Stock exchange alliances, access fees and competition

Oz Shy – Juha Tarkka
Research Department
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* Department of Economics, University of Haifa, 31905 Haifa, Israel. E-mail: ozshy@econ.haifa.ac.il.
** Research Department, Bank of Finland, P.O. Box 160, FIN-00101 Helsinki, Finland. E-mail: juha.tarkka@bof.fi.
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Oz Shy – Juha Tarkka
Research Department

Abstract

This paper investigates the market consequences of alliance formation among stock exchanges. These alliances enable brokers to match investors internationally at their local market, thereby eliminating the need for brokers to maintain memberships in foreign stock exchanges. We sort out the conditions under which alliance formation increases profits for stock exchanges and brokers, and how changes in fee structures affect investors’ participation rates and welfare. Finally, we examine several methods for implementing access fees and their welfare implications.

Key words: stock exchange alliances, access fees, competition among stock exchanges

JEL classification numbers: G2
Pörssien liittoutumat, liittymismaksut ja kilpailu

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1 Introduction

1.1 Observations and motivation

Stock exchanges in Europe and in the United States are in a transition period. Two major changes are taking place at the same time. First, many become public (for example, London Stock Exchange, and Deutsche Börse). Second, they seek to form alliances with other stock exchanges, thereby enhancing liquidity (for example, Euronext: the alliance among the Paris, Amsterdam, and Brussels bourses; Newex: Deutsche Börse with Vienna; and Norex: consisting of Copenhagen, Stockholm, Oslo, and Iceland).

The present paper deals with the second aspect characterising this transition period. It provides a comprehensive microeconomic analysis of alliances among stock exchanges. We attempt to answer the following questions: (1) How alliances affect the fees stock exchanges levy on security houses, and the fees security houses levy on investors, as well as their profits? (2) What would be the effect on investors’ participation rate, investors’ welfare, and social welfare? (3) What are the efficient and inefficient access fee mechanisms that would characterize the formation of alliances?

The recent wave of alliance formation among stock exchanges follows a large increase in cross-border equity flows which is estimated to exceed one-trillion dollars. In Europe, the launch of a single currency has facilitated the accounting side of cross-border transactions, and has increased the number of international investors. Equally important, alliances are triggered by technology changes stemming from innovations in the software and communication industries, which consist of technologies allowing trade in securities to become fully automated.\footnote{Other consequences brought about by the information revolution include a reduction in the information gap between institutions and investors, which further intensified the search for cost-reducing trading technologies such as Internet trading, see Madhavan (2000). In fact, historically, technological innovation has always played a role in the integration of trading service bringing down the number of stock exchanges in the U.S. from over a hundred in the nineteenth-century to five major stock exchanges.}
A natural question to ask is whether it is optimal to have several stock exchanges, rather than having a single market, given that the switch to electronic trading systems has probably removed or at least limited the diseconomies of scale which must be associated with very large floor-based systems. We argue that this is not the case for the following reasons. First, we would define stock exchange alliances as agreements to connect trading systems so that orders can flow from each participating exchange to the other. Actually, the alliances and exchange mergers are similar in the sense that alliances, just as outright mergers, allow trading services to be provided in a larger scale. The difference is that in alliances, ownership, decision making and pricing are not completely unified as they would be in a merger. Alliances may therefore combine the benefits of mergers (in terms of efficiency) with the advantages of having several geographically or otherwise specialized exchanges.

Secondly, investors would continue to prefer to place orders for equity in markets located in the proximity of the firms, simply because of better information, resulting from language and cultural barriers. Thirdly, Blume (2000) argues that investors have different needs in the form of preferences for speed of execution and anonymity.² Lastly, since we do not observe a single worldwide telephone company, neither we observe a single mail carrier or a single commercial bank, so we are unlikely to observe a single market for equity. The reason is that large organizations are operating under supply-side decreasing returns to scale, so entry of small firms (stock exchanges, in our case) always occurs.

In most European and Asian countries, stock exchanges have historically been local monopolies, whereas North-American exchanges compete with each other. Malkamäki and Topi (1999) report that the average cost per transaction at the end of 1996 was about three times higher in Europe than in North America. Since the value of cross-border transactions in Europe has increased substantially with the advent of the Euro, many measures take place to

²Although our paper does not analyze firms’ listing choice problems it should be mentioned that some firms find it beneficial to be listed in different markets. Pagano and others list a variety of reasons for cross listing, relating to companies’ characteristics and behavior (2001 a paper); and relating to differences in characteristics between destination and home markets, (2001b paper).

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maintain European stock exchanges globally competitive. As a result, security houses will want to keep their own cost down by avoiding paying membership fees to many European stock exchanges and to have dozens of different terminals for trading and settlement of trades. Indeed, the present paper models the real gain from alliance formation among stock exchanges as the reduction in real cost of having to maintain multiple memberships by each security house. Therefore, Malkamäki and Topi (1999) argue that at least in the short-run cooperation among European exchanges will continue to be based on alliances rather than mergers. Alliances are likely to persist in the long run given the fact that Europe is heterogeneous with respect to language, culture accounting principles, and bankruptcy legislation.

Finally, we should mention that we do observe alliances among banks in payment systems. The widely-used SWIFT (Society for Worldwide Interbank Financial Telecommunication established in 1973, see www.swift.com) links over 7000 financial institutions in 193 countries. The average daily value of payment messages is estimated to be above $5 trillion. In the U.S., CHIPS (Clearing House Interbank Payment System) executes transfers of funds. In Europe, the European Banking Association has established a payment clearing alliance called Euro1.

1.2 Access pricing

Access pricing is now widely recognized as the essential mechanism for deregulating and opening to competition of (what used to be called) natural-monopoly industries. In the past twenty years, the waves of deregulation and privatization of public utility companies has proven that there is no need to grant a monopoly power to a single firm merely because the service it provides requires a large investment in infrastructure. Instead, by utilizing access fee mechanisms, competition in this type of industries can be generated by requiring that all firms (incumbents, in particular) allow other competing firms to make use their infrastructure thereby granting access to consumers connected
to competing networks. The access fee mechanism could be regulated (as commonly observed in telecommunication markets) or negotiated (commonly observed in the airline industry in the form of code-sharing agreements). Access pricing is also observed in the banking industry (access to ATMs), credit cards and banks utilizing charge/debit cards, and railroad track sharing.

The industries mentioned above have been (and still are) in the process of transiting from a regulatory supervision to being subjected to competition policy under antitrust regulation. We anticipate that stock exchange markets may soon follow the same transition patterns.

1.3 Theoretical literature

We are not aware of any theoretical literature particularly dealing with the impact alliances among stock exchanges on market structures and equilibrium fees levied by stock exchanges and security houses. Several authors analyzed the implications of network externalities in securities markets. Economides and Siow (1988) have emphasized the tradeoff between network externalities and economies of scale versus spatial or localization advantages. Pagano (1989) analyzed how market asymmetric market access costs may lead to multiple equilibria, where large-quantity investors select markets with high access fees. Gehrig (1998) suggests a novel approach for modeling competition between market places that endogenously differentiates the interests of firms within a market place from the interests of outside firms. A comprehensive survey of this literature is given in Gehrig (2000).

1.4 Organization

Section 2 constructs a model of competing stock exchanges and brokerage firms competing on investors. Section 3 solves for the equilibrium fees brokers pay stock exchanges, and the fees investors pay the brokers and for the market
coverage when there is no cooperation between stock exchanges. Section 4 solves for the equilibrium fees and market coverage when stock exchanges form an alliance. Section 5 analyzes the implications of alliance formation. Section 6 explores a variety of access fee mechanisms. Section 7 concludes.

2 A model of stock exchanges and brokerage firms

Consider a world economy with two geographically-separated markets, say, two different countries. In each country there is one stock exchange and one major security house (a brokerage firm, or a broker, in what follows). We label stock exchanges by $A$ and $B$ and security houses by 1 and 2. Each security house maintains two memberships, one in the stock exchange located at the broker’s base market (country), and a second membership in the stock exchange located in the other market (foreign country).

On each exchange, stocks become available for purchase in fixed (exogenous) quantities and are purchased by investors. For simplicity, the supply of shares on the stock markets is modeled as a primary market, even though we do not see any reason why the results would not hold in the (more complicated) case of secondary markets.

We assume that investors do not have direct access to stock exchanges so all trade must be done via one of the two brokerage firms. In addition, each trade in stocks must be executed via one of the stock exchanges (that is, stock brokers cannot match buyers and sellers without utilizing a stock exchange).

2.1 Potential investors

There is a continuum of risk-neutral potential investors who are uniformly-distributed on the interval $[0, 1]$ with unit density according to increased preference for investing via broker 2. Let $V$ denote a investor’s basic aggregate value of the assets. $V$ could represent the value for a sale or an acquisition of
the assets. Also let $f_1$ and $f_2$ denote the fee each investor pays broker 1 or 2, respectively, for executing the trades. Formally, the utility function of an investor indexed by $x$ ($x \in [0,1]$) is given by

$$U_x \overset{\text{def}}{=} \begin{cases} V - f_1 - \tau x & \text{if trades via broker 1} \\ V - f_2 - \tau (1 - x) & \text{if trades via broker 2} \\ 0 & \text{if does not trade,} \end{cases} \quad (1)$$

where $\tau > 0$ is the differentiation parameter. The interval over which the investors are distributed could be given a geographic interpretation (of distance between two financial centers). However, other interpretations of why some investors prefer broker 1 to broker 2 or vice versa may also be given.

Let $x_1$ denote a potential investor who is indifferent between trading via broker 1 and not trading at all. Similarly, let $x_2$ denote a potential investor who is indifferent between trading via broker 2 and not trading at all. Figure 1 illustrates two possibilities. First, we say that the market is partially served if $x_1 < x_2$. Second, if $x_1 \geq x_2$ we say the the market is fully served. Suppose that the market is only partially served. Then, (1) implies that

$$x_1 = \frac{V - f_1}{\tau} \quad \text{and} \quad x_2 = -\frac{V - f_2 - \tau}{\tau}. \quad (2)$$

2.2 Traded assets

Investors place trade orders on a variety of assets, which we normalized to unity. We assume that a fraction $\theta$ of these assets is available for trade in
market $A$, whereas a fraction $1 - \theta$ is traded in market $B$.\(^3\)

One way to interpret an asymmetry between the stock exchanges, for example the case where $\theta > 1/2$, is that stock exchange $A$ is “larger” to than $B$, so trade is more likely to be realized at $A$ than at $B$. A second interpretation would be that $A$ has been in operation long before $B$, hence the asset is traded in $A$ more often than in $B$. The following assumption is needed to ensure that both brokers maintain strictly-positive market shares among potential investors.

**Assumption 1**

The fraction of shares traded in each market is bounded. Formally, $1/3 < \theta < 2/3$.  

### 2.3 Brokerage firms

In view of Figure 1, Broker 1 receives trade orders from $x_1$ investors, and broker 2 receives purchase orders from $1 - x_2$ investors. Both brokers match their investors with investors appearing either in market $A$ or market $B$.

Each broker incurs a cost of $\mu \geq 0$ per investor for each trade that takes place at the broker’s foreign market. The following assumption ensures that in this economy some potential investors will not participate.\(^4\)

**Assumption 2**

The brokers are sufficiently differentiated from investors’ point of view. Formally,

$$\tau > \max \left\{ \frac{2V - \mu}{6}, \frac{V}{4} \right\}.$$  

The following assumption restricts the value of $\mu$ to a range where in equilibrium each broker will serve some investors.

---

\(^3\)An equivalent formulation would be to assume that there is only one asset which is traded in market $A$ with probability $\theta$ and in market $B$ with probability $1 - \theta$.

\(^4\)A welfare analysis involving a fully-served market is not interesting since in that case, aggregate social welfare remains invariant to changes in market structures. When the market is fully served, alliances would only affect the distribution of rents among investors, brokers, and stock exchanges, but will not have any real effect.
ASSUMPTION 3

A broker’s cost of a trade at a foreign market is bounded. Formally,

$$\mu < \bar{\mu} \overset{\text{def}}{=} \min \left\{ \frac{V}{2 - 3\theta}, \frac{V}{3\theta - 1}, V \right\}.$$  

Notice that for symmetric markets, i.e., $\theta = 1/2$, Assumption 3 is reduced to the restriction that $\mu < \bar{\mu} = V$ which means that the benefit from a trade must exceed a broker’s cost of a match at a foreign market. Also, note that Assumption 1 implies that $\bar{\mu} > 3V/4$.

Let $f_1$ and $f_2$ denote the fees broker 1 and broker 2 charge investors for executing their trades. Broker 1 chooses the fee $f_1$ to maximize profit given by

$$\pi_1 \overset{\text{def}}{=} x_1 [f_1 - \theta f_A - (1 - \theta)(f_B + \mu)], \quad (3)$$

where $f_A$ and $f_B$ are the fees collected by stock exchange $A$ and $B$, respectively. The first term in (3) is the profit from matching the $x_1$ investors in both stock exchanges. The second and third terms are the costs generated by matching investors at stock exchanges $A$ and $B$, respectively. Similarly, broker 2 chooses the fee $f_2$ to maximize profit given by

$$\pi_2 \overset{\text{def}}{=} (1 - x_2) [f_2 - \theta(f_A + \mu) - (1 - \theta)f_B]. \quad (4)$$

Substituting (2) into (3) and (4), broker 1 takes $f_A$ and $f_B$ as given and chooses buyer’s fee that solves

$$\max_{f_1} \pi_1 = \left( \frac{V - f_1}{\tau} \right) [f_1 - \theta f_A - (1 - \theta)(f_B + \mu)]. \quad (5)$$

Similarly, broker 2 solves

$$\max_{f_2} \pi_2 = \left( 1 - \frac{V - f_2 - \tau}{\tau} \right) [f_2 - \theta(f_A + \mu) - (1 - \theta)f_B]. \quad (6)$$

2.4 Stock exchanges

Stock exchanges collect fees from brokers for matching their investors. We assume that the cost to a stock exchange from matching an additional investor is zero. Fixed costs are analyzed in Section 6.3. Therefore, the profit of each
stock exchange is the fraction of shares traded in its market, multiplied by
the number of investors (submitted by all brokers), and by the fee levied on
brokers for each match. Formally let,

\[
\pi_A \equiv \theta (x_1 + 1 - x_2) f_A \quad \text{and} \quad \pi_B \equiv (1 - \theta) (x_1 + 1 - x_2) f_B.
\]  

(7)

2.5 Timing

Figure 2 depicts the structure of our economy. The actions of the agents in

![Diagram of the economy structure]

**Figure 2:** The structure of the economy.

this economy are divided into three stages.

**Stage I:** Stock exchange A sets its fees on brokers, \( f_A \), and stock exchange B
sets \( f_B \).

**Stage II:** Broker 1 sets investors' fee \( f_1 \), and broker 2 sets \( f_2 \).

**Stage III:** Potential investors determine whether to trade via broker 1, via
broker 2, or not trade at all.
3 Equilibrium fees in the absence of alliances

We now solve for the subgame-perfect equilibrium fees levied by brokers and by stock exchanges. In stage III, investors choose which broker to utilize for their trades, or whether not to trade at all. The outcomes of these decisions are already summarized by (2). In stage II, brokerage firms solve (5) and (6) yielding unique fees given by

\[ f_1 = \frac{V + \theta f_A + (1 - \theta)(f_B + \mu)}{2}, \quad \text{and} \quad f_2 = \frac{V + \theta (f_A + \mu) + (1 - \theta)f_B}{2}. \]

Therefore, the fees brokers levy on investors increase with investors’ valuation of the transaction, \( V \), fees they have to pay stock exchanges, \( f_A, f_B \), and the cost of handling a transaction outside their home market, \( \mu \). Substituting (8) into (2) yields

\[ x_1 = \frac{V - \theta f_A - (1 - \theta)(f_B + \mu)}{2\tau}, \quad \text{and} \quad x_2 = \frac{V - \theta (f_A + \mu) - (1 - \theta)f_B - 2\tau}{2\tau}. \]

Finally, substituting (9) into (7), in stage I, stock exchanges set their fees on brokers to maximize (7). The best-response functions are given by

\[ f_A(f_B) = \frac{2V - \mu}{4\theta} - \frac{1 - \theta}{2\theta} f_B \quad \text{and} \quad f_B(f_A) = \frac{2V - \mu}{4(1 - \theta)} - \frac{\theta}{2(1 - \theta)} f_A. \]

**Proposition 1**

The fees stock exchanges levy on brokers are strategic substitutes. That is, an increase in the fee set by one stock exchange would reduce the fee set by the other exchange.

Proposition 1 is rather surprising since in the present model Figure 2 shows that there is no direct competition between the two brokers and between the two stock exchanges. In fact, our derivations are based on a partially-served market. However, each stock exchange confers an externality on the other exchange. The externality stems from the fact that all participating investors trade on both exchanges, so an increase in a fee levied by one stock exchange will reduce the total number of investors. This means that the other exchange must respond by lowering its fee in order to mitigate the reduction in the number of excluded investors.
Solving (10) yields the equilibrium fees stock exchanges levy on brokers. Hence,
\[ f_A = \frac{2V - \mu}{6\theta} \quad \text{and} \quad f_B = \frac{2V - \mu}{6(1 - \theta)}. \] (11)
Substituting (11) into (9) yields the equilibrium market share of each broker. Hence,
\[ x_1 = \frac{V - (2 - 3\theta)\mu}{6\tau} \quad \text{and} \quad 1 - x_2 = \frac{V - (3\theta - 1)\mu}{6\tau}. \] (12)
Clearly, the market is partially served if \( x_1 + 1 - x_2 < 1 \) which holds by Assumption 2. In addition, Assumption 3 implies that each broker serves a strictly positive number of investors, i.e., \( x_1 > 0 \) and \( x_2 < 1 \). Substituting (11) and (12) into (7) yields the equilibrium profit levels of the stock exchanges. Thus,
\[ \pi_A = \pi_B = \frac{(2V - \mu)^2}{36\tau}. \] (13)
Now, to find the equilibrium fees brokers charge their investors, substitute (11) into (8). Then,
\[ f_1 = \frac{5V + (2 - 3\theta)\mu}{6} \quad \text{and} \quad f_2 = \frac{5V + (3\theta - 1)\mu}{6}. \] (14)
The profits of the brokerage firms are found by substituting (11) into (5) and (6). Therefore,
\[ \pi_1 = \frac{[V - (2 - 3\theta)\mu]^2}{36\tau}, \quad \text{and} \quad \pi_2 = \frac{[V - (3\theta - 1)\mu]^2}{36\tau}. \] (15)

We conclude this section by investigating how changes in \( \theta \) affect the equilibrium fees levied by stock exchanges and the brokers. In the asymmetric case where \( \theta > 1/2 \), stock exchange A is larger than the exchange B in the sense that more stocks are traded in exchange A than in B. Figure 3 plots the equilibrium stock exchange fees, \( f_A \) and \( f_B \), as well as brokers’ fees \( f_1 \) and \( f_2 \), all as functions of \( \theta \). Figure 3 demonstrates that stock exchange A reduces the fee when \( \theta \) increases. This can be explained by observing that when \( \theta \) increases stock exchange A gains from reducing its fee (compared to B) in order to mitigate the reduction in the number of investors placing orders via broker 2 who is facing a higher cost (as long as \( \mu > 0 \)). This means that, as long as \( \mu > 0 \), as \( \theta \) increases the demand facing stock exchange A becomes
more elastic relative to the demand facing \( B \). Figure 3 also illustrates that \( f_1 \) decreases with \( \theta \) (\( f_2 \) increases with \( \theta \)) and this follows from the decrease in the fee levied by stock exchange \( A \) (increase in the fee levied by stock exchange \( B \)).

Equations (11)–(15) yield the following proposition which concludes our investigation of the no alliance equilibrium.

**Proposition 2**

Let \( \theta > 1/2 \) so that stock exchange \( A \) is “bigger” than \( B \), and suppose that \( \mu > 0 \). Then,

(a) Stock exchange \( A \) charges the brokers a lower fee than \( B \).

Formally, \( f_A < f_B \).

(b) Broker 1 (based in market \( A \)) charges investors a lower fee, maintains a higher market share, and earns a higher profit than broker 2 (based in market \( B \)). Formally, \( f_1 < f_2, x_1 > 1 - x_2, \text{ and } \pi_1 > \pi_2 \).

4 Equilibrium under the alliance

Suppose now that stock exchanges \( A \) and \( B \) sign an agreement on sharing their lists of investors. An alliance agreement between the stock exchanges
would permit each stock exchange to match an investor with another investor on the other stock exchange for a preannounced fee. Similar to alliances in the telecommunication industry, we call this fee an *access fee*, meaning that each stock exchange can access the list of investors appearing on the competing stock exchange.

4.1 Competition under the alliance agreement

Let \( a_A \) denote the fee stock exchange \( A \) levies on \( B \) for letting \( B \) match an \( A \)'s investor with a \( B \)'s investor. Similarly, let \( a_B \) be the fee that \( B \) levies on \( A \) for letting \( A \) match an \( A \)'s investor with a \( B \)'s investor. Modifying the timing structure described in Section 2.5, the interaction among stock exchanges, brokers, and investors is now given by the following four-stage game.

**Stage I:** Stock exchange \( A \) sets its access fee, \( a_A \) and stock exchange \( B \) sets its access fee, \( a_B \), noncooperatively.\(^5\)

**Stage II:** Stock exchange \( A \) sets its fee on brokers, \( f_A \), and stock exchange \( B \) sets \( f_B \).

**Stage III:** Broker 1 sets investors’ fee \( f_1 \), and broker 2 sets \( f_2 \).

**Stage IV:** Potential investors determine whether to trade via broker 1, broker 2, or not trade at all.

Comparing this timing structure to Section 2.5 reveals an additional step at which stock exchanges commit for access fees. A second difference is that each broker now sets a single buyer’s fee (as oppose to two fees) since under the alliance between the stock exchanges each broker always finds a seller in the stock exchange located near its base office. This clearly saves the per-match cost of \( \mu \) associated with a foreign match. That is, under this alliance, broker 1 trades in stock exchange \( A \) whereas broker 2 trades in stock exchange 2 only.

\(^5\)Section 6.1 analyzes the case where stock exchanges jointly determine access fees.
4.2 Equilibrium fees

We now solve for the subgame-perfect equilibrium fees set by brokers and stock exchanges. In stage III, the brokers solve

$$\max_{f_1} \pi_1 = x_1 (f_1 - f_A) \quad \text{and} \quad \max_{f_2} \pi_2 = (1 - x_2)(f_2 - f_B),$$

(16)

where $x_1$ and $x_2$ are given in (2). Comparing (16) with (3) and (4) demonstrates the effect of the alliance on the brokers, where under the alliance brokers trade only at their local stock exchanges thereby saving $\mu$ which is the cost executing a foreign trade. Solving (16), the unique fees brokers levy on investors, as functions of the fee levied by the corresponding stock exchange are

$$f_1 = \frac{V + f_A}{2} \quad \text{and} \quad f_2 = \frac{V + f_B}{2}.$$

(17)

Substituting (17) into (2) yields the brokers’ market shares as functions of stock exchange fees. Therefore

$$x_1 = \frac{V - f_A}{2\tau} \quad \text{and} \quad 1 - x_2 = \frac{V - f_B}{2\tau}.$$

(18)

In stage II, stock exchange $A$ sets a fee to be levied on the brokers to solve

$$\max_{f_A} \pi_A = \theta [x_1 f_A + (1 - x_2)a_A] + (1 - \theta)x_1 (f_A - a_B),$$

(19)

where $x_1$ and $x_2$ are given in (18). The first term is the profit collected by stock exchange $A$ in the event that the trade takes place at market $A$. This profit is composed of direct fees collected from broker 1 plus the access fees collected from stock exchange $B$ for matching $A$’s investors with $B$’s investors. The second term in (19) is the revenue stock exchange $A$ collects in the event that trade takes place at market $B$, in which case stock exchange $A$ must pay access fees to stock exchange $B$. Similar to (19), stock exchange $B$ sets its fee to be levied on brokers to solve

$$\max_{f_B} \pi_B = (1 - \theta) [(1 - x_2)f_B + x_1 a_B] + \theta(1 - x_2)(f_B - a_A).$$

(20)

Substituting (18) into (19) and (20), the profit-maximizing fees set by stock exchanges, as function of their access fees, are given by

$$f_A = \frac{V + (1 - \theta)a_B}{2} \quad \text{and} \quad f_B = \frac{V + \theta a_A}{2}.$$

(21)
In stage I, stock exchange A sets its access fee, \( a_A \), to maximize \( \pi_A \), and stock exchange B sets \( a_B \) to maximize \( \pi_B \). Substituting (18) and then (21) into (19) and (20) yields that stock exchange A chooses its access fee \( a_A \) to solve

\[
\max_{a_A} \pi_A = \frac{[V - (1 - \theta)a_B]^2 + 2a_A\theta V - 2(a_A)^2}{8\tau},
\]

and stock exchange B chooses \( a_B \) to solve

\[
\max_{a_B} \pi_B = \frac{(a_A)^2\theta^2 - 2a_A\theta V - 2(a_B)^2(1 - \theta)^2 + V[2(1 - \theta)a_B + V]}{8\tau}.
\]

Then, the equilibrium access fees of stock exchanges are given by

\[
a_A = \frac{V}{2\theta} \quad \text{and} \quad a_B = \frac{V}{2(1 - \theta)},
\]

Substituting (24) into (21) yields the fees stock exchanges charge brokers. Thus,

\[
f_A = f_B = \frac{3V}{4}.
\]

Therefore, the profits of the stock exchanges are given by

\[
\pi_A = \pi_B = \frac{3V^2}{32\tau}.
\]

Equations (24)–(26) yield the following proposition. As before, \( \theta > 1/2 \) means that stock exchange A trades more shares than B.

**Proposition 3**

(a) The larger stock exchange charges a lower access fee. Formally, \( a_A < a_B \), if and only if \( \theta > 1/2 \). However,

(b) For all admissible values of \( \theta \), both exchanges charge brokers equal fees and earn the same profit.

Proposition 3(a) demonstrates that under the alliance the large stock exchange faces a more elastic demand than the smaller stock exchange, since it has more to gain by lowering the fee thereby increasing the total number of investors. Proposition 3(b) reveals that the alliance serves as a mechanism by which stock exchanges “insure” each other against a shortage of sellers, thereby raising their profit.
On the brokers’ side, substituting (25) into (18) yields the equilibrium market share of each broker. Thus,

\[ x_1 = 1 - x_2 = \frac{V}{8\tau}, \tag{27} \]

Notice that the market is partially served if \( x_1 + 1 - x_2 < 1 \), hence if \( \tau > V/4 \), which is implied by Assumption 2. Substituting (25) into (17), and then (17) and (27) into (16) we obtain the equilibrium fees brokers levy on investors, and their profit levels. Hence,

\[ f_1 = f_2 = \frac{7V}{8} \quad \text{and} \quad \pi_1 = \pi_2 = \frac{V^2}{64\tau}. \tag{28} \]

5 The consequences of the alliance

In this section we analyze the effects of the formation of the alliance between the stock exchanges on fees, the profits of brokers and of stock exchanges, and on the welfare of investors, by comparing the equilibria of Section 3 with Section 4. In order to reduce the amount of writing, this section presents the results for the symmetric case only, where \( \theta = 1/2 \).

5.1 Profit comparison

Direct comparisons of the stock exchange profit functions (13) and (26), and of the brokers’ profit functions, (15) and (28), yield the following proposition.

**Proposition 4**

Let \( \theta = 1/2 \). The formation of an alliance between the stock exchanges

(a) Increases the profits earned by stock exchanges if and only if

\[ \mu \geq \frac{8 - 3\sqrt{6}}{4} V \approx 0.163 V. \]

(b) Increases the profits earned by brokers if and only if \( \mu \geq V/2 \).
Figure 4: Change of profits caused by the alliance as functions of brokers’ cost of a foreign match. Note: Figure assumes $\theta = 1/2$.

Figure 4 illustrates how profit levels are affected by the alliance for all admissible values of $\mu$ (see Assumption 3). Figure 4 reveals that for an intermediate range of $\mu$ the alliance between the stock exchanges shifts some rents from brokerage firms to the stock exchanges. This shift is at a larger magnitude than the cost saving to brokers resulting from the elimination of the need for a foreign match. Hence, all the associated cost saving are now captured by the stock exchanges and not by the brokers. Therefore, in this range of $\mu$ stock exchanges would benefit from the alliance whereas brokers would lose from the alliance. This result is important since it highlights the recently-debated issue of governance which questions whether stock exchanges can be owned by security houses, and how ownership affects the decision to form alliances among stock exchanges.

5.2 Fees and investors’ welfare comparisons

Direct comparisons of fees levied by stock exchanges (11) and (25), the fees levied by brokers (14) and (28), and their market shares (12) and (27) yield the following proposition.

**Proposition 5**

Let $\theta = 1/2$. The formation of an alliance between the stock exchanges

(a) Increases the fees, $f_A$ and $f_B$, levied by stock exchanges on the brokers.

(b) Decreases the fees, $f_1$ and $f_2$, levied by brokers on investors if and only if $\mu \geq V/2$. 
Proposition 5(a) and (b) highlights the rent-shifting effects associated with the alliance where stock exchanges increase their fees on brokers. In particular, if $\mu > V/2$ the alliance reduces the fees brokers levy on buyers whereas stock exchanges increase their fees, which means that the increase in the brokers’ cost is not rolled over the investors. Hence, the alliance extracts rents from brokers in favor of stock exchanges.

Proposition 5(b) provides the condition under which participating investors become better off under the alliance. Proposition 5(c) provides the condition under which potential investors become better off in the sense that the reduction in investors’ fees associated with the alliance induces them to participate in the market and trade. Therefore,

**Proposition 6**

All investors become strictly better off under the alliance if $\mu > V/2$.

5.3 Social welfare comparison

We define the (world) economy’s welfare function as the sum of the investors’ utility levels and the profits of stock exchanges and the brokers. However, since fees are merely transfers among stock exchanges, brokers, and the investors, social welfare becomes

$$W \overset{\text{def}}{=} \int_{x_1}^{x_2} U_x \, dx + \int_{x_2}^{1} U_x \, dx + \pi_A + \pi_B + \pi_1 + \pi_2$$

\[= (x_1 + 1 - x_2)V - \frac{\tau(x_1)^2}{2} - \frac{\tau(1 - x_2)^2}{2} - (1 - \theta)x_1\mu - \theta(1 - x_2)\mu. \tag{29}\]

Thus, after cancelling all fees with the corresponding revenue, social welfare is reduced to the sum of existing investors’ gain from trade (net of aggregate differentiation cost) minus the cost brokers incur for trading at a foreign market. Clearly, this cost vanishes when the alliance is formed. Setting $\mu = 0$ and substituting (27) into (29) yields the social welfare level after the alliance is
formed. Thus,
\[ W_{\text{alliance}} = \frac{15V^2}{64\tau}. \]  
(30)

Substituting (12) into (29) yields the social welfare level before the alliance is formed. Thus,
\[ W_{\text{no alliance}} = \frac{11(2V - \mu)^2}{144\tau}. \]  
(31)

Let \( \Delta \) denote the change in social welfare resulting from the alliance. Then, (30) and (31) imply that
\[ \Delta \overset{\text{def}}{=} W_{\text{alliance}} - W_{\text{no alliance}} = \frac{176\mu V - 44\mu^2 - 41V^2}{576\tau}. \]  
(32)

Our conclusions concerning the effect of the alliance on social welfare are summarized by the following proposition.

**Proposition 7**

Let \( \theta = 1/2 \). An alliance between the stock exchanges improves social welfare \( (\Delta \geq 0) \) if and only if
\[ \mu \geq \frac{44 - 3\sqrt{165}}{22} \approx 0.248V. \]

Notice that when \( \mu \geq V/2 \), not only that social welfare improves, but also that this improvement is in the Pareto sense as implied by Propositions 4 and 5. Figure 5 extends Figure 4 showing also how the alliance affects investors’ welfare and social welfare for different values of \( \mu \).

![Diagram](image)

**Figure 5:** Welfare consequences of the alliance. *Note:* Figure assumes \( \theta = 1/2 \).
5.4 Social optimum

We conclude our investigation of the welfare effects of alliance formation by solving for social optimum. The welfare function (29) reveals that the only real variable that affects social welfare is the investor’s participation rate, as measured by \( x_1 \) and \( 1 - x_2 \). Maximizing (29) therefore yields

\[
x_1^* = 1 - x_2^* = \frac{2V - \mu}{2\tau}, \quad \text{or} \quad m^* \overset{\text{def}}{=} x_1^* + 1 - x_2^* = \frac{2V - \mu}{\tau}
\]

(33)

where \( m^* (0 \leq m^* \leq 1) \) is the first-best investors’ participation rate “conditional” on the absence of an alliance. However, this optimum can be further improved when an alliance is formed. We can compute this “unconditional” first-best optimum by maximizing (29) setting \( \mu = 0 \). This yields the market participation rate given by

\[
x_1^{**} = 1 - x_2^{**} = \min \left\{ \frac{V}{\tau}, \frac{1}{2} \right\} \quad \text{or} \quad m^{**} \overset{\text{def}}{=} x_1^{**} + 1 - x_2^{**} = \min \left\{ \frac{2V}{\tau}, 1 \right\}
\]

(34)

Comparing (33) and (34) with (27) and (12) yields the expected result that

**Proposition 8**

*Both, the equilibrium participation rate in the absence of the alliance and in the presence of the alliance are below the corresponding socially-optimal participation rates.*

Figure 6 compares the market participation rates under the various regimes.

**Figure 6:** Equilibrium versus optimal Market participation rates.

*Top: \( \mu > V/2 \). Bottom: \( \mu < V/2 \).*
6 Access fee mechanisms

In this section we investigate parallels and differences between access pricing associated with alliances in the telecommunication industry and between stock exchanges. Competition bureaus often complain that alliances tend to lessen competition since access prices are being negotiated rather than set in a competitive way. We investigate this question by analyzing a variety of access fee mechanisms.

6.1 Collusion on access fees and social optimum

We now modify Stage I of Section 4.1 so that access fees are determined cooperatively, to maximize joint profits of the two stock exchanges. We assume that Stage II in which stock exchanges compete on fees $f_A$ and $f_B$ remains noncooperative, so stock exchanges manage to collude on access fees, but not on the fees they charge the brokerage firms.

In stage I, stock exchanges jointly determine $a_A$ and $a_B$ to maximize joint profit $\pi_A + \pi_B$, where $\pi_A$ is given in (22) and $\pi_B$ is given in (23). Then, we obtain

$$\frac{\partial(\pi_A + \pi_B)}{\partial a_A} = \frac{-\theta^2 a_A}{4\tau}, \quad \text{and} \quad \frac{\partial(\pi_A + \pi_B)}{\partial a_B} = \frac{-(1 - \theta)^2 a_B}{4\tau}. \quad (35)$$

Therefore,

**Proposition 9**

Stock exchanges maximizing joint profit would eliminate positive access fees.

Proposition 9 generates an opposite result to common results obtained in the telecommunication access pricing literature. This literature argues that collusion among phone companies would result in above-marginal-cost access fees, see for example Armstrong (1998), Laffont, Rey, and Tirole (1998), Laffont and Tirole (2000, §5.4), and Shy (2001, §5.3.2). By doing so, they mutually raise costs which are rolled-over to consumers, thereby raising their profits. The reason why we obtain the opposite result for the stock exchange industry
is that whereas phone companies market their services directly to consumers, stock exchanges market their services to brokers, who then sell their matching services to the investors. This means that, unlike the phone companies, stock exchanges do not benefit from artificially raising each other’s cost via access pricing in order to pass these artificial costs to consumers. Furthermore, Proposition 9 highlights the fact that if the operation of stock exchanges were subject to fixed costs (see Section 6.3), stock exchanges would cover the fixed cost by the fees they levy on brokers and not by access fees.

We now investigate how the collusion on zero access fees affects other fees and profit levels. Substituting $a_A = a_B = 0$ into (21), (22), and (23) yields

$$f_A = f_B = \frac{V}{2}, \quad \text{and} \quad \pi_A = \pi_B = \frac{V^2}{8}. \quad (36)$$

Thus, the collusion on zero access pricing reduces the fees stock exchanges levy on brokers, but increases their profits because this collusion increases market participation. To see this, substitute (36) into (18) to obtain

$$x_1 = 1 - x_2 = \frac{V}{4\tau}, \quad (37)$$

which is twice the participation rates (27) when stock exchanges do not collude on access fees. Substituting (36) into (17), yields

$$f_1 = f_2 = \frac{3V}{4}, \quad \text{and} \quad \pi_1 = \pi_2 = \frac{V^2}{16}. \quad (38)$$

Comparing (38) with (28) reveals that collusion on zero access pricing among stock exchanges also reduces the fees brokers levy on investors. However, the increase in market participation dominates so brokers end up with higher profits. We can now state the following proposition.

**Proposition 10**

Collusion on access fees among stock exchanges is Pareto improving.

Observe that the zero access fees do not induce the socially-optimal participation rate. Comparing (37) with (34) reveals that although collusion on access fees is Pareto improving, the investor participation rate is below the socially-optimal level. Therefore, we conclude that in order to induce the socially-optimal participation rate stock exchanges should set negative access
fees. That is, there should be cross subsidization between the stock exchanges for giving access to other exchanges. Laffont and Tirole (1998 Prop.3, 2000 §5.4.2) obtain a similar result where the socially-optimal termination charge (the fee local phone companies charge the long-distance companies) lies below the marginal cost of terminating access. These cross subsidies offset the markups imbedded in the fees charged to brokers and investors. Finally, notice that the first-order condition for collusive access fees (35) implies that stock exchanges would indeed use subsidies if the non-negativity restriction on access fees is removed.

6.2 The ECPR mechanism

A commonly-used access fee mechanism in the telecommunication industry requires entrants to compensate incumbents according an incumbent’s loss of profit associated with an entrant’s operation. This access fee, commonly-referred to as the Efficient Component Pricing Rule, or the Baumol-Willig rule, prescribes an access fee equals to the difference between an incumbent’s price and the incumbent’s cost.\(^6\)

So far, the present model did not incorporate costs of stock exchanges (costs are analyzed in Section 6.3 below), hence when there are no costs the ECPR mechanism becomes a simple mechanism by which each stock exchange compensates the other for “lost sales” associated with the alliance. In our model, \(\theta\) transactions take place in stock exchange \(A\), in which case this mechanism implies that stock exchange \(B\) should reimburse \(A\) an amount of \(f_A\) for each transaction originating at \(B\); that is a total of \(\theta(1 - x_2)f_A\). Similarly, stock exchange \(A\) reimburses \(B\) an amount of \(f_B\) for each transaction originating at \(A\); thus a total of \((1 - \theta)x_1f_B\). Formally, under this mechanism access fees are set to

\[
a_A = f_A, \quad \text{and} \quad a_B = f_B. \tag{39}
\]

\(^6\)This mechanism is “efficient” in the sense that it is designed so that those firms who choose to enter must be (weakly) more cost efficient than the incumbent. See, for example, Laffont and Tirole (1996; 2000, Ch.3).
Substituting (18) and (39) into (19) and (20), stock exchange $A$ chooses the fee it levies on brokers to solve

$$\max_{f_A} \pi_A = \frac{[(1 + \theta)V + (1 - 2\theta)f_B]f_A - (f_A)^2 - (1 - \theta)Vf_B}{2\tau},$$

and stock exchange $B$ chooses its fee to solve

$$\max_{f_B} \pi_B = \frac{f_A[f_B(2\theta - 1) - \theta V] - (f_B)^2 + (2 - \theta)Vf_B}{2\tau}.$$  

The best-response functions are given by

$$f_A = a_A = \frac{(1 + \theta)V - (2\theta - 1)f_B}{2}, \quad \text{and} \quad f_B = a_B = \frac{(2 - \theta)V + (2\theta - 1)f_A}{2}.$$  

Figure 7 plots the two best-response functions. Recall that one interpretation of having $\theta > 1/2$ is that stock exchange $A$ is “larger” or simply more established than stock exchange $B$. Thus, when $\theta = 1/2$ the stock exchanges pay the same amount to each other since they service equal volume of trade.

In contrast, when $\theta > 1/2$ stock exchange $A$ has higher trade volume and is therefore heavily affected by the global participation rate. For this reason, stock exchange $A$ reduces its fee and access fee in response to an increase in the fee and access fee of stock exchange $B$ (downward-sloping response function).

In contrast, when $\theta > 1/2$, stock exchange $B$ will increase its fee and access fee.
fee in response to increases in A’s fee and access fee (upward-sloping response function).

Solving (42) yields the equilibrium fees (also access fees) under the ECPR mechanism. Hence,

\[ f_A = a_A = \frac{(2\theta^2 - 3\theta + 4)V}{4\theta^2 - 4\theta + 5}, \quad \text{and} \quad f_B = a_B = \frac{(2\theta^2 - \theta + 3)V}{4\theta^2 - 4\theta + 5}. \]  

(43)

Clearly \( f_A \leq f_B \) if and only if \( \theta \geq 1/2 \), as depicted in Figure 7. Substituting (43) into (40) and (41) yields the equilibrium profit levels. Hence,

\[ \pi_A = \frac{(8\theta^2 - 16\theta^4 + 26\theta^3 - 18\theta^2 + 8\theta + 1)V^2}{2\tau(4\theta^2 - 4\theta + 5)} \]  

(44)

and

\[ \pi_B = \frac{(-8\theta^5 + 24\theta^4 - 42\theta^3 + 44\theta^2 - 26\theta + 9)V^2}{2\tau(4\theta^2 - 4\theta + 5)}. \]  

(45)

Figure 8 illustrates how the equilibrium fees and profit levels of the two stock exchanges vary with \( \theta \). Figure 8 indicates that as \( \theta \) increases stock exchange A lowers its fee and access fee in order to increase investors’ participation thereby increasing its profit. Notice that under the ECPR mechanism there are two contributing effects on market participation when a stock exchange lowers its fee. First, it lowers brokers’ fees which are partly rolled over the investors, thereby increasing investors’ participation. Second, it lowers the access fee on
the competing stock exchange, which would further enhance investors’ participation.

In order to compare the ECPR mechanism to the equilibria analyzed in previous sections, substitute $\theta = 1/2$ into (43), (44), and (45) to obtain

$$f_A = a_A = f_B = a_B|_{\theta = \frac{1}{2}} = \frac{3V}{4}, \quad \text{and} \quad \pi_A = \pi_B|_{\theta = \frac{1}{2}} = \frac{3V^2}{32\tau}. \quad (46)$$

Comparing (46) with (25) and (26) yields the following proposition.

**Proposition 11**

For the symmetric case where $\theta = 1/2$, the ECPR mechanism yields the same equilibrium allocation as the equilibrium where access fees are determined noncooperatively. Hence it is inefficient.

The last statement follows from Proposition 10 which demonstrates that colluding on zero access fees is Pareto improving over independently-determined access fees.

### 6.3 Fixed cost and the fully-distributed cost mechanism

So far, with no loss of generality, we have ignored the fixed cost of stock exchanges. In fact, as long as the fees they levy on brokers multiplied by the stock exchanges’ trade volume, are above their fixed cost, the analysis would not differ from our zero fixed cost analysis. In what follows, we assume that stock exchanges bear fixed cost (say, construction and infrastructure costs). Let $\phi_A$ denote the fixed cost borne by stock exchange $A$. $\phi_B$ is similarly defined.\(^7\)

The *fully-distributed cost mechanism* [also known as the *usage-proportional markup*, see for example Laffont and Tirole (2000, §4.2) or Shy (2001, §5.3.1)], prescribes an access fee in which the firm utilizing the infrastructure pays its share of the fixed cost according to its relative use of this infrastructure.

\(^7\)An interesting extension to our model would be to assume that fixed costs vary with the “size” of the stock exchange. In our model it would mean assuming that $\phi'_A(\theta) > 0$ and $\phi'_B(\theta) < 0$. 

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Accordingly, suppose that the access fees are now set to satisfy

\[ a_A = \left( \frac{1 - x_2}{x_1 + 1 - x_2} \right) \phi_A \quad \text{and} \quad a_B = \left( \frac{x_1}{x_1 + 1 - x_2} \right) \phi_B. \]  

(47)

Therefore, under the fully distributed cost compensation mechanism, stock exchange B pays A an access fee which equals to its relative share given by \((1 - x_2)/(x_1 + 1 - x_2)\) multiplied by \(\phi_A\). Similarly, stock exchange A pays an access fee equals its relative share given by \(x_1/(x_1 + 1 - x_2)\) multiplied by \(\phi_B\).

Substituting (18) into (47) we obtain

\[ a_A = \left( \frac{V - f_B}{2V - f_A - f_B} \right) \phi_A \quad \text{and} \quad a_B = \left( \frac{V - f_A}{2V - f_A - f_B} \right) \phi_B. \]  

(48)

Substituting (18) and (48) into (19) and (20), stock exchange A chooses the fee it levies on brokers, \(f_A\), to maximize (19), and stock exchange B chooses \(f_B\) to maximize (20). Since the general case does not have a closed-form solution (thus, requires numerical simulations), we display only the symmetric case where stock exchange have identical fixed cost \((\phi_A = \phi_B = \phi)\) and are of equal size \((\theta = 1/2)\). In this case, the unique equilibrium is given by

\[ f_A = f_B = \frac{2V + \phi}{4}, \quad x_1 = 1 - x_2 = \frac{2V - \phi}{8\tau}, \quad \text{and} \quad a_A = a_B = \frac{\phi}{2}. \]  

(49)

Thus, given that the two stock exchanges maintain equal market shares, each compensates the other by exactly half of the fixed cost incurred by the other. Furthermore, (49) reveals that fees increase and investors’ market participation declines when there is an increase in the fixed cost incurred by stock exchanges.

Finally, Comparing (49) with (36) and (37) reveals that as the fixed cost decline to zero \((\phi \to 0)\), the fully-distributed cost mechanism allocation approaches the collusive access fee allocation. This follows from the fact that the fully-distributed cost mechanism prescribes no access fee in the absence of fixed (and marginal) costs. Therefore, following Proposition 10, we can state

Proposition 12

Under the alliance between the stock exchanges, the fully-distributed cost mechanism supports an allocation which is Pareto superior to the ECPR mechanism and the independently-determined access fee equilibrium.
Our analysis has focused on the implications of alliances among stock exchanges. The process of alliance formations has only started so we expect that in the next few years competition bureaus will be forced to make decisions regarding the competitive implications of these alliances.

The analysis suggests that the welfare and distribution effects of stock exchange alliances depend crucially on the how much they are able to reduce transaction costs of foreign share purchases. The alliances are very likely to increase the profit of stock exchanges, and almost as likely to benefit general social welfare. However, brokers or investors may suffer as a result of the alliance even when total welfare is improved.

The regulatory implications of our analysis relate to access pricing. Generally, mutual agreements on access fees by stock exchanges do not seem to be as detrimental as in some other industries. In particular, we have shown that, whereas there are strong parallels between stock exchanges and the telecommunication industries, the utilization of access fees yields completely different market outcomes. The reason for these differences stems from the fact that phone companies that form alliances sell their services directly to consumers. In contrast, stock exchanges do not sell matching services directly to investors, as only security houses are allowed to hold memberships. However, we do foresee a possibility that in the future stock exchanges would permit investors to trade directly utilizing their electronic trading systems, thereby circumventing security houses that act as dealers. This development would seem to threat the existence of brokers as we know them. However, the brokers can, in principle, also start to match traders among their own customers without the participation of stock exchanges if the clearing and settlement institutions would permit it. The outcome of all this could be a convergence of the brokerage and stock exchange functions, brokers becoming more like exchanges and exchanges becoming more like brokers.
Appendix A  Market-dependent brokers’ fees

We now demonstrate how brokers’ fees can be decomposed into market-dependent fees levied on investors. Therefore, let $f_1^A$ and $f_1^B$ denote the fees broker 1 charges investors for executing a trade in markets $A$ and $B$, respectively. $f_2^A$ and $f_2^B$ are similarly defined. Then, the brokers’ profit functions (3) and (4) become

\[
\pi_1 = x_1 \left[ \theta(f_1^A - f_A) + (1 - \theta)(f_1^B - f_B - \mu) \right],
\]
\[
\pi_2 = (1 - x_2) \left[ \theta(f_2^A - f_A - \mu) + (1 - \theta)(f_2^B - f_B) \right].
\]

(50a)  

(50b)

where $f_A$ and $f_B$ are the fees collected by stock exchange $A$ and $B$, respectively. The first term in (50a) is the profit from matching the $x_1$ investors in stock exchange $A$. The second term is the profit generated by matching these investors at stock exchange $B$.

We now combine the two fees determined by each broker into the fee that investors pay their brokers by defining

\[
f_1 {\overset{\text{def}}{=}} \theta f_1^A + (1 - \theta)f_1^B \quad \text{and} \quad f_2 {\overset{\text{def}}{=}} \theta f_2^A + (1 - \theta)f_2^B.
\]

(51)

Substituting (51) into (50a) and (50b) yield (3) and (4), respectively.

Altogether, the brokers solve a two stage problem. First, they set investors’ fees, $f_1$ and $f_2$, to maximize (5) and (6), respectively. Then, they can arbitrarily decompose investors’ fee into the fee levied on matching at stock exchange $A$ and the fee for matching at $B$ according to (51). Figure 9 shows how a given fee, $f_i$, can be decomposed according to (51). Figure 9 illustrates that a decrease in $\theta$ to $\theta'$ would change brokers’ range of possible fees to include a higher fee on matching at exchange $A$, and vice versa. This is because, when $\theta$ is low, this fee is levied a smaller number of trades.
Figure 9: Broker $i$ investors’ fee settings, $f^A_i$ and $f^B_i$, given the broker’s fee, $f_i$. The two cases drawn satisfy $0 < \theta' < 1/2 < \theta < 1$. 
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