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Optimal monetary policy under bounded rationality*

Jonathan Benchimol[†] and Lahcen Bounader[‡]

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Abstract

Optimal monetary policy under discretion, commitment, and optimal simple rules regimes is analyzed through a behavioral New Keynesian model. Flexible price level targeting dominates under discretion; flexible inflation targeting dominates under commitment; and strict price level targeting dominates when using optimal simple rules. The optimality of a particular regime is found to be independent of bounded rationality and only regime's stabilizing properties condition its hierarchy. For every targeting regime, the policymaker's knowledge of agents' myopia is decisive in terms of policy reactions. Welfare evaluation of different targeting regimes reveals that bounded rationality is not necessarily associated with decreased welfare. Several forms of economic inattention can increase welfare.

Keywords: heterogeneous myopia, behavioral economics, bounded rationality, strict targeting, flexible targeting.

JEL Classification: C53, E37, E52, D01, D11.

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1 Introduction

Optimal monetary policy is widely analyzed in the literature through New Keynesian models (Clarida et al., 1999; Woodford, 2003). These models assume that agents are rational, meaning that agents' expectations about the future are also rational and somehow perfect. According to Blanchard (2009, 2018), this assumption is exaggerated and quite far from reality, even when considering an aggregation of agents. Who knows exactly what the inflation rate will be next month? What will the output gap be next quarter? Even perfectly informed people cannot be certain.

Despite this caveat, academics and practitioners widely consider the New Keynesian model to be the workhorse for studying optimal monetary policy, and it continues to provide conclusions that shape the monetary economics literature. As Stiglitz (2011) notes, one important underlying assumption of the traditional model is the rational behavior of the economy, but the real-world economy seems inconsistent with any model of rationality. Criticisms of policy prescriptions arising from such models, as well as the dimensions along which they fall short of capturing true economic behavior and their policy implications, should be scrutinized.

Many empirical studies note that an agent's knowledge of the future is bounded (Andrade and Le Bihan, 2013; Coibion and Gorodnichenko, 2015; Gennaioli et al., 2016). Economic models should relax the rationality assumption in favor of *bounded rationality*, whereby agents are assumed to be partially myopic and not to perfectly anticipate the future, an observation made long ago by Akerlof and Yellen (1987). They claim that theory fitting the real world has to be based on the assumption that economic agents are not fully rational, which is the main element characterizing the original Keynesian ideas in particular and monetary economics models in general.

Bringing non-rational elements to New Keynesian models and highlighting their impact on monetary policy prescriptions is essential for policymakers. In addition to their intensive use of rational expectation-based models for analyzing or forecasting the economy, policymakers have to educate and communicate to real (non-rational) economic agents. Policymakers also face different forms of bounded rationality by agents (such as output gap myopia, inflation myopia, and interest rate myopia), which raises questions about the optimality of monetary policy under such forms of inattention.

Inspired by Gabaix (2016) and his approach of bounded rationality, this paper studies optimal monetary policy within a behavioral New Keynesian model with boundedly rational agents. Here, bounded rationality means an agent's inattention to variables of interest in its decision-making. The plausibility of this approach finds its roots in the work of Kahneman (1973), who attributes attention to effort

and inattention, by deduction, to laziness. Consequently, it is more convenient to model *Homo sapiens* as inattentive agents.

Unlike many of the assumptions in the New Keynesian literature, including Gabaix (2016), we assume that an economy's production does not exhibit constant returns to scale, leading us to a new behavioral Phillips curve reflecting the importance of both inflation expectations and price inertia in the determination of current inflation. Additionally, our policy assessments incorporate a number of approaches considered in the literature, such as monetary policy within discretion or commitment designs and an instrument-rule framework,¹ in strict² or flexible³ senses of targeting. Moreover, the monetary policy literature has generally addressed such approaches by conducting numerical simulations. In addition to numerical simulations, this paper characterizes each central bank targeting rule with a tractable (analytical) insightful solution regarding the interactions between an agent's beliefs and the conduct of monetary policy.

We assess optimal monetary policy under different monetary policy designs: discretion, commitment and simple rules. Under both discretion and commitment, several central bank's loss functions are examined, and each refers to a specific monetary policy targeting regime. These are flexible and strict inflation, price level, nominal GDP growth and level targeting regimes. The instrument rules considered here reflect similar targeting regimes. All these monetary policy targeting regimes are examined through variants of the behavioral New Keynesian model that emphasize different forms of myopia:⁴ output gap, interest rate, inflation, general or full myopia.

The literature uses two methods to derive optimal policy. The first is the ad hoc linear quadratic problem (Clarida et al., 1999; Galí, 2015). The second is welfare-based utility maximization (Rotemberg and Woodford, 1998). Connections can be established between these two approaches (Woodford, 2003). However, these two methods are equivalent only when a central bank operates under flexible inflation targeting. According to the literature (Jensen, 2002; Garín et al., 2016; Billi, 2017) and given our objectives, we consider ad hoc loss functions that are not necessarily derived from household utility. The ad hoc methodology can be preferable to the microfounded methodology, as illustrated in Clarida et al. (1999). If some groups suffer more than others from certain distortions, then an agent's utility might not provide an accurate measure of cyclical fluctuations in welfare.

This paper is related to several strands of the literature. First, it extends

¹For optimal simple rules, see, e.g., Taylor (1993) or McCallum and Nelson (1999).

²When it is the only variable targeted by monetary policy, see, e.g., Guender (2007).

³When it is monetary policy's target variable together with an output objective, see, e.g., Svensson (1999).

⁴The terms myopia, inattention and bounded rationality are used interchangeably in this paper.

the monetary economics literature (Clarida et al., 1999; Woodford, 2003; Galí, 2015) by relaxing the rational expectations hypothesis. Second, it provides an alternative way to deviate from the rational expectation hypothesis compared to the learning (Evans and Honkapohja, 2012, 2013; Woodford, 2013) or the rational inattention (Sims, 2003; Maćkowiak and Wiederholt, 2009, 2015) literatures. This paper, based on Gabaix (2014), extends the behavioral New Keynesian model of Gabaix (2016) to focus on optimal monetary policy analysis.

Our findings demonstrate that bounded rationality has important implications for the conduct of monetary policy. Specifically, each type of myopia considered here affects policy reactions differently. When agents are boundedly rational *vis-à-vis* all the macroeconomic variables of interest (a case that we refer to as full myopia), the central bank's reactions are shown to be more aggressive after a cost-push shock. We show that bounded rationality involves, for the monetary authority, complete knowledge of the myopia characterizing agents. Our welfare evaluation indicates that the optimal monetary policy under discretion is flexible price level targeting. Optimal commitment implies the choice of flexible inflation targeting. However, optimal simple rules indicate that strict price level targeting delivers the lowest loss of household welfare. This result demonstrates that the choice of the optimal targeting regime, among all monetary policy designs, is independent of the form of myopia characterizing agents. However, the latter has important consequences for the amplitude of the central bank's reactions to a particular shock.

Under both discretion and commitment, flexible price level targeting involves a more aggressive reaction by the central bank in the case of full myopia compared to what would prevail in the rational case (by more than 1%). The case of general myopia yields a 0.5% greater reaction relative to the rational case. Intuitively, the extreme case of full myopia necessitates important intervention by the central bank to offset the shock's impact and counter agents' beliefs.

Interestingly, myopia does not necessarily make boundedly rational agents' welfare criterion (welfare compensating variation) worse from that of rational agents. Appropriate actions by the central bank could improve boundedly rational agents' welfare compared to the rational case, to the extent that the presence of some forms of myopia implies lower inflation volatility (for example, in the case of flexible price level targeting). In contrast, the case of output gap myopia implies significant welfare losses compared to the rational case, which is attributed to the output volatility implied by this type of myopia.

The remainder of the paper is organized as follows. Section 2 describes the behavioral New Keynesian model used for the study of optimal monetary policies. Section 3 presents optimal monetary policy under discretion, and Section 4 does so under commitment. Section 5 characterizes optimal simple rules and weights

within the same model. Section 6 interprets and discusses our findings, and Section 7 draws policy implications from our results. Section 8 presents the concluding remarks.

2 The model

Our model is based on the psychological foundations of bounded rationality brought by Gabaix (2014, 2016), among others (De Grauwe, 2012; Evans and Honkapohja, 2013; Woodford, 2013), to macroeconomic analysis.

In this framework, agents' representations of the economy are sparse, i.e., when they optimize, agents care only about a few variables that they observe with some myopia. Our model departs from Gabaix (2016) by not assuming constant returns to scale and allowing for specific types of myopia (other than general myopia). Our modified framework will serve later on to assess optimal monetary policy under different policy designs: discretion, commitment and optimal simple rules, through several monetary policy regimes.

2.1 Households

In the rational case, we assume an infinitely lived rational household that maximizes its utility subject to its resource constraint.

The utility function can be represented as

$$U(c_t, N_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\phi}}{1+\phi} \quad (1)$$

where c_t is real consumption and N_t labor supply. γ is the coefficient of the household's relative risk aversion (or the inverse of the intertemporal elasticity of substitution), and ϕ is the inverse of the Frish elasticity of labor supply (or the inverse of the elasticity of work effort with respect to the real wage).

The household maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, N_t) \quad (2)$$

where \mathbb{E} is the usual expectation operator. We denote by r_t the real interest rate and y_t the agent's real income, which is the sum of labor income, $w_t N_t$, and transfers, T_t .

The dynamics of the agent's wealth, k_t , can be expressed as

$$k_{t+1} = (1 + r_t)(k_t - c_t + y_t) \quad (3)$$

and the real income as

$$y_t = w_t N_t + T_t \quad (4)$$

The rational household's problem is to maximize period utility (Eq. 2) subject to the wealth evolution (Eq. 3).

The behavioral household maximizes the same lifetime utility function (Eq. 2) but does not pay full attention to all the variables in the budget constraints, as correctly processing information entails a cost. The behavioral agent perceives reality with some myopia, which is associated with this information cost. We assume that a behavioral agent's inattention is associated with deviations from the steady-state real interest rate, $\hat{r}_t = r_t - \bar{r}$, and real income, $\hat{y}_t = y_t - \bar{y}$.

The behavioral agent's budget constraint is

$$k_{t+1} = (1 + \bar{r} + m_r \hat{r}_t) (k_t - c_t + \bar{y} + m_y \hat{y}_t) \quad (5)$$

where m_r and m_y are inattention parameters⁵ in $[0, 1]$. For $m_r = m_y = 1$, we recover the rational household's budget constraint. Separately, m_r is the specific myopia related to the evolution of the interest rate, and m_y is real income myopia.

The future state vector of the whole economy populated by rational agents evolves as

$$S_{t+1} = f(S_t, \epsilon_{t+1}) \quad (6)$$

where f is a function of the current state vector of the economy (that may contain technological shocks, fiscal measures, etc.), S_t , and an innovation process vector in the next period, ϵ_{t+1} .

The future state vector of the whole economy populated by behavioral agents evolves as

$$S_{t+1} = \bar{m} f(S_t, \epsilon_{t+1}) \quad (7)$$

where $\bar{m} \in [0, 1]$ represents the general myopia of the agent regarding the economy's state. When $\bar{m} = 1$, we recover the rational agent's law of motion (Eq. 6).

Consequently, the problem of the behavioral household consists of maximizing period utility (Eq. 2) subject to the behavioral wealth (Eq. 5) and the behavioral state vector of the economy (Eq. 7).

By clearing the goods market, $y_t = c_t$, and solving the household's problem with respect to c_t , we can find the behavioral IS equation⁶ as

$$\tilde{y}_t = M \mathbb{E}_t [\tilde{y}_{t+1}] - \sigma (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n) \quad (8)$$

⁵In this paper, the inattention parameters are considered exogenous. The endogenous case may be obtained by specifying cost functions for the agents. See Section 2.3 for more details about these parameters.

⁶See Appendix A for a detailed derivation of the IS curve.

where \tilde{y}_t is the output gap expressed as deviations of output from its natural level, r_t^n is the natural level of real interest rate, $M = \bar{m}/(R - m_y r)$, $\sigma = m_r/(\gamma R(R - m_y r))$ and $R = 1/\beta$.

The solution with respect to N_t can be expressed as

$$w_t = \gamma c_t + \phi n_t \quad (9)$$

The rational IS curve, obtained as a particular case when $m_r = m_y = \bar{m} = 1$, is

$$\tilde{y}_t = \mathbb{E}_t [\tilde{y}_{t+1}] - \sigma_{re} (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n) \quad (10)$$

where $\sigma_{re} = 1/(\gamma R)$.⁷

By comparing the behavioral (Eq. 8) and the standard (Eq. 10) cases, future output appears to have less influence on current output in the behavioral equation. Moreover, monetary policy transmission to the real economy is stronger in the standard than in the behavioral case ($\sigma_{re} \geq \sigma$).

2.2 Firms

Our economy is populated by a continuum of firms. Each firm produces differentiated goods using the same technology, which is described by

$$Y_t = A_t N_t^{1-\alpha} \quad (11)$$

where A_t is the technological factor (identical across all firms) that evolves such that $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, where $a_t = \ln A_t$ and $\varepsilon_t^a \sim N(0; \sigma_a)$, *i.i.d.* over time.

Note that unlike [Gabaix \(2016\)](#), we assume decreasing returns to scale ($\alpha > 0$), allowing our inflation dynamics to depend on the elasticity of substitution between different goods, ε . Assuming constant returns to scale ($\alpha = 0$) in the production function removes the role of this elasticity of substitution in the Phillips curve.

Following [Galí \(2015\)](#), firms face [Calvo \(1983\)](#) pricing frictions and adjust their prices in each period with probability $1 - \theta_p$. The optimal price setting of the firm is given by P_t^* , which is the price that maximizes the current market value of the profits generated while that price remains effective.

The problem of the behavioral firm is to maximize

$$\sum_{k=0}^{\infty} \theta_p^k \mathbb{E}_t^{BR} [\Lambda_{t,t+k} (P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}))] \quad (12)$$

⁷To obtain the rational version of the IS equation (Eq. 10), we invite the reader to expand Eq. 53 in Appendix A, as we do for the behavioral case.

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \quad (13)$$

where boundedly rational agents have a subjective expectation⁸ denoted by the operator $\mathbb{E}_t^{BR}[\cdot]$, $\Lambda_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\gamma} (P_{t+k}/P_t)$ is the stochastic discount factor in nominal terms, $\Psi_{t+k}(\cdot)$ is the cost function, and $Y_{t+k|t}$ is the output in period $t+k$ for a firm that last reset its price in period t .

By expanding the first-order condition (FOC) of the firm's problem around the zero-inflation steady state,⁹ we obtain

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k \geq 0} (\beta\theta)^k \mathbb{E}_t^{BR} [\psi_{t+k|t}] \quad (14)$$

where μ is the (log) desired gross markup, and $\psi_{t+k|t} = \ln \Psi'_{t+k}$ the (log) marginal cost in $t+k$ of a firm that last reset its price at t .

The resulting behavioral Phillips curve is¹⁰

$$\begin{aligned} \pi_t &= \beta M^f \mathbb{E}_t [\pi_{t+1}] + \kappa \tilde{y}_t \\ &+ (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] \end{aligned} \quad (15)$$

where $M^f = \theta \bar{m}$ and $\kappa = (1 - \theta) (1 - \beta\theta) \Theta m_x^f (\gamma + \frac{\phi + \alpha}{1 - \alpha})$.

Our microfounded Phillips curve reflects the importance of both (inflation) expectations and (prices) inertia in the determination of current inflation.

The standard Phillips curve (rational expectation case), obtained by assuming $m_x^f = m_\pi^f = \bar{m} = 1$, is

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \kappa_{re} \tilde{y}_t \quad (16)$$

where $\kappa_{re} = \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta (\gamma + \frac{\phi + \alpha}{1 - \alpha})$.

The first contrast between the behavioral (Eq. 15) and the standard (Eq. 16) Phillips curves is the weight of future inflation in the determination of current inflation. This weight is more attenuated in the behavioral than in the standard equation. Second, the sensitivity of inflation to the output-gap in the standard case, κ_{re} , is bigger than the one in the behavioral case, κ . The behavioral Phillips curve contains an additional component composed of past and current prices reflecting a more complex inflation stickiness than in the standard Phillips curve.

⁸See Appendix B for the definition of the subjective expectation operator.

⁹See Eq. 62 in Appendix B for further details.

¹⁰See Appendix B for detailed derivations.

2.3 Note on myopia parameters

Our paper considers inattention parameters as exogenous but in $[0, 1]$. [Gabaix \(2014\)](#) argues that inattention is derived from information's cost minimization which yields to inattention parameters in $[0, 1]$. By construction, New Keynesian models have to obey to some heuristics, like convergence and stability, implying that the framework may not support all irrationality forms, like over-attention which is behaviorally plausible. Knowing these limitations, we use this type of model because it is highly tractable. The model is solved in order to derive analytical equations describing the behavior of agents and emphasizing the impact of bounded rationality on optimal monetary policies.

Although our model only focuses on under-reaction, it is also able to generate over-reaction (indirectly). As raised in [Gabaix \(2014\)](#), neglecting mitigating factors (i.e., negatively correlated additional effects) leads to overreaction. In other words, a consumer overreacts to an income shock if he pays too little attention to the fact that this shock is very transitory.

2.4 Welfare-relevant model

In the presence of nominal rigidities alongside real imperfections, the flexible price equilibrium is inefficient ([Galí, 2015](#)). Consequently, it is not optimal for the central bank to target this allocation, but it is optimal to target the efficient allocation. Our model has to be expressed in terms of deviations with respect to the efficient aggregates, and the resulting variables can be called *welfare-relevant* variables.

Let us define the welfare-relevant output gap such that $x_t = y_t - y_t^e$, where y_t is the (log) output, y_t^e is the efficient output and y_t^n is the natural output (flexible-price output). Since we have $\tilde{y}_t = y_t - y_t^n$, we can link the output gap and the welfare-relevant output gap such that $\tilde{y}_t = x_t + (y_t^e - y_t^n)$.

By exploiting this relationship, one can transform the IS curve in terms of the welfare-relevant output gap as

$$x_t = M\mathbb{E}_t x_{t+1} - \sigma (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^e) \quad (17)$$

where $M = \bar{m}/(R - m_y r)$, $\sigma = m_r/(\gamma R(R - m_y r))$, $R = 1/\beta$, and $r_t^e = r_t^n + (1/\sigma)[M\mathbb{E}_t(y_{t+1}^e - y_{t+1}^n) - (y_t^e - y_t^n)]$ is the efficient interest rate perceived by households.¹¹

The behavioral Phillips curve can be represented in welfare-relevant output gap

¹¹See Appendix D for technical details.

terms as

$$\begin{aligned}\pi_t &= \beta M^f \mathbb{E}_t [\pi_{t+1}] + \kappa x_t \\ &+ (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] + u_t\end{aligned}\tag{18}$$

where $M^f = \theta \bar{m}$, $\kappa = (1 - \theta) (1 - \beta\theta) \Theta m_x^f (\gamma + ((\phi + \alpha) / (1 - \alpha)))$, and $u_t = \kappa (y_t^e - y_t^n)$ is a cost-push shock evolving according to an $AR(1)$ process such that $u_t = \rho_u u_{t-1} + \varepsilon_t^u$ and $\varepsilon_t^u \sim N(0; \sigma_u)$, *i.i.d.* over time.

Expectations in Eq. 17 and Eq. 18 are augmented by M and M^f , respectively, reducing the exaggerated weight given to expectations (Blanchard, 2009).

3 Optimal monetary policy under discretion

According to Plosser (2007), when the central bank is “not bound by previous actions or plans and thus is free to make an independent decision every period”, monetary policy is called *discretionary*. In such a case, the central bank makes whatever decision is optimal in each period without committing itself to any future actions. In this section, we characterize the second-best solutions of the central bank’s optimization problem following a cost-push shock u_t under different policy targets, namely inflation targeting, price level targeting and nominal GDP targeting, in both the flexible and strict senses.

Our definition of flexible targeting is in line with Svensson (1999), where the central bank not only stabilizes its targeted variable but also addresses real fluctuations by stabilizing the output gap. Following the literature on central bank loss functions (Svensson, 1999; Vestin, 2000; Billi, 2017), we report in Table 1 our objective functions.

Section 3.1 characterizes the central bank’s targeting rules under different monetary policy targeting regimes by minimizing these loss functions (Table 1) subject to the welfare-relevant behavioral Phillips curve (Eq. 18). These monetary policy regimes are simulated and compared using the same welfare criterion (Section 3.2). We evaluate the incidence of each targeting regime on household welfare by calculating a household’s compensating variation (Section 3.3).

3.1 Targeting rules

The central bank’s optimization problem consists of minimizing its loss function subject to the welfare-relevant behavioral Phillips curve. Solving this problem for each targeting regime yields the following proposition.

Name	Targeting regime	Loss function
F1D	Flexible inflation	$L_t = \frac{1}{2} (\pi_t^2 + \alpha_x x_t^2)$
F2D	Flexible price level	$L_t = \frac{1}{2} (p_t^2 + \alpha_x x_t^2)$
F3D	Flexible nominal GDP growth	$L_t = \frac{1}{2} [(\pi_t + \Delta y_t)^2 + \alpha_x x_t^2]$
F4D	Flexible nominal GDP level	$L_t = \frac{1}{2} [(p_t + y_t)^2 + \alpha_x x_t^2]$
S1D	Strict inflation	$L_t = \frac{1}{2} \pi_t^2$
S2D	Strict price level	$L_t = \frac{1}{2} p_t^2$
S3D	Strict nominal GDP growth	$L_t = \frac{1}{2} (\pi_t + \Delta y_t)^2$
S4D	Strict nominal GDP level	$L_t = \frac{1}{2} (p_t + y_t)^2$

Table 1: Regime names, descriptions and loss functions. The coefficient α_x is the weight that the central bank places on the output gap objective in its loss function, and Δy_t is real output growth.

Proposition 1 *Under discretion, the central bank's targeting rules for the respective targeting regimes are*

$$F1D : \quad x_t = -\psi(\kappa, \alpha_x, \theta, m_\pi^f, m_x^f, \bar{m})\pi_t \quad (19)$$

$$F2D : \quad x_t = -\psi(\kappa, \alpha_x, \theta, m_\pi^f, m_x^f, \bar{m})p_t \quad (20)$$

$$F3D : \quad g_t = -\chi(\kappa, \alpha_x, \theta, m_\pi^f, m_x^f, \bar{m})x_t \quad (21)$$

$$F4D : \quad n_t = -\chi(\kappa, \alpha_x, \theta, m_\pi^f, m_x^f, \bar{m})x_t \quad (22)$$

$$S1D : \quad \pi_t = 0 \quad (23)$$

$$S2D : \quad p_t = 0 \quad (24)$$

$$S3D : \quad \pi_t = -\Delta y_t \quad (25)$$

$$S4D : \quad y_t = -p_t \quad (26)$$

where $\psi(\cdot) = \frac{\kappa}{\alpha_x(1-(1-\theta)[\beta\theta\bar{m}+(1-\beta\theta)m_\pi^f])}$, $\chi(\cdot) = \frac{\alpha_x[1-(1-\theta)(\beta\theta\bar{m}+(1-\beta\theta)m_\pi^f)]}{1-(1-\theta)(\beta\theta\bar{m}+(1-\beta\theta)m_\pi^f)+\kappa}$, n_t is the nominal GDP level, and g_t is nominal GDP growth.

Proof. See Appendix E. ■

Those targeting rules must be satisfied at each time t to minimize the associated loss functions. In the case in which all types of myopia are set to 1, we easily recover the targeting rules of the New Keynesian model with rational agents.¹²

A first look at the targeting rules reveals that we can group them into two categories. In the first, myopia is considered in the central bank's targeting rule, and in the second, the targeting rule is not affected by myopia. The first group includes the flexible targeting regimes (Eq. 19 to Eq. 22), and the second group contains the strict targeting regimes (Eq. 23 to Eq. 26).

In the first group, the policymaker's mission is more complex than in the second group. To achieve his target, the policymaker is required to have complete knowledge of agents' beliefs. This requirement is especially affected by the type of inattention and the extent to which agents are inattentive (i.e., the degree to which they are wrong about the exact state of the economy).

¹²See Appendix F for further details.

In the second group, the central bank's targeting rule is not impacted by bounded rationality and consists of simple rules regarding certain measures of economic activity (prices, inflation or output). However, the central bank must also assess the state of myopia to adjust the interest rate accordingly.

Targeting rules F1D (Eq. 19) and F2D (Eq. 20) state that, following a cost-push shock, the policymaker has to let the output gap deviate proportionally to the observed rise in inflation or price level, respectively. The presence of only output gap myopia ($m_x^f < 1$) implies that the amplitude of accommodation is smaller than what would prevail without myopia (i.e., under the rational agent assumption). Recall from the proposition that $\psi(\kappa, \alpha_x, \theta, m_\pi^f = 1, m_x^f = 1, \bar{m} = 1)$ is the amplitude of targeting rules F1D and F2D for the rational case, denoted ψ^R hereafter. Simple arithmetic yields $\psi(\kappa, \alpha_x, \theta, m_\pi^f = 1, m_x^f < 1, \bar{m} = 1) < \psi^R$. When firms are myopic regarding the output gap, the increase in inflation (due to the cost-push shock) is not fully reflected in a decline in the output gap of the same amplitude, as would be observed in the rational case. Moreover, under inflation ($m_\pi^f < 1$) or general ($\bar{m} < 1$) myopia, monetary policy is less accommodative than in the rational case because $\psi(\kappa, \alpha_x, \theta, m_\pi^f < 1, m_x^f = 1, \bar{m} = 1) < \psi^R$ and $\psi(\kappa, \alpha_x, \theta, m_\pi^f = 1, m_x^f = 1, \bar{m} < 1) < \psi^R$. Under inflation myopia, agents do not pay attention to the full increase in prices involving a structurally moderated decline in the output gap. The case of general myopia reveals that when agents ignore the state of the economy, this may reduce the amplitude of the decline in the output gap relative to the rational case. However, when comparing the amplitudes of inflation myopia and general myopia, we find that $\psi(\kappa, \alpha_x, \theta, m_\pi^f < 1, m_x^f = 1, \bar{m} = 1) < \psi(\kappa, \alpha_x, \theta, m_\pi^f = 1, m_x^f = 1, \bar{m} < 1)$, suggesting that when agents are boundedly rational over the economy's state, this implies that the shock is accommodated more aggressively than it would be if they failed to observe only a particular aspect of the economy, such as in the inflation myopia case.

The discussion above focused primarily on the amplitude of the targeting rules in both flexible inflation targeting and price level targeting while focusing little on the difference between them. In the first case, the output gap has to adjust proportionally to inflation, while in the latter case, it has to adjust to the price level. Consequently, inflation variance is proportional to the output gap variance in F1D, while inflation variance in F2D is proportional to the variance of output gap growth, thus implying that flexible price level targeting induces less inflation volatility.

How does bounded rationality come into play? Regarding the volatility properties of price level targeting, it appears that the presence of myopia implies less inflation volatility compared to the rational case, given the underlying assumption that the policymaker maintains his targeting rule.

Targeting rules F3D (Eq. 21) and F4D (Eq. 22) allow the nominal GDP growth and level to rise proportionally to the decline in the output gap following a cost-push shock. Note that, under the assumption of constant efficient output, the targeting rule (Eq. 21) can be rewritten as

$$\pi_t = x_{t-1} - [1 + \chi(\cdot)] x_t \quad (27)$$

which indicates that inflation under F3D is more persistent than under the previous targeting regimes (F1D and F2D). Myopia affects the feedback from the output gap to inflation through the coefficient $1 + \chi(\cdot)$. When agents are boundedly rational, the feedback from the output gap to inflation shrinks¹³ relative to the rational case except for the case of output gap myopia. In fact, when agents are boundedly rational *vis-à-vis* the output gap, the transmission of the shock from inflation to the output gap decreases. Then, the inverse of the transmission coefficient, which is $1 + \chi(\cdot)$, is greater than in the rational case. For the case of flexible nominal GDP level targeting (F4D), the targeting rule (Eq. 22) can be rewritten as

$$p_t = -y_t^e - [1 + \chi(\cdot)] x_t \quad (28)$$

which suggests that the price level must be held negatively proportional to the output gap. In this setup, the persistence of inflation also comes into play because its variance will be proportional to the variance of output gap growth. As in the previous case, bounded rationality impacts the feedback from the output gap to inflation through the coefficient $1 + \chi(\cdot)$.

Regarding the central bank's target, in both targeting regimes, the cost-push shock's impact is shared between prices and output, and thus, the impact on the targeted variable is mitigated and small. This implies that monetary policy conduct within F3D or F4D may be passive after a cost-push shock.

The central bank's targeting rules S1D (Eq. 23) to S4D (Eq. 26) state that the policymaker has to stabilize his target period-by-period. To achieve his objective, the policymaker is required to adjust the monetary policy stance depending on agent myopia.

For the case of the strict inflation targeting in S1D, by substituting the targeting rule (Eq. 23) into the behavioral Phillips curve (Eq. 17), we find the following:

$$x_t = -\frac{1}{\kappa} (1 - \theta) \left((1 - \beta\theta) m_\pi^f + \beta\theta\bar{m} - 1 \right) p_{t-1} - \frac{1}{\kappa} u_t \quad (29)$$

The absence of myopia reduces (Eq. 29) to $x_t = -\frac{1}{\kappa|_{m_x^f=1}} u_t$, meaning that the overall shock u_t has to be accommodated by the decline in the output gap. However,

¹³In the cases of inflation or general myopia, we can easily find that $\chi(\kappa, \alpha_x, m_\pi^f, m_x^f = 1, \bar{m}) < \chi^R$, where χ^R is the rational case's amplitude. When we allow for only output gap myopia, we have the opposite result: $\chi(\kappa, \alpha_x, m_\pi^f = 1, m_x^f, \bar{m} = 1) > \chi^R$.

the presence of myopia implies that the output gap must track with the previously prevailing level of prices, depending on the myopia related to inflation, the output gap or the economy's state.

Strict price level targeting (S2D) delivers a simpler feedback rule when substituting the targeting rule into the behavioral Phillips curve (Eq. 17)

$$x_t = -\frac{1}{\kappa}u_t \quad (30)$$

which suggests that the shock should be accommodated by engineering a proportional decline in the output gap. Recall that κ is a function of m_x^f , and thus, only output gap myopia can impact the output gap's reaction. In the presence of output gap myopia, the decline in the output gap is exacerbated to a greater extent than in the rational case.

For the remaining two targeting regimes, S3D and S4D, when following the targeting rules (Eq. 25) and (Eq. 26), the central bank is required to make output growth negatively proportional to the rise in inflation in the first case, while it has to keep output negatively proportional to the price level in the second case.

3.2 Simulation results

After characterizing the targeting rules and their interactions with bounded rationality, we now turn to a numerical assessment of the behavior of the central bank in the presence of bounded rationality under different monetary policy targeting regimes.¹⁴ Fig. 1 describes impulse response functions of the nominal interest rate with respect to a 1% cost-push shock for different types of bounded rationality (or inattention) and monetary policy targeting cases under discretion.

Whatever the targeting regime, the full myopia case induces large central bank reactions to a cost-push shock (Fig. 1). When agents are inattentive to macroeconomic developments, the central bank has to act aggressively to counter both agents' beliefs and the cost-push shock affecting the economy. The other cases in which the central bank intervenes aggressively, but less so than in the former case, are interest rate myopia and general myopia. Intuitively, whatever the central bank's action is, myopic (regarding the interest rate) agents underestimate interest rate movements. This is why the central bank has to overshoot. The remaining cases, output gap myopia and inflation myopia, show slightly larger responses than in the rational case.

¹⁴Analytical solutions for the interest rate reactions yield non-tractable expressions for the interest rate, and this is why we choose to report only the numerical solutions, which are more insightful regarding the role of bounded rationality. The calibration of our model is presented in Appendix G.

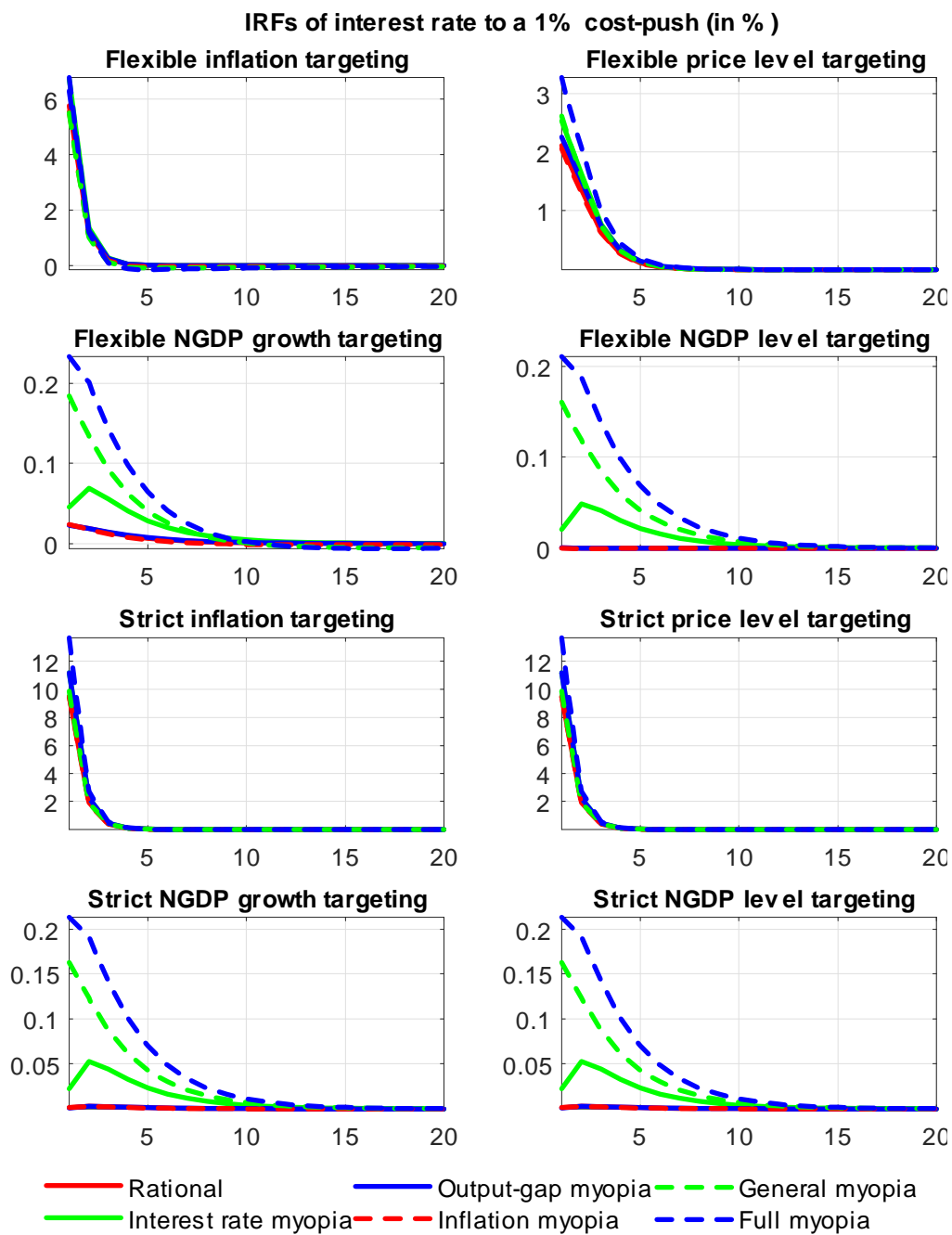


Figure 1: Impulse response functions of the nominal interest rate with respect to a 1% cost-push shock for different cases under discretion.

When comparing the targeting regimes considered here, it appears that F3D, F4D, S3D and S4D show identical and small interest rate responses. We can infer that, under discretion, whether one targets nominal GDP growth or the level does not matter. In addition, even the choice of targeting nominal GDP in the flexible or strict sense does not change the central bank’s reaction. All the targeting regimes provide small interest rate reactions following a cost-push shock. This shock implies, at the same time, a rise in prices and a decline in output, which stabilize nominal GDP. The S1D and S2D targeting regimes provide by far the largest interest rate increase following a cost-push shock. Given the central bank’s mandate (inflation stabilization only in the case of S1D and price level stabilization only in the case of S2D), the central bank has to accommodate the rise in inflation by reducing output significantly below its efficient level, which is realized by a significant interest rate increase. Although F1D and F2D display moderate reactions by the central bank compared to the previous policies, F2D delivers a lower reaction compared to F1D.

3.3 Which target to choose ?

To compare the performance of all targeting regimes considered previously, we compare their implications for household welfare. Following [Garín et al. \(2016\)](#), we calculate the compensating variation in terms of household consumption, which is the percent of consumption that makes the household indifferent between the flexible price equilibrium and the sticky price equilibrium. For each monetary policy targeting framework and type of myopia, the compensating variation can be expressed as¹⁵

$$CEV = 100 [\exp (\mathbb{E}W^{flexible} - \mathbb{E}W) - 1] \quad (32)$$

where $\mathbb{E}W^{flexible}$ is the expected household welfare under flexible prices, and $\mathbb{E}W$ is the expected household welfare when prices are sticky. Lower values for the CEV are associated with more desirable monetary policy targeting regimes.

The computed compensating variation values for the monetary policy frameworks considered here are reported in [Table 2](#).

It appears that flexible price level targeting delivers the best outcomes in terms of social welfare, whatever the assumed form of myopia. The impulse response

¹⁵Household welfare in the compensation variation analysis is the second-order approximation of household utility, as proven in [Galí \(2015\)](#)

$$W = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha x_t^2) \quad (31)$$

Myopia	F1D	F2D	F3D	F4D	S1D	S2D	S3D	S4D
No (rational)	1.64	1.21	2.30	2.29	2.55	2.55	2.28	2.28
Interest rate	1.64	1.21	2.30	2.29	2.55	2.55	2.28	2.28
Output gap	1.82	1.32	2.43	2.42	3.00	3.00	2.41	2.41
Inflation	1.59	1.19	2.14	2.13	2.55	2.55	2.12	2.12
General	1.50	1.23	2.16	2.15	2.55	2.55	2.14	2.14
Full	1.55	1.31	2.11	2.10	3.00	3.00	2.10	2.10

Table 2: Compensating variation values by regime and type of bounded rationality under discretion.

functions show that under this targeting regime, the cost-push shock induces a moderate rise in inflation, a small decline in the output gap, and smoother reactions by the central bank. However, as noted previously (Section 3.1), the flexible price level targeting regime implies that the central bank has complete knowledge of the myopia characterizing households and firms. This requirement constitutes a serious limitation for the policymaker when adopting such a targeting regime. Moreover, note that the rational and interest rate myopia cases exhibit the same welfare losses under all targeting regimes considered.¹⁶ As the presence of interest rate myopia only impacts the amplitudes but not the final outcomes (inflation and output) of the central bank’s reactions, and the central bank’s loss functions do not include an interest rate smoothing objective, this result seems intuitive.

4 Optimal monetary policy under commitment

The central bank is assumed to be credible and able to commit to a policy plan. The monetary authority must be able to choose a path for the output gap and inflation over the infinitely lived horizon to minimize a policy objective function. Policy objective functions similar to those in Table 1 are considered and presented in Table 3.

As above, Section 4.1 characterizes the central bank’s targeting rules, following a cost-push shock u_t , under different loss functions (Table 3) subject to the welfare-relevant behavioral Phillips curve (Eq. 18). These monetary policy targeting regimes are simulated (Section 4.2) and evaluated by calculating compensating variation for the households (Section 4.3).

¹⁶In addition to the inflation targeting regime, we also examined the speed limit policy, which is expressed as

$$L = \frac{1}{2} [\pi_t^2 + \alpha_x (x_t - x_{t-1})^2] \quad (33)$$

The conclusions are the same as those for flexible inflation targeting.

Name	Targeting regime	Loss function
F1C	Flexible inflation	$L_t = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha_x x_t^2)$
F2C	Flexible price level	$L_t = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (p_t^2 + \alpha_x x_t^2)$
F3C	Flexible nominal GDP growth	$L_t = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [(\pi_t + \Delta y_t)^2 + \alpha_x x_t^2]$
F4C	Flexible nominal GDP level	$L_t = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [(p_t + y_t)^2 + \alpha_x x_t^2]$
S1C	Strict inflation	$L_t = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2$
S2C	Strict price level	$L_t = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t p_t^2$
S3C	Strict nominal GDP growth	$L_t = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t + \Delta y_t)^2$
S4C	Strict nominal GDP level	$L_t = \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (p_t + y_t)^2$

Table 3: Regime names, descriptions and loss functions. The coefficient α_x is the weight that the central bank places on the output gap objective in its loss function, and Δy_t is real output growth.

4.1 Targeting rules

Minimizing the loss function (Table 3) subject to the welfare-relevant behavioral Phillips curve (Eq. 18) characterizes central bank's targeting rules presented below.

Proposition 2 *Under commitment, the central bank's targeting rules are*

$$F1C : \quad \begin{aligned} \pi_t &= \beta\pi_{t+1} + \frac{\alpha_x}{\kappa} M^f x_{t-1} + \beta\theta \frac{\alpha_x}{\kappa} x_{t+1} \\ &- \frac{\alpha_x}{\kappa} (1 + \beta M^f - (1 - \theta) [\beta\theta\bar{m} + (1 - \beta\theta) m_\pi^f]) x_t \end{aligned} \quad (34)$$

$$F2C : \quad \begin{aligned} p_t &= \frac{\alpha_x}{\kappa} M^f x_{t-1} + \beta \frac{\alpha_x}{\kappa} x_{t+1} \\ &- \frac{\alpha_x}{\kappa} (1 + \beta M^f - (1 - \theta) [\beta\theta\bar{m} + (1 - \beta\theta) m_\pi^f]) x_t \end{aligned} \quad (35)$$

$$F3C : \quad \begin{aligned} &\kappa(g_t - \beta g_{t+1}) - \beta\theta(g_{t+1} - \beta g_{t+2} + \alpha_x x_{t+1}) \\ &- M^f(g_{t-1} - \beta g_t + \alpha_x x_{t-1}) \\ &+ (1 + \beta M^f - (1 - \theta) [\beta\theta\bar{m} + (1 - \beta\theta) m_\pi^f]) (g_t - \beta g_{t+1} + \alpha_x x_t) = 0 \end{aligned} \quad (36)$$

$$F4C : \quad \begin{aligned} &n_t - \frac{\beta\theta}{\kappa} (n_{t+1} + \alpha_x x_{t+1}) - \frac{M^f}{\kappa} (n_{t-1} + \alpha_x x_{t-1}) \\ &+ \frac{1}{\kappa} (1 + \beta M^f - (1 - \theta) [\beta\theta\bar{m} + (1 - \beta\theta) m_\pi^f]) (n_t + \alpha_x x_t) = 0 \end{aligned} \quad (37)$$

$$S1C : \quad \pi_t = 0 \quad (38)$$

$$S2C : \quad p_t = 0 \quad (39)$$

$$S3C : \quad \begin{aligned} &\kappa(g_t - \beta g_{t+1}) - \beta\theta(g_{t+1} - \beta g_{t+2}) - M^f(g_{t-1} - \beta g_t) \\ &+ (1 + \beta M^f - (1 - \theta) [\beta\theta\bar{m} + (1 - \beta\theta) m_\pi^f]) (g_t - \beta g_{t+1}) = 0 \end{aligned} \quad (40)$$

$$S4C : \quad \begin{aligned} &[\kappa + \alpha_x (1 + \beta M^f - (1 - \theta) [\beta\theta\bar{m} + (1 - \beta\theta) m_\pi^f])] n_t \\ &- M^f n_{t-1} - \beta\alpha_x n_{t+1} = 0 \end{aligned} \quad (41)$$

where α_x is the weight attached by the central bank to the output objective, β is the discount factor, and κ is the sensitivity of inflation to output in the behavioral Phillips curve.

Proof. See Appendix H. ■

Except for S1C and S2C, all the targeting rules¹⁷ derived under commitment incorporate two key ingredients: history-dependency and the presence of myopia. First, due to the trade-offs created by the cost-push shock, the central bank stabilizes its target by adjusting the actual (i.e., in t) and future (i.e., in $t + 1$) output gap. The additional term associated with the previous period (i.e., in $t - 1$) indicates that there is some persistence in the central bank's reactions. This can

¹⁷Appendix I presents targeting rules for the rational case under commitment.

be interpreted as the central bank having to fulfill past promises regardless of the actual situation. This result overcomes shortcomings of the traditional New Keynesian model with respect to the persistence of the impact of monetary policy on the targeted variables, as raised by [Fuhrer and Moore \(1995\)](#) and [Walsh \(2017\)](#). Second, note that the coefficients associated with each term in the central bank’s targeting rule depend on our myopia parameters. The intuition is that the central bank weights its preferences over sooner or later adjustments while accounting for agents’ beliefs. This adjustment is not due solely to the central bank’s preferences, captured by α_x or β , but is a combination of the central bank’s preferences and the behavioral character of agents. This is in line with the fact that monetary policy is about expectations management ([King et al., 2008](#)).

When the central bank operates under strict inflation (S1C) or price level (S2C) targeting, the targeting rules to satisfy forever are quite simple. They state that the central bank has to keep inflation (Eq. 38) or the price level stable (Eq. 39). It is clear that the targeting rules do not include any myopia term. However, the mechanisms by which the central bank can achieve its objective depend on the agents’ myopia.

4.2 Simulation results

Fig. 2 describes impulse response functions of the nominal interest rate with respect to a 1% cost-push shock for different myopic and monetary policy targeting cases under commitment.¹⁸

The same observation about myopia’s impact on the central bank’s reactions under discretion applies here. The extreme case of full myopia implies a large central bank reaction after a cost-push shock. The other cases in which the central bank is acting aggressively, but less than in the extreme case, are interest rate and general myopia. The remaining cases, output gap and inflation myopia, show slightly larger responses than the rational case.

Compared to the discretionary case, F1C shows a considerably more moderate interest rate increase. Following a cost-push shock, inflation rises, and the central bank need not counter the shock in the same period. For the interest rate reactions under F2C, the interest rate starts increasing until deflation is realized, to keep the price level stable, and then the movement is reversed. Regarding nominal GDP targeting, F3C, F4C, S3C and S4C, the central bank does not react significantly in this case because of the quasi-neutral effect of the cost-push shock on nominal GDP. Inflation overreacts to a 1% percent increase in the cost-push shock. Indeed, the central bank lets inflation rise to the extent that its ultimate objective is the

¹⁸Simulations use the same calibration as in the case of discretion. See Appendix G for more details.

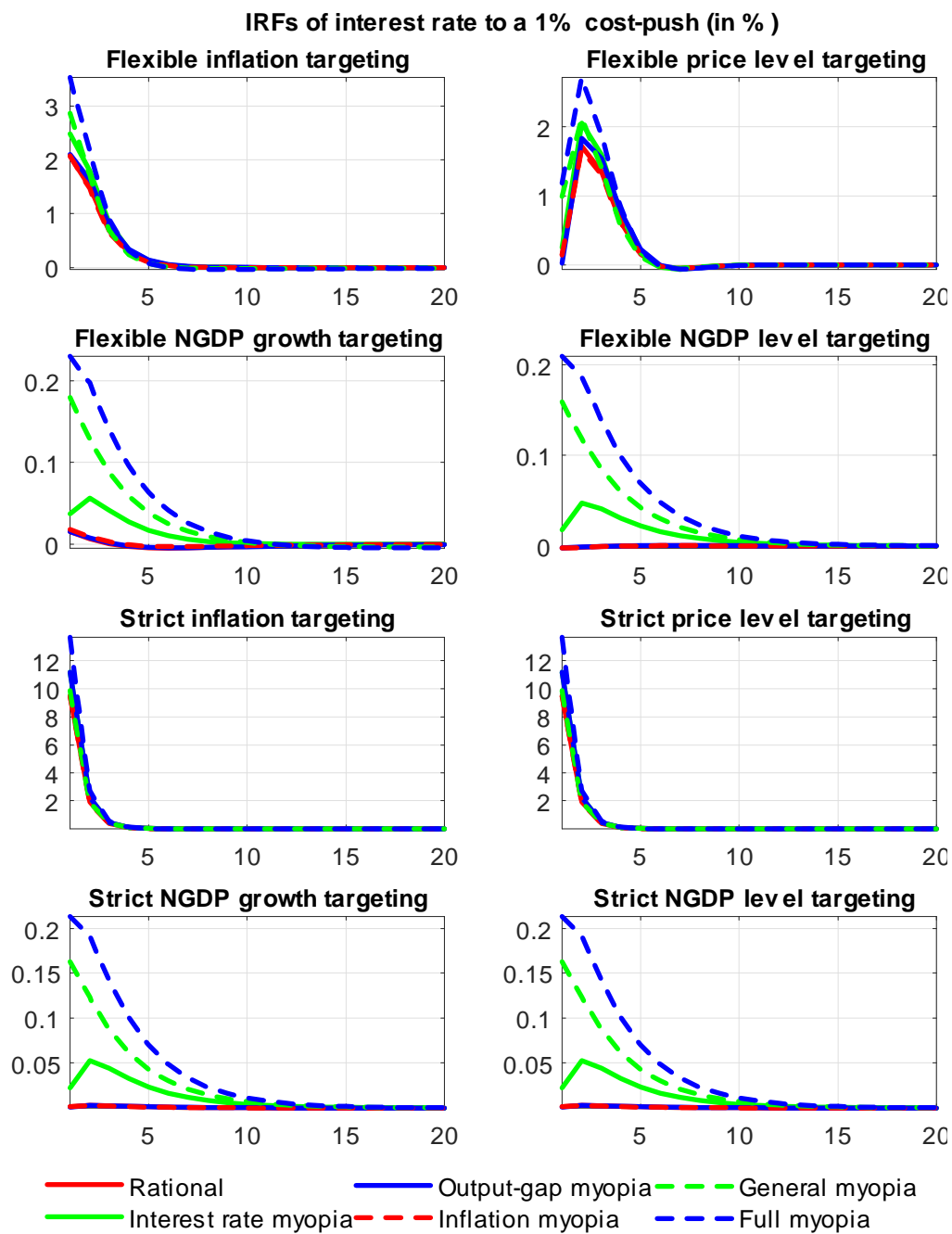


Figure 2: Impulse response functions of the nominal interest rate with respect to a 1% cost-push shock for different cases under commitment.

nominal GDP growth or level. At the same time, output declines, leaving the nominal GDP insulated from the shock. The remaining regimes, S1C and S2C, reveal the central bank’s caution *vis-à-vis* its objective. To do so, the central bank has to engineer a meaningful contraction of output to accommodate the rise in inflation. Consequently, the implied interest rate rise is also important.

4.3 Which target to choose ?

As in the case of discretion (Section 3.3), a comparison of our targeting regimes is performed by comparing the compensating variation values.

Table 4 reports the computed welfare loss values for different monetary policy targeting regimes and bounded rationalities.

Myopia	F1C	F2C	F3C	F4C	S1C	S2C	S3C	S4C
No (rational)	1.21	1.29	2.31	2.29	2.55	2.55	2.28	2.28
Interest rate	1.21	1.29	2.31	2.29	2.55	2.55	2.28	2.28
Output gap	1.32	1.41	2.43	2.42	3.00	3.00	2.41	2.41
Inflation	1.19	1.27	2.14	2.13	2.55	2.55	2.12	2.12
General	1.23	1.34	2.16	2.15	2.55	2.55	2.14	2.14
Full	1.31	1.42	2.11	2.10	3.00	3.00	2.10	2.10

Table 4: Compensating variation values by regime and bounded rationality under commitment.

The results show that the flexible inflation targeting regime, F1C,¹⁹ provides the lowest compensating variation in terms of household welfare, whatever the agents’ myopia is. Note that S1C and S2C deliver the same welfare losses. The nominal GDP targeting regimes, F3C, F4C, S3C and S4C, provide similar welfare losses. Thus, targeting the nominal GDP growth or level in a flexible or strict manner does not significantly change households’ welfare. Moreover, the rational and interest rate myopia models provide the same welfare losses in all considered targeting regimes. As interest rate myopia only impacts the amplitude of the central bank’s reaction, but not the final outcomes (inflation and output), and the central bank’s loss functions do not include an interest rate smoothing objective, this result seems intuitive.

¹⁹In addition to inflation targeting regime, we also studied the speed limit policy, which is expressed as

$$L = \frac{1}{2} \sum \beta^t [\pi_t^2 + \alpha_x (x_t - x_{t-1})^2] \quad (42)$$

The conclusions are the same as for flexible inflation targeting.

5 Optimal simple rules

In this section, we postulate some simple rules and attempt to numerically determine the optimal values of the associated coefficients. We consider eight rules, as described in the table below.

Name	Targeting regime	Instrument-rule
F1O	Flexible inflation	$i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + \varepsilon_t^{mp}$
F2O	Flexible price level	$i_t = \phi_p p_t + \phi_y \tilde{y}_t + \varepsilon_t^{mp}$
F3O	Flexible nominal GDP growth	$i_t = \phi_g (\pi_t + \Delta y_t) + \phi_y \tilde{y}_t + \varepsilon_t^{mp}$
F4O	Flexible nominal GDP level	$i_t = \phi_n (p_t + y_t) + \phi_y \tilde{y}_t + \varepsilon_t^{mp}$
S1O	Strict inflation	$i_t = \phi_\pi \pi_t + \varepsilon_t^{mp}$
S2O	Strict price level	$i_t = \phi_p p_t + \varepsilon_t^{mp}$
S3O	Strict nominal GDP growth	$i_t = \phi_g (\pi_t + \tilde{y}_t) + \varepsilon_t^{mp}$
S4O	Strict nominal GDP level	$i_t = \phi_n (p_t + y_t) + \varepsilon_t^{mp}$

Table 5: Regime names, descriptions and simple rules

The instrument rules described in Table 5 reproduce the same monetary policy targeting regimes addressed in previous sections (Section 3 and Section 4). The central bank's instrument reacts only to the targeted variable when operating in a strict sense (rules S1O to S4O) and reacts to real fluctuations and its target when operating in a flexible sense (rules F1O to F4O). Note that, in some cases, the central bank does not restrict its attention to the endogenous variables, which is why we add the monetary policy shock (ε_t^{mp}) to reflect the deviations of the central bank from its rule.

Hereafter, we provide the optimal parameter values in Section 5.1 and welfare losses in Section 5.2.

5.1 Monetary policy rules' optimal values

Table 6 reports the optimal values²⁰ of ϕ_π , the weight on inflation; ϕ_y , the weight on the output gap; ϕ_p , the weight on the price level; ϕ_g the weight on nominal GDP growth; and ϕ_n the weight on the nominal GDP level for different monetary policy rules.

	F1O		F2O		F3O		F4O		S1O	S2O	S3O	S4O
	ϕ_π	ϕ_y	ϕ_p	ϕ_y	ϕ_g	ϕ_y	ϕ_n	ϕ_y	ϕ_π	ϕ_p	ϕ_g	ϕ_n
No (rational)	3.7	0.87	3.9	0	1	2.5	0.68	0	5	3.7	5	0.68
Interest rate	3.9	1.10	4.7	0	1	2.5	0.82	0	5	4.8	5	0.82
Output gap	3.4	0.78	3.8	0	1	2.2	0.60	0	5	3.7	5	0.60
Inflation	1	0.92	3.8	0	1	1.7	0.63	0	*	3.7	*	0.63
General	1	0.88	3.6	0.1	1	1.5	0.69	0	5	4.4	5	0.69
Full	1	0.80	3.7	0.2	1	1.3	0.67	0	5	4.8	5	0.67

Table 6: Optimal values for the coefficients of simple rules F1O to S4O

As shown in the table above, the inflation coefficient in rule F1O is greater than 1.5 in the rational case and both the interest rate and output gap myopia cases, which is in line with the literature²¹ (Galí (2015)). Note that when agents are myopic with respect to the interest rate, the sensitivity of the policy instrument to inflation is greater than in other cases. The cases of inflation, general and full myopia yield a coefficient of 1 for inflation. Intuitively, myopia with respect to inflation implies that people do not perceive the exact rise in inflation, meaning that the central bank does not have to react aggressively. The same mechanics are operative when agents are myopic with respect to the whole state of the economy or when they are fully myopic. However, the rule (S1O) reflecting strict inflation targeting yields a coefficient of inflation that is approximately 5 in all models considered except that for inflation myopia, where the star indicates that the model does not yield any result due to the presence of unit roots. Note that those optimal values are bound-dependent,²² but more interestingly, the central bank is very sensitive to its target when operating under strict targeting.

The nominal income coefficient associated with rules F3O and F4O is higher for the growth rules than the level rules, across all types of myopia, a result in line with the literature (Rudebusch (2002)). The same table reveals that when the central bank targets nominal GDP growth, both in the strict and flexible senses (F3O) and (F4O), myopia has little effect on the central bank's behavior.

²⁰Optimizations are based on the calibration presented in Appendix G.

²¹See Benchimol and Fourçans (2017) for an empirical perspective under rational expectations.

²²We set bounds when calculating the optimal values. If we did not, the result would have been astronomical.

Moreover, the nominal GDP coefficient is the same when comparing nominal GDP level targeting in its strict and flexible forms. The latter depends on agent myopia, and it is clear that interest rate myopia delivers the most substantial amplitude compared to other types of myopia.

For the optimal values of ϕ_p in rules F2O and S2O, regardless of whether the central bank targets the price level in a flexible or strict way, the sensitivity of the policymaker's instrument to the price level does not vary significantly between the flexible and strict regimes. In the rational case and in the presence of interest rate, output gap or inflation myopia, F2O shows null values for output sensitivity. It becomes positive only for the general and full myopia cases.

The coefficient of the output gap varies across the different types of myopia and rules considered. First, the rule (F4O) reflecting flexible nominal GDP targeting shows null optimal values, which suggests that the central bank, when targeting the nominal GDP level, does not have to care about real fluctuations. The flexible price level rule (F2O) suggests the same conclusion except in the general and full myopia cases, where the central bank places some weight on stabilizing real activity (more weight is assigned here in the case of full myopia than in the case of general myopia).

5.2 Which rule to choose ?

The performance of policy rules is compared using the same criterion as in the previous sections 3 and 4. The calculations of the compensating variation for each rule are reported in the table below.

Myopia	F1O	F2O	F3O	F4O	S1O	S2O	S3O	S4O
No (rational)	20.73	12.61	33.51	13.69	2.40	1.84	5.21	2.0
Interest rate	20.73	12.61	30.98	13.82	2.41	1.86	5.44	2.0
Output gap	25.94	15.44	47.07	16.74	2.59	1.98	6.25	2.1
Inflation	15.97	11.92	29.25	12.63	*	1.79	*	1.93
General	14.81	12.55	27.21	12.93	2.28	1.88	4.92	1.96
Full	15.96	14.17	32.91	14.40	2.37	1.93	5.82	1.99

Table 7: Compensating variation values by regime and bounded rationality for different optimal simple rules. * denotes the presence of unit roots when considering such cases of myopia.

Table 7 demonstrates the superiority of the strict simple rules over flexible rules. Flexible targeting rules induce substantial welfare losses, while strict targeting rules deliver far more moderate values. In particular, rule S2O (strict price level targeting) is the more desirable in terms of household welfare.

Regarding bounded rationality, it is clear that across those targeting regimes, output gap myopia implies important welfare losses compared to the rational case. However, general myopia (and, to a lesser extent, full myopia), combined with the appropriate action by the central bank, yields smaller welfare losses compared to the rational case.

Unlike in previous sections (Section 3 and Section 4), where equivalence between particular monetary targeting regimes can be established, no equivalence can be established in the results for optimal simple rules.

6 Interpretation and discussion

Before discussing our results, recall that the specific model derived here highlights agents' inattentiveness to the full macroeconomic environment, which is reflected in the new Keynesian Phillips curve that links inflation to its past and future price dynamics in addition to the output gap (Eq. 18). Such a microfounded equation captures both price inertia and the role of expectations, which are lacking in many Phillips curves developed earlier, according to [Ball et al. \(2005\)](#). This feature provides valuable background for studying optimal monetary policy and provides a realistic dimension to their policy implications.

6.1 Discretion

Overall, the results regarding household welfare loss support the finding that flexible price level targeting (F2D) delivers the lowest social loss in the bounded rationality and rational setups. This performance is attributed to its stabilizing properties compared to other targeting regimes in terms of inflation, output and price level responses to a cost-push shock. More important, as mentioned above, inflation volatility under flexible price level targeting is smaller than under other targeting regimes in the rational case, which is in line with numerous studies of the rational case ([Guender and Tam, 2004](#); [Vestin, 2000](#); [Svensson, 1999](#)). Although bounded rationality does not impact the hierarchy of monetary policy targeting regimes, price level targeting emerges as a natural choice when agents are myopic. The presence of myopia reduces inflation volatility more than in the rational case, and naturally, flexible price level targeting will be better suited for inflation-averse households (the household welfare loss function weights inflation more than the output gap).

By deviating from the rational agent hypothesis and using price setters' information stickiness, [Ball et al. \(2005\)](#) finds that flexible price level targeting is optimal. [Honkapohja and Mitra \(2018\)](#) employs a nonlinear NK model under learning and finds that price level targeting might show a good performance de-

pending on the credibility of the central bank. By using different deviations from rationality, namely bounded rationality, we find similar results. Moreover, our behavioral take is supported by a real experiment led by [Amano et al. \(2011\)](#), who find that price level targeting is better suited to (real) agents' beliefs, which are presumably boundedly rational.

Although the behavioral approach used here is novel and provides some new insights into optimal policy relative to the previous literature, our model, as well as the new Keynesian literature broadly, is characterized by the 'representative' agent, which does not allow for the study of the impact of heterogeneous myopia on the optimal conduct of monetary policy. Recently, [Elbittar et al. \(2018\)](#) emphasizes that aggregation from the individual level to the macro level is not evident and that the outcomes in the two cases may shift significantly. Nevertheless, and unlike [Gabaix \(2016\)](#), the case of full myopia we present here is a mix of different types of myopia that may be seen as an aggregation of heterogeneous agents' beliefs.

6.2 Commitment

Note that relaxing the rational agent hypothesis contributes, under all the targeting regimes we consider, to addressing one of the critiques ([Walsh, 2017](#); [Fuhrer and Moore, 1995](#)) made of the New Keynesian model, namely, the persistence of macroeconomic variables with respect to monetary policy shocks. Indeed, proposition 2 (Section 4.1) implies that the optimal choice of inflation and output gap (and, implicitly, the policy instrument) depends on endogenous variables' past values. These findings, small deviations from rationality by assuming near-rational agents, are in line with [Woodford \(2010\)](#). One can infer that assuming more *realistic* agents in the New Keynesian model would provide a more accurate replication of the real world.

Regarding the hierarchy among different targeting regimes, the results are striking in several respects. From the household welfare criterion, it is clear that flexible inflation targeting appears to perform slightly better than flexible price level targeting for all forms of myopia. However, the results of the impulse response functions show that under flexible inflation targeting, the central bank acts like a price level targeter in the sense that, following a cost-push shock, it engineers smoother deflation to accommodate the observed rise in prices. Regarding the impact of myopia, it appears that both the general and full myopia cases, under inflation targeting, exhibit slower adjustment of prices after the shock compared to other cases, which move in line with the rational agent case. The smooth reactions observed in the presence of myopia echo our earlier finding that, under commitment, the central bank decides to adjust sooner or later according not only to its own preferences but also the form of myopia in place. Hence, this contrast between the results regarding household welfare loss and the impulse response functions leads to no

affirmative conclusion regarding whether inflation targeting or price level targeting is superior.

However, regardless of whether the central bank operates under flexible inflation targeting or flexible price level targeting, it is crucial for the policymaker to closely assess agents' myopia. Under commitment, the outcomes of the central bank's reactions depend on people's expectations. Bounded rationality distorts those private expectations, and consequently, monetary policy might be futile if people's expectations are ignored. Unlike [Eggertsson and Woodford \(2003\)](#), who demonstrate the crucial role of expectations management by the central bank in certain circumstances²³ (ZLB), our results extend this view even to normal times, as people's expectations do not match the usual assumptions of macroeconomic models.

6.3 Optimal simple rules

The evaluation of the instrument rules indicates the desirability of strict price level targeting over other monetary policy targeting regimes, which is in line with the literature surveyed by [Hatcher and Minford \(2016\)](#). A more general result that we derived concerns strict rules, which are more desirable (i.e., induce less welfare loss) than flexible rules.

When the policymaker reacts positively to output, tighter monetary policy implies a decline in the output gap, and in turn, the agents' welfare (which depends on output) decreases. Such a result, supported by [Schmitt-Grohé and Uribe \(2007\)](#), shows that our interest rate rules feature a positive reaction to output that implies significant welfare losses.

Regarding myopia, behavioral agents experience a significant welfare loss when they are output-gap-myopic compared to the cases of rational actors and some other types of myopia. Let us consider, for instance, monetary policy tightening. The reaction of output-gap-myopic agents is to reduce the output gap by more than a rational agent would. This effect will transmit to inflation. As a result, the outcomes of both inflation and output will be worse than in the rational case, and consequently, the associated welfare will decrease. The same mechanics are at work, but inversely, in the case of general or full myopia, where the welfare loss is smaller than in the rational case.

²³Mentioning the zero lower bound (ZLB) problem and the solutions engineered during the crisis lead us to the case of forward guidance policies. It appears from many central banks experiences that the implementation of forward guidance has had very limited impact compared with what the conventional wisdom (the New Keynesian model with rational agents) would suggest. Under the assumption of boundedly rational agents, this puzzle is resolved, as the impact of forward guidance is no longer powerful ([Gabaix, 2016](#)).

6.4 Bounded rationality

As monetary policy is about managing expectations (King et al., 2008), agents' perceptions of the economy become central to monetary policy analysis. Hereafter, we highlight, in two points, some results regarding bounded rationality that are common to all the targeting regimes and policy designs considered here.

First, deviation from the rational benchmark can take different forms. Each type of bounded rationality has particular properties regarding its impact on monetary policy conduct and households' welfare. Interest rate myopia does not differ substantially from the rational case, as it affects only the interest rate's amplitude but does not significantly affect output or inflation. Consequently, the interest-rate-myopic households' welfare is close to the rational households' welfare. The worst case appears to be output gap myopia. Even with optimal reactions by the central bank, it delivers a welfare loss that is greater than in the other cases, including the rational case. Moreover, inflation myopia is similar to the rational case in terms of central bank's reactions but not in terms of welfare, as our results show lower welfare losses compared to the rational benchmark. For the remaining cases, full or general myopia lead to substantial reactions by the central bank but welfare losses being reduced by more than the rational case. Overall, boundedly rational agents form their expectations based on misperceived representations of the economy, which suggests smoother current macroeconomic outcomes and, consequently, delivering moderate welfare losses compared to the rational case. In simple terms, ignoring some aspects of reality may be welfare-increasing.

Second, bounded rationality does not impact the choice of the monetary policy targeting regime. The way in which we model bounded rationality implies a quasi-symmetric impact on all the policy cases considered. Myopia is inherent to the agents, and under every monetary policy targeting regime, such an agent will behave according to his representation of the economy, which is independent of monetary policy. As Kahneman (2003) argues, boundedly rational agents behave intuitively; they do not take guidance from the policy targeting regimes in place and then act accordingly, but instead, they act on the basis of what they perceive at the moment. In every policymaking case, the central bank must assess the agents' bounded rationality.

7 Policy implications

To the extent that the optimal targeting rules consist of myopia terms, the type of bounded rationality at work and its amplitude necessitate close attention from

the central bank²⁴ (Section 3 and Section 4). Whatever monetary policy targeting regime is chosen by the central bank, bounded rationality significantly influences policy reactions relative to the rational case.

Under discretion, a flexible price level appears to deliver good stabilizing properties in terms of inflation variability. While commitment suggests the desirability, in terms of welfare, of flexible inflation targeting, monetary policy making under bounded rationality acts *as if* the central bank is price level targeter; “bygones are not bygones” when the economy is hit by a cost-push shock. Regarding the optimal simple rules, strict price level targeting delivers the best welfare outcomes. Note that in the presence of inflation myopia, a problem of “non-stationarity” in the price level arises when the central bank adopts strict inflation or strict nominal GDP growth targeting.

Overall, price level targeting appears to be a good candidate for the actual inflation targeting paradigm of a central bank. In an environment where agents are boundedly rational, such a targeting regime may enhance economic stabilization and thus societal welfare. After a cost-push shock, agents expect an increase in the price level path, which then contracts to return to its target, and this explains why they do not transmit this shock to current prices. This transmission effect is attenuated when agents are myopic, which entails substantial welfare gains. Another practical policy implication relates to agents’ expectations and central banks’ desire to educate them. Central banks have, for several decades, been educating agents in economics, to, among other objectives, increase public comprehension when communicating their monetary policies. Such a program may be perceived as an effort by policymakers to attenuate myopia, thus guiding actors to rationality. Even if bounded rationality is not a curable disease, and even not always a disease (as we demonstrated previously, myopia improves sometimes welfare), it is inherent to human functioning and should motivate central banks to act using the correct tools and analyze and use myopia for welfare-increasing purposes. Borrowing an analogy from [Thaler \(2016\)](#), the central bank should invest in studying the degree to which *Homo sapiens* are myopic and act consistently rather than educate people and attempt to transform humans into *Homo economicus*.

8 Conclusion

Following the suggestion of [Gabaix \(2016\)](#), optimal monetary policy under discretion, commitment and simple rules is assessed in a behavioral New Keynesian framework. Under both the discretion and commitment designs, several policy loss functions are examined, namely, flexible and strict inflation, price level and

²⁴In this paper, we do not model central bank myopia, which should be the subject of further research.

nominal GDP growth and level targeting regimes. The instrument rules considered reflect the same targeting regimes. All of these policy types are examined through variants of the behavioral New Keynesian model that emphasize different behaviors (output gap, interest rate, inflation, general or full myopia).

Bounded agent rationality matters for the conduct of monetary policy. We show that all monetary policy targeting regimes involve the central bank knowing the myopia characterizing the agents. Our welfare evaluation indicates that under discretion, the optimal monetary policy is flexible price level targeting, while under commitment, it is flexible inflation targeting (but the policymaker acts *as if* he employs flexible price level targeting). Optimal simple rules indicate that strict price level targeting delivers the lowest household welfare loss. This result demonstrates that price level targeting is a good candidate for the conduct of monetary policy and, importantly, that the choice of the targeting regime is independent of the form of myopia characterizing agents.

However, the latter has important consequences for the amplitude of the central bank's reactions to a cost-push shock. For instance, under both discretion and commitment, flexible price level targeting in the case of full myopia implies a larger reaction by the central bank than would prevail in the rational case (by more than 1%), and the case of general myopia yields to a greater reaction (by 0.5%) compared to the rational case. Intuitively, the extreme case of full myopia calls for important intervention by the central bank to offset the shock's impact. Surprisingly, myopia is not always associated with increases in welfare compensating variation. With the correct actions by the central bank, boundedly rational agents benefit from higher welfare compared to the rational case, to the extent that the presence of myopia implies lower inflation volatility. In contrast, the case of output gap myopia implies a significant loss in welfare, thus emphasizing the risk associated with inattention to measures of real economic activity.

This study offers a variety of further research ideas. Our basic model should be extended to include open economy and labor market features. One could further enrich the monetary policy analysis with such improvements. Additionally, non-linearities in monetary policy transmission channels and behavioral linkages cannot be addressed by our model. Our model is also silent with respect to central bank myopia and the endogeneity of myopia, which may impact monetary policy interactions under bounded rationality.

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9 Appendix

A Derivation of the IS curve equation

Deriving the Taylor expansion of the consumption deviations: Feynman-Kac method

The Lagrangian of the problem described is

$$L = \sum_{t=0}^{\infty} \beta^t u(c_t, N_t) + \sum_{t=0}^{\infty} \beta^t \lambda_t^k (k_t - (1 + r_t)(k_{t-1} - c_{t-1} + y_{t-1})) \quad (43)$$

where $r_t = \bar{r} + m_r \hat{r}_t$, $y_t = \bar{y} + m_y \hat{y}_t$, λ_t^k is the Lagrangian multiplier, which is equal to $V_k(k_t)$, the derivative of the value function with respect to k .

The value function is defined as²⁵ $V(k_t) = \max_c u(c) + \beta V(k_{t+1})$

At the optimum, the agent solves the following problem: $V(k) = \max_{c,k} L$.

The envelope theorem implies that

$$V_{r_t} = L_{r_t} = \beta^t [u_r(c_t) + \beta \lambda_t^k (k_t - c_t + y_t)] \quad (44)$$

By deriving this expression with respect to k_0 , we find that

$$V_{r_t, k_0} = \beta^t \frac{Dk_t}{Dk_0} D_{k_t} [u_r(c_t) + \beta \lambda_t^k (k_t - c_t + y_t)] \quad (45)$$

By applying this formula to the problem at hand, and taking into account the derivative of the value function in the default case, $\lambda_t^k = V_{k_t} = \left(\frac{\bar{r}k_t + R\bar{y}}{R}\right)^{-\gamma}$, we obtain

$$V_{r,k} = \beta^t D_k \left[\beta \left(\frac{\bar{r}k_t + R\bar{y}}{R} \right)^{-\gamma} \frac{k_t}{R} \right] \quad (46)$$

²⁵In this description, we omit the variable L_t , denoting labor supply, because we are considering the FOC with respect to consumption

By deriving and simplifying the expression above, we obtain

$$V_{r,k} = \frac{1}{R^{t+2}} c_0^{-\gamma-1} \left(-\gamma \frac{\bar{r}}{R} k_0 + c_0 \right) \quad (47)$$

Since $u_{c_0} = V_{k_0}$, we have $u_{cc} \partial_{\bar{r}} c_0 = \partial_{\bar{r}} V_{k_0}$, which implies

$$\partial_{\bar{r}} c_0 = \frac{\partial_{\bar{r}} V_{k_0}}{u_{cc}} = \frac{1}{R^{t+2}} \left(\frac{\bar{r}}{R} k_0 - \frac{1}{\gamma} c_0 \right) \quad (48)$$

which gives the expression for $b_r(k_t) = \frac{1}{R^{t+2}} \left(\frac{\bar{r}}{R} k_0 - \frac{1}{\gamma} c_0 \right)$.

Now, we take the derivative of the value function with respect to y_t . Applying the envelope theorem yields

$$V_{y_t} = L_{y_t} = \beta^t [u_y(c_t) + \beta \lambda_t^k (1 + r_t)] \quad (49)$$

By deriving this expression with respect to k_0 , we find the following expression:

$$V_{y_t, k_0} = \beta^t \frac{D k_t}{D k_0} D_{k_t} [u_y(c_t) + \beta \lambda_t^k (1 + r_t)] \quad (50)$$

Eq. 50 can be simplified as

$$V_{y_t, k_0} = \frac{1}{R^t} \left(-\gamma \frac{r}{R} c_0^{-\gamma-1} \right) \quad (51)$$

Since $u_{c_0} = V_{k_0}$, we have $u_{cc} \partial_{\bar{y}} c_0 = \partial_{\bar{y}} V_{k_0}$, which implies

$$\partial_{\bar{y}} c_0 = \frac{\partial_{\bar{y}} V_{k_0}}{u_{cc}} = \frac{r}{R^{t+1}} \quad (52)$$

Once we obtain Eqs. 48 and 52, the Taylor expansion of \hat{c} can be expressed as

$$\hat{c}_t = \mathbb{E}_t \sum_{\tau \geq t} \frac{b_r|_{k=0} \hat{r}_\tau + b_y \hat{y}_\tau}{R^{\tau-t+1}} \quad (53)$$

where $b_r = \frac{1}{R} \left(\frac{\bar{r}}{R} k_0 - \frac{1}{\gamma} c_0 \right)$ and $b_y = r$.

For the behavioral agent expression, 53 becomes

$$\hat{c}_t = \mathbb{E}_t^{BR} \sum_{\tau \geq t} \frac{b_r|_{k=0} \hat{r}_\tau + b_y \hat{y}_\tau}{R^{\tau-t+1}} \quad (54)$$

Recall from Gabaix (2016) the term structure of attention: $\mathbb{E}_t^{BR}[\hat{r}_{t+k}] = m_r \bar{m}^k \mathbb{E}_t[\hat{r}_{t+k}]$ and $\mathbb{E}_t^{BR}[\hat{y}_{t+k}] = m_y \bar{m}^k \mathbb{E}_t[\hat{y}_{t+k}]$, where \bar{m} , m_r and m_y are general, interest rate

and revenue myopia, respectively. By replacing those expressions in Eq. 54, we obtain

$$\widehat{c}_t = \mathbb{E}_t \sum_{\tau \geq t} \frac{\overline{m}^{\tau-t}}{R^{\tau-t+1}} (b_{r|k=0} m_r \widehat{r}_\tau + b_y m_y \widehat{y}_\tau) \quad (55)$$

Dividing Eq. 55 by \bar{c} , we find

$$\frac{\widehat{c}_t}{\bar{c}} = \mathbb{E}_t \sum_{\tau \geq t} \frac{\overline{m}^{\tau-t}}{R^{\tau-t+1}} \left(\frac{b_{r|k=0}}{\bar{c}} m_r \widehat{r}_\tau + b_y m_y \frac{\widehat{y}_\tau}{\bar{c}} \right) \quad (56)$$

The market clearing condition is $y_t = c_t$, and thus, $\frac{\widehat{c}_t}{\bar{c}} = \frac{\widehat{y}_t}{\bar{c}} = \widetilde{y}_t$ is the output gap. Moreover, $\frac{b_{r|k=0}}{\bar{c}} = \frac{1}{\bar{c}} \frac{1}{R} \left(-\frac{1}{\gamma} c_0 \right) = -\frac{1}{\gamma R}$. Then, Eq. 56 becomes

$$\widetilde{y}_t = \mathbb{E}_t \sum_{\tau \geq t} \frac{\overline{m}^{\tau-t}}{R^{\tau-t+1}} \left(-\frac{1}{\gamma R} m_r \widehat{r}_\tau + r m_y \widetilde{y}_\tau \right) \quad (57)$$

Expanding this expression yields

$$\widetilde{y}_t = -\frac{1}{\gamma R^2} m_r \widehat{r}_t + \frac{r}{R} m_y \widetilde{y}_t + \frac{\overline{m}}{R} \mathbb{E}_t \widetilde{y}_{t+1} \quad (58)$$

which can be simplified to

$$\widetilde{y}_t = M \mathbb{E}_t [\widetilde{y}_{t+1}] - \sigma \widehat{r}_t \quad (59)$$

where $M = \frac{\overline{m}}{R - r m_y}$, $\sigma = \frac{m_r}{\gamma R (R - r m_y)}$ and $R = 1/\beta$.

B Derivation of the Phillips curve

The problem of the behavioral firm is then to maximize

$$\sum_{k=0}^{\infty} \theta_p^k \mathbb{E}_t^{BR} [\Lambda_{t,t+k} (P_t^* Y_{t+k|t} - \Psi_{t+k} (Y_{t+k|t}))] \quad (60)$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \quad (61)$$

where $\Lambda_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\gamma} (P_{t+k}/P_t)$ is the stochastic discount factor in nominal terms, $\Psi_{t+k}(\cdot)$ is the cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t .

The first-order condition (FOC) of the problem is the following:

$$\sum_{k=0}^{\infty} \theta_p^k \mathbb{E}_t^{BR} [\Lambda_{t,t+k} Y_{t+k|t} (P_t^* - \mathcal{M} \psi_{t+k|t})] \quad (62)$$

where $\mathcal{M} = \frac{\varepsilon}{\varepsilon-1}$ is the desired or frictionless markup.

By expanding the FOC, Eq. 62, of the firm's problem around the zero-inflation steady state, we obtain Eq. 14.

The (log) marginal cost can be expressed as

$$\psi_{t+k|t} = \psi_{t+k} - \frac{\alpha \varepsilon}{1 - \alpha} (p_t^* - p_{t+k}) \quad (63)$$

By replacing Eq. 63 in Eq. 14, we obtain

$$p_t^* = \mu + (1 - \beta \theta) \sum_{k \geq 0} (\beta \theta)^k \mathbb{E}_t^{BR} \left\{ \psi_{t+k} - \frac{\alpha \varepsilon}{1 - \alpha} (p_t^* - p_{t+k}) \right\} \quad (64)$$

After writing μ as a geometric series and rearranging terms, we obtain

$$p_t^* = (1 - \beta \theta) \sum_{k \geq 0} (\beta \theta)^k \mathbb{E}_t^{BR} \left[\psi_{t+k} - \frac{\alpha \varepsilon}{1 - \alpha} (p_t^* - p_{t+k}) + \mu \right] \quad (65)$$

Using the fact that $\psi_{t+k} = p_{t+k} - \mu_{t+k}$ and $\widehat{\mu}_{t+k} = \mu_{t+k} - \mu$, we find the following expression:

$$p_t^* = (1 - \beta \theta) \sum_{k \geq 0} (\beta \theta)^k \mathbb{E}_t^{BR} \left[p_{t+k} - \widehat{\mu}_{t+k} - \frac{\alpha \varepsilon}{1 - \alpha} (p_t^* - p_{t+k}) \right] \quad (66)$$

which is equivalent to

$$p_t^* = (1 - \beta \theta) \sum_{k \geq 0} (\beta \theta)^k \mathbb{E}_t^{BR} \left[p_{t+k} + \frac{\alpha \varepsilon}{1 - \alpha} p_{t+k} - \widehat{\mu}_{t+k} - \frac{\alpha \varepsilon}{1 - \alpha} p_t^* \right] \quad (67)$$

By grouping the terms with p_{t+k} and separating the term with p_t^* from the sum, we obtain

$$p_t^* = (1 - \beta \theta) \sum_{k \geq 0} (\beta \theta)^k \mathbb{E}_t^{BR} \left[\frac{1 - \alpha + \alpha \varepsilon}{1 - \alpha} p_{t+k} - \widehat{\mu}_{t+k} \right] - \frac{\alpha \varepsilon}{1 - \alpha} p_t^* \quad (68)$$

Rearranging terms yields

$$\left(1 + \frac{\alpha \varepsilon}{1 - \alpha} \right) p_t^* = (1 - \beta \theta) \sum_{k \geq 0} (\beta \theta)^k \mathbb{E}_t^{BR} \left[\frac{1 - \alpha + \alpha \varepsilon}{1 - \alpha} p_{t+k} - \widehat{\mu}_{t+k} \right] \quad (69)$$

which simplifies to

$$p_t^* = (1 - \beta\theta) \sum_{k \geq 0} (\beta\theta)^k \mathbb{E}_t^{BR} \left[p_{t+k} - \frac{1 - \alpha}{1 - \alpha + \alpha\epsilon} \widehat{\mu}_{t+k} \right] \quad (70)$$

If we take the notation $\Theta = \frac{1 - \alpha}{1 - \alpha + \alpha\epsilon}$, the previous expression becomes

$$p_t^* = (1 - \beta\theta) \sum_{k \geq 0} (\beta\theta)^k \mathbb{E}_t^{BR} [p_{t+k} - \Theta \widehat{\mu}_{t+k}] \quad (71)$$

We recall the term structure of expectations from [Gabaix \(2016\)](#): $\mathbb{E}_t^{BR}[p_{t+k}] = m_\pi^f \bar{m}^k \mathbb{E}_t[p_{t+k}]$ and $\mathbb{E}_t^{BR}[\widehat{\mu}_{t+k}] = m_x^f \bar{m}^k \mathbb{E}_t[\widehat{\mu}_{t+k}]$, where \bar{m} is the general myopia associated with the inattention to the evolution of the economy's state, m_π^f is the myopia associated with inattention to prices, and m_x^f is the myopia related to inattention to output. Then, the Eq. 71 simply becomes

$$p_t^* = (1 - \beta\theta) \sum_{k \geq 0} (\beta\theta \bar{m})^k \mathbb{E}_t [m_\pi^f p_{t+k} - \Theta m_x^f \widehat{\mu}_{t+k}] \quad (72)$$

By removing the term associated with $k = 0$ from the sum, we have

$$\begin{aligned} p_t^* &= (1 - \beta\theta) \sum_{k \geq 1} (\beta\theta \bar{m})^k \mathbb{E}_t [m_\pi^f p_{t+k} - \Theta m_x^f \widehat{\mu}_{t+k}] \\ &\quad + (1 - \beta\theta) (m_\pi^f p_t - \Theta m_x^f \widehat{\mu}_t) \end{aligned} \quad (73)$$

As

$$\sum_{k \geq 1} (\beta\theta \bar{m})^k \mathbb{E}_t [m_\pi^f p_{t+k} - \Theta m_x^f \widehat{\mu}_{t+k}] = \beta\theta \bar{m} \mathbb{E}_t p_{t+1}^* \quad (74)$$

we obtain

$$p_t^* = \beta\theta \bar{m} \mathbb{E}_t [p_{t+1}^*] + (1 - \beta\theta) (m_\pi^f p_t - \Theta m_x^f \widehat{\mu}_t) \quad (75)$$

By using the relationship $\pi_t = (1 - \theta) (p_t^* - p_{t-1})$, which means that $p_t^* = \frac{1}{1 - \theta} \pi_t + p_{t-1}$, we find

$$\begin{aligned} \frac{1}{1 - \theta} \pi_t + p_{t-1} &= \beta\theta \bar{m} \mathbb{E}_t \left[\frac{1}{1 - \theta} \pi_{t+1} + p_t \right] \\ &\quad + (1 - \beta\theta) (m_\pi^f p_t - \Theta m_x^f \widehat{\mu}_t) \end{aligned} \quad (76)$$

which leads to the following inflation dynamics:

$$\begin{aligned} \frac{1}{1 - \theta} \pi_t &= \frac{\beta\theta \bar{m}}{1 - \theta} \mathbb{E}_t [\pi_{t+1}] - (1 - \beta\theta) \Theta m_x^f \widehat{\mu}_t \\ &\quad + [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] \end{aligned} \quad (77)$$

Now, we express the real marginal cost, μ_t , as a function of the output gap, \tilde{y}_t . Notice that the real marginal cost is defined in terms of the real wage and marginal productivity of labor:

$$\mu_t = -w_t - mpn_t \quad (78)$$

Using the facts that the real wage equals the marginal rate of substitution between consumption and labor and marginal productivity can be derived from Eq. 11, expression Eq. 78 can be written as

$$\mu_t = -(\gamma y_t + \phi n_t) - (y_t - n_t) - \ln(1 - \alpha) \quad (79)$$

Now we use the production function Eq. 11 to eliminate n_t from Eq. 79, and we obtain

$$\mu_t = -\left(\gamma + \frac{\phi + \alpha}{1 - \alpha}\right) y_t + \frac{1 + \phi}{1 - \alpha} a_t - \ln(1 - \alpha) \quad (80)$$

Writing Eq. 80 in the flexible price economy yields

$$\mu = -\left(\gamma + \frac{\phi + \alpha}{1 - \alpha}\right) y_t^n + \frac{1 + \phi}{1 - \alpha} a_t - \ln(1 - \alpha) \quad (81)$$

where y_t^n is the natural output. Finally, by subtracting Eq. 81 from Eq. 80, we obtain

$$\hat{\mu}_t = -\left(\gamma + \frac{\phi + \alpha}{1 - \alpha}\right) (y_t - y_t^n) = -\left(\gamma + \frac{\phi + \alpha}{1 - \alpha}\right) \tilde{y}_t \quad (82)$$

Finally, by replacing Eq. 82 in the price setting Eq. 77 we obtain the following expression:

$$\begin{aligned} \frac{1}{1 - \theta} \pi_t &= \frac{\beta \theta \bar{m}}{1 - \theta} \mathbb{E}_t[\pi_{t+1}] + (1 - \beta \theta) \Theta m_x^f \left(\gamma + \frac{\phi + \alpha}{1 - \alpha}\right) \tilde{y}_t \\ &\quad + [(1 - \beta \theta) m_\pi^f p_t + \beta \theta \bar{m} p_t - p_{t-1}] \end{aligned} \quad (83)$$

The resulting behavioral Phillips curve is the following:

$$\begin{aligned} \pi_t &= \beta M^f \mathbb{E}_t[\pi_{t+1}] + \kappa \tilde{y}_t \\ &\quad + (1 - \theta) [(1 - \beta \theta) m_\pi^f p_t + \beta \theta \bar{m} p_t - p_{t-1}] \end{aligned} \quad (84)$$

where $M^f = \theta \bar{m}$ and $\kappa = (1 - \theta) (1 - \beta \theta) \Theta m_x^f \left(\gamma + \frac{\phi + \alpha}{1 - \alpha}\right)$. Note that if we take the rational case where $m_x^f = m_\pi^f = \bar{m} = 1$, we end up with the usual Phillips curve as in Galí (2015).

C Derivation of the natural output

The marginal cost of a firm is defined as

$$\mu_t = -w_t - mpn_t \quad (85)$$

where mpn_t is the marginal productivity of labor. Recall that the marginal rate of substitution between labor and consumption equals the real wage, which can be expressed as

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \quad (86)$$

Taking logs, we obtain $w_t = \phi n_t + \gamma c_t$.

For the marginal productivity of labor in logs, we have

$$mpn_t = a - \alpha n_t + \ln(1 - \alpha) \quad (87)$$

and because the production function takes the form $y_t = a_t + (1 - \alpha)n_t$, we can express the marginal cost formula in terms of output and a technological factor as

$$\mu_t = -\left(\gamma + \frac{\phi + \alpha}{1 - \alpha}\right)y_t - \frac{1 + \phi}{1 - \alpha}a_t - \ln(1 - \alpha) \quad (88)$$

By expressing this formula in the flexible price economy, we obtain

$$\mu = -\left(\gamma + \frac{\phi + \alpha}{1 - \alpha}\right)y_t^n - \frac{1 + \phi}{1 - \alpha}a_t - \ln(1 - \alpha) \quad (89)$$

where $\mu = \ln\left(\frac{\varepsilon}{\varepsilon - 1}\right)$ is the marginal cost prevailing under flexible prices, and y_t^n is the natural output. By solving for y_t^n , we obtain the expression for natural output as follows:

$$y_t^n = \frac{1 + \phi}{\phi + \alpha + \gamma(1 - \alpha)}a_t + \frac{(1 - \alpha)(-\mu + \ln(1 - \alpha))}{\phi + \alpha + \gamma(1 - \alpha)} \quad (90)$$

D Derivation of the efficient interest rate

The IS curve Eq. 91 is written as

$$\hat{y}_t = M\mathbb{E}_t[\hat{y}_{t+1}] - \sigma(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) \quad (91)$$

Note that the definitions of the output gap, \hat{y}_t , and the relevant output gap, x_t , are

$$\hat{y}_t = y_t - y_t^n \quad (92)$$

$$x_t = y_t - y_t^e \quad (93)$$

where y_t^n is the natural output, and y_t^e is the efficient output.

By employing those definitions, we can write the IS curve Eq. 17 as

$$y_t - y_t^n = M\mathbb{E}_t [y_{t+1} - y_{t+1}^n] - \sigma (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n) \quad (94)$$

which is equivalent to

$$y_t - y_t^e + y_t^e - y_t^n = M\mathbb{E}_t [y_{t+1} - y_{t+1}^e + y_{t+1}^e - y_{t+1}^n] - \sigma (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n) \quad (95)$$

Now we can introduce the welfare-relevant output gap:

$$x_t + y_t^e - y_t^n = M\mathbb{E}_t [x_{t+1} + y_{t+1}^e - y_{t+1}^n] - \sigma (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n) \quad (96)$$

which leads us to the following expression

$$x_t = M\mathbb{E}_t [x_{t+1}] + M\mathbb{E}_t [y_{t+1}^e - y_{t+1}^n] - (y_t^e - y_t^n) - \sigma (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^n) \quad (97)$$

Hence, we obtain

$$x_t = M\mathbb{E}_t [x_{t+1}] - \sigma (i_t - \mathbb{E}_t [\pi_{t+1}] - r_t^e) \quad (98)$$

where

$$r_t^e = r_t^n + \frac{1}{\sigma} (M\mathbb{E}_t [y_{t+1}^e - y_{t+1}^n] - (y_t^e - y_t^n)) \quad (99)$$

The expression for efficient interest rate in deviation form

By taking Eq. 99 in deviation from its flexible price economy counterpart, we can write

$$\begin{aligned} r_t^e - r_t^n &= \left[r_t^n + \frac{1}{\sigma} (M\mathbb{E}_t [y_{t+1}^e - y_{t+1}^n] - (y_t^e - y_t^n)) \right] \\ &\quad - \left[r_t^n + \frac{1}{\sigma} (M\mathbb{E}_t [y_{t+1}^n - y_{t+1}^n] - (y_t^n - y_t^n)) \right] \end{aligned} \quad (100)$$

Considering the notation $\hat{v} = v - v^n$, Eq. 100 can be simplified to

$$\hat{r}_t^e = \frac{1}{\sigma} (M\mathbb{E}_t [\hat{y}_{t+1}^e] - \hat{y}_t^e) \quad (101)$$

E Proof of Proposition 1

E.1 Flexible inflation targeting under discretion (F1D)

To obtain targeting rule Eq. 19, we write the Lagrangian of the central bank's problem as

$$\begin{aligned} \mathfrak{L} = & \frac{1}{2} (\pi_t^2 + \alpha_x x_t^2) \\ & + \lambda (\pi_t - \kappa x_t - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] - G_t) \end{aligned} \quad (102)$$

where λ is the Lagrangian multiplier, and $G_t = \beta M^f \mathbb{E}_t \pi_{t+1} + u_t$ is taken as given for the policymaker who operates under discretion. By differentiating \mathfrak{L} with respect to p_t and y_t , we find the following FOCs:

$$\pi_t + \lambda (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta \bar{m}]) = 0 \quad (103)$$

$$\alpha_x x_t - \lambda \kappa = 0 \quad (104)$$

By combining those FOCs to eliminate λ , we obtain targeting rule Eq. 19.

E.2 Flexible price level targeting under discretion (F2D)

To obtain targeting rule Eq. 20, we write the Lagrangian of the central bank's problem as

$$\begin{aligned} \mathfrak{L} = & \frac{1}{2} (p_t^2 + \alpha_x x_t^2) \\ & + \lambda (\pi_t - \kappa x_t - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] - G_t) \end{aligned} \quad (105)$$

where λ is the Lagrangian multiplier, and $G_t = \beta M^f \mathbb{E}_t \pi_{t+1} + u_t$ is taken as given for the policymaker who operates under discretion. By differentiating \mathfrak{L} with respect to p_t and y_t , we find the following FOCs:

$$p_t + \lambda (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta \bar{m}]) = 0 \quad (106)$$

$$\alpha_x x_t - \lambda \kappa = 0 \quad (107)$$

By combining those FOCs to eliminate λ , we obtain targeting rule Eq. 20.

E.3 Flexible nominal GDP growth targeting under discretion (F3D)

We write the Lagrangian for the policy problem as follows:

$$\begin{aligned} \mathfrak{L} = & \frac{1}{2} [(\pi_t + \Delta y_t)^2 + \alpha_x x_t^2] \\ & + \lambda(\pi_t - \kappa x_t - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] - G_t) \end{aligned} \quad (108)$$

where λ and G_t are defined as previously. Deriving \mathfrak{L} with respect to p_t and y_t yields the following FOCs:

$$g_t + \lambda (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta \bar{m}]) = 0 \quad (109)$$

$$g_t + \alpha_x x_t - \lambda \kappa = 0 \quad (110)$$

The second FOC implies that

$$\lambda = \frac{1}{\kappa} (g_t + \alpha_x x_t) \quad (111)$$

By replacing the value of λ in the first FOC, we can easily find our targeting rule Eq. 36.

E.4 Flexible nominal GDP level targeting under discretion (F4D)

We write the Lagrangian for the policy problem as follows:

$$\begin{aligned} \mathfrak{L} = & \frac{1}{2} [(p_t + y_t)^2 + \alpha_x x_t^2] \\ & + \lambda(\pi_t - \kappa x_t - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] - G_t) \end{aligned} \quad (112)$$

where λ and G_t are defined as previously. Deriving \mathfrak{L} with respect to p_t and y_t yields the following FOCs:

$$n_t + \lambda (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta \bar{m}]) = 0 \quad (113)$$

$$n_t + \alpha_x x_t - \lambda \kappa = 0 \quad (114)$$

The second FOC implies that

$$\lambda = \frac{1}{\kappa} (n_t + \alpha_x x_t) \quad (115)$$

By replacing the value of λ in the first FOC, we can easily find our targeting rule Eq. 22.

E.5 Strict inflation targeting under discretion (S1D)

We write the Lagrangian of the central bank's problem as

$$\mathfrak{L} = \frac{1}{2}\pi_t^2 + \lambda (\pi_t - \kappa x_t - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] - G_t) \quad (116)$$

where λ is the Lagrangian multiplier, and $G_t = \beta M^f \mathbb{E}_t \pi_{t+1} + u_t$ is taken as given for the policymaker who operates under discretion. By differentiating \mathfrak{L} with respect to p_t and x_t , we find the following FOCs:

$$\pi_t + \lambda (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta \bar{m}]) = 0 \quad (117)$$

$$- \lambda \kappa = 0 \quad (118)$$

which implies that $\lambda = 0$, and hence, we find the targeting rule $\pi_t = 0$.

E.6 Strict price level targeting under discretion (S2D)

We write the Lagrangian of the central bank's problem as

$$\begin{aligned} \mathfrak{L} = & \frac{1}{2}p_t^2 \\ & + \lambda (\pi_t - \kappa x_t - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] - G_t) \end{aligned} \quad (119)$$

where λ is the Lagrangian multiplier, and $G_t = \beta M^f \mathbb{E}_t \pi_{t+1} + u_t$ is taken as given for the policymaker who operates under discretion. By differentiating \mathfrak{L} with respect to p_t and x_t , we find the following FOCs:

$$p_t + \lambda (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta \bar{m}]) = 0 \quad (120)$$

$$- \lambda \kappa = 0 \quad (121)$$

which implies that $\lambda = 0$, and hence, we find the targeting rule $p_t = 0$.

E.7 Strict nominal GDP growth targeting under discretion (S3D)

We write the Lagrangian of this problem as

$$\begin{aligned} \mathfrak{L} = & \frac{1}{2}(\pi_t + \Delta y_t)^2 \\ & + \lambda (\pi_t - \kappa x_t - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] - G_t) \end{aligned} \quad (122)$$

where λ is the Lagrangian multiplier, and $G_t = \beta M^f \mathbb{E}_t \pi_{t+1} + u_t$ is taken as given for the policymaker who operates under discretion. By differentiating \mathcal{L} with respect to p_t and y_t , we find the following FOCs:

$$\pi_t + \Delta y_t + \lambda (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}]) = 0 \quad (123)$$

$$\pi_t + \Delta y_t - \lambda\kappa = 0 \quad (124)$$

By combining those FOCs to eliminate λ , we obtain

$$\pi_t + \Delta y_t + (\pi_t + \Delta y_t) (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}]) \frac{1}{\kappa} = 0 \quad (125)$$

which is equivalent to

$$(\pi_t + \Delta y_t) \left[1 + \frac{1}{\kappa} (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}]) \right] = 0 \quad (126)$$

Because $1 + \frac{1}{\kappa} (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}])$ is different from zero, we obtain $\pi_t + \Delta y_t = 0$.

E.8 Strict nominal GDP level targeting under discretion (S4D)

We write the Lagrangian of this problem as

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} (p_t + y_t)^2 \\ & + \lambda (\pi_t - \kappa x_t - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta\bar{m} p_t - p_{t-1}] - G_t) \end{aligned} \quad (127)$$

where λ is the Lagrangian multiplier, and $G_t = \beta M^f \mathbb{E}_t \pi_{t+1} + u_t$ is taken as given for the policymaker who operates under discretion. By differentiating \mathcal{L} with respect to p_t and y_t , we find the following FOCs:

$$p_t + y_t + \lambda (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}]) = 0 \quad (128)$$

$$p_t + y_t - \lambda\kappa = 0 \quad (129)$$

By combining those FOCs to eliminate λ , we obtain

$$p_t + y_t + (p_t + y_t) (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}]) \frac{1}{\kappa} = 0 \quad (130)$$

which is equivalent to

$$(p_t + y_t) \left[1 + \frac{1}{\kappa} (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}]) \right] = 0 \quad (131)$$

Because $1 + \frac{1}{\kappa} (1 - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}])$ is different from zero, we obtain $p_t + y_t = 0$.

F Targeting rules under discretion in the rational case

Targeting regime	Central bank's targeting rule
F1D	$x_t = -\frac{\kappa_{re}}{\alpha_x} \pi_t$
F2D	$x_t = -\frac{\kappa_{re}}{\alpha_x} p_t$
F3D	$g_t = -\frac{\alpha_x}{1+\kappa_{re}} x_t$
F4D	$n_t = -\frac{\alpha_x}{1+\kappa_{re}} x_t$
S1D	$\pi_t = 0$
S2D	$p_t = 0$
S3D	$\pi_t = -\Delta y_t$
S4D	$y_t = -p_t$

Note that $\kappa_{re} = \frac{\kappa}{\theta}$, where κ_{re} is inflation's sensitivity to output in the (rational) Phillips curve in Galí (2015), and κ is our coefficient of output in the behavioral Phillips curve (Eq. 77)

G Calibration

Based on Galí (2015) and Gabaix (2016) for structural and behavioral parameters, respectively, Table 8 presents the calibration used to simulate our different models.

Note that for simplicity reasons, households revenue myopia (m_y) is not analyzed in this study.

H Proof of Proposition 2

H.1 Flexible inflation targeting under commitment (F1C)

The Lagrangian of the central bank's problem can be written as

$$\mathcal{L} = \sum_{t \geq 0} \beta^t \left[+\lambda_t \left(\begin{array}{c} \frac{1}{2} (\pi_t^2 + \alpha_x x_t^2) \\ \pi_t - \beta M^f \mathbb{E}_t \pi_{t+1} - \kappa x_t \\ -(1-\theta) [(1-\beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] \end{array} \right) \right] \quad (132)$$

Models	No myopia (Rational)	Interest rate myopia	Inflation myopia	Output gap myopia	General myopia	Full myopia
θ	2/3	2/3	2/3	2/3	2/3	2/3
γ	1	1	1	1	1	1
ϕ	1	1	1	1	1	1
β	0.99	0.99	0.99	0.99	0.99	0.99
α	1/3	1/3	1/3	1/3	1/3	1/3
ε	6	6	6	6	6	6
\bar{m}	1	1	1	1	0.85	0.85
m_r	1	0.85	1	1	1	0.85
m_y	1	1	1	1	1	0.85
m_x^f	1	1	1	0.85	1	0.85
m_π^f	1	1	0.85	1	1	0.85
ρ_a	0.75	0.75	0.75	0.75	0.75	0.75
ρ_u	0.2	0.2	0.2	0.2	0.2	0.2

Table 8: Calibration of the parameters used for the simulations

The FOCs with respect to p_t and x_t are the following:

$$\beta^t (\pi_t + \lambda_t [1 + \beta M^f - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}]]) + \beta^{t+1} (-\pi_{t+1} - \lambda_{t+1}\theta) + \beta^{t-1} (-\beta M^f \lambda_{t-1}) = 0 \quad (133)$$

$$\alpha_x x_t - \lambda_t \kappa = 0 \quad (134)$$

Eq. 134 implies

$$\lambda_t = \frac{\alpha_x}{\kappa} x_t \quad (135)$$

Replacing Eq. 135 in Eq. 133, we find our targeting rule 34.

H.2 Flexible price level targeting under commitment (F2C)

The Lagrangian of the central bank's problem can be written as

$$\mathcal{L} = \sum_{t \geq 0} \beta^t \left[+\lambda_t \left(\begin{array}{c} \frac{1}{2}(p_t^2 + \alpha_x x_t^2) \\ \pi_t - \beta M^f \mathbb{E}_t \pi_{t+1} - \kappa x_t \\ - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta\bar{m} p_t - p_{t-1}] \end{array} \right) \right] \quad (136)$$

The FOCs with respect to p_t and x_t are the following:

$$p_t + \lambda_t [1 + \beta M^f - (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m}]] - \lambda_{t+1} \beta \theta - M^f \lambda_{t-1} = 0 \quad (137)$$

$$\alpha_x x_t - \lambda_t \kappa = 0 \quad (138)$$

Eq. 138 implies

$$\lambda_t = \frac{\alpha_x}{\kappa} x_t \quad (139)$$

Replacing Eq. 139 in Eq. 137, we find our targeting rule 35.

H.3 Flexible nominal GDP growth targeting under commitment (F3C)

The Lagrangian of the central bank's problem can be written as

$$\mathfrak{L} = \sum_{t \geq 0} \beta^t \left[+\lambda_t \left(\begin{array}{c} \frac{1}{2} [(\pi_t + \Delta y_t)^2 + \alpha_x x_t^2] \\ \pi_t - \beta M^f \mathbb{E}_t \pi_{t+1} - \kappa x_t \\ -(1-\theta) [(1-\beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] \end{array} \right) \right] \quad (140)$$

The FOCs with respect to p_t and y_t are the following:

$$g_t - \beta g_{t+1} + \lambda_t [1 + \beta M^f - (1-\theta) [(1-\beta\theta) m_\pi^f + \beta\theta \bar{m}]] - \lambda_{t+1} \beta\theta - M^f \lambda_{t-1} = 0 \quad (141)$$

$$\beta^t (g_t + \alpha_x x_t - \lambda_t \kappa) + \beta^{t+1} (-g_{t+1}) = 0 \quad (142)$$

By solving for λ_t and substituting in the first FOC, we prove our result.

H.4 Flexible nominal GDP level targeting under commitment (F4C)

The Lagrangian of the central bank's problem can be written as

$$\mathfrak{L} = \sum_{t \geq 0} \beta^t \left[+\lambda_t \left(\begin{array}{c} \frac{1}{2} [(p_t + y_t)^2 + \alpha_x x_t^2] \\ \pi_t - \beta M^f \mathbb{E}_t \pi_{t+1} - \kappa x_t \\ -(1-\theta) [(1-\beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] \end{array} \right) \right] \quad (143)$$

The FOCs with respect to p_t and y_t are the following:

$$n_t + \lambda_t [1 + \beta M^f - (1-\theta) [(1-\beta\theta) m_\pi^f + \beta\theta \bar{m}]] - \lambda_{t+1} \beta\theta - M^f \lambda_{t-1} = 0 \quad (144)$$

$$\beta^t (n_t + \alpha_x x_t - \lambda_t \kappa) = 0 \quad (145)$$

By solving for λ_t and substituting in the first FOC, we prove our result.

H.5 Strict inflation targeting under commitment (S1C)

The Lagrangian of the central bank's problem can be written as

$$\mathfrak{L} = \sum_{t \geq 0} \beta^t \left[+\lambda_t \left(\begin{array}{c} \frac{1}{2} \pi_t^2 \\ \pi_t - \beta M^f \mathbb{E}_t \pi_{t+1} - \kappa x_t \\ -(1-\theta) [(1-\beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] \end{array} \right) \right] \quad (146)$$

The FOCs with respect to p_t and x_t are the following:

$$\pi_t + \lambda_t [1 + \beta M^f (1-\theta) [(1-\beta\theta) m_\pi^f + \beta\theta \bar{m}]] - \lambda_{t+1} \beta\theta - M^f \lambda_{t-1} = 0 \quad (147)$$

$$-\lambda_t \kappa = 0 \quad (148)$$

From Eq. 148, we can conclude that $\lambda_t = 0$, and plugging this into Eq. 147 yields the proposition's result.

H.6 Strict price level targeting under commitment (S2C)

The Lagrangian of the central bank's problem can be written as

$$\mathcal{L} = \sum_{t \geq 0} \beta^t \left[+\lambda_t \left(\begin{array}{c} \frac{1}{2} p_t^2 \\ \pi_t - \beta M^f \mathbb{E}_t \pi_{t+1} - \kappa x_t \\ - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] \end{array} \right) \right] \quad (149)$$

The FOCs with respect to p_t and x_t are the following:

$$p_t + \lambda_t [1 + \beta M^f (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta \bar{m}]] - \lambda_{t+1} \beta\theta - M^f \lambda_{t-1} = 0 \quad (150)$$

$$-\lambda_t \kappa = 0 \quad (151)$$

Eq. 151 implies that $\lambda_t = 0$, and when combined with Eq. 150, it yields the result.

H.7 Strict nominal GDP growth targeting under commitment (S3C)

The Lagrangian of the central bank's problem can be written as

$$\mathcal{L} = \sum_{t \geq 0} \beta^t \left[+\lambda_t \left(\begin{array}{c} \frac{1}{2} (\pi_t + \Delta y_t)^2 \\ \pi_t - \beta M^f \mathbb{E}_t \pi_{t+1} - \kappa x_t \\ - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] \end{array} \right) \right] \quad (152)$$

The FOCs with respect to p_t and y_t are

$$\begin{aligned} & (\pi_t + \Delta y_t) - \beta (\pi_{t+1} + \Delta y_{t+1}) \\ & + \lambda_t [1 + \beta M^f (1 - \theta) [(1 - \beta\theta) m_\pi^f + \beta\theta \bar{m}]] - \lambda_{t+1} \beta\theta - M^f \lambda_{t-1} = 0 \end{aligned} \quad (153)$$

$$(\pi_t + \Delta y_t) - \beta (\pi_{t+1} + \Delta y_{t+1}) - \kappa \lambda_t = 0 \quad (154)$$

which completes our proof.

H.8 Strict nominal GDP level targeting under commitment (S4C)

The Lagrangian of the central bank's problem can be written as

$$\mathcal{L} = \sum_{t \geq 0} \beta^t \left[+\lambda_t \left(\begin{array}{c} \frac{1}{2} (p_t + y_t)^2 \\ \pi_t - \beta M^f \mathbb{E}_t \pi_{t+1} - \kappa x_t \\ - (1 - \theta) [(1 - \beta\theta) m_\pi^f p_t + \beta\theta \bar{m} p_t - p_{t-1}] \end{array} \right) \right] \quad (155)$$

The FOCs with respect to p_t and x_t are the following:

$$p_t + y_t + \lambda_t \left[1 + \beta M^f (1 - \theta) \left[(1 - \beta\theta) m_\pi^f + \beta\theta\bar{m} \right] - \lambda_{t+1}\beta\theta - M^f \lambda_{t-1} \right] = 0 \quad (156)$$

$$p_t + y_t - \lambda_t \kappa = 0 \quad (157)$$

By solving Eq. 157 for λ

$$\lambda_t = \frac{1}{\kappa} (p_t + y_t) \quad (158)$$

Replace λ in Eq. 156 by the expression given in Eq. 158.

I Targeting rules under commitment in the rational case

Targeting regime **Central bank's targeting rule**

F1C $\pi_t = -\frac{\alpha_x}{\kappa_{re}}(x_t - x_{t-1})$

F2C $p_t = \frac{\alpha_x}{\kappa_{re}}x_{t-1} + \beta\frac{\alpha_x}{\kappa_{re}}x_{t+1} - \frac{\alpha_x}{\kappa_{re}}(\beta + 1)x_t$

F3C $\beta g_{t+1} - (1 + \kappa_{re} + \beta)g_t + g_{t-1} - \alpha_x(x_t - x_{t-1}) = 0$

F4C $-\beta n_{t+1} + (\kappa_{re} + 1 + \beta)n_t - n_{t-1} = \alpha_x \beta x_{t+1} - (1 + \beta)\alpha_x x_t + \alpha_x x_{t-1}$

S1C $\pi_t = 0$

S2C $p_t = 0$

S3C $\beta g_{t+1} - (1 + \kappa_{re} + \beta)g_t + g_{t-1} = 0$

S4C $\beta n_{t+1} - (1 + \kappa_{re} + \beta)n_t + n_{t-1} = 0$

Simplification of the (F1C) under the rational case

The targeting rule in the generalized case is the following:

$$\pi_t = \beta\pi_{t+1} + \frac{\alpha_x}{\kappa} M^f x_{t-1} + \beta\theta\frac{\alpha_x}{\kappa} x_{t+1} - \frac{\alpha_x}{\kappa} \left(1 + \beta M^f - (1 - \theta) \left[\beta\theta\bar{m} + (1 - \beta\theta) m_\pi^f \right] \right) x_t \quad (159)$$

For the rational case, we set $\bar{m} = m_\pi^f = m_x^f = 1$, and the targeting rule (Eq. 159) becomes

$$\pi_t = \beta\pi_{t+1} + \frac{\alpha_x}{\kappa}\theta x_{t-1} + \beta\theta\frac{\alpha_x}{\kappa}x_{t+1} - \frac{\alpha_x}{\kappa}\theta(1 + \beta)x_t \quad (160)$$

As $\kappa_{re} = \frac{\kappa}{\theta}$ and by rearranging the terms of (Eq. 160), we find

$$\pi_t - \beta\pi_{t+1} = -\frac{\alpha_x}{\kappa_{re}}(x_t - x_{t-1}) - \beta\left(-\frac{\alpha_x}{\kappa_{re}}\right)(x_{t+1} - x_t) \quad (161)$$

Solving (Eq. 161) as a polynomial equation for β , we obtain

$$\pi_t = -\frac{\alpha_x}{\kappa_{re}}(x_t - x_{t-1}) \quad (162)$$

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