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Bruno M. Parigi

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Bank of Finland
Research Unit

PO Box 160
FIN-00101 Helsinki

Phone: +358 9 1831

Email: research@bof.fi
Website: www.bof.fi/en/research

# Systematic Risk, Bank Moral Hazard, and Bailouts *

Marcella Lucchetta, Univ. Venice Michele Moretto, Univ. Padova
Bruno M. Parigi, Univ. Padova and CESifo

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**Abstract**

We show that the impact of government bailouts (liquidity injections) on a representative bank's risk taking depends on the level of systematic risk of its loans portfolio. In a model where bank's output follows a geometric Brownian motion and the government guarantees bank's liabilities, we show first that more generous bailouts may or may not induce banks to take on more risk depending on the level of systematic risk; if systematic risk is high (low), a more generous bailout decreases (increases) bank's risk taking. Second, the optimal liquidity policy itself depends on systematic risk. Third, the relationship between bailouts and bank's risk taking is not monotonic. When systematic risk is low, the optimal liquidity policy is loose and more generous bailouts induce banks to take on more risk. If systematic risk is high and the optimal liquidity policy is tight, less generous bailouts induce banks to take on less risk. However, when high systematic risk makes a very tight liquidity policy optimal, a less generous bailout could increase bank's risk taking. While in this model there is only one representative bank, in an economy with many banks, a higher level of systematic risk could also be a source of systemic risk if a tighter liquidity policy induces correlated risk taking choices by banks.

**JEL Classification**: G00, G20, G21

**Kewords:** bailout, bank closure, real option, systematic risk.

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# 1 Introduction

This paper revisits the relationship between government's bailouts of and bank's risk taking. The bulk of the literature argues that expectations of government bailouts increase banks' risk taking. Demirgüç-Kunt and Detragiache (1998) provide empirical evidence of this relationship. Two papers find evidence that government guarantees increase bank's risk taking in Germany: Dam and Koetter (2012) identify the moral hazard effect of bailout expectations by exploiting regional political factors; Gropp et al (2014) show that the banks that lost government guarantees lowered credit risk by cutting off riskiest borrowers. Similarly, Brandao Marques et al. (2013) in an international sample of rated banks find that government support is associated with more bank's risk taking, especially prior and during the 2008-2009 financial crisis. Among the recent papers see for example Acharya and Mora (2015).

On the theoretical side Farhi and Tirole (2012) argue that banks may find it optimal to take correlated risks if they believe that bailouts are more likely when many of them could fail simultaneously. Cordella, Dell'Ariccia and Marquez (2017) show that when bank capital is endogenous, public guarantees lead unequivocally to an increase in bank leverage and an associated increase in risk taking.

Other papers challenge these conclusions and suggest a complex relationship between prudential policy, the institutional framework governing bank resolution and bailouts, and bank's risk taking. In a theoretical paper Cordella and Yeyati (2003) show that a central bank announcing and committing ex-ante to bailout insolvent institutions, can create a risk-reducing 'value effect' that outweighs the moral hazard component of the policy. Dell'Ariccia and Ratnovski (2014) show that when a bank's success depends on both its effort and the overall stability of the banking system, bailouts that shield banks from contagion may increase their incentives to invest prudently.

In a model where intertemporal consumption risk and asset risk generate both panic runs and fundamental runs, Allen et al. (2014) study how government guarantees affect a bank's risk choice, measured by the amount of liquidity held by the bank. They show that broader government guarantees can be preferred to less generous ones if they lower the probability of both panic and fundamental crises, and that the guarantees do not always induce banks to take excessive risk.

To assess the impact of the expectations of bailouts on bank's risk taking we embed a representative bank's risk choice in a dynamic model where bank's output follows a geometric Brownian motion. The government decision to end liquidity injections when output is low entails the exercise of a real option, whose value depend on the level of systematic risk, which is outside the control of both the government and the bank.

In our model a representative bank, with its own capital and insured deposits, finances an investment (the "project" hereafter) of given size, which represents its loans portfolio. We assume that the project is subject to two sources of risks: the first is the risk that it aborts immediately yielding no output afterwards, a risk that can be lowered by the bank with costly monitoring and screening effort. This captures the *bank's risk taking* (See among others De Nicolò and Lucchetta 2009, and Carletti et al. 2016). The second is the underlying risk of the output once the project starts producing. We call this risk *systematic* because it is not diversifiable, it is outside the control both of the bank and of the government, and may reflect the conditions of the economy as a whole. A higher level of systematic risk could also be a source of systemic risk if the government liquidity policy induces banks to take correlated risk.

If the project starts producing, for an infinite time it yields an output that follows a geometric Brownian motion. If the output exceeds the coupon to pay to the depositors, the bank keeps the difference and consumes it. If the output is insufficient to pay the coupon, the government pays the difference and may close the bank. When it closes the bank, the government fully reimburses the depositors and sells the bank's assets. The bank chooses effort (i.e. risk taking) given the liquidity policy of the government (Lemma 1). The impact of liquidity policy on bank's risk taking depends on the level of systematic risk of the project: a more generous liquidity policy increases risk taking when systematic risk is low, and decreases it when systematic risk is high (Proposition 1). The reason is that the bank anticipates that high systematic risk increases the probability of a bailout, and to take advantage of this, it puts more effort to increase the chances of being in business to enjoy it. This effect, which is at the heart of our model, arises because we cast the bank's effort choice in a dynamic model where the government has the option to decide when to stop injecting liquidity and close an illiquid bank.

In a dynamic model the commitment of the government to guarantee deposits can be interpreted as a call-like option of the bank to obtain liquidity if needed. We measure the value of this option using a real option framework. The value of the option feedbacks into the bank's effort, and alters how the bank chooses to hedge the systematic risk with its own effort. However, hedging through effort is costly and the bank hedges more the systematic risk when the option value is higher. In fact, the longer the government allows the bank to stay in business (i.e. the looser is the liquidity policy), the higher the bank profits. When systematic risk is low, the probability of a bailout is low and thus the advantage that the bank obtains from a bailout is low; hence the incentive to put effort to enjoy future profits is small, and the standard effect highlighted by the literature that argues that bailouts induce risk taking, arises.

Our model has two frictions. First, bank closure entails a deadweight loss (e.g. fire sale of assets, negative externality). Second, the cost of liquidity injection to keep a bank open, increases with the amount of liquidity already injected. Since the two frictions work in opposite directions, the government determines its liquidity policy by trading off the deadweight loss and the liquidity costs. We compute how the government sets the optimal liquidity policy under full info (FI) (i.e. when the government chooses bank's effort) and under moral hazard (MH) (i.e. when the bank chooses its effort). We show that even under FI, and commitment to liquidity policy, it is optimal for the government to delay bank closure when the output falls short of the coupon; that is it is optimal to bailout an illiquid bank for a while (Proposition 2). Importantly, the optimal liquidity policy depends on the level of systematic risk. In particular, under FI the optimal liquidity policy becomes tighter as systematic risk increases (Proposition 3). However, for a given liquidity policy, when systematic risk is high the bank puts more effort under MH than under FI, and vice-versa when systematic risk is low (Proposition 4). The reason follows from Proposition 1: when the bank can choose its effort (i.e. under MH) a given liquidity policy is more valuable to the bank when systematic risk is high, and hence the bank puts more effort to enjoy this liquidity injection.

More generally, there is a positive (more effort than under FI) or negative misalignment between the effort chosen by the bank and that under FI. To lower this distortion, the government manipulates the incentive of the bank to put effort, and it does so by modifying its liquidity policy w.r.t. FI. Under MH when systematic risk is high the government tightens liquidity policy w.r.t. FI (Proposition 5). Liquidity policy, in turn, feedbacks into bank effort (Proposition 7): when systematic risk is low, the optimal liquidity policy is laxer under MH than FI, which induces the bank to put less effort than socially desirable. When both systematic risk is high and the optimal liquidity policy is tighter under MH than FI, the bank puts more effort than socially desirable, despite the restrictive policy; this is so because the bank prefers to bear the effort cost of hedging the systematic risk. If instead the systematic risk is high but the government adopts and even stricter liquidity policy w.r.t. FI, the incentive of the bank to hedge the systematic risk via effort could be reverted; the bank could be discouraged to exert high effort because the chances of receiving liquidity assistance are so low that it could choose a lower level of effort under moral hazard than socially desirable.

### 1.1 Related literature

Our paper is linked to several strands of literature.

A number of papers argue that the government may not be able to commit to a bailout strategy. Mailath and Mester (1994) investigate whether the threat to close a bank that has chosen risky assets is ex post credible. Absence commitment, for certain parameter values, this threat is not credible, for once the bank has selected risky assets it will not be in the best interest of the government to close it. Indeed under some parameter restrictions a bank that has chosen a risky asset in the first period is less likely to choose a risky asset in the second period. That is, a government may forbear by not closing an insolvent bank. Acharya and Yorulmazer (2007, 2008) argue that ex ante regulators would like to be tough to prevent excessive risk taking. However, during systemic crises the costs associated with not providing assistance can be so high that regulators may feel compelled to provide assistance. Bailing out banks at taxpayer's expenses as opposed to prompt corrective actions (i.e. early liquidation of collateral that may lose value over time) may also be socially optimal when the probability that the collateral loses value is low (Kocherlakota and Shim 2007). Shim (2011) shows that a combination of a risk-based deposit insurance premium and a book-value capital regulation with stochastic liquidation can implement a regulation akin to the Prompt Corrective Action in the USA. Morrison and White (2013) show that a regulator may prefer not to close a unsound bank because of the fear of inducing contagion. The action of promptly closing a weak bank reveals that the regulator has less skill in screening banks than previously expected. This revelation reduces confidence in other banks screened by the same regulator, and, in some circumstances, triggers financial contagion and the closure of these banks, even though their intermediation remains socially valuable.

Several studies have focused on the bailout strategies followed by regulators with different objective functions. In an empirical study on US commercial banks Agarwal et al. (2014) compare state and federal regulatory interventions. Exploiting an exogenous rotation between state and federal in-site bank inspections, they find a larger propensity to forbear of the state regulators rather than of the federal ones. Carletti, Dell'Ariccia, Marquez (2016) consider the different incentive to close insolvent banks faced by a National and Supranational regulator, in an architecture similar to that envisioned by the BRRD regulation in the eurozone. The Supranational regulator needs the local expertise of the National (local) regulator, which, however, may have less incentives to close an insolvent bank because of the local political cost. This reduces the incentive of the National regulator to acquire information about the bank that may be used by the Supranational regulator to take actions that the National regulator may dislike.

The rest of the paper is organized as follows: In Section 2 we introduce the model. In Section 3 we consider the bank's choice of risk. In Section 4 we study how the government chooses effort and liquidity policy under FI of bank effort. In Section 5 we study the problem that the government faces when he has to choose the liquidity policy without observing bank's risk taking and we compare the resulting liquidity policy and bank's risk taking with the FI case. The proofs are in Appendix.

## 2 Model set up

### 2.1 Output and effort

The economy consists of depositors, a representative bank, and a government, all risk neutral. At $t = 0$ the bank with its wealth $k$ and deposits $D$ makes loans. For short we will denote the loan portfolio as a single project. We normalize the size (cost) of the project to 1 so that $k + D = 1$. The bank is run by a shareholder-manager, that we will denote as the bank for short. Time is continuous and infinite. At $t = 0$ the bank puts unobservable effort $q$, which increases the probability that the project succeeds, e.g. because the bank screens projects more accurately ex ante. Once this action is undertaken it cannot be modified, that is the bank cannot change $q$ over time. Effort entails a disutility $\frac{a}{2}q^2$, $a > 0$, for the bank.[1]

If the project succeeds, in each instant $t$ it generates an output, a cash flow stream $v_t$, where $v_t$ evolves as a geometric Brownian motion:

$$dv_t/v_t = (\mu - \phi\sigma)dt + \sigma d\omega_t \text{ with } v_{t=0} = v_0 > 0, \tag{1}$$

and $\mu > 0$ and $\sigma > 0$ are the constant expected growth rate and standard deviation of the cash flow, respectively, $\phi > 0$ is the market price of risk associated with the project, and $\omega_t$ is a standard Wiener process under a risk-neutral measure.[2] We also assume that the bank is not able to affect the realizations of cash flow stream (1). If we interpret the investment as a portfolio of loans, this assumption captures a situation where after the investment is made, the bank no longer controls it. It describes well its banking book but not its trading book where the bank has more possibilities to modify its positions over time.

To simplify the notation, in what follows we set the risk-neutral drift for the diffusion in (1) equal to zero. By using the single-beta version of the Capital Asset Pricing Model (Merton 1973), this assumption implies a project's expected rate of return equal to $r+\mu$ where $r > 0$ is the constant risk-free interest rate (See Appendix 1). That is, on average the project pays a risk premium equal to $\mu$.

With complementary probability $1 - q$ the project does not succeed, that is it generates zero output forever. Therefore it is convenient to interpret a successful project, one that starts and generates an output stream in continuous time.

The cash flow is the sole state variable and we assume that it is observable by both the government and the bank. However, there is an important contracting friction in this economy: namely we assume that the bank cannot store the cash flow. This implies that over time the bank cannot build reserves to offset output short fall. Thus for a given project size 1 the only buffer is the initial capital $k$. The assumption that the bank cannot store output is not new in the banking literature. Parlour et al. (2012) assume that dividends must be consumed immediately and cannot be invested to become new capital. In a model with dynamic interactions between a banker and a regulator Shin (2011) assumes that output is either consumed by the banker or paid to the Deposit Insurance Fund. This assumption is also linked to the observation of Rajan and Myers (1998) that liquid reserves could be easily wasted or subject to absconding. For the same reason we assume that the bank keeps no liquidity at t=0, that is $k + D = 1$.

To recap there are two sources of risks: $1 - q$ is the risk that the project does not start and it can be lowered by bank effort; $\sigma$ is the underlying risk of the output once the project starts; we call this risk systematic because it is outside the control of the bank and may reflect the conditions of the economy as a whole. If the project starts, depositors receive a promised (per unit of deposit) return, a coupon $r(1 - k)$ per unit of time. The deposit market is perfectly competitive so that the bank will set $r(1 - k)$ at the level the depositors require to recover their opportunity cost of funds and to be willing to participate. Since output cannot be stored, the bank does not hold any liquidity buffer, that is when $v_t - r(1 - k) > 0$ the bank consumes it immediately.

[1] This is a standard way to capture how bank's effort affects the risk of its loan portfolio in a static setting. See for example De Nicolò and Lucchetta (2009) and Carletti et al. (2016).

[2] The process (1) is quite standard in the Real Options literature (see e.g. McDonald and Siegel, 1984). However, for the convenience of the reader, we provide a detailed derivation in Appendix 1. We remind the reader also that a world where the expected growth rate is set equal to $(\mu - \phi\sigma)$ is referred to as a "*risk-neutral*" world (see e.g. Cox and Ross, 1976; Constantinides, 1978; Harrison and Kreps, 1979).

## 2.2 Liquidity injection and probability of bank closure

We assume that the government acts on behalf of society, fully guarantees both the stock of deposits and their coupons, and provides liquidity to the bank until it closes it. Thus the government performs the functions of different agencies: bank supervisor and regulator, central bank, and deposit insurance fund. We assume away agency problems between any of these institutions and society, to focus only on the agency problem between the government and the bank.

The rationale for guaranteeing deposits is that they must be risk free to perform their function of payment instruments. As the bank does not hold a liquidity buffer, when the output falls below the coupon $r(1-k)$, the bank is illiquid and the only way to keep it open is for the government to inject liquidity to pay the coupon. Examples of liquidity injections include collateralized lending by the central bank, revolving credit lines, publicly-funded recapitalizations, government guarantees for new debt. As long as it injects liquidity the government bears the per period bank losses, $v_t - r(1-k) < 0$. As we will see later this entails a distortion.

Since deposits are fully insured, the stochastic process that we consider enables us to treat liquidity injection as a buffered stochastic flow**.** More specifically, the process $v_t$ has a lower barrier and the government wants to prevent the stochastic variable from falling below that barrier. In our model the lower barrier is the coupon $r\,(1-k)$. Accordingly, the government intervenes by means of instantaneous, infinitesimal liquidity injections never allowing $v_t$ to go below the threshold $r(1-k)$. The process $v_t$ is free to move as dictated by (1) as long as $v_t > r(1-k)$, but the instant $v_t$ crosses $r(1-k)$ from above, it is reflected at $r(1-k)$.[3] Letting $\tilde{v}_t$ be a version of the process $v_t$ reflected at $r(1-k)$ it can be defined as $\tilde{v}_t = v_t + L_t$, where $L_t$ indicates the process that tracks the cumulative amount of liquidity injected up to $t$. The process $L_t$ is non-decreasing and continuous with $L_0 = 0$, and increases only when $v_t = r(1-k)$. The bailout of the bank takes the form of these liquidity injections.[4]

To determine the optimal liquidity policy $\psi$ we introduce the time $\tau$ at which the government stops injecting liquidity. The government has discretion to stop injecting liquidity at any time which entails the closure of the bank. Formally, we define the closing time as:

$$\tau = \inf\left(t \geq 0 : \psi L_t = l\right). \tag{2}$$

By (2) the government closes the bank the first time that the process $\psi L_t$ hits the threshold $l$, where $\psi \in [0, \infty)$ is the weight that the government assigns to the cumulated liquidity injected up to date $t$. Now, assuming that the trigger $l$ is a random variable described by an exponential distribution independent of (1), we are able to calculate the probability of bank closure as:[5]

$$\Pr(\tau \leq t) = \Pr\left(\psi L_t > l\right) = 1 - e^{-\psi L_t}. \tag{3}$$

The process $\psi L_t$ depends on the amount liquidity already injected $L_t$ and the importance (the weight), $\psi$, that the government assigns to $L_t$. In other words, the probability that the government continues to inject liquidity after time $t$, declines the more liquidity $L_t$ it has already injected. If the government chooses $\psi = 0$, it guarantees liquidity forever and the bank will never be closed, i.e. $\Pr(\tau \leq t) = 0$. On the contrary, setting $\psi = \infty$ means that the government never injects liquidity

[3] A reflected process has the same dynamics as the original process but is required to stay above a given barrier whenever the original process tends to fall below it. See Harrison (2013) for a formal definition of these processes.

[4] Formally, we define the liquidity injection as the positive increment $dv_t$ to let $v_t$ stay at $r(1-k)$. That is: $\tilde{v}_t \equiv v_t U_t$, for $\tilde{v}_t \in [r(1-k), \infty)$, where: *i)* $v_t$ is given by (1); *ii)* $U_t$ is a non-decreasing continuous process, with $U_0 = 1$ if $v_0 \geq r(1-k)$, and $U_0 = r(1-k)/v_0$ if $v_0 < r(1-k)$, so that $\tilde{v}_0 = r(1-k)$; *iii)* $U_t$ increases only when $\tilde{v}_t = r(1-k)$. Then, the amount of liquidity up to $t$ can be represened as $\tilde{v}_t - v_t \equiv L_t \equiv (U_t - 1)v_t$.

[5] For the stopping time (2) see Harrison (2013, p. 159-160).

and closes the bank the first time that $v_t$ hits the boundary $r(1-k)$. This captures the notion that the government updates the probability of closing the bank as a function of both the liquidity injected and the state of the economy $v_t$.

We assume that the government announces its liquidity policy $\psi$ at $t=0$ before the bank chooses effort $q$, and commits to it.[6] Similarly, in Shim (2011) stochastic liquidation after output shortfall provides the banker with incentives to continue to act in the interest of the regulator.

When the government closes the bank, it reimburses depositors in full and cancels the shareholders's claims. When the bank is closed the government does not terminate the project, rather it sells the bank's assets, and receives the proceeds from the assets sale.

We also assume that the government incurs a deadweight loss $Z>0$, from closing a bank and that this cost, which is not internalized by the bank, is independent from the time of closure. This cost arises from several sources. First, a fire sale discount makes the resale value of the assets smaller than the expected present value of their output stream (Leland 1994). This is so because, for example, the incumbent bank management is more capable than anybody else to extract value from these assets (See for example Diamond and Rajan 2006), or because outsiders can observe the output only at a cost (Townsend 1979). Second, bank's closure generates a negative externality as it may induce financial instability by casting a doubt on the ability of the bank supervisor to screen other banks as in Morrison and White (2013), and there may be costs to layoff the bank employees.[7]

# 3 The bank's risk choice

The bank chooses the risk it takes, $1-q$, given the liquidity policy. Indicating with $R$ the present expected value of the project once it started, the problem that the bank faces at $t=0$ is:

$$V=\max_q qR+(1-q)(1-k)-1-\frac{a}{2}q^2. \quad (4)$$

Observe that (4) corresponds to a standard static objective function of the bank as in De Nicolò and Lucchetta (2009) and Carletti et al. (2016). In a dynamic model the present expected value of the project once it started is:

$$R=\mathbb{E}[\int_0^\tau e^{-rt}(\tilde{v}_t-r\,(1-k))dt+e^{-r\tau}\,(1-k)], \quad (5)$$

where $\mathbb{E}[\cdot]$ denotes the expectation of (1) conditional on the information at $t=0$, and $\tilde{v}_t$ is the reflected process. The interpretation of equation (4) is straightforward: At $t=0$ with probability (i.e. effort) $q$ the project starts and the bank receives a cash flow net of a coupon $\tilde{v}_t-r\,(1-k)$ until the government closes it. At closing time $\tau$, the government reimburses the current value of deposits $1-k$, which in present value terms is $e^{-r\tau}\,(1-k)$. With probability $1-q$ the project fails to start and at $t=0$ and the government reimburses the value of deposits $1-k$.

By simple algebra and using (2) and (5), we are able to write (4) as:

$$V=\max_q q\left(\mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}\left(\tilde{v}_t-2r\,(1-k)\right)dt\right)-\frac{a}{2}q^2-k. \quad (6)$$

[6] Similarly to Dell'Ariccia and Ratnovski (2014) we assume that the government can commit to a bailout strategy.

[7] More formally, when the bank is closed, the realized assets will revert to the government. The value of these assets is $(1-\xi)\frac{v_\tau}{r}$ where $\xi\frac{v_\tau}{r}$ $(\xi\in[0,1))$ measures a fire sale cost (Leland 1994). As at the closure time $\tau$ the cash flow is $v_\tau=r(1-k)$, the salvage value is $(1-\xi)\,(1-k)$. Thus the deadweight $Z$ loss is equal to $S-(1-\xi)\,(1-k)>0$ where $S$ is the closure cost, for example from staff layoff.

Formally, in (6) we can regard the project a infinitely-lived one, where the payoffs at each time are multiplied by the probability of bank closure that depends on the amount of liquidity injected up to $t$.[8] Thus, assuming that at $t = 0$ the project is viable, i.e. $v_0 > r(1-k)$, the bank chooses an effort level equal to:[9]

$$\begin{aligned} aq^B &= \mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}(\tilde{v}_t - 2r(1-k))dt \\ &= \frac{v_0 - 2r(1-k)}{r} + \underbrace{\mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}L_t dt}_{M}. \end{aligned} \tag{7}$$

Exploiting the properties of the reflecting process $\tilde{v}_t$, the second term on the R.H.S. of (7) measures the expected present value of total liquidity that the government injects. Since this term is positive, it increases bank effort. Substituting (7) into (6) it is easy to see that:

$$V = \frac{a}{2}(q^B)^2 - k,$$

so that maximizing $V$ is equivalent to maximize (7).

To compute the discounted expectation in the second term on the R.H.S (7) we use the dynamic programming decomposition. We may split the above conditional expectation into the contribution over the infinitesimal time interval 0 to $dt$ and the integral from $dt$ to $\infty$ with a particular condition at the (reflecting) barrier $r(1-k)$. The solution of (7) is given by the following Lemma.

**Lemma 1**: *The probability of success* (7) *is equal to:*

$$aq^B = \frac{v_0 - 2r(1-k)}{r} - \left(\frac{v_0}{r(1-k)}\right)^\beta (1-k)\frac{(1+\psi r(1-k))}{\beta - \psi r(1-k)}, \tag{8}$$

*where* $\beta = \frac{1}{2} - \sqrt{(\frac{1}{2})^2 + \frac{2r}{\sigma^2}} < 0$, *and* $\frac{\partial \beta}{\partial \sigma} > 0$.

**Proof:** See Appendix 2.

Lemma 1 addresses the incentive compatibility problem of the bank, that is how the bank chooses effort as a function of liquidity policy. By direct inspection of (7) and (8), denoting with $M$ the expected present value of the total liquidity supplied by the government this is equal to:

$$M = \mathbb{E}\left(\int_0^\infty e^{-rt}e^{-\psi L_t}L_t dt\right) = -\left(\frac{v_0}{r(1-k)}\right)^\beta (1-k)\frac{(1+\psi r(1-k))}{\underbrace{\beta - \psi r(1-k)}_{<0}} > 0, \tag{9}$$

which depends on the systematic risk $\sigma$, and the government's liquidity policy through the stopping rate $\psi$. In particular, since $\beta$, which recall is negative, is monotonically increasing in $\sigma$, the level of systematic risk may be equivalently characterized in terms of $\beta = \beta(\sigma)$. For the rest of the paper we say that a project is "high risk" when $\mid \beta \mid$ is very low, i.e. close to 0, and "low risk" when $\mid \beta \mid$ is very high. In both cases, however, unless otherwise specified, we exclude the extreme values

[8] Notice that the effect of (2) is similar to the case of calculating the value of an investment opportunity with an uncertain expiration date. If the expiration date is described by a Poisson process with parameter $\psi$, Merton (1971) shows that the investment opportunity is equal to a perpetual one with the discount rate subsituted by $r + \psi$.

[9] When it is not necessary, for the rest of the paper we drop the dependence of $q^B$ form the iniatial condition $v_0$

$\beta = -\infty$, where the cash flow becomes constant over time, and $\beta = 0$, where the cash flow volatility is infinite.[10]

By the real option theory (McDonald and Siegel 1984; Dixit and Pindyck 1994), equation (9) can be seen as a sum of infinite set of call-like options. At each time $t$ the bank has the option to use a unit of liquidity for free to prevent $v_t$ to fall below $r(1-k)$. Then, the present value of the total liquidity (9) can be calculated by valuing each of these options and summing these values by integrating over time $t$. In particular, it can be proved that $\left(\frac{v_0}{r(1-k)}\right)^{\beta} = \mathbb{E}(e^{-rT}) < 1$ where $T$ is the first time starting from $v_0$ that the process (1) hits the liquidity threshold $r(1-k)$, that is the first moment that the bank is illiquid. Thus the term

$$-(1-k)\frac{(1+\psi r(1-k))}{\beta - \psi r(1-k)}$$

indicates the payoff the bank expects to receive from exercising these options.[11] Keeping this in mind, we are able to write (8) as:

$$aq^B = \frac{v_0 - r(1-k)}{r} - (1-k)\left[\mathbb{E}(e^{-rT})\frac{(1+\psi r(1-k))}{\beta - \psi r(1-k)} + 1\right], \tag{10}$$

where the second term on the R.H.S. of (10) shows the contribution of the options to call for liquidity to increase the bank's effort above the value of insured deposits. Furthermore, as the value of these options increases with $\sigma$, the second term is higher when the project is high risk. That is, for a high risk project both the payoff of the options and the probability that they will be exercised increases. On the contrary they tend to $-(1-k)$ for low risk projects.

The constant $a$ in (8) serves as a normalization, for any given initial value $v_0$. Note, however, that if the initial valuation of the project is high, i.e. $v_0$ far exceeds the boundary $r(1-k)$, it could be always worth for the bank to exert the maximum level of effort, i.e. $q^B \to 1$. On the contrary if the initial valuation of the project is low, i.e. $v_0$ is close to the boundary $r(1-k)$, we obtain:

$$\lim_{v_0 \to r(1-k)} aq^B = -(1-k)\left[\frac{\beta+1}{\beta - \psi r(1-k)}\right],$$

and the probability of success is greater than zero only if $\beta + 1 > 0$. That is, if the initial condition on the cash flow is such that the project starts with low cash flow if it starts at all, the bank has an incentive to put effort only if the project volatility is "sufficiently" high to guarantee that the expected payoff from exercising the call-like options is positive.

Taking the derivative of (8) with respect to $\psi$, we are able to investigate the effect of the government's liquidity policy on the bank level of effort. In particular we prove the following proposition:

**Proposition 1.** *The sign of the effect on $q^B$ of the government's liquidity policy, is given by:*

$$sign\frac{\partial q^B}{\partial \psi} = sign[-r(1-k)(\beta+1)] \quad \text{for } \beta \in (-\infty, 0) . \tag{11}$$

[10] In particular the former case is of no interest for the aim of this paper. If $\beta = -\infty$ the model collapses to a static one without uncertainty as in Dell'Ariccia and Ratnovski (2014) and Carletti et al. (2016). The per unit opportunity cost of capital is $r$ with no differential cost between equity and debt financing. Provided that $v_0 - r(1-k) > 0$, the value of the project once it started $R$ is given by $\frac{v_0 - r(1-k)}{r}$ and the regulator is prefectly indifferent as to what policy to adopt. In this sense the liquidity policy is neutral w.r.t. the bank decision, as the liquidity policy does not affect bank choice.

[11] The expected present value $\mathbb{E}(e^{-rT})$ can be determined by using dynamic programming (see for example Dixit and Pindyck, 1994, pp. 315-316).

**Proof:** See Appendix 3.

Proposition 1 is crucial to understanding why liquidity injections may or may not induce banks to behave in a safer way depending on systematic risk. To understand the $sign[-r(1-k)(\beta+1)]$, recall that $\beta < 0$ and $\frac{\partial \beta}{\partial \sigma} > 0$. Thus, if the project is high risk, i.e. $\mid \beta \mid$ is close to zero, by (11) we have $\frac{\partial q^B}{\partial \psi} < 0$, that is the bank will increase effort if the government injects more liquidity. Conversely, if the project is low risk, i.e. $\mid \beta \mid$ is very high, $\frac{\partial q^B}{\partial \psi} > 0$, that is the bank will reduce effort if the government relaxes the liquidity policy.

Intuitively, the reason a loser liquidity policy increases the incentive of the bank to put effort when the project is high risk, is that the bank anticipates that a bailout is more likely when systematic risk is high, and that to take advantage of that it has to put more effort to increase the chances that the project starts. This effect arises because we consider a dynamic model where the longer the government allows the bank to stay in business, the higher is the expected value of the call-like options to obtain liquidity.

When instead the project is low risk, the probability of a bailout is low and thus the advantage that the bank obtains from a bailout is low. Indeed, in this case, the loser is the liquidity policy the lower the incentive to put effort to enjoy future profits. Therefore both the received view that argues that expectations of generous bailouts induce bank's risk taking, and the studies that challenge this conclusion, capture only a piece of the story, because they do not take into account how liquidity policy itself is optimally set. We now turn to determine the optimal liquidity policy as a function of the systematic risk. We do so first when the government has full information on bank effort, and then when the bank is free to set its own effort.

# 4 Full information regulation

## 4.1 How government chooses effort

When the government has full information on the effort of the bank, perhaps because of on-site inspections, it chooses both the level of effort and the liquidity policy to maximize its objective function which includes both the objective function of the bank, $V$, and the expected costs of the intervention.

Besides the reimbursement of deposits, which has no impact on welfare, the costs of intervention arise from two frictions with welfare implications: the deadweight cost of bank closure $Z$ discussed above, and the increased risk of the government's portfolio from liquidity injection. As for the latter, we assume that the cost that the government faces to inject liquidity grows with the amount of liquidity already injected. This reflects the deterioration of the portfolio of the authority providing liquidity to the bank in distress. A bank in distress may lose market access, and increase the use of central bank credit. Since the eligible collateral that central banks accept from banks in distress tends to be both of lower quality and more illiquid (Acharya et al. 2009, Nyborg 2015), as central bank lending becomes more concentrated on weaker counterparties, the average quality of the risks in the central bank's portfolio worsens (Bindseil and Jabłecki 2013). More generally, Hall and Reis (2015) argue that the unconventional monetary policies followed by the FED and the ECB after the crisis have exposed them to increased interest rate and default risks.

Recall that the government insures both the deposits and the coupons, and that the process $v_t$ is reflected at the lower barrier $r(1-k)$ by liquidity injection. This reflection is costly: in particular to model the cost of liquidity injection we assume that the government bears a cost of $c(\psi)$ units for each unit of liquidity injected. That is to say, liquidity supply has a time-invariant marginal cost $c(\psi)$ per unit of account. Formally this is equivalent to set:[12]

$$dC_t = c(\psi) \times dL_t, \tag{12}$$

where $dL_t$ is the increment of liquidity, if any, in the interval $(t, t+dt)$. We assume that the cost of liquidity is a decreasing function of $\psi$ with the properties $c'(\psi) < 0$, and $c''(\psi) < 0$. That is, the cost that the government bears for providing liquidity declines at a declining rate, if it commits to close a bank sooner. Furthermore we assume that $c(\psi) \geq 1/r$ and that there is no cost over the market rate $1/r$ only if the government closes the bank the first time that the output falls short of the coupon; i.e. $c(\infty) = 1/r$.

As in the previous section, indicating with $\tilde{v}_t$ a version of the process $v_t$ reflected at $r(1-k)$, the government's objective function is:

$$\begin{aligned}
W = & \underbrace{\max_q q\left[\mathbb{E}[\int_0^\tau e^{-rt}(\tilde{v}_t - r(1-k))dt + e^{-r\tau}(1-k)]\right] + (1-q)(1-k) - \frac{a}{2}q^2 - 1}_{V} \\
& - \left\{ q\left[\mathbb{E}[\int_0^\tau e^{-rt}dC_t + e^{-r\tau}((1-k)+Z)]\right] + (1-q)(1-k)\right\} \\
= & \max_q V - q\left[\mathbb{E}[\int_0^\tau e^{-rt}dC_t + e^{-r\tau}((1-k)+Z)]\right] - (1-q)(1-k).
\end{aligned} \tag{13}$$

Using the stopping time (2) and going through the same steps as before, equation (13) can be reduced to:

$$W = \max_q q\left[\mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}[(\tilde{v}_t - r(1-k))dt - dC_t] - Z\mathbb{E}\int_0^\infty e^{-rt}\psi e^{-\psi L_t}dL_t\right] - \frac{a}{2}q^2 - 1, \tag{14}$$

where $Z\mathbb{E}\left[\int_0^\infty e^{-rt}\psi e^{-\psi L_t}dL_t\right] = Z\mathbb{E}(e^{-r\tau})$ is the discounted expected value of the cost to close the bank. Thus, the government chooses a probability of success/effort equal to:

$$\begin{aligned}
aq^W &= \mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}[(\tilde{v}_t - r(1-k))dt - dC_t] - Z\mathbb{E}\int_0^\infty e^{-rt}\psi e^{-\psi L_t}dL_t \\
&= \frac{v_0 - r(1-k)}{r} + \underbrace{\mathbb{E}\int_0^\infty e^{-rt}e^{-\psi L_t}[L_t dt - (c(\psi) + \psi Z)dL_t]}_{H},
\end{aligned} \tag{15}$$

where the second term on the R.H.S. of (15), denoted by $H$, measures the expected net present value of the regulatory intervention, i.e. the difference between the present value of the liquidity injections $M$ and the expected value of additional payments, where the latter is given by the additional costs $dC_t$, minus the expected value of the closure cost $Z$. Thus, defining with $P \equiv E\int_0^\infty e^{-rt}e^{-\psi L_t}(c(\psi) + \psi Z)dL_t$ the expected value of additional payments, we obtain $H = M - P$.

Substituting (15) into (14) it is easy to see that:

$$W = \frac{a}{2}(q^W)^2 - k, \tag{16}$$

so that also for the government, maximizing $W$ is equivalent to maximize (15).

To compute the discounted expectation in (15) we repeat the arbitrage calculation of section 3. The solution of (15) is in the following Lemma.

[12] We express payments in present value terms, i.e. the liquidity cost has the dimension of the present value of one unit of account injected in the bank forever.

**Lemma 2:** *The probability of success under full information is given by:*

$$aq^W = \frac{v_0 - r(1-k)}{r} + \overbrace{\left(\frac{v_0}{r(1-k)}\right)^{\beta}(1-k)\underbrace{\frac{c(\psi)r - 1 + \psi r Z}{\beta - \psi r(1-k)}}_{<0}}^{H<0}. \quad (17)$$

**Proof:** See Appendix 4**.**

By direct inspection of (15) and (17), the expected net present value of the regulatory intervention $H$ is negative which induces the government to lower effort in (17). Also for the government, similarly to what happens for the bank, when the initial valuation of the project is high it is worth demanding high effort, i.e. $q^W = 1$. On the contrary, if $v_0$ is close to $r(1-k)$, we obtain:

$$\lim_{v_0 \to r(1-k)} aq^W = \frac{(1-k)[c(\psi)r - 1 + \psi r Z]}{\beta - \psi r(1-k)} < 0.$$

That is, if the initial condition on the cash flow is such that if the project starts, it will start with low cash flow, it will be never optimal for the government that the bank puts effort i.e. $q^W = 0$. Even if, at time zero, the expected cash flows are sufficient to cover the coupons, the government prefers that the project does not start producing.

## 4.2 Optimal liquidity policy

Having this in mind we can now derive the government's optimal liquidity policy under full information. Taking the derivative of (17) with respect to $\psi$, and collecting the results we obtain:

**Proposition 2:** *Under full information, there exists an optimal liquidity policy* $\psi$, *such that* $0 < \psi < \infty$, *given by:*

$$c'(\psi^W)(\beta - \psi^W r(1-k)) + (c(\psi^W)r - 1)(1-k) = -Z\beta. \quad (18)$$

**Proof:** See Appendix 5.

Several comments are in order. First, the result that the optimal $\psi^W$ is finite, means that the government chooses not to close a bank immediately after an output shortfall, that is forbearance is optimal under FI. This shows that lack of commitment is not necessary to establish that the government may find it optimal not to close an insolvent bank. Second, the L.H.S. of (18) is the net marginal benefit of tightening the liquidity policy, which is strictly positive.[13] The first term on the L.H.S. is the benefit from the lower cost to acquire liquidity with respect to the market discount rate $1/r$. The second term on L.H.S. is the insurance cost to guarantee deposits. The R.H.S. reflects the deadweight cost associated with bank closure. Third, to better grasp the role of the assumption on $c(\psi)$, it is useful to see what the optimal liquidity policy would be if the liquidity supply had instead a constant marginal cost equal to the market cost, $1/r$. By direct inspection of (17) and recalling that $\beta < 0$, one can see that the sign of $\frac{\partial q^W}{\partial \psi}$ is always negative. Thus, the government would maximize the welfare function (14) by providing liquidity forever, $\psi^W = 0$. The intuition is evident from (17). Under FI, the government faces a trade-off between the cost of intervention and the level of bank effort to increase the probability that the project starts producing. However, if

[13] Taking the derivative of the L.H.S. of (18), we get: $c''(\beta - \psi^W r(1-k)) > 0$.

the government's cost of liquidity is equal to the market cost, for any given level of systematic risk, the government is able to reduce the cost of the regulatory intervention by supporting the project forever.

As for the effect of systematic risk on bank's risk taking and optimal liquidity policy we are able to prove that:

**Proposition 3:** *Under full information, both the effort $q^W$ and the the stopping rate $\psi^W$ decrease as systematic risk increases:*

$$\frac{\partial q^W}{\partial \sigma} < 0 \quad \text{and} \quad \frac{\partial \psi^W}{\partial \sigma} < 0. \tag{19}$$

**Proof:** See Appendix 5.

The intuition for $\frac{\partial q^W}{\partial \sigma} < 0$ is straightforward. A higher level of systematic risk increases the probability that the government has to inject liquidity into the bank which, ceteris paribus, increases the cost of liquidity and makes $q^W$ (and $W$) lower.

The explanation of $\frac{\partial \psi^W}{\partial \sigma} < 0$ is less intuitive. If we interpret $\frac{\partial q^W}{\partial \psi}$ as the marginal productivity of a restrictive liquidity policy, that is how a unit reduction of liquidity is able to affect the probability that the project starts, then an increase in systematic risk lowers the marginal productivity of a restrictive liquidity policy (i.e. $\frac{\partial^2 q^W}{\partial \psi \partial \sigma}(\psi(\sigma), \sigma) < 0$; see Appendix 5). Therefore, when the projects are riskier, if the government wants that the productivity of liquidity increases, it has to increase the liquidity, i.e. $\frac{\partial \psi^W}{\partial \sigma} < 0$. To sum up, as systematic risk increases, the government reduces the probability of getting the pojects off, $\frac{\partial q^W}{\partial \sigma} < 0$, but it increases the liquidity to keep them alive once they start producing, $\frac{\partial \psi^W}{\partial \sigma} < 0$.

# 5 Moral hazard

## 5.1 The government problem

In the previous section we assume that the government can perfectly observe the bank effort and thus it can dictate its desired level of effort, $q^W$. Differently, in this section we take into account that the government cannot observe the effort that the bank chooses, $q^B$, and we explore how this affects the trade-off between bank's effort and the liquidity policy. The government problem becomes:

$$W^h = \max_{\tau} q^B \left[ \mathbb{E} \int_0^{\tau} e^{-rt} \left[ (\tilde{v}_t - r(1-k))dt - dC_t \right] - \mathbb{E}(e^{-r\tau})Z \right] - \frac{a}{2}(q^B)^2 - 1, \tag{20}$$

subject to the incentive compatibility constraint of bank's risk taking

$$aq^B = \mathbb{E} \int_0^{\tau} e^{-rt}(v_t - 2r(1-k))dt, \tag{21}$$

and the bank's participation constraint, $V \geq 0$.

Constraint (21) indicates that the bank chooses effort to maximize its payoff taking as given the liquidity policy. Substituting (21) in (20) and rearranging, we obtain:

$$W^h = \max_{\tau} \frac{a}{2}(q^B)^2 - 1 + q^B \left[ \mathbb{E} \int_0^{\tau} e^{-rt}[r(1-k)dt - dC_t] - \mathbb{E}(e^{-r\tau})Z \right]. \tag{22}$$

By using the stopping time (2) and going through the same steps as in the previous sections, we can write the last term in (22) as:

$$\mathbb{E}\int_0^{\tau} e^{-rt}[r(1-k)dt - dC_t] - Z\mathbb{E}(e^{-r\tau}) = 1 - k - \underbrace{\mathbb{E}\int_0^{\infty} e^{-rt}e^{-\psi L_t}[c(\psi) + \psi Z]dL_t}_{P}, \tag{23}$$

where the second term on the R.H.S. of (23) , is the expected value of additional payments $P$. Observe that by using (9) and (17) we obtain:

$$P = H - M = -\left(\frac{v_0}{r(1-k)}\right)^{\beta}(1-k)\left(\frac{rc(\psi) + \psi r(1-k) + \psi rZ}{\underbrace{\beta - \psi r(1-k)}_{<0}}\right) > 0. \tag{24}$$

Substituting (23) in (22), we are able to write the government's objective function as:

$$W^h = \max_{\psi} \underbrace{\frac{a}{2}(q^B)^2 - k}_{V} - \underbrace{[(1-q^B)(1-k) + q^B P]}_{\text{expected costs of government intervention}}. \tag{25}$$

The first term on the R.H.S. of (25) is the bank's ex-ante value of the project $V$, and the second and the third terms are the expected costs of regulatory intervention, evaluated at the level of effort $q^B$ chosen by the bank.

The government maximizes (25) by trading off the value of the project for the bank $V$ and the expected costs of the intervention $(1-q^B)(1-k) + q^B P$. Thus a high bank effort, on the one hand, increases the value of the project $V$ and lowers the probability to have to repay deposits, $(1-q^B)(1-k)$, but on the other hand it increases the expected cost of liquidity and the cost of closure, $q^B P$. The government determines $\psi$ in such a way that the effort that the bank chooses maximizes $V$ without leading to excessive costs of intervention.

Before maximizing (25), let's investigate the difference between $q^B$ with $q^W$ for given liquidity policy. From Lemma 1 and Lemma 2, it is easy to show that:

$$a[q^B - q^W] = -(1-k) + P. \tag{26}$$

Now, as the sign of the difference between $q^B$ with $q^W$, for a given $k$ is driven by the additional payments $P$, we are ready to prove the following result:

**Proposition 4:** *For any given liquidity policy $\psi$, under moral hazard the bank effort is greater (smaller) than the effort under full information when the project is high risk (low risk), i.e.:*

$$q^B - q^W > 0 \quad \textit{for high risk projects}$$
$$q^B - q^W < 0 \quad \textit{for low risk projects.}$$

**Proof:** See Appendix 6.

To understand Proposition 4 observe that for a given liquidity policy, the expected present value of the total liquidity supplied by the government to the bank, equation (9) , increases in $\beta$ and thus in $\sigma$. Therefore, from Proposition 1, when systematic risk is low, the expected value of liquidity injection is low, and thus the advantage that the bank obtains from a given liquidity policy is low. Hence the ex ante incentive to put effort to take advantage of liquidity injection is lower than socially desirable. When instead systematic risk is high, the expected value of liquidity injection is high, and from Proposition 1, the bank puts more effort than socially desirable to increase the chances that the project starts in order to enjoy the likely liquidity injection.

In both cases this misalignment with respect to the FI case generates a deadweight loss for society. In particular, using (26) we are able to rewrite $W^h$ in (25) as

$$W^h = \max_{\psi} \underbrace{\frac{a}{2}(q^W)^2 - 1}_{W} - \underbrace{\frac{a}{2}[q^B - q^W]^2}_{I}. \tag{27}$$

The first term $W$ in (27) is the government's objective function under FI, and the second term $I$ represents the distortion induced by MH. This distortion lowers the government's objective function both if the bank puts less effort and if the bank puts more effort than socially desirable. While the first case is in line with the literature, the second one may appear counterintuitive. However, if systematic risk is high, it is more costly for the government that the bank induces a higher probability of project success than under FI because it is likely that the government has to provide liquidity to that bank for some time to come.

Next step is to determine the government optimal liquidity policy under MH, that we denote $\psi^h$. If an optimal $\psi^h$ exists it should equate the marginal value of liquidity $\frac{\partial W}{\partial \psi}$ to its marginal cost $\frac{\partial I}{\partial \psi}$. Although a close form solution for $\psi^h$ is difficult to obtain, we are able to compare $\psi^h$ with $\psi^W$. In particular, since $W$ has an interior maximum, $\frac{\partial W}{\partial \psi}_{|\psi=\psi^W} = 0$, we obtain:

$$\psi^h < \psi^W \quad \text{if} \quad \frac{\partial I}{\partial \psi} > 0,$$
$$\psi^h > \psi^W \quad \text{if} \quad \frac{\partial I}{\partial \psi} < 0.$$

In addition from (26) and (27) we obtain $\frac{\partial I}{\partial \psi} = a[q^B - q^W]\frac{\partial P}{\partial \psi}$ so that the sign of $\frac{\partial I}{\partial \psi}$ is dictated by the role played by the government's liquidity policy in reducing the expected value of additional payments $P$. In this respect we are able to prove the following Proposition:

**Proposition 5:** *Under moral hazard, if the projects are high risk we obtain* $\psi^h > \psi^W$.

**Proof:** See Appendix 7.

Proposition 5 states that under MH the government wants to provide less liquidity than under FI if systematic risk is high. This result combines two effects. If systematic risk is high the difference $q^B - q^W$ is positive (Proposition 4), and a restrictive liquidity policy is always able to reduce the expected value of additional payments $P$. In other words, since when the projects are high risk the bank is eager to obtain liquidity (Proposition 1), the government is able to tune-in the liquidity policy in order to reduce the social distortion. That is from (26) we have $\frac{\partial a[q^B - q^W]}{\partial \psi} = \frac{\partial P}{\partial \psi} < 0$. Figure 1 represents this case. (See Appendix 7).

A separate analysis must be made for low risk projects. In general, in this case the government has no incentive to relax the liquidity policy (Proposition 3) and furthermore, the bank itself does not see a great advantage from an expansion of the bailout (Proposition 1). The overall effect on $P$ is uncertain. However, we have a clear cut result that we can summarize in the following proposition:

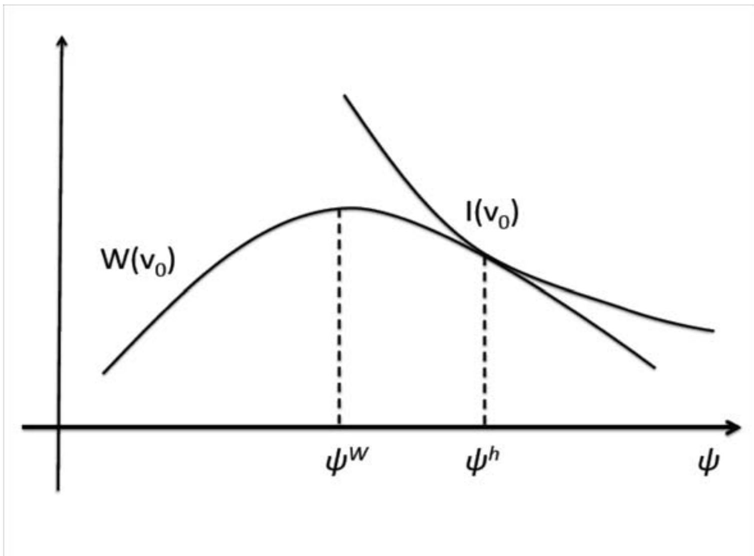


Figure 1: $\psi^W < \psi^h$ .

**Proposition 6:** *Under moral hazard, as $P$ is given by the expected value of additional payments $c(\psi)dL_t$ to guarantee liquidity, plus the expected discounted value of the deadweight cost of bank closure $Z$, if government savings from early closure are particularly large, i.e.*

$$-rc'(\psi) > r(1-k) + rZ, \tag{28}$$

*we can show that $\frac{\partial P}{\partial \psi} < 0$ irrespective of the level of systematic risk.*

**Proof:** See Appendix 7.

Thus, under MH, if condition (28) holds, and if projects are low risk the government may want to provide more liquidity than socially optimal ($\psi^h < \psi^W$). See Figure 2.

## 5.2 Effort under full information and moral hazard

Finally, we are able to shed some light on the question we posed at the beginning of this paper, namely whether the expectations of government bailouts contribute to bank's risk taking. We can now compare the bank's effort $q^B(\psi^h)$ under the MH optimal liquidity policy $\psi^h$ and the bank's effort $q^W(\psi^W)$ under the FI optimal liquidity policy $\psi^W$.

Denote $\Delta\psi = \psi^h - \psi^W$ and $\Delta q = q^B(\psi^h) - q^W(\psi^W)$. By taking the Taylor expansion of $q^B(\psi^h)$ around $\psi^W$ we obtain:

$$\Delta q \simeq q^B(\psi^W) - q^W(\psi^W) + \frac{\partial q(\psi^W)}{\partial \psi}\Delta\psi. \tag{29}$$

Thus, by (29) the difference $\Delta q$ can be approximated by the sum of two terms. The first one, $q^B(\psi^W) - q^W(\psi^W)$ from (27), measures the (social) net value of the liquidity. The last term is related to the bank's incentive compatibility constraint as it measures how the government's liquidity policy affects the bank's effort. If projects are low risk we know from Proposition 1 that $\frac{\partial q(\psi^W)}{\partial \psi} > 0$, from Proposition 4 that $q^B(\psi^W) - q^W(\psi^W) < 0$, and that $\Delta\psi < 0$ if the condition (28) in Proposition 6 holds. Then we conclude that if the projects are not risky, $\Delta q < 0$.

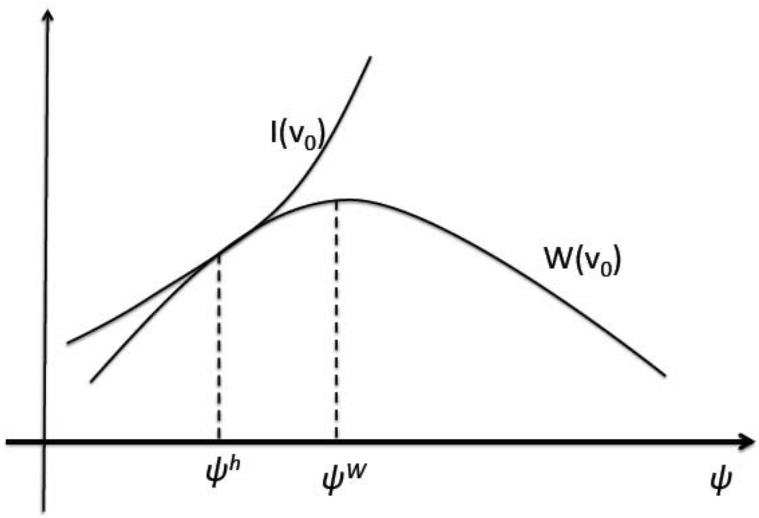


Figure 2: $\psi^W > \psi^h$ when savings on liquidity costs from early closure are large and the level of systematic risk is low.

On the contrary, if projects are high risk, from Proposition 1 we have that $\frac{\partial q(\psi^W)}{\partial \psi} < 0$, from Proposition 4 that $q^B(\psi^W) - q^W(\psi^W) > 0$, and from Proposition 5 that $\Delta\psi > 0$. Combining these three results, if the MH problem is not too severe, i.e. $\Delta\psi \simeq 0$, we obtain $\Delta q > 0$. On the contrary, with a severe moral hazard, i.e. $\Delta\psi >> 0$, the reverse can happen.

We resort to numerical examples to better illustrate the role played by MH in determining the sign of (29) when systematic risk is high. We replace the expressions for $q^B(\psi^W)$, $q^W(\psi^W)$ and $\frac{\partial q(\psi^W)}{\partial \psi}$ into (29) , and consider the case in which at the outset the value of the output is close to the value of the coupon, $\left(\frac{v_0}{r(1-k)}\right)^\beta \simeq 1$, that is the project is expected to repay the original investment. For the difference $\Delta q$ to be positive it must be[14]

$$\psi^h < f(\psi^W) \equiv \psi^W + \frac{-(\beta + rc(\psi^W) + \psi^W rZ)(\beta - \psi^W r(1-k))}{r(1-k)(\beta+1)}, \tag{30}$$

where, from Proposition 1, we assume that $\beta + 1 > 0$.

To gain some insight about the impact of systematic risk on the optimal liquidity policy, and thus on bank's risk taking, we calibrate the model. Let us that assume $r = 2\%$ while $\sigma$ can take on three values: $25\%, 30\%, 35\%$. In this case we obtain $\beta(25\%) = -0.443\,40$, $\beta(30\%) = -0.333\,33$, and $\beta(35\%) = -0.259\,30$. Furthermore let us assume that $Z = 0.5$, $k = 0.5$ and finally for the cost of liquidity we adopt the following function $c(\psi) = \frac{G-\psi^2}{rG}$, with $\psi \in [0, \sqrt{G}]$ and $G = 100$.

In Figure 3 we plot the functions $\psi^h = \psi^W$ and $\psi^h = f(\psi^W)$ for different values of $\sigma$. From (30) we know that $\Delta q > 0$ if $\Delta\psi < f(\psi^W) - \psi^W$ where, recall, $\Delta\psi$ measures the severity of the MH. We note that as $\sigma$ increases the difference $f(\psi^W) - \psi^W$ declines, which, for a given value of $\Delta\psi$, increases the probability that $\Delta\psi < f(\psi^W) - \psi^W$ does not hold anymore. That is, as difference $f(\psi^W) - \psi^W$ declines, the probability of having a liquidity policy $\psi^h$ that affects negatively the bank effort, $\Delta q < 0$, increases.

[14]This simplification reduces the complexity of numerical analysis wihtout affecting the quality of the results (See Appendix 8).

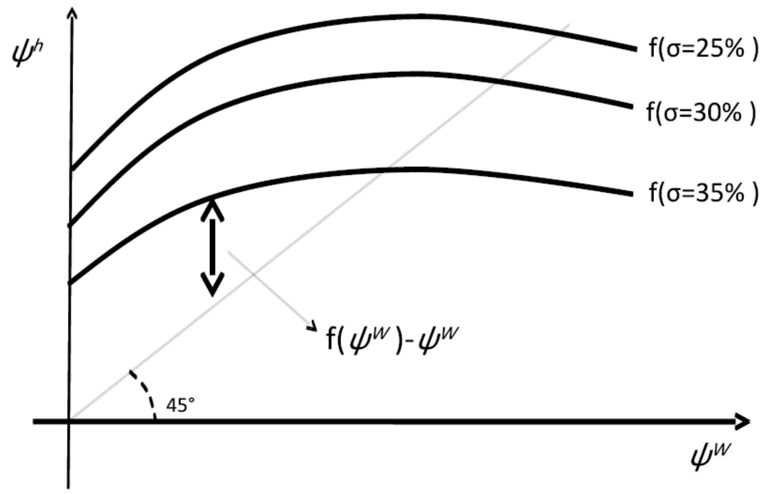


Figure 3: $f\left(\psi^W\right)$ for different values of $\sigma$.

We summarize the above observations in the following proposition.

**Proposition 7:** *Under moral hazard, the government's optimal liquidity policy has a different impact on bank's effort depending on the level of systematic risk. By collecting the results of the previous sections, we can state that:*

*a) when the project is low risk, and condition* (28) *holds* $\Delta q < 0$;

*b) when the project is high risk, but the optimal liquidity policy is not too restrictive,* $\Delta q > 0$;

*c) on the contrary, when the project is high risk and the optimal liquidity policy is very restrictive, it could be that* $\Delta q < 0$.

Proposition 7 establishes that the relationship between bailouts and bank's risk taking depends crucially on the level of systematic risk. When the latter is low and condition (28) holds (point a), it is optimal for the government to provide a more generous liquidity policy under MH than under FI, $\psi^h < \psi^W$. This induces an opportunistic behavior on the side of the bank which takes on more risk than socially desirable, $1 - q^B(\psi^h) > 1 - q^W\left(\psi^W\right)$. This is the effect stressed by the received view that argues that expectations of more generous bailouts induce a bank to take on more risk.

The opposite distortion arises when systematic risk is high (point b) and it is optimal for the government to establish a liquidity policy which is not too restrictive, albeit more stringent than under FI, $\psi^h > \psi^W$. The combination of high risk and a restrictive liquidity policy, induces the bank to take on less risk than socially desirable, $1 - q^B(\psi^h) < 1 - q^W(\psi^W)$. This counterintuitive result arises because when systematic risk is high, the bank has additional incentives to exploit the likely bailout, even if the government mitigates this effect by providing less liquidity than socially optimal.

A qualitatively new effect arises when the level of systematic risk is high and the optimal liquidity policy is very restrictive (point c). When the optimal liquidity policy under MH is much tighter than under FI $\left(\psi^h >> \psi^W\right)$, the bank could find it optimal to take on more risk than socially desirable, $1 - q^B(\psi^h) > 1 - q^W(\psi^W)$. This novel effect arises because the bank anticipates that an output shortfall is likely given the high level of systematic risk. However, the bank knows that it cannot count on generous bailouts, which could discourage it from being prudent. This illustrates the notion that a very tight liquidity policy, although optimal from a welfare standpoint, may back fire in terms of stability. This effect is similar to the one stressed by the studies that challenge the received view that generous bailouts induce bank risk taking, in particular Dell'Ariccia and Ratnovski (2014).

# 6 Conclusions

We have revisited the relationship between bank's risk taking and expectations of government bailouts, in the form of liquidity injections. At the center of our analysis is the notion that exogenous market conditions affect the optimal liquidity policy, which in turn affects the bank's risk taking behavior. By making liquidity policy endogenous as a function on external conditions, that we capture with systematic risk, our paper reconciles the studies that show that expectations of government bailouts increase bank risk taking, and the recent studies that challenge this view.

When systematic risk is taken into consideration and bailouts are optimally set, the relationship between bailouts and bank risk taking is not monotonic. In particular when systematic risk is low, the optimal liquidity policy is loose and more generous bailouts induce banks to take on more risk. When systematic risk is high and the optimal liquidity policy is tight, less generous bailouts induce banks to take on less risk. However, when high systematic risk makes a very tight liquidity policy optimal, a less generous bailout could increase bank's risk taking.

We can capture the complex interactions between systematic risk, liquidity policy and bank's risk, because we have cast the bank's and government's choices in a dynamic model. In our model bank output follows a continuous time process and the government decision to end liquidity injections entails the exercise of a real option, whose value crucially depends on systematic risk. Finally, observe that while in our model there is only one representative bank, in an economy with many banks, a higher level of systematic risk could also be a source of systemic risk if a tighter liquidity policy induces correlated risk taking choices by banks.

# 7 Appendix

## 7.1 Appendix 1. Project cash flow and its diffusion.

Assume that the stream $v_t$ follows the geometric Brownian motion:

$$dv_t/v_t = \mu dt + \sigma dz_t,$$

where $\mu$ is the drift rate, $\sigma$ is the constant instantaneous volatility, and $z_t$ is a standard Wiener process. Under the assumption of a complete capital market, a portfolio of tradeable securities $y_t$ capable of hedging the risk of the process $z_t$ exists. Assume that $y_t$ follows a stochastic differential equation of the form $dy_t/y_t = \eta dt + \xi dz_t$. Given the assumption of complete markets, the process $y_t$ can be written as (Harrison and Pliska, 1981):

$$\begin{aligned} dy_t/y_t &= rdt - rdt + \eta dt + \xi dz_t \\ &= rdt + \xi d\omega_t, \end{aligned} \tag{31}$$

where $r$ is the riskless interest rate, $d\omega_t = \phi dt + dz_t$ and $\phi = (\eta - r)/\xi$ is the market price of the risk class $z_t$. Under the new measure $\omega_t$, the process $v_t$ can be written as:

$$\begin{aligned} dv_t/v_t &= \mu dt + \sigma dz_t \\ &= (\mu - \phi\sigma)dt + \sigma d\omega_t. \end{aligned} \tag{32}$$

Note that $r + \phi\sigma$ represents the project's expected rate of return.

### 7.2 Appendix 2. Proof of Lemma 1.

The Bellman equation is:

$$rq^B = v_0 - 2r\,(1-k) + lim_{dt\to 0}\frac{1}{dt}\mathbb{E}[dq^B]. \tag{33}$$

Using the stochastic process (1) and Ito's Lemma on $\mathbb{E}[dq^B(v_0)]$, we obtain the following partial differential equation:

$$\frac{1}{2}\sigma^2 v^2 q^{B\prime\prime} - rq^B = -(v_0 - 2r(1-k)) \qquad \text{for } v_0 \in [r(1-k), \infty), \tag{34}$$

with boundary conditions:

$$\lim_{v_0\to\infty}[aq^B - \frac{v_0 - 2r\,(1-k)}{r}] = 0, \tag{35}$$

$$q^{B\prime}(r(1-k)) - \psi q^B(r(1-k)) = 0, \tag{36}$$

where $q^{B\prime}$ and $q^{B\prime\prime}$ represent the first and the second derivative of $q^B\,(v)$ w.r.t. $v$.

Equation (35) states that, when cash flows go to infinity the effort must be bounded. In fact, the second term in (35) represents the discounted present value of excess returns over an infinite horizon starting from $v_0$. The boundary condition (36) means that when the cash flows reach the lower boundary $r(1-k)$, to continue to keep the bank open the marginal value of one extra unit of liquidity must not fall below the bank's cost to increase effort by one unit represented by $\psi q^B(r(1-k))$.[15] By the linearity of the differential equation (34) and making use of (35), the general solution of (34) takes the form:

$$aq^B = \frac{v_0 - 2r(1-k)}{r} + A(v_0)^\beta, \tag{37}$$

where $A$ is a constant to be determined and $\beta$, with $-\infty < \beta < 0$, is the negative root of the characteristic equation $\frac{1}{2}\sigma^2\beta(\beta-1) - r = 0$. The boundary condition (36), yields the value of the constant $A$:

$$A = -\frac{(1-k)(1+\psi r(1-k))}{\underbrace{\beta - \psi r(1-k)}_{<0}}[r\,(1-k)]^{-\beta} > 0. \tag{38}$$

Then, the expected present value of the total liquidity supplied is equal to:

$$Av_0^\beta = -\left(\frac{v_0}{r(1-k)}\right)^\beta \frac{(1-k)(1+\psi r(1-k))}{\beta - \psi r(1-k)} > 0. \tag{39}$$

Finally, substituting (39) in (37), we are able to write the probability of success (the effort) as in the text.

[15] The boundary condition (36) requires a linear combination of the unknown function $q^B(v)$ and its first derivative $q^{B\prime}(v)$ at $v = r(1-k)$. In differential equation therory this condition is called *Robin* (or third type), boundary condition. See Harrison (2013) p 159-160 for an application of this condition in a context similar to ours.

### 7.3 Appendix 3. Proof or Proposition 1.

Recall that the level of effort is given by (8). Taking the derivative of (8) with respect to $\psi$, we obtain:

$$\begin{aligned}\frac{\partial q^B}{\partial \psi} &= -\left(\frac{v_0}{r(1-k)}\right)^{\beta}(1-k)\frac{r(1-k)(\beta-\psi r(1-k))+(1+\psi r(1-k))r(1-k)}{(\beta-\psi r(1-k))^2} \\ &= -\left(\frac{v_0}{r(1-k)}\right)^{\beta}\frac{(1-k)}{(\beta-\psi r(1-k))^2}\left[r(1-k)\beta-\psi(r(1-k))^2+r(1-k)+\psi(r(1-k))^2\right] \\ &= -\Gamma(\beta)r(1-k)\left[\beta+1\right],\end{aligned} \tag{40}$$

where

$$\Gamma(\beta)\equiv\left(\frac{v_0}{r(1-k)}\right)^{\beta}\frac{1-k}{(\beta-\psi r(1-k))^2}>0. \tag{41}$$

By (40) it is easy to show that:

$$\frac{\partial q^B}{\partial \psi}>0 \quad \text{if} \quad -\beta>1; \qquad \frac{\partial q^B}{\partial \psi}<0 \quad \text{if} \quad -\beta<1.$$

Moreover, taking the limits:

$$\lim_{\beta\to 0}\frac{\partial q^B}{\partial \psi}=-\frac{1}{(\psi r)^2}r<0; \qquad \lim_{\beta\to-\infty}\frac{\partial q^B}{\partial \psi}=0.$$

### 7.4 Appendix 4. Proof of Lemma 2.

The solution for $q^W$ is obtained by solving the following Bellman equation:

$$\frac{1}{2}\sigma^2v^2q^{W\prime\prime}-rq^W=-(v_0-r(1-k)) \qquad \text{for } v_0\in[r(1-k),\infty), \tag{42}$$

with boundary conditions:

$$\lim_{v_0\to\infty}[aq^W-\frac{v_0-r(1-k)}{r}]=0, \tag{43}$$

$$q^{W\prime}(r(1-k))-\psi[q^W(r(1-k))+Z]=c\,(\psi)\,. \tag{44}$$

While (43) is equal to (35), and has the same meaning, condition (44) replaces the boundary condition (36). In fact, since liquidity is costly for the government, at each liquidity injection the marginal value of continuing to keep the bank open must not fall below the marginal cost, that now includes both the cost of liquidity $c\,(\psi)$ as well as the deadweight cost of bank closure $Z$, i.e.:

$$c\,(\psi)+\psi[q^W(r(1-k))+Z].$$

Again, by the linearity of the differential equation (42) and making use of (43), the general solution takes the form:

$$aq^W=\frac{v_0-r(1-k)}{r}+B(v_0)^{\beta}, \tag{45}$$

where $B$ is a constant to be determined and $\beta < 0$ is still the negative root of the characteristic equation $\frac{1}{2}\sigma^2\beta(\beta-1) - r = 0$. Using (44) we obtain:

$$B = \frac{(1-k)[(c(\psi)r-1)+\psi rZ]}{\underbrace{\beta - \psi r(1-k)}_{<0}}(r(1-k))^{-\beta} < 0.$$

The expected net value of government intervention is negative and equal to:

$$B(v_0)^\beta = \left(\frac{v_0}{r(1-k)}\right)^\beta \frac{(1-k)[(c(\psi)r-1)+\psi rZ]}{\beta - \psi r(1-k)} < 0, \tag{46}$$

which, because of (45), concurs to lower the level of effort $q^W$. Finally, substituting (46) in (45) we obtain the expression in the text.

## 7.5 Appendix 5. Proof of Propositions 2 and 3.

Recall that from (16) maximizing $W$ is equivalent to maximize $q^W$ and that from (17) the probability of success is

$$aq^W = \frac{v_0 - r(1-k)}{r} + h(\psi, \beta),$$

and

$$H = \left(\frac{v_0}{r(1-k)}\right)^\beta (1-k)\frac{(c(\psi)r-1)+\psi rZ}{\beta - \psi r(1-k)} < 0.$$

To maximize $q^W$ we look for a $\psi^W$ that minimizes $H$. Let us consider the F.O.C.:

$$\frac{\partial H}{\partial \psi} = \left(\frac{v_0}{r(1-k)}\right)^\beta (1-k)\left[\frac{(c'(\psi)r + rZ)(\beta - \psi r(1-k)) + [(c(\psi)r-1)+\psi rZ]r(1-k)}{(\beta - \psi r(1-k))^2}\right] \tag{47}$$

$$= \underbrace{\left(\frac{v_0}{r(1-k)}\right)^\beta \frac{(1-k)}{(\beta - \psi r(1-k))^2}}_{\Gamma(\beta)>0}\left[c'(\psi)r\beta + rZ\beta - \psi r(1-k)c'(\psi)r + (c(\psi)r-1)r(1-k)\right] = 0.$$

Hence we obtain:

$$\frac{\partial H}{\partial \psi} = \Gamma(\beta)r[c'(\psi)(\beta - \psi r(1-k)) + (c(\psi)r-1)(1-k) + Z\beta] = 0. \tag{48}$$

Since the expression in the square bracket of (48) is (18) , this proves Proposition 2.

To prove Proposition 3 we first analyze the effect of $\sigma$ on the level of effort $q^W$. In particular:

$$\frac{\partial H}{\partial \sigma} = \frac{\partial H}{\partial \psi^W}\frac{\partial \psi^W}{\partial \sigma} + \frac{\partial H}{\partial \beta}\frac{\partial \beta}{\partial \sigma},$$

where taking the derivative of $H$ with respect to $\beta$, and observing that $\frac{\partial H}{\partial \psi^W} = 0$ by (48) we obtain:

$$\frac{\partial H}{\partial \beta}\frac{\partial \beta}{\partial \sigma} = \left(\frac{v_0}{r(1-k)}\right)^\beta (1-k)\frac{[(c(\psi)r-1)+\psi rZ]}{\beta - \psi r(1-k)}\left[\log\left(\frac{v_0}{r(1-k)}\right) - \frac{1}{(\beta - \psi r(1-k))}\right]\frac{\partial \beta}{\partial \sigma} < 0, \tag{49}$$

from which $\frac{\partial q^W}{\partial \sigma} < 0$.

To prove $\frac{\partial q^W}{\partial \psi} < 0$ observe that the S.O.S.C. of the maximization of minimizes $H$ w.r.t. $\psi$ is:

$$\frac{\partial^2 H}{\partial \psi^2} = \frac{\partial \Gamma(\beta)}{\partial \psi} \underbrace{[...]}_{=0 \text{ by FOC}} + \Gamma(\beta)\left[c''(\psi)r\beta - r(1-k)c'(\psi)r - \psi r(1-k)c''(\psi)r\right] \tag{50}$$

$$= \underbrace{\Gamma(\beta)\left[rc''(\psi)(\beta - \psi r(1-k)) - r(1-k)c'(\psi)r\right]}_{SOSC} > 0.$$

Totally differentiating (48) with respect to $\sigma$ and using (50) we obtain:

$$\frac{\partial \psi^W}{\partial \sigma} = -\frac{\frac{\partial(.)}{\partial \sigma}}{\frac{\partial(.)}{\partial \psi}} = -\frac{r[c'(\psi)+Z]\frac{\partial \beta}{\partial \sigma}}{SOSC} = -\frac{\overbrace{[\frac{-(c(\psi)r-1)r(1-k) - rZ\psi r(1-k)}{\beta - \psi r(1-k)}]}^{>0}\overbrace{\frac{\partial \beta}{\partial \sigma}}^{>0}}{SOSC > 0} < 0. \tag{51}$$

To gain the intuition for why $\frac{\partial \psi^W}{\partial \sigma} < 0$ observe that the regulator maximizes $W$ w.r.t. $\psi$ by maximizing $q^W$. So, at the maximum we have:

$$\frac{\partial q^W}{\partial \psi}(\psi(\sigma), \sigma) = 0. \tag{52}$$

Taking the total differential of (52) w.r.t. $\sigma$, we obtain:

$$\frac{\partial^2 q^W}{\partial \psi^2}(\psi(\sigma), \sigma)\frac{\partial \psi^W}{\partial \sigma} + \frac{\partial^2 q^W}{\partial \psi \partial \sigma}(\psi(\sigma), \sigma) = 0,$$

from which

$$\frac{\partial \psi^W}{\partial \sigma} = -\frac{\frac{\partial^2 q^W}{\partial \psi \partial \sigma}(\psi(\sigma), \sigma)}{\frac{\partial^2 q^W}{\partial \psi^2}(\psi(\sigma), \sigma)} < 0. \tag{53}$$

Now, since $\frac{\partial^2 q^W}{\partial \psi^2}(\psi(\sigma), \sigma) < 0$, from (53) we have $\frac{\partial \psi^W}{\partial \sigma} < 0$ if $\frac{\partial^2 q^W}{\partial \psi \partial \sigma}(\psi(\sigma), \sigma) < 0$. If we interpret $\frac{\partial q^W}{\partial \psi}$ as the marginal productivity of a restrictive liquidity policy, then $\frac{\partial^2 q^W}{\partial \psi \partial \sigma}(\psi(\sigma), \sigma) < 0$ denotes that an increase in systematic risk lowers the marginal productivity of a restrictive liquidity policy. Therefore, when the projects are riskier, if the government wants that the productivity of liquidity increases, it has to increase the liquidity, i.e. $\frac{\partial \psi^W}{\partial \sigma} < 0$.

### 7.6 Appendix 6. Proof of Proposition 4.

We proceed in steps.

First step. The difference $q^B - q^W$ is continuous in $\sigma$ (i.e. $\beta$), with limits:

$$\lim_{\beta \to 0}[q^B - q^W] = \frac{c(\psi) + \psi Z}{a\psi} > 0, \tag{54}$$

and

$$\lim_{\beta \to -\infty}[q^B - q^W] = -\frac{1-k}{a} < 0. \tag{55}$$

It is also worth noting that in the special case where the government does not deliver any liquidity, only the closure cost matters, i.e. $\lim_{\beta \to 0}[q^B - q^W] = \frac{Z}{a}$.

Second step. For any given value of $\psi$, the derivative of $q^B - q^W$ with respect to $\sigma$ is positive:

$$a\frac{\partial(q^B - q^W)}{\partial\sigma} = \frac{\partial P}{\partial\sigma} =$$
$$= P\left[\log\left(\frac{v_0}{r(1-k)}\right) - \frac{1}{\beta - \psi r(1-k)}\right]\frac{\partial\beta}{\partial\sigma} > 0. \quad (56)$$

Third step. Combining (56), (54) and (55), there exists a value of $\sigma$ in the region $[0, \infty)$ such that the difference $q^B - q^W$ changes sign.

### 7.7 Appendix 7. Proof of Propositions 5 and 6.

We proceed in steps.

First step. Recall that $W^h = W - I$. The FOC is:

$$\frac{\partial W^h}{\partial\psi} = \frac{\partial W}{\partial\psi} - \frac{\partial I}{\partial\psi} = 0. \quad (57)$$

As $W$ is concave with $\frac{\partial W}{\partial\psi}_{|\psi=\psi^W} = 0$, if a value $\psi^h$ that satisfies (57) exists, for $\psi^h > \psi^W$ it must be that $\frac{\partial I}{\partial\psi} < 0$. In particular, since $a[q^B - q^W] = -(1-k) + P$ we have $\frac{\partial I}{\partial\psi} = a[q^B - q^W]\frac{\partial P}{\partial\psi}$. Then, comparing $\psi^h$ with $\psi^W$, we obtain:

$$\psi^h < \psi^W \quad \text{if} \quad q^B - q^W > 0 \quad \text{and} \quad \frac{\partial P}{\partial\psi} > 0 \quad \text{or} \quad q^B - q^W < 0 \quad \text{and} \quad \frac{\partial P}{\partial\psi} < 0$$
$$\psi^h > \psi^W \quad \text{if} \quad q^B - q^W > 0 \quad \text{and} \quad \frac{\partial P}{\partial\psi} < 0 \quad \text{or} \quad q^B - q^W < 0 \quad \text{and} \quad \frac{\partial P}{\partial\psi} > 0.$$

We then analyze separately the sign of $\frac{\partial P}{\partial\psi}$ and $q^B - q^W$.

Second step. Consider the term $\frac{\partial P}{\partial\psi} = a\frac{\partial[q^B - q^W]}{\partial\psi}$. This is given by:

$$\frac{\partial P}{\partial\psi} = -\left(\frac{v_0}{r(1-k)}\right)^{\beta} r(1-k)\left(\frac{c'(\psi)(\beta - \psi r(1-k)) + c(\psi)r(1-k) + ((1-k)+Z)\beta}{(\beta - \psi r(1-k))^2}\right) \quad (58)$$
$$= -\underbrace{\Gamma(\beta)}_{>0} r[c'(\psi)(\beta - \psi r(1-k)) + c(\psi)r(1-k) + ((1-k)+Z)\beta],$$

where $\Gamma(\beta)$ is defined in (41). Expression (58) is continuous in $\sigma$ (i.e. $\beta$), with limits:

$$\lim_{\beta\to 0}\frac{\partial P}{\partial\psi} = -\frac{-c'(\psi)\psi r(1-k) + c(\psi)r(1-k)}{\psi^2 r(1-k)} < 0 \quad \text{and} \quad \lim_{\beta\to -\infty}\frac{\partial P}{\partial\psi} = 0.$$

In addition, $\frac{\partial P}{\partial\psi}$ from (58) is negative if:

$$-[c'(\psi) + ((1-k)+Z)]\beta < c(\psi)r(1-k) - c'(\psi)\psi r(1-k)). \quad (59)$$

Since the the R.H.S. of (59) is always positive, there exists a value of $\sigma$ in the region $[0, \infty)$ that satisfies (59). In the specific, if the projects are high risk (i.e. $\mid \beta \mid$ is close to 0), (59) is easily satisfied for any acceptable range of $\psi$.

Third step. From Proposition 4 we know that $q^B - q^W$ is increasing in $\sigma$ and that $q^B - q^W$ is negative for low $\sigma$ and positive otherwise. Then, combining Proposition 4 with $\frac{\partial P}{\partial\psi} < 0$ for high risk projects we obtain Proposition 5.

Fourth step. For low risk projects (i.e. when $\mid \beta \mid$ is high), the sign of $\frac{\partial P}{\partial \psi}$ is harder to determine. However, from (59) a sufficient condition for $\frac{\partial P}{\partial \psi} < 0$ regardless of the project's riskiness is (28), that is

$$\begin{aligned} c'(\psi) + (1-k) + Z < 0 \Leftrightarrow \\ -rc'(\psi) > r(1-k) + rZ. \end{aligned} \tag{60}$$

Thus, if (60) holds then $\psi^h > \psi^W$ for high $\sigma$ and $\psi^h < \psi^W$ for low $\sigma$.

### 7.8 Appendix 8. Analysis of equation (30).

Replacing the expressions for $q^B(\psi^W)$, $q^W(\psi^W)$ and $\frac{\partial q(\psi^W)}{\partial \psi}$ into (29), we obtain:

$$q^B(\psi^h) - q^W(\psi^W) =$$
$$(1-k)\left[-1-\left(\frac{v_0}{r(1-k)}\right)^{\beta} \cdot \frac{\psi^W r(1-k)) + c(\psi^W)r + \psi^W rZ + \frac{r(1-k)[\beta+1]\Delta\psi}{\beta-\psi^W r(1-k)}}{\beta - \psi^W r(1-k)}\right].$$

Considering $\left(\frac{v_0}{r(1-k)}\right)^{\beta} \simeq 1$, for the difference $q^B(\psi^h) - q^W(\psi^W)$ to be positive it must be:

$$\begin{aligned}
&-\frac{\psi^W r(1-k)) + c(\psi^W)r + \psi^W rZ + \frac{r(1-k)[\beta+1]\Delta\psi}{\beta-\psi^W r(1-k)}}{\beta - \psi^W r(1-k)} > 1 \\
&r(1-k)\,[\beta+1]\,\Delta\psi < (-\beta - rc(\psi^W) - \psi^W rZ)(\beta - \psi^W r(1-k)) \\
&\psi^h - \psi^W < -\frac{(\beta + rc(\psi^W) + \psi^W rZ)(\beta - \psi^W r(1-k))}{r(1-k)\,[\beta+1]},
\end{aligned}$$

where the last step follows from the fact that if systematic risk is high, from Proposition 1 we have $\beta + 1 > 0$.

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