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Bank of Finland Research
Discussion Paper
20 • 2017

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Abstract

We model a network of linked permit markets to examine efficiency and dependencies between the markets in a competitive equilibrium. Links enable the participants of one emissions trading system to use the permits of another system. To improve the cost-efficiency of the international policy architecture, the Paris climate agreement set out a framework for linking local policies. International trade in permits reduces costs by merging markets, but in a large network it is generally not obvious which markets end up linked in the equilibrium. Also, indirect links might allow foreign regulators to undermine domestic policy outcomes. We apply graph theory to study dependencies between markets and to determine how the network is partitioned into separate market areas. Our main theorem characterizes the dependency structure of the equilibrium in an exogenous trading network. We show that markets merge when they are connected by a particular pattern of links. The results help to identify potential sources of both cost reductions and foreign interference, and to secure the efficiency of climate change policies.

Keywords: Networks, graph theory, emissions trading, trade theory
JEL codes: L14, F13, Q54, Q58, D41

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Acknowledgements: Many thanks to Panu Poutvaara, Klaus Kultti, Essi Eerola, Hannu Vartiainen, Marko Terviö, Pauli Murto, Markku Ollikainen, Roland Magnusson, Corrado Di Maria, Beat Hintermann and Matti Liski for helpful comments and discussion. Funding and support from HECER is gratefully acknowledged.
The author declares he has no conflicts of interest.

Mitigating climate change requires reductions in global greenhouse gas emissions. To minimize the cost of reducing emissions, international coordination of climate policies is needed. In theory, costs could be minimized by agreeing on a top-down mechanism, which would put a legally binding cap on emissions and equalize marginal abatement costs in all countries. However, attempts to build such a mechanism have been unsuccessful. A key innovations in the Paris Agreement in 2015 was to adopt a more feasible bottom-up approach. Parties to the agreement made non-binding commitments to implement domestic climate policies, but established a framework to improve cost efficiency by linking policies (Stavins and Stowe, 2016).

Emissions trading has been one of the most important instruments used to reduce emissions (Stavins, 2010; Grubb, 2012; Goulder, 2013). Several countries and regions have created local emissions trading systems (ETS), and already a handful of ETSs have been linked (Ranson and Stavins, 2015; Haites, 2016)(Figure 1).

Linking means that the regulator of one ETSs allows its participants to use the emissions permits of another system. Trade tends to equalize permit prices and marginal abatement costs, which is necessary to minimize the cost of reducing emissions (Montgomery, 1972). Instead of imposing a global cap-and-trade system, the bottom-up approach aims to improve cost-efficiency by linking local ETSs together.

An important concern with linking, however, is that it makes domestic policy outcomes dependent on the decisions of foreign regulators. For example, a foreign regulator might unilaterally issue new permits and sell them to the linked market. This would profit the issuer but increase total emissions.

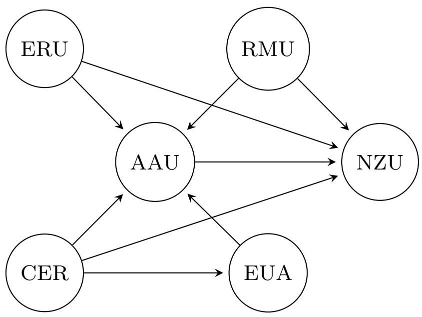


Figure 1: The simplified graph depicts links between ETSs related to the the Kyoto Protocol: Emission Reduction Unit (ERU), Removal Unit (RMU), Assigned Amount Unit (AAU), New Zealand Unit (NZU), Certified Emission Reductions (CER), EU emission allowance (EUA).

Dependencies between markets may also arise indirectly. When two systems are linked with a common third system, all three permit markets may become dependent of each other (Anger, 2008; Flachsland et al., 2009a; Newell et al., 2013). In the general case, where ETSs are linked in an arbitrary network, dependencies could be conveyed through several links. To our knowledge, the general case has not been studied previously. It would be intuitive to assume that any path of links between two systems is sufficient to make them dependent. We find that this is not the case.

These dependencies are also important for cost-efficiency. The free movement of permits through links create a common market, where ETSs are dependent. Gains from trade will be exhausted among those that are dependent in equilibrium. New links are required to connect independent parts of the network and to further improve cost-efficiency.

Our main research question is the following: Given an arbitrary network of linked ETSs, which systems will be dependent in the competitive market equilibrium? This information is important to the policymaker for two reasons. First, it tells how cost-efficiency could be improved by forging new links. Second, it tells how domestic policy outcomes could be influenced through indirect links. Furthermore, we ask how marginal changes in exogenous variables of one system (e.g. the endowment of permits) will affect endogenous variables of other systems (e.g. price of permits).

We use partial equilibrium analysis, which allows us to focus on the permit markets, and we set up a model with greenhouse gas emitting firms that participate in an ETS. Firms have an incentive to trade because production technologies and endowments differ. ETSs can be cap-and-trade systems (firms need to buy permits to produce emissions) or baseline-and-credit systems (firms receive credits if they reduce emissions). Links between systems are represented by a directed graph.

We use graph theoretic tools to derive a dependency structure from the equilibrium conditions. We show that in equilibrium the network of ETSs is partitioned into subsets, which we call *supply and demand components*. The members of these components have the same equilibrium price and are connected by a specific pattern of links, which we call *an alternating path of supply and demand*. Members need not be linked directly to each other. Each supply component has a matching demand component and vice versa. All members of a supply component effectively face the same demand which is generated by the matching demand component. Similarly, all members of a demand component effectively share the same supply generated by the matching supply component. The answer to our main research question is given in Theorem 7.

Furthermore, comparative statics results are derived in a corollary, which shows how exogenous changes impact market outcomes. A change in the endowments or the production technology will affect the price of the matching supply component and the emissions of the matching demand component.

The theory provides a simple heuristic for identifying when markets are connected: If an alternating supply or demand path can be found between two systems, they belong to the same component. All members belonging to same component or its matching counterpart essentially share the same market.

The theory could be useful beyond emissions trading. Considering trade theory, the framework generalizes the comparison between free trade (all markets are linked) and autarky (no markets are linked) by allowing bilateral trade between some markets while denying it between others. For example, the theory tells whether a given set of links is sufficient to reach the same efficient outcome as free trade. Similarly, embargoes could be studied to determine which links need to be removed in order for it to have an effect?[1]

To the best of our knowledge, this is the first study dedicated to the structure of dependencies in a competitive equilibrium with an exogenous network of trade restrictions. Our model is related to spatial price equilibrium models, in which goods are sold between network nodes under perfect competition while each link entails a specific transport cost (Enke, 1951; Samuelson, 1952; Takayama and Judge, 1964). Our setting can be viewed as a special case, where the transport cost is either zero or infinite. However, within the spatial price equilibrium literature, we know of no study that would have attempted to provide a characterization of the equilibrium's dependency structure.

The economic model we use is rather conventional (e.g. Baumol and Oates, 1975), if not for the network of constraints. This simple setting opens a host of new questions, and we believe it allows us to contribute rather fundamental results. Our main theoretical contribution to the literature on networked markets is to characterize the equilibrium's dependency structure assuming perfect competition and an exogenous trading network.

Section 1 surveys related literature. Section 2 defines necessary graph-theoretic concepts and sets up the economic model. Section 3 solves the equilibrium and shows some of its basic properties. Section 4 describes the equilibrium's dependency structure and defines concepts to analyse it. Section 5 analyses comparative statics. Section 6 discusses the interpretation and the assumptions of the model, and Section 7 concludes. The appendixes give proofs, examples and an illustrative map outlining the theory.

[1]The theory implies that the embargoed country would lose gains from trade only if all alternating paths of supply and demand are removed between the embargoed country and the rest of the world. This would separate the embargoed country into its own components of supply and demand.

1 Review of the Literature

The idea of trading emissions is old. Coase (1960) was first to view legal rights as factors of production that had market value. Dales (1968) and Crocker (1966) refined the idea into permits markets, and Montgomery (1972) gave a proof for efficiency in a partial equilibrium model.

Linking ETSs has been a topical issue ever since the Kyoto Protocol established multiple emissions trading mechanisms. The Protocol allowed the use of permits from different mechanisms for the purpose of complying with the agreed emissions targets. Linking has been studied extensively in policy papers, which cover various issues relevant for the implementation of linking policies, e.g. cost-efficiency, distributional effects, and the compatibility of design features (Haites, 2001; Ellis and Tirpak, 2006; Jaffe and Stavins, 2007).[2] Studies have also looked at legal issues (Jaffe et al., 2009), sectoral perspectives (Aasrud et al., 2009; Anger, 2010; Marschinski et al., 2012), and linking as a part of the international policy architecture (Flachsland et al., 2009a; Hare et al., 2010; Olmstead and Stavins, 2012). Even laboratory experiments have been performed to test the efficiency of different linking structures (Cason and Gangadharan, 2011). Metcalf and Weisbach (2012) show that linking may also be feasible between ETSs and other types of policy instruments.

Linking has been framed as an application of trade theory by Copeland and Taylor (2005), who emphasise that benefits can be ambiguous due to general equilibrium effects (see also Chichilnisky, 1994; Marschinski et al., 2012). Studies have used numerical simulations to analyse the costs and benefits of linking both in general equilibrium (Böhringer et al., 2005; Klepper and Peterson, 2006) and in partial equilibrium models (Anger, 2008). Linking is also known to affect the incentives of regulators to uphold emissions targets (Rehdanz and Tol, 2005; Itkonen, 2009).

Recent work related to trading in networks has mostly focused on the interaction between strategic agents and link formation (whereas our focus is on the interaction between exogenously linked markets which consist of agents without bargaining power). In an early paper, Jackson and Wolinsky (1996) investigated link formation with strategic individual while focusing on the stability and efficiency of networks. In related studies, Kranton and Minehart (2000, 2001) and Elliott (2015) analysed exchange networks where individual buyers and sellers need to form a link in order to trade. In their models buyers cannot freely resell their assets to other buyers (see also Nava,

[2] See also Itkonen (2009); Mehling and Haites (2009); Flachsland et al. (2009b); Tuerk et al. (2009); Hare et al. (2010); Newell et al. (2013).

2015). Such models are less relevant for permit markets as nothing prevents firms from buying and selling foreign permits—merely the use of foreign permits is not allowed by default. Miettinen and Poutvaara (2014) studied link formation in a setting where strategic interaction is ruled out by assuming a uniform market price for links. Corominas-Bosch (2004), Polanski (2007) and Manea (2011) assume an exogenous network but focus on strategic interaction.

The existence of equilibria in trade networks has been studied with matching models where agents have predetermined roles and goods are indivisible (Ostrovsky, 2008; Hatfield and Kominers, 2012; Hatfield et al., 2013). When goods are divisible and the optimization problem is convex, as is in our case, the existence of an equilibrium is easy to show, and we can focus on the structure of dependencies. Hatfield and Kominers (2014) also studied divisible goods but focused on complement goods, whereas permits are perfect substitutes.[3]

2 Preliminaries

Next, we define necessary graph theoretic concepts and set up the economic model. Graph theory not only helps to visualize dependencies between markets but it also provides a tool for rigorous deduction. Its usefulness will become evident as we prove key propositions in the following sections.[4]

2.1 Graphs and connectivity

A *graph* (S, A) consists of a set of *vertices* S and a set of *edges* $A \subset \{\{s, r\} \mid s, r \in S\}$. A *directed graph* (S, A) consists of a set of vertices S and a set of *arcs* $A \subset \{(s, r) \mid s, r \in S\}$. A (directed) graph (S', A') is a *subgraph* of (directed) graph (S, A) if $S' \subset S$ and $A' \subset A$. If (S', A') is a subgraph of (S, A), then (S, A) is said to be a *supergraph* of (S', A').

Vertices s_0 and s_k are *connected* by a path in graph (S, A) if there is a sequence of vertices $s_1, \ldots, s_{k-1} \in S$, $k \in \mathbb{N}$ such that $(s_{i-1}, s_i) \in A$ for all $i = 1, \ldots, k$. Subgraph (S', A') of graph (S, A) is a *connected component* if (1) any two vertices $s, r \in S'$ are connected by a path in (S', A') when $s \neq r$, (2) there are no $s \in S'$ and $r \in S \setminus S'$ which are connected by a path in (S, A), and (3) if $s, r \in S'$ and $(s, r) \in A$ then $(s, r) \in A'$. We call the set of

[3] Our setup might look similar to a network flow problem, studied in operations research (Boyd and Vandenberghe, 2004), but it is very different at a closer look.

[4] For a similar approach, see De Benedictis and Tajoli (2011).

vertices in a connected component and the component itself with same name when no confusion can arise.

2.2 The model

We construct a partial equilibrium model that focuses on the dependencies between permit markets. We keep the production side very simple in order to make the analysis tractable.

Consider a set of emissions trading systems S. Each system $s \in S$ has an endowment of permits $\omega_s > 0$ and regulates a set of firms which are called participants. Each system's permits have a perfectly competitive market. We assume that each system $s \in S$ can be described by a representative firm, to focus on the relationships between the systems and not on what happens inside the the systems. Representative firm's variables are defined as the sum of variables of all firms that participate in the system. The representative firm produces output y_s using emissions $c_s \geq 0$ as an input.[5] The production technology of the representative firm s is described by a strictly concave and twice continuously differentiable production function $f_s \colon \mathbb{R}_+ \to \mathbb{R}$ for which $y_s = f_s(c_s)$.

The regulator of an ETS obligates the participants to acquire permits if they wish to emit. Firm's emissions must not exceed the number of permits it owns. When several ETSs exist, the regulator might allow its participants to use the permits of other systems, i.e. ETSs may be linked. Note that firms can buy and sell permits of any variety with whomever they wish. For regulatory compliance, however, they can only use permits that are accepted by their system's regulator.

Together, the ETSs and the links form a trading network:

Definition. A *trading network* is a directed graph (S, A), where the set of vertices $S = \{1, \ldots, n\}$ consists of n emissions trading systems and the set of arcs $A \subset S \times S$ consists of links, i.e. $(s, r) \in A$ means that the participants of system r may use permits of system s. We use binary variable $a_r^s \in \{0, 1\}$ to indicate that $(s, r) \in A$ and vector $a_s = (a_s^1, \ldots, a_s^n)$ to summarise which permits are allowed by system $s \in S$.

The *emissions permit vector* of a representative firm describes how many permits of each system the firm holds. It is denoted by $e_s = (e_s^1, \ldots, e_s^n)$, where $e_s^r \geq 0$ is the number of permits of system r that firm s has.

[5] It is not uncommon in the literature to view emissions as a factor of production, instead of an output with a negative price. This has no effect on the results other than simplification.

Two constraints need to hold in equilibrium: The *regulation constraint* requires that a firm's emissions cannot exceed the number of allowed permits it has acquired. That is,

$$c_s \leq a_s e_s, \tag{1}$$

for all s. The *resource constraint* requires that the sum of permits used cannot exceed the number of permits issued. That is,

$$\sum_{r \in S} e_r^s \leq \omega_s$$

for all s. In practice, the initial endowment ω_s is usually either auctioned or allocated freely to participants.

The prices of a trading network are denoted by a non-negative vector $p = (p_1, \ldots, p_n)$. The price of output is 1. Due to perfect competition, firms take prices as given.

The *profit maximization problem* of representative firm $s \in S$ of trading network (S, A) is to choose the emissions $c_s \in \mathbb{R}_+$ and the emissions permit vector $e_s \in \mathbb{R}_+^n$ to maximize its profits

$$f_s(c_s) + p_s \omega_s - p e_s,$$

subject to the regulation constraint (1) while prices $p \in \mathbb{R}_+^n$ are taken as given. Note that the value of the endowment $p_s \omega_s$ is also taken as given.

The Karush-Kuhn-Tucker theorem can be used to derive the sufficient and necessary conditions for the equilibrium, since the optimization problem is convex. These conditions imply descriptive properties of firms' behaviour:

Lemma 1. *A profit-maximizing representative firm*

1. *uses only the cheapest permits among the allowed,*
2. *never emits beyond a saturation point, and*
3. *holds unallowed permits only if their price is zero.*

All proofs are presented in Appendix A.

3 Equilibrium

We assume the standard market clearing condition, which states that each resource constraint is binding if the price of permits is nonzero. That is, $\sum_{r \in S} e_r^s = \omega_s$ if $p_s > 0$ for all $s \in S$.

Now, an *equilibrium* of the model is a (n^2+3n)-dimensional vector

$$(e_1^1, \ldots, e_n^1, \ldots, e_1^n, \ldots, e_n^n, c_1, \ldots, c_n, \lambda_1, \ldots, \lambda_n, p_1, \ldots, p_n)$$

which solves the profit maximization problems of the representative firms while satisfying the resource constraints and the market clearing condition. Shadow prices $\lambda_1, \ldots, \lambda_n$ are Lagrangian multipliers related to the regulation constraints, and can be interpreted as the marginal costs of regulation. Permit prices $p_1, \ldots, p_n$ are the perfect competition equilibrium prices which are related to the resource constraints.

Lemma 2. *The perfect competition equilibrium outcome of the trading network is efficient.*

The proof of Lemma 2 contains the Karush-Kuhn-Tucker conditions that characterize the equilibrium. First, we note that the Karush-Kuhn-Tucker conditions include inequality constraints, and not all equilibrium variables appear in each system's constraint because of the network structure. This suggest that a subset of equilibrium variables could perhaps be solved with a subset of the equilibrium conditions. In such cases, the value of the equilibrium variables depends only on the exogenous variables that appear in the subset of equations. We will show that these subsets are the key to answer our main question: which systems are interdependent in equilibrium.[6]

Second, we note that the profit maximization problem does not generally have a unique solution with respect to the emissions permit vectors e_s. The solution for emissions c_s, however, is unique. This is because permits are perfect substitutes. Firms are indifferent between permits that are allowed.

Equilibrium emissions $c_s > 0$ can be expressed as a function of the lowest price available. We denote by p_s^* the *minimum price available* to system s. First, note that because the production function f_s is strictly concave, its derivative f_s' is also strictly decreasing and it has an inverse function $f_s'^{-1}$. Now, equation (9) in Appendix A gives the equilibrium emissions:

$$c_s = f_s'^{-1}(p_s^*) = f_s'^{-1}\left(\min_{(r,s)\in A} p_r\right) \quad \forall s \in S. \tag{2}$$

Equation (2) can be used to illustrate the complex network of dependencies that emerges from the equilibrium. The demand for emissions by firm s depends on the price of the least expensive permits among those that are linked to s. The minimal price, however, depends on the demand generated

[6] With a fully parametrized model we could solve the dependencies between systems numerically using the equilibrium conditions.

by all those firms that use the same permits that s does. This demand, again, depends on the minimal price among the permits they are allowed to use.

But everything does not necessarily depend on everything. In principle, we could continue recursively to find all relevant equilibrium conditions. If this procedure would result in a subset of all conditions, we would have found a subset of the network where the market outcome is determined independently of the other parts of the network. In the next two sections we characterize these subsets.

4 Equilibrium network

In this section, we apply graph theoretic concepts to analyse the dependencies between the systems in equilibrium. Part 1 of Lemma 1 implies that links from more expensive systems to less expensive systems will not be used. This suggest we can focus on a set of links that is smaller than the set in the trading network. We define this subgraph as follows:

Definition. The *equilibrium network* of trading network (S, A) is the directed subgraph (S, M), where $M = \{(r, s) \in A \mid p_r = p_s^*\} \subset A$ and p_s^* is the lowest price available to system s.

Part 1 of Lemma 1 also implies that transactions will occur only via links in the equilibrium network, i.e. $e_r^s = 0$ if $(s, r) \notin M$ and $p_s > 0$. When a link exist in the trading network but not in the equilibrium network, it means that participants of one system are allowed use another system's permits but will not do so, because they have access to cheaper permits.[7]

The equilibrium network is a function of the equilibrium, which in turn depends on the specification of the underlying economic model. Therefore, changes in the trading network, endowments, or production functions may alter the equilibrium network. The equilibrium network's link structure is altered when changes in exogenous variables are large enough to change the ranking of equilibrium prices. This, however, occurs at a rather limited set of crossing points. In Section 5.1, we will introduce the property of nonzero equilibrium, which guarantees the equilibrium network remains unchanged for small changes.

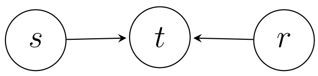
[7] Note that, in equilibrium, the system's own permits might be too expensive for its participants to buy, i.e it is possible that $p_s > p_s^*$ and hence $(s, s) \notin M$ even if $(s, s) \in A$.

4.1 Adjacent sellers and buyers

Next, we represent price equalization properties of the equilibrium as relationships in the equilibrium network. We begin by defining two adjacency relationships between systems in the equilibrium network. Then we show that these relationships imply an equivalence relation for prices. Finally, these relationships will be extended transitively to the whole network.

Definition. Systems s and r are *adjacent sellers* in equilibrium network (S, M) if there is a system t such that $(s, t) \in M$ and $(r, t) \in M$. Similarly, s and r are *adjacent buyers* in equilibrium network (S, M) if there is a system t such that $(t, s) \in M$ and $(t, r) \in M$.

That is, systems s and r are adjacent sellers if the equilibrium network has a subgraph



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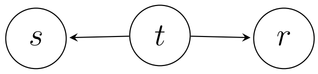
for some t. Similarly, systems s and r are adjacent buyers if there is a subgraph



for some t. Note that s, r, and t need not be distinct.

When two firms sell to the same market or two firms buy from the same markets, under perfect competition, prices will be the same. The assumption of perfect competition rules out price discrimination.

Lemma 3. *In an equilibrium network, the permits of adjacent sellers have an equal price, and adjacent buyers use permits with an equal price.*

Lemma 3 is a price equalization result for pairs only. Next, we develop new concepts so that Lemma 3 can be applied transitively to prove a similar price equalization result for wider sets. We aim to find the largest set of system among which prices are equated.

First, note that the adjacency of seller and buyer is a symmetric relationships. Such relationships can be summarized as an undirected graph.

Definition. The *adjacent seller graph* of equilibrium network (S, M) is the undirected graph (S, M_S) where $M_S = \{\{s, r\} \mid s, r \in S \text{ are adjacent sellers}\}$. The *adjacent buyer graph* is the undirected graph (S, M_D) where $M_D = \{\{s, r\} \mid s, r \in S \text{ are adjacent buyers}\}$.

4.2 Supply and demand components

The adjacent seller and buyer graphs can be used to partition the trading network into connected components. Basic graph theory tells that the connected components of a graph induce a unique partition for the set of vertices.

Definition. A *supply component* of equilibrium network (S, M) is a connected component in adjacent seller graph (S, M_S). A *demand component* of (S, M) is a connected component in adjacent buyer graph (S, M_D).

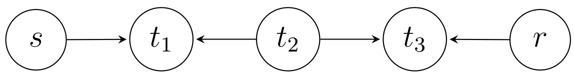
We call them supply and demand components because they generate unified sources of supply and demand in equilibrium. All permits of a supply component are sold at the same price, and all firms in a demand component pay the same price for their permits.

Proposition 4. *In equilibrium, all permits of a supply component have an equal price. All firms of a demand component use permits with an equal price*

We denote the *price of permits in supply component* $S_i \subset S$ of equilibrium network (S, M) by p_{S_i}. Thus $p_{S_i} = p_s$ for all $s \in S_i$.

4.3 Alternating supply and demand paths

Next, we look closer at the links that bind supply and demand components together. To illustrate, consider the following subgraph,



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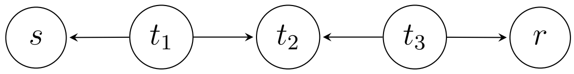
where pairs (s, t_2) and (t_2, r) are adjacent sellers. Both pairs have a common system that buys their permits, t_1 and t_3, respectively. This means (s, t_2) and (t_2, r) are edges in the adjacent seller graph. Therefore, systems s, t_2, and r belong to the same supply component. Similarly, consider subgraph



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where pairs (s, t_2), and (t_2, r) are adjacent buyers. These pairs buy permits from t_1 and t_3, respectively. Thus (s, t_2) and (t_2, r) are edges in the adjacent buyer graph. Systems s, t_2 and r belong to the same demand component.

These illustrations indicate that supply and demand components are connected by a sequence of links going back and forth between systems. By generalizing this observation, supply and demand components can be characterized using the link sequences contained within the equilibrium network.

Definition. Let m be an even number, and $i = 1, \dots, m-1$. Vertices s_0 and s_m are connected by an *alternating supply path* in equilibrium network (S, M) if there is a sequence $s_1, \dots, s_{m-1}$ such that (s_{i-1}, s_i) and (s_{i+1}, s_i) are in M for every odd i. Similarly, vertices s_0 and s_m are connected by an *alternating demand path* in equilibrium network (S, M) if there is a sequence $s_1, \dots, s_{m-1}$ such that (s_i, s_{i-1}) and (s_i, s_{i+1}) are in M for every odd i.

In essence, these alternating paths are composed of sequences of adjacent sellers and buyers. Based on the definitions, it is clear that two systems are members of the same supply (demand) component if and only if they are connected by an alternating supply (demand) path.

4.4 Matching supply and demand components

We can show that each supply component is related to a particular demand component, and conversely each demand component is related to a particular supply component. We say that such components are *matching*.

Lemma 5. *Let (S, M) be an equilibrium network.*

1. *If S_i is a supply component of (S, M) and $D_i = \{r \in S \mid \exists s \in S_i \colon (s, r) \in M\}$, then D_i is a demand component of (S, M).*
2. *If D_i is a demand component of (S, M) and $S_i = \{s \in S \mid \exists r \in D_i \colon (s, r) \in M\}$, then S_i is a supply component of (S, M).*

Lemma 5 shows that for each supply component there is a unique matching demand component, and vice versa. We indicate that supply and demand components S_i and D_i are matching by using the same index $i = 1, \dots, k$, where k is the number of components.

Now, we can show that all links, which start from some supply component, end up in the matching demand component, and the other way around.

Proposition 6. *If S_i and D_i are matching supply and demand components of equilibrium network (S, M), then*

$$\{(s, r) \in M \mid s \in S_i, r \in S\} = \{(s, r) \in M \mid s \in S, r \in D_i\}.$$

The result means that all permits, which are sold from a supply component, are bought by a firm somewhere in the matching demand component. Likewise, all permits, which are bought by some firm in some demand component, are sold from the matching supply component.

5 Comparative statics

In this section we will study the comparative statics of the equilibrium. First, we define a concept which describes non-trivial equilibria. Second, we apply Propositions 4 and 6 to derive a system of equations that characterizes the equilibrium locally and enables comparative statics analysis. Finally, two comparative statics results are presented.

5.1 Nonzero equilibrium

Some constraints might be non-binding in equilibrium. This occurs when the endowment of permits is large enough to eliminate scarcity or small enough to stop firms from emitting altogether (i.e. to exit the market). Non-binding constraints complicate the study of comparative statics. Exogenous variables that appear only in non-binding constraints have no marginal effect. Also, variables that appear in constraints that are binding only in one direction have a one-sided marginal effect.

These problems only occur, however, in rather rare and pathological cases. We will focus on equilibria where all constraints are binding within an open set, which we call nonzero equilibria:

Definition. An equilibrium is *nonzero* for matching supply and demand components S_i and D_i if $c_r > 0$, $\lambda_r > 0$, $p_{S_i} > 0$, and $e_r^s > 0$ for all $s \in S_i$ and $r \in D_i$ when $(s, r) \in M$.

The nonzero property relates to the complementary slackness conditions of the optimization problem. It guarantees that relevant resource and regulation constraints are binding.

5.2 Subset of equations

Finally, we have developed all the necessary concepts and propositions in order to derive the theorem that gives a subset of equilibrium conditions that are necessary and sufficient to describe the equilibrium locally.

Theorem 7. *Consider an equilibrium that is nonzero for matching supply and demand components S_i and D_i. Variables $(c_r)_{r \in D_i}$ and $(p_s)_{s \in S_i}$ satisfy the equilibrium constraints if and only if*

$$\sum_{r \in D_i} c_r - \sum_{s \in S_i} \omega_s = 0 \tag{3}$$

$$f_r'(c_r) - p_{S_i} = 0 \quad \forall r \in D_i, \tag{4}$$

where $p_{S_i} = p_s = \lambda_r$ for all $s \in S_i$ and $r \in D_i$.

Theorem 7 states that equations (3) and (4) characterize the nonzero equilibrium for matching supply and demand components S_i and D_i. Equation (3) can be interpreted as a market clearing condition and equation (4) as an optimality condition. The equilibrium is unique with respect to p_s and c_r, since f_r' is strictly decreasing.

Theorem 7 answers our main research question: Only systems in matching supply or demand components affect each other. All firms within the same demand component face the same permit price and pay the same marginal abatement cost.

5.3 Marginal effects

With Theorem 7, comparative statics can be used to assess the effects of a change in the endowments and the production technology. A change in the production technology is modelled using an additional factor of production β_s, which acts as a perfect substitute for emissions. The firm's production function is now $y_s = f_s(c_s + \beta_s)$, where $\beta_s \in \mathbb{R}$ is taken as given. An increase in β_s allows the firm to produce its output y_s with fewer emissions c_s.

Corollary 8. *Consider an equilibrium that is nonzero for matching supply and demand components S_i and D_i.*

1. *An increase in the endowment ω_s of system $s \in S_i$ will decrease price p_{S_i} and increase emissions c_r for all $r \in D_i$.*

2. *An increase in the non-emitting factor β_r of system $r \in D_i$ will decrease price p_{S_i}, decrease emissions c_r, and increase $c_{r'}$ for all $r' \in D_i \setminus \{r\}$.*

Part 1 of Corollary 8 means that increasing the endowment of permits in one system will (i) lower prices within its supply component, and (ii) increase emissions of firms within the matching demand component. Part 2 of Corollary 8 means that introducing cleaner technology to one firm will (i) lower prices in the matching supply component, and (ii) reallocate emissions from cleaner firm to other firms in the demand component.[8]

A key observation in Corollary 8 is that, on the margin, exogenous effects are limited to the matching supply and demand components. Other systems in the network will remain unaffected.

[8] The quantity of emissions is unaffected as it is determined by the number of permits.

6 Discussion

Our model can be interpreted as a generalization of a simple trade model. If all systems are linked, the model is reduced to a simple partial equilibrium model of production with open trade. On the other hand, if none of the systems are linked, the model corresponds to autarchy. Our theory is of no use in these extreme cases, but it can be used to study the intermediate cases. We can ask, for example, will the the single market break up, if certain links are removed from the trade network.

Our model could also be formulated as a variational inequality problem (Nagurney, 1993), which have several numerical solving algorithms (Noor, 2004). Still, we find that a traditional optimization problem provides better economic intuition.

Alternating paths may be viewed as pathways for price signals, which are needed to equate prices and to achieve gains from trade. Our theory shows that agents need not operate directly in the same market to equate prices. The essential requirement is that there is a proper sequence of agents who share buyers and sellers. Without such paths the competitive market will end up fragmented and inefficient.

Alternating paths of supply and demand are useful for detecting whether markets are interdependent. If we observe permits being sold from one system to another, we know they are connected in the equilibrium network. If an alternating path of connections is found between two systems, we can infer that they belong to same component.

To avoid the illicit printing of new permits, linking agreements should forbid linking partners from making new links without the consent of other. This would prevent unwarranted entry into the supply component. However, third parties could still unilaterally allow the of use of foreign permits and thus connecting to the demand component. Fortunately, this is less harmful as it would merely make the domestic emissions target more stringent at the cost of the third party.

The model formulation has the useful feature of allowing for credit-and-baseline systems (in addition to cap-and-trade systems). The production function has a saturation point $\hat{c}_s$, a point above which there is no gains from further emissions, that is $f'_s(\hat{c}_s) = 0$. This point is often called the baseline emissions level. The credit-and-baseline systems are special cases where endowments equal baseline emissions. The number of credits equals the difference between the baseline and emissions, $\omega_s - c_s$, while emissions themselves require no permits.[9] Note that credits need an external source of

[9]Economically it makes no difference whether (a) firms receive $\omega_s - c_s$ credits to sell,

demand, e.g. a link to a cap-and-trade system.

Two key assumptions limit the applicability of the model. First, the model assumes that permit markets are competitive, which implies that firms take prices as given. In reality, however, firms might have market power, which is known to cause problems (Hahn, 1984). Market power might result from poor ETS design, and weaken cost-efficiency (Hahn and Stavins, 2011; Liski and Montero, 2011). Beside diminished efficiency, it is unclear how the results would differ under imperfect competition. It is worth noting that linking increases market size and therefore reduces the market power of individual firms. Also, price discrimination is unlikely to occur when the secondary markets for permits are liquid.

Second, we have assumed that links and endowments are exogenous. This allows to focus on the dependency structure of the trading network but it rules out strategic behaviour by the regulators. We have implicitly assumed that regulators are willing to set up costly limits on emissions and link systems together. However, linking is a strategic choice and links change the incentives for choosing emissions targets. These considerations are beyond the scope of this study, but nonetheless our theory provides a vital intermediate step towards that goal. The model indicates what payoffs would result from given strategy profile.

7 Conclusions

Our simple model of trade in a network of linked markets has shown that in equilibrium the network is split into separate market areas. These areas are characterized by matching supply and demand components that have a common price. Supply and demand components function much like a single market, and exogenous changes affect the equilibrium as conventional economic theory predicts.

The theory suggests that links between ETSs should be made with caution to avoid unanticipated dependencies. However, these dependencies are unavoidable if policymakers want to improve cost-efficiency.

Future research should examine the incentives regulators have for adding new links and tightening emissions targets when the policy architecture consists of a network of linked markets. We hope our theory is be a good starting point for such work.

The next decades are likely to see a considerable rise in both the ambition and the cost of climate policies. This will highlight the need to improve cost-efficiency and link policies internationally. Our theory shows that markets

or (b) they receive ω_s permits, use c_s permits, and sell the remaining $\omega_s - c_s$.

could become fragmented and points where potential gains from trade may be found. If links succeeds in uniting permit markets and reducing costs, they will take us closer to the goals set out in the Paris Agreement.

A Proofs

Proof of Lemma 1

Proof. Let $s \in S$ and $p_r \geq 0, \omega_r > 0$ for all $r \in S$. The necessary and sufficient conditions for the solution are given by the Karush-Kuhn-Tucker theorem, since the optimization problem is convex and satisfies appropriate regularity conditions (e.g. Slater's condition). The conditions can be expressed with the help of a Lagrangian function

$$L_s(c_s, e_s^1, \ldots, e_s^n, \lambda_s) = f_s\left(c_s\right) + p_s\omega_s - pe_s - \lambda_s(c_s - a_s e_s).$$

The Karush-Kuhn-Tucker conditions are as follows:
for all $r \in S$ such that $a_s^r = 1$

$$\frac{\partial L}{\partial e_s^r} = \lambda_s - p_r \leq 0, \tag{5a}$$

$$e_s^r \frac{\partial L}{\partial e_s^r} = e_s^r\left(\lambda_s - p_r\right) = 0, \tag{5b}$$

$$e_s^r \geq 0, \tag{5c}$$

for all $r \in S$ such that $a_s^r = 0$

$$\frac{\partial L}{\partial e_s^r} = -p_r \leq 0, \tag{6a}$$

$$e_s^r \frac{\partial L}{\partial e_s^r} = e_s^r(-p_r) = 0, \tag{6b}$$

$$e_s^r \geq 0, \tag{6c}$$

and

$$\frac{\partial L}{\partial c_s} = f_s'(c_s) - \lambda_s \leq 0, \tag{7a}$$

$$c_s \frac{\partial L}{\partial c_s} = c_s\left(f_s'(c_s) - \lambda_s\right) = 0, \tag{7b}$$

$$c_s \geq 0, \tag{7c}$$

$$\frac{\partial L}{\partial \lambda_s} = c_s - a_s e_s \le 0, \tag{8a}$$

$$\lambda_s \frac{\partial L}{\partial \lambda_s} = \lambda_s \left(c_s - a_s e_s\right) = 0, \tag{8b}$$

$$\lambda_s \ge 0. \tag{8c}$$

Formally, the first part of the lemma states: if $c_s > 0$, then for all $r' \in S$ such that $(r', s) \in A$ and $p_{r'} > \min_{(r,s)\in A} p_r$ applies that $e_s^{r'} = 0$.

By rearranging (5a) and (7a), we see that the marginal product $f_s'(c_s)$ is a lower bound for all prices among the allowed permits, that is

$$f_s'(c_s) \le \lambda_s \le p_r$$

for all r such that $(r, s) \in A$.

Since $c_s > 0$, (7a) is binding, i.e. $f_s'(c_s) = \lambda_s$. Inequality (8a) implies $a_s e_s \ge c_s > 0$, so there is some r for which $e_s^r > 0$ and $a_s^r = 1$. For such r inequality (5a) must be binding, because of (5b). Now $\lambda_s = p_r$, therefore $f_s'(c_s) = \lambda_s = p_r$. Because $f_s'(c_s)$ is a lower bound,

$$f_s'(c_s) = \min_{(r,s)\in A} p_r. \tag{9}$$

For all r' for which $p_{r'} > p_r = \lambda_s$, that is $\lambda_s - p_{r'} < 0$, (5b) implies $e_s^{r'} = 0$.

Second, let there be a saturation point $\hat{c}_s$. Suppose, contrary to our claim, that $c_s > \hat{c}_s > 0$. Strict convexity and the definition of a saturation point imply $f_s'(c_s) < f_s'(\hat{c}_s) = 0$. Now (7b) and (7a) yield $\lambda_s = f_s'(c_s) < 0$, which is a contradiction with (8c). Thus $c_s \le \hat{c}_s$.

Third, let $e_s^r > 0$ for some $r \in S$ such that $(r, s) \notin A$. Then (6b) shows that $p_r = 0$. $\square$

Proof of Lemma 2

Proof. We first construct the social planner's problem, and then show that the solution is equivalent with the decentralized perfect competition equilibrium in a trading network.

The social planner's problem is to choose emissions c_s for each $s \in S$ and emissions permits e_s^r for each $s, r \in S$ to maximize the sum of outputs while satisfying the resource and regulation constraints. That is, it solves

$$\max_{\substack{c_1,\dots,c_n,\\ e_1,\dots,e_n}} \sum_{s\in S} f_s\left(c_s\right),$$

so that resource constraints

$$\sum_{r \in S} e_r^s \leq \omega_s$$

and regulation constraints

$$c_s \leq a_s e_s$$

are satisfied for all $s \in S$.

As with Lemma 2, the Karush-Kuhn-Tucker theorem gives the necessary and sufficient conditions for the solution. The conditions can be expressed with the help of a Lagrangian function

$$\begin{aligned} & L(e_1, \ldots, e_n, c_1, \ldots, c_n, \lambda_1, \ldots, \lambda_n, p_1, \ldots, p_n) \\ & = \sum_{s \in S} \left(f_s\left(c_s\right) - \lambda_s (c_s - a_s e_s) - p_s \left(\sum_{r \in S} e_r^s - \omega_s \right) \right), \end{aligned}$$

where $\lambda_1, \ldots, \lambda_n, p_1, \ldots, p_n$ are the non-negative Lagrangian multipliers.

The Karush-Kuhn-Tucker conditions consists of conditions (5–8) and the market clearing condition

$$\frac{\partial L}{\partial p_s} = \sum_{r \in S} e_r^s - \omega_s \leq 0, \tag{10a}$$

$$p_s \frac{\partial L}{\partial p_s} = p_s \left(\sum_{r \in S} e_r^s - \omega_s \right) = 0, \tag{10b}$$

$$p_s \geq 0, \tag{10c}$$

for all $s \in S$.

If all firms' problems are solved individually while taking prices as given and the market clearing conditions (10) is assumed, we get the same set of conditions as in the social planner's case. Therefore the equilibrium for the representative firms is also efficient. □

Proof of Lemma 3

Proof. First, let s and r be adjacent sellers. Now there is a system t such that $(s, t) \in M$ and $(r, t) \in M$, and by definition $p_s = p_t^*$ and $p_r = p_t^*$. Hence $p_s = p_r$.

Second, let s and r be adjacent buyers. Now there is a system t such that $(t, s) \in M$ and $(t, r) \in M$, and by definition $p_t = p_s^*$ and $p_t = p_r^*$. Hence $p_s^* = p_r^*$. Now, Part 1 of Lemma 1 implies that s and r use permits with the same price. □

Proof of Proposition 4

Proof. First, let s and r be members of a supply component. Now there is a path of adjacent sellers between s and r. According to Lemma 3, each consecutive pair in the path must sell at the same price. Due to the transitivity of the equivalence relation, s and r sell at the same price. The proof for demand components is similar. □

Proof of Lemma 5

Proof. Let (S_i, M_i) be a supply component of (S, M), and $D_i = \{r \in S \mid \exists s \in S_i \colon (s, r) \in M\}$. Let $s, r \in D_i$. Now there are vertices $t_0 \in S_i$ and $t_l \in S_i$ such that $(t_0, s) \in M$ and $(t_l, r) \in M$. If $t_0 = t_l$, then s and r are adjacent buyers. Suppose $t_0 \neq t_l$. Since S_i is a supply component, t_0 and t_l are connected by a path of adjacent sellers, which we denote by sequence of vertices $(t_0, t_1, \ldots, t_{l-1}, t_l)$. For each $j = 1, \ldots, l$, subsequent system (t_{j-1}, t_j) are adjacent sellers, and hence there exists $q_j \in D_i$ such that $(t_{j-1}, q_j) \in M$ and $(t_j, q_j) \in M$. Now vertices q_{j-1} and q_j are adjacent buyers for all $j = 2, \ldots, l$, as are (s, q_1) and (q_l, r). Thus s and r are connected by a path in the adjacent buyer graph.

Vertices in D_i are not connected to other vertices of the supergraph outside of D_i, because if $s \in D_i$ were connected to $r \in S \setminus D_i$, we could use the same strategy as above to show, that $r \in D_i$, which would be a contradiction.

The proof for part 2 is similar. □

Proof of Proposition 6

Proof. Let S_i and D_i be matching supply and demand components of equilibrium network (S, M).

Suppose $(s, r) \in \{(s, r) \in M \mid s \in S_i, r \in S\}$. Now $s \in S_i$ and $r \in S$, and part 1 of Lemma 5 implies $r \in D_i = \{r \in S \mid \exists s \in S_i \colon (s, r) \in M\}$. Therefore $(s, r) \in \{(s, r) \in M \mid s \in S, r \in D_i\}$.

Suppose $(s, r) \in \{(s, r) \in M \mid s \in S, r \in D_i\}$. Now $s \in S$ and $r \in D_i$, and part 2 of Lemma 5 implies $r \in S_i = \{s \in S \mid \exists r \in D_i \colon (s, r) \in M\}$. Therefore $(s, r) \in \{(s, r) \in M \mid s \in S_i, r \in S\}$. □

Proof of Theorem 7

Proof. Consider a nonzero equilibrium, where S_i and D_i are matching supply and demand components.

Let variables $(c_r)_{r\in D_i}$ and $(p_s)_{s\in S_i}$ satisfy the equilibrium constraints. The nonzero property and Proposition 4 imply that $p_{S_i} = p_s > 0$ for each $s \in S_i$. Now, the related resource constraints (10a) are binding, i.e. $\sum_{r\in S} e_r^s = \omega_s$, for all $s \in S_i$. Part 1 of Lemma 1 implies $e_r^s = 0$ for all r such that $(s,r) \notin M$. Therefore we can rewrite

$$\sum_{(s,r)\in M} e_r^s = \omega_s \quad \forall s \in S_i. \tag{11}$$

Similarly, the nonzero property means that $\lambda_r > 0$, thus the related regulation constraints (8a) are binding for each $r \in D_i$. That is, $a_r e_r \equiv \sum_{(s,r)\in A} e_r^s = c_r$ for all $r \in D_i$. Again, Part 1 of Lemma 1 implies $e_r^s = 0$ for all s such that $(s,r) \notin M$. Therefore we can rewrite

$$\sum_{(s,r)\in M} e_r^s = c_r \quad \forall r \in D_i. \tag{12}$$

Now, we can derive (3) using (11) and (12) together with Proposition 6:

$$\sum_{r\in D_i} c_r = \sum_{r\in D_i} \sum_{(s,r)\in M} e_s^r = \sum_{s\in S_i} \sum_{(s,r)\in M} e_s^r = \sum_{s\in S_i} \omega_s.$$

For all $s \in S_i$ and $r \in D_i$ such that $(s,r) \in M$ the nonzero property implies $e_r^s > 0$. Thus constraint (5a) is binding, so that $\lambda_r = p_s$. Furthermore, Proposition 4 implies $\lambda_r = p_s = p_{S_i}$ for all $s \in S_i$ and $r \in D_i$. Constraint (7a) is also binding, because $c_r \geq e_r^s > 0$. This gives $f_r'(c_r) = \lambda_r = p_s = p_{S_i}$ for all $s \in S_i$ and $r \in D_i$, which is equivalent to equation (4).

Necessity is trivial. □

Proof of Corollary 8

Proof. Let S_i and D_i be matching supply and demand components of a nonzero equilibrium. Let the production functions now be of the form $y_s = f_s(c_s + \beta_s)$, where $\beta_s \in \mathbb{R}$ is given. We rewrite (3) and (4) as

$$F((c_r)_{r\in D_i}, p_{S_i}; (\beta_r)_{r\in D_i}, (\omega_s)_{s\in S_i}) = 0, \tag{13}$$

where $F\colon \mathbb{R}^{2n_D+n_S+1} \to \mathbb{R}^{n_D+1}$ is an implicit function and n_D and n_S indicate the number of elements in sets D_i and S_i. Note that the second derivatives of the production functions f_r are continuous and strictly negative, and the Jacobian matrix of F with respect to the endogenous variables is invertible. The implicit function theorem asserts that (13) defines a unique continuously differentiable function from the exogenous variables $(\beta_r)_{r\in D_i}$ and $(\omega_s)_{s\in S_i}$ to the endogenous variables $(c_r)_{r \in D_i}$ and p_{S_i} in some neighbourhood of given exogenous variables.

First, consider that system $s \in S_i$ increases its endowment ω_s. Partially derivating (13) with respect to ω_s yield a system of equations:

$$\sum_{r \in D_i} \frac{\partial c_r}{\partial \omega_s} - 1 = 0 \tag{14}$$

and

$$f_r''(c_r) \frac{\partial c_r}{\partial \omega_s} - \frac{\partial p_{S_i}}{\partial \omega_s} = 0 \quad \forall r \in D_i. \tag{15}$$

Equation (15) can be rearranged to get

$$\frac{\partial c_r}{\partial \omega_s} = f_r''(c_r)^{-1} \frac{\partial p_{S_i}}{\partial \omega_s} \quad \forall r \in D_i \tag{16}$$

and plugged into (14) to get

$$\frac{\partial p_{S_i}}{\partial \omega_s} = \left(\sum_{r \in D_i} f_r''(c_r)^{-1} \right)^{-1} < 0, \tag{17}$$

which is negative because the second derivative of a strictly concave function is negative. Plugging this into (16) gives

$$\frac{\partial c_r}{\partial \omega_s} = f_r''(c_r)^{-1} \left(\sum_{r' \in D_i} f_{r'}''(c_{r'})^{-1} \right)^{-1} > 0 \quad \forall r \in D_i. \tag{18}$$

Second, consider that system $r \in D_i$ increases β_r. Partially derivating (13) with respect to β_r yield a system of equations:

$$\sum_{r' \in D_i} \frac{\partial c_{r'}}{\partial \beta_r} = 0, \tag{19}$$

$$f_{r'}''(c_{r'}) \frac{\partial c_{r'}}{\partial \beta_r} - \frac{\partial p_{S_i}}{\partial \beta_r} = 0 \quad \forall r' \in D_i \setminus \{r\}, \tag{20}$$

$$f_r''(c_r + \beta_r) + f_r''(c_r + \beta_r) \frac{\partial c_r}{\partial \beta_r} - \frac{\partial p_{S_i}}{\partial \beta_r} = 0, \quad r \in D_i. \tag{21}$$

We evaluate the derivative at $\beta_r = 0$, and rearrange (20) and (21) to get

$$\frac{\partial c_{r'}}{\partial \beta_r} = f_{r'}''(c_{r'})^{-1} \frac{\partial p_{S_i}}{\partial \beta_r} \quad \forall r' \in D_i \setminus \{r\} \tag{22}$$

and

$$\frac{\partial c_r}{\partial \beta_r} = f_r''(c_r)^{-1} \frac{\partial p_{S_i}}{\partial \beta_r} - 1, \quad r \in D_i.$$

Plugging these into (19) gives

$$\frac{\partial p_{S_i}}{\partial \beta_r} = \left(\sum_{r' \in D_i} f_{r'}''(c_{r'})^{-1} \right)^{-1} < 0, \tag{23}$$

which is negative as the second derivatives of a strictly concave function is negative. Plugging (23) into (22) gives

$$\frac{\partial c_{r'}}{\partial \beta_r} = f_{r'}''(c_{r'})^{-1} \left(\sum_{r' \in D_i} f_{r'}''(c_{r'})^{-1} \right)^{-1} > 0 \quad \forall r' \in D_i \setminus \{r\}, \tag{24}$$

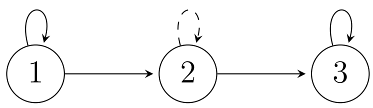
while (19) is equivalent to

$$\frac{\partial c_r}{\partial \beta_r} = - \sum_{r' \in D_i \setminus \{r\}} \frac{\partial c_{r'}}{\partial \beta_r} < 0, \tag{25}$$

where the expressions are negative due to inequality (24). □

B Numerical example

Consider emissions a trading network (S, A) with systems $S = 1, 2, 3$ and arcs $A = \{(1,1), (1,2), (2,2), (2,3), (3,3)\}$. The corresponding graph is depicted below.



Let $f_s(c_s) = \theta_s c_s - c_s^2$ and $\omega_s = 1/4$ for $s = 1, 2, 3$, and $\theta_2 = \theta_2 = 2$ and $\theta_3 = 5$. We can derive the derivatives $f_s'(c_s) = \theta_s - 2c_s$ and the saturation points $\hat{c}_s = \theta_s/2$. Given the arcs, the resource constraints are $e_1^1 + e_2^1 \leq \omega_1$, $e_2^2 + e_3^2 \leq \omega_2$, and $e_3^3 \leq \omega_3$.

Consider the equilibrium of the specified model. First, note that part 2 of Lemma 1 implies that in equilibrium $f_s'(c_s) > 0$ when $c_s < \hat{c_s}$. Because $c_s \leq a_s e_s \leq \sum_{r \in S} \omega_r = 3/4 < \theta_s/2 = \hat{c}_s$ for all $s \in S$, it follows that $f_s(c_s) > 0$. Now (7a) implies $\lambda_s > 0$, therefore (8b) implies that the regulation constraint (8a) is binding, and (5a) implies that $p_r > 0$ and the resource constraint (10a) is binding for all $s, r \in S$.

Given that (8a) is binding, (5a) and (7a) imply that

$$2 - 2e_1^1 \leq p_1,$$

$$2 - 2\left(e_2^1 + e_2^2\right) \leq p_1 \quad \text{and} \quad 2 - 2\left(e_2^1 + e_2^2\right) \leq p_2,$$

and

$$5 - 2\left(e_3^2 + e_3^3\right) \leq p_2 \quad \text{and} \quad 5 - 2\left(e_3^2 + e_3^3\right) \leq p_3.$$

If $p_3 > p_2$, then $e_3^3 = 0$ and $e_3^3 \neq \omega_3$, which would contradict the binding resource constraint for system 3. If $p_3 < p_2$, then $e_3^2 = 0$. As $e_2^1 \geq 0$ and $c_2 \geq e_2^2 = \omega_2 = 1/4$, which means $p_2 = \lambda_2 = f_2(c_s)$, we get

$$3/2 \geq 2 - 2\left(e_2^1 + e_2^2\right) = p_2 > p_3 = 5 - 2\left(e_3^2 + e_3^3\right) = 9/2,$$

which is a contradiction. Therefore $p_3 = p_2$.

If $c_2 = 0$, then $c_1 = e_1^1 = 1/4 > 0$ and (5a) and (7a) imply that $3/2 = 2 - 2e_1^1 = p_1 \geq 2 - 2\left(e_2^1 + e_2^2\right) = 2$, which is a contradiction. If $c_1 = 0$, then $c_2 \geq e_2^1 = 1/4 > 0$ and (5a) and (7a) imply that $2 = 2 - 2e_1^1 \leq p_1 = 2 - 2\left(e_2^1 + e_2^2\right) \leq 3/2$, which is a contradiction. Therefore $c_1 > 0$ and $c_2 > 0$. Also $c_3 \geq e_3^3 = \omega_3 > 0$

Now (7b) implies that $f_s(c_s) = \lambda_s$ for $s = 1, 2, 3$.

Suppose $p_1 = p_2 = p_3$. Now $e_1^1 = c_1 > 0$, $e_3^3 > 0$, and either $e_2^1 > 0$ or $e_2^2 > 0$, which implies $\lambda_1 = \lambda_2 = \lambda_3 = p_1 = p_2 = p_3$. Hence the marginal products must be equal:

$$2 - 2e_1^1 = 2 - 2\left(e_2^1 + e_2^2\right) = 5 - 2\left(e_3^2 + e_3^3\right).$$

By plugging in the resource constraints, we get

$$2 - 2(\omega_1 - e_2^1) = 2 - 2\left(e_2^1 + e_2^2\right) = 5 - 2\left((\omega_2 - e_2^2) + \omega_3\right). \tag{26}$$

Both sides of the first equation in (26) can be reduced by 2 and divided by -2 to get $\omega_1 - e_2^1 = e_2^1 + e_2^2$ and solved to get $e_2^1 = \frac{1}{2}\omega_1 - \frac{1}{2}e_2^2$. The second equation in (26) can be rearranged to get, $e_2^1 = \omega_2 + \omega_3 - \frac{3}{2} - \frac{3}{2}e_2^2$. Equating these gives $\frac{1}{2}\omega_1 - \frac{1}{2}e_2^2 = \omega_2 + \omega_3 - \frac{3}{2} - \frac{3}{2}e_2^2$, which can be solved to get

$$e_2^2 = \omega_2 + \omega_3 - \frac{3}{2} - \frac{1}{2}\omega_1 = -\frac{9}{8} < 0,$$

which is a contradiction.

Suppose $p_1 > p_2 = p_3$. Now $e_2^1 = 0$, $e_1^1 = \omega_1$, and $p_1 = \lambda_1$, which implies $3/2 = 2 - 2e_1^1 = p_1 > p_2 = 5 - 2\left(e_3^2 + e_3^3\right) \geq 4$, which is a contradiction.

Therefore in equilibrium the prices must have order $p_1 < p_2 = p_3$. Now, because $p_2 > \lambda_2$, (5b) implies that $e_2^2 = 0$. As the marginal products of systems 1 and 2 are equal, $2 - 2e_1^1 = 2 - 2e_2^1$, we can solve their permit usage: $e_1^1 = e_2^1 = c_1 = c_2 = \omega_1/2 = 1/8$. The price of permit 1 is set to the marginal product, $p_1 = 2 - \frac{2}{8} = \frac{7}{4}$. Given the resource constraints $e_3^2 = e_3^3 = \omega_2 = \omega_3 = 1/4$ and emissions are $c_3 = 1/2$. Also the prices of permit 2 and 3 are set to the marginal product of system 3:

$$p_2 = p_3 = 5 - \frac{2}{2} = 4.$$

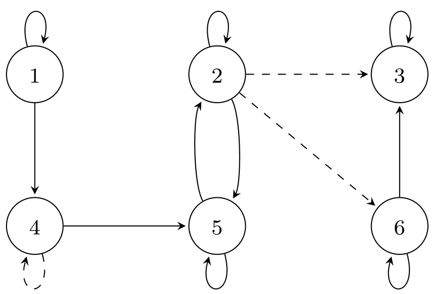
The equilibrium network is determined as (S, M), where $S = 1, 2, 3$ and $M = \{(1, 1), (1, 2), (2, 3), (3, 3)\}$. This depicted by the solid arcs in the previous graph. Note that arc $(2, 2)$ is missing which indicates that system 2 is not using its own permits in equilibrium.

The equilibrium network contains 2 pairs of demand and supply components: components with low price, $D_1 = \{1, 2\}$ and $S_1 = \{1\}$, and components with high price, $D_2 = \{3\}$ and $S_1 = \{2, 3\}$. We can denote the supply components' prices by $p_{S_1} = p_1$ and $p_{S_2} = p_2 = p_3$, respectively.

Note that this is a rather pathological situation for system 2 as it is not using its own permits. Furthermore, its supply and demand components are not matching. So supply component S_1 is matched with its demand component D_1, and demand components D_2 is matched with its supply component S_2. So according to Corollary 8, changes of supply in S_1 would affect emissions in S_1, while changes in demand in D_2 would affect the price of the permits in D_2.

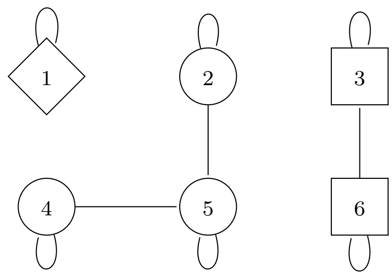
C Graphical example

Consider a trading network of six systems whose links can be depicted by the following directed graph:

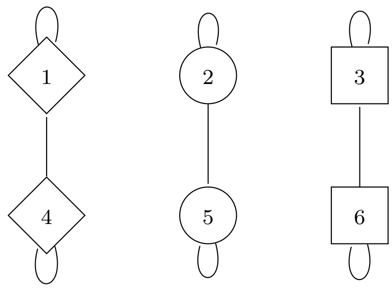


Suppose the dashed arcs are part of the trading network but not of the equilibrium network. That is, the dashed links are removed because at the arc's terminal vertex the lowest available price is cheaper than the price at the initial vertex.

Now we can determine which vertices are adjacent sellers to construct the adjacent seller graph:

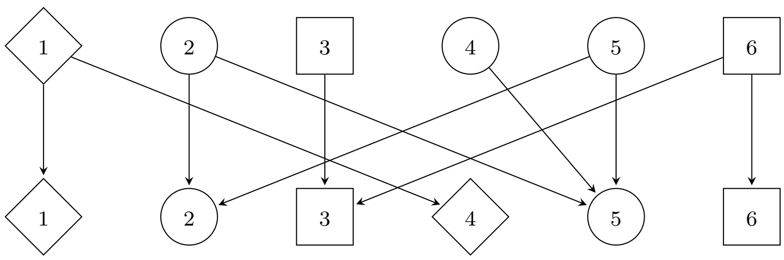


Here the different vertex shapes identify the three supply components that emerge. Similarly we can determine the adjacent buyers to construct the adjacent buyer graph:



We identify the matching demand component with the corresponding shapes. Sets $\{1\}$, $\{2, 4, 5\}$, and $\{3, 6\}$ indicate supply components and sets $\{1, 4\}$, $\{4, 5\}$, and $\{3, 6\}$ indicate their matching demand components, respectively.

Equivalently, the equilibrium network can be formulated as a bipartite graph where one set represents supply and the other demand, and where all nodes are included in both sets.

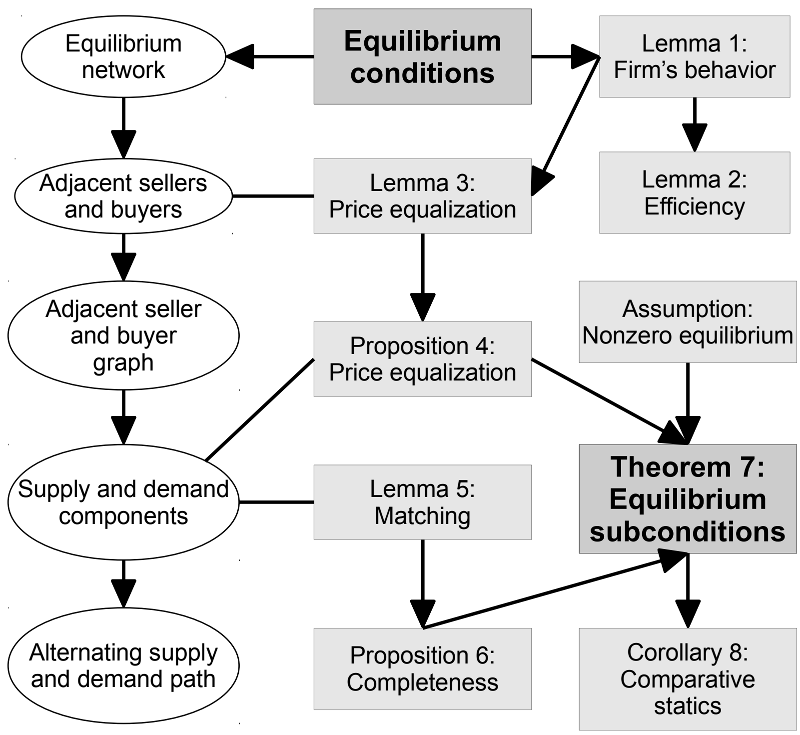


From this formulation we can identify matching supply and demand components. For example, the circle nodes $\{2, 4, 5\}$ on the upper row form a supply component. Its matching demand component can be found on the lower row, the circle nodes $\{2, 5\}$. The reader might find it useful to confirm that Lemma 5 and Proposition 6 apply for this graph.

Finally, we can easily determine which systems will be affected by changes in the exogenous variables. For example, a change in the endowment of system 4, which is a member of the ”circle component”, will affect the price of permits 2, 4, and 5 and emissions of systems 2 and 5. Similarly, cleaner technology in system 4 will decrease the price of permit 1, decrease emissions in system 4 and increase emissions in system 1.

D Outline of the theory

To help the reader, the following map outlines the structure of the theory and new concepts laid out in this paper. The theory starts from the equilibrium conditions and leads to our main result in Theorem 7. The oval nodes depict the key concepts which we have defined and used to present the theory.



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