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an Investment,
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Owner-occupied Housing as an Investment, Regional House Price Cycles and Residential Sorting*

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Abstract

We develop a dynamic multi-region model, with fluctuating regional house prices, where an owner-occupied household's location choice depends on its current wealth and its current type and involves both consumption and investment considerations. The relative strength of the consumption motive and the investment motive in the location choice determines the equilibrium pattern of residential sorting, with a strong investment (consumption) motive implying sorting according to the type (wealth). The model predicts a negative relation between the size of house price fluctuations and the degree of residential sorting in the type dimension. Also, movers should be more sorted than stayers in the type dimension. These predictions are consistent with evidence from US metropolitan areas when income, education and age are used as proxies for household type.

Keywords: Residential sorting, House prices, Consumption motive, Investment motive, Incomplete markets, Household mobility

JEL Classification: D52, G11, R13, R21, R23

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1 Introduction

When a household buys a home in a certain location, the choice it makes has major implications for the composition of its wealth portfolio. If the household buys a home in an expensive area, it has less net wealth left to allocate to other assets. Also, regional house prices fluctuate over time, and as investments, different houses and locations offer different prospects. The success of today's investment in a house will, in part, determine what kind of home the household will be able to buy in the future.

This paper studies how the investment aspect of housing affects households' location choices, the socioeconomic make-up of local jurisdictions, and the pattern of residential sorting. In particular, we address the following question: If investment considerations and regional house price dynamics play a role in households' location choices, does this make local jurisdictions internally more or less homogenous, and externally more or less distinct from each other?

The double nature of housing as a consumption good and as an investment is a central theme in housing economics. Questions addressed in this literature include the role of investment considerations in housing choices, and the interaction between housing investments and investments in other assets.¹ However, the implications for the socioeconomic make-up of local jurisdictions are not examined in this branch of research.

The aggregate spatial implications of households' housing and location choices is a key theme in the urban and regional economics literature that studies residential sorting.² In the sorting models households' location choices typically involve a trade-off between, say, a better quality of public goods and amenities in a desirable location, and lower housing costs in a less desirable location. Then in equilibrium households with a high income and / or a strong taste for the public goods and amenities tend to live in the desirable areas. In many of these models also the quality of local public goods is endogenously

¹See e.g. Ranney (1981), Henderson and Ioannides (1983, 1987), Poterba (1984), Brueckner (1997), Flavin and Yamashita (2002), Cocco (2005), Sinai and Souleles (2005), Li and Yao (2007), and Bayer, Ellickson and Ellickson (2010).

²See e.g. Ellickson (1971), Eberts and Gronberg (1981), Epple, Filimon and Romer (1984), Epple and Romer (1989, 1991), Henderson (1991), Wheaton (1993), Epple and Platt (1998), Epple and Sieg (1999), Kessler and Lulfesman (2005), and Bayer and McMillan (2011). For a survey see Ross and Yinger (1999).

determined: people vote in local elections, as well as with their feet. However, since the sorting literature typically uses static general equilibrium models, it largely abstracts from the investment aspect of housing.

In this paper we develop a dynamic multi-region model, with fluctuating regional house prices, where both the consumption aspect and the investment aspect of housing affect households' location choices and the pattern of residential sorting. In the model, some locations are more desirable and popular than others, in the sense that the majority of households derives a higher utility from residing in a desirable location. In each period a household chooses its housing location based on its current wealth and its current "match," or current type (we use the terms match and type interchangeably). Empirically, the type may be interpreted as reflecting various socioeconomic characteristics (household size, the age of household members, education etc.) that affect the household's preferences and location choices. Current wealth, on the other hand, determines whether the household can afford a house in a desirable location.

Residential sorting takes place in two dimensions, wealth and the match. The pattern of sorting that emerges in equilibrium depends on the relative strength of the consumption motive and the investment motive of housing. If the consumption motive is strong, the households mainly care about current benefit streams and seek to live in a desirable location in each period. However, some of them cannot do so, because they are borrowing constrained. Then the equilibrium pattern of residential sorting boils down to differences in wealth, with wealthy households residing in desirable locations and less wealthy households living in less desirable locations. A household's current wealth depends, in part, on its past fortunes in the housing markets: the capital gains or losses the household has made. Thus, when combined with a strong consumption motive in location choice, the asset aspect of housing (i.e., housing market related wealth dynamics) gives rise to residential sorting according to wealth, rather than household type.

However, there is also a second effect, that works in the opposite direction. When choosing its housing location a household may think of the house as an investment and as a stepping stone to a future home. In particular, buying an expensive home in a currently popular location is a risky investment. In the future, when the household may

want to resell its home, the location may no longer be so popular, and the previously expensive house may have lost (a part of) its value. If these investment considerations play an important role in households' location choices, only households with a truly good current match with (i.e., a strong preference for) a desirable location choose to buy a house there. Thus a strong investment motive in location choice results in sorting according to household type.

In sum, the more the households care about their expected future wealth and the resale value of the home, the less the housing market related wealth dynamics and the aggregate wealth distribution mould the pattern of residential sorting. Thus there is an inverse relation between the importance of investment considerations at the household level and the importance of the wealth aspect of housing at the aggregate level.

The main empirical prediction of our model links the degree of sorting according to household type to the size of regional house price fluctuations: these two should be negatively correlated. Using income, age and education as proxies for household type, we provide some empirical evidence from US metropolitan areas in support of this prediction. In addition to the main empirical prediction, the model predicts that among owner-occupied households, movers should be more sorted in the type dimension than stayers. Finally, there should be a non-linear relation between wealth and mobility, so that households with intermediate wealth levels are more mobile than the poor and the wealthy. We present some evidence in support of these predictions as well.

Our paper is closest to the recent study by Ortalo-Magné and Rady (2008). In a two-period framework, Ortalo-Magné and Rady examine tenure choice and income heterogeneity in booming cities, where house prices rise, and home-owners, who make capital gains, may choose to stay put, even when newcomers typically earn higher incomes. In this model setup the asset aspect of housing (the capital gains of home-owners) decreases the degree of income sorting. The empirical results of the paper indicate that in locations that have experienced strong growth in house prices, home-owners who have recently moved in have higher incomes than their neighbors. Furthermore, in these locations, there is a positive correlation between the dispersion of home-owners' incomes and the dispersion of the times since they bought their homes. Also the related work by Ortalo-Magné and

Rady (2006) on house price dynamics and housing choices shares common themes and features with our paper, although here households choose between different apartment types (“flats” and “houses”) rather than between different locations. In particular, in Ortalo-Magné and Rady (2006), as well as in our paper, capital gains and losses made in the housing market are the key driver of household wealth dynamics, and borrowing constraints may limit the set of feasible housing options.

There are also a number of other papers that analyze sorting in a dynamic framework. Glaeser and Gyourko (2005) study the joint process of falling house prices and neighborhood change in declining cities. Due to the durability of housing, a negative shock leads to a sharp fall in housing prices, but only a slow and gradual decline in city size. Low housing costs in a city attract low-income households. In the model, however, households are assumed to be renters, so that investment considerations and realized capital losses do not affect residential location choices. Also in the earlier dynamic models of Bénabou (1996) and Fernandez and Rogerson (1996), (who study sorting, the quality of local schools and the accumulation of human capital) households are assumed to be renters, and they choose their location once and for all (in the first period), so that the wealth aspect of housing does not shape the equilibrium pattern of residential sorting.

Finally, the themes of the paper are related to recent empirical findings in the residential sorting literature. In particular, evidence from the US indicates that local jurisdictions are internally more heterogeneous, and less distinct from each other, than many standard theories of residential sorting predict: In a typical American neighborhood, neighbors tend to differ significantly in terms of income, age and education (Ioannides 2004). Furthermore, heterogeneity across US municipalities and counties, measured with respect to income and a number of other socioeconomic variables (including age, education, race, nativity, religion, owner-occupancy rate and party vote shares in presidential elections) has not increased over time, despite falling migration costs, which should have made sorting easier (Rhode and Strumpf 2003). Finally, while the extent of sorting is generally quite small, it also varies widely across metropolitan areas (Davidoff 2005). The predictions of the present paper may help in explaining a part of these observations as well.

The plan of the paper is as follows. The basic model is developed in Section 2. Section 3

shows how the equilibrium pattern of residential sorting reflects the relative strength of the consumption motive and the investment motive of housing. The section also establishes a link between the size of house price fluctuations and the pattern of residential sorting, and analyzes the degree of sorting among movers and stayers. Section 4 develops some extensions to the basic model. Empirical evidence is presented in Section 5. Section 6 concludes.

2 The model

2.1 Some background and motivation

We develop a dynamic model of residential sorting, based on the following main ideas: (i) For owner-occupied households, housing is both a consumption good and an asset, and residential location choices involve not only consumption but also investment considerations; essentially, expected resale value matters. (ii) Regional house prices fluctuate, and the capital gains and losses made in the housing market play an important role in determining how a household's wealth evolves over time. (iii) Borrowing constraints may narrow the set of feasible housing options, and impair a household's ability to move.

It is natural to include these elements in a framework which tries to understand households' location choices and residential sorting. In most developed countries, owner-occupied housing is the single most important investment for a typical household. For example, in the late 1990's (i.e. before house prices ballooned in the housing boom of the early and mid 2000's), single family owner-occupied housing composed 2/3 of household wealth in the UK, 1/3 of household wealth in the US, and 2/3 of the assets of a US household with median wealth.³ Given the importance of housing as an asset, it is reasonable to assume that investment considerations may also play a role when people choose where to buy a home. One simple way to motivate this assumption is to conduct an internet search. Our Google search with key words "location", "home" and "resale value" produced over six million hits, with phrases such as "Buying a home with a resale

³Banks et al. (2002), *Federal Reserve's 2001 Survey of Consumer Finances*.

value: location, location, location” abounding.

Second, house prices are often highly volatile, and in different regions property values tend to rise and fall asynchronously, so that relative regional prices may vary considerably over time. Figures 1 and 2 illustrate this finding with price data from UK regions and US metropolitan areas.⁴ Relative prices can fluctuate significantly even at a more local level. In London, for example, the borough of Greenwich was 3% more expensive than the borough of Hackney in 1995, but in 2001 prices were 20% higher in Hackney than in Greenwich⁵; see also Iacoviello and Ortalo-Magné (2003). For similar findings on the Boston metropolitan area, see Case and Mayer (1996).

The capital gains and losses made in the housing market can be remarkably large in comparison with typical household incomes and savings, and empirical studies reveal that falling home equity value may seriously constrain a household’s ability to move.⁶ To illustrate the size of the wealth shocks, Table 1 shows maximum and minimum house-price-to-income ratios in four major US cities over the period 1979-1996. In the UK, the average annual capital gain in the London market between 1983 and 1988 corresponded to 72% of the mean annual disposable household income in the UK over that period, and exceeded by the factor of 7.8 average yearly household savings. Between 1989 and 1992, the annual capital loss of a typical London homeowner was equivalent to 77% of average disposable household income, and 8.4 times average household savings.

As a general rule, these housing market risks are uninsurable. Shiller (1993, 2003), for example, lists home equity insurance as one of the key financial markets currently missing.^{7,8} Nevertheless, location choices and the timing of transactions can affect the

⁴According to Shiller (1993, Ch 5 p. 79) real estate booms and busts in US cities have been regionally asynchronized and prize movements often dramatic. Del Negro and Otrok (2007) find that, with the exception of the boom of the early 2000s, US house price dynamics have been mainly driven by local or regional, rather than national, shocks. For further evidence on US prices, see also Case and Shiller (1989), Malpezzi (1999), Capozza et al (2004) or Himmelberg, Mayer and Sinai (2005). For British evidence, see Muellbauer and Murphy (1997), or Cook (2003).

⁵Source: Land Registry, <http://www.landreg.gov.uk>.

⁶See Chan (1996, 2001), Ferreira, Gyourko and Tracy (2010, 2011), Henley (1998).

⁷Shiller (1993), and Shiller and Weiss (1999) discuss the potential problems, both economic and psychological, involved in providing hedging against house price swings, as well as ways to overcome these problems. See Shiller (1993, 2003), and Iacoviello and Ortalo-Magné (2003) for discussion on some real life experiments in the US and the UK.

⁸While CDSs on MBSs could in principle provide protection against deteriorating (local) housing market conditions, these products have been designed for large-scale institutional investors and they

distribution of risks that a household faces. While house price fluctuations include an important random component, they also display certain regularities. In particular, regional house prices tend to exhibit mean-reversion in time horizons of one year and longer; possible explanations include lags in housing construction, mean-reversion in underlying economic fundamentals, and the interaction of borrowing constraints and wealth effects, which gives rise to temporary overshooting of prices.⁹ There is also some evidence on long-run equilibrium relationships between house prices in different areas: if prices in a particular location are currently above the equilibrium level, they are likely to fall, in relative terms, some time in the future; if relative prices are above the equilibrium level, the opposite is likely to happen.¹⁰

2.2 The basics of the economy

The economy has two locations. Each location has an equal, fixed, stock of identical houses. Each house is occupied by a single household and no one household is ever homeless. All households are owner-occupiers and there is no rental housing. For convenience, assume that the stock of houses and the mass of households each comprises a continuum of size unity.

There are infinite discrete time periods indexed by $t = 0, 1, \dots$. In each period, one of the locations is deemed to be “desirable” while the other one is “less desirable”. When a period changes, the relative ranking of the locations is reversed with probability $\pi \in (0, 1)$.¹¹

have not been available to ordinary households. Also, arguably CDSs could not even in principle provide protection to all households: for the owner of a CDS to receive a payment, some households have to be unable to service their mortgage.

⁹See Glaeser and Gyourko (2007), Ortalo-Magné and Rady (2006), Evenson (2003), Lamont and Stein (1999).

¹⁰That is, regional house prices are cointegrated. For evidence from British regions, see MacDonald and Taylor (1993), Alexander and Barrow (1994) or Cook (2003). For evidence from US census regions, as well as for a comparison between the US and the UK, see Meen (2002). More recent econometric analyses are also consistent with regional house price cycles. For example, Holly, Pesaran and Yamagata (2011) find that house price shocks to dominant UK regions, such as London, are propagated across other dominant regions, but have impact on other regions with a delay. Furthermore, lagged effects are found to echo back to the dominant regions. These findings are consistent with our assumption that house prices fluctuate asynchronously across regions. For related evidence on the US, see Holly, Pesaran and Yamagata (2010).

¹¹The regional shock may reflect e.g. altering labor market conditions, changes in the supply of public

We also consider a small region interpretation of the model, with a continuum of locations. Then in each period, one half of the locations are “desirable” while the remaining locations are “less desirable”, and when a period changes, a measure π of the locations is hit by a regional shock. The long-run equilibrium of the model is essentially identical under both interpretations. Section 4 shows that the small region (interpretation of the) model can be generalized to cover the asymmetric case where the shares of desirable and undesirable locations are not equal (they differ from 50/50).

The households differ in the utility premium they derive from residing in the desirable location. The household specific component of the premium is captured by the match, θ : a high realization of θ implies a good match with the currently desirable location, while a low (negative) realization implies a good match with the less desirable location.¹² The aggregate heterogeneity of households is unchanged over time, and θ has a stationary distribution, with a cumulative distribution function $G(\theta)$, on some support $[\theta_L, \theta_H]$. Without loss of generality, we assume that the median match $\theta_m = 0$, i.e. $G(0) = \frac{1}{2}$.

A household with current match θ receives per period utility $\frac{1}{2}\varepsilon + \theta$, when living in the currently desirable location. The per period utility of anyone household living in the less desirable location is $-\frac{1}{2}\varepsilon$. Here the parameter $\varepsilon > 0$ measures regional welfare differences. ε also gauges the size of regional shocks: if a location is hit by a shock, the utility stream it offers to the (median) household changes from $\frac{1}{2}\varepsilon$ to $-\frac{1}{2}\varepsilon$, or vice versa.

Given these assumptions, all households with a current match $\theta > -\varepsilon$ derive a positive utility premium from residing in the desirable location. The measure of these households is $1 - G(-\varepsilon) > \frac{1}{2}$. In particular, if $\theta_L > -\varepsilon$ and $G(-\varepsilon) = 0$, all households would rather live in the popular area. Since the measure of houses in the desirable location is one half, housing is in short supply in the popular region.

A household’s match may change over time. First, if the neighborhood or jurisdiction goods and services, or the evolution in the tastes and the needs of the population. Alternatively, the resulting house price dynamics may be interpreted as reflecting (in a reduced form) the interaction between housing demand and supply. According to this interpretation, an area is currently expensive, because housing supply has not yet increased to absorb a positive demand shock.

¹²As will become clear below, even households with low realizations of θ may derive a positive premium from the desirable location. However, even if this is the case, households with low θ lose less if they reside in the undesirable location than households with higher realizations of θ .

where the household resides is hit by a regional shock, the match between the household and the location is broken, and a new match is independently drawn from the distribution function $G(\theta)$.¹³ Second, even if the overall popularity of the jurisdiction remains unaltered, between periods the match may change for some idiosyncratic, or household specific, reason¹⁴, with probability $\lambda \in [0, 1]$, and the new match is independently drawn from the distribution $G(\theta)$. As discussed in Section 4, the assumption of independent draws can be dropped: the main results of the paper generalize to the case where the match is allowed to follow a general Markov process.

Finally, the households live forever and discount future utilities by a common factor $\beta \in (0, 1)$.

In any period, the aggregate welfare is maximized, if all households with $\theta > \theta_m = 0$ are allocated to the (currently) desirable location, those with $\theta < 0$ live in the less desirable location, and the group (always of measure zero, if G is continuous) with $\theta = 0$ is divided between the locations so that capacity constraints on housing are not violated. In other words, there is perfect sorting according to the match.

2.3 Wealth dynamics

In the market outcome, the location choice depends on not only the match, but also on wealth. In this section, we study how a household's wealth evolves over time.

A household cannot sell a home without buying another one, and vice versa.¹⁵ We choose the minimum level of housing wealth as the origin and fix the value of a cheap home to 0. We also normalize the house price in a popular location to 1. This normalization means that house price swings are always of size unity. However, we shall below show

¹³An underlying premise is that a location which was popular (unpopular) in period t and another location which is popular (unpopular) in period $t + 1$ are likely to be “desirable” (“undesirable”) in different ways; thus it is plausible to assume that the match that the household had with the period- t desirable (undesirable) location does not carry over to the period- $(t + 1)$ desirable (undesirable) area.

¹⁴The match changes for similar reasons as in the search models by Wheaton (1990, 1993) and Williams (1995). Examples include change of household size or educational status and evolution in tastes when members of the household age.

¹⁵This follows from our basic assumptions: (i) no household can be homeless (being homeless would result in very large negative utility), (ii) there is no rental housing, and (iii) the measure of homes equals the measure of households.

how their magnitude can be measured in a meaningful way, by comparing them with the value of financial assets, and with average household wealth.

Consistent with empirical evidence, we assume that capital gains and losses made in the housing market are uninsurable.¹⁶ The incomplete markets setting we consider here is the simplest possible one. In addition to owning a home, the households can carry wealth to the future by holding a single risk-free, non-interest bearing financial asset, which can be interpreted as outside money. The real supply of money is M/p , where M is the fixed nominal supply, and $1/p$ is the price of money, in terms of housing (in desirable locations).¹⁷

Denote financial asset holdings by a and let h be housing. h is equal to 1, if the household owns a house in a desirable location, and equal to 0, if the house is in an undesirable location. We also define a household's total wealth (n), which consists of both financial wealth (money) and housing wealth

$$n_t = a_t + h_t. \tag{1}$$

In any given period t , the household's budget constraint is

$$h_t + a_t = a_{t-1} + (1 - s_t)h_{t-1} + s_t(1 - h_{t-1}), \tag{2}$$

where s_t is an indicator function which is equal to 1 if there is a regional shock between periods $t - 1$ and t , and 0 otherwise. Combining (1) and (2) yields

$$n_{t+1} = n_t + s_{t+1}(1 - 2h_t). \tag{3}$$

The household's wealth position (n) changes if and only if the household makes a capital gain or suffers a capital loss in the housing market. This stark way to model wealth dynamics is motivated by the observation that wealth shocks realized in the housing market can be remarkably large compared with typical household incomes and savings.

¹⁶Clearly, also changes in the "match" are uninsurable.

¹⁷We could also easily introduce pure credit, or inside money, and allow the households to borrow up to a certain limit without changing any of the results. See the discussion at the end Section 2.5.

If, prior to the regional shock, the household owned a property in a then unpopular location, ($h_t = 0$) the household makes a capital gain and climbs one rung in the wealth ladder; if the house was in an expensive area ($h_t = 1$) before the change of fortunes, the household suffers a loss and falls one rung down.

There is a lower limit for financial asset holdings a_{\min} , that a household is not allowed to exceed. A simple and fairly natural normalization is adopted here by fixing the minimum balance to be zero, $a_{\min} = 0$, but allowing a negative minimum balance would just involve a change of origin, without altering the analysis or any of the results.¹⁸ Since the minimum wealth level is $n = 0$ (the minimum level of housing wealth is 0, and the minimum level of financial asset holdings is 0) and since households make capital gains and losses of size unity, we can now assume, without loss of generality, that wealth only takes non-negative integer values $n = 0, 1, 2, \dots$. At wealth levels $n \geq 1$, a household may freely choose its housing location, and its wealth portfolio may consist of n units of financial assets and a cheap house ($h = 0$), or $n - 1$ units of financial wealth and an expensive home ($h = 1$). If $n = 0$, the household owns a house in an undesirable location, $h = 0$, and since it has no money, $a = a_{\min} = 0$, it cannot afford a house in a desirable location: choosing $h = 1$ would imply $a = -1 < a_{\min}$, and this is not allowed. The borrowing constraint that limits a household's location choices can be expressed as follows:

$$h_t = 0 \text{ if } n_t = 0. \quad (4)$$

2.4 The household's problem

At each time t a household chooses its location $h_t \in \{0, 1\}$ so as to maximize the expected discounted utility stream

$$E_\theta \sum_{t=0}^{\infty} \beta^t \left[h_t \left(\frac{1}{2} \varepsilon + \theta_t \right) - (1 - h_t) \frac{1}{2} \varepsilon \right],$$

¹⁸This is because the interest rate is zero. See Aiyagari (1994) or Ljungqvist and Sargent (2004, Ch. 17.10). See also the discussion at the end of Section 2.5.

subject to (3) and (4). The problem can be conveniently presented in a recursive form. Let $V(\theta, n)$ be the (ex post) value function of a household with current type θ and current wealth n . Also define the household's ex ante value function $V(n) = E_\theta [V(\theta, n)]$, which describes the household's expected prospects when the household faces a shock (idiosyncratic or regional) and does not yet know its new match. The value function $V(\theta, n)$ satisfies the Bellman equation

$$V(\theta, n) = \max_{h \in \{0,1\}} h \left(\frac{1}{2}\varepsilon + \theta \right) - (1-h) \frac{1}{2}\varepsilon + \beta \left\{ (1-\pi) [(1-\lambda)V(\theta, n) + \lambda V(n)] + \pi [(1-h)V(n+1) + hV(n-1)] \right\} \quad (5)$$

subject to (4). In the current period, the household's utility is $-\frac{1}{2}\varepsilon$ or $\frac{1}{2}\varepsilon + \theta$, depending on its location choice. Its prospects for the next period are discounted by β and are given inside the curly brackets. With probability $(1-\pi)(1-\lambda)$ the household is not exposed to any shocks, and it will face the same value function $V(\theta, n)$ as today. With the complementary probability $[1 - (1-\pi)(1-\lambda)]$ the match is broken and the household's prospects are captured by the ex ante value function. If the match changes for household specific reasons, the wealth of the household remains unaltered and future welfare is given by $V(n)$. If there is a regional shock, not only the match changes, but also house prices rise or fall, and depending on housing location, the household makes a capital gain or suffers a capital loss, resulting in expected future welfare $V(n+1)$ or $V(n-1)$.

At each unconstrained wealth level $n \geq 1$, the household chooses the desirable location if and only if

$$\theta + \varepsilon > \pi\beta [V(n+1) - V(n-1)]. \quad (6)$$

The condition (6) involves a useful decomposition of the decision problem into the consumption motive, figuring on the left-hand side, and the investment motive, visible on the right-hand side. The strength of the consumption motive depends on the current match θ and the measure of regional disparities ε . If there were no need to care about the future, all households with $\theta > -\varepsilon$ would choose the currently desirable region, while only those with $\theta < -\varepsilon$ would (voluntarily) live in the less popular area. The downside of choosing a currently popular and expensive location is that a household may suffer

capital losses, if regional house prices fall, and may then be borrowing constrained in the future, when the match θ with an expensive location is better than today. By contrast, opting for a currently less popular and less expensive area entails the chance of making capital gains. These considerations are captured by the investment motive. Due to the investment motive, even some households with $\theta > -\varepsilon$, i.e. households whose immediate benefits are higher in the desirable location, may voluntarily choose the unpopular area.

At each wealth level n , there is then a critical value of the match

$$\theta_n^* = \begin{cases} \theta_H & \text{if } n = 0 \\ -\varepsilon + \pi\beta [V(n+1) - V(n-1)] & \text{if } n \geq 1 \end{cases} \quad (7)$$

and the household's location choice rule assumes a simple threshold form:

$$h(\theta, n) = \begin{cases} 1 & \text{if } \theta > \theta_n^* \\ 0 & \text{if } \theta \leq \theta_n^* \end{cases} \quad (8)$$

Figure 3 shows the critical match θ_n^* with different values of n when θ is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\varepsilon = 1$, $\beta = .95$, and $\pi = .3$. Clearly, θ_n^* decreases with n , and wealthier households are ready to choose the desirable location even with a more modest match. This is a general property of θ_n^* , and it stems from the fact that the ex ante value function is concave. (Concavity is proved in the appendix.) Also, this finding has a natural interpretation. Assets are valued since they provide the option to make unconstrained choices in the future. However, if a household is wealthy, additional assets are of less value: the more assets the household has, the more distant is the prospect of being borrowing constrained at some point in the future. To put it differently, the investment motive is more important for poor households than for wealthy households.

The appendix shows that at very high wealth levels, the investment motive all but vanishes, and as a consequence $\lim_{n \rightarrow \infty} \theta_n^* = -\varepsilon$. That is, the majority of sufficiently wealthy households live in expensive locations. This property is needed, when we establish the equilibrium of the model. In particular, if $\theta_L > -\varepsilon$ – and all households prefer the desirable location from the consumption point of view – there is a finite wealth level \bar{n} ,

such that all households with $n \geq \bar{n}$ choose a desirable location. In Figure 3, $\theta_L = -\frac{1}{2} > -1 = -\varepsilon$, and $\bar{n} = 3$.

2.5 Equilibrium

The previous section showed how a household chooses its location, and its asset portfolio, based on its current wealth and its current match. On the other hand, a household's current wealth depends on its past fortunes in the housing market, and its past location and portfolio choices. Then the long-run wealth distribution is induced by the households' policy rule. Location choices, the stationary wealth distribution, and the relative price of housing and financial assets (p) together constitute the long-run equilibrium of the model.

Denote by $f(n)$ the size of wealth class n . Given the households' location choice rule (8),

$$f_n^0(n) = G(\theta_n^*) f(n) \quad (9)$$

is then the frequency of households at wealth level n , with a cheap home ($h = 0$) and n units of financial assets. Similarly,

$$f_{n-1}^1(n) = (1 - G(\theta_n^*)) f(n) \quad (10)$$

is the frequency of households at wealth level n , owning an expensive home ($h = 1$) and $n - 1$ units of financial assets. If there is a regional shock, all $f(n)$ households which were previously in wealth class n either go up to $n + 1$ or fall to $n - 1$, depending on their house location. They are replaced by $f_{n-1}^0(n - 1)$ class $n - 1$ households which have made a capital gain and $f_n^1(n + 1)$ class $n + 1$ households which have suffered a capital loss. The wealth distribution is stationary if and only if

$$f(n) = (1 - s) f(n) + s (f_{n-1}^0(n - 1) + f_n^1(n + 1)) \quad (11)$$

for all n , where s is an indicator variable which is 1 if there is a regional shock and 0 otherwise. We also consider the model version, with a continuum of atomistic regions. Between any periods, a measure π of the locations is hit by a regional shock, and the

wealth distribution is stationary if and only if

$$f(n) = (1 - \pi) f(n) + \pi (f_{n-1}^0(n-1) + f_n^1(n+1)). \quad (12)$$

It is easy to conclude that (11) and (12) both reduce to

$$f(n) \equiv f_n^0(n) + f_{n-1}^1(n) = f_{n-1}^0(n-1) + f_n^1(n+1). \quad (13)$$

As a consequence, both model variants have the same long-run wealth distribution.

There are no wealth classes below 0 (i.e., $f(n) = 0$ for $n < 0$) and at wealth level 0 the households can only choose an unpopular location (i.e., $f_{-1}^1(0) = 0$). These restrictions and (13) then imply the set of equations

$$f_n^1(n+1) = f_n^0(n), \quad n = 0, 1, \dots \quad (14)$$

Equations (14) imply that the distribution of financial assets is identical in both locations - or location types. This symmetry property means that in steady state the asset side of the economy, as defined by the joint distribution of housing wealth and financial wealth, looks exactly the same at the end of any given period and at the beginning of the subsequent period even if the popularity ranking of the locations is reversed.

Next, plugging (9) and (10) into (14) shows that the long-run wealth distribution is implicitly characterized by the sequence

$$f(n)/f(n-1) = \gamma(n), \quad n = 1, 2, \dots, \quad (15)$$

where

$$\gamma(n) \equiv \frac{G(\theta_{n-1}^*)}{1 - G(\theta_n^*)}. \quad (16)$$

For the stationary distribution to exist, the sequence (15) has to converge. Convergence is guaranteed since $\lim_{n \rightarrow \infty} \theta_n^* = \lim_{n \rightarrow \infty} \theta_{n-1}^* = -\varepsilon$ and

$$\lim_{n \rightarrow \infty} \frac{f(n+1)}{f(n)} = \frac{G(-\varepsilon)}{1 - G(-\varepsilon)} < 1 \quad (17)$$

The inequality (17) holds since $G(-\varepsilon) < \frac{1}{2}$: the majority of wealthy households live in desirable location. Then the long-run wealth distribution can be presented in the form

$$f(n) = \frac{\prod_{i=0}^n \gamma(i)}{\sum_{j=0}^{\infty} \prod_{k=0}^j \gamma(k)}$$

where $\gamma(0) \equiv 1$ and $\gamma(n)$, $n = 1, 2, \dots$ are given by (16).

The wealth distribution is single-peaked, with wealth classes in the middle having more mass than those on the tails, and the right tail can be approximated by a power series.¹⁹ These properties are consistent with observed empirical wealth distributions. In the hump of the wealth distribution $\gamma(n) \approx 1$,²⁰ meaning that the critical match θ_n^* tends to be relatively close to θ_m : interestingly, in the hump the households' location choice rule (summarized by θ_n^*) tends to be relatively close to the socially optimal rule, while in the tails location choices deviate more from the socially optimal policy. (A similar property applies in the model with asymmetric locations; see Section 4.1.)

The shape of the wealth distribution and the pattern of location choices in different parts of the distribution both follow from the fact that the distribution is induced by households' location choices. Poor households tend to choose a currently cheap location, and they tend to make capital gains and move up in the distribution. On the other hand wealthy households typically choose a currently expensive location, and they are more prone to suffer capital losses. Thus wealth transitions tend to happen from the tails of the distribution (where households make socially non-optimal location choices) towards the middle (where location choices are closer to the socially optimal rule).

Households' location choices together with the endogenously arising long-run wealth distribution also guarantee that housing markets clear. Essentially, if few households willingly choose the less desirable location, in the long-run equilibrium many households

¹⁹These properties hold, since $\gamma(1) = \frac{G(\theta_0^*)}{1-G(\theta_1^*)} = \frac{1}{1-G(\theta_1^*)} \geq 1$, $\gamma(n)$ is decreasing in n and $\lim_{n \rightarrow \infty} \gamma(n) = \frac{G(-\varepsilon)}{1-G(-\varepsilon)} < 1$.

²⁰The mode of the distribution is a wealth level n_{mod} such that $\gamma(n_{\text{mod}} - 1) > 1$ and $\gamma(n_{\text{mod}}) < 1$.

end up living there because they are borrowing constrained. More formally, using the fact that $f_{-1}^1(0) = 0$, and summing both sides of (14) over all wealth classes yields $\sum_{n=0}^{\infty} f_n^0(n) = \sum_{n=0}^{\infty} f_{n-1}^1(n)$. Given that the aggregate mass of households is unity, it follows that

$$\sum_{n=0}^{\infty} f_{n-h}^h(n) = \frac{1}{2}, \quad h \in \{0, 1\}. \quad (18)$$

These equations indicate that the demand for housing, on the left-hand side, is equal to the supply of housing ($\frac{1}{2}$), in both locations, or location types.

In addition to the households' location choice rule and the wealth distribution, the third constituent of the equilibrium is the relative price of housing and financial assets, p . To solve for p , consider the asset market clearing condition $E[a] = \frac{M}{p}$, where the left-hand side is the aggregate demand for financial assets and the right-hand side is the net supply, equal to real outside money.²¹ Using (1) and the housing market equilibrium $E[h] = \frac{1}{2}$, the asset market equilibrium condition can be rewritten as $E[n] = \frac{1}{2} + \frac{M}{p}$, and the relative price of housing and financial assets is²²

$$p = \frac{M}{E[n] - \frac{1}{2}}. \quad (19)$$

Notice that p also measures the monetary size of house price fluctuations.

We end this section with a few clarifying notes. First, there appear to be three markets in the model - housing markets in the desirable location and in the undesirable location, plus the market for financial assets - but there is only one relative price, p . However, since the households have to live somewhere, in each period they actually face a single choice: whether to own a home in a desirable location or in an undesirable location (given this choice, financial asset holdings are then determined by the budget constraint (2)). Thus one relative price is enough to clear the markets.

Second, while outside money is the only financial asset in the model, we could also

²¹The equilibrium we establish here essentially resembles the equilibrium of the simple Bewley-type model considered by Ljungqvist and Sargent (2004, Ch 17.10.4), where outside money and inside money (credit) are perfect substitutes, and the interest rate is zero.

²²Notice that $\lim_{n \rightarrow \infty} \frac{n+1}{n} \frac{f(n+1)}{f(n)} = \lim_{n \rightarrow \infty} \gamma(n) = \frac{G(-\epsilon)}{1-G(-\epsilon)} < 1$. Thus the sum $E[n] \equiv \sum_{n=0}^{\infty} n f(n)$ converges, and $E[n]$ is always finite.

introduce pure credit, or inside money, and allow the households to borrow up to a certain limit, without changing any of the results. In the steady state of our simple economy, the interest rate is zero, so that inside and outside money are perfect substitutes (see e.g. Ljungqvist and Sargent (2004, Ch. 17.10)).²³ Then if the borrowing limit, denoted in monetary terms, is $-B$, the asset market equilibrium condition reads $E[a] = \frac{M+B}{p}$, and $p = (M+B) / (E[n] - \frac{1}{2})$. This condition is essentially identical to (19) (and the housing market equilibrium is exactly the same as in the basic model).

Finally notice the simple recursive structure of the equilibrium: Households' location choices and the endogenously arising wealth distribution together equilibrate the housing markets and also determine the real demand for financial assets. The relative price of financial assets p then adjusts so that the real supply of financial assets equals the demand. This simple recursive structure makes the equilibrium of the model easy to solve. This simple structure also allows us to analytically characterize the pattern of residential sorting under different circumstances.

3 Residential sorting

3.1 Consumption motive, investment motive and residential sorting

This section studies how the equilibrium pattern of residential sorting reflects the relative strength of the investment motive and the consumption motive of housing. Furthermore, we show that the size of house price fluctuations depends on these motives as well, and finally establish a relation between house price fluctuations and residential sorting.

We begin by analyzing how the relative strength of the investment motive and the consumption motive depends on the characteristics of the households, and on the envi-

²³Assume by contrast, that the interest rate is positive and only inside money is held in equilibrium. Then in any (non-degenerate) equilibrium of a pure credit economy, with zero net supply of financial assets (see e.g. Huggett (1993)) some households must have negative positions. But, since the households have no income sources outside the housing market, a household with negative initial financial asset holdings exceeds any finite debt limit with a positive probability. Thus there cannot be a stationary equilibrium with a positive rate of interest.

ronment where they operate. We focus on three aspects: the weight households give to their future welfare (β), the size (ε) of regional shocks and the frequency (π) of regional shocks.

Remember from the households' location choice rule (6) that the strength of the consumption motive is given by the term $\theta + \varepsilon$, while the investment motive is captured by the term $\pi\beta[V(n+1) - V(n-1)]$. Now we have the following proposition:

Proposition 1 *The investment motive of housing becomes stronger, compared to the consumption motive, when (i) the households become more patient (β increases), (ii) the regional shocks become more frequent (π increases), or (iii) the size of the regional shocks (ε) decreases. Then at any unconstrained wealth level $n \geq 1$ a household is more likely to choose an undesirable location. That is $\frac{d\theta_n^*}{d\beta} > 0$, $\frac{d\theta_n^*}{d\pi} > 0$ and $\frac{d\theta_n^*}{d\varepsilon} < 0$.*

Proof See the appendix. ■

If the households become more patient (β increases), they care more about their future wealth and the expected resale value of their homes. Then the investment motive becomes stronger, and at any unconstrained wealth level, a household is more willing to live in a currently undesirable location, and it needs a better match in order to choose a currently desirable location.

Likewise, an increase in the frequency of regional shocks, π , strengthens the investment motive to choose a currently less popular and less expensive location in the current period. The higher π , the more likely a household that buys an expensive house suffers a capital loss, while the more likely a household living in an unpopular area makes a capital gain.

An increase in the size of regional shocks, ε , has two effects. While larger interregional welfare differences strengthen the consumption motive to choose a desirable location in the current period, they also reinforce the incentives to accumulate assets (investment motive) since a household loses more if it faces the borrowing constraint in the future. However, since future utility losses are discounted and occur by chance, the effect on the consumption motive dominates. Hence, the larger the regional shocks are, the less likely an unconstrained household chooses a currently undesirable area.

Since the stationary wealth distribution is induced by households' location and wealth

portfolio choices, it also depends on the relative strength of the consumption motive and the investment motive. Together with Proposition 1, the following lemma is a key to understanding various subsequent results.

Lemma 1 *When the investment motive becomes stronger compared to the consumption motive (see Proposition 1), the wealth distribution shifts to the right, in the sense of first-order stochastic dominance. In particular, the size of the borrowing constrained group decreases.*

Proof Define the cumulative distribution function $F(n; \varepsilon, \pi, \beta) = \sum_{i=0}^n f(i)$. By Proposition 1, the θ_n^* -schedule shifts up when ε decreases, or when π or β increases. This then increases $\gamma(n) \equiv \frac{G(\theta_{n-1}^*)}{1-G(\theta_n^*)}$, so that by (15) the ratio $f(n)/f(n-1) = \gamma(n)$ goes up for all $n = 1, 2, \dots$. It follows that $dF(n; \varepsilon, \pi, \beta)/d\varepsilon \geq 0$, $dF(n; \varepsilon, \pi, \beta)/d\pi \leq 0$ and $dF(n; \varepsilon, \pi, \beta)/d\beta \leq 0$ for each $n = 0, 1, \dots$ ■

Notice the interplay between households' location choices and the wealth distribution in clearing the housing market under different circumstances. When the investment motive becomes stronger, at any given wealth level n households are more willing to live in a less desirable location (the θ_n^* -schedule shifts up, see Proposition 1). But then in the long-run equilibrium more households reach the higher rungs of the wealth distribution (as shown in Lemma 1), where they are more likely to choose a desirable location (since the θ_n^* -schedule is downward sloping).

Let us proceed to studying how the balance between the consumption motive and the investment motive affects social welfare. Addressing this normative issue will then allow us to characterize the pattern of sorting, since in the present model high social welfare is associated with location choices based on household type, rather than wealth.

Consider any given period. Since the households choose their location according to the threshold rule (8), the average utility at wealth level n is

$$u(n) = -G(\theta_n^*) \frac{1}{2} \varepsilon + (1 - G(\theta_n^*)) \left(\frac{1}{2} \varepsilon + E[\theta \mid \theta \geq \theta_n^*] \right).$$

Summing over all wealth classes, and using the housing market equilibrium (18), yields

the overall welfare in any given period:

$$w = \sum_{n=0}^{\infty} f(n) u(n) = \sum_{n=1}^{\infty} f_{n-1}^1(n) E[\theta \mid \theta \geq \theta_n^*] = \frac{1}{2} E[\theta \mid h = 1]. \quad (20)$$

That is, the average quality of the match in the desirable location, $E[\theta \mid h = 1]$, is a measure of social well-being. Notice that w does not depend directly on the parameters β , π and ε ; there is only an indirect link, through households' location choices (Proposition 1, Lemma 1).

The next two propositions describe how social welfare and sorting in the match dimension depend on the relative strength of the investment motive and the consumption motive in location choice.

Proposition 2 *When the investment motive becomes stronger compared to the consumption motive (see Proposition 1), social welfare increases. That is $\frac{dw}{d\beta} > 0$, $\frac{dw}{d\pi} > 0$, $\frac{dw}{d\varepsilon} < 0$.*

Proof See the appendix. ■

Proposition 3 *When the investment motive becomes stronger compared to the consumption motive (see Proposition 1), the degree of residential sorting in the match dimension increases in the following sense. (a) In each location $h \in \{0, 1\}$, the average match $E[\theta \mid h]$ becomes more distinct from the economywide average $E[\theta]$. (b) The locations become more distinct from each other and the between-locations variance of the match increases. (c) The locations become internally more homogenous in the sense that the within-location variance of the match decreases.*

Proof When the investment motive becomes stronger, compared to the consumption motive, it follows from Proposition 2 and eq. (20) that $E[\theta \mid h = 1]$ increases. (a) Then, since $\frac{1}{2}E[\theta \mid h = 1] + \frac{1}{2}E[\theta \mid h = 0] = E[\theta]$, and $E[\theta]$ is a constant, it follows that $E[\theta \mid h = 0]$ decreases. Thus the difference $|E[\theta \mid h] - E[\theta]|$ increases for $h \in \{0, 1\}$. (b) Item (a) implies that the between-locations variance $Var(E[\theta \mid h]) = \frac{1}{2}(E[\theta \mid h = 0] - E[\theta])^2 + \frac{1}{2}(E[\theta \mid h = 1] - E[\theta])^2$ increases. (c) The economywide variance of the match $Var(\theta)$ can be decomposed $Var(\theta) = Var(E[\theta \mid h]) + E[Var(\theta \mid h)]$. Since $Var(\theta)$ is a con-

stant, it follows from item (b) that the within-locations component $E[\text{Var}(\theta | h)]$ must decrease. ■

To understand these results, recall that the basic allocation problem in the economy arises since there is not enough housing capacity in desirable locations to accommodate all households with a positive utility premium. There are two main ways in which this problem can be solved: self-selection according to household type (and the size of the utility premium), and borrowing constraints that prevent some households from living in an expensive area.

When the investment motive is strong, and the households care a lot about their future prospects, many households willingly choose a currently less desirable location. Then housing markets are mainly cleared through self-selection, which results in a high degree of sorting according to household type, and high social welfare. When the consumption motive dominates, few households willingly choose a less desirable location, and in equilibrium wealth determines who lives where. A household's current wealth reflects its past fortunes in the housing market, rather than some inherent characteristics of the household. Thus in equilibrium there is little sorting according to household type, and the level of social welfare is low.

Let us have a closer look at sorting in the wealth dimension. Above we noted that the distribution of financial assets is identical in both location types; see eq. (14). Then interregional wealth differences derive entirely from different house values. This applies to average wealth in different locations as well as to the quantiles of regional wealth distributions

$$E[n | h = 1] - E[n | h = 0] = E[h | h = 1] - E[h | h = 0] = 1 \quad (21)$$

$$n_q^1 - n_q^0 = 1, \quad (22)$$

where n_q^h is the q th wealth quantile in region $h \in \{0, 1\}$.²⁴ To assess the magnitude of

²⁴Equation (21) holds since $E[a | h = 0] = E[a | h = 1]$. Likewise eq. (22) holds since we know that the quantiles of the regional distributions of financial assets a_q^h , $h \in \{0, 1\}$ are equal $a_q^1 = a_q^0$ (given that the distributions are identical) and since $n_q^h = a_q^h + h$.

these interregional wealth differences in a meaningful way, we compare them with typical household wealth in the economy:

Proposition 4 *When the investment motive becomes weaker compared to the consumption motive (see Proposition 1), in equilibrium interregional wealth differences become larger compared with typical household wealth, as measured by average wealth $E[n]$, median wealth, or any other quantile n_q of the economywide wealth distribution, where $n_q = \min n, s.t. q \leq F(n)$.*

Proof The result follows from equations (21) and (22) and Lemma 1. ■

Remarkably, there is an inverse relation between the importance of investment considerations at the household level, and the importance of the wealth aspect of housing at the aggregate level. The less the households see the home as an investment, the larger is the role of wealth in residential sorting.

The following proposition is about polar cases.

Proposition 5 *(a) When $\varepsilon \rightarrow 0$ or $\delta \equiv \frac{\pi\beta}{1-\beta(1-\pi)} \rightarrow 1$, there is perfect sorting in the match dimension and no sorting in the wealth dimension. In any given period, a household chooses a desirable location if and only if $\theta > \theta_m$. (b) If $\theta_L + \varepsilon > \pi\beta \frac{E[\theta] - \theta_L}{1-\beta}$, there is perfect sorting in the wealth dimension and no sorting in the match dimension. A household resides in a less desirable location if and only if it is borrowing constrained.*

Proof See the appendix. ■

The equilibrium pattern of residential sorting, with different values of ε , is illustrated in Figure 4. A similar set of figures could also be presented with different values of π and β .

Next we turn to house price fluctuations. One way to gauge the magnitude of house price fluctuations is to use their monetary size p . According to equation (19), $p = M/(E[n] - \frac{1}{2})$ essentially depends on average wealth in the economy (M is an exogenously given constant). Then more generally we can measure the price fluctuations (the size of which is normalized to unity) by comparing them to average wealth, median wealth or any other quantile of the economywide wealth distribution.

Remark 1 *Assume that the investment motive becomes stronger compared to the consumption motive (see Proposition 1). Then in equilibrium (i) the monetary size of house price fluctuations, p , decreases, and (ii) the price fluctuations become smaller compared with household wealth, measured by average wealth, median wealth, or any other quantile of the wealth distribution.*

Proof The result follows from eq (19) and Lemma 1. ■

When the consumption motive dominates the investment motive, most households are willing to allocate the bulk of their wealth in housing, rather than financial assets, although buying a home in an expensive location is a risky investment. Given this demand for different assets, in equilibrium the relative price of housing, in terms of financial assets, p is high. Furthermore, the price changes that follow a regional shock are large, and capital gains and losses made in the housing market are likely to affect a household's ability to buy a home in an expensive location. After a capital loss, a household is often borrowing constrained. When the investment motive dominates, and many households willingly choose a currently less desirable location, we have the opposite situation. House price fluctuations are small in monetary terms and compared to typical household wealth. Then a housing market related wealth shock has only a relatively small impact on a household's (relative) wealth position, and a typical household is well equipped to withstand capital losses.

Combining Remark 1 with Propositions 1, 3, 4 and 5 allows us to establish a connection between the size of house price fluctuations and the pattern of residential sorting.

Corollary 1 *Large (small) house price fluctuations are associated with (i) a low (high) degree of residential sorting according to household type and (ii) a high (low) degree of sorting according to wealth.*

For an illustration, see Figure 4. In Section 5 we present some empirical evidence on house price fluctuations and the degree of sorting according to household type.

3.2 Movers and stayers

The asset aspect of housing has also implications for household mobility, and for residential sorting among movers and stayers.

We begin by demonstrating a simple humpshaped relation between wealth and mobility. Take any given wealth class n . At the beginning of any period, the portion $1 - G(\theta_n^*)$ of households own a house in the desirable location; since equations (14) hold in the steady state, this is true even after a regional shock. Between any two periods, $(1 - s)\lambda + s$ households are hit by a shock, which breaks their match. Then the share $((1 - s)\lambda + s)G(\theta_n^*)$ of the households, which are in the popular area at the beginning of the period, get a realization $\theta < \theta_n^*$ and move to the unpopular area. Therefore, mobility from the desirable to the undesirable location in wealth class n is equal to $((1 - s)\lambda + s)G(\theta_n^*)[1 - G(\theta_n^*)]$. Likewise, it is easy to conclude that mobility from the undesirable to the desirable location equals the same measure. Then overall mobility in wealth class n is $\tilde{\mu}(n; s) = ((1 - s)\lambda + s)\mu(n)$, where

$$\mu(n) = 2G(\theta_n^*)(1 - G(\theta_n^*)). \quad (23)$$

Clearly, there is more mobility in those periods when the economy is hit by a regional shock and $s = 1$. Under the atomistic locations interpretation, in any given period, mobility at wealth level n is $\hat{\mu}(n) = ((1 - \pi)\lambda + \pi)\mu(n)$. Notice also that in the two-region case, $\hat{\mu}(n)$ is the long-run average mobility at wealth level n .

Essentially, $\tilde{\mu}(n; s)$ or $\hat{\mu}(n)$, defines a humpshaped relation between wealth and mobility:

Proposition 6 *Assume that there is at least one wealth class n with positive mobility (i.e. $\theta_n^* \in (\theta_L, \theta_H)$). Then mobility is increasing in wealth at low wealth levels, and decreasing in wealth at high wealth levels, so that households at intermediate wealth levels are more mobile than the poor and the wealthy.*

Proof Equation (23) implies that $\mu(G)$ is a downward opening parabola, with its peak at $G(\theta_m) = \frac{1}{2}$. Also $\mu(G) = 0$ at the extreme points $G = 0$ and $G = 1$. Now the result follows from three observations. (i) $G(\theta_0^*) = 1$; (ii) the critical match θ_n^* is decreasing in

n , and thus also $G(\theta_n^*)$ is decreasing in n ; (iii) $\lim_{n \rightarrow \infty} G(\theta_n) = G(-\varepsilon) < \frac{1}{2}$. ■

This pattern of mobility essentially reflects the varying strength of the investment motive at different wealth levels. Rich households, with a weak investment motive, typically want to live in a popular location, and only rarely find it optimal to move. Poor households tend to reside in a cheap location; for the borrowing constrained this is obviously the only alternative. At intermediate levels of wealth, the investment motive is neither extremely strong nor very weak; when the match is broken, these households often find it optimal to change location. Maximum mobility is attained, if the location choice rule θ_n^* corresponds to the socially optimal median rule θ_m . As discussed in Section 2.5, in the mode of the wealth distribution, households location choices tend to deviate relatively little from the socially optimal policy. Then, typically, the most mobile households are found in the hump of the wealth distribution, while the least mobile are in the tails.

Remarkably, the relationship between wealth and mobility established in Proposition 6 is essentially the same as empirically documented by Henley (1998) for the UK; see especially Figure 2 in Henley (1998). According to Henley (1998, p.425), "levels of housing wealth are an important factor in explaining mobility, and the relationship between the two is not linear." British households with large negative housing equity are virtually immobile. Also very wealthy households tend to move relatively little. Households with intermediate levels of wealth are the most mobile.

Next we proceed to comparing the degree of residential sorting among movers and stayers. In any given period, we classify as a mover a household which has moved during that period. The following results are proved in the appendix.

Proposition 7 *(a) In both locations, movers have a better match with their (new) home region than stayers, in the sense of first order stochastic dominance. In an expensive location, stayers (old residents) are wealthier than movers (newcomers) in the sense of first order stochastic dominance, while in a cheap location the opposite is true. (b) Movers are more sorted than stayers in the type dimension. Stayers are more sorted than movers in the wealth dimension. (c) The difference between the sorting patterns of movers and stayers (both in the type dimension and in the wealth dimension) is more pronounced in*

periods when house prices change, due to a regional shock ($s = 1$), than in periods when house prices do not change ($s = 0$).

When interpreting item (a) of the proposition, remember that a good match with a cheap location means that a household has a low realization of θ .

Item (a) reflects the fact that those who move from one location to another tend to have rather strong match-related reasons to make that choice, while those who stay put may do so largely because they have been lucky or unlucky in the housing market. For example, households which move from a desirable location to an undesirable location, choose a cheap area, although they could afford a more expensive house (their former home). By contrast, at least a part of the old residents live in a cheap location because they have been locked in by falling home equity values. Likewise, in an expensive region, newcomers from cheaper locations tend to have a good match with the area they have chosen, whereas old residents, who may have bought their home before the rise of local house prices, often stay put even with a more modest match. Item (b) is a rather straightforward corollary of item (a). Since movers are better matched with their home region than stayers in both location types, movers are obviously more sorted than stayers in the type dimension. On the other hand, interregional wealth differences are larger among stayers than among movers, and stayers are more sorted in the wealth dimension. Finally, item (c) indicates that the housing-market based mechanism, that gives rise to different sorting patterns among movers and stayers, has a stronger effect in periods when regional house prices change. The empirical work on movers and stayers reported in Section 5 is based on items (b) and (c).

4 Extensions

In this section we consider three extensions to the basic model. In subsection 4.1 we show how the model can be generalized to cover the asymmetric case where the shares of desirable and undesirable locations are not equal (they differ from 50/50). In subsection 4.2 we consider the possibility that the degree of sorting according to household type enters the households' utility function; for example, the households may dislike the idea

of living in a completely segregated area. Finally, in subsection 4.3 we relax the assumption of independent draws from the type distribution, and allow the household type to follow a general Markov process. In subsections 4.1 and 4.3, we adopt the small region interpretation of the model.

This section serves two purposes. First, we show that the main results derived from the basic model continue to hold in somewhat different, and more general, settings. Second, especially the extension with more general match dynamics (subsection 4.3) also provides some new insights.

4.1 Asymmetric locations

In this subsection we consider a model specification where in each period the share of desirable areas is $\phi \in (0, 1)$, while the remaining locations are undesirable. In the main text we explain how the basic model is adjusted to allow for unequal shares of different locations, and show how the equilibrium is established. The appendix then shows that the main results of the paper (Propositions 1-4, and Corollary 1) apply also in this model version.

To study the extension with asymmetric locations, we need to alter some of the assumptions of the basic model. First, we now assume that between any periods, a measure $\phi\pi$ of currently undesirable locations is hit by a regional shock, while the measure of desirable location that are hit by a regional shock is $(1 - \phi)\pi$. This process of regional shocks is consistent with the stationary shares ϕ and $1 - \phi$ of desirable and undesirable locations.

Second, while the probability of regional shocks is different in different locations, we think that it is still reasonable to assume that the probability that household type changes is independent of location choice. Thus we assume that the probability that a new type is drawn (due to a regional shock or for household specific reasons) is the same, no matter where the household lives. We denote this probability by σ . This then means that in a desirable location the match changes with probability $(1 - \phi)\pi$ due to a regional shock, and with probability $\sigma - (1 - \phi)\pi$ due to a household specific shock. In an undesirable

location, the corresponding probabilities are $\phi\pi$ and $\sigma - \phi\pi$, respectively.

We also slightly alter the payoff structure. A household with current match θ receives per period utility $(1 - \phi)\varepsilon + \theta$, when living in a currently desirable location while the per period utility of anyone household living in a less desirable location is $-\phi\varepsilon$. These modifications are made to guarantee that, just like in the basic model, the measure of social welfare w reflects the degree of sorting in the match dimension. Indeed, with these payoffs

$$w = \phi \{(1 - \phi)\varepsilon + E[\theta | h = 1]\} + (1 - \phi)(-\phi\varepsilon) = \phi E[\theta | h = 1] \quad (24)$$

depends on the average quality of the match in the desirable locations.²⁵

What is important for households' location choices is the difference between the payoffs available in a desirable and in an undesirable location. Just like in the basic model, this difference is $\theta + \varepsilon$. Thus, all households with $\theta > -\varepsilon$ derive a positive utility premium from living in a desirable location; again this is exactly as in the basic model.

We assume that the distribution of θ is such that $G(0) = 1 - \phi$. This assumption implies that housing is scarce in the desirable locations: the measure of households deriving a positive utility premium from living in a desirable location is $1 - G(-\varepsilon) > \phi$, while the measure of houses in these locations is ϕ . Finally, in any period, aggregate welfare is maximized, if all households with $\theta > \theta_{1-\phi} = 0$ (where $\theta_{1-\phi}$ is defined by $G(\theta_{1-\phi}) = 1 - \phi$) are allocated to the (currently) desirable locations, while those with $\theta < 0$ live in the less desirable locations.

With these assumptions, the household's problem boils down to the Bellman equation

$$V(\theta, n) = \max_{h \in \{0,1\}} h((1 - \phi)\varepsilon + \theta) - (1 - h)\phi\varepsilon + \beta \{(1 - \sigma)V(\theta, n) + \sigma V(n) + \pi [(1 - h)\phi(V(n + 1) - V(n)) + h(1 - \phi)(V(n - 1) - V(n))]\} \quad (25)$$

subject to (4). At any unconstrained wealth level $n \geq 1$, the household chooses a currently

²⁵By contrast, the payoff structure of the basic model would yield $w = \phi [\frac{1}{2}\varepsilon + E[\theta | h = 1]] + (1 - \phi)(-\frac{1}{2}\varepsilon) = \phi E[\theta | h = 1] + (\phi - \frac{1}{2})\varepsilon$. In this setting w would not be directly applicable as a measure of matchwise sorting (since here w depends on the parameter ε as well as on the average match $E[\theta | h = 1]$).

desirable location if and only if

$$\theta + \varepsilon > \beta\pi \{ \phi [V(n+1) - V(n)] + (1 - \phi) [V(n) - V(n-1)] \}. \quad (26)$$

In addition to the households' location - and asset portfolio - choice rule, the second component of the long-run equilibrium is the long-run wealth distribution. The wealth distribution is stationary if and only if

$$\begin{aligned} f(n) \equiv f_n^0(n) + f_{n-1}^1(n) &= (1 - \phi\pi) f_n^0(n) + (1 - (1 - \phi)\pi) f_{n-1}^1(n) \\ &+ \pi (\phi f_{n-1}^0(n-1) + (1 - \phi) f_n^1(n+1)). \end{aligned} \quad (27)$$

It is easy to conclude that (27) reduces to

$$\phi f_n^0(n) + (1 - \phi) f_{n-1}^1(n) = \phi f_{n-1}^0(n-1) + (1 - \phi) f_n^1(n+1). \quad (28)$$

There are no wealth classes below 0 (i.e., $f(n) = 0$ for $n < 0$) and at wealth level 0 the households can only choose an unpopular location (i.e., $f_{-1}^1(0) = 0$). These restrictions and (28) then imply the set of equations

$$\frac{f_n^1(n)}{f_n^0(n-1)} = \frac{\phi}{1 - \phi}, \quad n = 1, 2, \dots \quad (29)$$

Using the equations (29) (and the fact that the aggregate mass of households is unity) and summing over all wealth classes yields

$$\sum_{n=0}^{\infty} f_n^0(n) = 1 - \phi, \quad \sum_{n=0}^{\infty} f_{n-1}^1(n) = \phi.$$

These equations imply that housing demand, on the left-hand side, is equal to housing supply, on the right hand side, in both location types. Finally, plugging (9) and (10) into (29) yields the sequence

$$\frac{f(n)}{f(n-1)} = \frac{\phi}{1 - \phi} \gamma(n), \quad n = 1, 2, \dots \quad (30)$$

(where $\gamma(n)$ is given by (16)) that determines the stationary wealth distribution. Since $G(-\varepsilon) < 1 - \phi$ and $1 - G(-\varepsilon) > \phi$ (this follows from the assumption that housing is scarce in the desirable locations), we have

$$\lim_{n \rightarrow \infty} \frac{f(n)}{f(n-1)} = \frac{\phi}{1-\phi} \frac{G(-\varepsilon)}{1-G(-\varepsilon)} < 1$$

and the sequence converges. This then guarantees the existence of a stationary wealth distribution. It is also worth noting that in the hump of the wealth distribution $\gamma(n) \approx \frac{1-\phi}{\phi}$,²⁶ meaning that in the hump, the households' location choice rule (summarized by θ_n^*) tends to be relatively close to the socially optimal rule (summarized by $\theta_{1-\phi}$); a similar result holds in the basic model.

Finally, the asset market equilibrium condition is $E[a] = \frac{M}{p}$. Using (1) and the housing market equilibrium $E[h] = \phi$, the asset market equilibrium condition can be rewritten as $E[n] = \phi + \frac{M}{p}$. Then the relative price of housing and financial assets is

$$p = \frac{M}{E[n] - \phi}. \quad (31)$$

This concludes our treatment of asymmetric locations. The appendix then shows that the main results of the paper (Propositions 1-4, and Corollary 1) hold also in this model version.²⁷

4.2 Sorting in the utility function

In the basic model perfect sorting according to household type is the socially optimal outcome. In this subsection we consider the possibility that the households may appreciate some diversity, and dislike the idea of living in a completely segregated area. Then perfect sorting in the type dimension may no longer be socially optimal. Nevertheless, the descriptive results of the paper (linking the investment aspect of housing, the size

²⁶The mode of the distribution is a wealth level n_{mod} such that $\frac{\phi}{1-\phi} \cdot \gamma(n_{\text{mod}} - 1) > 1$ and $\frac{\phi}{1-\phi} \cdot \gamma(n_{\text{mod}}) < 1$.

²⁷Also Propositions 5, 6, and 7a and 7b continue to hold. The proofs are very similar to those in the basic model. Proposition 7c applies to the two-locations version of the model, and it cannot be generalized to the case with asymmetric locations, where we assume that there is a continuum of atomistic locations.

house price fluctuations, and the pattern of sorting) still hold in this setting.

Assume that the utility that a household with current type θ derives from living in a desirable location is

$$\theta + \frac{1}{2}\varepsilon + u(\text{sorting in the type dimension})$$

while utility in an undesirable location is

$$-\frac{1}{2}\varepsilon + u(\text{sorting in the type dimension}).$$

The new term $u(\cdot)$ captures the fact that while households may like having enough households like themselves around, they may dislike residing in a highly segregated area - a ghetto. Thus a high degree of sorting in the type dimension may lower the households' welfare.

Three observations are in order. First, the new term $u(\cdot)$ depends on the aggregate state of the economy, which is outside the control of an individual household. Second, the term $u(\cdot)$ affects the level of utility in either location. Third, it is also easy to conclude that there is a level shift in the value function, so that the new (ex post) value function is $V^*(\theta, n) = V(\theta, n) + u(\cdot)$, (where $V(\theta, n)$ is the value function of the basic model), and the ex ante value function $V^*(n) \equiv E_\theta[V^*(\theta, n)] = V(n) + u(\cdot)$.

Importantly, households' location choices depend on the differences between the prospects available in the locations. The difference between immediate benefit streams (the consumption motive in equation (6))

$$\theta + \frac{1}{2}\varepsilon + u(\cdot) - \left(-\frac{1}{2}\varepsilon + u(\cdot)\right) = \theta + \varepsilon$$

is exactly the same as in the basic model. Also the investment motive (see the right-hand side of eq. (6))

$$\pi\beta [V^*(n+1) - V^*(n-1)] = \pi\beta [V(n+1) - V(n-1)]$$

is the same as in the basic model. Thus the new element we have introduced into the model does not change households' location choices. Finally, since households' location choices give rise to the equilibrium pattern of residential sorting, all the descriptive results of the paper (Propositions 1 and 3-7, and Corollary 1) remain intact.

The only aspect of the analysis, that has to be modified, is the interpretation of the result established in Proposition 2. The measure $w = \frac{1}{2}E[\theta | h = 1]$ (see eq (20)) still reflects the degree of residential sorting in the type dimension. However, w no longer gauges social welfare, which is now measured by

$$w^* = w + u(\text{sorting in the type dimension}).$$

Proposition 2 now reads:

Proposition 2* *When the investment motive becomes stronger, compared to the consumption motive (see Proposition 1), the measure of residential sorting in the type dimension, w , increases. That is $\frac{dw}{d\beta} > 0$, $\frac{dw}{d\pi} > 0$ and $\frac{dw}{d\varepsilon} < 0$.*

(The proof of the proposition is exactly the same as in the basic model.)

A stronger investment motive in location choice still translates into more residential sorting in the type dimension. However, this does not necessarily mean that social welfare increases.

4.3 More general match dynamics

In this subsection, we drop the assumption that, after a shock, the new match is independently drawn, and allow the match to follow a general Markov process. This extension introduces two new features to the model. First, the strength of the investment motive may reflect expected tenure length and household specific moving plans. Second, in equilibrium there is endogenous correlation between wealth and the match.

There are $J \geq 2$ different match realizations. If the match changes for idiosyncratic, or household specific, reasons ($s = 0$), the transition probabilities from one match to another are given by a transition matrix Λ_0 . If there is a regional shock ($s = 1$), the transitions are

governed by a (possibly) different matrix Λ_1 . To guarantee the existence of a stationary joint distribution for wealth and the match, we adopt the small region interpretation of the model, and assume that there is a continuum of atomistic locations. In each period, a measure π of the matches is broken due to regional shocks, and a measure λ for household specific reasons. Let $\pi = \xi\sigma$ and $\lambda = (1 - \xi)\sigma$, where $\sigma \in (0, 1)$ is the overall probability that the match is broken, and $\xi \in (0, 1]$ measures the relative frequency of regional and idiosyncratic shocks. The parameter σ can be interpreted as reflecting the overall degree of turbulence in the economy. The stationary marginal distribution of the match is defined as the eigenvector associated with a unit eigenvalue of Λ' , where $\Lambda \equiv (1 - \xi)\Lambda_0 + \xi\Lambda_1$.²⁸ Notice that if the frequency of shocks (σ) changes, but the relative probabilities of regional and idiosyncratic shocks (ξ and $1 - \xi$) remain constant, the stationary match distribution is unaltered.

Next we proceed to studying households' location choices. The value function $V(\theta, n)$ satisfies the Bellman equation

$$V(\theta, n) = \max_{h \in \{0, 1\}} h \left(\frac{1}{2}\varepsilon + \theta \right) - (1 - h) \frac{1}{2}\varepsilon + \beta \left\{ (1 - \sigma) V(\theta, n) + \lambda E_{\tilde{\theta}} \left[V(\tilde{\theta}, n) \mid \theta, s = 0 \right] + \pi E_{\tilde{\theta}} \left[(1 - h) V(\tilde{\theta}, n + 1) + h V(\tilde{\theta}, n - 1) \mid \theta, s = 1 \right] \right\}, \quad (32)$$

subject to (4). At any unconstrained wealth level $n \geq 1$, the household chooses a currently desirable location if and only if

$$\theta + \varepsilon > \pi\beta E_{\tilde{\theta}} \left[V(\tilde{\theta}, n + 1) - V(\tilde{\theta}, n - 1) \mid \theta, s = 1 \right]. \quad (33)$$

Importantly, the investment motive, figuring on the right-hand side of (33) now depends on not only the household's wealth position, but also on the current match θ (and on the distribution of future matches $\tilde{\theta}$, conditional on the current match). Intuitively, the connection between the match and the investment motive may be interpreted as reflecting the household's expected tenure length, and future moving plans. The investment motive

²⁸We assume that the matrix Λ is indecomposable, so that it induces a unique long-run match distribution, but otherwise we do not impose any restrictions on the structure of the stochastic matrices Λ_0 and Λ_1 .

tends to be weak, if the household is attached to the home area, and wants to live there even when the area is unpopular: it does not matter, if local house prices fall, since the household has no intentions to sell.²⁹ Attachment to home can be modelled by letting the match be correlated with regional shocks: the household is likely to draw a high realization of θ , when the home area is “desirable,” and a low realization of θ , when the home area is “undesirable.” Conversely, the investment motive tends to be strong, if the household buys a home knowing that it will probably not live there for a long time. Then a major function of the current house is to serve as a springboard to the future home. In particular, if the household is planning to move to a popular and expensive area in the future³⁰, it has an incentive to avoid housing market risks, which might jeopardize these plans. In sum, condition (33) indicates that a household is likely to buy a home in an expensive location (i) if it has a good match with that location, (ii) if it is wealthy and (iii) if it is planning to stay in the location for a long time.

In addition to households’ location choices, the second component of the long-run equilibrium is the endogenous stationary joint distribution of wealth and the match. The vector difference equation, which implicitly defines the long-run joint distribution is presented in the appendix. The appendix also establishes the equilibrium of the model.

Unlike in the basic model, wealth and the match are typically not independently distributed.³¹ If households are attached to a home region, a positive correlation between the value of the match, θ , and household wealth naturally arises. This is illustrated in Figure 5. In equilibrium, those households, which derive the highest utility premium from residing in an expensive location, also tend to be wealthy. Typically, these households have seen the value of their house go up, as their home region has become more popular and more expensive. This coevolution of housing costs and household wealth is one of the advantages of owner occupation, discussed by Sinai and Souleles (2005).

While attachment to home, and the resulting positive correlation between wealth an

²⁹In a similar vein, Sinai and Souleles (2005) argue, that owner occupation is not risky, if a household intends to stay put for a long time

³⁰Formally, the household expects to draw a high realization of θ in the future.

³¹Correlation arises, since (i) current wealth depends on past location and portfolio choices (and luck), (ii) past choices were influenced by past match realizations, and (iii) the current match is correlated with past match realizations.

the match, may seem a rather natural case to consider, the model is flexible enough to allow for many other alternatives as well. For example, if some households constantly derive a high utility premium from residing in a currently popular and expensive area, a different pattern arises. Those who insist on living in a fashionable location in every period, have to move against the tide, from an area of fading popularity and falling prices to an area of high prices. Then in equilibrium, the size of the utility premium and household wealth tend to be negatively correlated.

Overall, since expected tenure length and future moving plans may affect households' location choices, and since wealth and the match tend to be correlated, the equilibrium is typically more complex than in the basic model. Nevertheless, the main message of the paper carries over: The pattern of residential sorting reflects the relative strength of the consumption motive and the investment motive. In particular, there is a negative correlation between the size of house price fluctuations and the degree of sorting in the match dimension. This is illustrated in Figure 5, where panel *a* corresponds to a situation with small regional shocks, a strong (in relative terms) investment motive, small house price fluctuations, and a high degree of sorting in the match dimension. In panel *b* regional shocks are larger, and the consumption motive dominates; then price fluctuations are more pronounced, and sorting takes place mainly in the wealth dimension.

More formally, the appendix proves that the main results of the paper, Propositions 1, 2 and 3, and Corollary 1, still hold, with the exception that π is substituted by σ . (A change in π would also alter the stationary match distribution.) If $\lambda = 0$, so that there are no idiosyncratic shocks, these results hold verbatim.

5 Empirical evidence

In this section we present some empirical evidence on regional house price fluctuations and residential sorting. We consider observations from the US metropolitan statistical areas (MSAs) and local municipalities (so called Minor Civil Divisions or MCDs). The data are from the 1990 decennial census (see the appendix for details).

We first examine how house price variations in the MSAs are related to the degree of

residential sorting of the MCDs within each MSA. By Corollary 1, we expect that MCDs within MSAs that experience large house price fluctuations have diverse populations in the sense that the shares of different demographic groups of the MCDs by and large correspond to the population structure of the underlying MSA. On the other hand, MCDs in areas where prices are less volatile should have a less diverse population, with certain demographic groups under- or overrepresented, compared with the MSA average.³²

We proxy household types by characteristics such as income, education and age (cf. Rhode and Strumph (2003)). We apply conventional sorting measures: the dissimilarity index, D , the Gini coefficient, GC , and the Theil’s information theory index, T , defined as

$$D = \frac{1}{2} \frac{\sum_m \sum_i N_i |S_{mi} - S_m|}{N \sum_m S_m (1 - S_m)} \quad (34)$$

$$GC = \frac{1}{2} \frac{\sum_m \sum_i \sum_j N_i N_j |S_{mi} - S_{mj}|}{N^2 \sum_m S_m (1 - S_m)} \quad (35)$$

$$T = \frac{\sum_i \sum_m N_i S_{mi} \ln(S_{mi}/S_m)}{N \sum_m S_m \ln(1/S_m)} \quad (36)$$

where S_{mi} is the share of age, education or income group m in the population of the MCD i , S_m is the corresponding share at the MSA level, N_i is the population of MCD i and N is the population of the MSA. The indices vary between zero and one. We have $D, GC, T = 0$ when each type is equally represented in each community (no sorting), and $D, GC, T = 1$ when the types are completely sorted across municipalities.³³ Basically, the indices D , GC and T rank the MSAs by the degree of residential sorting.

As a “benchmark” measure of house price fluctuations, (henceforth “house price volatility”) we use the standard deviation of the monthly house price p_{it} of the MSA i over the period Jan 1975 to Dec 2000, where $p_{it} = \log(PI_{it}/PI_t)$, PI_{it} is the house price index in MSA i in month t , and PI_t is the US house price index in month t .³⁴ Basically, this measure ranks MSAs by the degree at which their house prices have fluctuated against

³²Here we adopt an interpretation of the model, where a location corresponds to a MCD, while the entire economy is the MSA.

³³For additional properties of the indices see Reardon and Firebaugh (2002) and Rhode and Strumph (2003).

³⁴We employ the Freddie Mac House Price Index data (see <http://www.freddiemac.com/finance/fmhpi/>).

the US average.

A potential problem in our benchmark measure of house price volatility is that it does not distinguish MSAs where relative house prices have been volatile, but stationary, from MSAs where relative house prices have been upward or downward trending. To address this issue, we consider an alternative measure of house price volatility obtained as the standard deviation of the detrended house price series $p_{it} - p_{it}^{HP}$, where the trend p_{it}^{HP} is computed by applying the Hodrick-Prescott filter.³⁵

In addition to the above defined measures of house price volatility, we examine house price fluctuations using turning point analysis. The turning points - peaks and troughs - of relative house prices p_{it} are identified with the Bry-Boschan algorithm (Bry and Boschan (1971), see also Harding and Pagan (2002)).³⁶ The minimum number of turning points in our sample is 2 (there are 5 such MSAs in the sample) while the maximum number is 13. Typically, there are 3-8 turning points: the mean is 5.8, the median as well as the mode is 4. Thus (since peaks and troughs alternate), on a whole in our sample period, the relative house price of a MSA tends to move up and down. (In the whole sample of 275 MSAs, relative house prices are in a downward phase 57% of the time, and in an upward phase 43% of the time.) Our turning point analysis based measures of house price volatility are the average and the maximum amplitude of upward and downward phases of house prices p_{it} in the sample period. These measures attempt to gauge the size of house price fluctuations the MSA has experienced during the sample period.

Table 2 reports sample correlations between the alternative sorting measures of the three type characteristics and the four measures of house price volatility. Consistent with our theory, each sorting measure is negatively correlated with each measure of house price volatility. Thus, independent of the applied measures, MCDs within MSAs subject to high house price volatility tend to be less sorted than MCDs in MSAs with little house price variation, and vice versa.

According to our theory, the degree of residential sorting should be associated with (expected) future regional house price fluctuations, as well as past fluctuations. Thus it

³⁵We use the value $\lambda = 14400$, for monthly series.

³⁶See Claessens et al. (2009, 2011) for recent applications of turning point analysis to financial cycles, including cycles in real house prices.

is arguably a rather natural choice to use price data from a period following, as well as preceding, our 1990 cross-section, when measuring the size of house price fluctuations. However, as a robustness check, we also computed the volatility measures using the subsample 1975-1990, preceding our cross section. Also these measures of the size of house price fluctuations are negatively correlated with all the sorting measures (D , GC , T) for income, age and education, see Table 2A in the appendix.

A potential concern is that the observed correlation between residential sorting and house price volatility might arise from factors beyond the mechanism suggested by our theory. Therefore, to examine the robustness of the correlations, we run OLS regressions of different sorting measures on house price volatility and selected covariates. Our baseline regressions are reported in Table 3. In all the regressions, the coefficient estimate of house price volatility is negative, and the estimate is also statistically significant at the 1% level in all the regressions, save one. (The exception is column (3) where the estimate is not statistically significant.) Therefore, residential sorting and house price volatility appear to be (negatively) correlated even if we partial out the applied covariates.

The coefficient estimate of the number of MCDs is positive and statistically significant in columns (1) and (3) of Table 3. This is consistent with the idea that a large number of MCDs offer more opportunities for forming different homogeneous income groups than a small number of MCDs.³⁷ The coefficient estimate of the average population size of MCDs is negative in all regressions. This is in line with the idea that a large population in an MCD can encompass a larger range of households than a small population, and thus, *ceteris paribus*, tends to reduce sorting across regions. This effect appears statistically significant in regressions regarding sorting by income and education. We expect that sorting may be more beneficial in urbanized areas with high population density than in rural areas with low population density. We also expect that larger MSAs are likely to provide more opportunities for beneficial sorting than small MSAs (cf. Hoxby (2000)).

³⁷In particular, if the number of MCDs is less than the number of different types, it is not possible to achieve maximal sorting in the sense that each type resides in a separate region (cf. Eberts and Gronberg (1981)). In our case, the number of income groups (25) exceeds the number of MCDs in many metropolitan areas. As a robustness check, we recomputed the sorting indices with four income groups (formed by merging the original groups). In our baseline regression the coefficient of the number MCDs was no longer statistically significant. Otherwise, however, the results were qualitatively the same.

In line with these assertions, the coefficient estimates of the density of the MSA and the area size of the MSA are positive (with one exception in column (4)) when they are statistically significant. Finally, the negative (and weakly significant) coefficient estimate of the number of families in MSA in columns (1) and (3) suggests that it is harder to obtain homogeneous income groups from a large population than from a small population, *ceteris paribus*.

Various additional covariates can be justified in our regressions. Recent literature indicates that physical and regulatory constraints, which hinder housing construction, may have significant implications for the house price dynamics and the development of the MSAs (See Gyourko, Saiz, and Summers (2008) and Saiz (2010)). Obviously, such constraints might induce correlation between residential sorting and house price volatility. To account for such effects, we consider the land topographic unavailability measure (“Physical constraints”) of Saiz (2010) and the Wharton Residential Urban Land Regulation Index (“Regulatory constraints”) of Gyourko, Saiz, and Summers (2008). These variables are available for 225 MSAs in our original sample.³⁸ Arguably, the characteristics of the built environment may affect the pattern of residential sorting (see Nechyba (2000) and the recent paper by Bayer and McMillan (2011)). If, say, the housing stock is very different in different parts of a MSA, one expects that the degree of sorting in the MSA should be relatively high, *ceteris paribus*. To measure heterogeneity of the housing stock across municipalities in an MSA, we compute the dissimilarity index for two aspects of the housing stock, the age of housing units, D_{HoAge} , and the number of housing units in a residential building³⁹, D_{HoUnit} . The interpretation of these measures is as above: the larger the value of D_{HoAge} (or D_{HoUnit}), the more the MCDs within the MSA differ from each other. Finally, given that our theory applies to owner-occupied households, it is reasonable to include the share of rental housing as a covariate.

Table 4 reports OLS regressions that augment our baseline regressions with all of the aforementioned additional covariates. Again, in all of the regressions, the coefficient estimate of house price volatility is negative, and the estimate is also statistically significant

³⁸The data are obtained from <http://real.wharton.upenn.edu/~saiz/SUPPLYDATA.zip>.

³⁹This measure essentially tells whether there are detached houses, semi-detached houses or blocks of flats in an area.

at the 1% level in all regressions but one. (The exception is column (3) where the estimate is not statistically significant.) The coefficient estimates of the measures of physical and regulatory constraints as well as those of the dissimilarity measures of housing, D_{HoAge} and D_{HoUnit} , are positive whenever they are statistically significant. This is as one would expect. The diversity of housing units, D_{HoUnit} , seems particularly important, as it is statistically significant at least at 1% level in all regressions. The coefficient estimates of the rental share is statistically significant in all regressions. The estimates indicate that a larger rental sector in a MSA is associated with a higher degree of sorting in terms of age and education, and a lower degree of sorting in terms of income. The latter observation may reflect the presence of rent control in a number of metropolitan housing markets: under rent control, the allocation of housing is not determined by the willingness to pay, but by some other mechanisms, such as queueing (cf. Glaeser and Luttmer (2003)). Finally, the coefficient estimates of the remaining regressors are similar to those of Table 3.

The results of Tables 3 and 4 are not sensitive to the measure of house price volatility. Table 5 (6) reports regressions with alternative house price volatility measures (one based on HP filtered relative house prices and two based on turning point analysis) and the covariates of Table 3 (4). The dependent variables, the sorting measures for income, education and age in Tables 5 and 6 are based on the dissimilarity index. In the regressions of Tables 5 and 6, the coefficient estimate of house price volatility is always negative and in most cases also statistically significant. Furthermore, the estimation results on the covariates are largely in line with those of Tables 3 and 4.

As a further robustness check we run the above regressions by using house price volatility measures computed from the subsample 1975-1990, preceding our cross section. The regression results are qualitatively similar, see Tables 3A-6A in the Appendix.

We turn to comparing residential sorting of movers to that of stayers across (so called) Public Use Microdata Areas (PUMAs) in different MSAs.⁴⁰ This part is related to the work by Ortalo-Magné and Rady (2008), who study income distributions among movers and stayers.

According to Proposition 7, stayers should be less sorted than movers in the type

⁴⁰Each PUMA has a population of approximately 100 000. For further information, see the appendix.

dimension. Furthermore, our theory suggests that the low degree of sorting among stayers is related to capital gains and losses made in the housing market. To investigate the above predictions, we classify an individual as a mover, if (s)he has resided in his/her current home for less than five years; otherwise the individual is a stayer. Then, for each characteristic (age, education, income) and each group (movers and stayers), we compute the three sorting indices (D , GC , T) across PUMAs in each MSA.⁴¹ The PUMA data allows us to compute separate indices for owner-occupied households and households that live in rental housing (the predictions of our model should hold for owner-occupiers.)

The results on owner-occupied movers and stayers are reported in the first two columns of Table 7, and in Table 8. Consistent with the prediction of our model, the degree of sorting is lower among stayers than movers. Moreover, the differences between the sorting patterns of (owner-occupied) movers and stayers are more pronounced in those metropolitan areas that had experienced large house price swings prior to our 1990 cross section; see Table 8.⁴² This observation is consistent with Proposition 7, item c ⁴³, and it lends support to the notion that the low degree of sorting among owner-occupied stayers could be related to capital gains and losses made in the housing market.

As a further piece of evidence on the role of house price fluctuations in moulding the pattern of sorting among owner-occupied households, we compare owners to renters, who do not face wealth shocks in the housing markets. We find that among renters, the pattern of sorting is quite different: stayers are actually more sorted than movers; see columns (3) and (4) in Table 7, and Table 9. Furthermore, among renters, the differences between the sorting patterns of movers and stayers tend to be less pronounced in those metropolitan areas that have experienced large house price fluctuations.

As an additional piece of evidence, we compare “short distance movers,” i.e. households which have moved within the same metropolitan area, and “long distance movers,”

⁴¹In our data set there are 103 MSAs, each of which contains at least two PUMAs.

⁴²When examining the hypothesis presented in Proposition 7c, it is natural to use price data from a period preceding the 1990 cross-section. As a measure of the size of house price fluctuations we use here the maximum amplitude of $p_t = \log(PI_t) - \log(PI_t^{US})$ over the period 1975-1990.

⁴³Here we have in mind the following - admittedly somewhat rough - correspondence between the observations and Proposition 7c: MSAs that had experienced large house price fluctuations (the 4th quartile) are locations with $s = 1$, while the remaining MSAs are locations with $s = 0$.

i.e. households, which have moved from another metropolitan area.⁴⁴ Because “long distance movers” have more likely moved between two uncorrelated markets (so that the prices of the old and the new home may have evolved very differently), they should be more sorted than “short distance movers.” The sorting measures reported in Table 10 indicate that “long distance movers” are indeed more sorted than “short distance movers,” according to all three type criteria and whether we look at owner-occupiers or renters. While this finding holds for both owner-occupiers and renters, the differences between the sorting patterns of “long distance movers” and “short distance movers” are more significant among owner-occupiers than among renters.

6 Conclusions

This paper examined how the asset aspect of housing affects households’ location choices and the socioeconomic make-up of local jurisdictions. Our theoretical analysis suggests that a strong investment motive is associated with a high degree of residential sorting according to household type. Since buying an expensive home in a currently popular location is a large and risky investment, in equilibrium only households with a high current utility premium reside in these areas, while other households choose a cheaper location.

If investment considerations only play a minor role, and the consumption motive dominates, there will be less sorting in the type dimension and more sorting in the wealth dimension. Typically, a household resides in an unpopular location if and only if its wealth is low and it cannot afford a more expensive home. Since current wealth depends, in part, on past luck in the housing market, households residing within the same area may then have little in common, except for the value of their home.

To sum up, there is an inverse relation between the importance of investment considerations at the household level, and the importance of the wealth aspect of housing at the aggregate level. The less the households see the home as an investment, the more

⁴⁴We also use data on people that have moved *from* or *to* a non-MSA region. See the appendix for more details.

the asset aspect of housing moulds the socioeconomic make-up of jurisdictions and the pattern of sorting.

Empirically, the model predicts that the size of regional house price fluctuations should be negatively correlated with the degree of residential sorting according to household type. To examine this hypothesis, we computed measures of residential sorting for income, age and education. In a sample of US metropolitan areas, we documented a negative relationship between the degree of sorting and the size of house price fluctuations.

Appendix A: Proofs of propositions

Location choice

The household's decision problem boils down to the choice of the sequence of optimal thresholds θ_n^* . Since $x_n \equiv G(\theta_n^*)$ is a monotonous function of θ_n^* , also x_n can be treated as a choice variable. Using the threshold rule (8) and integrating (5) over all θ shows that the household's decision problem can be summarized by the Bellman equation⁴⁵

$$V(n) = \max_{x_n} u(x_n) + \beta \{(1 - \pi) V(n) + \pi [x_n V(n+1) + (1 - x_n) V(n-1)]\}, \quad (37)$$

subject to $x_0 = 1$, where

$$u(x_n) \equiv \left(\frac{1}{2} - x_n\right) \varepsilon + \int_{x_n}^1 G^{-1}(x) dx$$

is the expected utility stream at wealth level n . Notice that $\frac{d^2 u(x_n)}{dx_n^2} = -\frac{1}{G'(\theta_n^*)} < 0$. Thus (37) defines a maximization problem with a concave objective function and linear constraints. As a consequence the value function $V(n)$ is concave.

We also show that $\lim_{n \rightarrow \infty} \theta_n^* = -\varepsilon$. If not, then $\lim_{n \rightarrow \infty} \theta_n^* = \widehat{\theta}^* > -\varepsilon$. Since θ_n^* is a non-increasing sequence, and, by assumption, the feasible values of θ_n^* lie on a finite interval, $\theta_n^* \in [\widehat{\theta}^*, \theta_H]$, we have $\lim_{n \rightarrow \infty} (\theta_{n+k}^* - \theta_n^*) = 0$ for all finite, positive integers $k \geq 1$. But then $\lim_{n \rightarrow \infty} (u_{n+k} - u_n) = 0$ for all $k \geq 1$. As a consequence, $\lim_{n \rightarrow \infty} [V(n+1) - V(n-1)] = 0$, and $\lim_{n \rightarrow \infty} \theta_n^* = -\varepsilon$. A contradiction.

Next, let $v(n) \equiv V(n+1) - V(n-1)$, and $\Delta x_n \equiv x_{n+1} - x_{n-1}$; since θ_n^* is a non-increasing sequence, $\Delta x_n \in [-1, 0]$. Also let $\Delta u_n \equiv u(x_{n+1}) - u(x_{n-1}) = \int_{x_{n+1}}^{x_{n-1}} G^{-1}(x) dx - \Delta x_n \varepsilon$. Finally define the operator L

$$L[z(n)] \equiv (1 - \pi) z(n) + \pi [x_{n+1} z(n+1) + (1 - x_{n-1}) z(n-1)],$$

where $z(n)$ is a generic function of n . Since $V(n)$ satisfies the recursive equation (37),

⁴⁵Differentiating (37) with respect to x_n shows that the optimal thresholds are characterized by (7).

$v(n)$ satisfies the recursive equation

$$v(n) = \Delta u_n + \beta L[v(n)]. \quad (38)$$

Finally, the expression for θ_n^* , eq. (7), can be rewritten as

$$\theta_n^* = Q(n; \varepsilon, \pi, \beta) \equiv -\varepsilon + \pi\beta v(n) \text{ for } n \geq 1.$$

Proof of Proposition 1

(i) Define $q^\varepsilon(n) \equiv \frac{dv(n)}{d\varepsilon}$. Differentiating (38) yields $q^\varepsilon(n) = -\Delta x_n + \beta L[q^\varepsilon(n)]$. (Notice that indirect effects can be ignored due to the envelope theorem.) Let $q_{\max}^\varepsilon \equiv \max q^\varepsilon(n)$ and $n^\varepsilon \equiv \arg \max q^\varepsilon(n)$. Now $q_{\max}^\varepsilon \leq -\Delta x_{n^\varepsilon} + \beta q_{\max}^\varepsilon (1 + \pi \Delta x_{n^\varepsilon})$, and $q_{\max}^\varepsilon \leq \frac{-\Delta x_{n^\varepsilon}}{1 - \beta(1 + \pi \Delta x_{n^\varepsilon})} \leq \frac{1}{1 - \beta(1 - \pi)}$. Finally

$$\frac{d\theta_n^*}{d\varepsilon} = \frac{dQ(n; \varepsilon, \pi, \beta)}{d\varepsilon} = -1 + \pi\beta q^\varepsilon(n) \leq -1 + \pi\beta q_{\max}^\varepsilon \leq -\frac{1 - \beta}{1 - \beta(1 - \pi)} < 0.$$

(ii) Define $q^\pi(n) = \frac{d[\pi v(n)]}{d\pi}$. Multiplying both sides of (38) by π and differentiating the resulting equation by π , yields $q^\pi(n) = \Delta u_n + \beta L[v(n)] - \beta v(n) + \beta L[q^\pi(n)]$. Using eq (38), this simplifies to $q^\pi(n) = (1 - \beta)v(n) + \beta L[q^\pi(n)]$. Let $q_{\min}^\pi \equiv \min q(n)$ and $n^\pi \equiv \arg \min q(n)$. Now $q_{\min}^\pi \geq (1 - \beta)v(n^\pi) + \beta q_{\min}^\pi (1 + \pi \Delta x_{n^\pi})$, and $q_{\min}^\pi \geq \frac{(1 - \beta)v(n^\pi)}{1 - \beta(1 + \pi \Delta x_{n^\pi})} \geq \frac{(1 - \beta)v(n^\pi)}{1 - \beta(1 - \pi)} > 0$. Finally

$$\frac{d\theta_n^*}{d\pi} = \frac{dQ(n; \varepsilon, \pi, \beta)}{d\pi} = \beta q^\pi(n) \geq \beta q_{\min}^\pi > 0.$$

(iii) Define $q^\beta(n) = \frac{d[\beta v(n)]}{d\beta}$. Multiplying both sides of (38) by β and differentiating the resulting equation by β , yields $q^\beta(n) = \Delta u_n + \beta L[v(n)] + \beta L[q^\beta(n)]$. Using eq (38), this simplifies to $q^\beta(n) = v(n) + \beta L[q^\beta(n)]$. Let $q_{\min}^\beta \equiv \min q(n)$ and $n^\beta \equiv \arg \min q(n)$. Now $q_{\min}^\beta \geq v(n^\beta) + \beta q_{\min}^\beta (1 + \pi \Delta x_{n^\beta})$, and $q_{\min}^\beta \geq \frac{v(n^\beta)}{1 - \beta(1 + \pi \Delta x_{n^\beta})} \geq \frac{v(n^\beta)}{1 - \beta(1 - \pi)} > 0$. Finally

$$\frac{d\theta_n^*}{d\beta} = \frac{dQ(n; \varepsilon, \pi, \beta)}{d\beta} = \pi q^\beta(n) \geq \pi q_{\min}^\beta > 0.$$

Proof of Proposition 2

(i) In the main text we used the welfare measure w , defined in (20). An alternative way to approach social welfare is to imagine that a new household enters the economy. The entrant is assigned to wealth class n with probability $f(n)$, and its expected intertemporal prospects are then given by the value function $V(n) = E_\theta[V(\theta, n)]$. The household's

prospects ex ante, i.e. before it knows its wealth and its match, are

$$W = \sum_{n=0}^{\infty} f(n) V(n). \quad (39)$$

We first establish a relationship between the welfare measures w and W ; this relationship is needed when we prove the proposition.

Using vector notation, equation (37) can be rewritten as follows

$$V = \max_{\{x_n\}} u + \beta [(1 - \pi) I + \pi A] V \quad (40)$$

for $n \geq 1$ (and $x_0 = 1$) where V is the (ex ante) value function, stacked as a column vector, u is a column vector with elements $u_n = u(x_n)$, and A is a transition matrix, with elements $A_{i,j} = 1 - x_i$ if $j = i - 1$, $A_{i,j} = x_i$ if $j = i + 1$ and $A_{i,j} = 0$ otherwise. Premultiplying both sides of (40) by the stationary wealth distribution f' yields $f'V = f'u + f'\beta [(1 - \pi) I + \pi A] V$. The distribution f is induced by the transition matrix A , and it satisfies the equation $f'A = f'$. But then we have

$$w = f'u = (1 - \beta) f'V = (1 - \beta) W \quad (41)$$

Essentially, W is the present value of a program with a (constant) per-period payoff w .

(ii) As proving the proposition with respect to ε , π and β involves the same steps, we introduce a generic parameter ρ , where $\rho \in \{\varepsilon, \pi, \beta\}$. Also, let x be the vector with the n th element x_n . Now

$$\frac{dw}{d\rho} \stackrel{(a)}{=} \frac{\partial w}{\partial \rho} + \frac{\partial w}{\partial x} \frac{dx}{d\rho} \stackrel{(b)}{=} \frac{\partial w}{\partial x} \frac{dx}{d\rho} \stackrel{(c)}{=} (1 - \beta) \frac{\partial W}{\partial x} \frac{dx}{d\rho} \stackrel{(d)}{=} (1 - \beta) V' \frac{df}{dx} \frac{dx}{d\rho}$$

Equality (a) involves a decomposition into the direct effect and the indirect effect. (b) follows from the fact that w does not depend directly on β , π and ε (see (20)), and thus $\frac{\partial w}{\partial \rho} = 0$. (c) follows from equality (41). (d) uses the definition of W , (39), and the envelope theorem: since the threshold θ_n^* , and thus also x_n , is optimally chosen in all wealth classes $n \geq 1$, a small policy change does not affect the value function $V(n)$.

By Lemma 1 we know that the wealth distribution shifts to the right, in the sense of first-order stochastic dominance, when ε decreases, and when π or β increases. As the value function $V(n)$ is increasing in n this shift in the stationary distribution translates into higher social welfare:

$$\frac{dw}{d\varepsilon} = (1 - \beta) V' \frac{df}{dx} \frac{dx}{d\varepsilon} \leq 0, \quad \frac{dw}{d\pi} = (1 - \beta) V' \frac{df}{dx} \frac{dx}{d\pi} \geq 0, \quad \frac{dw}{d\beta} = (1 - \beta) V' \frac{df}{dx} \frac{dx}{d\beta} \geq 0. \blacksquare$$

Proof of Proposition 5

(a) Sorting in the match dimension. When $\varepsilon \rightarrow 0$, the basic allocation problem vanishes, and the result is obvious. Next consider the case $\delta \rightarrow 1$. The household chooses

$\{x_n\}$, so as to maximize the value function V , where V satisfies the recursive equation $V = \delta AV + (1 - \delta) \frac{u}{1 - \beta}$. (This equation follows directly from (40).) Iterating forward, we get $V = (1 - \delta) \sum_{t=0}^{\infty} (\delta A)^t \frac{u}{1 - \beta}$. Next notice that $\lim_{t \rightarrow \infty} A^t = \mathbf{1} \otimes f'$ (where \otimes is Kronecker product). Thus when $\pi \rightarrow 1$ and $\beta \rightarrow 1$, so that $\delta \rightarrow 1$, maximizing V becomes essentially equivalent to maximizing $f'u = w = \frac{1}{2} E[\theta | h = 1]$. The objective function $w = \frac{1}{2} E[\theta | h = 1]$ is maximized iff there is perfect sorting in the match dimension.

(b) Sorting in the wealth dimension. The putative equilibrium strategy is of the following form: $h(0, \theta) = 0$ for all θ (due to the borrowing constraint), $h(n, \theta) = 1$ for all θ and $n \geq 1$. Then in equilibrium $f(0) = f(1) = \frac{1}{2}$ and $f(n) = 0$ for all $n \geq 2$.

Given this strategy, it is easy to calculate the ex ante values of the program $V(n)$ at different wealth levels $n \geq 0$. In particular, one can show that $V(2) - V(0) = (1 - \delta) \frac{\varepsilon + E[\theta]}{1 - \beta}$. Given the optimal location choice rule (6), the putative strategy is optimal for the household iff it always prefers the desirable location at wealth level $n = 1$, i.e., iff

$$\theta + \varepsilon > \pi\beta [V(2) - V(0)] = \pi\beta(1 - \delta) \frac{\varepsilon + E[\theta]}{1 - \beta} \text{ for all } \theta. \quad (42)$$

In particular, the condition (42) must hold for the lowest possible realization of the match θ_L . Inserting $\theta = \theta_L$, and slightly manipulating (42), yields the condition for residential sorting in the wealth dimension: $\theta_L + \varepsilon > \pi\beta \frac{E[\theta] - \theta_L}{1 - \beta}$. ■

Proof of Proposition 7

(a) *Wealth dimension.* We need to study wealth distributions conditional on housing location ($h \in \{0, 1\}$), and mobility ($m = 1$ for the households that have moved during the current period, and $m = 0$ for the households that have not moved). In the case of stayers ($m = 0$), we also need to condition on the realization of the regional shock ($s \in \{0, 1\}$). The main objective is to establish a first-order stochastic dominance relation between movers and stayers in each location.

Let $f(n | h, m; s)$ denote the frequency mass of households with wealth n conditional on h , m and s . The conditional cumulative distribution function of wealth is

$$F(n | h, m; s) = \frac{\sum_{i=0}^n f(i | h, m; s)}{\sum_{j=0}^{\infty} f(j | h, m; s)}$$

Now, from the discussion and analysis conducted at the beginning of Section 3.2 it follows that

$$f(n | h, m = 1; s) = \frac{1}{2} \tilde{\mu}(n; s) f(n), \quad h \in \{0, 1\}, \quad n \geq 1 \quad (43)$$

where $\tilde{\mu}(n; s)$ is mobility at wealth level n . (In the small region interpretation of the model, $\tilde{\mu}(n; s)$ is replaced by $\hat{\mu}(n)$.) Essentially, equation (43) tells that the wealth distribution of movers is identical in both locations. Notice that since $\tilde{\mu}(n; s) = (s + \lambda(1 - s)) \mu(n)$, $F(n | h, m = 1) = \mu(n) f(n) / \sum_{j=0}^{\infty} \mu(j) f(j)$ does not depend on the realization of s . Hence

$$F(n | h = 0, m = 1) = F(n | h = 1, m = 1), \quad n = 0, 1, \dots \quad (44)$$

where we have dropped s from the conditioning set.

Next, let $f(n | h)$ denote the frequency mass of households with wealth n conditional on the location h (evidently $f(n | h) = f_{n-h}^h(n)$, using the notation of Section 2.5). We denote the corresponding cumulative wealth distribution function by $F(n | h)$. Using equations (9), (10) and (14) we get $\frac{f(n|h=0)}{f(n-1|h=0)} = \hat{\gamma}(n) \equiv \frac{G(\theta_n^*)}{1-G(\theta_n^*)}$, $\frac{f(n|h=1)}{f(n-1|h=1)} = \hat{\gamma}(n-1)$ for $n = 0, 1, 2, \dots$, while using equations (15), (16), (23) and (43) we get $\frac{f(n|h \in \{0,1\}, m=1)}{f(n-1|h \in \{0,1\}, m=1)} = \tilde{\gamma}(n) \equiv \frac{G(\theta_n^*)}{1-G(\theta_{n-1}^*)}$ for $n = 0, 1, 2, \dots$. Now, since $G(\theta_n^*) \leq G(\theta_{n-1}^*)$ it is clear that $\hat{\gamma}(n) \leq \tilde{\gamma}(n) \leq \hat{\gamma}(n-1)$ and we can conclude that

$$F(n | h = 1) \leq F(n | h \in \{0, 1\}, m = 1) \leq F(n | h = 0) \quad (45)$$

That is, movers (in either location) are wealthier than (all) households living in cheap locations, but less wealthy than (all) households residing in expensive locations. Next, $F(n | h)$ is a convex combination of the wealth distributions of movers and stayers living in location h

$$F(n | h) = \bar{\mu}(s) F(n | h, m = 1) + (1 - \bar{\mu}(s)) F(n | h, m = 0; s), \quad h \in \{0, 1\}, \quad (46)$$

where $\bar{\mu}(s) = \sum_{n=0}^{\infty} \tilde{\mu}(n; s)$ is the aggregate share of movers in the economy. Then it follows from (45) and (46) that

$$F(n | h = 1, m = 0; s) \leq F(n | h \in \{0, 1\}, m = 1) \leq F(n | h = 0, m = 0; s) \quad (47)$$

That is, stayers living in the expensive location form the wealthiest group while stayers living in the cheap location is the least wealthy group. Movers are then in between.

Finally notice that the wealth distribution of stayers depends on the realization of the regional shock s . Since $\bar{\mu}(s = 1) > \bar{\mu}(s = 0)$, it follows from (45), (46) and (47) that

$$\begin{aligned} F(n | h = 1, m = 0; s = 1) &< F(n | h = 1, m = 0; s = 0) \\ F(n | h = 0, m = 0; s = 1) &> F(n | h = 0, m = 0; s = 0) \end{aligned} \quad (48)$$

In periods of a regional shock ($s = 1$) there is lots of mobility in the intermediate wealth groups; thus the borrowing constrained in the cheap location (who have been locked in by falling house prices) and the very wealthy in the expensive location make up a larger proportion of stayers than otherwise (when $s = 0$). (By contrast the distributions $F(n | h)$ - see equation (14) - and $F(n | h, m = 1)$ do not depend on the realization of the shock.)

Match dimension. To prove the proposition we need to construct, and compare, cumulative match distribution functions $G(\theta | h, m; s)$, contingent on housing location $h \in \{0, 1\}$ and mobility $m \in \{0, 1\}$ (and the regional shock $s \in \{0, 1\}$).

As a *first step*, we characterize the match distributions of households living in the desirable and in the undesirable location, conditional on wealth class n . Given the threshold location choice rule (8), the distribution in the desirable location $G(\theta | h = 1, n) = G(\theta | \theta \geq \theta_n^*) = \frac{G(\theta) - G(\theta_n^*)}{1 - G(\theta_n^*)}$ for $\theta \geq \theta_n^*$ (and 0 for $\theta < \theta_n^*$) is left-truncated, while the dis-

tribution in the undesirable location $G(\theta | h = 0, n) = G(\theta | \theta < \theta_n^*) = \frac{G(\theta)}{G(\theta_n^*)}$ for $\theta \leq \theta_n^*$ (and 1 for $\theta > \theta_n^*$) is right-truncated. Now, two properties follow from the threshold rule: (i) $G(\theta | h = 1, n = n_1) \leq G(\theta | h = 0, n = n_2)$ for all θ , and for all n_1, n_2 . That is, the match distribution of households living in a desirable location stochastically dominates the match distribution of households living in an undesirable location; the “typical” match at *any* wealth level n_1 in location $h = 1$ is higher than the “typical” match at *any* wealth level n_2 in location $h = 0$. (ii) It is easy to see that $\frac{\partial G(\theta | \theta \geq \theta_n^*)}{\partial \theta_n^*} \leq 0$ and $\frac{\partial G(\theta | \theta \leq \theta_n^*)}{\partial \theta_n^*} \leq 0$ for all θ . Now, since the threshold θ_n^* is decreasing in n (wealthier households are more likely to choose a desirable location), we get the first-order stochastic dominance relation

$$G(\theta | h, n'_1) \leq G(\theta | h, n'_2) \text{ for all } \theta, \text{ when } n'_1 < n'_2 \text{ and } h \in \{0, 1\}. \quad (49)$$

That is, the lower the wealth level n , the higher is the “typical” match in either location (desirable or undesirable).

As a *second step*, notice that the conditional match distributions $G(\theta | h, m; s)$ are convex combinations of location-contingent distributions $G(\theta | h, n)$ at different wealth levels n

$$G(\theta | h, m; s) = \sum_n \hat{f}(n | h, m; s) G(\theta | h, n), \text{ for } h, m, s \in \{0, 1\} \quad (50)$$

where $\hat{f}(n | h, m; s) = f(n | h, m; s) / \sum_{i=0}^{\infty} f(i | h, m; s)$ is the relative size of wealth class n in the group (m, h) (given the realizations of the shock). Now it follows from (50) and property (i) (see step 1 above) that

$$G(\theta | h = 1, m; s) \leq G(\theta | h = 0, m'; s), \text{ } m, m' \in \{0, 1\}, \text{ } s \in \{0, 1\}. \quad (51)$$

That is, households (both movers and stayers) living in an expensive location have higher match realizations, in the sense of first order stochastic dominance, than households (both movers and stayers) living in a cheap location. Next, combining equation (50) the stochastic dominance relationship (47) and the (stochastic dominance) property (ii) (see step 1 above) gives

$$\begin{aligned} G(\theta | h = 0, m = 0; s) &\leq G(\theta | h = 0, m = 1) \\ G(\theta | h = 1, m = 0; s) &\geq G(\theta | h = 1, m = 1) \end{aligned} \text{ for all } \theta \quad (52)$$

The expressions (52) mean that in a currently cheap location, the match distribution of old residents stochastically dominates the match distribution of newcomers, while in the currently expensive location the opposite is true. Thus we have proved that in both areas movers (with $m = 1$) tend to have a better match with the location than stayers ($m = 0$).

As a *third step*, we combine (51) and (52), and establish a stochastic dominance relation that covers all four groups:

$$\begin{aligned} G(\theta | h = 1, m = 1) &\leq G(\theta | h = 1, m = 0; s) \leq G(\theta | h = 0, m = 0; s) \\ &\leq G(\theta | h = 0, m = 1) \text{ for all } \theta, \text{ } s \in \{0, 1\} \end{aligned} \quad (53)$$

As a *final step*, notice that the match distributions of stayers depend on the realization of the regional shock. From (48), (49) and (50) we get

$$\begin{aligned} G(\theta \mid h = 0, m = 0; s = 1) &\leq G(\theta \mid h = 0, m = 0; s = 0) \\ G(\theta \mid h = 1, m = 0; s = 1) &\geq G(\theta \mid h = 1, m = 0; s = 0) \end{aligned} \quad \text{for all } \theta \quad (54)$$

In a period of a regional shock ($s = 1$), households that have been locked in by falling property values form a large portion of stayers in (currently) cheap locations; among this group there are many who have a high realization of the match θ . A similar logic holds for the expensive regions: when house prices rise in the region, many households become wealthy, due to the capital gains, and decide to stay put although they have a rather modest match θ . (By contrast, the match distributions of movers - in either location - do not depend on the realization of the shock.)

(b) *Wealth dimension.* It follows from (47) that interregional wealth differences are larger among stayers than among movers. Thus stayers are more sorted than movers in the wealth dimension. *Match dimension.* It follows from (53) that interregional match differences are larger among movers than among stayers. Thus movers are more sorted than stayers in the match dimension.

(c) *Wealth dimension.* It follows from (47) and (48) that the degree of sorting among stayers is greater when there is a regional shock ($s = 1$), while the degree of sorting among movers is not affected by the shock. Since stayers are anyway more sorted than movers, the difference between the sorting patterns of movers and stayers is more pronounced, when house prices change. *Match dimension.* It follows from (54) and (53) that the degree of sorting among stayers is smaller when there is a regional shock ($s = 1$), while the degree of sorting among movers is not affected by the shock. Since stayers are anyway less sorted than movers, the difference between the sorting patterns of movers and stayers is more pronounced, when house prices change. ■

Asymmetric locations

Using the threshold rule (8) and integrating (25) over all θ shows that the household's decision problem can be summarized by the Bellman equation

$$V(n) = \max_{x_n} u(x_n) + \beta \{V(n) + \pi(\phi x_n [V(n+1) - V(n)] + (1 - \phi)(1 - x_n) [V(n-1) - V(n)])\} \quad (55)$$

subject to $x_0 = 1$, where

$$u(x_n) \equiv (1 - \phi - x_n)\varepsilon + \int_{x_n}^1 G^{-1}(x) dx.$$

Next, let $\tilde{v}(n) \equiv V(n) - V(n)$, and $\tilde{\Delta}x_n \equiv x_n - x_{n-1}$; since θ_n^* is a non-increasing sequence, $\tilde{\Delta}x_n \in [-1, 0]$. Also let $\tilde{\Delta}u_n \equiv u(x_n) - u(x_{n-1}) = \int_{x_n}^{x_{n-1}} G^{-1}(x) dx - \tilde{\Delta}x_n \varepsilon$.

Next, define the operator \tilde{L}

$$\begin{aligned}\tilde{L}[z(n)] &= (1 - \pi(1 - \phi)(1 - x_n) - \pi\phi x_{n-1})z(n) \\ &\quad + \pi[\phi x_n z(n+1) + (1 - \phi)(1 - x_{n-1})z(n-1)]\end{aligned}$$

Since $V(n)$ satisfies the recursive equation (55), $\tilde{v}(n)$ satisfies the recursive equation

$$\tilde{v}(n) = \tilde{\Delta}u_n + \beta\tilde{L}[\tilde{v}(n)] \quad (56)$$

Finally, the critical match θ_n^* is given by

$$\theta_n^* = \tilde{Q}(n; \varepsilon, \pi, \beta) \equiv -\varepsilon + \pi\beta[\phi\tilde{v}(n+1) + (1 - \phi)\tilde{v}(n)] \text{ for } n \geq 1.$$

Proposition 1' For all $n \geq 1$, (i) $\frac{\partial\tilde{Q}(n; \varepsilon, \pi, \beta)}{\partial\varepsilon} < 0$, (ii) $\frac{\partial\tilde{Q}(n; \varepsilon, \pi, \beta)}{\partial\pi} > 0$ and (iii) $\frac{\partial\tilde{Q}(n; \varepsilon, \pi, \beta)}{\partial\beta} > 0$.

Proof (i) Define $\tilde{q}^\varepsilon(n) \equiv \frac{d\tilde{v}(n)}{d\varepsilon}$. Differentiating (56) yields $\tilde{q}^\varepsilon(n) = -\tilde{\Delta}x_n + \beta\tilde{L}[\tilde{q}^\varepsilon(n)]$. Let $\tilde{q}_{\max}^\varepsilon \equiv \max \tilde{q}^\varepsilon(n)$ and $\tilde{n}^\varepsilon \equiv \arg \max \tilde{q}^\varepsilon(n)$. Now $\tilde{q}_{\max}^\varepsilon \leq -\tilde{\Delta}x_{n^\varepsilon} + \beta\tilde{q}_{\max}^\varepsilon(1 + \pi\tilde{\Delta}x_{n^\varepsilon})$, and $\tilde{q}_{\max}^\varepsilon \leq \frac{-\tilde{\Delta}x_{n^\varepsilon}}{1 - \beta(1 + \pi\tilde{\Delta}x_{n^\varepsilon})} \leq \frac{1}{1 - \beta(1 - \pi)}$. Finally

$$\begin{aligned}\frac{d\theta_n^*}{d\varepsilon} &= \frac{d\tilde{Q}(n; \varepsilon, \pi, \beta)}{d\varepsilon} = -1 + \pi\beta[\phi\tilde{q}^\varepsilon(n+1) + (1 - \phi)\tilde{q}^\varepsilon(n)] \\ &\leq -1 + \pi\beta\tilde{q}_{\max}^\varepsilon \leq -\frac{1 - \beta}{1 - \beta(1 - \pi)} < 0.\end{aligned}$$

(ii) Define $\tilde{q}^\pi(n) = \frac{d[\pi\tilde{v}(n)]}{d\pi}$. Multiplying both sides of (56) by π and differentiating the resulting equation by π yields $\tilde{q}^\pi(n) = \tilde{\Delta}u_n + \beta\tilde{L}[\tilde{v}(n)] - \beta\tilde{v}(n) + \beta\tilde{L}[\tilde{q}^\pi(n)]$. Using eq. (56), this simplifies to $\tilde{q}^\pi(n) = (1 - \beta)\tilde{v}(n) + \beta\tilde{L}[\tilde{q}^\pi(n)]$. Let $\tilde{q}_{\min}^\pi \equiv \min \tilde{q}^\pi(n)$ and $\tilde{n}^\pi \equiv \arg \min \tilde{q}^\pi(n)$. Now $\tilde{q}_{\min}^\pi \geq (1 - \beta)\tilde{v}(\tilde{n}^\pi) + \beta\tilde{q}_{\min}^\pi(1 + \pi\tilde{\Delta}x_{\tilde{n}^\pi})$, and $\tilde{q}_{\min}^\pi \geq \frac{(1 - \beta)\tilde{v}(\tilde{n}^\pi)}{1 - \beta(1 + \pi\tilde{\Delta}x_{\tilde{n}^\pi})} \geq \frac{(1 - \beta)\tilde{v}(\tilde{n}^\pi)}{1 - \beta(1 - \pi)} > 0$. Finally

$$\frac{d\theta_n^*}{d\beta} = \frac{d\tilde{Q}(n; \varepsilon, \pi, \beta)}{d\beta} = \beta[\phi\tilde{q}^\pi(n+1) + (1 - \phi)\tilde{q}^\pi(n)] \geq \beta\tilde{q}_{\min}^\pi > 0.$$

(iii) Define $\tilde{q}^\beta(n) = \frac{d[\beta\tilde{v}(n)]}{d\beta}$. Multiplying both sides of (56) by β and differentiating the resulting equation by β yields $\tilde{q}^\beta(n) = \tilde{\Delta}u_n + \beta\tilde{L}[\tilde{v}(n)] + \beta\tilde{L}[\tilde{q}^\beta(n)]$. Using eq (56), this simplifies to $\tilde{q}^\beta(n) = \tilde{v}(n) + \beta\tilde{L}[\tilde{q}^\beta(n)]$. Let $\tilde{q}_{\min}^\beta \equiv \min \tilde{q}^\beta(n)$ and $\tilde{n}^\beta \equiv \arg \min \tilde{q}^\beta(n)$. Now $\tilde{q}_{\min}^\beta \geq \tilde{v}(\tilde{n}^\beta) + \beta\tilde{q}_{\min}^\beta(1 + \pi\tilde{\Delta}x_{\tilde{n}^\beta})$, and $\tilde{q}_{\min}^\beta \geq \frac{\tilde{v}(\tilde{n}^\beta)}{1 - \beta(1 + \pi\tilde{\Delta}x_{\tilde{n}^\beta})} \geq \frac{\tilde{v}(\tilde{n}^\beta)}{1 - \beta(1 - \pi)} > 0$. Finally

$$\frac{d\theta_n^*}{d\beta} = \frac{dQ(n; \varepsilon, \pi, \beta)}{d\beta} = \pi[\phi\tilde{q}^\beta(n+1) + (1 - \phi)\tilde{q}^\beta(n)] \geq \pi\tilde{q}_{\min}^\beta > 0.$$

■

Lemma 1' Let $F(n; \varepsilon, \pi, \beta) = \sum_{i=0}^n f(i)$. For all $n = 0, 1, \dots$, we have $dF(n; \varepsilon, \pi, \beta)/d\varepsilon \geq 0$, $dF(n; \varepsilon, \pi, \beta)/d\pi \leq 0$ and $dF(n; \varepsilon, \pi, \beta)/d\beta \leq 0$.

Proof The result follows from Proposition 1' and equation (30). See the proof of Lemma 1. ■

Proposition 2' Let $w = \sum_{n=0}^{\infty} f(n) u(x_n)$. Then $\frac{dw}{d\varepsilon} < 0$, $\frac{dw}{d\pi} > 0$, $\frac{dw}{d\beta} > 0$.

Proof The proof consists of the same steps, (i) and (ii), as the proof of Proposition 2. Step (i): Using vector notation, the Bellman equation (55) can be rewritten as

$$V = \max_x u + \beta \tilde{A}V \quad (57)$$

where \tilde{A} is a transition matrix, with elements $\tilde{A}_{i,i} = 1 - \pi(1 - \phi)(1 - x_i) - \pi\phi x_i$, $\tilde{A}_{i,j} = \pi(1 - \phi)(1 - x_i)$ if $j = i - 1$, $\tilde{A}_{i,j} = \pi\phi x_i$ if $j = i + 1$ and $\tilde{A}_{i,j} = 0$ otherwise. Premultiplying both sides of (57) by the stationary wealth distribution f' yields $f'V = f'u + f'\beta\tilde{A}V$. The distribution f is induced by the transition matrix \tilde{A} , and it satisfies the equation $f'\tilde{A} = f'$. But then we have

$$w = f'u = (1 - \beta)f'V = (1 - \beta)W \quad (58)$$

Essentially, $W = f'V$ is the present value of a program with a (constant) per-period payoff w . Step (ii). See step (ii) in the proof of Proposition 2. ■

Proposition 3' When (i) the size of regional shocks (ε) decreases, (ii) the regional shocks become more frequent (π increases), or (iii) the households become more patient (β increases) the degree of residential sorting in the match dimension increases in the sense stated in Proposition 2.

Proof When conditions (i),(ii) and/or (iii) hold, it follows from Proposition 2' that $E[\theta | h = 1]$ increases.

(a) Then, since $\phi E[\theta | h = 1] + (1 - \phi)E[\theta | h = 0] = E[\theta]$, and $E[\theta]$ is a constant, it follows that $E[\theta | h = 0]$ decreases. Thus the difference $|E[\theta | h] - E[\theta]|$ increases for $h \in \{0, 1\}$.

(b) Item (a) implies that the between-locations variance $Var(E[\theta | h]) = (1 - \phi)(E[\theta | h = 0] - E[\theta])^2 + \phi(E[\theta | h = 1] - E[\theta])^2$ increases.

(c) The economywide variance of the match $Var(\theta)$ can be decomposed $Var(\theta) = Var(E[\theta | h]) + E[Var(\theta | h)]$. Since $Var(\theta)$ is a constant, it follows from item (b) that the within-locations component $E[Var(\theta | h)]$ must decrease. ■

Proposition 4' When (i) the size of regional shocks (ε) increases, (ii) the regional shocks become less frequent (π increases), or (iii) the households become less patient (β increases) the degree of residential sorting in the wealth dimension increases in the sense stated in Proposition 3.

Proof Notice that equations (30) imply that the distribution of financial assets is identical in both location types. Then the result follows from Lemma 1'. See Proof of Proposition 4. ■

Remark 1' Assume that regional shocks become smaller (ε decreases) or more frequent

(π increases), or that households become more patient (β increases). Then the size of house price fluctuations decreases in the sense stated in Remark 1.

Proof The result follows from equation (31) and Lemma 1'. ■

Corollary 1' The size of house price fluctuations and the pattern of residential sorting are related in the sense stated in Corollary 1.

Proof The result follows from Propositions 3' and 4' and Remark 1'. ■

More general match dynamics

Let $v(\theta, n) \equiv V(\theta, n+1) - V(\theta, n-1)$ and $\Delta h(\theta, n) \equiv h(\theta, n+1) - h(\theta, n-1)$. Also define the operator \widehat{L}

$$\begin{aligned} \widehat{L}[z(\theta, n)] &\equiv (1 - \sigma)z(\theta, n) + \lambda E_{\tilde{\theta}} \left[z(\tilde{\theta}, n) \mid \theta, s = 0 \right] \\ &+ \pi E_{\tilde{\theta}} \left[h(\theta, n-1)z(\tilde{\theta}, n-1) + (1 - h(\theta, n+1))z(\tilde{\theta}, n+1) \mid \theta, s = 1 \right], \end{aligned}$$

where $z(\theta, n)$ is a generic function of θ and n . Since $V(\theta, n)$ satisfies the Bellman equation (32), the function $v(\theta, n)$ satisfies the recursive equation

$$v(\theta, n) = \Delta h(\theta, n)(\varepsilon + \theta) + \beta \widehat{L}[v(\theta, n)]. \quad (59)$$

For all θ and all $n \geq 1$, the household's location choice rule assumes the form $h(\theta, n) = 1$ iff $\theta \geq \widehat{Q}(\theta, n; \varepsilon, \sigma, \beta)$ (and 0 otherwise), where $\widehat{Q}(\theta, n; \varepsilon, \sigma, \beta) \equiv -\varepsilon + \pi \beta E_{\tilde{\theta}} \left[v(\tilde{\theta}, n) \mid \theta, s = 1 \right]$.

Proposition 1'' For all θ and $n \geq 1$, (i) $\frac{d\widehat{Q}(\theta, n; \varepsilon, \sigma, \beta)}{d\varepsilon} < 0$, (ii) $\frac{d\widehat{Q}(\theta, n; \varepsilon, \sigma, \beta)}{d\sigma} > 0$ and (iii) $\frac{d\widehat{Q}(\theta, n; \varepsilon, \sigma, \beta)}{d\beta} > 0$.

Proof (i) Define $\widehat{q}^\varepsilon(\theta, n) \equiv \frac{dv(\theta, n)}{d\varepsilon}$. Differentiating (59) with respect to ε shows that $\widehat{q}^\varepsilon(\theta, n)$ satisfies the equation $\widehat{q}^\varepsilon(\theta, n) = \Delta h(\theta, n) + \beta \widehat{L}[\widehat{q}^\varepsilon(\theta, n)]$. Next define $\widehat{q}_{\max}^\varepsilon \equiv \max \widehat{q}^\varepsilon(\theta, n)$ and $\{\widehat{\theta}^\varepsilon, \widehat{n}^\varepsilon\} \equiv \arg \max \widehat{q}^\varepsilon(\theta, n)$. Then

$$\widehat{q}_{\max}^\varepsilon \leq \Delta h(\widehat{\theta}^\varepsilon, \widehat{n}^\varepsilon) + \beta \widehat{q}_{\max}^\varepsilon (1 - \pi \Delta h(\widehat{\theta}^\varepsilon, \widehat{n}^\varepsilon)),$$

and we can conclude that

$$\widehat{q}_{\max}^\varepsilon \leq \frac{\Delta h(\widehat{\theta}^\varepsilon, \widehat{n}^\varepsilon)}{1 - \beta(1 - \pi \Delta h(\widehat{\theta}^\varepsilon, \widehat{n}^\varepsilon))} \leq \frac{1}{1 - \beta(1 - \pi)}.$$

Finally,

$$\begin{aligned} \frac{d\widehat{Q}(\theta, n; \varepsilon, \sigma, \beta)}{d\varepsilon} &= -1 + \pi \beta E_{\tilde{\theta}} \left[\widehat{q}^\varepsilon(\tilde{\theta}, n) \mid \theta, s = 1 \right] \\ &\leq -1 + \pi \beta \widehat{q}_{\max}^\varepsilon \leq -\frac{1 - \beta}{1 - \beta(1 - \pi)} < 0. \end{aligned}$$

(ii) Define $\hat{q}^\sigma(\theta, n) \equiv \frac{d[\sigma v(\theta, n)]}{d\sigma}$. Multiplying both sides of (59) by σ and differentiating with respect to σ yields $\hat{q}^\sigma(\theta, n) = \Delta h(\theta, n)(\varepsilon + \theta) + \beta \hat{L}[v(\theta, n)] - \beta v(\theta, n) + \beta \hat{L}[\hat{q}^\sigma(\theta, n)]$. Using (59), this simplifies to $\hat{q}^\sigma(\theta, n) = (1 - \beta)v(\theta, n) + \beta \hat{L}[\hat{q}^\sigma(\theta, n)]$. Next define $\hat{q}_{\min}^\sigma \equiv \min \hat{q}^\sigma(\theta, n)$ and $\{\hat{\theta}^\sigma, \hat{n}^\sigma\} \equiv \arg \min \hat{q}^\sigma(\theta, n)$. Then

$$\hat{q}_{\min}^\sigma \geq (1 - \beta)v(\hat{\theta}^\sigma, \hat{n}^\sigma) + \beta q_{\min}^\sigma (1 - \pi \Delta h(\hat{\theta}^\sigma, \hat{n}^\sigma)),$$

and

$$\hat{q}_{\min}^\sigma \geq \frac{(1 - \beta)v(\hat{\theta}^\sigma, \hat{n}^\sigma)}{1 - \beta(1 - \pi \Delta h(\hat{\theta}^\sigma, \hat{n}^\sigma))} \geq \frac{(1 - \beta)v(\hat{\theta}^\sigma, \hat{n}^\sigma)}{1 - \beta(1 - \pi)} > 0.$$

Finally,

$$\frac{d\hat{Q}(\theta, n; \varepsilon, \sigma)}{d\sigma} = \xi \beta E_{\tilde{\theta}} \left[\hat{q}^\sigma(\tilde{\theta}, n) \mid \theta, s = 1 \right] \geq \xi \beta \hat{q}_{\min}^\sigma > 0.$$

(iii) Define $\hat{q}^\beta(\theta, n) \equiv \frac{d[\beta v(\theta, n)]}{d\beta}$. Multiplying both sides of (59) by β , and differentiating with respect to β yields $\hat{q}^\beta(\theta, n) = \Delta h(\theta, n)(\varepsilon + \theta) + \beta \hat{L}[v(\theta, n)] + \beta \hat{L}[\hat{q}^\beta(\theta, n)]$. Using (59), this simplifies to $\hat{q}^\beta(\theta, n) = v(\theta, n) + \beta \hat{L}[\hat{q}^\beta(\theta, n)]$. Next define $\hat{q}_{\min}^\beta \equiv \min \hat{q}^\beta(\theta, n)$ and $\{\hat{\theta}^\beta, \hat{n}^\beta\} \equiv \arg \min \hat{q}^\beta(\theta, n)$. Now

$$\hat{q}_{\min}^\beta \geq v(\hat{\theta}^\beta, \hat{n}^\beta) + \beta q_{\min}^\beta (1 - \pi \Delta h(\hat{\theta}^\beta, \hat{n}^\beta)),$$

and

$$\hat{q}_{\min}^\beta \geq \frac{v(\hat{\theta}^\beta, \hat{n}^\beta)}{1 - \beta(1 - \pi \Delta h(\hat{\theta}^\beta, \hat{n}^\beta))} \geq \frac{v(\hat{\theta}^\beta, \hat{n}^\beta)}{1 - \beta(1 - \pi)} > 0.$$

Finally,

$$\frac{d\hat{Q}(\theta, n; \varepsilon, \sigma, \beta)}{d\beta} = \pi E_{\tilde{\theta}} \left[\hat{q}^\beta(\tilde{\theta}, n) \mid \theta, s = 1 \right] \geq \pi \hat{q}_{\min}^\beta > 0.$$

■

Stationary distribution. Let $\hat{f}_n(\theta_j)$ denote the long-run frequency mass of households with match θ_j and wealth n , and let \hat{f}_n be a $J \times 1$ vector, with the j th element $\hat{f}_n(\theta_j)$. Also let H_n , $n \geq 1$, be a $J \times J$ diagonal matrix, with the j th diagonal element $h(\theta_j, n)$ (and all off-diagonal elements equal to 0), and let $B_n = I - H_n$. The stationary distribution satisfies the following set of recursive equations

$$\hat{f}_n' = (1 - \sigma) \hat{f}_n' + \lambda \hat{f}_n' \Lambda_0 + \pi \left(\hat{f}_{n-1}' B_{n-1} + \hat{f}_{n+1}' H_{n+1} \right) \Lambda_1$$

for all $n = 0, 1, \dots$. Simplifying yields

$$\hat{f}_n' = (1 - \xi) \hat{f}_n' \Lambda_0 + \xi \left(\hat{f}_{n-1}' B_{n-1} + \hat{f}_{n+1}' H_{n+1} \right) \Lambda_1 \quad (60)$$

Notice in particular that the parameters ε and σ do not appear in (60), and thus the joint distribution of wealth and the match depends on these parameters only indirectly, through changes in policies.

Equilibrium. Postmultiplying both sides of equation (60) by the unit vector $\mathbf{1}$, and taking into account the fact that $\Lambda_0\mathbf{1} = \Lambda_1\mathbf{1} = \mathbf{1}$, yields a set of recursive equations for the marginal distribution of wealth

$$f(n) = f_{n-1}^0(n-1) + f_n^1(n+1) \quad (61)$$

where $f(n) = \widehat{f}_n^1\mathbf{1}$ is the frequency mass of households at wealth level n , $f_n^0(n) = \widehat{f}_n^0 B_n\mathbf{1}$ is the mass of households at wealth level n residing in an unpopular location, and $f_{n-1}^1(n) = \widehat{f}_n^1 H_n\mathbf{1}$ is the mass of households at wealth level n residing in a popular location. But equation (61) is identical to equation (13) so that equilibrium follows in the same way as in Section 2.5.

Lemma 1' Define the cumulative distribution function $\widehat{F}(\theta_j, n; \varepsilon, \sigma, \beta) = \sum_{i=0}^n \widehat{f}_i(\theta_j)$. Then $\frac{d\widehat{F}(\theta_j, n; \varepsilon, \sigma, \beta)}{d\varepsilon} \geq 0$, $\frac{d\widehat{F}(\theta_j, n; \varepsilon, \sigma, \beta)}{d\sigma} \leq 0$ and $\frac{d\widehat{F}(\theta_j, n; \varepsilon, \sigma, \beta)}{d\beta} \leq 0$ for all n and θ_j .

Proof Define a history as a collection of match realizations and regional shock realizations $\mathcal{H}_t = \{(\theta_\tau, s_\tau)\}_{\tau=0}^t$. Notice that histories are exogenous in the sense that they do not depend on the households' location choices. Denote a state by $y = (\theta, n)$. Consider two location choice rules h^0 and h^1 such that for some state \widehat{y} , $h^0(\widehat{y}) = 0$ and $h^1(\widehat{y}) = 1$, and for all other states $y \neq \widehat{y}$, $h^0(y) = h^1(y) = h(y)$ (where $h(y)$ is the common policy).

Next notice that there is a mapping from histories \mathcal{H}_t to states y_t , conditional on policy h^i , $i \in \{0, 1\}$ (and initial state). That is, at any date t , the household's wealth $n_t^i = n^i(\mathcal{H}_t)$ and the state $y_t^i = y^i(\mathcal{H}_t)$, where $i \in \{0, 1\}$ refers to the policy that the household follows.

Consider two households. Household 0 follows policy h^0 , while household 1 follows policy h^1 . Assume the households have the same history \mathcal{H}_t . Define $\nu_t \equiv n_t^0 - n_t^1$ and notice that by equation (3) it obeys the law of motion $\nu_{t+1} = \nu_t + 2s_{t+1}(h^1(y_t^1) - h^0(y_t^0))$. Obviously,

$$\Delta\nu_t \equiv \nu_{t+1} - \nu_t = 2s_{t+1}(h^1(y_t^1) - h^0(y_t^0)) \in \{-2, 0, 2\}. \quad (62)$$

Assume that for some period t , $\nu_t = 0$ so that also $y_t^0 = y_t^1$. Given the properties of h^0 and h^1 it is evident that

$$\Delta\nu_t \in \{0, 2\}, \text{ if } \nu_t = 0. \quad (63)$$

($\Delta\nu_t = 2$ iff $y_t^0 = y_t^1 = \widehat{y}$ and $s_{t+1} = 1$). Next, assume the households have the same initial wealth, $\nu_0 = 0$. From (62) and (63) it follows that $\nu_t = 2k$, $k \in \{0, 1, 2, \dots\}$ for all $t = 0, 1, 2, \dots$. The essential finding is that, given identical histories and equal initial wealth, household 1 cannot be wealthier than household 0.

Assume that there is a population of households following policy h^0 , and another population following policy h^1 . Also assume that all households, in either population, have the same initial wealth. As above, we refer to a household belonging to population 0 (1) as household 0 (1). Now, the proof of the lemma derives from the following observations. (i) After any given (common) history \mathcal{H}_t , household 0 is at least as wealthy as household 1. (ii) After any given (common) history \mathcal{H}_t , household 0 and household 1 have the same

match. (iii) The probability distribution over the histories does not depend on policy. (iv) In any period t , and for any given current match, the wealth distribution under policy h^0 stochastically dominates the wealth distribution under policy h^1 . (v) When $t \rightarrow \infty$, the joint distribution of wealth and the match converges to the stationary distribution. Thus stochastic dominance applies to the stationary distribution. Finally, Proposition 1" and Lemma 1" imply that when ε increases, or when σ or β decreases, the households may shift from policy h^0 to policy h^1 , but the opposite shift (from policy h^1 to policy h^0) never happens. ■

Proposition 2" Social welfare grows, when ε decreases, or when σ or β increases,

Proof Let us define a KJ state Markov chain y , where the $(nJ + j)$ th state is given by the pair (θ_j, n) . Notice that K (the number of wealth levels) is $\bar{n} + 1$, if $\theta_L > -\varepsilon$, and otherwise $K = \infty$. Let \mathbf{h} be a $KJ \times 1$ vector, with the $(nJ + j)$ th element $h(\theta_j, n)$. Further define a $KJ \times KJ$ diagonal matrix H , with the vector \mathbf{h} on the diagonal (and all off-diagonal elements equal to zero), and let the $KJ \times KJ$ matrix \hat{A} be the transition matrix of the Markov chain y .

The value function can be presented as a $KJ \times 1$ vector \hat{V} , where the $(nJ + j)$ th element is the value of the household's program in state (θ_j, n) . \hat{V} satisfies the Bellman equation

$$\hat{V} = H(\mathbf{1}_K \otimes \boldsymbol{\theta}) + \left(\mathbf{h} - \frac{1}{2} \mathbf{1}_{KJ} \right) \varepsilon + \beta \left[(1 - \sigma) I + \sigma \hat{A} \right] \hat{V}, \quad (64)$$

where $\boldsymbol{\theta}$ is the $J \times 1$ vector of types θ_j . The stationary distribution of y is a $KJ \times 1$ vector \hat{f} . The distribution is induced by the transition matrix \hat{A} and it satisfies the equation $\hat{f}' = \hat{f}' \hat{A}$. Now define the measures of social welfare

$$\begin{aligned} \hat{w} &\equiv \sum_n \sum_j \hat{f}_n(\theta_j) h(\theta_j, n) \theta_j = \hat{f}' H(\mathbf{1}_K \otimes \boldsymbol{\theta}) = \frac{1}{2} E[\theta \mid h = 1] \\ \hat{W} &\equiv \sum_n \sum_j \hat{f}_n(\theta_j) V(\theta_j, n) = \hat{f}' \hat{V} \end{aligned}$$

Next we premultiply both sides of (64) by \hat{f}' . Then using the fact that $\hat{f}' = \hat{f}' \hat{A}$, and noting that $\hat{f}' (\mathbf{h} - \frac{1}{2} \mathbf{1}_{KJ}) = 0$, by the housing market equilibrium, yields

$$\hat{W} = \hat{w} + \beta \hat{W} \Leftrightarrow \hat{W} = \hat{w} / (1 - \beta). \quad (65)$$

Given the equation (65), and Lemma 1", Proposition 3" can be proved following the same steps as in the proof of Proposition 3. See part (ii) of the proof. ■

Proposition 3" When ε increases, or σ or β decreases, the degree of residential sorting in the match dimension decreases in the sense explained in Proposition 2.

Proof The result follows from Proposition 2'. See the proof of Proposition 3. ■

Proposition 4" When ε increases, or σ or β decreases, the degree of residential sorting in the wealth dimension increases in the sense explained in Proposition 4.

Proof The results follows from Lemma 1'. See the proof of Proposition 4. ■

Remark 1” Assume that regional shocks become smaller (ε decreases) or more frequent (π increases), or that households become more patient (β increases). Then the size of house price fluctuations decreases in the sense stated in Remark 1.

Proof The result follows from equation (19) and Lemma 1”. ■

Corollary 1” The size of house price fluctuations and the pattern of residential sorting are related in the sense stated in Corollary 1.

Proof The result follows from Propositions 3” and 4” and Remark 1”. ■

Appendix B: Data description

Description of variables of Tables 2-6

The “house price volatility” is described in the main text.

Except the house price volatility measures, the variables in Tables 2, 3 and 5 are computed from extraction of data from the 1990 decennial Census, published in the ICPSR study 2889 (1990). We apply the data set 2 (DS2) where each variable is aggregated to the municipality (MCD) level. Because MCDs are geographically comprehensive, our MSA level observations are formed by summing up all relevant MCD level data. We apply the MSA classification defined by the variable “v7” in the DS2 data set (see ICPSR study 2889 (1990)). For each of the MSAs, we find a corresponding MSA in the MSA classification of the house price data.

The sorting measures for age, education and income are based on the following groups of types. Five groups for age: (1) “children” (those of 0-15 years old), (2) “youth” (16-24 years old), (3) “adults, early career” (25-44 years old), (4) “adults, late career” (45-64 years old), and (5) “seniors” (those at least 65 years old). Three groups for education: (1) less than a high school degree, (2) at least a high school degree but not a college degree, and (3) a college degree or more. The Census defines the education groups for only those who are at least 25 years old. This age category is used to normalize the education groups within each region. Finally, for income we apply all the 25 income groups available in the ICPSR study 2889. In each of the cases, the US level groups are obtained by a population weighted average of the MSA level groups. The education and income categories applied here are similar to those of the dissimilarity indices and Gini coefficients considered by Rhode and Strumpf (2003, p. 1660) (see also their Data Appendix at www.unc.edu/~cigar/ or www.unc.edu/~prhode/).

With reference to the original variable symbols in the DS2 data set (see ICPSR study 2889 (1990)) we use: “v9” for “Number of municipalities”; “v103” and “v9” for “Average population of municipalities”; “v103” and “v121” for “Population density in MSA”; “v103” and “v121” for “Land area of MSA”; “v103” for “Number of families in MSA.”; “v1804” and “v1801” for “Rent share” (i.e., the share of people that live in rental housing). Finally, the dissimilarity index for the age of housing units (“ D_{HoAge} ”) assumes three groups: houses build (1) “at most 5 years ago,” (2) “6-10 years ago,” and (3) “at least 11 years ago.” The corresponding measure for the number of housing units in a residential building (“ D_{HoUnit} ”) assumes three groups: (1) “1-unit structures,” (2) “2-4

unit structures” and (3) “5 or more unit structures.”

The additional regressors in Tables 4 and 6, “Physical constraints” (the land topographic unavailability measure of Saiz (2010)) and “Regulatory constraints” (the Wharton Residential Urban Land Regulation Index of Gyourko, Saiz, and Summers (2008)), are obtained from <http://real.wharton.upenn.edu/~saiz/SUPPLYDATA.zip>.

The samples of the regressions in Tables 2-6 constitute all those MSAs for which the applied variables are available.

Description of sorting measures of Tables 7-10

The data applied in Tables 7-10 are from the Census data provided at www.ipums.org. The web site provides detailed definitions for each variable in the data. For each observation unit (i.e., person) in the 1% sample from the 1990 Census, we downloaded household id (SERIAL), age (AGE), educational attainment (EDUC99), household income (FTOTINC), tenure (OWNERSHP), migration information (MIGRATE5, MIGMET5, MIGPLAC5) and location indicators (PUMA, STATEFIP, METAREA). These data include observations on 2,479,568 persons from 1760 different PUMAs. The actual number of people in each PUMA is also obtained from www.ipums.org.

To compute the sorting measures applied in Tables 7-9, we classify each sample person into a mover (MIGRATE5 = 2) or a stayer (MIGRATE5 = 1). Furthermore, we classify a person as an owner, if OWNERSHP = 10 and a renter, if OWNERSHP = 20. Persons with missing observations on MIGRATE5 or OWNERSHP are excluded from the calculations. The sorting measures for age, education and income apply similar categories as in Tables 2-6. For age, we estimate the shares of “children,” “youth,” etc. in each PUMA (and MSA) by computing the relative shares of the sample persons belonging to the relevant age category (for “children” the share of those 0-15 years old, etc.). For education, we restrict the sample to those at least 25 years old. The three education groups (consistent with those in Tables 2-6) are formed by (1) $EDUC99 \leq 9$, (2) $10 \leq EDUC99 \leq 11$, and (3) $12 \leq EDUC99$. Finally, to compute the index for income, we first restrict the sample to household heads only (SERIAL = 1). Then we employ FTOTINC to classify each household into one of the 25 income ranges used in the ICPSR data, and compute the corresponding relative shares in each PUMA (and MSA).

To compute the sorting measures applied in Table 10, we first restrict the sample into persons that have moved recently (MIGRATE5 = 2). Within this subsample, we classify a person as a “short distance mover,” if his current MSA is the same as five years ago, i.e., if METAREA and MIGMET5 match; otherwise the person is classified as a “long distance mover.” In addition to data on persons that have moved from one MSA region to another, we also use data on persons that have moved *from* or *to* a non-MSA region. If a person has moved from an MSA region to a non-MSA region, or vice versa, he or she is recorded as a “long distance mover,” while a person that has moved between two non-MSA regions is recorded as a “long distance mover” only, if his or her current state of residence (STATEFIP) is different from that five years ago (MIGPLAC5). Otherwise, the sorting measures with respect to income, education and age are formed by applying the same procedures as in Tables 7-9.

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Table 1. Maximum and minimum house-price-to-income ratios, 1979-1996

	House-price-to-income ratio	
	min	max
Boston	5.4	12.0
New York	5.3	12.0
Los Angeles	6.7	11.1
San Francisco	6.4	11.4

Source: Malpezzi, 1999

Table 2. Correlation between sorting measures and house price volatility

		Measure of house price volatility			
		<i>benchmark</i>	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>
Income	<i>D</i>	-.257	-.154	-.136	-.210
	<i>GC</i>	-.253	-.158	-.131	-.209
	<i>T</i>	-.189	-.158	-.129	-.181
Education	<i>D</i>	-.282	-.291	-.206	-.250
	<i>GC</i>	-.293	-.296	-.202	-.254
	<i>T</i>	-.281	-.295	-.226	-.270
Age	<i>D</i>	-.243	-.080	-.141	-.166
	<i>GC</i>	-.268	-.130	-.143	-.186
	<i>T</i>	-.186	-.053	-.139	-.144

Notes: Correlations are reported between measures of residential sorting and house price volatility. For each of the types (income, education, and age), the applied sorting measures are the dissimilarity index D in (34), the Gini coefficient GC in (35), and the Theil's information theory index T in (36). See Appendix B for definitions of the applied income, educational and age categories. The applied measures of house price volatility are computed from monthly house price indices Jan 1975 through Dec 2000. "*Benchmark*" is the standard deviation (SD) of the MSA house price relative to the US house price; "*filter*" is the SD of (HP-filter) detrended relative house price, while "*a.ampl.*" ("*m.ampl.*") is the average (maximum) difference between two consecutive turning points of relative house price. The sample size is 275.

Table 3. OLS regressions of sorting measures on house price volatility and selected covariates

Dependent variable: Sorting index of	Income			Education			Age		
	<i>D</i> (1)	<i>GC</i> (2)	<i>T</i> (3)	<i>D</i> (4)	<i>GC</i> (5)	<i>T</i> (6)	<i>D</i> (7)	<i>GC</i> (8)	<i>T</i> (9)
Independent variable									
House price volatility	-.103*** (.035)	-.128*** (.048)	-.016 (.010)	-.222*** (.056)	-.278*** (.068)	-.088*** (.022)	-.151*** (.037)	-.167*** (.040)	-.037*** (.010)
Number of municipalities	.012* (.006)	.022** (.009)	.007*** (.002)	-.007 (.007)	-.007 (.009)	-.002 (.003)	.000 (.005)	.006 (.007)	.000 (.002)
Average population of municipalities	-.070*** (.009)	-.106*** (.013)	-.019*** (.003)	-.036*** (.010)	-.060*** (.012)	-.012*** (.004)	-.014 (.009)	-.024** (.012)	-.003 (.003)
Population density in MSA	.203*** (.035)	.286*** (.046)	.034*** (.012)	.148*** (.054)	.252*** (.065)	.035* (.021)	.000 (.032)	.057* (.034)	-.015 (.009)
Land area of MSA	.103*** (.021)	.159*** (.028)	.022*** (.007)	.139*** (.044)	.213*** (.054)	.053*** (.016)	.049** (.024)	.096*** (.026)	.012 (.009)
Number of families in MSA	-.040** (.019)	-.063* (.027)	-.013* (.007)	.032 (.033)	.037 (.046)	.018 (.015)	-.004 (.017)	-.021 (.020)	.000 (.006)
Constant	.143*** (.008)	.191*** (.010)	.026*** (.003)	.154*** (.012)	.194*** (.014)	.039*** (.005)	.097*** (.009)	.113*** (.009)	.017*** (.003)
R^2	.410	.479	.388	.223	.299	.227	.087	.156	.063

Notes: The dependent variable, a sorting measure, varies by type (“Income,” “Education,” “Age”) and the applied measure (“*D*,” “*GC*,” “*T*”). “House price volatility” is the “benchmark” measure (see notes to Table 2). “Number of municipalities” is the number of MCDs divided by 100, “Average population of municipalities” is the average population of MCDs (divided by 10000), “Population density in MSA” is the number of families in MSA per square kilometer (divided by 1000), “Land area of MSA” is the size of MSA area in squared kilometers (divided by 10000). “Number of families in MSA” is the size of MSA population (in millions). The White’s robust standard errors for the coefficient estimates are reported in parentheses. The ***, ** and * indicates statistical significance at 1%, 5% and 10% level, respectively. Sample size is 275.

Table 4. OLS regressions of sorting measures on house price volatility and selected covariates

Dependent variable: Sorting index of	Income			Education			Age		
	<i>D</i> (1)	<i>GC</i> (2)	<i>T</i> (3)	<i>D</i> (4)	<i>GC</i> (5)	<i>T</i> (6)	<i>D</i> (7)	<i>GC</i> (8)	<i>T</i> (9)
House price volatility	-.079*** (.032)	-.101*** (.042)	-.013 (.009)	-.234*** (.060)	-.287*** (.073)	-.094*** (.024)	-.101*** (.039)	-.110*** (.040)	-.031*** (.011)
Number of municipalities	-.006 (.006)	-.002 (.008)	.002 (.003)	-.015* (.009)	-.020* (.011)	-.006 (.004)	.000 (.005)	.002 (.006)	.000 (.001)
Average population of municipalities	-.048*** (.010)	-.076*** (.012)	-.015*** (.002)	-.008 (.016)	-.022 (.019)	-.002 (.006)	-.011 (.012)	-.016 (.014)	-.003 (.004)
Population density in MSA	-.027 (.058)	.002 (.070)	-.017 (.022)	-.127* (.077)	-.100 (.089)	-.063* (.033)	-.184*** (.046)	-.164*** (.044)	-.055*** (.015)
Land area of MSA	.023 (.025)	.058* (.032)	.004 (.010)	.051 (.048)	.095 (.060)	.020 (.018)	.003 (.026)	.036 (.027)	.004 (.010)
Number of families in MSA	.015 (.023)	.003 (.029)	.002 (.009)	.074** (.036)	.090* (.047)	.034** (.016)	.019 (.017)	.009 (.019)	.005 (.006)
Physical constraints	.001 (.010)	.001 (.013)	-.003 (.004)	.008 (.020)	.011 (.022)	.001 (.008)	.016 (.013)	.019 (.012)	.004 (.004)
Regulatory constraints	.002 (.003)	.001 (.005)	.001 (.001)	.002 (.005)	.002 (.006)	.001 (.002)	.008*** (.003)	.008*** (.003)	.002* (.001)
Diversity of the age of housing units (D_{HoAge})	.160*** (.030)	.210*** (.036)	.029*** (.008)	.119** (.055)	.196*** (.064)	.051*** (.021)	-.014 (.035)	.023 (.036)	-.015 (.010)
Diversity of the number of housing units ($D_{HoUnits}$)	.201*** (.033)	.275*** (.044)	.052*** (.009)	.270*** (.063)	.356*** (.075)	.099*** (.028)	.244*** (.043)	.273*** (.042)	.062*** (.014)
Rent share	-.103*** (.038)	-.128*** (.054)	-.032* (.018)	.140*** (.050)	.156*** (.059)	.044** (.021)	.135*** (.042)	.128*** (.044)	.043*** (.014)
Constant	.121*** (.018)	.155*** (.025)	.025*** (.008)	.053** (.025)	.062** (.030)	.004 (.011)	.014 (.015)	.021 (.016)	-.004 (.005)
R^2	.613	.666	.516	.368	.453	.348	.358	.421	.283

Notes: Notes to Table 3 apply with the following addition. “Physical constraints” is the land topographic unavailability measure of Saiz (2010). “Regulatory constraints” is the Wharton Residential Urban Land Regulation Index of Gyourko, Saiz, and Summers (2008). “Diversity of the age of housing units” and “Diversity of the number of housing units”, respectively, is the dissimilarity index (34) computed for the age of the housing units and the number of housing units in a residential building, see appendix for the definition of the groups in both cases. “Rent share” denotes the share of households that live in rental housing in the MSA. Sample size is 225.

Table 5. OLS regressions of sorting measures on alternative house price volatility measures and selected covariates

Dependent variable: Sorting index (D) of	Income			Education			Age		
	<i>filter</i> (1)	<i>a.ampl.</i> (2)	<i>m.ampl.</i> (3)	<i>filter</i> (4)	<i>a.ampl.</i> (5)	<i>m.ampl.</i> (6)	<i>filter</i> (7)	<i>a.ampl.</i> (8)	<i>m.ampl.</i> (9)
Independent variable									
House price volatility (meas. by col.)	-.361 (.220)	-.077*** (.023)	-.029*** (.011)	-1.047*** (.296)	-.157*** (.036)	-.063*** (.017)	-0.163 (.190)	-.069*** (.025)	-.028** (.014)
Number of municipalities	.013** (.006)	.011* (.006)	.011* (.006)	-.004 (.007)	-.008 (.007)	-.008 (.007)	.001 (.005)	.000 (.005)	.000 (.005)
Average population of municipalities	-.073*** (.009)	-.074*** (.009)	-.071*** (.009)	-.042*** (.010)	-.044*** (.010)	-.039*** (.010)	-.019* (.010)	-.020** (.010)	-.017* (.010)
Population density in MSA	.211*** (.038)	.226*** (.035)	.217*** (.034)	.164*** (.055)	.196*** (.053)	.177*** (.052)	.014 (.033)	.027 (.032)	.018 (.031)
Land area of MSA	.100*** (.024)	.113*** (.021)	.106*** (.021)	.125*** (.042)	.160*** (.042)	.143*** (.042)	.052** (.024)	.061** (.024)	.053** (.024)
Number of families in MSA	-.045** (.020)	-.044*** (.019)	-.042** (.019)	.019 (.037)	.024 (.032)	.029 (.033)	-.007 (.017)	-.008 (.016)	-.006 (.016)
Constant	.139*** (.010)	.143*** (.008)	.141*** (.008)	.152*** (.012)	.152*** (.012)	.150*** (.011)	.081*** (.008)	.089*** (.009)	.088*** (.009)
R^2	.400	.412	.409	.213	.224	.219	.044	.061	.060

Notes: Notes to Table 3 apply with the following addition. The applied dependent variable, indicated by “Income”, “Education” and “Age”, is computed by using the dissimilarity index (34) for each type (see Appendix B for definitions of the groups in each case). The column titles “*filter*”, “*a.ampl.*” and “*m.ampl.*” indicate the applied measure of house price volatility; “*filter*”, the standard deviation of (HP-filter) detrended MSA house prices, “*a.ampl.*” (“*m.ampl.*”) the mean (the maximum) difference between two consecutive turning points of MSA house prices.

Table 6. OLS regressions of sorting measures on alternative house price volatility measures and selected covariates

Dependent variable	Income			Education			Age		
	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
House price volatility (meas. by col.)	-.333** (.149)	-.060*** (.021)	-.023** (.009)	-1.089*** (.315)	-.195*** (.041)	-.072*** (.018)	-.183 (.181)	-.078*** (.027)	-.027** (.011)
Number of municipalities	-.005 (.006)	-.007 (.006)	-.006 (.006)	-.012 (.009)	-.018** (.008)	-.016* (.008)	.000 (.005)	-.002 (.005)	.001 (.005)
Average population of municipalities	-.051*** (.010)	-.054*** (.010)	-.049*** (.010)	-.018 (.017)	-.028* (.017)	-.011 (.016)	-.014 (.013)	-.019 (.013)	-.012 (.012)
Population density in MSA	-.028 (.056)	-.018 (.057)	-.019 (.057)	-.130 (.075)	-.096 (.072)	-.102 (.076)	-.185*** (.047)	-.171*** (.045)	-.175*** (.046)
Land area of MSA	.017 (.027)	.027 (.026)	.023 (.025)	.032 (.046)	.067 (.045)	.052 (.047)	-.001 (.026)	.009 (.026)	.003 (.026)
Number of families in MSA	.012 (.022)	.016 (.023)	.015 (.023)	.063 (.041)	.077** (.035)	.072** (.036)	.016 (.018)	.020 (.017)	.018 (.017)
Physical constraints	.006 (.011)	.006 (.011)	.002 (.011)	.025 (.021)	.024 (.021)	.013 (.020)	.020 (.014)	.023 (.014)	.018 (.013)
Regulatory constraints	.003 (.003)	.004 (.003)	.003 (.003)	.005 (.005)	.007 (.005)	.004 (.005)	.009*** (.003)	.010*** (.003)	.009*** (.003)
Diversity of the age of housing units (D_{HoAge})	.163*** (.030)	0.157*** (.030)	.159*** (.030)	.127** (.055)	.110* (.057)	.115** (.057)	-.013 (.036)	-.018 (.036)	-.016 (.036)
Diversity of the number of housing units (D_{HoUnit})	.201*** (.034)	.201*** (.033)	.202*** (.033)	.269*** (.069)	.268*** (.063)	.272*** (.064)	.249*** (.046)	.244*** (.042)	.246*** (.043)
Rent share	-.107*** (.037)	-.100*** (.037)	-.104*** (.038)	.129*** (.051)	.151*** (.050)	.138*** (.049)	.128*** (.042)	.139*** (.042)	.133*** (.042)
Constant	.120*** (.019)	.112*** (.018)	.120*** (.018)	.053** (.025)	.060*** (.025)	.052** (.024)	.006 (.016)	.015 (.014)	.012 (.014)
R^2	.609	.614	.613	.355	.383	.373	.341	.361	.355

Notes: For the definitions of the applied variables see notes to Tables 4 and 5.

Table 2A. Correlation between sorting measures and house price volatility

		Measure of house price volatility			
		<i>benchmark</i>	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>
Income	<i>D</i>	-.164	-.145	-.186	-.189
	<i>GC</i>	-.151	-.147	-.184	-.189
	<i>T</i>	-.097	-.153	-.185	-.157
Education	<i>D</i>	-.255	-.281	-.257	-.237
	<i>GC</i>	-.255	-.284	-.254	-.243
	<i>T</i>	-.255	-.285	-.268	-.252
Age	<i>D</i>	-.203	-.075	-.087	-.156
	<i>GC</i>	-.223	-.123	-.110	-.177
	<i>T</i>	-.164	-.051	-.071	-.128

Notes: Notes to table 2 apply except that the applied measures of house price volatility are computed from monthly house price indices Jan 1975 through Dec 1990.

Table 3A. OLS regressions of sorting measures on house price volatility and selected covariates

Dependent variable: Sorting index of	Income			Education			Age		
	D (1)	G (2)	T (3)	D (4)	G (5)	T (6)	D (7)	G (8)	T (9)
Independent variable									
House price volatility	-.106*** (.031)	-.133*** (.042)	-.016** (.008)	-.214*** (.047)	-.273*** (.059)	-.082*** (.019)	-.122*** (.030)	-.142*** (.034)	-.029*** (.008)
Number of municipalities	.012* (.006)	.021** (.010)	.007*** (.002)	-.007 (.007)	-.008 (.009)	-.002 (.003)	.000 (.005)	.005 (.007)	.000 (.002)
Average population of municipalities	-.074*** (.009)	-.111*** (.013)	-.020*** (.003)	-.044*** (.010)	-.071*** (.012)	-.015*** (.004)	-.020** (.009)	-.031*** (.011)	-.004 (.003)
Population density in MSA	.217*** (.034)	.302*** (.044)	.036*** (.011)	.177*** (.052)	.288*** (.063)	.047*** (.020)	.019 (.031)	.077** (.033)	-.010 (.009)
Land area of MSA	.104*** (.022)	.159*** (.029)	.022*** (.008)	.141*** (.040)	.215*** (.049)	.054*** (.015)	.051** (.025)	.098*** (.026)	.012 (.009)
Number of families in MSA	-.042** (.018)	-.064*** (.027)	-.013* (.007)	.029 (.032)	.034 (.044)	.017 (.014)	-.006 (.017)	-.023 (.019)	-.001 (.006)
Constant	.145*** (.008)	.193*** (.010)	.026*** (.003)	.154*** (.011)	.196*** (.013)	.039*** (.004)	.094*** (.008)	.111*** (.008)	.017*** (.002)
R^2	.415	.483	.390	.229	.307	.230	.078	.151	.056

Notes: Notes to Table 3 apply except that the house price volatility measure is computed by using house price data from 1975 to 1990.

Table 4A: OLS regressions of sorting measures on house price volatility and selected covariates

Dependent variable: Sorting index of	Income			Education			Age		
	<i>D</i> (1)	<i>GC</i> (2)	<i>T</i> (3)	<i>D</i> (4)	<i>GC</i> (5)	<i>T</i> (6)	<i>D</i> (7)	<i>GC</i> (8)	<i>T</i> (9)
House price volatility	-.072*** (.029)	-.097*** (.038)	-.011 (.008)	-.206*** (.050)	-.261*** (.063)	-.079*** (.021)	-.077*** (.032)	-.095*** (.034)	-.024*** (.009)
Number of municipalities	-.006 (.006)	-.002 (.008)	.002 (.002)	-.015* (.008)	-.020* (.011)	-.006 (.004)	.000 (.005)	.002 (.006)	.000 (.001)
Average population of municipalities	-.051*** (.010)	-.080*** (.012)	-.015*** (.002)	-.017 (.016)	-.034* (.019)	-.006 (.006)	-.015 (.012)	-.020 (.014)	-.005 (.004)
Population density in MSA	-.016 (.058)	.017 (.071)	-.015 (.022)	-.095 (.076)	-.061 (.090)	-.051 (.033)	-.172*** (.047)	-.149*** (.045)	-.051*** (.016)
Land area of MSA	.024 (.026)	.059* (.032)	.004 (.011)	.054 (.045)	.099* (.055)	.021 (.017)	.004 (.026)	.037 (.027)	.004 (.010)
Number of families in MSA	.014 (.023)	.001 (.029)	.002 (.009)	.070** (.035)	.085* (.046)	.033** (.016)	.017 (.017)	.007 (.019)	.004 (.006)
Physical constraints	.002 (.011)	.002 (.013)	-.003 (.004)	.011 (.020)	.015 (.022)	.002 (.008)	.018 (.013)	.020 (.013)	.004 (.004)
Regulatory constraints	.002 (.003)	.001 (.005)	.001 (.002)	.003 (.005)	.003 (.006)	.001 (.002)	.008*** (.003)	.009*** (.003)	.002* (.001)
Diversity of the age of housing units (D_{HoAge})	.159*** (.030)	.209*** (.036)	.029*** (.008)	.116** (.056)	.192*** (.065)	.050*** (.021)	-.015 (.035)	.021 (.037)	-.016 (.010)
Diversity of the number of housing units ($D_{HoUnits}$)	.200*** (.033)	.273*** (.044)	.052*** (.009)	.267*** (.064)	.352*** (.076)	.098*** (.028)	.244*** (.044)	.272*** (.042)	.062*** (.015)
Rent share	-.102*** (.039)	-.127*** (.055)	-.032* (.018)	.141*** (.050)	.158*** (.059)	.044** (.021)	.135*** (.042)	.129*** (.044)	.043*** (.014)
Constant	.121*** (.018)	.156*** (.025)	.025*** (.008)	.051*** (.025)	.061*** (.029)	.003** (.011)	.011 (.015)	.020 (.015)	-.004 (.005)
R^2	.614	.667	.516	.368	.456	.345	.353	.420	.279

Notes: Notes to Table 4 apply except that the house price volatility measure is computed by using house price data from 1975 to 1990.

Table 5A. OLS regressions of sorting measures on alternative house price volatility measures and selected covariates

Independent variable	Income			Education			Age		
	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
House price volatility (meas. by col.)	-.270 (.170)	-.064*** (.021)	-.028*** (.010)	-.786*** (.228)	-.154*** (.030)	-.059*** (.016)	-.114 (.149)	-.029 (.025)	-.026*** (.013)
Number of municipalities	.013** (.006)	.011* (.006)	.012* (.006)	-.004 (.007)	-.009 (.007)	-.008 (.007)	.001 (.005)	.000 (.005)	.001 (.005)
Average population of municipalities	-.073*** (.009)	-.072*** (.009)	-.072*** (.009)	-.042*** (.010)	-.040*** (.010)	-.040*** (.010)	-.019* (.010)	-.019* (.010)	-.018* (.010)
Population density in MSA	.211*** (.038)	.217*** (.035)	.218*** (.034)	.166*** (.055)	.180*** (.054)	.181*** (.052)	.014 (.033)	.021 (.032)	.020 (.031)
Land area of MSA	.101*** (.024)	.107*** (.021)	.106*** (.021)	.126*** (.042)	.147*** (.042)	.144*** (.042)	.053** (.024)	.056*** (.024)	.054** (.024)
Number of families in MSA	-.045** (.020)	-.041** (.019)	-.042** (.019)	.020 (.037)	.031 (.031)	.028 (.033)	-.007 (.017)	-.006 (.016)	-.007 (.016)
Constant	.138*** (.010)	.141*** (.008)	.140*** (.008)	.150*** (.012)	.152*** (.011)	.147*** (.011)	.080*** (.008)	.082*** (.009)	.087*** (.008)
R^2	.400	.409	.408	.211	.231	.216	.043	.046	.058

Notes: Notes to Table 5 apply except that the house price volatility measure is computed by using house price data from 1975 to 1990.

Table 6A. OLS regressions of sorting measures on alternative house price volatility measures and selected covariates

Dependent variable	Income			Education			Age		
	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>	<i>filter</i>	<i>a.ampl.</i>	<i>m.ampl.</i>
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
House price volatility (meas. by col.)	-.251** (.117)	-.055*** (.020)	-.021** (.009)	-.820*** (.245)	-.189*** (.035)	-.066*** (.017)	-.126 (.143)	-.044* (.025)	-.024*** (.010)
Number of municipalities	-.005 (.006)	-.007 (.006)	-.006 (.006)	-.012 (.009)	-.019** (.008)	-.015* (.008)	.000 (.005)	-.001 (.005)	-.001 (.005)
Average population of municipalities	-.051*** (.010)	-.052*** (.010)	-.050*** (.010)	-.018 (.017)	-.023 (.016)	-.013 (.016)	-.014 (.013)	-.015 (.013)	-.013 (.012)
Population density in MSA	-.028 (.056)	-.023 (.057)	-.018 (.057)	-.128* (.076)	-.111 (.073)	-.098 (.076)	-.184*** (.047)	-.180*** (.046)	-.173*** (.046)
Land area of MSA	.017 (.027)	.024 (.025)	.024 (.025)	.032 (.046)	.062 (.045)	.054 (.046)	-.001 (.026)	.004 (.026)	.004 (.026)
Number of families in MSA	.012 (.022)	.018 (.023)	.014 (.022)	.064 (.041)	.074*** (.034)	.071 (.071)	.016 (.018)	.020 (.017)	.018 (.017)
Physical constraints	.006 (.011)	.005 (.011)	.002 (.011)	.024 (.022)	.023 (.021)	.012 (.020)	.020 (.014)	.020 (.014)	.018 (.013)
Regulatory constraints	.003 (.003)	.004 (.003)	.003 (.003)	.005 (.005)	.007 (.005)	.004 (.005)	.009*** (.003)	.010*** (.003)	.009*** (.003)
Diversity of the age of housing units (D_{HoAge})	.163*** (.030)	.157*** (.030)	.159*** (.031)	.127** (.055)	.106* (.055)	.114** (.057)	-.013 (.036)	-.017 (.036)	-.016 (.036)
Diversity of the number of housing units ($D_{HoUnits}$)	.202*** (.034)	.203*** (.033)	.202*** (.033)	.271*** (.069)	.278*** (.063)	.274*** (.064)	.250*** (.046)	.249*** (.043)	.247*** (.043)
Rent share	-.108*** (.037)	-.104*** (.037)	-.104*** (.038)	.127*** (.051)	.140*** (.048)	.138*** (.049)	.128*** (.042)	.131*** (.043)	.133*** (.042)
Constant	.119*** (.019)	.122*** (.019)	.119*** (.018)	.051** (.025)	.061*** (.023)	.048* (.024)	.006 (.016)	.010 (.013)	.010 (.014)
R^2	.608	.614	.612	.352	.393	.367	.341	.348	.352

Notes: Notes to Table 6 apply except that the house price volatility measure is computed by using house price data from 1975 to 1990.

Table 7. Sorting among movers and stayers

		Owners		Renters	
		Movers	Stayers	Movers	Stayers
Income	<i>D</i>	.190	.137	.163	.243
	<i>GC</i>	.218	.159	.189	.278
	<i>T</i>	.045	.021	.035	.085
Education	<i>D</i>	.137	.112	.133	.145
	<i>GC</i>	.161	.132	.154	.167
	<i>T</i>	.029	.021	.028	.032
Age	<i>D</i>	.077	.071	.080	.117
	<i>GC</i>	.089	.083	.093	.138
	<i>T</i>	.010	.008	.010	.022

Notes: The entries of the table report the means across 103 MSAs of sorting measures of movers and stayers. The measures are reported separately for owner-occupiers and renters.

Table 8: Differences in the sorting patterns of movers and stayers: owner-occupiers

	Income			Education			Age		
	$D_m - D_s$	$GC_m - GC_s$	$T_m - T_s$	$D_m - D_s$	$GC_m - GC_s$	$T_m - T_s$	$D_m - D_s$	$GC_m - GC_s$	$T_m - T_s$
MSAs classified by house price volatility:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
All MSAs	.053*** (.003)	.059*** (.004)	.024*** (.002)	.025*** (.005)	.030*** (.005)	.008*** (.001)	.006*** (.003)	.007** (.003)	.0017*** (.0006)
1st quartile	.050*** (.006)	.055*** (.007)	.018*** (.002)	.030*** (.011)	.036** (.012)	.008** (.003)	.003 (.005)	.005 (.006)	.0011 (.001)
2nd quartile	.043*** (.005)	.049*** (.007)	.017*** (.003)	.015** (.007)	.018** (.008)	.005** (.002)	.000 (.005)	.000 (.005)	.0006 (.001)
3rd quartile	.045*** (.007)	.053*** (.009)	.023*** (.004)	.015** (.007)	.021** (.008)	.008*** (.003)	-.003 (.004)	-.004 (.005)	-.0003 (.0013)
4th quartile	.074*** (.006)	.080*** (.008)	.038*** (.005)	.041*** (.012)	.044*** (.012)	.013*** (.003)	.024*** (.007)	.026*** (.008)	.0055*** (.0015)

Notes: The entries of the table report the differences of the means across 103 MSAs of sorting measures of movers and stayers. The results are for owner-occupiers. The White's robust standard errors are reported in parentheses. The comparisons are reported for all MSAs and subgroups defined by the quartiles of the house price volatility. The measure of house price fluctuation is the maximum amplitude of the relative house prices (see text) over the period 1975-1990.

Table 9. Differences in the sorting patterns of movers and stayers: renters

	Income			Education			Age		
	$D_m - D_s$	$GC_m - GC_s$	$T_m - T_s$	$D_m - D_s$	$GC_m - GC_s$	$T_m - T_s$	$D_m - D_s$	$GC_m - GC_s$	$T_m - T_s$
MSAs classified by house price volatility:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
All MSAs	-.079*** (.007)	-.089*** (.007)	-.050*** (.004)	-.012* (.007)	-.014* (.008)	-.0037 (.0023)	-.037*** (.004)	-.045*** (.005)	-.012*** (.002)
1st quartile	-.066*** (.010)	-.075*** (.012)	-.043*** (.006)	-.013 (.013)	-.016 (.014)	.000 (.003)	-.042*** (.009)	-.050*** (.010)	-.012*** (.004)
2nd quartile	-.090*** (.010)	-.101*** (.013)	-.055*** (.007)	-.018 (.014)	-.026 (.017)	-.011** (.005)	-.042*** (.007)	-.049*** (.007)	-.014*** (.002)
3rd quartile	-.117*** (.014)	-.134*** (.015)	-.072*** (.008)	-.032*** (.009)	-.033*** (.011)	-.010*** (.0035)	-.047*** (.010)	-.059*** (.011)	-.017*** (.004)
4th quartile	-.043*** (.014)	-.046*** (.014)	-.031*** (.008)	.015 (.016)	.020 (.017)	.007 (.005)	-.017*** (.006)	-.021*** (.007)	-.0054*** (.0016)

Notes: Notes to Table 8 apply except that the applied sorting measures concern renters.

Table 10. Differences in the sorting patterns of long and short distance movers

	Income			Education			Age		
	$D_{lo} - D_{sh}$	$GC_{lo} - GC_{sh}$	$T_{lo} - T_{sh}$	$D_{lo} - D_{sh}$	$GC_{lo} - GC_{sh}$	$T_{lo} - T_{sh}$	$D_{lo} - D_{sh}$	$GC_{lo} - GC_{sh}$	$T_{lo} - T_{sh}$
Owners	.117*** (.010)	.121*** (.010)	.087*** (.009)	.033*** (.007)	.037*** (.008)	.016*** (.003)	.037*** (.006)	.041*** (.006)	.016*** (.003)
Renters	.046*** (.006)	.050*** (.007)	.028*** (.004)	.026*** (.007)	.029*** (.008)	.012*** (.003)	.031*** (.005)	.034*** (.006)	.012*** (.003)

Notes: The table reports, separately for owner-occupiers and renters, the differences of the means across 103 MSAs of sorting measures of long and short distance movers. The White's robust standard errors are reported in parentheses. All differences are statistically significant at 1% level.

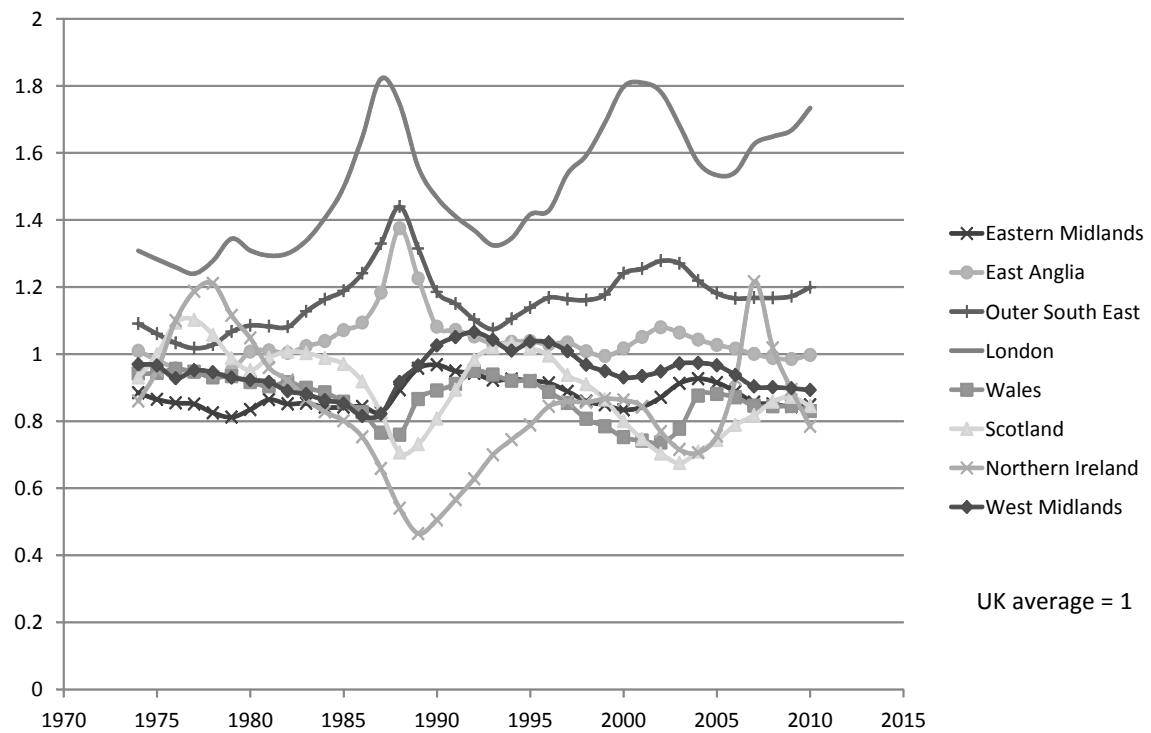


Figure 1: Relative house prices in the UK (Source: Nationwide Building Society)

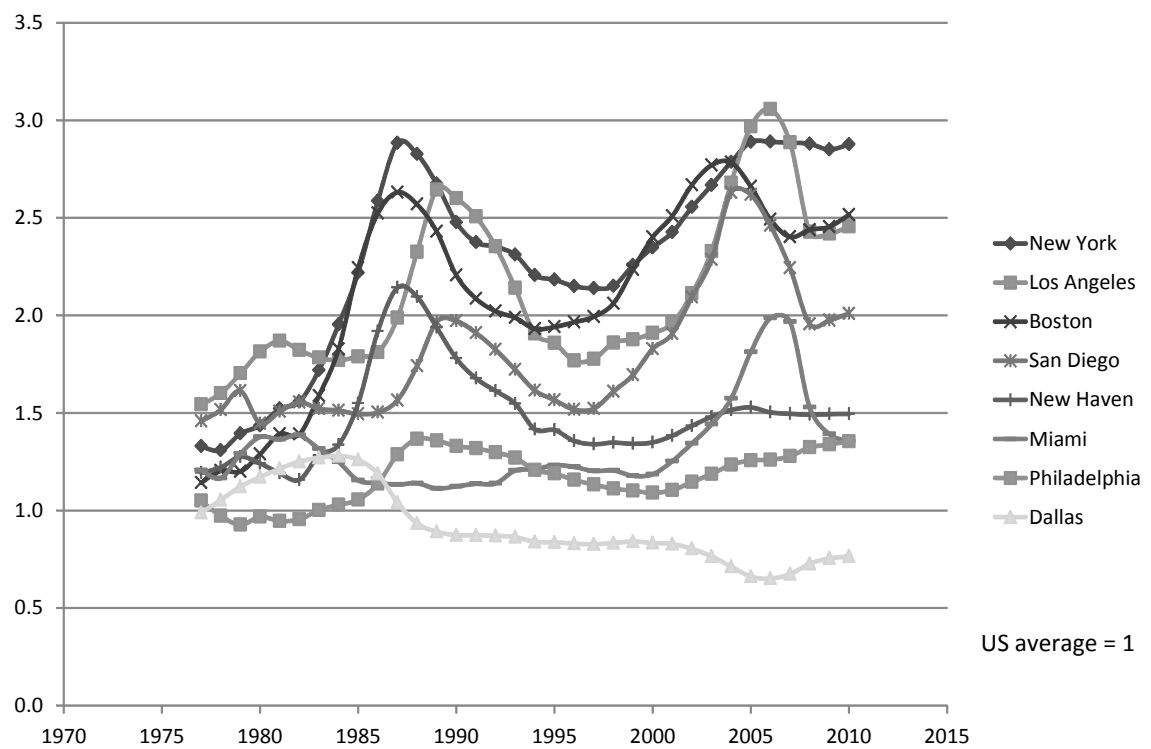


Figure 2: Relative house prices in the US (Source: Federal Housing Finance Agency)

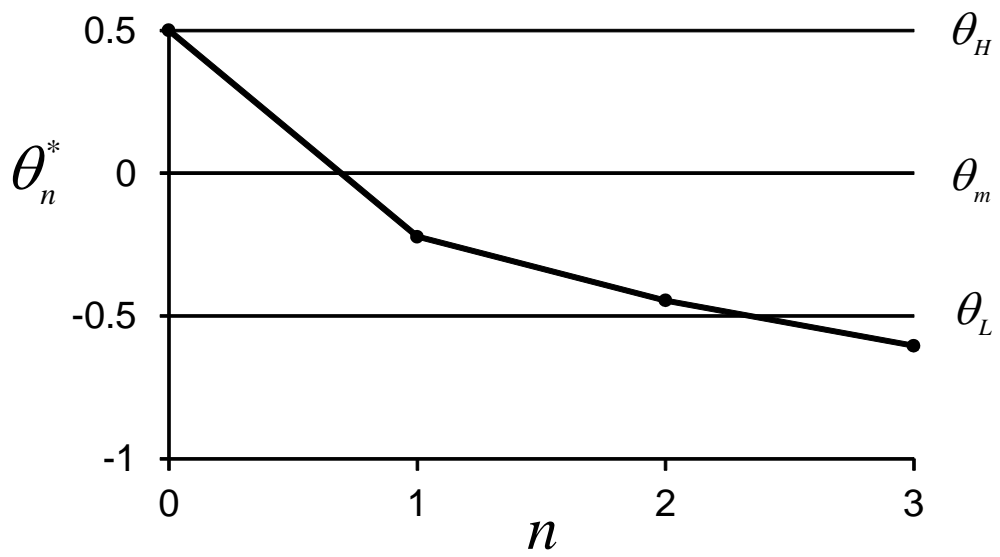


Figure 3: The θ_n^* -curve when θ is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\varepsilon = 1$, $\beta = .95$, and $\pi = .3$.

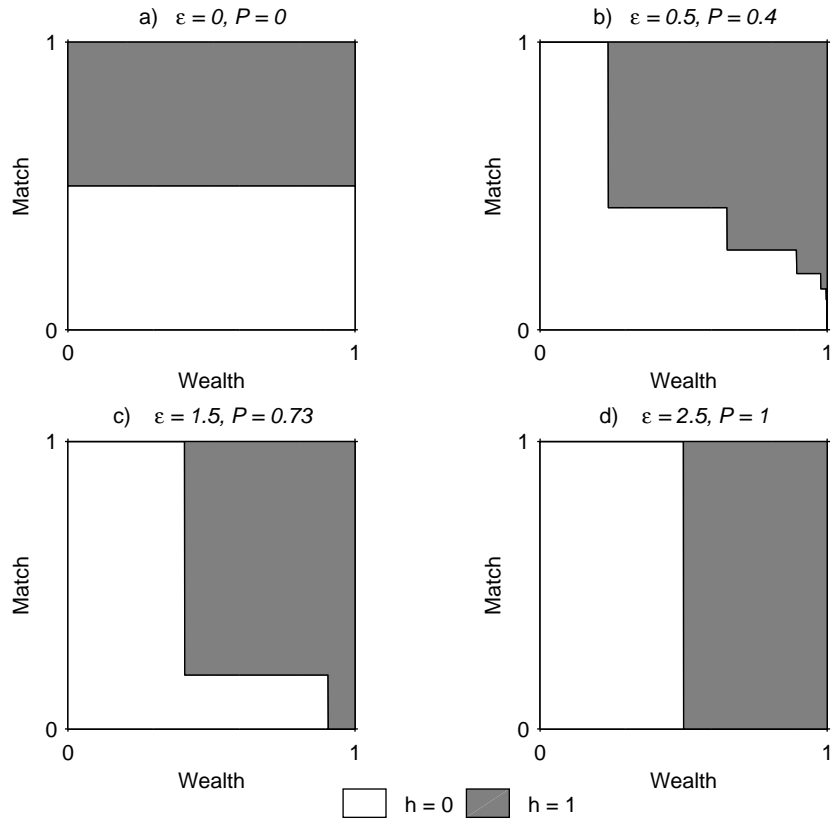


Figure 4: Equilibrium pattern of residential sorting with different values of the regional shock, ε , when the match, θ , is uniformly distributed on $[-\frac{1}{2}, \frac{1}{2}]$, $\beta = .95$, and $\pi = .2$. In each panel, the cumulative wealth (match) distribution is measured on the horizontal (vertical) axis. The measure of house price fluctuations is $P = \frac{1}{2} \frac{1}{E[n]}$, $P \in [0, 1]$.

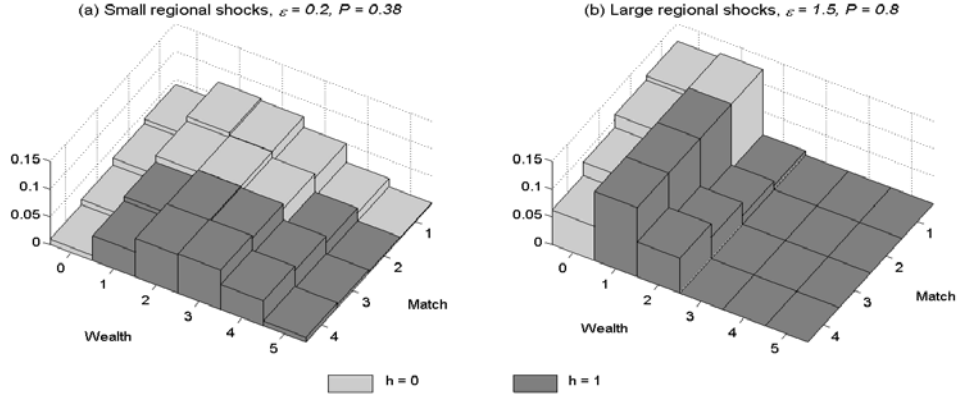


Figure 5: Equilibrium distribution of wealth, match and location, with two values of the regional shock, ε , when the match, θ , is governed by a four-state Markov process. The match realizations are $\theta_1 = -\frac{1}{2}$, $\theta_2 = -\frac{1}{6}$, $\theta_3 = \frac{1}{6}$, $\theta_4 = \frac{1}{2}$ and the associated transition

matrices are $\Lambda_0 = \begin{bmatrix} .5 & .5 & 0 & 0 \\ .5 & 0 & .5 & 0 \\ 0 & .5 & 0 & .5 \\ 0 & 0 & .5 & .5 \end{bmatrix}$ (when the match changes for household specific reasons) and $\Lambda_1 = \begin{bmatrix} 0 & 0 & .3 & .7 \\ 0 & .3 & .4 & .3 \\ .3 & .4 & .3 & 0 \\ .7 & .3 & 0 & 0 \end{bmatrix}$ (when the match changes due to a regional shock).

In steady state the mass of each realization is $\frac{1}{4}$. The remaining parameters of the model (see text) are $\pi = .3$, $\lambda = .6$ and $\beta = .94$. The measure of house price fluctuations is $P = \frac{1}{2} \frac{1}{E[n]}$, $P \in [0, 1]$.

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