Stabilisation bias in monetary policy under endogenous price stickiness
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The views expressed are those of the author and do not necessarily reflect the views of the Bank of Finland.

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Stabilisation bias in monetary policy under endogenous price stickiness

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Abstract

This paper investigates the consequences of introducing endogenous price stickiness into a standard monetary policy model. We find that the modification reduces the optimal degree of inflation stabilization to which the central bank should commit. The reason is that less inflation stabilization encourages firms to review their prices more frequently. The economy becomes more flexible and the Phillips-curve tradeoff is improved, making it easier for the central bank to control inflation. This reduces, and may even reverse, the stabilization bias that is present in models with exogenous price stickiness and that recommends that the central bank generally commit to tighter stabilization of inflation than it would in a discretionary policy regime.

Key words: price stickiness, monetary policy, stabilization bias

JEL classification numbers: E 52, E 58
Rahapolitiikan stabilisaatioiharha endogeenisten hintajäykkyysien vallitessa

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1 Introduction

Recent papers by Svensson (1997) and Clarida et al (1999) have suggested that there may be a stabilisation bias in monetary policy. They show that there is a bias in discretionary policy-making towards under-stabilisation of inflation. Optimal monetary policy would require the central bank to be tougher in controlling inflation but the central bank is unable to credibly commit to such a policy. Although the central bank can promise to be tough in controlling inflation in the future, there is always an incentive to accommodate inflation shocks once they actually occur. In discretionary equilibrium, the incentive to accommodate dominates and the policy of the central bank does not stabilise inflation sufficiently.

In this paper, we develop arguments showing that the stabilisation bias can actually go in the other direction. When firms are able to choose the frequency of their price changes, there may be a bias for over-stabilisation of inflation in discretionary equilibrium. We argue that the central bank should commit to a policy that stabilises inflation less, because this encourages firms to make more frequent price changes. As the economy becomes more flexible, the Phillips curve becomes steeper and it is easier for the central bank to control inflation. Again, such a commitment is not credible because once firms have decided on the frequency of their price changes, the central bank has an incentive to renege on its promise and stabilise inflation more.

To establish our result we construct a model of endogenous price stickiness, in which firms are able to choose the expected frequency of their price adjustments. We introduce a fixed cost of reviewing prices into the quadratic loss function model of Rotemberg (1987), making our model essentially a variant of Kiley (1996). The direction of the stabilisation bias has important policy implications. If the bias is towards under-stabilisation then there may be welfare gains from delegating monetary policy to a weight-conservative central banker, as originally suggested by Rogoff (1985). In contrast, if the bias goes in the other direction then monetary policy should be delegated to a central banker who is less averse than society to inflation variability.

The paper is organised as follows. In Section 2 we derive the conditions under which there is a stabilisation bias in the economy and show when the bias is towards under or over-stabilisation. Our model of endogenous price stickiness is described in Section 3 and calibrated in Section 4. The results in Section 5 show quantitative evidence of the likely direction and importance of the stabilisation bias. A final section concludes.

2 Stabilisation bias

To characterise the nature of the stabilisation bias problem we use a standard dynamic general equilibrium model with temporary nominal price rigidities. For a detailed derivation of the model we refer readers to Clarida et al (1999) and the references therein. The key aggregate relationship in the model is
the Phillips curve (1), which is derived by aggregating the pricing decisions of individual firms in the economy. It relates inflation \( \pi_t \) positively to the output gap \( x_t \) and expectations of future inflation. The slope parameter \( \lambda \) depends on the probability with which each firm reviews and changes its price. \( u_t \) is a cost-push shock which evolves according to equation (2), where \( \rho \leq 1 \) and \( \hat{u}_t \) is an i.i.d. random variable with mean zero and variance \( \sigma_u^2 \). There is no asymmetric information in the model so the push shock \( u_t \) is observed by both firms and the central bank.

\[
\begin{align*}
\pi_t &= \lambda x_t + \beta E_t \pi_{t+1} + u_t \\
u_t &= \rho u_{t-1} + \hat{u}_t
\end{align*}
\] (1) (2)

The central bank objective function maps economic outcomes into a measure of social welfare. Following Clarida et al (1999) and a large number of other studies, we assume that this objective function is over the output gap and inflation, and takes the form of equation (3). Since the output gap appears in the objective function, the central bank implicitly takes potential output as the target and there is no inflation bias. \( \alpha \) reflects the relative weight placed on output deviations.

\[
W_t = -\frac{1}{2} E_t \sum_{j=0}^{\infty} \beta^j (\alpha x_{t+j}^2 + \pi_{t+j}^2)
\] (3)

We analyse the direction of the stabilisation bias in this model by looking at monetary policies with and without credible commitment. In the terminology of the established literature, we examine “rules versus discretion”.

2.1 Monetary policy without commitment

In the absence of commitment, the central bank sets the output gap each period to maximise the objective function (3), subject to the Phillips curve (1). Since there is no mechanism to credibly manipulate beliefs, the central bank takes private sector expectations as given. Clarida et al (1999) derive the optimal monetary policy under discretion (4).

\[
x_t = -\frac{\lambda}{\lambda^2 + \alpha(1 - \beta \rho)} u_t
\] (4)

According to equation (4), the central bank contracts demand \( x_t \) whenever there is a positive cost-push shock. The strength of the contraction depends on the slope of the Phillips curve \( \lambda \) and the relative weight \( \alpha \) placed on output losses.
2.2 Monetary policy with commitment

In analysing central bank behaviour under commitment, we restrict ourselves to simple monetary policy rules of the type (5), in which the output gap reacts linearly to the cost-push shock through the feedback parameter $\omega$. The rule includes the optimal monetary policy without commitment as a special case, with $\omega^d = \lambda/(\lambda^2 + \alpha(1 - \beta \rho))$.

$$x_t = -\omega u_t$$  \hspace{1cm} (5)

To derive the direction of the stabilisation bias, we calculate the gains from the central bank committing to a policy rule. We evaluate the effect on welfare of committing to a rule that marginally increases the feedback coefficient $\omega$ from its discretionary level $\omega^d$. If the effect is positive then there are gains to be made from increasing $\omega$ and committing to a policy that is tougher in controlling inflation. This is the case for the exogenous price stickiness models of Svensson (1997) and Clarida et al (1999), which lead to the optimal rule given by equation (6) and a bias in discretionary policy towards under-stabilisation. If the effect is negative then there are gains from reducing $\omega$ and committing to a policy that stabilises inflation less, implying a bias in discretionary policy towards over-stabilisation.

$$x_t = -\frac{\lambda}{\lambda^2 + \alpha(1 - \beta \rho)^2} u_t$$  \hspace{1cm} (6)

If the level of price stickiness in the economy is endogenous then changing the policy of the central bank will also affect the probability with which firms review and change their prices. When the central bank gets tougher in controlling inflation, firms generally need to review and change their prices less frequently. To allow this we permit the slope of the Phillips curve to depend on central bank policy, i.e. $\lambda = \lambda(\omega)$. The responsiveness of the Phillips curve to changes in policy is measured by $\xi_{\lambda \omega}$, the elasticity of the slope $\lambda$ with respect to the feedback parameter $\omega$.

The effect on welfare of marginally increasing the feedback coefficient from its discretionary level $\omega^d$ can be calculated from the objective function (3). After substituting for the output gap and inflation using the policy rule (5) and the Phillips curve (1), the first derivative of the objective function with respect to $\omega$ is shown in equation (7). The derivative is evaluated at the discretionary policy $\omega^d$.

$$\frac{d W_i}{d \omega} \bigg|_{\omega^d} = \frac{\alpha \lambda (\beta \rho + \xi_{\lambda \omega})}{(\lambda^2 + \alpha(1 - \beta \rho))(1 - \beta \rho)}$$  \hspace{1cm} (7)

The direction of the stabilisation bias depends on the sign of the expression in equation (8). Since $\beta \rho < 1$ and all other parameters are positive, we can simplify the determinants of the direction of the stabilisation bias to equation (8).
\[
\text{sgn} \left. \frac{dW_i}{d\omega} \right|_{\omega_d} = \text{sgn}(\beta \rho + \xi_{\lambda \omega}) \tag{8}
\]

Economic theory suggests that the elasticity of the Phillips curve slope \(\xi_{\lambda \omega}\) is negative\(^1\), so the overall direction of the stabilisation bias is ambiguous. However, we can identify two special cases. When price stickiness is exogenous \(\xi_{\lambda \omega} = 0\), the expression in equation (8) is positive, and the bias is always towards under-stabilisation of inflation. The model reduces to that of Clarida et al (1999) and unambiguously recommends that the central bank commits to a policy which is tougher in controlling inflation. Alternatively, if the cost-push shocks are not persistent \(\rho = 0\), the expression in equation (8) is negative and the bias is always towards over-stabilisation of inflation. The central bank should then unambiguously commit to a policy stabilising inflation less.

The precise direction of the stabilisation bias depends on the relative size of the elasticity of the Phillips curve slope and the persistence of the cost-push shocks. In the following section we develop a model of endogenous price stickiness which generates quantitative estimates for the elasticity \(\xi_{\lambda \omega}\). It allows us to determine the direction of the stabilisation bias for a wide range of parameter values.

3 A model of endogenous price stickiness

There is a continuum of monopolistically competitive firms producing differentiated goods in the economy. The differentiated goods are aggregated into a single composite good used for consumption.

Firms face constraints on the frequency with which they review and adjust their prices. We employ a variant of the Calvo (1983) model and assume that firms review their prices infrequently, with opportunities to review arriving according to a Poisson process. Each period the firm has a probability \(1 - \theta\) of having the opportunity to review its prices. The expected frequency of price reviews is \(1 - \theta\) and the interval between price reviews is a random variable.

Price reviews occur infrequently because of the presence of a review cost, \(F\), incurred whenever the firm has an opportunity to review its price. The timing of the model is shown in Figure 1. Once the opportunity to review prices arises, the firm has to pay the review cost \(F\). The price review reveals current market conditions and the desired price \(p^*_i\) that would maximise short-run profits. The firm then sets its price \(p_{it}\). In our model, once the review cost has been paid the firm incurs no further cost in changing its price. Price reviews are consequently always followed by price changes and the two expressions are synonymous.

---

\(^1\)Increasing the feedback parameter \(\omega\) leads to better stabilisation of inflation, which discourages firms from making frequent price changes and flattens the Phillips curve. Such a relationship is predicted by many models of endogenous rigidities, e.g. Ball and Romer (1990).
Figure 1: Timing of the model

Optimal firm behaviour involves choosing an *ex ante* probability $1 - \theta$ of making a price review and then determining the actual price, $p_{it}$, to set if the opportunity to review prices arises. As in Kiley (1996), the firm trades off the flexibility of frequent price reviews against the higher expected costs. Equation (9) defines the value function $V_j$ for a firm which set its price $j$ periods ago. Following Rotemberg (1987), we assume that the per-period return is quadratic in the difference in logarithms between the firm’s actual price $p_{it}$ and the desired price $p^*_j$. The continuation value in the Bellman equation is a weighted average of the expected value if the firm is able ($V_0$) or unable ($V_{j+1}$) to review its price next period. If the firm is able to review its price it also incurs the review cost $F$.

$$V_j = -E_t (p_{it} - p^*_j)^2 + \beta E_t [(1 - \theta)(V_0 - F) + \theta V_{j+1}]$$  \hspace{1cm} (9)

The optimisation problem of the firm is solved backwards to ensure subgame perfection. We first focus on optimal price setting and then proceed in reverse to derive the optimal probability of price reviews.

### 3.1 Optimal price setting

The price-setting decision of the firm takes the probability of price reviews as fixed. When the opportunity to review prices arises, the firm maximises its value $V_0$ by setting price $p_{it}$. Equation (10) is obtained by solving equation (9) forwards. The probability of not being able to review prices in the future enters exogenously into the discount factor.

$$V_0(1 - \beta) = E_t \left[ -(1 - \beta \theta) \sum_{j=0}^{\infty} (\beta \theta)^j (p_{it} - p^*_j)^2 - \beta (1 - \theta) F \right]$$  \hspace{1cm} (10)

The desired price $p^*_j$ can be approximated by the flexible price equilibrium under monopolistic competition. In the standard framework of Dixit and Stiglitz (1977), each individual monopolistically competitive firm faces a downward-sloping demand curve of the form $Y_{it} = (P_{it}/P)^{-\eta} Y_t$. In the flexible
price equilibrium, prices are set as a constant mark-up over real marginal costs $mc_t$. The desired price in logarithms is defined by equation (11).

$$p_t^* = p_t + mc_t$$  \hspace{1cm} (11)

We assume that real marginal costs are related to the output gap by a linear relationship and are subject to cost-push shocks. The proportional relationship and the shocks can be justified by making assumptions about preferences, technology, and the structure of labour markets, as discussed in more detail in Gali and Gertler (1999). Equation (12) summarises the definition of real marginal costs. $\gamma$ is the output elasticity of marginal cost and $u_t$ is the cost-push shock, scaled by $\lambda^{-1}$ so the aggregate Phillips curve has the form in equation (1).

$$mc_t = \gamma(x_t + \lambda^{-1}u_t)$$  \hspace{1cm} (12)

In optimal price setting under Calvo-type constraints, the firm maximises its value (10) subject to the definition of the desired price (11) and the given probability of price reviews. The problem is solved in Appendix A.1 and leads to a linear pricing rule of the form (13). The firm bases its price on the aggregate price level $p_t$ and passes on a fraction of its current marginal cost. The fraction $\phi$ is a function of the probability of price reviews $1 - \theta$ and the parameters $\lambda$, $\beta$, $\rho$ and $\gamma$.

$$p_t = p_t + \phi mc_t$$  \hspace{1cm} (13)

Aggregate prices in the economy evolve according to a weighted average of the new price that is set by firms able to review their price and the old price that other firms continue to charge. Clarida et al (1999) derive the familiar aggregate Phillips curve relationship (1) already presented in Section 2. The slope of the Phillips curve $\lambda$ depends on the probability with which firms review their prices, according to equation (14).

$$\lambda = \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \gamma$$  \hspace{1cm} (14)

3.2 Optimal probability of price reviews

We now introduce a fixed cost $F$ that has to be paid every time the firm has an opportunity to review its price. This “review cost” can be interpreted as the cost of acquiring the information needed to determine the optimal price to set.

The firm chooses the probability of price reviews $ex\ ante$, before any pricing decisions are made. The optimisation problem of the firm is therefore to
maximise its unconditional expected value (15), subject to the optimal price-setting rule (13) and the definition of the desired price (11). $\theta^i (1 - \theta)$ is the unconditional probability that the firm has a price set $j$ periods ago.

$$V = E \sum_{j=0}^{\infty} \theta^j (1 - \theta)V_j$$  \hspace{1cm} (15)

In Appendix A.2 we show that the unconditional expected value (15) can be written in terms of the volatility of marginal costs, the volatility of inflation, and expected review costs.\(^2\) Equation (16) shows the decomposition, where $A_1$ and $A_2$ are functions of the probability of making price reviews $1 - \theta$ and all other parameters. Marginal cost volatility creates problems because the firm is unwilling to pass all the current cost variation on to prices, in the knowledge that the price may remain valid in the future even though costs have changed. Inflation is similarly problematic since it drives an ever-increasing wedge between the firm’s price and the desired price until the next opportunity for the firm to review its price arises. More infrequent price adjustments are typically associated with increasing expected losses due to inflation.

$$V = -\frac{A_1 \sigma_{mc}^2}{\pi} - \frac{A_2 \sigma_\pi^2}{\pi} - F(1 - \theta)$$  \hspace{1cm} (16)

The optimisation problem of the firm is illustrated in Figure 2. The firm faces a trade-off between the benefits of price flexibility and the high costs of frequent price reviews. The firm chooses the probability of price reviews $1 - \theta$ that minimises expected losses and maximises the value of the firm.

\(^2\)There is also a term in the expected covariance of marginal costs and inflation. Without loss of generality, we subsume this term into inflation variability costs.
The first order condition of the firm’s optimisation problem is given by equation (17). The optimal probability of price reviews is a function of the review cost and the expected volatilities of marginal costs and inflation.

$$-\frac{\partial A_1}{\partial \theta} \sigma_{mc}^2 - \frac{\partial A_2}{\partial \theta} \sigma_x^2 + F = 0$$  \hspace{1cm} (17)

A set of comparative static results can be derived from total differentiation of the first order condition (17) and the second order condition for a maximum. In general, an increase in the review cost or a decrease in either marginal cost or inflation volatility leads to more infrequent price reviews and hence a longer expected duration for individual prices.

3.3 Calculation of elasticity $\xi_{\lambda\omega}$

Our endogenous price stickiness model enables us to quantify the effect of central bank policy on the slope of the Phillips curve. To evaluate the elasticity $\xi_{\lambda\omega}$ at the discretionary point $x_d$, we begin by calculating the effect of the feedback parameter $\omega$ on aggregate volatilities in the economy. The discretionary policy (4) and the Phillips curve (1) define the volatilities of marginal cost and inflation respectively. Our model of endogenous price stickiness provides the link between volatilities and the Phillips curve slope. The volatilities of marginal costs and inflation determine the probability of price reviews $1 - \theta$ by the firm’s first order condition (17), and translate into the slope $\lambda$ of the Phillips curve through equation (14).

4 Calibration

The baseline calibration of the model matches the quantitative features of the Rotemberg and Woodford (1997) analysis of U.S. monetary policy. They find that the responses of inflation and output to a monetary shock in the model fit best to those estimated empirically when the output elasticity of marginal costs $\gamma$ is 0.134 and the probability of making a price review $1 - \theta$ is 0.34. They also calibrate the relative weight $\alpha$ on output volatility in the central bank objective function to 0.048 by a Taylor series expansion of the expected utility of the representative firm.

The persistence $\rho$ of the cost-push shocks is crucial for the properties of the model. Woodford (1999) suggests a value of 0.35 is most appropriate on the basis of the volatility observed in U.S. interest rates. We also perform sensitivity analysis with other values.

We calibrate the review cost $F$ such that, if the central bank follows the discretionary policy, firms will endogenously choose to review their prices with the probability $1 - \theta$. In the baseline calibration, we set $F$ to 37.139 to support
a value for $1 - \theta$ of 0.34. By changing the review cost we are able to support a wide range of different probabilities. We assume the discount factor $\beta$ is 0.99 and normalise the variance of the i.i.d. component of cost-push shocks to unity.

5 Results

Table 1 shows the value of the elasticity $\xi_{\lambda\omega}$ in our model, evaluated at the discretionary policy $\omega^d$. In the baseline calibration, a 1% increase in the feedback parameter $\omega$ flattens the slope $\lambda$ of the Phillips curve by 0.32%. The implication of this for the direction of the stabilisation bias depends on the sign of $\beta \rho + \xi_{\lambda\omega}$ as explained in Section 2. A positive value for the baseline calibration indicates a slight bias towards under-stabilisation of inflation by the central bank.

<table>
<thead>
<tr>
<th>Persistence</th>
<th>$\rho$</th>
<th>$\xi_{\lambda\omega}$</th>
<th>$\beta \rho + \xi_{\lambda\omega}$</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.35</td>
<td>-0.32</td>
<td>0.03</td>
<td>Under-stabilisation</td>
</tr>
<tr>
<td>High</td>
<td>0.90</td>
<td>-0.73</td>
<td>0.16</td>
<td>Under-stabilisation</td>
</tr>
<tr>
<td>Low</td>
<td>0.10</td>
<td>-0.25</td>
<td>-0.15</td>
<td>Over-stabilisation</td>
</tr>
</tbody>
</table>

Table 1: Calculations of the stabilisation bias in the model

The values of $\xi_{\lambda\omega}$ for different persistence parameters $\rho$ are also presented in Table 1. The results suggest that the stabilisation bias is more pronounced if the persistence of the cost-push shocks differs from the baseline calibrated value. For high persistence, $\beta \rho + \xi_{\lambda\omega} > 0$ and there is a bias towards under-stabilisation of inflation by the central bank. This supports the conclusions of Svensson (1997) and Clarida et al (1999), who rely on persistent cost-push shocks to derive their results. However, for low persistence, $\beta \rho + \xi_{\lambda\omega} < 0$ and the bias goes in the opposite direction. The effect of endogenous price stickiness dominates and there is a bias towards over-stabilisation of inflation.

In Figure 2 we show the direction of the stabilisation bias for all combinations of the persistence parameter $\rho$ and the probability of price reviews $1 - \theta$. The output elasticity of marginal cost $\gamma$ and the relative weight on output stability $\alpha$ are both held fixed but the review cost $F$ is re-calibrated at each point to support the desired probability of price reviews. Our model of endogenous price stickiness is well defined for a large proportion of the parameter space. Only if $\theta$ is very high and $\rho$ very low is it impossible to calibrate $F$ to support the desired probability of price reviews.3

3In the region where the model is undefined, the firm’s second order condition for the optimal probability of price reviews is violated. Rather than review their prices with the desired frequency, the firm switches to a policy of perfect flexibility with $\theta = 0$. 

15
For high values of $\theta$ and $\rho$, the bias is towards under-stabilisation. In other words, the conclusions of Svensson (1997) and Clarida et al (1999) remain valid when there is a high degree of nominal price rigidity in the economy and cost-push shocks are persistent. Otherwise, the bias goes in the other direction. Our baseline calibration based on Rotemberg and Woodford (1997) is very close to the border between the two regions, suggesting that in practice the overall magnitude of the stabilisation bias is likely to be small.

6 Conclusions

The results of this paper suggest taking endogenous price stickiness into account has important consequences for the direction of the stabilisation bias in monetary policy. Contrary to the conclusions of exogenous price stickiness models, we show that the direction of the bias is ambiguous. There is an under-stabilisation effect as in Svensson (1997) and Clarida et al (1999), but also an over-stabilisation effect because there are gains to committing to a policy that stabilises inflation less. Such a policy encourages firms to make more frequent price changes and hence promotes price flexibility. The Phillips curve trade-off improves and it is easier to control inflation.

The overall direction of the stabilisation bias depends on the relative size of the incentives towards under and over-stabilisation. Our numerical calculations reveal a bias to under-stabilisation if there are high nominal price rigidities and persistent shocks. In contrast, inflation is likely to be over-stabilised in a flexible economy with shocks of low persistence. Our baseline calibration, based on Rotemberg and Woodford (1997), is close to the border between under and over-stabilisation. In this case, the gains to commitment largely
offset the losses and the overall magnitude of the stabilisation bias is likely to be small.
References


Appendix 1. Optimal price setting

The optimal price-setting problem is to maximise the expected value of the firm (10), subject to the definition of the desired price (11) and the given frequency of price reviews. Equation (A.1) shows the first order condition for the optimisation problem. The current price is set as a weighted average of the current and all future desired prices.

\[ p_t = E_t(1 - \beta \theta) \sum_{j=0}^{\infty} (\beta \theta)^j p_{t+j}^* \]  
(A.1)

Expanding equation (A.1) using the definition of the desired price (11) gives an expression (A.2) in terms of the current aggregate price, expected future inflation, and current and future expected costs.

\[ p_t = p_t + E_t \left[ \sum_{j=1}^{\infty} (\beta \theta)^j \pi_{t+j} + (1 - \beta \theta) \sum_{j=0}^{\infty} (\beta \theta)^j m_{t+j} \right] \]  
(A.2)

Inflation in the model is given by equation (A.3), as in the exogenous price stickiness model of Clarida et al (1999).

\[ \pi_t = \frac{1 - \omega \lambda}{1 - \beta \rho} u_t \]  
(A.3)

Combining equations (A.2), (A.3) and the definition of the cost-push shock (2) leads to the optimal pricing rule (A.4). This is of the form required for equation (12).

\[ p_t = p_t + \left[ \frac{\lambda \beta \theta \rho + \gamma(1 - \beta \theta)(1 - \beta \rho)}{\gamma(1 - \beta \rho)(1 - \beta \theta \rho)} \right] m_{t} \]  
(A.4)
Appendix 2. Unconditional expected value of the firm

We show that it is possible to write the unconditional expected value of the firm (15) in terms of the volatilities of marginal cost and inflation. Begin by substituting the firm’s value function (9) into the unconditional expectation (15) and solving forward to obtain equation (A.5).

\[ V(1 - \beta) = E \left[ - (1 - \theta) \sum_{j=0}^{\infty} \theta^j (p_t - p^*_t)^2 - \beta(1 - \theta) F \right] \quad \text{(A.5)} \]

The summation term in (A.5) is analysed in Table A.1. We decompose the expected deviation of a firm’s price from its desired level and use the optimal price-setting rule (13).

<table>
<thead>
<tr>
<th>$j$</th>
<th>Discount factor</th>
<th>$p_t - p^*_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>$\phi mc_t - mc_t$</td>
</tr>
<tr>
<td>1</td>
<td>$\theta$</td>
<td>$\phi mc_t - \pi_{t+1} - mc_{t+1}$</td>
</tr>
<tr>
<td>2</td>
<td>$\theta^2$</td>
<td>$\phi mc_t - \pi_{t+1} - \pi_{t+2} - mc_{t+2}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$n$</td>
<td>$\theta^n$</td>
<td>$\phi mc_t - \pi_{t+1} - \cdots - \pi_{t+n} - mc_{t+n}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Table A.1: Expected deviation of price from desired price

Inflation and marginal costs are both AR(1) variables with persistence parameter $\rho$ in the model. On this basis, it is possible to use Table A.1 to calculate the expectation of the summation in equation (A.5). The full expression for the unconditional expected value of the firm is shown in equation (A.6).

\[
V(1 - \beta) = - \left[ \frac{(\phi^2 + 1)(1 - \theta \rho) - 2\phi(1 - \theta)}{1 - \theta \rho} \right] \sigma_{mc}^2
- \left[ \theta \frac{1 + \theta \rho}{(1 - \theta)(1 - \theta \rho)} \right] \sigma^2 - \left[ 2\theta \frac{1 - \phi \rho}{1 - \theta \rho} \right] \sigma_{mc,\pi}
- \beta(1 - \theta) F \quad \text{(A.6)}
\]

This can be expressed in the form required for equation (16) by applying the transformation $\sigma_{mc,\pi} = (\lambda / (1 - \rho \lambda)) \sigma^2$, derived from the definition of inflation (A.3) and the policy rule (5).
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