



Mikael Bask

Instrument rules in monetary policy under heterogeneity in currency trade



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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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Instrument rules in monetary policy under heterogeneity in currency trade

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Abstract

We embed different instrument rules into a New Keynesian model for a small open economy that is augmented with technical trading in currency trade to examine the prerequisites for monetary policy. Specifically, this paper focuses on conditions for a determinate, least-squares learnable rational expectations equilibrium (REE). Under an interest rate rule with only contemporaneous macroeconomic data, the intensity of technical trading or trend-seeking in currency trade does not affect these conditions, except in the case of an extensive use of trend-seeking. On the other hand, if the central bank uses only forward-looking information in its interest rate rule, a determinate and learnable REE is a less likely outcome when trend-seeking in currency trade becomes more popular. The interest rate rule followed by the central bank in the model incorporates interest rate smoothing.

Keywords: determinacy, DSGE model, interest rate rule, least-squares learning, technical trading

JEL classification numbers: C62, E52, F31, F41

Rahapolitiikan korkosäännöt, talouden tasapaino ja heterogeenisuus valuuttamarkkinakaupassa

Suomen Pankin keskustelualoitteita 22/2007

Mikael Bask
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tutkimuksessa tarkastellaan, miten vaihtoehtoiset korkosäännöt vaikuttavat rahapolitiikan toimintaehtoihin pienessä avotaloudessa, jonka valuuttamarkkinakaupan toimijoista osa on teknisiä kaupankävijöitä. Työn teoreettiset tarkastelut perustuvat sentyyppiseen uuskeynesiläiseen avotalouden makromalliin, jollaisia on suhteellisen yleisesti käytetty modernissa rahapolitiikan teoreettisessa analyysissä. Työssä tarkastellaan erityisesti ehtoja, joiden vallitessa rationaalisten odotusten tasapaino on hyvin määritelty ja opittavissa tilastollisin menetelmin ja joita taloudenpitäjät käyttävät muodostaessaan keskeisten makrotaloudellisten muuttujien ennusteita. Opittavuudella tarkoitetaan tässä yhteydessä tilastollisen mallien tuottamien ennusteiden harhattomuutta eli yhtäläistymistä rationaalisten odotusten kanssa. Keskuspankki perustaa ohjauskorkoa koskevat päätöksensä joko käytettävissään olevaan tilastoituun makrotaloudelliseen informaatioon tai käyttämäänsä ennakoivaan tietoon. Tämän lisäksi se pyrkii välttämään korkotason liiallisia vaihteluita, eli se haluaa tasata korkomuutoksia ajan suhteen. Tulosten mukaan valuuttamarkkinoiden tekninen kaupankäynti eli trendien etsintä vaikuttaa rationaalisten odotusten tasapainon määräytyneisyyteen ja opittavuuteen vain ääritapauksissa, joissa tekninen kaupankäynti on valuuttamarkkinoita hallitseva, ja kun keskuspankin korkosääntö perustuu käytettävissä olevaan tilastoinformaatioon. Ennuste- tai ennakoivaan informaatioon perustuvan korkosäännön tapauksessa teknisen kaupankäynnin yleistyminen valuuttamarkkinoilla sen sijaan vaikuttaa talouden asettumista yksikäsitteiseen ja opittavissa olevaan rationaalisten odotusten tasapainoon.

Avainsanat: määrittynisyys, DSGE-malli, korkosääntö, pienimmän neliösumman oppiminen, tekninen kaupankäynti

JEL-luokittelu: C62, E52, F31, F41

Contents

Abstract.....	3
Tiivistelmä (abstract in Finnish).....	4
1 Introduction.....	7
2 A small open economy.....	9
2.1 Baseline model.....	9
2.2 Expectations formations in currency trade.....	10
3 Instrument rules in policy-making.....	11
3.1 Instrument rules investigated.....	11
3.2 Determinacy.....	11
3.2.1 Complete model.....	12
3.2.2 Numerical findings.....	12
3.3 Least squares learning.....	19
3.3.1 MSV solution.....	19
3.3.2 Learning the MSV solution.....	20
4 Conclusions.....	21
References.....	22
Appendix.....	25

1 Introduction

Interest rate rules

In 1993, Taylor (1993) demonstrated that Federal Reserve's policy could be described by the following interest rate rule

$$r_t = 0.04 + 1.5 (\pi_t - 0.02) + 0.5 (y_t - \bar{y}) \quad (1.1)$$

where r_t is Federal Reserve's operating target for the funds rate, π_t is the inflation rate according to the GDP deflator, y_t is the logarithm of real GDP, and \bar{y} is the logarithm of potential real GDP. In particular, the rule in (1.1) prescribes setting the funds rate in response to the inflation rate and the output gap, where the latter is the difference between the two measures of GDP. Taylor (1999) also argues that since the rule in (1.1) describes Federal Reserve's policy during a successful period, one should adopt a rule like this in policy-making.

The so-called Taylor rule in (1.1) is an example of an instrument rule since the funds rate, which is Federal Reserve's instrument in policy-making, is an explicit function of the information available to the central bank. The rule in (1.1) is also named a simple rule since the funds rate is a function of a small subset of this information (see Svensson, 2003). See Clarida et al (1999) for a review of interest rate rules in the new Keynesian model, and Zimmermann (2003) for a more introductory text on the same topic. Woodford's (2003) seminal work on rules in policy-making should also be part of the reading list.

Clarida et al (2000) estimate different interest rate rules to evaluate Federal Reserve's policy during 1960–1996 utilizing a new Keynesian model similar to the one that we analyze in this paper, and they found that the policy during the Volcker-Greenspan period was more successful to stabilize the economy than the policy during the pre-Volcker period. Even though their evaluation of Federal Reserve's policy is somewhat simplistic, it is very intriguing.

A determinate and learnable REE?

Typically, in the literature, conditions for uniqueness of the rational expectations equilibrium (REE) are investigated since the policy-maker would like to avoid coordination problems in the economy. For instance, without imposing additional restrictions into a rational expectations model, it is not known in advance which of the REE that agents will coordinate on, if there will be any coordination at all. To give an example, the effects of changes in monetary policy may not be known beforehand: is it the case that agents will coordinate on a REE that has undesirable properties, like a very high inflation rate, or on a REE in which the price level is stable?

Another problem in this context is the actual computations of the time-paths of economic variables when agents have rational expectations since one cannot expect that they have perfect knowledge of the economy's law of motion. For example, it is a well-known fact among economists that the transmission mechanism for monetary policy has a complicated structure, and this also means that there are disagreements about the exact nature of this mechanism. The following question arises, however: may agents eventually

learn the REE, if they can make use of data generated by the economy itself to improve their knowledge of its law of motion?

The concept of learning that we make use of in this paper is least squares learning, and to have a REE that is least squares learnable, the parameter values in the perceived law of motion (PLM) of the economy have to converge to the economy's actual law of motion (ALM), and this happens when the REE is characterized by expectational stability. See Evans and Honkapohja (2001) for an introduction to this literature, and Bullard (2006) and Evans and Honkapohja (2003) for two reviews of interest rate rules in the new Keynesian model from a learning perspective.

Heterogeneity in currency trade

Questionnaire surveys made at currency markets around the world reveal that currency trade to a large extent not only is determined by an economy's performance or expected performance. Indeed, a non-negligible fraction is guided by technical trading, meaning that past exchange rates are assumed to provide information about future exchange rate movements.

For this reason, we embed different instrument rules into Galí and Monacelli's (2005) new Keynesian model for a small open economy that is augmented with technical trading in currency trade to examine the prerequisites for monetary policy. Specifically, conditions for a determinate and least squares learnable REE are in focus, where the following rules are examined: (i) a contemporaneous data specification that includes the output gap, the CPI inflation rate, the exchange rate change, and the interest rate in the previous time period to have inertia in policy-making; and (ii) a forward expectations specification with the same variables as well as policy-inertia.

See Oberlechner (2004) for an in-depth discussion of two large questionnaire surveys conducted at the European and the North American markets, Gehrig and Menkhoff (2006) for a recent survey on trading behavior that includes references to several other surveys made at currency markets (cf., Cheung and Chinn, 2001, Lui and Mole, 1998, Menkhoff, 1997, Oberlechner, 2001, and Taylor and Allen, 1992), and Neely (1997) for a layman's guide on technical trading. Note that other terms for technical trading are chartism and technical analysis.

Relation to the literature

Since the model in this paper nests other models that previously have been examined, we have a neat relation to this literature. Examples of papers include Bullard and Mitra (2002)–(2006) and Evans and Honkapohja (2003). It goes without saying that we replicate the results in their papers since they focus on conditions for least squares learnability of a unique REE, like we do in this paper, but for a closed economy.

Bullard and Schaling (2006) and Llosa and Tuesta (2006) are papers that do the same exercise, but for an open economy. The former paper is built around the two-country model in Clarida et al (2002), whereas the latter paper uses the same model as in this paper, namely, the Galí and Monacelli (2005) model. However, there is no technical trading in Llosa and Tuesta (2006) nor

in the other papers. Lubik and Marzo (2007) should also be mentioned in this context, even though they neglect from learning issues.

Despite the fact that there are not too many papers that incorporate technical trading into a theoretical framework, there are a few important papers that should be mentioned. Frankel and Froot (1986) implement technical and fundamental analyses into a foreign exchange model, and their model was among the first that utilized this setup when focusing on currency trade. Other seminal papers include Brock and Hommes (1997), De Long et al (1990), Kirman (1993), and Zeeman (1974). See Hommes (2006) for a survey of the literature on heterogeneous agent models in economics and finance, and De Grauwe and Grimaldi (2006) for an introduction to exchange rate determination in a behavioral finance framework.

Outline of the paper

The theoretical framework is outlined in Section 2, whereas the prerequisites for monetary policy when an instrument rule is used in policy-making are in focus in Section 3. Section 4 concludes the paper.

2 A small open economy

A dynamic stochastic general equilibrium (DSGE) model with imperfect competition and nominal rigidities is presented in Galí and Monacelli (2005) for a small open economy, and we outline their model in Section 2.1. In Section 2.2, we present the expectations formations in currency trade that we augment this model with.

2.1 Baseline model

The baseline model consists of the following four equations: (i) a dynamic IS-type equation; (ii) a new Keynesian Phillips curve; (iii) a condition for uncovered interest rate parity (UIP); and (iv) a stochastic process for the natural rate of interest

$$\begin{cases} x_t = x_{t+1}^e - \alpha \left(r_t - \frac{1}{1-\delta} \cdot (\pi_{t+1}^e - \delta (\Delta e_{t+1}^{e,m} + \pi_{t+1}^{e,*})) - \bar{r}_t \right) \\ \pi_t = \beta \pi_{t+1}^e + \gamma (1 - \delta) x_t + \delta (\Delta e_t - \beta \Delta e_{t+1}^{e,m} + \pi_t^* - \beta \pi_{t+1}^{e,*}) \\ r_t - r_t^* = \Delta e_{t+1}^{e,m} \\ \bar{r}_t = \rho \bar{r}_{t-1} + \varepsilon_t \end{cases} \quad (2.1)$$

where x_t is the output gap, r_t is the nominal interest rate, π_t is the CPI inflation rate, e_t is the nominal exchange rate, and \bar{r}_t is the natural rate of interest. Moreover, the superscripts e and e, m denote expectations in general and market expectations in currency trade, respectively, and an asterisk in the superscript denotes a foreign quantity. See the Appendix for the derivation of the first two equations in (2.1) using equations that are derived in Galí and Monacelli (2005). The third equation in (2.1) can also be found in Galí and

Monacelli (2005), and the fourth equation in (2.1) can be found in Woodford (1999).

Turning the focus to the structural parameters in the baseline model, $\beta > 0$ is the discount factor that is used when the representative household in the home country maximizes a discounted sum of instantaneous utilities derived from consumption and leisure, and $\delta \in [0, 1]$ is the share of consumption in the home country allocated to imported goods, meaning that δ is an index of openness of the economy. Moreover, $0 \leq \rho < 1$ is the serial correlation in the stochastic process, and $\varepsilon_t \in IID(0, \sigma_\varepsilon^2)$.

The other two parameters in the model, α and γ , are functions of structural parameters in the Galí and Monacelli (2005) model. First, α depends on four parameters: (i) the openness index, δ ; (ii) the intertemporal elasticity of substitution in consumption; (iii) the elasticity of substitution between domestic and foreign goods in consumption; and (iv) the elasticity of substitution between foreign goods in consumption. Second, γ depends on α as well as three other parameters: (i) the discount factor, β ; (ii) the intertemporal elasticity of substitution in labor supply; and (iii) the share of firms that set new prices in each time period (see Calvo, 1983).

Since we examine the properties of the model numerically,¹ the exact relationships between the parameters in the baseline model and the parameters in the Galí and Monacelli (2005) model are not presented. Of course, to fully grasp the microeconomic foundations in their model, it is necessary to consult Galí and Monacelli (2005).

2.2 Expectations formations in currency trade

There are two types of trading behavior in the currency market: (i) trend following in currency trade; and (ii) trading that is based on fundamental analysis.

When trend following is used in currency trade, it is believed that the exchange rate will increase (decrease) between time periods t and $t + 1$, if it increased (decreased) between time periods $t - 1$ and t . Moreover, to minimize the number of structural parameters in the model, these two consecutive increases (decreases) in the exchange rate are believed to be of the same size.

When fundamental analysis is used in currency trade, agents have rational expectations regarding the next time period's exchange rate change, meaning that market expectations can be summarized as

$$\Delta e_{t+1}^{e,m} = \omega \Delta e_{t+1}^{e,c} + (1 - \omega) \Delta e_{t+1}^{e,f} = \omega \Delta e_t + (1 - \omega) \Delta e_{t+1}^e \quad (2.2)$$

where $\omega \in [0, 1]$ is the degree of trend following in currency trade, and e, c and e, f denote expectations according to chartism and fundamental analysis, respectively. Even though the superscript e denotes rational expectations, we will also think of it as possibly non-rational expectations when we focus on learning issues in Section 3.3.

¹ All MATLAB routines that are used are available on request from the author.

More sophisticated trading rules, like the moving averages (MA) technique, would also be desirable to examine in the Galí and Monacelli (2005) model. However, this would complicate the analysis considerably, and it is not certain that the dynamics is affected that much compared to when simple trend following is used in technical trading. This conclusion comes from the asset pricing model in Bask (2006), where it was found that the exchange rate in time periods $t - t_0$, $t_0 \geq 2$, had a second-order effect on the current exchange rate, whereas the exchange rate in the previous time period had a first-order effect. See Bask (2007) for the MA technique in a Dornbusch-style model.

It could be mentioned in this context that since extrapolative expectations in a general equilibrium model has been rather successful to resolve the equity premium puzzle (see Choi, 2006), we feel somewhat encouraged to implement trend following in currency trade in a DSGE model since trend following is a form of extrapolative expectations.

3 Instrument rules in policy-making

The Taylor rules investigated are presented in Section 3.1, and in Section 3.2, we examine whether there is a determinate REE, given a specific parametrization of the Taylor rule and calibrated values of the structural parameters. Thereafter, in Section 3.3, we look into the possibility that agents using fundamental analysis, which includes the central bank, may learn the REE when it is unique.

3.1 Instrument rules investigated

The central bank is using a Taylor rule when setting the interest rate, and responds to the output gap, the CPI inflation rate, the exchange rate change, and the interest rate in the previous time period when making its policy-decision. Specifically, two specifications of the Taylor rule are investigated; a contemporaneous data specification of the rule

$$r_t = \zeta_r r_{t-1} + \zeta_x x_t + \zeta_\pi \pi_t + \zeta_e \Delta e_t \quad (3.1)$$

and a forward expectations specification of the rule

$$r_t = \zeta_r r_{t-1} + \zeta_x x_{t+1}^e + \zeta_\pi \pi_{t+1}^e + \zeta_e \Delta e_{t+1}^e \quad (3.2)$$

Note that the central bank has rational expectations regarding the variables in the rule in (3.2).

3.2 Determinacy

The complete model under both specifications of the Taylor rule is summarized in matrix form in Section 3.2.1, and numerical findings when it comes to conditions for a unique REE are presented in Section 3.2.2.

3.2.1 Complete model

Contemporaneous data in the rule

The complete model in matrix form is (see the Appendix for derivations)

$$\mathbf{\Gamma} \cdot \mathbf{y}_t = \mathbf{\Theta} \cdot \mathbf{y}_{t+1}^e + \mathbf{\Lambda} \cdot \mathbf{y}_{t-1} + \mathbf{\Xi} + \mathbf{\Pi} \cdot \bar{r}r_t \quad (3.3)$$

where

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & \frac{\alpha\delta\omega}{1-\delta} & \alpha \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) & 0 \\ 0 & 0 & -\omega & 1 \\ -\zeta_x & -\zeta_\pi & -\zeta_e & 1 \end{bmatrix} \quad (3.4)$$

$$\mathbf{\Theta} = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta(\omega-1)}{1-\delta} & 0 \\ 0 & \beta & \beta\delta(\omega-1) & 0 \\ 0 & 0 & 1-\omega & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3.5)$$

$$\mathbf{\Lambda} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \zeta_r \end{bmatrix} \quad (3.6)$$

and

$$\mathbf{y}_t = [x_t, \pi_t, \Delta e_t, r_t]' \quad (3.7)$$

Forward expectations in the rule

The complete model in matrix form is (3.3) and (3.6)–(3.7), where (see the Appendix for derivations)

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & \frac{\alpha\delta\omega}{1-\delta} & \alpha \\ \gamma(\delta-1) & 1 & \delta(\beta\omega-1) & 0 \\ 0 & 0 & -\omega & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.8)$$

and

$$\mathbf{\Theta} = \begin{bmatrix} 1 & \frac{\alpha}{1-\delta} & \frac{\alpha\delta(\omega-1)}{1-\delta} & 0 \\ 0 & \beta & \beta\delta(\omega-1) & 0 \\ 0 & 0 & 1-\omega & 0 \\ \zeta_x & \zeta_\pi & \zeta_e & 0 \end{bmatrix} \quad (3.9)$$

3.2.2 Numerical findings

To be able to determine whether the complete model has a determinate REE, a first step is to rewrite the model into first-order form, and, then, to compare the number of pre-determined variables in the model with the number of

eigenvalues of a certain matrix that are outside the unit circle (see Blanchard and Kahn, 1980).

We make use of the following variable vector when rewriting the model

$$\mathbf{y}_t = [x_t, \pi_t, \Delta e_t, r_t, r_t^L]' \quad (3.10)$$

where

$$r_t^L \equiv r_{t-1} \quad (3.11)$$

This means that the relevant coefficient matrices are

$$\mathbf{\Gamma}_d = \begin{bmatrix} & & \mathbf{\Gamma} & & -\mathbf{\Lambda}_4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (3.12)$$

and

$$\mathbf{\Theta}_d = \begin{bmatrix} & & & & 0 \\ & & \mathbf{\Theta} & & 0 \\ & & & & 0 \\ & & & & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (3.13)$$

where $\mathbf{\Lambda}_4$ is the fourth column in matrix $\mathbf{\Lambda}$, because the complete model in matrix form is now

$$\mathbf{\Gamma}_d \cdot \mathbf{y}_t = \mathbf{\Theta}_d \cdot \mathbf{y}_{t+1}^e + \mathbf{\Xi}_d + \mathbf{\Pi}_d \cdot \bar{r}r_t \quad (3.14)$$

Thus, since there are two variables in (3.10) that are pre-determined, r and r^L , exactly two eigenvalues of the matrix $\mathbf{\Gamma}_d^{-1} \cdot \mathbf{\Theta}_d$ must be outside the unit circle to have a unique REE.

However, deriving analytical conditions for determinacy is not meaningful for practical reasons since these expressions would be too large and cumbersome to interpret. Consequently, we adopt the same strategy as in other papers within this area and illustrate our findings for determinacy using calibrated values of the structural parameters. Specifically, the following parameter values, or range of values, are used in the analysis that are based on quarterly data

$$\begin{cases} \alpha = \frac{1}{0.157}, & \beta = 0.99, & \gamma = 0.024, & \delta = 0.2, 0.4, \\ \rho = 0.35, & 0 \leq \zeta_x \leq 3, & 0 \leq \zeta_\pi \leq 3. \end{cases} \quad (3.15)$$

See Woodford (1999)–(2003) for the closed economy parameters α , β , γ and ρ . Moreover, when the index of openness of the economy is $\delta = 0.2$, the index is slightly larger than the import/GDP ratio in the US, and when the index of openness of the economy is $\delta = 0.4$, which is the parameter setting in Galí and Monacelli (2005), the index corresponds roughly to the import/GDP ratio in Canada.

Contemporaneous data in the rule

See *Figures 1a–b* for findings when the central bank is using contemporaneous data in the Taylor rule. In both figures, the regions in the parameter space of $(\delta, \omega, \zeta_r, \zeta_x, \zeta_\pi, \zeta_e)$ for which we have a determinate REE are shown. Specifically, since δ, ω, ζ_r and ζ_e are given in the figures, it is the combinations of ζ_x and ζ_π that are in light areas that give rise to a unique REE.²

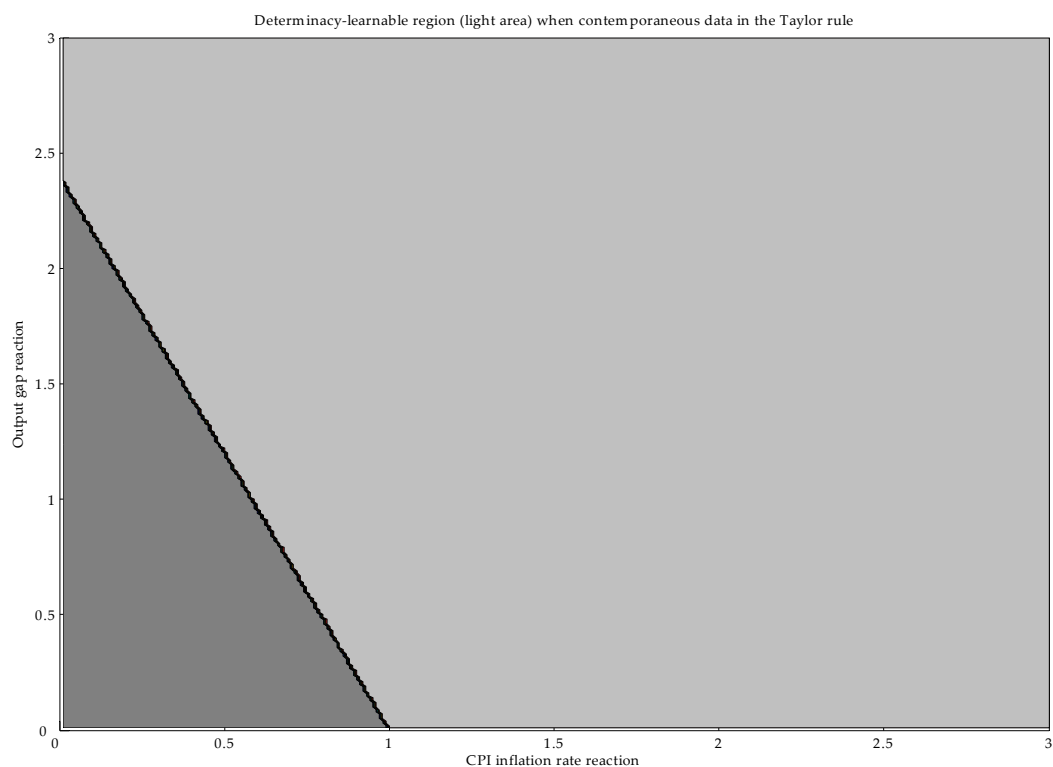


Figure 1a: ω is not too high and $\zeta_r + \zeta_e = 0$.

² To minimize the number of figures in the paper, the regions in the figures are not only regions for determinacy, but also regions for least squares learnability of the REE. As will be clear in Section 3.3, the reason is that when there is a unique REE, agents will also learn this REE.

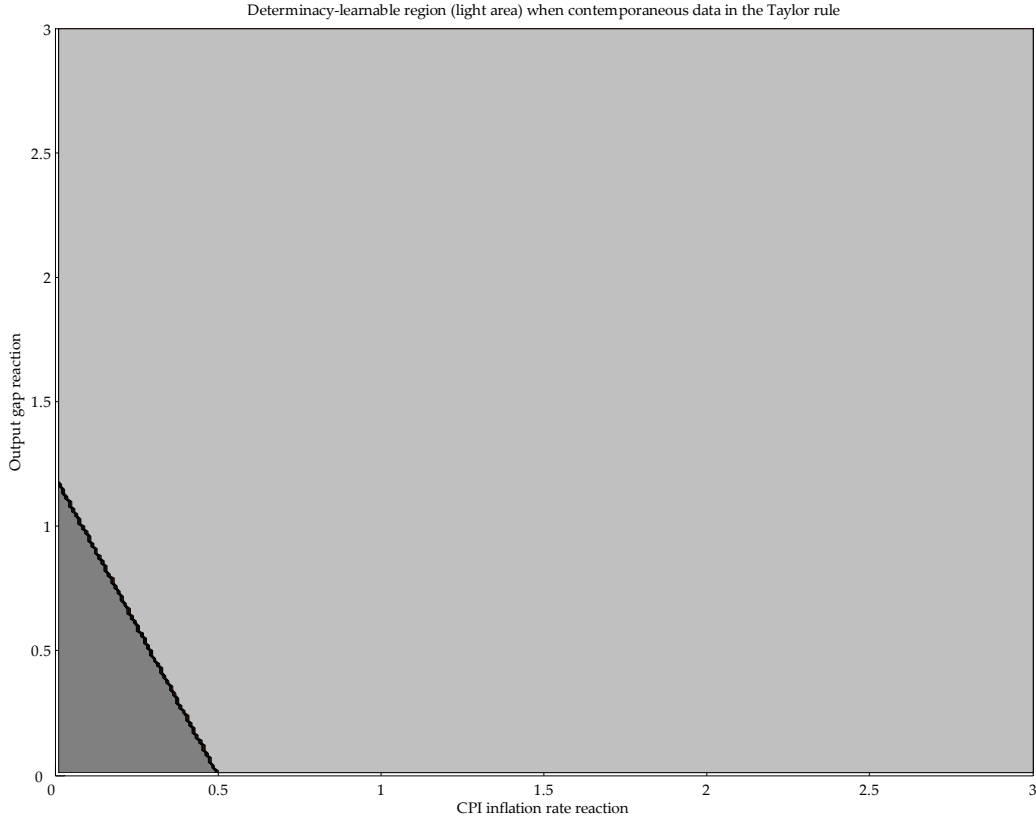


Figure 1b: ω is not too high and $\zeta_r + \zeta_e = 0.5$.

Figure 1a can also be found in Bullard and Mitra (2002), meaning that we replicate the finding in their paper that is for a closed economy. In fact, the determinacy region in the figure is not affected by the openness index, δ , as long as $\zeta_r + \zeta_e = 0$. Note that this condition does not preclude inertia, $\zeta_r \neq 0$, nor an exchange rate change reaction, $\zeta_e \neq 0$, in policy-making. Moreover, numerical findings show that these two tools in monetary policy are perfect substitutes when it comes to affecting the shape of the determinacy region.

In *Figure 1b*, the relationship between inertia and an exchange rate change reaction to have a unique REE is $\zeta_r + \zeta_e = 0.5$. Thus, compared to *Figure 1a*, the determinacy region is larger in size. One consequence of this is that the Taylor principle no longer is a necessary condition to have a unique REE, meaning that the central bank does not have to change the interest rate more than one-to-one to a change in the inflation rate for determinacy. Note again that the determinacy region in the figure is not affected by the openness index.

When $\zeta_r + \zeta_e \geq 1$, we always have a unique REE no matter the value of the openness index, the degree of inertia and the strength of the exchange rate change reaction in policy-making. Of course, this finding does not mean that the outcome from a welfare perspective is necessarily desirable since it might be the case that the unique REE is associated with a very high inflation rate.

That the degree of trend following in currency trade, ω , cannot be too high to have the determinacy regions shown in *Figures 1a–b* should be made more precise. Thus, when the openness index is $\delta = 0.2$, the maximum amount of

trend following is $\omega = 0.44$, whereas the same figure has decreased to $\omega = 0.38$ when the openness index is $\delta = 0.4$. Obviously, the maximum amount of trend following depends on the values of the structural parameters in the model (see (3.15)).

Forward expectations in the rule

See *Figures 2a–e* for findings when the central bank is using forward expectations of the variables in the Taylor rule.

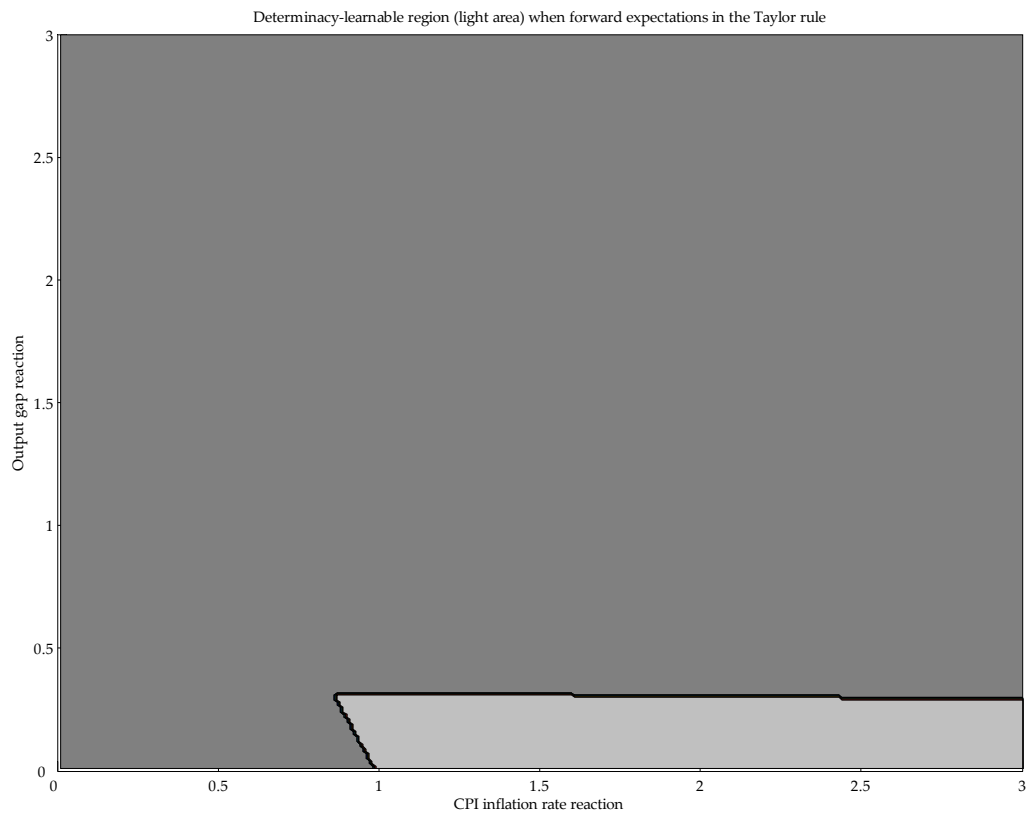


Figure 2a: $\delta = 0$ and $\zeta_r = 0$.

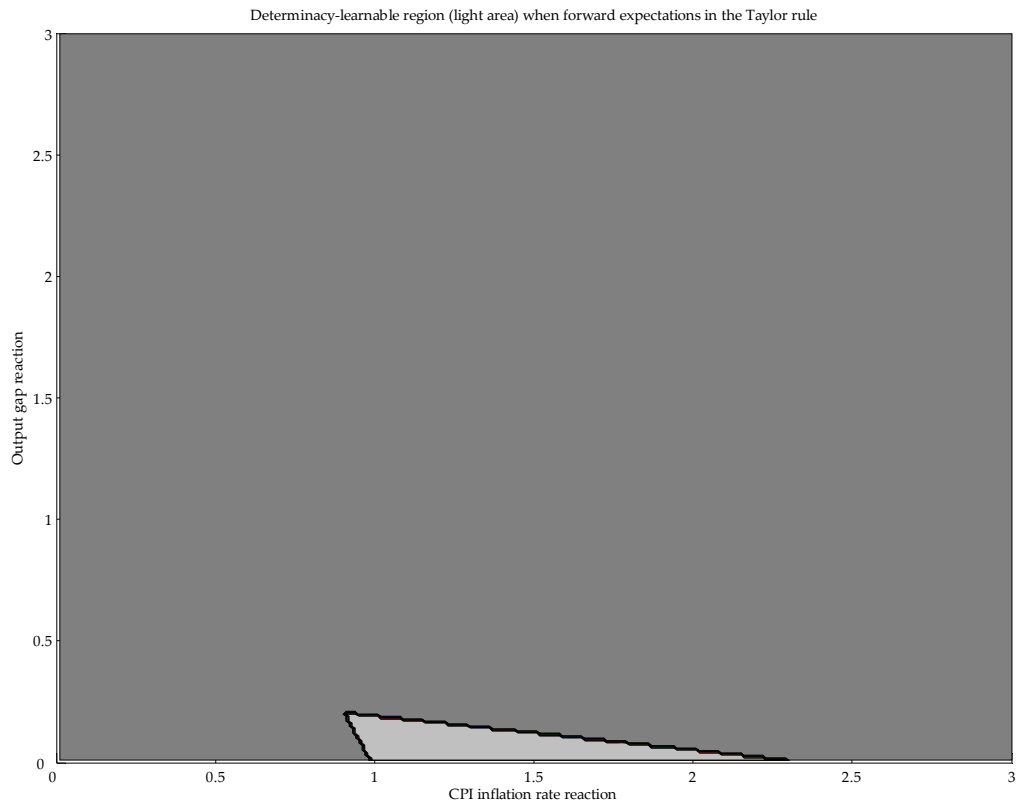


Figure 2b: $\delta = 0.4$, $\omega = 0$, $\zeta_r = 0$ and $\zeta_e = 0$.

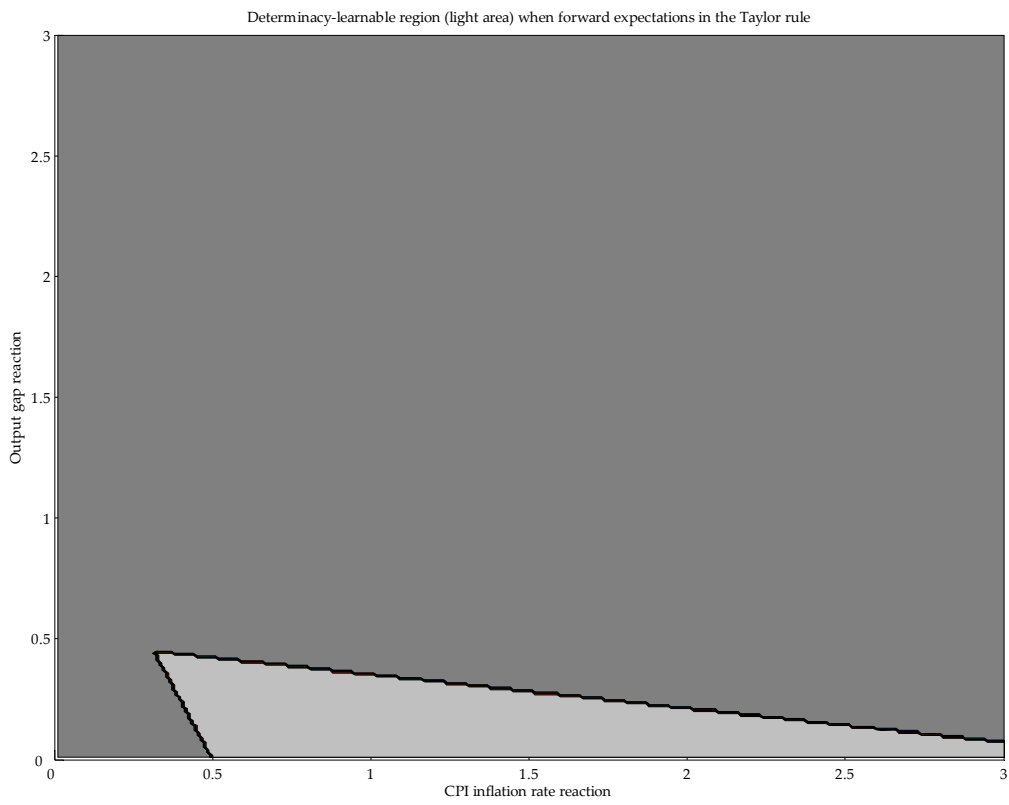


Figure 2c: $\delta = 0.4$, $\omega = 0$, $\zeta_r = 0.5$ and $\zeta_e = 0$.

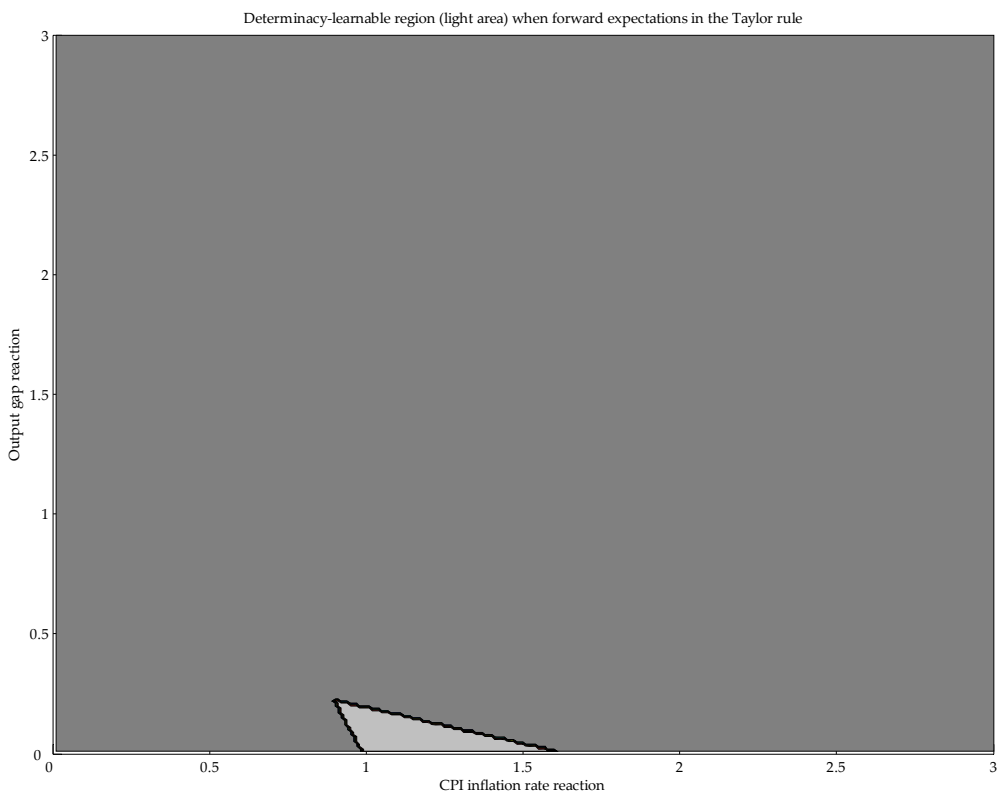
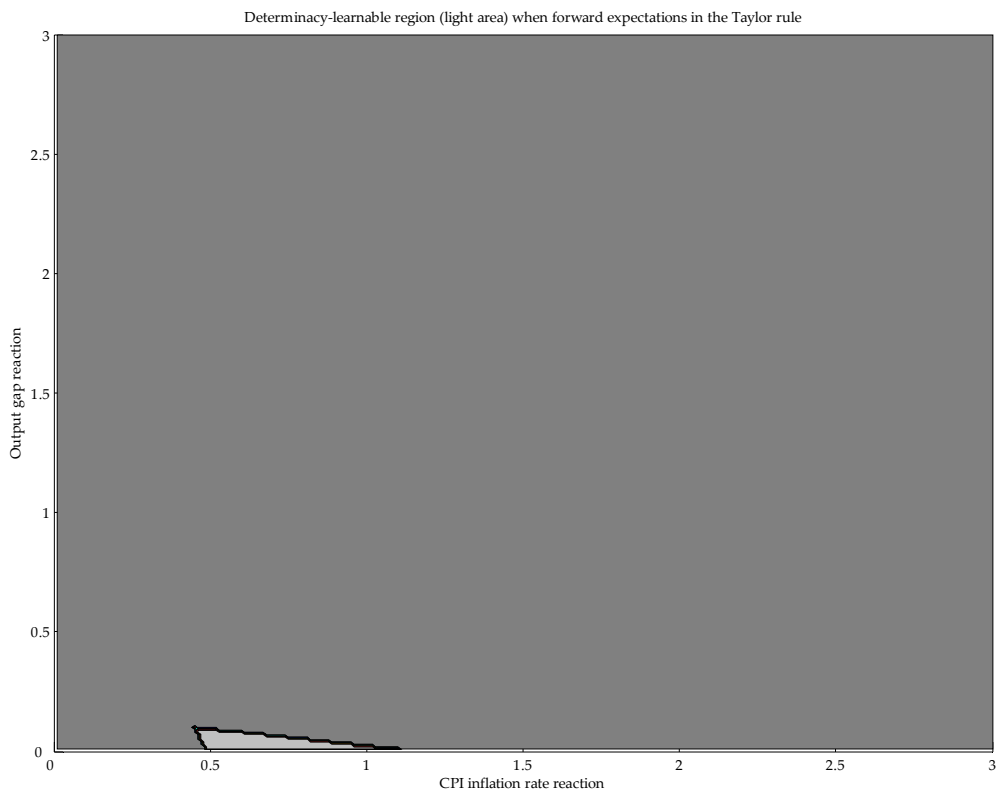


Figure 2a can also be found in Bullard and Mitra (2002), meaning that we again replicate the finding in their paper. A difference between contemporaneous data in the Taylor rule and forward expectations of the variables in the rule is that the openness index matters for the shape of the determinacy region. Specifically, the determinacy region is smaller in size when the economy is more open. For example, in *Figure 2b*, the openness index has increased to $\delta = 0.4$.

Inertia and an exchange rate change reaction are no longer substitutes in policy-making when it comes to affecting the shape of the determinacy region. Specifically, inertia makes the determinacy region larger in size (see *Figure 2c*), whereas an exchange rate change reaction makes the determinacy region smaller in size (see *Figure 2d*). Moreover, an increased degree of trend following in currency trade decreases the size of the determinacy region more or less in the same way as an exchange rate change reaction in policy-making does (see *Figure 2e*).

The last finding should, however, not be taken to an extreme, because when the degree of trend following is high enough, the determinacy region starts to increase in size when an even smaller amount of currency trade is determined by fundamental analysis. Thus, it seems that there is always a unique REE no matter the degree of trend following in currency trade.

3.3 Least squares learning

When there is a determinate REE, we make use of the minimal state variable (MSV) solution that is the solution of a linear difference equation that depends linearly on a set of variables such that there does not exist a solution that depends linearly on a smaller set of variables (see McCallum, 1983). Therefore, in Section 3.3.1, we derive the MSV solution of the model in (3.3), whereas least squares learnability of the MSV solution is under scrutiny in Section 3.3.2.

3.3.1 MSV solution

The MSV solution of the model in (3.3), which applies for both specifications of the Taylor rule in (3.1)–(3.2), is

$$\mathbf{y}_t = \widehat{\Phi} \cdot \mathbf{y}_{t-1} + \widehat{\Psi} + \widehat{\Omega} \cdot \overline{r\overline{r}}_t \quad (3.16)$$

where Φ , Ψ and Ω are coefficient matrices to be determined with the method of undetermined coefficients. Hence, calculate the mathematically expected state of the economy in the next time period

$$\mathbf{y}_{t+1}^e = \widehat{\Phi} \cdot \mathbf{y}_t + \widehat{\Psi} + \widehat{\Omega} \cdot \overline{r\overline{r}}_{t+1}^e = \widehat{\Phi} \cdot \mathbf{y}_t + \widehat{\Psi} + \widehat{\Omega} \cdot \rho \overline{r\overline{r}}_t \quad (3.17)$$

where the fourth equation in (2.1) is used in the second step in (3.17), and the dating of expectations is time period t . Thereafter, substitute (3.17) into (3.3)

$$\Gamma \cdot \mathbf{y}_t = \Theta \cdot \left(\widehat{\Phi} \cdot \mathbf{y}_t + \widehat{\Psi} + \widehat{\Omega} \cdot \rho \overline{r\overline{r}}_t \right) + \Lambda \cdot \mathbf{y}_{t-1} + \Xi + \Pi \cdot \overline{r\overline{r}}_t \quad (3.18)$$

or, if solved for the contemporaneous values of the model's variables

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \left(\widehat{\mathbf{\Phi}} \cdot \mathbf{y}_t + \widehat{\mathbf{\Psi}} + \widehat{\mathbf{\Omega}} \cdot \rho \overline{r}_t \right) + \\
&\quad \mathbf{\Gamma}^{-1} \cdot \mathbf{\Lambda} \cdot \mathbf{y}_{t-1} + \mathbf{\Gamma}^{-1} \cdot \mathbf{\Xi} + \mathbf{\Gamma}^{-1} \cdot \mathbf{\Pi} \cdot \overline{r}_t \\
&= \left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \widehat{\mathbf{\Phi}} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \mathbf{\Lambda} \cdot \mathbf{y}_{t-1} + \\
&\quad \left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \widehat{\mathbf{\Phi}} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \left(\mathbf{\Theta} \cdot \widehat{\mathbf{\Psi}} + \mathbf{\Xi} \right) + \\
&\quad \left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \widehat{\mathbf{\Phi}} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \left(\mathbf{\Theta} \cdot \widehat{\mathbf{\Omega}} \cdot \rho + \mathbf{\Pi} \right) \cdot \overline{r}_t
\end{aligned} \tag{3.19}$$

Finally, by comparing the parameter values in (3.16) with those in (3.19), we can solve the following equation system for the MSV solution

$$\begin{cases} \widehat{\mathbf{\Phi}} = \left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \widehat{\mathbf{\Phi}} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \mathbf{\Lambda} \\ \widehat{\mathbf{\Psi}} = \left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \widehat{\mathbf{\Phi}} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \left(\mathbf{\Theta} \cdot \widehat{\mathbf{\Psi}} + \mathbf{\Xi} \right) \\ \widehat{\mathbf{\Omega}} = \left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \widehat{\mathbf{\Phi}} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \left(\mathbf{\Theta} \cdot \widehat{\mathbf{\Omega}} \cdot \rho + \mathbf{\Pi} \right) \end{cases} \tag{3.20}$$

where $(\widehat{\mathbf{\Phi}}_{MSV}, \widehat{\mathbf{\Psi}}_{MSV}, \widehat{\mathbf{\Omega}}_{MSV})$ constitute the solution of the equation system. Of course, the MSV solution depends on the type of Taylor rule that is used in policy-making since the elements in the matrices are partly different for the two rules.

3.3.2 Learning the MSV solution

Now, is the MSV solution characterized by least squares learnability? To have a REE that is learnable, the parameter values in the PLM of the economy have to converge to the parameter values in the economy's ALM. In fact, the MSV solution in (3.16) is also the PLM of the economy, which is emphasized by the "hat"-symbols since $\widehat{\mathbf{\Phi}}$, $\widehat{\mathbf{\Psi}}$ and $\widehat{\mathbf{\Omega}}$ are coefficient matrices that are estimated, and the solution in (3.19) is the economy's ALM.³

Observe that there is a mapping from the parameter values in the PLM to the parameter values in the ALM

$$\mathbf{M} \begin{pmatrix} \widehat{\mathbf{\Phi}} \\ \widehat{\mathbf{\Psi}} \\ \widehat{\mathbf{\Omega}} \end{pmatrix} = \begin{pmatrix} \left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \widehat{\mathbf{\Phi}} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \mathbf{\Lambda} \\ \left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \widehat{\mathbf{\Phi}} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \left(\mathbf{\Theta} \cdot \widehat{\mathbf{\Psi}} + \mathbf{\Xi} \right) \\ \left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \widehat{\mathbf{\Phi}} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \left(\mathbf{\Theta} \cdot \widehat{\mathbf{\Omega}} \cdot \rho + \mathbf{\Pi} \right) \end{pmatrix} \tag{3.21}$$

and consider the following matrix differential equation

$$\frac{\partial}{\partial \tau} \begin{pmatrix} \widehat{\mathbf{\Phi}} \\ \widehat{\mathbf{\Psi}} \\ \widehat{\mathbf{\Omega}} \end{pmatrix} = \mathbf{M} \begin{pmatrix} \widehat{\mathbf{\Phi}} \\ \widehat{\mathbf{\Psi}} \\ \widehat{\mathbf{\Omega}} \end{pmatrix} - \begin{pmatrix} \widehat{\mathbf{\Phi}} \\ \widehat{\mathbf{\Psi}} \\ \widehat{\mathbf{\Omega}} \end{pmatrix} \tag{3.22}$$

³ To be more precise, to have the economy's ALM, a possibly non-rational forecast of the next time period's state of the economy should be substituted into the model in (3.3) allowing for non-rational expectations. However, since the mathematical expression in (3.19) would not be affected by this, (3.19) is also the economy's ALM.

where τ is artificial time, and $(\Phi_{MSV}, \Psi_{MSV}, \Omega_{MSV})$ is the fix point of the mapping. Then, the MSV solution is expectational stable, or E-stable, meaning that the solution is characterized by least squares learnability, if the coefficient matrices $\hat{\Phi}$, $\hat{\Psi}$ and $\hat{\Omega}$ are locally asymptotically stable under (3.22). Specifically, the MSV solution is E-stable, if all eigenvalues of the following matrices have real parts less than 1 (see the Appendix for a derivation)

$$DM_{\hat{\Phi}}(\Phi_{MSV}) = \begin{pmatrix} (\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \Phi_{MSV})^{-1} \cdot \Gamma^{-1} \cdot \Lambda \\ (\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \Phi_{MSV})^{-1} \cdot \Gamma^{-1} \cdot \Theta \end{pmatrix}' \otimes \quad (3.23)$$

and

$$\begin{cases} DM_{\hat{\Psi}}(\Phi_{MSV}) = (\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \Phi_{MSV})^{-1} \cdot \Gamma^{-1} \cdot \Theta \\ DM_{\hat{\Omega}}(\Phi_{MSV}) = (\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \Phi_{MSV})^{-1} \cdot \Gamma^{-1} \cdot \Theta \cdot \rho \end{cases} \quad (3.24)$$

Note the correspondence with Proposition 10.3 in Evans and Honkapohja (2001).

Be aware that Φ_{MSV} determines Ψ_{MSV} and Ω_{MSV} (see the last two equations in (3.20)). Moreover, since there are multiple solutions of the equation that determines Φ_{MSV} (see the first equation in (3.20)), there are multiple solutions of the model in (3.3). This means that we have to plug in each Φ_{MSV} into the coefficient matrices in (3.23)–(3.24), and, in each case, investigate whether all eigenvalues have real parts less than 1.

However, we will not do this exercise since we are only interested in the behavior of the economy when there is a unique REE. This simplifies the analysis considerably since it is shown in McCallum (2007) that for a general class of linear rational expectations models, a determinate solution is E-stable when the dating of expectations is time period t . Thus, the regions in the figures in Section 3.2 are not only regions for determinacy, but also regions for least squares learnability of the unique REE.⁴

4 Conclusions

We have embedded different instrument rules into Galí and Monacelli's (2005) new Keynesian model for a small open economy that was augmented with technical trading in currency trade to examine the prerequisites for monetary policy. Specifically, conditions for a determinate and least squares learnable REE were in focus. When a contemporaneous data specification of the rule is used in policy-making, the degree of trend following in currency trade does not affect these conditions, except in case of an extensive use of trend following, whereas a forward expectations specification makes it less likely to have a determinate and learnable REE when the degree of trend following is increasing. We allowed for interest rate inertia in the analysis.

⁴ When the dating of expectations is time period $t - 1$, a determinate solution does not have to be E-stable. See footnote 18 in McCallum (2007) for an example.

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Appendix

Derivation of (2.1)

The Galí and Monacelli (2005) model can after extensive derivations be reduced to a dynamic IS-type equation and a new Keynesian Phillips curve

$$\begin{cases} x_t = x_{t+1}^e - \alpha (r_t - \pi_{H,t+1}^e - \bar{r}) \\ \pi_{H,t} = \beta \pi_{H,t+1}^e + \gamma x_t \end{cases} \quad (\text{A.1})$$

where $\pi_{H,t}$ is the domestic inflation rate. However, (A.1) is not in an appropriate form since there are no expected exchange rate terms in the equations. These terms are necessary when incorporating the expectations formations in currency trade that are presented in Section 2.2. Fortunately, it is possible to use the following equations, which are derived in Galí and Monacelli (2005), to rewrite (A.1) into a suitable form

$$\begin{cases} \pi_t = \pi_{H,t} + \delta \Delta s_t \\ s_t = e_t + p_t^* - p_{H,t} \end{cases} \quad (\text{A.2})$$

where s_t is the terms of trade, p_t^* is the index of foreign goods prices, and $p_{H,t}$ is the index of domestic goods prices. Specifically, shift the first equation in (A.2) one time period forward in time, and rearrange terms

$$\pi_{H,t+1}^e = \pi_{t+1}^e - \delta \Delta s_{t+1}^e \quad (\text{A.3})$$

Thereafter, shift the second equation in (A.2) one time period forward in time, and take differences

$$\Delta s_{t+1}^e = \Delta e_{t+1}^{e,m} + \Delta p_{t+1}^{e,*} - \Delta p_{H,t+1}^e = \Delta e_{t+1}^{e,m} + \pi_{t+1}^{e,*} - \pi_{H,t+1}^e \quad (\text{A.4})$$

Then, substitute (A.4) into (A.3), and solve for $\pi_{H,t+1}^e$

$$\pi_{H,t+1}^e = \frac{1}{1-\delta} \cdot (\pi_{t+1}^e - \delta (\Delta e_{t+1}^{e,m} + \pi_{t+1}^{e,*})) \quad (\text{A.5})$$

and shift (A.5) one time period backward in time

$$\pi_{H,t} = \frac{1}{1-\delta} \cdot (\pi_t - \delta (\Delta e_t + \pi_t^*)) \quad (\text{A.6})$$

Thereafter, substitute (A.5) into the first equation in (A.1), and the first equation in (2.1) is derived. Finally, substitute (A.5)–(A.6) into the second equation in (A.1), solve for π_t , and the second equation in (2.1) is derived.

Derivation of (3.3)–(3.7)

Firstly, substitute market expectations at the currency market in (2.2) into the dynamic IS-type equation in the first equation in (2.1), and rearrange terms

$$\begin{aligned} & x_t + \frac{\alpha \delta \omega}{1-\delta} \cdot \Delta e_t + \alpha r_t \\ &= x_{t+1}^e + \frac{\alpha}{1-\delta} \cdot \pi_{t+1}^e + \frac{\alpha \delta (\omega - 1)}{1-\delta} \cdot \Delta e_{t+1}^e + \frac{\alpha \delta}{\delta - 1} \cdot \pi_{t+1}^{e,*} + \alpha \bar{r} \end{aligned} \quad (\text{A.7})$$

Secondly, substitute market expectations at the currency market in (2.2) into the new Keynesian Phillips curve in the second equation in (2.1), and rearrange terms

$$\begin{aligned} & \gamma(\delta - 1)x_t + \pi_t + \delta(\beta\omega - 1)\Delta e_t \\ &= \beta\pi_{t+1}^e + \beta\delta(\omega - 1)\Delta e_{t+1}^e + \delta(\pi_t^* - \beta\pi_{t+1}^{e,*}) \end{aligned} \quad (\text{A.8})$$

Thirdly, substitute market expectations at the currency market in (2.2) into the UIP condition in the third equation in (2.1), and rearrange terms

$$-\omega\Delta e_t + r_t = (1 - \omega)\Delta e_{t+1}^e + r_t^* \quad (\text{A.9})$$

Fourthly, rearrange the terms in the Taylor rule in (3.1)

$$-\zeta_x x_t - \zeta_\pi \pi_t - \zeta_e \Delta e_t + r_t = \zeta_r r_{t-1} \quad (\text{A.10})$$

Finally, put (A.7)-(A.10) into matrix form, where

$$\Xi = \left[\frac{\alpha\delta}{\delta - 1} \cdot \pi_{t+1}^{e,*}, \delta(\pi_t^* - \beta\pi_{t+1}^{e,*}), r_t^*, 0 \right]' \quad (\text{A.11})$$

and

$$\Pi = [\alpha, 0, 0, 0]' \quad (\text{A.12})$$

in (3.3), and the derivations are completed.

Derivation of (3.3) and (3.6)–(3.9)

Rearrange the terms in the Taylor rule in (3.2)

$$r_t = \zeta_x x_{t+1}^e + \zeta_\pi \pi_{t+1}^e + \zeta_e \Delta e_{t+1}^e + \zeta_r r_{t-1} \quad (\text{A.13})$$

Thereafter, put (A.7)–(A.9) and (A.13) into matrix form, where the coefficient matrices Ξ and Π in (3.3) are given by (A.11)–(A.12), respectively, and the derivations are completed.

Derivation of (3.23)

Making use of Magnus and Neudecker (1999), the differential of the first element in the mapping in (3.21) with respect to $\widehat{\Phi}$ is

$$\begin{aligned} d\mathbf{M}_{\widehat{\Phi}} &= - \left(\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \widehat{\Phi} \right)^{-1} \cdot d \left(\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \widehat{\Phi} \right) \cdot \\ &\quad \left(\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \widehat{\Phi} \right)^{-1} \cdot \Gamma^{-1} \cdot \Lambda \\ &= - \left(\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \widehat{\Phi} \right)^{-1} \cdot \left(-\Gamma^{-1} \cdot \Theta \cdot d\widehat{\Phi} \right) \\ &\quad \left(\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \widehat{\Phi} \right)^{-1} \cdot \Gamma^{-1} \cdot \Lambda \end{aligned} \quad (\text{A.14})$$

or

$$dvec\mathbf{M}_{\widehat{\Phi}} = \left(\begin{array}{c} \left(\left(\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \widehat{\Phi} \right)^{-1} \cdot \Gamma^{-1} \cdot \Lambda \right)' \otimes \\ \left(\mathbf{I} - \Gamma^{-1} \cdot \Theta \cdot \widehat{\Phi} \right)^{-1} \cdot \Gamma^{-1} \cdot \Theta \end{array} \right) \cdot dvec\widehat{\Phi} \quad (\text{A.15})$$

or

$$DM_{\hat{\Phi}} = \left(\left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \hat{\Phi} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \mathbf{\Lambda} \right)' \otimes \left(\left(\mathbf{I} - \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \cdot \hat{\Phi} \right)^{-1} \cdot \mathbf{\Gamma}^{-1} \cdot \mathbf{\Theta} \right) \quad (\text{A.16})$$

where the derivative at $\hat{\Phi} = \Phi_{MSV}$ is of interest, and the derivation is completed.

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