Juha-Pekka Niinimäki

Screening in the credit market when the collateral value is stochastic
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The views expressed in this paper are those of the author and do not necessarily reflect the views of the Bank of Finland.

* E-mail: juha-pekka.niinimaki@bof.fi

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Screening in the credit market when the collateral value is stochastic

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Juha-Pekka Niinimäki
Monetary Policy and Research Department

Abstract

This theoretical paper explores screening with loan collateral when both the collateral value and the probability of project success fluctuate. Some model versions challenge the classic findings of Bester (1985) by showing that high-risk borrowers may in such case be more willing to pledge collateral than low-risk borrowers. Abundant collateral then would not signal low risk. The results may help explain the mixed empirical findings on the role of collateral. The paper also extends the analysis of the topical subprime crises and risky real estate collateral.

Keywords: banking, collateral, screening, signalling, subprime lending

JEL classification numbers: G21, G22, G28
Lainanhakijoiden erottelu lainamarkkinoilla vakuuksien arvon vaihdellessa

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Juha-Pekka Niinimäki
Rahapolitiikka- ja tutkimusosasto

Tiivistelmä

Tässä keskustelualoitteessa tutkitaan lainanhakijoiden erottelua vakuuksien avulla (screening) tapauksessa, jossa sekä lainan takaisinmaksun todennäköisyyteen että lainan vakuuden tulevaan arvoon kohdistuu epävarmuutta.


Avainsanat: pankit, vakuudet, lainanhakijoiden erottelu

JEL-luokittelut: G21, G22, G28
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1 Introduction

Collateral plays a crucial role in lending. According to Blinks et al. (1993) and De Meza and Southey (1996) the ratio of collateral to loan exceeded unity for 85% of loans in Britain. Moreover, business startups and investment activity are strongly influenced by the supply and value of collateral (Black et al., 1996; Bernanke and Gertler, 1989; Gan, 2009). Given the costs of collateral and its large and broad-based impacts, it is vital to examine the role of collateral in detail.

The cornerstone of the theoretical research on collateral is the study of Bester (1985). Lenders screen borrowers by offering a menu of loan contracts. Low-risk borrowers choose secured loans at lower interest premiums, whereas high-risk borrowers prefer unsecured loans with larger premiums. Alternatively, borrowers may signal their types via collateral. In both scenarios, a high level of collateral is associated with low risk.

Bester’s (1985) visions have triggered abundant empirical research, but the findings are mixed. His visions are supported by the empirical findings of Jimenez et al. (2006) and Berger et al. (2007): low-risk borrowers pledge more collateral than high-risk borrowers. Cressy and Toivanen (2001) find no significant relationship between risk and collateral pledging. A few researchers find that high-risk loans are more secure than low-risk loans: e.g., Neuberger (2001) and Brick and Palia (2007). Consequently, the results are a bit conflicting.

This paper offers an explanation for the disagreement. In contrast to Bester (1985), our model reveals that in some cases a high-risk borrower may be more willing to pledge collateral than a low-risk borrower. As a result, the best secured loans may prove to be the most risky ones. The findings are based on the fluctuating value of collateral and fluctuating probability of project success. The findings may help to explain the mixed results of the empirical research and in this way suggest new ideas for research.

The type of the collateral proves to be important, and it is necessary to clarify the difference between costly collateral and non-costly collateral. Costly collateral entails costs to a borrower. If his project fails and yields no income for loan repayment, the borrower loses the collateral or a part of it. It is natural that outside collateral represents costly collateral. Yet, inside collateral may also be costly. Suppose an established firm finances a new project with a new bank loan. If the project fails, its value is zero. If the loan is collateralized by the old property of the firm, the failure of the project entails costs to owner of the firm. Non-costly collateral entails no cost to the borrower if he is unable to repay the loan. In subprime lending, for example, banks granted mortgage loans without down payments. The purchase price of a house was funded with loan capital which was secured by the house. If the borrower was unable to repay the mortgage, the bank
was able to seize the house, but the borrower did not incur any costs, because he
had not invested his personal funds.

The following findings are obtained when both the future value of collateral and the future probability of project success fluctuate.

1. The borrower’s expected cost from collateral decreases with the correlation between the value of collateral and the probability of project success. Suppose that they are strongly positively correlated. When the collateral value is high, the borrower’s project is likely to succeed and the expected cost from collateral is small. When a project is likely to fail, the collateral value is low and thus the borrower’s loss is small. Hence, the expected cost from collateral is smaller than the expected value of collateral. In contrast, if the probability of project success is negatively correlated with the collateral value, the expected cost from collateral exceeds the expected value of collateral, which is equal to the current value of collateral.

2. The bank’s expected collateral proceeds are negatively related to the borrower’s cost from collateral. If the collateral value and the probability of project success are negatively (positively) correlated, the bank’s expected proceeds from collateral are larger (smaller) than the current value of collateral.

3. A high-risk borrower may be more willing to pledge collateral than a low-risk borrower if: i) the value of his collateral is more highly correlated with the probability of project success than is the collateral of a low-risk borrower or ii) the variance in the high-risk type’s collateral value is sufficiently large compared to the collateral of the low-risk type.

4. The high-risk borrower’s expected costs from collateral may be zero even if the current value and the expected value of his collateral are clearly positive.

5. Non-costly collateral never entails costs to a borrower. Thus, it does not offer the same screening / signalling ability as costly collateral. A high amount of non-costly collateral does not signal low risk even when a lower amount of costly collateral may signal it.

6. Although non-costly collateral never entails cost to a borrower, it may generate income to him. If the collateral value appreciates above the loan repayment, the borrower can keep the surplus. A few subprime borrowers, for example, purchased houses without down payments in order to gamble with the appreciating value of real estate.1

Consequently, the findings depart somewhat from Bester (1985). A high-risk borrower’s expected cost from the same initial amount of collateral may be lower than for a low-risk borrower. A large amount of collateral does not signal low risk

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1 For subprime lending, see Chomsisengphet and Pennington-Cross (2006).
if the variance of the collateral value is large or if the collateral value is strongly correlated with the probability of project success, or if the collateral is non-costly. These findings may help explain the mixed results of the empirical research.


The paper is organized as follows. Section 2 studies costly collateral and Section 3 non-costly collateral. Section 4 gives evidence and examples. This material is presented after the models are set out, because the meaning of the evidence becomes clearer. Section 5 concludes.

2 Costly collateral

Consider a risk-neutral economy with entrepreneurs (borrowers) and banks. A bank is funded with deposits on which it pays interest at rate $r$. Two types of entrepreneurs exist, $i = L, H$, where $L$ denotes a low-risk entrepreneur and $H$ marks a high-risk one. Each entrepreneur can undertake an investment project which lasts for one period and requires one unit of investment input. If successful, with expected probability $t_i$, a project produces output $Y_i$, but if the project fails the output is zero. It is assumed that $t_L > t_H$, $Y_L < Y_H$, $t_L Y_L = t_H Y_H > r$.

The probability of project success is assumed to be stochastic and to depend on the future phase of the economy, which is unknown when the project is started. In prosperity (with probability $g$), the probability of success is high, $t_i \geq t_i$, but in depression, (with probability $1−g$) the probability of success is lower, $t_i \leq t_i$. Here we have $g t_i + (1−g) t_i = t_i$.

A credit contract consists of loan interest rate, $R$, and collateral, $C$. Collateralization may entail transaction costs, which are assumed to be proportional to the amount of collateral at the factor $k \geq 0$. The future value of the collateral is stochastic. With probability $h$, the collateral value appreciates during the loan period and it is $\alpha C$ units at the end of the period. With probability $1−h$, the collateral value depreciates to $\alpha C$, $\alpha < 1 < \alpha$. Collateral is priced correctly, with current value equal to expected value: $C = h\alpha C + (1−h)\alpha C$. If a project fails, the bank can seize the collateral. The following assumption is made.

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2 In this paper, either the positive transaction cost or the availability of collateral will reduce the use of collateral.
**Assumption 1.** The bank’s income from collateral is, at maximum equivalent to the promised loan repayment.

Thus, a bank cannot benefit from a project failure. The correlation between collateral value and the probability of success is

\[
P(\overline{r}|h) - g \sqrt{(1-g)(1-h)g/h} \]

(2.1)

The borrower’s expected return on a loan contact (loan interest rate, collateral combination) is

\[
\pi_i(R, C) = hP(\overline{r}, h)(\overline{r}_i(Y - R) - (1 - \overline{r}_i)\text{Min}(\overline{C}, R)) + h(1 - P(\overline{r}, h))(\overline{r}_i(Y - R) - (1 - \overline{r}_i)\text{Min}(\overline{C}, R)) + (1 - h)P(\overline{r}, l - h)(\overline{r}_i(Y - R) - (1 - \overline{r}_i)\text{Min}(\overline{C}, R)) + (1 - h)(1 - P(\overline{r}, l - h))(\overline{r}_i(Y - R) - (1 - \overline{r}_i)\text{Min}(\overline{C}, R)) - kC
\]

(2.2)

The R.H.S consists of four lines, which represent the borrower’s return in the four phases of the world: 1) collateral value is high together with high probability of loan success, 2) collateral value is high but the probability of loan success is low, 3) collateral value is low but the probability of success is high, 4) Both factors are low. The last term indicates the transaction cost of collateral. Besides, \(P(\overline{r}, h)\) denotes the probability of loan success when the collateral value is high, and \(P(\overline{r}, l - h)\) is the probability of success when the collateral value is low. Since the economy booms with probability \(g\) we must have

\[
hP(\overline{r}, h) + (1 - h)P(\overline{r}, l - h) = g
\]

(2.3)

In addition, the probabilities in (2.2) must satisfy

\[
t_i = hP(\overline{r}, h)\overline{r}_i + h[1 - P(\overline{r}, h)]\overline{r}_i + (1 - h)P(\overline{r}, l - h)\overline{r}_i + (1 - h)[1 - P(\overline{r}, l - h)]\overline{r}_i
\]

(2.4)

Some manipulation gives

\[
t_i - t_i = hP(\overline{r}, h)(\overline{r}_i - \overline{t}_i) + (1 - h)P(\overline{r}, l - h)(\overline{t}_i - t_i)
\]

(2.5)

This implies
\[
P(\bar{t}_i|l - h) = \frac{t_i - t_1 - hP(\bar{t}_i|h)(\bar{t}_i - t_1)}{(1 - h)(\bar{t}_i - t_1)}
\]  \hspace{1cm} (2.6)

which is utilized in the proofs. Obviously, the entrepreneur’s expected profit in 
(2.2) must be positive or zero. The profit can be restated as

\[
t_i(Y - R) - hP(\bar{t}_i|h)(1 - \bar{t}_i)Min(\bar{t}C, R) - h[1 - P(\bar{t}_i|h)(1 - \bar{t}_i)Min(\bar{t}C, R)] - (1 - h)P(\bar{t}_i|l - h)(1 - \bar{t}_i)\bar{t}C - (1 - h)[1 - P(\bar{t}_i|l - h)](1 - \bar{t}_i)\bar{t}C - kC
\]  \hspace{1cm} (2.7)

Two cases appear depending on the appreciated value of loan collateral.

2.1 Case \( \text{Min}(\bar{t}C, R) = \bar{t}C \)

The appreciated value of collateral does not cover the loan interest payment and 
(2.7) simplifies to

\[
t_i(Y - R) - h(1 - t_1)\bar{t}C + hP(\bar{t}_i|h)(\bar{t}_i - t_1)\bar{t}C - (1 - h)(1 - t_1)\bar{t}C + (1 - h)P(\bar{t}_i|l - h)(\bar{t}_i - t_1)\bar{t}C - kC
\]  \hspace{1cm} (2.8)

Inserting \( P(\bar{t}_i|l - h) \) from (2.6) into this implies

\[
t_i(Y - R) - h(1 - t_1)\bar{t}C + [t_i - 1 + h(1 - t_1)]kC + hP(\bar{t}_i|h)(\bar{t}_i - t_1)(\bar{t} - \alpha)C - kC
\]  \hspace{1cm} (2.9)

Under the fixed level of profit, it is possible to calculate the entrepreneur’s 
marginal rate of substitution between loan interest rate and collateral

\[
\frac{dR}{dC} = -\frac{1 - t_i - (t_i - t_1)\alpha - hP(\bar{t}_i|h)(\bar{t}_i - t_1)(\bar{t} - \alpha) + k}{t_i}
\]  \hspace{1cm} (2.10)

The marginal rate of substitution gives the required reduction in loan interest rate 
when the amount of collateral increases by one unit. In Bester (1985), where the 
collateral value and the probability of project success are fixed, the marginal rate 
of substitution is\(^3\)

\(^3\) More precisely, (2.11) represents a discrete version of the Bester’s formula.
It is easy to see that in Bester (1985) a low-risk borrower is more willing to pledge collateral than a high-risk borrower. This makes it possible to screen borrowers via collateral. In equilibrium, a bank offers two contracts to borrowers. Low-risk borrowers optimally self-select a contract with low loan interest rate and a positive amount of collateral, whereas high-risk borrowers prefer a contract with higher loan interest but no collateral. In our model, the following result can be seen from (2.10).

**Proposition 1.** The marginal rate of substitution is increasing in \( P(\bar{t},|h) \).

Intuitively, when \( P(\bar{t},|h) \) is large, the fluctuations are highly positively correlated. When the collateral value is high, the probability of success is high and thus the borrower’s expected collateral losses are modest. The borrower is likely to lose collateral if the collateral value is low. Again, the borrower’s expected costs from collateral are modest and therefore his marginal rate of substitution is high. A borrower is willing to pledge a lot of additional collateral if the loan interest rate declines a bit, because his expected costs from collateral are modest. The opposite holds if \( P(\bar{t},|h) \) is low.

**Proposition 2.** The marginal rate of substitution is the same as in Bester (1985) if
i) Only the probability of success fluctuates or
ii) Only the collateral value fluctuates or
iii) Both fluctuate independently

**Proof:** To show the first result, assume \( \alpha = \bar{\alpha} = 1 \). Inserting this into (2.10) gives (2.11). Secondly, substituting \( t_i = \bar{t}_i = t \) into (2.10) yields (2.11). Thirdly, (2.10) implies

\[
\frac{dR}{dC} = \frac{1 - t_i + k}{t_i} \tag{2.11}
\]

Since \( 1 - \alpha = h(\bar{\alpha} - \alpha) \) it is possible to restate this as

\[
\frac{dR}{dC} = \frac{1 - t_i + (g - P(\bar{t},|h))h(\bar{t}_i - t_i)(\bar{\alpha} - \alpha) + k}{t_i} \tag{2.13}
\]
When the fluctuations are independent, \( g = P(\bar{t}, \bar{h}) \), which simplifies to (2.11). Q.E.D

Thus, the model includes many cases with the same marginal rate of substitution as in Bester (1985). A comparison follows

**Corollary 1.** If the probability of loan success and the collateral value are positively (negatively) correlated, the marginal rate of substitution is higher (lower) than in Bester (1985). With the given loan interest rate and amount of collateral, the borrower’s returns are increasing and the bank returns are decreasing in \( P(\bar{t}, \bar{h}) \).

**Proof:** The first part of the corollary is based on Proposition 1, Proposition 2iii, and (2.1). For the second part, recall the borrower’s returns from (2.9), and note that they are increasing in \( P(\bar{t}, \bar{h}) \). Alternatively, the borrower’s returns can be restated as \( \pi_t = \pi_t Y - \pi_b \), where \( \pi_b \) denotes the bank’s expected returns (loan interest income and collateral) from a financed project. Since the borrower’s returns are increasing in \( P(\bar{t}, \bar{h}) \), the bank’s returns are decreasing in it. Q.E.D

At first sight one might presume that the expected bank returns from collateral are equal to the expected value of collateral, which is equal to the current value of collateral. Corollary 1 shows that this is not true. The correlation between collateral value and probability of loan success has a crucial influence on the bank’s expected collateral proceeds as well as on the borrower’s expected costs from collateral.

**Proposition 3.** If \( P(\bar{t}, \bar{h}) = 1, \ \alpha = 0, \ \bar{t} = 1 \) the borrower’s expected costs from collateral are equal to the transaction costs of collateralization, \(-kC\), and the marginal rate of substitution is \(-k/t\). The borrower never loses the collateral.

**Proof:** This is easy to see from (2.9) and (2.10). Note that when \( \alpha = 0 \), we have \( h\bar{\alpha} = 1 \). Q.E.D

A project is always successful when the collateral value is high. It fails with a positive probability, but only when the collateral value is zero. Hence, the expected costs from collateral amount to the initial transaction cost. If this is zero, collateral entails no costs to high-risk borrowers, even when the initial amount of collateral and its expected value are high.
Corollary 2. Suppose that a high-risk borrower has more collateral than a low-risk borrower, the value of the low-risk type’s collateral is non-stochastic, and the value of the high-risk type’s collateral is stochastic. Then, the expected costs from collateral may be higher for the low-risk type.

Proof: Suppose \( k = 0 \). Given (2.11) or Proposition 2, we have \( \frac{dR}{dC} = -(1 - t_L + k)/t_L \) for the low-risk type. Suppose that the project and collateral of the high-risk type satisfy \( P(\tilde{t}_H|h) = 1, \quad \alpha = 0, \quad \tilde{t}_H = 1 \). Given Proposition 2, his costs from collateral are \( \frac{dR}{dC} = 0 \). Q.E.D

The intuition is the same as above. The result is interesting, because it departs from Bester (1985). When \( k \) is sufficiently small, a high-risk borrower is more willing to pledge collateral than a low-risk one. Yet, the types of the collateral are different. It is, however, possible to derive a stronger result. In the following, borrowers have the same kind of collateral. Again, collateral is more costly for a low-risk borrower.

Corollary 3. Suppose that a high-risk borrower pledges more collateral than a low-risk borrower, the type of collateral is the same and that its value is stochastic. It is possible, that the expected costs from collateral are higher for the low-risk type, because its collateral is less correlated with the probability of project success.

Proof: Suppose that \( k, \alpha = 0, \tilde{t}_H = 1 \). Suppose that, for the low-risk type, \( P(\tilde{t}_L|h) = g \) (collateral and the probability of project success are independent) and, for the high-risk borrower, \( P(\tilde{t}_H|h) = 1 \). Obviously, for the high-risk borrower, the expected cost from collateral is zero (Proposition 3), but for the low-risk borrower it is the same as in Bester (1985) (Proposition 2). Therefore, the cost is higher for the low-risk borrower when \( k \) is sufficiently small Q.E.D

Corollary 4. Suppose that the NPV of a high-risk project is negative. It is possible that a rational borrower is willing to pledge full collateral for the project and bears no cost from the collateral. Thus, full collateralization does not screen from the loan market projects with negative NPV.

Suppose: \( k = 0, \quad h = g = 1, \quad \alpha = 0, \quad \bar{\alpha} = 2, \quad \tilde{t}_H = 1, \quad t_H = 0, \quad P(\tilde{t}_H|h) = 1, \quad r = 1.05, \quad R = 1.1 = C, \quad Y = 2 \). Then, collateral is priced correctly \( (\frac{1}{2}\bar{\alpha} + \frac{1}{2}\alpha = 1) \), the expected probability of project success is 50\% \( (g\tilde{t}_H + (1-g)t_H = \frac{1}{2} = t_H) \) and the NPV of the project is negative \( (t_H Y < r) \). Given proposition 3, the expected cost from collateral is zero. Q.E.D
In contrast to a standard theory with non-stochastic collateral, a borrower is willing to pledge full collateral for a project with negative NPV.

In sum, we have observed that the expected costs from collateral may be lower for the high-risk type, which makes screening and signaling impossible. The high-risk borrower’s expected costs from both collateral and loan interest are lower than for the low-risk borrower.\(^4\) Obviously, there are no such loan contracts (collateral & loan interest rate combinations), which are correctly self-selected by entrepreneurs according to their true risk types.

Furthermore, when both collateral and loan interest payments yield lower expected costs to high-risk borrowers, raising the required amount of collateral could make the loan contract unprofitable to low-risk borrowers and so only the high-risk borrowers would apply for a loans. Hence, the collateral requirement could have negative effects on the average quality of the loan portfolio.

Interpreting a large amount of collateral to surely signal low risk or to screen high-risk borrowers from the loan contract may lead to erroneous conclusions by bankers or researchers. It is necessary to find out the variance and correlation between collateral value and probability of project success.

So far we have assumed that the type of collateral is given. Its variance and its correlation with the probability of loan success have been given. Alternatively, a borrower might be able to choose the type of collateral so that his expected returns are maximized: \(P(\bar{f}, h)\) is as high as possible. Obviously, this would lower the bank’s expected returns (Corollary 1). Therefore, the bank must pay close attention to the type of collateral, its variance and correlation with the probability of project success.

In the following, it is assumed that the collateral type can be chosen by the bank, which seeks the optimal type of collateral. Above we found that in some cases screening / signalling is impossible. Next we shall illustrate an opposite-type case. It is possible to separate borrowers using collateral, which entails more costs for high-risk borrowers than for low-risk ones. The result is similar to standard models. Two alternative contracts are available. In the high-risk contract, the loan interest rate is high, because the loan is repaid with a low probability, but the required amount of collateral is zero. The low-risk contract includes a low interest rate, but the amount of collateral is positive. Although the risk types of borrowers are unobservable, their characteristics are assumed to be known. Mainly, the correlations between collateral and the probabilities of project success are known. This knowledge is utilized when the collateral type in chosen so that the costs of

\(^4\) The high-risk borrower’s expected costs from the loan interest are lower than for the low-risk borrower because the high-risk borrower pays the loan interest with lower probability. His costs from collateral are lower than the costs for the low-risk borrower if corollary 2 or 3 is satisfied.
screening are minimized. This is achieved when the amount of collateral is as small as possible. The following result is obtained.

**Proposition 4.** Suppose that the type of collateral can be chosen. Then the optimal type of collateral is such that it minimizes (maximizes) the costs of the low-risk (high-risk) type: \( P(\tilde{t}_h|h) \) is small and \( P(\tilde{t}_l|h) \) is high.

**Proof:** In equilibrium, a high-risk agent prefers his contract (loan interest rate \( R_h \), no collateral) to a low-risk contract (loan interest rate \( R_l \), collateral \( C \)) when it entails lower costs to him

\[
t_h R_h \leq t_h R_L + h(1 - t_h)\alpha C - [t_h - 1 + h(1 - t_h)]\alpha C - hP(\tilde{t}_h|h)(\tilde{t}_h - t_h)(\alpha - \alpha)C + kC
\]

(2.14)

The L.H.S expresses the high-risk agent’s expected payments from his contract and the R.H.S his payments from the low-risk contract. The optimal level of collateral can be chosen from (2.14)

\[
\frac{t_h(R_h - R_L)}{1 - t_h - g(\tilde{t}_h - t_h)\alpha - hP(\tilde{t}_h|h)(\tilde{t}_h - t_h)(\alpha - \alpha) + k} = C^*
\]

(2.15)

This is increasing in \( P(\tilde{t}_h|h) \). The expected costs of the low-risk agent are

\[
t_l R_L + (1 - t_l - g(\tilde{t}_L - t_L)\alpha - hP(\tilde{t}_L|h)(\tilde{t}_L - t_L)(\alpha - \alpha) + k)C^*
\]

(2.16)

The costs are decreasing in \( P(\tilde{t}_L|h) \), but increasing in \( C^* \). As a result, the optimal type of collateral is such that \( P(\tilde{t}_L|h) \) is high, but \( P(\tilde{t}_h|h) \) is low. Q.E.D

2.2 Case \( \min(\alpha C, R) = R \)

In this section the appreciated value of collateral is so high that it exceeds the loan interest rate. As a result, when a project fails and the collateral value appreciates during the loan period, collateral covers the whole loan repayment. Thus, the bank does not bear a loan loss even if the project produces no output. In addition, the borrower can keep the surplus, \( \alpha C - R \). Given (2.7), the borrower’s expected returns can now be restated as

\[
16
\]
\[ \pi_i = t_i(Y_i - R) + hP(\bar{t}_i|h)(\bar{t}_i - \hat{t}_1)R - h(1 - \hat{t}_1)R \]
\[ (t_i - \hat{t}_1 - hP(\bar{t}_i|h)(\bar{t}_i - \hat{t}_1))\alpha C - (1 - h)(1 - \hat{t}_1)\alpha C - kC \]  \hspace{1cm} (2.17)

From this it is possible to solve for the marginal rate of substitution

\[ \frac{dR}{dC} = \frac{(t_i - 1 + h(1 - \hat{t}_1))\alpha - hP(\bar{t}_i|h)\alpha - k}{t_i - hP(\bar{t}_i|h)(\bar{t}_i - \hat{t}_1) + h(1 - \hat{t}_1)} \]  \hspace{1cm} (2.18)

**Proposition 5.** When \( \alpha C - R > 0 \), the marginal rate of substitution is always lower than in Bester (1985). The marginal ratio of substitution is zero if \( k, \alpha = 0 \).

**Proof:** Note that (2.18) is negative and it is minimized when \( P(\bar{t}_i|h) = 1 \). Then, (2.18) gives

\[ \frac{dR}{dC} = -\frac{(1 - t_i)\alpha + h(1 - \bar{t}_i)\alpha - k}{t_i + h(1 - \bar{t}_i)} \]  \hspace{1cm} (2.19)

which is negative, but exceeds (2.11). The latter of Proposition 5 can be seen from (2.18). Q.E.D

A borrower requires a smaller decrease in the loan interest rate than in Bester (1985), because he benefits from the low loan interest rate if his project succeeds and even if it fails, provided the collateral value is high (recall that the borrower can keep the surplus, \( \alpha C - R > 0 \)). Since he benefits from the low loan interest rate relatively often, he is willing to pledge more collateral even when the loan interest rate declines a bit.

**Proposition 6.** Suppose \( \alpha C - R > 0 \). If \( P(\bar{t}_i|h) = 1, \alpha = 0 = k, \bar{t}_1 = 1 \) the borrower’s expected cost from collateral is zero. More commonly, the expected cost from collateral is decreasing in its variance. The marginal rate of substitution is also decreasing in variance. \(^5\)

**Proof:** The first result can be easily seen from (2.7). For the second result, see (2.7)

\[ \frac{\partial \pi_i}{\partial \alpha} = -((1 - h)P(\bar{t}_i|h) - h)(1 - \bar{t}_1)C - (1 - h)[1 - P(\bar{t}_i|h) - h]k(1 - \bar{t}_1)C \frac{d\alpha}{d\alpha} d\alpha > 0 \]  \hspace{1cm} (2.20)

\(^5\) The average value of collateral does not change due to the increased variance, \( h\bar{C} + (1 - h)d \alpha = 0 \).
For the third result, we use (2.7) to restate the marginal rate of substitution as

$$\frac{\partial R}{\partial C} = -\frac{(1-h)(1-t_1)\alpha + \alpha(1-h)P(t_i|h)(\bar{t}_i - h)P(t_i|h)(\bar{t}_i - t_i) - k}{t_i - hP(t_i|h)(\bar{t}_i - t_i) + h(1-t_i)} < 0$$

(2.21)

Since $d\alpha/d\alpha < 0$, we obtain $dR/dC\alpha > 0$. Q.E.D

The intuition of the first result is the same as in the context of Proposition 3. The project is always successful if the collateral value is high. When a project fails, the collateral value is zero. Hence, the borrower never loses the collateral and the costs of collateral consist of the initial transaction costs. Let us now turn to the second and the third result. Suppose that the variance increases and that the collateral value is high. The higher variance has no effect on the costs of collateral, because the collateral value is already at the upper limit (= loan interest rate). Suppose now that the collateral value is low. The increased variance pushes down the lower limit, $\alpha$. Thus, the total effect is asymmetric and the increased variance erodes the costs from collateral. Put differently, when the collateral value appreciates, a borrower can keep the surplus, $\alpha C - R$. Obviously, the higher the variance, the higher the surplus of the borrower. The following corollary, which follows from Proposition 6, highlights this point.

**Corollary 5.** Suppose that a low-risk borrower and a high-risk borrower pledge the same kind of collateral. For both borrower types, the probability of project success is independent of the collateral value. Although the high-risk borrower has more collateral, his expected costs from collateral may be lower than for the low-risk type, if the variance of the high-risk type’s collateral is higher than the variance of the low-risk type’s collateral.

The loan collateral may, for example, consist of real estate. Suppose that a high-risk borrower operates in an environment where the value of real estate fluctuates more widely than in the environment of the low-risk type. Then, even when the collateral value and the probability of project success are independent, and the high-risk borrower pledges more collateral than the low-risk one, the high-risk borrower’s expected costs from collateral may be lower than the costs of the low-risk borrower. Hence, the high initial level of collateral does not signal low risk.
So far we have investigated costly collateral, but in this section collateral entails no costs to borrowers because it is purchased with loan capital. It is natural that such collateral consists of inside collateral. Consider, for instance, a homebuyer with a subprime loan without a down payment. He borrows a unit from a bank and purchases a house, which is pledged as collateral. If the borrower earns income for the loan repayment, he repays the loan and can keep the house. If he is unable to earn money, the bank can seize the collateral.\(^6\)

The same kind of loan, without down payment, has been used in commercial projects. Lamm and O’Keefe (1998a, p. 342) give an example from the S&L crisis: ‘These problems arose in part because the Massachusetts Miracle had lured novice developers – many with weak business plans often based on little or poor market research – into the real estate game. Some commercial projects were 100 per cent financed and based on such unrealistic expectations as the continuation of 10 per cent annual price hikes into the 1990s’.

This section explores screening with non-costly collateral. Suppose a firm with no capital of its own that seeks a bank loan. The loan size is 1 unit, and \(\gamma\) units of it is used to cover the out-of-pocket expenses of the project and the rest of the loan is spent on production facilities, eg factory area or office building, \(0 \leq \gamma < 1\). The production facilities represent loan collateral. The initial value of the collateral is, \(C = 1 - \gamma\), but its value fluctuates during the loan period. At the end of the period, the collateral value is either \(C\alpha\) units (with probability \(h\)) or \(\alpha C\) units (with probability \(-h\)), \(\alpha < 1 < \bar{\alpha}\). Again, the current value of the collateral is equal to its expected value. Banks fund loans by attracting deposits at the interest rate of the economy, \(r\).

The economy has two types of firms (borrowers): low risk and high risk. With probability \(t_L\), a low-risk borrower can earn income \(Y_L\), but with probability \(1 - t_L\) it earns nothing. A high-risk borrower is unable to earn income. It is assumed that \(0 < t_H < t_L\). Again, the expected probability of repayment is stochastic and its future value is unknown at the start of the period when the lending decision is made. With probability \(g\), the economy later booms and the expected probability of success is relatively high, \(\bar{t}\), with probability \(1 - g\), the economy slumps and the probability of success is lower, \(t_i \leq t_i\). Obviously, we have \(g\bar{t}_i + (1 - g)t_i = t_i\). Now the firm’s expected returns are

\(^6\) For subprime mortgages with 100% loan finance, see Chomsisengphet and Pennington-Cross (2006).
\[ \pi_i = hP(\bar{\tau}, h)(Y_i - R + \bar{C})(1 - \bar{\tau}_i)\text{Max}(\bar{C}C - R, 0)) + \\
(1 - h)P(\bar{\tau}, h)(Y_i - R + \bar{C})(1 - \bar{\tau}_i)\text{Max}(\bar{C}C - R, 0)) + \\
(1 - h)(1 - P(\bar{\tau}, h)(Y_i - R + \bar{C})(1 - \bar{\tau}_i)\text{Max}(\bar{C}C - R, 0)) \]

Note that (3.1) differs from (2.2) in an important way. In (2.2), collateral entails costs to a borrower if his project fails. Here collateral entails no cost. On the contrary, the value of the collateral may appreciate so strongly during the loan period that it exceeds the loan interest rate. Then, the borrower obtains positive returns from the project, \( \alpha C - R \), even if the underlying project fails. Since \( C \leq 1 \) and \( R \geq r > 1 \), it is known that \( \alpha C < R \). Some manipulation yields

\[ \pi_i = t_i(Y_i - R) + \Omega + h\text{Max}(\bar{C}C - R, 0)(1 - \bar{\tau}_i - P(\bar{\tau}, h)(\bar{\tau}_i - \bar{\tau})) \]  

where

\[ \Omega = \bar{\tau}_i(hP(\bar{\tau}, h)\bar{C} + (1 - h)P(\bar{\tau}, h| - h)\alpha)C \]

\[ \bar{\tau}_i(h(1 - P(\bar{\tau}, h))\bar{C} + (1 - h)(1 - P(\bar{\tau}, h| - h)\alpha))C \]

is the expected value of the production facilities for a successful firm. In (3.2), we have three terms. The first one indicates the profits of a successful firm, and the second term shows the expected value of the production facilities of a successful firm. The last term expresses the profits when a project fails but collateral value appreciates. Two cases can occur.

### 3.1 Max (\( \bar{C}C, R \)) = \( \bar{C}C \)

The appreciated value of collateral does not cover the loan repayment. Thus, only a low-risk type seeks a loan. A high-risk type does not seek for a loan, because his project fails with certainty and the appreciated value of collateral does not exceed the loan interest rate.

When a project fails, a loan defaults and the bank seizes the collateral. Thus, we do not directly see \( C \) in firm profits, \( \pi_i = t_i(Y_i - R) \). In a competitive economy collateral has an indirect effect because it lowers the loan interest rate. Collateral cannot be used to screen borrowers, and it has no direct effect on their incentives to seek a loan. The screening effect, however, obtains even without collateral because the high-risk types cannot earn positive income.
3.2 Max \((\overline{\alpha}C, R) = R\)

The appreciated value of collateral is now so high that it covers the loan repayment. The expected returns are given by (3.2). For the low-risk firm this is positive, and it will seek a loan. For a high risk type, we obtain

\[ h(\overline{\alpha}C - R) \]  \hspace{1cm} (3.4)

This is positive, and a high-risk firm seeks a loan. Although it cannot operate successfully, the value of its production facilities – for example, real estate – may rise so rapidly during the lending period that the value exceeds the loan repayment. The high-risk borrower’s expected returns are positive. Both types seek a loan, and the problem of asymmetric information is present. If the proportion of high-risk borrowers is high, their existence can destroy the loan market: nobody receives a loan.

**Proposition 7.** Suppose that \( \overline{\alpha} > R \). Then high-risk types seek loans, and the problem of asymmetric information is present only if the initial level of collateral is sufficiently high.

**Proof:** The initial value of loan collateral is \( C = 1 - \gamma \). If \( \gamma = 0 \), we have \( \overline{\alpha}C - R > 0 \). High-risk types seek loans, and the problem of asymmetric information is present. Alternatively, suppose that \( \gamma \) approaches one, so that \( \text{Max}(\overline{\alpha}C - R, 0) = 0 \). Now high-risk types cannot gamble with collateral, and they do not seek loans. The problem of asymmetric information is avoided. Q.E.D.

Non-costly collateral never causes losses to a borrower, but it may provide benefits. First, in the competitive economy collateral pushes down the loan interest rate, because unsuccessful loans yield income to banks. Secondly, if a borrower earns enough income for the loan repayment, he can keep the collateral. Thirdly, if the collateral value appreciates sufficiently during the loan period, the borrower obtains a positive return, \( \overline{\alpha}C - R \), even if his project fails.

The initial value of the collateral is given by the production process of the sector, and the variance of the collateral value is affected by the characteristics of the economy. These factors may make a project lucrative to borrowers who gamble on the future value of collateral. A firm may, for example, purchase an office building and not begin any business at all. If the value of the building appreciates sufficiently during the loan period, the firm can sell the office building, pay back the loan and make a handsome profit.
Given Proposition 7, if the initial value of collateral is low, only low-risk types seek loans, but if the initial value of collateral is high, both types seek loans. Hence, a large initial amount of collateral does not signal a low risk. As a matter of fact, the case is totally the opposite.  

With costly collateral, a different screening equilibrium is likely to occur: low-risk borrowers have secured loans, but the loans of high-risk types are without collateral (Bester, 1985). However, even low-risk borrowers may pledge less collateral than what both borrower types have in the pooling contract with non-costly collateral. Thus, the initial amount of collateral does not reveal the true borrower type. It is necessary to analyze whether collateral consists of costly or non-costly collateral. Non-costly collateral may provide good liquidation proceeds to a bank, if a borrower defaults. Yet, since collateral is funded with loan capital, it does not offer screening and signalling advantages like those of costly collateral.

4 Evidence and examples

The results of the previous sections are based on the assumption of a stochastic value of collateral. In addition, the correlation between collateral value and probability of project success proves to be important. This section cites examples which show that the assumptions are realistic. Collateral values fluctuate widely and are often highly positively correlated with the probability of project success.

It is natural that an outside shock which reduces the probability of project success in a sector also reduces the values of production factors in the same sector. If collateral consists of production factors, the outside shock reduces the probability of project success and the collateral values. Lamm and O'Keefe (1998b) give the following example. In 1980s the fall of the oil price led to a banking crisis in the Southwest of USA. In 1973 the import price of crude oil was $2.75 per barrel, but the price rose gradually to $36.95 per barrel in April 1981. Numerous new oil wells, which could not operate profitably with oil prices below $15 a barrel, were established. Several banks, for example Penn Square, specialized in financing oil business. These loans were secured by the borrowers’ oil and gas reserves. Lenders were anticipating the price of oil to rise to $60 a barrel, but it slid to $10 per barrel, making numerous oil wells unprofitable. Lamm and O'Keefe (1998b, p. 325) report: ‘The energy loans in which Penn

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7 We have simplified the analysis by assuming that the project of the high-risk borrower never succeeds. Alternatively, it is possible to assume that the project succeeds with a positive probability, but the project causes a participation cost. Then, it is possible that the high-risk borrower undertakes the project only if the collateral value can exceed the loan repayment. Again, the option to gamble with the future value of collateral makes the project profitable.
Square was so heavily invested had been based on extremely high oil and gas prices. When the energy markets deteriorated, a huge volume of loans defaulted and the value of the supporting collateral was minimized, leading to Penn Square’s failure. Consequently, the drop in the oil price simultaneously devastated the borrowers’ earnings and the value of collateral, causing a surge of bank failures.

Furthermore, when collateral consists of production factors, the highest valuation potential buyers of collateral are likely to be other firms in the sector. The price of the collateral may fall below value in best use if many of the sector’s firms become insolvent due to the shock and the rest of the firms are also damaged by the shock. Then, the potential buyers also encounter severe financial difficulties, their ability to pay for the production factors of the failed firms is weak, and the price of the production factors (collateral) will be low (Shleifer and Vishny, 1992).

Real estate and firms’ stocks represent the most common forms of collateral. The variance of their prices is high and the prices are usually positively correlated with the probability of project success. During the Asian crisis in the 1990s, for example, stock prices appreciated sharply during the boom phase: 405% in Hong-Kong, 352% in Philippines, and over 155% in Malaysia. The depreciation was dramatic during the economic downturn: –56% in Hong-Kong in a year, –53% in Philippines in 18 months and –76% in Malaysia in 18 months. The collapse of stock prices was accompanied by a drop in the value of real estate collateral. In Hong-Kong, commercial real estate prices rose 100% from 1993 to 1997, but dropped back to the original level by the end of 1998 (Collyns and Sennadji, 2002). As to the real estate, there are many examples of wide fluctuations. In Stockholm, inflation-adjusted real estate prices rose 450% in the 1980s. From 1989 to 1993, inflation-adjusted prices depreciated below the 1982 level (Herring and Wachter, 1999). In Japan, commercial property prices rose by over 300% in the 1980s, but declined again to the initial level over the next 5 years (Hilbers et al, 2001). Consequently, there is abundant evidence that the prices of the most common types of collateral fluctuate widely and are positively correlated with firms’ probability of repaying loans. Hence, our extensions to the Bester’s article are realistic.

8 Schleifer and Vishny (1992, p. 1355) give the following example: ‘…when Eastern and Pan Am put their assets up for sale at time when other airlines were themselves losing money, the potential buyers could not borrow money as easily and assets appeared to be selling at or “distressed prices”. For example, in December 1991 United bought bankrupt Pan Am’s Latin American routes for $135 million compared to the $215 million it had offered in late August and $342 million paid earlier to Eastern by American for similar routes.’

9 According to Borio (1996), the portion of loans secured by real estate is high: 59% in Great Britain, 56% in Canada and 66% in United States.
5 Conclusions

This paper extends the classic study of Bester (1985). With full agreement on the importance of his contributions, it is shown that if his model framework is modified a bit (in a realistic way), it is possible to present cases in which the findings depart from Bester’s findings.

1. When both high-risk and low-risk borrowers pledge the same type of collateral and the variance of the collateral is the same, the expected costs from the collateral may be smaller for the high-risk type if the value of the collateral is more highly correlated with the success probability of his project than with the success probability of a low-risk project.

2. When both high-risk and low-risk borrowers pledge the same type of collateral and the collateral value is independent of the project success, the expected costs from the collateral may be smaller for a high-risk type, if he operates in an environment in which the variance of the collateral value is higher than in the environment of a low-risk type.

3. When collateral consists of non-costly collateral, a large initial ratio of collateral to loan capital may attract high-risk borrowers to take loans in order to gamble on the appreciation of collateral value.

Consequently; lenders, borrowers and researchers might find it useful to highlight the difference between costly and non-costly collateral and pay more attention to the variance of collateral value and correlation between collateral value and probability of project success. These factors have a strong influence on incentives as well as on costs and proceeds from collateral.
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