Estimating the Behavioural Equilibrium Exchange Rate of EUR/USD using a Monetary Model with Labour Productivity

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**ABSTRACT:**  
This study estimates the behavioural equilibrium exchange rate (BEER) (Clark & McDonald, 1998) of the Euro-US Dollar currency pair in the period Q1 2000-Q4 2011. The BEER is based on the standard monetary model (Frenkel, 1976) with an interest rate differential (Frankel, 1979). Additionally, it includes a labour productivity regressor, defined as the GDP per hour work (Hilferding, 1981) of the two economies.

Two separate cointegration regressions are estimated. The one with an exogenous labour productivity represents the main model. It shows that +1% change in the money supply differential decreases the euro with -0.23%. The relative future CPI differential (in USD) decreases the dollar, respectively increases the euro, with 1.3% in the current period. The VECM yields that +1% change in the productivity differential would increase the euro with +0.46% in a short-run. The interest rate differential is estimated to be positive, but insignificant.

The case of comparison represents a cointegration with endogenous labour productivity. It shows that +1% change in its differential affects the long-run EUR/USD with +0.10%. The variable, however, is insignificant.

Based on the BEER estimation from the main model, the euro is found to be overvaluated in 2003-2005 and 2008; undervalued in 2001-2003, 2005-2007, 2009- onwards; and close to its equilibrium in end-2009 and mid-2011.

**KEYWORDS:** EURUSD, BEER, equilibrium, misalignment, cointegration, monetary model, labour productivity, overvaluation, undervaluation
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LIST OF ABBREVIATIONS

AIC- Akaike Information Criterion
ACF- Autocorrelation Function
ADF-Augmented Dickey-Fuller Test
BEER –Behavioural Equilibrium Exchange Rate
CA- Current Account
CB- Central bank
CPI-Consumer Price Index
DM- Deutsche Mark
ECB- European Central Bank
ECT- Error-Correction Term
EMU- European Monetary Union
EUR- Euro
EZ- Eurozone (17countries)
FX- Foreign Exchange
GDP – Gross Domestic Product
HEGY - Hylleberg-Engle-Granger-Yoo Test
Log- Logarithm
PACF- Partial Autocorrelation Function
PPP- Purchasing Power Parity
QE- Quantitive Easing
RER- Real Exchange Rate
UIP- Uncovered Interest Rate Parity
US- the United States
USD- US Dollar
VAR- Vector Autoregression
VECM- Vector Error-Correction Model
INTRODUCTION

The euro decreased sharply right after it was physically circulated in the market on 01.01.2000 (De Grauwe & Grimaldi, 2006). This fact, together with the euro-dollar fluctuations in the following years, generated an increased academic interest towards estimating the true value of the euro. The broad economic research attempted to estimate whether the euro is correctly priced, whether its movements corresponded to the underlying fundamentals, or whether they simply represented misalignments. These types of questions required the introduction of some measure of sustainable equilibrium. The attention towards the concept of equilibrium rate increased during the first years of the euro`s circulation (MacDonald, 2000).

This study estimates the behavioural equilibrium exchange rate (BEER), developed by Clark & MacDonald (1999), in an attempt to examine the deviation of the actual EUR/USD exchange rate from its sustainable equilibrium values. In other words, it investigates whether this pair converges to its equilibrium level. If the exchange rate is misaligned, then this estimation would reveal how the euro deviates.

The variables in the econometric estimations are the ones defined by the standard monetary model (Frenkel, 1976) with interest rates (Frankel, 1979). Additionally, this study introduces the labour productivity as a new variable. It is defined as the output of one country divided by the hours spent for its realization. The novelty of this approach corresponds to the fact that both- prices and productivity- are presented in the monetary equation. The distinction between values originating from work and prices is essential. This alternative method (Basso, 2003) is based on the concept of currency value inspired by the work of Rudolf Hilferding (1981).

In his major study Das Finanzkapital, Hilferding attempts to explain the loss of importance of the gold and silver standards after the World War I. His economic views were promptly against the demolishment of the gold standard, because the latter allows transparent price creation between countries based on hours of work for their production. Thus, faced with the growing significance of paper currencies (and hyperinflation in Germany), Hilferding proposes the concept of currency value. It represents a division of the output of one country by the sum of hours of work spent to produce it. This study will use the productivity variable as suggested by Hilferding in an attempt to better estimate the equilibrium exchange rate of the EUR/USD. As mentioned above, the monetary model does not incorporate in itself any measure of
productivity. There is, as well, a growing need of suitable models, aiming to better-explain the exchange rate behaviour.

The study is structured as follows. Chapter 1 provides a brief overview of the basic concepts regarding the exchange rate between two currencies. Chapter 2 discusses the different methods for modeling the real equilibrium exchange rate. Chapter 3 introduces the theoretical framework of this study. Chapter 4 provides a brief overview of the econometric methodology (unit root test, cointegration), later to be used in the estimations. Chapter 5 provides the actual econometric estimations. The BEER and the total misalignment are calculated in Chapter 6. This section also includes the concluding discussion of the results.
1 DEFINING THE EXCHANGE RATE

1.1 NOMINAL EXCHANGE RATE

The nominal exchange rate represents the price of a domestic currency measured in terms of a foreign currency. It indicates how much foreign money units would be received for a domestic money unit (or vice versa), but it does not show the purchasing power of neither currencies. The Central Bank (CB) expresses the nominal exchange rate with an official quote which is always shown bilaterally in a pair. This results from the fact that in every foreign exchange (FX) transaction, there is simultaneously buying of one currency and selling of another. The bilateral nominal exchange rate is established on the FX market as a result of matching demand and supply, or via interbank transactions. In this latter case, the CB acts usually as one of the counterparties of this relationship (Piana, 2001).

The FX rate can be expressed in two ways, every time in which the first currency would represent the base for the trading. It is a constant - always set to be equal to 1. The second currency is called “term,” ”counter”, ”payable” or “quote”, because it varies during the time. The only two possible quotations are:

- Direct (also called American) – the domestic currency is expressed per unit foreign currency
- Indirect – how many units of foreign currency are used for the purchasing of one unit domestic currency.

Their relationship is expressed below.

\[ \text{Indirect quotation} = \text{Direct quotation}^{-1} \]

The FX market has no natural numeraire currency. Therefore, there are no strictly defined rules for determining the order of quotation. This choice is purely market convention (Clark, 2011) and depends on the CB`s preferences of how to express the domestic currency. It can be argued, that the indirect quotation is more practical. Considering Occam`s razor principle, holding the domestic currency as base per se, provides a direct insight of how its relative value changes over time. Another benefit of

\[ \text{Indirect quotation} = \text{Direct quotation}^{-1} \]

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1 CBs compute also multilateral exchange rates between a domestic and a weighted- average basket currency of several other countries, which are usually important trading partners.

2 The principle states that other things being equal, one should proceed to simpler theories until the simplicity can be traded for greater explanatory power.
using the indirect quoting is that it allows an instant conversion of financial results into domestic currency unit, i.e. all business profits and losses are directly priced into home money.

The indirect quotation is used by the European Central Bank (ECB) for all exchange cross-rates with the euro since its actual introduction within the European Monetary Union (EMU)\(^3\). Throughout this study, therefore, the nominal exchange rate will be expressed indirectly as the price of one euro in terms of dollars.

\[ \text{EUR/USD } 1.4745 \]

\( \text{Source: Oanda} \)

In this perspective, an appreciation of the EUR/USD currency pair would mean that the USD depreciates, implying that more dollar units would be required for the purchase of 1 euro unit. Simultaneously, this would result into a nominal appreciation of the euro.

The EUR/USD graph will exhibit an increasing pattern. Reciprocally, if the FX rate changes in a way that 1 EUR purchases less USD units, this would represent a depreciation of the euro relative to the dollar (Mankiw, 2011). The latter would manifest itself as a decreasing EUR/USD graph.

1.2 REAL EXCHANGE RATE

The real exchange rate (RER) between two countries represents the combined values of all domestic against foreign production at the prevailing (nominal) FX rate. Stated differently, the RER measures the rate at which all home goods and services can be traded relative to the foreign ones. Expressed in formula, this relationship would be as follows

\[ \text{RER} = \frac{P_h}{P_f} \]

\(^3\) According to the established heuristic rules for currency quotation on the FX market, the hierarchy for determining the base currency is EUR>GBP>AUD>NZD>USD>CAD>CHF>JPY (Clark, 2011)
\[ q = \frac{s p}{p^*} \text{ or } \log q = \log s + \log p - \log p^* \tag{1.1} \]

where \( q \) is the RER, \( s \) is the nominal exchange rate, \( p \) and \( p^* \) are the relative price of domestic and foreign consumption baskets.

The RER’s crucial difference with the nominal exchange rate lies within the observed item of trade. In this case, the buyer is interested in what can be bought with the purchased currency. Thus considering the RER, it is not the currency itself, but the prices of foreign goods per domestic ones (Mankiw, 2011). Therefore, higher domestic prices would mean an appreciation in the RER of the home currency, other things equal (Piana, 2001). This observation is valid for higher nominal exchange rate as well.

The correctly priced RER is essential. When the RER diverge, the currencies also face pressure to change. It is often the case that significantly deviated currencies tend to be pressed to shift in an opposite direction (overvalued currencies depreciate, whereas undervalued- appreciate). In reality, there are some major factors like transportation, trade tariffs, government policies etc. that impact greatly the FX and RER rates, and therefore, disrupt a potential straight price comparison (Catao, 2007). These expenditures shift additionally a currency away from its equilibrium level, which could incorrectly set its purchasing power and the prices in a country as a whole.

The RER between two countries could also be measured by only a single good in order to minimize the effects of the above-mentioned disturbances. It, however, must be an offered by only one company product which is widely traded and also relatively standardized on the different geographical markets. A suitable good that fulfills all these assumptions is the Big Mac™ burger. It represents a standardized bundle of ingredients, offered in an identical version by only one producer in 120 countries worldwide. This particular good may be regarded as a palatable purchase power measurement tool \(^4\) (Pakko & Pollard, 2002).

The Big Mac™ Index was introduced for the first time in 1986 by the British magazine “The Economist”. Ever since, it is published annually as a comparison of Big Mac™ burger`s prices in various countries worldwide. In this way, it allows a systematic evaluation of the prevailing RERs. The Big Mac™ Index is based on the

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\(^4\) Other examples may include Starbucks’ Tall latte index and Bloomberg’s Billy Index, tracking the Ikea’s Billy bookshelf.
purchasing power parity (PPP) concept. It states that in a long-run the RER should adjust to one price between the countries, all things equal. Thus, a burger expressed in a single currency is expected to cost the same on both markets in consideration. It is assumed that the concept of PPP holds true in such an ideal “one-product-world” where prices represent the exchange rates and the RER equals 1.

The last set of data was provided by “The Economist” on 11th of January 2012. It indicates that a burger costs 4.43$ in the Eurozone (EZ) and only 4.20$ in the United States (US). The implied PPP of the dollar is calculated to be only 1.21. Compared to the actual FX rate of 1.27 EUR/USD at that time, this implies that the euro is approximately 6% overvalued against the dollar. The cost of buying a Big Mac™ burger in the EZ, therefore, would be 6% higher. According to the PPP theory, if conditions for arbitrage appear in the market, i.e. if speculators buy burgers in the US in order to sell them for a higher price in the EZ, this would increase the USD nominal exchange rate because more dollars would be demanded for the trade. The price in EZ and the US would equalize and the RER would return to 1 (Catao, 2007).

This one-product-world is oversimplified. In reality, countries produce and export more than one good. When the whole economy is taken into account, the items of interest would not be individual prices but their constituting price indices. These consumer baskets usually contain the very basic goods and services of the households. Their comparison tracks the price movements, expressed in consumer price index (CPI), among different countries. Measuring the RER in indices is a very flexible procedure. At any given moment in time, it allows benchmarking of overall price changes relative to a base year. In this perspective, it differs from the Big Mac™ measurement which provides only an absolute price comparison. The concept of relative PPP holds true when the RER between countries do not change over time. If the average prices grow linearly, the constituting RER indices should also increase accordingly during the observed time span in order to justify the PPP (Catao, 2007).

The RERs have a significant impact on global markets. More precisely, the RER is a key determinant of the international trade. It impacts the net exports- and therefore- the trade account of a country. For example, considering the EZ as a home region- when the euro depreciates, domestic goods become cheaper than the imported ones. This leads to an increased consumption of local products at home, as well as abroad.

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5 See Section 2.1 for more details on the PPP theory
6 See http://www.economist.com/blogs/graphicdetail/2012/01/daily-chart-3
Due to the two-fold correlation, the demand of US products inevitably decreases. As a result in the EZ, the imports fall and the exports rise, which leads to an increased net EZ current account (CA). This is a consequence of the Marshall-Lerner condition which states that a change in the domestic currency rate would have a positive effect on the trade balance only if the sum of the price elasticities of the export and import (in an absolute values) are greater than 1.

Andrew Rose (1991) proposes the so-called imperfect substitutes model in order to view the FX effects on the trade flows. It is a “two-country” trade model that considers imported and domestic products as imperfect substitutes. Although in his analysis Rose (1991) estimates the partial derivatives when holding real incomes constant, he cannot reveal any strong short-run relationship between exchange rates and the trade balance. However, he is also unable to reject the hypothesis that the generalized Marshall-Lerner condition does not hold in medium- or long-term.

For a simplification of the analysis in this study, like in most other macroeconomic works (Rose, 1991), it is assumed that the strengthening of the domestic currency exhibits a negative effect on the trade balance at home. This is due to the fact that domestically produced goods become more expensive on the global market, and therefore, lose their competitiveness. It will be also assumed that when the RER depreciates, this would increase exports as the aggregate demand for cheaper domestically produced goods would also increase. The international trade is a zero-sum sphere and an incorrectly priced currency can produce serious political and economic issues. Such example of an unfair pricing that could lead to “currency wars” is the undervalued Chinese currency. China runs a massive trade surplus particularly against the US and Europe as a consequence of this policy. According to Thorbecke (2010), China’s trade surplus against the world was 9% of the GDP in 2006. During 2007 this percentage rose to 11%. The surplus against the US alone was +152.7 bn USD in 2006. He estimates that 10% appreciation in the Chinese renminbi would reduce the exports by 12%. In 2007, Dias & MacDonald estimate as well that the renminbi’s adjustment against a basket of the major world currencies is supposed to range between +27.3 and +46.6%. Interesting comparison is that in 2012 the Big Mac™ Index shows also a 42% undervaluation of the Chinese currency against the dollar alone.
1.3 THE EUR/USD CURRENCY PAIR

The EUR/USD is the most traded spot currency pair, according to the latest Bank of International Settlements (BIS) triennial survey (2010). It accounts for 28% of the entire 4 trln USD daily turnover. Thus, its volume equals 1,101 trln USD every trading day. The US dollar is involved in 84.9% of all transactions, followed by the euro with 39.1%. Thus, their combined share of all FX market turnover is 120.5% (out of 200% as each transaction involves two currencies). According to the International Monetary Fund (IMF), in 2009 the dollar accounted for 60% of all government and institutional FX reserves because of its status as a reserve currency of the world. The euro’s share was 25% and it is growing gradually ever since its introduction as an accounting currency of the EMU in January 1999. It circulates physically since 01.01.2002. Figure 2 shows the plotted EUR/USD nominal exchange rate:

Figure 2: EUR/USD exchange rate

Source: Saxobank
In the first 2 years of existence, the euro depreciated significantly against the US dollar compared to the FX movements prior to its actual inception\(^7\). The dollar increased more than 70% although the fundamental news about the EUR were relatively more favorable than the ones about the USD (De GRauwe & Grimaldi, 2006). After the initial depreciation, the common currency consolidated in a range of 0.85-0.95 USD per one euro before returning to the same levels from the end of 1998. The euro started appreciating steadily after 2002 and reached all times high in mid-2008.

The euro’s initial depreciation was unforeseen by many analysts. On the contrary, it was broadly expected that exactly the opposite would occur due to the increased euro transactions in the EZ, as well as globally (De GRauwe & Grimaldi, 2006). The depreciation phase was explained by the higher US GDP growth rate in the GDP differential between the two economies, whereas the subsequent appreciation was seen as a consequence of the large US account deficit (Schmidt, 2004).

After the second half of 2008 and onwards, extreme fluctuations can be observed in the development of the EUR/USD exchange rate. Within 5 month in the period July 2008 - Nov. 2008 the euro decreased by -21% in terms of USD. According to BIS (2009), this was a result from the collapse of Lehman Brothers on 15 September 2008. The banks’ prior accumulation of US dollar assets\(^8\) signified their cross-currency funding requirements. After Lehman, many of them faced severe difficulties securing short-term USD funding on the interbank market, or via FX swaps with bank and-particularly for the EZ banks- non-bank counterparties (like money market funds for example). Suppliers of funds withdrew, the global interbank market deteriorated, and the FX swaps seized up as all large banks were employing similar funding strategies. Thus, all these implications resulted in an increased demand of USD. After this phase, the Euro increased with +19% from March 2009 to Oct. 2009, and again, it depreciated with -20% until June 2010 due to concerns related to the troubled EZ sovereign countries (McKinsey & Co research). The amplitude of the movements correlates strongly with the increased uncertainty in the market.

\(^7\) Before 1999 the euro did not exist. The DM/USD fixed rate of 1.95583 is typically used for converting Deutsche Marks into euros and observing the exchange rate behaviour.

\(^8\) This was due to financial innovations that expanded banks’ balance sheets prior to 2008 such as structured finance, “universal banking”, proprietary trading, growing hedge fund industry to which banks offer prime brokerage etc.
2 EQUILIBRIUM EXCHANGE RATE MODELING

“Equilibrium therefore means different things to different people and this is no less true in the context of exchange rates than it is for any other field in economics.”

Driver & Westaway (2004)

Williamson (1985) describes the concept of an equilibrium exchange rate as a market equilibrium that balances the demand and supply of a currency in an absence of official intervention, but nevertheless, might be distorted by other policies. It is consistent with the sustainable rate of the internal balance, and occurs when capacity utilization and external balance are at their stationary means— a situation in which the investors are indifferent between domestic and foreign assets. It can be argued that at every specific moment, the rate is determined continuously on the FX market, which is very liquid and dynamic. Therefore, it is important to segregate the time frames as short-, medium- and long-term horizons when evaluating the equilibrium. This will be done later in the chapter.

Converging towards the equilibrium level can occur either through a change in the nominal exchange rate, or by changes in the relative price indexes (Giannellis, 2010). One of the CB’s primary functions is to stabilize the inflation levels. Therefore, it is interested in the concept of equilibrium FX rates because of the following points:

i) A depreciating nominal exchange rate may lead to inflation if the RER’s equilibrium is unchanged. One is required to tighten the monetary policy in order to counteract the inflationary pressures in this situation. These policy measures, however, are not necessary to be implemented in a situation when the nominal depreciation is produced only by a decreasing RER. Determining the real cause, therefore, is of significant importance for any future counteraction.

ii) Considering an expanding monetary union similar to the EMU recently, it is important to calculate how the equilibrium value of the common currency would be affected by a possible admission of new members (Stein, 2001).

The exchange rate represents a relationship between the domestic and foreign economy. Testing its volatility and misalignment, therefore, comprises one of the most important priorities for policy makers. The exchange rate volatility represents the currency pair’s short-run fluctuations oscillating around long-run trends. Its
misalignment, on the other hand, comprises the deviations from the equilibrium. There are two types of such deviations—over- and undervaluation.

- An overvaluation occurs when the currency of interest, which is the euro in this study, is priced above its equilibrium rate. Overpriced EUR will reduce the competitiveness of the EZ products on the global market, particularly in terms of US consumers. The implication is based on the fact that in the aggregate normal case, rising prices lead consequently to decreased demand, which itself may cause a growing unsustainability of the current account.

- Undervaluation, on the other hand, prices the euro below its equilibrium level. This deviation may cause inflationary pressures because the economy faces overheating caused by the increased demand for cheap EZ products. The ECB may counteract with raising the main interest rates. However, this measure might attract even more foreign funds, and therefore, might boost the money supply at home further. This scenario is particularly harmful to the countries with pegged currencies.

Significant exchange rate misalignments may impose major costs and negative pressures on the home economy, because a highly deviated FX rate would oscillate greatly in a short- or long-run to converge to its equilibrium (Giannellis, 2010). Therefore, it is important to estimate the real equilibrium levels. Thorough the time, several models have been developed in order to calculate it.

PPP is the first traditional theory for equilibrium exchange rate determination. Its model has been discussed widely in the academia overtime. A growing number of research papers have challenged its propositions and exposed some flaws in the assumptions. By departing from the existing traditional theories for exchange rate determination, these studies contribute greatly to the development of new approaches which measure the equilibrium exchange rate (Rogoff, 1996). Some of them are the fundamental, natural and behavioural equilibrium exchange rate models. They will be discussed accordingly in the subsequent sections.

Figure 3 illustrates a comparison between the above-mentioned equilibrium exchange rate theories, grouped by the horizon they cover. The longest one is the PPP which may include a time span of 30 to approximately 100 years for the FX rate to

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9 In various studies these models are regarded as different approaches as well, particularly the BEER is defined primarily as an econometric approach (Clark & MacDonald, 1998).
converge to its equilibrium. According to its propositions, the RER for similarly
developed countries is supposed to equal 1. Different tax levels, wages, trade barriers
and other factors in the economy, however, affect the equilibrium rate which may
additionally fluctuate with \( \pm \mu \) amplitude. Therefore, the entire PPP range is \( 1 \pm \mu \) and is
represented by the shaded area in the figure (note that the FX rate is quoted directly).

**Figure 3: Equilibrium exchange rate theories**

*Source: Egert, Halpern & MacDonald (2004)*

When the FX rate between two countries at different development stages is set,
initially the more developed one experiences an overvalued RER. This is represented by
point A in the figure. By initiating higher productivity and growth in the less developed
country, its price levels and currency appreciate. The equilibrium exchange rate adjusts
accordingly to points B and C, and eventually, converges to the PPP zone of \( 1 \pm \mu \). This
whole process ranges in a corridor and not as a single linear line due to transactional
costs and different dual productivity levels of the two countries. An illustrating example
is point A`. It shows an undervalued exchange rate that both in terms of PPP, as well as
productivity levels. This requires an initial undervaluation of the domestic currency
that could accelerate the real appreciation given by the productivity. On the other hand,
point A`` shows a situation in which the RER is overvalued only in terms of
productivity.

In a relatively long-term, the exchange rate would converge to 1, in line with the
PPP theory. For a shorter period, however, the PPP has still far too long span to
interpret the equilibrium real exchange rate for policy purposes. Other more
appropriate measures are required. In this perspective, the FEER, BEER and NATREX
models include additional variables that could estimate the equilibrium exchange rate
in the medium-run (Egert, Halpern & MacDonald, 2004).

\section*{2.1 PURCHASING POWER PARITY (PPP)}

The concept of PPP was developed by Cassel in 1916 and highly discussed
afterwards in attempts to restore the financial collapse of the World War I. The gold
standard from prior years enabled countries to value their currencies with an exchange
rate that reflected their relative gold possessions. Maintaining it after the War,
however, seemed no longer possible due to speculation\textsuperscript{10}. Consequently, the countries
faced the serious issue of how to restore the exchange rate without deeper disruption of
domestic prices and government finances. The solution was proposed by Cassel in 1922.
He calculated the exchange rate that would maintain the same PPP between the
countries by using the inflation differentials from 1914 (Rogoff, 1996).

The PPP states that in a world with functioning efficient markets, all identical
goods would be priced equally between two countries, i.e. converted into a common
currency they will have a single price. These characteristics comprise the Law of one
price, which can be expressed as follows

\[ s_t = \frac{P_t}{P_t^*} \] (2.1)

\textsuperscript{10} Defined by Graham and Dodd (1934), a speculation act offers profit possibilities in return for
risk (p.230) by employing “when to buy” rather than “what to buy” strategy (p.3)
where \( s_t \) denotes the exchange rate, \( p_t \) is the domestic and \( p_t^* \) - the foreign currency price.

The equation (2.1) in dynamic terms, stating the proportional change of variables in consideration, is

\[
\Delta s_t = p_t - p_t^*
\]

where \( \Delta s_t = \frac{\Delta s_t}{s_t} \), \( \Delta p_t = \frac{\Delta p_t^*}{p_t} \) and reciprocally \( \Delta p_t = \frac{\Delta p_t^*}{p_t^*} \).

When all identical goods cost the same between two countries, their RER is constant and its equilibrium is defined in a long run. PPP is an important theoretical concept with various implications today, although there are some factors that can create deviations and complicate prices between two countries like trade tariffs, transport costs or other barriers. The concept of PPP is used, for instance, for defining the initial exchange rate of an independent country, forecasting RER in medium- to long-term run, making adjustments to price differentials and others.

It is necessary to highlight the fact that in general both prices and nominal exchange rates are variables that are determined endogenously and simultaneously, thus discussing their cointegration provides not enough insights when analyzing the RER (Frenkel, 1976). Many years research exposed difficulties to support the PPP hypothesis in reality. Numerous empirical studies show that RER tends toward PPP in an extended long run, making this convergence a very slow process, if it occurs at all (Rogoff, 1996). They have also exposed that short-run deviations are significant and volatile. Even the price volatility of homogeneous highly traded goods within one country is surprisingly large, implying that there are high differentials between states. These empirical findings constitute the so-called PPP puzzle. It states that RERs are more volatile than most models can account for (Chari, Kehoe & McGrattan, 2002). In summary, PPP is found not to hold in the short-run. Additionally, the RER`s convergence towards it is a very slow process in time.

Figure 4 presents a quick visual cointegration test between EUR/USD and PPP as originally performed by Rogoff (1996). The relative CPI ratio in logs and the log EUR/USD exchange rate are plotted in the period Q12000-Q42011. It can be observed that the variance of the exchange rate is much greater than the variance of the relative CPIs. This suggests that PPP does not hold true in the period for the observed
economies. Its failure could be due to sticky prices\footnote{The term “sticky price” is applied to economic models that exhibit some sort of barrier to the adjustment of prices. In this perspective, it represents a variable resistant to respond to innovations in other variables, and despite the prevailing changes in the economy which require a different optimal price (Farmer, 1990).} or because the nominal FX rate is affected by some economic shocks which also affect the RER over time.

As previously mentioned, the PPP’s failure invokes a necessity to employ different methodic for estimating the equilibrium exchange rate. The conventional view when the PPP prevails is that the exchange rate of a currency depends on its amount in circulation at any given moment. This asset view of FX rate determination was dominating during the years. It implies that a currency is viewed as a stock, rather than a flow. Frenkel (1976) notes that exchange rate’s general equilibrium is rather determined by the both features:

“Being a relative price of two assets (moneys), the equilibrium exchange rate is attained when the existing stocks of the two moneys are willingly held. It is reasonable, therefore, that a theory of the determination of the relative price of two moneys could be stated conveniently in terms of the supply of and the demand for these moneys.”

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{EUR_USD_CPIs}
\caption{EUR/USD against the relative CPIs}
\label{fig:EUR_USD_CPIs}
\end{figure}

Source: Own calculations
Dornbusch (1976) also argues that the exchange rate is determined in the market. These rather different views gave rise to new approaches in the equilibrium exchange rate determination. Some of them are presented accordingly as different subsections.

2.2 FEER

The Fundamental Equilibrium Exchange Rate (FEER) was developed by Williamson (1985, 1994) in order to determine the equilibrium rate through a medium-term analysis. It is called “fundamental” because it emphasizes the factors that are important in the medium- instead of only short-run period. FEER assumes that the exchange rate converges to its equilibrium level when the internal and external balances are in equilibrium simultaneously- a condition, consistent with the non-accelerating inflation rate of unemployment\(^{12}\) (NAIRU). This means that the economy operates under full employment, low inflation (internal balance) and a sustainable current account (external balance).

Figure 5 represents the relationship between the CA and the RER as units of foreign per one unit of the domestic currency. It is downward sloping for any given level of output Y, meaning that the current account improves as the RER depreciates and the exports increase/imports decrease. If the domestic output Y rises, the curve will shift leftwards as exports improve. Similarly, if the foreign output Y* increases, imports improve and the shift is rightwards.

Internal balance is achieved when the current account equalizes savings less investments. This is satisfied when Y is optimal (also called capacity output) domestically and abroad, i.e. \(Y=Y^*\) (Stein, 1999). External balance is conditional on the internal balance and represents the S-I line. Under such circumstances, the equilibrium is sustainable in medium and long run. In the figure, it represents point B where FEER is also determined.

\(^{12}\) Milton Friedman (1968) describes the “natural” rate of unemployment to be consistent with the real wages equilibrium. Below this threshold level, an excess demand for labor would occur which would produce upward pressures on real wage rates. This process would lead to an increase of the inflation (p.8).
Williamson (1994) characterizes this model as an equilibrium exchange rate that would be consistent with the above-mentioned “ideal” economic conditions. They are desired but may never get realized, and thus, are not necessarily the ones that will inevitably occur at some time in the future. The FEER describes a model in which the RER is based on assumptions that all variables are in their steady-state. The exchange rate under this model remains the same as long as the internal and external balances are undisturbed. Thus, it does not account for all the other variables that could possibly affect the rates at this very same time. Under FEER, it is not particularly certain that the exchange rate is in equilibrium in the behavioural sense (Clark & MacDonald, 1998). In order to confirm this issue, one is required to perform a direct econometric analysis. This estimation constitutes, in fact, the BEER approach. A FEER might be preferred when estimating volatile and unpredictable exchange rates or by policymakers’ assessment regarding sustainable current account positions.

2.3 NATREX

Another method for estimating FX equilibrium is the Natural Equilibrium Exchange Rate (NATREX). It was developed by Stein (1995) as a variation of FEER, comprising both medium- and long-term analysis. The NATREX is a growth model, directly amendable to empirical analysis. According to Stein (1995):

“The NATREX is the RER which equates the current account to ex ante social savings less social investments generated by the fundamentals, productivity and
thrift, when the fundamentals are corrected for cyclical variations in the rate of capacity utilization and incomplete adjustments in asset markets.”

NATREX follows the internal-external balance prepositions which are very similar to the FEER ones. In a medium-term, the equilibrium point is sustainable and ensures (i) internal equilibrium with a capacity output in the economy; and (ii) external equilibrium with a balance of payments without cyclical factors, capital or international reserves movements. In a long-run, the net foreign debt to GDP ratio is stable, i.e. the country finance itself internally, and the interest rates at home and abroad are equal.

The NATREX differs from FEER with respect to its assumptions about the speculative capital flows and changes in the official reserves. It considers them unsustainable. Thus, they cannot be determinants of NATREX model. The exchange rate becomes sustainable only when the speculative capital flows and changes in the official reserves are zero (Stein, 1995).

In figure 6 the investment-savings-current account, or ISCA line, has a negative slope, because a possible appreciation in the RER would worsen the trade balance as discussed in the previous subsection. It would also reduce the aggregate demand which itself requires a lower interest rate \( r \) for encouraging the investments and restoring the internal balance. Accordingly, the foreign economy`s ISCA* line has a positive slope. When the speculative capital flows are absent, the external equilibrium requires the interest rate between home and abroad to be equal in the long run, i.e. \( r = r^* \).

![Figure 6: NATREX determination](source: Serge Rey (2009))
Considering the initial level of RER at $R_0$, the real domestic interest rate $r_0$ is higher than the one abroad ($r_0 > r_0^*$). Thus, according to the definition above this situation cannot represent a medium-term equilibrium rate. When the interest rates at home and abroad differ among each other, there will be certain conditions for arbitrage on the financial market. The investors would prefer to buy the more expensive home securities and sell the cheaper foreign ones. This arbitrage would lower the domestic interest rate and increase the foreign one until the equality is restored. These implications would lead to an appreciation of the domestic currency, depreciation of the foreign one, and achieving the conditions for external and internal balance. Thus, the medium term equilibrium point $E_{MT}$ is simultaneously achieved. This cross point itself corresponds to the medium term NATREX, denoted with $R_{Natrex}^{MT}$.

2.4 BEER

The Behavioral Equilibrium Exchange Rate (BEER) represents one of the latest approaches for determining the equilibrium FX rate. It was developed by Clark & MacDonald (1998) as an alternative to FEER for assessing the current value of the exchange rate. BEER is primarily an econometric analysis. It estimates the misalignments that occur due to deviations of the actual exchange rate from its estimated values. The BEER is derived from the long-run cointegration relationship of the macroeconomic fundamentals which are separated from any cyclical components with the Hodrick-Prescott filter.\(^{13}\)

Unlike the NATREX model that relies on a structural equilibrium concept, the BEER is based upon a statistical notion of equilibrium. It attempts to explain the actual behaviour of the RER based on a set of relevant explanatory regressors. These fundamentals are selected according to what the economic theory prescribes as variables that have an impact on the RER in medium- to short-run period (Makrydakis, de Lima, Claessens & Kramer, 2000). The principle difference between BEER and NATREX is that the latter has a specific theoretical dynamic stock-flow model as a point of departure, whereas the equilibrium RER depends upon relative thrift and relative differentials (Stein, 2001).

\(^{13}\) This filter is a mathematical tool for separating the cyclical components of a time-series from the raw macroeconomic data (Hodrick & Prescott, 1997)
Since BEER measures the above-mentioned deviations, it differs from FEER which estimates only that RER, consistent with the macroeconomic balance. In many cases the FEER calculations do not incorporate some variables that have been found to affect the actual FX behaviour. The advantages of BEER, therefore, are that it is highly tractable, represents the underlying data generating process and is flexible in constructing different estimations. These differences with the preceding methods are achieved through “the use of an estimated reduced-form equation that explains the behavior of the real effective exchange rate over the sample period” (Clark & MacDonald, 1998).

When estimating BEERs, Clark and MacDonald based their approach on the uncovered interest rate parity (UIP) condition that links the interest and exchange rates in the domestic and foreign economy. According to Frenkel (1976), interest parity theory also provides a fundamental relationship for deriving the market measure of inflationary expectations. UIP represents an equilibrium states in which the interest rates on bank deposits between two countries are insignificant to the investors, implying that no arbitrage opportunities occur

\[ E_t(\Delta s_{t+k}) = i - i^* + r_p t \]  
(2.2)

where \( i \) and \( i^* \) are the domestic and foreign nominal interest rates, \( r_p \) is the country’s risk premium with a time-varying component and \( E_t(\Delta s_{t+k}) \) is the expected nominal exchange rate in time \( t \) for the period \( t + k \). Its first difference operator equals the upcoming period nominal exchange rate less the current one.

\[ E_t(\Delta s_{t+k}) = s_{t+k} - s_t \]  
(2.3)

In order to calculate the RER, the home and foreign expected inflation rates should be subtracted as follows

\[ q_t = E_t(\Delta q_{t+k}) + (r - r^*) - r_p t \]  
(2.4)

where \( q_t \) is the RER, \( E_t(\Delta q_{t+k}) \) is the expected RER, \( r \) and \( r^* \) represent the domestic and foreign real interest rates. Each of them equals the difference between nominal interest rate and the expected inflation in the economy, i.e. \( r = i - E_t(\Delta \pi_{t+k}) \).

Equation (2.4) states that the current RER is a function of its future expected value plus the real interest rate differential and subtracting the risk premium for the
domestic country. The latter has a negative impact on the RER, implying that when the risk rises in the EZ the euro depreciates in real terms.

Assuming that the RER’s expectations in time $t$ for the period $t+k$ equal the long-term RER that is defined by some current economic fundamentals, the equation (2.4) is transformed further.

$$ q_t = (fundefamentals)_t + (r - r^*) \quad (2.5) $$

Considering the UIP condition of no-arbitrage,

$$ q_t = q_t(fundamentals_{LT}, fundamentals_{MT}) \quad (2.6) $$

the equation indicates that the RER is a function of the long- and medium-term fundamentals. Equation (2.6) represents in fact the BEER’s theoretical fundament. Since it is primarily an econometric analysis, Clark and MacDonald introduce a notation $Z_{LT}$, representing a vector of current macroeconomic fundamentals that affect persistently the FX rate over a long-term. Additionally, $Z_{MT}$ is denoted as a vector of economic fundamentals that affect the RER over a medium-term, whereas the short-term transitory factors are represent with $T_{ST}$ vector of variables. Consequently, the nominal exchange rate $s_t$ is defined as follows

$$ s_t = \beta_1 Z_{LT,t} + \beta_2 Z_{MT,t} + \tau T_{ST,t} + \epsilon_t \quad (2.7) $$

where $\beta_1$, $\beta_2$ and $\tau$ represent reduced form coefficients and $\epsilon_t$ is the random disturbance.

The current equilibrium exchange rate $\tilde{s}_t$ is the rate given by the current values of the medium- and long-term economic fundamentals. It also represents the current medium-run BEER when all transitory components are zero.

$$ \tilde{s}_t = \beta_1 Z_{LT,t} + \beta_2 Z_{MT,t} \quad (2.8) $$

By subtracting equation (2.8) from (2.7) the current misalignment is obtained. It is the difference between the actual RER and the one given by all current economic fundamentals.

$$ s_t - \tilde{s}_t = \beta_1 Z_{LT,t} + \beta_2 Z_{MT,t} + \tau T_{ST,t} + \epsilon_t - \beta_1 Z_{LT,t} - \beta_2 Z_{MT,t} \quad (2.9) $$

$$ s_t - \tilde{s}_t = \tau T_{ST,t} + \epsilon_t \quad (2.10) $$
Equation (2.10) shows that the current misalignment is defined only by the short-run economic fundamentals and some white noise.

The FEER discussion above notes that the current values of the economic variables themselves may be a subject of bias, and consequently, depart from their sustainable levels. Therefore, the concept of total misalignment $TM$ is introduced. It is what actually matters in estimating the BEER. The total misalignment in equation (2.11) represents a deviation of the actual exchange rate from the total equilibrium exchange rate calculated by filtering the fundamentals from speculative and cyclical factors (Giannellis, 2011). Thus, the sustainable long-run values of the economic fundamentals $\tilde{Z}_{LT,t}$ and $\tilde{Z}_{MT,t}$ enter the equation (2.12) as replacements of the underlying vectors from equation (2.8).

$$TM_t = s_t - \tilde{s}_t \quad (2.11)$$

$$TM_t = s_t - \beta_1 \tilde{Z}_{LT,t} - \beta_2 \tilde{Z}_{MT,t} \quad (2.12)$$

The source of the total misalignment is further decomposed by substituting the current equilibrium exchange rate $\tilde{s}_t$ in the right-hand side of (2.12), yielding the econometric form.

$$TM_t = (\tau T_t + \epsilon_t) + [\beta_1 (Z_{LT,t} - \tilde{Z}_{LT,t}) - \beta_2 (Z_{MT,t} - \tilde{Z}_{MT,t})] \quad (2.13)$$

Equation (2.13) shows that at any point in time, the total exchange-rate misalignment can be decomposed into:

- short-term effecting transitory factors
- random disturbance term
- deviation of the macroeconomic fundamentals from their equilibrium long-run values (Giannellis, 2011).

The BEER is primarily an econometric study. Therefore, it may or may not employ an underlying theoretical model (Giannellis, 2011). By definition, it is an econometric technique that links a large pool of macroeconomic fundamentals and the RER in a single equation, aiming to explain the past movements of the exchange rate. BEER is said to be behavioural because it is based on ad-hoc selected variables in accordance to their ability to explain these past movements. In the literature, a large set
of fundamentals have been used, ranging from oil price to public debt and unemployment rate. Table 1 summarizes the subjectively chosen by the researchers variables for estimating the euro-dollar BEER in some of the major preceding studies.\textsuperscript{14}

\begin{table}[!h]
\centering
\caption{Fundamental variables in EUR/USD BEER estimations}
\begin{tabular}{|l|l|l|l|}
\hline
Article & Period & Variables\textsuperscript{15} & Main Results \\
\hline
de Ubide (1999) & (Quarterly) & - net foreign assets & undervalued in Q4 1998 \\
\hline
Clostermann & 1975-1998 & – relative price (traded and non-traded) differential, & euro is -7%
& Schnatz (2000) & (Quarterly) & - real price of oil, & undervalued in Q3 1999 \\
& & - public expenditure differential, & & & - real interest rate differential, & \\
& & - real interest rate differential, & & & - net foreign assets & \\
& & - net foreign assets & & & & \\
& & – terms of trade differential & & & & \\
& & – relative price (traded and non-traded) differential, & & & & \\
& & – non-working to working & & & & \\
& & – output differential & & & & \\
& & – public expenditure differential, & & & & \\
Koen, Boone, deSerres & 1976-1999 & – output per worker differential, & euro is overvalued
& Fuchs (2001) & (Semi- & - total consumption/GDP & before 1999 and
annual) & & & -15% undervalued & \\
& & & & & & towards the end & \\
& & & & & & of 2000 & \\
Stein (2001) & 1971- & – output per worker differential, & euro is -10%
& Q12000 & (Quarterly) & - relative price (traded and non-traded) differential, & undervalued in Q1 2000 \\
& & - real price of oil, & & & - real interest rate differential, & \\
& & - real interest rate differential, & & & - public expenditure differential, & \\
& & - net foreign assets & & & - real price of oil, & \\
& & & & & - net foreign assets (proxy) & \\
Maeso-Fernandez, Osbat & 1975-1998 & – labour productivity differential & euro is slightly
& & (Quarterly) & - real interest rate differential, & overvalued in the 1970s and
& & & & & - real price of oil, & the first half of 1990s; \\
& & & & & - government expenditures & undervalued in the first half of 1980 and 1999 \\
& & & & & & & \\
& Osbat (2003) & (Quarterly) & & & & \\
& & & & & & & \\
\hline
\end{tabular}
\end{table}

\textsuperscript{14} No distinction is made between the long-, short-term or other variables used in the econometric estimations. For more details see Sonja Costa (2005), p.56
<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Period</th>
<th>Variables</th>
<th>Currency Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>MacDonald &amp; Dias (2007)</td>
<td>1988-2006 (Quarterly)</td>
<td>net exports, terms of trade differential, output differential, real interest rate differential</td>
<td>dollar is 5-11% overvalued; no adjustments required for the euro</td>
</tr>
<tr>
<td>Heimonen &amp; Vataja (2008)</td>
<td>01.1989-08.2004 (Monthly)</td>
<td>CPI to PPI ratio differential, real oil price, stock index to CPI differential, real interest rate differential, public expenditure differential, net foreign assets</td>
<td>euro is undervalued in the early days of EMU, overvalued in1995; undervalued again 1999-2001 due to the US and the EZ equity markets</td>
</tr>
</tbody>
</table>

As seen from the table, one of the commonly used variables is the relative price of non-tradable and tradable products from the BEER model of Alberola et al. (1999). Some studies are based on the balance of payments approach. Mussa (1984) argues that foreign asset position of the country has a substantial effect on equilibrium exchange rate in the long-run. Other studies test the Balassa-Samuelson effect. They use the CPIs and productivity. However, the latter variable can be defined differently. Stein (2001) includes the ratio of the real GDP to total employment, whereas Bénassy-Quéré et al. 16 The BEER estimation in this study is based on the Real Effective Exchange Rate (REER), which is the weighted average of a country’s currency relative to an index of other major currencies adjusted for relative changes in CPIs. 17 This study is based on a unit-root and cointegration tests with structural breaks in the data
(2008) employ the productivity differentials in non-traded against the traded sectors. Schnatz et al. (2003) use several different measures of labour productivity in order to estimate the BEER of the euro-dollar rates:

- non-farm business sector output per hour worked for the United States and GDP per person employed for the EZ,
- GDP per person employed,
- GDP per hour worked which according to them is the most consistent and therefore their preferred measure,
- relative prices of traded and non-traded goods and services an indirect measure, which is commonly used as a proxy for relative sectoral productivity developments.

According to Schnatz et al. (2003) “our results indicate that the extent to which productivity can explain the euro depreciation varies with the productivity proxy used: readily available measures explain most, our new, preferred measure least”. However, they highlight that the hours worked data is difficult to obtain for the EZ countries. In the period 1981-2001, they are available with a rather long lag, and in most cases only on an annual basis. Schnatz et al. (2003) also conclude that “although economists generally agree that GDP per hour worked is a better measure of labour productivity, the most readily available proxies remain predominant in the literature due to these practical constraints”.

Very few of the existing euro-dollar BEER studies base their estimations on all monetary model differentials. So far, none of them has used a homogenous productivity variable, in addition to the monetary-based BEER estimation. Therefore, this study will assume that the fundamentals affecting the long-run exchange rate are the money supply, real interest rate, inflation and output per hour work differentials between the EZ and the US. They will be discussed in the subsequent chapter.
3 THEORETICAL FRAMEWORK AND DERIVATION OF THE BEER MODEL

“Some topics lie dormant in our minds for several years, and we do not attribute immediate importance to many of them until the day arrives when the reality that surrounds us makes the solution to these issues preeminent.”

Leonardo Cruz Basso (1999)

3.1 THE MONETARY MODEL AND ITS COMPONENTS

The monetary model for exchange rate determination was at first estimated by Frenkel (1976) and Mussa (1976, 1978). The model defines the exchange rate as a price of two currencies and attempts to determine it in terms of their relative demand and supply. Assuming that prices are flexible, employment is at full level in the economy and the UIP and PPP conditions hold true all the time, the domestic and foreign monetary equilibria are described with the following two equations

\[ m_t - p_t = \beta_1 y_t - \beta_2 i_t \]  
\[ m^*_t - p^*_t = \beta_1 y^*_t - \beta_2 i^*_t \]  

where \( m_t \) represents the domestic real money supply, \( p_t \) is the domestic price level, \( y_t \) is the domestic real income, \( i_t \) is the domestic nominal interest rate and \( \beta \) is a coefficient. The sign * denotes the foreign variables respectively.

The equations above show that the money demand in a country depends on the output and the nominal interest rate. The money supply, on the other hand, is determined by the money in circulation as well as price levels.

It is assumed that the money market equilibrium holds in each country \( m^D = m^S = m \) and \( m^{D*} = m^{S*} = m^* \).

Thus, the two equations (3.1) and (3.2) could be combined.

\[ (m_t - m^*_t) - (p_t - p^*_t) = \beta_1 (y_t - y^*_t) - \beta_2 (i_t - i^*_t) \]  

Assuming that the PPP condition holds true, i.e. the FX rate equals domestic less foreign prices, the equation (3.3) shows that the nominal exchange rate is dependent on the relative money supply, output and the difference in the interest rates. Equation
(3.4) represents this relationship. The sign in front of the interest rate differential can vary (see Table 2 below).

\[ s_t = (m_t - m^*_t) - \beta_1 (y_t - y^*_t) \pm \beta_2 (i_t - i^*_t) \]  (3.4)

According to Mussa (1984), this form of the monetary model has an important deficiency. It does not incorporate the crucial role of expectations for future economic developments that are shaping the determination of the current exchange rate. The impact of new information is also not taken into consideration. Apparently, there is no variable in this form of the model that could explain the randomness and unpredictability of the future exchange rate (Brook, 2002).

The above-mentioned deficiency was corrected in the second type of monetary models proposed by Dornbusch (1976) and Frenkel (1976). They include the expected rate of change of the FX rate. This is achieved through a combination of the UIP condition with Fisher’s open hypothesis. The latter states that the long-run interest rates equal the inflation rates. The nominal interest differentials between similar assets, denominated in different currencies, can be explained fully by the exchange rate’s expected change over the period (Cumby & Obstfeld, 1980). Although in their study, these authors conclude that there is a risk premia at equilibrium, it is generally assumed that when the UIP condition in equation (2.2) holds true, the risk premium is rather negligible. In this way, the equation (3.4) becomes

\[ s_t = (m_t - m^*_t) - \beta_1 (y_t - y^*_t) \pm \beta_2 (i_t - i^*_t) + \beta_3 E_t(\Delta s_{t+1}) \]  (3.5)

where \( E_t(\Delta s_{t+1}) \) is the expected rate of change of \( s_t \).

Since it was assumed that the PPP is valid, the expectations of domestic and foreign inflation rates shape the expected future exchange rate.

\[ E_t(\Delta s_{t+1})=E_t(\pi_{t+1})-E_t(\pi^*_{t+1}) \]  (3.6)

Hence, the current equilibrium exchange rate equation is transformed below.

\[ s_t = (m_t - m^*_t) - \beta_1 (y_t - y^*_t) \pm \beta_2 (i_t - i^*_t) + \beta_3 [E_t(m_{t+1}) - E_t(m^*_{t+1})] \]  (3.7)

The estimation of this form requires employing proxies for the expected domestic and foreign inflations because they are not directly observable (Frenkel, 2004). It is assumed that the expectations of economic agents are consistent with the market development. In other words, they are close to be rational. In this case, Frenkel (1976)
assumes that they are incorporated in the forward FX market. This is an important implication for the so-called forward rate unbiased hypothesis (FRUH) which states that the current forward FX rates are predictors of the future spot FX rate (Fama, 1984). Thus, the current equilibrium exchange rate becomes a function of monetary supply, industrial output, interest rates and the expected future inflation level of the counterparty’s countries.

\[ s_t = (m_t - m_t^*) - \beta_1 (y_t - y_t^*) \pm \beta_2 (i_t - i_t^*) + \beta_3 (\pi_{t+1} - \pi_{t+1}^*) \] (3.8)

Equation (3.8) shows how the exchange rate between two countries is determined by the relative demand and supply of their currencies. The monetary differential does not have a coefficient because in a long-run it is assumed to be unity. It, therefore, equals 1. The standard model assumes that the coefficient \( \beta_1 \) is negative, \( \beta_2 \) may have both signs, whereas the \( \beta_3 \) is positive. Table 2 provides a summary of the previous studies that estimate the direction in which these variables are expected to influence the exchange. It shows the 4 variations of the monetary model. The differentials will be analyzed separately in the subsequent sections.

Table 2: The effect of monetary variables on a directly quoted exchange rate

<table>
<thead>
<tr>
<th>Source</th>
<th>Money Differential</th>
<th>Output Differential</th>
<th>Nominal Interest Rate Differential</th>
<th>Expected Inflation Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frenkel</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>Bilson</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>Dornbusch</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Frankel</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>


3.1.1 Money supply

Money supply in one country is determined through the monetary policy of the CB. It represents one of the exclusive functions of this authority. The primary objective of a CB could be inflation, nominal GDP tracking, interest rate targeting, currency stabilization or other long-term goals. In this perspective, the main goal of the ECB, as outlined in the Treaty on the Functioning of the European Union, is “to maintain price stability with an inflation target of 2 % over a medium-term”. On the other hand,
FED’s primary objective is “to promote effectively the goals of maximum employment, stable price, and moderate long-term interest rates” and it is laid down in the Federal Reserve Act.

A CB can employ different policy instruments and procedures to achieve its main goal. Determining the monetary base in circulation is one of them. It represents an important policy instrument because i) it has a relatively stable relationship with the economy and ii) the CB is its monopoly supplier. The effects of monetary aggregates on the equilibrium exchange rate are illustrated in figure 7.

![Figure 7: Effect of the money supply on the equilibrium exchange rate](Source: Viswanath (2006))

In the figure, the initial condition is a demand curve DD and a supply curve SS. The equilibrium exchange rate, therefore, is \( e_0 \). Suppose that the US government decides to lower the interest rate, the money supply then will increase from \( Q_0 \) to \( Q_1 \). This would cause the supply curve to shift inwards from S to \( S' \). It is due to the fact that at a higher level of money supply, the aggregate US prices will start to move up as well. The dollar price for one euro will increase, constituting a depreciation of the USD relative to the EUR. Hence, the investors will be willing to supply fewer euros at this new given exchange rate, but their demand will be greater. Consequently, the new curves will be DD' and SS' with a new equilibrium exchange rate in \( e_1 \).
The differential \((m_t - m^n_t)\) in equation (3.8) means that if the foreign monetary base increases more relative to the domestic one the domestic currency is expected to appreciate. The foreign economy would face an increased flow of money which would lead to a higher consumption and higher consumer prices. The production from abroad would become less competitive than the domestic one, thus making it less attractive for the domestic buyers. The imports would decrease.

This definition seems to be conventional in normal times, but the Global Financial Crisis\(^{18}\) poses some significant challenges to such mainstream conclusions about the CB’s money supply (see BIS, 2009). Any further analysis in this study has to account for the fact that this severe recession requires substantially higher monetary base in circulation to cover the reduced liquidity on the financial markets. The troubled functioning of the banking systems in both economies implies an excessive supply of currency by the FED and the ECB to mitigate the negative effects. As a result of that, both CBs step-in as intermediaries to supply the financial sectors at home and abroad with dollars, resp. euros.

Different unconventional measures are being implemented by the FED and the ECB in order to counteract the decreased liquidity, and to prevent the financial market distress to hit the real economy. Majority of them depart from the traditional pre-2007 narrowed and well-defined CB policy towards overlapping fiscal with other measures. These more expansive monetary policies include a severe balance sheet increase to the CB’s toolkit which is aimed to counteract the liquidity problem. Consequently, both CBs have significantly increased their balances by acting as lenders and market makers of last resort in the recent years (Repullo, 2010). According to the data provider Thomson Reuters Datastream, the ECB’s balance sheet was 14% of the GDP in December 2007, compared to 33% in January 2012. The FED increased its balances from 6% in late 2007 to 19% in early 2012. The unconventional measures, undertaken from both CBs in the period 2007- onwards, are summarized below.

\(^{18}\) A term used for describing the Financial crisis of the late-2000s, also known as the “Great Recession”. 
Table 3: Examples of Central Bank Unconventional Balance Sheet Policies

<table>
<thead>
<tr>
<th>Financial stability</th>
<th>Fed</th>
<th>ECB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquidity provision to funding and credit markets</td>
<td>Active use of repo operations, TAF, TSLF, CPFF, AMLF, and TALF purchases, Security Market Program</td>
<td></td>
</tr>
<tr>
<td>Foreign exchange liquidity provision to local markets</td>
<td>USD operations by many central banks</td>
<td>Euro operations by Sweden, Denmark, Poland and Hungary</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Macroeconomic stability</th>
<th>Fed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase of long-term public securities</td>
<td>Large-scale purchase of Agency MBS, Agency debt and US Treasury</td>
</tr>
</tbody>
</table>

Source: Stone et. Al (2011)

Most of the above-mentioned unconventional monetary measures appear to be effective and positive in the short-run. However, the costs and risks of their implementation would inevitably impact the future periods. Purchasing of long-term bonds is a suitable policy in a low main interest rate environment. Its effectiveness, though, is limited and it overlaps with the fiscal policy. These large-scale FX interventions (especially the FED`s quantitative easing, or QE) may appreciate the domestic nominal rate in the short-run. Nevertheless, their implementation is tricky as they must be fitted in the broad macroeconomic policy (Stone et. al, 2011).

Conclusions about the effect of this variable on the EUR/USD exchange rate would not be made on this stage of the study. As mentioned above, it is not particularly correct to speculate with the sign of the money supply differential considering such a volatile and unpredictable macroeconomic environment. The effect will reveal itself in the actual econometric analysis, which is to be performed in Chapter 5.

3.1.2 GDP and Interest rates

The Gross Domestic Product (GDP) is another one of the monetary model, and thus, equilibrium exchange rate`s determinants. As the value of the total output of goods and services, it represents a measure of the economic activity in a country. The domestic markets are in equilibrium when nationally produced output equalizes the existing demand in the market. This relationship is expressed in equation (3.9).

\[ Y = C + i(Y, i) + G + NX(Y, Y*, e) \]  

(3.9)
This equation states that the gross domestic output $Y$ is dependent on consumption of the population $C$, investments in the economy $I$, state spending $G$ (which would be considered as given) and net difference between imports and exports $\text{NX} = \text{IM} - \text{X}$.

Investments are influenced positively by the output. This impact results from the substitution of consumer goods with capital goods, which would optimize further the process of production. On the other hand, increased interest rates may hinder the investment process within the country. They may make the borrowing on the interbank market more expensive, and therefore, the lending to customers may decrease\textsuperscript{19} (Hasset, 2008). Higher interest rates may also raise the costs of investing in this particular country in comparison to the rest countries with similar profile but lower interest rates. Investments- both domestic and foreign- would decrease\textsuperscript{20}, therefore the output would be reduced. The exchange rate decreases\textsuperscript{21}.

There is never a simple relationship between interest and exchange rates. The FX rate is influenced by expectations about future interest rates and by any unexpected current change. Provided that investors expect the interest rates to rise, they may increase the amount they invest in a currency before this actually occurs. This would result in an appreciation of the exchange rate of the home currency (Brooks, 2002). Thus, $i$ is viewed as an exogenous variable which affects the RER via its effect on the demand for money. The $(i - i^*)$ differential may affect the exchange rate in two ways. Therefore, its sign has to be determined empirically similar to the money supply variable.

Imports, as part of the net import-export balance, are correlated positively with the home output and the RER. An increase in the value of the home currency will reduce the price of imports and this will have a direct influence on inflation as many imported goods are included in the CPI. In addition, a higher local currency will tend

\textsuperscript{19}The relationship is not easy to be interpreted without the context of a particular case study.
\textsuperscript{20}It is only in the case when no carry trade strategy is not employed. Carry trades are considered speculative and involve taking “long positions in high-interest currency against short positions in low interested currencies”. In this way, a speculator profits from the existing arbitrage on the global FX market. This strategy is considered to have a negative effect when unwinding, because it causes major depreciation of the high-interest currency, which is not driven by fundamentals, but by speculative actions, also called speculative attack.
\textsuperscript{21}This is not a direct effect from the increased interest rate, as usually an increased interest rate would increase a demand for money (Frenkel & Koske, 2004)
to reduce the demand abroad for home goods and services, which, in turn, will reduce output (Education Group HO-M, Bank of England, 2011).

Exports, on the other hand, depend positively on the foreign output and negatively on the RER. If there is a reduction of the domestic demand, it would trigger a decrease in the output, which would in its turn, result in decreased exports. This may shift the demand towards foreign products, especially when there is an increase in the domestic RER. The domestic currency will be stronger, implying more imported goods. The citizens would be willing to consume cheaper foreign products, instead of buying the more expensive home production.

A recent case illustrates this relationship accurately. When the EZ sovereign crisis escalated, investors previewed the Swiss Franc (CHF) as a “safe heaven” investment. This strengthened its value relative to the euro. As a consequence, the Swiss consumers reduced the demand for local goods in favour of the cheaper European ones. Also, the foreign demand for Swiss products decreased, hurting the local business. The Swiss National Bank (SNB) was forced to introduce a floor of 1.2 for the EUR/CHF pair. In other words, the minimum exchange rate was set to CHF 1.20 per 1 euro. This represents an extreme measure. Before implementing it, the SNB has also expanded the sight deposits, the supply of liquidity via purchasing SNB Bills and has employed FX swaps. All these attempts target one purpose - to devaluate the Frank, because as the Bank put it itself in a press release from 6 September 2011: “The current massive overvaluation of the Swiss franc poses an acute threat to the Swiss economy and carries the risk of a deflationary development.”

Based on the logic above, the combined overall impact of the output differential \((y - y^*)\) on the domestic RER is expected to be positive. An increase in the domestic output would inevitably appreciate the local currency and depreciate the foreign one. Linked to the increased production, the demand for money would rise as well. If the money supply is unchanged, the market equilibrium will be shifted and the domestic currency would appreciate. The equilibrium is restored when customers decrease their consumption. The domestic prices will be lowered and - based on the PPP condition – the exchange rate will depreciate as well.

3.1.3 Inflation expectations

Inflation expectations are another one of the factors that influence the exchange rate. They are of a significant importance when considering the inflation-targeting
monetary policy of the CB. Its decision-making process is done via analyzing the current and predicted future economic developments and their determinants. The inflation forecasts and economic growth, therefore, represent key objectives of the CB’s future monetary policy.

A rise in the domestic interest rate implies higher future domestic inflation expectations (Giannellis, 2011). This scenario has an opposite effect on the FX rate, compared to the discussion about the increased domestic output above. A rise in the interest rate will decrease the money demand. This instrument represents one of the most common tools of the CB to address increased inflation in the economy. Provided that money supply is unchanged, the domestic price level will rise (Giannellis, 2011). Thus, the cheaper foreign goods would substitute the domestic ones, implying deterioration in the trade balance.

Considered from the viewpoint of an EZ consumer, a higher inflation in US will depreciate the dollar relative to the euro, i.e. the euro will appreciate relative to the dollar. Therefore, a higher inflation in the foreign economy would increase the imports and reduce the exports abroad, leading to a new equilibrium exchange rate. Consequently, when there are expectations for an increased inflation in future periods at home, the domestic currency would depreciate in the current moment.

3.2 THE INCLUSION OF PRODUCTIVITY

“In a world without currency productivity is what counts. In a world with currency, productivity and some monetary variables are important. All this because the value of currency, a relation between the Gross Domestic Product and the hours spent to produce it, cannot be different between two countries (as a long run proposition).”

Basso (1999)

The monetary models have been criticized due to some inadequacy of their derivation and assumptions. There is, for example, a broad empirical research dating back from the 1970s on the PPP condition. Additionally, many other fundamental factors that shift the FX rates constantly cannot be accounted fully into the empirical version of the monetary equation. According to Mussa (1984), however, these issues cannot represent a leading reason for rejecting equation (3.8) as a model of exchange rate determination. In reality, the FX rate configuration is very dynamic and influenced
by a diversity of factors to be fully replicated by a single equation. Therefore, it is essential for the researcher to make a choice of which other variables to include in the monetary model in order to increase the overall explanatory power.

The methodology of introducing productivity in equation (3.8), defined as the output per hour work, is justified by the following logical steps.

1. Considering a world without money would imply that the domestic, i.e. Eurozone product consists of a set of goods and services, and equivalently does the US GDP. In such an environment, the productivity would play a major role into the FX determination. When these two countries engage in exchanging goods, the one with the less productive power, i.e. number of output units produced by their hours of work, would lose market share unless in attempts to increase its productivity.

2. In a world with currency, on the other hand, the scenario is more complex as the exchange rate is affected by several other factors. Additionally, the money market equilibrium’s conditions are more relevant to shape the exchange rate than the price levels in the two countries.

3. What determines the behaviour of national price levels is an important issue which persists under the PPP conditions. Attempting to resolve it is essential because it constitutes a considerable part of the exchange rate’s determination.

A theory developed by Basso (2003) and based on the work of Hilferding (1981) proposes a supplementary structure to the PPP doctrine which aims to explain better the FX movements. This alternative method differs from the paradigms developed in the previous research papers of Samuelson (1964) and Balassa (1984). The mentioned studies employ a different approach. They argue that the distribution of productivity between economies and across tradable and non-tradable goods sectors in each country is important for the RER’s determination. The standard Balassa-Samuelson framework refers to the total factor productivity. This measure, however, entails some practical data problems because it is difficult to be calculated directly\textsuperscript{22} (Schnatz et.al, 2003).

\textsuperscript{22} For example, one issue is how to measure the technological advances in capital goods properly.
Basso’s alternative theory attempts to resolve the above-mentioned issue in point 3 by including both prices and productivity in the monetary equation. This is contrary to the PPP theory which includes productivity within prices. Thus according to PPP, the change in the FX rate would be triggered entirely by price movements, and not by productivity changes. It totally omits the productivity factors and focuses primarily on the money supply movements. The PPP states also that the prices of two goods would be similar in a long-run. Basso (2003), on the other hand, expects the price-equality to be based on the quantity of time social work. Thus, this new approach supplements the PPP doctrine by relating labour values to the prices and estimating these two effects separately on the determination and movements of the exchange rates. This theory is based on Hilferding’s concept of currency value. It represents a macroeconomic relationship, calculated by dividing the domestic product of a country to the hours of work spent to produce it.

\[
\text{Currency value} = \frac{GDP}{\text{Hours of work to produce it}} (3.10)
\]

The concept of productivity, therefore, is defined by the sum of the hours work. It can also be transformed into a physical productivity by multiplying the number of workers with their average amount of hours of work (Basso, 1999). However, the physical productivity index in not a variable in this econometric study, and therefore, it would not be explained in details.

Considering the Law of one price, one can re-write equation (1.1) as follows:

\[
\frac{GDP}{Wh} = s_t \frac{GDP^*}{Wh^*} (3.11)
\]

where \(GDP\) is the EZ’s domestic product, \(Wh\) is the working hours spent to produce it and the star denotes the foreign equivalents accordingly. The nominal exchange rate between the countries is denoted with \(s_t\).

Assuming the GDP consists of final goods, equation (3.11) can be expressed as a product of price \(P\) and quantity \(Q\) index.

\[
\frac{PQ}{Wh} = s_t \frac{P^*Q^*}{Wh^*} (3.12)
\]

The ratio \(\frac{Q}{Wh}\) represents in fact the productivity of the EZ. Reciprocally, \(\frac{Q^*}{Wh^*}\) is the US one. If the productivity is denoted with \(od\), and \(prod = \frac{Q}{Wh}\), than the equation between the two countries is further transformed.
\[ P_\text{prod} = s_t_\text{prod}^* \Rightarrow s_t = \frac{P_\text{prod}}{P_\text{prod}^*} \quad (3.13) \]

This equation is the cornerstone of the alternative theory and in dynamic term it becomes (3.14).

\[ s_t = (P - P^*) + (\text{prod} - \text{prod}^*) \quad (3.14) \]

This equation shows that the exchange rate between the two economies is affected by the productivity, in addition to the price levels. If productivity alone determines the prices, higher US in respect to the EZ one would be offset by higher EZ prices and this connection would not alter the exchange rate. Important feature is that the productivity is measured by the homogeneous variable \( W_h \) and not by the physical productivity, which itself is the quantity of output produced by one unit of production input. Additionally, prices are obviously altered by other than productivity factors. The equation (3.14) gives a separate explanatory power to the productivity differential between both economies, and therefore, it can be used in the BEER estimation for strengthening the monetary equation (3.8).

### 3.3 DERIVATION OF THE ECONOMETRIC BEER MODEL

Since its inception, the euro is always quoted indirectly by the ECB for all bilateral pairs, i.e. as units of foreign currency per one euro. Therefore, the variable \( s_t \) is expressed in EUR/USD\(^{23} \). Thus, the exchange rate is tracking primarily the value of the USD while holding the EUR normalized to 1. The theoretical studies, summarizing the different monetary models in Table 2, base their estimations on a directly quoted domestic exchange rate. Therefore, the variables` impact in this econometric study will be opposite in signs. Again, this results from the officially accepted by the ECB way of quoting the euro. All theoretical applications are valid accordingly, just with an opposite sign from the ones summarized by Frenkel (2004) in subsection 3.1.

Since the productivity variable contains information about the output in itself, the GDP differential will be excluded from the equation in order to reduce any double-counting in the cointegration equation. In the process of empirical estimations, the logged GDP differential is also found to be a stationary variable, thus, it cannot have a long-term effect on the RER (see Fig. A5 in APPENDIX A).

\(^{23}\) See subsection 6.1 for more information
The combined equation for deriving the long-term equilibrium exchange rate, according to the flexible-price monetary model and the alternative theory of labour productivity is shown below.

\[ s_t = -\beta_1(m_t - m_t^*) - \beta_2(\pi_{t+1} - \pi_{t+1}^*) \pm \beta_3(r - r^*) + \beta_4(prod - prod^*) \] (3.15)

A basic assumption of (3.15) is that changes in the relative supply of M2 in the EZ lead to prices adjustments, and thereby, influence the euro negatively. The effect on the US M2 is positively correlated with \( s_t \) - a rise of the US money supply is expected to increase the USD value in terms of EUR. As a result of these two features, the money supply differential would have an overall negative effect on the euro. However, these are only the theoretical implications; the note from the end of subsection 3.1.1 is still valid. One is required to run the actual econometric estimation to calculate the effect.

An increase in the domestic inflation expectations - ceteris paribus- causes a depreciation of the euro because the higher interest rate reduces domestic demand for money (Schröder & Dornau, 2000). Respectively, more US inflation expectations for \( t + 1 \) period would decrease the value of the USD in the current period. Thus, the proxy for growing inflation expectations differential would depreciate the euro. The value of the USD will strengthen in terms of 1 euro and the graph of EUR/USD will depreciate.

An increased labour productivity at home will affect positively the euro, whereas a rise in the US productivity will reduce it accordingly. The differential is negatively correlated with the USD movements- the bigger it is, the more USD will depreciate in terms of EUR and the FX curve will go up. The differential will have a positive effect on EUR/USD.

The final econometric model for estimating the long-term equilibrium exchange rate contains also a constant, trend, as well as seasonal dummies due to the nature of the variables (see Chapter 5).

\[ s_t = \pm b_0 - b_1(m_t + m_t^*) - b_2(cpi_{t+1} - cpi_{t+1}^*) \pm b_3(R_t - R_t^*) + \beta_4(prod - prod^*) \pm \gamma t \pm \text{seasonal dummies} + \epsilon_t \] (3.16)

This equation has the following assumptions based on the above-mentioned effects

\[ b_1, b_2 < 0 \]

\[ b_4 > 0 \]
and the sign of $b_3$ would have to be defined empirically.

A long-run neutrality of money is imposed and the monetary restriction is

$$b_1 = 1.$$  

The tested hypotheses are:

**H0:** The alternative monetary model with productivity does not explain the EUR/USD exchange rate

**H1:** The alternative monetary model with productivity explains the EUR/USD exchange rate.

Much of the subsequent analysis centers on the concept of the exchange rate’s total misalignment from its fundamental value. It is a generic representation of the long-run FX equilibrium, derived by the modern theories for its determination. In other words, it will be tested whether the long-term and the actual exchange rates are cointegrated, using the econometric form above and the techniques in the following Chapter 4. Furthermore, if the long-run and short-run fundamentals $Z_{LT,t}$, $Z_{MT,t}$ and the RER are cointegrated, their equation can be rearranged according to the Granger Theorem (1987) into an error-correction model that would show the short-term disequilibrium. Based on the estimation equation, afterwards, the BEER values will be derived empirically and plotted on a single graph with the actual RER. Thus, the main goal of this econometric study is to evaluate whether the euro is under-, overvaluated or in line with the estimated equation, based on the monetary variables and the productivity.


4 ECONOMETRIC METHODOLOGY

This chapter explains the basic terms like time series, stochastic processes, stationarity and autoregressive moving average (ARIMA) models. The unit root methodology is represented by Dickey-Fuller test. Furthermore, the concepts of cointegration and vector error correction are introduced.

4.1 TIME SERIES AND STOCHASTIC PROCESSES

A time series is a sequence of data points representing the movement of a variable at uniform time interval. Referring to series of observations as a time series, some regularity of observation frequency is assumed, in the case of this study- it is a quarterly data. A stochastic process \( Y_t \) is a sequence of random discreet variables belonging to a domain \( R \) which represents a common probabilistic space such that all joined probabilities exist. If \( Y_t \) is defined as a stochastic process, \( Y_1, ..., Y_t \) will denote the realization of discreet-time stochastic process, which is finite \( R \). Time series analysis is used to address a number of economic issues in the reality, valid for many different macroeconomic estimations and forecasts. The general idea is to explain the possible relationship in a generated model between some presumably related variables, or ideally to uncover any cointegration in an economic system (Lutkepohl, 2006).

4.1.1 Stationary stochastic processes

A stochastic process is said to be stationary if its mean and variance are constant over time and the value of the covariance between two periods depends only on the distance between the two time periods and not on the actual time, at which the covariance is computed (Gujarati, 2004, p. 797). It represents a “basic model” of time series analysis, characterized by a zero mean and constant variance.

Mean: \( E(Y_t) = \mu = 0, \forall t \)

Variance: \( \text{var}(Y_t) = E(Y_t - \mu)^2 = \sigma^2 < \infty, \forall t \)

Covariance: \( \text{cov}= E[(Y_t - \mu)(Y_{t+k} - \mu)] \)

and since \( \mu = 0 \), the equation can be re-written:

\( E(Y_t Y_{t+k}) = 0, \forall t \neq t + k \)
Such stationary time series will tend to return to their mean (also called mean reversion). The fluctuations around this mean have constant amplitude. The time series data would not be scattered in a different uncorrelated episodes, but will represent a whole process. This is very important as it has a particular practical value for forecasting processes.

A stochastic process is called a random walk or white noise when its first differences have a zero mean, constant variance and are serially uncorrelated.

\[ Y_t - Y_{t-1} = \varepsilon_t \tag{4.1} \]

\[ E(\varepsilon_t) = 0; E(\varepsilon_t^2) = \sigma^2; E(\varepsilon_t \varepsilon_s) = 0, s \neq t \]

The term “random walk” was first introduced by Karl Pearson in 1905 and is often described as a “drunkard’s walk” (Dixon, 2004), i.e. randomly taken steps constituting arbitrary developed trajectory.

### 4.1.2 ARIMA models

These autoregressive processes are very frequently used and represent the most relevant class of time series models. The general AR(p) order model is expressed as follows with \( p \) as the number of autoregressive order.

\[ Y_t = \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \ldots + \beta_p Y_{t-p} + e_t \tag{4.2} \]

The moving averages processes MA(q) are a weighted sum of the error terms which are always stationary.

\[ Y_t = e_t + \theta_1 e_{t-1} + \ldots + \theta_q e_{t-q} \tag{4.3} \]

### 4.2 INTEGRATED PROCESSES AND UNIT ROOT TESTS

A time series process is said to be integrated of order \( d \) and denoted I(\( d \)) if it becomes stationary after \( d \)-times differencing. A random walk process is I(1). If the time series is stationary, the standard method for estimating the coefficients will be the ordinary least squares (OLS) (Brook, 2002). However, if the series are integrated of the same order, the OLS can be misleading. In such a case, when the series are integrated of the same order and there is a cointegration relationship amongst them, the
parameters can be estimated by a vector autoregression model (VAR) or a vector error correction model (VECM). Therefore, it is essential to establish a simple test for determining the order of integration and the cointegration relationship.

### 4.2.1 Dickey-Fuller Unit Root Test

The most widely used statistical test for testing order of integration is Dickey-Fuller (D) and Augmented Dickey-Fuller tests (Brook, 2002). The DF test assumes that the process is AR (1). As described in the previous chapter, a time series is said to be a random walk if it has a stable variance.

\[
Y_t = Y_{t-1} + \epsilon_t \quad (4.4)
\]

This equation shows that the value of \( Y \) in time \( t \) is equal to its value in the previous period plus a random error, called white noise. This constitutes an autoregressive model 1 or AR(1) (Brook, 2002).

Testing for a unit root involves adding a lag of one period and equation can be re-written into (4.5).

\[
Y_t = \rho Y_{t-1} + \epsilon_t \quad \text{and} \quad -1 < \rho < 1 \quad (4.5)
\]

The procedure to estimate if the process is a nonstationary or stationary is to regress the \( Y_t \) on its own period lagged value \( Y_{t-1} \) and find out the value of the \( \rho \) coefficient. For manipulative reasons \( Y_{t-1} \) is subtracted from both sides.

\[
Y_t - Y_{t-1} = \rho Y_{t-1} - Y_{t-1} + \epsilon_t = (\rho - 1) Y_{t-1} + \epsilon_t \quad (4.6)
\]

This equation can be further re-written as follows

\[
\Delta Y_t = \delta Y_{t-1} + \epsilon_t \quad (4.7)
\]

where \( \delta = (\rho - 1) \) and \( \Delta \) is the first difference operator and the equation expresses a random walk with no drift.

The null hypothesis is \( \delta = 0 \), which yields \( \rho = 1 \), meaning that the series is nonstationary. In this case the first difference of a random walk is stationary because as previously assumed \( \epsilon_t \) is stationary (uncorrelated) (Brook, 2002).

\[
\Delta Y_t = Y_t - Y_{t-1} = \epsilon_t \quad (4.8)
\]
In case when the error term $\epsilon_t$ is correlated, the Dickey and Fuller have developed an augmented Dickey-Fuller test (ADF).

The procedure of implementing the normal DF test is to estimate the nature of the unit root process. A random walk process may not have a trend.

$$\Delta Y_t = \delta Y_{t-1} + \epsilon_t \quad (4.9)$$

In case there is a distinguished drift, the equation changes.

$$\Delta Y_{t-1} = \beta + \delta Y_{t-1} + \epsilon_t \quad (4.10)$$

When the constant variable $\beta > 0$, the process will exhibit a positive trend. In case $\beta < 0$ the trend will be downwards. The trend is called stochastic and in the econometrics a time series with such trend is not predictable. A predictable time series has a deterministic trend and represents a random walk with drift oscillating around a stochastic trend.

$$\Delta Y_{t-1} = \beta_1 + \beta_2 t + \delta Y_{t-1} + \epsilon_t \quad (4.11)$$

The mean of $Y_t$ is $\beta_1 + \beta_2 t$, which is not a constant, but nevertheless its variance is a constant.

The ADF test is conducted by augmenting the above mentioned 3 equations by adding the lagged value of the dependent variable $Y_t$ and estimating the following equation

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \delta \sum_{i=1}^{w} \Delta Y_{t-i} + u_t \quad (4.12)$$

where $\Delta Y$ denotes the first differenced value of $Y$, $\alpha$ is a constant and $\beta$ - a time trend coefficient, $Y_{t-1}$ is the first-lagged value of $Y$, $\sum_{i=1}^{w} \Delta Y_{t-i}$ are the 1st, 2nd, 3rd,..., nth-lagged of 1st-differenced values of $Y$, $i$ is the lag order of the autoregressive process, $u_t$ is a pure white noise term (Brook, 2002). The main idea of this technique is to add as many empirically determined terms in order to make the error term $u_t$ serially uncorrelated. The null hypothesis tests for non-stationarity, i.e. $H_0: \gamma = 0$, whereas the alternative $H_1: \gamma < 0$ tests for stationarity of the time-series. If the series is integrated of some order, the lagged level of the series $Y_{t-1}$ will provide no relevant information for a prediction in $Y_t$, besides the one obtained in the lagged changes $\Delta Y_{t-1}$ In that case the null hypothesis is not rejected.
4.2.2 HEGY unit root test for seasonality

A Hylleberg-Engle-Granger-Yoo (HEGY) tests for seasonal unit roots in quarterly time series, based on the following model:

\[
\Delta^4 Y_t = \pi_1 z_{1,t-1} + \pi_2 z_{2,t-1} + \pi_3 z_{3,t-1} + \pi_4 z_{4,t-1} + \delta \sum_{i=1}^{p} \alpha_i \Delta^4 Y_{t-i} + u_t \tag{4.13}
\]

where \( z_{1t} = (1 + L + L^2 + L^3)Y_t \), \( z_{2t} = -(1 - L + L^2 - L^3)Y_t \) and \( z_{3t} = -(1 - L^2)Y_t \) with \( L \) being the lag operator. The null hypotheses are \( \pi_1 = 0 \) (regular), \( \pi_2 = 0 \) (semiannual) and \( \pi_3 = \pi_4 = 0 \) (annual) unit roots, considering the relevant t- and F-tests of the regression. The latter is used for the joint null hypotheses that some \( \pi_i \) are all zero and that all four \( \pi_i \) are jointly zero (Lutkepohl & Krätzig, 2006). If the HEGY statistic is smaller than the critical values, the H0 for seasonality cannot be rejected. In this case, in the ADF model of the particular variable will be added 3 seasonal dummies and not 4 in order to avoid the multicollinearity.

4.2.3 Residuals` test

The residual test procedure is required in order to decide upon the truthfulness of the performed unit root estimations. It involves the following tests.

- ARCH-LM test for autocorrelation

The ARCH-LM tests the hypothesis of no series autocorrelation in the residuals from the regressions. The approach was proposed by Engle (1982) and involves a regression of the squared residuals on a constant and their \( k \)-lagged value. From the results of this auxiliary regression, a test statistic is calculated as \( (n - k) R^2 \), where \( n \) is number of observation and \( k \) is the degrees of freedom. It follows \( \chi^2 \) critical values for the used degrees of freedom. There is evidence to reject the null hypothesis if the ARCH-LM test statistic exceeds the critical value from a chi-square distribution with \( q \) degrees of freedom. The diagnostic tests for the AR(?) model are given together with p-values. The latter represents the probability of getting a test value greater than the observed one if the null hypothesis is true. Hence, the null hypothesis is actually rejected only for p-values smaller than 0.1 or 0.05. If all p-values are relatively large, the Ho cannot be rejected (Luetkepohl, 2004)

- Jarque-Bera test for normality

\[\text{The multcollinearity problem occurs when two or more regressors are highly correlated in a multiple regression.}\]
Jarque-Bera (JB) test for normality analyzes whether the residuals from regressions are normally distributed, since this is one of the properties of OLS estimators under the normality assumption (Brooks, 2002). In order to make conclusions on coefficients’ truthfulness and the estimated model, the residuals are required to follow a standard normal distribution. The test computes the skewness and kurtosis of the OLS residuals. Kurtosis is a measure of peakedness with less density in the middle, whereas the skewness is associated with the fat-tails of a normal distribution. The latter has ideally a kurtosis of 3.0 and skewness of 0. Therefore, the joint hypothesis, where the JB statistic is expected to be 0, so $H_0: S=0$ and $K=3$. In other words, the $H_0$ tests whether the residuals are normally distributed. It follows a Chi-square with 2 degrees of freedom distribution (Brooks, 2002). If the computed JB statistic is smaller, that indicates that the residuals are normal.

### 4.3 COINTEGRATION ANALYSIS

Testing macroeconomic variables almost always involves nonstationarity and trending series. Price levels and exchange rates could be some examples of them. Notwithstanding that differentiation and other operations transform the series into stationary, unfortunately differentiating I(1) nonstationary series limits the scope of the questions to be answered. There are also other techniques to estimate the trending behavior of a variable. For example the error terms in the simple regression model $y_t = \beta x_t + e_t$ are assumed to be stationary. This, however, may not be true if the variables $y_t, x_t$ are integrated series. Regression of a nonstationary on another nonstationary time series may produce a problem with spurious regression.

If two series are integrated of different orders, the logical conclusion would be that a linear combination of them would be integrated of a higher order, not I(0) (i.e. not stationary) (Brooks, 2002). If they are both trending upwards for example, the difference between them would also grow with time. The model is inconsistent. On the other hand if both series are I(1) there must be a beta coefficient allowing that $e_t = y_t - \beta x_t$ is stationary, i.e. I(0), and the difference is stable at a fixed mean (Brooks. 2002). The series are drifting together at the same rate and they are said to be cointegrated. In this perspective, the correct econometric methodic would be to regress the cointegrated of the same order variables on each other, which would itself produce a meaningful result and avoid the problem of spurious regression. This methodological
meaningfulness only appears if there is a long-term equilibrium relationship existing between the estimates. In this perspective, a cointegration between two variables also determines not only their short-run relationship, but also the long-run trend.

4.3.1 Johansen`s test for cointegration

Johansen (1991) proposed a test for cointegration of several time series. The major advantages using Johansen`s methodology are: (i) it allows testing for number of cointegrating vectors when their number is \( n > 2 \), and (ii) it provides a joint procedure of testing and maximum likelihood estimation of the VECM and long-run equilibrium relations (Hjalmarsson & Österholm, 2007). Considering a VAR of order \( p \)

\[
Y_t = \mu + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \ldots + \beta_p Y_{t-p} + \gamma X_t + e_t \quad (4.14)
\]

where \( Y_t \) is a \( k \)-vector of non-stationary I(1) variables, \( X_t \) is a deterministic variable and \( e_t \) is the error term. The VAR may be re-written as

\[
\Delta Y_t = \mu + \Pi Y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + BX_t + e_t \quad (4.15)
\]

where \( \Pi = \sum_{i=1}^{p} A_i - I \) and \( \Gamma_i = -\sum_{j=i+1}^{p} A_j \)

If the coefficient matrix \( \Pi \) has a reduced rank \( r<n \), then there exist \( n*r \) matrices \( \alpha \) and \( \beta \) each with rank \( r \) such that \( \Pi = \alpha \beta' \) and \( t \beta' \gamma \) is stationary (Hjalmarsson & Österholm, 2007). \( r \) is the number of cointegrating relationships, the elements of \( \alpha \) are known as the adjustment parameters in the VECM and each column of \( \beta \) is a cointegrating vector (Brooks, 2002).

Johansen proposes two different likelihood ratio tests of the significance of these canonical correlations, and thereby, the reduced rank of the \( \Pi \) matrix: trace and maximum eigenvalue tests (Hjalmarsson & Österholm, 2007). The method is to estimate the matrix from an unrestricted VAR and to check whether one can reject the restrictions implied by the reduced rank of \( \Pi \) (Hjalmarsson & Österholm, 2007). Thus, the tested hypothesis for the trace test is the number of cointegration vectors \( r \leq n \), on the other hand the null hypothesis for the eigenvalue test is \( r = n \). The estimated equation is as follows

\[
H_1(r): \Pi Y_{t-1} + BX_t = \alpha (\beta' Y_{t-1} + \rho_0 + \rho_t t) + \alpha_1 Y_0 + \text{seasonal dummies} \quad (4.16)
\]

where \( \alpha_1 \) represent deterministic terms “outside” the cointegrating relations.
### 4.3.2 ECM

A VECM is a restricted VAR, designed to use nonstationary series that are known to be cointegrated. The VEC has cointegration relations, which are built into its specification so that it restricts the long run behavior of the endogenous variables (Kasibhatla et al., 2006). Thus, this is making them to converge to their cointegrating relationships, and at the same time, allowing for short-run adjustment dynamics. The cointegration term is known as the error correction term since the deviation from long-run equilibrium is corrected gradually through a series of partial short-run adjustments (Kasibhatla et al., 2006). In case there is disequilibrium in the short-run this would mean that the error term should be used to stabilize the short-run behavior of the regressand to its long-run value. This is called error-correction mechanism (ECM). ECM states that the regressand depends on the regressor and the equilibrium error-term. In case the latter has a non-zero value, then the model is out of equilibrium. If the error term indeed is statistically 0 this would mean that the regressand adjust itself to changes in the regressor in the same period (Gujarati, 2004, p.825).

Considering the simple example of a two variable cointegration equation with no lagged differences is equation (4.17).

\[ y_{2,t} = \beta y_{1,t} \]  
\[ (4.17) \]

The corresponding VECMs are presented below.

\[ \Delta y_{1,t} = \alpha_1 (y_{2,t-1} - \beta y_{1,t-1}) + \varepsilon_{1,t} \]  
\[ (4.18) \]

\[ \Delta y_{2,t} = \alpha_2 (y_{2,t-1} - \beta y_{1,t-1}) + \varepsilon_{2,t} \]  
\[ (4.19) \]

The right-side of this equation represents the error correction term, which in long-run equilibrium is 0. However, if the parameters \( y_1 \) and \( y_2 \) deviate from the long-run equilibrium the error correction term will be non-zero and each variable adjust in order to restore the equilibrium. The coefficient \( \alpha \) measures the speed of adjustment of the \( i \)-th endogenous variable towards equilibrium.

Considering an autoregressive distributed lag model without an intercept and rearranging it, one obtains the equation below.

\[ y_t = \gamma_1 y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + \varepsilon_t \]  
\[ (4.20) \]
Thus, the ECM is shown below.

\[ \Delta y_t = \beta_0 x_t + (y_t - 1) \beta_1 (y_{t-1} - \theta x_{t-1}) + \epsilon_t \]  \hspace{1cm} (4.21)

In the context of the BEER estimation in this study, the equation

\[ \ddot{s}_t = \beta_1 Z_{LT,t} + \beta_2 Z_{MT,t} \]  \hspace{1cm} (4.22)

can be rearranged in the VECM according to the Granger Theorem which has the following form

\[ \Delta s_t = -\alpha (s - \beta Z)_{t-1} + \sum_{k=0}^{\infty} \alpha_k \Delta Z_{t-k} + \sum_{k=0}^{\infty} \gamma_k \Delta s_{t-k} + \epsilon_t \]  \hspace{1cm} (4.23)

where \( \alpha \) is the speed of adjustment parameter, \( (s - \beta Z)_{t-1} \) is the BEER and the rest is the short-run dynamics plus the error term.

The model shows the equilibrium relationship and the equilibrium error, accounting for the deviation of these variables from the denoted equilibrium. A change in the in the previous period consists of movement changes in along a long-run path plus a part of the deviation from the denoted equilibrium (Greene,p.690).
5 ESTIMATION RESULTS

5.1 DATA

All time-series, used for performing this econometric study, are in quarterly frequencies for the period Q12000 – Q42011. The year 1999 from the implementation of the euro as an accounting currency is omitted. This is because a data on the productivity per hour work for the 17 Eurozone countries in 1999 is not available from any source. Therefore, the overall time frame consists of 48 total observations. The estimations are made with statistical programs Gretl, JMulti and EViews 7, because none of these packages provides a fully integrated statistical package for all the required tests. The estimations are preformed, and thus, outputs are presented on a consistent basis.

The variables are examined in logarithms, because in levels they are either very big numbers, or indices, and therefore, this common statistical approach attempts to make their variance more constant. The only exception is the interest rate differential as it is in percentages and do not vary much in time. Therefore, it is analyzed in levels and is denoted with big letters in the econometric models of this study. The observations are obtained on a consistent basis from several sources.

- The nominal exchange rate is quoted indirectly as EUR/USD with the EUR as normalized to 1, i.e. the time-series represent the movements of the USD relative to the value of 1 EUR. The data is obtained from the data provider Thompson Reuters Datastream AFO, with the code USEURSP.

- The money supply figures, used for both areas, represent the monetary aggregate M2. ECB defines it as an intermediate monetary aggregate, comprising the narrow money in circulation and in overnight deposits plus the deposits up to 2 years and redeemable at notice up to 3 months. Whereas, according to the FED, M2 consists of M1 in addition to the savings accounts, time deposits under 100,000 USD and the balances in retail money market mutual funds. Both figures, used for calculating the money supply differential, are obtained from Thomson Reuters Datastream. The EZ code is EMM2B, given in bn Eur, whereas US code is USM2B, in bn USD. The differential is calculated accordingly.

- The relative consumer price indices for the US and the EZ (17) are in index values with base year 2005(=100%). They are obtained from the OECD
database and have no particular source codes. They track the change in a country's CPI (converted into US dollars) to a weighted average of changes in its competitors' CPIs (also expressed in US dollars). Their differential, therefore, follows the movement of the dollar. Due to the fact that the model requires a use of proxies for the inflation expectations, similarly to the methodology of Giannellis (2011), this study assumes that the future one period \( t+1 \) values of the both indices are representing the current estimates of the expected inflation rates in time \( t \). For example, the data point for Q2 2000 CPI would be the proxy for the Q1 2000 inflation expectations. Their differential is calculated accordingly. Using these particular relative indices results from the fact that the HICPs are producing very high standard errors; the inflation differential, on the other hand, is an I(0) variable and is also not suitable for this study.

- The interest rate series are obtained from EconStats with no particular coding. They represent the average-over-a-quarter 3 months EUR and USD LIBOR rates. These are the average interest rates at which a selection of banks in London are prepared to lend to one another in euros, respectively US dollars, with a maturity of 3 months. In this study, exactly these market rates are preferred over the 3 months US Treasury Bills and their euro equivalents, in order to obtain comparable data, consisting of figures traded on a single market. Their differential is calculated accordingly.

- The productivity data is obtained from Eurostat for the 17 EZ countries and The Bureau for Labor Statistics for the US. The unit measure for both is an index with a base year 2005 (=100%). The EZ data represents the quarterly real labour productivity per hour work with a source code INDIC_NA. The US data is the statistics “output per hour” with a series number PRS84006093. Their differential is calculated accordingly.

### 5.2 TESTING FOR UNIT ROOTS

Testing for stationarity of time-series is the initial procedure when performing an econometric analysis, but prior to its actual implementation, a visual plot of the data is usually the preceding step (Gujarati, 2004). The most widely discussed visual technics for analyzing the data in the econometric literature are the graphical analysis and the

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25 From the beginning of 2012, this data is provided only on annual basis.
correlogram tests. The latter will be discussed in the following subsection as it is a procedure for testing the lagged differences of the ADF test for autocorrelation (Brooks, 2002).

5.2.1 Graphical inspection for a trending behaviour

Stationary time-series have a time-varying mean and variance that are not subject to a major change. Graphically, this implication comprises a sideways trend with no major fluctuations and random movements. It reflects economic conditions in which the variable is moving back and forth between narrowly ranging levels. On the FX market for instance, the sideways trend would show no major difference between the currency pair’s price values from the beginning to the end of the observed period. When the first impression from the graphical inspection is that the time-series exhibits some trending behaviour, therefore, this would indicate that there is a presence of a unit root in the data. Contrary to the definition of trendless market, an upward trend represents a systematic increase in the variable. It increases more than it falls over the period. A downward sloping trend possesses, on the other hand, the opposite characteristics, expressed in a reversed direction. The variable exhibits, in general, more falling that increasing behavior.

There is another main issue when working with time series data which does not follow any of the above-mentioned descriptions. It occurs when the values of the parameters change in opposite directions several times thorough the period. This would result into having one or more levels, or trend structural changes in the relationship between $Y_t$ and the regressors. A structural break in the time series data may be caused by exogenous factors like policy changes aimed to reduce economic shocks. Therefore, its discussion is very much applicable for the data used in this study, considering the volatile macroeconomic situation. Giannellis (2011), for example, analyzes the EUR/USD between 1999 and 2008. His estimations show that the pair has two structural breaks, as follows:

- Early 2002 – This structural shift accompanies the period after 11th September 2001, when the FED chairmen Allan Greenspan cut the federal fund rate and kept it below its normal value for several years. The period was also characterized with a significant increase in the government spending aiming to finance the US military actions.
• Early 2005 – According to the above mentioned authors, this period shows a continuous depreciation of the USD against the EUR due to the large US current account deficit.

His study, nevertheless, does not estimate the period after 2008, when the major recession became evident. Using the statistical package JMulti, which provides technical assistance to test for structural breaks, one may estimate that there is another break date in the EUR/USD in Q4 2008. The unit root test for two and more structural breaks in a time-series, initially introduced by Lumsdaine and Papell (1997) however, is far beyond the scope of this essay as it requires specialized econometric knowledge. Therefore, this study will employ only the standard ADF procedure.

As a fundamental concept not only on the FX market, but also in econometrics, the trend analysis test is conducted for all of the variables in the model. Each series of interest is plotted on the vertical axis against the date, which is always set horizontally. All graphics are available in FIGURES Appendix.

• The log EUR/USD line exhibits an upwards slope over the whole period apart from Q1 2000-Q2 2002, when the value of the euro systematically decreased. From Q2 2002-Q3 2008, the trend was clearly upwards with a small correction in the period Q1 2005- Q1 2006. After this bullish stage, the euro`s exchange rate declined slightly and consolidated a ranging behavior from mid-2008 until presently. This is a result of the global financial crisis when the fundamental macroeconomic news from both regions were negative, investors adopted a risk averse behaviour. Thus, the volatility on the FX market increased to levels never seen before as a consequence from the uncertain environment. Notwithstanding this consolidation, the EUR/USD exchange rate exhibits clear trend pattern during the period. One might be tempted to conclude that there is a non-stationarity in the data. If this is the case, the upcoming ADF test(s) would define the order of integration of this variable.

• In the first years, the M2 differential between the two economies exhibits a slight downward trending. This was due to the fact that the money supply in the US increased considerably in order to stimulate the recessed economy after the events of 9/11 and the busted Dot-com bubble. The money supply variable

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26 A trading jargon, denoting the rising value of a particular instrument on a market.
27 A speculative bubble period in the information technology sector covering the years 1995-2000.
exhibits a clear upward trend behavior from Q3 2003 until Q2 2008. This is in line with the steady increase of the ECB`s balance sheet due to the admittance of some new members to the currency union. This trend persisted until the Lehman Brothers` collapse when the subprime crises turned into a liquidity one and the FED responded with the first quantitative easing, MBS and other stimulating programs that pumped dollars in the domestic and world economy (see Table 3). The action of printing USD and supplying the world with liquidity increased further the difference between the balance sheets of the FED and the ECB. The visual analysis of the graph, suggests that there is a clear structural break in Q3 2008. Therefore, this variable is most probably non-stationary.

- As visible on the graphs, the CPI differential has an upward trending.
- The interest rate differential exhibits an interesting patter. No trend is visible in the figure, suggesting that presumably this variable is stationary as it revolves around the zero meant.
- The differential of both economies` output per hour work exhibits a downward pattern over the whole period, except between Q32003 until Q32008, when the indices (2005=100%) between the two regions show no major differences and fluctuate steadily around the zero mean.

The conclusions from all graphical tests for stationarity before the actual econometric analysis are that on a first glance only the R-R* and the productivity differential show no particular trend behaviour. All the other variables in the model might be individually integrated of a higher order. A time series is said to be integrated of order d, denoted ~I(d), if it becomes stationary after differentiating it d-times. The following unit-root tests will show precisely whether the series under consideration are stationary and/or how many times they would have to be differenced in order to obtain stationarity.

### 5.2.2 Unit root test procedure

The ADF test involves an estimation of autoregressive process that consists of lagged variables. The Ho is that there is a unit-root. It is tested against the alternative that the series is stationary (Brook, 2002). The ADF test in this study will comprise the following general procedure:

1) At first, it is essential to define the correct ADF model, which may or may not include a constant, trend and seasonal dummies (Brook, 2002). Testing economic data
without an intercept is not a very common procedure in econometrics. Therefore, a constant will be always present in the estimations. The trending has already been examined in the subsection 5.2.1. Furthermore, all variables will be test empirically for seasonality.

2) The lags included in the ADF tests have to be selected. There are many different approaches to lag length selection in the econometric literature (Koop, 2009). Moreover, some statistical programs provide the best lag order. For example JMulti performs automatically and suggests a lag order according to the following criteria-AIC, BIC, HQC and final prediction error. Gretl, on the other hand, sets always 1 lag as default, and thus, is not the optimal program to estimate the ADF test.

3) The entire unit root testing will be performed with JMulti with an automatically suggested optimal lag length after searching for up to 10 lags in the differences. Thereafter, another ADF test will be performed with the suggested lag, which would provide the t-statistic for the final decision-making. If the series are nonstationary in levels, they will be differentiated one or several times in order to achieve stationarity.

4) In order to have an unbiased estimator in each equation, the normality of the residuals has to be checked. Therefore, the autocorrelation function (ACF) and partial ACF (PACF) from the equations are plotted (Brooks, 2002). If there are significant spikes in the graphs, one may conclude that there is an autocorrelation. When all spikes are within the limits, this problem does not exist. Additionally, a numerical ARCH-LM test for autocorrelation and a Jarque-Bera test for normality will be performed as well. Only when there is a model with normally distributed random residuals, it is reliable to decide upon stationarity.

5.2.2.1 Defining the correct ADF model

Since it is decided upon the inclusion of an intercept and trend already, another issue left to be resolved is the presence of seasonality. The seasonal component of many macroeconomic time series is an important aspect in econometrics and it is best explained by seasonal dummies. Hylleberg (1992) defines it as “the systematic, although not necessarily regular, intra-day movement caused by changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by the agents of the economy”. The critical values for the test are obtained with reference P.H. Franses and B. Hobijn (1997).
The HEGY test is performed with the statistical package JMulti with automatically suggested lag length according to AIC. The residuals are controlled for normality. Table 4 provides the results.

The results detect a presence of seasonal unit roots in all series, apart from the interest rate differential. This variable does not indicate seasonality on any of the tested hypotheses. Therefore, it will be tested with no seasonal dummies. All the other variables will include them in the ADF estimation. This is done in an attempt to reduce the seasonal effect of the time series.

5.2.2.2 ADF testing

After defining the correct model, an ADF test is performed with the suggested lag lengths on levels. The results are summarized below, with the star denoting stationarity on 5% level. All relevant ADF tests are presented fully in APPENDIX A.
Table 6: ADF test on levels

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constant, Trend, Seasonal dummies</th>
<th>Lags</th>
<th>ADF statistic</th>
<th>MacKinnon’s (1993) critical distribution</th>
<th>I(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>constant, trend, seasonal dummies</td>
<td>0</td>
<td>-1.7968</td>
<td>-3.96, -3.41, -3.13</td>
<td>not 0</td>
</tr>
<tr>
<td>m-m*</td>
<td>constant, trend, seasonal dummies</td>
<td>3</td>
<td>-0.2062</td>
<td>-3.96, -3.41, -3.13</td>
<td>not 0</td>
</tr>
<tr>
<td>cpi-cpi*</td>
<td>constant, trend, seasonal dummies</td>
<td>6</td>
<td>-2.5692</td>
<td>-3.96, -3.41, -3.13</td>
<td>not 0</td>
</tr>
<tr>
<td>R-R*</td>
<td>constant</td>
<td>1</td>
<td>-3.4424</td>
<td>-3.43, -2.86, -2.57</td>
<td>I(0)*</td>
</tr>
<tr>
<td>prod-prod*</td>
<td>constant, trend, seasonal dummies</td>
<td>10</td>
<td>-4.6968</td>
<td>-3.96, -3.41, -3.13</td>
<td>I(0)</td>
</tr>
</tbody>
</table>

Source: Own calculations

The table summarizes the results from the ADF tests on the log levels of the variables. The only exception is the interest rate differential which is tested in levels. McKinnon’s critical values (McKinnon, 1993) are used for the decision-making upon the Ho. The results suggest that the interest rate and productivity differentials are integrated of order 0, i.e. they are stationary, because their t-statistics exceed the critical values. The Ho for non-stationarity can be rejected on 5% level for both variables. On the other hand, the exchange rate, the money supply and the CPI differentials are nonstationary. This means that they exhibit trend behaviour and will be differentiated further until stationarity is reached. Thus, the first differences of these variables are obtained. They are tested with another ADF model. The best suitable lag is selected as previously discussed. Figure 7 presents the ADF results on their first differences. The sign * denotes again a significance on the 5% level.

Table 7: ADF test on the first differences of the variables in logs

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constant, Trend, Seasonal dummies</th>
<th>Lags</th>
<th>ADF statistic</th>
<th>MacKinnon’s critical distribution</th>
<th>I(d) of the log levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(s)</td>
<td>constant, trend, seasonal dummies</td>
<td>10</td>
<td>-4.2887</td>
<td>-3.96, -3.41, -3.13</td>
<td>I(1)</td>
</tr>
<tr>
<td>D(m-m*)</td>
<td>constant, trend, seasonal dummies</td>
<td>0</td>
<td>-3.6298</td>
<td>-3.96, -3.41, -3.13</td>
<td>I(1)*</td>
</tr>
<tr>
<td>D(cpi-cpi*)</td>
<td>constant, trend, seasonal dummies</td>
<td>10</td>
<td>-3.7422</td>
<td>-3.96, -3.41, -3.13</td>
<td>I(1)*</td>
</tr>
</tbody>
</table>

Source: Own calculations
The ADF test results show that there are no more unit roots in the data as the Ho is rejected on 5% significance. This implies that the log levels on these variables are integrated of first order, because after differentiating ones the log levels became stationary. The following procedure is to test the residuals of all relevant for the cointegration regression ADF tests.

- **Stationarity of the productivity differential**

At this stage, it is important to point out that when the productivity differential is tested with only 1 lag, as suggested by the Schwarz Criterion, it becomes non-stationary. In other words, the 10 included lags, suggested by the other 3 criteria, can compensate its deviations from the zero mean. Using only 1 lag and transformed into its first difference, therefore, the variable becomes stationary.

**Table 8: Stationary productivity differential**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constant, Trend, Seasonal dummies</th>
<th>Lags</th>
<th>ADF statistic</th>
<th>MacKinnon`s (1993) critical distribution</th>
<th>I(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>prod-prod*</td>
<td>constant, trend, seasonal dummies</td>
<td>1</td>
<td>-1.7955</td>
<td>-3.96, -3.41, -3.13</td>
<td>not I(0)</td>
</tr>
<tr>
<td>D(prod-prod*)</td>
<td>constant, trend, seasonal dummies</td>
<td>0</td>
<td>-5.2666</td>
<td>-3.96, -3.41, -3.13</td>
<td>I(1)</td>
</tr>
</tbody>
</table>

*Source: Own calculations*

When only one lag is included, however, the residuals show some signs of autocorrelation in the 8th lag. Therefore, this result will enter the subsequent discussion only as a point of comparison, in which the productivity differential can enter the long-term cointegration.

**5.2.2.3 Residual analysis on the significant ADF tests**

The ACF and PACF of all relevant ADF tests have no major significant spikes. They are presented in APPENDIX B. Table 9 below summarizes the ARCH-LM and JB test statistics.

**Table 9: Residual tests**

<table>
<thead>
<tr>
<th>Variable</th>
<th>ARCH-LM p-Value($\chi^2$)</th>
<th>JB statistic p-Value($\chi^2$)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(s)</td>
<td>11.0363</td>
<td>0.3547</td>
<td>0.1227</td>
<td>-0.0944</td>
</tr>
<tr>
<td>D(m-m*)</td>
<td>0.1600</td>
<td>0.6892</td>
<td>0.9894</td>
<td>-0.3262</td>
</tr>
</tbody>
</table>
The null hypothesis for no autocorrelation from the ARCH-LM test cannot be rejected for none of the variables as all the p-values are larger than 0.05.

The critical values for the JB test will be obtained from the Monte Carlo distribution of the Lagrange multiplier test of normality of Deb & Sefton (1996). JB test was originally developed for large samples with >2000 observations. The sample in this study contains only 48 observations. Deb and Sefton’s critical values for 50 observations at 5% levels are 5.0002. When $J_B < LM_{50}$, one can reject the Ho that the residuals are normally distributed. Table 9 shows that all residuals from the ADF tests are within the critical values apart from the interest rate. Its JB statistic is higher than 5.0002. This might be due to the fact that there was only one lag length included in the ADF estimation. It was suggested by all criteria. However, its non-normality does not influence the validity of the ADF test’s asymptotics. Also, the variable is stationary, meaning that it cannot affect the exchange rate in the long-run in the BEER estimation. The next step is to estimate the cointegration of the variables.

5.3 COINTEGRATION TEST

Essential part of the cointegration estimation is to decide upon which variables should be included in the model. Johansen’s methodology is typically used in a setting where all variables in the system are I(1) (Hjalmarsson & Österholm, 2007). Nevertheless, having stationary I(0) variables is theoretically not an issue. Johansen (1995) states himself that there is a little need to pre-test the variables in the system to establish their order of integration (Hjalmarsson & Österholm, 2007). If a single variable is I(0) instead of I(1) it will reveal itself separately in the beta cointegrating vector(s).

As noted in section 2.4, the BEER model ignores per se the shorter term transitory I(0) effects for the long-run RER’s determination. Nevertheless, some studies use different stationary variables, like the interest rate differential for example (Stein, 2001). Omitting these I(0) variables will reduce the explanatory power of the model. In this study it is empirically proven, that the interest rate and productivity differentials are stationary variables, integrated of order I(0). They will be included in

<table>
<thead>
<tr>
<th>Variable</th>
<th>4.9099</th>
<th>0.8971</th>
<th>2.1986</th>
<th>0.3331</th>
<th>-0.5398</th>
<th>3.5480</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-R*</td>
<td>0.1842</td>
<td>0.6678</td>
<td>14.8223</td>
<td>0.0006</td>
<td>1.1391</td>
<td>4.5947</td>
</tr>
<tr>
<td>prod-prod*</td>
<td>4.4253</td>
<td>0.9261</td>
<td>1.1901</td>
<td>0.5515</td>
<td>0.4308</td>
<td>2.7737</td>
</tr>
</tbody>
</table>

Source: Own calculations
the model as unrestricted exogenous variables because they carry important information regarding the RER formation.

Before proceeding to the actual cointegration test, one is required to a VAR lag selection test perform at first. Afterwards, Johansen`s Trace and Maximum eigenvalue tests are estimated in order to test the long-run cointegration relationship between the RER and the fundamentals in the monetary model. Economically speaking, two variables are cointegrated if they have a long-term relationship between each other (Brooks, 2002). A potential cointegration vector will reveal itself carrying a p-value higher than >0.05. According to the Granger Representation Theorem, the cointegration is a necessary condition for the VECM. The latter is an important econometric model. According to Gujarati (2004), it allows the error term to be treated as an “equilibrium error”. The model as well as ties the short-run behavior of the variable to the long-run value of the regressand (or vice versa). Therefore, the last step comprises estimating a VECM.

5.3.1 VAR lag selection test

Before the actual cointegration estimation, a test for the appropriate lag-length is employed on the log variables in the VAR system. The endogenous variables are \( s_t \), \((m - m^*)\) and \((cpi - cpi^*)\). The exogenous variables include \((R - R^*)\) and \((prod - prod^*)\). In the matrix estimations are included also a constant, restricted trend and seasonal dummies. The lag length is chosen to be 4 because it is somehow reasonable to assume that the effects on the exchange rate, carried by the regressors, last approximately 1-1.5 years on such a liquid market like the FX (also regarded as the closest example to Fama`s theoretical concept of efficient markets).The results from the VAR lag test, performed with Gretl, could be seen below.

**VAR system, maximum lag order 4**

The asterisks below indicate the best (that is, minimized) values of the respective information criteria, AIC = Akaike criterion, BIC = Schwarz Bayesian criterion and HQC = Hannan-Quinn criterion.

<table>
<thead>
<tr>
<th>lags</th>
<th>loglik</th>
<th>p(LR)</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>358.01854</td>
<td>0.000412</td>
<td>-14.500843</td>
<td>-12.919402</td>
<td>-13.914368</td>
</tr>
<tr>
<td>3</td>
<td>364.94794</td>
<td>0.12744</td>
<td>-14.496724</td>
<td>-12.460336</td>
<td>-13.649410</td>
</tr>
<tr>
<td>4</td>
<td>382.48647</td>
<td>0.00006</td>
<td>-14.794839*</td>
<td>-12.483503</td>
<td>-13.937684*</td>
</tr>
</tbody>
</table>

*Figure 8: VAR lag selection test*
5.3.2 Johansen`s cointegration tests

The procedure for cointegration estimation with more than two variables is testing for more than two cointegration vectors (Brooks, 2002). Thus, the Johansen`s test is employed. In each case, the vector $Y_t$ contains the logs of the exchange rate, money supply differential and the inflation expectations differential. The interest rate and log production per hour work differentials are included as exogenous to the system. A constant, trend and seasonal dummies are added as well. The Johansen cointegration test is performed on the log levels of the variables (and ($R-R^*$) levels) because according to Gonzalo (1997) “the multico integration between more than two variables implies cointegration between the cumulated errors at one level of cointegration with the original variables”. The included lag length is 4 as suggested by two of the VAR lag selection criteria. The output of the regression can be preview on figure 9.

**Figure 9: Johansen`s cointegration estimation**

**Source: Own calculations**
As seen in the figure, all tests suggest unilaterally a Pi matrix with 1 rank, i.e. there is at least one cointegrating vector between the variables in the model. In the case of the Trace test corrected for small samples, for instance, the p-value on the first vector is 0.2763 which is much higher than 0.05. An Engle-Granger test cannot be performed for double-checking the results due to the different integration of the tested variables. There is also no need for it as all Johansen’s tests show quite strong evidence of existing cointegration. Therefore, the following procedure is to perform a VECM estimation.

5.3.3 VECM

After the established cointegration relationship, a VECM is estimated with 4 endogenous lags and 1 cointegration rank as suggested by the Johansen’s procedure. It includes the same variables. The VECM regressions are performed with Eviews 7 and Gretl in order to obtain the full set of data, as well as for comparison. Gretl, for example, does not show the constant coefficient in the long-term equation separately, whereas EViews does not present the lagged short-term vectors in a user-friendly way. They are consistent with each other and are presented in APPENDIX C. The long-run cointegration vector is shown below.

```
Vector Error Correction Estimates
Date: 04/29/12   Time: 17:27
Sample (adjusted): 2001Q1 2011Q4
Included observations: 44 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:       CointEq1
L_S(-1)               1.000000
M(-1)                -0.234098
            (0.11936)
            [-1.96136]
CPI(-1)              -1.308015
            (0.04321)
            [-30.2739]
TREND(00Q1)           0.000946
            (0.00081)
            [1.17296]
C                -0.255297
```

Figure 10: VECM

Source: Own calculations
As noted previously, the interest rate and productivity differentials have been found to have only a short-term effect on the RER. Thus, they are included as exogenous variables. The variables, affecting equilibrium EUR/USD in a long-run, are presented with the following equation

\[ s_t = 0.26 + \frac{0.23(m - m^*)_t}{(0.1194)} + \frac{1.31(cpi - cpi^*)_t}{(0.0432)} - \frac{0.009t}{(0.008)} \]  

(5.1)

where the terms in () represent the standard deviation. The coefficients enter the equation with an opposite sign as this is one of the mathematical properties of the theoretical ECM with a normalized non-zero coefficient on \( s_t \) equal to 1.

It can be observed that the coefficients of the trend term and the CPI differential in the cointegration relation have an asymptotic normal distribution under the standard assumptions - clearly they are significant at the 5% level. The money supply coefficient is also broadly accepted at 10% levels. The \( H_0 \) that these variables do not affect the rate in a long-run can be rejected.

The money supply differential, as mentioned, is significant. An important point to be made regarding this variable is that its levels are negative throughout the whole period except in Q32008. Thus, the sign of the M2 differential in the estimated equation is positive but the variable exhibits an overall negative influence on the euro, due to its negative values. A simple methodology to test this implication on the real market is to plot the log(m-m*) against the log EUR/USD in order to observe the way they correlate with each other.
In the figure above, the M2 difference is plotted on the left scale and is the entire time negative, with Q3 2008 as the only outlier. The log $s_t$ is plotted on the right axis and is positive all the time since 2002. Thus, the graph illustrates that when this negative difference grows negatively, the value of the euro depreciates. This is observed very clearly in the period 2000-2002. Afterwards, in the period Q2 2003-Q2 2008 the money supply differential between the two countries diminishes as there is a demand for more euros. At the same time the value of the euro increases. After the Lehman’s bankruptcy and the US QE1, this differential started growing negatively again and the value of the euro became lower in general. This is due to the fact that there was a significant demand for dollars because of the liquidity issues in the global economy, which increased the price of the dollar on the FX market (see the discussion in subsection 3.1.1).

The hypothesis that an increased money supply at home could raise the domestic currency is not new in the academia. The Austrian School of Economics makes a distinction between inflation as expansion of money supply from its subsequent immanent effect of rising prices (von Mises, 1980). This represents an important viewpoint, where two presumptions can be made.

1) Departing from the quantity equation of money
velocity \times \text{money supply} = \text{real GDP} \times \text{GDP deflator}

one may conclude that if (i) the money velocity is reduced and/or (ii) GDP grows, there will be an increased demand for domestic money in the economy. So, if (iii) the banking system is troubled, the economy enters a liquidity trap situation. This is an abnormal economic phenomenon, described by the Keynesian economics with near zero interest rates and cash injections from the CB that fails to produce price fluctuations- and therefore- to stimulate the economic growth. This is precisely what is observed on the financial markets after the subprime crisis from 2007, and the sovereign troubles in the EZ. Thus as a summary, according to the Austrian School of Economics, in normal conditions the stimulated GDP after a recession bottom would “neutralize” the money supply effect on the inflation.

2) Higher inflation would, therefore, lead to stronger currency in such an environment. Observations on the FX market show that the currency usually rises when the CB releases inflation data; particularly when it increases the main interest rate in order to tackle the rising inflation. This could be an implication of carry trade strategies in the market. Thus, a higher interest rate may be beneficial for the investors, in general. It, at the same time, certainly harms the population in the domestic economy.

The money supply differential has also a p-value, higher than the rest of the variables. This might be due to the fact that there is a structural change in the trend after Q42008. It cannot be captured by neither omitting this period, not by introducing a shift dummy variable with the following properties

\[ d_{Q42008} = \begin{cases} 1, & t < Q42008 \\ 0, & t > Q42008 \end{cases} \]

A regression involving such a dummy biases the estimation towards results, that are not only theoretically, but also logically inconsistent (a very high ECT for instance). In result, the effect of this major trend shift cannot be dismissed. As discussed in subsection 6.2.1, a unit-root and cointegration procedure with structural breaks are both required to account for shifts in the data. Their properties, however, are not covered at this academic level. Thus, due to the above-mentioned facts and because a
re-estimation of the model without this variable leads to non-significant results, at this stage the M2 differential is left as it is.

The sign of the CPI differential is compatible with the theory. This is because both indices represent the relative consumer prices of the two economies, denominated in US dollars. They are also used as indicators of competitiveness. Therefore, their difference resembles closely the movements of the USD (see the FIGURES Appendix). When the CPI differential increases, this would decrease the dollar’s value and increase the value of the euro. The resulting EUR/USD graph will increase. Thus, the equation shows that an additional 1% change in inflation expectations’ differential (in dollars) for the \( t + 1 \) period would decrease the USD a time \( t \), and thus increase the EUR’s values, with 1.31% in the current period.

Since the exchange rate, based on the CPI differential, is broadly expected to provide a good reflection of the change in the purchasing power of the dollar, the estimation should follow the Law of one price in equation (1.1). It states that converted in one currency the identical goods in the two economies would have a single price. Nevertheless, it is seen that the estimation in equation (5.1) differs from it.

\[
s_t = 1.31(cpi - cpi^*)_t \tag{5.2}
\]

This means that the coefficient is 0.31 points higher than the theory predicted number of 1. The exchange rate does not converge to its PPP in this period. This might be due to transaction costs and/or different productivity levels.

It has to be pointed out that this long-term estimation would not be the same if the exogenous variables are absent from it. In other words, this equation is unique precisely because the interest rate and the productivity differentials are included in the system. The difference between a first differenced VAR and VECM is merely the one period lagged error-correction term (ECT), derived from the cointegrating vectors (Brooks, 2002). It represents the short-run equilibrium of the system.
The error-correction term is the difference between the actual and estimated exchange rate at time $t - 1$. Its coefficient measures the adjustment speed towards its long-run values. The EC1 in this regression has a p-value of 0.0021, therefore it is highly significant at 1% level, supporting once again the validity of the long-run equilibrium relationship. Theoretically, the ECT is considered meaningful when it is a negative number, ranging from 0 to 1 and no more than 2. If it is positive, this would mean that the system is explosive, and therefore, not reasonable. This model produces a negative ECT of -1.1. It represents the short-run misalignment of the system and will shift the regressand $s_t$ to return to its long-term equilibrium.

Based on the output above, the short run equation with the significant variables becomes equation (5.3).

$$\Delta s_t = 0.30 - \frac{0.82(cpi-cpi^*)_{t-1}}{0.0541} + \frac{0.46(prod-prod^*)_{t-1}}{0.0037} - \frac{1.11ECT_{t-1}}{0.008}$$ (5.3)

Thus, the significant variables in this short-run equilibrium are the first lag of the differenced CPI and the productivity. When the last period’s CPI increase with 1%, the exchange rate would decrease with 0.82%. No effects are seen from the past values of...
the RER itself as the macroeconomic conditions are fast changing, and therefore, the rate is adjusting itself quickly.

The productivity differential enters the cointegration as an exogenous variable due to its stationarity. It is significant, unlike the interest rates. This means that it carries also an impact on the system. A 1% positive change in the productivity differential would correct the disequilibrium by increasing the exchange rate with 0.46% in the short term. The finding is significant and compatible with the real market developments. In the FX market, the productivity news can be interpreted as a proxy that is released in advance of the actual GDP. Thus, they are used to predict its growth without the biasing impact of an increased inflation, for example. The productivity also affects the general employment outlook. In addition to being an indicator of labor market’s efficiency, the employment figures represent also one of the leading news on the FX market. The US Non-Farm Payroll (NFP) report is considered very important according to the FX participants. The FX market moves considerably if there is an unanticipated shift in the US employment figures. Thus, the market participants do follow the productivity numbers when making their decisions very closely. Influenced by them, the exchange rate fluctuates in the short-run.

The lagged money supply difference is not particularly significant in the estimation above. This may be due to the fact that the real effects of an increased monetary base may in reality manifest themselves not immediately, but in later periods. The interest rate differential is insignificant as well, perhaps due to its stationary nature and small amplitude of change.

- **VECM residual analysis**

The last procedure is to test the residuals of the VECM. At first, this is done by using some descriptive statistics of the residuals- plots, ACF, PACF or Kernel density of the individual series.

---

28 In the FX jargon, the NFP is considered to be “the king of the reports”. See its latest release on [http://bls.gov/news.release/empsit.nro.htm](http://bls.gov/news.release/empsit.nro.htm)
The plots of the standardized residuals look to be stationary. A unit root procedure is performed in order to test for autocorrelation and normality (Brooks, 2002). The tests are performed on each residual series and results are presented below.

Table 10: VECM residuals’ test

<table>
<thead>
<tr>
<th>Variable</th>
<th>ARCH-LM</th>
<th>P-Value($\chi^2$)</th>
<th>JB statistic</th>
<th>P-Value($\chi^2$)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>u1</td>
<td>0.6285</td>
<td>0.7303</td>
<td>1.3949</td>
<td>0.4978</td>
<td>-0.3319</td>
<td>3.5814</td>
</tr>
<tr>
<td>u2</td>
<td>0.8771</td>
<td>0.6450</td>
<td>1.1334</td>
<td>0.5674</td>
<td>0.3861</td>
<td>2.8094</td>
</tr>
<tr>
<td>u3</td>
<td>0.9914</td>
<td>0.6091</td>
<td>2.1850</td>
<td>0.3354</td>
<td>-0.5294</td>
<td>2.6863</td>
</tr>
</tbody>
</table>

Source: Own calculations

The residuals are normally distributed as all JB test statistics are smaller than chi-square with 2 df = 5.0002. There is also no sign for autocorrelation as all the computed ARCH-LM test statistics indicate p-values greater than 0.05. Thus, the Ho for no autocorrelation cannot be rejected.
5.4 PRODUCTIVITY AS AN ENDGENOUS VARIABLE – A CASE FOR COMPARISON

In section 5.2.2.2 was mentioned that the productivity differential could be included as an endogenous regressor in the cointegration relation, if the Schwarz Criterion for 1 lag from the ADF test on levels is taken into consideration. The estimation results including the productivity as a long-term affecting variable are presented below. The entire VECM is to be found in the APPENDIX C.

The VAR lag selection based on AIC shows 4 lags as optimal.

<table>
<thead>
<tr>
<th>lags</th>
<th>loglik</th>
<th>p(LR)</th>
<th>AIC</th>
<th>BIC</th>
<th>HQC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>508.86995</td>
<td>-21.312270</td>
<td>-19.690280*</td>
<td>-20.710758*</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>529.33790</td>
<td>0.00057</td>
<td>-21.515359</td>
<td>-19.244572</td>
<td>-20.673242</td>
</tr>
<tr>
<td>3</td>
<td>543.36834</td>
<td>0.03109</td>
<td>-21.425834</td>
<td>-18.506250</td>
<td>-20.343111</td>
</tr>
<tr>
<td>4</td>
<td>567.31342</td>
<td>0.00005</td>
<td>-21.786974*</td>
<td>-18.218594</td>
<td>-20.463647</td>
</tr>
</tbody>
</table>

*Figure 14: VAR lag selection for endogenous productivity

Source: Own calculations

The Johansen`s tests show all a stable cointegration as rank 1.

Case 4: Restricted trend, unrestricted constant
Exogenous regressor(s): R

Log-likelihood = 692.18 (including constant term: 567.313)

Cointegration tests, ignoring exogenous variables

<table>
<thead>
<tr>
<th>Rank</th>
<th>Eigenvalue Trace test</th>
<th>p-value</th>
<th>Imax test</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.62023</td>
<td>73.829</td>
<td>0.0048</td>
<td>42.600</td>
</tr>
<tr>
<td>1</td>
<td>0.29866</td>
<td>31.228</td>
<td>0.4371</td>
<td>15.609</td>
</tr>
<tr>
<td>2</td>
<td>0.21667</td>
<td>15.619</td>
<td>0.5931</td>
<td>10.745</td>
</tr>
<tr>
<td>3</td>
<td>0.10485</td>
<td>4.8737</td>
<td>0.6207</td>
<td>4.8737</td>
</tr>
</tbody>
</table>

Corrected for sample size (df = 22)

Rank Trace test p-value

<table>
<thead>
<tr>
<th>Rank</th>
<th>73.829</th>
<th>0.0372</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.228</td>
<td>0.5615</td>
</tr>
<tr>
<td>2</td>
<td>15.619</td>
<td>0.5941</td>
</tr>
<tr>
<td>3</td>
<td>4.8737</td>
<td>0.6301</td>
</tr>
</tbody>
</table>

*Figure 15: Johansen`s cointegration estimation for endogenous productivity

Source: Own calculations
The VECM estimation produces the following long-run equation (5.4).

\[ s_t = c + 0.34(m - m^*)_t + 1.28(cpi - cpi^*)_t + 0.10t(prod - prod^*) - 0.001t + 0.2386 + 0.0479 + 0.3731 - 0.0018 \]

(5.4)

As seen from the regression, the productivity differential has only a minor effect on the long-run determination of the exchange rate. A +1% change in it would increase the value of the euro with +0.10%. The standard deviation is high and the variable cannot be considered as significant (Brooks, 2002). Observable is also the fact, that the p-values of all other variables are much higher.

Basso (1999) estimates also the labour productivities and CPIs of the US and Germany in the long-run cointegration with their exchange rate between 1966 and 1990. Assuming that the DM is as a proxy for the euro, one can make a simple comparison. It has to be noted, however, that he does not use the monetary model. In his paper, Basso (1999) finds that the productivity of labor “plays an important role on the long term” for the 6 countries that he estimates. His results show that +1% change in the German productivity increases the DM’s value with +1.60%. Unfortunately, the labour productivities, together with the CPI variables, show high standard errors, ranging from 0.36 to 0.41. Thus, they cannot be considered significant. His estimation shows also that the US productivity and the German prices have wrong signs. According to Basso (1999), this could be because “the small sample period, with only 25 annual observations, jeopardizes the quality of the tests, since it limits the number of degrees of liberty – this problem recurs in this work”.

These implications suggest that, indeed, the productivity within as endogenous variable in the long-term cointegration with the exchange rate does not represent the optimal model. The productivity differential, defined as the output per hour work, should not enter the long-run cointegration regression for the RER determination.
6 BEER ESTIMATION, TOTAL MISALIGNMENT AND CONCLUDING DISCUSSION

As discussed in subsection 2.4, the main purpose of this study is to estimate the behavioural equilibrium exchange rate with the monetary model of exchange rate determination including the productivity; and to compare it to the actual EUR/USD. The BEER is estimated according to the following procedure:

1. Obtaining the long term BEER which is the equation (5.1)

2. Estimating the long-run components of the fundamental variables by isolating the speculative and cycle components with the Hodrick-Prescott filter

3. Substituting the smoothened macroeconomic values in equation (5.1) for the observed period

4. Plotting the log of the actual EUR/USD and the estimated BEER against time

After performing the above-mentioned steps, the BEER is estimated and presented accordingly on the figure below.

Figure 16: BEER against the actual RER

Source: Own calculations
Figure 16 presents the estimated in this econometric study behavioural equilibrium exchange rate, in comparison to the actual exchange rate on the FX market for the period Q12000-Q42011. The shaded areas present the recessions as recorded by the NBER Committee.

The smoothened BEER line is consistent with the general trend of actual EUR/USD. Nevertheless, it appears that the common currency exceeds several times the indications, suggested by the macroeconomic fundamentals. These periods occur when the actual FX rate is above the BEER line. They carry the meaning that the common currency is overvalued. In this case, a higher amount of US dollars is required for the exchange of 1 euro. Additionally, there are several other periods, in which the actual rate is below the estimated BEER. This means that the euro is undervalued during these quarters.

The euro became undervalued immediately after it started to circulate in the EZ (Bénassy-Quéré et.al, 2008). This could be seen in the figure between the period 2001-2003 when the actual movement of the USD per 1 EUR is below the BEER line. Some researchers argue that this outcome occurred as a combined result from the positive expectations about the US economy and the excess supply of euros in the EZ (Rosenberg, 2003). Table 1 presents a brief summary on why the euro was undervalued.

From 2003 to 2005, the euro was overvalued (Bénassy-Quéré et.al, 2008). This is possibly due to the not very symmetrically distributed productivity across the US sectors from the previous years, a higher US current account deficit and excessive US investment spending (Rosenberg, 2003). Between Q3-Q42004, the actual exchange rate briefly approaches the BEER line as the euro depreciated due to an increased growth in the US output. It can be argued that in this particular moment the EUR/USD actual exchange rate is close to its behavioural equilibrium exchange rate value as estimated in this study.

The subsequent undervaluation of the EUR against the USD begins again from Q3 2005, as suggested by the BEER line. In 2008, the USD was strengthening as mentioned in subsection 3.1.1. Therefore, the euro becomes undervalued as the BEER lies below the actual FX line. Another period of euro’s undervaluation is Q22010, when

29 The National Bureau of Economic Research (NBER) business cycles records from 1923 (see http://www.nber.org/cycles.html)
it was announced that Greece would receive a financial rescue package from the rest EZ
countries. The exchange rate between the two economies seems to approach its total
equilibrium BEER line two times - in Q32009 and Q32011. The euro was particularly
undervalued again in Q32011, when the Italian and Spanish government bonds yielded
over 6%. This rate is viewed as an unsustainable for the long-run financing of a state.

The above-mentioned estimation results seem to be supported by the
macroeconomic development of the two economies in the observed period. In his study
for example, Giannellis (2011) concludes that the euro is overvaluated almost all the
time from 1999 until 2008, apart from the depreciation period in 2002. He considers
this particular exception as a “disequilibrium correction movement”, but nevertheless,
provides some macroeconomic explanations for it. In other words, Giannellis (2011)
does not attribute this move entirely to his estimated BEER model, but argues that
there are, indeed, fundamental reasons behind this undervaluation. It has to be pointed
out, however, that his estimation does not include the interest rate and productivity
differentials. Giannellis (2011) calculates his BEER only based on the CPIs, the output
and money supply differentials and uses the unit-root and cointegration tests with two
structural breaks in the time-series. He also uses the industrial production as a
variable for the output. According to him, productivity is found to have only some
positive short-run effects on the euro’s exchange rate behaviour. In his work, the
money supply differential is also found to enter the estimation with an opposite sign.
This outcome provides, indeed, a stable basis to conclude that ECB and FED’s
monetary mass in circulation do not behave exactly as the theory predicts. This might
be due to the fact that the dollar and the euro constitute the major currencies in the
world. Their demand is exceptionally high, as noted previously. This is particularly
valid for the dollar as it is the world’s reserve currency.

The total misalignment of the EUR/USD can also be calculated in comparison to
the BEER estimation above. It is the deviation of the actual FX rate from its estimated
BEER values and it shows the alignment varying around this equilibrium mean. The
periods over the zero line suggest overvaluation and the ones below- undervaluations.
The total misalignment is calculated according to equation (2.11) and graphical results
are presented in the figure 17.
Based on the figure, one may draw the very same conclusions as from the BEER comparison above. The periods of undervalued euro are: 2001-2003, 2005-2007, 2009, Q3 2010 and Q3 2011. The overvaluated euro is observed in 2003-2005 and 2008. Observed also are the three periods of near-equilibrium in Q3-Q4 2004, Q3 2009 and Q3 2011 when the TM is very close to its zero mean.

6.1 Conclusions

This empirical study attempted to estimate whether the euro is over-, undervaluated or in line with its explanatory variables in the BEER model. In other words, its aim was to verify whether the EUR/USD rate, money supply, CPI, interest rate and labor productivity differentials are cointegrated. A long-term relationship was, indeed, found to exist between the M2, the CPIs and the exchange rate. Consequently, it can be concluded that the supply of money and the expected inflation in the two economies are affecting the exchange rate in a long-term.

The money supply is estimated to enter the monetary equation with a positive sign. However, this differential is negative thorough all the time. In the end, this variable practically affects the regressand as predicted in equation (3.16). The (negatively) growing money supply differential affects the euro negatively. It was estimated, that 1% change in M2 differential would decrease the euro with 0.23%. The sign in the econometric equation is positive but the entire underlying time-series is
negative. Interestingly, Giannellis (2011) also estimates that the money supply differential enters the long-run cointegration with a wrong sign according to the common logic. In his work, it is -1.05 (note, that he uses a directly quoted exchange rate, i.e. as USDEUR). Therefore, additional explanation from the Austrian School of Economics was provided in order to account for the phenomenon when an increased home money supply increases the domestic currency’s value. This is only aiming to support the case, when the reader decides to interpret the positive sign of this study’s cointegration, while disregarding the negative time-series of the variable at the same time. This regressor also has a standard error of 0.119 which can be broadly accepted on the 10% significance level. It was empirically tested that no dummies – slope shifting or period omitting - could account for the substantial trend shift in Q3 2008.

The relative consumer price index, denominated in dollars, is found to have a compatible with the logic sign. It resembles closely the movement of the USD as it is expressed in this currency. A 1% change in the future CPI differential, denominated in USD, would decrease the dollar and increase the euro with 1.3% in the current period. The coefficient of this significant logarithmic variable is estimated to be 1.3, which is 0.3% higher than the propositions of the Law of one Price. It indicates that this currency pair does still not converge to its PPP during the observed period.

The interest rate differential is found to be a stationary variable. It, therefore, cannot enter the long-run cointegration with the exchange rate. This variable is found to be very small, but positive. Its coefficient in the short-run VECM is only +0.0018. Thus, the variable is negligible.

The labour productivity was tested both- as an endogenous, and exogenous variable in the BEER estimation. This is based on the lag selection criteria in the ADF test. When the variable is tested with 10 lags, as suggested by 3 of the 4 criteria, it becomes stationary. Therefore, it can enter the cointegration relation only as an exogenous regressor. It does influence the exchange rate cointegration but only in a short-run. The productivity is found to correlate positively with the domestic currency value. A +1% change in the productivity differential would increase the euro with 0.46% in a short-run. The hours of work spent for producing the EZ output do influence decision-making of the investors in the FX market in a short-term.

If the productivity differential is tested with only 1 lag in the ADF test, the variable is non-stationary. Therefore, it can be included in the long-run cointegration estimation. The model with endogenous productivity estimates that this variable’s
effect on the long-run EUR/USD is only +0.10%. This is a very small impact considering the long-term cointegration. The estimation also shows that the other variables have higher standard errors. A comparison with the cointegration test of Basso (1999) was presented as well. It shows similar results, however, also with insignificant variables. Thus, the endogenous productivity within the long-term cointegration regression does not represent the best fitted model, and therefore, is discussed only as a case of comparison in this study.

Finally, the BEER and the total misalignment are estimated. Figures 16 and 17 at the very end of the study show the exchange rate deviations from its fundamental non-cyclical values. Observed are periods of over- and undervaluated euro, but in general, the actual exchange rate seems to fluctuate with no major amplitudes around the linear equilibrium BEER line. The periods of under- and overvaluation of the euro are found to be compatible with the previous research papers. The euro was undervalued euro in 2001-2003, 2005-2007, 2009, Q3 2010 and Q3 2011. An overvaluated euro is observed in 2003-2005 and 2008.
APPENDIX A

ADF TESTS ON THE VARIABLES IN THE MODEL

ADF Test for series: 5
sample range: [2000 Q2, 2011 Q4], T = 47
lagged differences: 0
intercept, time trend, seasonal dummies
asymptotic critical values
Oxford University Press, London
1% 5% 10%
-3.96 -3.41 -3.13
value of test statistic: -1.7968
regression results:

<table>
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<tr>
<th>variable</th>
<th>coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(-1)</td>
<td>-0.1688</td>
<td>-1.7968</td>
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<tr>
<td>constant</td>
<td>0.0288</td>
<td>1.2698</td>
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<tr>
<td>trend</td>
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<td>1.4065</td>
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<tr>
<td>sdummy(2)</td>
<td>0.0243</td>
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<td>sdummy(3)</td>
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<td>-0.1910</td>
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<tr>
<td>sdummy(4)</td>
<td>0.0114</td>
<td>0.5164</td>
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<tr>
<td>RSS</td>
<td>0.1189</td>
<td></td>
</tr>
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</table>

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA
sample range: [2002 Q4, 2011 Q4], T = 37
optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion: 0
Final Prediction Error: 0
Hannan-Quinn Criterion: 0
Schwarz Criterion: 0

ARCH-LM TEST with 1 lags:
test statistic: 1.1178
p-value(Chi^2): 0.2904
F statistic: 1.1456
p-Value(F): 0.2903

JARQUE-BERA TEST:
test statistic: 2.3725
p-value(Chi^2): 0.3054
skewness: -0.3045
kurtosis: 2.0832

Fig. A1 ADF test on the log EUR/USD
ADF Test for series: s_dr
sample range: [2003 Q1, 2011 Q4], T = 36
lagged differences: 10
intercept, time trend, seasonal dummies
asymptotic critical values
Oxford University Press, London
1% 5% 10%
-3.96 -3.41 -3.13
value of test statistic: -4.2887
regression results:

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<td>3.0490</td>
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<td>2.9295</td>
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OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA
sample range: [2003 Q1, 2011 Q4], T = 36

optimal number of lags (sought up to 10 lags of 1. differences):
Akaike Info Criterion: 10
Final Prediction Error: 10
Hannan-Quinn Criterion: 10
Schwarz Criterion: 0

ARCH-LM TEST with 10 lags:
test statistic: 11.0363
p-value(Chi^2): 0.3547
F statistic: 1.9176
p-value(F): 0.1233

JARQUE-BERA TEST:
test statistic: 0.1227
p-value(Chi^2): 0.9405
skewness: -0.0944
kurtosis: 3.2148

Fig. A2 ADF test on the first difference of the log EUR/USD
ADF Test for series: \( m-m^* \)

Sample range: [2001 Q1, 2011 Q4], \( T = 44 \)

Lagged differences: 3

Intercept, time trend, seasonal dummies

Asymptotic critical values


1% 5% 10%
-3.96 -3.41 -3.13

Value of test statistic: -0.2062

Regression results:

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<th>t-statistic</th>
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<td>( dx(-3) )</td>
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<tr>
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Optimal endogenous lags from information criteria

Sample range: [2002 Q4, 2011 Q4], \( T = 37 \)

Optimal number of lags (searched up to 10 lags of 1. differences):

Akaike Info Criterion: 3
Final Prediction Error: 3
Hannan-Quinn Criterion: 3
Schwarz Criterion: 1

ARCH-LM test with 3 lags:

Test statistic: 0.4850
p-value (chi^2): 0.9222
F statistic: 0.1636
p-value (F): 0.9202

Jarque-Bera test:

Test statistic: 0.6548
p-value (chi^2): 0.7208
Skewness: -0.2581
Kurtosis: 2.6989

Fig. A3 ADF test on the log (m-m*)
ADF Test for series: \((m-m*)_{dl}\)

Sample range: [2000 Q3, 2011 Q4], \(T = 46\)

Lagged differences: 0

Intercept, time trend, seasonal dummies

Asymptotic critical values


1% 5% 10%
-3.96 -3.41 -3.13

Value of test statistic: -3.6298

Regression results:

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<th>t-statistic</th>
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Optimal endogenous lags from information criteria

Sample range: [2003 Q1, 2011 Q4], \(T = 36\)

Optimal number of lags (searched up to 10 lags of 1. differences):

Akaike Info Criterion: 2
Final Prediction Error: 2
Hannan-Quinn Criterion: 2
Schwarz Criterion: 0

ARCH-LM test with 1 lags:

Test statistic: 0.1600
p-Value(Chi^2): 0.8892
F statistic: 0.1606
p-Value(F): 0.6906

Jarque-Bera test:

Test statistic: 0.9894
p-Value(Chi^2): 0.6097
Skewness: -0.3262
Kurtosis: 2.6989

Fig. A4 ADF test on the first difference of the log \((m-m*)\)
**ADF Test for series: \((y-y^*)\)**

**sample range:** [2002 Q3, 2011 Q4], \(T = 38\)

**lagged differences:** 7

**intercept, seasonal dummies, no time trend**

**asymptotic critical values**

<table>
<thead>
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<th></th>
<th></th>
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<tbody>
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<td>-3.43</td>
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<td>-2.57</td>
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**value of test statistic:** -4.2962

**regression results:**

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<th>t-statistic</th>
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</tbody>
</table>

**OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA**

**sample range:** [2003 Q2, 2011 Q4], \(T = 35\)

**optimal number of lags (searched up to 10 lags of 1. differences):**

- **Akaike Info Criterion:** 7
- **Final Prediction Error:** 7
- **Hannan-Quinn Criterion:** 7
- **Schwarz Criterion:** 0

**ARCH-LM TEST with 7 lags:**

**test statistic:** 3.2995

**p-Value(Chi^2):** 0.8560

**F statistic:** 0.5275

**p-Value(F):** 0.8046

**JARQUE-BERA TEST:**

**test statistic:** 0.3585

**p-Value(chi^2):** 0.8359

**skewness:** -0.0264

**kurtosis:** 2.5271

---

*Fig. A5 ADF test on the log \((y-y^*)\)*
ADF Test for series: cpi-cpi*

- sample range: [2001 Q4, 2011 Q4], T = 41
- lagged differences: 6
- intercept, time trend, seasonal dummies
- asymptotic critical values

1% 5% 10%
-3.96 -3.41 -3.13

- value of test statistic: -2.5692

Regression results:

<table>
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<tr>
<th>variable</th>
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<th>t-statistic</th>
</tr>
</thead>
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Optimal Endogenous Lags from Information Criteria

- sample range: [2002 Q4, 2011 Q4], T = 37

- optimal number of lags (searched up to 10 lags of 1. differences):
  - Akaike Info Criterion: 6
  - Final Prediction Error: 6
  - Hannan-Quinn Criterion: 6
  - Schwarz Criterion: 4

ARCH-LM Test with 6 lags:

- test statistic: 3.0324
- p-value(Chi^2): 0.8048
- F statistic: 0.5533
- p-value(F): 0.7632

Jarque-Bera Test:

- test statistic: 1.6863
- p-value(Chi^2): 0.4303
- skewness: -0.4951
- kurtosis: 2.9196

Fig. A6 ADF test on the log (cpi-cpi*)
ADF Test for series: \((cpi-cpi^*)\_d1\)
sample range: [2003 Q1, 2011 Q4], \(T = 36\)
lagged differences: 10
intercept, time trend, seasonal dummies
asymptotic critical values
Oxford University Press, London
1% 5% 10%
-3.96 -3.41 -3.13
value of test statistic: -3.7422
regression results:

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<tr>
<th>variable</th>
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OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA
sample range: [2003 Q1, 2011 Q4], \(T = 36\)

optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion: 10
Final Prediction Error: 10
Hannan-Quinn Criterion: 10
Schwarz Criterion: 3

ARCH-LM TEST with 10 lags:

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<td>F statistic</td>
<td>0.6053</td>
</tr>
<tr>
<td>p-Value(F)</td>
<td>0.7868</td>
</tr>
</tbody>
</table>

JARQUE-BERA TEST:

<table>
<thead>
<tr>
<th>test statistic</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-Value(Chi^2)</td>
<td>0.3331</td>
</tr>
<tr>
<td>skewness</td>
<td>-0.5398</td>
</tr>
<tr>
<td>kurtosis</td>
<td>3.5480</td>
</tr>
</tbody>
</table>

Fig. A7 ADF test on the first difference of the log (cpi-cpi^*)
ADF Test for series: R-R*
sample range: [2000 Q3, 2011 Q4], T = 46
lagged differences: 1
intercept, no time trend
asymptotic critical values
1% 5% 10%
-3.43 -2.86 -2.57
value of test statistic: -3.4424
regression results:

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>x(-1)</td>
<td>-0.1171</td>
<td>-3.4424</td>
</tr>
<tr>
<td>dx(-1)</td>
<td>0.7348</td>
<td>7.6106</td>
</tr>
<tr>
<td>constant</td>
<td>0.0196</td>
<td>0.4329</td>
</tr>
<tr>
<td>RSS</td>
<td>3.9243</td>
<td></td>
</tr>
</tbody>
</table>

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA
sample range: [2002 Q4, 2011 Q4], T = 37
optimal number of lags (searched up to 10 lags of 1. differences):

<table>
<thead>
<tr>
<th>Information Criterion</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Akaike Info Criterion</td>
<td>1</td>
</tr>
<tr>
<td>Final Prediction Error</td>
<td>1</td>
</tr>
<tr>
<td>Hannan-Quinn Criterion</td>
<td>1</td>
</tr>
<tr>
<td>Schwarz Criterion</td>
<td>1</td>
</tr>
</tbody>
</table>

ARCH-LM TEST with 1 lags:

t-test statistic: 0.1842
p-value(Chi^2): 0.6678
F statistic: 0.1850
p-value(F): 0.6693

JARQUE-BERÁ TEST:

test statistic: 14.8223
p-value(Chi^2): 0.0006
skewness: 1.1391
kurtosis: 4.5947

Fig. A8 ADF test on the level (R-R*)
ADF Test for series: prod-prod*
sample range: [2003 Q1, 2011 Q4], T = 36
lagged differences: 10
intercept, time trend, seasonal dummies
asymptotic critical values
1% 5% 10%
-3.96 -3.41 -3.13
value of test statistic: -4.6968
regression results:

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>x(-1)</td>
<td>-0.5623</td>
<td>-4.6968</td>
</tr>
<tr>
<td>dx(-1)</td>
<td>0.5565</td>
<td>3.5788</td>
</tr>
<tr>
<td>dx(-2)</td>
<td>0.2858</td>
<td>1.5808</td>
</tr>
<tr>
<td>dx(-3)</td>
<td>0.2454</td>
<td>1.5439</td>
</tr>
<tr>
<td>dx(-4)</td>
<td>0.1874</td>
<td>1.1862</td>
</tr>
<tr>
<td>dx(-5)</td>
<td>0.0757</td>
<td>0.5126</td>
</tr>
<tr>
<td>dx(-6)</td>
<td>0.5560</td>
<td>3.7806</td>
</tr>
<tr>
<td>dx(-7)</td>
<td>0.1839</td>
<td>1.1668</td>
</tr>
<tr>
<td>dx(-8)</td>
<td>0.0157</td>
<td>0.1018</td>
</tr>
<tr>
<td>dx(-9)</td>
<td>0.3658</td>
<td>2.4119</td>
</tr>
<tr>
<td>dx(-10)</td>
<td>0.3349</td>
<td>1.9869</td>
</tr>
<tr>
<td>constant</td>
<td>0.0030</td>
<td>1.1359</td>
</tr>
<tr>
<td>trend</td>
<td>-0.0015</td>
<td>-4.4390</td>
</tr>
<tr>
<td>sdummy(2)</td>
<td>0.0010</td>
<td>0.3414</td>
</tr>
<tr>
<td>sdummy(3)</td>
<td>-0.0044</td>
<td>-1.3586</td>
</tr>
<tr>
<td>sdummy(4)</td>
<td>-0.0029</td>
<td>-0.9551</td>
</tr>
<tr>
<td>RSS</td>
<td>0.0006</td>
<td></td>
</tr>
</tbody>
</table>

OPTIMAL ENDOGENOUS LAGS FROM INFORMATION CRITERIA
sample range: [2003 Q1, 2011 Q4], T = 36
optimal number of lags (searched up to 10 lags of 1. differences):
Akaike Info Criterion: 10
Final Prediction Error: 10
Hannan-Quinn Criterion: 10
Schwarz Criterion: 1

ARCH-LM TEST with 10 lags:
test statistic: 4.4253
p-Value(Chi^2): 0.9261
F statistic: 0.5333
p-Value(F): 0.8411

JARQUE-BERA TEST:
test statistic: 1.1901
p-Value(Chi^2): 0.5515
skewness: 0.4308
kurtosis: 2.7737

Fig. A9 ADF test on the log (prod-prod*)
AFPPENDIX B

ACF, PACF AND RESIDUALS FROM THE FINAL ADF TEST

Fig.B1 ACF, PACF and Residuals plot from the ADF test on the first difference of EUR/USD
Fig.B2 ACF, PACF and Residuals’ plot from the ADF test on the first difference of log \((m-m^*)\)

Fig.B3 ACF, PACF and Residuals’ plot from the ADF test on the first difference of level \((cpi-cpi^*)\)
Fig.B4 ACF, PACF and Residuals’ plot from the ADF test on the level (R-R*)

Fig.B5 ACF, PACF and Residuals’ plot from the ADF test with 10 lags on the level (prod-prod*)
APPENDIX C

VECM

VECM system, lag order 4
Maximum likelihood estimates, observations 2001:1-2011:4 (T = 44)
Cointegration rank = 1
Case 4: Restricted trend, unrestricted constant
beta (cointegrating vectors, standard errors in parentheses)

\[
\begin{align*}
\text{l}_s & & 1.0000 \\
& & (0.00000) \\
\text{m} & & -0.22626 \\
& & (0.12793) \\
\text{cpi} & & -1.3086 \\
& & (0.046279) \\
\text{trend} & & 0.00087089 \\
& & (0.00086244)
\end{align*}
\]

alpha (adjustment vectors)

\[
\begin{align*}
\text{l}_s & & -1.1094 \\
\text{m} & & -0.060288 \\
\text{cpi} & & 2.1991
\end{align*}
\]

Log-likelihood = 372.46505
Determinant of covariance matrix = 8.9098766e-012
AIC = -14.4757
BIC = -12.2860
HQC = -13.6636

Equation 1: d_l_s

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.296691</td>
<td>0.0879718</td>
<td>3.3726</td>
</tr>
<tr>
<td>d_l_s_1</td>
<td>-0.193181</td>
<td>0.257721</td>
<td>-0.7496</td>
</tr>
<tr>
<td>d_l_s_2</td>
<td>-0.118986</td>
<td>0.146255</td>
<td>-0.8136</td>
</tr>
<tr>
<td>d_l_s_3</td>
<td>0.143744</td>
<td>0.0852545</td>
<td>1.6861</td>
</tr>
<tr>
<td>d_m_1</td>
<td>-0.302045</td>
<td>0.535881</td>
<td>-0.5636</td>
</tr>
<tr>
<td>d_m_2</td>
<td>-0.666036</td>
<td>0.564213</td>
<td>-1.1805</td>
</tr>
<tr>
<td>d_m_3</td>
<td>-0.132317</td>
<td>0.531501</td>
<td>-0.2489</td>
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<tr>
<td>d_cpi_1</td>
<td>-0.820327</td>
<td>0.407313</td>
<td>-2.0140</td>
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<tr>
<td>d_cpi_2</td>
<td>-0.0303438</td>
<td>0.364505</td>
<td>-0.0832</td>
</tr>
<tr>
<td>d_cpi_3</td>
<td>0.218382</td>
<td>0.283161</td>
<td>0.7712</td>
</tr>
<tr>
<td>R</td>
<td>0.00175782</td>
<td>0.00405513</td>
<td>0.4335</td>
</tr>
<tr>
<td>prod</td>
<td>0.464327</td>
<td>0.145971</td>
<td>3.1809</td>
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<tr>
<td>S1</td>
<td>0.0152265</td>
<td>0.0110907</td>
<td>1.3729</td>
</tr>
<tr>
<td>S2</td>
<td>0.0014885</td>
<td>0.0106555</td>
<td>0.1397</td>
</tr>
<tr>
<td>S3</td>
<td>0.00442687</td>
<td>0.0107975</td>
<td>0.4100</td>
</tr>
<tr>
<td>EC1</td>
<td>-1.10938</td>
<td>0.326863</td>
<td>-3.3940</td>
</tr>
</tbody>
</table>

Mean dependent var | 0.009298 | S.D. dependent var | 0.053587
Equation 2: \( d_m \)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
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<td>0.030882</td>
<td>0.5057</td>
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<tr>
<td>( d_l_s_1 )</td>
<td>0.079648</td>
<td>0.090471</td>
<td>0.8804</td>
</tr>
<tr>
<td>( d_l_s_2 )</td>
<td>0.0343506</td>
<td>0.051341</td>
<td>0.6691</td>
</tr>
<tr>
<td>( d_l_s_3 )</td>
<td>-0.0287048</td>
<td>0.029928</td>
<td>-0.9591</td>
</tr>
<tr>
<td>( d_m_1 )</td>
<td>0.358459</td>
<td>0.188118</td>
<td>1.9055</td>
</tr>
<tr>
<td>( d_m_2 )</td>
<td>-0.140976</td>
<td>0.198064</td>
<td>-0.7118</td>
</tr>
<tr>
<td>( d_m_3 )</td>
<td>0.294632</td>
<td>0.18658</td>
<td>1.5791</td>
</tr>
<tr>
<td>( d_cpi_1 )</td>
<td>0.00955273</td>
<td>0.142985</td>
<td>0.0668</td>
</tr>
<tr>
<td>( d_cpi_2 )</td>
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<td>0.127957</td>
<td>-0.6985</td>
</tr>
<tr>
<td>( d_cpi_3 )</td>
<td>-0.00627229</td>
<td>0.099402</td>
<td>-0.0631</td>
</tr>
<tr>
<td>( R )</td>
<td>-0.00266712</td>
<td>0.001424</td>
<td>-1.8736</td>
</tr>
<tr>
<td>prod</td>
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<td>0.051242</td>
<td>1.1319</td>
</tr>
<tr>
<td>S1</td>
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<td>0.003893</td>
<td>0.0106</td>
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<tr>
<td>S2</td>
<td>0.000493711</td>
<td>0.003741</td>
<td>0.1320</td>
</tr>
<tr>
<td>S3</td>
<td>0.000805071</td>
<td>0.003790</td>
<td>0.2124</td>
</tr>
<tr>
<td>EC1</td>
<td>-0.0602875</td>
<td>0.114743</td>
<td>-0.5254</td>
</tr>
</tbody>
</table>

Mean dependent var          0.000700  S.D. dependent var  0.010476
Sum squared resid           0.001948  S.E. of regression  0.008494
R-squared                   0.587276  Adjusted R-squared  0.342699
rho                        0.181937  Durbin-Watson       1.588475

Equation 3: \( d_cpi \)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
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<td>0.115447</td>
<td>-5.0793</td>
</tr>
<tr>
<td>( d_l_s_1 )</td>
<td>-1.18984</td>
<td>0.338213</td>
<td>-3.5180</td>
</tr>
<tr>
<td>( d_l_s_2 )</td>
<td>-0.521042</td>
<td>0.191933</td>
<td>-2.7417</td>
</tr>
<tr>
<td>( d_l_s_3 )</td>
<td>-0.065817</td>
<td>0.111881</td>
<td>-0.5871</td>
</tr>
<tr>
<td>( d_m_1 )</td>
<td>1.09522</td>
<td>0.703248</td>
<td>1.5574</td>
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<tr>
<td>( d_m_2 )</td>
<td>0.550206</td>
<td>0.740429</td>
<td>0.7431</td>
</tr>
<tr>
<td>( d_m_3 )</td>
<td>2.26781</td>
<td>0.6975</td>
<td>3.2513</td>
</tr>
<tr>
<td>( d_cpi_1 )</td>
<td>2.97812</td>
<td>0.534525</td>
<td>5.5715</td>
</tr>
<tr>
<td>( d_cpi_2 )</td>
<td>1.79948</td>
<td>0.478348</td>
<td>3.7619</td>
</tr>
<tr>
<td>( d_cpi_3 )</td>
<td>1.25002</td>
<td>0.371598</td>
<td>3.3639</td>
</tr>
<tr>
<td>( R )</td>
<td>-0.00109947</td>
<td>0.00532163</td>
<td>-2.0666</td>
</tr>
<tr>
<td>prod</td>
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<td>-3.265</td>
</tr>
<tr>
<td>S1</td>
<td>0.00692733</td>
<td>0.0145546</td>
<td>0.4760</td>
</tr>
<tr>
<td>S2</td>
<td>-0.00648171</td>
<td>0.0139835</td>
<td>-0.4893</td>
</tr>
<tr>
<td>S3</td>
<td>0.0103183</td>
<td>0.0141698</td>
<td>0.7282</td>
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<tr>
<td>EC1</td>
<td>2.19907</td>
<td>0.42895</td>
<td>5.1266</td>
</tr>
</tbody>
</table>

Mean dependent var          0.006887  S.D. dependent var  0.039925
Sum squared resid           0.027221  S.E. of regression  0.031752
R-squared                   0.602848  Adjusted R-squared  0.367499
Cross-equation covariance matrix:

\[
\begin{pmatrix}
  l_s & m & cpi \\
  l_s & 0.00035923 & -2.4968e-006 & 4.3879e-005 \\
  m & -2.4968e-006 & 4.4269e-005 & -4.8644e-005 \\
  cpi & 4.3879e-005 & -4.8644e-005 & 0.00061866
\end{pmatrix}
\]

determinant = 8.90988e-012

Fig.C1 VECM estimation by Gretl

Vector Error Correction Estimates
Date: 04/29/12  Time: 17:27
Sample (adjusted): 2001Q1 2011Q4
Included observations: 44 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq: CointEq1

\[
\begin{align*}
L_S(-1) & = 1.000000 \\
M(-1) & = -0.234098 \
    & \quad (0.11936) \\
    & \quad [-1.96136] \\
CPI(-1) & = -1.308015 \
    & \quad (0.04321) \\
    & \quad [-30.2739] \\
@TREND(00Q1) & = 0.000946 \
    & \quad (0.00081) \\
    & \quad [1.17296] \\
C & = -0.255297
\end{align*}
\]

Fig.C2 VECM estimation by Eviews7

VECM system, lag order 4
Maximum likelihood estimates, observations 2001:1-2011:4 (T = 44)
Cointegration rank = 1
Case 4: Restricted trend, unrestricted constant
beta (cointegrating vectors, standard errors in parentheses)

\[
\begin{align*}
l_s & = 1.0000 \
    & \quad (0.00000) \\
m & = -0.34259 \
    & \quad (0.23855)
\end{align*}
\]
XXIII

cpi  -1.2827  
     (0.047913)
prod -0.10125  
      (0.037309)
trend 0.017388  
      (0.0018835)

alpha (adjustment vectors)

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>l_s</td>
<td>-1.0518</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>-0.098219</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>2.3320</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>prod</td>
<td>-0.072653</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Log-likelihood = 551.69929
Determinant of covariance matrix = 1.5107598e-016
AIC = -21.2591
BIC = -17.8529
HQC = -19.9959

Equation 1: d_l_s

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-ratio</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>0.315039</td>
<td>0.110678</td>
<td>2.8465</td>
<td>0.00870 ***</td>
</tr>
<tr>
<td>d_l_s_1</td>
<td>-0.213523</td>
<td>0.291427</td>
<td>-0.7327</td>
<td>0.47057</td>
</tr>
<tr>
<td>d_l_s_2</td>
<td>-0.11493</td>
<td>0.160887</td>
<td>-0.7144</td>
<td>0.48163</td>
</tr>
<tr>
<td>d_l_s_3</td>
<td>0.128698</td>
<td>0.0877727</td>
<td>1.4663</td>
<td>0.15504</td>
</tr>
<tr>
<td>d_m_1</td>
<td>-0.39828</td>
<td>0.685895</td>
<td>-0.5807</td>
<td>0.56666</td>
</tr>
<tr>
<td>d_m_2</td>
<td>-0.97114</td>
<td>0.696651</td>
<td>-1.3940</td>
<td>0.17558</td>
</tr>
<tr>
<td>d_m_3</td>
<td>0.199184</td>
<td>0.679174</td>
<td>0.2933</td>
<td>0.77173</td>
</tr>
<tr>
<td>d_CPI_1</td>
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<td>d_CPI_2</td>
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<td>d_CPI_3</td>
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<td>0.316175</td>
<td>0.9279</td>
<td>0.36232</td>
</tr>
<tr>
<td>d_prod_1</td>
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<td>0.627351</td>
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<tr>
<td>d_prod_2</td>
<td>0.306514</td>
<td>0.71443</td>
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<td>0.757074</td>
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<tr>
<td>R</td>
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<td>0.00484066</td>
<td>0.0927</td>
<td>0.92685</td>
</tr>
<tr>
<td>S1</td>
<td>0.0114529</td>
<td>0.0123451</td>
<td>0.9277</td>
<td>0.36242</td>
</tr>
<tr>
<td>S2</td>
<td>-0.00405959</td>
<td>0.0118928</td>
<td>-0.3413</td>
<td>0.73569</td>
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Mean dependent var 0.009298  S.D. dependent var 0.053587
Sum squared resid 0.015667  S.E. of regression 0.025033
R-squared 0.873122  Adjusted R-squared 0.781769
rho 0.083639  Durbin-Watson 1.772363

Equation 2: d_m

<table>
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<th>p-value</th>
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### Equation 3: d_CPI

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Mean dependent var 0.000700  S.D. dependent var 0.010476
Sum squared resid 0.001152  S.E. of regression 0.006787
R-squared 0.755970  Adjusted R-squared 0.580268
rho 0.076060  Durbin-Watson 1.724117

### Equation 4: d_prod

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Mean dependent var 0.000700  S.D. dependent var 0.010476
Sum squared resid 0.001152  S.E. of regression 0.006787
R-squared 0.755970  Adjusted R-squared 0.580268
rho 0.076060  Durbin-Watson 1.724117
<table>
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<th>p-value</th>
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Mean dependent var: -0.003636  
S.D. dependent var: 0.007504  
R-squared: 0.326267  
Adjusted R-squared: -0.158821  
rho: -0.085349  
Durbin-Watson: 2.142470  

Cross-equation covariance matrix:

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<th>prod</th>
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</tbody>
</table>

determinant = 1.51076e-016

Fig. C3 VECM estimation with endogenous productivity by Gretl
Fig. 1 Plot of $s$

Fig. 2 Plot of $(m-m^*)$
Fig. 3 Plot of \((y - y^*)\)

Fig. 4 Plot of \((\text{cpi - cpi}^*)\)
**Fig. 5** Plot of $(R-R^*)$

**Fig. 6** Plot of $(prod-prod^*)$
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