Yehning Chen – Iftekhar Hasan

The transparency of the banking industry and the efficiency of information-based bank runs
Yehning Chen* – Iftekhar Hasan**

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* National Taiwan University
** Rensselaer Polytechnic Institute and Bank of Finland
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Yehning Chen – Iftekhar Hasan
Monetary Policy and Research Department

Abstract

In this paper, we investigate the relationship between the transparency of banks and the fragility of the banking system. We show that information-based bank runs may be inefficient because the deposit contract designed to provide liquidity induces depositors to have excessive incentives to withdraw. An improvement in transparency of a bank may reduce depositor welfare through increasing the chance of an inefficient contagious bank run on other banks. A deposit insurance system in which some depositors are fully insured and the others are partially insured can ameliorate this inefficiency. Under such a system, bank runs can serve as an efficient mechanism for disciplining banks. We also consider bank managers’ control over the timing of information disclosure, and find that they may lack the incentive to reveal information about their banks.

Key words: bank run, contagion, transparency, market discipline, deposit insurance

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1 Introduction

Imposing market discipline to alleviate the banks’ moral hazard problems has become an important part of bank regulation policies around the world. In the new Basel Capital Accord, market discipline is recognized as one of the three ‘pillars’ of the new regulation framework. As stated in a consultative document of the Basel Committee, ‘…market discipline has the potential to reinforce capital regulation and other supervisory efforts to promote safety and soundness in banks and financial systems. Market discipline imposes strong incentives on banks to conduct their business in a safe, sound, and efficient manner.’

Although market discipline has the benefit of alleviating the banks’ incentive problems, it may increase the fragility of the banking industry. To implement market discipline, banks must become transparent so that market participants have precise information about banks. However, as depositors learn more about their banks, they may react to adverse information and start bank runs more frequently. As suggested by articles in the bank run literature such as Diamond and Dybvig (1983), Chari and Jaganathan (1987), and Chen (1999), bank runs have the ‘panic’ feature in the sense that a run can happen even if it reduces depositor welfare. If improvements in transparency of banks may lead to inefficient bank runs, then welfare losses caused by this effect should be taken into consideration when regulators design information disclosure regulations in the banking industry.

The purpose of this paper is to investigate the relationship between the transparency of banks and the efficiency of bank runs. Specifically, we ask the following questions.

(1) Is it possible that, by raising the chance of an inefficient bank run, an improvement in the transparency of the banking system will reduce rather than improve social welfare?

(2) If the answer to question (1) is yes, is there anything that regulators can do to alleviate this problem?

(3) How will bank managers who dislike bank runs use their influence on the banks’ information disclosure decisions to affect the efficiency of bank runs?

To answer these questions, we build a simple model with two banks and atomistic depositors. In this model, the banks’ returns are positively correlated. Interim information about the banks’ returns will be revealed, so information-based bank runs can serve as a disciplining mechanism for liquidating poor banks. However, as in Diamond and Dybvig (1983) and Chen (1999), in our model depositors have

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2 For the definitions of inefficient bank runs, please see Subsection 3.2.
excessive incentives to withdraw because the sequential service constraint in the deposit contract creates negative payoff externalities among them. As a result, welfare-reducing bank runs may occur. The inefficient bank run problem becomes worse when information about banks arrives sequentially. Since the banks’ returns are positively correlated, depositors of a bank will use information about the other bank to evaluate their own bank. With the excessive incentives to withdraw, they may jump on adverse information about the other bank and start a bank run even if they know that information about their own bank will be revealed soon. In other words, inefficient contagious runs may occur in our model.

Under the above setting, we obtain the following results, which provide answers to the questions we raise.

(1) By increasing the chance of an inefficient contagious run, an improvement in transparency may reduce depositor welfare.

(2) There is a deposit insurance system that can eliminate the inefficient bank run problem. In this deposit insurance system, some depositors are fully insured and the remaining ones are partially insured. When this system is imposed, improvements in transparency of banks always increase depositor welfare.

(3) If we assume that bank managers who dislike bank runs can control the timing of information disclosure, then contagious runs will disappear. However, efficient bank runs may also be eliminated under this assumption.

The above results can be explained as follows. For the first result, consider depositors of a bank. An improvement in the transparency of the banking system has two effects on their incentives to respond to information about the other bank. On the one hand, as information about their own bank becomes more precise, depositors become more patient and are less likely to respond to information about the other bank. On the other hand, as information about the other bank becomes more precise, depositors have a stronger incentive to respond to it because it contains more information about their own bank. We show that, if the correlation between the banks’ returns is high and information about banks is relatively precise, the second effect will dominate, so an improvement in transparency will result in a higher chance of a contagious run and a reduction in depositor welfare.

As to our second result, the purpose of fully insuring some depositors is to remove the negative payoff externalities among depositors by reducing the number of depositors who will rush to the bank when a bank run occurs, and the purpose of partially insuring the others is to induce these depositors to have the right incentive to withdraw. Under this system, bank runs can be an efficient mechanism for enforcing market discipline. This result suggests that, when regulators require banks to reveal more information, they should also adopt mechanisms that can induce depositors to use information efficiently.
The first two results are derived under the assumption that bank managers have no control over the timing of information disclosure. To reflect the bank managers’ influence on information disclosure decisions, we relax this assumption to obtain our third result. Under the new assumption, contagious runs disappear because bank managers can avoid them by simultaneously revealing the information. On the other hand, if the prospects of the banking industry are so favorable that no bank run will occur when no new bank-specific information is revealed, run-averse bank managers may delay the timing of information disclosure so that depositors cannot base their withdrawing decisions on the new information. In this case, efficient runs are also eliminated and depositors may become worse off. This result provides a rationale for regulators to impose mandatory information disclosure requirements in the banking industry.

In the banking literature, Cordella and Levy Yeyati (1998) show that full transparency of bank risks may increase the chance of a bank failure through raising the deposit interest rate that banks have to pay in the riskier state. Hyytinen and Takalo (2002) propose that the costs of information disclosure will reduce the banks’ franchise values, thus increase their risk-taking incentives. Complementing to these papers, our paper suggests another channel through which information transparency can affect banking fragility.

The model and the deposit insurance system in our paper are similar to those in Chen (1999). There are two differences between the two papers. First, Chen (1999) assumes that the informed depositors receive perfect information about bank returns, so his model cannot be used to study the impacts of information transparency on the stability of the banking system. Second, in the deposit insurance system proposed by Chen (1999), depositors are either fully insured or not insured at all. By contrast, in our paper depositors are either fully insured or partially insured. As mentioned, when information about banks is imperfect, the partial deposit insurance can induce depositors to have the right incentive to withdraw.

Our paper is also related to articles in the information disclosure literature. Admati and Pfleiderer (2000) show that, when the firms’ returns are positively correlated, information disclosure by one firm generates positive spillover effects because investors can use this information to evaluate other firms. In our model, information disclosure by banks also has spillover effects. However, depending on whether the revealed information will trigger contagious runs, the spillover effects may be either positive or negative. Boot and Thakor (2001) propose that agency problems provide a justification for disclosure regulation. Similar to their idea, we show that the reluctance of bank managers to reveal information can justify mandatory disclosure requirements in the banking industry.

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3 For an excellent review of this literature, please see Boot and Thakor (2001).
4 In their paper, an agency problem arises because managers may have a vested interest in withholding information to preserve personal control rents or to profit from inside trading.
The rest of the paper is organized as follows. Section 2 describes the basic model. Section 3 analyzes the model and shows that an improvement in transparency may increase the chance of a contagious run. Section 4 demonstrates that inefficient bank runs can be eliminated by deposit insurance. The case where bank managers can affect the timing of information disclosure is investigated in Section 5. Section 6 contains concluding remarks.

2 The basic model

Consider a three-date (dates 0, 1, and 2) model. There are two banks, bank 1 and bank 2, located in different geographical areas. Each bank is owned and controlled by its manager. For each bank, there are numerous atomistic depositors living in the area where the bank is located. Depositors are risk-neutral. At date 0, each depositor receives an endowment of one dollar. Depositors face liquidity shocks. Some of them die at date 1, so have to consume before they die. The others die at date 2, and can consume at either date 1 or date 2. For $T = 1, 2$, we will call those who die at date $T$ type-$T$ depositors. The fraction of type-1 depositors is denoted by $t$. The liquidity shocks are realized at date 1. At date 0, depositors do not know whether they will die early, and each depositor has the same chance of becoming a type-1 depositor. If a type-1 depositor consumes less than $r$ at date 1, she will suffer a liquidity loss $X$, where $r$ and $X$ are constants with $r > 1$ and $X > 0$. Let $U_T$ denote the utility function of a type-$T$ depositor, and $c_l$ denote her consumption at date $l$. The depositors’ utility functions can be written as

$$U_1(c_1,c_2) = \begin{cases} c_1 - X & \text{if } c_1 < r, \\ c_1 & \text{if } c_1 \geq r, \end{cases}$$

and $U_2(c_1,c_2) = c_1 + c_2$.

At date 0, a depositor can either deposit her endowment at the bank in her neighborhood, or stores the endowment herself without any storage cost. Depositors make deposits at date 0, and can withdraw at either date 1 or date 2. For each dollar deposited at date 0, the bank promises to pay $d_1$ if a depositor withdraws at date 1, and pay $d_2$ if a depositor withdraws at date 2, where $d_1$ and $d_2$ are determined by the bank. A bank’s deposit contract can be denoted by the pair $(d_1, d_2)$. When serving depositors, banks cannot distinguish between type-1 and type-2 depositors. The sequential service constraint is imposed, which means depositors are served according to the time they arrive at the bank. Convertibility suspension is not allowed; a bank has to keep operating at date 1 unless it runs out
of money. For now, assume there is no deposit insurance. This assumption will be relaxed in Section 4.

Banks do not have capital. For each bank, if depositors deposit their endowments at the bank at date 0, the bank can invest these endowments in a long-term project that matures at date 2. A bank’s project will succeed with probability $p$ and will fail with probability $1 - p$, where $0 < p < 1$. For each dollar invested, a project yields $R$ at date 2 when it succeeds and yields nothing when it fails, where $R > r$. A bank can liquidate any proportion of its project at date 1. When early liquidation occurs, only the initial investment can be returned. That is, for each dollar invested at date 0, early liquidation yields one dollar. Given these assumptions, a bank will invest all the deposits in a long-term project at date 0, and liquidate part of their projects to repay the early withdrawing depositors at date 1. Also note that because $R > r > 1$, we have

$$pR + (1 - p)l > 1$$

which implies that the net present value of a bank’s project is positive if it is continued to date 2 when it will succeed and is liquidated at date 1 when it will fail.

The banking industry is competitive. Therefore, when determining the deposit contract, a bank’s manager maximizes the depositors’ payoff subject to the bank’s zero-profit constraint. Given $d_1$, the bank’s zero-profit condition implies that

$$d_1 = \frac{(1 - td_1)R}{1 - t}$$

(2.1)

To induce type-2 depositors not to withdraw at date 1, $d_2$ must be higher than $d_1$. We assume

$$r < \frac{R}{1 + t(R - 1)}$$

(2.2)

which implies that $d_1 < d_2$ when $d_1 = r$. If (2.2) does not hold, there is no deposit contract with $d_1 \geq r$ that can induce type-2 depositors to wait until date 2.

The returns of the two banks’ projects are positively correlated. Assume that the probabilities that ‘both banks’ projects succeed’, ‘both banks’ projects fail’, and ‘one bank’s project succeeds and the other’s project fails’ are $p^2 + \varepsilon$,

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5 Suppose that only type-1 depositors withdraw at date 1. In this case, the fraction of the project that the bank has to liquidate at date 1 is $t d_1$. Let $N$ denote the total number of depositors who make deposits at date 0. If the bank’s project succeeds, at date 2 the total amount of money left in the bank is $N(1 - td_1)R$. Therefore, each type-2 depositor can get $N(1 - td_1)R/N(1 - t) = (1 - td_1)R/(1 - t)$.
\[(1 - p)^2 + \varepsilon, \text{ and } p(1 - p) - \varepsilon, \text{ respectively, where } 0 < \varepsilon < p(1 - p). \text{ Let } \rho \text{ denote the correlation coefficient between the two projects’ returns. It can be easily shown that}
\[
\rho = \frac{\varepsilon}{p(1 - p)}
\]

(2.3)

By (2.3), the higher the \( \varepsilon \), the higher the correlation between the two banks’ returns is.

If banks invest at date 0, then for each bank a public signal about its project will be revealed at date 1. Signals arrive sequentially. Suppose that date 1 can be divided into three subdates (subdates 1.1, 1.2, and 1.3). After banks invest at date 0, the signal of one bank is revealed at subdate 1.1, and the signal of the other bank is revealed at subdate 1.2. At subdate 1.3, the liquidity shocks of both banks’ depositors are revealed, and type-1 depositors consume and die.\(^6\) Let bank A denote the bank whose information is revealed at subdate 1.1, and bank B denote the bank whose information is revealed at subdate 1.2. Ex ante, each bank has an equal chance to become bank A. Depositors and bank managers learn whether their banks will become bank A at date 0 after the investments are made.

For \( i = A, B \), let \( s_i \) denote the public signal about bank \( i \)’s project. If bank \( i \)’s project will succeed, \( s_i = H \) with probability \( q \) and \( s_i = L \) with probability \( 1 - q \), where \( q \) is a constant and \( q > 0.5 \). On the other hand, if bank \( i \)’s project will fail, \( s_i = H \) with probability \( 1 - q \) and \( s_i = L \) with probability \( q \). The \( q \) can be explained as the quality of the signals; the larger the \( q \), the more precise the public information about bank assets is. The public signals are the only information that depositors receive, and all the parties can observe the signals when the signals are revealed. The information structure and the time line of the model are shown in Figures 1 and 2, respectively.\(^7\)

\(^6\) It is assumed that, for each dollar deposited at date 0, a bank promises to pay a depositor \( d_1 \) if she withdraws at any of the three subdates of date 1.

\(^7\) We are grateful to an anonymous referee for pointing out that the correlation structure in our model is similar to the correlation setup adopted in Acharya and Yorulmazer (2005). The focus of their paper is to explain how limited liability will lead to bank herding, which is different from ours.
In this paper, we will say that a bank run occurs to a bank if all of its depositors withdraw at date 1. Note that the assumption $r > 1$ plays an important role in the model. To provide liquidity, that is, to allow type-1 depositors to avoid the liquidity loss $X$, banks have to set $d_1 \geq r$. Since only the initial investment can be returned when early liquidation occurs, $d_1 \geq r$ implies that the early withdrawing depositors receive more than the liquidation values of their deposits. In this case, if all the depositors try to withdraw at date 1, those who go to the bank late will get nothing. In other words, a deposit contract with $d_1 \geq r$ creates negative payoff externalities among depositors. As will be shown, the negative payoff externalities induce depositors to have excessive incentives to withdraw and result in inefficient bank runs.
Figure 2. The time line of the basic model

- Banks offer deposit contracts. Depositors decide whether to deposit.
- For each bank, if depositors deposit, then (i) the manager invests, and (ii) both the manager and depositors learn whether their bank will become bank A or bank B.

- Depositors of both banks decide whether to withdraw.
- $s_A$ is revealed.
  Having observed $s_A$, depositors of both banks who have not withdrawn yet decide whether to withdraw.

- $s_B$ is revealed.
  Having observed $s_B$, depositors of both banks who have not withdrawn yet decide whether to withdraw.

- For both banks, the depositors’ liquidity shocks are revealed.
  Depositors who have not withdrawn yet decide whether to withdraw.
  - Type-1 depositors consume and die.

- For each bank, its project matures if a bank run does not occur at date 1.
  Banks pay off the depositors who have not withdrawn yet.
3 The analysis of the basic model

This section analyzes the depositors’ behavior in the basic model. In this section, we first introduce assumptions that help us simplify the analysis. We then investigate the equilibria of the basic model in Section 3.1. Section 3.2 discusses the efficiency of bank runs, and demonstrates that an improvement in transparency may increase the probability of an inefficient run. Finally, we end this section with a discussion on the optimal deposit contract.

To simplify exposition, we study only the symmetric pure-strategy subgame-perfect Nash equilibria, that is, the equilibria in which depositors of the same type adopt the same pure strategy in each subgame. In addition, we make the following assumptions. First, \( X \) is large so that the optimal \( d_1 \) is no less than \( r \), which implies that type-1 depositors can avoid the liquidity loss \( X \) if no bank run occurs. Second, a depositor will not withdraw when she is indifferent between withdrawing and not withdrawing. Third, depositors will choose the Pareto dominant equilibrium when there are multiple equilibria. The purpose of making this assumption is to illustrate the point that information-based bank runs can still be inefficient even if depositors always choose the Pareto dominant equilibrium.

As shown in Chen (1999), under these assumptions (i) a bank run is always an equilibrium outcome in all the date 1 subgames, and (ii) a bank run equilibrium is always the Pareto dominated one when there are multiple equilibria. Given our equilibrium selection rule, in our model a bank run will occur if and only if the bank run equilibrium is the only subgame-perfect Nash equilibrium.

3.1 The equilibrium given a deposit contract \((d_1, d_2)\)

Let us start from subdate 1.3. Suppose that no depositor withdraws before subdate 1.3. When liquidity shocks are revealed, type-1 depositors will withdraw and consume. On the other hand, whether type-2 depositors will withdraw depends on the information they receive. For \( i = A, B \), let \( p_i(s_A, s_B) \) denote the probability that...
bank i’s project will succeed given $s_A$ and $s_B$. The following lemma states important features of $p_i(s_A, s_B)$.

**Lemma 1.**
(a) $p_A(L,L) < p_A(L,H) < p < p_A(H,L) < p_A(H,H)$.
(b) $p_B(H,H) = p_A(H,H)$, $p_B(L,L) = p_A(L,L)$, $p_B(H,L) = p_A(L,H)$, and $p_B(L,H) = p_A(H,L)$.

**Proof.** Please see the Appendix.

The results in Lemma 1 are intuitive. For part (a), other things being constant, the probability that bank A’s project will succeed increases when $s_A$ or $s_B$ equals H, and decreases when $s_A$ or $s_B$ equals L. Therefore, $p_A(L,L) < p_A(L,H) < p < p_A(H,L)$ and $p_A(H,L) < p_A(H,H)$. Moreover, for calculating $p_A$, the impact of $s_A$ is greater than that of $s_B$ because $s_A$ contains more information about bank A’s project. Therefore, we have $p_A(L,H) < p < p_A(H,L)$. Part (b) of the lemma reflects the fact that banks A and B are symmetric.

Consider a type-2 depositor of bank i who believes that no other type-2 depositor of her bank will withdraw at subdate 1.3. Given $s_A$ and $s_B$, her payoff for withdrawing is $d_1$ and her payoff for not withdrawing is $p_i(s_A, s_B)d_2$. Therefore, she will withdraw at subdate 1.3 if and only if

$$p_i(s_A, s_B) < \frac{d_1}{d_2} \quad (3.1)$$

When (3.1) holds, ‘no type-2 depositor withdraws at subdate 1.3’ cannot be sustained as an equilibrium, and the only symmetric Nash equilibrium is the bank run equilibrium. On the other hand, when (3.1) is violated, ‘no type-2 depositor withdraws at subdate 1.3’ can be sustained as an equilibrium. According to our equilibrium selection rule, it will be the realized equilibrium. This discussion implies that, if no depositor withdraws before subdate 1.3, a bank run will occur at subdate 1.3 if and only if (3.1) holds.

Now go back to subdate 1.2 when $s_B$ is revealed. For $i = A, B$, suppose that a bank i’s depositor assumes that no other depositor will withdraw at subdate 1.2. If (3.1) holds, a bank run will occur at subdate 1.3 suppose it did not occur at
In this case, the depositor’s payoff for not withdrawing at subdate 1.2 is
\[
V_{BR} = \frac{1}{d_1} d_1 + (1 - \frac{1}{d_1})[t(0 - X) + (1 - t)0] = 1 - (1 - \frac{1}{d_1})tX \tag{3.2}
\]

By (3.2) and the fact that \(d_1 \geq r\), \(V_{BR} < d_1\), which implies that the depositor will withdraw at subdate 1.2 if (3.1) holds.

On the other hand, if (3.1) is violated, no bank run will occur at subdate 1.3. The depositor’s payoffs for withdrawing and not withdrawing at subdate 1.2 become \(d_1\) and \(W_2(s_A, s_B)\) respectively. In this case, she will withdraw at subdate 1.2 if and only if \(W_2(s_A, s_B) < d_1\), or equivalently, (3.1) holds. Applying the same logic we use to analyze the subgame of subdate 1.3, we know that a bank run will occur at subdate 1.2 if and only if (3.1) holds. The above results are summarized in the following proposition.

**Proposition 1.** Suppose that no depositor withdraws before subdate 1.2.

(a) For \(i = A, B\), if (3.1) does not hold, no bank run will occur to bank \(i\) at either subdate 1.2 or subdate 1.3. If (3.1) holds, a bank run will occur to bank \(i\) at subdate 1.2.

(b) At subdate 1.2, the equilibrium payoff for bank \(i\)’s depositors is
\[
V_2'(s_A, s_B) \equiv \begin{cases} 
W_2(s_A, s_B) & \text{if } p_i(s_A, s_B) \geq d_1/d_2 \\
V_{BR} & \text{if } p_i(s_A, s_B) < d_1/d_2
\end{cases} \tag{3.4}
\]

In the rest of the paper, we will consider only the case where

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11 Equation (3.2) can be explained as follows. When a bank run occurs, the fractions of depositors who successfully withdraw and who receive nothing are \(1/d_1\) and \((1 - 1/d_1)\), respectively. The type-1 depositors who receive nothing suffer the liquidity loss \(X\). The fraction of these depositors is \((1 - 1/d_1)t\). Therefore, the depositor’s payoff in a bank run equilibrium equals the liquidation value of her deposits (which is one dollar) minus the expected liquidity loss (which is \((1 - 1/d_1)tX\)).

12 A depositor becomes a type-1 depositor with probability \(t\) and becomes a type-2 depositor with probability \(1 - t\). Given that a bank run will not occur at subdate 1.3, a depositor’s payoff is \(d_1\) if she becomes a type-1 depositor, and is \(p_i(s_A, s_B)d_2\) if she becomes a type-2 depositor. Using these facts, we can get equation (3.3).

13 In this paper, the proofs of Proposition 1, Proposition 2, and Lemma 4 are omitted because they are obvious from discussions in the paper. The proofs of the other propositions and lemmas are in the Appendix.
By Lemma 1 and Proposition 1, condition (3.5) implies two things. First, because \( \frac{d_1}{d_2} < p_A(H,L) = p_B(L,H) \), for \( i = A, B \), a bank run will never occur to bank \( i \) at subdate 1.2 when \( s_i = H \). Second, because \( p_A(L,L) < \frac{d_1}{d_2} \), a bank will occur to bank \( i \) when \( s_A = s_B = L \). If \( p_A(L,L) \geq \frac{d_1}{d_2} \), bank runs will never happen in our model.

We next study what will happen at subdate 1.1 when \( s_A \) is revealed. For \( s = H, L \), let \( \mu_s \) denote the probability that \( s_B = H \) given \( s_A = s \). For \( i = A, B \), suppose that a bank \( i \)'s depositor assumes that no other depositor will withdraw at subdate 1.1. Given \( s_A = s \), her payoff for withdrawing and not withdrawing are \( d_1 \) and 14

\[
W_i^1(s) = \mu_s V_i^1(s, H) + (1 - \mu_s) V_i^1(s, L) \tag{3.6}
\]

The depositor will withdraw if and only if \( W_i^1(s_A) < d_1 \), which implies that a bank run will occur to bank \( i \) at subdate 1.1 if and only if \( W_i^1(s_A) < d_1 \). This result is documented in the following proposition.

**Proposition 2.** Suppose that no depositor withdraws before subdate 1.1.
(a) For \( i = A, B \), a bank run will occur to bank \( i \) at subdate 1.1 if and only if \( W_i^1(s_A) < d_1 \).
(b) For \( i = A, B \), at subdate 1.1, the equilibrium payoff for bank \( i \)'s depositors is

\[
V_i^1(s_A) = \begin{cases} 
W_i^1(s_A) & \text{if } W_i^1(s_A) \geq d_1 \\
V_{BR} & \text{if } W_i^1(s_A) < d_1 
\end{cases} \tag{3.7}
\]

Back to date 0 when depositors learn whether their banks are bank A or bank B. For \( i = A, B \), let \( W_0^i \) denote the depositors’ payoff when they learn their bank is bank \( i \). At date 0, \( s_A \) equals \( H \) with probability \( pq + (1-p)(1-q) \) and equals \( L \) with probability \( p(1-q) + (1-p)q \). Therefore, we have

\[
W_0^i = [pq + (1-p)(1-q)] V_i^1(H) + [p(1-q) + (1-p)q] V_i^1(L) \tag{3.8}
\]

We assume that

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14 Given \( s_A = s \), the depositor’s payoff for not withdrawing is \( V_i^1(s, H) \) if \( s_B = H \) and is \( V_i^1(s, L) \) if \( s_B = L \). The probabilities for \( s_B = H \) and \( s_B = L \) are \( \mu_s \) and \( 1 - \mu_s \), respectively. Using these facts, we can get (3.6).
which implies that no depositor will withdraw at date 1 before \( s_A \) is revealed.

Because each bank has an equal chance to become bank A, a depositor’s date 0 payoff for making deposits is \( 0.5(W_0^A + W_0^B) \). On the other hand, if she stores the endowment herself, her payoff is \( 1 - tX \) because she suffers the liquidity loss \( X \) with probability \( t \). By (3.9) and the fact that \( 1 - tX < r \leq d_1 \), depositors will always deposit at date 0.

### 3.2 The efficiency of bank runs and the optimal deposit contract

As mentioned, information-based bank runs may involve inefficiencies because depositors have excessive incentives to withdraw. In our model, two kinds of inefficiencies may arise when a bank run occurs.

First, a bank run may occur at subdate 1.2 even though the depositors’ payoff would have been higher if it did not happen. At subdate 1.2, the payoff for bank i’s depositors is \( W_2^i(s_A, s_B) \) when a bank run does not occur and is \( V_{BR} \) when it occurs. If depositors could act as a group to determine when they would withdraw together, a bank run would occur at subdate 1.2 if and only if \( W_2^i(s_A, s_B) < V_{BR} \). However, by Proposition 1, a bank run will occur to bank i if and only if \( W_2^i(s_A, s_B) < d_1 \). Therefore, when

\[
V_{BR} < W_2^i(s_A, s_B) < d_1 \tag{3.10}
\]

a bank run occurs even though depositors would have been better off if it did not happen.

The second kind of inefficiency involved in bank runs is that depositors may be too hasty when they make the withdrawing decisions at subdate 1.1. That is, a bank run may occur at subdate 1.1 even though depositors would not have withdrawn if they waited and learned more information at subdate 1.2. This kind of inefficient bank runs can happen to both bank A and bank B. For \( i = A, B \), if \( d_1/d_2 \leq p_i(L, H) \) and \( W_1^i(L) < d_1 \), a bank run will occur at subdate 1.1 when \( s_A = L \). However, depositors would not have withdrawn if they waited one more subdate and learned that \( s_B = H \).\(^{15}\)

In this paper, we call a bank run that occurs to bank B at subdate 1.1 a contagious bank run since it is triggered by information about bank A. In the

\(^{15}\) By (3.5) and Lemma 1, \( d_1/d_2 < p_B(L, H) \), so depositors would not have withdrawn if they waited until subdate 1.2 and learned that \( s_B = H \).
literature, contagious runs attract a lot of attention because they provide evidence for the argument that bank runs are inefficient. In our model, a contagious run is inefficient because depositors of bank B give up the information about their bank which will be revealed soon.

One may think that bank runs are inefficient because depositors lack precise information when they make the withdrawing decisions. If banks can be more transparent, bank runs should become a more efficient tool for disciplining banks. Against this intuition, we will show that, through increasing the probability of a contagious bank run, an improvement in transparency may reduce depositor welfare. To see this, note that a low \( s_A \) triggers a contagious run at subdate 1.1 if \( W_i^B(L) < d_1 \). The following proposition states that an increase in \( q \) may reduce \( W_i^B(L) \), thus make a contagious run more likely to occur.

**Proposition 3.** If the public signals are precise and the correlation between the banks’ returns is high, an increase in the precision of public signals will result in a decrease in \( W_i^B(L) \). That is, there exist an \( \overline{\varepsilon} < p(1-p) \) and a \( \overline{q} < 1 \) such that \( W_i^B(L) \) is decreasing in \( q \) if \( \varepsilon > \overline{\varepsilon} \) and \( q > \overline{q} \).

**Proof.** Please see the Appendix.

Proposition 3 can be explained as follows. Given (3.5), \( W_i^B(L) \) can be written as

\[
W_i^B(L) = \mu_L [td_i + (1-t)p_B(L,H)d_2] + (1-\mu_L)V_{BR}
\]

(3.11)

Differentiate \( W_i^B(L) \) with respect to \( q \), we have

\[
\frac{dW_i^B(L)}{dq} = \frac{d\mu_L}{dq} [td_i + (1-t)p_B(L,H)d_2 - V_{BR}] + \frac{dp_B(L,H)}{dq} [\mu_L (1-t)d_2]
\]

(3.12)

Recall that \( \mu_L \) is the conditional probability that \( s_B = H \) given \( s_A = L \). When \( \varepsilon \) is high, the two projects are likely to have the same outcome. When \( q \) is high, \( s_i \) is likely to reflect the true outcome of bank \( i \)’s investment. Therefore, given \( s_A = L \), an increase in \( q \) will lower the probability that \( s_B = H \) when both \( \varepsilon \) and \( q \) are high. That is, \( d\mu_L/dq < 0 \) when both \( \varepsilon \) and \( q \) are high.

---

Suppose that \( s_A = L \) and depositors wait until subdate 1.2. Because (3.5) and Lemma 1 imply \( p_B(L, L) < d_1/d_2 < p_B(L, H) \), in this case a bank run will occur to bank B at subdate 1.2 if and only if \( s_B = L \). Therefore, the depositors’ payoff is \( td_1 + (1-t)p_B(L, H)d_2 \) if \( s_B = H \) and is \( V_{BR} \) if \( s_B = L \). Moreover, given \( s_A = L, s_B = H \) with probability \( \mu_L \) and \( s_B = L \) with probability \( 1-\mu_L \). Using these results, we can get (3.11).
On the other hand, $dp_B(L, H)/dq$ approaches 0 when $\varepsilon$ approaches $p(1 - p)$. This is because when $\varepsilon$ is high, $s_A$ contains almost as much information about bank B’s project as $s_B$ does, so the effect of a high $s_B$ and that of a low $s_A$ cancel out. In this case, $p_B(L, H)$ is very close to $p$, and a change in $q$ has almost no impact on the value of $p_B(L, H)$.

Finally, it can be verified that the terms in both brackets of (3.12) are strictly positive. Combining these results, we know that an increase in $q$ will reduce $W_i^B(L)$ when both $\varepsilon$ and $q$ are high.

The following numerical example illustrates Proposition 3. Assume that $d_1 = r = 1.035$, $R = 1.1$, $t = 0.2$, $X = 0.8$, $\varepsilon = 0.03072$, and $p = 0.96$. In this case, as shown in Panel A of Figure 3, $W_i^B(L)$ is less than $d_1$ when $q$ is either lower than $q'$ or higher than $q''$. From the figure, an increase in $q$ may lead to a contagious run when $s_A = L$. Panel B of Figure 3 demonstrates that, because of the contagious run problem, depositor welfare $(0.5(W_0^A + W_0^B))$ is not monotonically increasing in $q$.

Having investigated the inefficiencies of bank runs, we now study the optimal deposit contract. Intuitively, because depositors have excessive incentives to withdraw, the optimal deposit contract should minimize this problem. To reduce the depositors’ incentives to withdraw early, $d_1$ should be minimized subject to the constraint that $d_1 \geq r$ and $d_2$ should be maximized. By (2.2), $d_2$ is decreasing in $d_1$. Therefore, the bank mangers will set $d_1 = r$. This result is documented in the following proposition.

**Proposition 4.** The optimal deposit contract that banks will offer at date 0 is

$$(d_1, d_2) = \left( r, \frac{(1 - tr)R}{1 - t} \right)$$

(3.13)

**Proof.** Please see the Appendix.
Figure 3. \( W_i^B(L) \) and \( 0.5(W_0^A + W_0^B) \) as functions of \( q \) when \( r = 1.035, R = 1.1, t = 0.2, X = 0.8, \varepsilon = 0.03072, \) and \( p = 0.96 \).
4 Deposit insurance

This section studies the possibility of using deposit insurance to improve the efficiency of bank runs. Suppose that all the assumptions in Sections 2 and 3 hold except that the following deposit insurance system is introduced. In this system, some depositors are fully insured and the others are partially insured. Let \( m \) denote the fraction of fully insured depositors at each bank. The deposit contract is modified when this system is introduced. For the fully insured depositors, \( d_1 = r \) and \( d_2 = d_{2f} \); for the partially insured depositors, \( d_1 = r \) and \( d_2 = d_{2p} \). Both \( d_{2f} \) and \( d_{2p} \) are greater than \( r \), and their values will be determined endogenously.

For a fully insured depositor, if the bank is unable to pay her the full amount specified in the deposit contract, the insurer will pay her the difference. For a partially insured depositor, the insurer guarantees that she receives no less than \( z \), where \( 0 < z < r \). For example, suppose that a partially insured depositor withdraws at date 2, and the bank pays her \( x \), where \( x < z \). In this case, the insurer will give this depositor \( z - x \). Deposit insurance is not free. Each bank has to pay the insurer an insurance premium at date 2 if no bank run occurs and its project succeeds. Deposit insurance is fairly priced in the sense that the insurer breaks even on average.

Since fully insured depositors withdraw at date 1 only when they become type-1 depositors, it is impossible that all the depositors of a bank withdraw at date 1. Therefore, in this section we will say that a bank run occurs when all the partially insured depositors of a bank withdraw at date 1.

According to the above description, in our model a deposit insurance system can be represented by the pair \((m, z, d_{2f}, d_{2p})\). In the following, we will first explain how the values of \( m \) and \( z \) are determined, and then solve the optimal \( d_{2f} \) and \( d_{2p} \). Let us start with the determination of the value of \( m \). In this system, the purpose of fully insuring some depositors is to reduce the number of depositors who may withdraw at date 1 so that the partially insured depositors can be patient when they make the withdrawing decisions. Since fully insured depositors withdraw at date 1 only when they become type-1 depositors, the fraction of depositors who withdraw at date 1 is no larger than \( mt + 1 - m \). We will set

\[
m = \frac{1 - \frac{1}{r}}{1 - t} \tag{4.1}
\]

When (4.1) holds, the fraction of depositors who withdraw at date 1 is no larger than \( 1/r \), which implies that all the depositors who withdraw at date 1 can receive \( r \). Knowing this, the partially insured depositors will not rush to the bank at subdate 1.1 or 1.2. They will make their withdrawing decisions at subdate 1.3
after all the relevant information is revealed, so the second kind of inefficient bank runs stated in Section 3 no longer occur.

As to $z$, it is used to induce the type-2 partially insured depositors to have the right incentive to withdraw at subdate 1.3. Given $s_A$ and $s_B$, the payoff for a type-2 partially insured depositor of bank $i$ is $d_1 = r$ if she withdraws at subdate 1.3, and is $p_i(s_A, s_B)d_2p + [1 - p_i(s_A, s_B)]z$ if she withdraws at date 2. She will withdraw at subdate 1.3 if and only if $p_i(s_A, s_B)d_2p + [1 - p_i(s_A, s_B)]z < r$, or equivalently,

$$p_i(s_A, s_B) < \frac{r - z}{d_2p - z} \quad (4.2)$$

We will set

$$z = \frac{rR - d_2p}{R - 1} \quad (4.3)$$

Given (4.3), condition (4.2) implies that type-2 partially insured depositors of bank $i$ will withdraw at subdate 1.3 if and only if

$$p_i(s_A, s_B) < \frac{1}{R} \quad (4.4)$$

Note that for each dollar invested at date 0, the date 1 liquidation value is one dollar and the continuation value is $p_i(s_A, s_B)R$. Therefore, the liquidation rule imposed by (4.4) is efficient, which means the first kind of inefficient bank runs mentioned in Section 3 no longer occur either.

We now turn to the optimal $d_2f$ and $d_2p$. Let $P_{\text{RUN}}$ denote the probability that a bank run will occur under the deposit insurance system, $P_{\text{SUC}}$ denote the probability that a bank run does not occur and a bank’s project succeeds, and $P_{\text{FAIL}}$ denote the probability that a bank run does not occur and a bank’s project fails. The $d_2f$ and $d_2p$ have to satisfy two constraints. First, the deposit insurance system is fairly priced and the bank has no excess profits. This condition is equivalent to

$$(1 - t)[P_{\text{RUN}}md_{2f} + P_{\text{FAIL}}[md_{2f} + (1 - m)z]]$$

$$= P_{\text{SUC}}[(1 - tr)R - (1 - t)[md_{2f} + (1 - m)d_{2p}]]. \quad (4.5)$$
The left hand side of (4.5) is the insurer’s expected expenditure,\(^{18}\) and the right hand side is the bank’s expected profit before it pays the insurance premium. When deposit insurance is offered, our interpretation of the bank’s zero-profit condition is that the bank manager gets nothing at date 2 after the bank pays the insurance premium. Therefore, equation (4.5) should hold.

The second constraint that \(d_{2f}\) and \(d_{2p}\) have to satisfy is that the fully and partially insured depositors should have the same payoff at date 0, otherwise the depositors with the lower payoff will complain. This constraint can be written as

\[
tr + (1 - t)d_{2f} = tr + (1 - t)(P_{\text{RUN}}r + P_{\text{SUC}}_d d_{2p} + P_{\text{FAIL}}z) \tag{4.6}
\]

The left and right hand sides of (4.6) are the payoffs for the fully and partially insured depositors, respectively.\(^{19}\) Let \((d^*_{2f}, d^*_{2p})\) denote the \((d_{2f}, d_{2p})\) that satisfy (4.5) and (4.6). We have the following proposition.

**Proposition 5.** Suppose that \(d^*_{2p} < rR\) and \(\min\{d^*_{2f}, d^*_{2p}\} > r\).

(a) When the deposit insurance system stated above is offered, depositors will deposit at date 0. No bank run will occur before subdate 1.3, and a bank occurs to a bank at subdate 1.3 if and only if (4.4) holds.

(b) The deposit insurance system increases the depositors’ payoff. Moreover, when it is offered, an improvement in transparency always increases depositor welfare.

**Proof.** Please see the Appendix.

Proposition 5 implies that, with an adequately designed deposit insurance system, enforcing a stricter information disclosure policy of the banking system is always welfare improving. Figure 4 shows the depositors’ equilibrium payoffs before and after deposit insurance is offered when \(r = 1.035\), \(R = 1.1\), \(t = 0.2\), \(X = 0.8\), \(\varepsilon = 0.03072\), and \(p = 0.96\). As illustrated in the figure, deposit insurance improves depositor welfare. Moreover, when it is offered, depositor welfare is always increasing in \(q\).

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\(^{18}\) When a bank run occurs, the fully insured depositors who become type-2 depositors will wait until date 2, and all the remaining depositors successfully withdraw at date 1, so the insurer has to pay \((1 - t)m_{2f}\). If a bank run does not occur and the bank’s project fails, the insurer has to pay all the type-2 depositors (both fully and partially insured). His expenditure in this case is \((1 - t)[m_{2f} + (1 - m)z]\). Using these facts, we can get the left-hand side of (4.5).

\(^{19}\) Because all the depositors who withdraw at date 1 receive \(r\), the payoff for a partially insured depositor is \(r\) if she becomes a type-1 depositor. If she becomes a type-2 depositor, her payoff is \(r\) in case a bank run occurs. If she becomes a type-2 depositor and a bank run does not occur, her payoff is \(d_{2p}\) if the bank’s project succeeds, and is \(z\) if the bank’s project fails. Using these facts, we can get the right-hand side of (4.6).
Figure 4. Depositor welfare before and after deposit insurance system is offered when \( r = 1.035, R = 1.1, t = 0.2, X = 0.8, \varepsilon = 0.03072, \) and \( p = 0.96 \)

* The solid line is the depositors’ equilibrium payoff before deposit insurance is offered, and the dashed line is the depositors’ equilibrium payoff after deposit insurance is offered.

One caveat about our results in this section is that we do not consider the costs of deposit insurance. In the real world, deposit insurance incurs at least two kinds of costs. First, when deposit insurance is provided, depositors have less incentive to collect information about their banks. A reasonable conjecture is that the quality of the public signals will decline when deposit insurance is provided. If this is a serious concern, the merit of deposit insurance in improving the depositors’ incentives to withdraw has to be weighted against its costs of reducing the precision of the information.

The second kind of costs of deposit insurance is that the funds of the insurer may be costly. For example, suppose that deposit insurance is offered by the government, and the government has to levy a tax to pay off the depositors of failed banks when the deposit insurance fund is depleted. If tax collection incurs welfare losses, then deposit insurance can be justified only when the welfare costs of tax collection are smaller than the gain from avoiding inefficient bank runs.
The case where bank managers can control the timing of information disclosure

The previous sections analyze the efficiency of bank runs. An important assumption imposed in the analysis is that bank managers cannot influence the public signals about their projects. In the real world, bank managers have various alternatives to affect both the quality of the information and the timing of information disclosure. If their abilities to affect information disclosure are taken into consideration, our results may change because bank managers can use their influence to avoid bank runs.

To explore this issue, we modify the assumptions of our model. Suppose that all the assumptions in Sections 2 and 3 hold except those about the timing for when public signals are revealed. Assume that a bank’s manager can decide when to disclose the information about his bank. At date 0, bank managers simultaneously make the timing decisions after depositors make deposits and the investments are undertaken. A bank manager can reveal the information at subdate 1.1, subdate 1.2, or date 2. If the two managers reveal the signals at the same subdate of date 1, the two signals arrive simultaneously. Also, because depositors cannot base their withdrawing decisions on a public signal revealed at date 2, we will say that a bank manager does not reveal the information if he discloses the public signal about his bank at date 2.

It is assumed that bank managers dislike bank runs, so a manager minimizes the probability that a run will occur to his bank when he makes the timing decision. This assumption does not contradict our previous assumption that a bank manager maximizes depositor welfare when designing the deposit contract. When designing the deposit contract, the bank manager has to maximize depositor welfare due to the potential competition. On the other hand, since the timing decisions are made after depositors make deposits, bank managers no longer need to care about depositor welfare when they make the timing decisions.

According to the bank managers’ timing decisions, there are four types of equilibria under the new setting.

1. The two managers reveal the signals at the same subdate of date 1.
2. The two managers reveal the signals at different subdates of date 1.
3. One manager reveals the signal about his bank at a subdate of date 1, while the other manager does not reveal the information.
4. Neither manager reveals the information.

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20 Allowing bank managers to reveal the signals at subdate 1.3 will not change the results. However, the exposition will become more complicated.
The equilibria of the second type have been analyzed in Section 3. In the rest of this section, we will first discuss the remaining types of equilibria holding the managers’ timing decisions as given. We will then determine the conditions under which each type of equilibria can be sustained, and discuss the implications of the results. To simplify exposition, we will focus on the case where

\[ p_A(L,H) < \frac{d_1}{d_2} \leq p_A(H,L) \]  

(5.1)

Condition (5.1) is stricter than (3.5). It means that when both signals are revealed at the same time, a run will occur to a bank if and only if the signal about the bank’s project equals L.

Let us start with the case where the managers reveal the signals at the same subdate of date 1. In this case, depositors will base their decisions on both signals. For j = 1, 2, let \( s_j \) denote the public signal about bank j’s project, and let \( p_j(s_1, s_2) \) denote the probability that bank j’s project will succeed given \( s_1 \) and \( s_2 \). The equilibrium outcome in this case is documented in the following lemma.

**Lemma 2.** For \( T = 1.1, 1.2 \), suppose that both banks reveal the signals at subdate \( T \).

(a) Depositors will make deposits at date 0.
(b) For \( j = 1, 2 \), a bank run will occur to bank j at subdate \( T \) if \( p_j(s_1, s_2) < \frac{d_1}{d_2} \), and no bank run will occur to bank j otherwise.

**Proof.** Please see the Appendix.

Next consider the case where neither manager reveals the information. The following Lemma states the equilibrium outcome in this case.

**Lemma 3.** Suppose that neither bank reveals the information.

(a) Depositors will make deposits at date 0.
(b) No bank run will occur at date 1 if \( p \geq \frac{d_1}{d_2} \), and bank runs will occur to both banks at subdate 1.1 otherwise.

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21 Because now the timing for when the public signals are revealed becomes endogenous, the banks A and B defined in Section 2 are no longer appropriate. Therefore, we call the two banks bank 1 and bank 2, as in the beginning of Section 2. Note that the assumptions about the information structure of the public signals are unchanged.

22 In our model, self-storage is a strictly dominated strategy for depositors even if a bank run will occur with probability one. To see this, note that a depositor receives \( 1 – tX \) if she stores the endowment herself. If the bank offers \( (d_1, d_2) = (r, (1 – t)R / (1 – t)) \) and a bank run occurs at date 1 with probability one, the depositors’ date 0 payoff is \( V_{BR} = 1 – (1 – t/r)tX > 1 – tX \). Therefore, depositors deposit even if they know that a bank run will always occur at date 1. This result may look unreasonable. However, we do not think it is a problem because in equilibrium the probability of a bank run is always strictly smaller than one.
Proof. Please see the Appendix.

Now suppose that one bank manager reveals the information at date 1 while the other does not. Let bank I and N denote the banks that reveals and does not reveal the information, respectively. Let \( s_I \) denote the signal revealed by bank I’s manager, and for \( k = I, N \), let \( p_k(s_I) \) denote the probability that bank k’s project will succeed given \( s_I \). It can be easily shown that \( p_k(L) < p < p_k(H) \). For \( k = I, N \), denote the equilibrium payoff for bank k’s depositors by \( V^k \). It can be verified that

\[
V^k = \begin{cases} 
  t d_1 + (1 - t) p d_2 & \text{if } d_1 / d_2 \leq p_k(L), \\
  W^k_0 & \text{if } p_k(L) < d_1 / d_2 \leq p_k(H) \text{ and } W^k_0 \geq d_1, \\
  V_{BR} & \text{otherwise.}
\end{cases} 
\]  
(5.2)

where

\[
W^k_0 = [p q + (1 - p)(1 - q)] [t d_1 + (1 - t) p_k(H) d_2] + [p(1 - q) + (1 - p) q] V_{BR} 
\]  
(5.3)

The following lemma states the equilibrium outcome in this case.

**Lemma 4.** For \( T = 1.1, 1.2 \), suppose that one bank manager reveals the public signal about his bank at subdate \( T \) while the other does not reveal the information, and that \( \min\{V^I, V^N\} \geq d_1 \).

(a) Depositors will make deposits at date 0.

(b) For \( k = I, N \),

(i) if \( d_1 / d_2 \leq p_k(L) \), no bank run will occur to bank k at date 1;

(ii) if \( p_k(L) < d_1 / d_2 \leq p_k(H) \), a bank run will occur to bank k at subdate T if \( s_I = L \), and no bank run will occur to bank k at date 1 otherwise.

Using Lemmas 2 to 4 and the results in Section 3, we can discuss the bank managers’ timing decisions. The following proposition states the conditions under which each possible equilibrium candidate can be sustained.

\[\text{23} \quad \text{Equation (5.2) can be explained as follows. If } d_1 / d_2 \leq p_k(L), \text{ a bank run will not occur to bank k at subdate 1.2 even if } s_I = L, \text{ so no bank run will occur to bank k and the depositors’ payoff is } t d_1 + (1 - t) p d_2. \text{ If } p_k(L) < d_1 / d_2 \leq p_k(H) \text{ and a bank run has not occurred to bank k before subdate 1.2, it will occur to bank k at subdate 1.2 if and only if } s_I = L. \text{ The } W^k_0 \text{ defined in (5.3) is a depositor’s payoff if she does not withdraw at subdate 1.1 given the belief that no others will withdraw at subdate 1.1. Given } p_k(L) < d_1 / d_2 \leq p_k(H), \text{ a bank run will not occur to bank k before subdate 1.2 if and only if } W^k_0 \geq d_1. \text{ The assumption } \min\{V^I, V^N\} \geq d_1 \text{ guarantees that a bank run will never occur before } s_i \text{ is revealed. Also note that } \min\{V^I, V^N\} \geq d_1 \text{ implies that both } p_k(H) \text{ and } p_k(H) \text{ are larger than } d_1 / d_2. \]

\[\text{24} \quad \text{The assumption } \min\{V^I, V^N\} \geq d_1 \text{ guarantees that a bank run will never occur before } s_i \text{ is revealed. Also note that } \min\{V^I, V^N\} \geq d_1 \text{ implies that both } p_k(H) \text{ and } p_k(H) \text{ are larger than } d_1 / d_2. \]
Proposition 6. Suppose that bank managers can disclose the public signals about their banks at subdate 1.1, subdate 1.2, or date 2. Moreover, (5.1) holds and \( \min\{V^l, V^N\} \geq d_1 \).\(^{25}\)

(a) There exists an equilibrium in which neither bank reveals the information if and only if \( p \geq d_1/d_2 \).

(b) There exists an equilibrium in which one bank reveals the information at a subdate of date 1 and the other does not reveal the information if and only if \( p < d_1/d_2 \).

(c) There exists an equilibrium in which both banks reveal the signals at the same subdate of date 1 if and only if \( p_N(L) < d_1/d_2 \).

(d) There exists an equilibrium in which banks reveal the signals at different subdates of date 1 if and only if \( p_N(L) < d_1/d_2 \) and \( B_1 d_l(L) \geq 1 \).\(^{25}\)

Proposition 6 have interesting implications. First, contagious runs no longer occur when bank managers can control the timing of information disclosure. As shown in part (d) of the proposition, a necessary condition for the two banks to reveal the signals at different subdates of date 1 is \( W^B_t(L) \geq d_1 \), which implies that a contagious run does not occur to bank B in the sequential information arrival case when \( s_A = L \). Intuitively, when bank managers have full control over the timing of information disclosure, they can avoid contagious runs by simultaneously revealing their information. This result implies that, when contagious bank runs is a problem, allowing bank managers to have more discretion on the timing of information disclosure may be welfare improving.

The second implication of proposition 6 is that bank managers may lack the incentive to reveal the bank-specific information when the prospects of the banking industry are favorable. As shown in part (a) of the proposition, when \( p \) is greater than \( d_1/d_2 \), neither bank manager reveals the information can be sustained as an equilibrium.\(^{26}\) This result is intuitive: if no bank run will occur when no new information is revealed, bank managers who dislike bank runs will not bother to disclose any information. When bank managers do so, both efficient and inefficient bank runs are eliminated. The elimination of bank runs may not be good for depositors. If public signals are precise so that bank runs are beneficial on average, that is,\(^{27}\)

\[
0.5(W^A_0 + W^B_0) > t d_1 + (1-t)p d_2
\]

\(^{25}\)Because depositors and bank managers may prefer different equilibria, the equilibrium selection rule that a Pareto dominant equilibrium will be the realized equilibrium does not work when multiple equilibria exist.

\(^{26}\)In fact, from Proposition 6, neither bank manager reveals the information is the only equilibrium when \( d_1/d_2 > p_B(L) \).

\(^{27}\)When no bank run occurs, a type-1 depositor’s payoff is \( d_1 \), and a type-2 depositor’s payoff is \( p d_2 \). Therefore, the depositors’ payoff in the no run equilibrium is \( t d_1 + (1 - t)p d_2 \).
then compared with the case where information arrives sequentially, depositors will become worse off when bank managers choose not to reveal the information. By demonstrating that bank managers may lack the right incentive to disclose information, Proposition 6 provides a justification for regulators to impose mandatory information disclosure requirements in the banking industry.

To sum up, compared with the case where information is revealed sequentially, depositors may become either better or worse off when bank managers have the ability to decide the timing of information disclosure. They will be better off if bank managers use this ability to eliminate only contagious runs (but not efficient bank runs), and they may become worse off if bank managers eliminate all the bank runs by not revealing any information.

6 Concluding remarks

In this paper, we show that improvements in the transparency of the banking system may increase the chance of a contagious bank run. We also discuss the possibility of using deposit insurance to improve the efficiency of bank runs. In addition, we illustrate how our results will change when bank managers can control the timing of information disclosure.

Although our paper focuses on policy and welfare implications, it also has empirical implications. For example, Propositions 1 and 2 imply that a bank run will occur to a bank when the probability of success for the bank’s project is low. This result is consistent with the empirical findings of Calomiris and Mason (1997, 2003) that bank failures in the United States during the great depression period can be well explained by bank fundamentals. Also, Proposition 3 predicts that contagious runs are more likely to occur when the correlation between the banks’ returns is higher. In addition, our model provides an explanation for the empirical result documented in Drwyer and Hasan (2005) that suspension of payments reduces the probability of a bank closure by twenty five percent. In our basic model, if the manager of bank B can suspend convertibility until $s_B$ is revealed when he observes that $s_A = L$, then a contagious run will never happen and depositors of bank B will use both $s_A$ and $s_B$ to make their withdrawing decisions. In other words, by forcing depositors to wait until more precise bank-specific information is revealed, convertibility suspension can improve the efficiency of bank runs, thus reduce the number of bank failures.

There are several directions to extend our paper. First, the information structure of our model can be enriched so that more subtle issues about information disclosure can be discussed. In our model, an improvement in transparency refers to an increase in the precision of the public signals. There
exist other definitions for improvements in transparency. One alternative is that the transparency of the bank industry improves when depositors know better whether the problems of the failed banks are systematic in nature or idiosyncratic in nature. If we define improvements in transparency in this way, an improvement in transparency may always reduce the chance of a contagious run.\textsuperscript{28} This extension will have policy implications on how to construct the optimal disclosure rule for financial institutions.

Second, this paper investigates how to use deposit insurance to eliminate inefficient bank runs. Our model can be extended to study the relationship between other bank regulation policies and the efficiency of bank runs. For example, bank capital regulations can affect the depositors’ withdrawing behaviors in several ways. As the bank capital requirement framework becomes more risk sensitive, depositors will have more precise information about banks. On the other hand, if bank capital regulations are accompanied by prompt corrective actions, bank regulators will play a more important role in disciplining banks, so depositors may need to worry less about the soundness of banks. This will reduce the depositors’ incentives to acquire information about banks. It will be interesting to study how different bank regulation policies affect the efficiency of bank runs.

Finally, to focus on how depositors respond to information, in this paper we assume that bank risk is exogenously determined. In the real world, bank managers have great influence on the choice of bank risk. In the banking literature, many papers discuss how to design regulation policies to reduce the bank managers’ incentives to pursue unsound risks.\textsuperscript{29} If we assume that bank risk is chosen by bank managers, our model will have implications on how the transparency of banks affects the bank managers’ risk-taking behaviors. We conjecture that, if the inefficient bank run problem can be controlled, bank managers will have less incentive to pursue risks when banks become more transparent.

\textsuperscript{28} We are grateful to an anonymous referee for raising this point.

\textsuperscript{29} For example, see Cordella and Levy Yeyati (1998, 2002), Hellmann, Murdock, and Stiglitz (2000), and Matutes and Vives (1996, 2000).
References


Appendix

**Proof of Lemma 1.** Let $v_{SS}$ denote the probability that both banks’ projects succeed, $v_{FF}$ denote the probability that both banks’ projects fail, $v_{SF}$ denote the probability that bank A’s project succeeds and bank B’s project fails, and $v_{FS}$ denote the probability that bank A’s project fails and bank B’s project succeeds. Our assumptions imply that $v_{SS} = p^2 + \varepsilon$, $v_{FF} = (1-p)^2 + \varepsilon$, $v_{SF} = v_{FS} = p(1-p) - \varepsilon$. Also, for $s' = H, L$ and $s'' = H, L$, let $\pi_{s's''}$ denote the prior probability that $s_A = s'$ and $s_B = s''$. We have

\[
\pi_{HH} = v_{SS}q^2 + v_{SF}q(1-q) + v_{FS}(1-q)q + v_{FF}(1-q)^2 \quad (A1)
\]
\[
\pi_{HL} = v_{SS}q(1-q) + v_{SF}q^2 + v_{FS}(1-q)^2 + v_{FF}(1-q)q \quad (A2)
\]
\[
\pi_{LH} = v_{SS}(1-q)q + v_{SF}(1-q)^2 + v_{FS}q^2 + v_{FF}q(1-q) \quad (A3)
\]
\[
\pi_{LL} = v_{SS}(1-q)^2 + v_{SF}(1-q)q + v_{FS}q(1-q) + v_{FF}q^2 \quad (A4)
\]

Because $v_{SF} = v_{FS}$, (A2) and (A3) imply that $\pi_{HL} = \pi_{LH}$. Define $x \equiv q/(1-q)$ and apply the Bayesian updating rule, it can be easily shown that

\[
\frac{1}{p_A(H,H)} = 1 + \frac{v_{FF}/x + v_{SF}}{v_{SS}x + v_{SF}} \quad (A5)
\]
\[
\frac{1}{p_A(H,L)} = 1 + \frac{v_{FF} + v_{SF}/x}{v_{SS} + v_{SF}x} \quad (A6)
\]
\[
\frac{1}{p_A(L,H)} = 1 + \frac{v_{FF} + v_{SF}x}{v_{SS} + v_{SF}x} \quad (A7)
\]
\[
\frac{1}{p_A(L,L)} = 1 + \frac{v_{FF}x + v_{SF}}{v_{SS}/x + v_{SF}} \quad (A8)
\]

Given $q > 0.5$, $x = q/(1-q) > 1$. Moreover, it can be verified that $v_{SS}v_{FF} > v_{SF}^2$. Using these facts, it can be verified that $p_A(L,L) < p_A(L,H) < p_A(H,L) < p_A(H,H)$. Also, because $x$ is increasing in $q$, by (A5) to (A8) we know that $p_A(H,L)$ is increasing in $q$, $p_A(L,H)$ is decreasing in $q$, and $p_A(H,L) = p_A(L,H) = p$ when $q = 0.5$ (so $x = 1$). Therefore, $p_A(L,H) < p < p_A(H,L)$. This completes the proof of part (a).
As to part (b) of the Lemma, because banks A and B are symmetric, we know that \( p_A(s_A, s_B) = p_B(s_B, s_A) \), which implies part (b) of Lemma 1. Q.E.D.

**Proof of Proposition 3.** From the definitions of \( \mu_L \) and \( W^B_1(L) \)

\[
\mu_L = \frac{\pi_{LH}}{\pi_{LH} + \pi_{LL}} = \frac{[pq + (1 - p)(1 - q)] - \frac{(2q - 1)^2}{p(1 - q) + (1 - p)q}\varepsilon}{(A9)}
\]

\[
W^B_1(L) = \mu_L[td_1 + (1 - t)p_A(H, L)d_2] + (1 - \mu_L)V_{BR} \quad (A10)
\]

Differentiate \( W^B_1(L) \) in (A14) with respect to \( q \), we have

\[
\frac{dW^B_1(L)}{dq} = \frac{d\mu_L}{dq}[td_1 + (1 - t)p_A(H, L)d_2 - V_{BR}] + \frac{dp_A(H, L)}{dq}[\mu_L(1 - t)d_2] \quad (A11)
\]

The term in the first bracket of the right hand side of (A11) is positive because \( W^B_2(L, H) > V_{BR} \) by (3.5), and obviously the term in the second bracket of the right hand side of (A11) is also positive. By (A9), when \( q = 1 \) and \( \varepsilon = p(1 - p) \),

\[
\frac{d\mu_L}{dq} = (2p - 1) - \left[ \frac{4}{1 - p} + \frac{2p - 1}{(1 - p)^2} \right]p(1 - p) = \frac{-1}{1 - p} < 0 \quad (A12)
\]

Because \( d\mu_L/dq \) is continuous in both \( q \) and \( \varepsilon \), the above result implies that \( d\mu_L/dq < 0 \) when \( q \) is close to 1 and \( \varepsilon \) is close to \( p(1 - p) \). Also, by rearranging (A6) and differentiating \( p_A(H, L) \) with respect to \( q \), we have

\[
\frac{dp_A(H, L)}{dq} = \frac{dx}{dq} v_{SF} \left[ v_{SS} + v_{SF} \left( x + \frac{1}{x} \right) + v_{FF} - (v_{SS} + v_{SF}x) \left( 1 - \frac{1}{x^2} \right) \right] \quad (A13)
\]

When \( \varepsilon = p(1 - p) \), \( v_{SF} = p(1 - p) - \varepsilon = 0 \). Therefore, (A13) implies that \( dp_A(H, L)/dq \) approaches 0 when \( \varepsilon \) approaches \( p(1 - p) \).

By the above results, when \( q \) is close to 1 and \( \varepsilon \) is close to \( p(1 - p) \), \( d\mu_L/dq < 0 \) and \( dp_A(H, L)/dq \) is close to 0. By (A15), \( W^B_1(L) \) is decreasing in \( q \) in this case. This completes the proof of Proposition 3. Q.E.D.

**Proof of Proposition 4.** Consider the deposit contracts with \( d_1 \geq r \) and \( d_2 = (1 - td_1)R/(1 - t) \). By the above equation, \( d_2 < R \) when \( d_1 \geq r \), which implies \( d_1/d_2 > 1/R \).

We first show that \( V^i_2(s_A, s_B) \) is decreasing in \( d_1 \). By (6),

...
\[
\frac{dW_2^i(s_A, s_B)}{dd_1} = t + (1 - t)p_i(s_A, s_B) \frac{dd_1}{dd_1} = t[1 - p_i(s_A, s_B)R]
\]  
(A14)

When \( p_i(s_A, s_B) \geq d_1/d_2 \), \( p_i(s_A, s_B) > 1/R \), so \( 1 - p_i(s_A, s_B)R < 0 \). By (A14), \( W_2^i(s_A, s_B) \) is decreasing in \( d_1 \) when \( p_i(s_A, s_B) \geq d_1/d_2 \). Also, by (3.2), \( V_{BR} \) is decreasing in \( d_1 \). In addition, an increase in \( d_1 \) (and a corresponding decrease in \( d_2 \)) increases the chance of a bank run through raising \( d_1/d_2 \), and a bank run reduces the depositors’ payoff when \( p_i(s_A, s_B) \) is close to \( d_1/d_2 \). These results imply that an increase in \( d_1 \) reduces \( W_2^i(s_A, s_B) \).

Given this result, an increase in \( d_1 \) also decreases \( W_1^i(s_A) \) because \( W_1^i(s_A) \) is a weighted average of \( V_2^i(s_A, H) \) and \( V_2^i(s_A, L) \). Applying the same logic, it follows that \( W_1^i(s_A) \) and \( W_0^i \) are also decreasing in \( d_1 \), which means that an increase in \( d_1 \) will reduce depositor welfare. Therefore, bank managers will set \( d_1 = r \) and \( d_2 = (1 - tr)R/(1 - t) \) to maximize depositor welfare. \textbf{Q.E.D.}

**Proof of Proposition 5.** Suppose that \( d_{zp}^2 < rR \) and \( \min\{d_{2f}^1, d_{2p}^1\} > r \). In this case, \( z > 0 \) and no depositor will withdraw before subdate 1.3 when the deposit insurance system is offered. Part (a) of the proposition is obvious from the discussion preceding the proposition and the fact that self-storage is a strictly dominated strategy for depositors.

For part (b) of the proposition, when deposit insurance is offered, no depositor will suffer the liquidity loss \( X \), and a bank’s project is liquidated if and only if its liquidation value is lower than the continuation value. Therefore, depositors are better off. We next prove that depositor welfare is increasing in \( q \) in this case. When deposit insurance is offered, the depositors’ equilibrium payoff can be written as

\[ tr + (1 - tr)V_2 \]

where

\[ V_2 \equiv \pi_{HH} \text{Max}\{1, p_A(H, H)R\} + \pi_{HL} \text{Max}\{1, p_A(H, L)R\} + \pi_{LH} \text{Max}\{1, p_A(L, H)R\} + \pi_{LL} \text{Max}\{1, p_A(L, L)R\} \]  
(A15)

By (A15), \( V_2 \) is the value of a call option. The exercise price of the option is one, and the value of the underlying asset for the option is \( p_A(s_i, s_j)R \). It can be shown that an increase in \( q \) raises the variance of the underlying asset’s value without changing its expected value. Therefore, \( V_2 \) is increasing in \( q \). This completes the proof of the proposition. \textbf{Q.E.D.}
**Proof of Lemma 2.** Part (b) of the lemma is obvious from the discussions in Section 3. To show part (a), let \( V_{SI} \) denote the depositors’ payoff for depositing and waiting until subdate 1.2. Given (5.1)

\[
V_{SI} = \pi_{HH} W_2^A (H, H) + \pi_{HL} W_2^A (H, L) + (\pi_{LH} + \pi_{LL}) V_{BR}
\]  

(A16)

Comparing (3.8) with (A16), it can be verified that when (5.1) holds, \( V_{SI} \) equals \( W_0^A \), and \( V_{SI} > W_0^B \) if \( W_1^B (L) < d_1 \) and \( V_{SI} = W_0^B \) otherwise. From (3.9) and the fact that \( V_{SI} \geq \max \{W_0^A, W_0^B\} \), depositors will make deposits at date 0, and no depositors will withdraw before subdate 1.2. This completes the proof of the lemma. **Q.E.D.**

**Proof of Lemma 3.** Given that no information is revealed, the depositors’ payoff is \( td_1 + (1-t)p_d2 \) when a bank run does not occur and is \( V_{BR} \) when a bank run occurs. Note that \( td_1 + (1-t)p_d2 \geq d_1 \) if and only if \( p \geq \frac{d_1}{d_2} \), so a bank run will occur at subdate 1.1 if and only if \( p < \frac{d_1}{d_2} \). This proves part (b) of the lemma. For part (a), at date 0, if a depositor stores her endowment, her payoff is \( 1-tX < V_{BR} < d_1 \). Therefore, she will deposit at date 0. **Q.E.D.**

**Proof of Proposition 6.** To prove the proposition, we need to calculate the probabilities that a bank run will occur under different conditions. For convenience of exposition, define \( P_L \equiv p(1-q) + (1-p)q \).

Let \( P^0 \) denote the probability that a bank run will occur to a bank when neither bank reveals the information. By Lemma 3, \( P^0 = 0 \) if \( \frac{d_1}{d_2} \leq p \) and \( P^0 = 1 \) if \( \frac{d_1}{d_2} > p \). For \( k = I, N \), let \( P^k \) denote the probability that a bank run will occur to bank \( k \) when only one bank reveals the information. By Lemma 4, given \( \min \{V_I, V_N\} \geq d_1 \), \( P^k = 0 \) if \( p_k (L) \geq \frac{d_1}{d_2} \) and \( P^k = P_L \) otherwise. In case both banks reveal the information at the same subdate of date 1, Lemma 2 implies that the probability of a bank run is \( P_L \) for each bank. Finally, for \( i = A, B \), let \( P^i \) denote the probability that a bank run will occur to bank \( i \) in the sequential information arrival case. Given (5.1), \( P^A = P_L \); \( P^B = P_L \) if \( W_1^B (L) \geq d_1 \) and \( P^B > P_L \) if \( W_1^B (L) < d_1 \). Proposition 6 can be easily shown using the above results. **Q.E.D.**


