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The Power of Forward Guidance and the Fiscal Theory of the Price Level

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Abstract

Standard New Keynesian models predict implausibly large and favorable responses of inflation and output to expansionary forward guidance on interest rates. We find that the introduction of permanent or recurring active fiscal policy dampens the response of output and inflation to forward guidance in the New Keynesian model. Moreover, the presence of regime-switching policy introduces expectation effects that cause forward guidance to be less stimulative in our regime-switching model's active money, passive fiscal policy regime. Finally, the introduction of long-term debt affects the magnitude of the stimulus resulting from forward guidance in models with active fiscal policy.

Keywords: Forward Guidance; Monetary-Fiscal Policy Interactions; Markov-Switching

JEL Classification: E63, D84, E50, E52, E58, E60

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1 Introduction

A large literature offers strong theoretical support for the use of expansionary forward guidance on interest rates, particularly when interest rates are constrained by the zero lower bound (see, for example, Eggertsson and Woodford, (2003)). Despite the effectiveness of forward guidance in theory, the predictions of workhorse New Keynesian models do not accord well with empirical studies of the effects of forward guidance in the U.S. (e.g. Del Negro et al (2015), D'Amico and King (2015)). That is, while New Keynesian models predict large responses of inflation and output to forward guidance on short-term rates, the empirical evidence points to responses that are positive but modest. This shortcoming of the New Keynesian model is dubbed "The Forward Guidance Puzzle" (Del Negro et al. (2015)), and it calls into question the ability of the standard New Keynesian model to predict the effects of anticipated monetary policy.

According to Del Negro et al. (2015), McKay et al. (2016), Carlstrom et al. (2012), Chung et al. (2015), Kiley (2014), the implausible responsiveness of output and inflation to forward guidance stems from three signature features of the New Keynesian model. First, consumption is excessively responsive to changes in interest rates. Second, the lack of a discount factor in the household's log-linearized Euler equation implies a strong response of consumption to long-run interest rates. Because forward guidance is designed to influence long-run rates, forward guidance naturally generates a large response in consumption through the Euler equation. Third, "front-loading" in the New Keynesian Phillips Curve renders inflation particularly sensitive to changes in current and future output. Together, the lack of discounting in the Euler equation and front-loading in the Phillips Curve generate a feedback loop that exacerbates the rise in inflation and output implied by forward guidance.

Several papers have addressed this puzzle by limiting the importance of these three features of the New Keynesian model. For instance, McKay et al. (2016) mute the response of agents to forward guidance by introducing borrowing constraints that prevent agents from drawing down their savings over the forward guidance horizon. Gabaix (2016) introduces an explicit discount factor into the Euler equation and an additional discount factor into the Phillips Curve to model myopic agents. Del Negro et al. (2015) show that a positive probability of death generates effective discounting in the Euler equation when they introduce a perpetual youth structure into the New Keynesian model. Chung et al. (2015) and Kiley (2014) introduce "sticky information" in the spirit of Mankiw and Reis (2002) to mitigate feedback effects from the Phillips Curve. Cole (2015) replaces rational expectations with a model of adaptive learning to demonstrate that bounded rationality lessens the effectiveness of forward guidance in specific policy experiments.

Unlike these attempts to explain the exaggerated response of inflation and output to forward guidance – which focus primarily on the specification of private sector behavior – this paper examines how the joint conduct of monetary and fiscal policy influences the effects of expansionary forward guidance in the New Keynesian model. Specifically, we show that the above mentioned exaggerated response of output and inflation to forward guidance may hinge on two assumptions (in addition to the three model features highlighted above): (1) the monetary authority employs an interest rate rule that satisfies the "Taylor Principle"; (2) fiscal policy is conducted in such a way that variation in fiscal surpluses acts to stabilize government debt, thereby rendering fiscal policy Ricardian.

Our approach is most closely related to Cochrane (2017a, 2017b, 2018a, 2018b), and Caramp and Silva (2018), which also study fiscal implications for anticipated monetary policy changes. Cochrane (2017b) suggests that fiscal considerations may help select equilibria with smaller initial price jumps in response to anticipated policy announcements.[1] Cochrane (2017b, 2018a, 2018b) demonstrate wealth effects of long term debt on forward guidance, but the focus of these analyses is not on the excessive responsiveness of output and inflation under passive fiscal policy. Caramp and Silva (2018) argues that the responsiveness of fiscal transfers to monetary policy generates wealth effects that explain the forward guidance puzzle, but their analysis also abstracts from fiscal policy stances on the debt.[2] In contrast to these works, we explicitly characterize fiscal policy regimes and study how wealth effects arising in these regimes may reduce the responses of inflation and output to forward guidance. Moreover, we model recurring fiscal regimes to capture how uncertainty about future fiscal policy impacts the effectiveness of forward guidance.

Our paper, of course, attempts to contribute to a broader monetary-fiscal policy interactions literature that examines regime-switching expectation effects and fiscal constraints on the effectiveness of unconventional monetary policy. Here we mention only a few recent papers that contribute to this literature. Chung et al. (2007), Davig and Leeper (2011), Ascari et al. (2017), Bianchi and Ilut (2017), Bianchi and Melosi (2017), Corhay et al. (2017) are all recent examples of papers that examine expectation effects of recurring fiscal and monetary policy regimes on current policy outcomes. In particular, Corhay et al. (2017) and Bianchi and Melosi (2017) examine these expectation effects in Great Recession environments or at the zero lower bound. Additionally, Sims (2013), Leeper and Leith (2016), Corhay et al. (2017), all examine unconventional policy and the maturity structure of debt in economies with non-Ricardian fiscal policymakers. Ascari et al. (2017) discusses forward guidance on future policy *regimes* and its implications for the efficacy of monetary and fiscal policy at the zero lower bound. Unlike the papers mentioned in this paragraph, we study the

[1]Cochrane (2017b) uses a bond valuation equation, which we introduce in (2), to point out the idea that the large price adjustments predicted in the standard "forward-stable" model equilibrium must be supported by changes in the present value of expected future surpluses.

[2]The wealth effects they attribute their results to would naturally arise under a passive fiscal policy. Similarly, active fiscal policy naturally eliminates said wealth effects.

forward guidance puzzle defined above. Our distinct contribution to this literature is straightforward: permanent or recurring active fiscal policy regimes can dampen the responsiveness of output and inflation to forward guidance on short-term nominal interest rates. We demonstrate this using an experiment that resembles recent Federal Reserve policy, and we include long-term debt to demonstrate a degree of flexibility in our impulse responses.

This works borrows heavily from the Fiscal Theory of the Price Level literature, which models inflation as the outcome of both monetary and fiscal policy (see Leeper and Leith (2016) or Cochrane (2018b) for a review of the Fiscal Theory of the Price Level). Work in this literature distinguishes between "passive" policymakers who are constrained to stabilize the government debt, and "active" policymakers who determine inflation. Intuitively, a monetary policy regime is active (passive) when interest rates respond strongly (weakly) to inflation, and a fiscal policy regime is passive (active) when a permanent implementation of said policy regime satisfies (violates) Ricardian equivalence. Careful consideration of the passive-active dichotomy reveals a number of channels through which the fiscal policy stance impacts the response of inflation and output to policy shocks.

This paper focuses on a specific channel through which active fiscal policy affects agents' perception of bond wealth. To illustrate this channel, we restrict attention to our model's intertemporal household budget constraint (assuming that all government debt is single-period debt):

$$E_t\left(\sum_{T=t}^{\infty} R_{t,T} P_T C_T\right) = E_t\left(\sum_{T=t}^{\infty}\left\{R_{t,T}[P_T Y_T - P_T \tau_T]\right\}\right) + B_{t-1} \tag{1}$$

where $R_{t,T}$ is the stochastic discount factor from time t to T, C is consumption, τ is the government's real primary surplus, P_T is the price level at T, B_{t-1} is the government debt stock that matures at t, and Y is income. Under the assumption that $Y_T = C_T\ \forall T$, and after substituting for $R_{t,T} = \beta^{T-t} u'(Y_T)P_t/u'(Y_t)P_T$, this equation reduces to

$$\frac{B_{t-1}}{P_t} = E_t\left(\sum_{T=t}^{\infty} \beta^{T-t}\frac{u'(Y_T)}{u'(Y_t)}\tau_T\right) \tag{2}$$

Equation 2 is the bond valuation equation in Cochrane (2001)[3], and it asserts that today's price level is determined by the real present value of expected future surpluses, $E_t\left(\sum_{T=t}^{\infty}\beta^{T-t}(u'(Y_T)/u'(Y_t))\tau_T\right)$, and the predetermined debt stock, B_{t-1}. From (1) it follows that any variation in B_{t-1} affects the household's consumption path, all else constant. When fiscal policy is passive, however, all else is not constant – any change in B_{t-1} induces an offsetting response in $\{\tau_T\}$ that leaves the households choice set intact. In other words, fiscal policy satisfies Ricardian equivalence. In an active fiscal

[3]We allow for stochastic discount factors in our equation.

policy regime, variations in bond wealth are not totally offset by changes in the stream of expected surpluses, and this implies that the change in bonds has wealth effects.

One source of the aforementioned variation in bond wealth is monetary policy. For example, a reduction in interest rates might force bonds to a lower equilibrium path as lower rates alleviate the burden of rolling over existing debt. In a model with active fiscal policy, households are not compensated for their lower bond holdings with tax cuts, which causes the household to feel nominally constrained through (1). Moreover, the fall in bonds places downward pressure on prices through (2). The resulting effect of this monetary expansion is an eventual fall in output and prices. Much of what follows in this paper depends on the fact that forward guidance on short-term interest rates appears in the model as a series of anticipated interest rate cuts.

We illustrate the role played by monetary-fiscal policy interactions in determining the effects of forward guidance by allowing fiscal (monetary) policy to be permanently or recurrently active (passive). Our results are twofold. First, we find that the presence of active fiscal policy allows for forward guidance to have wealth effects that dampen the response of output and inflation to forward guidance. This result depends on the fact that agents view government debt as net wealth in a regime with active fiscal policy. Hence, an anticipated reduction in interest rates which places downward pressure on agents' nominal bond returns causes agents to feel more constrained today. This mutes agents' responses to lower long-run real interest rates and induces firms to lower prices.

Second, the presence of switching in fiscal and monetary policy has expectation effects that may cause forward guidance to be less stimulative in the switching model's passive fiscal, active monetary policy regime. In such a setting, the *possibility* that fiscal policy may become active during the forward guidance horizon causes agents to become less optimistic about the effects of forward guidance in an economy where monetary and fiscal policy are *currently* active and passive, respectively. Interestingly, these spillover effects always attenuate the short-term effects of forward guidance, but can lead to more persistent responses of output and inflation, as we demonstrate in one specific case. Our Markov-switching approach helps to highlight the role that expectations play in generating a response of inflation and output to forward guidance, and builds on other papers that study expectation effects in similar modeling environments.[4]

Additionally, the presence of long-term government debt in a model with active fiscal, passive monetary policy introduces "revaluation effects" that mitigate the deflationary effects observed in the corresponding model with only short-term debt. We observe these effects because an anticipated reduction in short-term interest rates

[4]For example, see Davig and Leeper (2011), Chung et al. (2007), Ascari et al. (2017), Bianchi and Ilut (2017), Bianchi and Melosi (2017), Corhay et al. (2017).

raises the market value of outstanding debt. Thus, while a reduction in interest rates lowers aggregate demand due to lower interest rate receipts, it can also raise aggregate demand by raising the price of the debt that households own. Such an effect cannot be observed in a model without long-term debt. We note that the long-term debt effects observed here are hardly novel; these effects are very closely related to the "stepping on a rake" effects studied by Sims (2011) and Cochrane (2018a). We therefore only include long-term debt in our discussion to help show how variation in all fiscal policy variables, including the average maturity of debt, can affect empirical impulse response functions.

The paper is organized as follows: section 2 develops the model we employ; section 3 explores the effects of forward guidance in active fiscal, passive monetary policy regimes without switching; section 4 extends these results to economies that experience switching in fiscal and monetary policy parameters; section 5 concludes.

2 Model

We use a basic New Keynesian model of the kind Woodford (1998) uses, and augment this model to allow for (1) a richer maturity structure of debt as in Woodford (2001), Eusepi and Preston (2018), and Leeper and Leith (2016); (2) Markov-switching in policy parameters as in Davig and Leeper (2011). The model is derived in Appendix A.1. This model features a representative household and firm, monopolistic competition in the production of intermediate goods, and price stickiness a la Calvo (1983) according to which $1-\theta$ fraction of firms can change their prices each period. The model also allows the government to issue both bond portfolios, B_t^m, that have a geometrically decaying maturity structure, and short-term debt, B^s, which is held in net-zero supply. The government collects lump-sum taxes in accordance with an endogenous primary surplus rule, τ_t, and government purchases are assumed to equal 0, so that income, Y_t, equals C_t in equilibrium.

The model is linearized around the non-stochastic steady state with zero inflation. Let $\hat{z}_t \equiv ln(z_t) - ln(\bar{z})$ where $\bar{z}$ is the value of z in steady state. The behavior of households and firms then reduces to two equations:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \sigma^{-1}(\hat{i}_t - E_t\hat{\pi}_{t+1}) + r_t^n \tag{3}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{y}_t + \mu_t \tag{4}$$

where y is the output gap, π is inflation, β is the household discount factor, σ^{-1} is the intertemporal elasticity of substitution and κ is defined in Appendix A.1. β, κ, and σ are positive by assumption, and β is also bounded above by 1. Moreover, r_t^n and μ_t are autoregressive exogenous processes. Monetary policy is given by:

$$\hat{i}_t = \phi_y(s_t)\hat{y}_t + \phi_\pi(s_t)\hat{\pi}_t + \epsilon_t^{MP} + \upsilon_{1,t-1} \tag{5}$$

where ϵ^{MP} is an i.i.d monetary policy shock, and $v_{1,t}$ is a linear combination of L forward guidance shocks that obeys

$$v_{1,t} = v_{2,t-1} + \epsilon^R_{1,t} \quad (6)$$

$$v_{2,t} = v_{3,t-1} + \epsilon^R_{2,t} \quad (7)$$

$$\vdots$$

$$v_{L,t} = \epsilon^R_{L,t} \quad (8)$$

such that $v_{1,t-1} = \sum_{l=1}^{L} \epsilon^R_{l,t-l}$, where $[\epsilon^R_{1,t}, \epsilon^R_{2,t}, ..., \epsilon^R_{L,t}]$ are the L forward guidance shocks announced at time t. This model of short-term interest rate guidance is borrowed from Laséen and Svensson (2011) and is widely used in the forward guidance literature. Intuitively, $\epsilon^R_{l,t}$ is a shock announced at time t that affects interest rates at time $t+l$. The general structure of forward guidance shocks given by (8)-(10) ensure that shocks announced at t are actually realized as intended at $t+l$. As shown in Appendix A.2. and A.3., policymakers can use $(\epsilon^{MP}_t, \epsilon^R_{1,t}, \ldots, \epsilon^R_{L,t})$ to announce an interest rate peg between time t and $t+L$. To model recent instances of forward guidance we will peg i at or near the zero lower bound on i. Our specification allows for switching in policy parameters: s_t follows a S-state Markov chain, and the value of s_t determines ϕ_π and ϕ_y. Fiscal policy is characterized by the following linearized rule for primary surpluses:

$$\hat{\tau}_t = \gamma(s_t)(\hat{b}^m_{t-1} + \beta\rho\hat{P}^m_t) + z_{ft} \quad (9)$$

$$z_{ft} = \rho_F z_{f,t-1} + \epsilon_{ft} \quad (10)$$

where $\hat{b^m}_t$ is the percentage deviation of real bonds from steady state, z_{ft} is an exogenous fiscal policy shock, and ϵ_f is an exogenous mean-zero i.i.d innovation. γ is the fiscal authority's policy parameter and it follows the same Markov process as ϕ_y and ϕ_π. Fiscal policy must also satisfy the following budget constraint:

$$\hat{b}^m_{t-1} = \beta(1-\rho)\hat{P}^m_t + \beta\hat{b}^m_t + (1-\beta)\hat{\tau}_t + \hat{\pi}_t \quad (11)$$

where $\hat{P}^m_t$ is the price of the bond portfolio at time t and $\rho \in [0,1]$ captures the maturity structure of the government debt. While we relegate the derivation of this equation to Appendix A.1., the intuition behind the bond portfolio is fairly simple: the government issues $\hat{b}^m_t$ units of a nominal debt portfolio at time t that pays 1 unit of nominal income at time $t+1$, ρ units at time $t+2$, ρ^2 units at $t+3$ and so forth. This is the sense in which the maturity of debt is geometrically decaying. This structure allows us to introduce long-term debt into our model by using a single state variable that captures the average maturity of debt, ρ. The limiting cases of ρ illuminate how larger values of ρ correspond to longer average maturities: when

$\rho = 0$, all debt is short term, and when $\rho = 1$, all debt is in the form of consols. As demonstrated in Appendix A.1., $\hat{P}_t^m$ satisfies

$$\hat{P}_t^m = -\hat{i}_t + \rho\beta E_t \hat{P}_{t+1}^m \tag{12}$$

The system given by (3)-(14) yields a solution for $x_t = (\hat{y}_t, \hat{\pi}_t, \hat{i}_t, \hat{b}_t, \hat{\tau}_t, \hat{P}_t^m)'$. We use Sims' (2002) method to solve the fixed regime model, and the forward method in Cho (2016) to solve the switching model. A rational expectations equilibrium assumes the form:

$$x_t = \Omega(s_t)x_{t-1} + \Gamma(s_t)u_t$$

Parameters are selected so that the model under study is determinate. That is, we apply tractable conditions for determinacy developed in Cho (2016) to ensure that we are always studying the unique equilibrium responses to forward guidance.[5] While there are no simple analytical conditions for determinacy in our switching model, Woodford (1998) gives simple conditions for determinacy in the case of non-switching (see Table 1):[6]

Table 1: Fixed Coefficient Model Determinacy Conditions

	$\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_y$	$\phi_\pi < 1 - \frac{1-\beta}{\kappa}\phi_y$
$\gamma \in (1, \frac{\beta^{-1}+1}{\beta^{-1}-1})$	determinate	indeterminate
$\gamma \notin (1, \frac{\beta^{-1}+1}{\beta^{-1}-1})$	no stable solution	determinate

We say that the economy is in Regime M when $\phi_\pi > 1 - (1-\beta)\phi_y/\kappa$ and $\gamma \in (1, (\beta^{-1}+1)/(\beta^{-1}-1))$. and that the economy is in Regime F when $\phi_\pi < 1 - (1-\beta)\phi_y/\kappa$ and $\gamma \notin (1, (\beta^{-1}+1)/(\beta^{-1}-1))$. In Regime M, fiscal policy is passive while monetary policy is active. This is the standard assumption in most New Keynesian research. In Regime F, fiscal policy is active while monetary policy is passive.

3 Fixed Coefficient Exercises

We now examine the effectiveness of forward guidance in the presence of fixed policy regimes (i.e. we constrain all policy parameters to be permanent). Our analysis involves three different model parameterizations: (1) a Regime M parameterization; (2) a Regime F parameterization with short-term debt ($\rho = 0$); (3) a Regime F parameterization with long-term debt ($\rho > 0$). Table 2 in Appendix A.4. contains

[5] As our parameterizations in Appendix A.4. suggest, determinacy allows for persistent and non-trivial active money (passive fiscal) and passive money (active fiscal) regimes.

[6] We assume that $\phi_\pi(s_t) \geq 0$ for all s_t.

the parameter values used in each of the three configurations, though our results are robust to different parameterizations.[7] Our analysis in this section also involves two distinct policy exercises that are commonly used in the literature. First, we examine the impulse responses of output and inflation to a single one unit k-period ahead forward guidance shock to the nominal interest rate. This exercise gives us useful intuition for the second policy experiment, which is the main result in this section. In that exercise, we examine the impulse responses of output and inflation to an announced 12-quarter interest rate peg that mimics aspects of the Federal Reserve's calendar-based forward guidance announcements in August 2011, January 2012, and September 2012 (see Del Negro et al. (2015) for more details).

3.1 Exercise 1: Inspecting the Mechanism

In order to better understand the mechanism driving our result in section 3.2, we examine the effects of a one-time expansionary forward guidance shock to the short-term nominal interest rate under all three parameterizations. The exercise takes place as follows: at time t the central bank announces a negative one unit shock to i_{t+k} where $k \geq 0$. Agents respond at time t by adjusting hours worked, consumption, prices and bond holdings, and this generates paths for inflation and output that are plotted in Figure 1 for the cases where $k = 7$.[8]

Overall, output and inflation respond less favorably to forward guidance shocks in a Regime F economy. In a Regime M economy, the negative shock at the k-horizon causes long run real rates to drop, which induces positive responses in output and inflation. These responses are magnified by the lack of a discount factor in the linearized Euler equation, which causes consumption and therefore output to be highly responsive to changes in long-run real rates. The response of inflation to the shock is driven, in part, by the front-loading in the Phillips curve and the presence of nominal rigidities. These effects combine to cause a large stimulus. Much of this is made possible by passive fiscal policy: the lower debt-service costs and resulting lower debt path implied by forward guidance generate a fiscal expansion as governments rebate their savings to households. That is, bondholders do not feel nominally constrained by the lower path for bonds and the rate of returns on bonds, and Regime M movements in output and inflation do not reflect changes in bond wealth.

In Regime F, however, output and inflation do respond to the changes in bond

[7] There is one exception: for small σ and as ϕ_π approaches 1 in Regime F, the Regime F impulse responses of output and inflation are strictly above the Regime M impulse responses before the realization of the shock in exercise 1 (after the shock, the Regime M impulse responses are above the Regime F responses). This applies only to our results in section 3.1 and we regard this as an unrealistic parameterization of the model. Our results in section 3.2 are robust to reasonable parameterizations.

[8] Qualitatively similar results obtain for different choices of k.

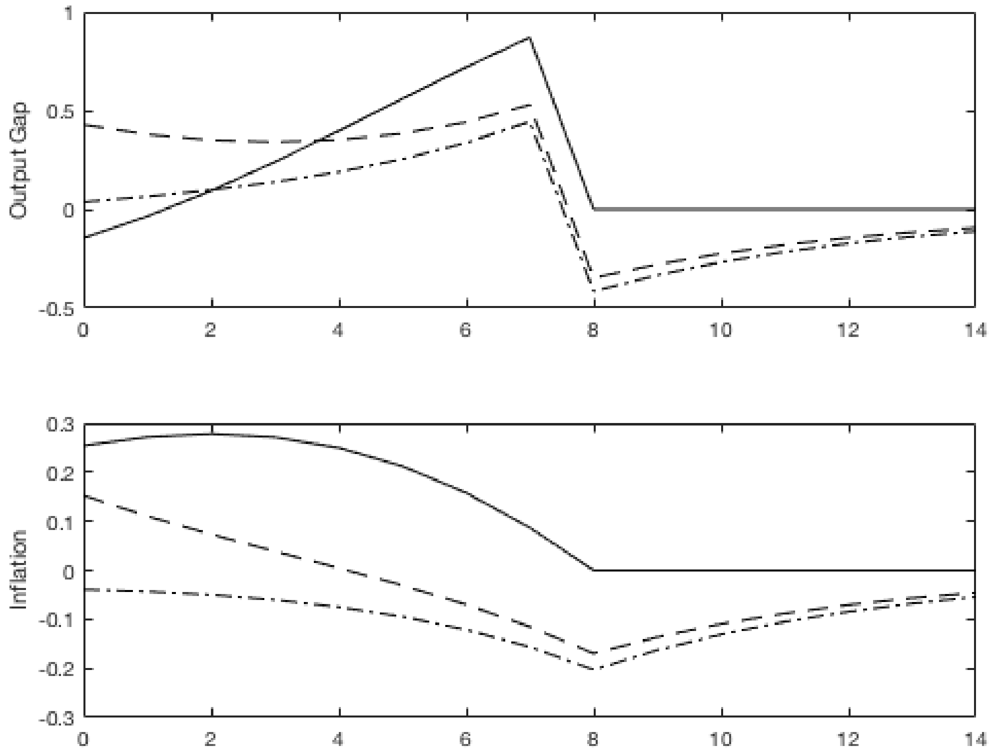

Figure 1: the impulse responses of output and inflation to a one-unit anticipated shock to i_{t+7} at t. The solid line shows impulse responses in the Regime M model; the dashed line shows impulse responses in the Regime F model with long-term debt; the dashed-dotted line shows impulse responses in Regime F with only short-term debt.

wealth. Since active fiscal policy fails to compensate households for the lower rate of return implied by lower interest rates, agents in a Regime F economy are nominally budget-constrained by expansionary forward guidance. We may therefore describe expansionary forward guidance as having wealth effects that counteract the stimulating effects of lower long-run real interest rates in a Regime F economy. In this framework, a k-period ahead shock initially lowers long-run real interest rates and raises consumption. At the time of the shock's realization, nominal wealth declines and puts downward pressure on prices through the bond valuation equation. After the shock is realized, agents reduce consumption and replenish bond holdings and this puts downward pressure on consumption and output. As with the Regime M case, firms respond by changing prices well in advance of the anticipated deflationary pressure. The overall effect of this price setting behavior is a large and persistent deflation.

Figure 1 also reveals that the presence of long-term debt (i.e. $\rho > 0$) in Regime F leads to higher paths of output and inflation than in a Regime F economy without long-term debt (i.e. $\rho = 0$). This is because the presence of long-term debt introduces yet another channel through which forward guidance impacts output and inflation: the anticipated decline in short-term interest rates raises the price of outstanding debt, and therefore raises the market value of outstanding debt held by the household. This is a debt revaluation effect, and it leans against the aforementioned negative wealth effects. We note that this result is in no way novel: it is the "stepping on a rake" effect studied by Sims (2011) and Cochrane (2018a). We only include it because it may help empirical impulse response functions.

One notable feature of the impulse response functions is that output responds more favorably to forward guidance on impact, i.e. at the time of the announcement, in the Regime F economies. We attribute this to one feature of the Regime M economy: monetary policy satisfies the Taylor Principle such that the increase in inflation observed on impact corresponds with higher real interest rates on impact. If we allow ϕ_π in all regimes to approach 1 from both directions, we observe similar responses in all economies on impact.

3.2 Exercise 2: The Fixed Regime Forward Guidance Experiment

Our second policy experiment in the fixed coefficient model assesses the effects of forward guidance on a specific path for interest rates. Using methods inspired by Del Negro et al. (2015), and Cole (2015), we study what happens when the central bank announces an interest rate target, $\bar{i}$, between time T and $T + L$.[9] We chose $L = 12$ to mimic the September 2012 FOMC statement that called for low interest rates through mid-2015. Additionally, $\bar{i} = 0$ is chosen as a target, but any interest rate target between 0 and 25 basis points may reasonably approximate the path implied by the September 2012 statement.[10] The economy is simulated for $T - 1$ periods prior to announcement, and the simulations are repeated 10000 times. Figure 2 and 3 report the mean impulse responses of output, inflation and interest rates to the $L + 1$ period anticipated interest rate peg. For simplicity's sake, we shut down shocks after time t so that $i_{t+l} = E_t i_{t+l}$ for $0 \leq l \leq L$. If shocks are present, monetary policymakers use some combination of unanticipated and anticipated monetary policy shocks at $t + 1$ to $t + L$ to maintain the peg and agents' expectation of the peg over the forward guidance horizon. As such, we regard this simplification as innocuous.

As with the previous exercises, Figures 2 and 3 demonstrate that output and inflation respond less favorably to expansionary forward guidance on interest rates

[9] See appendix A.2. for further details.

[10] The main qualitative results in this section are robust to any $\bar{i}$ below steady state, $i^* = \beta^{-1} - 1$.

under the assumption of active fiscal, passive monetary policy. In contrast to previous exercises, the forward guidance shocks do not induce a dramatic fall in output in the Regime F economies. This result may be driven by an important feature of the impulse responses in section 3.1: each expansionary forward guidance shock raises output before the shock is realized, and depresses output after the shock is realized. Therefore, when the economy is hit by a sequence of such shocks, as in this section, the contractionary effects of realized forward guidance shocks are partially offset by the expansionary effects of unrealized shocks. In general equilibrium, this leads to a relatively flat trajectory for output (see Figure 3). Also in contrast to results from previous exercises, inflation responds much more positively to forward guidance on the interest rate path in the presence of long-term debt. We attribute this result to a particular strong revaluation effect, as forward guidance on $L+1$ future short-term interest rates has a huge impact on $\hat{P}_t^m$ (which is simply a weighted sum of expected future short-term interest rates) .

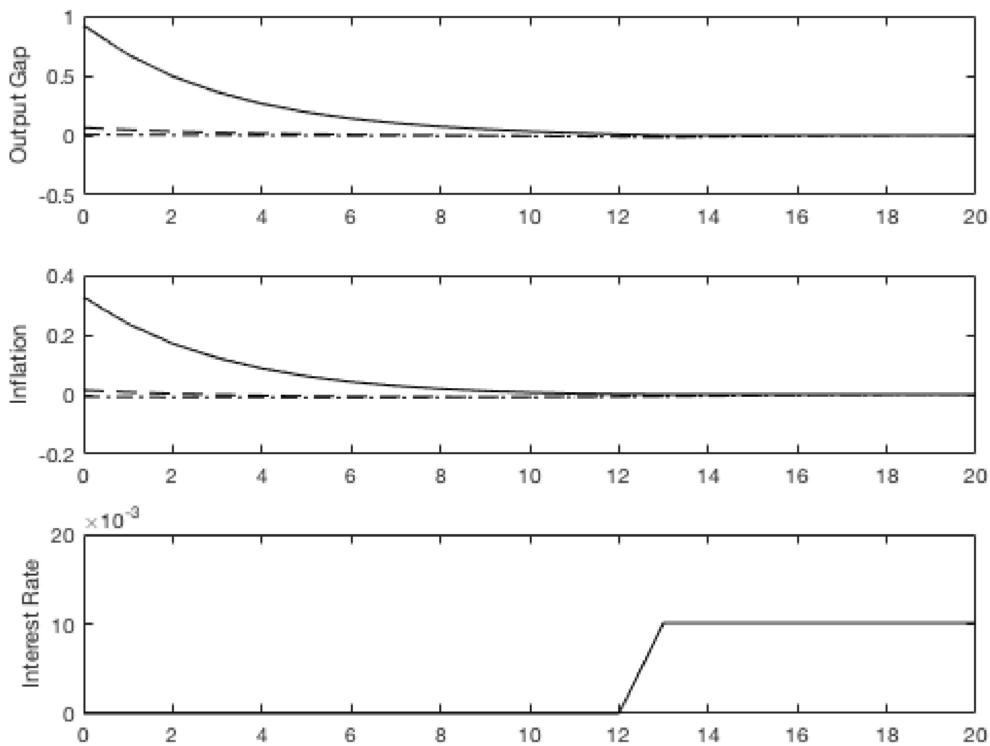

Figure 2: The 12-quarter Forward Guidance Horizon Experiment. The solid line shows impulse responses in the Regime M model; the dashed line shows impulse responses in the Regime F model with long-term debt; the dashed-dotted line shows impulse responses in Regime F with only short-term debt.

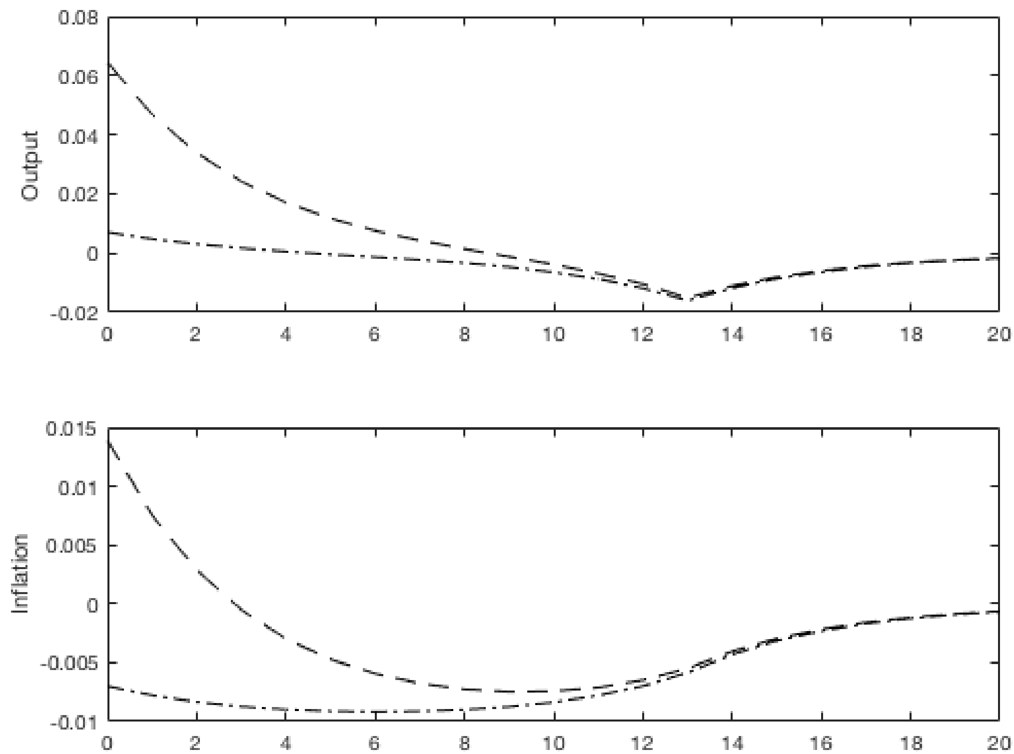

Figure 3: The 12-quarter Forward Guidance Horizon Experiment. The dashed line shows impulse responses in the Regime F model with long-term debt; the dashed-dotted line shows impulse responses in Regime F with only short-term debt.

We emphasize that the strong responses of output and inflation in Regime M are a reflection of the forward guidance puzzle. Also note that Figure 3 uses a different vertical scale than Figure 2.

4 Markov-Switching Forward Guidance Experiment

We now employ the full model developed in section 3 and allow the policy stances of the monetary and fiscal authorities to periodically and recurrently change. Specifically, we assume that the economy switches between a Regime F configuration ($s_t = F$) and a Regime M configuration ($s_t = M$). This assumption is restrictive, but it allows us to get at one important mechanism: expectations of changing responses to forward guidance cause agents to behave differently today. These expectational spillovers shock the impulse responses of inflation and output in Regime M away from

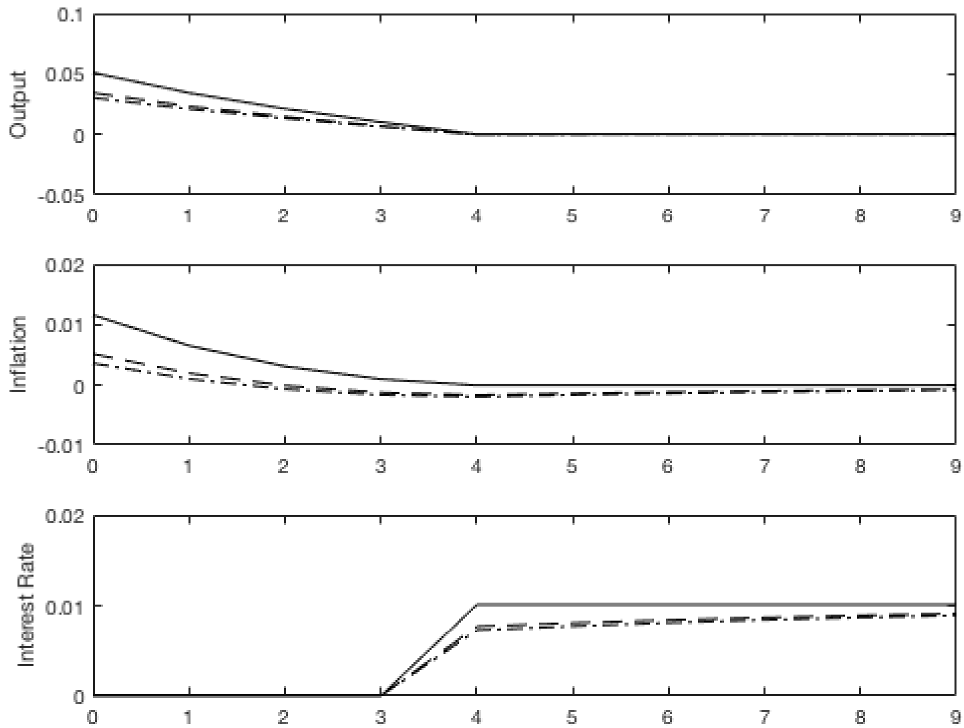

Figure 4: The 3-quarter Forward Guidance Horizon Experiment, Parameterization 1. The solid line shows impulse responses in the fixed coefficient model Regime M; dashed line shows impulse responses in the switching model Regime M with long-term debt; the dashed-dotted line shows impulse responses in the switching model Regime M with only short-term debt.

the paths implied by the corresponding fixed coefficient models, and may therefore help the impulse responses agree with the data.

To illustrate this idea, we conduct the policy experiment from section 3 in the switching model. Specifically, we first assume that the economy is in Regime M when the central bank announces a sequence of shocks at time T such that $i_T = E_T i_{T+1} = ... = E_T i_{T+L} = \bar{i}$. We then assume that the economy remains in Regime M at $T+1$, when another sequence of shocks is announced such that $i_{T+1} = E_{T+1} i_{T+2} = ... = E_{T+1} i_{T+L} = \bar{i}$. This process is repeated until $T+L$. This experiment shows how the switching economy responds to an announced $L+1$ period interest rate peg in Regime M. Figures 4-6 show the Regime M effects of this experiment when $L = 3$ using a parameterization inspired by a similar model in Ascari et al. (2017) (see Table 3 in Appendix A.4. for the parameter values contained in Figures 4-6; see Appendix A.3. a derivation for the policy experiment).

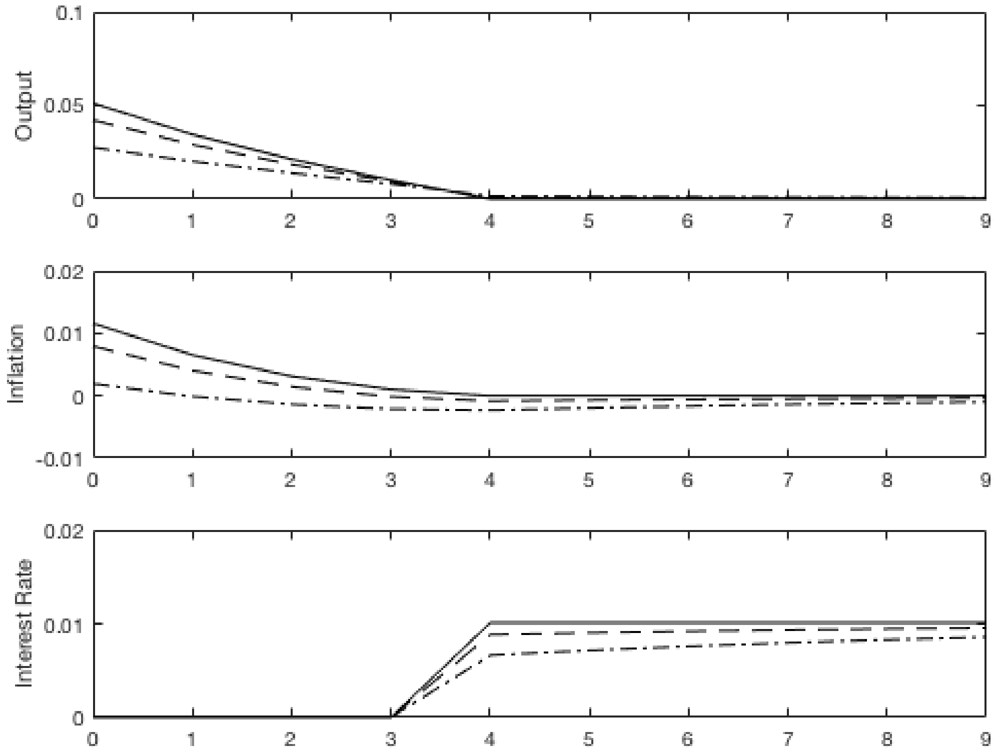

Figure 5: The 3-quarter Forward Guidance Horizon Experiment, Parameterization 2. The solid line shows impulse responses in the fixed coefficient model Regime M; dashed line shows impulse responses in the switching model Regime M with long-term debt; the dashed-dotted line shows impulse responses in the switching model Regime M with only short-term debt.

We emphasize that agents do not expect the economy to remain in Regime M throughout the forward guidance horizon. Agents form rational expectations using the true transition probabilities (e.g. $E_T(s_{T+1} = M|s_T = M) = p_{MM}$ where p_{MM} is the probability of remaining in Regime M). We only hold M fixed to compare the Regime M impulse responses in the switching model, to the Regime M impulse responses in the fixed regime model. More generally, we could allow for regime changes during our forward guidance experiment, but this would only strengthen our argument that the wealth effects of active fiscal policy are always at play. Relative to the fixed regime cases, expansionary forward guidance appears to be less stimulative in the switching model's Regime M. In Regime M, this is driven by the positive probability that the economy will switch to a state where the expansionary shock has negative wealth effects.

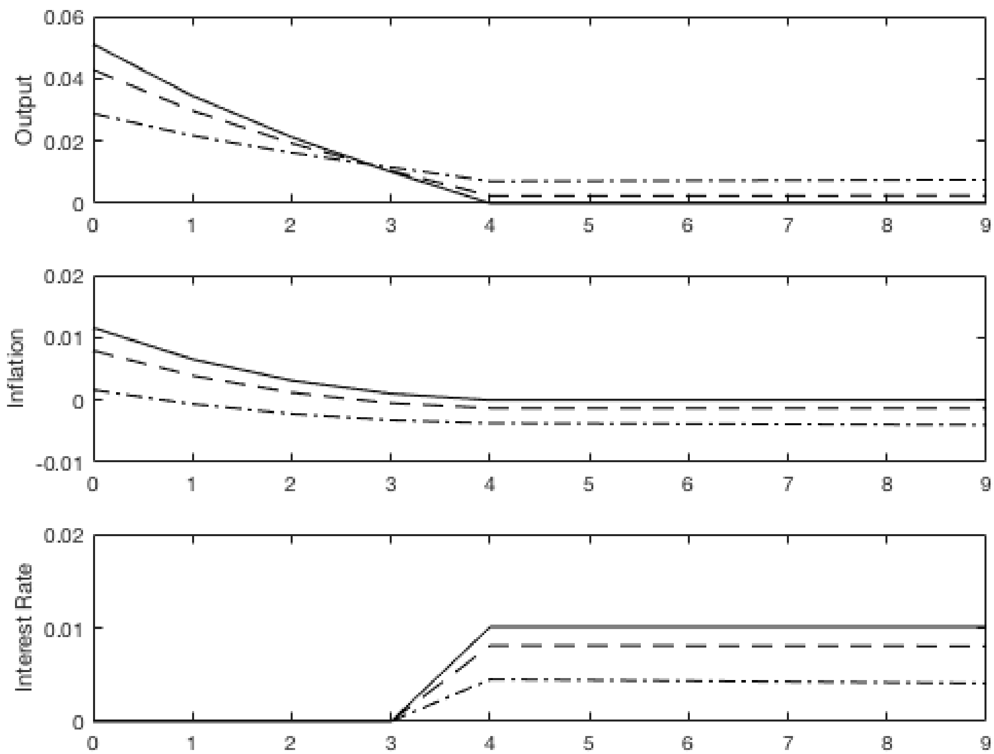

Figure 6: The 3-quarter Forward Guidance Horizon Experiment, Parameterization 3. The solid line shows impulse responses in the fixed coefficient model Regime M; dashed line shows impulse responses in the switching model Regime M with long-term debt; the dashed-dotted line shows impulse responses in the switching model Regime M with only short-term debt.

While the qualitative results in Figures 4-5 are robust to different policy coefficients, structural parameters, transition probabilities,[11] and forward guidance horizons, Figure 6 shows that the output and inflation impulse responses in the switching model can barely overshoot and undershoot the fixed regime responses after the interest rate peg is over when inflation reaction coefficients are high and the fiscal policy parameter in Regime M is relatively low.[12] We have two remarks about this particular result. First, a regime switch quickly eliminates the persistent output and inflation gaps. In the calibrated model, these switches occur every 20 periods on average. Second, lower inflation and higher output reaction coefficients in the interest rate rule

[11]We note that fiscal policy parameters are always chosen so that fiscal policy is non-Ricardian in section 4 exercises. Otherwise, the current fiscal policy stance would be irrelevant for inflation and output.

[12]See Appendix A.4. for parameter values.

help to raise i faster and close the gaps.

5 Conclusion

Standard New Keynesian models predict implausibly large and favorable responses of inflation and output to forward guidance on interest rates. This paper investigates the effects of forward guidance in a New Keynesian model with active fiscal and passive monetary policy regimes. Our specific contribution is to show that permanent or recurring active fiscal policy regimes can eliminate the above mentioned excessive responsiveness. The intuition underlying our results is simple: passive fiscal policy rules induce fiscal expansions in response to a lower path for interest rates, and this can cause expansionary forward guidance to be very stimulative. Under active fiscal policy, this fiscal expansion does not occur, which means that expansionary forward guidance constrains agents via lower expected paths for bonds and the rate of return on bond holdings. We demonstrate the general equilibrium implications of these ideas in a policy experiment developed by Del Negro et al. (2015). In that experiment, the assumption of a permanent active fiscal policy virtually eliminates the responses of output and inflation relative to the responses in an economy with permanent passive fiscal policy.

For our results to hold, we do not need heroic assumptions about the stance of fiscal policy on debt during the forward guidance horizon. Using a regime-switching New Keynesian model, we show that expectations of future active fiscal policy can significantly dampen the response of output and inflation to forward guidance, even while fiscal (monetary) policy is currently passive (active). Expectations of future active fiscal policy always attenuate the short-term effects of forward guidance, and sometimes lead to interesting persistent dynamics. The addition of long-term debt to the model may also help to improve the fit of impulse responses.

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Appendix

A.1. Model Derivation

In this paper we study an economy that is populated by a large number of infinite-lived identical household-firms indexed by $j \in [0,1]$. Each household-firm is a monopolistically competitive producer of a unique product variety indexed by $i \in [0,1]$, where $i = j$ denotes the product of household-firm j. Household-firm j engages in a decision-making process to maximize the following objective:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t^j)^{1-\sigma}}{1-\sigma} - \omega(y_t(j)) \right)$$

subject to

$$\begin{aligned} \int_0^1 p_t(i) c_t^j(i) di + E_t(R_{t,t+1} W_{t+1}^j) &\leq W_t^j + p_t(j) y_t(j) + P_t(z_t - \zeta_t) \\ C_t^j &= \left(\int_0^1 c_t^j(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \end{aligned}$$

where $c^j(i)$ is household-firm j's consumption of good i; W_t^j denotes the nominal value of the bond portfolio that the household holds at the beginning of t and W_0 is given; $R_{t,T}$ is the stochastic discount factor between time t and T; $y(j)$ is the quantity of product variety j produced by the household-firm; z is a lump-sum transfer from the government; ζ is a lump-sum tax; ω is a strictly convex function; $p_t(j)$ and P_t are the price of product variety j and the price level, respectively. To preclude arbitrage opportunities, we assume that all asset prices are determined by stochastic discount factors. This implies, for example, that $Q_{t,t+1} = 1/(1+i_t) = E_t(R_{t,t+1})$ where $Q_{t,t+1}$ is the price of a single-period government bond at time t, and i_t is the nominal interest rate on a riskless one-period bond. Furthermore, market completeness is assumed. The sequence of flow constraints implied by the household budget constraint yields the following intertemporal constraint:

$$\sum_{t=0}^{\infty} E_0\{R_{0,t} \int_0^1 p_t(i) c_t^j(i) di\} \leq \sum_{t=0}^{\infty} E_0\{R_{o,t}\big(p_t(j) y_t(j) + P_t(z_t - \zeta_t)\big)\} + W_t^j$$

Since each household-firm is identical and markets are complete, we assume that each household-firm has the same initial wealth level. This induces agents to engage in a process of perfect risk-sharing that generates identical equilibrium paths for household consumption and so forth. As a result, we can drop the j subscript and treat household-firm j as the representative household and firm.

The household-firm chooses (1) how to allocate its expenditures among the product varieties; (2) how much to consume or save in each period; (3) how much to produce in each period. We study these three decision processes in turn. In making these decisions, the representative household-firm acts as a "price-taker" by taking the actions of other household-firms as given (i.e. the household-firm takes P_t and Y_t as given). Additionally, the household-firm faces a price rigidity when solving its producer problem, and we discuss this in greater detail below.

In this environment, a rational expectations equilibrium is a collection of stochastic processes such that each household-firm chooses sequences of consumption, asset portfolios, and prices that maximizes its objective given $\{P_t, Y_t, z_t, \tau_t\}$ and a specification for fiscal and monetary policy; such that net demand of assets by private household-firms equals the supply of government debt. By studying the aforementioned three decision-making processes of the representative household-firm, we uncover conditions that characterize such an equilibrium.

We present the first of these problems–the problem of maximizing C_t subject to a given level of expenditure–in the form of a Lagrangean:

$$L = \left(\int_0^1 c_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{1-\epsilon}} - \mu\left(\int_0^1 p_t(i)c_t(i)di - X_t\right)$$

where X_t is the minimum level of expenditure. Differentiating with respect to $c_t(z)$ yields the following optimality condition:

$$\frac{\epsilon}{\epsilon-1}\left(\int_0^1 c_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}-1}\left(1-\frac{1}{\epsilon}\right)c_t(z)^{-\frac{1}{\epsilon}} - \mu p_t(z) = 0$$

which can be combined with the first-order condition for any other product variety (e.g. product variety i) to obtain:

$$\left(\frac{c_t(z)}{c_t(i)}\right)^{-\frac{1}{\epsilon}} = \frac{P_t(i)}{P_t(z)}$$

Now, we can substitute this into the expenditure function and solve for $c_t(i)$:

$$\begin{aligned} X_t &= \int_0^1 p_t(i)c_t(i)di = \int_0^1 p_t(i)\left(\frac{p_t(i)}{p_t(z)}\right)^{-\epsilon} c_t(z)di \\ &= \frac{c_t(z)}{p_t(z)^{-\epsilon}}\int_0^1 p_t(i)^{1-\epsilon}di \end{aligned}$$

Because $P_t = (\int_0^1 p_t(i)^{1-\epsilon}di)^{1/(1-\epsilon)}$, this last equation implies:

$$c_t(z) = \frac{X_t}{P_t}\left(\frac{p_t(z)}{P_t}\right)^{-\epsilon} \tag{13}$$

We can then substitute the analogous equation for $c_t(i)$ this into the definition for C_t:

$$
\begin{aligned}
C_t &= \left(\int_0^1 c_t(i)^{1-\frac{1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} = \frac{X_t}{P_t^{1-\epsilon}} \left(\int_0^1 p_t(j)^{1-\epsilon} dj\right)^{\frac{-\epsilon}{1-\epsilon}} = \frac{X_t}{P_t^{1-\epsilon}} P_t^{-\epsilon} \quad (14)\\
\therefore P_t C_t &= X_t = \int_0^1 p_t(i) c_t(i) di \quad (15)
\end{aligned}
$$

Equations (15) and (17) therefore imply the following demand schedule for good i:

$$
c_t(i) = C_t \left(\frac{p_t(i)}{P_t}\right)^{-\epsilon}
$$

Henceforth, let government purchases equal 0 in every period. This assumption delivers the following market-clear condition:

$$
Y_t = C_t \tag{16}
$$

Accordingly, the demand schedule for product variety i may be expressed as:

$$
y_t(i) = Y_t \left(\frac{p_t(i)}{P_t}\right)^{-\epsilon} \tag{17}
$$

The optimal consumption-savings plan of the representative household must satisfy: (1) $Y_t = C_t$ for all t; (2) the intertemporal household budget constraint (with equality) in each period; (3) a consumption Euler-equation that can be derived through a variational argument:

$$
\beta E_t \{ \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} \} = \frac{1}{1+i_t} \tag{18}
$$

Condition (2) along with complete risk-sharing imply [13]:

$$
\sum_{T=t}^{\infty} E_t R_{t,T} P_t C_T = \sum_{T=t}^{\infty} E_t \{ R_{t,T} [P_T Y_T + P_T (z_T - \zeta_T)] \} + W_t \tag{19}
$$

where $W_t = B_{t-1}^m (1 + \rho P_t^m)$ is nominal outstanding government debt at beginning of t. Combining with the government flow constraint,

$$
B_{t-1}^m (1 + \rho P_t^m) = P_t(\zeta_t - z_t) + P_t^m B_t^m \tag{20}
$$

yields the transversality condition:

$$
\lim_{t\to\infty} E_t [R_{t,T} W_T] = 0
$$

[13] Under full insurance, identical households with identical initial wealth levels choose identical optimal consumption paths. Moreover, $\int_0^1 B^j(t) dj = B_t$ follows from the assumption that net demand for assets by private households equals supply of government debt

We now turn to the pricing-production decision of the household-firm. Because price determines quantity through the demand schedule, we assume that the household-firm chooses price when solving for the optimal production schedule. The firm is constrained by a price friction of the form developed in Calvo (1983). Each period, $1-\theta$ fraction of firms are randomly allowed to reset prices, while the remaining θ fraction continue to charge last period's price. This means that a firm expects its price to persist for $1/(1-\theta)$ periods into the future each time it resets prices. As a result, it is natural for the household-firm to treat θ as a discount factor and choose a single price that maximizes the discounted sum of future profits:

$$\sum_{k=0}^{\infty}\theta^k\big\{\Lambda_t E_t[R_{t,t+k}\mathcal{P}y_{t+k}(\mathcal{P})]-\beta^k E_t[\omega(y_{t+k}((P)))]\big\}$$

where Λ_t is the marginal utility of household income at t and $y_{t+k}(\mathcal{P})$ is defined in (19). As in Woodford (1998), we treat Λ_t as a constant, and proceed to the first-order condition:

$$\sum_{k=0}^{\infty}\theta^k E_t\big\{\Lambda[R_{t,t+k}\left(\frac{\mathcal{P}}{P_{t+k}}\right)^{-\epsilon}Y_{t+k}(1-\epsilon)-\beta^k\omega'(y_{t+k}((P)))(-\epsilon)\frac{\mathcal{P}^{-\epsilon-1}}{P_{t+k}^{-\epsilon}}Y_{t+k}\big\}=0$$

Multiply both sides of the first-order condition by $\frac{\mathcal{P}}{\Lambda(1-\epsilon)}$ to obtain

$$\begin{aligned}\sum_{k=0}^{\infty}\theta^k E_t\big\{R_{t,t+k}\left(\frac{\mathcal{P}}{P_{t+k}}\right)^{-\epsilon}\mathcal{P}Y_{t+k}-\beta^k\Lambda^{-1}\omega'(y_{t+k}((P)))(\frac{\epsilon}{\epsilon-1})\frac{\mathcal{P}^{-\epsilon}}{P_{t+k}^{-\epsilon}}Y_{t+k}\big\} &= 0\\ \sum_{k=0}^{\infty}\theta^k E_t\big\{[R_{t,t+k}\left(\frac{\mathcal{P}}{P_{t+k}}\right)^{-\epsilon}Y_{t+k}\big(\mathcal{P}-\frac{\beta^k}{R_{t,t+k}\Lambda}\omega'(y_{t+k}(\mathcal{P}))(\frac{\epsilon}{\epsilon-1})\big\} &= 0\end{aligned}$$

To further simplify the first-order condition, consider the following two equations:

$$\begin{aligned}\beta^k\frac{u'(Y_{t+k})}{y'(Y_t)}\frac{P_t}{P_{t+k}} &= R_{t,t+k}\\ \Lambda_t &= u'(Y_t)/P_t\end{aligned}$$

The first equation is a necessary and sufficient condition for household optimization, while the second equation is an expression for the marginal utility of income. We substitute these equations into the first-order condition to yield:

$$\sum_{k=0}^{\infty}\theta^k E_t\big\{[R_{t,t+k}(\frac{\mathcal{P}}{P_{t+k}})^{-\epsilon}Y_{t+k}\big(\mathcal{P}-S_{t+k,t}(\frac{\epsilon}{\epsilon-1})\big\} = 0 \qquad (21)$$

$$\frac{\omega'(y_{t+k}(\mathcal{P}))}{u'(Y_{t+k})}P_{t+k} = S_{t+k,t} \qquad (22)$$

$S_{t+k,t}$ captures the household's expected marginal costs at time $t+k$. A sufficient but not necessary condition for optimality is that $\mathcal{P} = \frac{\epsilon}{\epsilon-1} S_{t+k,t}$ for all $t+k$. In this case, the optimal price is always a mark-up of $\frac{\epsilon}{\epsilon-1}$ over marginal costs, which is essentially what condition (23) tells the household-firm to do. Since $\mathcal{P}$ is the same for all firms who change price at t it is straightforward to show that

$$P_t = [\theta P_{t-1}^{1-\epsilon} + (1-\theta)\mathcal{P}]^{\frac{1}{1-\epsilon}} \tag{23}$$

We are now in a position to characterize the non-policy aggregate demand (AD) and aggregate supply (AS) blocks of the model. The non-policy AD block is given by equations (18)-(21), and the AS block is given by equations (23)-(25). The AD equations give us consumption demand, bond holdings and rates of return subject to monetary and fiscal policy and a path for prices. The AS equation gives us a path for the price index and optimal prices subject to AD. To complete the model, we discuss simple fiscal and monetary policy arrangements. First, the monetary authority uses the linearized interest rate rule presented in section 2. The fiscal authority only issues a bond portfolio, B_t^m, with a maturity that declines at a rate $\rho \in [0, 1]$. Under this maturity structure, the quantity of government debt issued at $t-1$ that matures at $t+j$ is:

$$B_{t-1}(t+j) = B_{t-1}^m \rho^j$$

The evolution of the government's bond portfolio satisfies that following budget constraint:

$$B_{t-1}^m (1 - \sum_{j \geq 0} Q_t(t+j)\rho^j) = P_t(\zeta_t - z_t) + B_t^m \sum_{j \geq 0} Q_t(t+j)\rho^{j-1}$$

where $Q_t(t+j)$ is the price of debt that matures at time $t+j$ and is sold at t. To simplify the government budget constraint, we define the price of the bond portfolio, P_t^m, as:

$$P_t^m = E_t \sum_{j \geq 0} Q_t(t+j)\rho^{j-1}$$

which allows us to rewrite the government budget constraint as

$$B_{t-1}^m (1 + \rho P_t^m) = P_t(\zeta_t - z_t) + P_t^m B_t^m$$

Furthermore, we can show that bond prices follow a recursive formulation:

$$P_t^m = Q_t(t+1)(1 + \rho E_t P_{t+1}^m)$$

given B_{-1}^m. The government also implements a rule that adjusts real primary surpluses in response to the market value of real debt. If we let $\tau_t = \zeta_t - z_t$ denote the real primary surplus, then we can describe the surplus rule using the linearized equations in section 2.

We add these policy equations to the non-policy AD block to completely characterize aggregate demand. To better analyze the equilibrium dynamics of the model, we linearize the equations of the AD and AS blocks. The linearized AD equations appear in equations (3), (5), (7)-(14). To arrive at equation (4) which is the linearized AS curve, we linearize equations (23)-(25):

$$\hat{\mathcal{P}}_t = (1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t\{\hat{s}_{t+k,t} + \sum_{s=t+1}^{t+k}\hat{\pi}_s\} \tag{24}$$

$$\hat{s}_{t+k,t} = (\omega^{-1} + \sigma^{-1}\hat{y}_t - \theta\omega^{-1}[\hat{\mathcal{P}}_t - \sum_{s=t+1}^{t+k}\pi_s]) \tag{25}$$

$$\hat{\pi}_t = \frac{1-\theta}{\theta}\hat{\mathcal{P}}_t \tag{26}$$

where $\hat{\mathcal{P}}_t$ is the percentage deviation of optimal price over the price index from its steady state value of 1, $\hat{S}_{t+k,t}$ is the percentage deviation of marginal costs over the price index from its steady state value of 1 over the markup and $\omega = (\omega'(Y^*))/(\omega''(Y^*)Y^*)$. To arrive at the linearized AS curve, we substitute equation (27) into equation (26) and quasi-difference to obtain:

$$\hat{\mathcal{P}}_t = \frac{\kappa\theta}{1-\theta}\sum_{k=0}^{\infty}(\theta\beta)^k E_t\hat{y}_{t+k} + \sum_{k=1}^{\infty}E_t\hat{\pi}_t \tag{27}$$

where

$$\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{\omega+\sigma}{\sigma(\omega+\theta)}$$

Substituting equation (29) into equation (28) yields the linearized AS equation in (4).

A.2. Section 3.2 Policy Experiment

This appendix shows how to implement an anticipated interest rate peg at time T (i.e. $i_T = E_T i_{T+1} = ... = E_T i_{T+L} = \bar{i}$) using the forward guidance shocks introduced in section 2. First, it helps to rewrite the equilibrium relationships as

$$Y_t = GY_{t-1} + \bar{\Psi}\bar{\epsilon}_t + \tilde{\Psi}\tilde{\epsilon}_t$$

where $Y_t = (\hat{y}_t, \hat{\pi}_t, \hat{i}_t, r_t^n, \mu_t, v_{1,t}, v_{2,t}, \ldots, v_{L,t}, \hat{\tau}_t, \hat{b}_t^m, \hat{P}_t^m)'$, $\bar{\epsilon}_t = (\epsilon_t^n, \epsilon_t^\mu, \epsilon_t^f)'$, and $\tilde{\epsilon}_t = (\epsilon_t^{MP}, \epsilon_{1,t}^R, \ldots, \epsilon_{L,t}^R)'$. It follows that the equilibrium process for $\hat{i}$ is given by an equation of the form:

$$\hat{i}_t = G_i Y_{t-1} + \bar{\Psi}_i\bar{\epsilon}_t + \tilde{\Psi}_i\tilde{\epsilon}_t$$

Assume $\bar{\epsilon}_T = 0$, for simplicity (we can relax this). Then:

$$\begin{aligned} i_T &= G_i Y_{T-1} + \tilde{\Psi}_i \tilde{\epsilon}_T \\ E_T i_{T+1} &= G_i^2 Y_{T-1} + (G\tilde{\Psi})_i \tilde{\epsilon}_T \\ &\vdots \\ E_T i_{T+L} &= G_i^{L+1} Y_{T-1} + (G^L \tilde{\Psi})_i \tilde{\epsilon}_T \end{aligned}$$

where G_i^k and $(G^k\tilde{\Psi})_i$ denote the rows of G^k and $(G^k\tilde{\Psi})$ that correspond to the nominal interest rate for $k = 1, ..., L+1$. If we set $i_s = \bar{i}$ for all $s \in \{T, T+1, \ldots, T+L\}$, then we have a system of $L+1$ equations in $L+1$ unknowns which are the elements of $\tilde{\epsilon}_T$. The solution of this system is given by:

$$\begin{pmatrix} \tilde{\Psi}_i \\ (G\tilde{\Psi})_i \\ \vdots \\ (G^L\tilde{\Psi})_i \end{pmatrix}^{-1} \left(\bar{i} 1_{L+1\times 1} - \begin{pmatrix} G_i \\ G_i^2 \\ \vdots \\ G_i^{L+1} \end{pmatrix} Y_{T-1} \right) = \tilde{\epsilon}_T$$

where $1_{L+1\times 1}$ is a $L+1\times 1$ vector of ones. We implement the interest rate peg by announcing $\tilde{\epsilon}_T$ at T. If we also suppose that $\bar{\epsilon}_s = 0$ for all $s \in \{T, T+1, \ldots, T+L\}$ and $\tilde{\epsilon}_s = 0$ for all $s \in \{T+1, \ldots, T+L\}$ then $\tilde{\epsilon}_T$ will also successfully implement the interest rate *ex post* (i.e. $i_T = i_{T+1} = ... = i_{T+L} = \bar{i}$). If we relax this last assumption (e.g. if $\bar{\epsilon}_s \neq 0$ for some some $s \in \{T+1, ..., T+L\}$), then the central bank will have to announce shocks after T to defend the peg. Regardless of whether shocks are present, the central bank can always use these shocks to defend an L-horizon peg.

A.3. Section 4 Policy Experiment

First, it helps to rewrite the equilibrium relationships as

$$Y_t = G(s_t)Y_{t-1} + \bar{\Psi}(s_t)\bar{\epsilon}_t + \tilde{\Psi}(s_t)\tilde{\epsilon}_t$$

where $Y_t = (\hat{y}_t, \hat{\pi}_t, \hat{i}_t, r_t^n, \mu_t, v_{1,t}, v_{2,t}, \ldots, v_{L,t}, \tau_t, \hat{b}_t^m, \hat{P}_t^m)'$, $\bar{\epsilon}_t = (\epsilon_t^n, \epsilon_t^\mu, \epsilon_t^f)'$, and $\tilde{\epsilon}_t = (\epsilon_t^{MP}, \epsilon_{1,t}^R, \ldots, \epsilon_{3,t}^R)'$. It follows that the equilibrium process for $\hat{i}$ is given by an equation of the form:

$$\hat{i}_t = G(s_t)_i Y_{t-1} + \bar{\Psi}(s_t)_i \bar{\epsilon}_t + \tilde{\Psi}(s_t)_i \tilde{\epsilon}_t$$

We now suppose that $\bar{\epsilon}_s = 0$ for all $s \in \{T, T+1, \ldots, T+L\}$ and $\tilde{\epsilon}_s = 0$ for all $s \in \{T+1, \ldots, T+L\}$. As in Appendix A.2. we can relax this assumption and allow the central bank to defend the peg using shocks after T. Next, we define the following matrices:

$$
\begin{aligned}
K^1_{s_t} &= (p_{s_t1}G(1)_i + p_{s_t2}G(2)_i) \\
K^2_{s_t} &= (p_{s_t1}p_{11}G(1)_iG(1) + p_{s_t1}p_{12}G(2)_iG(1) + p_{s_t2}p_{21}G(1)_iG(2) + p_{s_t2}p_{22}G(2)_iG(2)) \\
K^3_{s_t} &= (p_{s_t1}p_{11}^2G(1)_i(G(1))^2 + p_{s_t1}p_{11}p_{12}G(2)_i(G(1))^2 + p_{s_t1}p_{12}p_{21}G(1)_iG(2)G(1) + \\
&\quad p_{s_t1}p_{12}p_{22}G(2)_iG(2)G(1) + p_{s_t2}p_{21}p_{11}G(1)_iG(1)G(2) + p_{s_t2}p_{21}p_{12}G(2)_iG(1)G(2) + \\
&\quad p_{s_t2}p_{22}p_{21}G(1)_i(G(2))^2 + p_{s_t2}p_{22}^2G(2)_i(G(2))^2)
\end{aligned}
$$

where $G(s_t)_i$ is the row of $G(s_t)$ corresponding to the nominal interest rate, i, for $s_t \in \{1,2\}$ and p_{s_tk} is the probability of transition from state s_t to state k for $k \in \{1,2\}$. Under these assumptions:

$$
\begin{aligned}
i_T &= G(s_T)_iY_{T-1} + \tilde{\Psi}(s_T)_i\tilde{\epsilon}_T \\
E_Ti_{T+1} &= K^1_{s_T}(G(s_T)Y_{T-1} + \tilde{\Psi}(s_T)\tilde{\epsilon}_T) \\
E_Ti_{T+2} &= K^2_{s_T}(G(s_T)Y_{T-1} + \tilde{\Psi}(s_T)\tilde{\epsilon}_T) \\
E_Ti_{T+3} &= K^3_{s_T}(G(s_T)Y_{T-1} + \tilde{\Psi}(s_T)\tilde{\epsilon}_T)
\end{aligned}
$$

If we set $i_s = \bar{i}$ for all $s \in \{T, T+1, T+3\}$, then we have a system of 4 equations in 4 unknowns which are the elements of $\tilde{\epsilon}_T$. The solution to this system is the set of shocks that sets the interest rate peg. The solution is given by:

$$
\begin{pmatrix} \tilde{\Psi}(s_T)_i \\ K^1_{s_T}\tilde{\Psi}(s_T) \\ K^2_{s_T}\tilde{\Psi}(s_T) \\ K^3_{s_T}\tilde{\Psi}(s_T) \end{pmatrix}^{-1} \left(\bar{i}1_{4\times1} - \begin{pmatrix} G(s_T)_i \\ K^1_{s_T}G(s_T) \\ K^2_{s_T}G(s_T) \\ K^3_{s_T}G(s_T) \end{pmatrix} Y_{T-1} \right) = \tilde{\epsilon_T}
$$

where $1_{L+1\times1}$ is a 4×1 vector of ones.

Since $E_Ti_{T+1} \neq i_{T+1}$ is typically true in the presence of switching coefficients and non-absorbing states, the central bank will typically have to announce a new sequence of shocks at $T+1$ to defend the interest rate peg (i.e. the central bank issues new monetary shocks to ensure that $i_{T+1} = E_{T+1}i_{T+2} = E_{T+1}i_{T+3} = \bar{i}$). Define $\tilde{\epsilon_{T+1}} = (\epsilon^{MP}_{T+1}, \epsilon^R_{1,T+1}, \epsilon^R_{2,T+1}, 0)'$. Then at time $T+1$:

$$
\begin{aligned}
i_{T+1} &= G(s_{T+1})_iY_T + \tilde{\Psi}(s_{T+1})_i\tilde{\epsilon}_{T+1} \\
E_{T+1}i_{T+2} &= K^1_{s_{T+1}}(G(s_{T+1})Y_T + \tilde{\Psi}(s_{T+1})\tilde{\epsilon}_{T+1}) \\
E_{T+1}i_{T+3} &= K^2_{s_{T+1}}(G(s_{T+1})Y_T + \tilde{\Psi}(s_{T+1})\tilde{\epsilon}_{T+1})
\end{aligned}
$$

If we set $i_s = \bar{i}$ for all $s \in \{T+1, T+3\}$, then we have a system of 3 equations in 3 unknowns, which we solve for $\epsilon^{MP}_{T+1}, \epsilon^R_{1,T+1}, \epsilon^R_{2,T+1}$ as before. This process repeats itself in $T+2$ where we use equations for i_{T+2} and i_{T+3} to solve for the pair $(\epsilon^{MP}_{T+2}, \epsilon^R_{1,T+2})$ that sets $i_{T+2} = E_{T+2}i_{T+3} = \bar{i}$. Then again at $T+3$ we use the equilibrium equation for i_{T+3} to solve for the ϵ^{MP}_{T+3} that sets $i_{T+3} = \bar{i}$

A.4. Tables

Table 2: Fixed Coefficient Model Parameterization

	Description	**Regime M**	**Regime F (short-term debt)**	**Regime F (long-term debt)**
σ	CRRA parameter	1	1	1
β	Discount Factor	.99	.99	.99
κ	Slope of Phillips Curve	.1	.1	.1
ϕ_π	Feedback Inflation	1.5	0	0
ϕ_y	Feedback Output	0	0	0
ρ_n	AR(1) natural rate parameter	.5	.5	.5
ρ_μ	AR(1) cost-push parameter	.5	.5	.5
ρ	Average Maturity of Debt	0	0	.93
γ	Feedback Debt	2	.1	.1

Table 3: Regime-Switching Model Parameterizations

	$\gamma(M)$	$\gamma(F)$	$\phi_\pi(M)$	$\phi_\pi(F)$	p_{MM}	p_{FF}
Figure 4	20	-5	1.5	0	.95	.95
Figure 5	20	-5	1.5	.8	.95	.95
Figure 6	5	-5	1.5	.8	.95	.95

Section 4 parameterizations same as Section 3 parameterizations except for the above values.

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