TUOMAS SAARENHEIMO

Studies on Market Structure and Technological Innovation

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Helsinki, September 1994

Tuomas Saarenheimo
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1 Introduction

There is no need to expound on the central role of technological innovation in the continuing rise in the material well-being of mankind. Technological progress is essential to the improvements in efficiency that are necessary for sustained economic growth. There is overwhelming evidence that this progress does not occur merely in a random manner (see Schmookler, 1962) but is governed by the interaction of two distinct forces. On the one hand, there are technological opportunities supplied by Mother Nature and by the current state of technological knowledge. On the other hand, there are the economic opportunities and market forces: changes in demand create new profit opportunities for those quick enough to exploit them. The same distinction exists in the literature as well: economists tend to treat technological innovation as a black box and to concentrate on the economic side of the problem. Technologists, on the other hand, have been interested in the contents of the black box and have neglected the economic environment within which the innovating firm must operate.

The present work takes an approach closer to the former, economics-oriented, tradition, although it gets into certain technological issues that have previously not received much attention from economists. The main body of the work consists of three theoretical studies each of which examines the effects of market structure — defined as the degree of concentration, size, and competitive organization of the market — on different aspects of the innovative process. The three studies analyze issues such as the choice of research strategy and risk taking in R&D, the incentives to patent innovations, and research joint ventures. Each of these topics is reviewed later in this chapter. The chapter begins with a brief overview of the nature of the innovation process, followed by a review of the development and present state of the economics of technological progress. The emphasis is on microeconomic stud-
ies. However, as the interest in the micro analysis of technological progress stems from the realization of its importance at the macro level, a brief look at the macroeconomic aspects will be provided as well.

1.1 On The Nature of Innovation

1.1.1 A Taxonomy for R&D

The production of a new technology is very different from the production of a normal economic good. The production technology for an innovation depends on the technological opportunities, the extent of which is often only verified ex post, that is to say, after the project is finished — or has failed. Indeed, often it is not known whether the innovation is possible at all. To understand the stochastic nature of R&D it is instructive to start with a slightly modified version of the familiar taxonomy of the innovation process, which divides the production of technology into three stages (see Gomulka, 1990): research, invention, and innovation.

Research involves the basic and applied sciences, with the primary concern being on the discovery of new facts or principles about nature and society. An example of pure research is the search for laws of physics, for example, in electromagnetics. The research stage of R&D is undertaken almost solely by scientists.

The next stage in the R&D process is invention, which may be the creation of a new product or a new method of organizing an economic activity. It may include the creation of a prototype, usually followed by a lengthy and costly process of improvement. The latter part of the inventive stage will be called development, as opposed to pure invention. Loosely, a pure invention can be thought of as a completely new solution or approach to a problem, qualitatively different from what has been used before. Thus, while the creation of semiconductor technology as a whole can be classified as an invention, the realization that silicon chips can be used to process information — the pure invention — is separated
from the development process, during which the number of transistors packed on a single chip has steadily grown tenfold every five years. These two parts of inventive work often have quite distinctive characteristics, as will be discussed shortly.

The output of inventive work may never find its way to a commercial application. In case it does, the R&D process moves into its third stage, innovation. A technology becomes an innovation only after it has been commercialized. Because innovation is the end product of R&D, the term 'innovation process' is frequently used as a synonym for R&D process.

In practice, of course, the R&D process is much more complex than this, and in many cases the different stages cannot be separated from each other. Still, this taxonomy provides a useful tool for structuring the analysis, and we will make use of it later in this work.

1.1.2 Uncertainty and Divisibility

The two key dimensions of innovation are the degree of uncertainty and the degree of divisibility. As will be shown below, these two are closely related and can be fit into the taxonomy presented above.

Success in science is said to be one per cent inspiration and 99 per cent perspiration. Still, the research part of the process is undeniably stochastic. Nobody knows whether there will ever be a general and accurate theory of superconductivity, and whether it will be one that can be implemented in practical applications. In many fields, the uncertainties related to basic research are so great that research tends to be less of a search for the solution to a particular problem and more of a search for solvable problems. In private firms such a research strategy is seldom viable. Therefore in commercial innovative work, research is considered to be the last resort — it is undertaken only if no other solution to the problem is available. Most scientific research relies on pub-
lic funding. In this work, the research part of the R&D process receives less attention.

Saying that the first part of the invention stage — pure invention — also involves a high degree of uncertainty borders on the tautological. By definition, pure invention means doing something nobody has done before. Creating something completely new necessarily involves elements that one does not comprehend at the beginning and so must involve some degree of uncertainty. An example of a class of highly uncertain research programs which aim at creating pure inventions is a branch of physical chemistry called explanatory synthesis (or ‘shake and bake’ chemistry, depending on the speaker’s attitude towards the approach in question). In explanatory synthesis, one chooses (based on theory, experience, and guess work) mixtures from the periodic table of elements, cooks them up, and hopes something useful will come into being. Sometimes this approach works: a scientist team at IBM decided to have another look at a ceramic, ‘shaked and baked’ together by a group of French chemists more than a decade earlier. The ceramic turned out to be superconductive at a temperature far higher than any other known superconductor, and it brought the team the Nobel Prize in physics. A less encouraging example of a pure innovation, apparently not yet realized, is cold fusion. No one knows if it can be achieved, and even the best experts can only guess where to start the search. In this case it might be more accurate to use the term ignorance instead of uncertainty to underline the complete lack of knowledge of the densities of the relevant probability distributions.

The development stage, which follows the pure innovation, is typically characterized by less uncertainty. Once the basic properties of semiconductor technology were known, designing new more efficient microprocessors seems to have been a fairly determinis-

1Exception exist. In some countries, especially in Japan, a considerable share of basic scientific research is undertaken in private enterprises. This, however, is more a result of the modest amount of public funding devoted to basic scientific research than anything else. Japan’s overall investment in scientific research is small compared to that in Europe or the USA.

2This example is adopted from Romer (1992).
tic process. Every two years the major microprocessor producers bring out another generation of more advanced microprocessors, doubling or tripling the speed of its predecessor. When Intel announces — two years before the projected introduction — its plans to build a microprocessor containing 3.1 million transistors, there is little reason to doubt the company's ability to fulfill its promise.\(^3\) Even though for a single engineer working on a single clearly defined problem the solution may consist of a fortunate series of clever small design innovations, at the aggregate level, the micro-level stochasticity more or less cancels out, making the cost and duration of the process highly predictable. As another example of very expensive R&D projects, improvements in car design also seem to include little technical uncertainty. Designing a more aerodynamic body, raising horsepower, improving suspension or fuel energy are all routine development projects to car makers.\(^4\)

The degree of uncertainty in an R&D program is often related to another distinctive characteristic of the program, namely the divisibility of the inventive process. Pure inventions are typically inherently discrete — one either succeeds in producing cold fusion or one does not — although even in extreme cases the outcome is seldom precisely binary (succeeds/failure). For example, even though success in producing cold fusion would in itself be a great invention, the usefulness of it would depend crucially on whether and to what extent the created process produces more energy than it consumes. Still, with pure inventions it is rather easy to single out discrete steps of progress — from vacuum tube to transistor to microprocessor. Typically, the degree of uncertainty related to a certain invention is directly related to the size of the technological step it involves.

In development work, technological reality does not usually limit R&D projects to discrete steps. A product almost always goes through a series of incremental improvements over its life cy-

\(^3\)Whether the announced schedule will hold is another issue. Strategic factors may tempt the firm to announce an unrealistically tight schedule.

\(^4\)This does not mean that there is no uncertainty involved in designing and introducing a new car. It means that the uncertainty is mostly related to the market response instead of to the actual technological obstacles.
The incremental nature of development work is easily blurred by the fact that a number of incremental improvements may be grouped together and introduced at the same time. Regardless of whether improvements are implemented one by one or are grouped together and introduced in a large scale remodeling, the distinctive feature of development work is that a project can be chosen from a large number of possible projects with essentially incremental differences. For example, the reason why, after designing a microprocessor with 1.2 million transistors (i486), Intel builds another with 3.1 million transistors (Pentium) is not that these two are the only technologically feasible alternatives. Intel could just as well have built a microprocessor with two or four million transistors; it could probably design a series of microprocessors adding one transistor at a time, if it made economic sense. The reason for building a microprocessor with 3.1 million transistors is that Intel estimates that such a processor can be built with reasonable costs within a reasonable time frame so as to hit the market before Intel's old line of processors has lost too much market share to competitors, that it represents a sufficiently large improvement to compete with its present and future competitors long enough to recover the fixed costs involved in the R&D process, and finally, that it is clearly differentiated from Intel's existing products—all economical, not technical, considerations.  

To sum up, there are many types of inventive work: at one end of the continuum are projects characterized by high uncertainty and big technological steps, at the other projects involving low uncertainty and incremental technological steps. There is a tendency to identify technological progress with the first, that is, with clearly visible major inventions. Economists do not avoid this pitfall either; most theoretical analyses of the economics of innovation concentrate on major inventions and emphasize the role of uncertainty. In reality, however, technological change consists of a steady stream of improvements and modifications to

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5The reason for this lengthy discussion is that economists often tend to overemphasize the role of uncertainty in R&D work. Some aspects of R&D involve more uncertainty than others but the majority of commercial R&D falls into a category characterized by less uncertainty.
existing technologies, dotted with infrequent occurrences of major innovations. Commercially, small improvements play a much greater role than major innovations.

It is clear that the economics of these two types of innovation are considerably different and cannot be captured within a single model. One cannot analyze major innovations without introducing uncertainty. For development work, on the other hand, divisibility of the outcome is the characterizing feature, and the introduction of uncertainty into the model is usually an unnecessary complication.

Rather than uncertainty, the feature shared by the different types of R&D processes is that they all produce outcomes that are diverse and hard to measure. Innovation is usually thought of as the creation of a new product, but it may also be one of the following (see Kline and Rosenberg, 1986):

- a new production process;
- the substitution of a cheaper material in an essentially unaltered product;
- the reorganization of production or distribution, leading to increased efficiency; or
- an improvement in the instruments or methods of producing innovations.

Product innovation is the most visible form of innovation. A product is usually easy to identify and patent. Innovations related to processes, methods, or organization are typically less tangible. These kind of innovations are often hard to patent, which is why the innovator instead of patenting such innovation may be inclined to keep it secret. As a result, patents do not measure all kinds of technological innovation. Given the diverse nature of innovation, it is no wonder that no satisfactory measure of innovative output it has been found.

A final technical feature of the innovative process worth recognizing is that it is cumulative in nature. Each sequential step
provides knowledge useful in the next step. Research projects may fail, but yet they almost always provide some useful information. If a research project searching for superconductive materials ends after experimenting with a thousand different mixtures, none of which proves to be of any use, then at least the vast space of possibly useful mixtures not yet tried is infinitesimally smaller.

1.1.3 Economic Properties of Technology

The properties of technological innovation that set it apart from other economic goods can be summed up with two terms: technology is shareable (or nonrival) and partially excludable. Shareability means that the expenditures in R&D are sunk costs. Producing the technical information required by an innovation may involve a large investment, but once it is produced the cost of reproducing of it is trivial. Sharing a technology does not reduce the initial owner’s ability to use it. In this respect, technology resembles a public good.

The second feature, partial excludability, means that, although shareable, the owner of the technology has some control over the use of it. This control may be partly due to the legal system (patent legislation), partly because of secrecy, and partly in consequence of complementarity with another asset, for example, when the use of the technology requires some firm specific know how. Whereas shareability distinguishes technology from private goods, partial excludability sets it apart from public goods; it makes technology a tradable product.

The degree of excludability varies over different kinds of innovations from practically complete excludability to almost no excludability. An example of the latter could be an innovation including a simple application based on a very fundamental finding of the properties of the physical world; the laws of physics cannot be patented. Also, some important innovations, once found, are so trivial that they cannot be patented. The most important innovation in the transport industry since World War II, containerization, was — for obvious reasons — never patented. On the other hand, the molecular structure of a new synthetic drug is
easy to patent and the property rights straightforward to enforce. In such a case, monopoly power can only be eroded by creating a substitute with a different chemical structure.

Patenting is not always a prerequisite for excludability. In fact, since patenting necessarily involves some degree of disclosure, which may be used by competitors to create differentiated products, patenting can be downright harmful to the owner of the innovation. If the use of a certain innovation by competitors cannot be monitored — as is often the case with process innovations — then difficulties in verifying patent violations may lead the innovator to conceal the innovation and use it internally instead of patenting it.

The combination of shareability and partial excludability has important ramifications on the market for technology. Since the marginal cost of reproducing technology is zero, the socially optimal price of technology is also zero — all information should be shared immediately and with no charge. Evidently, if this was the case, there would be little incentive to innovate in a perfectly competitive industry. Competition would dissolve any monopoly profits and thus such an industry would produce no innovation. In order for costly R&D to be sustainable, a deviation from marginal cost pricing — from the first best solution — is necessary.

The partial excludability of technology enables a non-zero price for technology. On the other hand, that excludability is only partial implies another reason why the competitive market does not attain the first best; there are positive externalities related to technological progress. The innovator can seldom extract the social value of the innovation and tends therefore to invest too little in R&D.\(^6\)

To summarize this section, technology is a diverse product with characteristics that make market failure likely. Whether this market failure leads to too much or too little innovation, too many or too few firms in the market, and what further effects it has on firms’ R&D strategy are questions that have become the focus of

\(^6\)This is often referred to as the appropriability problem.
economic analysis in the last two or three decades. This issue is addressed in the next section.

1.2 Economic Analysis of Innovation

1.2.1 History

Considering that technological progress is a phenomenon that — with the advantage of hindsight — should have been clearly observable at least two hundred years ago during the industrial revolution, the start of microeconomic analysis on the subject took place surprisingly late. One explanation (although not a very satisfactory one) is that it was not until the turn of the century that macroeconomists began to understand the extent of the impact that the changing products and production methods were having on the people’s lives. During the 19th century and well into the 20th, growth was understood as the result of the accumulation of the physical factors of production. Not until World War II and the spurt in technological progress that it ushered in did economists grasp the importance of the role played by technological progress. Starting with the work by Abramowitz (1956), a series of studies showed that the accumulation of capital explained only 15-20 percent of the total growth in the labor productivity. The remaining 80-85 percent was labeled “the extent of our ignorance”.

In the era of neoclassical growth theory, from the mid 1950s until late 1970s, this residual term, “Solow residual” — i.e. what was left after taking into account the accumulation physical factors of production — was modeled as “exogenous technological progress.” Technological progress was considered exogenous, i.e. as something unaffected by economic forces and so beyond the scope of economic science. It was not until the mid 1980s that economists took the step from the recognition of the phenomenon of technological progress to the first attempt to genuinely model and explain it at the macroeconomic level. In the new branch of growth theory, the theory of endogenous growth, technological progress and topics like human capital accumulation, technolog-
ical externalities, and learning-by-doing play a central role (for classic articles, see Lucas 1988, and Romer 1990).

The microeconomic analysis of innovation took off somewhat earlier, in the works of Joseph Schumpeter. His central thesis was that to promote technological progress society must sacrifice perfect competition and static efficiency (Schumpeter 1942). According to Schumpeter, monopoly power and a sufficient scale of R&D was a prerequisite for technological innovation. Since much of the later formal microeconomic analysis has focused on Schumpeter’s propositions, they will be reviewed in the next subsection.

1.2.2 Schumpeter’s Hypothesis

Schumpeter’s work “Capitalism, Socialism, and Democracy” pointed the direction for later microeconomic analysis of innovation. He emphasized the central role of innovation in modern capitalist economies as well as the roles of the entrepreneur and the market structure in the innovation process. One of Schumpeter’s central arguments was that perfect competition suppresses innovation and is therefore “... not only impossible, but inferior, and has no title to being set up as a model of ideal efficiency” (Schumpeter 1942, p. 106). Schumpeter justified his argument in several ways. First, he suggested that large firms are a natural breeding ground for innovations because increasing returns are prevalent in R&D — two scientists or engineers work more efficiently if they combine efforts and share results. Secondly, he argued that large firms are better able to diversify the risks that are inherent in R&D. Thirdly, large firms are more eager to innovate because they can implement innovations in a larger scale. All these factors are related to the size of the firm rather than monopoly power. But Schumpeter also argued that bigness alone is not sufficient to bring about innovation and that some degree of monopoly power is essential for promoting R&D. Here, one can distinguish two arguments: The first was the old notion that the expectation of monopoly power encourages firms to innovate. More original was his second argument that an existing monopoly position was an ideal platform for undertaking innova-
tions, because a monopolist does not have competitors ready to imitate his innovations or to circumvent a patent on the innovation. Thus, one needs to distinguish between the pre-innovation and post-innovation market structures.

As Geroski (1990) points out, if the Schumpeterian hypothesis were solely an assertion that innovation occurs only when the post-innovation monopoly profits at least cover the innovator’s costs, it would be relatively uncontroversial. Some kind of post-innovation rent (or the expectation thereof) is crucial in order to cover the fixed costs involved in creating the innovation — otherwise no firm is willing to undertake R&D. However, few early participants in the debate were willing to go as far as Schumpeter in arguing that pure monopoly in the pre-innovation market is optimal for R&D. Galbraith (1952), for example, suggested that oligopoly market structures were more conducive to innovation than either monopoly or pure competition. One argument for this view that is often associated with the work of Leibenstein (see, e.g. Leibenstein 1966 or earlier Fellner 1951) is that competition acts as a “stick” that promotes innovative activity: when the rivals catch up, the profits of the incumbent drop, thus enhancing its incentive to introduce a new set of innovations. A highly concentrated market structure protected from entry may create little incentive for innovation if the group of established firms is content with its old oligopoly position and faces no threat of competition. In contrast, Schumpeter’s interpretation of the effect of potential entry on the innovative activity of the incumbent was that the threat of entry deters innovation by forcing the incumbent to lower its price, thereby diminishing the resources left for innovation. These two opposing views illuminate the complex nature of the question and underlines the need of a formal framework to structure the analysis. The interaction of these two forces, the “carrot” of post-innovation monopoly power and the “stick” of pre-innovation competition, is one of the central themes in the literature of market structure and innovation.

In a way, the dispute over the role of pre-innovation and post-innovation market structures is a rather academic one. This division is meaningful only in a stylized, one-shot model of R&D
competition, in which firms compete over a single innovation. The separation of pre-innovation and post-innovation market structures looses its meaning if — as is the case in most R&D work — technological progress is a never-ending sequence of innovations of different magnitude. The post-innovation market structure for one innovation is the pre-innovation market structure for the next. The typical patent race paradigm in which the pre-innovation market is competitive and the post-innovation market essentially monopolized is very rarely observed in reality.

1.2.3 Theoretical Approaches

There are several thorough surveys of the theoretical work on the relationship between market structure and innovation (see Kamien and Schwartz 1982, Baldwin and Scott 1987, Gomulka 1990, and also Reinganum 1989). Therefore, this section does not attempt to cover all the theoretical work in the area. Rather, it attempts to isolate and classify the most frequently used approaches. Regardless of the approach, most economic analysis of market structure and innovation has sought to answer two questions. The first is one inspired by Schumpeter: what is the optimal market structure for promoting innovation? The second question is: does the industry equilibrium involve too much or too little R&D compared to the social optimum. It is no surprise that the answer depends on the framework chosen.

The formal theoretical analysis of market structure and innovation started slowly in the 1960s, beginning with the seminal work by Arrow (1962). Arrow analyzed the polar cases of pure monopoly versus pure competition. In his model, R&D activity is monopolized: a single innovator, completely protected from competition, produces a cost-reducing innovation. Arrow examined two cases. In the first case, after producing an innovation the monopolist licenses it to a competitive industry; in the second case he uses the innovation himself and (in the case of drastic innovation) also monopolizes the product market. In Arrow’s model, the incentive for either drastic or nondrastic innovation is greatest in the case of a competitive industry.
Arrow's contribution inspired a large body of subsequent work which showed that the incentive to innovate is likely to depend on variables from which Arrow had abstracted. These studies introduced new elements by allowing for bilateral monopoly (Fixler 1983), uncertainty (Fixler 1983 and Donnenfeld 1983) and a more general n-firm structure in the production market (Kamien and Schwartz 1982). All of them followed Arrow in assuming that the production of technology is monopolized. Of course, this is not a completely satisfactory assumption. In most industries, there are a number of firms undertaking R&D simultaneously. Arrow's approach may be suitable in particular applications, but it is not useful to serve as a basis for a more general discussion of the economics of innovation.

*Patent races* are probably the most thoroughly analyzed class of R&D models. In a patent race, a number of firms compete for a single, well defined innovation. The winner of the race, i.e., the firm that innovates first, receives a patent and a perpetual monopoly generating a perfectly anticipated flow of rewards, while other firms receive nothing. Some kind of uncertainty is an essential part of this type of model. It may come in the form of uncertainty about the R&D intensity chosen by other firms (as in Kamien and Schwartz 1976a), or it may be related to the stochasticity in R&D technology (as in Kamien and Schwartz 1976b, Loury 1979, Lee and Wilde 1980, and many others).

In a typical patent race model firm i invests a constant stream $x_i$ in R&D. This investment produces a constant instantaneous probability $h(x_i) \geq 0$ that the firm succeeds in innovation. In other words, if the innovation is not ready at time $t$, then the probability that firm $i$ completes the innovation by time $t + dt$, where $dt$ is an infinitesimal increment of time, is $h(x_i)dt$. This formulation implies that the probability of innovation is independent of the past. The amount of R&D undertaken in the past
plays no role; all that matters is the instantaneous rate of R&D investment.\footnote{Because of this assumption, the optimization problem of a firm can be treated as a static one. Reinganum (1979) offers a genuinely dynamic model of R&D in which the factor determining the instantaneous probability success is a function of the cumulative R&D effort instead of the instantaneous effort.}

In patent races, competition takes place in the pre-innovation market, whereas the post-innovation market is monopolized (there are some exceptions, as in Delbono and Denicolo 1991). This assumption has some important implications. Since competition (measured by the number of competing firms) reduces pre-innovation profits but has no effect on post-innovation profits, one would expect that the incentive to innovate is positively related to the degree of competition. This is indeed the case. The standard result is that the industry level R&D expenditure is higher — and therefore the expected date of innovation is earlier — the greater the number of firms (Loury 1979). If the function $h(x_i)$ is concave, atomistic competition is the socially optimal market structure. Loury argues that it is more realistic to assume that $h$ is convex for small values of $x_i$, so that there are initial scale economies in R&D, in which case an intermediate number of firms is optimal. Lee and Wilde (1980) elaborate this view by assuming that there is a fixed cost involved in R&D activity.

Another result shared by most studies is that given a fixed market structure in the post-innovation market the industry equilibrium entails each firm investing more in R&D than is socially optimal. This result is quite intuitive: when the post-innovation market is monopolized and hence the appropriability problem does not arise, the departure from optimality is caused by the negative externality each firm imposes on its competitors by participating in the race. In a more realistic setting, post-innovation profits should also be related to market structure. The greater the number of pre-innovation competitors the more likely it is that competing products will appear side by side in the post-innovation market. Thus, patent races abstract from an important characteristic of R&D competition: the incentive to innovate depends
on the difference between the winner's profit and the loser's profit, which should both be determined by the market structure.

A further shortfall of patent races is that they are based on a narrow-minded view of technological change, in which technological progress comes solely in the form of distinct, major innovations. This view comes close to the definition of a pure invention, as classified above. The patent race framework is clearly not appropriate for analyzing incrementally accumulating technological progress. Furthermore, even in the case of pure inventions, the assumption of a memoryless R&D process is clearly unrealistic.

A third, much more heterogeneous category of models concentrates on the development part of the R&D process. Technological progress comes in the form of a reduction in production cost, and the relation between R&D expenditure and cost reduction is assumed to be deterministic. A seminal and representative work is the paper by Dasgupta and Stiglitz (1980). In their static model, firms choose simultaneously the level of cost-reducing investment and the level of production. Dasgupta and Stiglitz concentrate on symmetric equilibria, in which the number of firms is endogenous and determined by the zero-profit condition. Since both the number of firms and R&D expenditure are endogenous, the results cannot be directly compared to models in which an exogenous market structure determines R&D. However, their model implies that in a cross-section study of different industries one should observe a positive relationship between research intensity and concentration. Several studies extend the work of Dasgupta and Stiglitz by analyzing spillovers (Spence 1984), diffusion of innovation (Mokherjee and Ray 1990), uncertainty (Clemenz 1992) and dynamic aspects of R&D competition (Flaherty 1980, Tandon 1984).

Apart from the (typical) lack of uncertainty, the feature that sets these models apart from patent races is that in the former R&D is usually redundant. Every firm is working on the same cost-reducing investment and adding more firms just adds to the duplication. A single firm completes the innovation in the same time as \( n \) firms working parallelly but with a fraction \( 1/n \) of the
cost. Wasteful duplication is an inherent feature of these models. It implies that atomistic competition which disseminates all profits in the post-innovation market cannot induce any R&D activity. The maximum amount of R&D is reached either with a monopoly or, more frequently, with an intermediate number of firms. Given the market structure, industry equilibrium, as a rule, leads to R&D expenditure.

1.3 Three Aspects of Market Structure and Innovation

This section outlines the work presented in the three studies that follow this chapter. Each of these studies considers a distinct feature of the innovative process that has so far received little or no attention in the literature. The first study recognizes that the decision on a firm’s R&D strategy involves more than deciding how much money to invest. Equally important is how the money is invested. This study allows firms to choose from a large set of research strategies and compares the outcome of the competitive equilibrium with the social optimum. The second study proposes an explanation for two empirical findings: large firms produce fewer patents per unit of R&D input than small firms and seem to patent a smaller fraction of their innovations than their smaller competitors. The final study contributes to recent discussion of the relative merits of research joint ventures and cartelized R&D.

1.3.1 Market Structure and the Choice of Research Strategy

Decision making regarding R&D is a complicated process. As Kamien et al. put it, R&D “...is a multidimensional heuristic rather than a one-dimensional algorithmic process.” Nevertheless, an overwhelming majority of the theoretical work on innovation...

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8Some models involve spillovers, which change the situation somewhat, as in Spence (1984).
has concentrated on a single dimension of the process, namely the size of the R&D budget. The first essay approaches the issue from another direction. Instead of analyzing the effect of market structure and rivalry on the scale of the R&D work, it takes the scale as given and analyzes the effect of market structure on the choice of research strategy.

The work on this aspect of the R&D process has received very limited attention. The studies by Dasgupta and Stiglitz (1980b), Klette and de Meza (1986), Battaharya and Mokherjee (1986), and Dasgupta and Maskin (1987) shed some light on the issue. The latter two studies use a similar approach in which each firm chooses a distribution function from which it can draw the value of its innovation. The computation is simplified considerably by assuming a winner-take-all framework — only the single firm that draws the highest value actually receives the money. Under certain severe restrictions on the shape of the distributions, these two studies find that in a symmetric market equilibrium firms choose a weakly riskier distribution (in the sense of second-degree stochastic dominance) than is socially desirable. In the work of Battaharya and Mokherjee, the distributions are restricted to (i) being symmetric around the mean, (ii) having identical means, and (iii) being such that for any two of them the distribution functions intersect only at their common mean. These assumptions suffice to produce the stated result. Dasgupta and Maskin only restrict the distribution functions to having a common mean. In their analysis, market equilibrium is riskier, if the elasticity of each density function is everywhere greater than $-1$, i.e. if there is sufficient mass in the upper end of the distributions.

Dasgupta and Stiglitz (1980b), and Klette and de Meza (1986) use a slightly different approach. They state the model in terms of a patent race in which the firms choose a distribution of innovation dates. The first firm to innovate wins the patent. Dasgupta and Stiglitz find that market equilibrium is excessively risky. However, Klette and de Meza argue that this conclusion is erroneous in that their model has a tendency toward corner solutions in which both the market and the social optimizer choose the most risky strategy. With a modified version of the original model they
obtain the result that the symmetric market equilibrium is riskier than optimal. The result relies on strong assumptions. The distributions, as in Battaharya and Mokherjee, are assumed to have identical means, to be symmetric around this mean, and be such that their distribution functions intersect only at their common mean.

The analysis in Chapter 2 covers both approaches. In both cases, it offers results opposite to those reached in the earlier studies. Using a measure of compensated increase in risk it shows that for a wide class of plausible distribution functions market equilibrium, whether symmetric or asymmetric, is less risky than the socially optimal choice. The analysis is in several ways more general than that of the earlier studies. First, the restrictions on the distributions are considerably less severe: they need not be symmetric or have a common mean. Second, the analysis is not limited to symmetric equilibria: it is shown that in any asymmetric equilibrium there is a social gain if any single firm shifts to a marginally riskier strategy. Finally, the analysis allows for an arbitrary number of competing firms, whereas Battaharya and Mokherjee (1986) and Dasgupta and Maskin (1987) restrict their analyses to the two-firm case.

There are several reasons for the difference in the results. Most importantly, the studies that obtain results opposite to those of Chapter 2 postulate a one-to-one correspondence between the riskiness and the cost of a strategy: The riskier a strategy, the higher is the cost involved. As a consequence, the tendency of the market to overinvest in R&D, characteristic to patent race models, dominates any possible biases in the risk choice. Secondly, the concepts of riskiness employed differ slightly. In the earlier studies, an increase in risk is defined as a mean preserving spread in the distribution. This definition entails the problem of yielding corner solutions, as pointed out by Klette and de Meza (1986). Chapter 2 employs instead a compensated measure of risk: distribution A is riskier than distribution B if and only if the premium a person requires for choosing A instead of B is always higher the more risk averse he is. The definition incorporates second-degree stochastic dominance, as used in Battaharya and Mokherjee, and
Klette and de Meza. Finally, the results of Chapter 2 require that the distributions have sufficient mass at low payoffs. The earlier literature, e.g. Dasgupta and Maskin (1987), assumes essentially the opposite.

1.3.2 The Propensity to Patent

Schumpeter's central thesis was that large firms and concentrated market structures provide a natural breeding ground for innovation. This thesis has been challenged by a number of empirical studies which show that R&D output, measured by the number of patented innovations, increases less than proportionately with R&D input. This has been interpreted as evidence of decreasing returns in R&D.

The second study (Chapter 3) seeks an alternative interpretation of these findings. It argues that these results may be the consequence of systematic differences in the propensity to patent innovations. The work of Schmookler (1966), Branch (1973) and Scherer (1984) provides empirical support for this view. According to these studies, there has been a growing discrepancy between patenting and the actual pace of innovation, especially in the post-war period. In many industries the propensity to patent seems to be low particularly in larger firms.

The model framework is a two-stage patent race in which firms have the choice between patenting the first-stage innovation, in which case it can be licensed forward, or keeping it secret and using it to relative advantage in pursuing the final innovation. Licenses can be sold only to a patented innovation. The motivation for not patenting the first-stage innovation is that patenting reveals information and makes it easier for competitors to create their own technologies.

It is found that in concentrated markets firms have a tendency to keep their intermediate innovations secret, whereas in highly competitive markets they tend to patent and license forward. The relation between firm size and patenting is less straightforward and depends on informational assumptions. It is shown that under complete information, large firms may be more or less prone
to patent than small firms. However, when information is incomplete, in the sense that the potential licensee cannot verify the state of the licensor’s R&D project at the time of the licensing agreement, then the propensity to patent is always inversely related to the size of firm.

These results provide an alternative explanation to results that could be interpreted as evidence against Schumpeter. Yet they cannot be characterized as “Schumpeterian” since they do not support the Schumpeterian recommendation to promote large firms and concentrated market structures. Instead, they point to another potentially serious problem related to such markets. Large firms operating in concentrated markets tend to keep their intermediate innovations secret, which results in unnecessary and inefficient duplication in R&D — each firm has to create its own intermediate innovations, which under a more competitive market structure would be shared.

1.3.3 Research Joint Ventures and Cartels or Competitive R&D?

The last study (Chapter 4) joins recent literature in analyzing the relative merits of research joint ventures (RJVs). An RJV is defined as a collection of firms which agree to share the results of their R&D work. Forming an RJV changes the incentive structure of the firms in several ways. The firms in an RJV may be able to internalize a larger fraction of any positive technological externalities, which tends to increase R&D effort. On the other hand, if innovation merely redistributes (rather than increasing) industry profits, an RJV helps firms to avoid competition and tends to decrease R&D spending.

The two most frequently cited recent contributions in the field are the papers by d’Aspremont and Jacquemin (1988) and Kamien et al. (1992). The framework for these studies is a two-stage game. In the first stage, firms undertake cost-reducing investment. The cost level of an individual firm depends not only on its own R&D investment but, through technology spillovers, also on the R&D investment of other firms. In the second stage,
firms engage in a Cournot competition in the product market where the demand structure is linear. The studies agree that an RJV, in which the individual R&D inputs are chosen to maximize the joint profits instead of individual profits, is the most efficient way of organizing R&D. It provides both a higher rate of R&D investment and higher social welfare than does uncooperative competition.

Chapter 4 demonstrates that this result, besides depending on the specific demand function, depends crucially on the assumption that there is no market for the R&D output. It is shown that if firms are allowed to license their technologies to each other, competition may well lead to higher R&D and higher welfare than a RJV cartel. More precisely, competition is more likely to entail a higher level of R&D when technology spillovers are small and substitutability between goods is high. Also, the larger the number of firms in the market the better uncooperative competition performs. On the other hand, an RJV performs better with high spillovers, low substitutability, and a concentrated market. It is also shown that while an RJV cartel cannot overinvest in R&D, this is not the case under uncooperative competition. There is nothing to prevent competitive market from investing more than is socially desirable.
2 The Choice of Research Strategy in a Patent Race

2.1 Introduction

The patent race model, in which a number of firms compete for a given prize which a single winner captures completely, has been a central tool in the analysis of the relationship between market structure and R&D (Loury 1979, Lee and Wilde 1980). A typical patent race concentrates on the dependence of R&D scale on market structure. In its basic form, the patent race paradigm represents a rather one-dimensional view of innovation: there is only one path leading to the innovation, and a firm needs only to decide the size of its R&D budget, which determines how fast it will proceed along the path. In reality, making a decision on R&D is much more complicated than this. First, the goal of an R&D process is not generally a single, well-defined innovation but rather to meet the demand for a new product or process, which demand can be satisfied by a number of ways, or to exploit a technological opportunity, from which a variety of widely different usable innovations may flow. Second, even if the goal of R&D were a single innovation, identical for all competing firms, there would almost always be more than one way of achieving it.

Aside from the mainstream analysis, a small thread of articles has approached the R&D process from another direction. These papers employ more or less the same stylized framework as in the case of ordinary patent races: a given number of firms compete for a single prize. The fact that a competing firm is only interested in its own chance of winning the race, whereas society is indifferent between which individual firm is the winner, creates a distortion. However, instead of examining how this distortion affects the scale of R&D, these papers pose another question: how does the distortion affect the risk-taking behavior of firms? More precisely, when each firm can choose from a set of research strate-
gies, each providing a different distribution for the discovery date, is there a bias in the market solution, as compared to the social optimum?

The first to address this question were Dasgupta and Stiglitz (1980a). They argued that if riskiness involves a mean-preserving spread in the distribution and the cost of an R&D project is constant, then the market solution is safer, at the margin, than the social optimum. Klette and de Meza (1986) claim that this conclusion is erroneous. They show that although in a symmetric competitive equilibrium an incremental shift towards riskiness by all firms would leave the society better off, such a shift is usually not possible. Klette and de Meza show that if (in addition to the assumptions of Dasgupta and Stiglitz) the probability distributions are symmetric, then the competitive equilibrium as well as the social optimum coincide with the riskiest available strategy. In their own analysis, Klette and de Meza facilitate interior solutions by allowing the cost of R&D to differ between strategies or, effectively, to increase with riskiness. They go on to demonstrate that for symmetric distributions which are such that their cumulative distribution functions intersect only once (at their common mean), the competitive equilibrium involves a riskier strategy than the social optimum. They also show (by example) that if the distributions are sufficiently skewed, the market may be biased against riskiness. Two related papers by Battacharya and Mokherjee (1986) and Dasgupta and Maskin (1987) obtain similar results in a slightly different framework.

Klette and de Meza interpret their result as an indication that the market is biased toward riskiness. The central theme of this chapter is to show that this interpretation is misleading. In their model, the choice is essentially between a risky project, which provides higher expected payoff, and a safe project, which involves lower cost. Hence, the question is not one of a pure risk choice but rather a decision regarding both the scale and the strategy of R&D investment. The departure from optimality found in their paper is a combination of the tendency of the market to overinvest in patent races and, possibly, a bias in the choice of research strategy.
The analysis by Klette and de Meza does not offer an answer as to whether the latter bias exists and in what direction it works.

In order to focus on the pure risk choice, this chapter restores the assumption of Dasgupta and Stiglitz (1980) that all strategies have the same cost. In order to obtain interior solutions, the analysis departs from the definition of riskiness based on mean-preserving spread in the distribution, and instead employs a measure of compensated risk similar to the concept of mean utility preserving increase in risk as defined in Diamond and Stiglitz (1974). The definition allows the means of the distributions to differ and assumes nothing regarding the symmetry of the distributions. It is shown that for a wide class of distributions, including practically all commonly used ones, competitive equilibrium leads to a less risky strategy than is socially optimal. This is shown both for the unique symmetric equilibrium as well as for all arbitrary asymmetric equilibria.

These changes in the framework also change the interpretation of the model. It is not evident how to interpret a model in which cost and riskiness of R&D are positively linked and expected outcome is kept constant, as, for example, in Dasgupta and Maskin (1987) and Klette and de Meza (1986). If risk is something one needs (and wants) to pay for, the interpretation is certainly not very intuitive. In the present model, interpretation is easy. In the short run, a firm’s R&D resources are largely fixed. Drastic changes in the size of R&D investment generally necessitate major organizational rearrangements, which are likely to be distributed over a longer period. The relevant decision in the short run is therefore not how big a research lab to build or how many researchers to employ but rather how to use the existing research staff in the most productive fashion. This decision is the focus of this study.

The chapter is organized as follows. The next section introduces a definition of risk which is applicable to distributions with arbitrary means. In section 2.3 the relation between market equilibrium and the social optimum is analyzed on the basis of this definition. Section 2.4 applies the same approach within the frame-
work used by Dasgupta and Maskin (1987). Section 2.5 concludes the chapter.

2.2 The Definition of Risk

In the patent race studies mentioned above, increasing risk was defined as a mean preserving spread in the distribution of the discovery date. Diamond and Stiglitz (1974) introduced an alternative definition which they referred to as mean utility preserving increase in risk. A change in a distribution is a mean utility preserving increase in risk if it keeps the mean utility constant for some individual and decreases (increases) utility for all more (less) risk averse individuals. Greater risk aversion is defined in the familiar Arrow-Pratt sense (Pratt 1964, Arrow 1970).¹ This chapter adopts the definition of risk by Diamond and Stiglitz in a somewhat stronger form:

**Definition 2.1** Let $H_1$ and $H_2$ be cumulative distribution functions of random variables in $\mathbb{R}^+$ such that there exist $x_1, x_2 \in \mathbb{R}^+$ for which $H_1(x_1) > H_2(x_1)$ and $H_1(x_2) < H_2(x_2)$ and for some nondecreasing bounded utility function $u_j(x)$

\[
\int_0^\infty u_j(x) dH_1(x) = \int_0^\infty u_j(x) dH_2(x).
\]  

If for all utility functions $u_i(x)$ showing more risk aversion than at least one such $u_j(x)$

\[
\int_0^\infty u_i(x) dH_1(x) \leq \int_0^\infty u_i(x) dH_2(x),
\]

then $H_1$ is riskier than $H_2$. If the inequality is strict for some $u_i(x)$, then $H_1$ is strictly riskier than $H_2$.

Hence, if $H_1$ is riskier than $H_2$ and the shift from $H_2$ to $H_1$ is mean utility preserving (as defined by Diamond and Stiglitz 1974)

¹Recall that by the Arrow-Pratt definition, individual $i$ is more risk averse than individual $j$ if there exists a function $\phi$, $\phi' > 0$, $\phi'' < 0$, such that $u_i(x) = \phi(u_i(x))$. 

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for an individual \( j \), then the shift must decrease expected utility for all individuals that are more risk averse than individual \( j \). This corresponds well to our intuition: individuals that are more risk averse choose less risky assets. Notice that Definition 2.1 is stronger than that in Diamond and Stiglitz (1974); in their paper it is sufficient that (2.2) holds for all \( u_i \)'s that are more risk averse than some \( u_j \) satisfying (2.1), whereas here it is required that the same holds for all such \( u_j \).

The following theorem establishes that the single crossing property, which was sufficient for an increase in risk by the definition of Diamond and Stiglitz, is now both sufficient and necessary for ordering by Definition 2.1 (see the Appendix):

**Theorem 2.1** Distributions \( H_1 \) and \( H_2 \) fulfill the conditions of Definition 2.1 if and only if there exists \( x^* \in \mathbb{R} \) such that

\[
[H_1(x) - H_2(x)](x - x^*) \leq 0 \quad \text{for all } x \in \mathbb{R}.
\] (2.3)

Hence, two distributions can be ranked if and only if their cumulative distribution functions intersect exactly once.\(^2\) The distribution with the lower slope at the intersection point is the riskier. As is the case with second-order stochastic dominance, Definition 2.1 provides a partial ordering. This ordering is consistent in the sense that if \( H_1 \) is strictly riskier than \( H_2 \) and \( H_2 \) in turn is riskier than \( H_3 \), then \( H_3 \) cannot be riskier than \( H_1 \). However, unlike second-order stochastic dominance, the ordering is not transitive, since \( H_1 \) and \( H_3 \) do not necessarily satisfy the single crossing condition. Another interesting observation is that second-order stochastic dominance is not sufficient for ordering by Definition 2.1; that is, if \( H_1 \) dominates \( H_2 \) in the second-order sense, and their cumulative distribution functions intersect more than once, one can always construct a situation in which one individual prefers the less risky asset \( H_1 \) while another individual with

\(^2\)Strictly speaking, the distribution functions need not intersect. Since distribution functions need not be continuous, one distribution function may discretely jump from below to above the other. For lack of a better term, the term "intersection" is stretched here to cover all cases where the difference of the distribution functions changes sign.
greater risk aversion chooses the riskier asset $H_2$. Only if the difference of the cumulative distribution functions changes sign exactly once can such a counterintuitive situation be excluded.

Figure 2.1 illustrates some distribution functions that can be ranked according to this measure. Distributions $H_3$ is riskier than $H_2$, while both are riskier than distribution $H_1$.

2.3 The Model

The market consists of $n \geq 1$ identical risk-neutral firms that compete for a single, well defined innovation. The value of the innovation at the beginning of the game (at time $t = 0$) is normal-

\footnote{In order for this to occur, it is necessary that these individuals have partly concave and partly convex utility functions.}
ized to unity, and it declines over time at the exogenous rate $r$. Thus, at time $t$ the value of the innovation is $e^{-rt}$. The discount rate $r$ can be interpreted as the sum of the interest rate and an additional term which can be labelled the "rate of obsolescence," by which we mean either the steady rate at which the value of the patent is eroded by further technical progress, or the instantaneous probability at any moment of time that the innovation will be made completely obsolete by another innovation. In the spirit of patent races, the value of the innovation is rewarded to the single firm that innovates first. The model follows Dasgupta and Stiglitz (1980a) in assuming that the scale of R&D is fixed and the choice of research strategy is the only control variable.

Each firm can choose its research strategy from a possibly infinite or innumerable set of research strategies $K$. A research strategy $k \in K$ is characterized by its distribution function $F(t, k)$, which gives the probability that using this research strategy the firm completes the innovation on or before time $t$. All members of $K$ are proper distributions in that $\lim_{t \to \infty} F(t, k) = 1$. The corresponding density function is denoted by $f(t, k)$. As in previous studies, the probability distributions of the different firms are assumed to be independent of each other.

Before applying Definition 2.1 in the present context, some discussion about the application of risk measures in patent races is in order. As mentioned above, previous studies (i.e. Dasgupta and Stiglitz 1980a, Klette and de Meza 1986) define an increase in risk as a mean preserving spread in the distribution of the discovery date. This definition is problematic for various reasons. First, as demonstrated by Klette and de Meza, it tends to lead to corner solutions if the cost of R&D is kept constant. Secondly, the way Klette and de Meza apply mean preserving increase in risk disconnects the concept of risk from utility theory. When applied to distributions over payoffs, mean preserving spread in distribution can be termed an increase in risk in a well justified sense: no risk-averse individual chooses the asset with the riskier distribution. However, applying mean-preserving spread to distributions over time does not have the same justification; a mean preserving spread in the distribution of the discovery date is not
mean preserving in terms of distributions of payoffs. Translating the definition, a mean preserving spread in the distribution of payoffs (or second order stochastic dominance) in terms of distributions over time yields the following condition for strategy $k_1$ to be riskier than strategy $k_2$:

$$\int_t^\infty e^{-ru} [F(u, k_1) - F(u, k_2)] \, du \leq 0 \quad \text{for all } t \geq 0, \quad (2.4)$$

with equality holding at $t = 0$. This condition guarantees that the two strategies produce the same expected payoff and that the relevant integral conditions are satisfied. The drawback of condition (2.4) is immediately evident: the ordering of strategies depends on the interest rate. Two strategies that can be ordered for a certain $r$ cannot generally be ordered when $r$ changes.

Finally, a definition of risk that requires the mean of the distribution to be constant over strategies is overly restrictive and unrealistic. The probability distributions are given by the technological reality; there is no reason to believe that different strategies would have the same expected discovery date, nor that the management of the firm would seek to adjust R&D inputs to keep this date constant, as the approach of Klette and de Meza can be interpreted.

Using the compensated measure of risk instead of second-order stochastic dominance solves these problems: the ordering retains a sound basis in utility theory and the means of the distributions are not restricted in any way. It also turns out that the interest rate does not affect this ordering: the single crossing condition for the distributions of payoffs translates to an equivalent single

\[ \int_0^x [H(u, k_1) - H(u, k_2)] \, du \leq 0 \quad \text{for all } x \geq 0, \]

where the $H$ functions are the cumulative distribution functions over the payoffs $x$. This is the condition for strategy $k_1$ to stochastically dominate $k_2$. To get the equivalent condition for distributions over time one needs to substitute $x = e^{-rt}$ and $F(t, k_i) = 1 - H(e^{-rt}, k_i)$, which gives the above result.

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\[ \text{This condition is obtained by starting from the definition of second order stochastic dominance} \]

\[ \int_0^x [H(u, k_1) - H(u, k_2)] \, du \leq 0 \quad \text{for all } x \geq 0, \]

where the $H$ functions are the cumulative distribution functions over the payoffs $x$. This is the condition for strategy $k_1$ to stochastically dominate $k_2$. To get the equivalent condition for distributions over time one needs to substitute $x = e^{-rt}$ and $F(t, k_i) = 1 - H(e^{-rt}, k_i)$, which gives the above result.
crossing condition for the distributions of discovery dates. That is, strategy $k_1$ is riskier than $k_2$ if and only if $F(t, k_1)$ intersects $F(t, k_2)$ exactly once and from above. Here, it is assumed that all elements in $K$ can be ordered pairwise by this measure and that the index $k$ arranges the strategies in increasing order of riskiness: if $k_1 > k_2$, then $F(t, k_1)$ is riskier than $F(t, k_2)$.

2.4 The Analysis

2.4.1 Symmetric Equilibrium

The strategy of the individual firm $i$ is denoted by $k_i$. When choosing its strategy, each firm takes the actions of the other firms as given. The problem of firm $i$ is to choose $k_i \in K$ to maximize the expected revenue

$$\pi_i(k) = \int_0^\infty e^{-rt} f(t, k_i) \prod_{j \neq i} (1 - F(t, k_j)) \, dt, \quad k = (k_1, \ldots, k_n).$$

Thus, firm $i$ weights the payoff $e^{-rt}$ at time $t$ by the probability density of innovation $f(t, k_k)$ and the probability that no other firm has innovated by time $t$, given by $\prod_{j \neq i} (1 - F(t, k_j))$.

The market equilibrium will be compared with what would be obtained under joint profit maximization. The symmetric cooperative optimum is determined by the maximum of

$$\pi_s(k) = \int_0^\infty e^{-rt} f(t, k)(1 - F(t, k))^{n-1} \, dt \quad (2.5)$$

over all strategies $k$. If the social value of the innovation is equal to its private value, as is the case if the winner can establish a perfectly discriminatory monopoly, then the cooperative equilibrium

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The relationship between the distribution over discovery dates and the distribution over payoffs is

$$F(t, k_1) = 1 - H(x, k_1),$$

where $x = e^{-rt}$. If $F(t, k_1) - F(t, k_2)$ changes sign exactly once at the point $t = t'$, then $H(x, k_1) - H(x, k_2)$ changes sign exactly once at the point $x = e^{-rt'}$. 

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is also the social optimum. The strategy that maximizes (2.5) is referred to below as the social optimum.

Generally, the competitive game may possess several equilibria, symmetric and asymmetric. Here, the focus is first on symmetric equilibria, for which the following proposition is established in the appendix.

**Proposition 2.1** There exists at most one symmetric competitive equilibrium.

Proposition 2.1 applies very generally — it is not limited to distributions that can be ranked according to riskiness. It also holds in situations where there is nonzero probability that the innovation is never achieved, i.e., when \( \lim_{t \to \infty} F(t, k) < 1 \). Notice also that Proposition 2.1 does not guarantee the existence of a symmetric equilibrium. Indeed, it is easy to construct counterexamples in which no symmetric equilibrium exists.

In the comparison of market equilibrium with the social optimum, a crucial role will be played by the function

\[
h(t, k) \equiv \frac{f(t, k)}{1 - F(t, k)}.
\]

The function \( h(t, k) \) is often referred to as the hazard rate. It gives the instantaneous probability of innovation; the probability that the innovation takes place in the next small time increment \( \Delta t \) is \( h(t, k) \Delta t \). The results presented later are based on the following assumption:

**Assumption 2.1** For all strategies in \( K \), the hazard rate \( h(t, k) \) is nondecreasing in \( t \).

How restrictive is it to assume a nondecreasing hazard rate? Not very. There are both statistical and intuitive reasons for this assumption. A nondecreasing hazard rate is exhibited by not only all the probability distributions that exist in the patent race litera-
ture but by virtually all other distributions as well. For example, the gamma distribution (which is often the preferred probability model for waiting times) and the exponential distribution and the normal distribution (as limiting cases of the gamma distribution) fall into this class. The Weibull, extreme value, and uniform distributions also have nondecreasing hazard rates.

Intuitively, an increasing hazard rate can be viewed as resulting from knowledge accumulation. Suppose the instantaneous probability of success of a research team is a function not only of the flow of resources devoted to the project, but also of the experience of the group and the relevant knowledge on the subject. Then as experience and knowledge accumulate over the course of research, the hazard rate should increase accordingly — at least when the resources are kept constant, as is assumed here.

The analysis has so far implicitly assumed that when choosing a strategy, a firm commits itself to following that strategy to the end of the game. With this restriction, the game is essentially a static one. What happens if firms are allowed to switch to another

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6 The most widely used parametric probability distribution is the exponential distribution, for which the hazard rate is constant (see, for example, Loury 1979, Dasgupta and Stiglitz 1980a). The reason for the frequent use of the exponential distribution has probably more to do with computational simplicity than anything else. In models with knowledge accumulation, the hazard rate is typically increasing (Reinganum 1979). In models with more general distributions, the assumption of constant or increasing hazard rate is commonplace — so much so that it is often made without providing any justification (see the survey by Kamien and Schwartz 1982, p. 115, 180). Grossman and Shapiro (1986) offer a discussion of the role of the hazard rate.

7 Having infinitely long tails, the normal distribution is not feasible in the current framework. One can, however, easily construct a sequence of feasible distributions with nondecreasing hazard rates that converges to the normal distribution.

8 Actually, Reinganum (1979) shows that if firms are allowed to change their R&D investment over the course of the race and knowledge accumulates, then it is optimal for the firms to increase also the rate of R&D investment over time. This serves to further strengthen the upwardness of the hazard rate.

9 This practice is also followed in each of the previous papers mentioned above.
strategy at any time during the game? The problem is, that even if all strategies in $K$ can be ranked by the definition of risk given in the previous section, the same is not generally true for the new strategies formed by switching from one strategy to another at some point of the game. Therefore, commitment will be assumed throughout the paper. The following proposition demonstrates that this assumption is not always restrictive. For the proof of this proposition, see the Appendix.

Proposition 2.2 Let Assumption 2.1 hold and let $k_c$ be the symmetric competitive equilibrium given the strategy space $K$. Then $k_c$ is a subgame perfect equilibrium also when switching to other strategies is allowed.

Proposition 2.2 establishes that if hazard rates increase over time, then in a symmetric equilibrium, sticking to the original strategy throughout the game is optimal at each point of time even if switching is allowed.

The following proposition states the main result of this chapter. (Also proved in the appendix.)

Proposition 2.3 Let Assumption 2.1 hold and let $k_c$ be a symmetric competitive equilibrium and $k_s$ be the socially optimal (cooperative) solution for $n \geq 2$ firms. Then $k_s$ is riskier than $k_c$.

Hence, whether the market is biased against riskiness depends on the behavior of the hazard rate. If the hazard rate is everywhere nondecreasing for both the market equilibrium strategy and the socially optimal strategy, then the market equilibrium is always less risky than the social optimum. Intuitively, nondecreasing hazard rate guarantees that there is sufficient mass in the right tale of the distribution over $t$ (i.e. at low payoffs) so that when a competitor shifts to a riskier strategy, the gain from the increased probability of winning at high $t$'s more than outweighs the decreased probability of winning at low $t$'s. Notice that a nondecreasing hazard rate does not imply that the distribution is left skewed; it is consistent with positive as well as negative skewness.
The result in Proposition 2.3 differs markedly from the findings of Klette and de Meza (1986). The reason for this difference is straightforward. In the model of Klette and de Meza an increase in risk, i.e. a mean preserving spread in the distribution, is always desirable. A firm gains more risk only by investing more in R&D. Increasing the riskiness of a firm's strategy always exerts a negative externality on its competitors. Since the firm does not take this into account, it "buys" too much risk, from society's viewpoint. Thus, in the model of Klette and de Meza, the result that market equilibrium involves a riskier strategy than the social optimum is a consequence of the tendency of the market to overinvest in R&D rather than evidence of a bias against risk.

In the present model, the new definition of risk makes it unnecessary to resort to variable costs in order to obtain interior solutions. A spread in the distribution may be accompanied by a change the mean value as well, and hence risk is not inherently desirable. The choice concerns only the riskiness of the project — the overinvestment effect does not exist.

As noted above, the compensated measure of risk used here and the definition based on mean preserving spread are not mutually exclusive. It may be that the social optimum and the competitive equilibrium can be ranked using both measures. How does the result in Proposition 2.3 relate to previous results in this case? First, notice that the result of Klette and de Meza applies only for symmetric distributions. If both in the socially optimal solution and in the competitive equilibrium the distributions are symmetric, have identical means, and are such that their distribution functions intersect exactly once, then both Proposition 2.3 and the result by Klette and de Meza apply. This means that one distribution is riskier by Definition 2.1 and the other by the definition based on mean preserving increase in risk. But since the two definitions never contradict the solutions have to be identical. For asymmetric distributions, the two solutions either coincide or the social optimum is riskier by both measures.
2.4.2 Asymmetric Equilibria

The above analysis has followed previous papers by assuming that the equilibrium is symmetric; that is, every firm chooses the same strategy. When both the market equilibrium and the social optimum are assumed symmetric, one gains not only mathematical but also conceptual simplicity; the comparison of the two solutions is identical to comparison of two distributions. If one wants to allow for asymmetric equilibria, the comparison of the solutions becomes much more difficult. The market equilibrium as well as the social optimum may consist of \( n \) different distributions instead of just one. Only in the case where every strategy played in the market equilibrium is riskier than all strategies involved in the social optimum, could the two solutions be definitively ranked. This certainly is not the case generally. Comparing the joint probability distributions of the discovery date is not possible either; the joint distributions cannot generally be ranked by a measure of risk even if the individual distributions can. If one wants to further generalize the analysis by allowing firms to be of different size or to choose the R&D investment level endogenously, the complications involved make a general comparison of the two solutions, for all practical purposes, impossible.

To allow for the introduction of such asymmetries in the model, this subsection examines the risk choice from a partial equilibrium point of view. Each firm chooses both its level of R&D investment and research strategy. It is assumed that for any given level of R&D investment, the distribution functions of any two strategies intersect exactly once, and thus they can be ordered by the compensated measure of risk as defined above. The question asked is: Given the resources invested in R&D and all the choices made by other firms, could welfare be improved by adopting a strategy other than the individual optimum for the particular firm and, if so, is there a systematic bias in the riskiness of the individual optimum? As before, the level of R&D investment is taken as given. Allowing the social planner to adjust both the strategy and the scale of the R&D activity would make general comparison between risk choices impossible — first,
because an explicit specification of the R&D production function would be needed and secondly, because it would not be reasonable to expect any measure of risk to apply across all strategies and all levels of R&D investment.\textsuperscript{10}

It turns out that using the partial approach, very strong results follow. The following proposition is proved in the Appendix:

**Proposition 2.4** Let Assumption 2.1 hold. Then, given the strategies played by the other firms and the R&D investments of all firms, the individually optimal strategy $k_i$ for firm $i$ is less risky than the socially optimal choice $k_{is}$.

Proposition 2.4 applies very generally. It holds for any asymmetric equilibrium as well as in the case where the competitors' choices are not optimal. It does not depend on the sizes of the R&D projects. The two crucial elements are that the strategies played by the competitors have a nondecreasing hazard rate and that the distribution function of the socially optimal strategy intersects that of the individually optimal strategy exactly once. From these assumptions it follows that society always prefers a (weakly) riskier strategy than is individually optimal for firm $i$.

Overall, the results of this section are quite strong. It has been shown that if each firm chooses its strategy to maximize individual expected profit, the symmetric market equilibrium is less risky than the socially optimal strategy, defined as the strategy that maximizes industrywide expected profits. This bias against risk applies also in the level of an individual firm: the strategy that maximizes an individual firm's profit is always less risky than the strategy that maximizes industrywide profits, independently of whether or not the other firm's choices are optimal in any sense.

\textsuperscript{10}To illustrate the difficulty of ranking the riskiness of R&D projects of different size, consider the comparison of a project of size zero with another, very large project. It would be implausible to assume that the distribution functions for the discovery date always intersect; i.e. that there always exists some range of $t$ for which the infinitesimally small R&D lab is more likely to attain the solution than the very large one.
2.5 A Modification: Stochastic Payoffs

In this section, the results of the previous section are reproduced using the framework of Dasgupta and Maskin (1987) and Battacharya and Mokherjee (1986). Whereas in the patent race framework the choice was over different distributions of the discovery date, here the game is cast directly in terms of distributions of payoffs. Firm \( i \) chooses its strategy \( k_i \in K \), which gives rise to a probability distribution \( H(x, k_i) \), where \( x \in [0, \bar{x}] \) is the value of the innovation. All the strategies in \( K \) can be ordered by riskiness according to Definition 2.1. Again, risk comparison reduces to the single-crossing condition; strategy \( k_1 \) is riskier than strategy \( k_2 \) if \( H(x, k_1) \) intersects \( H(x, k_2) \) once and from above.

Only the firm that realizes the most valuable innovation actually receives the payoff — i.e. the winner-take-all assumption is retained. This framework can be interpreted, for example, as the reduced form of a game in which firms first undertake cost reducing investment and then engage in Bertrand competition in the product market. If there is a small fixed cost involved in the production, only the single firm with lowest production cost enters the market. The discounted value of the monopoly profits the firm gets can be interpreted as the value of the innovation \( x \).

Hence, firm \( i \) seeks to maximize

\[
\int_0^{\bar{x}} x h(x, k) \prod_{j \neq i} H(x, k_j) \, dx
\]  

(2.6)

over all strategies \( k \). The function \( h(x, k) \) is the probability density function corresponding to the distribution \( H(x, k) \). Expression (2.6) is the expected payoff where each \( x \) is weighted by \( \prod_{j \neq i} H(x, k_j) \), which is the probability that no other firm realizes a more valuable innovation.

Here the role of hazard rate is taken by the elasticity of the cumulative distribution function

\[
z(x, k) \equiv \frac{x h(x, k)}{H(x, k)}.
\]

Assumption 2.1 is modified accordingly:
Assumption 2.2 For all strategies $k$ in $K$, the elasticity function $z(x, k)$ is nonincreasing in $x$.

It can be shown that $z$ decreases weakly in $x$ for almost all common nonnegative distributions, such as the uniform, exponential, log-normal and gamma distributions. The normal distribution also falls into this class as a limiting case. Notice that while in the previous section nondecreasing hazard rate means that the right tail has sufficient mass, here it is the left tail that needs to be sufficiently thick.\footnote{That nonincreasing elasticity of the distribution function is equivalent to nondecreasing hazard rate in the previous section can be shown as follows. The relationship between the payoff and the discovery date is $x = e^{-rt}$, and the distribution over the discovery date is related to the distribution over the payoff according to}

Proposition 2.5 Let Assumption 2.2 hold. Then the symmetric social optimum is riskier than the symmetric market equilibrium strategy.

Proposition 2.6 Let Assumption 2.2 hold. Let the strategies played by firms other than firm $i$ be given. The socially optimal choice of firm $i$ is riskier than the individually optimal strategy.

These propositions can be directly contrasted with those in Dasgupta and Maskin (1987). They obtained the result that if
distributions can be ordered according to second-order stochastic dominance, if the cost of R&D increases with riskiness, and if
\[ 1 + \frac{x h(x, k)}{h(x, k)} \geq 0, \tag{2.7} \]
then the market solution is excessively risky. On the other hand, the condition for decreasing elasticity in Assumption 2.2 can be written as
\[ 1 + \frac{x h(x, k)}{h(x, k)} - \frac{x h(x, k)}{H(x, k)} \leq 0. \tag{2.8} \]

The effect of the different assumptions can be explained as follows. A move toward riskiness inflicts a twofold externality on its rivals: it decreases the competitors' chances of winning at low payoffs but increases them at high payoffs. Condition (2.7) requires that probability distributions have sufficient mass at the high payoffs so that in the model of Dasgupta and Stiglitz, the latter effect dominates the former. For the present model, on the other hand, condition (2.8) guarantees that what happens at low payoffs is more important, and thus the externality caused by a move towards riskiness is positive.

Again, this explains only part of the difference. The two conditions are not mutually exclusive; both hold, for example, in the case of the uniform distribution. As in the previous section, this is explained by the tendency of the market to overinvest in R&D. In the model of Dasgupta and Stiglitz, the only way a firm can increase its probability of winning is by shifting to a riskier strategy.

2.6 Conclusions

Several studies analyzing the relation of market structure and the choice of research strategy conclude that the market is biased toward riskiness. The analysis of this chapter shows that under certain plausible assumptions the opposite is true: for practically all common distributions of payoffs, risk-neutral firms choose a more conservative strategy than is socially optimal.
This disparity in the results originates from two differences in the framework. First, it is assumed here that the probability distributions have sufficient mass at the low payoffs so that the externality from a firm shifting to a riskier strategy is positive. In the case of patent races, a sufficient condition for this is that the hazard rate increases over time. This distributional assumption is relatively weak and is fulfilled by all common distributions.

Secondly, and more importantly, the earlier studies defined risk in a way that makes it desirable; in the class of symmetric distributions a risky strategy is always preferred to a safe strategy by individual firms as well as by society. Interior solutions are then obtained only by letting the cost of the project increase with riskiness. The outcome of such a model is that firms tend to “buy more risk” than is socially desirable. It is argued above that since the possible bias in the risk choice intermingles with the tendency of firms to overinvest in patent races, this result cannot be interpreted as evidence of a bias toward risk.

In this paper, a measure of compensated risk is adopted; an increase in risk is compensated with a change in the mean of the distribution. This approach facilitates interior solutions without having to resort to variable cost of R&D. It is shown that when the risk choice is analyzed separately the original results of Dasgupta and Stiglitz (1980) are retained in a stronger form: market equilibrium is biased against riskiness in both symmetric and asymmetric situations.

The policy interpretation of the analysis is that in a competitive setting similar to the one described in the present chapter, risk taking should be encouraged. Indeed, R&D subsidies that effectively decrease the risk involved in R&D projects are being used in many countries; in Japan, for example, the government provides designated loans for R&D projects with the condition that the loan is paid back only to the extent that the profits created by the project cover the loan.
Appendix to Chapter 2

The following two lemmas are used in the proofs:

Lemma A2.1: Let $\mu(x)$ and $\xi(x)$ be two functions, not identically zero, on the nonnegative real axis with the following properties:

(i) $\mu(x) \geq 0$, if $0 < x \leq x'$
(ii) $\mu(x) \leq 0$, if $x \geq x'$
(iii) $\int_0^\infty \mu(x) \, dx \geq 0$
(iv) $\xi(x) \geq 0$ for $x \geq 0$
(v) $\xi'(x) \leq 0$ for $x \geq 0$.

Then $\int_0^\infty \mu(x) \xi(x) \, dx \geq 0$. This inequality is strict if the inequality in (iii) is strict.

Proof: Notice that these properties imply that $\int_0^s \mu(x) \, dx \geq 0$ for all $s > 0$. Integration by parts yields

$$\int_0^\infty \mu(x) \xi(x) \, dx = \lim_{x \to \infty} \xi(x) \int_0^\infty \mu(s) \, ds - \int_0^\infty \xi'(s) \int_0^s \mu(x) \, dx ds.$$

The first term on the r.h.s. is positive by properties (iii) and (iv). The integrand in the second term is negative by property (v) and the fact that $\int_0^s \mu(x) \, dx \geq 0$. Thus the expression is positive and the lemma is proved.

Lemma A2.2: For all positive unequal real numbers $x$ and $y$ and integer $n \geq 2$,

$$x^n - y^n > \frac{n}{n-1} y(x^{n-1} - y^{n-1})$$

Proof: The Lemma is an intermediate result in the proof of Theorem 41 in Hardy, Littlewood and Pólya (1952).

Proof of Theorem 2.1. Sufficiency is shown first. Let condition (2.3) hold; then $H_1$ and $H_2$ intersect exactly once at point $x^*$, where $H_1 - H_2$ changes sign from positive to negative. Let $u_j$ be a nondecreasing bounded utility function for which (2.1) holds.
Integrating equation (2.1) by parts produces an equivalent condition
\[ \int_{0}^{\infty} u'_j(x) (H_1(x) - H_2(x)) \, dx = 0. \]

Notice that the integrand changes sign exactly once (at the intersection point of the distribution functions) and qualifies as the function \( \mu(x) \) as defined in Lemma A2.1.

It then holds for any increasing and concave transformation \( \phi \) of \( u_j \) that
\[
\int_{0}^{\infty} \phi(u_j(x))dH_1(x) - \int_{0}^{\infty} \phi(u_j(x))dH_2(x) \\
= [\phi(u_j(x))(H_1(x) - H_2(x))]_0^{\infty} \\
- \int_{0}^{\infty} \phi'(u_j(x))u'_j(x)(H_1(x) - H_2(x)) \, dx \\
= - \int_{0}^{\infty} \phi'(u_j(x))u'_j(x)(H_1(x) - H_2(x)) \, dx.
\]

Since the function \( \phi'(u_j(x)) \) qualifies as function \( \xi(x) \) as defined in Lemma A2.1, the expression is nonpositive. This proofs the sufficiency part.

To prove the necessity part, notice that as the difference of two distribution functions, \( H_1 - H_2 \) is continuous from the right. It follows that if condition (2.3) fails to hold, it must fail in a set of nonzero size. Thus, if condition (2.3) is violated, there exist two convex sets \( A \subset \mathbb{R}^+ \) and \( B \subset \mathbb{R}^+ \) of nonzero size such that \( \sup A \leq \inf B \) and \( H_1(x) - H_2(x) < 0 \) for \( x \in A \) and \( H_1(x) - H_2(x) > 0 \) for \( x \in B \). Define \( z_A \) and \( z_B \) as
\[
z_A = \int_A H_1(x) - H_2(x) \, dx \\
z_B = \int_B H_1(x) - H_2(x) \, dx.
\]

Clearly, \( z_A, z_B > 0 \). Define the utility function \( u_j \) as follows: let \( u_j(0) = 0 \) and \( u_j(x) \) be the integral of
\[
u'_j(x) = \begin{cases} 
z_B & \text{if } x \in A \\
z_A & \text{if } x \in B \\
0 & \text{otherwise.}
\end{cases}
\]

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It then holds that
\[
\int_0^\infty u_j(x) dH_1(x) - \int_0^\infty u_j(x) dH_2(x) \\
= - \int_0^\infty u'_j(x) (H_1(x) - H_2(x)) dx \\
= - \int_A u'_j(x) (H_1(x) - H_2(x)) dx \\
\quad - \int_B u'_j(x) (H_1(x) - H_2(x)) dx \\
= -z_B \int_A (H_1(x) - H_2(x)) dx \\
\quad -z_A \int_B (H_1(x) - H_2(x)) dx \\
= z_B z_A - z_A z_B \\
= 0.
\]

Thus, condition (2.1) holds and the shift from $H_1$ to $H_2$ is mean utility preserving for $u_j$. For any increasing and strictly concave transformation $\phi$

\[
\int_0^\infty \phi(u_j(x)) dH_1(x) - \int_0^\infty \phi(u_j(x)) dH_2(x) \\
= - \int_0^\infty \phi'(u_j(x)) u'_j(x) (H_1(x) - H_2(x)) dx.
\]

The function $-u'_j(x) (H_1(x) - H_2(x))$ changes sign only once from positive to negative, and its integral over $\mathbb{R}^+$ is zero, so it satisfies the conditions required of function $\mu(x)$ in Lemma A2.1. On the other hand, $\phi'(u_j(x))$ decreases weakly in $x$ so it qualifies as the function $\xi(x)$. It then follows from the lemma that the expression in (A2.1) is positive, violating condition (2.2). Hence, $H_1$ cannot be riskier than $H_2$ by Definition 2.1 and necessity is established.

Proof of Proposition 2.1. The proposition is proved by contradiction. Let $k_1$ be a competitive equilibrium of an $n$-firm race and $k_2$ any other strategy such that $F(t, k_2) \neq F(t, k_1)$ for some $t$. From the definition of an equilibrium we know that

\[
\int_0^\infty e^{-rt} f(t, k_1)(1 - F(t, k_1))^{n-1} dt \\
\geq \int_0^\infty e^{-rt} f(t, k_2)(1 - F(t, k_1))^{n-1} dt.
\]

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It will be shown that for all strategies $k_2$ it holds that if $n-1$ other firms play $k_2$, then the $n$th firm prefers strategy $k_1$ to $k_2$, and hence $k_2$ cannot constitute a symmetric equilibrium.

Suppose the contrary holds; i.e.

$$\int_0^\infty e^{-rt} f(t, k_2)(1 - F(t, k_2))^{n-1} dt \geq \int_0^\infty e^{-rt} f(t, k_1)(1 - F(t, k_2))^{n-1} dt.$$ (A2.3)

Combining the inequalities (A2.2) and (A2.2) yields

$$\int_0^\infty e^{-rt}(f(t, k_1) - f(t, k_2)) \times [(1 - F(t, k_2))^{n-1} - (1 - F(t, k_1))^{n-1}] dt \leq 0.$$ (A2.4)

The expression on the l.h.s. of this inequality can be written

$$\int_0^\infty (f(t, k_1) - f(t, k_2)) \times [(1 - F(t, k_2)) - (1 - F(t, k_1))] G(t) dt,$$

where

$$G(t) \equiv [(1 - F(t, k_1))^{n-2} + (1 - F(t, k_1))^{n-3}(1 - F(t, k_2)) + \ldots + (1 - F(t, k_1))(1 - F(t, k_2))^{n-3} + (1 - F(t, k_2))^{n-2}] e^{-rt}$$

The function $G(t)$ is positive and strictly decreasing and approaches zero as $t$ grows without bound. Hence,

$$\int_0^\infty (f(t, k_1) - f(t, k_2)) [(1 - F(t, k_2)) - (1 - F(t, k_1))] G(t) dt$$

$$= -\int_0^\infty (f(t, k_1) - f(t, k_2)) \times [(1 - F(t, k_2)) - (1 - F(t, k_1))] \int_t^\infty G'(u) du dt$$

$$= -\int_0^\infty G'(u) \int_0^u (f(t, k_1) - f(t, k_2)) \times [(1 - F(t, k_2)) - (1 - F(t, k_1))] dtdu$$

$$= -\int_0^\infty G'(u) \left[\frac{(F(u, k_1) - F(u, k_2))^2}{2}\right] du$$

$$> 0.$$
where the last inequality follows from the fact that $G'(t) < 0$. Thus
\[
\int_0^\infty e^{-rt}(f(t, k_1) - f(t, k_2))
\times [(1 - F(t, k_2))^{n-1} - (1 - F(t, k_1))^{n-1}] dt > 0.
\]

This contradicts with (A2.4); inequality (A2.3) cannot hold. If $k_1$ constitutes a symmetric equilibrium, a firm would still prefer $k_1$ to $k_2$ even if the other firms were playing $k_2$. This means that $k_2$ cannot constitute a symmetric equilibrium. Hence, $k_1$ must be the unique equilibrium. The proof is complete.

**Proof of Proposition 2.2.** The formal proof of Proposition 2.2 is straightforward but lengthy. The outline of the proof goes as follows:

Let $k_c$ constitute a symmetric competitive equilibrium. Denote (the current value of) the aggregate expected profit of all firms at time $s$ by $\bar{v}(s, k_c)$. It can be shown that since the hazard rate increases over time, $\bar{v}(s, k_c)$ increases over time as well. It follows that the individual expected profit, equal to $\bar{v}(s, k_c)/n$, also increases over time.

Denote the expected profit at time $s$ of a firm that at that time shifts from the equilibrium strategy $k_c$ to another strategy $k_j$ by $d(s, k_j)$. Since the hazard rate of the competitors increases over time, $d(s, k_j)$ decreases over time. The shift from strategy $k_j$ is profitable only if $d(s, k_j) > \bar{v}(s, k_c)/n$.

By the definition of equilibrium $d(0, k_j) \leq \bar{v}(0, k_c)/n$. But since $d$ decreases and $\bar{v}$ increases in $s$, it must be that $d(s, k_j) \leq \bar{v}(s, k_c)/n$ for all $s > 0$. Hence, deviating is never strictly profitable and playing $k_c$ is a subgame perfect equilibrium.

**Proof of Proposition 2.3.** Assume the contrary: Let $k_c$ be the symmetric competitive equilibrium for $n$ firms, and let the socially optimal strategy $k_s$ be strictly less risky than $k_c$.

The social optimizer maximizes the expected present value of the patent over all symmetric solutions. The problem can be written as one of maximizing
\[
n \int_0^\infty e^{-rt} f(t, k)(1 - F(t, k))^{n-1} dt
\]
(A2.6)
over all strategies $k$. Thus

$$\int_{0}^{\infty} e^{-rt} f(t, k_s)(1 - F(t, k_s))^{n-1} dt > \int_{0}^{\infty} e^{-rt} f(t, k_c)(1 - F(t, k_c))^{n-1} dt.$$  

It was shown in the proof of Proposition 2.1 that if strategy $k_c$ is a symmetric equilibrium, then for any other strategy, if all $n-1$ other firms play that strategy, then the $n$th firm still prefers strategy $k_c$ to it. The same applies to strategy $k_s$, so

$$\int_{0}^{\infty} e^{-rt}(f(t, k_c) - f(t, k_s))(1 - F(t, k_s))^{n-1} dt > 0.$$  

Combining these two inequalities gives

$$\int_{0}^{\infty} e^{-rt} f(t, k_c) \times [(1 - F(t, k_s))^{n-1} - (1 - F(t, k_c))^{n-1}] dt > 0. \tag{A2.7}$$

Integrating (A2.6) by parts shows that the social objective can be represented equivalently as one of minimizing

$$\int_{0}^{\infty} e^{-rt}(1 - F(t, k))^{n} dt$$

over all strategies $k$. Thus, it must hold that

$$\int_{0}^{\infty} e^{-rt} [(1 - F(t, k_s))^n - (1 - F(t, k_c))^{n}] dt \leq 0. \tag{A2.8}$$

Multiplying this by $(n - 1)/n$ and using Lemma A2.2, equation (A2.8) implies that for nonidentical functions $F(t, k_s)$ and $F(t, k_c)$

$$\int_{0}^{\infty} e^{-rt} (1 - F(t, k_c)) \times [(1 - F(t, k_s))^{n-1} - (1 - F(t, k_c))^{n-1}] dt < 0. \tag{A2.9}$$

The negative of the integrand can be verified to satisfy the conditions for function $\mu$ in Lemma A2.1. By assumption, the hazard rate $f(t, k_c)/(1 - F(t, k_c))$ increases in $t$, and thus qualifies as the function $\xi$. According to the lemma, (A2.9) then implies that

$$\int_{0}^{\infty} e^{-rt} f(t, k_c) \times [(1 - F(t, k_s))^{n-1} - (1 - F(t, k_c))^{n-1}] dt < 0. \tag{A2.10}$$
But this contradicts with A2.7; thus, \( k_c \) cannot be riskier than \( k_s \) and the proposition is established.

**Proof of Proposition 2.4.** Define the distribution function \( G(t) \) as

\[
1 - G(t) \equiv \prod_{j \neq i} (1 - F(t, k_j)), \tag{A2.11}
\]

where \( k_j \) is the strategy played by firm \( j \). Hence, \( G(t) \) is the distribution of the first innovation by firms other than \( i \). The corresponding density function \( g(t) \) is then given by

\[
g(t) \equiv \sum_{j \neq i} \left[ f(t, k_j) \prod_{h \neq i, j} (1 - F(t, k_h)) \right].
\]

It is immediately seen that \( g(t)/(1 - G(t)) \), the joint hazard rate of firms other than \( i \), is the sum of the individual hazard rates \( f(t, k_j)/(1 - F(t, k_j)) \), and thus nondecreasing by assumption.

Suppose that given \( G(t) \) firm \( i \) prefers strategy \( k_i \), which is strictly riskier than the socially optimum strategy \( k_s \). Then it holds that

\[
\int_0^\infty e^{-rt} (f(t, k_i) - f(t, k_s)) (1 - G(t)) dt > 0. \tag{A2.12}
\]

Again, given \( G(t) \) the social planner’s problem can be represented as one of choosing \( k_s \) to maximize

\[
\int_0^\infty e^{-rt} [(f(t, k_s)(1 - G(t)) + g(t)(1 - F(t, k_s))] dt, \tag{A2.13}
\]

or alternatively, to minimize

\[
\int_0^\infty e^{-rt} (1 - F(t, k_s))(1 - G(t)) dt. \tag{A2.14}
\]

Thus

\[
\int_0^\infty e^{-rt} [(f(t, k_s)(1 - G(t)) + g(t)(1 - F(t, k_s))] dt
> \int_0^\infty e^{-rt} [(f(t, k_i)(1 - G(t)) + g(t)(1 - F(t, k_i))] dt.
\]
Combining this with (A2.12) gives
\[ \int_0^\infty e^{-rt} g(t) \left[ (1 - F_{k_s}(t)) - (1 - F_{k_i}(t)) \right] dt > 0. \] (A2.15)

Since \( k_s \) minimizes (A2.14), the following holds
\[ \int_0^\infty e^{-rt} (1 - G(t)) \left[ (1 - F(t, k_s)) - (1 - F(t, k_i)) \right] dt \leq 0. \]

Multiplying the integrand of (A2.14) by the increasing function \( g(t)/(1 - G(t)) \) retains, by Lemma A2.1, the sign of the inequality
\[ \int_0^\infty e^{-rt} g(t) \left[ (1 - F(t, k_s)) - (1 - F(t, k_i)) \right] dt < 0. \]

This contradicts with (A2.15): \( k_i \) cannot be riskier than \( k_s \). The proof is established.

**Proof of Proposition 2.5.** Let the opposite hold: assume that \( k_c \) is riskier than \( k_s \). Then since \( k_c \) maximizes (2.6), the following holds:
\[ \int_0^{x} x \left( h(x, k_c) - h(x, k_s) \right) H(x, k_c)^{n-1} dx > 0. \] (A2.16)

Parallely to the proof of Proposition 2.1, it can be shown that \( k_s \) cannot be an equilibrium, and that a firm still prefers strategy \( k_c \) if other firms play \( k_s \):
\[ \int_0^{x} x \left( h(x, k_c) - h(x, k_s) \right) H(x, k_s)^{n-1} dx > 0. \] (A2.17)

The social planning problem is to maximize
\[ \int_0^{x} x h(x, k) H(x, k)^{n-1} dx. \] (A2.17)

Thus, it follows that
\[ \int_0^{x} x h(x, k_s) H(x, k_s)^{n-1} dx > \int_0^{x} x h(x, k_c) H(x, k_c)^{n-1} dx. \] (A2.18)

Combining (A2.16) and (A2.18) yields.
\[ \int_0^{x} x h(x, k_c) \left[ H(x, k_s)^{n-1} - H(x, k_c)^{n-1} \right] dx > 0. \] (A2.19)
The social optimizer's maximizing problem (A2.17) can be stated equivalently as one of minimizing

\[ \int_0^\infty H(x, k)^n \, dx. \]

Thus, it follows that

\[ \int_0^\infty [H(x, k_s)^n - H(x, k_c)^n] \, dx \leq 0, \]

which, according to Lemma A2.2, implies that

\[ \int_0^\infty H(x, k_c) \times [H(x, k_s)^{n-1} - H(x, k_c)^{n-1}] \, dx < 0. \]

Multiplying this by the nonincreasing function \( x h(x, k_c)/H(x, k_c) \) retains, by Lemma A2.1, the sign:

\[ \int_0^\infty x h(x, k_c) \times [H(x, k_s)^{n-1} - H(x, k_c)^{n-1}] \, dx < 0. \]

This contradicts with (A2.19) and establishes the proposition.

*Proof of Proposition 2.6.* The proof of this proposition follows the method used in the proof of Proposition 2.4 and is omitted.
3 Market Structure and
the Propensity to Patent:
Patenting or Secrecy?

3.1 Introduction

What is the optimal firm size for technological innovation? Is a large number of small R&D labs better than a few big ones? In other words, what are the scale economies of innovative work? A hypotheses most commonly associated with Schumpeter — although part of the credit belongs to J.K. Galbraith — is that large firms are the natural breeding ground for innovation. The natural advantage may be related to greater ability to diversify risk in a large firm, to the ability to implement new technologies on a larger scale, or to increasing returns to scale inherent in R&D.

This view has been challenged by a series of empirical studies examining the relation between firm size and R&D input and output (see Mansfield, 1968a, 1968b, 1977, and Scherer, 1965, 1983). Empirical research seems to indicate no systematic deviation from linearity in the relation between firm size and patenting (i.e. R&D output) but gives some evidence in favor of a convex relationship between firm size and R&D input. Thus, if anything, these studies suggest that there are decreasing returns to scale in innovation.

The problem with these empirical studies is, of course, the poor quality of the data. More precisely, the measurement of R&D output is extremely difficult. The most widely used proxy for innovative output is the number of patents. This poses at least two problems. First, patents are extremely heterogeneous products, with their market value varying from zero to very high. Another potential complication that seldom receives attention is that sometimes it may be beneficial for an innovator not to patent an innovation. Indeed, as Schmookler (1966) and Branch (1973)
found, there has been an increasing discrepancy between patenting and the actual pace of innovation, especially in the post-war period. Reduced incentive to patent innovations has been attributed to a more hostile political and legal attitude toward patents. Freeman (1982, p. 136) asserts that although there has been a tendency to assume that large firms have a higher propensity to patent than small firms and thus that studies measuring inventive output using patent statistics would understate the contribution of small firms, the opposite seems to be true. This view is supported by Schmookler (1966, p. 33), who presents convincing evidence that large firms in the United States have a lower propensity to patent than small ones. The work by Scherer (1984, p. 179) and, using British data, Smyth, Samuels and Tzannatos (1972) provide similar results, at least for many industries. Scherer suggests that this may be because the inventive output of larger firms includes a higher proportion of unpatentable contributions to pure knowledge or because patents afford less marginal benefit to large firms than their smaller and more vulnerable competitors. If it is true that larger firms have a lower propensity to patent their innovations, the effect on the interpretation of the empirical studies mentioned above would be considerable; instead of being an indication of diminishing returns on R&D, the relatively poor R&D output (measured by number of patents) of large firms might be the result of systematic measurement error.

This chapter builds a model of innovation that offers an explanation for this empirical observation. The point of departure is the assumption that patenting involves disclosure, which reveals useful information to the competitors. In the model, firms patent their intermediate innovations only if they want to license them to the competitors.\(^1\) If licensing is not desirable, firms keep their innovations secret. It is shown that the incentive to patent and license an intermediate innovation is lower the more concentrated is the market structure and, depending on the informational struc-

\(^1\)Actually, it is also a possible that a firm would use patenting as a signal of the state of its research project without licensing the innovation, as shown later in this chapter.
ture, large firms may have less incentive to patent innovations than their smaller competitors.

There has been practically no work done which seeks to connect the propensity to patent with firm size or market concentration. There is a rather extensive literature on when it is optimal for a firm to license a patented innovation (see e.g. Gallini 1984, Shapiro 1985, Hill 1992), but the role of firm size has been a mere footnote in this literature. One reason for this is that the typical framework, in which the innovation reduces the unit cost of production, does not readily lend itself to such analysis; there is no state variable that could be interpreted as size. One might consider adding capacity constraints to play this role, but the resulting calculations are likely to be messy. In the present model, "size" means the size of the R&D lab, which is assumed to remain constant throughout the game.

Although offering an alternative explanation for an empirical observation that seems to dispute Schumpeter's hypothesis, the results can hardly be interpreted as "Schumpeterian"; they do not support Schumpeter's central idea that concentrated market structure is optimal for technological progress. However, it is suggested that the reason why a concentrated market structure may perform worse than atomistic competition is not decreasing returns to scale in R&D, but the tendency of concentration to decrease the sharing of intermediate innovations and thereby to slow the dissemination of new knowledge. This explanation adds another dimension to the discussion of the optimal market structure for inducing technological progress. The model in this chapter shows that the relation between the degree of concentration and wasteful duplication is not necessarily monotonic. Instead, the degree of duplication first increases with the number of competitors, but decreases (and actually disappears) after some threshold point where the licensing of innovations becomes commonplace.

This chapter is organized as follows. The next section introduces the basic setup of the model. Since informational assumptions play a crucial role in many respects, the actual analysis is divided in two parts. Section 3.3 presents the results under complete information; that is, when a firm can monitor the stage of its
competitors' R&D programs. In section 3.4 information is incomplete; it is assumed that firms cannot observe the state variables. The last section concludes the chapter.

3.2 The Basic Setup

The market consists of \( n \) identical, risk-neutral firms. In the tradition of patent races, these firms compete for a single final innovation, the value of which is normalized to unity. This prize is captured entirely by the single firm that first patents the final innovation. This winner-takes-all assumption, although unrealistic, is standard in patent race literature and considerably simplifies the calculations.

The path to the final innovation consists of two stages. In the first stage, each firm tries to develop an initial technology. A firm needs to have access to an initial technology before it can proceed to develop the final innovation. Having an initial technology does not produce a direct payoff — the only benefit is that it enables a firm to proceed to the final stage of the game, i.e. to develop the final innovation. Nevertheless, being the first firm to develop an initial technology is valuable, not only because it gives the firm a head start, but also because the initial technology can be patented and licensed to other firms.

While there is only a single, well defined final innovation, the number of possible initial technologies is assumed to be large so that if two firms independently develop their own initial technologies, the probability that the two technologies are similar enough that patenting one would preclude the use of the other is zero. On the other hand, all initial technologies are perfect substitutes for each another. Thus, even though in a legal sense there is no duplication, from a technological point of view, developing more than one initial technology is wasteful.

Thus, after a firm has developed an initial technology, it has several options: It can keep the technology secret and move on to the second stage alone, it can patent the technology and retain exclusive rights to the use of the it, or it can patent the technology
and license it to one or more other firms. If the firm decides to keep the technology secret, then no other firm will know about its existence and the R&D efficiency of other firms is unaffected. If, on the other hand, the firm decides to patent the initial technology the situation is more complicated. An assumption that plays a crucial role in the analysis is that by patenting an initial technology, a firm produces knowledge spillovers that enhance the ability of competing firms to develop their own initial technologies. The rationale behind this assumption is that patenting always involves some degree of exposure. Although complete secrecy regarding an innovation is seldom possible, patenting certainly reveals more information about the technology than competitors would otherwise obtain. Moreover, when there are a large number of variations of the same innovation, it is easier to take an existing design and make the adjustments necessary to comply with patent laws than it would be to create a product from scratch. It is assumed that patenting is necessary in order to sell licenses for the use of the technology — if a firm licenses an unpatented technology to another firm, there is nothing to prevent the licensee from patenting the technology itself. Further, it is assumed that patenting is the only credible way to inform other firms that a firm has developed a technology. If a firm decides to keep its technology secret, then the other firms have no way to find out that the technology exists.

Thus, when making the decision whether to patent a technology or not, a firm faces two conflicting incentives. On one hand, patenting enables the firm to sell licenses and extract immediate revenues from the technology. On the other hand, spillovers connected to patenting the technology reduce the value of a license. If these spillovers are large, a firm may prefer to keep the technology secret and use it as an advantage in pursuing the final innovation.

Since patenting a technology without licensing it does not produce any revenue to the innovating firm but does help competing firms complete their own technologies, one is tempted to conclude that no firm would patent an initial technology unless it intended to license it to other firms. This, however, is not generally true. It will be seen later, that in some cases, it may be beneficial for
a firm to patent for strategic reasons without intending to license the innovation to anybody.

The game takes place in continuous time. The flow of R&D inputs per firm is constant, exogenous and, for the sake of simplicity, assumed to be identical for all firms. R&D expenses are contractual, i.e. independent of the duration of the game. A firm cannot save money by, for example, giving up the race when it realizes it is hopelessly far behind. Also, there is no discounting; the present value of the patent remains at unity over an infinite time period. Thus, firms are only interested in their probability of winning the game and do not care about the duration. Neither of these assumptions — contractual costs and zero discounting — is likely to have a substantial effect on the main results. Their role is to simplify the analysis by transforming the game into a zero-sum game.

R&D technology is stochastic at both stages. In the tradition of patent races, the elapsed time before completion of an innovation (either the initial technology or the final innovation) is assumed to be an exponentially distributed random variable. In the first stage, the time it takes a single firm to develop an initial technology, when no firm has patented one, is distributed according to

$$F_a(x, t) = 1 - e^{-ah(x)t}, \quad a > 0,$$

(3.1)

where $x$ is the R&D input of the firm and $h(.)$ is an increasing function determining the scale economies of R&D. Since $x$ is same for all firms, $h(x)$ will cancel out in all relevant subsequent calculations and, without loss of generality, $h(x)$ can be normalized to unity. Thus, (3.1) can be written as

$$F_a(t) = 1 - e^{-at}, \quad a > 0.$$  

(3.2)

The corresponding density function is

$$f_a(t) = ae^{-at}.$$  

The coefficient $a$ is the instantaneous (Poisson) probability of innovation, usually referred to as the hazard rate. If the initial technology has not been completed by time $t$, the probability that the
a firm will develop it before time \( t + dt \) approaches \( a \cdot dt \) as \( dt \) approaches zero.\(^2\) When one firm patents an initial technology, the R&D technology of those firms that do not have an initial technology changes to

\[
F_b(t) = 1 - e^{-bt}, \quad b > a.
\]

The assumption \( b > a \) means that this probability distribution stochastically dominates \( F_a \) in the first order sense; patenting an initial innovation improves the R&D technology of other firms.

After a firm has gained access to some initial technology, it begins to pursue the final innovation. Again, it is assumed that the moment of the final innovation is exponentially distributed:

\[
F_c(t) = 1 - e^{-ct}, \quad c > 0.
\]

From these distributions, it follows that the probability distribution for the duration of the whole two-stage R&D project, given that no other firm has patented or is expected to patent an initial technology (that is, the firm will not be able to benefit from spillovers connected to patenting), denoted by \( F_{ac}(t) \), is

\[
F_{ac}(t) = \int_0^t f_a(s) \int_s^t f_c(u) du ds
= \int_0^t ae^{-as} (1 - e^{-c(t-s)}) ds
= \begin{cases} 
1 - \frac{ae^{-ct} - ce^{-at}}{a - c}, & \text{if } a \neq c \\
1 - \frac{c}{(1 + at)e^{-at}}, & \text{if } a = c.
\end{cases}
\]

Similarly, the probability distribution for the duration of the R&D project for a firm without an initial technology, after another firm has patented one, is

\[
F_{bc}(t) = \begin{cases} 
1 - \frac{be^{-ct} - ce^{-bt}}{b - c}, & \text{if } b \neq c \\
1 - \frac{ce^{-bt}}{(1 + bt)e^{-bt}}, & \text{if } b = c.
\end{cases}
\]

Some additional informational assumptions are still needed to make the game well specified. We shall return to these assumptions in the next section in the context of the actual analysis, where they are easier to put into perspective.

\(^2\)With a constant hazard rate \( a \), both the expectation and the variance of the distribution is equal to \( 1/a \).
3.3 Concentration and Patenting: Complete Information

In this section, it is assumed that information is complete in the sense that a licensee can verify that, at the time of the licensing agreement, the licensor does not have the final innovation ready and waiting to be patented. That is, a firm cannot "cheat" its competitors by keeping the initial technology secret until it finishes the final innovation and then, just before patenting and publishing the final innovation, license the initial technology to its competitors — a technology that becomes effectively worthless a moment later. The effect of allowing such a possibility is studied later in this chapter.

The game may generally possess several equilibria, both symmetric and asymmetric. Here the analysis is restricted to symmetric equilibria. As will be seen, even with this restriction, more than one equilibrium may exist. The solution concept in the subsequent models is subgame perfect Nash equilibrium in pure strategies. First, the circumstances under which the immediate patenting of an initial technology is an equilibrium strategy is analyzed. Later, equilibria involving secrecy are examined.

3.3.1 Patenting Equilibrium

Let us denote the patenting strategy by $S_p$ and define it as follows.

1. At the time the initial technology is ready,
   
   (i) if no other firm has patented a technology before, patent the new technology immediately and license it to all other firms at the highest possible price;
   
   (ii) if one or more firms already have a patented initial technology, keep the new technology secret.

2. If before completing one's own initial technology, another firm offers a license to an initial technology, buy it if the price is lower than a threshold price $\tilde{\pi}$; otherwise, continue working on your own initial technology.
Strategy $S_p$ specifies rules for several contingencies. It tells what a firm does when it develops an initial technology, both in the case that no other firm has patented a technology before, and in the case when one or more firms have patented a technology. It also tells how to respond if another firm patents a technology and offers the firm a license for it. In the following, we examine the conditions under which strategy $S_p$ is the optimal response from firm $i$ when it expects all other firms to play that strategy as well.

To establish the conditions under which $S_p$ constitutes a symmetric equilibrium, one needs to show that no unilateral deviation from strategy $S_p$ is profitable. The space of possible deviations is very large and cannot be exhausted. Fortunately, however, this is not necessary; only a few candidates need to be considered. If we define the state of the game as the information about which firms have and do not have a patented technology, then strategy $S_p$ is a Markov (or state space) strategy with respect to this state variable; it specifies rules as a function of the current state of the game. When the state changes (a firm patents an initial technology), strategy $S_p$ specifies an immediate reaction. A general result of dynamic games states that if a player’s opponents use Markov strategies, that player has a best response which is a Markov strategy as well — a firm has nothing to gain by playing a more complicated strategy.\(^3\) Thus, all strategies that are contingent on time, i.e. of the form “if $A$ takes place, wait for a period $s$ and then do $B$”, can be ruled out as a direction in which to deviate. The optimal response to other firms playing strategy $S_p$ must involve immediate responses. The set of such strategies is small and straightforward to analyze.

In order to calculate the payoff from playing strategy $S_p$, one needs to specify the optimal licensing strategy. First, suppose that a firm succeeds in developing an initial technology before any other firm has patented one; then, knowing that other firms play strategy $S_p$ (which involves immediate patenting), it cor-

\(^3\)See e.g. Fudenberg and Tirole 1991.
rectly concludes that no other firm yet has an initial technology. Hence, all its competitors are potential licensees.

Following Katz and Shapiro (1986), we characterize the optimal licensing strategy among the following class of sales schemes. The licensor arranges a \( k \)-unit sealed bid auction, which may or may not involve a minimum bid. By arranging a \( k \)-unit auction, the licensor renounces its right to sell further licenses, even if not all \( k \) licenses are sold. All \( n-1 \) competitors are allowed to submit bids. Once the bids are submitted, the \( k \) licenses are sold to the \( k \) highest bidders for a fixed fee equal to the bid, provided those bids exceed the minimum bid. Ties are resolved by random choice.

As in Katz and Shapiro, it is easy to show that in such a setting the optimal strategy takes one of the following two forms. The licensor may offer \( k < n - 1 \) licenses for auction with no minimum bid. Alternatively, it may offer \( n - 1 \) licenses with a minimum bid equal to \( 1/n \) (the expected payoff to a firm if every firm has a license) minus the expected payoff to a firm if it is the only firm without a license. In the present context, this result can be strengthened further. The following lemma is proved in the Appendix:

**Lemma 3.1** The licensor maximizes its expected payoff by offering \( n - 1 \) licenses for sale and setting the minimum bid equal to \( \bar{\pi} \) defined by

\[
\bar{\pi} = \frac{n-1}{n} \frac{c}{b + c(n-1)} \quad (3.7)
\]

All licenses will be sold at that price.

It is shown in the Appendix that this strategy is not the unique maximum; the licensor would have the same expected payoff by instead auctioning \( n - 2 \) licenses without a minimum bid. It will be assumed from here on that the licensor always sells to every competitor, i.e. to \( n - 1 \) firms. Using the the alternative solution would not change the payoffs and would cause only trivial changes in the results.

Each of the \( n - 1 \) licence transactions creates a surplus to the parties of that transaction; a surplus that, in a zero sum
game, is offset by a loss if expected profit of other firms. By choosing the optimal sales strategy, the licensor extracts the total surplus from each bilateral licensing agreement. For each licensee, the expected payoff decreases as other firms buy licenses, but is independent of whether the licensee itself buys a license or not. In other words, this auction structure allocates all bargaining power to the licensor and maximizes the incentive to patent an innovation. The effect of shared bargaining power is examined later in this chapter.

An interesting observation can be made regarding result (3.7). The price of the license is not monotonic in the number of firms. For large \( n \), the price of a license decreases with the number of firms, as expected. However, for \( n \) smaller than \( 1 + \sqrt{b/c} \), the price of a license actually increases with the number of firms. The reason for this is that for \( n < 1 + \sqrt{b/c} \), the fallback profit of the licensee (i.e. its probability of winning if it does not buy the license) decreases in \( n \) faster than \( 1/n \), the probability of winning in a symmetric situation. This shows up as increasing price of a license.

The total expected payoff for the innovating firm, after selling the license to the \( n - 1 \) other firms, consists of its own probability of winning, \( 1/n \), and \( n - 1 \) times the price of a license \( \pi \):  

\[
\frac{1}{n} + \frac{(n-1)^2}{n} \frac{c}{b+c(n-1)}.
\]

The total payoff for the innovator and the price of a license both decrease with \( b/c \); increasing \( b \) relative to \( c \) makes it more likely that a firm, if not buying a license, finishes its own initial technology before any of the firms with a license can finish the final innovation, hence lowering the value of the license for the licensees. It can also be observed from (3.8) that the total revenue from licensing increases with the number of competing firms. For \( n > 1 + \sqrt{b/c} \), the net effect on the licensor’s payoff of an increase in \( n \) is positive; the increase in licensing revenue is strong enough to dominate the negative effect caused by tighter competition in the final stage of the game. The licensor’s maximum payoff is
always reached when \( n \) goes to infinity (payoff approaches unity). Hence, although it is true that as long as no firm has an initial technology, it is in each firm's interest to reduce competition; once a firm attains the initial technology, it prefers that the final stage of the innovation process be as close to atomistic competition as possible.\(^4\)

To establish \( S_p \) with the described auction structure as the best response to the playing of \( S_p \) by others, only two possible deviations need to be shown suboptimal. The first is that instead of patenting an innovation the innovator keeps it secret and uses it as an advantage in pursuing the final innovation. The second is that the innovator patents the innovation but does not license it to any competitor.

In order to analyze the first deviation strategy — keeping the technology secret — we need to make an additional assumption. What happens if firm \( i \) develops an initial technology and keeps it secret and later another firm (firm \( j \)) develops its own technology and patents it? It is assumed that in this case the two firms with licensable initial technologies cannot collude but instead face off in a Bertrand competition that drives the price of a license to the level of the marginal cost, i.e. to zero. Thus, in this case, all firms get a license gratis. From the point of view of firm \( i \), the consequence of this process is that it returns to par with other firms without gaining any licensing revenue.\(^5\)

\(^4\)Notice that the threshold value of \( n \), for which the licensor's payoff turns upward, is the value at which the price of a license turns downward. Thus, the rather peculiar result is that the number of competitors that maximizes the price of a license minimizes the revenue of the innovator.

\(^5\)The alternative outcome of this subgame would be that no trading takes place at all. Any strategy with nonzero price of licence could only be an equilibrium in a weak sense; if firm \( i \) is selling a licence for price \( p \), then — no matter what the strategy of firm \( i \) — by setting the price of its own license infinitesimally smaller than \( p \), firm \( j \) could do no worse and could often do better than by letting firm \( i \) take the trade.
Given this assumption, the probability that the innovator (firm \(i\)) wins the game if it keeps the technology secret is

\[
p_i = \int_0^\infty f_c(t)(1 - F_a(t))^{n-1} dt + \left(1 - \int_0^\infty f_c(t)(1 - F_a(t))^{n-1} dt\right) n^{-1}.
\]

The first integral in (3.9) is the probability that firm \(i\) completes the final innovation before any other firm develops an initial technology. The second term is the probability that the opposite takes place, multiplied by \(1/n\), which is the expected payoff to firm \(i\) conditional on that this scenario is realized. Substituting the specific distributions into (3.9), we obtain

\[
p_i = \int_0^\infty ce^{-ct}e^{-a(n-1)t} dt + (1 - \int_0^\infty ce^{-ct}e^{-a(n-1)t} dt)n^{-1}
\]

Comparing (3.8) and (3.10) it follows that patenting and licensing yields a higher expected payoff for the innovating firm if and only if

\[
\frac{1}{n} + \frac{(n - 1)^2}{n} \frac{c}{b + c(n - 1)} > \frac{1}{n} + \frac{n - 1}{n} \frac{c}{a(n - 1) + c}.
\]

By solving this inequality, we get the result that patenting and licensing yields a higher expected revenue if and only if

\[
b/a < (n - 1)^2.
\]

Hence, immediate patenting and licensing is preferred to keeping the innovation secret if \(b/a\) is lower than a threshold value which increases with the number of firms. The intuition behind this result is simple: A high \(b\) means that patenting reveals plenty of useful information to the other firms, which shows up as low price for the license. A low \(a\), on the other hand, means that the expected time before any of the other firms develops an initial technology is long and the probability that firm \(i\) completes the final innovation before that is high. Thus, high values of \(b\) and low values of \(a\) tend to reduce the incentive to patent.
The role of $n$ is related to the externalities involved in licensing. The price of a license is given by the increase in the probability of winning that the technology gives to the licensee. In a zero sum game, this improvement in the position of the licensee is counterbalanced by a deterioration in the position of the other players. The larger the number of the firms in the market the larger the portion of the improvement that comes at the expense of firms other than the licensor. In other words, when the number of firms is large, the price of the license represents mostly the negative externalities to other firms and only to a small extent a compensation for the weakening position of the licensee. At the other extreme, if there are only two firms in the market, no externalities are involved; the revenue the licensor receives for the license is fully offset by its weakening competitive position. As a consequence, a duopolist would be indifferent between patenting and not patenting if $b = a$, in which case patenting would reveal no useful information. Under the present assumption, $b > a$, a duopolist never patents its technology. On the other hand, if $n = 11$, then patenting is still preferred if $b = 100 \cdot a$; i.e. if patenting reveals so much information that the expected time it takes another firm to develop a competing technology drops to one-hundredth of its previous value.

The second possible deviation which needs to be considered was to patent the technology but not license it to anyone. The motivation for this kind of strategy could be that by patenting the technology the firm would send a signal to other innovators that there are no revenues to be gained by patenting their technologies since that would only lead to Bertrand competition and a zero price for the license. This threat would preclude the patenting of technologies and would slow down diffusion among other firms. However, that this strategy is not optimal in the present setting can be shown as follows. The expected profit of firm $i$ if it licenses its technology to all other firms was shown in the proof of Lemma 3.1 to be $1/n + (n-1)\bar{p} = 1 - (n-1)p_j$, where $p_j$ is the probability of winning for a firm without an initial technology when all other firms have a patented technology. Denote by $p_k$ the probability
of winning for a firm without technology when only one of the competing firms has a patented technology. Then

\[
p_k = \int_0^\infty f_{bc}(t)(1 - F_b(t))(1 - F_{bc}(t))^{n-2}dt \\
\geq \int_0^\infty f_{bc}(t)(1 - F_b(t))^{n-1}dt \\
= p_j,
\]

where the inequality follows from the fact that \( F_b(t) > F_{bc}(t) \) for all \( t \). The probability of winning for firm \( i \), if it patents its technology but does not license it to any other firm, is \( 1 - (n-1)p_k \). But according to (3.12), this is less than the expected revenue \( 1 - (n-1)p_j \) that the firm gets by licensing the technology to all other firms. Thus, patenting without licensing is not profitable.

The following proposition summarizes the results of this section:

**Proposition 3.1** The patenting strategy \( S_p \) constitutes a symmetric equilibrium if and only if

\[
b/a < (n-1)^2.
\]

**3.3.2 Equilibrium without Patenting**

In this subsection, the conditions under which an outcome in which no firm patents its technology can be sustained as an equilibrium. Suppose firm \( i \) expects other firms to play strategy \( S_n \), defined as follows:

1. Keep the initial technology secret until somebody else patents, in which case, patent and license the technology to all firms at the highest attainable price.

2. If, before developing your own technology, one or more competitors patent technologies and offer a license, buy the least expensive license if the price is lower than or equal to the threshold value \( \tilde{\pi} \).
The assumptions of the previous subsections are maintained. A firm has no way to verify which firms have developed their initial technology — it only knows the probability distributions given above. The only way a firm can let others know that it has developed an initial technology is to patent it. Thus, patenting is the only way of signaling. Also, information is complete; a firm cannot "cheat" by selling a worthless technology.

If firm $i$ develops an initial technology at time $t$ and decides to keep it secret, it has the probability of winning the game

$$p^s_i(t) = \int_t^\infty f_c(s - t) \left( \frac{1 - F_{ac}(s)}{1 - F_{ac}(t)} \right)^{n-1} ds.$$  \hspace{1cm} (3.13)

The second term inside the integral is the probability that none of the $n - 1$ other firms completes the final innovation on or before time $s$, conditional on that none of them have completed it on or before time $t < s$. If, on the other hand, the firm decides to patent the innovation and sell licenses to other firms, it has the expected payoff

$$p^p_i(t) = \frac{1}{n} + \left( \frac{1 - F_a(t)}{1 - F_{ac}(t)} \right)^{n-1} (n - 1)\bar{\pi}. $$  \hspace{1cm} (3.14)

Here, $1/n$ is the probability that after selling licenses to all firms the firm itself wins the race for the final innovation. The second term is the expected revenue from selling the licenses. If no other firm has a secret technology at the time firm $i$ patents its own technology, the revenue from licensing is $(n - 1)\bar{\pi}$. However, since other firms play strategy $S_n$, one or more of the competitors may already possess a secret technology, in which case the price of a license is driven to zero. The term $((1 - F_a(t))/(1 - F_{ac}(t)))^{n-1}$ is the probability that at time $t$ no other firm has a technology, conditional on the information that no firm has completed the final innovation.

Substituting the distributions directly into expressions (3.13) and (3.14) would lead to considerably complicated expressions. Fortunately, this is not necessary, as is established in the following lemma. The lemma is proved in the Appendix.
**Lemma 3.2** If the payoff \( p^*_i(t) \) is greater than or equal to the payoff \( p^*_j(t) \) at time \( t = 0 \), then the same is true for every \( t > 0 \). Hence, if \( p^*_i(0) \geq p^*_j(0) \), then nonpatenting is an equilibrium.

According to Lemma 3.2, \( p^*_i(0) \geq p^*_j(0) \) is a sufficient condition for nonpatenting to be an equilibrium. It is, of course, also a necessary condition, for if this condition fails, then a firm would be better off by patenting if it succeeds in developing the initial technology very quickly when it can be relatively sure that no other firm yet has an initial technology.\(^6\)

Hence, only the conditions under which \( p^*_i(0) \geq p^*_j(0) \) holds need to be checked. Substituting \( t = 0 \) and the exponential distributions in (3.13) and denoting \( p^*_i(0) = p^*_i \), we derive for \( a \neq c \)

\[
p^*_i = \int_0^\infty ce^{-cs} \left( \frac{c e^{-as} - ae^{-cs}}{c-a} \right)^{n-1} ds
\]

By straightforward but rather tedious integration, this yields

\[
p^*_i = \frac{c}{(c-a)^{n-1}} \sum_{i=1}^{n} \frac{(n-i)c^{n-i}(-a)^{i-1}e^{-(a(n-i)+ci)s}}{a(n-i) + ci} \bigg|_{s=0} ^{\infty}
\]

\[
= \frac{c}{(c-a)^{n-1}} \sum_{i=1}^{n} \frac{(n-i)c^{n-i}(-a)^{i-1}}{a(n-i) + ci}
\]

This can be represented as a function of \( c/a \):

\[
p^*_i = \frac{c/a}{(c/a - 1)^{n-1}} \sum_{i=1}^{n} \frac{(n-i)(c/a)^{n-i}(-1)^{i-1}}{n + (c/a - 1)i}
\]

\(^6\)One should not conclude from this that a strategy in which the firm patents if it succeeds in developing the final innovation before some time \( t' \), but otherwise keeps the technology secret, could constitute an equilibrium. It can be shown that this kind of time contingent strategy cannot form a symmetric equilibrium.
For \( a = c \), this simplifies to\(^7\)

\[
p_i^e = \int_0^\infty c(1 + cs)^{n-1}e^{-cns}ds = \sum_{i=1}^n \frac{(n - 1)!}{(i - 1)!n^{n+1-i}}. 
\]

The expected payoff when patenting, given by expression (3.14), can be solved as in (3.8) to obtain

\[
p_i^p = \frac{1}{n} + \frac{(n - 1)^2}{n} \frac{c}{b + c(n - 1)}. 
\]

Thus, the condition for nonpatenting to dominate patenting is

\[
p_i^e - p_i^p \geq 0, 
\]

which can be solved for \( b/a \) to obtain

\[
b/a \geq \frac{n(n - 1)(1 - p_i^e)}{np_i^e - 1}c/a, 
\]

where \( p_i^e \) is as in (3.15). Since \( p_i^e \) is a function of \( c/a \) and \( n \), the expression on the right hand side of (3.18) is a highly nonlinear function of these variables. Numerical simulations shows that this function increases in both \( n \) and \( c/a \) over the relevant range.

It is easy to see that for \( n = 2 \), inequality (3.18) holds for any parameter values, whereas for \( n \to \infty \) it always fails. For any set of parameter values, there exists a threshold value \( n' \) such that for \( n < n' \) nonpatenting is an equilibrium, whereas for \( n > n' \) it is not.

Figure 3.1 illustrates the two equilibria, patenting and non-patenting, for the special cases \( n = 3 \) and \( n = 4 \). The area with diagonal hatching consists of the parameter combinations for which a patenting equilibrium exists. It is bounded from above by the line \( b/a = (n - 1)^2 \). The area with vertical hatching gives the parameter combinations for which a nonpatenting equilibrium exists. This area is bounded from below by the inequality in (3.18). In both cases the two areas overlap. This is a general feature: It can be shown that when \( c/a \) goes to infinity, the right hand side of

\(^7\)The following integral can be solved by a recursive application of integration by parts. To save space, the intermediate results are omitted.
Figure 3.1  
Existence of equilibria  
for cases \( n = 3 \) and \( n = 4 \)

(3.18) approaches (from below) \( n(n - 1)/2 \), which is smaller than \( (n - 1)^2 \), except at \( n = 2 \), where the two coincide. Thus, for any \( n \), if patenting equilibrium exists for any parameter combination, then for some parameter combinations both kinds of equilibria exist.

Are the two equilibria analyzed above the only possible types of symmetric equilibria? The answer is that given the assumptions, they are the only symmetric equilibria that exist in a set of parameter values that has a measure greater than zero. An example of a symmetric equilibrium that exists in a set of parameter values of size zero is one in which a firm patents and licenses its technology immediately if it manages to develop it before some point of time \( t' \), but keeps it secret if the developing time is longer than that. This constitutes a symmetric equilibrium for any (common) value of \( t' \) if and only if (3.18) holds with equality. A number of asymmetric equilibria may also exist but they are not examined in the present setting.

It was assumed above that patenting is the only way for a firm to let other firms know that it has an initial technology. What happens if this assumption is relaxed and firms can signal to each others about the current state of their research project? More precisely, suppose that a firm can at any time announce that it
has developed an initial technology, and that the other firms have no means to verify the truthfulness of the announcement. It is easy to show that this does not provide a basis for the exchange of credible information. If a firm finds it profitable to patent an innovation, then in a zero sum game, it must necessarily leave its competitors worse off. Consequently, a firm always wants to prevent any other firm from patenting. If a firm can do this by announcing the existence of its own technology (at zero cost), it will do so. Thus, each firm, regardless of whether it has an initial technology or not, has an incentive to claim to have an initial technology and thereby prevent other firms from patenting their innovations later in the game.

It can be concluded that the incentive to patent intermediate innovations grows rapidly as the number of firms in the market increases. This shows up as increasing probability for the existence of a patenting equilibrium and decreasing probability for the existence of a nonpatenting equilibrium. For a fixed set of parameters, the general picture is the following: First, for a small number of firms no patenting takes place. When the number of firms increases, the amount of wasteful duplication increases up to the point where patenting equilibrium appears. For some range of \( n \), both patenting and nonpatenting equilibria exist. Finally, when \( n \) exceeds some upper threshold value, only the patenting equilibrium remains and no wasteful duplication takes place.

### 3.3.3 A Generalization: Shared Surplus

It was assumed above that the innovating firm captures all the surplus from the trade of a license. What happens if another sharing rule is adopted? In this subsection, the surplus is divided between the seller and the buyer in exogenously fixed proportions.

Let \( p_j \) denote the probability of winning of a firm without an initial technology when all other firms have a technology. It was shown in the proof of Lemma 3.1 that

\[
p_j = \frac{b}{n+c(n-1)} \cdot \frac{1}{n}.
\]
It was also shown in the same proof that the highest possible price for a license, when one is sold to each firm, is \( \pi^u = 1/n - p_j \). On the other hand, the lowest possible price, denoted by \( \pi^l \), is equal to

\[
\pi^l = \frac{1 - p_j}{n - 1} - \frac{1}{n}.
\]

Here the first term is the probability that a firm with an initial technology wins when one of its competitors does not have a technology. The lower bound \( \pi^l \) is the difference between this probability and \( 1/n \), the latter being the probability of the firm winning when every firm has a technology. If the price of the license is \( \pi^l \), then the innovating firm is just indifferent between selling and not selling the license to the last firm; the revenue from the license exactly offsets the loss the licensee suffers as its competitive position weakens.

It is assumed that the actual price of the license is a weighted average of the two extremes \( \pi^u \) and \( \pi^l \):

\[
\bar{\pi} = \alpha \pi^u + (1 - \alpha) \pi^l, \quad 0 \leq \alpha \leq 1.
\]

Substituting and arranging terms gives

\[
\bar{\pi} = \frac{c \alpha(n - 2) + 1}{n b + c(n - 1)}.
\]

The total expected payoff of the innovating firm if it sells \( n - 1 \) licenses is therefore

\[
\frac{1}{n} + (n - 1) \bar{\pi} = \frac{1}{n} + \frac{(n - 1)c \alpha(n - 2) + 1}{n b + c(n - 1)}.
\]

Again, the firm compares this to the payoff it gets if it keeps the innovation secret, given in (3.10). Patenting is preferred if

\[
\frac{1}{n} + \frac{(n - 1)c \alpha(n - 2) + 1}{n b + c(n - 1)} \geq \frac{1}{n} + \frac{n - 1}{n} \frac{c}{\alpha(n - 1) + c}.
\]

Solving for \( b/a \) gives

\[
b/a \leq (n - 1)(\alpha(n - 2) + 1) - (1 - \alpha)(n - 2)c/a.
\]
For $\alpha = 1$, condition (3.20) simplifies to condition (3.11), which is independent of $c$. However, for $\alpha < 1$, the threshold value of $b/a$ for which the innovating firm is indifferent between patenting and not patenting is linear and decreasing in $c/a$. The greater $c/a$ the less likely it is that patenting is an equilibrium. Notice that the negative effect of $c/a$ on the threshold value of $b/a$ is stronger the greater the number of firms in the market. Is it possible that this effect dominates the first term so that some parameter combinations that support a patenting equilibrium may not do so if the number of firms is increased? It can be shown that although there are combinations of $b/a$ and $c/a$ for which (3.20) holds for $n+1$ firms and fail for $n$ firms, this can only happen when $b/a < 1$, which is infeasible given the assumptions. Thus, the earlier result generalizes; if for a feasible parameter combination, patenting is preferred for $n$ firms, then it is also preferred for any number of firms greater than $n$.

The major difference between this case and the one analyzed in section (3.3.1) is that whereas in the previous version the existence of a patenting equilibrium depended solely on $b/a$ (i.e. the extent of spillovers related to patenting), here the relative importance of the initial technology, reflected in the term $c/a$, also plays a role in determining the existence of an equilibrium. Condition (3.20) shows that the larger the step represented by the initial technology is (i.e. the smaller the value of $c/a$) the more attractive the patenting alternative. This latter effect is especially strong when $\alpha$ is small, i.e. when the negotiating power of the licensor is small.

It was shown in the previous section that if $\alpha = 1$, then patenting without licensing cannot be an equilibrium. However, if $\alpha < 1$, things may be different. Suppose a firm patents its technology but decides not to sell licenses. By doing this, it sends a credible message to other firms that further patenting can only lead to a price war in the market for licenses and therefore cannot be profitable. Thus, patenting serves as a means to block any attempts by other firms to license their future innovations. The probability of winning for the innovating firm if it patents but does not license is
then

\[ p_i = \int_0^\infty f_c(t)(1 - F_{bc}(t))^{n-1} dt. \]

Substituting for the distributions and solving the integral as in equation (3.15) gives

\[ p_i = \int_0^\infty ce^{-ct} \left( \frac{be^{-ct} - ce^{-bt}}{b - c} \right)^{n-1} dt \]

\[ = \frac{c}{(b - c)^{n-1}} \sum_{i=1}^n \frac{(n-1)!}{(n-i)!} b^{n-i} (-c)^{i-1} \left( b(i-1) + c(n - i + 1) \right). \]

The innovating firm compares this payoff to the one given in equation (3.19). Licensing is preferred if

\[ \frac{1}{n} + \frac{(n-1)c \alpha(n-2) + 1}{b + c(n-1)} \geq \frac{c}{(b - c)^{n-1}} \sum_{i=1}^n \frac{(n-1)!}{(n-i)!} b^{n-i} (-c)^{i-1} \left( b(i-1) + c(n - i + 1) \right) \]

or, alternatively, if

\[ \frac{1}{n} + \frac{(n-1) \alpha(n-2) + 1}{b/c + n - 1} - \frac{1}{(b/c - 1)^{n-1}} \sum_{i=1}^n \frac{(n-1)!}{(n-i)!} (b/c)^{n-i} (-1)^{i-1} \left( (b/c)(i-1) + n - i + 1 \right) \geq 0. \]

Condition (3.22) defines the feasible values of \( b/c \) as a function of \( n \) and \( \alpha \). This expression cannot generally be solved for \( b/c \) (in terms of algebraic functions). Numerically, it can be verified that for given \( n \) the expression on the right hand side of (3.22) is negative or positive depending on whether \( b/c \) is smaller or greater than a threshold value and that this threshold value decreases as the number of firms increases. If \( \alpha = 1/2 \), then for \( n = 3 \) the threshold value is 1/2; for \( n = 4 \) it is approximately 0.434 and for \( n = 6 \) approximately 0.345. Intuitively, this can be explained as follows: When \( b \) increases relative to \( c \), creating an initial technology becomes easier relative to completing the final innovation and consequently the price of a license decreases. However, the probability that the innovating firm wins the game if it does not license
Existence of patenting equilibrium when $\alpha = 1/2$ for cases $n = 3$ and $n = 4$

- Patenting equilibrium exists for $n = 3$.
- Patenting equilibrium exists for $n = 4$.

As $\alpha$ decreases even more rapidly, making licensing more profitable relative to patenting without licensing.

Another result that can be easily seen from condition (3.22) is that the larger is $\alpha$ the more likely it is that the condition holds. This is, of course, plausible: a large $\alpha$ means that the innovating firm can negotiate a high price for the licenses, which makes licensing more attractive.

Figure 3.2 illustrates the situation in the case where $\alpha = 1/2$ for $n = 3$ and $n = 4$. The area in which an equilibrium exists is bounded from above by the downward sloping line given by condition (3.20) and from below by the upward sloping line given by (3.22). When the number of firms increases, the set of parameter combinations that support patenting equilibrium increases.

Naturally, also non-patenting equilibria exist when $\alpha < 1$. As can be expected, the effect of decreasing $\alpha$ (changing the negotiation power in favor of the licensee) makes secrecy more attractive, and hence, the smaller is $\alpha$, the larger is the set of parameter combinations supporting non-patenting equilibrium. Non-patenting
equilibria with $\alpha < 1$, however, will not differ qualitatively from the equilibrium for $\alpha = 1$, and thus are not analyzed here.

3.4 Concentration and Patenting: Incomplete Information

The game above was one of complete information. It was assumed that when a firm offers a license to its competitors, the potential licensees can verify whether the licensor, at the time of the offer, already has the final innovation developed. In this subsection, this assumption is changed. If a firm, after developing an initial technology, chooses to keep it secret and manages to finish the final innovation before anyone else has developed a competing initial technology, it can postpone the patenting of the final innovation for a while, sell licenses to its initial technology, and publish the final innovation only after that. It is assumed that this process takes only an infinitesimal increment of time, during which the probability that another firm will finish the final innovation is zero. The potential licensees do not know whether the licensor, at the time of the offer, has the final innovation ready—thus, we have a game of incomplete information. If the licensor does not have the final innovation ready and everybody buys a license, then all firms stand an equal chance of $1/n$ to win the final innovation. On the other hand, if the licensor has the final innovation ready, then immediately after the trade the licensor will publish and patent the final innovation, making the licenses worthless to the licensees. Figure 3.3 plots the relevant part of the game in extensive form for the case of two players $A$ and $B$.

The first node represents the choice made by Nature, $N$, which determines whether firm $A$ or $B$ succeeds first in developing an initial technology. Only that part of the game tree that follows firm $A$'s success is drawn here. At the second level, firm $A$ has

8 The case $n = 2$ is not a very good example since patenting can then never be an equilibrium in it, as will be seen below. However, it is the only case for which an extended form game tree can be drawn and serves thereby to clarify the decision process.
Figure 3.3 Extended form subgame when $A$ is the first to develop an initial technology

the technology and the decision is whether to license it or keep it secret. If $A$ keeps the technology secret, then in the next move Nature determines whether $A$ succeeds in finishing the final technology before $B$ creates an initial technology. The probability of this happening is denoted by $p = \int_0^\infty f_c(t)(1 - F_a(t))dt$. With probability $1 - p$, $B$ creates its own initial technology first, after which both firms have an equal probability of winning the game.

Let us consider the decision problem of $B$ when $A$ offers an initial technology to license. Firm $B$ does not know whether $A$ has the final innovation ready, i.e. whether the game is at node $b'$ or at node $b$. If the game is at node $b'$, in which case $A$ has the final innovation, $B$ knows it will lose the price it pays for the license since $A$ will publish and patent the final innovation immediately after the trade. On the other hand, if $A$ does not have the final innovation (i.e. $b$ is the true location) and $B$ approves the offer, then both will have an equal probability of winning. If
B rejects the offer, the payoffs are either \((q, 1 - q)\), where \(q = \int_0^\infty f_c(t)(1 - F_b(t))dt\), if \(A\) does not have the final innovation or \((1, 0)\) if \(A\) has it.

Suppose \(A\) expects \(B\) to always accept its offer. Then it is straightforward to see that \(A\) chooses immediate patenting if and only if \(1/2 + \pi \geq p(1 + \pi)\). For firm \(B\), accepting is an equilibrium if it expects \(A\) to patent immediately and \(1/2 - \pi \geq 1 - q\). Combining these two inequalities gives the condition for the existence of a patenting equilibrium.

The equivalent condition is straightforward to derive in the general case. Suppose each firm expects others to play strategy \(S_p\), as defined in the previous section, and firm \(i\) is the first to create an initial technology. Then if firm \(i\) deviates from \(S_p\) and instead keeps the technology secret, the probability \(p\) that it will also finish the final innovation before any other firm develops an initial technology is

\[
p = \int_0^\infty f_c(t)(1 - F_a(t))^{n-1}dt
\]

\[
= \frac{c}{c + a(n - 1)}.
\]

Thus, \(p\) is the probability that deviating pays off and firm \(i\) gets both the final innovation with value equal to unity and, in addition, \(n - 1\) times the price of the license \(\bar{\pi}\) given by (3.7). On the other hand, with probability \(1 - p\), one of the other firms develops its initial technology and offers it to everybody before \(i\) has the final technology, in which case each firm has the probability \(1/n\) to win. Hence, by “cheating” firm \(i\) gets the expected payoff

\[
p(1 + (n - 1)\bar{\pi}) + (1 - p)n^{-1}
\]

\[
= \frac{c}{c + (n - 1)a} \left[ \frac{1 + (n - 1)^2}{n} \frac{c}{b + (n - 1)c} \right]
\]

\[
+ \left[ 1 - \frac{c}{c + (n - 1)a} \right] \frac{1}{n}.
\]

Setting this smaller than or equal to the payoff obtained by playing strategy \(S_p\) (i.e. by not deviating), given in (3.8), and manip-
ulating gives the condition for the existence of patenting equilibrium

\[
b/a \leq (n - 1)^2 - (n - 1)c/a. \tag{3.23}
\]

Compared to condition (3.11), the effect of incomplete information shows up in the new term \((n - 1)c/a\) on the r.h.s. of the inequality. Thus, unlike under complete information, the existence of a patenting equilibrium now depends on the importance of the initial information. The easier it is to complete the final innovation compared to the initial technology (the larger is \(c/a\)) the more likely it is that by choosing the cheating strategy the firm actually gets an opportunity to cheat — to sell an obsolete technology — and hence the less likely it is that patenting can be sustained as an equilibrium. The set of parameter values for which a patenting equilibrium exists is uniformly smaller than in the complete information case. As before, this set is larger the larger the number of firms.

The effect of incomplete information on the existence of the nonpatenting equilibria is more drastic; in this case, an equilibrium essentially identical to the nonpatenting equilibrium exists for all parameter combinations. To see this, consider a strategy in which the developer of an initial technology keeps the technology secret and tries to license it only after it has the final innovation. Suppose all firms expect each other to play this strategy. Then, if a firm is offered a licence to a technology, it correctly takes this as an indication that the licensor has already finished the final innovation and thus the license would have zero value. Hence, the buyer would not accept any offer with a price greater than zero. But knowing that no revenue can be extracted by selling licenses, no firm ever tries to patent before it has completed the final innovation and has nothing to loose. Hence, all firms playing this strategy constitutes an equilibrium. 9

\[\text{9 Actually, since the innovating firm knows that no firm will buy the license anyway, it is indifferent between offering and not offering licenses after finishing the final innovation. However, the equilibrium is unique in the trembling hand sense, i.e. offering licenses is preferred if there is an infinitesimally small probability that some competitor accidentally approves the offer.}\]
The result that the nonpatenting equilibrium exists for any parameter combination depends crucially on the limited set of negotiation options allowed for the firms. If, for example, the licensor could credibly commit itself to exit from the game after selling the licenses, then the set of parameter values for which a nonpatenting equilibrium exists would shrink considerably. There are a number of other strategies that would have the same effect on the outcome. They will not be elaborated on further.

3.5 Firm Size and Patenting: Complete Information

The analysis above compared the propensity to patent under different degrees of concentration. It does not address the question in interpreting the empirical studies referred to earlier: Is the superior patenting performance of small firms, as compared to their larger competitors, a true indication of decreasing returns to scale in R&D or just a result of large firms’ lower propensity to patent their innovations. This section approaches the question by analyzing the relationship between firm size and the propensity to patent in a market with firms of different size.

In abandoning the assumption of symmetric firms, the dimensionality of the problem increases by an order of magnitude: the incentives to patent depend not only on a firm’s own relative size but also on the sizes of all other firms. Hence, simple analytical conditions like those provided above cannot always be obtained. The answers provided here are partial and depend on the informational assumptions. The first part of this section concentrates on complete information. In the following subsection, incomplete information is analyzed.

The focus of the analysis is on patenting equilibria — i.e. every firm expects others to play strategy $S_p$ as specified in section (3.3.1). The R&D output of firm $k$ is denoted by $x_k$. Throughout this section, the ‘size’ of a firm refers to the size of its R&D output. Without loss of generality, R&D outputs can be normalized to sum to unity: $\sum_{k=1}^{n} x_k = 1$. The probability distribution for the
The date of discovery of the initial technology is as in equation (3.1) but with \( h(\cdot) \) a linear function:

\[
F_a(x, t) = 1 - e^{-axt},
\]

and similarly for distributions \( F_b(x, t) \) and \( F_c(x, t) \). Given this probability function and the normalizations, the probability of firm \( k \) being the first to develop an initial technology is simply \( x_k \). The combined probability function \( F_{ac} \) is then

\[
F_{ac}(x, t) = 1 - \frac{axe^{-xt} - cxe^{-xt}}{a - c}, \quad a \neq c.
\]

Function \( F_{bc} \) can be derived similarly.

For simplicity, it is again assumed that the licensor can arrange an auction and thus effectively set the price of a license. To solve for the equilibrium price of a license, consider again the situation where all firms except firm \( j \) have a license. If firm \( j \) buys a license from another firm, it has the probability \( x_j \) of winning the game. If, on the other hand, it decides to pursue the initial technology by itself, it has the probability of winning

\[
\hat{p}(x_j) = \int_0^\infty f_{bc}(x_j, t) \prod_{i \neq j} [1 - F_c(x_i, t)] dt.
\]

Substituting the distributions, this becomes

\[
\hat{p}(x_j) = \int_0^\infty \frac{bcx_j e^{-c x_j} - bx_j e^{-c(1 - x_j) t}}{b - c} dt
= \frac{bcx_j}{b - c} \left[ \frac{1}{c} - \frac{1}{bx_j + c(1 - x_j)} \right]
= \frac{bx_j}{bx_j + c(1 - x_j)}.
\]

By Taylor expansion, \( \hat{p}(x_j) \) can be shown to behave like \( (b/c)x_j^2 \) for near zero values of \( x_j \). For \( x_j \) close to unity, \( \hat{p}(x_j) \) is approximately \( x_j \). Thus, the fallback probability of the licensee is sensitive to the size of the firm and diminishes with the square of the market.
share for small firms. The highest price firm $j$ is willing to pay for the license is

$$\pi(x_j) = x_j - \hat{p}(x_j) = \frac{c(1 - x_j)x_j}{bx_j + c(1 - x_j)}.$$ (3.24)

As before, the price of a license is not monotonic in the size of the buyer. Instead, it increases with $x_j$ for small firm sizes, reaches a maximum at $x_j = \sqrt{c/(c + b)}$, and decreases thereafter. For $x_j$ close to zero, equation (3.24) is approximately $\pi(x_j) = x_j$.

The total expected revenue of firm $i$, if it licenses the technology to all other firms, is

$$v^p(x_i) = x_i + \sum_{j \neq i} \pi(x_j) = x_i + \sum_{j \neq i} \frac{c(1 - x_j)x_j}{bx_j + c(1 - x_j)}.$$ (3.25)

Not surprisingly, the expected profit from licensing the initial technology does not depend solely on the market share of firm $i$ but also on the distribution of the rest of the market over its competitors. It can be verified that the price of a license is a concave function of $x$. This has several immediate consequences. First, given the number and the aggregate market share of the licensees, the revenue from selling the licenses is maximized when all the licensees are of equal size. Secondly, the value of the initial technology rises when a licensee is split into smaller firms, each of which buys a license. It follows from the second point that given the size of firm $i$, the revenue in (3.25) is maximized when the rest of the market is shared by an infinite number of atomistic firms. Actually, in this case the expected total revenue is equal to one, regardless of the size of the licensor.

The strategy alternative to patenting is to keep the technology secret. The expected profit of the innovating firm $i$ if it decides to follow this strategy is

$$v^*(x_i) = \int_0^\infty f_c(x_i, t) \prod_{k \neq i}(1 - F_a(x_k, t)) dt + \left[ 1 - \int_0^\infty f_c(x_i, t) \prod_{k \neq i}(1 - F_a(x_k, t)) dt \right] x_i,$$
which renders

\[ v^p(x_i) = \int_0^\infty cx_i e^{-cx_i t} e^{-a(1-x_i)t} dt + \left[ 1 - \int_0^\infty cx_i e^{-cx_i t} e^{-a(1-x_i)t} dt \right] x_i \]

\[ = \frac{cx_i}{cx_i + a(1-x_i)} + \frac{1}{cx_i + a(1-x_i)} \frac{c(1-x_i)x_i}{x_i} \]

\[ = x_i + \frac{c(1-x_i)x_i}{cx_i + a(1-x_i)}. \] (3.26)

Comparing this with \( v^p(x_i) \) in (3.25) gives the condition for patenting to be an equilibrium:

\[ \sum_{j \neq i} \bar{\pi}(x_j) - \frac{c(1-x_i)x_i}{cx_i + a(1-x_i)} \geq 0. \] (3.27)

Expression (3.27) is of a dimensionality to large to allow for general results. However, two special cases shed light on the problem. Suppose that in addition to firm \( i \) the market consists of \( n-1 \) symmetric firms; i.e. \( x_j = (1-x_i)/(n-1), j \neq i \). Using this identity, the definition of \( \bar{\pi}(x_j) \), and some algebra one obtains the following condition for firm \( i \) to prefer patenting strategy:

\[ x_i < \frac{n-2}{b/a - 1}. \] (3.28)

Inequality (3.28) provides an upper bound for \( x_i \) under which patenting is an equilibrium. This upper bound increases with the number of competitors and decreases when \( b/a \) (the externality connected to patenting) increases. A useful result is that for any \( b > a > 0 \), if the rest of the market is atomistic (\( n \) is infinite), then firm \( i \) finds it profitable to patent. On the other hand, comparing (3.25) and (3.26) for \( x_i \) approaching zero shows that if the total number of firms in the market is greater than 2, then regardless of the structure of the rest of the market, a very small firm always finds it profitable to patent. Combining these two results shows that in a market consisting of a single dominant firm and a large number of very small firms patenting is an equilibrium. In contrast, it is easy to show that if there are two dominant firms
of equal size and a large number of small firms, then if the joint market share of the dominant firms is large enough, they prefer keeping their intermediate innovations secret. This example shows that the generalization of the results from the symmetric case is not straightforward; a diffuse market structure does not always provide greater incentive to patent than a concentrated one. To achieve complete licensing and quick dissemination of technological knowledge, it seems that either a large number of small firms or a single dominant firm and many small firms are desirable whereas a market with a handful of dominant firms is not.

To examine the relationship between firm size and patenting, the following setting will be used. There are two firms, denoted by \( A \) and \( B \), with \( x_A > x_B \). The sizes of the rest of the firms are taken as given. The aggregate market share of all firms other than \( A \) and \( B \) is denoted by \( z \) so that \( x_A + x_B + z = 1 \). Substituting into expression (3.27) yields the following condition for firm \( A \) to find patenting profitable:

\[
\sum_{j \neq A,B} \bar{\pi}(x_j) + \frac{c(1 - x_B)x_B}{bx_B + c(1 - x_B)} - \frac{c(1 - x_A)x_A}{cx_A + a(1 - x_A)} \geq 0
\]

or, after taking into account the identity \( x_A + x_B = 1 - z \),

\[
\sum_{j \neq A,B} \bar{\pi}(x_j) + \frac{c(1 - x_A - z)(x_A + z)}{b(1 - x_A - z) + c(x_A + z)} - \frac{c(1 - x_A)x_A}{cx_A + a(1 - x_A)} > 0
\]  

(3.29)

It is easy to see that for any \( z > 0 \), this inequality holds for small enough \( x_A \). Thus, as established before, a very small firm always finds it profitable to patent. Unfortunately, the dimensionality of the problem again prevents more general analytical results. Numerical examples, however, are easy to compute, and they show that under complete information the relationship between firm size and propensity to patent is not monotonic. Depending on the parameters, either of the two firms may have the greater incentive to patent.
Figure 3.4 plots two examples. Besides the two big firms, $A$ and $B$, the market is assumed to consist of an infinite amount of infinitesimal firms which together have the aggregate size $z = 0.1$. In the first case, the sizes of the large firms are $x_A = 0.6$ and $x_B = 0.3$, and in the second, $x_A = 0.8$ and $x_B = 0.1$. In both plots, the area denoted by $E$ is the one for which both firms find it profitable to patent. In the area I, the smaller firm $B$ would choose the patenting strategy but deviation by the larger firm $A$ breaks down patenting equilibrium. In area II, the converse is true: the large firm finds patenting profitable while the small firm deviates. Finally, in the area above both curves, both firms choose to deviate.

The figure reveals a property that applies quite generally to asymmetric cases under complete information. The cases in which the large firm has a greater incentive to patent than the small firm fall into the region where $c/a$ is high relative to $b/a$; that is, when the relative importance of the initial technology is large and the externalities from patenting are small. Similarly, relatively unimportant initial technology (small $c/a$) and high spillovers (large $b/a$) make it likely that the small firm finds patenting more attractive. The reason for this result is not immediately evident. The dominant effect producing this result is the strong reaction
of \( v^*(x) \), the expected payoff when the technology is kept secret, to changes in \( c/a \). As an example, take the latter case in Figure 3.4 \((x_A = 0.8 \text{ and } x_B = 0.1)\). When \( c/a \) falls from 8 to 1 (i.e. the relative importance of the initial technology decreases to one eight of its previous value), \( v^*(x_A) \) is reduced from approximately 0.99 to 0.96 — a decrease of a mere 3 per cent. For the smaller firm, the corresponding decrease in the expected payoff is approximately from 0.52 to 0.19 or about 64 per cent. Thus, if the innovation is kept secret, the sensitivity of the expected payoff to changes in \( c/a \) is much smaller for the larger firm. The revenue from patenting and licensing also decreases as \( c/a \) decreases but the effect depends much less on the size of the firm.

3.6 Firm Size and Patenting: Incomplete Information

Suppose the licensees cannot verify at the time of the licensing agreement whether the licensor has the final innovation. Then each firm has the option to deviate from the patenting strategy \( S_p \) and keep its technology secret until it has the final innovation and then offer licenses if no other firm has patented before. Suppose firm \( i \) has the initial technology. If it keeps the innovation secret, the probability that it completes the final innovation before any other firm has the initial technology is

\[
\bar{p}(x_i) = \int_0^\infty f_c(x_i, t) \prod_{k \neq i} (1 - F_a(x_k, t)) \, dt
\]

\[
\frac{c x_i}{c x_i + a(1 - x_i)}.
\]

In this case, firm \( i \) gets the payoff from the final innovation, normalized to unity, and the revenue \( \sum_{k \neq i} \pi(x_k) \) from licensing the initial technology. With probability \( 1 - \bar{p}(x_i) \), some other firm develops an initial technology before firm \( i \) has the final innovation, in which case firm \( i \) has an expected payoff equal to its probability
of winning $x_i$. The total payoff $v^d(x_i)$ ($d$ for 'deviate') if firm $i$ plays this strategy is therefore

$$v^d(x_i) = \bar{p}(x_i) \left[ 1 + \sum_{k \neq i} \bar{\pi}(x_k) \right] + [1 - \bar{p}(x_i)]x_i. \quad (3.30)$$

When deciding on its strategy, firm $i$ compares $v^d(x_i)$ with $v^p(x_i)$ in (3.25). Substituting for $\bar{p}(x_i)$ and rearranging, the condition for immediate patenting to be optimal for firm $i$ can be written as

$$\sum_{k \neq i} \bar{\pi}(x_k) - cx_i/a \geq 0. \quad (3.31)$$

Unlike in the complete information case, the implications of condition (3.31) are straightforward. They are presented in the following two propositions, the first of which states the effect of merging and splitting up firms, and the second the relationship between firm size and patenting for asymmetric market structures in general. The propositions are proved in the Appendix.

**Proposition 3.2** Let information be incomplete and let all firms other than firm $i$ prefer the patenting strategy $S_p$.

(i) If firm $i$ also prefers playing strategy $S_p$; then if firm $i$ is split into any number of smaller firms, an outcome in which each of the small firms plays $S_p$ is an equilibrium.

(ii) If firm $i$ prefers deviating from patenting strategy, then a larger firm formed by merging firm $i$ with any number of its competitors also prefers deviating.

**Proposition 3.3** Let information be incomplete and firm $A$ larger than firm $B$. Then if firm $A$ prefers patenting, so does firm $B$. Conversely, if firm $B$ finds it profitable to deviate, so does firm $A$.

Hence, under incomplete information, the effect of concentration and firm size are both clear. A concentrated market is always less likely to patent its intermediate innovations than a diffuse one. Splitting up firms can never decrease the propensity to patent and
merging them never increases it. Furthermore, a small firm always has a greater incentive to play patenting strategy than its larger competitor. This holds irrespective of the number or size of other firms.

To understand the difference between the results under the two informational assumptions, let us compare the expressions (3.27) and (3.31), which give the final conditions for patenting to be equilibrium under complete and incomplete information respectively. The Taylor approximation of condition (3.27) around $x_i = 0$ shows that for $x_i$ close to zero, the condition is approximately

$$\sum_{k \neq i} \pi(x_k) - cx_i/a - O(x_i^2) \geq 0.$$ 

Thus, for small $x_i$, the two conditions differ only by a term that is of the order $x_i^2$; that is, small firms are almost unaffected by the informational assumptions. For larger firms, incomplete information makes deviating much more attractive.

Heuristically, the difference can be seen as follows. If a small firm is the first to develop an initial technology, it knows that the chance that it would succeed in finishing also the final innovation before any of its competitors gets an initial technology is very small. The possible gain from this gamble is not very large; even in the best case, succesful cheating could at most double the revenue (i.e. raise the revenue from 1 to 2). In case the cheating fails (another firm gets an initial technology first), the firm's chance to win becomes very small. Therefore, the firm prefers the sure revenue from selling the licenses immediately to the small chance of getting a bigger prize later. That is, if the small firm beats the odds by being the first to develop an initial technology, it will not gamble its revenue further. For a large firm, the chance to gain by cheating is larger. The chance that the cheating strategy will succeed is greater, as is the expected payoff when the cheating strategy fails. Hence, the larger firm has both less to loose and a smaller chance to loose than does the small firm.
3.7 Conclusions

According to Schumpeter, one of the reasons that big firms and concentrated market structure are the natural environment for technological innovation is that big firms are better able to utilize their innovations. As such, this statement is quite plausible. The ability to utilize innovations internally has, however, several consequences, all of which are not necessarily beneficial. A less desirable consequence is that it tends to slow down the diffusion of those innovations. When the intermediate innovations are kept secret, each firm has to go through all the necessary steps by itself, essentially duplicating the work of its competitors. Hence, aggregate technological progress is determined by the most successful of the individual R&D programs. If intermediate innovations were licensed, duplication could be avoided and technological progress would be determined by the joint R&D output of the firms.

The analysis in this chapter used the framework of a two-stage patent race to examine how concentration and firm size affect the propensity to patent intermediate innovations. The results can be summarized as follows:

1. The degree of concentration is inversely related to the propensity to patent. This holds irrespective of the informational assumptions and negotiating powers.

2. If the licensees can monitor the state of the licensor’s R&D project, then the propensity to patent is lowest when a small number of large firms dominate the market. The propensity to patent is high if there is a single dominant firm or if the market is divided between a large number of small firms. Small firms have a higher propensity to patent when spillovers are large and the relative importance of the innovation is small. Large firms find patenting more attractive if the spillovers are small and the innovation is important.

3. If the licensees cannot monitor the state of the licensor’s R&D project, then the firm size is always inversely related
to the propensity to patent. Merging decreases and splitting up firms increases the propensity to patent.

4. Patenting (nonpatenting) is more (less) often an equilibrium under complete information; that is, the incentive to license an innovation is greater when the potential licensees can monitor the true status of the licensor’s R&D programs.

Although the results depend on informational assumptions, generally large firms tend to have weaker incentive to patent their innovations than small firms. Hence, the number of patented innovations is a biased proxy for R&D output and may lead to incorrect conclusions when used in the analysis of the relationship between R&D performance and firms size. Another (and more robust) result is that concentrated oligopolistic market structures are undesirable because they discourage patenting and increase unnecessary duplication; both more diffuse market structures and a markets with a single dominant firm do better in this respect.
Appendix to Chapter 3

Proof of Lemma 3.1: Suppose firm \( i \) offers \( k \leq n - 1 \) licenses to auction. After the \( k \) licenses are sold, there are \( n - k - 1 \) firms that do not have the technology. The probability of winning the final innovation and the expected payoff for each of these firms, if they keep any possible future innovations secret, is

\[
p(k) = \int_0^\infty f_{bc}(t)(1 - F_c(t))^{k+1}(1 - F_{bc}(t))^{n-k-2} dt, \quad (A3.1)
\]

for \( k = 1, \ldots, n - 2 \). If the number of licenses offered is equal to the number of potential buyers, then if one firm does not buy the license, it has the probability of winning

\[
p(n - 1) = \int_0^\infty f_{bc}(t)(1 - F_c(t))^{n-1} dt. \quad (A3.2)
\]

Thus, \( p(n - 1) = p(n - 2) \). Since by identity the probabilities sum to unity, the probability of winning for each firm that has a license is

\[
\frac{1 - (n - k - 1)p(k)}{k + 1}, \quad (A3.3)
\]

again conditional on the firms keeping their future innovations secret. The maximum price a licensee is willing to pay for a license is the difference of the two, namely

\[
\bar{\pi}(k) = \frac{1 - (n - k - 1)p(k)}{k + 1} - p(k). \quad (A3.4)
\]

The expected payoff \( v(k) \) of the licensor is \( k \) times the price \( \bar{\pi}(k) \) plus its own probability of winning, also given by (A3.3). Using (A3.3) and (A3.4), \( v(k) \) can be solved as

\[
v(k) = 1 - (n - 1)p(k). \quad (A3.5)
\]

It is easy to see that \( p(k) \) decreases with \( k \) up to \( k = n - 2 \), i.e. the probability of winning for firms which do not have a license decreases with the number of firms that have a license. Thus, \( v(k) \) is maximized by \( k = n - 2 \) and \( k = n - 1 \). Calculating the
integral in (A3.2) and substituting into (A3.4) yields the desired expression (3.7) for the price $\bar{p}(n - 1)$.

Allowing the firms without a license to patent and sell licenses to their possible future innovations does not change the situation. For $k = n - 2$ and $k = n - 1$, there is never more than one firm without a license. For $k \leq k - 3$, allowing this would increase the fallback probability, $p(k)$, which would make these alternatives even less attractive to the licensor.

**Proof of Lemma 3.2:** Let all other firms except firm $i$ play the nonpatenting strategy $S_n$. At the moment firm $i$ finishes its initial technology, there are $n$ possible states of the game. We index these states by the variable $k = 0, \ldots, n - 1$, where $k$ indicates the number of firms, other than firm $i$, that have a (secret) initial technology at that moment. Thus, the contingency $k = 0$ corresponds to the case where no other firm has an initial technology and the contingency $k = n - 1$ indicates that all other firms have one. When calculating the payoffs, firm $i$ considers each of these contingencies. If $k = n - 1$, it makes no difference whether firm $i$ patents its innovation or not — every firm has an initial technology and the expected payoff is $1/n$ regardless of what firm $i$ does. If $1 \leq k \leq n - 2$, i.e. at least one but not all of the firms have a technology, then firm $i$’s expected payoff when patenting is still $1/n$ — no revenue could be extracted by patenting because the price of the license were driven to zero in Bertrand competition. On the other hand, if firm $i$ keeps its technology secret, it is in a better position than the $n - 1 - k$ firms that do not have an initial technology and therefore has an expected payoff greater than $1/n$. The only case where patenting may produce a higher expected payoff is when no other firm has a technology, i.e. when $k = 0$. If firm $i$ innovates at time $t = 0$, the probability of this contingency is one. If keeping the technology secret is the preferred strategy at time $t = 0$, then the payoff of not patenting is greater than that of patenting also under the contingency $k = 0$. If this is the case, then nonpatenting dominates patenting and must be the equilibrium irrespective of the probabilities of the contingencies, i.e. for all $t \geq 0$. 

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Proof of Proposition 3.2: Part (i): If a firm of size \( x_i \) prefers strategy \( S_p \), then by (3.31) its revenue from patenting exceeds \( cx_i/a \). If the firm is split into two firms of sizes \( x_1 \) and \( x_2 = x - x_1 \), then the revenue from patenting for firm 1 exceeds that of the original firm by the price firm 2 is willing to pay for the license. Since \( x_1 < x \) implies \( cx_1/a < cx/a \), it follows that (3.31) also holds for firm 1. The same reasoning applies to firm 2. Hence, splitting a firm into two smaller firms cannot decrease the propensity to patent. Generalization to an arbitrarily fine split follows trivially.

Part (ii): Suppose firm \( i \) finds it profitable to deviate. This implies that its revenue from patenting is smaller than \( cx_i/a \); i.e. condition (3.31) fails for firm 1. If firm \( i \) is merged with an arbitrary competitor \( k \), then the licensing revenue for the new firm is smaller by \( \pi(x_k) \) than what firm \( i \) alone would get. Since \( cx_1/a < c(x_1 + x_2)/a \), condition (3.31) must fail also for the new firm. Again, generalization to merging with an arbitrary number of firms is trivial.

Proof of Proposition 3.3: If firm \( A \) prefers patenting and the smaller firm \( B \) does not, then by condition (3.31) the following inequalities must hold:

\[
\sum_{k \neq A,B} \pi(x_k) + \frac{c(1-x_B)x_B}{bx_B + c(1-x_B)} - cx_A/a \geq 0
\]

\[
\sum_{k \neq A,B} \pi(x_k) + \frac{c(1-x_A)x_A}{bx_A + c(1-x_A)} - cx_B/a < 0
\]

This implies that

\[
\frac{(1-x_A)x_A}{bx_A + c(1-x_A)} + x_A/a < \frac{(1-x_B)x_B}{bx_B + c(1-x_B)} + x_B/a.
\]

Define the function \( h(x) \) as

\[
h(x) = \frac{(1-x)x}{bx + c(1-x)} + x/a.
\]
For inequality (A3.6) to hold, this function has to decrease in \( x \) at least over some range of values. Differentiating \( h(x) \) gives

\[
\begin{align*}
h'(x) &= \frac{1 - 2x}{bx + c(1 - x)} - \frac{b(1 - x)x}{[bx + c(1 - x)]^2} + \frac{c(1 - x)x}{[bx + c(1 - x)]^2} + 1/a \\
&> \frac{b(1 - x)x}{[bx + c(1 - x)]^2} + 1/a \\
&= -\frac{x}{bx + c(1 - x)} + \frac{1}{c(1 - x)^2} + 1/a \\
&> -\frac{1}{bx + c(1 - x)} + 1/a \\
&> -1/b + 1/a \\
&> 0.
\end{align*}
\]

Thus, \( h(x) \) increases in \( x \). But then \( h(x_A) > h(x_B) \) and (A3.6) cannot hold and the proposition follows.
4 Research Joint Ventures vs. R&D Competition

4.1 Introduction

In standard economic theory, cartels and collaboration between producers are (mostly) bad; they result in higher prices and lower output than does perfect competition. Since the times of Schumpeter, economists have known that things are less simple in the production of technology. There are a number of reasons why cooperation in R&D may be desirable. If the private returns to R&D fall short of the social returns due to technology spillovers, cooperation will be socially beneficial if it helps to increase appropriability. Cooperation may also improve coordination in R&D work. Since technology is a shareable good, producing the same piece of technology more than once is waste of resources from the social point of view. Such duplication may be avoided with cooperation. Finally, cooperation may increase the diffusion of innovations.

These arguments have worked their way into the legislation in many countries. In the United States, where the attitude toward cartels has generally been strict, antitrust laws were rewritten as a result of lobbying by computer firms, and in 1984 the National Cooperative Research Act practically removed constraints on collaboration in research (see Brodley, 1990). In the same year, the European Commission adopted Regulation No. 418/85, which further extended the favorable antitrust treatment of R&D from what had been expressed already in 1968 “Notice of Cooperation between Enterprises”. Similar practices have emerged in Japan as well.\(^1\) Consequently, it has come commonplace for companies that compete in a product market to undertake joint R&D projects.

\(^1\)For a discussion of the practice followed by the Fair Trade Commission of Japan, which is responsible for executing and enforcing the Antimonopoly Act of 1947, see Jorde and Teece (1990).
One of the most prominent examples of such projects is Sematech, the successful Research Joint Venture (RJV) of the big American microelectronics manufacturers, was established in 1987 and is heavily subsidized by the federal government. Other frequently cited examples are the German machine tool industry and the R&D collaboration between American and Japanese automobile companies.

However, whether RJVs are desirable from the social point of view is not a straightforward issue. While cooperation in research may provide the advantages outlined above, the familiar risks of reduced competition remain. Eliminating competition in R&D may lead to excessive cuts in R&D spending, particularly if innovation redistributes rather than increases industry profits. Collusion in R&D may also facilitate collusion in production, for example, through license payments.

The work by Grossman and Shapiro (1986) and Ordover and Willing (1985) isolated some of the fundamental forces that determine the effect of an RJV. A later line of papers, starting with the work of d’Aspremont and Jacquemin (1988) and further represented in the works of Henriques (1990), Kamien et al. (1992), Suzumura (1992), and, in a continuous time framework, Stenbacka and Tombak (1993), approaches the question using a rigorous two-stage model of oligopolistic competition. In the first stage, a number of firms first choose their R&D investments and then engage in a Cournot competition, taking R&D investments as given. A number of different scenarios have been employed regarding how the R&D stage is arranged. Firms may share their R&D output or try to keep it secret, and the decision of R&D investment may be made either in a centralized fashion or in the individual firms. Particularly interesting is the comparison between a cartelized RJV, in which firms cooperatively choose R&D investment to maximize their joint profits and share their R&D output, and non-cooperative competition, where firms compete in both stages and R&D output is not shared. The pa-

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2For profound nontheoretical discussion of the merits and weaknesses of research as well as production joint ventures, see Jorde and Teece (1990), Brodley (1990) and Shapiro and Willig (1990).
pers by d’Aspremont and Jacquemin (1988) and Kamien et al. (1992) agree that an RJV cartel leads to a higher level of effective R&D and guarantees a higher level of welfare than does the non-cooperative equilibrium. Thus, these papers conclude that firms should be encouraged to form R&D joint ventures, provided they coordinate their R&D inputs.

The superior performance of the RJV cartel in those models relies on the assumed elastic demand structure; innovation increases industry profits instead of just redistributing them. It would be trivial to show that with inelastic demand the conclusion would not hold. Even with elastic demand, the result may not be robust to the inclusion on some features of the real innovation processes. For example, in those models, it is assumed that forming an RJV is the only way that firms can share their R&D output. This chapter augments the stylized framework used in the papers by d’Aspremont and Jacquemin (1988) and Kamien et al. (1992) by allowing firms to share their technologies by selling licenses to each other and analyzes whether the superiority of the RJV cartel still holds. It is shown that this type of modification changes the results significantly. Whether the RJV cartel or competition with licensing leads to higher R&D investment depends on several model parameters, such as the extent of spillovers and duplication in R&D, substitutability of products, and the outcome of the bargaining between seller and buyer in the market for technology. It is also found that while an RJV cartel cannot, from the social point of view, overinvest in R&D, there is no mechanism preventing firms in competition with licensing from doing so.³

To understand the way the market for technology changes the situation, one needs to separate two types of externalities that firms exert on each other. First, there is the direct technological externality, by which one firm’s R&D investment improves the production technology of others. An RJV cartel internalizes this

³That an RJV cartel never overinvests in R&D holds in this particular framework. The same is not generally true, as a counterexample by Suzumura (1992) shows.
positive spillover and thereby increases R&D investment. Allowing firms to trade their R&D does not correct this externality, but instead introduces an additional spillover — or strategic interaction — that works in the opposite direction. This spillover is created when a firm licenses its existing technology to another firm; due to demand interdependencies, the transaction causes a deterioration in the relative position of all third party firms. The licensor does not take this effect into account when deciding on its R&D investment. As shown below, this externality may or may not outweigh the technological externality, depending on the parameters.

In order for the market for technology to play any role, two conditions must be met. First, the R&D undertaken in the individual firms must not be completely overlapping. This is not likely to be a problem in real life. As Kamien et al. (1992) state, R&D "...is a multidimensional heuristic rather than a one-dimensional algorithmic process." There is good reason to believe that two firms, each undertaking R&D independently, could not come up with identical R&D outcomes even if they tried — first, because the probability of this happening by pure chance is negligible, and secondly, because the firms have an economic incentive not to try; if there is a market for R&D output, then the firms want to differentiate their products, and the multidimensionality of the R&D process makes this possible. Even if the firms do not directly cooperate in R&D, they can keep each other informed about the general area of their R&D work — and each has an incentive to tell the truth. Alternatively, each firm can sell its R&D output to others in small pieces over the course of the R&D process and thereby avoid duplication. In what follows, it is assumed that even when firms do not cooperate they are able to coordinate their R&D efforts so as to completely avoid duplication. It would be straightforward to introduce an additional parameter, "degree of duplication", in the model, but this would add little insight and would not change the qualitative implications.

The second consideration is related to appropriability in the market for technology; that is, how a technology is priced. Once a firm has produced a technology, the marginal cost of reproducing
it and licensing it to another firm is essentially zero. An indirect cost arises from the lost market share in the product market that results from the increased competitiveness of the licensee, if the products of the two firms are substitutes. If the benefit to the licensee from competitiveness gained exceeds the corresponding loss to the licensor, then licensing creates a surplus to the partners and there is room for negotiation.

The standard approach to model this negotiation would be to find the threat points and postulate a bargaining process. In this particular case, the threat point for the licensee is straightforward to find (it is simply the profit it would get without the technology), but the threat point of the licensor is harder to find. It is not simply the profit it would get by not selling the technology, because in this case the licensor might do better by choosing a completely different strategy for selling licenses. It could, for example, avoid the negotiation process entirely by arranging an auction for a prespecified number of licenses, say \( n - 2 \), i.e. one less than the number of its competitors, but any number smaller than the number of its competitors would do. The price of a license would then be driven up to the point where a competitor is indifferent between getting a license and not getting it, and the licensor would extract the total surplus from \( n - 2 \) firms. If the number of firms is large enough, the profit the licensor can extract using this strategy is close to what it would get if it were able to set the price of a license and sell them to all \( n - 1 \) competitors.

Here, the complications related to this multidimensional bargaining problem are sidestepped by assuming the following structure for the trade of licenses: the owner of a technology chooses the number of licenses \( k \) it is going to offer for sale. It then sends a licensing proposal to \( k \) competitors, offering the right to the use of its technology for a fixed fee. The receivers of the proposal must approve or reject it; no counteroffers can be made. The licensor does not have any other chance to sell its technology. If one or more of the firms reject the proposal, the number of licenses
sold is reduced accordingly.\footnote{This pricing scheme is differs slightly from the optimal one used by Katz and Shapiro (1986) and applied in the previous chapter. In the optimal pricing scheme, an auction is run for a prespecified number of licenses. Here, it is the number of offers, not licenses, that is prespecified. The price of a license coincides in the two schemes if the licenses are sold to every competitor. If the number of licenses sold is smaller than the number of competitors, the two schemes differ. The reason for this choice is that it allows for clear limits for complete cross-licensing to be an equilibrium (see Proposition 4.1).} This scheme effectively allocates all bargaining power to the licensor and maximizes the income he gets for the technology. Consequently, the incentive to undertake R&D is also maximized. A less extreme assumption regarding the sharing of the surplus would produce less pronounced but qualitatively similar results, at the expense of more complicated calculations. Also, for the reasons mentioned above, price setting by the licensee should be a fair approximation of the outcome of a negotiation process if the number of firms is large enough. The assumption that each firm chooses to license its technology to all other firms is usually not restrictive, as is shown later in the paper.

Finally, it is assumed that a fixed fee is paid for the license. Aside from the relative simplicity it provides, this assumption can be justified by several studies showing that charging a fixed fee is more profitable to the licensor than royalties (see Kamien an Tauman 1986). Finally, the restriction to fixed fees is also natural when the licensor cannot monitor licensees' output levels. In this case a scheme involving royalties could not be enforced. Since this particular model is concerned with the trading of a technology that lowers the production cost, a lump sum payment seems more appropriate than would be the case if the object of trade were a license to manufacture a product.\footnote{In general, a licensing contract can be designed so that the outcome is identical to a fully collusive outcome [see Shapiro (1985)]. Most likely, a scheme consisting of a combination of royalties and a lump sum payment would also result in collusion in the product market. How much incentive to innovate such a situation would create could be an interesting topic for future research.}
4.2 The Model

The model used here follows, in many respects, the framework used in Kamien et al. (1992), which is based on the model presented in d'Aspremont and Jacquemin (1988). Where possible, the notation also follows that of Kamien et al. (1992). The market consists of $n$ identical firms, each producing a single product. The products of different firms are symmetric; that is, substitutability is constant over the products. The quantity produced by firm $i$ is denoted by $Q_i$ and the price it realizes is given by the inverse demand function identical to the one in Kamien et al. (1992), i.e.

$$P_i = a - Q_i - \gamma \sum_{j \neq i} Q_j.$$  

(4.1)

Here $\gamma \leq 1$ is the substitutability parameter. The values $\gamma = 0$ and $\gamma = 1$ correspond to no substitutability and perfect substitutability, respectively.

The unit cost of firm $i$'s production is $c - f(X_i)$, where $X_i$ is firm $i$'s effective stock of technology. Firm $i$'s effective stock of technology is the sum of the technology produced by the firm in-house $x_i$, and the technology it receives from the competing firms through involuntary technology spillovers, voluntary disclosures of technology, or by buying licenses to the use of technologies. The cost of producing in-house technology is assumed to be an increasing function $g(x_i)$ of the technology output $x_i$. To guaran-
tee the existence of solutions later in this chapter, function $g$ is assumed to be sufficiently convex.\(^6\)

Three scenarios will be examined, each with a different assumption as to the arrangement of the R&D stage and the diffusion of technology:

**Scenario RJV (R&D Joint Venture):** Firms form an industry-wide cartellized R&D joint venture. Individual investments in R&D are chosen jointly to maximize the joint profit of the firms. All R&D output is voluntarily disclosed to other firms, so that the representative firm $i$'s stock of technology is the sum of all technology outputs:

$$X_i = \sum_{j \in N} x_j \quad i \in N = \{1, ..., n\}. \quad (4.2)$$

**Scenario CN (R&D competition without licensing):** Firms compete in the R&D stage and technology is not tradable. Each firm chooses its R&D investment to maximize its individual profit. The only technology that a firm receives from its competitors comes through involuntary spillovers. Hence, firm $i$'s stock of technology is given by

$$X_i = x_i + \beta \sum_{j \neq i} x_j, \quad i \in N \quad (4.3)$$

where $\beta \in [0, 1]$ is the spillover parameter.

---

\(^6\)This formulation combines features from the models of d'Aspremont and Jacquemin (1988), who use a quadratic cost function and assume the function $f$ linear, and Kamien et al. (1992), who assume a linear cost function. Both the concavity of $f$ (see Assumption 4.1) and the convexity of $g$ serve to convexify the model. However, the interpretation of these two approaches is somewhat different. A concave $f$ means *decreasing returns to scale* in R&D — additional units of technological knowledge produce ever smaller decreases in production cost — while a convex $g$ can be interpreted as *decreasing returns to concentration* — a given amount of R&D input produces the higher R&D output the larger the number of firms among which the R&D effort is distributed. Here, both are assumed to be present to a sufficient degree to ensure that a unique equilibrium exists.
Scenario CL (R&D competition with licensing): Firms compete in the R&D stage and technology is tradable. Each firm chooses its R&D investment to maximize its individual profit and can sell licenses to that part of its technology which does not leak out through involuntary spillovers. The effective stocks of technology are somewhere between (4.3), if no licenses are sold, and (4.2), if cross-licensing is complete — i.e. each firm licenses its technology to every other firm.

Hence, if the individual investments are the same in all three scenarios, RJV and CL are equivalent and efficient in the sense that every firm can make use of all technology created by the firms (provided cross-licensing is complete in the latter), whereas scenario CN is inefficient in that each firm can access only a part of the technology created in the market. The main question is which of the first two scenarios leads to a higher level of R&D investment.

**Assumption 4.1** The function $f$ is increasing, concave and twice continuously differentiable. It is bounded from above by $c$ and $f(0) = 0$.

**Assumption 4.2** $\lim_{X \to \infty} f(X) < (a - c)/(n - 1)$.

**Assumption 4.3** $f'(X)^2 + f(X)f''(X) < 0$ for all $X \geq 0$.

These assumptions follow closely those of Kamien et al. (1992). Assumption 4.1 contains some standard assumptions about R&D production technology and guarantees that the cost of production is always positive. Assumption 4.2 is needed to guarantee that every firm finds it profitable to participate actively in production. Notice that for any given nontrivial function $f(X)$ and parameters $a$ and $c$ there exists an $n$ large enough so that Assumption 4.2 fails to hold. Assumption 4.3 sets a lower bound for the concavity of function $f$. An important implication of this assumption is that an individual firm's profit is a strictly concave function of its own stock of technology. This is stronger requirement than
that made by Kamien et al. (1992) who only assumed monopoly profit to be strictly concave in technology.\textsuperscript{7} The stronger form is needed to guarantee the existence of an equilibrium in the market for technology under competition with licensing. It also provides a sufficient condition for equilibrium in an RJV cartel. It does not necessarily guarantee the existence of an equilibrium in R&D competition without licensing. This is not a major consideration since the focus of interest here is on the comparison between the RJV cartel and competition with licensing.

It is established in earlier papers that given the stocks of technology firm $i$'s profit from production (i.e. not counting R&D costs or payments related to the trade of technology) is $\pi_i = Q_i^2$, where

$$Q_i = \frac{a - c + f(X_i) - \frac{\gamma}{2} \sum_{j \neq i} [f(X_j) - f(X_i)]}{2 + \gamma(n - 1)} \quad (4.4)$$

and $X_i$ is given by either (4.3) or (4.2), depending on whether or not firms share or trade their technologies. It is shown in the Appendix that given Assumption 4.2, $Q_i$ is positive. Hence, all firms participate in production.

4.3 The Analysis

Following the earlier papers, the focus here is on symmetric Nash equilibria. Asymmetric equilibria are not considered. Before proceeding to the analysis, it is convenient to define an additional function $\pi(X_i, X_j)$ to denote the profit a firm gets in the second

\textsuperscript{7}The weaker form of Assumption 4.3 used by Kamien et al. does not guarantee the existence of an equilibrium under R&D competition. It is only sufficient to establish the uniqueness of an equilibrium, provided one exists. It is unclear whether Kamien et al. intended to assume or prove the existence of the equilibrium. In footnote 1 (p. 1298) they state that they assume existence, but in the Appendix (p. 1306) they (incorrectly) claim to have proven it.
period if its stock of technology is $X_i$ and $X_k = X_j$ for all $k \neq i$; i.e. all other firms have identical stocks $X_j$. Thus,

$$\pi(X_i, X_j) \equiv [2 - \gamma(n - 1)]^{-2} \times \left[ a - c + f(X_i) - \frac{\gamma}{2}(n - 1)(f(X_j) - f(X_i)) \right]^2. \quad (4.5)$$

The following properties of the function $\pi$ are established in the Appendix.

**Lemma 4.1**

(i) $\frac{\partial}{\partial X_i} \pi(X_i, X_j) > 0$

(ii) $\frac{\partial}{\partial X_j} \pi(X_i, X_j) < 0$

(iii) $\frac{\partial}{\partial X_i} \pi(X_i, X_j) + \frac{\partial}{\partial X_j} \pi(X_i, X_j) > 0$, for $X_i < X_j$

(iv) $\frac{\partial^2}{\partial^2 X_i} \pi(X_i, X_j) < 0$

(v) $\frac{\partial^2}{\partial X_i \partial X_j} \pi(X_i, X_j) < 0$

(vi) $\frac{\partial^2}{(dX)^2} \pi(X, X) < 0$.

Thus, given the demand function and Assumptions 4.1-4.3, profit function $\pi$: (i) increases in own stock of technology; (ii) decreases in the competitors’ stock of technology; (iii) increases if everybody’s stock of technology is increased; (iv) is concave in own stock of technology; (v) has negative cross derivative; and (vi) is concave in $X$ in a symmetric solution.

**Case RJV: R&D joint venture cartel.**

The RJV’s problem can be written as one of maximizing

$$n\pi(X, X) - ng(X/n)$$

with respect to $X$. Denoting the symmetric solution by $X^R$, the first-order condition for a maximum becomes

$$n \frac{d\pi(X^R, X^R)}{dX^R} - g'(X^R/n) = 0. \quad (4.6)$$

By convexity of $g$ and property (vi) of Lemma 4.1, the objective function for an RJV cartel is globally concave. Thus, the equilibrium is unique. Existence requires low enough $g'(0)$, which will be assumed.
Case CN: R&D competition without licensing.

The maximization problem for the individual firm $i$, given that all other firms invest $x_j$, is

$$\max_{x_i} \pi [x_i + \beta(n - 1)x_j, (1 + \beta(n - 2))x_j + \beta x_i] - g(x_i).$$

(4.7)

Differentiating the objective function yields the first order condition

$$\pi_1 [x_i + \beta(n - 1)x_j, (1 + \beta(n - 2))x_j + \beta x_i]
+ \beta \pi_2 [x_i + \beta(n - 1)x_j, (1 + \beta(n - 2))x_j + \beta x_i]
-g'(x_i) = 0,$$

(4.8)

where $\pi_1$ and $\pi_2$ are the partial derivatives. Increasing R&D investment has two opposite effects on firm $i$'s profits. The first term represents the direct effect a reduction in firm $i$'s production cost has on its profits. The second term is the negative externality firm $i$'s R&D investment has on its own profits through the decrease in its competitors' production cost, caused by the spillover effect. Substituting the symmetric solution $x_i = x_j$ and denoting $X_{CN} = (1 + \beta(n - 1))x_i$ gives

$$\pi_1 [X_{CN}, X_{CN}] + \beta \pi_2 [X_{CN}, X_{CN}]
-g' \left[ X_{CN}/(1 + \beta(n - 1)) \right] = 0.$$

(4.9)

By the definition of $\pi$, the following can be seen to hold

$$\pi_1(X, X) = \frac{d\pi(X, X)}{dX} \left[ 1 + \frac{\gamma}{2 - \gamma}(n - 1) \right]$$
$$\pi_2(X, X) = -\frac{d\pi(X, X)}{dX} \frac{\gamma}{2 - \gamma}(n - 1).$$

(4.10)

Substituting these into equation (4.9) yields the final form for the first-order condition

$$\frac{d\pi(X_{CN}, X_{CN})}{dX_{CN}} \times \left[ 1 + (1 - \beta) \frac{\gamma}{2 - \gamma}(n - 1) \right]
-g' \left[ X_{CN}/(1 + \beta(n - 1)) \right] = 0.$$

(4.11)

The objective function (4.7) can be shown to be concave in $x_i$ in the neighborhood of the symmetric solution, so (4.11) provides
at least a local maximum. Assuming that the function $g$ is sufficiently convex guarantees that the solution is global. Uniqueness of the symmetric solution to (4.11) follows from the convexity of $g$ and property (vi) of Lemma 4.1.

Case CL: R&D competition with licensing

This case is slightly more complicated than the previous two. Here each firm first decides on its own R&D effort independently, after which it has an opportunity to sell to other firms licenses to that part of its R&D in-house technology that has not already become common knowledge through involuntary spillovers. Before analyzing the choice of R&D output, one has to know what happens at the licensing stage of the game. The assumption is that the price of the license is set by the licensor at the highest price the licensee will accept. The fact that any two firms both buy each other's technology is assumed not to affect the outcome; that is, the price firm $i$ charges firm $j$ for its technology does not depend on the price firm $j$ charges for its own technology. Hence, the strategic variables are the prices themselves, not the pricing rules.

The equilibrium solved for is one in which every firm licenses its technology to every other firm. To establish this as the equilibrium of the licensing subgame, consider the maximization problem of a representative licensor firm $i$. Firm $i$ knows that the other firms license their technologies to everybody. If firm $i$'s individual R&D output is $x_i$ and the aggregate stock of technology $X = \sum_{j \in N} x_j$, then the stock of technology firm $i$'s competitors have access to without firm $i$'s technology is $X - (1 - \beta)x_i$; that is, the aggregate stock of technology less that part of firm $i$'s technology that can be kept secret. Suppose firm $i$ also decides to offer its technology to every competitor. If all firms license the technology, then their profit in the production stage is $\pi(X, X)$. If a single firm rejects the offer, it gets the profit $\pi(X - (1 - \beta)x_i, X)$. The maximum fee firm $i$ can get for each license, denoted here $\rho(x_i, X)$ is the difference of these two, i.e.

$$\rho(x_i, X) = \pi(X, X) - \pi(X - (1 - \beta)x_i, X).$$  \hspace{1cm} (4.12)
By Lemma 4.1, $\rho(x_i, X)$ is positive. The value of the license $\rho(x_i, X)$ is here derived under the assumption that all other firms purchase licenses to all available technologies. A natural question is whether firm $i$ is still willing to pay that fee if cross licensing among competitors is less than complete. Using property (v) of Lemma 4.1 it is straightforward to show that this is indeed the case. Decreasing the level of the effective stock of technology of a firm's competitors makes each new technology more valuable to the firm itself. Hence, $\rho(x_i, X)$ gives the minimum value of the license for the licensee over all feasible prospects.

The following proposition states the condition under which it is optimal for a firm to sell the license to every competitor and charge the fee given by equation (4.12). The proposition is proved in the Appendix.

**Proposition 4.1** (i) Given the fee $\rho$ in (4.12), each firm maximizes its profits by buying a license from every competitor. (ii) Each firm maximizes its profits by selling its technology to all other firms if

$$ \frac{f(X - (1 - \beta)x_i)}{f(X)} > \frac{n - 1}{3n - 1}. $$

(4.13)

Hence, under this condition, mutual licensing at price $\rho$ is an equilibrium in this subgame.

Part (i) of the proposition is based on Assumption 4.3, which gives a lower bound for the concavity of $f$. It ensures that profit is a concave function of technology. As a result, the fee for a license, which is based on its marginal contribution on the buyer's profit, is smaller than the average contribution of several licenses, and thus the buyer gains by buying from all sellers.

Part (ii) of the proposition states that it is optimal for a firm to sell to all of its competitors, provided its contribution to the cost reduction is not too large. More precisely, if the cost reduction with firm $i$'s technology is no more than $(3n - 1)/(n - 1) = 3 + 2/(n - 1)$ times the cost reduction without it, then firm $i$ finds it optimal to sell to all its competitors. This condition holds if
the \(x_i\)'s are sufficiently symmetric; it also holds if the spillover parameter \(\beta\) is high enough, as established in the following corollary:

**Corollary 4.1** Let

\[
\beta > \left( \frac{n-1}{3n-1} \right)^2.
\]

Then complete cross-licensing is an equilibrium.

Based on Corollary 4.1, it can be verified that condition (4.13) holds for any \(n\) if \(\beta > 1/9\). For a duopoly, the requirement is even weaker; if \(\beta > 1/25\), then mutual licensing is an equilibrium. Only if a firm’s innovation is sufficiently drastic and \(\beta\) is small, can it be optimal for the firm to sell only to a subset of competitors. Higher oligopoly profits for those firms would, in turn, drive the price of a license sufficiently higher to compensate for the smaller number of licenses. In the rest of the paper, condition (4.13) is assumed to hold so that all technologies are traded.

Firm \(i\) has two sources of income: its own profits in the production stage and the licensing income which is equal to \(n-1\) times the fee given by (4.12). On the negative side, it has to buy technology from \(n-1\) competitors. Since the competitors are symmetric, the R&D investment of each firm \(j \neq i\) can be denoted by \(x_j\). Then the price firm \(i\) has to pay for each license is \(\rho(x_j, X)\). Finally, firm \(i\) has to pay for its R&D investment. Thus, firm \(i\) chooses \(x_i\) to maximize

\[
\pi(X, X) + (n-1)\rho(x_i, X) - (n-1)\rho(x_j, X) - g(x_i)
\]

subject to \(X = x_i + (n-1)x_j\).  

(4.14)

Substituting from (4.12) to (4.14) and differentiating gives the first order condition

\[
\pi(X, X) = g'(x_i)
\]

(4.15)

\[
-(n-1)\frac{d}{dx_i} \pi(X - (1-\beta)x_i, X)
\]

\[
-(n-1) \left[ \frac{d}{dx_i} \pi(X, X) - \frac{d}{dx_i} \pi(X - (1-\beta)x_j, X) \right]
\]

\[= 0.\]
The first line gives the marginal change in the joint net profits from an additional unit of R&D investment. In the RJV cartel, this is set equal to zero. The second line gives the change in the competitors' threat point resulting from a change in firm i's R&D investment, i.e. the change in a competitor's profit in case the competitor does not buy firm i's technology. This effect may be positive or negative, depending on the size of the spillover effect. The third line represents the change in the price firm i pays for its competitors' technology. Properties (iii) and (v) of Lemma 4.1 imply that the net effect of this term on firm i's profit is positive: by investing more in R&D, firm i decreases the price it has to pay for other firms' technology.

Putting in the fixed point condition $x_i = x_j = X^{CL}/n$ and using the definition of function $\pi$ gives, after some manipulation, the final condition for a symmetric equilibrium under competition with licensing:

$$\begin{align*}
\frac{d\pi(X^{CL}, X^{CL})}{dX^{CL}} &+ (n-1)(1-\beta)\pi_1(X^{CL} - (1 - \beta)X^{CL}/n, X^{CL}) \\
-g'(X^{CL}/n) &= 0.
\end{align*}
$$

(4.16)

The existence and uniqueness of the solution again relies on small $g'(0)$ and sufficiently convex $g$, which will be assumed.

### 4.4 Comparison of the Scenarios

Using the definition of the function $\pi$ yields the following forms for the first order conditions for the first two scenarios:

$$2n \left[ \frac{a - c + f(X^R)}{2 + \gamma(n - 1)} \right] f'(X^R) - g'(X^R) = 0$$

(4.17)

in the RJV cartel and

$$2 \left( 1 + \frac{\gamma - (1 - \beta)(n - 1)}{2 - \gamma(n - 1)} \right) \left[ \frac{a - c + f(X^{CN})}{2 + \gamma(n - 1)} \right] f'(X^{CN}) - g'(X^{CN}) = 0$$

(4.18)
under competition without licensing. It follows from Assumption 4.3 that the expression

$$\left[ \frac{a - c + f(X)}{2 + \gamma(n - 1)} \right] f'(X)$$

decreases in $X$. Since the term multiplying this expression is smaller in (4.18) than in (4.17), the following holds:

**Proposition 4.2** (Kamien et al., 1992). $X^R > X^{CN}$ except when $\beta = 0$ and $\gamma = 1$, in which case $X^R = X^{CN}$.

Equally straightforward is the comparison between competition with and without licensing (see the Appendix):

**Proposition 4.3** $X^{CL} > X^{CN}$ except when $\beta = 1$, in which case $X^{CL} = X^{CN}$.

The comparison between the RJV cartel and competition with licensing is more complicated. Necessary and sufficient conditions for this comparison would require specification of the function $f$ and practically all parameters. However, a simple sufficient condition is easy to establish (see Appendix).

**Proposition 4.4** Let

$$\beta < -\frac{(n - 1)\gamma}{(n - 2)\gamma + 2}.$$

(4.19)

Then $X^{CL} > X^R$.

Inequality (4.19) provides a sufficient condition under which the change in firm $i$'s total profit (firm $i$'s profit from production plus the revenue from selling its technology minus the expenditure on other firm’s technology) resulting from an additional unit of investment is greater than the change in the joint profit of all firms (the objective function in the RJV cartel). If this is the case, then the marginal effect of one firm’s R&D investment on its
competitors' net profits is negative. As a consequence, investment is higher in competition with licensing.

It can be immediately seen that competition with licensing leads to a higher level of R&D investment if the spillover parameter $\beta$ is not too large and if the substitutability parameter $\gamma$ is high enough. The intuition behind the effect of $\beta$ is straightforward: the larger the spillover effect the smaller the portion of a firm's technology that remains to be licensed and hence the lower the price of a license.

The role of the substitutability parameter $\gamma$ is less evident. It is related to the strength of the strategic interaction between the firms, which is reflected in the responsiveness of competitors to changes in a firm's production cost. If substitutability between the products is zero, the demands are independent and the loss to firm $i$ from a small increase $dc_i$ in its unit cost is simply $dc_i \cdot Q_i$. On the other hand, if the products are close substitutes, then an increase in $c_i$ (that is, a decrease in $X_i$) has an additional effect: other firms react to their improved competitiveness by increasing their production, which, in turn, decreases the price of firm $i$'s good. The cost disadvantage hurts firm $i$'s profits more the greater the substitutability of the goods. Hence, a high value of $\gamma$ contributes to a high price of the license and thereby increases the incentive to invest in R&D. Note also that for any fixed parameters $\beta < 1$ and $\gamma > 0$, condition (4.19) holds for large enough $n$.

Figure 4.1 illustrates the comparison between competition with licensing and the RJV cartel. It plots the threshold parameter combinations for which (4.19) holds with equality for the cases $n = 2, 3, 5, 10$ and $20$. Below the threshold line, competition with licensing leads to higher R&D investment. Above the threshold line the ranking is unclear, but the further above the line one moves the larger the set of functions $f$ for which an RJV cartel invests more.

A higher level of R&D investment does not necessarily correspond to a higher level of welfare. Rigorous welfare comparison is not possible at the present level of generality. However, some specific results can be extracted. Since a higher stock of technol-
ogy, in this particular model, leads to higher production and lower price in the second stage (see Kamien et al. 1992), consumer welfare increases with R&D investment. On the other hand, producer surplus is always maximized in the RJV cartel. It is easy to see that since the RJV cartel leads to higher investment than R&D competition without licensing it must also provide higher welfare (producer and consumer surplus combined). Comparison of the welfare properties of the RJV cartel and competition with licensing is less clear cut. For parameter values such that $X^R > X^{CL}$, both producer and consumer surplus (and hence also welfare) are higher under the RJV cartel. On the other hand, since in the RJV cartel solution $X^R$, producer surplus is at the maximum and consumer surplus is increasing in R&D investment, the total surplus increases in $X$ at values close to $X^R$. Thus, for $X^{CL}$ greater than, but close to $X^R$, competition with licensing leads to higher welfare. It is possible that in some cases, competition with licensing leads to overinvestment in R&D. Exactly when this happens and under what conditions the social loss from this overinvestment is
greater than the loss from underinvestment under a RJV cartel are questions that can only be answered for specific forms of the functions $f$ and $g$.

4.5 Conclusions

Several recent papers conclude that a research joint venture cartel that maximizes the joint profits of the participating firms is likely to produce higher R&D investment and welfare than does R&D competition. This result has been used to recommend policies in which firms are encouraged to form RJV cartels. The analysis of this paper shows that this result, and therefore the policy recommendations implied by it, do not necessarily hold if technology is tradeable; that is, if firms are allowed to license their R&D outputs to each other.

To understand the results of this chapter one has to examine the spillovers involved. The most evident one is the technology spillover that is built in the basic structure of the model. Firms do not take into account the positive effect of their R&D input on other firms' technology and therefore undertake a socially inadequate amount of R&D investment. Making technology tradeable introduces another spillover into the game. When two firms trade their technologies, the competitive position of all third party firms deteriorates. Hence, by investing in R&D, a firm exerts also an indirect negative externality on its competitors. Whether the direct technological spillover or the indirect spillover from the trading of technology is dominant determines which one — the RJV cartel or competition with licensing — yields the highest level of R&D. It is shown in this chapter that the closer competitors the firms are — i.e. the closer substitutes they produce — the more likely it is that competition with licensing provides a faster pace of technological progress.

The model of this chapter builds on several assumptions. Most importantly, it is assumed that the seller of a license can extract all the surplus created by the trade. While this is probably a reasonable approximation when the number of firms in the market
is large, it may be less so in a typical real-world oligopolistic R&D setting. Further, the analysis excluded the possibility that information asymmetries may lead to duplication in R&D, which the RJV cartel may be able to correct. Still, this chapter serves to show that even under elastic demand it is not inherent of an RJV cartel to lead to higher R&D investment than does competition.
Appendix to Chapter 4

Derivation of (4.4): Firm i maximizes

\[ Q_i \left[ a - Q_i - \gamma \sum_{j \neq i} Q_j - (c - f(X_i)) \right]. \quad \text{(A4.1)} \]

The first order condition is

\[ a - 2Q_i - \gamma \sum_{j \neq i} Q_j - (c - f(X_i)) = 0 \]

or

\[ a - (2 - \gamma)Q_i - \gamma \sum_j Q_j - (c - f(X_i)) = 0. \quad \text{(A4.2)} \]

Summing over \( i = 1, \ldots, n \) and rearranging gives

\[ \sum_i Q_i = \frac{n(a - c) + \sum_i f(X_i)}{2 + \gamma(n - 1)}. \quad \text{(A4.3)} \]

Substituting from (A4.3) to (A4.2) and solving (A4.2) for \( Q_i \) gives the desired result.

**Proof that \( Q_i > 0 \).** \( Q_i \) takes its minimum value when firm \( i \)’s production cost is \( c \) (\( X_i = 0 \)) and other firms’ production cost is \( c - \lim_{X \to \infty} f(X) \). In this case,

\[ Q_i = \frac{a - c - \frac{\gamma}{2(1 - \gamma)}(n - 1) \lim_{X \to \infty} f(X)}{2 + \gamma(n - 1)}. \quad \text{(A4.4)} \]

The numerator is greater or equal to \( a - c - (n - 1) \lim_{X \to \infty} f(X) \), which, by Assumption 2, is positive.

**Proof of Lemma 4.1:** Differentiating \( \pi_i \) w.r.t. \( X_i \)

\[ \frac{d\pi_i(X_i, X_j)}{dX_i} = \frac{2Q_i}{2 + \gamma(n - 1)} \left[ 1 + \frac{\gamma}{2 - \gamma(n - 1)} \right] f'(X_i), \quad \text{(A4.5)} \]

where

\[ Q_i = \left[ \frac{a - c + \left(1 + \frac{\gamma}{2 - \gamma(n - 1)} \right) f(X_i) - \frac{\gamma}{2 - \gamma(n - 1)} f(X_j)}{2 + \gamma(n - 1)} \right] \]
was proved above to be positive. Since \( f \) is an increasing function, the right hand side of (A4.5) is also positive. Thus, property (i) is established.

Differentiating \( \pi_j \) w.r.t. \( X_j \):

\[
\frac{d\pi_i(X_i, X_j)}{dX_j} = -2Q_i \left[ \frac{\gamma}{2 - \gamma} (n - 1) \right] f'(X_j), \tag{A4.6}
\]

which is negative by the same argument as above. Property (ii) follows.

Summing the previous two results yields

\[
\frac{d\pi_i(X_i, X_j)}{dX_i} + \frac{d\pi_i(X_i, X_j)}{dX_j} = 2Q_i \left\{ \left[ 1 + \frac{\gamma}{2 - \gamma} (n - 1) \right] f'(X_i) - \frac{\gamma}{2 - \gamma} (n - 1) f'(X_j) \right\}. \tag{A4.7}
\]

If \( X_i \leq X_j \), then \( f'(X_i) \geq f'(X_j) \) and the term inside the curly brackets is positive. Property (iii) follows.

Differentiating (A4.5) w.r.t. \( X_i \) renders

\[
\frac{d^2\pi_i(X_i, X_j)}{(dX_i)^2} = 2 \left\{ \frac{a - c + \left( 1 + \frac{\gamma}{2 - \gamma} (n - 1) \right) f(X_i) - \frac{\gamma}{2 - \gamma} (n - 1) f(X_j)}{[2 + \gamma(n - 1)]^2} \right\} \times \left[ 1 + \frac{\gamma}{2 - \gamma} (n - 1) \right] f''(X_i)
\]

\[
+ \left[ 1 + \frac{\gamma}{2 - \gamma} (n - 1) \right]^2 f'(X_i)^2
\]

\[
< \left[ 1 + \frac{\gamma}{2 - \gamma} (n - 1) \right]^2 \left[ f(X_i) f''(X) + f'(X)^2 \right]
\]

\[
\leq 0,
\]

where the last inequality holds by Assumption 4.3. Thus, property (iv) holds.

Finally, differentiating (A4.5) w.r.t. \( X_j \) yields

\[
\frac{d^2\pi_i(X_i, X_j)}{dX_i dX_j} = -2Q_i \times \left\{ \frac{1 + \frac{\gamma}{2 - \gamma} (n - 1)}{[2 + \gamma(n - 1)]^2} \right\} f'(X_i) \frac{\gamma}{2 - \gamma} (n - 1) f'(X_j), \tag{A4.9}
\]
which is easily seen to be negative. This establishes property (v).

Substituting $X_i = X_j = X$ into the definition of $\pi(X_i, X_j)$ gives

$$\pi(X, X) = [2 - \gamma(n - 1)]^{-2} [a - c + f(X)]^2.$$ 

Differentiating this twice yields

$$\frac{d^2 \pi(X, X)}{(dX)^2} = [2 - \gamma(n - 1)]^{-2} \times [(a - c)f''(X) + 2(f'(X)^2 + f(X)f''(X))].$$

which is negative by Assumption 4.3 and the concavity of $f$. Thus property (vi) holds.

Proof of Proposition 4.1: Part (i) of the proposition follows directly from Assumption 3, which makes profit concave in the firm's own stock of technology. Since technology is priced according to the marginal contribution to the profit of the licensee provided by the last license, the average contribution of all licenses must exceed the total fees. Hence, part (i) of the proposition holds.

(ii) Denote the stock of technology of a firm other than $i$ which does not buy firm $i$'s technology by $X_{-i} = X - (1 - \beta)x_i$. Suppose firm $i$ decides to license its technology to $k \in \{1, \ldots n - 1\}$ firms. The profit it receives is its own profit in the production stage plus $k$ times the fee for a license. With a slight abuse of notation, let us write the fee as a function of $k$:

$$\rho(k) = \left[ \frac{a - c + f(X) - \frac{\gamma}{2} (n - 1 - k)(f(X_{-i}) - f(X))}{2 + \gamma(n - 1)} \right]^2 - \left[ \frac{a - c + f(X_{-i}) - \frac{\gamma}{2} k(f(X) - f(X_{-i}))}{2 + \gamma(n - 1)} \right]^2.$$ (A4.10)

The first term on the r.h.s is the profit a licensee gets if it, together with $k - 1$ other firms, buys the license. The second term is its if it does not buy the license and consequently the license is sold only to $k - 1$ firms. Firm $i$'s total profit consists of $k$ times the fee plus its own profit in the production period, which is equal to
the first term on the r.h.s of (A4.10). Thus firm \( i \) chooses \( k \) to maximize

\[
\pi(k) = (k + 1) \left[ \frac{a-c+f(X)-\frac{2}{2+\gamma(n-1)}(n-1-k)(f(X_{-i})-f(X))}{2+\gamma(n-1)} \right]^2

- k \left[ \frac{a-c+f(X_{-i})-\frac{2}{2+\gamma(n-1)}k(f(X)-f(X_{-i}))}{2+\gamma(n-1)} \right]^2.
\]

(A4.11)

It can be checked that \( \pi(k) \) is a downward opening parabola in \( k \). Therefore, it suffices to find the conditions under which \( \pi(n-1) \) is greater than \( \pi(n-2) \); if this is the case, then \( \pi(k) \) increases in \( k \) over the whole feasible range from 2 to \( n-1 \). Thus,

\[
\pi(n-1) - \pi(n-2) = \frac{f(X) - f(X_{-i})}{(2 + \gamma(n-1))^2(2 - \gamma)^2} Z
\]

(A4.12)

where

\[
Z = \left[ -2(a-c)(n-3) - (3n^2 - 10n + 7)f(X) \right.
\]

\[+ (3n^2 - 12n + 13)f(X_{-i}) \gamma^2
\]

\[+ 4 [(a-c)(n-4) - nf(X) + 2(n-2)f(X_{-i})] \gamma
\]

\[+ 4 [2(a-c) + f(X) + f(X_{-i})].
\]

(A4.13)

(A4.14)

Substituting \((n-1)f(X)\) for \( a - c \) makes the r.h.s smaller (by Assumption 2). Rearranging terms yields

\[
Z_{\gamma=1} > -f(X)(n-1)^2 + f(X_{-i})(n-1)(3n-1),
\]

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where the r.h.s. is positive if and only if the condition in Proposition 4.1 holds. Thus, part (ii) of the proposition holds if \( n \geq 3 \).

Similarly, for the case \( n = 2 \), substituting \( n = 2 \), \( f(X) < a - c \) and \( f(X)/5 < f(X_{i}) \) (by the condition in Proposition 4.1) into the expression for \( Z \) yields

\[
Z > \frac{16}{5}(\gamma^2 - 5\gamma + 4)f(X).
\]

The r.h.s. of this expression is greater or equal to zero for any feasible \( \gamma \). Hence, part (ii) of Proposition 4.1 holds also for \( n = 2 \).

**Proof of Corollary 4.1:** Define the function \( h \) as

\[
h(X) = \frac{f(X)}{f'(X)} - 2X.
\]

By Assumption 4.1, \( h(0) = 0 \). Differentiating \( h \) gives

\[
h'(X) = -\frac{f''(X)f(X)}{f'(X)^2} - 1 > 0,
\]

where the inequality follows from Assumption 4.3. Thus, \( h(X) > 0 \) for all \( X \geq 0 \), or equivalently,

\[
\frac{f'(X)}{f(X)} - \frac{1}{2X} < 0.
\]

Integrating this shows that the function

\[
\log(f(X)) - \frac{\log(X)}{2},
\]

or equivalently

\[
\log \left[ \frac{f(X)}{X^{1/2}} \right]
\]

decreases in \( X \). This implies that \( f(\beta X)/f(X) \geq \beta^{1/2} \), for \( \beta \in [0,1] \). Since \( X - (1 - \beta)x_i \geq \beta X \), it holds that

\[
\frac{f(X - (1 - \beta)x_i)}{f(X)} \geq \frac{f(\beta X)}{f(X)} \geq \beta^{1/2}.
\]
Thus, for condition (4.13) to hold it suffices that

$$\beta > \left(\frac{n-1}{3n-1}\right)^2.$$

**Proof of Proposition 4.3:** The first-order condition for R&D competition without licensing was

$$\frac{d\pi(X^CN, X^CN)}{dX^CN} \times \left[1 + (1 - \beta)\frac{\gamma}{2-\gamma}(n-1)\right]$$

$$-g'[X^CN/(1 + \beta(n-1))] = 0. \quad (A4.15)$$

The first-order condition under competition with licensing can be written

$$\frac{d\pi(X^{CL}, X^{CL})}{dX^{CL}} + (n-1)(1-\beta)\pi_1(X, X^{CL})$$

$$-g'(X^{CL}/n) = 0, \quad (A4.16)$$

where \(X \equiv X^{CL} - (1 - \beta)X^{CL}/n \leq X^{CL}\). If \(\beta = 1\), conditions (A4.15) and (A4.16) are identical, so that in this case \(X^{CL} = X^{CN}\).

Suppose next that \(\beta < 1\). Using property (iv) of Lemma 4.1 and equation (4.10),

$$\pi_1(X, X^{CL}) > \frac{\pi_1(X^{CL}, X^{CL})}{d\pi(X^{CL}, X^{CL})} \times \left[1 + \frac{\gamma}{2-\gamma}(n-1)\right].$$

Substituting this into (A4.16) yields

$$\frac{d\pi(X^{CL}, X^{CL})}{dX^{CL}} \times \left\{1 + (n-1)(1 - \beta)\left[1 + \frac{\gamma}{2-\gamma}(n-1)\right]\right\}$$

$$-g'(X^{CL}/n) < 0. \quad (A4.17)$$

By the convexity of \(g\), condition (A4.15) implies

$$\frac{d\pi(X^{CN}, X^{CN})}{dX^{CN}} \times \left[1 + (1 - \beta)\frac{\gamma}{2-\gamma}(n-1)\right]$$

$$-g'[X^{CN}/n] > 0. \quad (A4.18)$$
The expression on the l.h.s. of this condition differs from the one in (A4.17) only by the term multiplying $d\pi/dX$. It can be verified that for $\beta < 1$, this multiplier is greater in (A4.17). Since $d\pi/dX$ is positive and decreasing and $g'$ is increasing in $X$, it follows that $X^{CL} > X^{CN}$.

Proof of Proposition 4.4: Comparing (A4.17) to the first-order condition for an RJV cartel in (4.6) shows by the argument used in the previous proof that $X^{CL} > X^{R}$ if

$$ \left\{ 1 + (n - 1)(1 - \beta) \left[ 1 + \frac{\gamma}{2 - \gamma} (n - 1) \right] \right\} > n. $$

Solving this for $\beta$ gives Proposition 4.4.
Chapters 2–4 of this work have shed light on some specific issues in the relationship between market structure and innovation. They do not attempt to give a complete picture of the subject, nor would that be possible. Various aspects of technology make it very different from a normal economic good and, one is inclined to say, more difficult to model. The shareable and partially excludable nature of technology introduces elements which, although familiar from the economic analysis of ordinary goods, are much more common in the production of technology. Phenomena such as externalities and duplication are an integral part of the economics of innovation. The heterogeneous nature of technology makes the process of innovation even harder to model. Even the basic task of specifying a production function for technology is anything but simple. The list of stylized facts concerning the production of technology is remarkably short.

A complete formal theory of the economics of technological innovation, if created, would not be compact. It would necessarily consist of a large number of cases, each of which describe to a particular type of innovation. As it is, the economics of technological innovation still has a long way to go. The main body of related research has concentrated mainly on a few very specific and stylized cases, such as patent races, which apply to a minuscule subset of actual innovative work. The purpose of this work has been to expand these stylized frameworks in various directions. The fact that each of the three studies included here adopts a very different framework describes the polymorphous nature of innovation; no single framework is adequate in all situations.

The first study (Chapter 2) concentrates on how rivalry affects the riskiness of a firm’s research strategy. The framework is a traditional patent race model, extended to allow firms to choose from a set of research strategies, each of which yields a different probability distribution for the discovery date. The conventional
wisdom seems to be that in such a case, under noncooperative competition, firms tend to choose wider distributions than is socially optimal and thus, the market is biased towards riskiness. It is shown in Chapter 2 that this conclusion is misleading and is based on a confusion of the tendency of firms to overinvest in patent races with the attitude toward risk. When analyzed separately, the bias is against, not towards, riskiness.

Chapter 3 maintains the patent race framework in a modified form. The race now consists of a sequence of two innovations, the first of which can be licensed to competitors. In order to sell licenses to its innovation, a firm must first patent it, which creates externalities. When deciding on whether to patent an innovation or not, a firm weighs the revenue from licensing against the consequent loss of competitive advantage. This framework is used to analyze the incentives of firms to patent their innovations under different market structures. It is shown that the degree of concentration is inversely related to the propensity to patent. The relationship between firm size and patenting depends on the informational assumptions, but in general big firms seem more often to find it profitable to keep their innovations secret and employ them internally whereas small firms have a greater incentive to sell licenses to their innovations and hence more often seek to protect them by patenting. These differences create a wedge between patenting and the actual pace of technological innovation. This has immediate implications on the interpretation of studies that measure R&D output by the number of patents. Such studies have found out that in many industries patenting increases less than proportionally with firm size and interpret this as evidence for decreasing returns to scale in R&D. The analysis of Chapter 3 suggests that this may instead be a consequence of a systematic bias in the data.

Finally, Chapter 4 challenges the view of several recent papers that firms should be allowed to form R&D cartels in order to internalize technological spillovers related to technological innovation. These papers have shown that an R&D cartel leads to both higher R&D as well as higher consumer welfare than noncooperative R&D competition. It is shown in this work that this result
depends partly on the assumption that firms are not able to license their innovations to each other. Removing this restriction introduces another spillover which, unlike technological spillover, tends to increase the R&D investment and consumer surplus. Which of the two spillovers is larger and hence whether the R&D cartel or noncooperative competition with licensing leads to higher R&D investment depends on model parameters. Specifically, it is shown that if the firms produce goods that are close enough substitutes, then the latter leads to faster technological progress.

Each of the studies points to possible directions for future research. The obvious way to go in order to expand the analysis of Chapter 2 is to include both the risk choice and the scale choice in the model and examine the interaction of the two. Also, the way the prize is determined is perhaps overly simple. Imagine, for example, a situation in which the R&D outcome determines the cost of production of a good that is later produced in the Bertrand fashion. In such a setting, the profit of the winner would depend not only on the outcome of its own R&D project, but also on the outcome of the second best firm, which determines the profit margin the winner is able to extract in the product market.

In the second study (Chapter 3), it is assumed that once a firm develops the final innovation, all initial technologies become worthless. This winner-take-all assumption is somewhat unsatisfactory. More realistically, one would expect that by selling the license to a competitor the licensor not only increases that firm’s probability of being the first to reach the final innovation but also enhances its ability to develop a substitute for the final innovation. A possible framework for analyzing this situation might be one in which there is a (possibly infinite) sequence of innovations each of which enables a firm to produce a better quality (or lower cost) product. The question is then under what circumstances will a firm that has gotten one step ahead of its competitors use that advantage to try to stay permanently ahead and when will it license the technology to its competitors. The same dynamic model could be used to analyze the question posed in Chapter 4 as well; that is, what is the optimal competitive arrangement in
such a market. In general, the idea of tradable technology might provide interesting insights within a wide variety of frameworks.

Each of the studies in this work should be seen more as a beginning than as an end. The end is so distant that it would be an overstatement to say that this work brings us any closer to it; an infinity can be approached, but it will always stay equally far away. Still, this work may take us a few steps further from the beginning.
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