Dynamics of inflation expectations in the euro area

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Maritta Paloviita
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Abstract

The present study examines empirically the inflation dynamics of the euro area. The focus of the analysis is on the role of expectations in the inflation process. In six articles we relax rationality assumption and proxy expectations directly using OECD forecasts or Consensus Economics survey data. In the first four articles we estimate alternative Phillips curve specifications and find evidence that inflation cannot instantaneously adjust to changes in expectations. A possible departure of expectations from rationality seems not to be powerful enough to totally explain the persistence of euro area inflation in the New Keynesian framework. When expectations are measured directly, the purely forward-looking New Keynesian Phillips curve is outperformed by the hybrid Phillips curve with an additional lagged inflation term and the New Classical Phillips curve with a lagged expectations term. The results suggest that the euro area inflation process has become more forward-looking in the recent years of low and stable inflation. Moreover, in low inflation countries, the inflation dynamics have been more forward-looking already since the late 1970s. We find evidence of substantial heterogeneity of inflation dynamics across the euro area countries. Real time data analysis suggests that in the euro area real time information matters most in the expectations term in the Phillips curve and that the balance of expectations formation is more forward- than backward-looking. Vector autoregressive (VAR) models of actual inflation, inflation expectations and the output gap are estimated in the last two articles. The VAR analysis indicates that inflation expectations, which are relatively persistent, have a significant effect on output. However, expectations seem to react to changes in both output and actual inflation, especially in the medium term. Overall, this study suggests that expectations play a central role in inflation dynamics, which should be taken into account in conducting monetary policy.

Keywords: inflation expectations, Phillips curve, vector autoregressive models, euro area

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1 Introduction

Inflation dynamics have continuously been one of the most debated issues in macroeconomics. Among other things, the puzzling combination of low inflation and high real growth rates in many industrialised countries in the 1990s has underlined the importance of examining price changes. Moreover, establishment of the European Central Bank, with the explicit mission of price stability, has highlighted the importance of understanding the determinants and dynamics of inflation. During the last several decades, price developments have changed a great deal in the euro area and inflation histories have been quite divergent across the EMU countries. This has underscored the need to understand the inflation process. Recent methodological advances have also increased the level of interest in studying the nature of inflation dynamics, which are crucial for designing optimal monetary policy.

Reducing inflation seems to be costly. Changes in monetary policy affect employment and production in the short run; many countries have over the years experienced disinflation along with output and employment losses. For example, in the 1970s and 1980s, a tightening of US monetary policy was typically followed by a period of substantial decline in production and employment (Romer and Romer 1989). Moreover, since the 1960s, moderate-inflation OECD countries have experienced a substantial decline in production together with disinflation (Ball 1994).

Many empirical studies find evidence that the inflation process is persistent, i.e., that inflation is strongly correlated with its own lagged values (see e.g., Gordon 1997, Angeloni et al. 2006). Inflation persistence has received much attention in recent monetary policy research. It is still a mystery, whether inflation persistence is structural, which would mean that its sluggishness is due to deep parameters or institutional constraints (such as indexation) in the economy. On the other hand, inflation may be persistent because of more transient factors such as expectations or policy regimes. A deep understanding of the patterns and determinants of inflation persistence is important for the conduct of monetary policy. For example, the appropriate response of monetary policy to shocks depends on the degree to which the inflationary effects of shocks are persistent. Inflation expectations are important determinants of inflation persistence. If inflation expectations are successfully anchored, inflation expectations are less dependent on past inflation and actual
inflation is less persistent. By anchoring inflation expectations, the central bank can ensure that actual inflation does not persistently deviate from the central bank’s medium-term inflation target.

Structural models can be used in empirical analysis of inflation dynamics. With these models, one can explore the short-run inflation process and how changes in monetary policy regime and other structural changes affect price developments. Since the late 1950s research on inflation dynamics has been largely based on the Phillips curve, the economic modelling of which has changed considerably over the years. Originally Phillips (1958) and Samuelson and Solow (1960) hypothesised a stable negative relationship between unemployment and inflation without paying special attention to the role of expectations. About ten years later, Phelps (1967) and Friedman (1968) developed the expectations-augmented Phillips curve, which ascribed a central role to expectations in the inflation process, via wage bargaining and price setting. In the 1970s, when rational expectations was a major theme of macroeconomic research, Lucas (1976) presented the rational expectations hypothesis, which holds that inflation expectations cannot systematically differ from actual inflation. In later empirical work on the expectations-augmented Phillips curve – nowadays often called the New Classical Phillips curve – rational expectations are typically assumed and real economic activity is measured by actual output relative to potential, ie by the output gap. Other measures have also been used such as the capacity utilisation rate and the unemployment gap, which is the difference between the actual unemployment rate and the non-accelerating inflation rate of unemployment (NAIRU). Alternative specifications of the New Classical Phillips curve may include additional lags of excess demand. In applied work, oil price and other additional variables have often been incorporated into the model in order to capture the supply shocks of the 1990s.

Recent advances in theoretical modelling of inflation dynamics have led to dynamic equilibrium models of price adjustment, which are based on optimisation of a representative agent, differentiated goods and monopolistic competition. As regards price setting, these models assume state-contingent or time-contingent rules. Under state-contingent rules, the timing of price changes is endogenous, as it is assumed that changes in economic conditions affect firms’ incentives to adjust prices. Firms are constantly monitoring cost conditions and are able to adjust their prices in every period if this is deemed profitable, given the costs of adjustment (see Caplin and Spulber 1987, Caplin and Leahy 1991, Dotsey et al 1999).
alternatively assume that costs of price changes are due to costs of collecting information and making decisions.

When a firm follows time-contingent rules in its pricing behaviour, it resets prices according to a timetable. Scheduled price changes are often seen as reasonable for low inflation economies, where price changes are actually quite rare. The New Keynesian Phillips curve is based on the assumption of time-contingent price setting. It can be derived using Taylor’s overlapping contracts model (Taylor 1980), Rotemberg’s model of quadratic costs of price adjustment (Rotemberg 1982) or the Calvo (1983) model with random price adjustment. In Taylor’s staggered wage model, only a fraction of wages are reset in each period, and contract wages are assumed to be fixed for a certain number of periods. In Rotemberg’s model it is costly to adjust prices, which means that firms do not change prices in every period. Typically, in that model, prices change frequently and in small amounts. Costs of changing prices, even if small for individual price setters, can cause stickiness in the aggregate price level. In the Calvo model, firms care about relative prices, since price setting is staggered. Even if individual prices are adjusted frequently, the aggregate price level may change slowly because of staggering. All these models yield the same reduced-form equation, which relates current inflation to currently expected future inflation and the current driving variable.

The New Keynesian Phillips curve is purely forward-looking and based explicitly on microfoundations. It suggests that prices are sticky and that inflation depends entirely on current and expected future economic conditions. The New Keynesian Phillips curve relationship is appealing, since it has features of both the ‘old’ Keynesian models and dynamic equilibrium models. It assumes, in the Keynesian tradition, that competition is imperfect and that there are nominal rigidities in price adjustment. In the New Keynesian theory, excess demand enters through real marginal costs, the empirical measure of which is often the output gap or real unit labour cost (labour income share). In empirical studies the New Keynesian Phillips curve relationship and rational expectations are typically jointly assumed.

Many studies suggest that when rational expectations are imposed, the empirical performance of the New Keynesian Phillips curve is poor and, contrary to the theory, the estimated coefficient of the driving variable is insignificant or incorrectly signed. Moreover, a drawback of the purely forward-looking New Keynesian model is its inability to explain inflation persistence. Some studies find evidence that the observed stickiness in the inflation process cannot be
explained by the New Keynesian Phillips curve with a reasonable length of wage contracts (Fuhrer and Moore 1995). In order to better capture the observed inflation dynamics, the New Keynesian Phillips curve has been typically modified in one of two ways: by experimenting with alternative empirical measures of real marginal costs or by incorporating some backward-lookingness in the price setting behaviour.

Empirical studies of the New Keynesian Phillips curve have not so far been able to firmly establish which of the two alternatives is a more appropriate measure of real marginal costs. Several studies suggest that, under rational expectations, real unit labour cost is a better proxy for real marginal cost than is the output gap. The superiority of real unit labour cost is based on the idea that real marginal costs and output gap are not closely related and that labour market rigidities must be taken into account in modelling short-run inflation dynamics. On the other hand, it has been argued that labour market rigidities are not important in inflation dynamics and that the output gap can be used with the New Keynesian Phillips curve, if it is measured correctly. Thus, the choice of empirical proxy for real marginal cost in the New Keynesian Phillips curve relationship is not unambiguous.

The hybrid specification of the New Keynesian Phillips curve includes elements of both forward-looking and backward-looking price setting, since it has the lagged inflation term as an additional explanatory variable. The Hybrid Phillips curve has been widely used in recent studies of inflation dynamics. The basic idea in the Hybrid model is that only some price setters behave optimally when adjusting prices while the rest use rules of thumb or indexation. Price setting of backward-looking firms is based on recent history of aggregate prices. Empirical studies of the Hybrid Phillips curve have yielded conflicting results. Many studies have found evidence of a predominant role for forward-looking expectations in the inflation process, but the importance of backward-looking expectations has also gathered support in the data. The ad hoc interpretation of backward-looking behaviour in the Hybrid Phillips curve has been questioned in the recent literature. It has been argued that the lagged inflation term in the hybrid model need not reflect backward-looking behaviour in price setting. Instead, it has been conjectured that persistence in the Hybrid Phillips curve is closely related to inflation expectations: the lagged inflation term could reflect a departure of expectations from rationality (Woodford 2007).
The central role of expectations in macroeconomics was already recognised by Keynes (1936). However, until recently, there has been only little interest in modelling expectations formation and typically rational expectations have been imposed in empirical studies. Under rational expectations, agents use all available information and do not make systematic errors in forming their expectations. Rationality of expectations has been questioned. It may be an unrealistic and overly restrictive assumption. Already in the 1970s Lucas (1972) presented the famous islands model, where agents do not share the same information and have different inflation expectations, since they live on separate islands. After a change in money supply, an agent witnesses a price change in his/her product but is not able to determine how much the relative price has changed and how much the general price level has changed. In the mid-1970s deviation from rational expectations was also discussed by Sargent (1976) and McCallum (1976), who presented the idea of ‘partly rational expectations’.

Many recent studies of inflation dynamics have resorted to other assumptions regarding expectations formation. For example, the learning approach is a new and rapidly growing area of macroeconomics (Evans and Honkapohja 2001, 2003 and Milani 2007). Limited information channels have also been modelled (Woodford 2002 and Adam 2007) and sticky information models have been derived (Mankiw and Reis 2002, 2003). One can also assume that agents have heterogeneous expectations (Branch 2004) or that information spreads slowly through the economy (the epidemiology approach; see Carroll 2001).

In some recent studies survey estimates have been used to proxy inflation expectations. This approach is appealing since when applying to the data, one need not assume any specific form of non-rationality in expectations. Contrary to the US, the availability of survey data for Europe is quite poor. For empirical analysis, it is not possible to find sufficiently long series of quantitative survey estimates for inflation expectations for the all euro area economies. However, the European Commission provides monthly survey estimates, which are qualitative, for all the member states. This information, available since 1986, has been quantified using statistical methods in some empirical studies. Moreover, the European Central Bank has conducted a quarterly Survey of Professional Forecasters since 1999 (see Garcia 2003). Monthly survey estimates for future inflation rates provided by Consensus Economics are also available for euro area countries since the late 1980s. Inflation expectations can alternatively be derived from financial asset prices, but the derivation
is necessarily based on certain non-testable assumptions on ex ante real interest rates.

The present study examines inflation dynamics in the euro area. In the empirical analysis, alternative Phillips curve specifications and vector autoregressive models are applied to the euro area data. Instead of imposing rational expectations, direct measures of inflation expectations are used as empirical proxies for inflation expectations. More specifically, inflation expectations are measured using OECD inflation forecasts or Consensus Economics survey data. In studying euro area inflation dynamics, data availability is one reason for using OECD forecasts to proxy inflation expectations. It is not possible to obtain any other series of forecasts of similar length and coverage from any other source (Gerlach 2004).

It is a challenging task to question the Phillips curve relationship under the hypothesis of rational expectations. In principle, we may be able to improve the empirical performance of the New Keynesian Phillips curve by allowing a departure from rational expectations that allows for slow adjustment of expectations. In using directly measured expectations, we do not impose the hypothesis of rational expectations but we do maintain appealing microfoundations for price adjustment behaviour from the New Keynesian theory. Moreover, only by using this approach can we examine empirically whether the timing of expectations matters in the Phillips curve relationship, i.e., we can compare empirically the New Classical Phillips curve and the New Keynesian Phillips curve. Using directly measured expectations, we may be better able to analyse the effects of monetary policy regime changes on inflation dynamics.

This study provides evidence that inflation cannot instantaneously adjust to changes in expectations in the New Keynesian framework. When expectations are measured directly, the purely forward-looking New Keynesian Phillips curve is outperformed by the hybrid Phillips curve with an additional lagged inflation term and the New Classical Phillips curve with a lagged expectations term. The results suggest that the euro area inflation process has become more forward-looking in the recent years of low and stable inflation. In addition, in low inflation countries, the inflation dynamics have been more forward-looking already since the late 1970s. We find evidence of substantial heterogeneity of inflation dynamics across the euro area countries. Real time data analysis provides evidence that in the euro area real time information matters most in the expectations term of the Phillips curve and that the balance of expectations formation is more forward-than backward-looking. The vector autoregressive (VAR) analysis of
this study suggests that inflation expectations are relatively persistent and have a significant effect on output. However, expectations seem to react to changes in both output and actual inflation, especially in the medium term. Overall, this study indicates that expectations play a central role in inflation dynamics, which should be taken into account in conducting monetary policy.

The rest of this introduction is structured as follows. In section 2, some alternative price setting models are derived in a common framework. The different Phillips curves are derived in section 3. Section 4 discusses related empirical studies of inflation dynamics, and section 5 reviews the articles. Finally, six articles on euro area inflation dynamics are presented.

2 Alternative models for optimal price setting

In this section we present alternative optimal price setting models using a production function with one variable factor of production, labour. Optimising models with endogenous supply and nominal rigidities enable analysis of the real effects of monetary policy. Due to nominal rigidities, prices do not adjust in every period, so that in some periods certain factors of production, such as labour, may be under- or over-utilized and aggregate output may differ from its potential level. Nominal rigidities can be due to sticky prices or sticky wages. Since the aim is to examine inflation dynamics in a simple framework, only sticky prices are considered. The optimal price setting models of this section are needed to derive microfoundations for different Phillips curve relationships.

Price setting models are based on the representative agent’s optimisation. They assume differentiated goods and monopolistic competition, which means that firms have some monopoly power in setting their prices. If prices are fully flexible, they can be adjusted in every period on the basis of full information about current demand and cost conditions, and equilibrium output is completely independent of monetary policy. In more realistic models, where monetary policy affects real economic activity, there are nominal rigidities in the economy, i.e., firms are not able to change their product prices in every period. Delayed price adjustment may be due to many factors, and prices effective in a given period must be announced in an earlier
period or suppliers will have to decide the price on the basis of old information.

This section follows Woodford (2003) and proceeds as follows. First, we develop the basic framework using very general forms of the maximisation problem, profit function and first order conditions. Necessary and sufficient conditions for the optimal price are presented. Also, nominal and real marginal supply costs are derived from the production function with labour as the only factor of production. Second, we discuss the maximisation problem of the representative agent and the first order conditions in a model with fully flexible prices and two alternative models with price stickiness. In all the models we use linear approximations, which are needed for deriving the different Phillips curves in the next section. Linear approximations are relevant in a moderate inflation environment. As will be shown, expectations play a central role in optimal price setting with nominal rigidities.

2.1 Basic model with endogenous supply

The basic model for optimal price setting is based on the representative agent, who obtains utility from consumption and real money balances and disutility from supplying labour. Supply of goods is assumed to be endogenous, as one factor of production, labour, is variable. In addition, goods are assumed to be differentiated and competition monopolistic, following the New Keynesian literature of Rotemberg (1982), Mankiw (1985) and Svensson (1986). In monopolistic competition, suppliers of goods have some market power, which means that they are able to make price setting decisions.

The problem is to maximise

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ u(C_t, M_t, P_t, \xi_t) - \int_0^t v(h_t(i), \xi_t) di \right] \right\}
\]

(2.1)

where

- \( E_0 = \) expected value in period 0
- \( C_t = \) index of household consumption of goods supplied in period \( t \)
- \( P_t = \) index of prices of goods supplied in period \( t \)
- \( M_t = \) nominal money balances in period \( t \)
- \( h_t(i) = \) quantity of labour of type \( i \) supplied in period \( t \)
- \( \xi_t = \) vector of aggregate shocks in period \( t \)
u(.) = utility in period t
v(.) = disutility in period t
$\beta^t$ = discount factor for period t.

It is assumed that labour of type i is the only variable input used in the production of differentiated good i. The representative agent supplies all the types of labour, as indicated by the integral in the above equation.

For optimal price setting, we need the first-order conditions for optimal supply of each type of labour. It is assumed that the consumption index, $C_t$, is a constant-elasticity-of-substitution aggregator

$$C_t \equiv \left[ \int_0^1 c_t(i)^{(\theta-1)/\theta} \, di \right]^{-\theta/(\theta-1)} \tag{2.2}$$

where $\theta > 1$, and the corresponding price index can be expressed as

$$P_t \equiv \left[ \int_0^1 p_t(i)^{1-\theta} \, di \right]^{1/(1-\theta)} \tag{2.3}$$

(see Dixit and Stiglitz 1977). When individual goods prices are given, the price index, $P_t$, gives the minimum cost of a unit of aggregate consumption, $C_t$. Nominal money balances, $M_t$, deflated by $P_t$, gives the purchasing power of the representative agent.

The agent’s intertemporal budget constraint can be written in the form

$$E_0 \left\{ \sum_{t=0}^{\infty} Q_{0,t} \left[ \int_0^1 p_t(i)c_t(i) \, di + \Delta_t M_t \right] \right\} \leq$$

$$W_0 + E_0 \left\{ \sum_{t=0}^{\infty} Q_{0,t} \left[ \int_0^1 w_t(i)h_t(i) \, di + \int_0^1 \Pi_t(i) \, di \right] \right\} \tag{2.4}$$

where

$Q_{0,t}$ = general stochastic discount factor
$\Delta_t$ = interest rate differential between non-monetary and monetary assets
$W_0$ = financial wealth (excluding money) in period 0
We need to consider three different requirements in order to obtain necessary and sufficient conditions for household optimisation. First, in every period consumption must be allocated optimally across differentiated goods. This means that, as the household maximises (2.2) given the level of total expenditure, the composition of goods must satisfy

\[ c_t(i) = C_t \left( \frac{P_t(i)}{P_t} \right)^\theta \]  

(2.5)

where the consumption of good i is related to its relative price. This implies that total expenditure is equal to \( P_t C_t \) and the utility and budget constraints can be expressed using aggregate consumption and the aggregate price index, without individual goods and individual prices.

Second, optimal behaviour requires that, with a given optimal allocation of consumption expenditure and amount of labour supply, the agent must choose in each period the optimal levels of total consumption and money balances, as well as the amount of financial wealth and its composition of available bonds. In equilibrium, the necessary and sufficient conditions must be satisfied also for these choices of the agent.

The third requirement is the optimal choice of labour supply of each type, given the wage rates and value of additional income. The first order condition for optimal supply of type i labour in period t is given by

\[ \frac{v_h(h_t(i);\xi_t)}{u_c(C_{it},m_{it};\xi_t)} = \frac{w_t(i)}{P_t} \]  

(2.6)

As shown in equation (2.6), in period t, with given exogenous disturbances, the ratio of disutility of supplying type i labour to utility of consumption and money balances is equal to the real wage rate of type i labour.

Next, the production function is used to derive real marginal cost, which is the basic element in endogenous supply decisions. The production function for good i is given by
where labour is the only factor of production, $A_t$ is a positive time-varying exogenous technology factor and $f$ is an increasing, concave function. It can be shown that the variable cost of supplying a quantity $y_t(i)$ of good $i$ can be expressed as

$$w_t(i)h_t(i) = w_t(i)f^{-1}(y_t(i)/A_t).$$

Differentiating the (variable) cost function yields the nominal marginal cost function of supplying good $i$

$$S(i) = \frac{w_t(i)}{A_t} \Psi(y_t(i)/A_t) \tag{2.8}$$

where

$$\Psi(y) \equiv \frac{1}{f'(f^{-1}(y))} \tag{2.9}$$

is an increasing, positive function. Equation (2.8) determines how much nominal supply costs change when the quantity of supplied good $i$ is increased by one unit. It is assumed that all producers of differentiated goods take wage rates as given. In order to obtain the real marginal cost function, we insert the labour-supply function (2.6) into the wage equation (2.8), which yields

$$s_t(i) \equiv S_t(i)/P_t = s(y_t(i), Y_t; \xi_t) \tag{2.10}$$

where the real marginal supply cost is related to the supplied quantity of good $i$. Thus the real marginal cost function is given by

$$s(y, Y; \xi) = \frac{v_h(f^{-1}(y/A); \xi)}{u_c(Y; \xi)A} \Psi(y/A) \tag{2.11}$$

where $\xi$ is the vector of exogenous shocks and $C_t$ has been replaced by $Y_t$, since aggregate consumption is equal to aggregate output in every period. For simplicity, real balances have been suppressed from $u_c$.

We have thus derived the maximisation problem and real marginal cost function in a model with endogenous supply. These can be used to derive optimal price setting models with fully flexible or sticky prices.
2.2 Optimal price setting models

Next, we examine maximisation problems for alternative forms of optimal price setting, the first order conditions of which are needed to obtain microfoundations for different Phillips curve relationships. First, we discuss optimal price setting with fully flexible prices and present a specific example. Then we introduce two alternative forms of price stickiness. For each case, we derive the maximisation problem of the representative agent and the first order conditions for optimal price adjustment. In order to express all the models within a common framework, very general forms of profit functions are used.

Alternative models for optimal price setting can be used in monetary policy analysis. As will be shown, with fully flexible prices, equilibrium output is totally independent of monetary policy decisions. If prices are set one period in advance, monetary policy has only unexpected and temporary effects on real economic activity. With staggered price setting, we are able to model longer-lasting real effects of monetary policy.

2.2.1 Optimal price setting with fully flexible prices

Goods prices are perfectly flexible if all suppliers of goods are able to set their goods prices in every period. Past price changes do not constrain price setting decisions in period \( t \) and suppliers always have perfect knowledge of current demand and cost conditions. In period \( t \) the demand function for good \( i \) is given by

\[
y_t(i) = Y_t \left( \frac{p_t(i)}{P_t} \right)^{\theta}
\]

which shows that demand is related to the relative price of the good, given aggregate demand, \( Y_t \), and the aggregate price index, \( P_t \). With optimal price setting, the equation for the price of good \( i \) is given by

\[
p_t(i) = \mu S(i), \quad \mu = \frac{\theta}{\theta - 1} > 1
\]

is the desired markup of the supplier of good \( i \). Dividing by the aggregate price level yields the optimal relative price as a function of real marginal cost

\[
\frac{p_t(i)}{P_t} = \mu S(y_t(i), Y_t; \bar{\xi})
\]
Using equation (2.12), we obtain the relative supply of good i as

\[
\left( \frac{y_t(i)}{Y_t} \right)^{-1/\theta} = \mu s(y_t(i), Y_t, \xi_t)
\]  

(2.14)

Real marginal costs increase with the supply of good i, which means that, given aggregate output $Y_t$, there is a unique solution for the supply of good i. In equilibrium, the same quantity of each goods is supplied and that common quantity is equal to $Y_t$. Thus equilibrium output can be presented by $Y_t = Y^0(\xi_t)$, where the RHS term refers to the natural rate of output (Friedman 1968), i.e. the solution to the equation

\[
s(Y^n_t, Y^n_t, \xi_t) = \mu^{-1}
\]

(2.15)

Thus the natural output level is the level at which real marginal cost is equal to the reciprocal of desired markup. Real marginal cost also increases with aggregate output, which means we can solve for the unique optimal supply of good i for any vector of exogenous shocks, $\xi_t$. In spite of endogenous supply and monopolistic competition, with perfectly flexible prices, we get the neutrality result, where equilibrium output is completely independent of monetary policy, as seen from equation (2.15).

The above neutrality result can also be applied to a broad range of models with different cost and demand structures by using the following very general form of the nominal profit function of the supplier of good i in period t

\[
\Pi_t(i) = \Pi(p_t(i), p^I_t, P_t; Y_t, \xi_t)
\]

(2.16)

where

- $p_t(i)$ = price of good i in period t
- $p^I_t$ = price index for industry I
- $P_t$ = aggregate price index.

It is assumed that in a given industry all suppliers adjust goods prices at the same time. Following the previous example, the profit function takes the form
\[ \Pi(p, p_i^1, P; Y, \xi) \equiv pY(p/P)^{-\theta} \]
\[ - \frac{v_h(f^{-1}(Y(p^1/P)^{-\theta}/A; \xi))}{u_h(Y - G; \xi)} pf(Y(p/P)^{-\theta}/A) \]  

(2.17)

We assume that each supplier face wage, which depends on total labour demand in its industry. It is assumed that the profit function \( \Pi \) is homogeneous of degree one in its first three arguments (price of good i, price index for industry I and aggregate price index). It is also a single-peaked function of the price of good i, with a maximum value at a positive price for any values of its other arguments.

The first order condition for the optimal price of good i is given by

\[ \Pi_i(p_i(i), p_i^1, P; Y, \xi) = 0 \]  

(2.18)

which can be solved uniquely, as implied by the previous assumption. With fully flexible prices, the first order condition can be written as

\[ \Pi_i(P_i, P_i^1, P; Y, \xi) = 0 \]  

(2.19)

The LHS of equation (2.19) is independent of the aggregate price index, \( P_i \) and the equilibrium output is independent of monetary policy. For each good, the level of equilibrium output is given by

\[ y_t(i) = Y_i^n, \]  

where the natural rate of output is defined by the equation

\[ \Pi_i(1, 1, 1; Y_i^n, \xi) = 0 \]  

(2.20)

The first order condition (2.18) is needed for deriving the New Classical Phillips curve, since a fraction of goods are assumed to be fully price-flexible in this aggregate supply relationship.

2.2.2 Optimal non-overlapping price setting with nominal rigidities

A simple form of price stickiness can be modelled by assuming that all goods prices must be predetermined one period in advance. Thus, when the goods price \( p_t(i) \) is set in period t-1, the supplier knows all the exogenous shocks in period t-1 and earlier but not the cost and
demand conditions in period $t$. With price predetermined, we assume
that the supplier uses the amount of labour needed to fill all the orders.

With delayed price adjustment, expectations are important, since
the firm’s maximisation problem can be written as

$$E_{t-1}[\Pi_t(l_t, p_t^i, p_t^1, Y_t, \xi_t)] = 0$$  \hspace{1cm} (2.21)

where the present value of period $t$ profits is maximised. In equation
(2.21), the profit function has the same general form as in the previous
section with fully flexible prices. The additional term $Q_{t-1,t}$ denotes the
stochastic discount factor. The choice of optimal price is based on the
expected values of random variables $Q_{t-1,t}$, $Y_t$, $p_t^1$, $p_t$ and the vector of
exogenous shocks, $\xi_t$.

The first order condition for optimal price setting takes the form

$$E_{t-1}[\Pi_{t-1}^1((p_t(i), p_t^1, p_t^1, Y_t, \xi_t))] = 0$$  \hspace{1cm} (2.22)

In equilibrium, all firms set a common price, as they face the same
maximisation problem. This implies the first order condition

$$E_{t-1}[\Pi_{t-1}^1((p_t^1, p_t^1, p_t^1, Y_t, \xi_t))] = 0$$  \hspace{1cm} (2.23)

This condition is independent of $p_t$, which implies

$$E_{t-1}[\Pi_{t-1}((l_t, l_t, Y_t, \xi_t))] = 0$$  \hspace{1cm} (2.24)

When using the New Classical Phillips curve to analyse inflation
dynamics, we assume that a fixed share of goods prices are set one
period in advance and the rest are fully flexible. Thus we need the first
order condition (2.22) to derivate the New Classical Phillips curve.

With the general profit function (2.17), equation (2.24) can be
written as

$$E_{t-1}\{u(Y_t; \xi_t) Y_t [\mu^{-1} - s(Y_t, Y_t, \xi_t)]\} = 0$$  \hspace{1cm} (2.25)

Thus output is equal to the natural rate only on average – not in every
period. By contrast, as was shown in the previous subsection, when
prices are fully flexible, output is always at the natural level. In other
words, with fully flexible prices, the equation \( s(Y^n_t, Y^n_t, \xi_t) = \mu^{-1} \) holds in every period.

Suppose that the monetary policy target can be expressed in terms of the path of nominal GDP, denoted by \( \Gamma_t = P_t Y_t \). Solving for real GDP and inserting it into the first order conditions (2.25) yields

\[
E_{t-1}\{u_c(\Gamma_t/P_t; \xi_t)\Gamma_t[\mu^{-1} - s(\Gamma_t/P_t, \Gamma_t/P_t; \xi_t)]\} = 0
\]  
(2.26)

Thus the price level in period \( t \) is determined by the joint distribution of nominal GDP and aggregate shocks in period \( t \), conditional upon information in period \( t-1 \). In order to present this result in a simpler form, we can use a log-linear version of the equation \( E_{t-1}[\Pi_t(1, 1, 1; Y_t, \xi_t)] = 0 \). When the natural rate of output is defined by \( \Pi_t(1, 1, 1; Y^n_t, \xi_t) = 0 \), we can use the log-linear approximation

\[
\Pi_t(P_t, P_t, Y_t, \xi_t) = \psi_y (\hat{Y}_t - \hat{Y}_t^n)
\]  
(2.27)

where \( \psi_y \) is a positive coefficient in the case of a moderate shock.

Thus the aggregate-supply relation \( E_{t-1}[\Pi_t(1, 1, 1; Y_t, \xi_t)] = 0 \) simplifies to

\[
E_{t-1}[\hat{Y}_t - \hat{Y}_t^n] = 0
\]

which means that the first order condition can be written as the expected deviation of real output from its natural value, i.e. the expected output gap. By solving \( \Gamma_t = P_t Y_t \) for real output and inserting it into \( E_{t-1}[\hat{Y}_t - \hat{Y}_t^n] = 0 \) we obtain

\[
\log P_t = E_{t-1} \log \Gamma_t - E_{t-1} \log Y^n_t
\]  
(2.28)

which can be solved for real output

\[
\log Y_t = E_{t-1} \log Y^n_t + [\log \Gamma_t - E_{t-1} \log \Gamma_t]
\]  
(2.29)

The first term on the RHS of equation (2.29) refers to the expected natural rate of output, which can be forecast one period in advance.
and which is independent of monetary policy decisions. The last term is the unexpected part of nominal GDP. Thus, contrary to the model with fully flexible prices, where equilibrium output is completely independent of monetary policy, monetary policy has real effects when prices are set one period in advance. In this model, monetary policy generates unexpected variation in nominal GDP due to unexpected variation in nominal spending.

2.2.3 Optimal overlapping price setting with nominal rigidities

In this section we assume that the intervals over which goods prices remain fixed overlap, as was first suggested by Phelps (1978) and Taylor (1980). Following Calvo (1983), we examine a discrete staggered price setting model, where in every period a fraction $0 < \alpha < 1$ of goods prices remain unchanged and the rest are adjusted. It is assumed that $1 - \alpha$ denotes the probability that a given price will be adjusted in a given period. The probability is independent of the timing of the last price change and of the current price.

Again, all suppliers who are able to adjust prices in period $t$ choose the common price $p_{i}^{*}$ because of the common decision problem. In every period, the fraction $\alpha$ of prices that remain unchanged comprise a subset of the previous period’s prices. In period $t$, the Dixit-Stiglitz price index is given by

$$p_{i}^{1-\theta} = \int_{0}^{1} p_{t}(i)^{1-\theta} \, di = (1 - \alpha)p_{t}^{*1-\theta} + \alpha \int_{0}^{1} p_{t-1}(i)^{1-\theta} \, di$$

(2.30)

which implies that

$$p_{i} = [(1 - \alpha)p_{t}^{*1-\theta} + \alpha p_{t-1}^{1-\theta}]^{1/(1-\theta)}$$

(2.31)

The time path of the aggregate price index can be determined by using the initial value and the common new price chosen in every period. The optimal choice, $p_{i}^{*}$, depends on current and expected-future demand conditions of the individual good. The demand function (2.12) implies that other prices have an impact on the demand curve for good $i$ only through the aggregate price index, $P_{t}$. If the supplier is able to set the new price in period $t$, the optimal choice is based on the maximisation of
where the term $\alpha^{T-t}Q_{t,T}$ is the probability that the price chosen in period $t$ will still be charged in period $T$. Again, expectations are important in optimal price setting, since the pricing decision is based on the expected present value of future profits. The first order condition is given by

$$E_t\left\{ \sum_{T=t}^{\infty} \alpha^{T-t}Q_{t,T} \Pi_t(p_t(i), p_t^1, P_T; Y_T, \bar{Y}_T) \right\} = 0$$  \hspace{1cm} (2.33)$$

When all suppliers in a given industry change prices at the same time, the first order condition reduces to

$$E_t\left\{ \sum_{T=t}^{\infty} \alpha^{T-t}Q_{t,T} \Pi_t(p_t^*, p_t^1, P_T; Y_T, \bar{Y}_T) \right\} = 0$$  \hspace{1cm} (2.34)$$

As shown above, the first order condition for optimal staggered price setting with nominal rigidities can be expressed by using the same general form of the profit function as in the two previous sections. This condition is needed for deriving the New Keynesian Phillips curve.

With staggered price setting, the general price level and first order condition (equations 2.31 and 2.34) can be approximated using the log-linear expressions

$$\log P_t = \alpha \log P_{t-1} + (1-\alpha)\log p_t^*$$  \hspace{1cm} (2.35)$$

and

$$\sum_{T=1}^{\infty} (\alpha\beta)^{T-t}E_t[\log p_t^* - \log P_T - \zeta(\hat{Y}_T - \hat{Y}_T^n)] = 0$$  \hspace{1cm} (2.36)$$

In this model the evolution of the aggregate price level is smooth, despite frequent adjustments of individual prices. In the empirical analysis, the output gap is proxied using detrended output. It has been
argued, however, that the output gap does not necessarily capture productivity shocks, taste shocks and other real disturbances.

As an alternative measure, it has been suggested that variation in production costs should be included in (2.36) instead of variation in output (Sbordone 2002 and Gali and Gertler 1999). Theory-based proxies for the output gap have also been used in empirical studies: hours worked (McCallum and Nelson 1999) and the real-interest rate gap (Nelson and Nikolov 2004).

3 Three Phillips curve relationships

In this section we follow Woodford (2003) and derive three different Phillips curve relationships. These models are based on alternative optimal pricing models presented in section 2. As will be shown, the New Classical Phillips curve, the New Keynesian Phillips curve and the hybrid modification of the New Keynesian Phillips curve involve very different assumptions about the role of expectations in the inflation process. In the New Classical Phillips curve, previously expected current inflation is an important determinant of inflation dynamics, since only a certain portion of prices are fully flexible and the rest are set one period in advance. By contrast, in the New Keynesian Phillips curve, based on staggered price setting, inflation is determined by currently expected future inflation. In modelling inflation dynamics with the Hybrid Phillips curve, we assume that only some firms are forward-looking and set their prices optimally; the rest are assumed to be backward-looking and to adjust their prices mechanically by rules of thumb.

Whether price setting is synchronised or staggered is important for the effects of monetary policy. If all firms change prices at the same time, relative prices do not matter for optimal pricing decisions. When price setting is staggered, firms care about relative prices when readjusting their prices. When firms reset their goods prices on a staggered basis, they know that some of the other firms have already adjusted prices in earlier periods and that some will change prices in later periods. Thus, when raising goods prices, firms try to adjust only slightly, since large price increases change relative prices substantially, and a firm will not want to loose customers. Other firms behave in exactly the same way: since some firms have already made moderate price changes, it is reasonable for a firm not to make a big price change. Thus, with staggered price setting, the aggregate price
Alternative Phillips curve relationships are based on time-dependent optimal price setting. When price adjustment is delayed due to nominal rigidities, firms cannot know the consequences of their pricing decisions for sales and profits. This means that optimal pricing decisions are based on the present value of expected profits. Thus expectations play a crucial role in pricing decisions. Due to differences in the nature of optimal price setting and in the role of expectations, alternative Phillips curves have clearly different policy implications. The New Classical Phillips curve implies that monetary policy has only temporary effects on real economic activity. By contrast, longer-lasting real effects of monetary policy can be modelled using the New Keynesian Phillips curve. The Hybrid model is able to explain inflation persistence due to delayed inflation-effects of monetary policy.

3.1 The New Classical Phillips curve

The New Classical Phillips relation was originally presented in the late 1960s by Phelps (1967) and Friedman (1968). It can be derived in the general framework of the previous section by combining two optimal price setting models. In the New Classical Phillips curve it is assumed that a fraction, $0 < \iota < 1$, of goods prices are fully flexible and the rest are fixed one period in advance. Thus, in order to derive microfoundations for this aggregate supply relation, we need the first order conditions derived in section 2, i.e. equations (2.18) and (2.22) with minor modifications:

$$\Pi_1(p_{it}, p_{it}, P_t, Y_t, \tilde{\xi}_t) = 0$$

(3.1)

and

$$E_{t-1}[Q_{t-1}, \Pi_1(p_{2t}, p_{2t}, Y_t, \tilde{\xi}_t)] = 0$$

(3.2)

Condition (3.1) applies to goods prices which can be adjusted in every period. Condition (3.2) applies to sticky prices, which must be predetermined on the basis of information available in period t-1. A
common price is assumed to be chosen for all flexible-price goods and for all sticky-price goods.

When a log-linear approximation of $\Pi_t$ close to the values $(1, \bar{Y}, 0)$ is inserted into $\Pi_t(p_t, p_t, P_t; \bar{Y}; \bar{\xi}_t) = \psi_y(\bar{Y}_t - \bar{Y}_t^n)$, we obtain

$$\Pi_t(p_t, p_t, P_t; \bar{Y}, \bar{\xi}_t) = \psi_p \log(p_t / P_t) + \psi_y(\bar{Y}_t - \bar{Y}_t^n)$$  \hspace{1cm} (3.3)$$

where $\psi_p < 0$. This can be written as

$$\Pi_t(p_t, p_t, P_t; \bar{Y}, \bar{\xi}_t) = \psi_p[\log(p_t / P_t) - \zeta(\bar{Y}_t - \bar{Y}_t^n)]$$  \hspace{1cm} (3.4)$$

where $\zeta = -\psi_y / \psi_p > 0$. Log-linear approximations of the first order conditions (3.1) and (3.2) are given by

$$\log p_{1t} = \log P_t + \zeta(\bar{Y}_t - \bar{Y}_t^n) \quad \text{and} \quad \log p_{2t} = E_{t-1}[\log P_t + \zeta(\bar{Y}_t - \bar{Y}_t^n)] = E_{t-1} \log p_{1t}.$$  \hspace{1cm} (3.5)$$

These approximations are valid if price and output changes are moderate. A log-linear approximation of the general price index, aggregated from two kinds of prices, is given by

$$\log P_t = t \log p_{1t} + (1 - t) \log p_{2t}.$$  \hspace{1cm} (3.6)$$

We can show that

$$\pi_t - E_{t-1}\pi_t = \log P_t - E_{t-1} \log P_t = \frac{1}{1 - t}(\log p_{1t} - \log P_t).$$  \hspace{1cm} (3.7)$$

Inserting equation (3.5) into equation (3.7) yields the New Classical Phillips curve

$$\pi_t = E_{t-1}\pi_t + \varphi(\bar{Y}_t - \bar{Y}_t^n)$$  \hspace{1cm} (3.8)$$

where $\varphi \equiv \frac{1}{1 - t} \zeta > 0$. As shown in equation (3.8), the New Classical Phillips curve relates current inflation rate to the previously expected

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1 See a more detailed derivation in Appendix 1.
current inflation rate and to current excess demand. The slope of the New Classical Phillips curve depends on the parameters \( \iota \) and \( \zeta \), i.e. on the share of fully flexible prices and the slope parameter in the two price equations. The New Classical Phillips curve is the steeper, the larger the parameter \( \iota \), i.e. the larger the portion of firms with fully flexible prices. The New Classical Phillips curve suggests that monetary policy has only temporary and unexpected effects on real activity. These effects are completely non-forecastable, since monetary policy causes unexpected variation in nominal spending and thus in nominal GDP. Some empirical studies indicate that the observed long-lasting real effects of monetary policy cannot be explained by the New Classical Phillips curve unless one assumes that prices are fixed for an unrealistically long time. Next, we derive an alternative Phillips curve relationship with longer-lasting real effects of monetary policy, i.e. the New Keynesian Phillips curve.

### 3.2 The New Keynesian Phillips curve

The New Keynesian Phillips curve is also based on nominal rigidities, but price setting is staggered rather than synchronised. Optimal price setting behaviour is strongly affected by staggering, and thus firms pay attention to relative prices of goods. If the fixed time intervals between price changes overlap, the model produces more realistic real effects of monetary policy than with synchronised price setting, but it can still be assumed that individual prices are reset optimally quite frequently. The New Keynesian Phillips curve implies that monetary policy effects on real activity are forecastable.

The New Keynesian Phillips curve can be derived following the Calvo model (Calvo 1983), which assumes that in every period a fraction \( 0 < \alpha < 1 \) of the goods prices are unchanged and the remaining prices are adjusted. When goods prices are reset, the price setters take into account that a price may be fixed for many periods. Each price has an equal probability of being revised in any given period, and this probability is independent of the timing of the last price change. We assume that a stochastic discount factor for profits is on average \( 0 < \beta < 1 \) and that the profit function has the same general form as in the two previous sections: 

\[
\Pi_t(i) = \Pi(p_t(i), p_t^*, P_t; Y_t, \xi_t).
\]

The aggregate price index can be expressed as a weighted average of the lagged aggregated price index and the optimal price in period \( t \):

\[
\log P_t = \alpha \log P_{t-1} + (1 - \alpha) \log p_t^*.
\]

The optimal price, \( p_t^* \), takes the form
\[ \sum_{T=1}^{\infty} (\alpha \beta)^{T-t} E_t [\log p_t^* - \log P_T - \zeta (\hat{Y}_T - \hat{Y}_T^n)] = 0. \]  
When making a pricing decision, it is optimal for a firm to set the price equal to the weighted average of prices it would have expected to set in future periods, in the absence of price rigidities. Solving the optimal solution for \( p_t^* \) and modifying, we get the following form, where current optimal price is a function of expected next-period optimal price\(^2\)

\[ \log p_t^* = (1 - \alpha \beta) \sum_{T=1}^{\infty} (\alpha \beta)^{T-t} E_t [\log P_T + \zeta (\hat{Y}_T - \hat{Y}_T^n)] \]

\[ = (1 - \alpha \beta) [\log P_t + \zeta (\hat{Y}_t - \hat{Y}_t^n)] + \alpha \beta E_t \log p_{t+1}^*. \]  

(3.9)

Subtracting \( \log P_t \) from both sides, we obtain the optimal relative price for period \( t \), \( \hat{p}_t^* = \log(p_t^*/P_t) \)

\[ \hat{p}_t^* = (1 - \alpha \beta) \zeta (\hat{Y}_t - \hat{Y}_t^n) + \alpha \beta E_t [\log p_{t+1}^* - \log P_t] \]

\[ = (1 - \alpha \beta) \zeta (\hat{Y}_t - \hat{Y}_t^n) + \alpha \beta E_t [\hat{p}_{t+1}^* + \pi_{t+1}] \]  

(3.10)

Finally, combining equation (3.10) with the aggregate price index, we obtain the New Keynesian Phillips curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa (\hat{Y}_t - \hat{Y}_t^n) \]

where

\[ \kappa = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \zeta > 0 \]  

(3.11)

Thus, in the New Keynesian Phillips curve, the current inflation rate is a function of the currently expected future inflation rate and current excess demand. The parameter \( \kappa \) is positive since, with excess demand, inflation tends to increase. Under excess demand, inflationary pressures are due to the fact that readjusted prices are higher on average. Iterating equation (3.11) forward, we obtain

\(^2\) A more detailed derivation is given in Appendix 2.
\[\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t (\hat{Y}_{t+k} - \hat{Y}_{t+k}^n)\]  

(3.12)

Since the current inflation rate is equal to the weighted, discounted stream of current and future output gaps, it is entirely forward-looking. Hence there is no persistence in the inflation process. Current pricing decisions are less related to cost and demand conditions in the far future than in the near future. This is due to the fact that, at the micro level, individual price setters are the more likely to make another price adjustment, the farther off the future period in question.

In empirical work, real marginal cost can be proxied by real unit labour cost (labour income share). We assume that in minimising costs the firm sets nominal marginal cost equal to the ratio of nominal wage to marginal product of labour: \(MC^n_t = W_t / MPL_t\). In the simplest model we assume a Cobb-Douglas production function of the form \(Y_t = K_t^b N_t^{1-b}\), which implies that nominal marginal cost can be expressed by

\[MC^n_t = W_t / [(1-b)Y_t / N_t]\]  

(3.13)

which can be modified to yield real marginal cost

\[MC_t = W_t N_t / [(1-b)P_t Y_t]\]  

(3.14)

Thus real marginal cost is proportional to labour’s share of income or equivalently to real unit labour cost

\[MC_t \propto S_t = W_t N_t / P_t Y_t\]  

(3.15)

The use of labour income share as an empirical proxy for real marginal cost is not unambiguous, since it is based on aggregate variables, not on the theoretical firm-level marginal cost. Since average and marginal cost are not equal, they may move in opposite directions. For example, during a recession, average cost of production and thus labour income share may increase, if employed labour is underutilised. However, overtime compensation declines in recessions, which reduces both average cost and labour income share.
3.3 The Hybrid Phillips curve

It may not be reasonable to assume that in the Calvo model prices are unchanged between optimising periods. Instead, we can assume that firms can save costs if prices are changed between price adjustment periods according to a mechanical rule. Under this assumption we are able to derive the Hybrid Phillips curve, which implies persistence in the inflation process. The Hybrid model relates current inflation to both currently expected future inflation and the lagged inflation rate. Only some firms are assumed to be forward-looking and to set their prices optimally. The rest are assumed to be backward-looking in their pricing decisions. Thus, in this aggregate supply relationship, the lagged inflation term is needed as an additional explanatory variable.

The same implications of the Hybrid Phillips curve can be derived in two different ways. One can assume that prices remain fixed between optimising periods and that, when prices can be adjusted, only some of them are reset optimally and the rest are changed according to backward-looking rules of thumb (see eg Gali and Gertler 1999). Alternatively, in indexation models, all prices can be assumed to be adjusted in every period. When every price is reset in every period, price setting is based on optimisation or backward-looking indexation. Different forms of indexation models have been developed by several authors. Yun (1996) proposed a model in which backward-looking indexation between optimising periods automatically follows the long-run average inflation rate

\[ \pi_t = (1 - \beta) \pi + \kappa (\hat{Y}_t - \hat{Y}_t^n) + \beta E \pi_{t+1}. \]  

(3.16)

We can also assume that backward-looking indexation is based on the most recent inflation rate. In principle, indexation of the current inflation rate is possible, but this assumption is unrealistic, since it is too costly to firms and would lead to simultaneity problems.

Next, we assume that in every period a fixed portion, \((1-\alpha)\), of all goods prices are re-optimised and the rest are changed mechanically according to the rule

\[ \log p_t(i) = \log p_{t-1}(i) + \gamma \pi_{t-1}, \]  

(3.17)

where the term \(0 \leq \gamma \leq 1\) determines the rate of indexation. In this case, the maximisation problem and first order condition can be expressed as
maximise

\[ E_1 \left[ \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[ \prod_{t} (P_t(i)(P_{T-1}/P_{t-1}))^\gamma \right] \right] \]  \hspace{1cm} (3.18)

and

\[ E_1 \left[ \sum_{T=1}^{\infty} (\alpha\beta)^{T-1} u_c(\bar{Y}_T, \tilde{\xi}_T) Y_T P_T^0 \left( \frac{P_{T-1}}{P_{t-1}} \right)^{(1-\theta)} \times \left[ \mu P_t s(\bar{Y}_T, P_t^*)^{-\theta}(P_{T-1}/P_{t-1})^{-\theta}, Y_T, \tilde{\xi}_T ) \right] \right] = 0 \]  \hspace{1cm} (3.19)

The aggregate price index can be expressed as

\[ P_t = \left[ (1-\alpha)p_t^{\gamma-\theta} + \alpha \left( \frac{P_{t-1}}{P_{t-2}} \right)^{\gamma-\theta} \right]^{1/(1-\theta)} \]  \hspace{1cm} (3.20)

Using log-linear approximations of equations (3.19) and (3.20), we can make similar modifications as in the derivation of the New Keynesian Phillips curve to obtain the Hybrid Phillips curve

\[ \pi_t - \gamma\pi_{t-1} = \kappa(\hat{Y}_t - \hat{Y}_t^n) + \beta E_t(\pi_{t+1} - \gamma\pi_t), \]  \hspace{1cm} (3.21)

where \( \beta, \kappa \) and \( \hat{Y}_t^n \) are defined as in the purely forward-looking New Keynesian model. Solving equation (3.21) forward, we obtain

\[ \pi_t = \gamma\pi_{t-1} + \sum_{j=0}^{\infty} \beta^j E_t[\hat{Y}_{t+j} - \hat{Y}_{t+j}^n] \]  \hspace{1cm} (3.22)

According to equation (3.22), the current inflation rate depends not only on the expected path of the output gap but also on the lagged inflation rate, \( \pi_{t-1} \). Therefore, the hybrid specification of the New Keynesian Phillips curve implies persistence in inflation. The inflation process is more persistent, the larger the indexation parameter \( \gamma \).

The indexation parameter \( \gamma \) varies across different studies. For example, in Christiano et al (2005), the indexation parameter is equal to one, which means full indexation. On the other hand, Smets and
Wouters (2002) have proposed that a reasonable rate of indexation is 0.64. If the parameter $\beta$ is equal to one in the indexation model, the model is identical to Galí and Gertler’s (1999) hybrid model, where backward-looking behaviour is based on rule-of-thumb behaviour, and to Fuhrer and Moore’s (1995) two-sided Phillips curve, where workers are concerned about their relative wages.

4 Related studies

The empirical relevance of the New Keynesian Phillips curve has not been firmly established. In many studies the empirical fit of the New Keynesian Phillips curve under rational expectations has been poor and, moreover, studies of the Hybrid Phillips curve have yielded conflicting results. There has also been disagreement on the best proxy for the real marginal cost variable and on methodological issues.

In empirical studies that find support for the New Keynesian Phillips curve, real unit labour costs are typically used to proxy real marginal costs. Real unit labour costs take into account the impact of productivity gains on inflation and adjust sluggishly for output variation. Galí and Gertler (1999) obtain such results for the US and Galí, Gertler and López-Salido (2001) for the euro area. These studies indicate that the New Keynesian Phillips curve provides a reasonably good approximation of inflation dynamics and that most agents are forward-looking when setting their prices. In Galí and Gertler (1999), about sixty to eighty per cent of individual price setters exhibit forward-looking behaviour in adjusting goods prices. Backward-looking price setting is also statistically important, but it is not quantitatively very important. The New Keynesian Phillips curve can be formally rejected against the Hybrid model with some mild backward-lookingness in Galí, Gertler and López-Salido (2001). However, it still describes euro area inflation dynamics quite well. Both Galí and Gertler (1999) and Galí, Gertler and López-Salido (2001) find strong evidence against the output gap-based New Keynesian Phillips curve.

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3 This section is not intended to be a comprehensive survey of all relevant issues. See Walsh (2003) and Woodford (2003) for a more detailed discussion.
The dominant role of the forward-looking expectations term in explaining US inflation dynamics has also been stressed in studies using a two-step estimation procedure (Sbordone 2002). When real marginal cost is proxied by unit labour cost and backward-looking firms are allowed to index their prices to the lagged inflation rate, the hybrid model results indicate a relatively small weight for the backward-looking term. Sbordone (2002) argues that inflation dynamics can even be fairly well captured with a purely forward-looking model. By contrast, McAdam and Willman (2004) find evidence that the New Keynesian Phillips curve fits euro area data poorly. The New Keynesian Phillips curve has also been applied to data on individual European countries. For example, Balakrishnan and López-Salido (2002) show that the New Keynesian and Hybrid Phillips curves both perform poorly with the UK data. Benigno and López-Salido (2006) find that the marginal cost-based New Keynesian Phillips curve fits the German data.

The dominance of forward-looking expectations is not unambiguous, since several studies suggest that the Hybrid model outperforms the New Keynesian Phillips curve. In Jondeau and Le Bihan (2005), the Hybrid specification fits better with US and euro area data, and the relative weights of forward- and backward-looking components are roughly equal. However, the driving variable is typically insignificant. Adding lags and leads to a model seems to improve its empirical performance, although this lacks theoretical foundation. In spite of the dominance of forward-looking expectations, the relative weight of the backward-looking component is still significant in Jondeau and Le Bihan (2005). Benigno and López-Salido (2006) find that, for France, Italy and the Netherlands, the Hybrid model fits the data better than does the New Keynesian Phillips curve. On the other hand, the results are mixed for Spain.

Sondergaard (2003) studies the inflation process in European countries. He argues that the New Keynesian Phillips curve is rejected in favour of the Hybrid model for France and Italy, but the results are mixed for Spain. European countries being more open than the US economy, Sondergaard (2003) derives real marginal cost measures using a production function that includes (imported) intermediate inputs. However, open-economy considerations do not significantly improve the empirical performance of the New Keynesian Phillips curve for France, Italy or Spain.

The output gap-based New Keynesian models, with rational expectations, have also received empirical support, but to a lesser extent. It has been shown that, for the US, United Kingdom and
Australia, inflation dynamics can be modelled using the output gap as the driving variable (see Neiss and Nelson 2005). On the other hand, it has been argued that replacing the output gap with real unit labour cost does not improve the empirical fit of the New Keynesian Phillips curve for the US (Rudd and Whelan 2002). Jondeau and Le Bihan (2005) show that the output gap-based Hybrid model is favoured for the euro area, Germany and Italy. By contrast, the Hybrid Phillips curve results for France are poor, with both real marginal cost and the output gap.

The reason for the dominant role of forward-looking expectations in the US inflation process is not clear. According to Fuhrer (1997) forward-looking expectations are essentially unimportant in US data. Rudd and Whelan (2003) find little evidence of the rational forward-looking behaviour implied by the New Keynesian theory. The New Keynesian model may instead suffer from reliance on a strict form of rational expectations. Although agents may not be rational, price setters may behave optimally. Rudd and Whelan (2003) suggest that further research on how agents process information and improve their forecasts would be worthwhile.

Roberts (1997 and 1998) analyses inflation dynamics in the US using the New Keynesian specification and survey estimates of inflation expectations. These studies provide evidence that inflation expectations are not rational, which appears to be connected with the poor empirical fit of the New Keynesian theory. Roberts (1997) argues that, with survey-based expectations, one can distinguish between the structural and expectational sources of lagged inflation. The need for the lagged inflation rate may be due to non-rationality of expectations rather than to the underlying structure of the economy. Roberts (1998) considers the case where survey expectations are a weighted average of forward- and backward-looking expectations. The empirical evidence suggests that forty per cent of price setters use a simple univariate rule for forecasting inflation, and sixty per cent have rational expectations. Robert (1998) argues that backward-looking expectations are important in US inflation dynamics.

Survey-based measures of inflation expectations are used in Adam and Padula (2003), which examines US inflation dynamics in 1968–2003. The empirical results show that the New Keynesian Phillips curve fits the data, since the estimated parameters are significant and plausible. Reasonable results are obtained with both the output gap and unit labour cost. Although possible non-rationality in survey-based expectations seems to improve the empirical fit of the New
Keynesian Phillips curve, the lagged inflation rate, i.e., the Hybrid model, is needed to adequately capture the inflation dynamics.

The empirical success of the New Keynesian Phillips curve suggested in Gali and Gertler (1999) and Gali, Gertler and López-Salido (2001) has been questioned for methodological reasons. For example, Rudd and Whelan (2001) and (2002) find evidence that the instrumental variable estimation procedure is sensitive to small specification errors. Also, Lindé (2005) argues that there may be specification bias associated with GMM and that full information maximum likelihood (FIML) may be more robust than GMM. Gali, Gertler and López-Salido (2005) have responded to the criticism and provided evidence that the importance of forward-looking behaviour is robust to a variety of estimation methods.

Mavroeidis (2002) discusses possible identification and misspecification problems in his examination of forward-looking monetary models. He argues that the possible low power of the Hansen test in these models is due to ‘over-instrumenting’ and ‘over-correction’. In other words, using too many instruments and too much autocorrelation correction reduces the power of the Hansen test. Ma (2002) investigates problems with identification and weak instruments, in estimating the New Keynesian Phillips curve with GMM. He argues that GMM is an inadequate methodology for estimating of New Keynesian models, since the purely forward-looking New Keynesian Phillips curve suffers from observational equivalence and the hybrid model results are poor due to problems of weak identification. Søndergaard (2003) shows that using GMM with small samples produces results for the New Keynesian Phillips curve that may be highly sensitive to normalisation. He provides evidence that, in the context of Monte Carlo simulations, the normalisation applied in Gali and Gertler (1999) suggests that some firms are backward-looking, although there are no backward-looking firms by construction. Thus, the normalisation tends to cause overestimation of the portion of backward-looking firms. Regarding instrumental variable methods, possible small sample problems like normalisation are also discussed in Fuhrer, Moore and Schuh (1995).

The New Keynesian Phillips curve has been estimated using exact methods (Khalaf and Kichian 2004). With these methods, which are valid for small samples and immune to the presence of weak instruments, one can assess how well the model is identified. Although the results reject the Galí and Gertler (1999) estimates, the New Keynesian Phillips curve is supported by the data. Moreover, in
Khalaf and Kichian (2004) the share of backward-looking firms seems to be the least-well-determined parameter in the Hybrid model.

Cogley and Sbordone (2005) study whether the New Keynesian Phillips curve is structurally invariant. Using a VAR approach, they investigate US inflation since the early 1960s and interpret shifts in trend inflation as changes in monetary policy regimes. They find evidence that the New Keynesian Phillips curve is structurally invariant, since parameter restrictions implied by the theoretical model are not at odds with the unrestricted VAR model based on drifting parameters. Cogley and Sbordone (2005) argue that changes in the parameters of VAR model reflect changes in actual inflation. If shifts in trend inflation are not taken into account, we obtain a large coefficient for the lagged term in the hybrid model. But, when these shifts are taken into account, US inflation history is captured with a purely forward looking New Keynesian Phillips curve.

Castelnuovo and Surico (2006) examine the empirical evidence on the price puzzle, i.e. the mystery of the initial upward response of prices to a contractionary monetary policy shock. In analysing the United States and the United Kingdom, they use structural VARs and two different identification strategies based on zero restrictions and sign restrictions. Castelnuovo and Surico (2006) argue that the price puzzle is limited to sub-samples associated with a weak interest rate response to inflation. Using a New Keynesian monetary policy model, they show that the price puzzle is present if we omit expected inflation from the VAR. Expected inflation is clearly more persistent when expectations are not fully stabilized by monetary policy. According to Castelnuovo and Surico (2006), changes in expectations are essential for identifying structural shocks in the economy.

Tillmann (2005) examines euro area inflation dynamics and uses VAR-based forecasts to generate a series of model-consistent or ‘fundamental’ inflation. Fundamental inflation is supposed to explain actual inflation well if the model is correct. However, a set of bias-corrected bootstrapped confidence bands reveals that there is a considerable degree of uncertainty surrounding VAR forecasts. Even with the hybrid model, there seems to be a high degree of estimation uncertainty.

It has been shown that explicit inflation targeting has an effect on inflation expectations and inflation persistence. Levin and Piger (2002) and Levin, Natalucci and Piger (2004) measure long term inflation expectations using surveys and compare inflation dynamics in inflation-targeting and other countries. Comparison indicates that inflation persistence is lower in countries that employ an explicit
inflation target. The same evidence is found by Gürkaynak et al (2006) from long-term bond yields in the United States, the United Kingdom and Sweden. In that study, inflation expectations are measured as the difference between far-ahead forward rates on nominal versus inflation-indexed bonds. However, Ball and Sheridan (2003) found opposing evidence. They argue that an explicit inflation target has no effect on inflation dynamics in OECD countries.

Piger and Rasche (2006) study inflation history in the United States since the beginning of the 1960s. They examine how much long term inflation expectations, the output gap and supply shocks contribute to actual inflation. They provide evidence that most of the changes in actual inflation can be explained by changes in inflation expectations. Moreover, they argue that inflation expectations were better anchored in the 1990s than in the earlier decades. Piger and Rasche (2006) emphasize that long term inflation expectations play a key role in the inflation process.

5 Review of the articles

This thesis deals with the relationship between inflation, inflation expectations and output. Technically, this issue is analysed using the Phillips curve, which is one of the main tools in monetary policy analysis. In addition to estimation of alternative specifications of the Phillips curve, inflation dynamics are examined in a general vector autoregressive (VAR) framework. The focus of the analysis is on inflation expectations, which are measured directly using two alternative proxies, OECD inflation forecasts and Consensus Economics survey data.

In Paloviita (2006), the New Keynesian Phillips curve and its hybrid specification are applied to aggregated and pooled euro area data since the late 1970s. The estimation results suggest that once the rational expectations hypothesis is relaxed and OECD inflation forecasts are used to proxy expectations, the forward-looking New Keynesian Phillips curve is outperformed by the hybrid Phillips curve with an additional lagged inflation term. Sub-period analysis provides evidence that the euro area inflation process has become more forward-looking in the recent years of low and stable inflation. Moreover, the inflation dynamics seem to have been forward-looking in those countries that have had more stable inflation continuously since the late 1970s. According to Paloviita (2006), the output gap
turns out to be at least as good as labour income share as a proxy for real marginal cost.

Paloviita (2008) continues the comparison of alternative Phillips curve specifications by estimating the New Classical Phillips curve, the New Keynesian Phillips curve and the Hybrid Phillips curve for pooled euro area data since the late 1980s. In all cases, the Consensus Economics survey data is used to measure inflation expectations. The results indicate that the New Classical Phillips curve with a lagged expectations term has satisfactory statistical properties. Moreover, the purely forward-looking New Keynesian Phillips curve is clearly outperformed by the New Classical and Hybrid Phillips curves. Thus this study, using survey-based inflation expectations, confirms the results in Paloviita (2006), according to which inflation cannot instantaneously adjust to changes in expectations.

Possible heterogeneity of inflation dynamics across the euro area countries is examined in Paloviita and Mayes (2006). In this study, inflation expectations are proxied by OECD forecasts and competing Phillips curves are compared using econometric tests. In the empirical analysis, Paloviita and Mayes (2006) assume that the output gap could be determined by a simple autoregressive process. The results suggest that there could be substantial differences in the slopes across the different Phillips curves specifications. Moreover, under each specification, there seems to be remarkable variation across the Phillips curve slopes of individual countries.

Directly measured expectations belong to the category of real time variables. Without the benefit of hindsight, they measure expectations formed on the basis of information available at the time. Using OECD panel data for the euro area countries over the period 1977–2003, Paloviita and Mayes (2005) explore how information available at the time affects the performance of alternative Phillips curve specifications, in which real time information is used for actual inflation, inflation expectations and the output gap. In GMM estimations, real time instruments are also used. The empirical analysis indicates that the real time information matters most in the expectations terms of the Phillips curve: real time data indicate that the bulk of expectations formation was more forward- than backward-looking; revised data suggest less forward-looking and less well-determined behaviour.

Paloviita and Virén (2008a, 2008b) estimate vector autoregressive (VAR) models of actual inflation, inflation expectations and the output gap for the euro area. When treating inflation expectations as endogenous variables in VAR models, it is possible to examine the
way in which inflation expectations affect both inflation and output and, on the other hand, how inflation expectations reflect developments in these variables. Paloviita and Virén (2008a, 2008b) find evidence that inflation expectations are the key ingredient for the euro area inflation process and that they have a significant effect on output. Moreover, the authors find that inflation expectations are relatively persistent, almost as persistent as output. However, expectations seem to adapt to developments in both output and actual inflation, especially in the medium term.
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Appendix 1

Derivation of New Classical Phillips curve

The New Classical Phillips curve is based on the assumption that a fraction, $0 < \iota < 1$, of goods prices are fully flexible and the rest are fixed one period in advance. A log-linear approximation of the first order condition of fully flexible prices can be written as

$$\log p_{1t} = \log P_t + \zeta (\hat{Y}_t - \hat{Y}^n_t).$$  \hspace{1cm} (A1.1)

A log-linear approximation of the first order condition of prices that are set one period in advance takes the form

$$\log p_{2t} = E_{t-1}[\log P_t + \zeta (\hat{Y}_t - \hat{Y}^n_t)] - \zeta + E_{t-1} \log p_{1t}. \hspace{1cm} (A1.2)$$

The same optimal price is chosen by all firms, with fully flexible prices. Firms which set their goods prices one period in advance make the same optimal pricing decision. The aggregate price level is a convex combination of the two prices

$$\log P_t = \iota \log p_{1t} + (1-\iota) \log p_{2t}$$ \hspace{1cm} (A1.3)

where $\iota$ and $(1-\iota)$ are the portions of firms with fully flexible prices and those that adjust their prices one period in advance.

Subtracting $\log P_{t-1}$ and $E_{t-1} \log P_t$ from both sides of equation (A1.3) yields

$$\pi_t - E_{t-1} \log P_t = \iota \log p_{1t} + (1-\iota) \log p_{2t} - \log P_{t-1} - E_{t-1} \log P_t$$ \hspace{1cm} (A1.4)

which can be written as

$$\pi_t - E_{t-1} \pi_t = \iota \log p_{1t} + (1-\iota) \log p_{2t} - E_{t-1} \log P_t$$ \hspace{1cm} (A1.5)

Inserting $E_{t-1} \log P_t = \iota E_{t-1} \log p_{1t} + (1-\iota) E_{t-1} \log p_{2t}$ into equation (A1.5), yields
\( \pi_t - E_{t-1}\pi_t = t \log p_{tt} - tE_{t-1} \log p_{tt} \) \hspace{1cm} (A1.6)

Combining equations (A1.2) and (A1.3) yields

\[ E_{t-1} \log p_{tt} = \frac{1}{1-t} \log P_t - \frac{1}{1-t} \log p_{tt} \] \hspace{1cm} (A1.7)

which simplifies equation (A1.6) to

\[ \pi_t - E_{t-1}\pi_t = \frac{1}{1-t} (\log p_{tt} - \log P_t) \] \hspace{1cm} (A1.8)

Finally, inserting equation (A1.1) into equation (A1.8), we obtain the New Classical Phillips curve

\[ \pi_t = E_{t-1}\pi_t + \varphi(\hat{Y}_t - \hat{Y}_t^n) \] \hspace{1cm} (A1.9)

where \( \varphi = \frac{1}{1-t} \zeta. \)
Appendix 2

Derivation of New Keynesian Phillips curve

The Calvo model is based on the idea that in every period a fraction, $0 < \alpha < 1$, of goods prices are unchanged and the remaining prices are adjusted. Accordingly, the aggregate price index can be expressed as

$$\log P_t = \alpha \log P_{t-1} + (1 - \alpha) \log P_t^*$$  \hspace{1cm} (A2.1)

where the optimal price, $p_t^*$, takes the form

$$\sum_{T=i}^{\infty} (\alpha \beta)^{T-i} E_t [\log p_t^* - \log P_T - \zeta (\hat{Y}_T - \hat{Y}_T^n)] = 0$$ \hspace{1cm} (A2.2)

Subtracting $\alpha \log P_t$ from both sides of equation (A2.1), enables expression of the optimal relative price in period $t$, $p_t^* = \log(p_t^*/P_t)$, as a function of the inflation rate in period $t$

$$(1 - \alpha) \log P_t = -\alpha (\log P_t - \log P_{t-1}) + (1 - \alpha) \log p_t^*$$

$\Leftrightarrow \alpha \log (P_t/P_{t-1}) = (1 - \alpha) \log (p_t^*/P_t)$

$\Leftrightarrow \alpha \pi_t = (1 - \alpha) \hat{p}_t^*$

$\Leftrightarrow \hat{p}_t^* = \frac{\alpha}{1 - \alpha} \pi_t.$

Solving equation (A2.2) for $\log p_t^*$, we obtain

$$\sum_{T=i}^{\infty} (\alpha \beta)^{T-i} E_t [\log p_t^* = \sum_{T=i}^{\infty} (\alpha \beta)^{T-i} E_t [\log P_T + \zeta (\hat{Y}_T - \hat{Y}_T^n)]$$

$\Leftrightarrow \log p_t^* = (1 - \alpha \beta) \sum_{T=i}^{\infty} (\alpha \beta)^{T-i} E_t [\log P_T + \zeta (\hat{Y}_T - \hat{Y}_T^n)]$ \hspace{1cm} (A2.4)

Alternatively, the optimal price in any period can be expressed using the expected optimal price in the next period.
\[ \log p_t^* = (1 - \alpha \beta)\log P_t + \zeta(\hat{Y}_t - \hat{Y}_t^n) \]
\[ + (1 - \alpha \beta) \sum_{T=t+1}^{\infty} (\alpha \beta)^{T-(t+1)} E_t[\log P_T + \zeta(\hat{Y}_T - Y_T^n)] \]
\[ = (1 - \alpha \beta)\log P_t + \zeta(\hat{Y}_t - \hat{Y}_t^n) + \alpha \beta E_t \log p_{t+1}^* . \]

Subtracting \( \log P_t \) from both sides yields the optimal relative price in period \( t \)
\[ \log p_t^* - \log P_t = (1 - \alpha \beta)\log P_t + \zeta(\hat{Y}_t - \hat{Y}_t^n) \]
\[ + \alpha \beta E_t \log p_{t+1}^* - \log p_t \iff \]
\[ \hat{p}_t^* = (1 - \alpha \beta)\zeta(\hat{Y}_t - \hat{Y}_t^n) + \alpha \beta E_t[\log p_{t+1}^* - \log p_t] \]
\[ (A2.6) \]

Next, adding and subtracting \( \alpha \beta \log P_{t+1} \) from the RHS of equation (A2.6) yields
\[ \hat{p}_t^* = (1 - \alpha \beta)\zeta(\hat{Y}_t - \hat{Y}_t^n) \]
\[ + \alpha \beta E_t \log p_{t+1}^* - \log P_{t+1} + \log P_{t+1} - \log p_t \]
\[ = (1 - \alpha \beta)\zeta(\hat{Y}_t - \hat{Y}_t^n) + \alpha \beta E_t[\hat{p}_{t+1}^* + \pi_{t+1}] \]
\[ (A2.7) \]

Expressing \( \hat{p}_t^* \) and \( \hat{p}_{t+1}^* \) in terms of inflation (see equation A2.3) and inserting them into equation (A2.7), we obtain the New Keynesian Phillips curve
\[ \frac{\alpha}{1 - \alpha} \pi_t = (1 - \alpha \beta)\zeta(\hat{Y}_t - \hat{Y}_t^n) + \alpha \beta E_t\left[\frac{\alpha}{1 - \alpha} \pi_{t+1} + \pi_{t+1}\right] \]
\[ \iff \pi_t = \frac{(1 - \alpha)(1 - \alpha \beta)\zeta(\hat{Y}_t - \hat{Y}_t^n) + \alpha \beta E_t \pi_{t+1} + (\beta - \alpha \beta)E_t \pi_{t+1}}{\alpha} \]
\[ (A2.8) \]
\[ \iff \pi_t = E_t \pi_{t+1} + \kappa(\hat{Y}_t - \hat{Y}_t^n), \]

where \( \kappa = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \zeta > 0. \)
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