Luis H. R. Alvarez & Rune Stenbacka

TAKEOVERS AND IMPLEMENTATION UNCERTAINTY:
A REAL OPTIONS APPROACH
Takeovers and Implementation Uncertainty: A Real Options Approach

Key words: Takeover incentives, market for corporate control, implementation uncertainty, real options.

JEL Classification: G34, L20

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Luis H. R. Alvarez & Rune Stenbacka
Swedish School of Economics and Business Administration
P.O.Box 479
00101 Helsinki, Finland

Distributor:

Library
Swedish School of Economics and Business Administration
P.O.Box 479
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Finland

Phone: +358-9-431 33 376, +358-9-431 33 265
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Takeovers and Implementation Uncertainty: 
A Real Options Approach

Luis H. R. Alvarez∗ Rune Stenbacka†

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Abstract

We design a compound real options model, which determines the timing of takeovers and characterizes the distribution of the associated surplus. We delineate a relationship between the imperfections in the market for corporate control and the takeover incentives. We characterize a critical bargaining power below which the compound takeover option is never exercised. This critical threshold is a decreasing function of the expected primary takeover gain and the embedded divestment gain and an increasing function of the implementation uncertainty. With implementation uncertainty the relationship between volatility and takeover timing depends on the functional form of the profit flow.

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∗Department of Economics, Quantitative Methods in Management, Turku School of Economics and Business Administration, FIN-20500 Turku, Finland, e-mail: luis.alvarez@tukkk.fi

†Swedish School of Economics and Business Administration, Department of Economics, P.O. Box 479, FIN-00101 Helsinki, Finland, e-mail: rune.stenbacka@hanken.fi
1 Introduction

On a fundamental level a takeover means that one firm, the bidder (B), acquires the assets of another firm, the target (T). Such a takeover tends to promote economic efficiency if the bidder is able to reorganize the assets so as to create synergy gains. The bidder might, for example, be able to create merger synergies through the exploitation of complementarities or through economies of scale or scope. However, the acquisition decisions are typically sizeable irreversible investments to which the bidding firm has to commit itself in the presence of substantial uncertainty. In particular, the issue of whether the corporate cultures, the governance systems or the produced brands of the merger partners are compatible represents the source of an important type of implementation uncertainty. This can be exemplified by the experiences associated with the merger leading to the formation of DaimlerChrysler, where we are still waiting for the synergies anticipated in the merger prospect to be realized.

A takeover decision can be seen as an irreversible investment, which creates a real option, which incorporates a substantial degree of implementation uncertainty. This implementation uncertainty might mean that the takeover decision generates a conditional probability distribution regarding the timing of when the benefits from a merger arrive. Such an approach captures the impact of unpredictability of the time needed to make different corporate cultures, governance systems or established brands fit together. Alternatively, the implementation uncertainty could also capture merger gains of stochastic magnitude. In this paper we will focus on the first type of implementation uncertainty.

In many situations takeovers also incorporate an embedded option to divest those activities, which do not belong to the core business of the acquiring firm. We can exemplify this by Kone's acquisition of Partek in May 2002 at a price of 1,45 Billion Euro (see, Kauppalehti Optio, 11 December 2003). Kone's core business is focused on elevators and escalators as well as cargotec. The acquisition target, Partek, was a conglomerate with a spectrum of activities belonging to business areas like container and cargo handling, production of machines for forestry and agriculture like tractors as well as mining technology. In 2003 Kone divested most of those activities which did not belong to its core business and those divestment operations were estimated to yield 1,1 Billion Euro. At the end of 2003, the container and cargo handling business acquired from Partek was estimated to represent a value of 960-1140 Million Euro (see, Kauppalehti Optio, 11 December 2003) and the profitability of this activity has developed very positively under the new owner (see, Kauppalehti, 21 July 2004). Thus, Kone's takeover of Partek incorporated an embedded option of divesting those activities, which did not belong to Kone's core business, and this divestment was executed before the end of 2003. In our present analysis of takeovers we assume these takeovers to incorporate divestments of non-core business activities as an embedded option.

In the present article we design a model, which endogenously determines the timing of a takeover and characterizes the distribution of the surplus created through the asset restructuring. The timing of the takeover is determined in recognition of an embedded option, whereby the takeover offers the opportunity of divesting those target firm activities, which do not belong to the acquiring firm's core business. In particular, our model is able to characterize the relationship between the timing of a takeover and the bargaining power of the bidder. We interpret the bargaining power of the bidder as a measure of the market imperfections, which prevail in the market of corporate control relevant from the point of view of the primary takeover. This bargaining power may be different from the bargaining power of the merged firm relative to outside firms in determining the terms at which the divestment takes place as the prevailing imperfections in the market for corporate control are likely to be industry-specific.

We develop a model which is able to address the following questions in a formal analytical sense. How will increased imperfections in the market for corporate control impact on the equilibrium timing of a takeover? What is the effect of increased implementation uncertainty on the timing of a takeover as well as on the distribution between the bidder and the target of the gains
from an asset restructuring? How will increased volatility of the underlying economic environment affect the timing of corporate restructuring and the distribution of gains from takeovers? What is the relationship between the potential gains from a corporate restructuring and the equilibrium timing and price of a takeover?

The present study shares the real options approach to the analysis of the timing and terms of takeovers with two other recent studies, namely Lambrecht (2004) and Morelli and Zhdanov (2004). Both these studies find that economic expansions promote takeover activities. Lambrecht (2004) focus on friendly mergers motivated by economies of scale and on acquiring firms with or without market power in the product market. Within such a framework he finds that increased market power of the acquiring firm speeds up the merger activity. However, in hostile takeovers he finds that the restructuring of assets is delayed in comparison with the equilibrium timing of mergers. Our analysis differs from that of Lambrecht (2004) along several dimensions. First, we focus on more general representations of the synergy gains and these are, in our analysis, subject to uncertainty. Second, we focus on direct imperfections in the market for corporate control rather than on potentially related market power in the product market. Third, we focus on compound takeover options, which include the embedded divestment options, which seems to represent a highly relevant feature in most cases of conglomerate mergers.

Morelli and Zhdanov (2004) analyze the determination of timing and terms of takeovers in the presence of a stock market with incompletely informed investors. They find that part of the uncertainty will not be resolved until the timing of the takeover, which leads to abnormal announcement returns. They also find that bidding competition among acquiring firms, potentially endowed with different estimates regarding the synergies associated with a takeover, speeds up the takeover process and may generate negative returns to the shareholders of the bidding firms. Our analysis is focused on the takeover decisions and the interactions between the firms involved in the takeovers and not on the issue of how a stock market comprised of incompletely informed investors transmits inside information regarding potential synergy gains.

We find that there is an intimate relationship between the magnitude of the imperfections in the market for corporate control and the takeover incentives. In particular, sufficiently strong imperfections in the market for corporate control represent a necessary condition for the acquiring firm to find it optimal to exercise a takeover option. Furthermore, under such circumstances favorable states of nature promote the takeover activities. Overall, decreased implementation uncertainty, in the sense of a lower variance of the delay required until merger synergies are realized, tends to stimulate the takeover activities. In addition, the presence of implementation uncertainty makes the relationship between underlying market volatility and the optimal takeover timing dependent on the functional form of the profit flow. In the absence of implementation uncertainty, our model would unambiguously predict increased volatility to decelerate takeovers. Finally, as the optimal threshold for the exercise of the takeover is determined in relationship to a compound real option, which incorporates the divestment opportunity as an embedded option, we find that increased bargaining power of the acquiring firm at both the takeover stage and at the divestment stage stimulates takeover activity.

Overall our theoretical predictions are broadly in line existing empirical evidence. Maksimovic and Phillips (2001) present empirical evidence in support of the view that merger and acquisition (M&A) activities are more likely during booms. Furthermore, their study can be seen as an empirical defense of transactions in the asset markets whereby M&A-activities serve as a facilitator of productivity gains and redeployment of assets from firms with a lower ability to firms with a higher ability to exploit assets for productive purposes. Mitchell and Mulherin (1996) present evidence for the accumulation of takeovers and restructuring in industries, which are subject to a large degree of uncertainty. Audrade, Mitchell and Stafford (2001) present evidence that the target firm is typically able to obtain a substantial share of the gains from mergers. This feature can be captured in a tractable way by the Nash bargaining approach, which we apply for the determination of the takeover terms.

Our study proceeds as follows. Section 2 presents the model. In section 3 we analyze the
takeover timing and terms for general profit flows. In section 4 we present explicit character-
izations of the exercise threshold for the class of profit flows with a multiplicatively separable 
stoctastic component. Finally, we offer some concluding comments in section 5.

2 The Model

Consider a situation where one firm, denoted firm A, faces the opportunity to buy out another 
firm, firm T. Firm T could be a rival firm as in the case of a horizontal merger, but it could 
also be a conglomerate with activities in industries different from that or those of firm A. These 
firms operate in an environment where the market uncertainty is captured by an underlying state 
variable which evolves on $\mathbb{R}_+$ according to a standard geometric Brownian motion described by the Itô-stochastic differential equation

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 = x,$$

where both the drift coefficient $\mu \in \mathbb{R}$ and the diffusion coefficient $\sigma \in \mathbb{R}_+$ are assumed to be 
exogenously determined constants and $W_t$ denotes standard Brownian motion.

Prior to the takeover the business activities of firm A generate a profit flow $\pi_A(x)$, whereas 
those of firm T yield the profit flow $\pi_T(x)$. Through the takeover firm A acquires the right to 
reorganize the assets of T so as to create synergy gains. For example, the merger might make 
possible for firm A to exploit complementarities or to exploit economies of scale or scope. 
Through the reorganization of the assets the merged firm can achieve the profit flow

$$\pi_c^A(x) + \pi_d^A(x) \geq \pi_A(x) + \pi_T(x) \quad (2.2)$$

which captures the idea that the synergies associated with the merger increases the profit flows 
from $\pi_A(x) + \pi_T(x)$ to $\pi_c^A(x) + \pi_d^A(x)$. The post-merger profit flow is decomposed into two parts. 
$\pi_c^A(x)$ captures the profit flow associated with the core business of the merged firm, whereas 
$\pi_d^A(x)$ denotes the profit flow associated with the activities outside the core competence of the 
merged firm.

We assume that the activities outside the firm’s core business can potentially be sold to an 
outside firm O, which can make more efficient use of these resources. Firm A has the option to 
divest the activities outside its core business and the outside firm can transform these assets into 
a profit flow $\pi^0(x)$. Formally, we assume that

$$\pi^0(x) \geq \pi_d^A(x). \quad (2.3)$$

Consequently, the takeover makes it possible to exploit synergy gains, and in addition to this 
primary benefit, it also incorporates the embedded option represented by the potential divestment 
gains.

We assume that the absence of speculative bubbles condition is satisfied and, therefore, that 
the expected cumulative present values of all the relevant cash flows exist and are well-defined. In 
the subsequent analysis we denote, for the sake of notational simplicity, the expected cumulative 
present value of the cash flow $f(x)$ by $(R_s f)(x)$, i.e.,

$$(R_s f)(x) = \mathbb{E}_x \int_0^\infty e^{-rt} f(X_t) dt.$$ 

The acquisition decisions are typically sizeable irreversible investments to which the bidding 
firm has to commit itself in the presence of substantial uncertainty. In particular, the issue of 
whether the corporate cultures, the governance systems or the produced brands of the merger 
partners are compatible represents the source of an important type of implementation uncertainty. 
In this model we formally capture the implementation uncertainty associated with takeovers in 
the following way. Suppose that the takeover takes place at time $\tau$. Then a consolidation phase
of random length \( T \) is needed until the synergies captured by (2.2) can be realized. We will assume that \( T \) is exponentially distributed with parameter \( \lambda \), i.e. \( T \) is randomly distributed with the density function \( f(T) = \lambda e^{-\lambda T}, T \geq 0 \). Thus, the synergies associated with a merger require an expected implementation lag of \( \lambda^{-1} \).

As shown by (2.2), the takeover yields a surplus, which has to be divided by the owners of the acquiring firm A and the target firm T. We model the determination of the takeover price as the outcome of bargaining. For reasons of tractability, the negotiation is captured by the traditional Nash bargaining solution. We assume that the bargaining power of firm A is \( \beta_A \) \((0 \leq \beta_A \leq 1)\), whereas that of firm T is \( 1 - \beta_A \).

In general, we can view the bargaining power of the acquiring firm as a measure of the imperfections in the market for corporate control. Alternatively, in a particular sense the bargaining power can also be seen as capturing the intensity of competition in the market for corporate control. In fact, in the literature there is, in general, no unique way to characterize the intensity of competition. In traditional oligopoly models the consequences of intensified competition are often evaluated by increasing the number of competitors. Another approach, frequently applied in industrial organization, is to measure the intensity of competition by the degree of product differentiation. This approach can be exemplified by the Hotelling-type models of horizontal product differentiation. A third way of capturing the degree of imperfections in the market for corporate control is to identify these with the buyer’s (A) bargaining power relative to that of the seller (T), i.e. to apply the Nash bargaining approach, which we have done in the present analysis. For the present purposes this approach has two advantages: it both incorporates the polar market structures of monopoly and perfect competition as special cases and avoids incorporation of market-specific, and often controversial, institutional details of the market for corporate control. It should, however, be emphasized, that a change in the bargaining power of the acquiring firm need not be equivalent to a change in the competitiveness of the market for corporate control.

Once the merger synergies are realized, the merged firm has the option to divest those assets, which are associated with the activities outside the firm’s core business. The inequality (2.3) describes the surplus, which can potentially be achieved through the divestment. Formally, we assume that the price at which the ownership of the divested assets is shifted to the outside firm (O) is determined through a separate round of bargaining between the merged firm (A) and the outside firm (O). This bargaining takes place at time \( t = \tau + T \). Again, for reasons of tractability, we apply the Nash bargaining solution to model the negotiation. We assume that the bargaining power at the divestment phase of firm A is \( \beta_d \) \((0 \leq \beta_d \leq 1)\), whereas that of firm O is \( 1 - \beta_d \).

In our analysis of the timing and terms of the takeover and of the terms of the divestment we apply the principles of dynamic programming. Thus, we initially characterize the outcome at the last stage, the price whereby the non-core business activities are divested. This equilibrium is then anticipated at the next stage of our analysis: the determination of the price of the merger. This price determination is, in its turn, anticipated by firm A in its timing decision of when to take over firm T. The sequence of decisions and the associated profit flows are illustrated in Table 1.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Pre-takeover ([0, \tau])</th>
<th>Consolidation ([\tau, \tau + T])</th>
<th>Divestment (\tau + T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow of firm A</td>
<td>(\pi^A(x))</td>
<td>(\pi^A(x) + \pi^T(x))</td>
<td>(\pi^A(x))</td>
</tr>
<tr>
<td>Cash flow of firm T</td>
<td>(\pi^T(x))</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Cash flow of firm 0</td>
<td>-</td>
<td>-</td>
<td>(\pi^0(x))</td>
</tr>
</tbody>
</table>

Table 1: The sequence of decisions and associated profit flows

\(^1\)For example, in the context of credit markets, Bester (1995) associates a change in the degree of competitiveness with a change in the outside option of the project holder. Thus, if the credit market becomes more competitive in response to a larger number of banks the outside option available to borrowers improves. But, this does not mean that the bargaining power of the borrowers would have increased.
3 Takeover timing and terms

3.1 Divestment Phase

Assume that the takeover was initiated at time $\tau$ and that we have reached time $\tau + T$ so that the synergy gains have been realized. At this stage the expected cumulative present value of the future revenues for firm $A$ read as

$$V_3(x) = (R_r \pi_A^x)(x) + D^*(x),$$

where $D^*(x)$ denotes the price of the divested assets. The divestment process associated with the potential divestment of parts of the acquired firm is modelled as a Nash bargaining game. Thus, the terms of the divestment are determined as the solution of the optimization problem

$$\sup_{D \in \mathbb{R}^+} \left[ D - (R_r \pi_A^x)(x) \right]^\beta_d \left[ (R_r \pi^0)(x) - D \right]^{1 - \beta_d},$$

where $\beta_d \in [0, 1]$ is a known exogenously determined parameter measuring the bargaining power of the acquiring firm and $\pi^0(x)$ denotes the profit flow of the outside firm once this has transformed the divested assets into its business operation. By solving (3.2) we find that the Nash bargaining solution is given by

$$D^*(x) = \beta_d (R_r \pi^0)(x) + (1 - \beta_d) (R_r \pi_A^x)(x).$$

The Nash bargaining solution (3.3) exhibits the intuitively appealing feature that the negotiated price is a linearly increasing function of firm $A$’s bargaining power at the divestment stage. Combining (3.3) with the definition of the expected cumulative present value of the future revenues for the firm $A$ (3.1) yields that

$$V_3(x) = (R_r \pi_A^x)(x) + (R_r \pi_A^x)(x) + \beta_d ((R_r \pi^0)(x) - (R_r \pi_A^x)(x)),$$

which in turn implies that

$$(R_r \pi_A^x)(x) + (R_r \pi_A^x)(x) \leq V_3(x) \leq (R_r \pi_A^x)(x) + (R_r \pi^0)(x).$$

Naturally, we also find that $\partial V_3(x)/\partial \beta_d = \partial D^*(x)/\partial \beta_d = (R_r \pi^0)(x) - (R_r \pi_A^x)(x) > 0$ meaning that increased bargaining power of firm $A$ translates into a more valuable divestment opportunity.

3.2 Consolidation Phase

Having determined the expected cumulative present value of the revenues associated with the divestment phase for firm $A$, we now proceed to analyze the consolidation phase $[\tau, \tau + T]$.

We find that the expected cumulative present value of the future revenues from the takeover timing $\tau$ reads as (when conditioned with respect to the information at date $\tau$)

$$V_2(x) = \mathbb{E}_x \int_0^T e^{-rs} (\pi_A^x(X_s) + \pi_T(X_s)) ds + \mathbb{E}_x \left[ e^{-r \lambda T} V_3(X_T) \right],$$

where $T \sim \exp(\lambda)$ is an exponentially distributed random date. Applying the strong Markov property of diffusions implies that this expected cumulative present value can be expressed as

$$V_2(x) = (R_r \pi_A^x)(x) + (R_r \pi^T_T)(x) + (R_r T)(x) + (R_r \Delta)(x),$$

where

$$\Delta(x) = \pi_A^x(x) + \pi_A^x(x) - (\pi_A^x(x) + \pi_T(x)) + \beta_d ((\pi^0(x) - \pi_A^x(x))$$
measures the profit flow associated with the compound synergy gain. This flow consists of two components: the primary takeover gain \( \pi^A(x) + \pi^A(x) - (\pi^A(x) + \pi^T(x)) \) and the value of the subsequent embedded divestment opportunity. The compound synergy gain is naturally positive, since by assumption it holds that \( \pi^A(x) + \pi^A(x) \geq \pi^A(x) + \pi^T(x) \) and \( \pi^A(x) \geq \pi^A(x) \). Thus, we have found that the value of the opportunities represented by the consolidation and the embedded divestment can be re-expressed as

\[
V_2(x) = (R_r \pi^A(x) + (R_r \pi^A(x) + \beta_d((R_r \pi^A(x) - (R_r \pi^A(x)))/(R_r + \Delta))(x).
\]

### 3.3 Takeover Phase: Timing and Price

Having derived the value associated with an optimal post-takeover management of the acquiring firm, we now proceed to characterize the timing and price of the takeover. We first consider the determination of the takeover price. For that purpose we again apply the Nash bargaining approach, whereby the takeover price is determined as the solution of the optimization problem

\[
\sup_{P \in R^+} [P - (R_r \pi^T(x))]^{1-\beta_A} [V_2(x) - P]^{\beta_A},
\]

where \( \beta_A \in [0, 1] \) is a known exogenously determined parameter measuring the bargaining power of the acquiring firm. The solution of this optimization problem reads as

\[
P^*(x) = \beta_A(R_r \pi^T(x) + (1 - \beta_A)V_2(x).
\]

From (3.7) we can directly conclude that the negotiated takeover price is a weighted average of the expected cumulative present value of the target firm’s future revenues and the expected value associated with an optimal post-takeover management of the acquiring firm’s assets. Further, it is worth observing that for all possible states \( x \in R^+ \) it holds that

\[
\frac{\partial P^*}{\partial \lambda}(x) = -(1 - \beta_A) \frac{\partial(R_r + \Delta)}{\partial \lambda}(x) > 0,
\]

since an increase in the intensity \( \lambda \) increases the risk-adjusted discount rate and, therefore, decreases the expected cumulative risk adjusted present value of the profit flow associated with the compound synergy gain. Given the negotiated takeover price (3.7), we now find that the optimal timing of the takeover is determined by the optimization problem

\[
V_1(x) = \sup_{\tau} E_x \left[ \int_0^\tau e^{-r_s \pi^A(X_s)} ds + e^{-r_T(V_2(X_T) - P^*(X_T) - C)} \right],
\]

which can be re-expressed as

\[
V_1(x) = (R_r \pi^A(x) + \sup_{\tau} E_x \left[ e^{-r_T(V_2(X_T) - P^*(X_T) - (R_r \pi^A(X_T) - C)} \right].
\]

It is worth observing that the formulation (3.9) implies that if \( x^* \) is a state at which the takeover opportunity is worth exercising then the classical balance identity

\[
V_1(x^*) + P^*(x^*) + C = V_2(x^*)
\]

holds. This balance identity states that, at the optimum, the value of the opportunity has to coincide with its full costs which, in the present case, are constituted by the sum of the sunk cost \( C \), the takeover price \( P^*(x) \), and the lost option value \( V_1(x^*) \). Given (3.7) we finally find that the timing problem characterizing the optimal date at which the takeover should be exercised reads as

\[
V_1(x) = (R_r \pi^A(x) + \sup_{\tau} E_x \left[ e^{-r_T \beta_A(V_2(X_T) - (R_r \pi^T(X_T)) - (R_r \pi^A(X_T) - C)} \right],
\]

6
which can be expressed in terms of the takeover price as

\[ V_1(x) = (R_r \pi^A)(x) + \sup_{\tau} \mathbb{E}_x \left[ e^{-\tau r} \left( \frac{\beta_A}{1 - \beta_A} (P^*(X_\tau) - (R_r \pi^T)(X_\tau)) - (R_r \pi^A)(X_\tau) + C \right) \right] \]

Applying the definition of the value of the opportunities represented by the consolidation and the embedded divestment finally implies that the value of the takeover opportunity (3.10) can be re-expressed as

\[ V_1(x) = (R_r \pi^A)(x) + \sup_{\tau} \mathbb{E}_x \left[ e^{-\tau r} (\Pi(X_\tau) - C) \right], \quad (3.11) \]

where \( \Pi(x) = \beta_A((R_r \Delta)(x) - (R_{r+\lambda} \Delta)(x)) - (1 - \beta_A)(R_r \pi^A)(x) \) denotes the expected cumulative present value of the profit flow associated with the compound real option constituted by the takeover and the subsequent divestment opportunities. \( \Pi(x) \) consists of two components. The first component \( \beta_A((R_r \Delta)(x) - (R_{r+\lambda} \Delta)(x)) \) measures the expected net gain associated with the acquisition of the opportunities of the primary takeover gain and the subsequent divestment gain. Second, the term \( (1 - \beta_A)(R_r \pi^A)(x) \) measures the direct costs associated with the takeover negotiations since the proportion \( (1 - \beta_A)(R_r \pi^A)(x) \) of the pre-takeover revenues are sacrificed in the negotiations.

Before proceeding in the analysis of the takeover, we explore the properties of \( \Pi(x) \) as a function of the bargaining power \( \beta_A \). We notice that \( \Pi(x) \) is a linear and increasing function of \( \beta_A \). In particular, we find that for all \( x \in \mathbb{R}_+ \) it holds that

\[-((R_r \pi^A)(x) + C) < \Pi(x) - C < (R_r \Delta)(x) - (R_{r+\lambda} \Delta)(x) - C.\]

Consequently, with a sufficiently low bargaining power and independently of the prevailing state of nature the compound takeover option has no value. Therefore, under such circumstances no takeover would take place. In light of the linearity and monotonicity of \( \Pi(x) \) as a function of \( \beta_A \), the compound real option represented by the takeover has a positive value whenever the condition

\[ \beta_A > \frac{C}{(R_r \pi^A)(x) - (R_{r+\lambda} \Delta)(x) - (R_r \Delta)(x) + 1} > 0 \quad (3.12) \]

is satisfied. Thus, condition (3.12) determines a critical bargaining power above which it may be worthwhile to exercise the compound takeover option, depending on the prevailing state of nature. This critical bargaining power can be seen as a Marshallian break-even trigger conditional on a given state of nature. Furthermore, from (3.12) we can conclude that the critical bargaining power for the exercise of the compound option is decreasing as a function of the expected sum of the primary takeover gain and the embedded divestment gain. In particular, we see that the presence of the embedded divestment opportunity lowers this critical bargaining power. Put somewhat differently, we can conclude that an increased bargaining power at the divestment stage decreases the critical bargaining power at the takeover stage by increasing the expected net gain associated with the acquisition of the opportunities of the primary takeover gain and the subsequent divestment gain.

From condition (3.12) we can conclude that there is an intimate relationship between the magnitude of the imperfections in the market for corporate control and the takeover incentives. With sufficiently small imperfections in the market for corporate control the acquiring firm has no incentive to exercise the takeover option independently of the prevailing state of nature. This would be the case if the market for corporate control would be sufficiently close to perfect competition. Furthermore, as condition (3.12) makes clear, the acquiring firm may find it optimal to exercise a takeover option, and thereby promote an efficiency-enhancing reallocation of corporate assets only if there are sufficiently strong imperfections in the market for corporate control. Under such circumstances there is a trigger with respect to the state of nature above which takeover
options are exercised. In light of these arguments we can conclude that the issue of whether a takeover option is exercised or not depends on the combined effect of the imperfections in the market for corporate control and the prevailing state of nature.

We next focus on cases where the acquiring firm has a sufficiently strong bargaining power so that condition (3.12) holds and, consequently, such that the market for corporate control enables a takeover to be realized. Below we state a set of reasonable sufficient conditions under which an optimal takeover policy and its value can be characterized. This takeover policy is expressed in terms of a unique exercise threshold (with respect to the prevailing state of nature) above which the takeover is exercised.

**Theorem 3.1.** Assume that \( \Pi(x) \) is non-decreasing and that there is a single threshold \( \bar{x} \in \mathbb{R}^+ \) such that

\[
\frac{1}{2} \sigma^2 x^2 \Pi''(x) + \mu x \Pi'(x) \geq r (\Pi(x) - C), \quad x \leq \bar{x}.
\]

Then there is a unique optimal takeover threshold

\[
x^* = \arg\max \{(\Pi(x) - C)x^{-\psi}\} > \bar{x}
\]

satisfying the ordinary first order condition \( \Pi'(x^*)x^* = \psi(\Pi(x^*) - C) \), where

\[
\psi = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 0
\]

denotes the positive root of the fundamental quadratic equation \( \sigma^2 a(a - 1) + 2 \mu a - 2r = 0 \). Moreover, the value of the optimal takeover policy reads as

\[
V_1(x) = \begin{cases} 
\Pi(x) - C & x \geq x^* \\
\frac{\Pi'(x^*)}{\psi} x^{1-\psi} & x < x^*.
\end{cases}
\]

**Proof.** See Appendix A.

The sufficient condition (3.13) included in Theorem 3.1 can be interpreted in terms of the expected rate of return (per unit invested) associated with the takeover. More precisely, since the expected rate of return from the takeover can be expressed as

\[
\frac{1}{dt} \mathbb{E}_x [d(\Pi(X_t) - C)] = \frac{1}{2} \sigma^2 X_t^2 \Pi''(X_t) + \mu X_t \Pi'(X_t),
\]

we observe that the sufficient condition (3.13) means that the expected rate of return per unit invested is required to exceed (fall short of) the risk free rate of return at unfavorable (favorable) states of nature. This condition essentially rules out the situations where the takeover opportunity would dominate the risk free investments at all states and vice versa. It is also worth noticing that this idea can be captured through the reformulation of (3.13) according to the condition

\[
(1 - \beta_A)\pi^A(x) + r C \geq \beta_A \lambda(R_{r+A} \Delta)(x), \quad x \leq \bar{x}.
\]

In line with both the theoretical and the empirical literature (for example, Lambrecht (2004), Morelli and Zhdanov (2004) and Maksimovic and Phillips (2001)), Theorem 3.1 establishes that favorable states of nature stimulate the takeover activities. In this respect our model predicts procyclical takeover activities.\(^2\) Furthermore, as we have made clear above, the takeover activities

\(^2\)However, except for this general procyclical feature our model is not designed to investigate merger waves. For updated studies of merger waves we simply refer to Holmström and Kaplan (2001) or Jovanovic and Rousseau (2004).
cannot be analyzed in isolation from the imperfections in the market for corporate control. The imperfections in the market for corporate control must be sufficiently strong, as determined by the threshold (3.12) for the acquiring firm’s bargaining power, for any takeover activities to potentially be initiated.

Let us further briefly characterize the bargaining power threshold (3.12), below which it is never optimal for the acquiring firm to exercise the compound takeover option. This critical value is lower, the higher is the potential net gain associated with the exploitation of the primary takeover option or the subsequent embedded divestment option. In particular, the shorter the expected lag \((\lambda - 1)\) or the lower the variance \((\lambda - 1)\) of the time required until the primary synergy benefits are realized, the lower is the critical threshold (3.12). Analogously, a higher bargaining power of the merged firm or higher potential divestment gains will, in general, induce a lower value of the threshold (3.12).

Finally, it should be emphasized that the impact of increased volatility \(\sigma\) on the optimal takeover timing is typically ambiguous. In general, it is hard to characterize the sign of the relationship between the optimal timing for the exercise of a compound option and the volatility of the underlying stochastic process. The major complication explaining this ambiguity has its roots in the fact that the values of the embedded options depend on the volatility due to the potential nonlinearity of the associated cash flows. In a somewhat related setting Alvarez and Stenbacka (2004) show how the optimal timing for the exercise of a compound option represents the interplay between two effects: 1. the real option effect associated with the ability to postpone the exercise of an investment opportunity and 2. the Jensen’s inequality effect associated with the complicated form of the expected cumulative present value of the profit flow associated with the compound real option constituted by the takeover and the subsequent divestment opportunities. We return to this issue in greater detail for a particular class of profit flows in the next section.

4 Profit Flows with a Multiplicatively Separable Stochastic Component

We now present an explicit version of our analysis by focusing on the class of profit flows with a multiplicatively separable stochastic component. Assume therefore that all considered cash flows are of the form 

\[
\pi^A_d(x) = \pi^A_d x^\eta, \quad \pi^A_c(x) = \pi^A_c x^\eta, \quad \pi^A(x) = \pi^A x^\eta, \quad \pi^T(x) = \pi^T x^\eta, \quad \text{and} \quad \pi^0(x) = \pi^0 x^\eta,
\]

where \(\eta \in \mathbb{R}_+\) is a known exogenously given parameter. If the absence of speculative bubbles condition \(r > \mu \eta + \frac{1}{2} \eta(\eta - 1)\sigma^2\) is met, the solution of the divestment game reads as

\[
D^*(x) = \left[\beta_d \pi^0 + (1 - \beta_d) \pi^A_d\right] \frac{x^\eta}{r - \delta(\eta)},
\]

where \(\delta(\eta) = \mu \eta + \frac{1}{2} \eta(\eta - 1)\sigma^2\) denotes the expected percentage growth rate of the cash flow \(x^\eta\). Under these circumstances, the value of firm A at the divestment stage can be expressed as

\[
V_3(x) = \left[\pi^A_c + \beta_d \pi^0 + (1 - \beta_d) \pi^A_d\right] \frac{x^\eta}{r - \delta(\eta)}.
\]

Having derived the value of the corporation at the divestment stage, we can now demonstrate that the value of the firm at the consolidation stage reads as

\[
V_2(x) = \left[(\pi^A + \pi^T) + \frac{\lambda \left(\pi^A_d \pi^A - (\pi^A + \pi^T) + \beta_d (\pi^0 - \pi^A_d)\right)}{(r + \lambda - \delta(\eta))}\right] \frac{x^\eta}{r - \delta(\eta)}.
\]

As is intuitively clear, the first term of this expression captures the expected cumulative present value of the revenues accrued during the consolidation phase. The second term measures the net gain from exercising the divestment opportunity at an exponentially distributed random date in the future.
We can now proceed to the analysis of the timing and terms of the takeover. We first determine the negotiated takeover price. We immediately observe that in the present case, the solution of the takeover game (i.e. the Nash bargaining solution) reads as

\[ P^*(x) = \frac{\beta_A x^\eta}{r - \delta(\eta)} + (1 - \beta_A)V_2(x) \]

implying that

\[ V_2(x) - P^*(x) = \beta_A \left[ \pi^A + \frac{\lambda (\pi_c^A + \pi_d^A - (\pi_c^A + \pi_d^A) + \beta_d(\pi_0^A - \pi_d^A))}{(r + \lambda - \delta(\eta))} \right] \frac{x^\eta}{r - \delta(\eta)}. \]

Hence, the optimization problem characterizing the optimal takeover timing reads as

\[ V_1(x) = \frac{\pi^A x^\eta}{r - \delta(\eta)} + \sup_{\tau} \mathbb{E}_x \left[ e^{-\tau r} \left( M(\eta, \lambda, \sigma, \beta_A, \beta_d) \frac{X^\eta}{r - \delta(\eta)} - C \right) \right], \]

where the multiplier \( M(\eta, \lambda, \sigma, \beta_A, \beta_d) \) is given by

\[ M(\eta, \lambda, \sigma, \beta_A, \beta_d) = \frac{\lambda \beta_A}{r + \lambda - \delta(\eta)} \theta - (1 - \beta_A)\pi^A, \]

and \( \theta = \pi_c^A + \pi_d^A - (\pi_c^T + \pi_d^A) + \beta_d(\pi_0^A - \pi_d^A) \) denotes the sum of the profit flows associated with the primary takeover gain and the embedded divestment gain. The multiplier \( M(\eta, \lambda, \sigma, \beta_A, \beta_d) \) captures that we have to give up the proportion \( 1 - \beta_A \) of the pre-takeover revenues in order to gain access to the proportion \( \beta_A \) of the net gain associated with the acquisition of the primary takeover option and the subsequent divestment option. In contrast to standard real option applications this multiplier now depends on the strategic pricing elements \((\beta_A, \beta_d)\), the measure of implementation uncertainty \((\lambda)\), and the volatility \((\sigma)\) of the underlying random state variable. Standard differentiation shows that this multiplier is an increasing function of the bargaining power of the acquiring firm both at the takeover stage and at the divestment stage as well as of the expected lag or the variance of the time required until the takeover gains are realized. Formally, we find that

\[
\frac{\partial M(\eta, \lambda, \sigma, \beta_A, \beta_d)}{\partial \lambda} = \frac{(r - \delta(\eta))\beta_A}{(r + \lambda - \delta(\eta))^2} (\pi_c^A + \pi_d^A - (\pi_c^T + \pi_d^A) + \beta_d(\pi_0^A - \pi_d^A)) > 0
\]

\[
\frac{\partial M(\eta, \lambda, \sigma, \beta_A, \beta_d)}{\partial \beta_A} = \frac{\lambda}{r + \lambda - \delta(\eta)} (\pi_c^A + \pi_d^A - (\pi_c^T + \pi_d^A) + \beta_d(\pi_0^A - \pi_d^A)) + \pi^A > 0
\]

\[
\frac{\partial M(\eta, \lambda, \sigma, \beta_A, \beta_d)}{\partial \beta_d} = \frac{\lambda \beta_A(\pi_0^A - \pi_d^A)}{r + \lambda - \delta(\eta)} > 0.
\]

Furthermore, it holds that

\[
\frac{\partial M(\eta, \lambda, \sigma, \beta_A, \beta_d)}{\partial \sigma} = \frac{\lambda \beta_A \theta \eta (\eta - 1)}{(r + \lambda - \delta(\eta))^2} \geq 0, \quad \eta \geq 1.
\]

This means that volatility increases (decreases) the multiplier when the profit flow is convex (concave) as a function of the state variable. In particular, in the absence of implementation uncertainty \((\lambda \rightarrow \infty)\) the multiplier is independent of the volatility.

The form of the multiplier implies that the exercise payoff associated with the optimal takeover decision is negative and, therefore, that the takeover option is valueless and never exercised as long as \( \beta_A < \beta_A \), where

\[
\beta_A = \frac{1}{1 + \frac{\lambda}{(r + \lambda - \delta(\eta))} \frac{\sigma^2}{2}}.
\]
Since $\theta$ is an increasing function of the bargaining power $\beta_d$, it follows that
\[ \frac{\partial \hat{\beta}_A}{\partial \beta_d} < 0. \]

Therefore, we can deduce that increased bargaining power in the divestment phase decreases the critical bargaining power $\hat{\beta}_A$. This feature is illustrated in Figure 1 where the marked region represents the set of bargaining powers where a takeover is ruled out. In addition, we find that the critical bargaining power $\hat{\beta}_A$ is a decreasing function of the implementation intensity $\lambda$ meaning that
\[ \frac{\partial \hat{\beta}_A}{\partial \lambda} < 0. \]

In other words, increased implementation uncertainty raises the required bargaining power of the acquiring firm for a takeover to potentially take place.

Our main result on the present example is now summarized in the following.

**Theorem 4.1.** Assume that $\beta_A > \hat{\beta}_A$. Then there is a unique optimal takeover threshold
\[
x^* = \left( \frac{\psi(r - \delta(\eta))C}{(\psi - \eta)M(\eta, \lambda, \sigma, \beta_A, \beta_d)} \right)^{1/\eta}
\]
satisfying the ordinary first order condition
\[
\frac{M(\eta, \lambda, \sigma, \beta_A, \beta_d)x^\eta}{(r - \delta(\eta))} = \psi \left( \frac{M(\eta, \lambda, \sigma, \beta_A, \beta_d)x^\eta}{(r - \delta(\eta))} - C \right).
\]

In particular, this optimal takeover threshold satisfies the comparative statics
\[
\frac{\partial x^*}{\partial \lambda} < 0, \quad \frac{\partial x^*}{\partial \beta_A} < 0, \quad \frac{\partial x^*}{\partial \beta_d} < 0.
\]

The value of the optimal takeover policy reads as
\[
V_1(x) = \frac{\pi^A x^\eta}{r - \delta(\eta)} + \begin{cases} \frac{M(\eta, \lambda, \sigma, \beta_A, \beta_d)x^\eta}{(r - \delta(\eta))} - C & x \geq x^* \\ \frac{M(\eta, \lambda, \sigma, \beta_A, \beta_d)x^\eta}{(r - \delta(\eta))} \left( \frac{x}{x^*} \right)^\psi & x < x^*. \end{cases}
\]

Theorem 4.1 demonstrates that the takeover threshold is a decreasing function of the acquiring firm’s bargaining power at the takeover stage and at the divestment stage. Thus, increased market imperfections in the market for corporate control, no matter whether this refers to the takeover stage or the divestment stage, raise the required bargaining power of the acquiring firm for a potential takeover to take place.
stage or the divestment stage, promotes the takeover activity. Also, from Theorem 4.1 we can infer that the takeover threshold is an increasing function of the expected lag or the variance of the time required until the takeover gains are realized. Thus, increased difficulties or uncertainty with respect to the implementation of the potential takeover gains tend to slow down the takeover activities.

As we saw in the previous section it is in general very hard, if possible at all, to characterize the sign of the relationship between increased volatility and the optimal takeover policy. For the particular class of profit flows considered in this section we can, however, establish the following.

**Lemma 4.2.** Assume that $\beta_A > \hat{\beta}_A$.

(i) If $\eta \geq 1$ then increased volatility increases the value of the takeover opportunity.

(ii) If $\eta < 1$ and $\delta(\eta) < r < \mu$ then increased volatility decreases the value of the takeover opportunity.

*Proof.* See Appendix B. □

It should be observed that Lemma 4.2 does not characterize the sign of the relationship between increased volatility and the value of the optimal takeover policy for all possible parameter configurations. More precisely, the impact of increased volatility on the value of the optimal takeover policy is ambiguous for $\eta < 1$ and $r > \mu$. Namely, under these conditions increased volatility decreases the factor $M(\eta, \lambda, \sigma, \beta_A, \beta_d)/(r - \delta(\eta))$ while it simultaneously increases the factor $(x/x^*)^\theta$ (measuring the price of a zero-coupon bond maturing at the takeover date) so that the overall effect is unclear.

In order to characterize the sensitivity of the optimal takeover threshold to changes in the volatility of the underlying stochastically fluctuating state variable we first observe that the optimal threshold can be re-expressed as

$$x^\eta = \left(1 - \frac{\eta}{\varphi}\right) \frac{rC}{M(\eta, \lambda, \sigma, \beta_A, \beta_d)},$$

where

$$\varphi = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$$

denotes the negative root of the fundamental quadratic equation $\sigma^2 a(a - 1) + 2\mu a - 2r = 0$. Thus, standard differentiation implies that

$$\frac{\partial x^\eta}{\partial \sigma} = -\frac{\eta r C}{\varphi^2 M(\eta, \lambda, \sigma, \beta_A, \beta_d)} \frac{\partial \varphi}{\partial \sigma} - \left(1 - \frac{\eta}{\varphi}\right) \frac{r CM(\eta, \lambda, \sigma, \beta_A, \beta_d)}{M^2(\eta, \lambda, \sigma, \beta_A, \beta_d)},$$

where

$$\frac{\partial \varphi}{\partial \sigma} = \frac{2\varphi(\varphi - 1)}{\sigma(\varphi - \varphi)} > 0.$$  

Consequently, we can decompose the total effect of increased volatility on the optimal takeover threshold as the sum of two separate effects. The first term is proportional to the positive effect $(\partial \varphi/\partial \sigma > 0)$ of increased volatility on the standard option multiplier. The second term captures the effect on the volatility-dependent multiplier, $M(\eta, \lambda, \sigma, \beta_A, \beta_d)$. As we have shown prior to Theorem 4.1 it holds that the sign of $M(\eta, \lambda, \sigma, \beta_A, \beta_d)$ is determined by whether $\eta$ exceeds or falls short of 1. Since the standard option multiplier $1 - \eta/\varphi$ is always positive we can draw the following conclusion.

**Theorem 4.3.** Assume that $\beta_A > \hat{\beta}_A$. The total effect of increased volatility on the optimal threshold can be decomposed into two potentially offsetting effects: (1) an effect proportional to the effect of the standard option multiplier $1 - \eta/\varphi$ and (2) an effect proportional to the volatility-dependent multiplier $M(\eta, \lambda, \sigma, \beta_A, \beta_d)$. In particular, increased volatility decelerates the takeover if
(a) the profit flow is concave ($\eta \leq 1$), or
(b) the implementation uncertainty disappears ($\lambda \to \infty$).

From Theorem 4.3 we can conclude that the effect of increased volatility converges towards that predicted by conventional real option models in the sense of always decelerating the takeover investment if the implementation uncertainty disappears. However, in the presence of implementation uncertainty we can see that the optimal takeover threshold is determined as the interplay between two effects. These effects operate in the same direction if the profit flow is concave, whereas they operate in different directions if the profit flow is strictly convex. Thus, the presence of implementation uncertainty, captured by the parameter $\lambda$, introduces a qualitatively important feature as it makes the effects of volatility dependent on the functional form (convexity or concavity) whereby the profit flow depends on the stochastic component. While increased volatility may increase the expected value of the primary takeover option it simultaneously may increase the value of the subsequent divestment option. Since the latter effect may dominate the former, the overall impact of increased volatility on the optimal takeover policy is, a priori, ambiguous. In light of the explicit analysis presented in this section it should be emphasized how the delay in the primary takeover gains and the embedded divestment option, and the negotiations over the associated surpluses, may result in situations where increased volatility makes the expected profitability of a takeover decrease despite its potentially positive impact on the value of waiting. To our best knowledge this is a novel finding contributing to the real options literature.

5 Concluding Comments

In this paper we have designed a compound real options model, which determines the timing of takeovers and characterizes the distribution of the surplus created through the asset restructuring. Relative to the existing literature we have emphasized the relationship between the imperfections in the market for corporate control and the takeover incentives within the framework of a model with two distinguishable aspects, which have been neglected in the literature so far. First, our model incorporates implementation uncertainty with respect to the time required until the merger synergies are realized. Second, the price and timing of the takeover is determined using a compound real option approach, which incorporates the divestment opportunity of non-core business activities as an embedded option.

We establish a relationship between the imperfections in the market for corporate control and the takeover incentives. More precisely, we characterize a critical bargaining power below which the compound takeover option is never exercised and above which the acquiring firm may find it optimal to exercise the compound option if the state of nature is sufficiently favorable. This threshold with respect to the bargaining power is a decreasing function of the expected net gain associated with the primary takeover gain and the embedded divestment gain and an increasing function of the expected lag or the variance of the time required until the takeover gains are realized. Further, we have demonstrated that the presence of implementation uncertainty has an important qualitative impact on the relationship between the optimal timing of takeovers and market volatility. Namely, with implementation uncertainty the effect of volatility on the optimal takeover timing depends on the way by which the stochastic component affects the profit flow. Finally, we find takeovers to be a procyclical activity.

In this paper we have analyzed the determinants of takeovers within the framework of a model which has emphasized the underlying process of industrial restructuring. However, our analysis is carried out in the absence of an institutional description of the capital market and the informational asymmetries prevailing therein. For that reason our study did not generate

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3This feature seems somewhat related to Alvarez and Stenbacka (2004), where it was established that the incentives for switching from one stochastic flow to a more volatile one are determined by how the stochastic component enters the profit flow of the firm.
predictions concerning phenomena like merger waves and announcement premia associated with
takeovers. As mentioned earlier these phenomena have been studied by, for example, Holmström

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A Proof of Theorem 3.1

**Proof.** Denote the proposed value function as $F(x)$. Since $F(x)$ can be re-expressed as

$$F(x) = \mathbb{E}_x \left[ e^{-\tau^*} (\Pi(X_{\tau^*}) - C) \right],$$

where $\tau^* = \inf\{t \geq 0 : X_t \geq x^*\}$, we naturally have that $V_1(x) \geq F(x)$. It is, therefore, sufficient to establish the opposite inequality. To accomplish this task, we first observe that the proposed value function is continuously differentiable on $\mathbb{R}_+$, twice continuously differentiable on $\mathbb{R}_+ \setminus \{x^*\}$, and satisfies the inequality $F''(x^\pm) < \infty$. Moreover, $(AF)(x) = rF(x)$ for all $x \in (0, x^*)$, where

$$A = \frac{1}{2} \sigma^2 x^2 \frac{d^2}{dx^2} + \mu x \frac{d}{dx}$$

denotes the differential operator associated with the underlying geometric Brownian motion. It is, therefore, sufficient to establish that $x^* \neq \bar{x}$ since in that case $(AF)(x) - rF(x) = (A\Pi)(x) - r(\Pi(x) - C) \leq 0$ for all $x > x^* > \bar{x}$. To prove that this is indeed the case, consider the mapping

$$G(x) = \Pi'(x)x^{1-\varphi} - \psi x^{-\varphi}(\Pi(x) - C) = x^{1+\varphi - \varphi} \frac{d}{dx} [\psi^{-1}(\Pi(x) - C)],$$

where $\psi$ denotes the positive and

$$\varphi = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$$

denotes the negative root of the fundamental quadratic equation $\sigma^2 a(a - 1) + 2\mu a - 2r = 0$. Since $x^{-\varphi} \downarrow 0$ as $x \downarrow 0$ we observe that our assumption on the monotonicity of $\Pi(x)$ implies that $\lim_{x \to 0} G(x) \geq 0$. Moreover, standard differentiation yields

$$G'(x) = \frac{2}{\sigma^2} x^{-\varphi - 1} \left[ \frac{1}{2} \sigma^2 x^2 \Pi''(x) + \mu x \Pi'(x) - r(\Pi(x) - C) \right] \leq 0 \quad x \leq \bar{x}$$

which, in turn, implies that if equation $G(x) = 0$ has a root it must be above the threshold $\bar{x}$. In order to establish that such a root indeed exists and is unique we notice that if $x > K > \bar{x}$ then the mean value theorem for integrals implies that

$$G(x) = G(K) + \frac{2}{\sigma^2} \int_K^x y^{-\varphi - 1} \left( (A\Pi)(y) - r(\Pi(y) - C) \right) dy$$

$$= G(K) + 2 \frac{((A\Pi)(\xi) - r(\Pi(\xi) - C))}{\sigma^2 \varphi} \left[ K^{-\varphi} - x^{-\varphi} \right],$$

where $\xi \in (K, x)$. Letting $x \to \infty$ then finally implies that $G(x) \downarrow -\infty$ and, therefore, that equation $G(x) = 0$ has a unique root on the set $(\bar{x}, \infty)$. These observations imply that $F(x)$ constitutes a $r$-excessive majorant of the exercise payoff $\Pi(x) - C$. Since $V_1(x)$ is the least of these majors, we find that $V_1(x) \leq F(x)$ which demonstrates that $V_1(x) = F(x)$. \qed

B Proof of Lemma 4.2

**Proof.** Standard differentiation yields

$$\frac{\partial \Pi(x)}{\partial \sigma} = \frac{(r - \delta(\eta)) M_x(\eta, \lambda, \sigma, \beta_A, \beta_B) + \sigma \eta (\eta - 1) M(\eta, \lambda, \sigma, \beta_A, \beta_B)}{(r - \delta(\eta))^2} x^\eta \quad \forall x > 0, \quad \eta \geq 1.$$

Consider now the strategies characterized by the threshold policy stopping times $\tau_y = \inf\{t \geq 0 : X_t \geq y\}$, where $\Pi(y) \geq C$, and denote the value of such takeover strategy as $J(x, y) =$
\( \mathbb{E}_x \left[ e^{-rt_x}(\Pi(X_{t_x}) - C) \right] \). It is now a standard exercise to establish that this value can be re-expressed as

\[
J(x, y) = \begin{cases} 
\Pi(x) - C & x \geq y \\
(\Pi(y) - C)(x/y)^\psi & x < y.
\end{cases}
\]

Standard differentiation now yields that

\[
\frac{\partial J(x, y)}{\partial \sigma} = \begin{cases} 
\frac{\partial \Pi(x)}{\partial \sigma} & x \geq y \\
\frac{\partial \Pi(y)}{\partial \sigma} (x/y)^\psi + (\Pi(y) - C)(x/y)^\psi \ln(x/y) \frac{\partial \psi}{\partial \sigma} & x < y,
\end{cases}
\]

where

\[
\frac{\partial \psi}{\partial \sigma} = \frac{2\psi(1 - \psi)}{\sigma(\psi - \varphi)} \geq 0, \quad r \leq \mu.
\]

If \( \eta \geq 1 \) then the absence of speculative bubbles condition \( r > \delta(\eta) \) implies that \( r > \mu \) and, therefore, that increased volatility increases the value \( J(x, y) \). Since \( J(x, x^*) = V_1(x) \) we find that increased volatility increases the value of the optimal takeover strategy as well. Establishing that increased volatility decreases the value of the optimal takeover strategy whenever the conditions \( \eta < 1 \) and \( \delta(\eta) < r < \mu \) are satisfied is analogous.
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