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Games with Externalities:  
The Case of the Baltic Sea Cod Fishery**



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# Sharing Rules and Stability in Coalition Games with Externalities:

## The Case of the Baltic Sea Cod Fishery

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**Abstract** *The paper studies cooperative sharing rules in fisheries coalition games and develops a new sharing rule which takes into account the stability of cooperation when externalities are present. The paper contributes to the literature by introducing a connection between cooperative games (sharing rules) and non-cooperative games (stability). For an illustrative example it sets up a discrete-time, deterministic, coalition game model among the major agents exploiting the cod stock in the Baltic Sea.*

**Key words** fisheries, cooperation, coalition game, Baltic Sea cod, Shapley value, characteristic function, nucleolus, sharing rules, stability of cooperation

**JEL codes** C62, C70, Q22, Q28

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## **1 Introduction**

What distinguishes a non-cooperative game from a cooperative game is the ability to make binding agreements. The non-cooperative game is often referred to as the competitive game, where the players act out of rational self-interest and they cannot communicate prior to the game. In a cooperative game the players have a binding agreement and the objective is to maximize the joint payoff of the game. The coalition game allows for a group smaller than all players (a coalition) to cooperate.

Non-cooperative games have the advantage that they are stable in nature if players are adapting their best response strategies. For cooperative and coalition games the stability has to be evaluated after the solution to the game is determined and is affected by the way the benefits inside the cooperation is shared among players. To evaluate whether a solution is stable or not requires determination of the distribution of benefits in the coalitions which is done by the characteristic function game and different sharing rules but it also requires determination of the solution to the non-cooperative game. The theory of games in coalitional form is not fully satisfactory, as it ignores the possibility of externalities. This means that typically the action available to a coalition is assumed to be independent of the actions chosen by non-members (Greenberg 1994). This paper deals with extraction of a renewable resource by more agents and thus externalities are present, therefore the paper reconsiders what a credible threat is. The ability to make credible threats is the basic premise of most solution concepts in games in coalitional form (Greenberg 1994).

The main motivation for this paper is a merger between non-cooperative (stability) and cooperative games (sharing rules). We define this merger as a coalition game. In the literature so far, the stability of the sharing rules as nucleolus or the Shapley value is unambiguous since they are not always stable to free riding when externalities are present. In fisheries coalition games this link

between non-cooperative games and cooperative games has not been studied, or has received too little attention.

The cooperative game approach is based on the fundamental assumption that players have already agreed to cooperate and that the model allows for transferable utility. The main references in setting up a model of cooperative game of fisheries include Kaitala & Lindroos (1998). They set up a cooperative game in characteristic function game framework and determine different one-point cooperative solution concepts. The model does, however, not take into account the externalities in the fisheries and it is a purely theoretical work.

Previous empirical work has included applying the coalition game approach to the Norwegian Spring-spawning Herring (Lindroos & Kaitala (2000) and Arnason, Magnusson & Agnarsson (2000)) and to the Northern Atlantic Bluefin Tuna (Duarte, Brasão & Pintassilgo 2000). These empirical studies do, however, not consider the important connection between the applied cooperative sharing rules and the stability of cooperation when externalities are present; therefore none of the determined sharing rules actually satisfy all the players. Brasão, Costa-Duarte & Chunha-e-Sá (2000) applies the coalition game to the Northern Atlantic Bluefin Tuna. They recognize the instability of the Shapley value due to free rider incentives; they find a stable non-cooperative feedback Nash with side payments, but are not able to determine the connection between the joint solution and its stability.

Pintassilgo (2003) sets up a theoretical work using a partition function approach and shows that the grand coalition is stand-alone stable if and only if no player is interested in leaving the cooperative agreement to adopt free rider behaviour. However, Pintassilgo does not study sharing of benefits.

The main contribution of this paper is the development of the new sharing rule which takes into account the stability of cooperation. It is also taken into account that externalities are present in

the fishery, since the strategy chosen inside a coalition also depends on the strategy of the players outside the coalition. In addition the paper also contributes to the literature by allowing all members of a coalition to be active in the fishery until the marginal benefits of the different technologies in the coalition are identical. In some of the previous work cost functions have been linear and only the most efficient coalition members have been active.

Section 2 describes the underlying bio-economic model and applied parameter values. Section 3 introduces the game theoretic setup, solves the game and comes to a solution for the inadequacies of the Shapley value and the nucleolus. It further discusses the stability of the sharing functions and defines a core which is stable to free riding. Section 4 involves a sensitivity analysis and section 5 concludes the paper.

## **2 The Bio-Economic Model**

### **2.1 The Baltic Cod**

The Baltic Sea is a sea shared among members of the European Union (EU) (Denmark, Finland, Germany, Sweden, Estonia, Latvia, Lithuania and Poland) and the Russian Federation. The Baltic Sea consists of the central Baltic Sea, the Gulf of Bothnia, the Gulf of Finland, the Sound and the Danish straits. It is a fairly remote area and it contains no international waters.

The most valuable fishery in the Baltic Sea is the cod fishery which is managed by the International Baltic Sea Fishery Commission (IBSFC).<sup>1</sup> All the parties exploiting the cod stock are members of the IBSFC who sets the total allowable catches (TACs) for the fishery.<sup>2</sup> Seemingly there is a coalition since TAC measures are agreed upon among all contracting parties in the IBSFC. The fact is, however, that TAC measures are often exceeded, thus free riding in the coalition exists. In 65% of the 20 years with TAC, from 1977 to 2003, catches exceeded the TACs.<sup>3</sup> In years where TACs are not exceeded catches has been close to TACs. In addition to the reported

landings it is well known that there exist illegal landings and discards too. Since all countries exploiting the Baltic Sea cod fishery are contracting parties of the IBSFC and the main objective of the fishery commission is ‘to cooperate closely’<sup>4</sup> we find it natural to apply a cooperative approach for the analysis. For example, Kronbak & Lindroos (2003) show that coalitions in the Baltic Sea cod fishery should be encouraged.

## 2.2 Population Dynamics

The population dynamics are described by a discrete time age-structured model. This is a standard type of cohort-model, where the numbers are determined as follows:

$$\begin{aligned}
 N_{2,y} &= R_y & y > y_1 \\
 N_{a+1,y+1} &= N_{a,y} e^{-m_a - S_a f_y} & a \in \{2,3,\dots,7\}, \\
 N_{a,y_1} &\text{ known} & a \in \{2,3,\dots,8\}
 \end{aligned} \tag{1}$$

where  $R_y$  describes the recruitment into the stock in year  $y$ ,  $m$  is the natural mortality,  $f$  is the total fishing mortality and  $S_a$  is the selectivity of the fishing year, that if an age class is not harvested, then the selectivity is zero, otherwise it is one. It is assumed that the initial abundance for all age classes in year  $y_1$ ,  $N_{a,y_1}$ , is known. The population dynamics is determined by 7 age classes,  $a=\{2,3,\dots,8\}$ . These age classes are chosen in accordance to the measure by the International Council for the Exploration of the Sea, abbreviated ICES, (ICES 2000) in which the recruits are aged 2 years before they become a part of the stock.  $y_1$  is the initial year for the simulation model. The biomass is determined as the sum of the number of fish multiplied by their stock weights at age over all age classes:

$$B_y = \sum_{a=2}^8 SW_a N_{a,y} , \tag{2}$$

where  $SW_a$  is the stock weights at age and  $B_y$  is the biomass in year  $y$ . The total spawning stock biomass is given by the sum of mature fish over all age classes:

$$SSB_y = \sum_{a=2}^8 MO_a SW_a N_{a,y}, \quad (3)$$

where  $MO_a$  is the proportion of mature fish in age class  $a$  and  $SSB_y$  is the spawning stock biomass in year  $y$ . We assume a Beverton-Holt stock-recruitment relationship, identical to the one used by ICES (2000), which is defined as follows:

$$R_y = \frac{cSSB_{y-1}}{1 + bSSB_{y-1}}, \quad (4)$$

where  $c$  and  $b$  are biological recruitment parameters;  $c$  is the maximum recruits per spawner in a low spawning stock size and  $c/b$  is the maximum number of recruits when the spawning stock biomass is very large.<sup>5</sup> The biological parameters of the stock recruitment relationship and other parameter values are summarized in table 1.

**Table 1.**

Table 2 identifies the initial biological parameters for the year classes.

**Table 2.**

### 2.3 The Yield

The catch in numbers for country  $i$  and for a specific cohort is given by:

$$C_{a,y}^i = \frac{S_a f_y^i}{m_a + S_a f_y} (N_{a,y} - N_{a+1,y+1}), \quad (5)$$

where  $f_y^i$  is the fishing mortality by country  $i$  and  $f_y$  is the total fishing mortality,  $C_{a,y}^i$  is the catch in number by country  $i$ , in year  $y$ , of a specific cohort  $a$ . The catch function is defined as the numbers of fish that do not survive to the next year and are not subject to natural mortality.

The yield (harvest) for a single country is defined by inserting the number of fish (1) into the catch in numbers (5) times the catch weights at age:

$$Y_y^i = \sum_{a=2}^8 CW_a N_{a,y} \frac{S_a f_y^i}{m_a + S_a f_y} (1 - e^{-m_a - S_a f_y}), \quad (6)$$

where  $Y_y^i$  is the total yield in kg for country  $i$  in year  $y$ .

## 2.4 The cost function

The cost function is assumed to follow the cost function for harvesting cod in the North Sea for Denmark, Iceland and Norway (Arnason *et al.* 2000):

$$Q_y(t) = \alpha^i \frac{Y_y^i{}^2}{B_y}, \quad (7)$$

where  $\alpha^i$  is a cost parameter and  $B_y$  is the total biomass in year  $y$ . The dependent variable, costs, is defined as total costs less depreciation, interest payments and skipper wage. This may be regarded as an approximation of the total variable costs. The cost function is defined such that if the total biomass is increased the cod are easier to locate and costs are therefore decreased, the effect of other players also exploiting the stock is included in changes in the biomass. It is further important to notice that the costs increase exponentially with the yield. Therefore, in a coalition it is most likely that all countries joining are active, otherwise the coalition is not competitive to countries being singletons.

## 2.5 The net present value

The net present value is defined as the functional where the control variable is the fishing mortality for player  $i$ ,  $f^i$ ,<sup>6</sup> and the state variable is the total number of fish in the stock,  $N$ ,  $\pi^i$  is the instantaneous profit for player  $i$ :

$$\pi_y^i = pY_y^i - Q_y^i, \quad (8)$$



where  $\pi_y^i$  is the instantaneous profit for country  $i$  in year  $y$ . The net present value of all future profit for a single player  $i$  is defined by the functional:

$$J^i(f^i, N) = \sum_{y=y_1}^{y_2} \frac{\pi_y^i}{(1+r)^{y-y_1}}. \quad (9)$$

The players chose their optimal fishing mortality or strategy by maximising the functional  $J$ . The model is assumed to have a finite horizon from  $y_1$  to  $y_2$ . We have chosen a period starting from 1997 to 2046, this yields a running period of 50 years. The starting and the ending points of the time horizon are of no importance for the model, what matters is the length of the running period.

## 2.6 Economic Parameter Values

We assume there is an open market for raw fish where the fishermen are all facing the same price for their landings. Furthermore, the Baltic Sea is a comparatively small supplier of cod to a global white fish market in which there are many substitute species and thus the price fishermen are facing is constant. The price, which is applied in this model, is 10.74 Dkr/kg, which was the average price received on landed cod on the Island of Bornholm in 1998 and 1999 (Fiskeridirektoratet, 1999 and 2000). Bornholm is located in a fairly central position in the Baltic Sea.

Cost parameters are calibrated for the year 1998. This is done by finding cost parameters which yields a fishing effort and a total biomass population equivalent to the arithmetic mean over the period 1966 to 1999 when fishermen were adapting a non-cooperative behaviour. This calibration reveals cost parameters at 9, 14 and 15 Dkr/kg. Since the calibration method is connected with some uncertainty the cost parameter will be subject for a sensitivity analysis at the end of the paper. It is assumed that there is no technological progress etc. for the simulated 50 years and thus cost parameters and prices remain unchanged throughout the model. So does the functional relationships and there are no stochastic jolts in the system.

Country group 1 is assumed to be the most efficient and country group 3 is the least efficient. It is possible beforehand to conclude that, with this type of cost function all countries in a coalition will apply effort until their marginal costs are equivalent. Parameter values for the economic parameters are summarised in table 3.

### **Table 3.**

The fishing mortality applied by the three groups of countries is assumed to be constant over the whole simulation period.<sup>7</sup> Fishermen are committing to their strategies only at the beginning of the game; this is a sort of open loop control. The open loop controls allow the players less rationality and flexibility compared to the closed loop but computing these solutions are a lot easier. There has been a tendency in the literature to resort to the use of open loop solution concepts (Sumaila 1999). The game has complete information since fishermen know all payoff functions but imperfect information since the fishermen are moving simultaneously. In the first year of the game the players in a sense face two stages. The first stage involves making a decision of which coalition to join, in the second stage players determine which fishing mortality to apply. When the model is solved backwards this endogenises the coalition formation.

The following section solves the game by determining the characteristic function and some of the corresponding one-point solution concepts. It further discusses the stability of the sharing rules.

## **3 Solving the Game**

The approach applied in this paper is the coalition game approach. The normalised characteristic function is defined and applied for determining two one-point sharing rules, the Shapley value and the nucleolus. The sharing rules are compared to the free rider values and the cooperative solution applying the respective sharing rules is stand-alone stable if and only if there are no free rider incentives (Pintassilgo 2003). This condition does not always hold for the Shapley

value and the nucleolus. Therefore a stable sharing rule is developed by this model which endogenises coalition formation and thus searches for equilibrium cooperation structures. This is done by taking the free rider incentives into account in developing a new sharing rule. It is thus a merger of the non-cooperative and the cooperative games.

To solve the game the normalised characteristic function is determined. The values from this function are applied for determining the Shapley and nucleolus sharing rules. The sharing rules are compared to the free ride values. Finally the satisfactory nucleolus is determined.

### 3.1 The Characteristic Function

The characteristic function (c-function) is determined by applying the definition of the characteristic function described in Mesterton-Gibbons (1992), which are the benefits of cooperation associated with the coalition. That is the difference between the benefits when members form a coalition and the sum of benefits of individual members' e.g. individual players' threat points.<sup>8</sup> We define the characteristic function as follows:

$$\bar{v}(i) = J^i(F^i, N) - J^i(F^i, N) = 0, \quad i = \{1, 2, 3\}$$

$$\bar{v}(i, j) = J^{i,j}(F^{i,j}, N) - \sum_{i,j} J^i(F^i, N), \quad i = \{1, 2, 3\}, j = \{1, 2, 3\}, \quad i \neq j \quad (10a)$$

$$\bar{v}(1, 2, 3) = J^{1,2,3}(F^{1,2,3}, N) - \sum_{i=1}^3 J^i(F^i, N),$$

where  $\bar{v}(i)$  is the value of singletons,  $\bar{v}(i, j)$  is the value of a two-player coalition, remark  $\bar{v}(i, j) = \bar{v}(j, i)$ , and  $\bar{v}(1, 2, 3)$  is the value of the grand coalition, that is the maximum payoff by the joint action of all players. The strategies are denoted by a cap  $F^K$  indicating it is the strategies chosen when coalition  $K$  plays a Nash game (Nash 1951) against players outside the coalition.  $K$  refers to the 7 possible coalition structures<sup>9</sup>,  $K = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$ ,  $F^{1,2,3}$  denotes the

full cooperative strategy.  $J$  is the functional. We apply a  $\gamma$ -type c-function, since we assume players outside the coalition are adopting Nash-strategies against the coalition (Chander & Tulkens 1997). The function is normalized by dividing the characteristic function with the benefits of the grand coalition:

$$\begin{aligned}
 v(i) &= \frac{J^i(F^i, N) - J^i(F^i, N)}{\bar{v}(1,2,3)} = 0, \quad i=\{1,2,3\} \\
 v(i, j) &= \frac{J^{i,j}(F^{i,j}, N) - \sum_{i,j} J^i(F^i, N)}{\bar{v}(1,2,3)}, \quad i=\{1,2,3\}, j=\{1,2,3\}, i \neq j \\
 v(1,2,3) &= \frac{J^{1,2,3}(F^{1,2,3}, N) - \sum_{i=1}^3 J^i(F^i, N)}{\bar{v}(1,2,3)} = 1,
 \end{aligned} \tag{10b}$$

The normalized characteristic function,  $v$ , has the properties that the value for a grand coalition is 1 and the value for a singleton is zero. The optimal fishing mortalities are determined for all possible coalition structures to determine the characteristic function.

When fishermen form a coalition we assume they are rational and therefore distribute effort between participants until the marginal profits of members of the coalition is identical. This distribution yields the highest possible benefits from a coalition. Since we assume fishermen face an identical and constant price on the landed fish, we conclude it is optimal to distribute effort in the coalition such that marginal costs from applying all technologies in the coalition are identical. This requires that a perfect redistribution of effort is possible.<sup>10</sup> The redistribution of effort inside the coalition is a contribution to Lindroos & Kaitala (2000) where it, due to the cost function, is only the most efficient player in the coalition that harvests. Solving the systems such that marginal costs are identical yields constant harvest shares in each year for the members of the coalition (Kronbak 2004).

The benefits from the possible coalitions and the optimal strategies are summarised in table 4.

#### **Table 4.**

The benefits from the grand coalition exceed the sum of benefits from free riding, therefore there are enough benefits in the grand coalition to be distributed in a satisfactory way, such that the grand coalition is stable (Pintassilgo 2003). By studying the benefits from the 7 possible coalitions, we clearly see the technological advantage player 1 has, since he receives significant higher benefits than player 2 and 3 both when acting as a singleton and when comparing the free rider values. If the players are all playing a non-cooperative game, then they will each choose a strategy (fishing mortality) optimizing own payoff as the best response to other two players fishing mortalities, this yields a Nash equilibrium, where the aggregate fishing mortality is at its highest level. The optimal strategies clearly indicate that the overall fishing mortality is reduced the more members included in the coalition.

The average population of the cod stock in the Eastern Baltic Sea in the period between 1966 and 1999 is 500000 tonnes. Our model suggests a long run population of app. 550000 tonnes in the non-cooperative scenario and app. 1200000 tonnes in the cooperative scenario. The initial population of the cod stock in this model is at the 1998-level, which is a very low level, only 174000 tonnes, therefore each of the scenarios presented starts with a period of rebuilding the population before the long run equilibrium is reached after app. 10 years (Kronbak 2004). In years with a high abundance of cod, the population has at its highest been 1023000 tonnes.<sup>11</sup> The cooperative equilibrium population might seem unreasonably high compared to the population levels in the record year, but in neither year has the stock been exploited corresponding to a cooperative behaviour. The non-cooperative simulation has a fishing mortality equivalent of 0.91 and the average fishing mortality between 1966 and 1999 (ICES 2000) was 0.89. The received total population is within 9 % of the average population estimated by scientists (ICES 2000).

The characteristic function and the normalised characteristic function are determined and their values are illustrated in table 5.

**Table 5.**

From the characteristic function in table 5 it is clearly seen that the two-player coalitions yield relatively small benefits compared to the grand coalition. It is also clear that it is relatively important, seen from an economic point of view, to have player 1 joining the coalition.

### 3.2 The Shapley Value and the Nucleolus

The Shapley value for a single player is defined as the potential to change the worth of the coalition by joining or leaving it, that is the expected marginal contribution. The Shapley value for player  $i$  is defined as follows (Dinar & Howitt 1997):

$$\phi_i = \sum_{s \in K, i \in s} \frac{(n-|s|)!(|s|-1)!}{n!} v(s) - v(s - \{i\}), \quad (11a)$$

where  $K$  is the 7 possible coalitions,  $n$  is the number of players in the game and  $|s|$  is the number of players in coalition  $s$ . Equation (11a) shows that the Shapley value is determined by the probability of the different coalitions times the marginal contribution to the coalition by player  $i$ .

In our specific case with 3 players and since we apply a normalised characteristic function the Shapley value becomes:

$$\phi_i = \frac{1-v(j,k)}{3} + \frac{v(i,j)}{6} + \frac{v(i,k)}{6}, \quad i=\{1,2,3\}, j=\{1,2,3\}, k=\{1,2,3\}, i \neq j \neq k \quad (11b)$$

Equations (11a) and (11b) describe player  $i$ 's expected marginal contribution.

The idea of the nucleolus is to minimize the dissatisfaction of the most dissatisfied coalition. This is done by finding the 'lexicographic center' of the core, which is the imputation maximizing the minimum gains to any possible coalition. The nucleolus has the advantage that it always lies in

the core, if the core exists. To determine the nucleolus, we define the reasonable set, the excess function and the core.

The reasonable set is defined as imputations satisfying three equations; First, a player receives no more than what he contributes to the coalition. Second, the imputation is individually rational, that is all players should be better off with cooperation. Third, the imputation is Pareto-optimal or group rational, that is, all benefits are distributed among players. The reasonable set determines the set of fair distributions of the benefits.

The excess is defined as the difference between the fraction of the benefits of cooperation that  $s$  can obtain for itself and the fraction of benefits of cooperation that the imputation allocates to  $s$ .

$$e(s, x) = v(s) - \sum_{i \in s} x_i, \quad (12)$$

where  $x = (x_1, x_2, x_3)$  is a three dimensional vector describing different shares (imputations), and  $x_i$  describes the share to player  $i$ . The core is defined by the excess being negative as an addition to the reasonable set, thus the core in our case with 3 players looks like the following:

$$\begin{aligned} v(i) - x_i &\leq 0 \quad \forall i = \{1, 2, 3\} \quad (\text{Individual rationality}) \\ x_1 + x_2 + x_3 &= 1 \quad (\text{Group rationality}) \\ v(1, 2) - x_1 - x_2 &\leq 0 \\ v(1, 3) - x_1 - x_3 &= v(1, 3) - x_1 - (1 - x_1 - x_2) = x_2 - 1 + v(1, 3) \leq 0. \\ v(2, 3) - x_2 - x_3 &= v(2, 3) - x_2 - (1 - x_1 - x_2) = x_1 - 1 + v(2, 3) \leq 0 \end{aligned} \quad (13)$$

The individual rationality is always satisfied since the  $v(i)$  is predefined to be zero according to eq. (10b). The core thus ensures that each player receives at least its payoff from playing singleton (the individual rationality). The core ensures that all the cooperative benefits are shared among the players (the group rationality). And finally the core ensures that the players receive at least what they could have had by joining a two-player coalition (the last three constraints of equation (13)). In

this specific case, the core does not narrow down the number of imputations since the core and the reasonable set coincide. The reason for this is that the contribution from all three players to the grand coalition is relatively high, or put in another way the two-player coalitions have relatively low payoffs. Thus, the gains from a grand coalition are significantly higher than the gains from a two-player coalition. The benefits from the coalitions are connected with the externalities. In a two-player coalition the players inside and outside the coalition are playing a Nash game against each other, thus externalities are present. Moving from a two-player coalition to the grand coalition the externalities then disappear since all players are joining together and acting as a sole owner.

The rational  $\varepsilon$ -core is determined by shrinking the boundaries of the core at the same rate until it collapses into either a line or a single point. The rational  $\varepsilon$ -core consist of the imputations  $x=(x_1,x_2,x_3)$  that satisfies:

$$\begin{aligned}
& -x_1 \leq \varepsilon, \quad -x_2 \leq \varepsilon, \quad -x_3 \leq \varepsilon \\
& v(1,2) - x_1 - x_2 \leq \varepsilon, \\
& x_2 - 1 + v(1,3) \leq \varepsilon, \\
& x_1 - 1 + v(2,3) \leq \varepsilon, \\
& x_1 + x_2 + x_3 = 1.
\end{aligned} \tag{14a}$$

Applying the values from the normalised characteristic function in table 5 yield a lower bound of  $\varepsilon$  which is -0.3333, setting  $\varepsilon$  to this lower bound yields the following specific equations:

$$\begin{aligned}
& 0.3333 \leq x_1 \leq 0.604475 \\
& 0.3333 \leq x_2 \leq 0.533355. \\
& 0.476084 \leq x_1 + x_2 \leq 0.66667
\end{aligned} \tag{14b}$$



Clearly, the only imputation satisfying these equations is  $x_1=x_2=x_3=0.3333$ , which is the least rational core. Since the boundary of the core collapses to a single point, this is identical to the nucleolus.

The Shapley value is determined from applying the characteristic function values to equation (11b), and after determining the Shapley values we easily confirm that in our case the Shapley values lie within the core. Table 6 summarises the distribution shares determined by the two one-point solution concepts and the shares received by free riding relative to the cooperative benefits.

**Table 6.**

The reason for the nucleolus to distribute benefits equally among the players is that none of the two-player coalitions has a very high value determined by the normalised characteristic function. Thus, the boundaries of the reasonable set which is identical to the core in this case are determined mainly by the individual rationality and the centre of this set then reveals an equal share to the players. The Shapley value is based on the average contribution to the coalition of the players by joining or leaving it and since player 1 has the lowest cost parameter it contributes on average more to the coalitions than the other players, therefore the Shapley value for player 1 exceeds the Shapley value of other players. The results from the game in table 6 clearly illustrate the difference between the two one-point solution concepts. Both results are in the core and are characterised as fair sharing rules. There is, however, still a problem since player 1 is not satisfied with any of the two sharing rules. With the nucleolus sharing rule, player 1 receives 33.3% of the cooperative benefits but when free riding on the grand coalition player 1 can receive what corresponds to 38.1% of the cooperative benefits. Player 1 is clearly better off by free riding. Therefore, the grand coalition applying this nucleolus sharing rule is not a stable cooperative solution. With the Shapley value player 1 receives 35.9% of the cooperative benefits which does neither exceed his free rider value.

The Baltic Sea does, at the moment, not face a cooperative harvest solution for the cod fishery. This can be explained as instability by our model if the benefits in a cooperative solution are distributed according to the Shapley or the nucleolus sharing rules. Some players do have an incitement to free ride and as such the cooperative agreement collapses. The problem is that the sharing rules do not take the stability of cooperation into consideration when externalities are present. This is a problem which has also occurred in previous empirical studies (Lindroos & Kaitala (2000), Arnason, Magnusson & Agnarsson (2000)) Duarte, Brasão & Pintassilgo (2000)) but it has not been recognized.<sup>12</sup> Brasão, Duarte & Cunha-e-Sá (2000) do identify the problem but do not suggest a cooperative solution to it.

### 3.3 The Satisfactory Nucleolus

Since the previous section showed us that the Shapley values and the nucleoli are not necessarily stable to free rider incentives we suggest another distribution, than has previously been applied in the literature, of the cooperative benefits solution concept. We define a new set, namely *the satisfactory core*. This is done by redefining the core by applying the concept *individual satisfaction*. The *individual satisfaction* ensures players are at least as well off as when free riding, this is a parallel to the individual rationality which ensures the players are as well off as when playing singletons. The breaking point is that players have already agreed to cooperate and if they should stick to this agreement, they must not be tempted to deviate, and hence the sharing rule should ensure all players receive at least their free rider value. Let us define the *satisfactory core* as follows:

$$\begin{aligned}
 e(s, x) &= v(s) - \sum_{i \in s} x_i \leq 0 \\
 x_i &\geq \frac{v(\text{freerider})}{v(M)} \quad (\text{Individual Satisfaction})
 \end{aligned}
 \tag{15}$$

$$x_1 + x_2 + x_3 = 1.$$

The satisfactory core deviates from the ordinary core by the individual satisfaction constraint. This constraint ensures that each player receives at least the amount the player could receive by free riding on the grand coalition. In our specific case the free rider values are credible threats since all two-player coalitions are stable, this means that if one player leaves the grand coalition the equilibrium will be such that there is a two-player coalition and a singleton. The individual satisfaction, applying values from the normalised characteristic function, looks like the following:

$$\begin{aligned} x_1 &\geq 0.3809 \\ x_2 &\geq 0.2823 \\ 1 - x_1 - x_2 &\geq 0.2714. \end{aligned} \tag{16}$$

When comparing the sharing rules from table 6 with equation (16) we clearly see, that both the Shapley value and the nucleolus violates the individual satisfaction for player 1.

We define another sharing rule, the *satisfactory nucleolus*, which is similar to the nucleolus in the sense that it is defined as the lexicographic center of the satisfactory  $\epsilon$ -core. The results of the satisfactory nucleolus are summarised in table 7.

**Table 7.**

Table 7 clearly shows that the satisfactory nucleolus is stable to free riding.

A graphic illustration of the difference between the core and the satisfactory core and the three applied sharing rules is available in figure 1. It should be emphasised that the proportions in the figure are not correct.

**Figure 1.**

The figure underlines again that the nucleolus and the Shapley values do not lie within the satisfactory core. The satisfactory nucleolus is a cooperative sharing imputation which is stable to free rider values, since it is defined with the aim of taking the stability into account. The satisfactory

nucleolus does therefore recognize the connection between non-cooperative and cooperative games. The satisfactory nucleolus takes into account that player 1 can make relatively large gains by free riding on the grand coalition, therefore this player receives a larger share of the cooperative benefits compared to the other two players. Our model suggests the satisfactory nucleolus as a sharing rule to be administered by the IBFSC for reaching a stable cooperative solution in the Baltic Sea cod fishery.

#### **4 Sensitivity Analysis**

The robustness of the results has been tested by varying the different economic and biological parameters. In particular we have focused on economic parameters such as the cost parameters, the discount rate and simulation length. Cost parameters have been increased, decreased and their mutual proportions have been changed, and we have tested another 6 different sets of cost parameters when testing the results. None of these changes has affected the theory that a stable grand coalition is a possible stable solution. The cost parameters mutual proportions and their level do, however, have an effect on the stability of the different sharing rules. If the cost parameters are all very low, then the players and the partial coalitions will apply full effort and the free rider values coincide with the singleton benefits. The core and the satisfactory core are identical and both Shapley and nucleolus are located in the core and are therefore stable sharing rules. Also, if countries have relatively identical cost parameters then they receive similar benefits by free riding which makes it more likely that the existing sharing rules are also located in the satisfactory core. If, however, one country is relatively more efficient compared to the others, then it receives relatively large benefits from free riding which can diminish the satisfactory core significantly, making it more unlikely that the Shapley value and the nucleolus are stable sharing rules. Increasing the discount rate to 5% and 8%, respectively, does not change the fact that it is possible to find a stable

cooperative solution. The Shapley value and the nucleolus are, however, unstable. Reducing the simulation length from 50 years to 25 years also shows that it is possible to find a stable cooperative solution, but again we have to search among other solutions than the Shapley value and the nucleolus to reach stability.

The Beverton-Holt stock-recruitment curve has been shifted up and down by increasing and decreasing the maximum recruits per spawner at low spawning stock size (the parameter  $c$  in equation (4)). It is still possible to find a stable cooperative solution, but the Shapley values and the nucleoli are again not among these stable solutions.

The initial conditions which include the initial abundance of cod, the stock weight at age, the catch weight at age and the proportion of the stock which is mature is changed (these initial conditions refers to table 2). The original scenario is based on data from the 1998-level, which is a year, together with the other more recent years, with a low abundance of cod. The initial level has therefore for the purpose of sensitivity been set to the 1982-level, which is the year on record with the highest abundance of cod (ICES 2000). These simulations do, however, show the same trend as the other results. There exists a stable grand coalition, but neither the Shapley value or the nucleolus are stable solutions.

We can thus conclude the grand coalition formed by our model is a rather robust solution since we can find a stable sharing rule in all the analysed cases, whether the Shapley values and the nucleoli are among these sharing rules are more parameter specific.

## **5 Conclusion**

Our model shows that there are enough benefits to make all players better off in the grand coalition compared to a non-cooperative or partly cooperative solution. This result is in stark contrast with the previous more pessimistic empirical coalitional game models. The critical point is how the

benefits received in a grand coalition are shared among the players in the game. Two different known one-point sharing rules, namely the Shapley value and the nucleolus are not taking the stability of the coalition into account even though they are both located in the core and are both characterised as fair sharing rules. If the benefits in the Baltic Sea cod fishery are shared according to these rules it is shown to be an unstable solution which does not satisfy all players of the grand coalition since one player is better off by free riding. We therefore suggest a new sharing rule connecting the cooperative and the non-cooperative game. The satisfactory core takes into account the stability of the grand coalition, by including the free rider values as threat points. The corresponding satisfactory nucleolus sharing rule ensures all players receive a share of the cooperative benefits which is at least as large as their free rider value, this yields a stable sharing imputation. A cooperative solution can be stable, but the Achilles heel is the distribution of the benefits and one should be aware that all players are satisfied compared to their cooperative benefits. If the satisfactory nucleolus sharing rule is applied to the Baltic Sea cod fishery we show that a stable solution can be achieved.

The current model is limited to three players. It can be argued that in some countries fishermen are members of producer organizations (POs) and these organizations act as a single group. The assumption might, however, be critical because not all countries have a high degree of membership in POs. If the number of players in a coalitional game increases, it most likely becomes more difficult to achieve a grand coalition solution. Olson (1965) discusses this as a general problem to collective goods, and Hännesson (1997) discusses it as a problem in fishery models, where he defines the critical number of fishermen for a full cooperative solution. A reason for not having a grand coalition in the Baltic Sea fisheries might be that having all fishermen joining a grand coalition may force higher transaction costs of planning and organising the grand coalition; also it decreases the likelihood of a stable grand coalition. The organizing of a grand coalition

should be performed in existing commissions such as the IBSEFC and a solution to the redistribution problem might be to introduce individual transferable quotas (ITQ). The number of players could be reduced by supporting memberships of POs and support the organisation of POs in developing countries.

The Shapley value, the nucleolus and the newly developed satisfactory nucleolus only consider how the sharing of benefits should be at the end of the game. It is beyond the scope of these sharing rules to discuss how the benefits of the grand coalition should be shared among members in time; this is, however, a very relevant point which should be subject to future research.

Our model is stable to changes in both economic and biological parameter values and for many of the tested scenarios the pattern is the same, namely that the nucleoli and the Shapley values are not stable solutions. This conclusion can also be drawn from the previous literature (Lindroos & Kaitala (2000), Arnason, Magnusson & Agnarsson (2000)) Duarte, Brasão & Pintassilgo (2000)) and we therefore find it particularly important to recognize the connection between the sharing rule and the free rider values, which is done in the development of the satisfactory nucleolus.

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**Table 1.**

Biological Parameter Values

Parameter	Value
Mortality	
$m_{2,3,\dots,8}$	0.2
Stock-Recruitment (B-H)	
$c$	0.9814216
$b$	0.000002340

Source: ICES (2000)

**Table 2.**

Initial biological parameters

	Age 2	Age 3	Age 4	Age 5	Age 6	Age 7	Age 8+
<i>MO</i>	0.14	0.32	0.84	0.94	0.98	0.96	1
<i>SW</i>	0.244	0.548	1.230	1.595	2.963	4.624	5.417
<i>CW</i>	0.662	0.773	1.127	1.448	2.337	3.485	4.647
<i>N0</i>	136493	71852	37621	15421	4332	2026	1452

Abbreviations: *MO*=Proportion Mature at the Start of the Year, *SW*=Mean Weight in Stock (kilograms), *CW*=Mean Weight in Catch (kilograms), *N0*=Initial Abundance (thousands)

Source: ICES (2000) 1998-estimates

**Table 3.**

## Economic Parameter Values

Parameter	Value
First fishing age, $a_1$	3
Selectivity $S_2$	0
Selectivity $S_{3,\dots,8}$	1
Cost parameter, country 1: $\alpha^1$	9 Dkr/kg
Cost parameter, country 2: $\alpha^2$	14 Dkr/kg
Cost parameter, country 3: $\alpha^3$	15 Dkr/kg
Discount rate, $r$	2%
Price, $p$	10.74 Dkr/kg
Max. fishing mortality, $f_{\max}^1$	0.35
Max. fishing mortality, $f_{\max}^i$ , $i=2,3$	0.3

Source: ICES (2000); Fiskeridirektoratet (1999 and 2000)

**Table 4.**

The benefits from the 7 possible coalition structures and the optimal strategies

Coalition	Strategy, $f$	Net-Benefit, Dkr	Free Rider Value, Dkr
1	0.35	$2.30694 * 10^{10}$	
2	0.29	$1.67376 * 10^{10}$	
3	0.27	$1.56076 * 10^{10}$	
1,2	0.457	$4.25624 * 10^{10}$	$2.02757 * 10^{10}$ ( $f^2=0.264$ )
1,3	0.457	$4.12502 * 10^{10}$	$2.10943 * 10^{10}$ ( $f^2=0.279$ )
2,3	0.407	$3.35437 * 10^{10}$	$2.84559 * 10^{10}$ ( $f^d=0.35$ )
1,2,3	0.351	$7.47167 * 10^{10}$	$6.98259 * 10^{10}$ (Sum of the above)

Numbers are subject to rounding.

**Table 5.**

The characteristic function & the normalised characteristic function

Coalition	Strategy, $f$	Characteristic function, Dkr	Normalized char. function
1	0.35	0	0
2	0.29	0	0
3	0.27	0	0
1,2	0.457	$2.7554 \cdot 10^9$	0.142751
1,3	0.457	$2.5732 \cdot 10^9$	0.133312
2,3	0.407	$1.1985 \cdot 10^9$	0.062092
1,2,3	0.351	$1.93021 \cdot 10^{10}$	1

Numbers are subject to rounding.

**Table 6.**

Sharing Functions

Player	Shapley	nucleolus	Free rider shares
1	35.9 %	33.3 %	38.1 %
2	32.3 %	33.3 %	28.2 %
3	31.8 %	33.3 %	27.1 %

Numbers are subject to rounding.



**Table 7.**

Satisfactory nucleolus

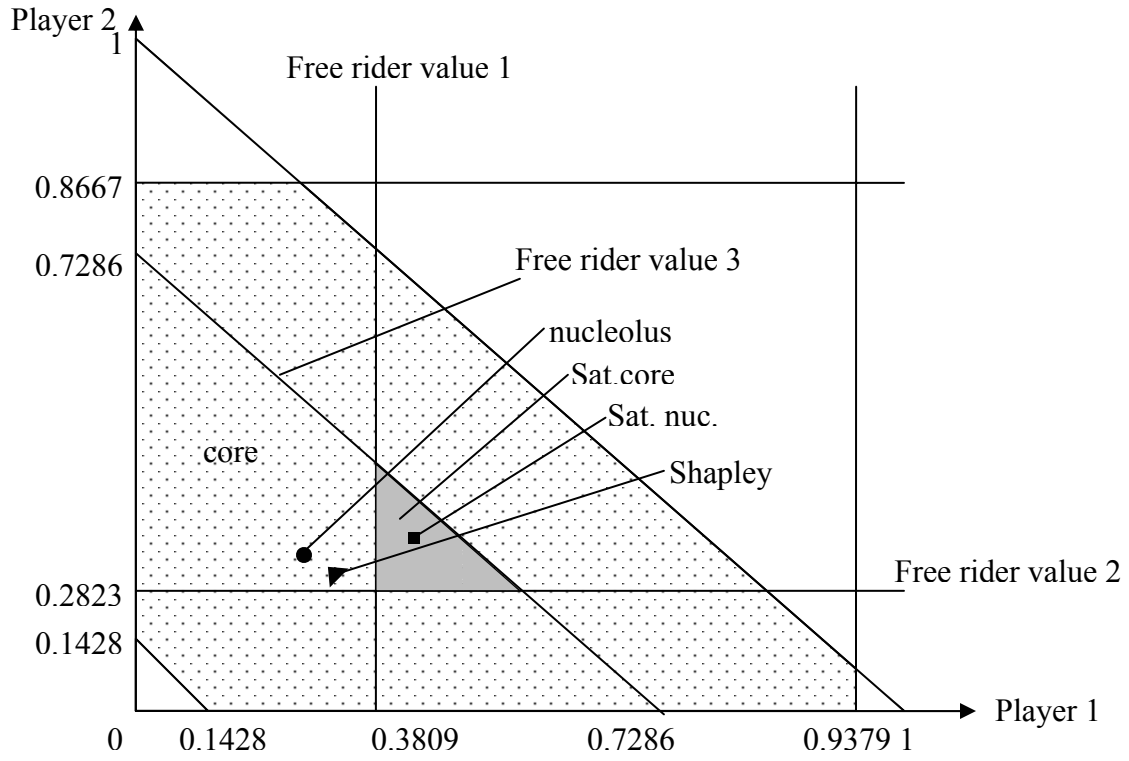
Player	Satisfactory nucleolus	Free rider shares
1	40.3 %	38.1 %
2	30.4 %	28.2 %
3	29.3 %	27.1 %

Numbers are subject to rounding.

**Figure 1.**

The reasonable set, the core and the satisfactory core.

The sharing rules; the nucleolus, the Shapley value and the satisfactory nucleolus.



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<sup>1</sup> The management issue is most likely subject to change in the nearby future since EU by the enlargement in May 2004 is the main actor in this sea with only the Russian Federation left outside.

<sup>2</sup> The TACs are the main regulatory tool for the Baltic Sea cod fishery.

<sup>3</sup> There has been 7 years with no TAC in the period in question.

<sup>4</sup> Source: Article 1 of the Gdansk Convention, IBSFC (2003), [www.ibsfc.org](http://www.ibsfc.org)

<sup>5</sup> The stock-recruitment estimated by ICES assumes that recruits are not entering the population before age 2, therefore the spawning stock biomass (SSB) is lagged two years in the Beverton-Holt recruitment function applied by ICES (2000). For reasons of simplicity, we apply only a one-year lag in our simulation model. We do not see this as a critical assumption since the SSB biomass is pretty similar for every two successive year periods.

<sup>6</sup> In the way this model is defined, there is a direct link between the fishing mortality and the yield; therefore the control variable might as well be the yield. The fishing mortality would then be determined as a residual.

<sup>7</sup> The constant fishing mortality over the whole period limits the model. The players have no possibilities to adapt to changes or fluctuations in, for instance, the stock. One way to cope with this would be to allow for renegotiations in the model.

<sup>8</sup> In Mesterton-Gibbons (1992) the benefits of cooperation is associated with a reduction in costs which explains why he applies the costs of singletons minus the costs of the coalition. Since we apply the increase in profits associated with cooperation we have the opposite sign in the characteristic function.

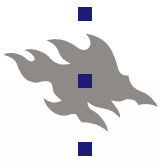
<sup>9</sup> We ignore the empty coalition, in which we assume the benefits are zero.

<sup>10</sup> Individual transferable quotas (ITQ) might be a possible way in solving the fact that an enormous amount of information is required to distribute effort perfectly.

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<sup>11</sup> We are considering only the years on record, 1966-1999 (ICES 2000).

<sup>12</sup> One could argue that free riding on the grand coalition is short-sighted since the grand coalition is more than likely to break down in the long term. If this is the case, then the stability should be discussed in the light of Trigger strategies. We prefer, instead, to search for a distribution of benefits among members, which is also stable compared to free rider value.



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