)} = (-/+)\,.$$

The last sign depends partly on the profitability of farming, partly on the unidentifiable sign of L_{bl}. Inside the brackets we have two negative and two positive terms. If the third and the fourth term dominate the first and the second, the equation in brackets is positive, and the overall sign is negative. Also, if the equation in brackets is negative but the profits are adequately high, the overall sign is (-). Only if the profits are close to zero the sign might be positive. Most probably, the sign is (-).

$$\frac{db}{d\gamma} = \frac{-L_{b\gamma}}{L_{bb}} = \frac{-\zeta\left[r_{0\gamma}(1 - l(b+1) - lr_0 z_\gamma + (1-bl) z_{b\gamma} r_0\right]}{(+)} = (+)$$

Whenever $l(b+1) > 1$, the overall sign is definitely (+). Even if it would be that $l(b+1) < 1$, we may assume that the two last terms in the brackets dominate the first one.

Appendix 3. Selected partial derivatives of PP-load and PP-loss functions.

PP-load (r), omitting the partial derivatives equal to zero:

$$r_{ll} = \underbrace{\frac{r_0 A_l}{l^2}}_{+} + \underbrace{\frac{r_0 A_l}{l^2}}_{+} + \underbrace{\frac{2r_0(1-A)}{l^3}}_{+} > 0$$

$r_l < 0$; $r_{lb} < 0$

PP-loss (L^{PP}), omitting the partial derivatives equal to zero:

$$L_l^{PP} = z_r r_l r_0(\cdot)(1-A) - z(\cdot) r_0(\cdot) A_l < 0$$

$$L_{ll}^{PP} = z_r r_{ll} r_0(\cdot)(1-A) - z_r r_l r_0(\cdot) A_l > 0$$

Appendix 4. The soil type of the target area.

The soil type is sandy clay which has the following shares of following textures:

Particle size fraction	**Percentage value**
Clay (<0.002mm)	30-60
Silt (0.002-0.02mm)	<20
Fine sand (0.02-0.2mm)	20-70

This soil type is one of the typical clay soils in Finland.

Appendix 5. The costs of farming.

Following fixed and variable costs were included in *C*, adopted from Lankoski et al (2006). The price of nitrogen is 1.2 € $kg^{-1}$and it is assumed to be applied 105 kg ha^{-1}. We use the LFA subsidy of the A category:

Cost category	**€ ha^{-1}**
Fixed costs	138.9
Variable costs:	
Tractors	19.5
Machinery	22.4
Labour	50.8
Seeds	40
Nitrogen	126
LFA subsidy	-150
Total costs	248.6

Appendix 6. Particulate phosphorus loss is surface runoff (37).

The PP-loss was defined in the analytical model as (equation 15):

$$L^{PP} = l * r(\gamma,l,b) * z(b,r(\cdot)) + L^{drain}, \qquad \text{A6.1}$$

where the PP-loss in surface runoff is the first term in the right hand side. It consists of three factors: $r(\gamma,l,b)$ defines the PP-load entering a point on a VFS as a function of the slope and the area under cultivation; $z(b,r(\cdot))$ is the fraction of the load that is not filtered by the VFS and l is the length of the VFS. In the theoretical model, the abatement percentage was assumed to be negatively correlated with the amount of erosion entering the VFS, hence the point-wise definition of the function.

The empirical counterpart is constructed from two sources. The relation of the slope and the erosion is taken from the ICECREAM simulation data. The retention of eroded materials is determined on the basis of the actual data from Uusi-Kämppä and

Kilpinen (2000), and simulation data for similar runoff values. The reason for this is that the empirical data left the interlinkages between the intensity of the erosion entering the VFS and the corresponding abatement percentage ambiguous. The simulation data suggested that the percentage reduction of eroded materials increases as the amount of incoming load increases, even for the narrowest VFSs. This contradicts with the theoretical model; and it is not supported by the measurement data provided by Uusi-Kämppä and Kilpinen (2000).

Uusi-Kämppä and Kilpinen (2000) provide a report on VFS abatement over a period of nine years. They report (among other things) the absolute values of surface runoff on monthly basis. The average concentrations of eroded materials in runoff waters are reported for three periods: spring, summer and autumn. All values are reported for six parcels, four of them having a 10 meter wide VFS. Two of these VFSs are covered with grass vegetation. For these VFSs, and for the two parcels without a VFS, we averaged and combined the values for erosion and the percentage retention.

The erosion reducing effect of the VFS in Uusi-Kämppä and Kilpinen (2000) – as well as in the present study – is comprised mainly of two effects. Firstly, the VFS reduces, or even removes, the erosion risk from the part of the field it covers. Secondly, it filters eroded material from runoff waters. However, Uusi-Kämppä and Kilpinen (2000) stress that these data does not distinguish between these two effects.[26]

At the target parcels of Uusi-Kämppä and Kilpinen (2000), the distance between the rear edge of the field and the water is 70 meters. Hence, a 10 meter wide VFS covers more than 14% of the parcel area. The VFS is located on a very steep part of the field (10-20%) whereas the slope in the remaining parts of the field is only few percentages. The authors therefore presume that the effect of erosion risk reduction *within* the strip plays a large role in overall retention. That is, the erosion on parcels with no VFS took mostly place on the last 10 meters of the parcels, exactly where the VFS was constructed on the other parcels. Therefore, if we were to regress the

[26] They did collect data from subsurface flow at the depth of 0.2 meters from various parts on the VFS which would enable the comparison. However, the authors considered these data unreliable due to difficulties in determining the absolute amount of runoff.

retention percentage against the values of absolute erosion on parcels with no strips, the coefficients would be (heavily) biased towards decreasing the negative effect of the load on the reduction percentage; or even making it positive.

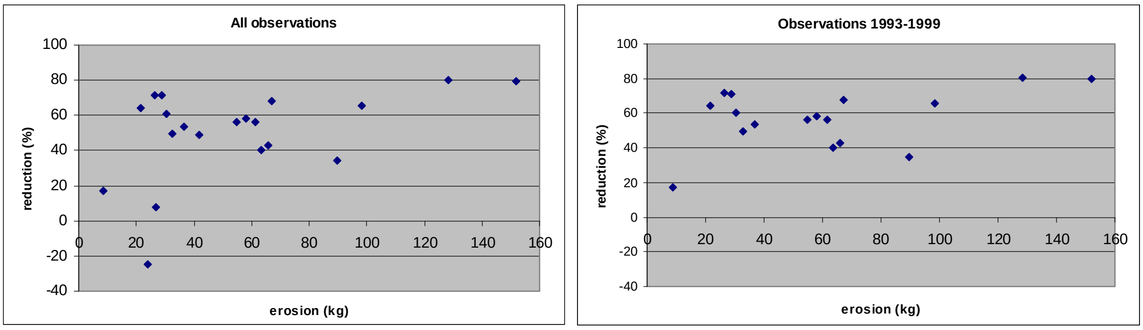
Table A6.1 presents average values for eroded material from the two parcels without a VFS, and the retention percentages from spring 1991 to autumn 1999.

Table A6.1. Erosion and retention.

Erosion (kg)	41.8	24.1	26.6	89.6	8.7	32.6	63.4	152.0	29.0	30.4	21.6	128.2	54.7	26.5	61.5	98.3	36.7	67.2	65.8	57.9
Retention (%)	48.9	-24.6	7.7	34.4	17.2	49.5	40.1	79.6	71.2	60.5	64.4	80.3	56.1	71.7	56.3	65.6	53.7	68.0	42.8	58.3

In a figure below, we plot these values against each other in two panels. The leftmost panel plots all values and the rightmost omits the first three values; according to authors, the grass strips did not operate properly during the first two autumns:

Figure A6.1. Erosion and retention.



Regressing the percentage reduction of eroded material against the absolute values of erosion in parcels with no VFS is not meaningful statistically. The coefficient for erosion (0.17) was statistically not significant at 5% level and the R^2 of the model including all observations is below 0.2 (omitting the first three observations drops R^2 to 0.07). And most importantly, the lack of proper counterfactual biases the estimates for the filtering effect of the strip. To be able to estimate this effect, one would need erosion data before and after the VFS and corresponding data from the parcels with no strips.

Altogether, based on the data from Uusi-Kämppä and Kilpinen (2000) one can not detect the effect the erosion load entering the VFS has on the abatement percentage.

However, one is able to say that the effect is not likely to be positive: despite a strong bias there was no statistically significant positive correlation. Presumably, for a given VFS width, the increase in erosion load decreases the percentage reduction but increases the absolute reduction. Due to lack of empirical data, however, we are imposed to assume that the amount of erosion entering the VFS *has no effect* on the percentage reduction of eroded material, and hence on PP-loss.

To derive the filtering effect of a VFS, we use the data in table A6.1 (excluding the three first observations) and the simulation data with the erosion load values closest to these, and regress the reduction percentage against the VFS width. The average total erosion from the parcels with no strip in Uusi-Kämppä and Kilpinen (2000) was 1156 kg ha^{-1}. The simulation data with erosion load of 1894 kg ha^{-1} for a field with no strip as closest to this. It consisted of the following observations:

Table A6.2. VFS width and retention in the ICECREAM model.

VFS (m)	**Retention (%)**								
1	32.2	52.7	37.0	41.6	53.5	35.8	26.9	52.0	52.6
3	40.0	53.8	39.6	43.7	56.4	41.2	29.8	55.2	54.1
15	58.8	58.9	50.8	49.9	63.4	60.6	42.4	68.5	58.0

Hence, together with the values in table A6.1 for a VSF of 10 meters, we have eventually 46 observation with four different VFS widths 1, 3, 10 and 15 meters. A logarithmic transformation of VFS width (+1) provides the following estimate for percentage reduction of eroded material as a function of the log of VFS width:

$$re = 25.57\ln(b+1). \qquad \text{A6.2}$$

The standard error of the coefficient is 1.3, i.e. it is highly significant. The R^2 of the regression is 0.89. Equation A6.2 thus determines the percentage reduction of eroded material – and in our model the percentage abatement of PP-loss – that enters the VFS in surface runoff. Given the nature of the data by Uusi-Kämppä and Kilpinen (2000) it is justified to think that for a VFS of 10 meters wide, the *percentage filtering of the* *strip can not exceed this value*. Most likely, it overestimates the filtering power for any given VFS width.

The following function eventually determines the PP-loss from parcels with various slopes and VFS widths:

$$L^{PP} = 0.244\gamma^2 (1 - \frac{25.57 \ln(b+1)}{100}) + L^{drain}, \qquad \text{A6.3}$$

In the empirical model, the PP-loss in surface runoff is thus defined as a product of the PP-load entering the VFS and the fraction of the load not filtered by the strip. That is, the length of the field edge adjacent to water is no longer affecting the PP-loss as suggested by the theoretical formulation. The shape of the parcel thus does not affect PP-loss in the empirical model. However, it continues to affect the phosphorus abatement costs.

Even though we were able to say something about the magnitude of filtering percentage estimated by A6.2 in comparison to the ‘true’ reduction percentage, we are not able to draw the associated conclusions from A6.3. This is because we do not know whether the simulation model under or overestimates the erosion for given slopes. Compared to absolute erosion values of Uusi-Kämppä and Kilpinen (2000), for instance, the values predicted by the simulation model seem fairly high. This might suggest, for instance, that the PP-loss function A6.3 is biased upwards for some parcels and downwards for others.

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