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Precision of Public Information Disclosures, Banks'Stability and Welfare*

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Abstract

We study the optimal precision of public information disclosures about banks' assets quality. In our model the precision of information affects banks' cost of raising funding and asset profile riskiness. In an imperfectly competitive banking sector, banks' stability and social surplus are non-monotonic functions of precision: an intermediate precision (or low-to-intermediate precision if banks contract their repayment promises on public information) maximizes stability, and also yields the maximum surplus when the social cost of bank failure c is large. When c is small and the banks' asset risk taking is not too sensitive to changes in the precision, the maximum surplus (and maximum risk) are reached at maximal precision. In a perfectly competitive banking sector in which banks' asset risk taking is not too sensitive to the precision of information, the maximum surplus (and maximum risk) are reached at maximal precision, while maximum stability is reached at minimal precision.

Keywords: financial stability, stress tests, bank transparency, banking regulation JEL codes: G21, G28, D83

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1 Introduction

In the aftermath of the global financial crisis, public information disclosure about banks' financial assets has emerged as a novel policy tool to promote financial stability. In a stress test, for example, a regulatory authority acquires information about the quality of a bank's assets, which is totally or partially disclosed to the public. It is well known that the disclosure of public information has complex effects on investors' confidence, and hence on the banks' costs of funds and risk-taking incentives – see, e.g., Goldstein and Sapra (2014). Thus, it is unclear how much information should be revealed by a regulator maximizing stability or welfare.

We study how the precision of public information about bank's asset quality affects both financial stability and social welfare. We show that some degree of opacity is conducive to stability, whereas maximal precision maximizes welfare when social costs of bank failures are small and banks' asset risk taking is not too sensitive to the precision of public information. These conclusions are robust to various assumptions about the structure of the banking sector and the contractibility of asset quality review outcomes, although these features affect optimal disclosure policies.

In our model, a bank chooses the level of risk of its assets' portfolio and, to be able to raise additional funding to complete the investments, offers investors a repayment promise. Before investors decide whether to provide funds to the bank, a regulatory authority discloses a signal of the quality of the bank's asset portfolio. This regulatory authority decides how informative (i.e., precise) is this signal, knowing that the precision of the signal affects the investors' willingness to invest, and hence the bank's cost of funds and asset risk profile.

The signal of the bank's asset quality may be interpreted as being generated by a stress test or a similar asset quality review. In practice, the purpose of regulatory stress testing is to determine whether the bank has enough capital to sustain adverse economic conditions – see, e.g., European Central Bank (2014) and Federal Reserve Board (2018) for the details on bank stress testing practices in the euro area and the United States, respectively. Banks that fail to pass these stress tests are forced to enhance their capital buffers, or even liquidated. In addition, public disclosures of information about a bank's assets quality aim to introduce market discipline and foster investors' confidence. Our model captures some of the main trade-offs associated with the precision of public disclosures of information about banks' asset quality while abstracting from the institutional details about the source of such information. Thus, the implications of our analysis extend to information disclosures about the quality of banks' asset portfolios from third parties such as credit-rating agencies, providing answers to the question of how such information disclosures should be regulated to promote stability and welfare.

We uncover two welfare effects of public disclosures of information about banks' asset quality: A direct effect improving investors' ability to separate solvent from insolvent banks, and an indirect effect on banks' risk-taking incentives. The direct effect, reducing both false positives (allowing insolvent banks to raise funding) and false negatives (preventing solvent banks from raising funding), improves the economy's resource allocation when the signal is sufficiently precise that affects investor decisions, and has no effect otherwise. The impact of the indirect effect on welfare and risk-taking incentives, however, depends on whether the environment is relatively opaque or transparent, and can be either positive or negative.

We find that when a bank operates in a relatively opaque environment, i.e., when the precision of public information about the quality of a bank's asset portfolio is below a certain threshold, the bank can refinance its investments irrespective of public information disclosure. As a result, the direct welfare effect of public information disclosures is absent. However, the indirect effect provides market discipline: since riskier banks pay more for their funding the more precise is the information, banks' risk choice decreases with information precision.

When a bank operates in a more transparent environment, i.e., when the precision of public information about the quality of the bank's asset portfolio is above this threshold, it becomes too costly for a bank to refinance its investments when the news about the asset's quality is bad: if public information signals that a bank's investments are unlikely to pay its return, the bank is unable to refinance and fails. Thus, a positive direct welfare effect of public information disclosures emerges, steering investments to successful assets. However, the more precise is the information the lower is the cost of funding for a sound bank and therefore, due to limited liability, the larger the bank's incentives to choose riskier portfolios. Hence, more precise information has an adverse indirect effect on welfare. In other words, in this relative transparent environment, more precise public information weakens market discipline, and leads to high asset risk levels.

In an opaque environment, a bank only fails when its assets do not pay their return, and asset riskiness decreases with the precision of public information. Hence, maximal stability is reached in this environment at the threshold level of precision. In a transparent environment, however, a bank fails either when the news about the quality of its asset is bad (because the bank is unable to raise funding), or when the news is good, but the asset pays no return. Moreover, a bank takes more risk than in an opaque environment. Therefore, an intermediate level of precision maximizes stability. Naturally, if the social costs of bank failures are sufficiently high, the precision that maximizes banking sector stability also maximizes welfare. However, we show that if the social costs of bank failures are small enough and the bank's asset risk choice is sufficiently insensitive to changes in the precision of public information, the benefits of more efficient resource allocation outweigh the costs of increased asset risk-taking. As a result, highly precise public information is optimal.

In our baseline model a bank does not condition its repayment promise on the public information disclosures about its asset quality review. Enlarging the set of contracts to allow a conditional repayment promise affects equilibrium outcomes in opaque environments, since banks receiving favorable asset quality reviews can raise funds at lower costs. This effect erodes market discipline, but also reduces the range of parameter values for which banks fail to raise funding.

We also study whether the results change if competition erodes the market power in the banking sector. We show how competition forces banks to gamble, i.e., to choose risky assets in order to be able to promise high returns to investors, simultaneously hoping a favorable outcome of the asset quality review. Since gambling is optimal irrespective of the precision of public information disclosures, the effects of changes in the precision of public information in a competitive banking sector are similar to those it has for a bank with market power operating in a relatively transparent environment.

Our work contributes to the emerging literature on stress test design (see, e.g., Orlov *et al.* 2017, Williams 2017, Goldstein and Leitner 2018, Inostroza and Pavan 2020, Quigley and Walther 2020), and also relates to the literature on optimal bank transparency (e.g., Chen and Hasan 2006, Bouvard *et al.* 2015, and Moreno and Takalo 2016), as well as to the literature studying the effects of information quality on financial stability (e.g., Vives 2014, and Iachan and Nenov 2015). In our model the optimal financing contract between banks and investors is a function of the precision of information about banks' asset quality as in Goldstein and Pauzner (2005), and our results support the finding of Dang et al. (2017) that an opaque banking system is conducive to investor confidence. Our results also contribute to the question of how competition affects the design of optimal disclosure policies, studied in the context of banking by, e.g., Matutes and Vives (2000) and Hyytinen and Takalo (2002) and, in the context of credit-rating agencies, by Goel and Thakor (2015).

The rest of the paper is organized as follows: Section 2 introduces the basic model, which features a bank, a group of investors, and a public disclosure of information about the bank's asset quality. In Section 3 we derive the main properties of the unique equilibrium of the game in which the investors can condition their decisions on the outcome of the asset quality review, but the bank cannot, and we study socially optimal disclosures of the bank's asset quality. Section 4 studies a variation of the model in which the bank can condition its promised repayment rate on the asset quality signal. Section 5 studies implications of competition among banks for the optimal disclosure policies. Section 6 provides numerical examples illustrating our main results, and Section 7 concludes. While we discuss the main arguments leading to our results in the body of the paper, we relegate to the Appendix technical proofs and calculations.

2 The Model

We consider a baseline setting in which a *bank* and a measure one of risk neutral *investors* interact. The bank selects an asset from a collection of assets $\{R(\sigma),\ \sigma \in$

$[\underline{\sigma}, \bar{\sigma}]\}$, where $0 < \underline{\sigma} < \bar{\sigma} \leq 1$, and offers to repay $\rho \in [0, \infty)$ monetary units for each monetary unit invested in the asset. Assets have constant returns to scale: $R(\sigma)$ pays $r(\sigma)$ per unit of investment if it is successful, which happens with probability σ, and pays 0 otherwise. Hence the expected return of asset $R(\sigma)$ is $\mathbb{E}[R(\sigma)] = \sigma r(\sigma)$. Note that σ serves as an inverse measure of the asset's risk. We impose the following mild assumption on the return function r.

Assumption 1. The return function $r : [\underline{\sigma}, \bar{\sigma}] \rightarrow [0, \infty)$ is twice differentiable, strictly decreasing, and such that the expected return $\mathbb{E}[R(\sigma)]$ is strictly concave and satisfies $\mathbb{E}[R(\tilde{\sigma})] > 1$, where $\tilde{\sigma} := \arg\max_{\sigma \in [\underline{\sigma}, \bar{\sigma}]} \mathbb{E}[R(\sigma)]$.

Assumption 1 implies that $\mathbb{E}[R(\sigma)]$ is twice differentiable and satisfies

$$\mathbb{E}'[R(\sigma)] = r(\sigma) + \sigma r'(\sigma) \gtreqless 0 \Leftrightarrow \sigma \lesseqgtr \tilde{\sigma},$$

and

$$\mathbb{E}''[R(\sigma)] = 2r'(\sigma) + \sigma r''(\sigma) < 0.$$

Assuming that $\mathbb{E}[R(\tilde{\sigma})] > 1$ is required for the exercise to be non-trivial.

Once the bank has selected an *asset* $\sigma \in [\underline{\sigma}, \bar{\sigma}]$ and *repayment promise* $\rho \in [0, \infty)$, an asset quality review (e.g., a stress test) is conducted, which yields a binary signal $S(q)$ of the likelihood that the asset will pay its return, where $q \in [1/2, 1]$ is the signal's *precision*. Specifically, regardless of the asset chosen by the bank, $S(q) = h$ (respectively, $S(q) = l$) with probability $q \in [1/2, 1]$ when the asset pays (does not pay) a return; i.e.,

$$\Pr[S(q) = h \mid R(\sigma) = r(\sigma)] = \Pr[S(q) = l \mid R(\sigma) = 0] = q.$$

Thus, the signal is correct with probability q, and is misleading with probability $1-q$. The signal $S(q)$ is truthfully disclosed to the investors. (Disclosing is often mandatory in practice. Even if it were voluntary, well known "unraveling" results – see, e.g., Grossman (1981), Milgrom (1981) – suggest that disclosure is the likely outcome when information cannot be manipulated and the investors know that information has been acquired. Also, hiding the signal $S(q)$ may be too costly.)

Each investor owns one monetary unit and, upon observing the bank's asset choice and repayment promise (σ, ρ) as well as the signal $S(q)$, chooses whether or not to invest it in the asset. (Alternatively, the investor's decision could be viewed as whether to rollover a loan whose payoff is endogenous, or withdraw and receive a monetary unit.) As we show in Section 3, assuming that σ is observed by investors is justified since the bank has incentives to disclose this information – see Remark 1. Also, we assume that the bank cannot condition its repayment promise ρ on the signal $S(q)$. (In Sections 4 and 5 we study a version of the model in which this assumption is relaxed.)

The payoff of an investor who chooses not to invest is 1, while the payoff of an investor who chooses to invest when the bank offers the *contract* $(\sigma, \rho) \in [\underline{\sigma}, \bar{\sigma}] \times [0, \infty)$ and the signal realization is $s \in \{h, l\}$ is

$$u(q, \sigma, \rho, s) = \Pr[R(\sigma) = r(\sigma) \mid S(q) = s] \min\{\rho, r(\sigma)\}, \tag{1}$$

The probabilities that an asset σ pays its return when the signal realization is either h or l are readily calculated as

$$\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = h\right] = \frac{q\sigma}{q\sigma + (1-q)(1-\sigma)} \tag{2}$$

and

$$\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = l\right] = \frac{(1-q)\sigma}{q\left(1-\sigma\right) + (1-q)\sigma}. \tag{3}$$

The following inequalities, which are strict for $q > 1/2$, are easily verified:

$$\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = h\right] \geq \sigma \geq \Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = l\right]. \tag{4}$$

Equation (1) presumes that investors have seniority if the bank cannot honor its contract. As we shall see, in equilibrium the bank is able to make the promised repayment unless the asset fails to pay its return. We describe investors' behavior by a vector (x_h, x_l) identifying the fractions of investors who choose to invest upon the high and the low signal, respectively.

The bank's expected payoff (profit) if it chooses $(\sigma, \rho) \in [\underline{\sigma}, \bar{\sigma}] \times [0, \infty)$ and investors' behavior is described by (x_h, x_l) is

$$B(q, \sigma, \rho, x_h, x_l) = \sigma\left(qx_h + (1-q)x_l\right)\left(r(\sigma) - \rho\right)_+, \tag{5}$$

where, and throughout the paper, for $z \in \mathbb{R}$ we write $z_+ := \max\{z, 0\}$. In equation (5), the term $(qx_h + (1-q)x_l)$ is the expected fraction of investors who invest, and the term $(r(\sigma) - \rho)_+$ is the bank's profit per unit of investment.

For $q \in [1/2, 1]$ we denote by $\Gamma(q)$ the dynamic game describing the bank and investors interactions. The game $\Gamma(q)$ proceeds in three-stages: in the first stage the bank chooses an asset and repayment promise, $(\sigma(q), \rho(q)) \in [\underline{\sigma}, \bar{\sigma}] \times [0, \infty)$, which is publicly observed. In the second stage, the realization of the signal $S(q)$ is disclosed, and investors choose whether to invest or not. Thus, investors' behavior when the signal is $s \in \{h, l\}$ is described by a mapping $x_s^*(q, \cdot)$ identifying for each contract offer $(\sigma, \rho) \in [\underline{\sigma}, \bar{\sigma}] \times [0, \infty)$ the fraction of investors who invest $x_s^*(q, \sigma, \rho) \in [0, 1]$. In the third stage the bank's asset return is realized and investors are compensated according to contract. The bank's repayments to investors are verifiable.

In an pure strategy perfect Bayesian equilibrium of $\Gamma(q)$ investors choose to invest whenever their expected payoff to investing is at least 1, and do not invest otherwise, while the bank chooses an asset and repayment promise that maximizes its expected payoff given the investors' behavior. A formal definition follows.

Definition 1. *A profile* $(\sigma^*(q), \rho^*(q), x_h^*(q, \cdot), x_l^*(q, \cdot))$ *is an equilibrium of* $\Gamma(q)$ *if it satisfies:*
(i) For all $(\sigma, \rho, s) \in [\underline{\sigma}, \bar{\sigma}] \times [0, \infty) \times \{h, l\}$*:* $u(q, \sigma, \rho, s) \geq 1$ *implies* $x_s^*(q, \sigma, \rho) = 1$*, and* $u(q, \sigma, \rho, s) < 1$ *implies* $x_s^*(q, \sigma, \rho) = 0$*; and*
(ii) $(\sigma^*(q), \rho^*(q)) \in \arg\max_{(\sigma,\rho) \in [\underline{\sigma}, \bar{\sigma}] \times [0,\infty)} B(q, \sigma, \rho, x_h^*(q, \sigma, \rho), x_l^*(q, \sigma, \rho))$.

While our model deals with a generic signal of the bank's asset quality, its implications extend to stress testing exercises whereby a regulator inquires about a bank's estimated assets returns in a certain adverse scenario, and assesses the bank's ability to absorb the losses it may incur in given its estimated capital. The signal h (l) is interpreted as meaning that the asset returns are sufficiently high (low), given the bank's capital, that it would (would not) survive such adverse scenario. The regulator publishes the bank's estimated asset returns directly or its capital ratio in the adverse scenario. Sometimes the key results published are coarse, e.g., the signals l and h may simply mean "fail" and "pass", respectively. However, even coarse the signals can be misleading. (Months after passing the 2011 European stress-tests, two large European banks, Bankia and Dexia, were on the verge of bankruptcy. The meltdown of one of the largest banks in Spain, Banco Popular, in June 2017, after passing its 2016 stress test, led Morgenson (2017) to note that "... there is much for investors to learn ... Lesson No. 1: Don't trust bank stress-test results.")

3 Analysis of the Model

In this section we study the equilibrium of the in the baseline setting, and show the it is a well defined mapping on the set of possible precisions of the public signal. Then we study the welfare properties of equilibrium.

3.1 Equilibrium

We show that the game $\Gamma(q)$ has a unique equilibrium,

$$(\sigma^*(q), \rho^*(q), x_h^*(q, \sigma^*(q), \rho^*(q)), x_l^*(q, \sigma^*(q), \rho^*(q)))$$

on $[1/2, 1]\backslash\{\bar{q}\}$. The level of precision $\bar{q}$ is a threshold identifying a change of regime: as q reaches $\bar{q}$ from below the bank changes the nature of the contract it offers, switching the objective from attracting investors whatever the signal, to attracting investors only when the signal is h. When the level of precision is exactly $\bar{q}$ the bank is indifferent between the two contracts, and hence two alternative equilibria arise. In what follows, we present the key equations and results.

In equilibrium, the investors' propensity to invest in a bank is increasing in the levels of asset quality signals and repayments – see Lemma 1 in the Appendix. Such investor behavior is intuitive and supported by laboratory evidence (see König-Kersting et al. 2020). Thus, an expected profit-maximizing bank offers the minimal repayment promise that attracts investors regardless of the signal,

$$\rho^*(q) = \frac{1}{\Pr\left[R\left(\sigma^*\right) = r\left(\sigma^*\right) \mid S(q) = l\right]},$$

and hence $x_h^*(q,\sigma^*,\rho^*) = x_l^*(q,\sigma^*,\rho^*) = 1$, or offers the minimal repayment promise that attracts investors only if the signal realization is h,

$$\rho^*(q) = \frac{1}{\Pr\left[R\left(\sigma^*\right) = r\left(\sigma^*\right) \mid S(q) = h\right]},$$

and hence $x_h^*(q,\sigma^*,\rho^*) = 1$ and $x_l^*(q,\sigma^*,\rho^*) = 0$. (For the proofs of these claims, see Lemmas 1 and 2 in the Appendix.) Building on these observations, we define two auxiliary functions,

$$\begin{aligned} B_l(q,\sigma) &= \sigma\left(r(\sigma) - \frac{1}{\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = l\right]}\right) \\ &= \mathbb{E}[R(\sigma)] - \frac{q\left(1-\sigma\right) + (1-q)\sigma}{1-q}, \end{aligned} \tag{6}$$

and

$$\begin{aligned} B_h(q,\sigma) &= q\sigma\left(r(\sigma) - \frac{1}{\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = h\right]}\right) \\ &= q\mathbb{E}[R(\sigma)] - q\sigma - (1-q)(1-\sigma), \end{aligned} \tag{7}$$

describing, respectively, the bank's expected profit when it offers the contract attracting investors regardless of the signal realization, which is given by

$$B(q,\sigma,1/\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = l\right],1,1) = B_l(q,\sigma)_+,$$

and when it offers the contract that attract investors only when $S(q) = h$, which is given by

$$B(q,\sigma,1/\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = h\right],0,1) = B_h(q,\sigma)_+.$$

Note that as q approaches 1, $B_l(q,\sigma)$ diverges to $-\infty$, while $B_h(q,\sigma)$ approaches $\mathbb{E}[R(\sigma)] - \sigma$. Further,

$$B_l(1/2,\sigma) = \mathbb{E}[R(\sigma)] - 1 = 2B_h(1/2,\sigma),$$

and therefore both $B_l(1/2,\cdot)$ and $B_h(1/2,\cdot)$ reach their maximum value at $\tilde{\sigma} \in (\underline{\sigma},\bar{\sigma})$ by Assumption 1. Also, both $B_l(q,\cdot)$ and $B_h(q,\cdot)$ are twice differentiable and strictly concave, and satisfy

$$\frac{\partial B_l(q,\sigma)}{\partial\sigma} = \mathbb{E}'[R(\sigma)] + \frac{2q-1}{1-q}, \tag{8}$$

and

$$\frac{\partial B_h(q,\sigma)}{\partial \sigma} = q\mathbb{E}'[R(\sigma)] - (2q-1). \tag{9}$$

Hence $\partial B_l(q,\sigma)/\partial\sigma > 0$ for $\sigma \in [\underline{\sigma}, \tilde{\sigma})$, and $\partial B_h(q,\sigma)/\partial\sigma < 0$ for $\sigma \in (\tilde{\sigma}, \bar{\sigma}]$.

We identify the bank's asset risk choice. Let us write

$$\sigma_l(q) := \arg\max_{\sigma\in[\underline{\sigma},\bar{\sigma}]} B_l(q,\sigma)$$

and

$$\sigma_h(q) := \arg\max_{\sigma\in[\underline{\sigma},\bar{\sigma}]} B_h(q,\sigma).$$

As noted above, $\sigma_l(1/2) = \sigma_h(1/2) = \tilde{\sigma}$. In the Appendix (Lemma 3), we establish that $\sigma_l > \sigma_h$ on $(1/2, 1]$, and that as q increases, σ_l (respectively σ_h) increases (decreases) until it reaches the value $\bar{\sigma}$ ($\underline{\sigma}$), remaining constant as q increases further.

The effect of the signal precision on the bank's asset risk-taking thus crucially depends on whether the equilibrium asset risk choice is identified by σ_l or by σ_h. Since both $B_l(1/2,\cdot)$ and $B_h(1/2,\cdot)$ reach their maximum at $\tilde{\sigma}$ and $2B_h(1/2,\tilde{\sigma}) = B_l(1/2,\tilde{\sigma}) = \mathbb{E}[R(\tilde{\sigma})] - 1 > 0$, when the signal's precision is close to 1/2 the equilibrium asset risk choice is $\sigma^*(q) = \sigma_l(q)$. Moreover, for q near 1, $B_l(q,\sigma) < B_h(q,\sigma)$, and therefore $\sigma^*(q) = \sigma_h(q)$. We show in the Appendix (Lemma 4) that there is $\bar{q} \in (1/2, 1)$ such that $\sigma^*(q) = \sigma_l(q)$ on $(1/2, \bar{q})$ and $\sigma^*(q) = \sigma_h(q)$ on $(\bar{q}, 1)$. At $\bar{q}$, there are two equilibria, one in which $\sigma^*(q) = \sigma_l(\bar{q})$ and another one in which $\sigma^*(q) = \sigma_h(\bar{q})$. Since $\sigma_l(\bar{q}) > \sigma_h(\bar{q})$, the mapping σ^* has a discontinuity at $\bar{q}$: as q crosses $\bar{q}$ from below, σ^* experiences a discrete jump downwards, from $\sigma_l(\bar{q}) > \tilde{\sigma}$ to $\sigma_h(\bar{q}) < \tilde{\sigma}$ – see Lemma 3 in the Appendix.

Consequently, the bank raises funding irrespective of the signal for levels of precision below $\bar{q}$, but raises funding only upon the high signal for levels of precision above $\bar{q}$. Further, the level of risk decreases with q below $\bar{q}$, but increases with q above $\bar{q}$. Thus, maximal (minimal) risk is reached at $q = 1$ (respectively, $q = \bar{q}$) – see Lemma 5 in the Appendix.

Write

$$B^*(q) := B(q, \sigma^*(q), \rho^*(q), x_h^*(q, \sigma^*(q), \rho^*(q)), x_l^*(q, \sigma^*(q), \rho^*(q)))$$

for the bank's equilibrium profit. This profit is positive irrespective of the signal's precision, i.e., $B^*(q) > 0$ for all q – see Lemma 2 in the Appendix. Moreover, $B^*(q)$ decreases below $\bar{q}$ and increases above $\bar{q}$, reaching its maximum value at $q = 1$, and its minimum value at $q = \bar{q}$ – see Lemma 5 in the Appendix.

Proposition 1 summarizes these findings and describes the effects of changes in the level of precision over the bank's payoff and risk choice. Figure 1 in Section 6 provides an illustration of these results for a linear return function r.

Proposition 1. *There is $\bar{q} \in (1/2, 1)$ such that in an equilibrium of $\Gamma(q)$ the bank offers the contract*

$$(\sigma^*(q), \rho^*(q)) = (\sigma_l(q), 1/\Pr\left[R\left(\sigma_l\left(q\right)\right) = r\left(\sigma_l\left(q\right)\right) \mid S(q) = l\right])$$

if $q \in [1/2, \bar{q})$, and offers the contract

$$(\sigma^*(q), \rho^*(q)) = (\sigma_h(q), 1/\Pr\left[R\left(\sigma_h\left(q\right)\right) = r\left(\sigma_h\left(q\right)\right) \mid S(q) = h\right])$$

if $q \in (\bar{q}, 1]$, whereas both these contracts may arise if $q = \bar{q}$. Both the bank's asset risk and its expected profit decrease with the level of precision on $(1/2, \bar{q})$, and increase on $(\bar{q}, 1)$, reaching a minimum at $q = \bar{q}$ and a maximum at $q = 1$.

The intuition for these result is as follows: Ignoring the cost, the bank would always prefer to raise funding to complete its project. When the environment is opaque (q is close to 1/2) the cost of raising funding upon the high and low signals are similar. As q increases, it becomes more (less) costly to raise funding upon the low (high) signal, because the likelihood that the asset will (will not) pay its return decreases. Eventually, when q reaches $\bar{q}$, the cost differential becomes too high, and the bank switches its choice, forgoing to raise funding when the signal is low, and increasing its asset riskiness.

In opaque environments (i.e., $q < \bar{q}$) increasing the precision imposes a market discipline effect, discouraging the bank's asset risk-taking: the bank partially compensates the impact of increasing q on the cost of raising funding by choosing a less risky asset. However, in transparent environments (i.e., $q > \bar{q}$) increasing the precision reduces the cost of raising funding. This feature together with the bank's limited liability encourages the bank to take more risk; i.e., this effect reverses the direction of market discipline. Proposition 1 thus reveals that in transparent environments there is a mismatch between stability concerns and bank's incentives to take risk: increasing the precision leads to both greater riskiness and greater profits.

We conclude our equilibrium analysis with an observation justifying our assumption that the bank's risk choice is observable: The bank's profit is larger when the risk choice is observable than when it is not. Thus, the bank has an incentive to commit and reveal its risk choice.

Remark 1. *The bank's profit is larger when the asset risk is observable than when it is unobservable.*

As shown in the Appendix, by revealing (i.e., committing) its risk choice the bank influences investors' decisions and increases its payoff. Revealing the risk choice gives the bank a first-mover advantage similar to that of a Stackelberg leader in a duopolistic industry of quantity competition, and results in a payoff increase – just as it does for the Stackelberg leader relative to its payoff in the Cournot equilibrium.

3.2 Welfare

Let us consider now the problem of a Regulator who can choose the precision of the bank's asset quality signal. For simplicity we assume that the set of feasible levels of precision is the interval $[1/2, 1]$. (If the set of feasible levels of precision is any closed subinterval of $[1/2, 1]$, then solving the Regulator's problem involves checking additional corners.)

The problem of a Regulator exclusively concerned with the stability of the banking sector is straightforward: the stability maximizing precision is that minimizing asset risk in the region where the bank can raise funding irrespective of the asset quality signal. Assuming that $(\sigma_l(\bar{q}), 1/\Pr[R(\sigma_l(\bar{q})) = r(\sigma_l(\bar{q})) \mid S(q) = l])$ is the equilibrium arising in $\Gamma(\bar{q})$, the stability maximizing precision is $\bar{q}$. (Since $(\sigma_h(\bar{q}), 1/\Pr[R(\sigma_h(\bar{q})) = r(\sigma_h(\bar{q})) \mid S(q) = h])$ is also an equilibrium of $\Gamma(\bar{q})$ by Proposition 1, the Regulator may alternatively choose a precision below but arbitrarily close to $\bar{q}$.)

Let us study next the problem of a Regulator who aims at maximizing social surplus rather than bank stability. Following the literature – see, e.g., Matutes and Vives (2000) and Freixas *et al.* (2007) – let us assume that the bank provides valuable services to the economy, so that its closure, due to either a failure to raise funding or a failure of its investments, creates an external social cost $c > 0$.

When the investors choose to invest regardless of the signal realization, the bank fails only if the investment does not pay its return, and therefore the surplus is the expected return net of the expected social cost of the bank failure, which given the bank asset choice σ is

$$W_l(c, \sigma) = \mathbb{E}[R(\sigma)] - (1-\sigma)c. \tag{10}$$

When the investors choose to invest only when the signal's realization is h and the bank selects the asset σ the surplus is

$$W_h(c, q, \sigma) = q\mathbb{E}[R(\sigma)] + [(1-q)\sigma + (1-\sigma)q] - (1-q\sigma)c. \tag{11}$$

The first term in the right-hand side of equation (11) is the expected return assuming that the investment is made only upon the signal realization $S = h$. The term in the middle is the expected surplus realized when the investment is not made because $S(q) = l$. The last term is the expected social cost of bank failure, which occurs except when $S(q) = h$ and the asset pays return.

By Proposition 1, for $(c, q) \in [0, \infty) \times [1/2, 1]$ the equilibrium surplus is

$$W(c, q) = \begin{cases} W_l(c, \sigma_l(q)) & \text{if } q \in [1/2, \bar{q}] \\ W_h(c, q, \sigma_h(q)) & \text{if } q \in [\bar{q}, 1]. \end{cases}$$

If the Regulator chooses a level of precision below $\bar{q}$, then the bank selects the asset $\sigma^*(q) = \sigma_l(q)$, and attracts the investors regardless of the signal. As equation (10) shows, in this relatively opaque environment the precision of public information affects welfare only indirectly via the bank's asset risk choice $\sigma_l(q)$. If the Regulator chooses a level of precision above $\bar{q}$, then the bank selects the asset $\sigma^*(q) = \sigma_h(q)$, and attracts the investors only if the signal is h. In this relatively transparent environment the precision of public information affects the surplus both directly, and indirectly via the bank's asset risk choice. The direct effect arises because an increase in the precision of public information increases the probability that investors can correctly separate a solvent bank from an insolvent one. This direct effect is absent in the relatively opaque environment where the bank can refinance its investments irrespective of the asset's quality signal S.

Let us identify the level of precision q_l that maximizes $W_l(c, \sigma_l(\cdot))$ on $[1/2, \bar{q}]$. Taking derivatives in equation (10) we get

$$\frac{\partial W_l(c, \sigma_l(q))}{\partial \sigma}\sigma_l'(q) = (\mathbb{E}'[R(\sigma_l)] + c)\,\sigma_l'(q). \tag{12}$$

Recall from Section 3 that $\sigma_l(q) \in [\tilde{\sigma}, \bar{\sigma}]$ and $\sigma_l'(q) > 0$ whenever $\sigma_l(q) < \bar{\sigma}$. Thus, if $\sigma_l(q) = \bar{\sigma}$, then $W_l(c, \cdot)$ is constant and maximal on $[q, \bar{q}]$. However, if $\sigma_l(q) < \bar{\sigma}$, then $\sigma_l(q)$ solves the equation $\partial B_l(q, \sigma)/\partial \sigma = 0$, which using equation (8) implies

$$\mathbb{E}'[R(\sigma_l)] = -\frac{2q-1}{1-q} := -\psi(q).$$

Therefore

$$\frac{\partial W_l(c, q)}{\partial \sigma}\sigma_l'(q) = (-\psi(q) + c)\,\sigma_l'(q).$$

Moreover, $\sigma_l'(q) > 0$ implies that either $c = \psi(q_l) < \psi(\bar{q})$, or $c \geq \psi(\bar{q})$ and $q_l = \bar{q}$. Since $\psi' > 0$, the level of precision that maximizes $W_l(c, \sigma(\cdot))$ on $[1/2, \bar{q}]$ is the mapping

$$q_l(c) = \min\{\psi^{-1}(c), \bar{q}\}, \tag{13}$$

where ψ^{-1} is readily calculated as

$$\psi^{-1}(c) = \frac{c+1}{c+2}.$$

Note $\psi^{-1}(0) = 1/2 < \bar{q}$, and hence $q_l(0) = 1/2$; that is, if there are no social externalities to bank failure, then maximizing $W_l(c, \sigma(\cdot))$ amounts to maximizing expected returns. Since $d\psi^{-1}(c)/dc = 1/(2+c)^2 > 0$, the precision that maximizes $W_l(c, \sigma(\cdot))$, $q_l(c)$, increases with c near $c = 0$: even though the signal has no effect on the investors' decision, increasing the precision reduces the level of risk, reducing the probability of bank failure.

We seek to identify the precision that maximizes $W_h(c, \cdot, \sigma_h(\cdot))$ on $(\bar{q}, 1]$. Taking derivatives in equation (11) we get

$$\frac{\partial W_h(c,q,\sigma)}{\partial q} + \frac{\partial W_h(c,q,\sigma)}{\partial \sigma}\sigma_h'(q) = \left(q\mathbb{E}'[R(\sigma_h)] + 1 - 2q + qc\right)\sigma_h'(q) + \sigma_h(q)c + H(q), \tag{14}$$

where

$$H(q) := \mathbb{E}[R(\sigma_h(q))] - \sigma_h(q) + 1 - \sigma_h(q) > 0.$$

Thus, the direct effect of an increase in q on welfare, captured by the two last terms of equation (14), is positive: the direct effect improves the resource allocation in the economy by reducing both false positives and false negatives. As a result, if the bank's asset choice is the corner solution $\sigma_h(q) = \underline{\sigma}$, then $\sigma_h' = 0$ above q. (Recall from Section 3 that $\sigma_h(q) \in [\underline{\sigma}, \tilde{\sigma}]$ and $\sigma_h'(q) < 0$ whenever $\sigma_h(q) > \underline{\sigma}$.) Hence

$$\frac{\partial W_h(c,q,\sigma)}{\partial q} + \frac{\partial W_h(c,q,\sigma)}{\partial \sigma}\sigma_h'(q) = \sigma_h(q)c + H(q) > 0,$$

i.e., $W_h(c, \cdot, \sigma_h(\cdot))$ is increasing on $(\bar{q}, 1]$, and therefore maximal precision of public information is optimal.

However, if the bank's asset choice is an interior solution $\sigma_h(q) \in (\underline{\sigma}, \bar{\sigma})$, then the Regulator also needs to take into account the indirect effect of q via the bank's asset risk choice $\sigma_h(q)$. The equilibrium asset risk choice solves the equation $\partial B_h(q,\sigma)/\partial\sigma = 0$ which, by using equation (9), can be written as

$$q\mathbb{E}'[R(\sigma_h)] = 2q - 1.$$

Substituting into equation (14) gives

$$\begin{aligned}\frac{\partial W_h(c,q,\sigma)}{\partial q} + \frac{\partial W_h(c,q,\sigma)}{\partial \sigma}\sigma_h'(q) &= \sigma_h'(q)qc + \sigma_h(q)c + H(q) \\ &= \left(\varepsilon_h(q) + 1\right)c\sigma_h(q) + H(q),\end{aligned} \tag{15}$$

where

$$\varepsilon_h(q) := \frac{q\sigma_h'(q)}{\sigma_h(q)}$$

denotes the elasticity of the bank's asset risk-taking with respect to the precision of information disclosure.

Write

$$\bar{e} := -1 - \frac{H(\bar{q})}{c\sigma_h(\bar{q})}. \qquad (16)$$

Since $r' < 0$ and $\sigma_h' < 0$,

$$\left(\frac{H}{\sigma_h}\right)' = \left(r + \frac{1}{\sigma_h} - 2\right)' = r'\sigma_h' - \frac{\sigma_h'}{\sigma_h^2} > 0,$$

and therefore $\varepsilon_h(q) > \bar{e}$ implies that the right-hand side of equation (15) is positive, and the surplus reaches its maximum on $(\bar{q}, 1]$ at $q = 1$. The intuition for this result is clear: In a relative transparent banking environment, in which the investment is made only upon a high signal, an increase in the precision of information disclosure allows investors to separate more accurately a solvent from an insolvent bank, which reduces the probability that either a solvent bank is liquidated or an insolvent bank receives funding. There is a cost to increasing transparency as it leads to increasing risk-taking. But unless the bank's risk-taking is highly sensitive to changes in the precision, the benefits of more efficient resource allocation outweigh the incremental cost of bank failure due to unsuccessful asset risk-taking. As a result, maximal precision is optimal.

The maximum surplus $W(c, \cdot)$ on $[1/2, \bar{q}]$ is $W_l^*(c) := W_l(c, q_l(c))$, while the maximum surplus $W(c, \cdot)$ on $[\bar{q}, 1]$ is $W_h^*(c) := W_h(c, 1)$ when $\varepsilon_h \geq \bar{e}$. We show in the proof of Proposition 2 in the Appendix that there exists $\bar{c} > 0$ such that $W_h^*(c) \lesseqgtr W_l^*(c)$ if and only if $c \gtreqless \bar{c}$.

Proposition 2 summarizes the implications of the results derived above.

Proposition 2: *In the setting described by the game $\Gamma(q)$ for $q \in [1/2, 1]$, the precision $\bar{q}$ maximizes stability. Moreover, if $c < \bar{c}$, then the maximum surplus is reached at the precision $q_l(c)$. If $c < \bar{c}$* and $\varepsilon_h(q) \geq \bar{e}$, *then the maximum surplus is reached at the maximal precision $q = 1$.*

Stability requires a relatively opaque environment (i.e., involves an intermediate level of precision no greater than $\bar{q}$) since in transparent environments (i.e., when $q > \bar{q}$) the bank both takes more risk and fails to raise funding if the asset quality signal is unfavorable. As for the surplus maximizing precision, when the social cost of bank failure is sufficiently small, i.e., $c < \bar{c}$, a maximally revealing asset quality signal generates an efficient allocation of resources: as q approaches 1 the bank raises funding cheaply and invests in an highly risky asset, but completes the investment when it is solvent, and is liquidated when it is not. The high riskiness of the bank's asset yields high returns in case of success and, even though the bank is more likely to fail, such failure is not too costly to society. In contrast, when the social cost of bank failure is sufficiently large, i.e., $c \geq (2\bar{q} - 1) / (1 - \bar{q})$, the the precision that maximizes surplus is $q_l(c) = \bar{q}$; i.e., for sufficiently high social costs of bank failure, choosing the stability-maximizing level of precision $\bar{q}$ becomes optimal as well from the surplus perspective.

Depending on the return function r, $\bar{c}$ can be smaller or larger than $\psi(\bar{q})$. If $\bar{c} < \psi(\bar{q})$, then for intermediate values of the cost of bank failure $c \in (\bar{c}, \psi(\bar{q}))$ a low precision of the asset signal, $q_l(c) = \psi^{-1}(c) = (c + 1) / (c + 2)$, is socially optimal: for these intermediate values of c the signal must be sufficiently imprecise to allow the bank to raise funding irrespective of the signal realization. Moreover, since in this regime the bank's asset risk-taking decreases with the precision of the signal, it is optimal to increase the precision the greater is c. If $\psi(\bar{q}) \leq \bar{c}$, however, this intermediate region vanishes, and the optimal precision is a binary variable, taking the value 1 for $c \leq \bar{c}$ and $\bar{q}$ for $c \geq \bar{c}$. Nonetheless, whether $\bar{c}$ is smaller or larger than $\psi(\bar{q})$ the main message of Proposition 3 is the same: as the cost of bank failure increases from zero, the socially optimal precision drops from maximal precision to an intermediate level of precision. Figure 2 in Section 6 provides a numerical example illustrating the results of Proposition 3 for the case $\psi(\bar{q}) > \bar{c}$.

Note that $\varepsilon_h \geq \bar{e}$ is a sufficient condition for $q = 1$ being optimal for $c \in [0, \bar{c}]$. If $\varepsilon_h < \bar{e}$, then an interior level of precision may be optimal for $c \in [0, \bar{c}]$. However, even in this case the optimal precision would be larger for $c \leq \bar{c}$ than for $c \geq \bar{c}$. Of course, were the right-hand side of equation (15) negative for all q on $[\bar{q}, 1]$, the optimal precision would be $q_l(c)$ for all c.

4 Contingent Contracts

In our baseline model a bank does not condition its repayment promise on the public information disclosure. While such contingent contracts are rare in practice, the assumption violates the informativeness principle (due to Holmström 1979) according to which informative signals should be included in contracts. We may also equivalently think of a bank as raising funds after the information disclosure. Thus, in this section we consider a variation of the setting of Section 2 in which the bank first selects the asset $\sigma \in [\underline{\sigma}, \bar{\sigma}]$, and then chooses the repayment promise $\rho \in [0, \infty)$ after the realization of the asset's signal $S(q)$ is observed. Although the payoffs in this modified setting are as in the previous setting, it is useful to describe them explicitly. If the bank chooses the asset $\sigma \in [\underline{\sigma}, \bar{\sigma}]$ and repayment promises $(\rho_h, \rho_l) \in [0, \infty)^2$, and the fractions of investors who choose to invest upon the high and low signals are $(x_h, x_l) \in [0, 1]^2$, then the bank's expected payoff when the signal is $s \in \{h, l\}$ is

$$\hat{B}_s(q, \sigma, \rho_s, x_s) = \Pr\left[R(\sigma) = r(\sigma) \mid S(q) = s\right] (r(\sigma) - \rho_s)_+ x_s, \tag{17}$$

whereas the bank's expected payoff before the signal is realized is

$$\hat{B}(q, \sigma, \rho_h, \rho_l, x_h, x_l) = q\sigma\left(r(\sigma) - \rho_h\right)_+ x_h + (1 - q)\sigma\left(r(\sigma) - \rho_l\right)_+ x_l. \tag{18}$$

For $q \in [1/2, 1]$ we denote by $\hat{\Gamma}(q)$ the game describing the bank and investors interactions in this modified setting. The game $\hat{\Gamma}(q)$ proceeds in four-stages: in the first stage the bank chooses an asset $\hat{\sigma}(q) \in [\underline{\sigma}, \bar{\sigma}]$, which is publicly observed. In the second stage the realization of the signal $S(q) = s$ is disclosed, and the bank offers a repayment promise ρ_s. Thus, the bank's behavior at this stage when the signal $s \in \{h, l\}$ is disclosed is described by a function $\hat{\rho}_s(q, \cdot)$ identifying for each asset choice $\sigma \in [\underline{\sigma}, \bar{\sigma}]$ the bank's repayment promise $\hat{\rho}_s(q, \sigma) \in [0, \infty)$. In the third stage, investors choose whether to invest or not upon observing the bank's asset choice $\hat{\sigma}(q)$, the signal $S(q) = s$, and the bank's repayment promise ρ_s. Thus, investors' behavior when signal $s \in \{h, l\}$ is disclosed is described by a function $\hat{x}_s(q, \cdot)$ identifying for each contract offer $(\sigma, \rho) \in [\underline{\sigma}, \bar{\sigma}] \times [0, \infty)$ the fraction of investors who invest $\hat{x}_s(q, \sigma, \rho) \in [0, 1]$. In the fourth stage the asset return is realized and investors are compensated according to the contract offer.

In an pure strategy perfect Bayesian equilibrium of $\hat{\Gamma}(q)$ investors choose to invest whenever their expected payoff to investing is at least 1, and do not invest otherwise, while the bank's choices at each stage maximize its expected payoff given investors' behavior or its own prior choices, or both. A formal definition follows.

Definition 2. *A profile* $(\hat{\sigma}^*(q), \hat{\rho}_h^*(q,\cdot), \hat{\rho}_l^*(q,\cdot), \hat{x}_h^*(q,\cdot), \hat{x}_l^*(q,\cdot))$ *is an equilibrium of* $\hat{\Gamma}(q)$ *if it satisfies:*
(i) For all $(\sigma,\rho,s) \in [\underline{\sigma},\bar{\sigma}] \times [0,\infty) \times \{h,l\}$*:* $u(q,\sigma,\rho,s) \geq 1$ *implies* $\hat{x}_s^*(q,\sigma,\rho) = 1$*, and* $u(q,\sigma,\rho,s) < 1$ *implies* $\hat{x}_s^*(q,\sigma,\rho) = 0$*.*
(ii) For all $(\sigma,s) \in [\underline{\sigma},\bar{\sigma}] \times \{h,l\}$*,* $\hat{\rho}_s(q,\sigma) \in \arg\max_{\rho\in[0,\infty)} \hat{B}_s(q,\hat{\sigma}(q),\rho,\hat{x}_s^*(q,\sigma,\rho))$*.*
(iii) $\hat{\sigma}^*(q) \in \arg\max_{\sigma\in[\underline{\sigma},\bar{\sigma}]} \hat{B}(q,\sigma,\hat{\rho}_h^*(q,\sigma),\hat{\rho}_l^*(q,\sigma),\hat{x}_h^*(q,\sigma,\hat{\rho}_h^*(q,\sigma),\hat{x}_l^*(q,\sigma,\hat{\rho}_l^*(q,\sigma))$*.*

We show that the game $\hat{\Gamma}(q)$ has a unique equilibrium on $[1/2,1]\backslash\{\hat{q}\}$. The level of precision $\hat{q}$ is a threshold identifying a change of regime: for q below $\hat{q}$ the bank selects the asset $\tilde{\sigma}$ and attracts investors whatever the signal, whereas above $\hat{q}$ the bank selects a more risky asset, and forgoes attracting investors when the signal is l. When the level of precision is $\hat{q}$ the bank is indifferent between the two contracts, and hence two alternative equilibria arise.

Given the bank's asset choice σ, upon observing the signal realization $S(q) = s$ it is a dominant strategy for the bank to offer the minimum repayment promise that induces investors to invest, $1/\Pr[R(\sigma) = r(\sigma) \mid S(q) = s]$. Moreover, this strategy is strictly dominant if

$$r(\sigma) > \frac{1}{\Pr[R(\sigma) = r(\sigma) \mid S(q) = s]}.$$

Hence for $s \in \{h,l\}$,

$$\frac{1}{\Pr[R(\hat{\sigma}^*) = r(\hat{\sigma}^*) \mid S(q) = s]} = \hat{\rho}_s^*(q,\hat{\sigma}^*).$$

Also, since

$$r(\tilde{\sigma}) > \frac{1}{\tilde{\sigma}} \geq \frac{1}{\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h]} = \hat{\rho}_h^*(q,\tilde{\sigma}),$$

where the first inequality follows from Assumption 1 and the second from inequality (4), the bank can secure a positive expected profit by choosing the asset $\tilde{\sigma}$ and repayment promise $\hat{\rho}_h^*(q,\tilde{\sigma})$. Hence, in equilibrium the bank's asset choice and repayment promises satisfy

$$r(\hat{\sigma}^*) > \frac{1}{\Pr\left[R\left(\hat{\sigma}^*\right) = r\left(\hat{\sigma}^*\right) \mid S(q) = h\right]} = \hat{\rho}_h^*(q, \hat{\sigma}^*),$$

and therefore $\hat{x}_h^* = 1$, since

$$\frac{1}{\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = h\right]} \leq \frac{1}{\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = l\right]}$$

would otherwise imply that the bank's expected profit is zero. Hence, either $r(\hat{\sigma}^*) < \hat{\rho}_l^*(q, \hat{\sigma}^*)$ and $\hat{x}_l^* = 0$, or $r(\hat{\sigma}^*) \geq \rho_l^*(q, \hat{\sigma}^*)$ and $\hat{x}_l^* = 1$, and therefore the bank's equilibrium expected profit is

$$\begin{aligned} \hat{B}(q, \hat{\sigma}^*, \hat{\rho}_h^*, \hat{\rho}_l^*, \hat{x}_h^*, \hat{x}_l^*) &= q\hat{\sigma}^* \left(r(\hat{\sigma}^*) - \frac{1}{\Pr\left[R\left(\hat{\sigma}^*\right) = r\left(\hat{\sigma}^*\right) \mid S(q) = h\right]} \right) \\ &\quad + (1-q)\hat{\sigma}^* \left(r(\hat{\sigma}^*) - \frac{1}{\Pr\left[R\left(\hat{\sigma}^*\right) = r\left(\hat{\sigma}^*\right) \mid S(q) = l\right]} \right)_+ . \end{aligned}$$

Let us now discuss the bank's asset choice. If $r(\hat{\sigma}^*) \geq \hat{\rho}_l^*(q, \hat{\sigma}^*)$, then

$$\frac{q\sigma}{\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = h\right]} + \frac{(1-q)\sigma}{\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = l\right]} = 1$$

(i.e., in expectation the bank pays to each investor exactly one monetary unit) implies

$$\hat{B}(q, \hat{\sigma}^*, \hat{\rho}_h^*, \hat{\rho}_l^*, \hat{x}_h^*, \hat{x}_l^*) = \hat{\sigma}^* r(\hat{\sigma}^*) - 1 = \mathbb{E}[R(\hat{\sigma}^*)] - 1,$$

and therefore $\hat{\sigma}^*$ maximizes $\mathbb{E}[R(\sigma)]$, i.e., $\hat{\sigma}^* = \tilde{\sigma}$. If $r(\hat{\sigma}^*) < \hat{\rho}_l^*(q, \hat{\sigma}^*)$, then

$$\hat{B}(q, \hat{\sigma}^*, \hat{\rho}_h^*, \hat{\rho}_l^*, \hat{x}_h^*, \hat{x}_l^*) = B_h(q, \hat{\sigma}^*),$$

where B_h is defined in equation (7) in Section 3. Hence $\hat{\sigma}^* = \sigma_h(q)$.

Finally, let us identify the mapping $\hat{\sigma}^*(q)$. Because

$$\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S\left(1/2\right) = h\right] = \Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S\left(1/2\right) = l\right] = \sigma,$$

and

$$\mathbb{E}[R(\tilde{\sigma})] - 1 > \frac{1}{2}\left(\mathbb{E}[R(\sigma)] - 1\right) = B_h(1/2, \sigma).$$

for all σ by Assumption 1, $\hat{\sigma}^*(q) = \tilde{\sigma}$ for q near $1/2$. As $\Pr\left[R\left(\tilde{\sigma}\right) = r\left(\tilde{\sigma}\right) \mid S(q) = l\right]$ decreases with q (reaching zero at $q = 1$), there exists $\tilde{q} \in (1/2, 1)$ such that

$$r\left(\tilde{\sigma}\right) = \frac{1}{\Pr\left[R\left(\tilde{\sigma}\right) = r\left(\tilde{\sigma}\right) \mid S(\tilde{q}) = l\right]},$$

and therefore

$$\mathbb{E}[R(\tilde{\sigma})] - 1 = B_h(\tilde{q}, \tilde{\sigma}) \leq B_h(\tilde{q}, \sigma_h(\tilde{q})).$$

Therefore, as q increases from $1/2$ to $\tilde{q}$, for some $\hat{q} \in (1/2, \tilde{q}]$ it becomes optimal for the bank to select the asset risk that maximizes B_h, i.e., $\hat{\sigma}^*(q) = \sigma_h(q) < \tilde{\sigma}$ for $q > \hat{q}$. (In the proof of Lemma 3 in the Appendix it is shown that B_h is continuous and increasing, while σ_h is continuous and decreasing.)

We summarize these results in Proposition 3.

Proposition 3. *There is $\hat{q} \in (1/2, 1)$ such that if $q \in [1/2, \hat{q})$ in the unique equilibrium of $\hat{\Gamma}(q)$ the bank chooses the asset $\tilde{\sigma}$, whereas if $q \in (\hat{q}, 1]$ in the unique equilibrium of $\hat{\Gamma}(q)$ the bank chooses the asset $\sigma_h(q) < \tilde{\sigma}$. Thus, the asset risk and the bank's expected profit are constant for levels of precision on $[1/2, \hat{q})$, while they increase with the level of precision on $(\hat{q}, 1)$, reaching their maximum at $q = 1$.*

The welfare analysis in this setting is analogous to that of Section 3 for the base model. Clearly, if the social objective is exclusively to maximize stability, then any precision below $\hat{q}$ achieves this objective. As for the surplus, it is given for $(c, q) \in [0, \infty) \times [1/2, 1]$ by

$$\hat{W}(c, q) = \begin{cases} W_l(c, \tilde{\sigma}) & \text{if } q \in [1/2, \hat{q}] \\ W_h(c, q, \sigma_h(q)) & \text{if } q \in [\hat{q}, 1], \end{cases}$$

where W_l and W_h are given in equations (10) and (11), respectively, in Section 3. Thus, in an opaque environment, i.e., when $q \in [1/2, \hat{q}]$, the signal's precision no longer affects the surplus, while in a transparent environment, i.e., when $q \in [\bar{q}, 1]$, the surplus reaches its maximum $W_h^*(c)$ at $q = 1$ provided $\varepsilon_h \geq \bar{e}$, as established in Proposition 2. We show in the Appendix that there exists a threshold on the cost of bank failure, $\hat{c} > 0$, such that for $c > \hat{c}$ any level of precision below $\hat{q}$ yields the maximum surplus, whereas if $c < \hat{c}$ and $\varepsilon_h \geq \bar{e}$, then maximal precision yields the maximum surplus. We summarize these results in Proposition 4.

Proposition 4. *In the setting described by $\hat{\Gamma}(q)$ for $\hat{q} \in (1/2, 1)$ any level of precision below $\hat{q}$ maximizes stability, and also yields the maximum surplus if the cost of bank* *failure c is above a threshold $\hat{c}$. When c is below $\hat{c}$ and $\varepsilon_h(q) \geq \bar{e}$, the maximum surplus is reached at the maximal precision $q = 1$.*

Considering repayment promises contingent on the signal realization, rather than unconditional repayment promises, leads to differing results only on relatively opaque environments, in which the level of risk, the bank's expected profit and the surplus are invariant to changes in the precision. Thus, in this setting the market discipline effect of increasing precision is absent. Also, it can be shown that $\hat{q} \geq \bar{q}$. i.e., the threshold level of precision is larger in this setting than when the repayment promise is unconditional. The example presented in Section 6 illustrates these findings.

5 Competitive Banks

In the previous sections we study the interactions of a bank that exercises market power and its investors. While the existence of market power in the banking sector may be realistic, we also explore whether our main findings arise in a (perfectly) competitive setting. Alternatively, interpreting the analysis of Sections 2-4 as corresponding to that of a banking sector in which depositors face prohibitively high bank switching costs, we examine in this section the consequences of removing those switching costs.

Assume that there is a measure one of banks identical to that described in the previous sections. Each bank asset choice $\sigma \in [\underline{\sigma}, \bar{\sigma}]$ is subject to a review generating a signal $S(q) \in \{l, h\}$ of the likelihood that the asset will pay its return. Bank's signals, which are independent and identically distributed, are then publicly disclosed. Since there are constant returns to scale to investments, investors will invest in the banks' assets who offer the best expected return given their signal.

As in Diamond and Dybvig's (1983) seminal paper, in our setting competitive pressure forces the banks to offer the contract that maximizes investors' payoff. Since there is a continuum of banks, by the law of large numbers an asset quality review will result in a positive fraction of banks signaling h. Further, since banks signaling h can match any repayment offer by banks signaling l, all banks must gamble that their asset will generate the signal h, and therefore choose the asset

$$\sigma_c(q) := \arg\max_{\sigma\in[\underline{\sigma},\bar{\sigma}]} \mathbb{E}[R(\sigma) \mid S = h], \tag{19}$$

and promise the repayment $\rho_c(q) = r(\sigma_c(q))$. Since

$$\mathbb{E}[R(\sigma_c(q)) \mid S(q) = h] \geq \mathbb{E}[R(\tilde{\sigma}) \mid S(q) = h] \geq \mathbb{E}[R(\tilde{\sigma})] > 1.$$

where the strict inequality is part of Assumption 1, all investors invest their unit in banks that generate the signal h, while banks that generate the signal l are unable to raise funding and fail. Hence in the this setting it is irrelevant whether repayment promises are unconditional or contingent on the signal.

Proposition 5 establishes the properties of the equilibrium contract in a competitive banking sector, which involves a level of risk that increases with the precision of the signal, regardless of whether the environment is opaque or transparent. Moreover, this level of risk is always larger than that arising in the monopolistic banking sector. (See the proof of Proposition 5 in the Appendix.) A numerical example illustrating these results is given in Section 6.

Proposition 5. *In a competitive banking sector asset risk is larger than in a monopolistic banking sector, i.e.,* $\sigma_c(q) < \sigma_h(q) < \tilde{\sigma} < \sigma_l(q)$*, and increases with the level of precision, i.e.,* $\sigma_c'(q) > 0$ *for all* $q \in [1/2, 1]$.

The inequalities of Proposition 5 show that competitive pressure increases the banks' asset risk-taking, a result well in line with the literature's consensus – see Vives (2016) for a survey. An often cited explanation for this result – see, e.g., Keeley (1990) – is that competition eliminates the banks' charter value, making banks *more willing* to gamble. In our setting, however, competition *forces* banks to gamble: to take a high risk so as to be able to promise high returns to investors and to hope that their asset choice will generate the signal h.

Proposition 5 also establishes that banks' risk-taking increases with the level of precision of information disclosures: the more informative is the signal, the more likely is that a bank's asset signaling h will pay its return. The banks betting on $S = h$ will then increase their risk level so as to offer a higher repayment in the case of success. Thus, a Regulator minimizing banks' asset risk-taking will keep information disclosure to a minimum.

Let us now identify the socially optimal precision under perfect competition. The expected surplus generated by a perfectly competitive banking sector is

$$W_c(c,q) = \mathbb{E}[R(\sigma_c(q)) \mid S(q) = h] - (1 - q\sigma_c(q))c. \tag{20}$$

The first term in the right hand side of equation (20) is the expected returns, which come from to the assets of banks signaling h. The second term captures the expected social losses resulting from bank failures. Note that only a fraction $q\sigma$ of banks generate signal h and pay its return, while the remaining fraction $1 - q\sigma$ of banks fail either because they signal l and are unable to raise funds, or because they pay no returns. As equation (20) shows, a change in q affects the surplus both directly and indirectly via the banks' asset risk choice, much as in the monopolistic setting when the environment is relatively transparent.

The Regulator's problem is to choose $q \in [1/2, 1]$ to maximize $W_c(c,q)$. Taking derivative with respect to q in equation (20), and using the implications of the Envelope Theorem applied to the problem described in (19) above, we get

$$\frac{\partial W_c(c,q)}{\partial q} = \frac{\partial \mathbb{E}[R(\sigma_c(q)) \mid S(q) = h]}{\partial q} + (\varepsilon_c(q) + 1)\,\sigma_c(q)\,c, \tag{21}$$

where, as in Section 3,

$$\varepsilon_c(q) := q\frac{\sigma_c'(q)}{\sigma_c(q)}$$

denotes the elasticity of a competitive bank's asset risk-taking with respect to the signal's precision.

All terms in the right-hand side of equation (21) are positive except for $\varepsilon_c(q)$. Thus, in a competitive banking sector in which asset risk-taking is not too sensitive to the precision of the asset quality signal, maximal precision is socially optimal. Proposition 6 establishes a result that uses an obvious bound on the elasticity ε_c. (While $\partial \mathbb{E}[R(\sigma_c(q)) \mid S(q) = h]/\partial q$ is positive, it may be non-monotonic.) The numerical example given in Section 6 illustrates these results.

Proposition 6. *In a competitive banking sector minimal precision maximizes stability. Maximal precision maximizes the surplus whenever* $\varepsilon_c(q) \geq -1$.

The banking technology of a perfectly competitive sector is quite different to that of a monopolistic sector, and hence the conclusions of Propositions 2 and 4 cannot be directly contrasted with those of Proposition 6. Nonetheless, the analysis offers qualitative similarities for levels of precision in the region $(\bar{q}, 1]$: In all cases, if asset risk-taking is not too sensitive to the changes in the signal precision q, then maximal precision of information disclosures is optimal. The mechanisms generating these results are also similar: the investment is made only upon a high signal and, as a result, an increase in the precision of information disclosure has both a positive direct effect and a negative indirect effect on surplus: the direct positive effect arises from the investors' ability to separate more accurately a good bank from a bad one; the indirect negative effect is the banks' increased risk-taking.

If the asset risk-taking is sufficiently insensitive to the changes in q, then the direct effect dominates. If the sufficient condition for maximal precision to be socially optimal in a competitive banking sector, $\varepsilon_c \geq -1$, fails to hold, then increased risk-taking considerations might dominate at least for some parameter values, and some $q < 1$ might be optimal. Moreover, the assumption implicit in the analysis, that the social cost of a bank failure is the same regardless of whether it is due to the inability to raise funds or because the asset pays no return, may not be innocuous.

6 An Example

In this section we study the implications of our analysis for a linear return function

$$r(\sigma) = b\,(a - \sigma/2)\,, \tag{22}$$

where $a \in (\underline{\sigma}, \bar{\sigma})$ and $b > 2/a$. Then $\mathbb{E}[R(\sigma)] = \sigma b\,(a - \sigma/2)$, $\mathbb{E}'[R(\sigma)] = b\,(a - \sigma)$, $\mathbb{E}''[R(\sigma)] = -b$, and $\tilde{\sigma} = a$. We derive the equilibrium contract for this example in the alternative settings studied in the previous sections, and discuss our main results. Detailed calculations are provided in the Appendix.

In the setting of Section 2, where a single bank interacts with a measure one of investors offering a contract unconditional to the signal realization, substituting equation (22) in the formulae of Section 3, we get

$$\sigma_l(q) = a + \frac{2q-1}{b\,(1-q)},\ \ \sigma_h(q) = a - \frac{2q-1}{bq}. \tag{23}$$

Recall that the bank's risk choice is $\sigma^*(q) = \sigma_l(q)$ if $q < \bar{q}$ and $\sigma^*(q) = \sigma_h(q)$ if $q > \bar{q}$, where $\bar{q}$ solves the equation $B_l(q, \sigma_l(q)) = B_h(q, \sigma_h(q))$. Consistently with Proposition 1, asset risk increases on $(1/2, \bar{q})$, and decreases on $(\bar{q}, 1)$. Using equations (22) and (23), one can readily calculate the bank's profit. Figures 1 to 4 provide a illustration of our results for the numerical example $a = 4/5$, $b = 120$.

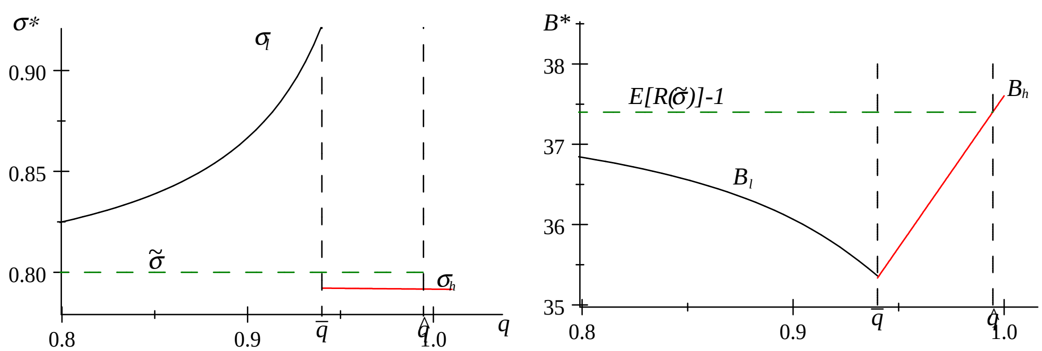

Figure 1. The left and right panels depict the bank's asset risk choice and profit, respectively, as a function of the signal's precision.

As Figure 1 shows, in the relatively opaque banking environment, i.e., when $q \in (1/2, \bar{q}]$, where $\bar{q} \simeq 0.94$, the bank chooses a low risk asset $\sigma^*(q) = \sigma_l(q)$, which results in the profit $B^*(q) = B_l(q)$.Both asset riskiness and profit decrease as the precision of the asset quality signal increases. In this regime the bank promises a high repayment that attracts investors regardless of the signal. There is a discontinuity in the asset risk at $\bar{q}$, when the bank switches from attracting investors regardless of the signal realization to attracting investors only when the signal is h. The maximal stability and minimum profit are reached at $\bar{q}$. In the relatively transparent environment, i.e., when $q \in (\bar{q}, 1]$, the bank promises a low repayment and chooses a more risky asset, $\sigma^* = \sigma_h$, the riskiness of which increases as the precision of asset quality signal increases. Moreover, the bank's profit also increases as the precision of the asset's quality signal increases. The asset risk and bank's profit are maximal when the signal is perfectly revealing. As established in Section 3, these features are general properties.

We calculate the precision that maximizes the surplus. From (23) we get

$$\varepsilon_h := q\sigma_h'(q)/\sigma_h = -(aqb + 1 - 2q)^{-1} > -1 > \bar{e}.$$

Thus, for the linear return function (22) the premise of Proposition 2 holds. Figure 2 provides graphs of the optimal precision and maximum surplus for $c \in [0, \infty)$. The surplus as a function of the signal's precision is readily calculated using equations (10), (11), (22), and (23). We obtain $\bar{c} = 5\sqrt{2} - 1 \simeq 6.07$ and $\psi(\bar{q}) \simeq 14.66$. Hence by Proposition 2, the optimal precision is 1 for $c \in [0, 6.07]$, it is $(c+1)/(c+2)$ for $c \in (6.07, 14.66]$, and it is $\bar{q} \simeq 0.94$ for $c > 14.66$.

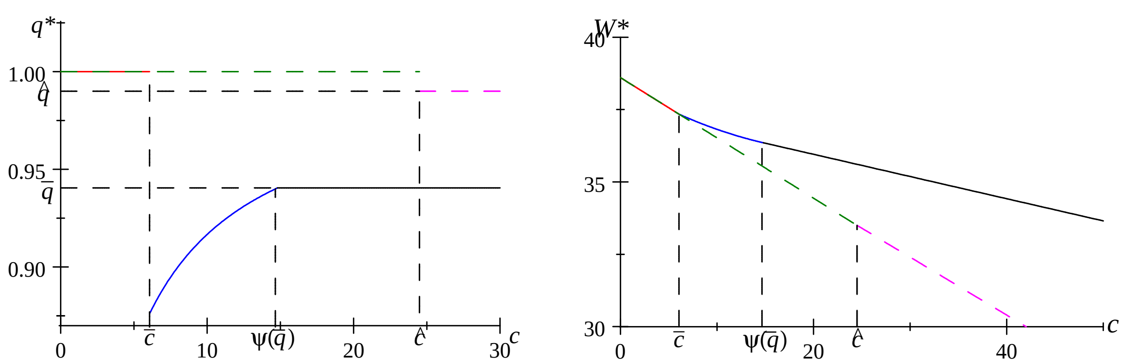

Figure 2. The left and right panels depict the optimal precision and surplus, respectively, as a function of the cost of bank failure.

Naturally, the maximum surplus decreases with the social cost of bank failure (the solid line in the right panel of Figure 2). The left panel depicts the socially optimal precision. The maximal precision $q = 1$ is socially optimal when the social cost of a bank failure is small. The socially optimal precision eventually drops to $q_l(c)$ when this cost reaches the value $\bar{c} = 6.07$, which increases with c on $(\bar{c}, \psi(\bar{q})]$, where $\psi(\bar{q}) \simeq 14.66$. (Thus, $\bar{c} < \psi(\bar{q})$ in this example.) Once the social cost of a bank failure reaches the value $\psi(\bar{q})$, the optimal precision settles on the stability maximizing value $\bar{q}$ and remains at this level for larger social costs. Thus, the socially optimal precision is a non-monotonic function of the social cost of a bank failure. (For this linear return function, taking $a = 1/2$ instead of $a = 4/5$, while keeping $b = 120$,we obtain $\psi(\bar{q}) < \bar{c}$ and, as a result, the socially optimal precision drops from $q = 1$ to $q = \bar{q}$ when the social cost of bank failure reaches the value $\bar{c} > (2\bar{q} - 1) / (1 - \bar{q})$.)

Figure 1 also shows the equilibrium for this numerical example in the setting of Section 4, where the bank's contract involves a repayment promise conditional on the signal realization: In the relatively opaque banking environment $q \in (1/2, \hat{q}]$, where $\hat{q} = 188/189 \simeq 0.9947$, the bank chooses the asset $\tilde{\sigma} = 4/5$ and is expected profit is $\hat{B}^*(q) = \mathbb{E}[R(\tilde{\sigma})] - 1 = 37.4$ (the horizontal green dash lines in the right and left panels of Figure 1, respectively); in the relatively transparent banking environment $q \in (\hat{q}, 1]$, the bank chooses a more risky asset, $\sigma^*(q) = \sigma_h(q)$, and obtains greater expected profits, $B_h(q)$, and both risk and payoffs increase with precision and are maximal at $q = 1,$ as shown in Figure 1.

In the setting of Section 4, if the social cost of bank failure c is below $\hat{c} = 24.5$, then maximal precision is $q = 1$ is socially optimal, and yields a maximum surplus equal to $\hat{W}^*(c) = W_h^*(c) = 1853/48 - (5/24)\,c$, which is the green dashed line displayed in the right panel of Figure 2. For larger values of c any level of precision below $\hat{q}$ yields the maximum surplus $\hat{W}^*(c) = \mathbb{E}[R(\tilde{\sigma})] - (1 - \tilde{\sigma})\,c = 38.4 - c/5$, which is the magenta line displayed in the right panel of Figure 2. Thus, for $c > \bar{c}$ the maximum surplus in this setting is below that in the base setting.

Finally, we calculate the equilibrium of perfectly competitive banking sector as described in Section 5, assuming the return function given in (22). Using (19) we calculate the equilibrium level of risk, which is given by

$$\sigma_c(q) = \frac{\sqrt{(1-q)(1-q+2a(2q-1))} - (1-q)}{2q-1}. \tag{24}$$

Figure 3 illustrates the results of Proposition 5 for the specific numerical example $a = 4/5$, $b = 120$. As Figure 3 shows, in a competitive banking sector asset risk increases (i.e., σ_c decreases) with the precision of the asset quality. Let us take $\underline{\sigma} = 1/5$ as a lower bound for σ, which σ_c reaches at $\underline{q} = 31/32 \simeq 0.97$. Then for q close to unity σ_c becomes independent of q. Comparing Figure 3 with the left panel of Figure verifies that $\sigma_c < \sigma_h < \tilde{\sigma} < \sigma_l$ for $q \in (1/2, 1]$.

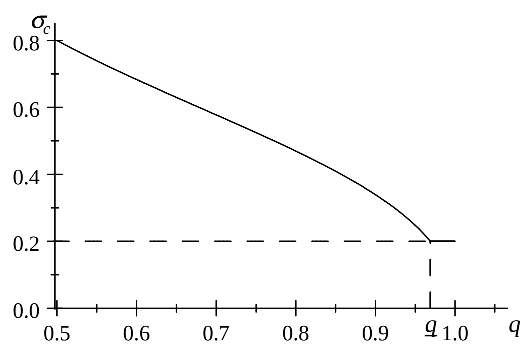

Figure 3. The level of risk in a competitive banking sector as a function of the signal's precision.

For this numerical example risk-taking is sensitive to the precision of the asset quality signal; specifically, $\varepsilon_c(q) < -1$ for $q \in \left(0.62, \underline{q}\right)$; hence the sufficient condition of Proposition 6 for maximal transparency to maximize surplus does not hold. Nonetheless, the left panel of Figure 4 shows that maximal precision maximizes surplus also in this example: Using equations (20) and (24) we provide graphs of the function $W_c(c, q)$ for $c = 1$ (black curve), $c = 10$ (red curve), $c = 20$ (green curve), $c = 40$ (blue curve), and $c = 80$ (magenta curve). These graphs suggest that $W_c(c, \cdot)$ is increasing in q, and therefore that maximal precision is socially optimal. Note that $W_c(c, \cdot)$ increases faster for $q > \underline{q}$ since banks' asset riskiness remains at $\underline{\sigma}$. The right panel shows a graph of the maximum surplus $W_c^*(c) = W_c(c, 1)$ as a function of the social cost of bank failure.

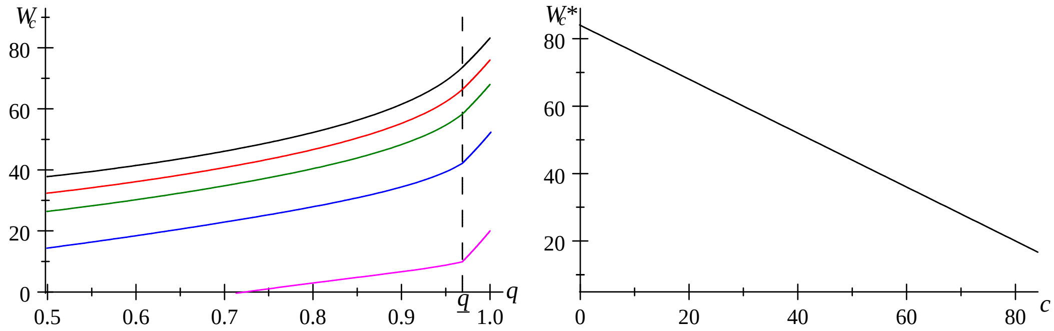

Figure 4. The left panel depicts graphs of $W_c(c, q)$ for $c \in \{1, 10, 20, 40, 80\}$. The right panel depicts the graph of $W_c^*(c) = W_c(c, 1)$.

7 Conclusions

We uncover two effects of public disclosures of information about banks' asset portfolio: A direct effect on investors behavior, resulting from their improved ability to distinguish between solvent and insolvent banks, and an indirect effect on the banks' risk-taking incentives. When information is imprecise, the direct effect of increasing precision is nil, whereas the indirect effect induces banks to choose less risky assets, providing market discipline and improving banking stability. When information is precise, however, the direct effect of increasing precision implies that banks refinance only when the news is good, and the indirect effect induces banks to choose more risky assets, impairing banking stability. Therefore, a certain degree of opacity might be conducive to banking sector stability and welfare. While we derive these results in a simple setting, the underlying effects we identify arise as well under alternative competitive environments and the contracting possibilities of public information.

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Appendix

A. Proofs

Proposition 1 follows from Lemmas 1 to 5. Lemma 1 identifies investors' equilibrium behavior.

Lemma 1. *Let* $(q,\sigma,s)\in[1/2,1]\times[\underline{\sigma},\bar{\sigma}]\times\{l,h\}$. *If* $r(\sigma)\geq 1/\Pr\left[R\left(\sigma\right)=r\left(\sigma\right)\mid S(q)=s\right]$, *then* $x_s^*(q,\sigma,\rho)=0$ *for* $\rho<1/\Pr\left[R\left(\sigma\right)=r\left(\sigma\right)\mid S(q)=s\right]$, *and* $x_s^*(q,\sigma,\rho)=1$ *for* $\rho\geq 1/\Pr\left[R\left(\sigma\right)=r\left(\sigma\right)\mid S(q)=s\right]$. *Otherwise,* $x_s^*(q,\sigma,\rho)=0$ *for all* ρ.

Proof: Assume that $r(\sigma)\geq 1/\Pr\left[R\left(\sigma\right)=r\left(\sigma\right)\mid S(q)=s\right]$. Let $\rho,\rho'\in[0,\infty)$ be such that $\rho<1/\Pr\left[R\left(\sigma\right)=r\left(\sigma\right)\mid S(q)=s\right]\leq\rho'$. Then $\min\{\rho,r(\sigma)\}=\rho$ and $\min\{\rho',r(\sigma)\}\geq 1/\Pr\left[R\left(\sigma\right)=r\left(\sigma\right)\mid S(q)=s\right]$, and therefore

$$
\begin{aligned}
u(q,\sigma,\rho,s) &= \Pr\left[R\left(\sigma\right)=r\left(\sigma\right)\mid S(q)=s\right]\rho \\
&< 1 \\
&\leq \Pr\left[R\left(\sigma\right)=r\left(\sigma\right)\mid S(q)=s\right]\min\{\rho',r(\sigma)\} \\
&= u(q,\sigma,\rho',s),
\end{aligned}
$$

and hence $x_s^*(q,\sigma,\rho) = 0$ and $x_s^*(q,\sigma,\rho') = 1$. Assume $r(\sigma) < 1/\Pr[R(\sigma) = r(\sigma) \mid S(q) = s]$. Then for all $\rho \in [0,\infty)$, $\min\{\rho, r(\sigma)\} < 1/\Pr[R(\sigma) = r(\sigma) \mid S(q) = s]$ and

$$u(q,\sigma,\rho,s) = \Pr[R(\sigma) = r(\sigma) \mid S = s] \min\{\rho, r(\sigma)\} < 1,$$

and hence $x_s^*(q,\sigma,\rho) = 0$. □

Lemma 2 shows that the bank's equilibrium payoff is positive, and identifies the repayment promises that may arise in equilibrium.

Lemma 2. *Let $q \in [1/2, 1]$. If $(\sigma^*(q), \rho^*(q), x^*(q))$ is an equilibrium of $\Gamma(q)$, then $r(\sigma^*(q)) > \rho^*(q)$ and $B^*(q) := B(q, \sigma^*(q), \rho^*(q), x^*(q)) > 0$. Moreover, either $\rho^*(q) = 1/\Pr[R(\sigma^*(q)) = r(\sigma^*(q)) \mid S(q) = h]$, or $\rho^*(q) = 1/\Pr[R(\sigma^*(q)) = r(\sigma^*(q)) \mid S(q) = l]$.*

Proof: By Assumption 1, $\mathbb{E}[R(\tilde{\sigma})] = \tilde{\sigma} r(\tilde{\sigma}) > 1$, where $\tilde{\sigma} := \arg\max_{\sigma \in [\underline{\sigma}, \bar{\sigma}]} \mathbb{E}[R(\sigma)]$. We show that the bank can secure a positive expected payoff by choosing $(\sigma, \rho) = (\tilde{\sigma}, 1/\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h])$. Since

$$\begin{aligned} r(\tilde{\sigma}) &> 1/\tilde{\sigma} \geq 1/\Pr[R(\sigma) = r(\sigma) \mid S(q) = s] \\ & 1/\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h], \end{aligned}$$

$x_h^*(q, \tilde{\sigma}, 1/\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h]) = 1$ by Lemma 1. Also,

$$u(q, \tilde{\sigma}, 1/\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h], l) = \frac{\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = l]}{\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h]} < 1,$$

and therefore $x_l^*(q, \tilde{\sigma}, 1/\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h]) = 0$. Hence

$$\begin{aligned} B^*(q) &= B(q, \sigma^*(q), \rho^*(q), x^*(q)) \\ &\geq B(q, \tilde{\sigma}, 1/\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h], x^*(q, \tilde{\sigma}, 1/\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h])) \\ &= q\tilde{\sigma}\left(r(\tilde{\sigma}) - 1/\Pr[R(\tilde{\sigma}) = r(\tilde{\sigma}) \mid S(q) = h]\right) \\ &> q\tilde{\sigma}\left(r(\tilde{\sigma}) - 1/\tilde{\sigma}\right) \\ &= q\left(\mathbb{E}[R(\tilde{\sigma})] - 1\right) \\ &> 0. \end{aligned}$$

Moreover,

$$B^*(q) = (q x_h^*(q) + (1-q) x_l^*(q))\, \sigma^*(q)\, (r(\sigma^*(q)) - \rho^*(q)) > 0$$

implies $r(\sigma^*(q)) > \rho^*(q)$.

Since $\Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = h\right] \geq \sigma \geq \Pr\left[R\left(\sigma\right) = r\left(\sigma\right) \mid S(q) = l\right],$ for $\rho < 1/\Pr\left[R\left(\sigma^*\left(q\right)\right) = r\left(\sigma\left(q\right)\right) \mid S(q) = h\right]$ Lemma 1 implies

$$x_h^*(q, \sigma^*(q), \rho) = x_l^*(q, \sigma^*(q), \rho) = 0,$$

and hence the bank's payoff is zero. Thus, $B^*(q) > 0$ implies $\rho^* \geq 1/\Pr\left[R\left(\sigma^*\left(q\right)\right) = r\left(\sigma\left(q\right)\right) \mid S(q) = h\right.$ Also, repayment promises $\rho \in (1/\Pr\left[R\left(\sigma^*\left(q\right)\right) = r\left(\sigma\left(q\right)\right) \mid S(q) = h\right], 1/\Pr\left[R\left(\sigma^*\left(q\right)\right) = r\left(\sigma\left(q\right)\right) \mid S(q\right.$ or $\rho > 1/\Pr\left[R\left(\sigma^*\left(q\right)\right) = r\left(\sigma\left(q\right)\right) \mid S(q) = l\right]$ are strictly dominated, since in either case the bank can increase its payoff by slightly lowering the repayment promise. Hence either $\rho^*(q) = 1/\Pr\left[R\left(\sigma^*\left(q\right)\right) = r\left(\sigma\left(q\right)\right) \mid S(q) = h\right]$ or $\rho^*(q) = 1/\Pr\left[R\left(\sigma^*\left(q\right)\right) = r\left(\sigma\left(q\right)\right) \mid S(q\right.$
□

Lemma 3 establishes some properties of the mappings $\sigma_h(q)$ and $\sigma_l(q)$.

Lemma 3. *The mappings σ_l and σ_h are well defined functions on $(1/2, 1)$ and satisfy $\sigma_l' \geq 0$ and $\sigma_h' \leq 0$. Moreover, $\sigma_l(1/2) = \sigma_h(1/2) = \tilde{\sigma}$ and $\sigma_l > \sigma_h$ for $q \in (1/2, 1]$.*

Proof: Recall that $\mathbb{E}[R(\sigma)]$ is strictly concave and is maximized at $\tilde{\sigma} \in (\underline{\sigma}, \bar{\sigma})$ by Assumption 1. Also, as shown in Section 3, $\sigma_h(1/2) = \sigma_l(1/2) = \tilde{\sigma}$.

Since $B_l(q, \cdot)$ is strictly concave, it has a unique maximizer. Hence σ_l is a function. Differentiating B_l we get

$$\frac{\partial B_l(q, \sigma)}{\partial \sigma} = \mathbb{E}'[R(\sigma)] + \frac{2q - 1}{1 - q}.$$

Because $\sigma_l(1/2) = \tilde{\sigma} \in (\underline{\sigma}, \bar{\sigma})$, $\sigma_l(q)$ is the solution the equation $\partial B_l(q, \sigma)/\partial \sigma = 0$ for q near $1/2$. Differentiating this equation and rearranging we get

$$\sigma_l'(q) = -\Big((1 - q)^2 \mathbb{E}''\left[R(\sigma)\right]\Big)^{-1} > 0,$$

i.e., σ_l is strictly increasing for all q such that $\sigma_l(q) \in (\underline{\sigma}, \bar{\sigma})$. If $\partial B_l(q, \sigma)/\partial \sigma > 0$ on $(\underline{\sigma}, \bar{\sigma})$ for some $q = q_0 < 1$, then $\sigma_l(q) = \bar{\sigma}$ and $\sigma_l'(q) = 0$ for all $q \in [q_0, 1]$. To summarize, σ_l increases with q on $[1/2, 1]$ from $\tilde{\sigma}$. If it reaches the value $\bar{\sigma}$ for some $q < 1$, then it remains constant at $\bar{\sigma}$ as q increases further.

Since $B_h(q,\cdot)$ is strictly concave, it has a unique maximizer. Hence σ_h is a function. Differentiating B_h we get

$$\frac{\partial B_h(q,\sigma)}{\partial \sigma} = q\mathbb{E}'[R(\sigma)] - (2q-1)$$

Because $\sigma_h(1/2) = \tilde{\sigma} \in (\underline{\sigma}, \bar{\sigma})$, $\sigma_h(q)$ is the solution the equation $\partial B_h(q, \underline{\sigma})/\partial \sigma = 0$ for q near $1/2$. Differentiating this equation and rearranging we get

$$\sigma_h'(q) = \left(q^2 \frac{d^2\mathbb{E}[R(\sigma)]}{d\sigma^2}\right)^{-1} < 0,$$

i.e., σ_h is strictly decreasing for all q such that $\sigma_h(q) \in (\underline{\sigma}, \bar{\sigma})$. If $\partial B_h(q,\sigma)/\partial\sigma < 0$ on $(\underline{\sigma}, \bar{\sigma})$ for some $q = q_1$, then $\sigma_h(q) = \underline{\sigma}$ and $\sigma_h'(q) = 0$ for all $q \in [q_1, 1]$. To summarize: σ_h decreases with q on $[1/2, 1]$ from $\tilde{\sigma}$. If it reaches the value $\underline{\sigma}$ for some $q < 1$, then it remains constant at $\underline{\sigma}$ as q increases further.

Therefore, for $q \in (1/2, 1]$

$$\sigma_h(q) < \sigma_h(1/2) = \sigma_l(1/2) < \sigma_l(q). \ \square$$

Lemma 4 identifies a threshold value for q below which the bank's repayment promise attracts investors whatever the realization of the signal, and above which the bank's repayment promise attracts investors only when the realization is h.

Lemma 4. *There exists $\bar{q} \in (1/2, 1)$ such that the unique equilibrium of $\Gamma(q)$ is*

$$(\sigma^*(q), \rho^*(q), x^*(q)) = (\sigma_l(q), 1/\Pr\left[R\left(\sigma^*\left(q\right)\right) = r\left(\sigma\left(q\right)\right) \mid S(q) = l\right], (1,1))$$

if $q \in [1/2, \bar{q})$, and it is

$$(\sigma^*(q), \rho^*(q), x^*(q)) = (\sigma_h(q), 1/\Pr\left[R\left(\sigma^*\left(q\right)\right) = r\left(\sigma\left(q\right)\right) \mid S(q) = h\right], (0,1))$$

if $q \in (\bar{q}, 1]$.

Proof: We first show that the function

$$g(q) := B_h(q, \sigma_h(q)) - B_l(q, \sigma_l(q))$$

is increasing. Differentiating B_h and noting that $\mathbb{E}[R(\sigma_h(q))] > 1$ and

$$\frac{\partial B_h(q,\sigma_h)}{\partial \sigma}\sigma_h'(q) = 0,$$

we get

$$\begin{aligned} \frac{dB_h(q,\sigma_h(q))}{dq} &= \frac{\partial B_h(q,\sigma_h)}{\partial q} + \frac{\partial B_h(q,\sigma_h)}{\partial \sigma}\sigma_h'(q) \\ &= \mathbb{E}[R(\sigma_h(q))] + 1 - 2\sigma_h(q) \\ &> 2 - 2\sigma_h(q) \\ &> 0. \end{aligned} \tag{25}$$

Differentiating B_l, and noting that

$$\frac{\partial B_l(q,\sigma_l(q))}{\partial \sigma}\sigma_l'(q) = 0,$$

we get

$$\begin{aligned} \frac{dB_l(q,\sigma_l(q))}{dq} &= \frac{\partial B_l(q,\sigma_l)}{\partial q} + \frac{\partial B_l(q,\sigma_l(q))}{\partial \sigma}\sigma_l'(q) \\ &= -\frac{1-\sigma_l(q)}{(1-q)^2} < 0. \end{aligned} \tag{26}$$

Hence g is increasing.

By Lemma 3,

$$g(1/2) = B_h(q,\sigma_l(q)) - B_l(q,\sigma_h(q)) = -\left(\mathbb{E}[R(\tilde{\sigma})] - 1\right)/2 < 0.$$

Hence $\sigma^*(q) = \sigma_l(q)$ for q near $1/2$. Also, because $\mathbb{E}[R(\sigma)]$ and σ are bounded, $g(q)$ becomes positive for q near 1, i.e., $\sigma^*(q) = \sigma_h(q)$ for q near 1. Since g is continuous and increasing there is $\bar{q}$ such that

$$g(q) \lesseqgtr 0 \Leftrightarrow q \lesseqgtr \bar{q},$$

and therefore $\sigma^*(q) = \sigma_l(q)$ for $q < \bar{q}$, and $\sigma^*(q) = \sigma_h(q)$ for $q > \bar{q}$. □

Lemma 5 derives some properties of the bank's equilibrium risk choice and payoff.

Lemma 5. *The functions σ^* and B^* satisfy $d\sigma^*/dq > 0$ and $dB^*/dq < 0$ on $(1/2, \bar{q})$ and $d\sigma^*/dq < 0$ and $dB^*/dq > 0$ on $(\bar{q}, 1)$. Moreover, σ^* (respectively, B^*) reaches its minimal (maximal) value at $q = 1$, and its maximal (minimal) value at $q = \bar{q}$.*

Proof: Because $\sigma_h(\bar{q}) < \sigma_l(\bar{q})$ and $\sigma_h' \leq 0$ and $\sigma_l' \geq 0$ by Lemma 3, the minimal (respectively, maximal) value of σ^* on $[1/2, 1]$ is reached at $q = 1$ $(q = \bar{q})$. Since $B^*(q) = B_l(q, \sigma_l(q))$ on $(1/2, \bar{q})$, equation (26) implies that $dB^*/dq < 0$ on $(1/2, \bar{q})$, and since $B^*(q) = B_h(q, \sigma_h(q))$ on $(\bar{q}, 1)$, equation (25) implies that $dB^*/dq > 0$ on $(\bar{q}, 1)$. In order to establish that B^* reaches its maximum value at $q = 1$ we show that $B^*(1) > B^*(1/2)$. We have

$$\begin{aligned}
B^*(1) - B^*(1/2) &= B_h(1, \sigma^*(1)) - B_l(\frac{1}{2}, \sigma^*(1/2)) \\
&= B_h(1, \sigma^*(1)) - (\mathbb{E}[R(\sigma^*(1/2))] - 1) \\
&\geq B_h(1, \sigma^*(1)) - (\mathbb{E}[R(\sigma^*(1/2))] - \sigma^*(1/2)) \\
&= B_h(1, \sigma^*(1)) - B_h(1, \sigma^*(1/2)) \\
&> 0,
\end{aligned}$$

where strict inequality follows since $\sigma^*(1)$ uniquely maximizes $B_h(1, \sigma)$ and $\sigma^*(1) < \sigma^*(1/2)$. □

Proof of Remark 1: For $q \in (1/2, 1]$ denote by $\bar{\Gamma}(q)$ the game the bank and the investors face when investors do not observe the bank's risk choice. The game $\bar{\Gamma}(q)$ is identical to $\Gamma(q)$ except that investors can only condition their decisions on the repayment promise ρ and on the realization of the signal, i.e., the mapping describing investors' behavior $y(q)$ associates with every repayment promise $\rho \in [0, \infty)$ a pair $y(q, \rho) = (y_h(q, \rho), y_l(q, \rho)) \in [0, 1]^2$. The formal definition of equilibrium in this game is obtained by replacing the function x with the function y in the definition of equilibrium in the end of Section 2.

Let $(\bar{\sigma}^*(q), \bar{\rho}^*(q), \bar{y}^*(q))$ and $\bar{B}^*(q)$ be the strategies and the payoff of the bank, respectively, in an equilibrium of the game $\bar{\Gamma}(q)$. It is easy to see that

$$\bar{y}^*(q, \bar{\rho}^*(q)) = x^*(q, \bar{\sigma}^*(q), \bar{\rho}^*(q)).$$

Therefore

$$\begin{aligned}
\bar{B}^*(q) &= B(q, \bar{\sigma}^*(q), \bar{\rho}^*(q), y^*(q, \bar{\rho}^*(q))) \\
&= B(q, \bar{\sigma}^*(q), \bar{\rho}^*(q), x^*(q, \bar{\sigma}^*(q), \bar{\rho}^*(q))) \\
&\leq B^*(q).
\end{aligned}$$

Furthermore, the inequality $\bar{B}^*(q) \leq B^*(q)$ is generally strict. $\square$

Proof of Proposition 2: The bank fails if either investors do not invest or the asset pays no return. For $q \in (1/2, \bar{q})$, $\sigma^*(q) = \sigma_l(q)$ by Proposition 1; hence $x^*(q) = (1,1)$, and therefore the bank fails only if the asset does not pay its return, i.e., it fails with probability $1 - \sigma^*(q)$. Since σ^* is decreasing on $(1/2, \bar{q})$, this probability is minimal at $\bar{q}$. For $q \in (\bar{q}, 1)$, $\sigma^*(q) = \sigma_h(q)$, and hence $x^*(q) = (1,0)$, by Proposition 1; therefore the bank fails either if $S = l$, or if $S = h$ and the asset pays no return, i.e., the bank fails with a probability greater than $1 - \sigma^*(q) = 1 - \sigma_h(q)$. Since $\sigma_h(q) < \sigma_l(q)$ for all $q \in (1/2, \bar{q})$ (see Lemma 3 in the Appendix), the stability maximizing precision is $\bar{q}$.

Next we show that there exists $\bar{c} > 0$, such that $W_h^*(c) \lesseqqgtr W_l^*(c)$ if and only if $c \gtreqqless \bar{c}$. Write

$$G(c) := W_h^*(c) - W_l^*(c).$$

Recalling $W_h^*(c) = W_h(c, 1, \sigma_h(1))$ and $W_l^*(c) = W_l(c, \sigma(q_l(c)))$ allows us to calculate $G(c)$ explicitly as

$$\begin{aligned} G(c) &= \mathbb{E}[R(\sigma_h(1))] + (1 - \sigma_h(1))(1 - c) - \mathbb{E}[R(\sigma_l(q_l(c)))] + (1 - \sigma_l(q_l(c)))c \\ &= \mathbb{E}[R(\sigma_h(1))] + 1 - \sigma_h(1) - \mathbb{E}[R(\sigma_l(q_l(c)))] - c\left(\sigma_l(q_l(c)) - \sigma_h(1)\right). \\ &= H(\sigma_h(1)) + (1+c)\sigma_h(1) - \mathbb{E}[R(\sigma_l(q_l(c)))] - c\sigma_l(q_l(c)). \end{aligned}$$

We next establish that $G(c)$ is decreasing in c. We have

$$G'(c) = -\left(\sigma_l(q) - \sigma_h(1)\right) - \left(\mathbb{E}'[R(\sigma_l)] + c\right)\sigma_l'(q_l(c))q_l'(c).$$

Note that $\sigma_h(1) < \sigma_l(q)$ and $\sigma_l' \geq 0$ by Lemma 3, and $q_l' \geq 0$ – see equation (10). Moreover, because q_l maximizes $W_l(c, \sigma_l(\cdot))$, $\mathbb{E}'[R(\sigma_l)] + c \geq 0$ – see equation (11). Hence

$$G'(c) \leq -\left(\sigma_l(q_l(c)) - \sigma_h(1)\right) < 0.$$

Now, since $\sigma_h(1)$ maximizes $\mathbb{E}[R(\sigma)] - \sigma$, we have

$$\begin{aligned} G(0) &= \mathbb{E}[R(\sigma_h(1))] - \mathbb{E}[R(\sigma_l)] + 1 - \sigma_h(1) \\ &= \mathbb{E}[R(\sigma_h(1))) - \sigma_h(1) - (\mathbb{E}[R(\sigma_l)] - 1) \\ &> \mathbb{E}[R(\sigma_h(1))] - \sigma_h(1) - (\mathbb{E}[R(\sigma_l)] - \sigma_l) \\ &> 0. \end{aligned}$$

We next show that for c large, $G(c) < 0$. We may write

$$\begin{aligned} G(c) &= \mathbb{E}[R(\sigma_h(1))] - \mathbb{E}[R(\sigma_l(q_l(c)))] + 1 - \sigma_h(1) - c\,(\sigma_l(q_l(c)) - \sigma_h(1)) \\ &= \mathbb{E}[R(\sigma_h(1))] - \sigma_h(1) - (\mathbb{E}[R(\sigma_l(q_l(c)))] - 1) - c\,(\sigma_l(q_l(c)) - \sigma_h(1))\,. \end{aligned}$$

Since

$$\mathbb{E}[R(\sigma_h(1))] - \sigma_h(1) - (\mathbb{E}[R(\sigma_l(q_l(c)))] - 1) < \mathbb{E}[R(\sigma_h(1))] - \sigma_h(1),$$

for all c, then $\sigma_l(q_l(c)) - \sigma_h(1) > 0$ and $\sigma_l \geq \sigma_l(1/2) > \sigma_h(1)$ by Lemma 3. Therefore for c such that

$$c(\sigma_l(1/2) - \sigma_h(1)) > \mathbb{E}[R(\sigma_h(1))] - \sigma_h(1)$$

we have $G(c) < 0$.

Let $\bar{c}$ be the unique solution to the equation $G(c) = 0$, i.e.,

$$\bar{c} = \frac{\mathbb{E}[R(\sigma_h(1))] - \mathbb{E}[R(\sigma_l(q_l(\bar{c})))] + 1 - \sigma_h(1)}{\sigma_l(q_l(\bar{c})) - \sigma_h(1)} > 0. \tag{27}$$

Equation (27) defines $\bar{c}$ implicitly as a function of q when $\psi(\bar{c}) < \bar{q}$, in which case $q_l(\bar{c}) = (\bar{c} + 1) / (\bar{c} + 2)$, as implied by equation (13). Whenever $\psi(\bar{c}) \geq \bar{q}$, equation (27) provides an explicit formula of $\bar{c}$, since in this case $q_l(\bar{c}) = \bar{q}$, as implied by equation (13). □

Proof of Proposition 4. First we show that there exists $\hat{c} > 0$, such that $W_h^*(c) \lesseqqgtr E[R(\tilde{\sigma})] + (1 - \tilde{\sigma})c$ if and only if $c \gtreqqless \hat{c}$. Write

$$\begin{aligned} \hat{G}(c) \quad &: \quad = W_h^*(c) - (\mathbb{E}[R(\tilde{\sigma})] + (1 - \tilde{\sigma})c) \\ &= (\mathbb{E}[R(\sigma_h(1))] + (1 - \sigma_h(1))(1 - c)) - (\mathbb{E}[R(\tilde{\sigma})] + (1 - \tilde{\sigma})c) \\ &= (\mathbb{E}[R(\sigma_h(1))] - \sigma_h(1)) - (\mathbb{E}[R(\tilde{\sigma})] - 1) - (\tilde{\sigma} - \sigma_h(1))c \\ &= \hat{G}(0) - (\tilde{\sigma} - \sigma_h(1))c \end{aligned}$$

Since $\sigma_h(1)$ maximizes $\mathbb{E}[R(\sigma)] - \sigma$, then

$$\begin{aligned}\hat{G}(0) &= (\mathbb{E}[R(\sigma_h(1))) - \sigma_h(1)) - (\mathbb{E}[R(\tilde{\sigma})] - 1) \\ &> (\mathbb{E}[R(\sigma_h(1))] - \sigma_h(1)) - (\mathbb{E}[R(\tilde{\sigma})] - \tilde{\sigma}) \\ &> 0.\end{aligned}$$

Moreover,

$$\left(\mathbb{E}[R(\sigma)] - \sigma\right)'\big|_{\sigma=\tilde{\sigma}} = -1,$$

implies $\tilde{\sigma} > \sigma_h(1)$, and therefore

$$\hat{G}'(c) = -(\tilde{\sigma} - \sigma_h(1)) < 0.$$

Further, $\hat{G}(c) < 0$ for $c > \hat{G}(0)/(\tilde{\sigma} - \sigma_h(1))$. Hence there is $\hat{c}$ such that $\hat{G}(c) \gtreqless 0$ for $c \gtreqless \hat{c}$.

Proof of Proposition 5: We proof the claims about asset risk. Let $q \in (1/2, 1]$. Since

$$\mathbb{E}[R(\sigma) \mid S(q) = h] = \Pr\left[R(\sigma) = r(\sigma) \mid S(q) = h\right] r(\sigma),$$

Using (2) and taking derivative we get

$$\begin{aligned}\frac{\partial \mathbb{E}[R(\sigma) \mid S(q) = h]}{\partial \sigma} &= \frac{\partial \Pr\left[R(\sigma) = r(\sigma) \mid S(q) = h\right]}{\partial \sigma} r(\sigma) \\ &\quad + \Pr\left[R(\sigma) = r(\sigma) \mid S(q) = h\right] r'(\sigma) \\ &= \frac{q}{q\sigma + (1-q)(1-\sigma)} \left(\frac{(1-q)\, r(\sigma)}{q\sigma + (1-q)(1-\sigma)} + \sigma r'(\sigma) \right).\end{aligned}$$

The first order condition for a solution to the competitive bank's problem (19), $\partial \mathbb{E}[\,R(\sigma)|\,S(q) = h]/\partial\sigma = 0$, yields the equation

$$\begin{aligned}0 &= \left(\frac{(1-q)\, r(\sigma)}{q\sigma + (1-q)(1-\sigma)} - r(\sigma) \right) + r(\sigma) + \sigma r'(\sigma) \qquad (28) \\ &= -\left(\frac{2q-1}{q\sigma + (1-q)(1-\sigma)} \right) \mathbb{E}[R(\sigma)] + \mathbb{E}'[R(\sigma)].\end{aligned}$$

Since $\mathbb{E}'[R(\tilde{\sigma})] = 0$ by Assumption 1, this equation implies $\sigma_c(q) < \tilde{\sigma}$ for all $q \in (1/2, 1]$. Also, $\tilde{\sigma} < \sigma_l(q)$ for all $q \in (1/2, 1]$ by Lemma 3. Now, noting that

$$\mathbb{E}[R(\sigma_c(q)) \mid S = h] \geq \mathbb{E}[R(\tilde{\sigma}) \mid S = h] \geq \mathbb{E}[R(\tilde{\sigma})] > 1,$$

equation (28) implies that for all $q \in (1/2, 1]$,

$$\begin{aligned} q\mathbb{E}'[R(\sigma_c(q))] &= (2q-1)\mathbb{E}[R(\sigma_c(q))]\left[\frac{q}{q\sigma_c(q)+(1-q)(1-\sigma_c(q))}\right] \\ &= (2q-1)\Pr\left[R\left(\sigma_c(q)\right) = r\left(\sigma_c(q)\right) \mid S(q)=h\right] r(\sigma_c(q)) \\ &= (2q-1)\mathbb{E}[R\left(\sigma_c\left(q\right)\right) \mid S(q)=h] \\ &> 2q-1 \\ &= q\mathbb{E}'[R(\sigma_h)], \end{aligned}$$

where the last substitution uses equation (9) derived in Section 3 and the condition $\partial B_h(\sigma_h, q)/\partial\sigma = 0$. Since $\mathbb{E}[R(\sigma)]$ is concave, $\mathbb{E}'[R(\sigma_c(q))] > \mathbb{E}'[R(\sigma_h)]$ implies $\sigma_c(q) < \sigma_h(q)$. Therefore, for $q \in (1/2, 1]$,

$$\sigma_c(q) < \sigma_h(q) < \tilde{\sigma} < \sigma_l(q).$$

Differentiating equation $\partial\mathbb{E}[\,R(\sigma)|\,S=h]/\partial\sigma = 0$ implicitly defining $\sigma_c(q)$ we get

$$\frac{d\sigma_c(q)}{dq} = -\frac{\partial^2\mathbb{E}[R(\sigma_c(q)) \mid S(q)=h]}{\partial\sigma\partial q}\left(\frac{\partial^2\mathbb{E}[R\left(\sigma_c\left(q\right)\right) \mid S(q)=h]}{\partial\sigma^2}\right)^{-1}.$$

Thus, if $\sigma_c(q)$ is an interior solution to the problem (19), then

$$\frac{\partial^2\mathbb{E}[R\left(\sigma_c\left(q\right)\right) \mid S(q)=h]}{\partial\sigma^2} < 0.$$

Moreover,

$$\frac{\partial^2\mathbb{E}[R\left(\sigma_c\left(q\right)\right) \mid S(q)=h]}{\partial\sigma\partial q} = \frac{-\sigma_c(q)r(\sigma_c(q))}{[q\sigma_c(q)+(1-q)(1-\sigma_c(q))]^2} < 0.$$

Hence $d\sigma_c(q)/dq < 0$. □

B. Example: Calculations

We begin with calculations underlying Figure 1. Substituting $b\left(a-\sigma/2\right)$ from equation (22) for $r(\sigma)$ in equation (6) gives

$$B_l(q,\sigma) = \sigma b\left(a-\frac{\sigma}{2}\right) - \frac{q\left(1-\sigma\right)+(1-q)\sigma}{1-q}. \tag{29}$$

From this equation (29) we get

$$\frac{\partial B_l(q,\sigma)}{\partial\sigma} = b\,(a-\sigma) + \frac{2q-1}{1-q},$$

and solving for σ the equation $\partial B_l(q,\sigma)/\partial\sigma = 0$ gives

$$\sigma_l(q) = a + \frac{2q-1}{b\,(1-q)}. \tag{30}$$

Likewise, substituting equation (22) for equation (7) gives

$$B_h(q,\sigma) = q\sigma b\,(a-\sigma/2) - q\sigma - (1-q)(1-\sigma), \tag{31}$$

from which we get

$$\frac{\partial B_h(q,\sigma)}{\partial\sigma} = qb\,(a-\sigma) - (2q-1).$$

Solving for σ the equation $\partial B_h(q,\sigma)/\partial\sigma = 0$ yields

$$\sigma_h(q) = a - \frac{2q-1}{bq}. \tag{32}$$

Equations (30) and (32) constitute the formulae in equation (23) of the main text. To identify $\bar{q}$, we first substitute equations (30) and (32) for (29) and (31), respectively, so as to obtain $B_l(q) = B_l(q,\sigma_l(q))$ and $B_h(q) = B_h(q,\sigma_l(q))$. Then, solving $B_l(q) = B_h(q)$ with $a = 4/5$ and $b = 120$ for q yields $\bar{q} \simeq 0.94$.

Using the values $a = 4/5$ and $b = 120$, the left panel of Figure 1 plots $\sigma_l(q)$ of equation (30) for $q \in [1/2, 0.94)$ and $\sigma_h(q)$ of equation (32) for $q \in (0.94, 1]$. Similarly, the right panel of Figure 1 plots $B_l(q)$ for $q \in [1/2, 0.94)$ and $B_h(q)$ for $q \in (0.94, 1]$ when $a = 4/5$ and $b = 120$.

Turning to calculations underlying Figure 2, the surplus as a function of the signal precision is readily calculated using the formulae developed in Section 4. For $q < \bar{q}$, using equations (10), (22), and (30), we get

$$\begin{aligned} W_l(c,\sigma_l(q)) &= \mathbb{E}[R(\sigma_l)] - (1-\sigma_l)c = \sigma_l r(\sigma_l) - (1-\sigma_l)c \\ &= b\left(a + \frac{2q-1}{b\,(1-q)}\right)\left(a - \frac{\left(a+\frac{2q-1}{b(1-q)}\right)}{2}\right) - (1 - a - \frac{2q-1}{b\,(1-q)})c. \end{aligned} \tag{33}$$

Similarly, for $q > \bar{q}$, using equations (10), (22) and (32), we get

$$\begin{aligned} W_h(c, q, \sigma_h(q)) &= q\mathbb{E}[R(\sigma_h)] + (1-q)\sigma_h + (1-\sigma_h)q - (1-q\sigma_h)c \\ &= q\sigma_h r(\sigma_h) + (1-q)\sigma_h + (1-\sigma_h)q - (1-q\sigma_h)c \\ &= qb\left(a - \frac{2q-1}{qb}\right)\left(a - \frac{1}{2}\left(a - \frac{2q-1}{qb}\right)\right) \\ &\quad +(1-q)\left(a - \frac{2q-1}{qb}\right) + (1 - \left(a - \frac{2q-1}{qb}\right))q - (1 - q\left(a - \frac{2q-1}{qb}\right))c. \end{aligned}$$

Hence,

$$W_h(c, 1, \sigma_h(1)) = \frac{1}{2b}\left(a^2b^2 - 1\right) - (1 - a + \frac{1}{b})(c-1). \tag{34}$$

As shown in the proof of Proposition 4, $\hat{c}$ is a solution of the equation

$$W_h(c, 1, \sigma_h(1)) = W_l(c, \sigma_l(q_l(c))),$$

where $q_l(c)$ is given by equation (13). Assume that $\hat{c} < \bar{c}$ in which case equation (13) implies that $q_l(c) = (c+1)/(c+2)$. Inserting $a = 4/5$ and $b = 120$ in equations (33) and (34) and solving $W_h(c, 11, \sigma_h(1)) = W_l(c, \sigma_l((c+1)/(c+2)))$ for c yields $\bar{c} = 5\sqrt{2} - 1 \simeq 6.07$. Since $\bar{q} \simeq 0.94$, we have $(2\bar{q}-1)/(1-\bar{q}) \simeq 14.67$. Thus the initial assumption that $\bar{c} < (2\bar{q}-1)/(1-\bar{q})$ is fulfilled.

The left panel of Figure 2 plots the socially optimal precision identifies in Proposition 2 for a case in which $\bar{c} < 14.67$. Using the values $a = 4/5$ and $b = 120$, the right panel plots the maximum surplus $W^*(c) = W_h(c, 1, \sigma_h(1))$ from equation (34) for $c \leq \bar{c}$, $W^*(c) = W_l(c, \sigma_l((c+1)/(c+2)))$ and $W^*(c) = W_l(c, \sigma_l(\bar{q}))$ from equation (33) for $c \in (6.07, 14.67]$ and $c > 14.67$, respectively.

Figure 3 is a result of the following calculations characterizing the equilibrium in a perfectly competitive banking sector: Since $\mathbb{E}[\,R(\sigma)|\,S = h] = \Pr[R(\sigma) = r(\sigma) \mid S(q) = h]\, r(\sigma)$,

$$\begin{aligned} \frac{\partial \mathbb{E}[R(\sigma) \mid S(q) = h]}{\partial \sigma} &= \frac{\partial \Pr[R(\sigma) = r(\sigma) \mid S(q) = h]}{\partial \sigma} r(\sigma) + \Pr[R(\sigma) = r(\sigma) \mid S(q) = h]\, r'(\sigma) \\ &= \frac{qb}{q\sigma + (1-q)(1-\sigma)}\left(\frac{(1-q)(a - \sigma/2)}{q\sigma + (1-q)(1-\sigma)} - \frac{\sigma}{2}\right), \end{aligned}$$

where the second equality uses equations (2) and (22). The definition (19) thus implies that the equilibrium level of asset risk solves the equation

$$\frac{(1-q)(a-\sigma/2)}{q\sigma + (1-q)(1-\sigma)} = \frac{\sigma}{2}.$$

Solving for σ we get

$$\sigma_c(q) = \frac{\sqrt{(1-q)(1-q+2a(2q-1))}-(1-q)}{2q-1}, \tag{35}$$

which is equation (24) of the main text. Note from equation (35) that $\sigma_c(1) = 0$. Thus, for q sufficiently large, $\sigma_c(q) = \underline{\sigma}$; otherwise $\sigma_c(q)$ is given by equation (35). Letting $\sigma_c(q) = \underline{\sigma} = 1/5$ and $a = 4/5$ in equation (35) and solving the resulting expression for q gives $q = 31/32$. Figure 3 plots $\sigma_c(q)$ of equation (35) for $q \in [1/2, 31/32)$ and $\sigma_c(q) = 1/5$ for $q \in [31/32, 1]$.

To construct Figure 3, we first observe from equation (20) that the surplus in the case of a competitive banking sector is given by

$$\begin{aligned} W_c(c,q) &= \mathbb{E}[\, R(\sigma_c(q)) |\, S = h] - (1 - q\sigma_c(q))c \\ &= \Pr\left[R\left(\sigma_c\left(q\right)\right) = r\left(\sigma_c\left(q\right)\right) \mid S(q) = h\right] r(\sigma_c(q)) - (1 - q\sigma_c(q))c. \end{aligned}$$

Substituting from equations (2) and (22) the formulae for $\Pr\left[R\left(\sigma_c\left(q\right)\right) = r\left(\sigma_c\left(q\right)\right) \mid S(q) = h\right]$ and $r(\sigma_c(q))$ in the above equation gives

$$W_c(c,q) = \frac{q\sigma_c(q)b\left(a - \sigma_c(q)/2\right)}{q\sigma_c(q) + (1-q)(1-\sigma_c(q))} - (1 - q\sigma_c(q))c. \tag{36}$$

The left panel of Figure 3 is plotted by letting $a = 4/5$ and $b = 120$ as follows: For $q \in [1/2, 31/32)$, we substitute equation (35) for $\sigma_c(q)$ in equation (36) and for $q \in [31/32, 1]$, we set $\sigma_c(q) = 1/5$ in equation (36).

Since taking $q = 1$, yields $\sigma_c(q) = \underline{\sigma}$, we obtain from equation (36) that

$$W_c^*(c) = W_c(c,1) = b(a - \underline{\sigma}/2) - (1 - \underline{\sigma})c.$$

For the numerical example $a = 4/5$, $b = 120$, and $\underline{\sigma} = 1/5$, we have $W_c^*(c) = 84 - 4c/5$, which is depicted by the right panel of Figure 3.

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