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Bidding in fixed rate tenders: theory and experience with the ECB tenders

Suomen Pankin keskustelualoitteita
Finlands Banks diskussionsunderlag

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Bidding in fixed rate tenders: theory and experience with the ECB tenders

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Abstract

This paper presents a model of the optimal bidding behaviour of a single bank in the context of fixed rate liquidity tenders. Banks' bidding is shown to depend crucially on the central bank's liquidity policy as regards tender allotments. The paper also analyses ECB liquidity policy in terms of the model. The ECB, while applying fixed rate tenders, appears to have been attempting stabilise the market interest rate at a level close to the main refinancing rate. However, this aim was at least partially overridden by that of stabilising total money market liquidity over the course of the reserve maintenance period – even more so when banks were expecting the ECB to raise the main refinancing rate in the near future. The banks' aggregate bids increased considerably during the period of fixed rate tenders. This was seen to result mainly from profit opportunities associated with a positive spread between market interest rate and main refinancing rate. The positive spread resulted from the combination of expectations of an interest rate hike and liquidity-oriented allotment policy.

Key words: bidding, money market tenders, liquidity policy, central bank operating framework

Tarjoukset kiinteäkorkoisissa huutokaupoissa: teoria ja kokemuksia EKP:n huutokaupoista

Suomen Pankin keskustelualoitteita 1/2002

Tuomas Välimäki
Tutkimusosasto

Tiivistelmä

Tutkimuksessa mallinnetaan yksittäisen pankin optimaalinen tarjoustenteko kiinteäkorkoisissa rahamarkkinahuutokaupoissa. Pankkien tarjousten osoitetaan ratkaisevasti riippuvan keskuspankin valitsemasta politiikasta, kun se päättää huutokaupassa jaettavan likviditeetin määrästä. Lisäksi tutkimuksessa analysoidaan EKP:n likviditeetinjakopolitiikkaa käytetyn mallin valossa. Saatujen tulosten mukaan EKP näyttäisi pyrkineen kiinteäkorkoisissa huutokaupoissaan vakauttamaan lyhyimmän rahamarkkinakoron perusrahoitusoperaatioissa sovelletun koron tasolle. Tätä päämäärää vahvempi vaikuttaisi kuitenkin osittain olleen pyrkimys tasata likviditeetin määrää kunkin vähimmäisvarantovelvoitteiden pitoajanjakson kuluessa. Näin näyttäisi käyneen etenkin tilanteissa, joissa pankit odottivat EKP:n nostavan perusrahoitusoperaatioiden korkoa lähitulevaisuudessa. Huutokauppoihin jätettyjen tarjousten kokonaismäärä kasvoi huomattavasti ajanjaksona, jona EKP toteutti operaationsa kiinteäkorkoisina huutokauppoina. Kasvun osoitetaan johtuvan pääasiassa voiton mahdollisuudesta, joka seuraa odotetun markkinakoron ja perusrahoitusoperaatioiden koron välisestä positiivisesta erotuksesta. Tällainen positiivinen erotus syntyy koronnosto-odotusten ja likviditeettimäärää korostavan likviditeettipolitiikan yhdistelmästä.

Asiasanat: huutokauppatarjoukset, rahamarkkinahuutokaupat, likviditeettipolitiikka, keskuspankin toimintakehikko

Contents

1 Introduction

The discussion over the European Central Bank's (ECB) operational framework during the first 18 months of operation was overwhelmingly characterized by the banks' behaviour in the main refinancing operations (MRO). These operations were conducted as fixed rate tenders, where the ECB announced the rate at which the banks could receive liquidity from it (*main refinancing rate*). After the announcements the banks informed the ECB over the amount of reserves they were willing to borrow at the given rate. Finally, the ECB decided on the actual amount of liquidity to be provided to the markets. In case the bid amount was larger than the allotment, each bank received equal proportion of its bid.[1] The banks seemed to place bids far above the amount needed to fulfill the reserve requirements. Thus, the allotment ratios (ie the liquidity allotted / the volume of aggregate bids) in the ECB weekly tenders averaged at 8%, varying from 100% (in 7 April 1999) to the low point of 0.87% (in 31 May 2000). Furthermore, the allotment ratio seemed to decline especially during the first half of 2000. This development was considered as a sign of severe overbidding by the banks, and finally it led to the adjustment of the tender procedure applied by the ECB.[2]

The bidding of the banks in the ECB main refinancing operations has received a variety of explanations in some recent papers. For example, Nautz and Oechssler (2000) and Ehrhart (2000) build up a simple model over the banks' bidding in the fixed rate tenders. They both claim that the overbidding phenomenon is produced as an optimal response to the fact that the central bank is supplying liquidity less than the banks demand for. However, these papers do not pay attention to the central bank incentives to act as proposed, they merely assume that the money market liquidity desired by the central bank is lower than the optimal amount to the banks (at the given rate). Furthermore, these papers abstract from the interbank market for bank reserves. However, it is just the money market rates that largely determine the bid behaviour in the tender operations, and these rates are strongly affected by the amount of reserves that is provided to the banks in the tenders. Ayuso and Repullo (2000) construct a model where the bidding of the banks is determined by the difference between the target rate of the central bank and the expected money market rate. They propose that the overbidding in the ECB fixed rate tenders was produced by an asymmetric preference function of the ECB. In their model the central bank provides the markets with liquidity that will on average keep the overnight rate above the tender rate, as it has a loss function that penalizes more heavily interbank rates below the target than those above it. However, the paper doesn't consider the motive for the ECB to have such an asymmetric loss function. The rationale behind the proposed asymmetricity is not at all trivial. The more so, if we consider that the ECB has stated that

[1] Eg if the aggregate bid were 1.500 units and the ECB allotted 1.000 units, a bank that bid for 300 units would have received 200 units.

[2] In 8 June 2000 the Governing Council of the ECB decided to swich to variable tender procedure as of 27 june 2000. In ECB press release (dated 8 June 2000) the new tender mechanism was announced to be "*a response to the severe overbidding problem which has developed in the context of the fixed rate tender procedure*".

"*The ECB tended to orient its allotment decisions towards ensuring an average interbank overnight rate close to the tender rate*" (ECB, 2000b). Finally, Ayuso and Repullo (2001) tests whether the overbidding of the banks resulted from the expectations of a future tightening of the monetary policy or from the existence of a positive spread between short term money market rates and the main refinancing rate that resulted from a contemporaneous restrictions in the supply of liquidity. They find empirical evidence supporting the latter option.

In this paper we propose an alternative explanation to the evolution of the bids in the ECB main refinancing operations. We show that it can be optimal to bid in excess one's neutral demand for liquidity even if the central bank has symmetric preferences over the interest rate variations in the interbank market. The incentive to "overbid" is enhanced, if the central bank pays attention to the deviations of liquidity from the level indicated by the reserve requirement. For example, when the banks expect the central bank to increase its policy rates during the remainder of the current reserves maintenance period, it is optimal for them to hold more reserves now (at the current rates) than after the rate change has occurred. A liquidity oriented central bank might want to curb such frontloading of the reserves from happening. The difference between this kind of liquidity oriented policy and the asymmetric preferences rationale suggested by Ayuso & Repullo is, that with a central bank interested in stable liquidity, the spread between the expected overnight rate and the tender rate should be affected by interest rate expectations, whereas with asymmetric preferences the spread should reflect only the expected asymmetry in preferences. We show that the liquidity orientation of the central bank both survives remarkably well in light of the empirical evidence we have from the ECB fixed rate tenders and is in line with the information the ECB has published over its liquidity policy.

The paper is organized as follows. In section 2 we model the optimal bidding strategy for a single bank. Section 3 describe some of the potential liquidity policies the central bank may choose to follow. In section 4 we consider what kind of paths the bidding will take under the various liquidity policies, and we also introduce the effects that arise with the collateral requirement. Section 5 reviews the evidence from the first 18 months of ECB operations. Finally section 6 concludes the discussion.

2 Model of the optimal bidding

The money market consists of a central bank (that is the monopoly supplier of liquidity) and of n homogenous banks that demand liquidity in order to fulfill the reserve requirements and to avoid being forced to use the standing facilities. The model money market liquidity consists of the net sum of autonomous liquidity factors[3] and the amount of reserves provided to the market in the tender operations. Let us denote the estimated amount of autonomous liquidity factors either by a^{CB} or a^{banks} depending on who makes the forecast ($a^{banks} = \sum_{i=1}^{n} a_{\text{bank } i}$), liquidity provided through tenders by q ($q = \sum_{i=1}^{n} q_i$), and the liquidity shock (ie the forecast error of the autonomous liquidity factors) either by ε^{CB} or ε^{banks}. We will divide the shock a single bank faces (ε_i) into two zero mean parts μ/n and ξ_i, where μ/n is bank i's share of the shock into the aggregate money market liquidity, whereas ξ_i is a liquidity distribution shock[4]. Furthermore, we'll assume that the aggregate liquidity shock is independent of the distribution shock (ie $\mu \perp \xi_i$, thus $\varepsilon_i = \mu/n + \xi_i$ and $\varepsilon = \sum_{i=1}^{n} \varepsilon_i = \mu$, as $\sum_{i=1}^{n} \xi_i = 0$). Note that even though the estimation of the autonomous liquidity factors made by the central bank doesn't have to equal that of the banks, we will always have $a^{CB} + \varepsilon^{CB} = a^{banks} + \varepsilon^{banks} =$ ex post amount of the autonomous liquidity factors. To ease the notation, we will drop out the superscripts of both a and ε whenever they aren't necessarily needed. The amount of liquidity allotted in a tender can't exceed the supply or the demand at the given price. Therefore, it will equal the minimum of the total amount of liquidity bid for by the banks in the tender ($\sum_{i=1}^{n} b_i = b$) and the amount the central bank is willing to provide to the markets (denoted by c). The total ex post money market liquidity is given by:

$$l = a + \varepsilon + q = a + \varepsilon + \min(b, c). \tag{1}$$

It can be shown that in a system with marginal lending facility, deposit facility and averaged reserve requirement, the interbank rate of interest with relevant maturity[5] is a monotonically decreasing function of money market liquidity and rates of the standing facilities (r^{SF}), and it is increasing in the expected future central bank rates (r^{ef}). Ie $r = r\left(l, r^{SF}, r^{ef}\right)$, where $\frac{dr}{dl} < 0$, $\frac{dr}{dr^{SF}} < 0$ $\wedge \frac{dr}{dr^{ef}} > 0$.[6] Now, the expected market rate of interest at the given central

[3]The autonomous liquidity factors are the balance sheet items of the central bank that are not affected by monetary policy operations. The most important autonomous factors affecting the euro area liquidity are net government deposits with the Eurosystem, banknotes and items in course of settlement. See ECB (2000c, 40–41) for a more detailed presentation of these factors.

[4]A distribution shock merely transfers liquidity from one bank to another. Thus, the distribution shocks must sum up to zero (a positive shock to bank i must always be accompanied by a negative shock of identical size to the rest of the banks, ie $\xi_i = -\xi_{-i}$, where ξ_{-i} denotes $\sum_{j=1}^{n} \xi_j - \xi_i$).

[5]The relevant maturity of the comparable market rate of interest is the same as that of the tender operation's. Note that if the tender operation is collateralized, also the comparable rate must be collateralized. The maturity of the ECB weekly tenders is two weeks, and the liquidity received from these operations must be covered with adequate collateral. We'll return to the questions that araise from collateral requirements in chapter 4.

[6]A detailed discussion on the specific functional forms of the demand for overnight liquidity can be found eg in Välimäki (2001).

bank rates (both current and expected future rates) is given by:

$$\mathrm{E}\left[r\left(l|r^{SF},r^{ef}\right)\right]=\int_{\varepsilon^{\min}}^{\varepsilon^{\max}} r\left(a+\varepsilon+\min[b,c]|r^{SF},r^{ef}\right)f\left(\varepsilon\right)d\left(\varepsilon\right),$$

where $f(\varepsilon)$ is the probability density function of the aggregate liquidity shock.[7]

We'll define the neutral amount of tendered reserves at the given central bank rates (both the current and the expected future rates) and autonomous liquidity factor estimate $(q^{neutral}|a, r^{CB})$[8] to be such that with it the expected market rate of interest will equal the tender rate[9]:

$$\begin{aligned}&\mathrm{E}\left[r\left(a+\varepsilon+q^{neutral}|r^{CB}\right)\right] \\ =& \int_{\varepsilon^{\min}}^{\varepsilon^{\max}} r\left(a+\varepsilon+q^{neutral}|r^{CB}\right)f\left(\varepsilon\right)d\left(\varepsilon\right)=r^{T}.\end{aligned} \quad (2)$$

As the market rate of interest is decreasing with liquidity, we know that the expected value of the rate is above (below) the tender rate, if $\min(b,c) < q^{neutral}$ $(\min(b,c) > q^{neutral})$.

Let us next consider the bidding of a single, risk neutral bank in three cases: i) $\min(b,c) < q^{neutral}$, ii) $\min(b,c) = q^{neutral}$ or iii) $\min(b,c) > q^{neutral}$.

The amount of liquidity allotted to bank i in a tender operation is either the amount it bid for (if $c > b$) or the bid amount scaled back by the allotment ratio c/b (if $c < b$):

$$q_i = \min\left(b_i, \frac{c}{b}b_i\right) \quad (3)$$

Thus, the expected amount to be received from the tender is given by:

$$\mathrm{E}\left(q_i\right)=b_i\mathrm{E}\left[\min\left(1,\frac{c}{b}\right)\right]$$

That is, the expected amount of reserves to be allotted to bank i will $i)$ equal the bid amount, if it's certain, that the total amount of bids will be lower than the central bank's target (ie if $p(c > b) = 1$), $ii)$ it will be the expected proportion $\mathrm{E}[c/b]$ of the bid amount, if it is certain that the banking sector as a whole to demand for more reserves than the central bank aims at providing (ie if $p(c > b) = 0$), and $iii)$ it will be smaller than b_i, if the bank can't be sure whether the bid will be scaled back (ie if $p(\frac{c}{b} > 1) \in (0,1)$).

Let us denote the private value of a specific amount of reserves for bank i by $r_i^{pv}(x)$ (ie $r_i^{pv}(x)$ is the value of x units of liquidity to bank i, when it does not participate the interbank market). The private value is decreasing in liquidity. Also, let l_i^T and l_i^m denote the amount of liquidity (with given current and expected future central bank rates) at which the private value of liquidity would equal the tender rate and the market rate respectively (ie $r_i^{pv}\left(l_i^T|r^{CB}\right) = r^T$

[7]Note that, if the distribution of liquidity shocks of the banks deviates from that of the central bank, the overnight rate expected by the banks does not necesserily have to coincide with that of the central bank.

[8]r^{CB} denotes the vector of current and future central bank rates.

[9]The main refinancing rate is the tender rate applied by the ECB.

and $r_i^{pv}\left(l_i^m|r^{CB}\right)=r$). Finally, let q_i^T and q_i^m denote the amount of liquidity, that has to be allotted to bank i, in order for the expected private value of liquidity to equal the tender rate or the expected market rate respectively (ie q_i^T and q_i^m are implicitly given by $\int_{\varepsilon_i^{\min}}^{\varepsilon_i^{\max}} r^{pv}\left(a_i+\varepsilon_i+q_i^T|r^{CB}\right) f\left(\varepsilon_i\right) d\left(\varepsilon_i\right)=r^T$ and $\int_{\varepsilon_i^{\min}}^{\varepsilon_i^{\max}} r^{pv}\left(a_i+\varepsilon_i+q_i^m|r^{CB}\right) f\left(\varepsilon_i\right) d\left(\varepsilon_i\right)=E[r]$.

Bank i can obtain the liquidity it desires either from the central bank tender operation or from the interbank market. Let's start the analysis of a single banks behaviour first by considering the bank's profit maximizing problem at the interbank market after the liquidity shock has realized. The problem of the risk neutral bank i is:

$$\max_{s_i} \Pi=\int_{a_i+\mu/n+\xi_i+q_i}^{a_i+\mu/n+\xi_i+q_i+s_i} r_i^{pv}(x)\,dx-s_i r, \tag{4}$$

where s_i is the net amount borrowed from the market. The first term at RHS of equation (4) is the change in the private value of the traded liquidity, and the second term is the direct cost of borrowing it from the market.[10] The FOC for the problem is:

$$\begin{aligned} \frac{\partial(\cdot)}{\partial s_i} &= -r+r_i^{pv}\left(a_i+\mu/n+\xi_i+q_i+s_i^*\right)=0 \\ \Rightarrow\quad & r_i^{pv}\left(a_i+\mu/n+\xi_i+q_i+s_i^*\right)=r. \end{aligned} \tag{5}$$

Equation (5) tells us that with the equilibrium borrowing, bank i adjusts its private value of liquidity to the level of the market rate. The explicit borrowing function is given by:

$$s_i^*=r_i^{pv^{-1}}(r)-a_i-\mu/n-\xi_i-q_i \tag{6}$$

Now, positive interbank borrowing by bank i must be met by negative borrowing (naturally with the same magnitude) by the rest of the banks (ie $s_i=-s_{-i}$)[11], and consequently the aggregate interbank borrowing must sum up to zero. Thus, the following holds for aggregated amounts:

$$\sum_{i=1}^{n} r_i^{pv^{-1}}(r)=a+\mu+q,$$

and as the banks are homogeneous, we can derive the following equation for the market rate of interest:

$$r(a+q+\mu)=r_i^{pv}\left(\frac{a+\mu+q}{n}\right). \tag{7}$$

Inserting equation (7) into equation (6) gives the optimal interbank borrowing of bank i in the following form:

$$s_i^*=\left(\frac{a+q}{n}\right)-a_i-\xi_i-q_i. \tag{8}$$

[10] Net lending to the interbank market is naturally denoted by negative borrowing.

[11] Throughout the paper we'll denote the aggregate value of any variable deducted by bank i's value for it by subscript $-i$. Ie $x_{-i}=\sum_{j=1}^{n} x_j-x_i$.

Ie in the interbank market, the banks equate their differences in the amount of liquidity they hold. The dissimilarities in the liquidity held before the interbank market operations result either from the (distributive) liquidity shock or from differences in the bid behaviour of the banks.

At the central bank tender, the banks are assumed to bid in order to maximize their expected profits. The cost of the liquidity acquired is naturally the tender rate times the amount allotted to the bank, while the expected income from the allotment is the expected change in the market value of the quantity traded at the interbank market and the expected change in the private value of the amount held by the bank. The bidding strategy of bank i must be based on maximizing the following equation:

$$\begin{aligned}\max_{q_i} \mathrm{E}\left[\Pi_i\right] &= \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ s_i\left(r\left(q_{-i},\mu\right),\mu,\xi_i\right) r\left(q_{-i},\mu\right) \right. \\ &\quad - s_i^*\left(r\left(q,\mu\right),\mu,\xi_i,q_i\right) r\left(q,\mu\right) \\ &\quad \left. + \int_{r_i^{pv^{-1}}(r(q_{-i},\mu))}^{r_i^{pv^{-1}}(r(q,\mu))} r_i^{pv}\left(x\right) dx \right\} f\left(\xi_i,\mu\right) d\xi_i d\mu - q_i r^T \\ \text{s.t. } s_i^* &= r_i^{pv^{-1}}\left(r\right) - a_i - \mu/n - \xi_i - q_i \text{ and } q_i \geq 0\end{aligned}$$

Now, we divide the analysis of optimal bidding in a tender into two parts according to the relative size of the aggregate bid versus the central bank's target amount (b vs. c). We'll first consider the case in which the central bank uses full allotment strategy, and after that we'll study the case where the central bank will scale the excess bids down according to its target.

Case 1: Full allotment

In full allotment the central bank will always provide the banks with all the liquidity bid for (ie $c = b \Rightarrow \frac{c}{b} = 1$). In this case, the profit maximizing problem of the bank i at the tender is:

$$\begin{aligned}\max_{b_i} \mathrm{E}\left[\Pi_i\right] &= \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ s_i|_{b_i=0} r\left(b_{-i},\mu\right) - s_i^*|_{b_i=b_i} r\left(b,\mu\right) \right. \\ &\quad \left. + \int_{r_i^{pv^{-1}}(r(b_{-i},\mu))}^{r_i^{pv^{-1}}(r(b,\mu))} r_i^{pv}\left(x\right) dx \right\} f\left(\xi_i,\mu\right) d\xi_i d\mu - b_i r^T \qquad (9) \\ \text{s.t. } s_i^* &= r_i^{pv^{-1}}\left(r\right) - a_i - \mu/n - \xi_i - b_i \text{ and } b_i \geq 0, \qquad (10)\end{aligned}$$

which can be transformed into the following Kuhn-Tucker formulation:

$$\begin{aligned}L(b_i,\nu) &= \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ \left(r_i^{pv^{-1}}\left(r\left(b_{-i},\mu\right)\right) - a_i - \mu/n - \xi_i \right) r\left(b_{-i},\mu\right) \right. \\ &\quad - \left(r_i^{pv^{-1}}\left(r\left(b,\mu\right)\right) - a_i - \mu/n - \xi_i - b_i \right) r\left(b,\mu\right) \qquad (11) \\ &\quad \left. + \int_{r_i^{pv^{-1}}(r(b_{-i},\mu))}^{r_i^{pv^{-1}}(r(b,\mu))} r_i^{pv}\left(x\right) dx \right\} f\left(\xi_i,\mu\right) d\xi_i d\mu - b_i r^T + \nu b_i.\end{aligned}$$

The first order conditions corresponding to the Lagrangian are:

$$\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ r(b,\mu) - \left[r_i^{pv^{-1}}(r(b,\mu)) - a_i - \mu/n - \xi_i - b_i \right] \right. \tag{12}$$

$$\left. \times \frac{\partial r(b^*,\mu)}{\partial b_i} \right\} f(\xi_i,\mu)\, d\xi_i d\mu - r^T + \nu = 0$$

$$\nu b_i^* = 0 \tag{13}$$

$$\nu \geq 0 \tag{14}$$

Equation (12) implicitly defines the optimal bid for bank i, and when the optimum bid is positive the condition can be reduced into:

$$\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} s_i^* \frac{\partial r(b^*,\mu)}{\partial b_i} f(\xi_i,\mu)\, d\xi_i d\mu = \mathrm{E}\left[r(b^*,\mu) \right] - r^T. \tag{15}$$

Ie with optimal central bank borrowing bank i *either* equates the expected change in the value of interbank borrowing with the expected difference between the market rate of interest and the tender rate *or* the bank doesn't participate the tender at all.

Let's next consider, under which conditions it is optimal for bank i to bid such an amount that it will bring the aggregate liquidity into its neutral level (ie when $b_i^* = q^{neutral} - b_{-i} \Leftrightarrow b_i^* + b_{-i} = b^* = q^{neutral}$). Now, $b_i = q^{neutral} - b_{-i}$ is the optimal bid, if the following holds:

$$\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left[r_i^{pv^{-1}}\left(r\left(q^{neutral},\mu\right)\right) - a_i - \mu/n - \xi_i - \left(q^{neutral} - b_{-i}\right) \right]$$

$$\times \quad \frac{\partial r\left(q^{neutral},\mu\right)}{\partial b_i} f(\xi_i,\mu)\, d\xi_i d\mu = 0. \tag{16}$$

Equation (16) holds only, if $b_{-i} = q_{-i}^T$, in which case $b_i^* = q^{neutral} - q_{-i}^T = q_i^T$.[12] Ie *it's optimal for bank i to bid the aggregate reserves up to the neutral level, when the aggregate bid of the rest of the banks is neutral, and its just the neutral demand of bank i that is needed to close the gap.* This incentive applies for all banks. Thus, *every bank bidding for its neutral liquidity is an equilibrium solution for the profit maximization problem.*

If the aggregate bid of the other banks is less than their neutral demand would be (ie $q_{-i}^T < q^{neutral} - q_i^T$), it will be optimal for the bank i to bid for more than the neutral demand is, but still less than what is needed to

[12] This comes from the fact that equation (16) can be rewritten as:

$$\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ \frac{a + q^{neutral}}{n} - \left[a_i + \left(q^{neutral} - b_{-i}\right) \right] \right\} \frac{\partial r\left(q^{neutral},\mu\right)}{\partial b_i} f(\xi_i,\mu)\, d\xi_i d\mu = 0. \tag{17}$$

Ie the liquidity of bank i after the tender must equal its share of the total neutral liquidity. Its easy to see that this holds only when the aggregate bid of all other (than i) banks is their neutral (aggregate) bid. Ie we must have $\frac{a+q^{neutral}}{n} = a_i + q_i^T = a_i + \left(q^{neutral} - b_{-i}\right)$, and consequently, it's optimal for bank i to bid the liquidity up to the neutral level only if $b_{-i} = q^{neutral} - q_i^T = q_{-i}^T$.

bring the aggregate bid up to the neutral level (ie $q_i^T < b_i^* < q^{neutral} - q_{-i}^T$). Intuitively, by this kind of bidding bank i increases its probability of being a lender at the interbank market with the expected value of the market rate being above the tender rate. As this incentive applies for all banks its not feasible to assume the rest of the banks bidding less than their neutral demand (ie the rest of the banks could increase their profits by increasing their bids). Consequently, we don't expect this kind of profit opportunity to open for bank i. Similarly, if the aggregate bid of the other banks is larger than their neutral demand (ie $q_{-i}^T > q^{neutral} - q_i^T$), it's optimal for bank i to place a smaller bid than the neutral demand would suggest, but still to bid for more than the amount which would take the aggregate liquidity down to the neutral level (ie $q_i^T > b_i^* > q^{neutral} - q_{-i}^T$). Now, with this kind of bid bank i will increase its probability of being a borrower at the interbank market while the expected value of the market rate will be below the tender rate. Again the incentive applies for all banks, and thus, we don't find the presumption of the rest of the banks bidding for too large liquidity (relative to the neutral demand) feasible (the rest of the banks could in this case increase their expected profits by lowering their bids). Consequently, under the full allotment *the equilibrium where every bank bids for its neutral liquidity is unique.*

Case 2: Proportional allotment, ie if $c < b \Rightarrow q_i = \frac{c}{b} b_i$

As the liquidity is determined (at least partly) by the preferences of the central bank, the expected value of the market rate of interest depends also on the central bank's target liquidity c. When $\min(c, b) < q^{neutral} \Rightarrow \mathrm{E}[r] > r^T$, $\min(c, b) > q^{neutral} \Rightarrow \mathrm{E}[r] < r^T$, and $\min(c, b) = q^{neutral} \Rightarrow \mathrm{E}[r] = r^T$. We'll analyze separately the case where the aggregate bid of other banks exceeds the central bank's target amount (ie $b_{-i} > c$) and the one in which the bids of the rest of the banks isn't enough for the target to be fulfilled (ie $b_{-i} < c$).

Let's start studying the case, where $b_{-i} \geq c$. Now, the market liquidity and hence the market rate of interest will depend only on the central bank's target (ie $\frac{\partial r(q,\varepsilon)}{\partial b_i} = 0$, as $\frac{\partial q}{\partial b_i} = 0$). Thus, bank i will choose its bid in order to maximize the expected profit that is simply the allotted amount of liquidity times the expected difference between the market rate of interest and the tender rate:

$$\begin{aligned} \max_{b_i} \mathrm{E}[\Pi_i] &= b_i \frac{c}{b} \left(\int_{\varepsilon^{\min}}^{\varepsilon^{\max}} r(c, \varepsilon) f(\varepsilon) \, d\varepsilon - r^T \right) \\ \text{s.t. } b_i &\geq 0. \end{aligned} \tag{18}$$

Now, we can formulate the following Lagrangian:

$$L(b_i, \nu) = b_i \frac{c}{b} \left(\int_{\varepsilon^{\min}}^{\varepsilon^{\max}} r(c, \varepsilon) f(\varepsilon) \, d\varepsilon - r^T \right) + \nu b_i,$$

from which we can derive the following FOCs:

$$\left(\frac{c}{b} - b_i c b^{-2} \right) \left(\mathrm{E}[r(c)] - r^T \right) + \nu = 0 \tag{19}$$

$$\nu b_i = 0 \text{ and } \nu \geq 0. \tag{20}$$

Now, it's easy to see that the optimal bid depends directly on the difference between the expected overnight rate with the central bank's liquidity target and the tender rate. The optimal bid is:

$$b_i^* = \begin{cases} b_i^{\max} & , \text{if } \mathrm{E}\left[r\left(c\right)\right] > r^T \\ \left[0, b_i^{\max}\right] & , \text{if } \mathrm{E}\left[r\left(c\right)\right] = r^T \\ 0 & , \text{if } \mathrm{E}\left[r\left(c\right)\right] < r^T \end{cases} \tag{21}$$

When $\mathrm{E}[r\left(c\right)] > r^T$, the optimal bid would be the maximum bid a bank can place in the tender ($b_i^* = b_i^{\max}$). As this applies for all banks, the presumption $b_{-i} > c$ is feasible.[13] However, if the target liquidity of the central bank is large enough to push the expected value of the market rate below the tender rate (ie $\mathrm{E}[r\left(c\right)] < r^T$), it would be optimal for bank i not to participate the tender (or to place a zero bid in it). As this applied for all banks, the presumption $b_{-i} > c$ wouldn't be feasible, and the central bank would not be in control of the liquidity. The case in which the market rate would equal the tender rate with the liquidity targeted by the central bank is, however, not so straight forward. The optimal bid for a single bank is anything from zero up to the maximum it can bid for, as long as the bank can be certain that the banking sectorwise aggregate bid will at least equal the target level of the central bank (ie $p\left(b^* \geq c\right) = 1$). Otherwise, the expected liquidity would be below neutral liquidity (ie $\mathrm{E}[\min\left(c, b\right)] < c$). Consequently, the expected value for the market rate of interest would increase above the tender rate (ie $\mathrm{E}[r\left(c\right)] = r^T < \mathrm{E}[r\left(\min\left(c, b\right)\right)]$), and therefore, the optimal bid would be $b_i^{\max}$.

Let's next consider the second possibility, ie the case in which the aggregate bid of the other banks is below the target of the central bank ($b_{-i} < c$). In this case the determination of the optimal bid for bank i is identical to the case of full allotment, as long as the aggregate bid of all banks remains below the target amount of the central bank (ie $b_i^* + b_{-i} < c$). Consequently, there will be a unique equilibrium in which all banks bid their neutral demand ($b_i^* = q_i^T$), if $q^{neutral} \leq c$. However, if $b_{-i} < c \leq q^{neutral}$, bank i's bid has an effect on the expected market liquidity, but the expected market rate of interest will be above the tender rate regardless of the size of b_i. Now, it can be shown that it'll be optimal for bank i to get as large portion of the liquidity allotted to the market as possible, and sell the liquidity in excess of its own need to the market with positive expected profits. The rest of the banks could also increase their expected profits by increasing their bids, thus, the presumption $b_{-i} < c$ wouldn't be feasible. Furthermore, with $q^{neutral} = c$, the optimal bid is $b_i^{\max}$ if $p\left(b \geq c\right) < 1$. Thus, the presumption $b_{-i} < c$ can hold only, if $q^{neutral} \leq c$.

Now, by combining the two cases above, we may conclude that the optimal bid of bank i depends on liquidity policy of the central bank. Under full allotment procedure the equilibrium bid is q_i^T, by which the expected money market liquidity will always equal the neutral demand of the banks, and the expected market rate of interest will be at the level of the tender rate. When the central bank applies proportional allotment procedure, the optimal bid depends over the difference between the target liquidity of the central bank

[13] With the natural implicit assumption that the possible limit for the bids is high enough, ie $b_i^{\max} > c$.

and the neutral liquidity; when $c \leq q^{neutral}$ the optimal bid is $b_i^{\max}$,[14] and the central bank chooses the expected money market liquidity, while the expected liquidity is chosen by the banks at $q^{neutral}$ when $c > q^{neutral}$. That is, with fixed rate tenders the central bank is able to raise the expected value for the market rate of interest (above the tender rate) by constraining the liquidity supply, but it can't lower the expected rate below the tender rate.

3 Liquidity policy of the central bank

Based on the analysis above we expect the banks' optimal bid to be depend on the difference between the central bank's liquidity target and the neutral liquidity. To understand why a particular path in the evolution of bids occurs, we must analyze what kind of liquidity policy the central bank applies. The alternative liquidity policy rules considered here are: full allotment, interest rate targeting (neutral liquidity policy), restricted liquidity supply, and liquidity targeting.

1. *Full allotment*

   The simplest procedure for the central bank to follow is the *full allotment* policy. With full allotment, the central bank always provides the market with all the liquidity bid for by the banks (ie $c = b \Rightarrow c/b = 1$). Under the full allotment, we know, that the equilibrium amount the banks bid for equals $q^{neutral}$, and consequently the expected market rate of interest will equal the current tender rate.

2. *Interest rate targeting rule (neutral liquidity policy)*

   In *interest rate targeting*, the central bank estimates the amount of liquidity demanded by the banking sector to bring the market rate to the level of the tender rate (ie $c = c^{irt} = q^{neutral}$; thus, we call this procedure also the neutral liquidity policy rule). Consequently, the expected market rate of interest equals the tender rate also with this procedure. From previous, we know that the equilibrium bidding depends on wether the banks can expect the central bank to always be in position to control the liquidity. If the answer is yes, the optimal bid of a single bank i would be anything from zero up to the maximum amount the bank is able to bid for. However, when the bank is not able to count on $p\left(c^{irt} \leq b\right) = 1$, the optimal bid is the maximum bid it can place without facing any extra costs.[15]

   What could motivate the central bank choosing interest rate targeting over the full allotment? Now, we have $\mathrm{E}\left[r | l = a^{banks} + d^{T} + \varepsilon^{banks}\right]$ $=\mathrm{E}\left[r | l = a^{CB} + c^{irt} + \varepsilon^{CB}\right]$ $=r^{T}$. That is, the expected market rate equals the tender rate with both these procedures. However, if either of these two parties (the banks or the central bank) possesses private information over the evolution of the autonomous factors or over the

[14] We will analyze, how the maximum bid should be defined in section 4.

[15] The determination of the maximum bid is analyzed in section 4.

functional form of the market rate as function of liquidity, the probability of the amount of neutral liquidity demanded by the banks equalling the neutral liquidity estimated by the central bank is below one (ie $p\left(q^{T,CB} = q^{T,banks}\right) < 1$). Now, if the central bank has superior knowledge over the development of the autonomous factors, it might be able to contain the stochastic volatility of the market rate by controlling the expected money market liquidity.[16] Thus, basically the selection between interest rate targeting policy and the full allotment procedure is one between restraining the *stochastic* volatility of the market rate versus having the banks bidding for more than their neutral demand is.[17]

3. *Restricted liquidity supply*

   As the third option, we consider a policy rule according to which the central bank provides the markets with less liquidity than is needed to keep the expected market rate of interest at the level of the tender rate (ie $c^{rls} < \mathrm{E}^{CB}\left[q^{neutral}\right]$). This *restricted liquidity supply* could be rationalized eg by asymmetric preferences of the central bank, as suggested by Ayuso and Repullo (2000). According to the asymmetric preferences argument, the central bank prefers the deviations of the market rate above the policy rate (here the tender rate) to deviations below the policy rate. Consequently, the true interest rate target of the central bank is above the policy rate. This can be achieved only by constraining the liquidity supply below the neutral liquidity level.

4. *Liquidity targeting rule*

   The last option we consider here is *liquidity targeting*. When applying this procedure, the central bank is not only interested in the expected market rate of interest, but it will also pay attention to the level of liquidity in the money market. According to this rule, the central bank wants to provide the markets with liquidity, that will on average equal the amount needed to fulfill the reserve requirement *and* is as stable as possible throughout the remaining maintenance period. This means, that even though the central bank provides the banks with an averaging possibility, it will try to prevent the banking sector as a whole from speculating over the interest rate developments during the rest of the reserves maintenance period by timing the reserve holdings. Therefore, the bid behaviour of the banks is largely depending on the expectations

[16]Note that the central bank can restrict the overnight volatility only, if its superior knowledge over autonomous factors is large enough to compensate the potentially inferior knowledge it has over the effect of interest rate expectations on the demand for liquidity. Furthermore, another way how the central bank might restrict the stochastic volatility of the overnight rate (with full allotment) would naturally be to make its private information public, and thus, increase the accuracy of the liquidity forecasts made by the banks.

[17]Nb that even if the stochastic volatility with full allotment were higher than with the interest rate targeting, this volatility will not be transmitted to longer interest rate periods as the expected value of the future overnight rates is not affected by this volatility. Consequently, the potential excess volatility resulting from full allotment does not interfere the signalling or tarnsmission of monetary policy.

of future interest rates during the rest of the period.[18] We'll denote the target liquidity of the liquidity targeting central bank by $c^{liquidity}$. Now, when the banks don't expect the central bank to change its interest rates during the rest of the current reserves maintenance period, we assume $q^{neutral}$ to be very close to $c^{liquidity}$ (ie there is no incentive for the banks either to front- or backload the reserve holdings). However, this does not need to be the case, especially if the reserve requirement is small relative to the standard deviation of the liquidity shocks.[19] If the banks expect the tender rate to be raised in the near future, we assume that $q^{neutral}|_{r^{ef}>r^T} > q^{neutral}|_{r^{ef}=r^T} \approx c^{liquidity}$, as the banks would like to postpone their reserve holdings until the rate change has taken place. Consequently, the expected market rate will raise above the tender rate, and the banks will have an incentive to overbid in the tender operations. When the banks expect the central bank to cut its rates in the near future, we assume that $q^{neutral}|_{r^{ef}<r^T} < q^{neutral}|_{r^{ef}=r^T} \approx c^{liquidity}$, as the banks would like to hold more reserves before the rate change occurs. Consequently, the banks will not bid enough for the central bank to be able to allot all the target amount. Thus, with expectations of a rate cut, we expect liquidity targeting to work as the full allotment rule does.

Let us next try to analyze more closely the bid behaviour of the banks with each of these liquidity policies, and let's also introduce the effect the collateral requirements have on the bids.

4 Collateral requirements, maximum bids and bid ratios

The dynamic path of the bid ratio (aggregate bids / allotted amount) will be different under each of the four liquidity policies described above. First, when the central bank uses *full allotment* procedure the bid ratio is naturally always 1. However, the path under the other liquidity policies is largely affected by how the size of the maximum bid is determined and/or what is the expected development of the tender rate during the rest of the maintenance period.

According to the General documentation on Eurosystem monetary policy instruments and procedures (ECB, 2000d), the ECB may impose a maximum bid limit in order to prevent disproportionately large bids. However, the ECB did not explicitly announce any such limit while conducting fixed rate tender operations between January 1999 and June 2000. Furthermore, *the ECB requires the counterparties to be in a position to cover the amount of liquidity they are allotted to by a sufficient amount of eligible collateral.* If a counterparty isn't able to provide the ECB with the required collateral, it may impose

[18] Note that the market rate of interest with given liquidity is the higher the higher is the expected future value for the central bank rates $\frac{\partial r(l|r^{ef})}{\partial r^{ef}} > 0$. Consequently, the higher the expected future rates are, the lower is the neutral bid for bank i (ie $\frac{\partial q_i^T}{\partial r^{ef}} > 0$).

[19] See Välimäki (2001) for a detailed discussion over this issue.

penalties to the counterparties. These sanctions may take the form of financial penalties or suspension of the counterparty from the subsequent tender operations for a given period.[20]

The profit for bank i from the liquidity it is allotted to and which it will sell at the overnight market is simply the traded amount times the difference between the tender rate and the (comparable) market rate, as long as the bank has collateral to cover the whole amount it receives from the tender. The cost of acquiring liquidity in excess of the collateral possessed is basically defined by the sanctions regime. However, the banks are allowed to borrow from the market the collateral they need. The collateral cost of allotment q_i is given by:

$$\int_{q_{-i}}^{q_{-i}+q_i} h(x)\,dx,$$

where $h(x)$ is the marginal cost of an additional unit of collateral. Now, let's denote the amount of collateral bank i has without any extra cost by k_i. Hence, there will be cost of acquiring the extra collateral, when the total allotment exceeds k. Furthermore, bank i has to submit the collateral to the central bank before rerceiving the liquidity. We assume that due to credit lines, each bank faces a limit (denoted by z_i) up to which it can borrow collateral. Hence, if the allotment for bank i is larger than the limit (ie if $q_i > z_i$), the bank will fail to comply with the tender rules and is sanctioned by $(q_i - z_i)\, r^{sanction}$. For the rest of the section we assume, that the limit for borrowing is always higher than the neutral liquidity for the banks (ie $z_i > q_i^T$). Therefore, the credit lines will reduce the banks incentive to bid only when it's optimal for the banks to "overbid" (ie to bid in excess of the neutral demand for liquidity).

Under *full allotment* if k is large enough to always cover the neutral demand of the banks (ie $q^{neutral} < k$), there isn't any extra cost from the collateral requirement for the bank, and naturally the collateral requirement doesn't affect the equilibrium bidding at all. When k is below $q^{neutral}$, the collateral requirement will affect the equilibrium bidding. In this case the banks will continue to place bids that with them the expected secured market rate of interest equals the tender rate. However, the equilibrium liquidity in this case is reduced to the level at which it the private value of liquidity for the banks will be the sum of market rate of interest and the marginal cost of collateral.[21] Thus, scarcity of collateral reduce the equilibrium liquidity, but it will not move the expected value of (collateralized) market rate of interest away form the tender rate. Now, if the marginal cost of collateral grows with the allotted amount, we expect the collateralization to reduce the banks' incentive to frontload reserve holdings when rate hike is expected.

Based on section 2, we know that under *interest rate targeting* the optimal bid for bank i is anything from 0 to $b_i^{\max}$, if it can be certain that the banking sector wide aggregate bid is larger than c^{irt}. Otherwise, it's optimal for the bank to bid $b_i^{\max}$. The certainty is achieved only by supplying a bid that is greater than $c^{irt} - b_{-i}$. As both c^{irt} and b_{-i} are unknown by the time the bid has

[20] A detailed description of the ECB's sanctioning regime in case event of non-compliance with counterparty obligations see ECB (2000 d, annex 6).

[21] The proof for this is presented in appendix A.

to be placed, the full certainty is achieved only by bidding atleast the maximum amount the central bank's neutral target can have ($b_i^* = [c^{itr,\max}, b_i^{\max}]$). If this amount isn't feasible (ie if $b_i^{\max} < c^{itr,\max}$), the optimal bid is $b_i^{\max}$.

Now, the question is, what defines the maximum bid with the ECB tender rules. As long as the interest rate targeting central bank is in control of liquidity ($b \geq c^{irt}$), bank i can't make expected profit between the market rate of interest and the tender rate.[22] Therefore, the bank should make such a bid, that it will not face any extra costs from the realizing allotment. Now, by bidding $b_i \leq z_i$, the maximum allotment for bank i is z_i ($q_i \leq k_i$), and it will under all circumstances avoid being short of collateral. If $z_i > c^{itr}$, bank i can be positive, that the control of liquidity is in the hands of the central bank by placing a bid in excess of the central bank's target ($c^{itr} \leq b_i \leq z_i$), hence, the equilibrium bid is anything from c^{irt} to z_i. However, if $z_i < c^{itr}$, the equilibrium bid of bank i depends on the probability at which the aggregate bid of the banks will be hogher than the target amount of the central bank ($p(b > c^{irt})$). If this probability is close to one, its very unlikely that there will be a positive expected spread between the market rate of interest and the tender rate. Thus, in such a case we expect bank i to bid z_i, as it's the maximum bid with zero probability of failing to meet the collateral requirement. Consequently, the aggregate bid will be $\sum_i z_i$, which would leave the central bank to be in charge of the expected liquidity (ie $p(\sum_i z_i > c^{irt}) = 1$). In this case, the we expect the bid ratio to be $\frac{z}{c^{itr}}$.

Under *restricted liquidity policy*, the optimal bid for bank i is always $b_i^{\max}$. Now, unlike in the case of liquidity targeting, bank i will make expected profit by trying to get as large share of the allotted liquidity as possible, as there is a positive expected spread between the market rate and the tender rate. If bank i's limit for borrowing is larger than the total allotted volume (ie $z_i > c^{rls}$), the optimal bid would be infinite or it would be bounded only by the requirement of the bid being a numerical value. Thus, we are interested here, how the maximum bid is determined when bank i would fail to comply with the collateral requirement after being allotted a large proportion of the total allotment (ie $z_i < c^{irt}$). Now, the expected income for bank i from the tender is the expected market rate of interest multiplied with the amount allotted to the bank, while the expected cost is the tender rate multiplied with the allotted amount and the expected cost from the non-compliance with the tender rules. When the bank estimates the allotment it receives with a given bid, the bank must make some assumptions on the bid behaviour of other banks and the total amount the central bank will provide to the market. We'll denote the subjective probalility density function over the bids of the other banks by $g(b_i)$. Now, the optimal bid for bank i is the outcome of the

[22] Note that the market rate of interest the central bank targets must be the collateralized rate. If the central bank's target was set on the unsecured rate and $k < c^{irt}$, the collateralized market rate would be below the tender rate. Hence, the banks would behave as under full allotment and the central bank wouldn't get enough bids to allot liquidity according to the target.

following maximization problem:

$$\max_{b_i}\left\{\left(\mathrm{E}\left[r|_{q=c^{rls}}\right]-r^T\right)\int_{\frac{c^{rls}-z_i}{z_i}b_i}^{b_{-i}^{\max}}\frac{c^{rls}}{b_{-i}+b_i}b_i g\left(b_{-i}\right)db_{-i}\right. \tag{22}$$

$$\left.-r^{sanction}\int_0^{\frac{c^{rls}-z_i}{z_i}b_i}\left(\frac{c^{rls}}{b_{-i}+b_i}b_i-z_i\right)g\left(b_{-i}\right)db_{-i}-S\int_0^{\frac{c^{rls}-z_i}{z_i}b_i}g\left(b_{-i}\right)db_{-i}\right\},$$

where $\int_0^{b_{-i}^{\max}}\frac{c^{rls}}{b_{-i}+b_i}b_i g\left(b_{-i}\right)db_{-i}$ is the expected allotment to bank i, $\frac{c^{rls}-z_i}{z_i}b_i$ is the minimum value of b_{-i} for bank i not to fail to comply with the tender rules, $r^{sanction}$ is the penalty rate that the central bank applies for the amount of bid that is not covered with collateral, and S denotes the fixed cost arising from the non-compliance[23]. Now, differentiating equation (22) w.r.t. b_i gives us the following FOC:

$$\left(\mathrm{E}\left[r|_{q=c^{rls}}\right]-r^T\right)\left[\int_{\frac{c^{rls}-z_i}{z_i}b_i^*}^{b_{-i}^{\max}}\frac{c^{rls}b_{-i}}{b^2}g\left(b_{-i}\right)db_{-i}-\left(c^{rls}-z_i\right)g\left(q_i^*\right)\right] \tag{23}$$

$$=r^{sanction}\left[\int_0^{\frac{c^{rls}-z_i}{z_i}b_i^*}\frac{c^{rls}b_{-i}}{b^2}g\left(b_{-i}\right)db_{-i}\right]+S\frac{c^{rls}-z_i}{z_i}g\left(q_i^*\right)$$

Equation (23) implicitly defines the optimal bid of bank i a function of both the expected interest rate spread and the expected bids of the the rest of the banks. Although the economic intuition of the FOC might be hard to get, we can derive the following conclusions based on it. The wider the expected (positive) interest rate spread becomes, the larger the optimal bid is. Ie the higher expected profit there is to be received from overbidding, the higher expected cost the bank is willing to face from the possibility of failing the tender. Similarly, bank i's optimal bid grows as the expectation over the aggregate bid of the other banks increases. Ie the possoibility of non-compliance with the tender rules with a given bid reduces when the rest of the banks bid for more liquidity, thus, bank i can increase its own bid to balance the expected gains with the expected losses. Furthermore, raising the sanctions (either the penalty rate or the fixed cost of failure) will naturally reduce the optimal bid.

With constant expected interest rate spread, the evolution of the bid amount, will depend mostly on the method of forming the expectations on the aggregate bid of the rest of the banks, based on which the bank also forms expectation over the forthcoming allotment ratio. Now, as $\frac{c^{rls}b_{-i}}{(b_{-i}+b_i)^2}$ is a convex function w.r.t. b_{-i}, we know by Jensen's inequality that the optimal bid of

[23] There is not any fixed cost mentioned in the ECB rules for non-compliance with the counterparty obligations, however, there is likely to be some sort of implicit reputational cost from the failure to cover the bid amount with eligible collateral. Eg the fact that Fed funds rate is sometimes below the discount rate is usually explained in the literature by the implicit cost related to the use of discount window. Furthermore, eg when discussing on the behaviour of the treasurer in the main refinancing operations of the ECB, Vergara (2000, p. 17) mentions the bank's willingness to protect its reputation vis-à-vis the central bank as the major constraint for overbidding.

bank i increases with the accuracy of its subjective PDF of the bids placed by the other banks. Now, we might expect the uncertainty over the bid behaviour of the other banks to be the greatest at the first operation. Thus, in it the allotment for each bank is likely to be below the ex post optimal amount (which is naturally z_i). The realization of the bid ratio gives the bank new information about the bid behaviour of the other bidders, and based on this information bank i can make a larger bid than it otherwise would have been able to make. However, the bank will expect the other banks also to behave in the same manner (ie to increase their bids), which itself leads to further increase in the optimal bid. Now, this kind of reasoning will lead us to expect the optimal bids to grow in time. Thus, the bid ratio is likely to grow from tender to tender, if the expected interest rate spread is constant, and the ratio is expected to make bigger swings when the expected spread changes.[24]

Finally, under *liquidity oriented allotment policy*, the bid behaviour was shown to depend on the expected future tender rate. From above we know, that when a rate cut is expected this policy will behave like full allotment, ie we expect the bid ratio to be one. When the central bank is not expected to change its rate, this policy should be similar to the interest rate targeting. When a rate increase is expected, there will be a positive expected spread between the market rate and the tender rate, in which case the optimal bid will be given by equation (23). Therefore, the development of the bid ratio in time depends largely over the expectations on the forthcoming tender rate. We would expect the ratio to grow with the faster pace the higher the expected market rate is relative to the current tender rate, and to collapse to unity when a rate cut is expected.

Let us next look at the data from the ECB tenders to analyze the liquidity policy of the ECB and the bid behaviour of the banks in these operations in light of the model built in chapters 2 to 4.

5 Experience with the ECB tenders

The allotment and bid ratios (ie (c/b) or (b/c)) from the ECB fixed rate tenders during 7.1.1999–21.6.2000 are given in figure 1. These charts show us undoubtedly, that the ECB did not use full allotment procedure in its FRT's. Furthermore, the bid ratio seems to grow (allotment ratio seems to decrease)

[24]Note that the spread doesn't need to be constant even if the asymmetry of the preferences is constant. Eg let's assume, that the central bank would like to see the expected spread being positive, but as small as possible subject to the requirement that more than 60% of the realizations of the spread should be positive. Now, it can be shown, that the difference between expected median of the market rate and its mean value may depend on the expectations over the development of the future tender rate. Eg with normally distributed shocks we expect that the market rate will be more often above its expected value than below it when an increase in the tender rate is expected. If a rate cut is expected we expect the opposite to be true. Thus, in this case, we would expect the spread between the market rate and the tender rate to be smaller when an increase is expected than when a rate cut is expected. Also, the amount of liquidity to be allotted is larger when an increase is expected, this could also decrease the rate of growth of the bid ratio relative to the case when a rat cut is expected.

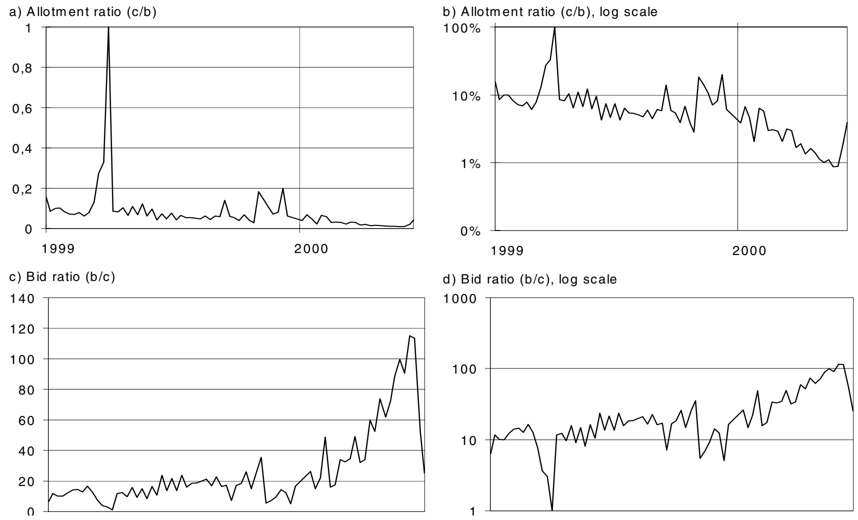

Figure 1: Allotment and bid ratios in ECB fixed rate tenders

in time. However, it is not very clear wether the growth rate of the bid ratio was increasing in time. We can not determine what made the banks tick in their bidding simply by analyzing the realized bid behaviour. As we saw in the previous chapter, the increasing bid ratio can be a result of various different liquidity policy rules used by the central bank. Thus, we must analyze these ratios together with the interest rate and liquidity data available.

Let us next try to assess what were the key factors affecting the ECB liquidity provision, ie what kind of liquidity policy rule the ECB seemed to have followed. After that we will turn to analyze, how the banks saw the ECB liquidity policy being driven (ie what could have caused the bid ratio to grow so considerably).

5.1 Liquidity provision of the ECB

5.1.1 On the EONIA spread

Figure 2 illustrates the overnight spread (ie EONIA[25] – main refinancing rate[26]) from the start of Stage Three until 23.6.2000. The figure calls attention for at least two separate features. First, we notice that there are regular spikes (both up and downwards) in the spread. These spikes reflect the increased volatility of the overnight rate that is associated with the ends of the reserves maintenance periods due to the bigger liquidity uncertainty during the last

[25] EONIA (Euro Overnight Index Average) is a measure of the effective interest rate prevailing in the euro interbank overnight market. It is calculated as a weighted average of the interest rates on unsecured overnight contracts on deposits in euro, as reported by a panel of contributing banks. (ECB, 2001)

[26] The *main refinancing rate* is the rate applied in ECB fixed rate tenders. Thus, if we use term *tender rate* in the empirical part of this paper, we refer to the main refinancing rate.

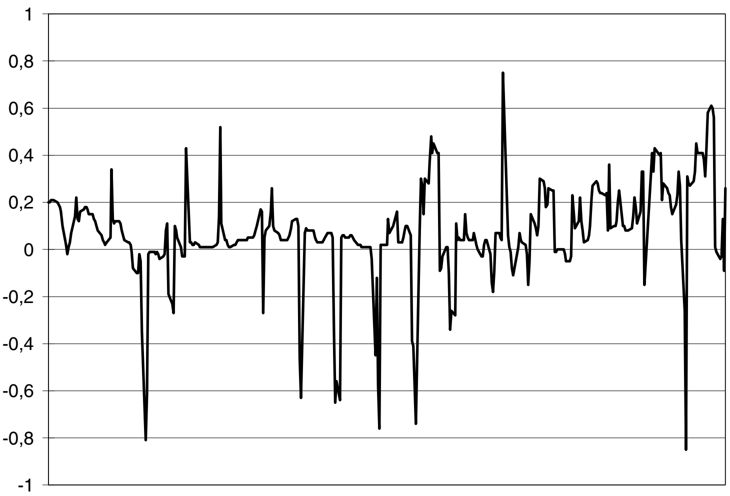

Figure 2: Overnight spread between 4.1.1999–23.6.2000

days of the period.[27] Another thing that calls for attention is that the (average) spread seems to grow in time. The average spread over the whole time period is 6.8 basis points (bps) while it is only 1.9 bps during the first half of the period and 12 bps during the second half of the period. The same figures are 10.0, 5.8 and 14.3 bps respectively, if we drop out the end of each maintenance period.[28] Furthermore, the difference in the size of the average overnight spread between the subperiods (5.1.–30.9.1999 and 1.10.1999–23.6.2000) is so large, that we can reject the hypothesis of it resulting from stochastic variations of the spread (the null-hypothesis of the two average spreads equalling each other will be rejected at every conventional confidence levels). Thus, there has been a shift (or several shifts) in the market conditions towards tighter supply of liquidity relative to its demand. Consequently, we expect to find a change (or several changes) *either* in the liquidity policy of the ECB *or* in the liquidity demand conditions (or both) during the 18 months period in question.

Does a positive average overnight spread indicate that the ECB has provided the markets with liquidity that was on average below the natural demand described in chapter 2? Before considering the question we must notice, that there are at least two flaws in using the EONIA as a "comparable market rate of interest" in the analysis. First, the maturity of EONIA is overnight whereas the maturity of the tendered liquidity is two weeks. The bias from the different maturities is perhaps not too drastic with interest rate being as

[27] The increase in the overnight volatility at the end of reserves maintenance period is a typical feature of the reserves averaging provision. This increase results from the fact that the interest rate elasticity of the demand for reserve balances grows as the banks' ability to average liquidity shocks diminishes towards the end of an averaging period (on the last day of the maintenance period there is no averaging possibility at all). A more strict statistical analysis on the day of the maintenance period can be found in Perez-Quiros (2000).

[28] In this case we have not included the spreads from the settlement day of the last tender operation until the end of each maintenance period. We have also left out the spread from the 30.12.1999 due to the millennium effect on the EONIA. The spread was 75 bps on that day.

low as it was during the analyzed period. Furthermore, this problem could be avoided by using effective interest rates. However, the second flaw might be more drastic. The EONIA is calculated from unsecured interbank deposits, whereas the tenders are fully collateralized operations. Thus, it can very well be the case that the EONIA should (on average) be a few basis points over the tender rate even with neutral liquidity. From now on we will call this kind of difference between the two rates *the natural spread*. Furthermore, the natural spread doesn't necessarily need to be constant over time. Consequently, we must be very careful in drawing any conclusions over the tightness of the ECB's liquidity policy by analyzing the EONIA-spread.

Now, the hypothesis of the EONIA-spread being zero will be rejected at every reasonable confidence level for the whole period as well as for the second half of the period. However, the hypothesis can't be rejected even at 10% confidence level for the first part of the period when the ends of periods are included. Still, it will be rejected even at 1% confidence level, if the ends of each reserve maintenance periods are dropped out of the sample. This results means that either the ECB didn't use interest rate targeting as its policy rule in liquidity allotment decisions or there exists a positive natural spread between the two rates. However, the spread for the second half of the period seems to be so far above zero (or the spread during the first subsample), that it probably can't be explained by the risk premium associated with these unsecured overnight deposits.[29] Ie we will reject the idea of ECB applying pure interest rate targeting rule (as defined in chapter 3) at least for the latter subsample. Still, we wouldn't feel very comfortable to say that the neutral liquidity policy rule should be rejected also for the first subsample.

The evolution of the overnight spread can be illustrated also with the average EONIA calculated from the five days following each tender operation (ie the average EONIA from the days whose liquidity a specific tender operation is aimed at affecting). This is done in figure 3. The grey bars in figure 3 show the EONIA-spreads calculated as average of the five EONIA-spreads following each tender operation (ie the average EONIA from the days whose liquidity a specific operation is aimed at affecting). The white bars represent the average EONIA-spreads following the last tender operation of each maintenance period, and the black bars show the changes in the tender rate.

The data behind figure 3 tells us, that on average the spread was positive and grew in time. The 18 months of fixed rate tenders can again be divide into two subperiods. The first 9 months is characterized by neutral or decreasing interest rate expectations (during this subsample there was only one rate cut and no increases), whereas the second half of the period is characterized by neutral or increasing interest rate expectations (there were 5 rate increases and no cuts during this subsample).

Now, if the liquidity policy of the ECB was restrictive (ie the liquidity

[29]The natural spread between the EONIA and the tender rate should be at least 8bps, for the nul-hypothesis (EONIA − tender rate = natural spread) not to be rejected even at 1% confidence level. Furthermore, the parameter estimate for allotment size is statistically insignificant when we regress the interest rate spread against the amount allotted in each tender. Hence, we don't expect the high spread between the unsecured market rate of interest and the main refinancing rate to originate from the scarcity of collateral.

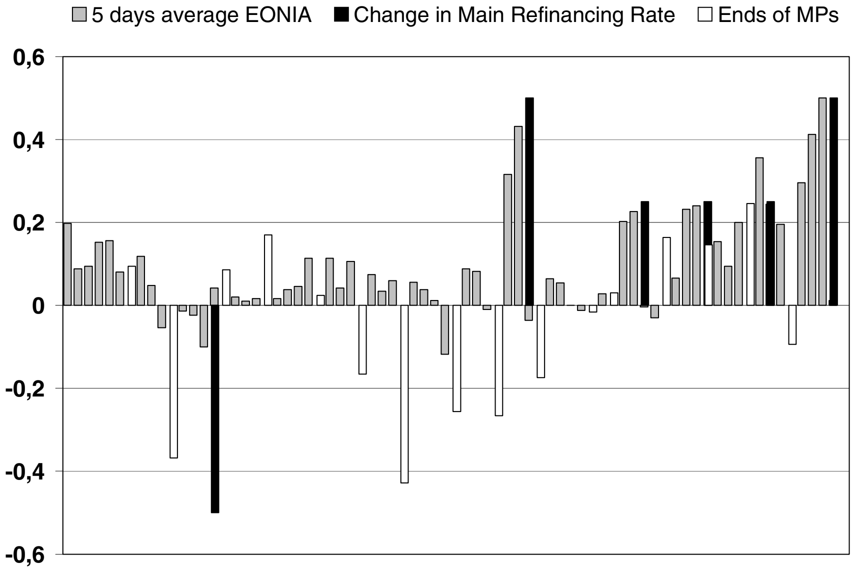

Figure 3: Average EONIA-spread and changes in the Main refinancing rate

provided was less than the neutral liquidity), both these subperiods should (according to the model described in chapter 2) carry positive spread between the tender rate and a comparable market rate of interest. As regards the second period it seems fair enough to conclude that on average the amount of liquidity provided to the market was smaller the neutral demand would have required; the natural spread between EONIA and the main refinancing rate should have been some 6.5 bps for the neutral liquidity policy not to be rejected at 10% confidence level. Furthermore, if the ends of reserve maintenance periods are omitted the natural spread should have been 11 bps for the same confidence level. Thus, we are again willing to reject the idea that the ECB used pure interest rate targeting (neutral liquidity policy rule) at least during the second half of the 18 months in question. However, these figures do not reject (with the same acceptance rules used above) the interest rate targeting hypothesis for the first part of the period. Still, the 10% confidence level would require the natural spread to be some 3 bps for the neutral liquidity policy not to be rejected, if the ends of the maintenance periods are omitted. Furthermore, the overnight spread between the two subsamples differs so much from each other, that we can with all reasonable confidence levels reject the assumption of the difference being a result of stochastic variations in liquidity. Hence, there must have been a change in the supply of liquidity relative to the demand for it. However, without analyzing the liquidity data, we can't say whether this change in the relative liquidity supply results from a change in the liquidity policy rule used by the ECB (eg from interest rate targeting into a asymmetric preferences rule à la Ayuso & Repullo) or from increased demand for liquidity under a liquidity targeting policy rule.

One thing suggesting that the liquidity targeting rule might be the correct interpretations of the reason behind the increase in the average overnight spread is, that the EONIA spread tends to grow significantly before the in-

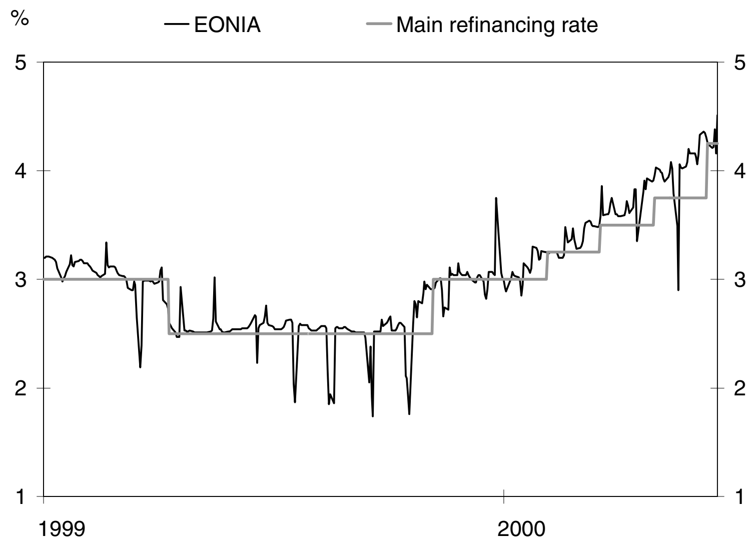

Figure 4: The EONIA and Main refinancing rates

creases in the main refinancing rate occur.[30] This feature is quite apparent if figure 4, that shows the EONIA and the main refinancing rate as levels instead of as a spread. We expect the demand for liquidity to be depending on the interest rate expectations; as indicated in chapter 2, the banks try to profit from the averaging provision by frontloading reserve holdings, when a rate increase is expected. Consequently, if the central bank doesn't increase the liquidity supply according to the increased demand (eg if the central bank has liquidity targeting policy rule in its liquidity provision) the EONIA-spread will react (by growing) to these interest rate expectations. Thus, the behaviour we have seen in the EONIA-spread could very well be a result of liquidity targeting policy rule.

We will next review the liquidity data from the period in which the ECB used fixed rate tenders.

5.1.2 On liquidity provision

Bindseil and Seitz (2001, 11) summarizes the logic of the ECB liquidity management as: "*The ECB attempts to provide liquidity through its open market operations in a way that, after taking into account the effects of autonomous liquidity factors, counterparties can fulfil their reserve requirement*". This indicates, that the ECB uses the reserve requirement as their benchmark for liquidity provision during the whole reserves maintenance period. However, this doesn't say anything on the timing of the liquidity provision. Ie this phrase doesn't necessarily mean, that the ECB attempted to hold the liquidity stable at the level of the requirement *within* the maintenance period. Furthermore,

[30] The average figure for the spread after the two operations before the interest rate increase was 32 bps, whereas it was only 3 bps after other operations. If we omit the final operation of each reserves maintenance period the corresponding averages are 32 bps and 6 bps.

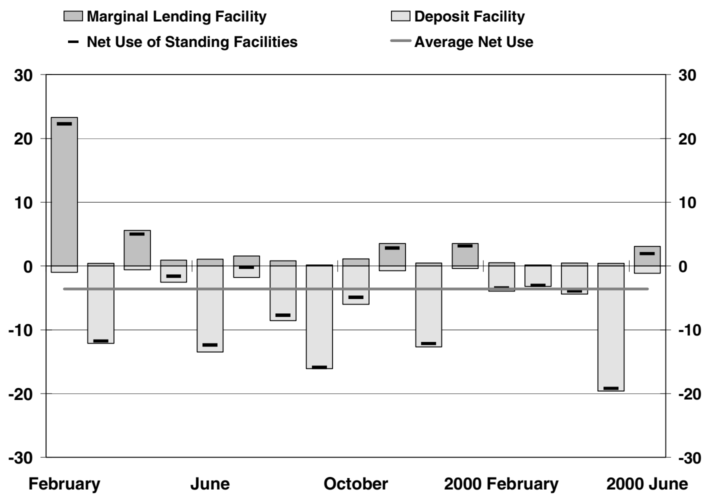

Figure 5: The use of standing facilities at the final day of each maintenance period

according to the ECB's Annual Report 1999, "*The ECB tended to orient its allotment decisions towards ensuring an average interbank overnight close to the tender rate*". Thus, we expect to find the ECB liquidity policy being aimed at controlling the price of liquidity in addition to the goal of providing the liquidity required by the reserve requirement at least during the 1999. Let us next analyze the evidence of the total liquidity provision during a reserve maintenance period. After that, we will move to analyze the timing of the reserve holdings.

The ECB seems to have provided the markets with at least fair amount of liquidity (relative to the reserve requirement) over the whole reserve maintenance period. This is illustrated in figure 5, that shows the use of standing facilities at the end of the reserves maintenance periods. The net use averaged at EUR -3.6 billions. Ie on average the amount of reserves deposited (on the final day of the reserves maintenance period) into the deposit facility (EUR 6.4 billions) was EUR 3.6 billions larger than the amount of liquidity credits acquired through the marginal lending facility (EUR 2.8 billions).This "loose" total liquidity provision is also shown in the end of period spikes of the EONIA-spread. The average spread calculated from the last banking day of each reserves maintenance period is -15 bps.

If the ECB provides the markets with liquidity that is more than enough for the banks to meet their reserve requirements, the positive average EONIA-spread (in excess of the neutral spread) must come from the banks' willingness to hold reserves earlier (during the maintenance period) than the central bank is willing to provide to them.

Figure 6 illustrates the timing of reserve holding during the reserves maintenance periods. On average, the ECB did allow for some frontloading of the reserves. Ie on average it provided the banks with more liquidity at the early days of the maintenance periods than at the later days. The level of reserve balances after the first operation (or first two operations when there was five

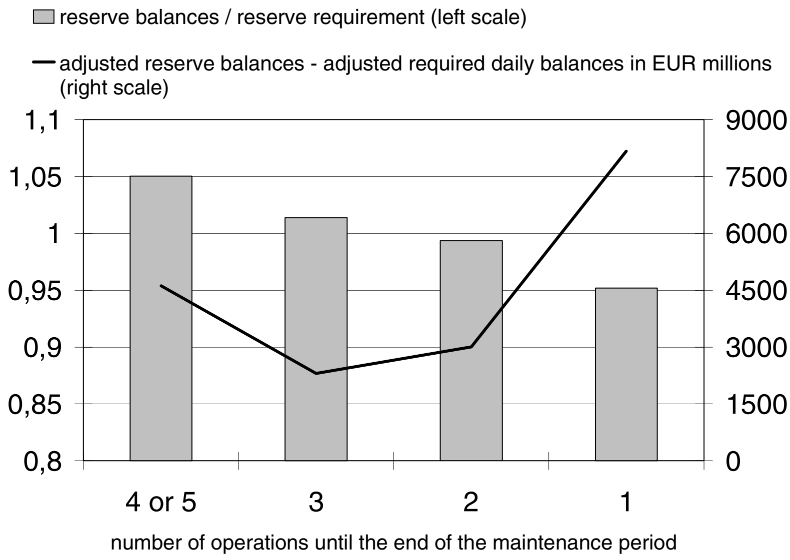

Figure 6:

operations in a maintenance period) was some 5% above the reserve requirement. After that, the amount of reserves on the market gradually declined from operation to another. This decline didn't mean that the level of reserves versus the amount needed to fulfill the requirement declined, as the need naturally declines, when there have been more reserves (than the requirement is) during the early days of the maintenance period. In figure 6 this is illustrated by the curve, that indicates the amount of reserves on the market in excess of the required daily balances.[31] We see, that the ECB did (on average) provide the markets with more reserves than needed in order to fulfill the reserve requirement in all operations during the reserves maintenance period. However, as the average EONIA-spread still was above its natural level (at least during the second half of the time period), the banks on average wanted to frontload reserve holding more than the ECB did allow for.

We will next analyze more closely the factors affecting the reserve provision of the ECB. We conducted a simple OLS regression to measure the relative importance of i) *the banks' liquidity need arising from the reserve requirement* and ii) *the banks' interest rate expectations in the ECB's decision* on amount of liquidity to be allotted. The regression equation will be of the following form:

$$\text{average liquidity supply} = b_1 RDB + b_2 spread + b_3 spread^2$$

[31] Required daily balances is the amount of liquidity that, if it was held daily (on average) until the end of the maintenance period, the reserve requirement would be just met (ie there would be no need for marginal lending or using the deposit facility). Ie

$$RDB_t = \frac{T \times RR - \sum_{j=1}^{t-1} RB_j}{T - (t-1)},$$

where T is the number of days in the maintenance period, RR is the reserve requirement and RB_j is the reserve balances held at day j.

The liquidity variable to be explained is the average amount of reserves on the five banking days following the tender operation.[32] The observations for the last operation of each maintenance period are omitted from the regression. By this construction we take into account, that there is only one weekly operation, and that the interest rate expectations affect mainly the demand for tender liquidity when there is still at least one operation remaining within the same maintenance period. The explanatory variables are the required daily balances for the remaining period (RDB) and the one week Euribor-spread (one week Euribor rate – the main refinancing rate). By the former we want to measure the demand for liquidity resulting from the reserve requirement. Thus, we expect b_1 to be close to one. The Euribor- spread is used as an indicator of the banks expectations over the average EONIA-spread until the following tender.[33] We allow the liquidity effect from the interest rate expectations to be nonlinear by adding the square of the spread into the equation. This formation should capture the possible concavity of the effect. The response of the central bank is not expected to be linear, as the effect the expectations have on the demand for reserves is expected to be nonlinear. Furthermore, we expect the effect of the interest rate expectations to be insignificant, when the central bank is applying *pure* liquidity targeting, and positive when the interest rate targeting is applied. However, it should be noted that, if the banks expect the central bank to follow pure interest rate targeting, there shouldn't be much variability in the expected value for the EONIA-spread. Furthermore, as the effect the interest rate expectations have on the demand for liquidity is monotonically increasing, the estimated effect on the supply of reserves should also be monotonically increasing (over the relevant range of the Euribor-spread), if the central bank applies pure interest rate targeting. The regression results are given in table 1.

Table 1: **Determinants of the supply of liquidity**

Dependent variable: average liquidity supply (in EUR bn)			
Variable	Coefficient	Standard deviation	t-probability
RDB	1.008	0.013	0.000
Euribor-spread	63.37	20.03	0.003
Euribor-spread^2	-185.8	49.92	0.001
Adj. R^2	0.63		
n	50		

[32] Note that this is ex post figure of the money market liquidity ($l = a^{CB} + q + \varepsilon^{CB}$). Ie this figure is not the amount of liquidity the ECB attempted to allot the markets, as it contains also the expected autonomous factors and the liquidity shocks. The reason for using this (publicly announced) ex post liquidity measure is simply, that we didn't have the figures for the desired liquidity supply nor for the liquidity shocks.

[33] We will return the question of the appropriateness of the one week Euribor rate as an indicator of the expected EONIA in the following section.

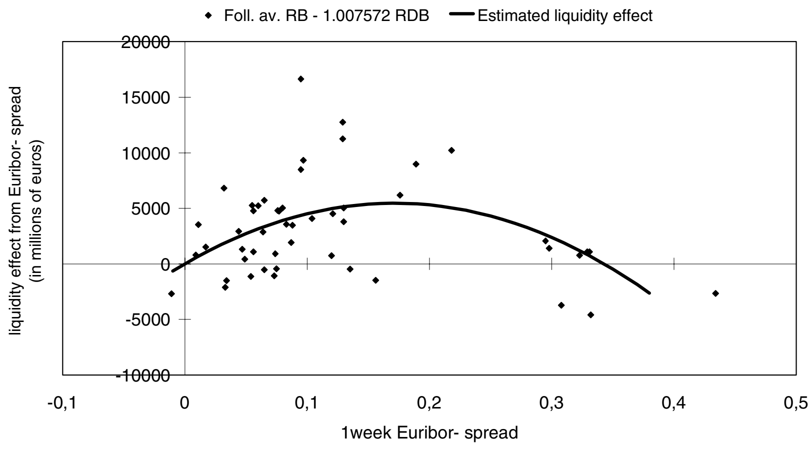

Figure 7:

The regression is based on the sample that includes 50 observations.[34] The parameter estimates are clearly statistically significant at 1% confidence level for both the required daily balances and for the two interest rate variables.

The required daily balances seems to be the starting point of the ECB liquidity provision, however, this amount is adjusted by the interest rate expectations of the banks. The estimated effect of the Euribor-spread on the liquidity provision was concave. An increase in the interest rate expectations raised the liquidity supply when the Euribor-spread was less than 17 bps. At this level the effect of the interest rate expectations reaches its peak value of EUR 5.5 billions (ie approximately 5% of the average liquidity). The effect of strong interest rate expectations on the liquidity provision vanishes when the spread reaches 35 bps. This result would suggest that the ECB didn't use pure liquidity nor pure interest rate targeting as its guiding liquidity policy rule. The regressors explain 64% of the variations in the liquidity supplied. Thus, one third of the liquidity variations result from both stochastic shocks and other variables that are not included in the regression equation.

The liquidity effect of the interest rate expectations is illustrated in figure 7, where the Euribor-spread is depicted in the horizontal axis. The diamonds in the figure are interest rate – liquidity observations, where the liquidity measure is the difference between the (average) liquidity supplied and the estimated liquidity provision stemming from the required daily balances ($1/5\sum_{1=t}^{t+5} RB_i - 1.007572 RDB_t$).

The estimated reaction curve for the ECB shows, that it did increase the supply of liquidity when the Euribor-spread was positive and below some 34 bps. Hence, we can reject the hypothesis of ECB using pure liquidity targeting policy. However, the reaction curve is far from being monotonically increasing

[34] There were 68 main refinancing operations between 24.2.1999 (the start of the first maintenance period with regular length) and 21.6.2000. 16 from which were last operations of the reserves maintenance periods. We have also excluded the one operation in which the allotment ratio aws 100%, as the ECB wasn't able to determine the allotted amount in that tender. Also the operation settled on 30.12.1999 was excluded due to the special circumstances of the change of the millennium.

over the range of the interest rate observations. Ie there seems to have been some kind of a loss function for the ECB, that included both a target for liquidity (the required daily balances) and for the interest rate (the tender rate). The ECB did smooth the interest rate deviations (from the target) by supplying extra liquidity (with regard to the liquidity target) when the EONIA was expected to be above the tender. However, with very strong interest rate expectations, the need for extra liquidity (to bring the EONIA closer to the tender rate) seems to have been so high, that the ECB reverted back to stricter liquidity orientation in its allotment policy.

The estimated concavity of the liquidity effect (from the interest rate expectations) depends largely on the observations with strong interest rate expectations (Euribor-spread at some 30 bps or more). If we excluded the (eight) observations with Euribor-spread above 25 bps from the regression, the maximum liquidity effect would raise to be some EUR 9.3 billions (by Euribor-spread of 32 bps). Furthermore, if we omit the observations with the strong interest rate expectations, a linear response to the expectations would fit the data better than the parabolic form. In this case, the estimated liquidity effect would be an increase of EUR 0.40 billions for an increase of one bps in the Euribor- spread.[35] The estimation for the eight observations with strong interest rate expectations showed, that very high Euribor-spread didn't have significant effect on the liquidity provided by the ECB.

In this section we have seen, that on average the ECB's liquidity supply over the whole duration of each reserves maintenance period has not been restrictive. Also, the liquidity supply of the ECB before the ends of the reserves maintenance periods was foremost driven by the liquidity need arising from the reserve requirement. However, the ECB seemed to allow for some frontloading of reserves when the interest rate expectations were not too strong, but when the expectations rose to very high level the ECB returned to simple liquidity targeting. Ie the ECB didn't use liquidity targeting in its purest form. It did pay some attention in keeping the EONIA close to the tender rate, but it did not allow the banks to speculate the interest rate changes by adjusting the timing of reserve holdings considerably.

Let us next turn into the banks' perception over the liquidity policy of the ECB.

5.2 Banks' perception over the ECB liquidity policy in light of the data

According to expectations hypothesis, we might use the one week Euribor rate as an indicator of the banks' expectations over the EONIA for the following week. Figure 8 illustrates the one week Euribor-spread (ie one week Euribor rate – main refinancing rate) with same settlement days the main refinancing operations have.

[35] The esimation with linear interest rate effect and exculding the observations with strong expectations yields the following result:

average liquidity supplied $= 1.012RDB + 40068\textit{Euribor-spread}.$

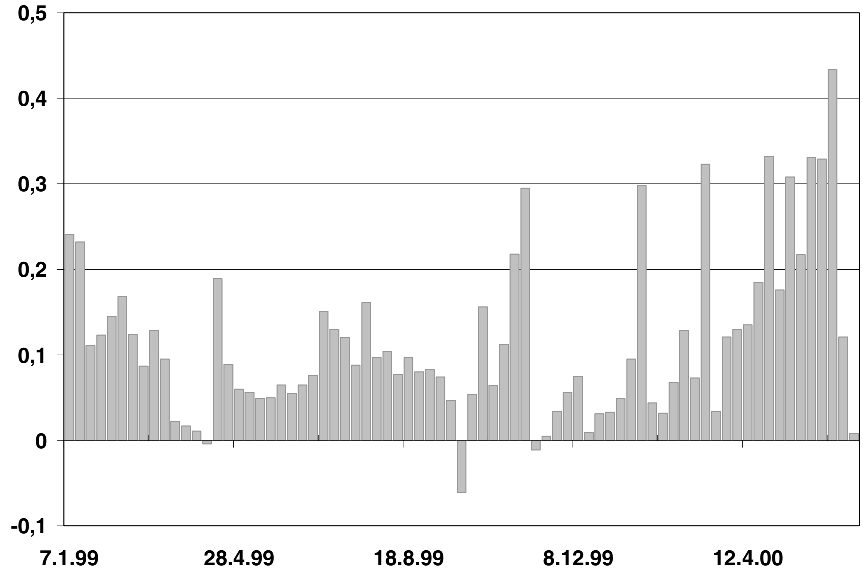

Figure 8: One week spread (one week Euribor – main refinancing rate) at days of the main refinancing operations

Figure 8 shows, that the spread was most of the times (significantly) above zero. Now, we must be careful not to compare apples with oranges while drawing conclusions about the significance of this spread. We saw earlier that there might be a natural positive spread between the EONIA and the main refinancing rate. Furthermore, there is no reason for which the spread should not be larger in case of a deposit has the maturity of one week than in case of the maturity is overnight. Thus, there can be a positive natural spread between the one week Euribor rate and the main refinancing rate. Consequently, a (small) positive average spread between the one week Euribor and the main refinancing rate doesn't necessarily need to indicate that the banks' assume the central bank to apply restrictive liquidity policy. Furthermore, this natural spread does not need to be constant over time.

When we take into account the rate changes made by the ECB, we will notice that the spread reacted to the policy rate changes in advance (see figure 9). The average Euribor-spread was 26 bps on the two tenders before each tender rate increase, whereas the average spread on other days with main refinancing operation was 9 bps. This indicates that the banks didn't expect the ECB to have been using pure interest rate targeting policy. The banks wanted to frontload their reserve holdings when an increase in the price of central bank reserves was expected, and they didn't expect the ECB to fully adjust the liquidity supply for the increased demand. Ie the banks expected the ECB to have liquidity oriented policy, that resulted in a (unusually high) positive spread between the overnight rate and the tender rate when an increase in the tender rate was expected.

Now, the bid amount is represented along with the banks interest rate expectations (one week Euribor- spread) in figure 10. This figure indicates that there is a close connection between the interest rate expectations and the bid

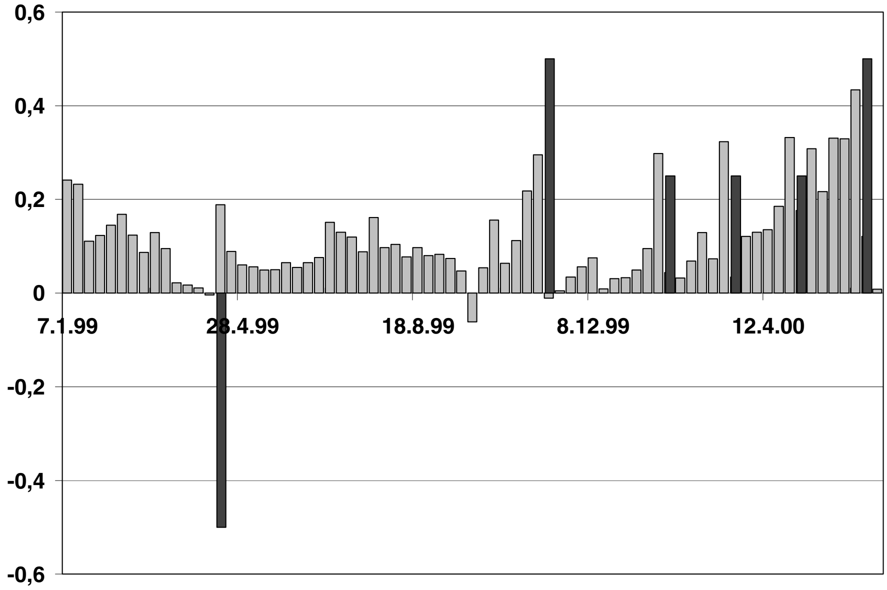

Figure 9: One week Euribor spread and the changes in the main refinancing rate

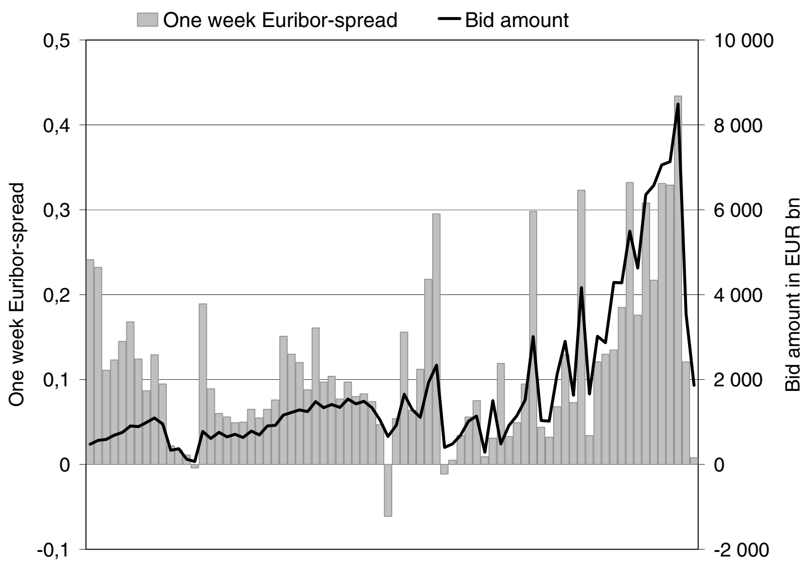

Figure 10:

behaviour – a phenomenon that we would expect to find when liquidity oriented policy is applied (or expected to be applied). Furthermore, the figure suggests, that there must have been some element that has restricted the growth rate of the bids. The figure clearly illustrates, that the bid amount isn't just a function of the interest rate spread, eg the bid amount is almost three times bigger at the tender settled at 3.11.1999 than in the one settled at 10.5.2000 even though the Euribor-spread is some 30bps during both operations. Most probably the element restricting the bid size has been the possibility of non-compliance with the tender rules, that originates from the collateral requirement, ie there seems to be an upper limit for a bank's ability to cover the allotted amount with eligible collateral. The banks seem to have been able to bid the more boldly (with a given interest rate spread) the higher has the bids been in recent operations. This is just the reaction we expect to see, if the restricting element in bidding is a limit in the possibility to borrow collateral from the market, and the banks use the past bid sizes as a benchmark when they from their expectations over the bid behaviour in the current tender.

We analyzed the bidding strategy of the banks when (according to the model built in chapter 2) the optimal bid is the maximum bid by explaining the aggregate bids at t with the average of the bid ratios applied in the four most recent tenders, the one week Euribor-spread and a trend component. According to the model, the banks should bid their neutral demand when the expected spread is negative. Consequently, we excluded the three observations with negative interest rate spread from the sample.

Now, in accordance with section 4, the optimal allotment for a bank depends positively on the expected spread between the market rate and the tender rate. As the actual allotment to a bank is the bid it places in the tender multiplied with the allotment ratio, the optimal bid (without uncertainty) would be the bid ratio times the optimal allotment. However, while preparing their bids, the banks are unaware of the bids of the other banks (as well as of the amount of liquidity to be allotted), thus, bid size is expected to grow with the product of the expected bid ratio and the expected interest rate spread. Its almost impossible to measure the banks' subjective probability density function over the expected bid ratio. Thus, we simply used the average bid ratio from the four previous tenders as an indicator of the expectation over the coming bid ratio.

Furthermore, we don't expect the interest rate spread and the expected allotment to be independent. When the interest rate spread increases, the rest of the banks are likely to increase their bid from the past, which bank i should take into account while preparing its bid. Thus, we included the product of the interest rate spread and the past average of the bid ratio into the set of explanators. Now, the functional form of this product term doesn't have to be linear. To capture the potential form of the non-linearity, we used figure 11, that is a scatter plot with the aggregate bid amount on the vertical axis and the product of the average past bid ratio and the interest rate spread on the horizontal axis. This figure suggested, that the effect of the product of the expected bid ratio and the interest rate spread is of second order. This indicated us that besides the direct product term we should include its square to capture the non-linearities of the term's effect on bid behavior. However,

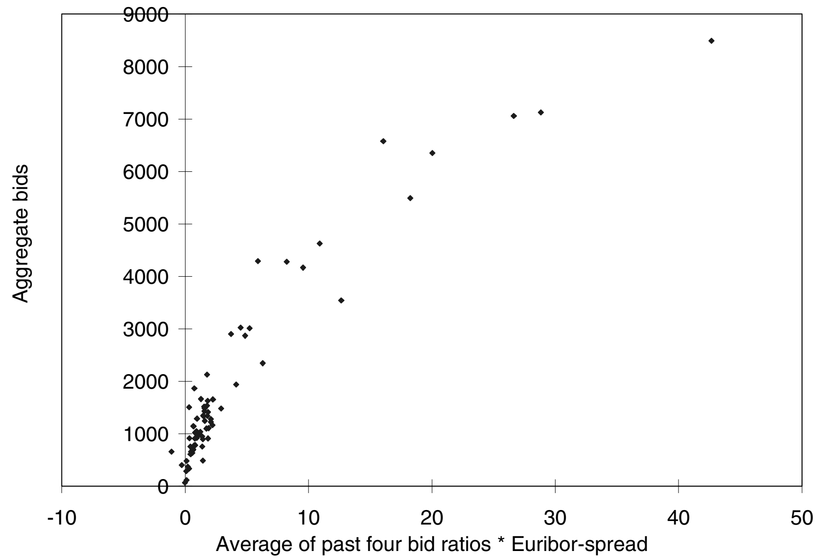

Figure 11:

the effect of the product term doesn't need to be independent of the level of the individual factors within the term. Thus, we included into the estimation equation *both* the interaction term the product term and the interest rate spread *and* the interaction of the product term and the past bid ratio. Finally, we also introduced a trend to capture both the potential effect of the banks expecting the bids to steadily increase in time and/or to allow the limits for borrowing collateral to increase in time.

The estimated OLS-regression took the following form[36]:

$$b_t = \beta_1 t + (\beta_2 + \beta_3 p_t + \beta_4 w_t + \beta_5 p_t w_t)\, p_t w_t,$$

where b_t is the aggregate bid of the banks at t, p is the average of four previous bid ratios (ie $p = \frac{\sum_{i=t-4}^{t-1}\left(\frac{b_i}{c_i}\right)}{4}$), and w is the one week Euribor-spread (ie $w = r^{one\ week\ Euribor} - r^T$). The estimation result are given in table 2.

Table 2.

Dependent variable: *Bids* [1]			
Variable	Coefficient	Std. Error[2]	t-probability
Trend	12.80	2.370	0.000
pw	787.1	89.93	0.000
pw^2	-1 308	274.6	0.000
p^2w	-5.012	0.8528	0.000
$(pw)^2$	10.54	2.557	0.000
adj R^2	0.974	n	69
DW	2.16		

[1] In billions of Euros
[2] White heteroskedasticity-consistent standard errors

[36]Note that besides this formulation, we estimated similar equation that contained in its set of explanators the direct interest rate spread (w) and the past average bid ratio (p), however, neither of the parameter estimates received a statistically signifficant value.

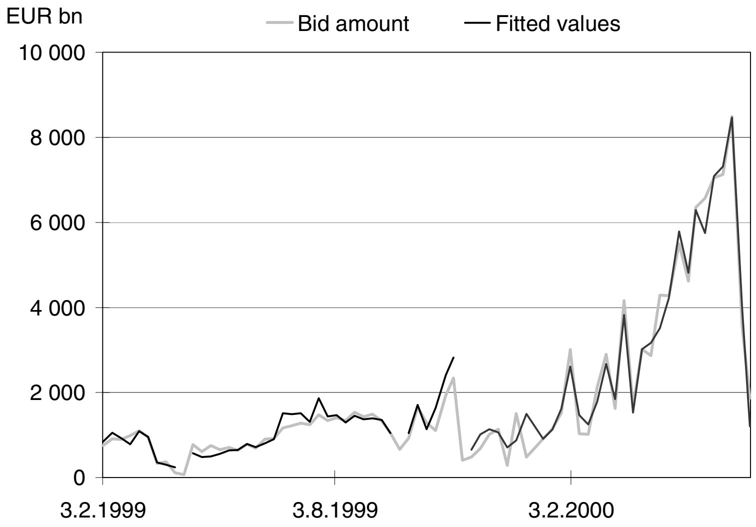

Figure 12:

All parameter estimates are highly significant in the regression. The variables also seem to explain fairly well the variations in the bid amount. The accuracy of the estimation results could be evaluated by figures 12 and 13, that present two illustrations of the realized bid amount and the fitted values.

According to the estimation, there was a positive trend in the evolution of the bids. The aggregate bid size tended to grow by some EUR 13 billion from tender to tender. This trend growth might result from the banks increased capacity of borrowing collateral. However, it could also result from the method the banks use in forming their expectations over the coming bid ratio (ie the banks might have expected the bid ratio just to increase slightly from tender to tender).

The estimation also shows that the product of average recent bid ratios and the expected interest rate spread has a very significant direct impact on the bid amount. However, this effect depends on its components p and w. The negative parameter estimates for pw^2, and p^2w could be interpreted as resulting from the growth of uncertainty about the coming bid ratio associated with wider interest rate spread and higher values of the past bid ratios. Figure 14 illustrates bid amount (in excess of the trend value) as a function of the expected interest rate spread for different values of past average bid ratios. The picture shows how much (according to the parameter estimates) the aggregate bid of the banks would have been above the trend bid, when the past bid ratio took the value of 5, 10 or 20. We can see that eg when the expected interest rate spread was doubled from 10 to 20 bps, the bid amount increased from 617 to 993 (ie some 60%). Also, the interest spread at 10 bps would (with these parameters) have lead to a bid of EUR 1 154 billion above the trend value when the past average bid ratio was 20 (instead of EUR 617 bn when the past bid ratio was 10).

To get some intuition how, the bid ratio evolves with this kind of parameters, figure 15 shows the interest rate spread that leads into bid ratio equal

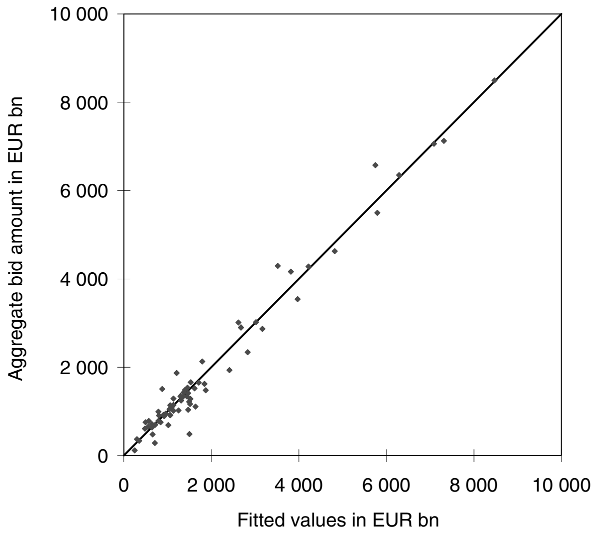

Figure 13:

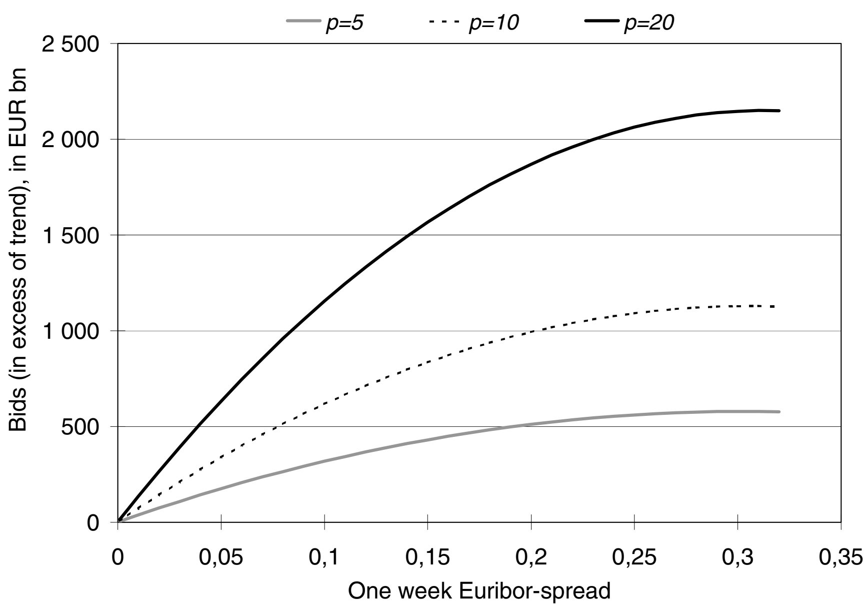

Figure 14:

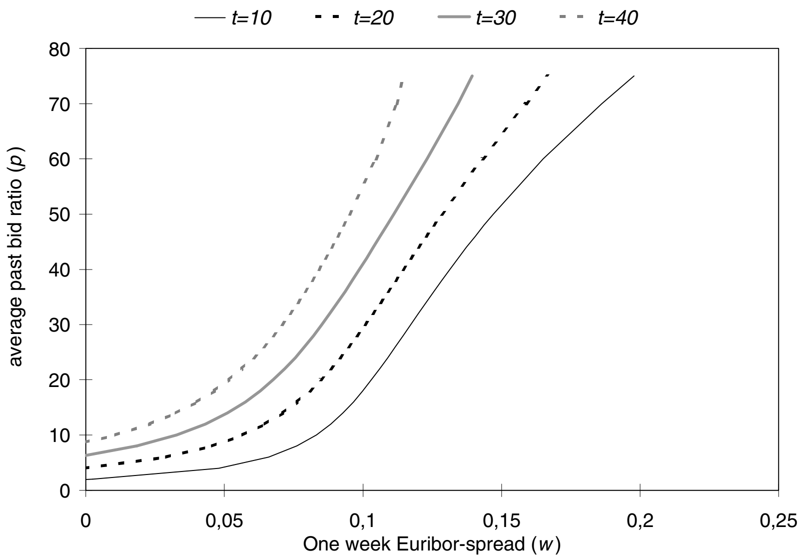

Figure 15:

to the past average ratio, when the allotted amount equals its average value (ie EUR 68 billion). This, spread depends on the trend value in the bidding, thus, we have calculated the equilibrium paths for four different points in time (namely, $t = 10$, 20, 30 and 40). The figure shows that the higher the past bid ratio was the higher the interest rate spread needed to be for the bid ratio to remain at the level of the past average. Eg with $t = 30$ the interest rate spread would have needed to be some 6.8 bps for $b_t/c_t = p_t = 20$, and 8.5 bps for $b_t/c_t = p_t = 30$. Furthermore, when the trend component in bidding increases as time goes by, the interest rate spread needed for $b_t/c_t = p_t$ (with given p_t) diminished. Eg the spread needed for $b_{20}/68 = p_{20} = 30$ was 10.1 bps, while it's only 8.5 bps for $b_{30}/68 = p_{30} = 30$. Note however, that when the time passes, the effect of the trend component into the bid ratio increases. Ie eg the effect of the trend on the bid is 128 at $t = 10$, while it's 384 at $t = 30$. Thus, with $c = 68$ the effect on the bid ratio is 1.9 and 5.6 at $t = 10$ and $t = 30$.

Finally, in order to analyze how well the banks' estimated the ECB's liquidity policy, figure 16 illustrates the difference between the one week Euribor rate with settlement at the settlement days of the tenders and the average of the EONIA-rates from the date until the next settlement day.[37] The solid lines in the figure illustrate the mean spreads for two sub periods. The break point dividing the total sample is at 20.10.1999, which is two operations before the first tender rate increase made by the ECB. The dashed lines give the two standard deviations bands for variations in the spread. During the first sub period, the spread between the one week Euribor-spread and the average of the following EONIAs was statistically significantly above zero, whereas we can't reject the null hypothesis for the spread being zero for the second subperiod. Furthermore, the difference between the two mean values is statistically

[37] Eg for the tender operation settled at 3.3.1999 we use the one week Euribor-rate quoted on 1.3.1999 and the five EONIAs between 3.3.–9.3.1999 such that the Friday quotation is weighted by three due it's effective throughout the weekend.

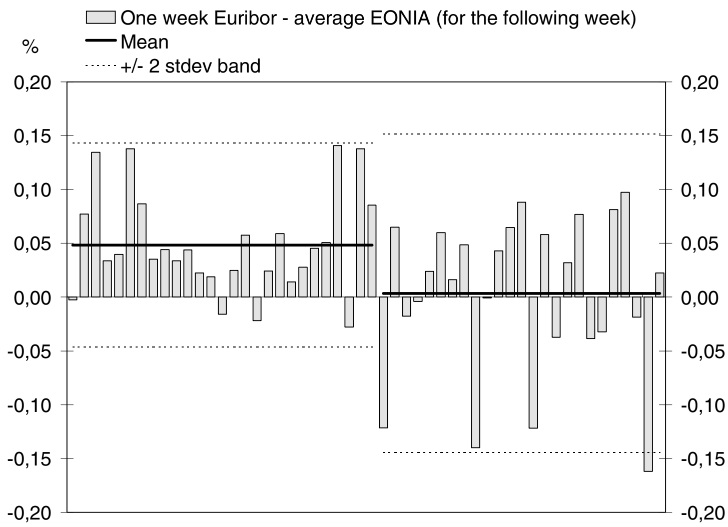

Figure 16:

significant at 5% level. If the neutral spread between these two rates were stable throughout the total sample period, it would seem that the liquidity policy of the ECB was not as tight as the banks expected it to be during the first subperiod, while the banks seem not to have such bias during the second subperiod. The bias might have disappeared due to tighter liquidity policy of the ECB under the interest rate hike expectations (ie during the second subperiod) than under the neutral expectations. Another possible explanation might be found from the banks' learning process over the liquidity policy of the ECB.

6 Summary and conclusions

In this paper we have constructed a model that describes the optimal bid of a single bank in money market tenders under various liquidity policies applied by the central bank. We saw that the bid amount depends crucially on the relation between central bank's liquidity target and the neutral liquidity of the banks. With neutral money market liquidity, the private value of the liquidity for the banks equals the tender rate. When the amount of liquidity targeted by the central bank is above the neutral liquidity level, the banks will place such bids that with them the money market liquidity will be neutral. If the target liquidity is at or below the neutral level, the banks will overbid. Ie they will bid in excess of the neutral liquidity.

In chapter 3 we introduced four potential liquidity policies for the central bank; full allotment, neutral liquidity policy rule, restricted liquidity supply and liquidity targeting rule. The banks will bid for neutral liquidity, if the central bank applies full allotment or if it uses liquidity targeting (ie the central bank aims at stable liquidity conditions on the money market) and the banks expect a interest rate cut in the near future. Overbidding will occur under interest rate targeting (at least when the target rate is not below the tender rate), restricted liquidity supply (eg due to Ayuso & Repullo (2000) kind of asymmetric central bank preferences) or under liquidity targeting when the central bank is expected not cut its rates in the near future.

In chapter 4 we saw, that when the liquidity allotted by the central bank in the tender needs to be covered with collateral, the amount of overbidding will be a function of the interest rate spread between the expected market rate of interest and the tender rate. Thus, the bid ratio (ie the aggregate bids / allotted amount) should behave differently under various liquidity policy rules, as the expected market rate of interest depends on the allotment decision rule applied by the central bank. With full allotment or when liquidity targeting policy is applied and the banks expect the tender rate to be cut, the expected market rate will be at the level of the tender rate and the central bank won't be rationing the allotted amount. Thus, under these conditions we don't expect to see overbidding by the banks. However, under neutral liquidity policy the bid amount will depend on the collateral borrowing capacity of the banks, even though the expected market rate of interest will equal the tender rate also in this case. Under restricted liquidity supply the extent to which the banks will overbid, depends on the restriction rule of the central bank. Eg if the limited liquidity supply is based on the preference asymmetry, the bid amount should reflect the effect of the asymmetry on the expected spread between the market rate and the tender rate. Finally, with liquidity oriented allotment policy the expected market rate will be a function of the expected future market rate, and for this reason the amount of bids in excess the neutral amount will also be positively correlated with the interest rate expectations.

Chapter 5 studied the liquidity policy of the ECB and the bidding of the banks against the model derived in the preceding chapters. We showed, that overall the liquidity provision of the ECB couldn't be considered as restricted. On average the ECB did provide the markets with liquidity that was quite loose compared to the reserve need based on the reserve requirement. Thus, we are not convinced by the argument of the ECB having had asymmetric preferences over the sign of interest rate differences between the market rate of interest and the tender rate. However, there still seems to have been significant positive spread between the market rate and the main refinancing rate, especially in the tenders preceding the tender rate increases by the ECB. Consequently, even though the overall liquidity policy of the ECB wasn't restrictive, the timing of the liquidity provision seems not to have met the demand of the banks. Furthermore, we saw that the reaction of the ECB to the interest rate expectations of the banks was not unambiguous. The ECB increased its allotment from the level indicated by the reserve requirements when there was moderate expectations of tighter future interest rate policy. However, when the expectations were considerably large (ie when the spread between the one week Euribor rate and the main refinancing rate was above 25 bps), the ECB seemed to have reverted to tighter control of liquidity (ie the allotted amount seems to have been based solely on the reserve requirements). This indicates that the liquidity policy applied by the ECB didn't fall under pure interest rate targeting or pure liquidity targeting, but it was something in between them. Ie the ECB put weight for both holding the market rate close to the main refinancing rate and trying to maintain the liquidity as stable. When the interest rate expectations became strong, the increase in the neutral amount of liquidity seems to have been so large from the view point of holding liquidity stable, that in such cases the ECB reverted to pure liquidity targeting policy. However, as the cases of strong interest rate increase expectations occurred all during the second half of the period, we couldn't rule out the possibility of the ECB having applied liquidity policy based on interest rate targeting until the fall of 1999.

The aggregate bid of the banks increased considerably during the period the ECB applied fixed rate tenders. This was seen to result from two factors. First, during the period from the beginning January 1999 until September 1999, the environment was characterized by neutral or falling interest rate expectations, whereas during the period from October 1999 until the change of the tender procedure in June 2000, the interest rate expectations were either neutral or an increase in the tender rate was expected. These expectations (of a rate hike) were reflected in the spread between one week Euribor-rate and the tender rate. Ie the banks didn't assume the ECB to adjust its liquidity supply (fully) to the increase in demand for liquidity during a rate hike expectations. Because the amount of liquidity the banks are willing to receive from the tender is the larger the wider is the spread between the market rate and the tender rate, each bank was willing to receive bigger share of the total allotment in many tenders during the second half of the period the with fixed rate tenders than during the first half of it. Secondly, to get a certain allotment from a tender a bank must place a bid that is the amount the bank is willing to receive times the bid ratio to be used in the tender. The expectation over the coming bid ratio in a tender was seen to depend positively on the bid ratios of the recent tenders. Thus, the aggregate bid at a given expected interest rate spread was considerably larger during the latter half of the period. However, the bid amount was seen to grow already during the first half of 1999. According to our model, this indicates the banks expecting a restricted liquidity supply during the period, when the ECB wasn't expected to raise its rates. This could mean either that for some reason the banks prefer frontloading in the reserve holding to the stable liquidity or that the banks did assumed the liquidity policy of the ECB to have been more restricted than it really was at the beginning of the Stage Three of the EMU.

Finally, the discrepancy that lies in simultaneously targeting the level of the market rate of interest and trying to hold the liquidity stable within the reserves maintenance period leads to ever increasing bid ratios during the expectations of a rate hike. The remarkable increase in the bid ratios (decline in the allotment ratios) that was seen to occur between October 1999 and June 2000, led the ECB to change the tender procedure into variable rate tenders. With variable rate tenders the expectations of a rate hike will be immediately reflected in the tender rate. Thus, the banks' incentive to overbid in the operations is diminished. According to the model of the paper alternative methods for the ECB to overcome the declining allotment ratios would have been to give up the aim of stabilizing liquidity holding within a reserves maintenance period. This could have been done either by applying the full allotment procedure or by moving to interest rate targeting in a stricter from.

References

[1] Ayuso, J. – Repullo, R. (2000) **A Model of the Open Market Operations of the European Central Bank**. *Banco de España – Servicio de Estudidios, Documente de Trabajo no 0014.*

[2] Bindseil, U. – Seitz, F. (2001) **Supply and Demand for Eurosystem Deposits the first 18 months.** European Central Bank Working Paper series 44.

[3] Ehrhart, K-M. (2000) **Repo Games: Experimental Investigation of the Fixed Rate Tender.** *An unpublished working paper presented in the ZEI Conference: Central Bank Operations: Theory and Practice in Frankfurt on 22–23 September 2000.*

[4] European Central Bank (2001) **The Monetary Policy of the ECB.** ECB 2001.

[5] European Central Bank (2000a) **Annual Report.** *Frankfurt am Main.*

[6] European Central Bank (2000b) **Monetary Policy Decisions.** ECB Press Release 8 June 2000.

[7] European Central Bank (2000c) **The switch to variable rate tenders in the main refinancing operations.** in *Monthly Bulletin*, July 2000 *Frankfurt am Main.*

[8] European Central Bank (2000d) **The Single Monetary Policy in Stage Three; General Documentation on ESCB Monetary Policy Instruments and Procedures.** *Frankfurt am Main* September 1998.

[9] Federal Reserve Bank of New York (2000) **Open Market Operations During 1999.** *Federal Reserve Bank of New York, Markets Group*, February 2000.

[10] Nautz, D. – Oechssler, J. (2000) **The Repo Auctions of the European Central Bank and the Vanishing Quota Puzzle.** *Humbold-University working paper No. 79, October 1999.*

[11] Vergara, P. (2000) **The performance of the Operational Framework of the Eurosystem since the Introduction of the Euro: A Market Appraisal.** *A conference paper prepared for the ECB Conference on "The Operational Framework of the Eurosystem and Financial markets", in Frankfurt 5.–6. May 2000.*

[12] Välimäki, T. (2001) **Fixed rate tenders and the overnight money market equilibrium**. *Bank of Finland Discussion Papers 8/01.*

A Full allotment with collateralization

The problem of the risk neutral bank i at the collateralized interbank market is:

$$\max_{s_i} \Pi = \int_{a_i+\mu/n+\xi_i+b_i}^{a_i+\mu/n+\xi_i+b_i+s_i} r_i^{pv}(x)\,dx - s_i r - \int_{b_i}^{b_i+s_i} h_i(x)\,dx, \tag{24}$$

The FOC for the problem is:

$$\begin{aligned} & r_i^{pv}(a_i+\mu/n+\xi_i+b_i+s_i^*) - r - h_i(b_i+s_i^*) = 0 \\ \Rightarrow\ & r_i^{pv}(a_i+\mu/n+\xi_i+b_i+s_i^*) = r + h_i(b_i+s_i^*). \end{aligned} \tag{25}$$

Ie the private value of liquidity after optimal interbank borrowing equals the sum of (collateralized) market rate of interest and the marginal cost of collateral.

Now, equation (25) can be rewritten as:

$$s_i^* = r_i^{pv^{-1}}(r + h_i(b_i+s_i^*)) - (a_i+\mu/n+\xi_i+b_i). \tag{26}$$

Aggregating over the whole banking sector will give the following equation:

$$\sum_{i=1}^{n} r_i^{pv^{-1}}(r + h_i(b_i+s_i^*)) = a+\mu+b, \tag{27}$$

from which we can derive the sum of market rate of interest and the marginal cost of collateral as:

$$r + h_i(b_i+s_i^*) = r_i^{pv}\left(\frac{a+\mu+b}{n}\right). \tag{28}$$

Now, substituting equation (28) back into equation (26) gives us:

$$s_i^* = \frac{a+b}{n} - (a_i+\xi_i+b_i),$$

that is identical to equation (8) in section 2.

The maximization problem of the bank at the tender becomes:

$$\begin{aligned} \max_{b_i} \mathrm{E}[\Pi_i] = & \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \Big\{ s_i|_{b_i=0}[r(b_{-i},\mu)] - s_i^*[r(b,\mu)] \\ & + \int_{a_i+\mu/n+\xi_i+s_i^*|_{b_i=0}}^{a_i+\mu/n+\xi_i+b_i+s_i^*} r_i^{pv}(x)\,dx \Big\} f(\xi_i,\mu)\,d\xi_i d\mu \\ & - \int_{s_i|_{b_i=0}}^{b_i+s_i^*} h(x)\,dx - b_i r^T \end{aligned} \tag{29}$$

$$\begin{aligned} \text{s.t. } & r_i^{pv}(a_i+\mu/n+\xi_i+b_i+s_i^*) \\ & -r - h(b_i+s_i^*) = 0 \text{ and. } b_i \geq 0 \end{aligned} \tag{30}$$

from which we can derive the following Lagrangian:

$$
\begin{aligned}
L &= \int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ s_i|_{b_i=0} \left[r\left(b_{-i}, \mu\right)\right] - s_i^* \left[r\left(b, \mu\right)\right] \right. \\
&\left. + \int_{a_i+\varepsilon_i+s_i^*|_{b_i=0}}^{a_i+\varepsilon_i+b_i+s_i^*} r_i^{pv}\left(x\right) dx \right\} f\left(\xi_i, \mu\right) d\xi_i d\mu - \int_{v_i|_{b_i=0}}^{b_i+v_i^*} h\left(x\right) dx - b_i r^T \\
&-\lambda \left[r_i^{pv}\left(a_i + \mu/n + \xi_i + b_i + s_i^*\right) - r - h\left(b_i + s_i^*\right)\right] - \nu b_i.
\end{aligned}
$$

Now, the FOCs for the maximization problem are:

$$
\begin{aligned}
&\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} \left\{ -s_i^* \frac{\partial r\left(b, \mu\right)}{\partial b_i} - r\left(b, \mu\right) \frac{\partial s_i^*}{\partial b_i} \right. \\
&\left. + r_i^{pv}\left(r_i^{pv^{-1}} \left[r\left(b, \mu\right) + h\left(b_i + s_i^*\right)\right]\right) \left(1 + \frac{\partial s_i^*}{\partial b_i}\right) \right\} f\left(\xi_i, \mu\right) d\xi_i d\mu \quad (31) \\
&-h\left(b_i + s_i^*\right)\left(1 + \frac{\partial s_i^*}{\partial b_i}\right) - r^T = 0,
\end{aligned}
$$

$$r_i^{pv}\left(a_i + \mu/n + \xi_i + b_i + s_i^*\right) - r - h\left(b_i + s_i^*\right) = 0 \text{ and} \quad (32)$$

$$b_i \geq 0, \quad (33)$$

that can also be represented as:

$$\int_{\mu^{\min}}^{\mu^{\max}} \int_{\xi_i^{\min}}^{\xi_i^{\max}} s_i^* \frac{\partial r\left(b, \mu\right)}{\partial b_i} f\left(\xi_i, \mu\right) d\xi_i d\mu = \mathrm{E}\left[r\left(b, \mu\right)\right] - r^T, \quad (34)$$

$$r_i^{pv}\left(a_i + \mu/n + \xi_i + b_i^* + s_i^*\right) - r - h\left(b_i^* + s_i^*\right) = 0 \text{ and} \quad (35)$$

$$b_i \geq 0, \quad (36)$$

where equation (34) is similar to equation (15).

Now, we have seen that at the equilibrium all banks will be bidding for neutral liquidity under full allotment even if we introduce collateral cost into the model. However, the neutral amount of liquidity (the amount that takes the market rate of interest to the level of the tender rate) is lower in case the borrowing is costly due to collateral requirements. This is obvious as if there are no collateral costs $r_i^{pv}\left(q_i^{T,\text{ no coll. req.}}\right) = r^T$, where as under costly collateral requirement we'll have $r_i^{pv}\left(q_i^{T,\text{ costly coll.}}\right) - h_i\left(q_i^{T,\text{ costly coll.}} + s_i^*\right) = r^T$, thus, the neutral liquidity is decreases due to the cost of collateral ($q_i^{T,\text{ no coll. req.}} > q_i^{T,\text{ costly coll.}}$).

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