EVALUATING THE PREDICTABILITY OF THE VOLATILITY SMILE USING RETURN DISTRIBUTIONS

- AN APPLICATION OF THE JARROW-RUDD MODEL
ON EUROPEAN INDEX OPTIONS MARKETS

DECEMBER 2000
Key words: Non-normality in returns, Skewness, Kurtosis, Option pricing, Volatility, Smile

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Distributor:

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Swedish School of Economics and Business Administration
P.O.Box 479
00101 Helsinki
Finland

Phone: +358-9-431 33 376, +358-9-431 33 265
Fax: +358-9-431 33 425
E-mail: publ@shh.fi
http://www.shh.fi/link/bib/publications.htm

SHS intressebyrå IB (Oy Casa Security Ab), Helsingfors 2000

ISBN 951-555-663-5
ISSN 0357-4598
Evaluating the Predictability of the Volatility Smile Using Return Distributions
- An Application of the Jarrow-Rudd Model on European Index Options Markets

Thomas Lindqvist

Reuters Trading Solutions
Nordic Desktop Consultancy Group
Yrjönkatu 23 B
00101 HELSINKI, FINLAND
phone: +358 9 680 501
thomas.lindqvist@reuters.com

Ronnie Söderman

Estlander & Rönnlund Financial Products Ltd.
Nedre Torget 1A
65100 VASA, FINLAND
phone: +358-6-3180600
ronnie.soderman@er-grp.com

Abstract

In this paper, we examine the predictability of observed volatility smiles in three major European index options markets, utilising the historical return distributions of the respective underlying assets. The analysis involves an application of the Black (1976) pricing model adjusted in accordance with the Jarrow-Rudd methodology as proposed in 1982. Thereby we adjust the expected future returns for the third and fourth central moments as these represent deviations from normality in the distributions of observed returns. Thus, they are considered one possible explanation to the existence of the smile. The obtained results indicate that the inclusion of the higher moments in the pricing model to some extent reduces the volatility smile, compared with the unadjusted Black-76 model. However, as the smile is partly a function of supply, demand, and liquidity, and as such intricate to model, this modification does not appear sufficient to fully capture the characteristics of the smile.

1 The authors would like to thank the people at Estlander & Rönnlund for encouraging company research, and professors Johan Knif and Kenneth Högholm for valuable comments and suggestions.
1 Introduction

After the crash in 1987, implied volatilities for out-of-the-money puts rocketed and never quite came down, generating the phenomenon commonly referred to as the volatility smile. The reason for the existence of the smile has been much debated and there is no simple explanation. However, as suggested e.g. by Cottle (1999) and by the crash-o-phobia as defined by Rubinstein (1994), it is very much a question of simple theory of supply and demand, as this intuitively represents the view of the market on probabilities for future returns in the underlying asset. More specifically, returns $\phi(j)$ over any period of length $j$ are generally not normally distributed,

$$\phi(j) \sim N(\mu_j, \sigma_j^2),$$  

with mean and variance that are linear in $j$, as presumed by the Geometric Brownian Motion (GBM) utilized by the Black & Scholes (1973) model (BSM), and hence a prerequisite for the assumption of constant volatilities across different strike prices.

In fact, as Bates (1996) and others have observed\(^2\), the existence of the smile is strongly suggestive of the presence of leptokurtosis (positive excess kurtosis) in the conditional return distribution. In 1982, Jarrow and Rudd presented a model that explores this theory of non-normal distribution in asset returns, i.e. when the skewness and excess kurtosis of $\phi(j) \neq 0$. Their model accounts for the smile by adjusting the original B&S model through implementation of the skewness and kurtosis coefficients for the distribution of the underlying asset. Corrado & Su (1997) uses this model to price S&P index options and finds that the model improves pricing accuracy, compared with the original B&S model. Hence, this study will explore the efficiency of the Jarrow & Rudd (J-R) model on European markets.

The shape as well as slope of the volatility smile has, among other factors, been found to vary across different underlying instruments. One way to analyze this phenomenon is to investigate differences between past return distributions. Hereby, it is fundamentally assumed that the volatility structure of any given instrument be dependent upon its respective historical return distribution, i.e. market expectations on future outcomes should be based on past performance. In the case of index options,\(^2\) Already in the early sixties Mandelbrot (1963) and Fama (1965), and more recently Lo & MacKinlay (1988, 1990), observe that asset returns exhibit e.g. excess kurtosis, especially when high frequency data is used.
we normally observe a more static shape to the smile than is the case for instance with equity options. This, in turn, could be interpreted as a result of the diversification effect in the underlying stock index, for which extreme returns are less frequent than for individual stocks. This is true in particular for large positive returns.

In this paper, we also suggest that the index decomposition is relevant regarding the shape of the smile. The logic behind this relies on the empirical observation that stock returns are generally more highly correlated during downtrends than during uptrends. Some empirical evidence on this phenomenon may be found in, e.g. Koutmos (1996). This implies that out-of-the-money call options on a well-diversified stock index such as the DJ EuroStoxx 50 intuitively should trade at lower volatilities compared with the at-the-money (ATM) volatility, than those on a less diversified index such as the SMI. The latter index is compounded of a smaller number of stocks, out of which a few major stocks have very heavy weights in the index.\(^3\) The issue of discrepancies between shapes of smiles of different underlying assets shall be discussed in further detail later.

The remainder of the paper is structured as follows. A brief review of the GBM is given in Chapter 2 and the Jarrow-Rudd method is presented in Chapter 3. The basic methodology is presented in Chapter 4, the data material in Chapter 5, and the empirical findings in Chapter 6. A summary with conclusions ends this study in Chapter 7.

### 2 The Geometric Brownian Motion\(^4\)

As mentioned in the introduction, the assumptions behind the GBM of zero skewness and excess kurtosis are constantly violated in most markets today. To understand how this affects the GBM, a brief review is appropriate. Let \(X_o\) denote the initial price of the asset. With respect to the GBM, the price at time \(t\) evolves according to

\[
X_t = X_o \exp\{\alpha t + \sigma W_t\}, \tag{2}
\]

where \(\alpha\) and \(\sigma\) are given constants, and \(W_t\) is a standard Brownian motion process. Let \(j > 0\) denote the length of time between observations of the price, and let \(\phi(j)\)

---

\(^3\) Individual stock weightings in the three indices may be found in Appendix A.

\(^4\) Following the outline presented by Das & Sundaram (1997)
denote the continuously compounded return form holding the asset over the time interval \([t, t + j]\). This by definition gives \(\phi(j)\) as

\[
\phi(j) = \log \left( \frac{X_{t+j}}{X_t} \right).
\]

From (2), \(\phi(j)\) is given by

\[
\phi(j) = \alpha_j + \sigma \left( W_{t+j} - W_t \right).
\]

Since \(W_t\) is a Wiener process, the difference \(W_{t+j} - W_t\) is normally distributed with a mean of zero and a variance of \(j\). It follows from (4), therefore, that the returns \(\phi(j)\) are themselves normally distributed with mean equal to \(\alpha_j\) and a variance equal to \(\sigma^2_j\), i.e.

\[
\phi(j) \sim N(\alpha_j, \sigma^2_j).
\]

As the normal distribution assumes zero skewness and kurtosis of 3, this implies that expression (5) carries two strong implications about the returns \(\phi(j)\). The returns have zero skewness and zero excess kurtosis and the skewness and kurtosis of the returns do not depend on \(j\). However, as the violation of these conditions on modern financial markets are argued to give rise to the existence of the volatility smile, it follows that the pricing models are to be adjusted to account for this. One model which accounts for the above parameters is the Jarrow-Rudd (1982) model (J-R).

### 3 The Jarrow-Rudd Method

Jarrow & Rudd (1982) proposes a method of valuing European options when the underlying security price at option expiration follows a distribution \(F\), known only through its moments. An option pricing formula is derived from an Edgeworth series expansion of the security price distribution \(F\) about an approximation distribution \(A\). The equation for a European call is

\[^5\text{As presented by Corrado & Su (1997)\text{}}\]
\[ C(F) = C(A) - e^{-\eta} \frac{\kappa_3(F) - \kappa_3(A)}{3!} \frac{da(K)}{dS_t} + e^{-\eta} \frac{\kappa_4(F) - \kappa_4(A)}{4!} \frac{d^2 a(K)}{dS_t^2} + \epsilon(K), \quad (6) \]

where \( C(F) \) denotes a call option price based on an unknown distribution \( F \) and \( C(A) \) is a call price based on the known distribution \( A \). The adjustment terms \( (\kappa_n) \) are similar to the moments of the distributions \( A \) and \( F \). The density of \( A \) is denoted by \( a(S_t) \) where \( S_t \) is a random stock price at option expiration. These derivatives are evaluated at the strike price \( K \). The factor \( \epsilon(K) \) continues the Edgeworth series with terms based on higher-order moments and derivatives. Assuming that stock returns are log-normally distributed and that higher order terms are negligible, \( C(A) \) becomes the familiar B&S model for a European call as

\[ C(A) = SN(d_1) - Ke^{-\eta} N(d_2), \quad (7) \]

where \( S \) is the price of the underlying asset, \( N \) is the cumulative normal distribution, \( K \) is the strike price, \( r \) is the interest rate and \( t \) is the time to maturity given in years. Furthermore,

\[ d_1 = \frac{\ln(S/K) + (r + \sigma^2/2) \cdot t}{\sigma \sqrt{t}} \quad \text{and} \quad d_2 = d_1 - \sigma \sqrt{t}, \quad (8a,b) \]

where \( \sigma^2 \) is the volatility. The Jarrow-Rudd methodology is based on the Black & Scholes pricing model but as the newer Black-76 version is commonly used for pricing European-style index options as the underlying hedge is a future, this is the equation used in this study. This means that (7) is replaced by

\[ C(A) = e^{-\eta} \left[ FN(d_1) - KN(d_2) \right]. \quad (9) \]

Measuring the log-normal density \( a(S_t) \) and its first two derivatives at the strike price \( K \) yields the following expressions:

\[ a(K) = (K\sigma \sqrt{2\pi})^{-1} \exp(-d_2^2/2), \quad \frac{da(K)}{dS_t} = \frac{a(K)(d_2 - \sigma \sqrt{t})}{K\sigma \sqrt{t}}, \quad (10a,b) \]

and
\[
\frac{d^2 a(K)}{dS_i^2} = \frac{a(K)}{K^2 \sigma^2} \left[ \left( d_2 - \sigma \sqrt{t} \right)^2 - \sigma \sqrt{t} \left( d_2 - \sigma \sqrt{t} \right) - 1 \right]. \tag{11}
\]

Jarrow & Rudd (1982) implies equality in the first order moments of \( F \) and \( A \), that is \( \kappa(F) = \kappa(A) = Se^\theta t \), and \( \kappa(F) = \kappa(A) \). Further, it is shown that when the distribution \( A \) is log-normal, the volatility parameter \( \sigma^2 \) is specified as a solution to the equality \( \kappa_2(F) = \kappa_1^2(A)(e^{\sigma^2 t} - 1) \). Dropping the remainder \( \varepsilon(K) \), the J-R option price equation in (6) is conveniently restated as

\[
C(F) = C(A) + \lambda_1 Q_3 + \lambda_4 Q_4, \tag{12}
\]

where

\[
\lambda_1 = \gamma_1(F) - \gamma_1(A), \tag{13a}
\]

\[
Q_3 = -\left( Se^\theta t \right)^3 (e^{\sigma^2 t} - 1)^{3/2} \frac{e^{-\theta t}}{3!} \frac{da(K)}{dS_i}, \tag{13b}
\]

\[
\lambda_2 = \gamma_2(F) - \gamma_2(A), \tag{13c}
\]

\[
Q_4 = -\left( Se^\theta t \right)^4 (e^{\sigma^2 t} - 1)^2 \frac{e^{-\theta t}}{4!} \frac{d^2 a(K)}{dS_i^2}. \tag{13d}
\]

The coefficients \( \gamma_1(F) \) and \( \gamma_1(A) \) are the skewness coefficients for distributions \( F \) and \( A \), while \( \gamma_2(F) \) and \( \gamma_2(A) \) represent the excess kurtosis. They are defined as (Stuart & Ord 1987, p. 107)

\[
\gamma_1(F) = \frac{\kappa_1(F)}{\kappa_2^{3/2}(F)} \quad \text{and} \quad \gamma_2(F) = \frac{\kappa_4(F)}{\kappa_2^2(F)}. \tag{14a,b}
\]

When the substitution \( q^2 = e^{\sigma^2 t} - 1 \) is used to simplify the algebraic expression, coefficients of skewness and excess kurtosis for the log-normal distribution \( A \) are defined as

\[
\gamma_1(A) = 3q + q^3 \quad \text{and} \quad \gamma_2(A) = 16q^2 + 15q^4 + 6q^6 + q^8. \tag{15a,b}
\]

Now, it is assumed that it is the non-normal skewness and kurtosis for \( \gamma_1(F) \) and \( \gamma_2(F) \) defined in (9a,b) that give rise to the implied volatility smile. By accounting for this in the pricing model, we should, in theory, obtain prices closer to the observed market prices than those generated by the unadjusted Black-76 model.
4 Methodology

The purpose of this study is to evaluate the efficiency of European index option pricing by accounting for deviations from normality in returns, using the Jarrow-Rudd method. Prices generated within the J-R framework will be compared with those calculated with the unadjusted Black-76 model, and with observed market prices. Given \( n \) observed market bid/ask-pairs, \( \{(bid_i, ask_i)\}_{i=1}^{n} \), with correspondence to strike prices and expiration dates \( \{(K_i, T_i)\}_{i=1}^{n} \), let the observed market prices

\[
\bar{m}_i = \frac{1}{2} (bid_i + ask_i), \quad i=1, \ldots, n.
\] (16)

Hence, market prices are estimated as the midpoint of observed bid/ask-spreads, applying a reasonability limit of 20% of the bid price as the maximum width of the market spread, i.e. \((bid_i - ask_i)/bid_i \leq 0.2\). In the case of the total spread exceeding 20% of the bid price, the observation is considered inadequate and is omitted in the material. As outlined by Corrado & Su (1997), the percentage of estimated theoretical prices, \( \hat{t}_i \), fitting within the market spread,

\[
bid_i \leq \hat{t}_i \leq ask_i, \quad i=1, \ldots, n,
\] (17)

is adapted as an indicator of the pricing accuracy of each model. When the level of the underlying is closer than 0.1% to an available strike, this strike is classed as the ATM level, and hence omitted. This procedure is introduced to avoid bias in the results as the differences between the prices generated with the models should be approximately zero for the ATM strike.

Where prices are available for both the call and the put option of the same strike price, comparison is made between the average implied volatility of the market call and put prices. If these are approximately similar, both bid/ask spreads are used in the study. If not, the abnormal observation (either the call or the put) is omitted, as the respective volatilities should be equal according to the put-call parity. If prices for either the call or the put are unavailable, or the market spread exceeds the preset
maximum width for one of these, market volatility is estimated on the basis of the one quoted option. Should there be no reasonable prices available for neither the call nor the put, the observation is omitted.

Further input variables required in order to calculate theoretical option prices, i.e. implied market at-the-money volatility and level of underlying, are collected from the market. The former is calculated as the average of market bid/ask implied volatilities for calls and puts with strikes currently closest-to-the-money. Where available, market volatilities for one call and one put on each side of the at-the-money level are used. Since options are typically listed in a greater number of maturities than are futures and forwards, the current future or forward price cannot consistently be used as the underlying level. It is therefore derived from the Black-76 put-call parity, and accordingly determined by market prices. To be able to calculate skewness and kurtosis for the return distribution, the returns on the indices are defined as

$$R_{t+1} = \ln(P_{t+1}) - \ln(P_t),$$

where $R_{t+1}$ is the change in the observed variable from day $t$ to $t+1$, $\ln$ is the natural logarithm and $P_t$ is the observation on the data for day $t$. The skewness and kurtosis coefficients, respectively, for these returns are then calculated as

$$\frac{\sum (X - \bar{X})^3}{n-1} \quad \text{and} \quad \frac{\sum (X - \bar{X})^4}{n-1},$$

where $\bar{X}$ is the mean of the returns.

The distributions are tested for normality using the Kolmogorov-Smirnov Z-statistic.

5 The Data

Option prices are collected for strike prices over a range of 10% around the present at-the-money level, as adequate market prices are less frequently available for option series further in- or out-of-the-money. These prices are collected each trading day for the duration of July 1 through July 30, 1999, and subsequently compared with the
theoretical prices supplied by the Black-76 model, \( \hat{I}_{t,76} \), on one hand, and the adjusted Jarrow-Rudd model, \( \hat{I}_{t,76} \), on the other. The instruments included in the study are options on the following stock indices, for which the abbreviations in brackets will be referred to henceforth; the German DAX (DAX), the Swiss SMI (SMI) and the Dow Jones EuroStoxx 50 (ESX). Whereas the latter is a pan-European blue chip index containing 50 highly capitalized stocks from the EMU zone, the two formers are both domestic general indices.

The skewness and kurtosis coefficients are calculated from the logarithmic returns on the respective indices. Since the DAX is the only dividend-adjusted index out of the three, we encounter some difficulties in synchronizing the correct past returns for the other two. In the case of EuroStoxx 50, appropriate data is available for a corresponding performance index, where paid dividends are continuously reinvested in the index on the ex-dividend day. However, due to limitations in data supply for the SMI, we are forced to use the SMI future contract as proxy for the index. Prices for these derivatives contracts are consistently discounted by the appropriate interest rate factor at every time \( t \). This means that the estimated dividend-adjusted index at \( t \) is determined by the traded future price discounted by the interest rate effect over the remaining time to maturity of the futures contract.

The interest rates used are three-month LIBOR interest rates. As derivative contracts on indices unadjusted for dividend payouts are traded based on market expectations of upcoming dividends, we thereby accept a minor potential error, as these may differ from factual payout figures. However, the effect of these inefficiencies should be negligible in the sample, and decidedly smaller than those of the alternative, using the unadjusted index. Admittedly, there may occur small estimation errors resulting from the different approaches applied. Any possible bias resulting from this choice of methodology should, however, be decidedly negligible in the sample.

Another issue one has to confront at this stage is that, given that the market smiles at present are based on the historical performance of the underlying asset, then on which historical performance? Does the market have a long memory, i.e. should historical distribution characteristics fed into a pricing formula stem back as long as possible, or is a shorter, more recent period of time more appropriate in trying to capture the current market situation? We hypothesize that the true case may be a bit of both, and for reasons of consistency we limit each data sample to a period of
approximately five years. This approach leaves us with a sufficient number of data observations, but does not heavily emphasize data from, e.g. the early 1990s, which was a period of comparatively much lower volatility levels than have been seen during more recent years. Table 1 contains a summary of observations used for the respective indices.

### Table 1.
Observations on returns on underlying indices.

<table>
<thead>
<tr>
<th>Index</th>
<th>Type of data</th>
<th>Period of observation</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>Index</td>
<td>Jun, 1994 – Jun, 1999</td>
<td>1,248</td>
</tr>
<tr>
<td>SMI</td>
<td>Index implied by future</td>
<td>Dec, 1994 – Jun, 1999</td>
<td>1,150</td>
</tr>
<tr>
<td>ESX</td>
<td>Index adjusted for dividends</td>
<td>Jan, 1993 – April, 1998</td>
<td>1,372</td>
</tr>
</tbody>
</table>

### 6 Empirical Findings

Estimating skewness and kurtosis coefficients for the returns on the three indices generates the results outlined in table 2.

### Table 2.
Estimated skewness, excess kurtosis, Kolmogorov-Smirnov Z-statistics and the annualized standard deviation for returns on the indices.

<table>
<thead>
<tr>
<th>Index</th>
<th>Skewness</th>
<th>Excess</th>
<th>Z</th>
<th>σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>- 0.277</td>
<td>2.151</td>
<td>2.362*</td>
<td>0.172</td>
</tr>
<tr>
<td>SMI</td>
<td>- 0.417</td>
<td>4.064</td>
<td>2.470*</td>
<td>0.195</td>
</tr>
<tr>
<td>ESX</td>
<td>- 0.360</td>
<td>3.969</td>
<td>1.848*</td>
<td>0.133</td>
</tr>
</tbody>
</table>

* denotes statistical significance on a 1% level at least

In order to support these findings, we have chosen to include distributions of these index returns in graphical form. These may be found in Appendix B, Figures B1 through B3. Table 3 presents the different option maturities and volatility ranges monitored, as well as a summary of the number of valid observations and observations omitted for each product, together with the maximum deviation between \( \hat{\mu}_{i,j,k} \) and \( \bar{\mu}_i \).
Table 3.
A summary of the monitored data material.
Deviation calculated as the absolute value of \( \left| \hat{m}_{JR,i} - \bar{m}_i \right| / \bar{m}_i \) representing the largest mispricing in percentage terms found in this study.

<table>
<thead>
<tr>
<th>Index</th>
<th>Maturities (days)</th>
<th>ATM-volatilities (range)</th>
<th>Omitted (n)</th>
<th>Max (%) deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>21 - 163</td>
<td>17.9% - 27.1%</td>
<td>45</td>
<td>19.6%</td>
</tr>
<tr>
<td>SMI</td>
<td>14 - 178</td>
<td>18.4% - 24.6%</td>
<td>28</td>
<td>27.8%</td>
</tr>
<tr>
<td>ESX</td>
<td>14 - 163</td>
<td>18.5% - 27.8%</td>
<td>26</td>
<td>24.0%</td>
</tr>
</tbody>
</table>

6.1 The models versus the market

The respective skewness and kurtosis coefficients are hence included in the Jarrow-Rudd formula, and the generated prices are compared to market prices, as described above. As the smile is documented to be highly asymmetric implying that its structure is different for out-of-the-money puts and calls and changing over maturities, the observations are divided into groups based on their moneyness (two different groups, \( K < \text{ or } > \text{ than the } \text{ATM-level} \)) and time-to-maturity (three different groups, see table 4).

Table 4.
A summary of the observed mean deviations between \( \hat{m}_{JR,i} \) and \( m_i \) for different maturities (10-40, 41-80 and 81-). N denotes the number of observations for the respective group.

<table>
<thead>
<tr>
<th>Index</th>
<th>10-40</th>
<th>41-80</th>
<th>81-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt;ATM (n)</td>
<td>&gt;ATM (n)</td>
<td>&lt;ATM (n)</td>
</tr>
<tr>
<td>DAX</td>
<td>-5.8% (99)</td>
<td>3.6% (109)</td>
<td>-4.0% (160)</td>
</tr>
<tr>
<td>SMI</td>
<td>-4.8% (97)</td>
<td>5.4% (104)</td>
<td>-3.0% (163)</td>
</tr>
<tr>
<td>ESX</td>
<td>-5.0% (62)</td>
<td>4.3% (71)</td>
<td>-4.1% (126)</td>
</tr>
</tbody>
</table>

The deviations between the prices generated with the Jarrow-Rudd method and the market prices seem rather obvious when looking at the results in table 4. As supported by figure 1, the Jarrow-Rudd methodology undervalues out-of-the-money puts and overvalues out-of-the-money calls, thus underestimating the smile.
Figure 1. 
**Implied volatility smiles**
Implied DAX-smile comparison (market vs. Jarrow-Rudd) for option series with 30 days to maturity and an implied ATM volatility of 16%.

To establish the accuracy of the J-R method, the additional comparison is made between market prices and prices generated with the Black-76 model, based on the same methodology as above.

**Table 5.**
A summary of the observed mean deviations between $i_{\text{B,76}}$ and $i_{\text{m}}$ for different maturities (10-40, 41-80 and 81-).

<table>
<thead>
<tr>
<th>Index</th>
<th>10-40</th>
<th>41-80</th>
<th>81-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>&lt;ATM (n)</td>
<td>&gt;ATM (n)</td>
<td>&lt;ATM (n)</td>
</tr>
<tr>
<td>DAX</td>
<td>-8.7% (99)</td>
<td>8.0% (109)</td>
<td>-5.5% (160)</td>
</tr>
<tr>
<td>SMI</td>
<td>-6.5% (97)</td>
<td>7.6% (104)</td>
<td>-4.4% (163)</td>
</tr>
<tr>
<td>ESX</td>
<td>-6.6% (62)</td>
<td>7.6% (71)</td>
<td>-5.8% (126)</td>
</tr>
</tbody>
</table>

It is rather obvious that the J-R pricing is significantly more accurate than the unadjusted Black-76 model, as prices determined by the latter deviate even more from the observed market prices. This is, of course, in line with expectations, as the Black-76 model uses a constant volatility, i.e. no smile at all. This means that the J-R method is more or less an average of the Black-76 model and the observed market prices. Another important aspect is to monitor the frequency of the J-R prices that are
actually between the observed market bid/ask spreads, to determine the usefulness of
the model considering e.g. intra-day quoting and/or scenario analysis simulations.

Table 6.
A summary of the observed \( \left( bid_i \leq \hat{b}_{i,JR} \leq ask_i \right) \). The total number of estimated
prices outside the bid/ask spreads are shown within brackets. The observations are again
divided into three different groups according to their respective time-to-maturity.

<table>
<thead>
<tr>
<th>Index</th>
<th>10-40 %</th>
<th>41-80 %</th>
<th>81- %</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>110 (99)</td>
<td>147 (171)</td>
<td>186 (87)</td>
</tr>
<tr>
<td>SMI</td>
<td>111 (85)</td>
<td>221 (117)</td>
<td>296 (56)</td>
</tr>
<tr>
<td>ESX</td>
<td>69 (58)</td>
<td>96 (141)</td>
<td>55 (74)</td>
</tr>
</tbody>
</table>

Table 7.
A summary of the observed \( \left( bid_i \leq \hat{b}_{i,76} \leq ask_i \right) \). The total
number of estimated prices outside the bid/ask spreads are shown within brackets.

<table>
<thead>
<tr>
<th>Index</th>
<th>10-40 %</th>
<th>41-80 %</th>
<th>81- %</th>
</tr>
</thead>
<tbody>
<tr>
<td>DAX</td>
<td>80 (129)</td>
<td>106 (212)</td>
<td>112 (161)</td>
</tr>
<tr>
<td>SMI</td>
<td>100 (96)</td>
<td>181 (157)</td>
<td>200 (152)</td>
</tr>
<tr>
<td>ESX</td>
<td>50 (77)</td>
<td>61 (176)</td>
<td>36 (93)</td>
</tr>
</tbody>
</table>

The results in table 7 support the conclusions above, i.e. that the prices
calculated using the original B-76 model and constant volatilities differ more from the
observed market prices. Only in the case of SMI are more than 50% of the calculated
prices fitted within the observed bid/ask spreads. A few comments may be made on
the differences between results in different instruments, as shown in table 6. For the
SMI, pricing accuracy measured by theoretical prices matching market spreads grows
seemingly higher with increasing time to maturity. However, this is partially
explained by exceptionally generous maximum quoting spreads that are allowed by
the exchange, which to a great extent are adopted by market makers particularly in
longer maturities.

Also, the underlying SMI index has a relatively high value, which means that
market prices available tend to be relatively closer-to-the-money than do market
prices in, e.g. the ESX. In the case of ESX, maximum quoting spreads permitted are
narrower, and hence we observe a lower frequency of modelled prices fitting those of
the market. Further, it may be pointed out that traded market volatility smiles are
generally steeper in the ESX than in the SMI. As we know that the J-R model does not sufficiently capture the entire smile effect, it follows inevitably that its pricing accuracy is likely to be poorer for instruments where market smiles are highly pronounced.

Figure 3.
Result summary
A summary of the J-R and B-76 prices fitting within observed bid-ask spreads for each index and maturity.

6.2 Implications

Based on the results of this study it is fair to say that the inclusion of deviations from normality in the return distribution enables the pricing model to capture the dynamics of the smile to a certain extent. It does not, however, remove the smile completely. Similar results are documented e.g in Peña et al. (1999). This could on one hand result from model risk, i.e. the J-R model fails to model the skewness and excess kurtosis correctly. Some light could be shed upon this topic by comparing the prices estimated with the J-R model with those estimated using similar models that account for skewness and excess kurtosis factors of the return distribution. Such models include the jump-diffusion models (see e.g. Jarrow & Rosefeld [1984] or Das & Faresi [1996]) and stochastic volatility models (Melino & Turnbull [1990], Heston [1993] or Hull & White [1998]). This topic is however outside the scope of this study.

Furthermore, the time period chosen was one of continuous growth in all stock markets examined, which means that there was to be an upward bias, or positive drift, in the returns used for calculating the central moments of the return distributions.
This, in turn, implies a lower skewness factor, resulting from a higher-than-normal
frequency of small positive returns. In hindsight, one might therefore consider
adjusting the historical returns by excluding small daily returns from the sample, thus
removing some of the positive drift now forcing the skewness factors for all markets
included into negative territory. Also, the smile is assumed to depend on other factors
than past returns, predominantly on market supply and demand. If this is accurate,
there are two major implications to this study. First, since supply and demand are not
instantly quantifiable in derivatives markets, any attempt to deterministically capture
all characteristics of the smile could be bound to fail. Accordingly, the results from
the evaluation of the J-R correction seem intuitively correct. By adjusting the pricing
model for non-normality in returns, we are able to reduce the smile effect in option
valuation. However, we are not able to fully capture it.

Another topic for questioning is the relatively short time-period used in the
study. We found, however, that monitoring additional market prices during the
autumn of 1999 proved only to further emphasize the current results without
providing any marginal information. Different results might be found if another type
of market was to emerge, generating a different structure to the smile. We do,
however, expect the basic implications to prevail across different market conditions.

7 Summary and Conclusions

The purpose of this study was to evaluate the efficiency of European index option
pricing by accounting for deviations from normality in returns, applying the Jarrow-
Rudd method. The skewness and kurtosis coefficients were calculated for three
different markets and tested for normality. Prices generated within the J-R framework
were then compared with those calculated with the original Black-76 model, and with
observed market prices. The results indicate that part of the smile can be captured by
accounting for the skewness and the excess kurtosis of the historical returns
distributions. However, comparing theoretical with market prices, we are merely in
part able to capture the smile effect. This implies that the volatility smiles displayed in
major European markets for index options examined build on factors outside return
distributions characteristics. We, therefore, argue that accounting for the non-normal
characteristics of stock market returns in general allows an improvement to the
pricing accuracy of the Black-76 model. However, it does not sufficiently capture the full extent of market volatility smiles.
References


Appendix A.

Respective index decompositions for the three markets included in the study may be found in *Tables A1* through *A3* below.

**Table A1.**
Decomposition of the DAX index as of 31.12.1999

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<tr>
<th>Stock</th>
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</thead>
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<tr>
<td>Adidas-Salomon</td>
<td>0.36 %</td>
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<tr>
<td>Allianz</td>
<td>7.47 %</td>
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<tr>
<td>Basf</td>
<td>3.44 %</td>
</tr>
<tr>
<td>Bayer</td>
<td>3.48 %</td>
</tr>
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<td>BMW</td>
<td>2.11 %</td>
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<td>Commerzbank</td>
<td>1.92 %</td>
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<tr>
<td>Deutsche Bank</td>
<td>4.54 %</td>
</tr>
<tr>
<td>DaimlerChrysler</td>
<td>7.71 %</td>
</tr>
<tr>
<td>Degussa-Huels</td>
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</tr>
<tr>
<td>Dresdner Bank</td>
<td>2.59 %</td>
</tr>
<tr>
<td>Deutsche Telekom</td>
<td>15.00 %</td>
</tr>
<tr>
<td>Fresenius Medical Care</td>
<td>0.68 %</td>
</tr>
<tr>
<td>Henkel</td>
<td>0.98 %</td>
</tr>
<tr>
<td>Bayerische Hypo-Vereinsbank</td>
<td>2.53 %</td>
</tr>
<tr>
<td>Karstadt Quelle</td>
<td>0.31 %</td>
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<table>
<thead>
<tr>
<th>Stock</th>
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<tbody>
<tr>
<td>Lufthansa</td>
<td>1.00 %</td>
</tr>
<tr>
<td>Linde</td>
<td>0.69 %</td>
</tr>
<tr>
<td>MAN</td>
<td>0.59 %</td>
</tr>
<tr>
<td>Metro</td>
<td>1.72 %</td>
</tr>
<tr>
<td>Mannesmann</td>
<td>12.34 %</td>
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<tr>
<td>Münchener Rückversicherung</td>
<td>3.88 %</td>
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<tr>
<td>Preussag</td>
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</tr>
<tr>
<td>RWE</td>
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<tr>
<td>SAP</td>
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<td>Schering</td>
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<td>Siemens</td>
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<td>Thyssen Krupp</td>
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<tr>
<td>Veba</td>
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<td>VIAG</td>
<td>1.24 %</td>
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<tr>
<td>Volkswagen</td>
<td>2.39 %</td>
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**Table A2.**

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<td>ABN Amro</td>
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<td>Alcatel Alsthom</td>
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<tr>
<td>Allianz</td>
<td>3.20 %</td>
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<tr>
<td>AXA</td>
<td>1.86 %</td>
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<tr>
<td>Banco Bilbao Vizcaya</td>
<td>1.13 %</td>
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<tr>
<td>Banco Santander</td>
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<tr>
<td>Basf</td>
<td>1.21 %</td>
</tr>
<tr>
<td>Bayer</td>
<td>1.25 %</td>
</tr>
<tr>
<td>BNP</td>
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<tr>
<td>Carrefour</td>
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</tr>
<tr>
<td>DaimlerChrysler</td>
<td>2.82 %</td>
</tr>
<tr>
<td>Deutsche Bank</td>
<td>2.04 %</td>
</tr>
<tr>
<td>Deutsche Telekom</td>
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</tr>
<tr>
<td>Dresdner Bank</td>
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<td>Electrabel</td>
<td>0.66 %</td>
</tr>
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<td>Endesa</td>
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<tr>
<td>Eni</td>
<td>1.60 %</td>
</tr>
<tr>
<td>Fortis</td>
<td>0.95 %</td>
</tr>
<tr>
<td>France Telecom</td>
<td>4.91 %</td>
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<tr>
<td>Generali Assicurazioni</td>
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<tr>
<td>Hypo-Vereinsbank</td>
<td>1.02 %</td>
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<tr>
<td>ING Group</td>
<td>2.26 %</td>
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<tr>
<td>Koninklijke KPN</td>
<td>1.74 %</td>
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<table>
<thead>
<tr>
<th>Stock</th>
<th>Weight</th>
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<tbody>
<tr>
<td>Koninklijke Ahold</td>
<td>0.69 %</td>
</tr>
<tr>
<td>L’Oréal</td>
<td>1.94 %</td>
</tr>
<tr>
<td>LVHM Moet Hennessy</td>
<td>1.58 %</td>
</tr>
<tr>
<td>Mannesmann</td>
<td>4.56 %</td>
</tr>
<tr>
<td>Metro</td>
<td>0.60 %</td>
</tr>
<tr>
<td>Münchener Rückversicherung</td>
<td>1.71 %</td>
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<tr>
<td>Nokia</td>
<td>7.79 %</td>
</tr>
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<td>Philips Electronics</td>
<td>1.69 %</td>
</tr>
<tr>
<td>Pinault-Printemps</td>
<td>1.17 %</td>
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<td>Repsol</td>
<td>1.04 %</td>
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<td>Aventis</td>
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<tr>
<td>Royal Dutch</td>
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<tr>
<td>RWE</td>
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<tr>
<td>Saint Gobain</td>
<td>0.58 %</td>
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<td>Sanofi</td>
<td>1.09 %</td>
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<td>Société Generale</td>
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<td>Suez Lyonnaise</td>
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<tr>
<td>Telecom Italia</td>
<td>2.81 %</td>
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<td>Telefonica de España</td>
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<td>Total Fina</td>
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<td>Unilever</td>
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<tr>
<td>Veba</td>
<td>0.93 %</td>
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<tr>
<td>Vivendi</td>
<td>2.00 %</td>
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### Table A3.

<table>
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<th>Stock</th>
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<td>ABB</td>
<td>5.62 %</td>
<td>Rentenanstalt</td>
<td>1.41 %</td>
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<td>Adecco</td>
<td>1.81 %</td>
<td>Roche Holding</td>
<td>14.35 %</td>
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<td>Alusuisse</td>
<td>1.45 %</td>
<td>Swissair Group</td>
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<td>Baloise Holding</td>
<td>0.95 %</td>
<td>SGS Geneva</td>
<td>0.21 %</td>
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<tr>
<td>Clariant</td>
<td>0.96 %</td>
<td>Sulzer</td>
<td>0.44 %</td>
</tr>
<tr>
<td>Ciba Spezialitätenchemie</td>
<td>1.19 %</td>
<td>Schweizerische Rückversicherung</td>
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<td>Swatch Group</td>
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<td>UBS</td>
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<td>Zürich Allied</td>
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<td>Novartis</td>
<td>19.87 %</td>
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### Appendix B.

Deviations from normality in the returns on the underlying stock indices present one plausible explanation to the occurrence of the volatility smile. *Figures B1 through B3* show the respective observed return distributions for each of the stock indices examined in the study. To visualize characteristics of non-normality, these are furthermore plotted against a normal distribution.

![Return distributions](image)

Figure B1.
**Return distributions**
Observed distribution of returns on the DAX, versus the corresponding normal distribution.
Figure B2.
**Return distributions**
Observed distribution of returns on the ESX, versus the corresponding normal distribution.

Figure B3.
**Return distributions**
Observed distribution of returns on the SMI, versus the corresponding normal distribution.