PRICING INDEX OPTIONS WITH STOCHASTIC VOLATILITY
- ON THE EFFICIENCY OF THE SQUARE ROOT MODEL

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Key words: Volatility Smiles, Stochastic Volatility, Parameter Estimation, the Square Root Process

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Ronnie Söderman & Daniel Djupsjöbacka
Department of Finance and Statistics
Swedish School of Economics and Business Administration
P.O.Box 287
65101 Vaasa, Finland

Distributor:

Library
Swedish School of Economics and Business Administration
P.O.Box 479
00101 Helsinki
Finland

Phone: +358-9-431 33 376, +358-9-431 33 265
Fax: +358-9-431 33 425
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Pricing Index Options with Stochastic Volatility
- On the Efficiency of the Square Root Model

Ronnie Söderman & Daniel Djupsjöbacka
ronnie.soderman@er-grp.com
ddjupsjo@abo.fi

Abstract

The objective of this paper is to investigate the pricing accuracy under stochastic volatility where the volatility follows a square root process. The theoretical prices are compared with market price data (the German DAX index options market) by using two different techniques of parameter estimation, the method of moments and implicit estimation by inversion. Standard Black & Scholes pricing is used as a benchmark. The results indicate that the stochastic volatility model with parameters estimated by inversion using the available prices on the preceding day, is the most accurate pricing method of the three in this study and can be considered satisfactory. However, as the same model with parameters estimated using a rolling window (the method of moments) proved to be inferior to the benchmark, the importance of stable and correct estimation of the parameters is evident.

Keywords: Volatility Smiles, Stochastic Volatility, Parameter Estimation, the Square Root Process
1 Introduction

The Black & Scholes (1973) model (BSM) has proven to be one of the most significant contributions to the theory of finance, as it still after more than twenty-seven years of presence and a growing number of subsequent developments, is a highly competitive and widely used option pricing model. As noted for example by Heston (1993) one reason for this success, besides its obvious simplicity, is that the BSM relates the distribution of spot returns to the cross-sectional properties of option prices.

However, after the stock market crash of 1987, it has become evident that the prices generated by the original BSM do not coincide with the notion of a fair price as perceived by traders and practitioners. These biases of the BSM led to emerging patterns commonly referred to as volatility smiles, or skews, and the term structure of implied volatility (see for example Rubinstein [1994] or Jackwerth & Rubinstein [1996]). The crash reminded the market of the fact that not only are markets risky, but extremely risky, and the risk itself is by all means variable. The constant volatility assumption and the assumption of log-normality in returns of the BSM simply do not hold. It is no coincidence that several academic papers after the crash were filled with a new generation of option pricing models that allowed the volatility to vary and the returns of the underlying asset to be non-normally distributed. This was the origin of stochastic volatility models.

Hull & White (1987), Wiggins (1987) and Scott (1987) all generalized the BSM to allow for stochastic volatility. However, these papers have the disadvantage of the models not having a closed-form solution and that they require extensive use of numerical techniques to solve two-dimensional partial differential equations, making them difficult to use in real-time trading. Further, Eisenberg & Jarrow (1991) and Stein & Stein (1991) assume that volatility is uncorrelated with the spot asset and use an average of BSM values over different volatility paths. However, this assumption of non-correlation between volatility and spot returns implies that these models cannot capture important skewness effects that arise from such a correlation.

These shortcomings are dealt with in Heston (1993), that introduces a closed-form solution for option pricing under stochastic volatility model where the volatility follows the Square Root Model (SRM). The solution also accounts for the correlation
between volatility and the spot asset, and thus enables a rich variety of pricing effects compared to the original BSM. The SRM is also used by Cox, Ingersoll & Ross (1985) to model stochastic interest rates. Whereas some stochastic volatility models have been tested under real circumstances (see for example Wiggins [1987], Chesney & Scott [1989], Melino & Turnbull [1990] or Scott [1992]), the SRM remains fairly untested in practice. The model is only proven in theory by for example Heston (1993) to provide option prices different from those generated by the BSM. Theoretical comparisons between prices generated by the BSM and stochastic volatility models can also be found in for example Hull & White (1987, 1988) and Stein & Stein (1991). It is also noted in for example Jiang & van der Sluis (1998) that empirical implications of stochastic volatility models on option pricing have not been adequately tested.

Thus the aim of this paper is to investigate the accuracy of the SRM on real market data (the German DAX index options market). Two different techniques of parameter estimation will be used, the method of moments (MoM) and implicit estimation by inversion. The equations for both methods are solved using a methodology denoted the Levenberg-Marquardt method (LMM). An efficient pricing model is essential for example considering risk management and associated scenario analysis, where the BSM is likely to provide biased figures. The corresponding BSM prices are also calculated to provide a benchmark. The rest of the study is structured as follows. The SRM is presented in Chapter 2 and the two different methods of parameter estimation in Chapter 3. Accordingly, the data is presented in Chapter 4 and the empirical findings in Chapter 5 while the results and conclusions are summarized in Chapter 6.

2 The Square Root Model

Following Heston (1993), assume that the spot asset at time $t$ follows the diffusion

$$dS(t) = \mu S(t)dt + \sqrt{V(t)}S(t)dZ_t,$$

where $Z_t$ is a Wiener process. If the volatility follows an Ornstein-Uhlenbeck process (used for example by Stein & Stein [1991])
\[ d\sqrt{V(t)} = -\beta \sqrt{V(t)} dt + \varphi dZ_2(t), \]  

then Ito’s lemma can be used to show that the variance, \( V(t) \), follows the process

\[ dV(t) = \left[ \omega^2 - 2\beta V(t) \right] dt + 2\varphi \sqrt{V(t)} dZ_2(t). \]  

This can be generalized into the familiar square root process used for example by Cox, Ingersoll & Ross (1985)

\[ dV(t) = \left[ \omega - \theta V(t) \right] dt + \xi \sqrt{V(t)} dZ_2(t), \]

where the Wiener processes \( Z_2(t) \) and \( Z_1(t) \) have the correlation \( \rho \).

Equation (4) is important as it is shown by Heston (1993) that there exists a closed-form solution for option prices, which is relatively easy to implement and displays the same qualitative properties that are expected in general time-homogenous cases. Here the volatility drift parameters \( \omega \) and \( \theta \) are assumed to be constants and capture the mean-reverting nature of the volatility process. The ratio \( \omega / \theta \) represents the long-term mean of the volatility process and as \( \theta \) has the dimensions of inverse time, \( 1/\theta \) represents the ‘half-life’ for volatility shocks. The parameter \( \xi \) represents the volatility of volatility.

It is shown in Lewis (2000) that if specialized to time-homogenous volatility processes of the form

\[ dV(t) = b(V(t)) + a(V(t))dZ(t), \]

meaning that the volatility changes in time only through the Brownian noise and level-dependent coefficients whilst there is no explicit time dependence, then the price for a call option is satisfied by

\[ C(S,V,\tau) = Se^{-\delta \tau} - Ke^{-\tau r} \int \frac{H(k,\nu,\tau)}{k^2 - ik} \, dk. \]
Here $S$ is the level of the underlying asset, $K$ is the strike price, $\tau$ is the time to maturity ($T - t$), $\delta$ is the dividend yield and $r$ is the risk-free interest rate. Further,

$$X = \ln \left( \frac{Se^{-\delta\tau}}{Ke^{-r\tau}} \right). \quad (7)$$

while $\max\{a, \alpha\} < \ln k < \min\{I, \beta\}$ where $\alpha$ and $\beta$ are real numbers and $\ln$ stands for the imaginary number. In typical examples $\alpha < 0$ and $\beta > 1$. For the SRM, $\hat{H}$ is given by

$$\hat{H}(k, \nu, \tau) = \exp\left[ f_1(t) + f_2(t)\nu \right], \quad (8)$$

where

$$f_1(t) = \tilde{a} \left[ t \ln \left( \frac{1 - h \exp(\nu t)}{1 - h} \right) \right], \quad f_2(t) = g \left( \frac{1 - \exp(\nu t)}{1 - h \exp(\nu t)} \right), \quad (9a,b)$$

and

$$d = \left[ \hat{\theta}^2 + 4c \right]^{1/2}, \quad g = \frac{1}{2} (\hat{\theta} + d), \quad h = \frac{\hat{\theta} + d}{\theta - d}. \quad (10a,b,c)$$

Further

$$\hat{\theta}(k) = \frac{2}{\xi_k^2} \left[ (l - \gamma + ik)p\xi_k + \sqrt{\theta^2 - \gamma(l - \gamma)\xi_k^2} \right]. \quad (11)$$

and

$$t = \frac{1}{2} \xi_k^2 \tau, \quad \tilde{a} = \frac{2}{\xi_k^2} \omega, \quad (12a,b)$$

$$\tilde{c} = \frac{2}{\xi_k^2} c(k), \quad c(k) = (k^2 - ik)/2. \quad (13a,b)$$

Finally, $\gamma$ represents the constant proportional risk-aversion parameter with constraints $\gamma \leq I$ and $\gamma(I - \gamma)\xi_k^2 \leq \theta^2/2.$
3 Parameter Estimation

As the model is given, it is a correct estimation of the parameters, as well as their stability in time, that is the main challenge when using stochastic volatility models (see for example Fouque et al. [2000]). Within this study, the parameters of the SRM are estimated using two different methods, the MoM and implicit estimation by numerical inversion.

3.1 The method of moments (MoM)

Suppose that we have a probability distribution equal to \( f_Y(y) \) and that its characteristics are given by the parameters \( \eta_1, \eta_2, \ldots, \eta_n \). Following Larsen & Marx (1986), the set of unknown parameters, using this method, is estimated by equating the theoretical moments of \( Y \) to its corresponding sample moments from

\[
\begin{align*}
\hat{\mu}_1 &= \mu_1(\eta_1, \eta_2, \ldots, \eta_n) \\
\hat{\mu}_2 &= \mu_2(\eta_1, \eta_2, \ldots, \eta_n) \\
&\vdots \\
\hat{\mu}_n &= \mu_n(\eta_1, \eta_2, \ldots, \eta_n)
\end{align*}
\]

Here \( \mu_i \) are the analytical expressions for the moments and \( \hat{\mu}_i \) the corresponding sample estimations. The mean of the SRM depends upon the parameters \( \omega, \theta, V \) and \( t \) according to

\[
E(V(t)) = \frac{\omega}{\theta} + e^{-\theta t} \left( V(0) - \frac{\omega}{\theta} \right),
\]

whilst the second central moment is given by

\[
E(V^2(t)) = \frac{\omega^2}{2\theta^2} + \frac{\omega^2}{\theta^2} + \left( V(0) - \frac{\omega}{\theta} \right) \left( \frac{\xi^2}{\theta} + \frac{2\omega}{\theta} \right) e^{-\theta t} + \\
\left( V(0) - \frac{\omega}{\theta} \right)^2 e^{-2\theta t} + \frac{\xi^2}{\theta} \left( \frac{\omega}{2\theta} - V(0) \right) e^{-2\theta t}.
\]
See Shreve (1997). The third moment is given by

\[ E(V^3(t)) = k \left( \frac{\omega^2}{\theta^3} \right) (1 - e^{-3\theta t}) + k \left( \frac{\omega^2}{3\theta^3} \right) (1 - e^{-3\theta t}) + \]

\[ \frac{k}{2\theta} \left( \frac{V(0) - \omega}{\theta} \left( \frac{\xi^2}{\theta} + \frac{2\omega}{\theta} \right) (e^{-\theta t} - e^{-3\theta t}) + \frac{k}{\theta} \left( \frac{V(0) - \omega}{\theta} \right)^2 \right), \quad (17) \]

\[ (e^{-2\theta t} - e^{-3\theta t}) + k \left( \frac{\xi^2}{\theta} \right) \left( \frac{\omega}{2\theta} - V(0) \right) (e^{-2\theta t} - e^{-3\theta t}) - V^2(0) e^{-3\theta t} \]

where \( k = 3(\omega + \xi^2) \). Further, the variance and skewness of the volatility distribution are given by

\[ \text{Var}(V(t)) = E(V^2(t)) - E(V(t))^2, \quad (18) \]

\[ \text{Skew}(V(t)) = \frac{E(V^3(t)) - 3E(V(t))E(V^2(t)) + 2E(V(t))^3}{\text{Var}(V(t))^{3/2}}. \quad (19) \]

Hence, the moments of the volatility sample distribution are set up to equal (15), (18) and (19). This equation is solved using the LMM, which through (17) gives the estimates regarding the desired parameters. Finally, the correlation, \( \rho \) is estimated by using the technique pioneered by Wiggins (1987), where

\[ \hat{\rho} = \frac{1}{\xi} \sqrt{\frac{1}{2} \sum_{t=2}^{T} \ln \left( \frac{\sigma_t}{\sigma_{t-1}} \right) \text{sgn}(R_t)} \]

(20)

and \( \xi \) is the estimated value of the standard deviation in the volatility process, \( \sigma_t \) is the observed volatility at time point \( t \), and \( R_t \) is equal to \( \ln \left[ S_t / S_{t-1} \right] \). The function \( \text{sgn} \) equals -1 if \( R_t \) is negative and 1 if \( R_t \) is positive.

The parameters are estimated using a one-month rolling window, which is recalculated each day. These parameters are hence used to price options on the following day. As pointed out, the stability of the parameters in time is one cause of problems when estimating these.
3.2 Implicit parameter estimation using numerical inversion

As noted by Bates (1996), option pricing-models are premised upon the underlying parameters and distributional structure being known with certainty, so that implicit parameters should in principle be a matter of inversion rather than estimation. The use of inversion is also suggested in Heston (1993). In the BSM world there is of course only one non-observable parameter, the volatility, which easily can be measure by calculating its implicit value. However, in the case of stochastic volatility models the number of non-observable parameters is several, which complicates the procedure. Here, five observed option prices for each day are used to estimate the parameters \((\omega, \theta, \xi, \rho, \gamma)\) by minimizing the maximum error of \(\left| P_{market} - P_{SRM}(\hat{\omega}, \hat{\theta}, \hat{\xi}, \hat{\rho}, \hat{\gamma}) \right|\), where \(P\) is the respective vector of prices.

The prices are collected around the at-the-money (ATM) level \((\pm 3\%)\), motivated by the fact that these prices are constantly available on most markets and that this information should enable the pricing of options deeper out-of-the-money, for the pricing model to be efficient. The prices are chosen so that one is approximately at-the-money, whilst the four others are spread out equally on both sides of this observation. The parameters perceived are hence used to price the options on the following day.\(^4\) The advantage of using a numerical inversion technique is of course that no historical data is needed. Further, as pointed out by Foque et al. (2000), modeling volatility as a random process might capture effects of market inefficiencies like transaction costs \textit{et cetera}. Here, implicit parameter estimation is not expected to give solely the parameters that drive the volatility path, but rather to a certain extent contain such market inefficiencies. Hence, this estimation technique is in practice expected to enable a correct pricing even in fairly inefficient markets.

However, there are also a few problems. The large number of parameters to be estimated makes non-linear estimates a bit unreliable as there might occur problems with distinguishing between local and global solutions. This might lead to parameter estimates that are quite unstable throughout time. To solve this problem, the LMM method is re-run with new starting values on occasions when the solutions for the parameters are closed to one of their defined boundary values. These are; \(\hat{\omega} \in [0,2]\), \(\hat{\theta} \in [0,30]\), \(\hat{\xi} \in [0,2.5]\), \(\hat{\rho} \in [-1,1]\), \(\hat{\gamma} \in [-10,1]\), respectively.
3.3 The sensitivity of the SRM to parameter changes

A relevant issue at this stage is of course how a change in the parameters affects the estimated prices using the SRM. This is efficiently illustrated by plotting the estimates of the SRM (compared with BSM prices) with respect to moneyness (deduced as \(\text{ln}[K/S]\)), maturity and different parameter values. The initial parameter values used are as follows; \(r = 0\), \(\tau = 90\), \(\omega = 0.625\), \(\theta = 10\), \(\xi = 0.5\), \(\rho = -0.5\) and \(\gamma = 0\). The ATM volatility is set at 25%. These parameters are chosen in accordance with values presented in finance literature (see for example Hull & White [1988]).

The correlation between spot and volatility, \(\rho\), is important as mentioned in Heston (1993), because it enables the modeling of deviations from normality in returns as the parameter positively affects the skewness of spot returns. This implies that a positive correlation results in higher variance when the spot asset rises, which of course spreads the right tail of the probability density. Accordingly, the left tail is associated with low variance and is thus not spread out. This phenomenon results in higher prices for out-of-the-money calls and lower prices for in-the-money calls, compared with BSM pricing, which is visualized in Figure 1.

![Figure 1](image)

**Figure 1**

**The difference between the SRM and the BSM**

A graphical illustration of the estimated pricing differences between the SRM and BSM with different estimates of correlation.
By plotting the effect of different values for the mean reverting parameters, $\omega$ and $\theta$, it is shown that a higher $\theta$ estimate results in a smile of less magnitude (see Figure 2). As there is less uncertainty involved the faster the process returns to its long-term mean, this would reduce the smile effect.

The volatility of volatility-parameter, $\xi$, is also of great importance in stochastic volatility models as this represents the magnitude of a random change in the level of volatility. Intuitively, a smaller value for $\xi$ should result in a reduced smile as there is less uncertainty or risk, which is supported by the graph in Figure 3. When $\xi$ is zero as in the BSM-case, the volatility is deterministic and the continuously compounded spot returns have a normal distribution.

![Figure 2](image)

**Figure 2**

**The smile and the mean reverting parameters**

A graphical illustration of how different values for the mean reverting parameters $\omega$ and $\theta$ affects the shape of the volatility smile.

Otherwise, as argued for example in Heston (1993), $\xi$ increases the kurtosis of spot returns, resulting in fatter tails in the distribution of spot returns. This, of course, results in higher prices for options out-of-the-money, whilst lowering the price for options near-the-money.
The smile and the volatility of volatility

A graphical illustration of how different values for the volatility of volatility affects the shape of the volatility smile.

The proportional risk-aversion is perceived as \((1 - \gamma)\), which implies that the lower the constant proportional risk-aversion parameter \((\gamma)\) is, the higher is the degree of risk-aversion (see for example Lewis [2000]). Thus, a risk parameter value equal to 1 indicates risk-neutrality. As visualized in Figure 4, a higher degree of risk-aversion results in higher volatilities across strike prices.

The smile and the risk-aversion parameter

A graphical illustration of how different values for the risk-aversion parameter affects the shape of the volatility smile.
4 The Data

The data consists of daily observations on call option settlement prices provided by the EUREX™ exchange, for a period ranging from 15.3 to 22.6.1999. As one month of data is needed to establish the rolling window mentioned above, the testing of the SRM is performed from 15.4 and onwards. The moneyness of the options is restricted to ±12% and the maturity to 10 - 100 days. Longer maturities are considered inappropriate as the magnitude of the smile is known to decrease with increasing maturity (see for example Derman [1996]). This gives a vector of observed prices with the size of (3,642 × 1) in the case of the DAX. The corresponding performance index (daily closing price) is used as the underlying asset and the three-month EURIBOR as proxy for the risk-free interest rate. The level of implied ATM volatility is iterated from the call option price closest to the level of the closing spot index (see Figure A1 [Appendix A] for graphical illustration).

5 Empirical Findings

The rolling window for the parameter estimation using the MoM is set up from 15.3 to 14.4 and used to price options on 15.4. This window is recalculated each day, enabling new and supposedly efficient pricing using the SRM. However, in this case the same set of parameters is adapted to all maturities. Using inversion, the parameters are initially inversely calculated from prices available 14.4 and used to price the respective options 15.4; again a procedure repeated for each maturity and trading day onwards. The initial starting values are chosen in accordance with literature. The descriptive statistics with regard to these estimates can be found in Table 1 and graphical illustrations in Figure B1 and Figure B2 (Appendix B).

As the MoM does not provide estimates for the constant proportional risk-aversion parameter, this parameter is assumed to equal -1. For notational clarity, the prices estimated with the parameters generated by the MoM and by inversion, will henceforth be referred to as $\text{SRM}_{\text{MoM}}$ and $\text{SRM}_{\text{INV}}$, respectively.
Table 1

The results of the parameter estimations using the two methods
The descriptive statistics concerning the sample of estimated parameters generated by
the two methods employed (MoM and inversion).

<table>
<thead>
<tr>
<th></th>
<th>MoM</th>
<th>Inversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega )</td>
<td>Average</td>
<td>0.3863</td>
</tr>
<tr>
<td></td>
<td>Stddev</td>
<td>0.0788</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Average</td>
<td>6.0223</td>
</tr>
<tr>
<td></td>
<td>Stddev</td>
<td>0.1291</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Average</td>
<td>0.4287</td>
</tr>
<tr>
<td></td>
<td>Stddev</td>
<td>0.2873</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Average</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Stddev</td>
<td>-</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Average</td>
<td>-0.1140</td>
</tr>
<tr>
<td></td>
<td>Stddev</td>
<td>0.0742</td>
</tr>
</tbody>
</table>

The results from using these two methods of parameter estimation, together
with the benchmark BSM prices, are visualized in Table 2. The deviations are
established using

\[
de\text{v}(\%) = \left| \frac{\hat{P}_{\text{model}} - \bar{P}_{\text{market}}}{\hat{P}_{\text{model}}} \right|
\]  

(21)

where \( P \) stands for the respective prices.

It is rather obvious that the SRM_{INV} is the methodology providing the most
accurate pricing patterns. A bit more surprising is the fact that the SRM_{MoM}
provides pricing estimates that are inferior to those generated with the standard BSM. This is
however likely to be due to the fact that the same parameters from the rolling window
are used to price all maturities, which might give this methodology a slight
disadvantage as the pricing performance of the SRM is fairly sensitive to the values of
its parameters. This is however in line with the original BSM assumptions where all
maturities are priced using the same parameters (except for maturity, of course). Another problem might be the length of the rolling window, as a one-month window might not be sufficient.

Table 2
Testing the SRM and BSM against market prices
The estimated deviations of the models compared with market prices from 15.4 to 22.6. The character $n$ denotes the number of observations. Moneyness is defined as $\ln(K/S)$.

<table>
<thead>
<tr>
<th></th>
<th>Moneyness ($\psi$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All $n = 2,481$</td>
</tr>
<tr>
<td><strong>BSM</strong></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>10.86%</td>
</tr>
<tr>
<td>Max</td>
<td>91.06%</td>
</tr>
<tr>
<td>Stddev</td>
<td>14.63%</td>
</tr>
<tr>
<td><strong>SRM_{MoM}</strong></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>11.40%</td>
</tr>
<tr>
<td>Max</td>
<td>94.01%</td>
</tr>
<tr>
<td>Stddev</td>
<td>15.60%</td>
</tr>
<tr>
<td><strong>SRM_{INV}</strong></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>4.01%</td>
</tr>
<tr>
<td>Max</td>
<td>85.02%</td>
</tr>
<tr>
<td>Stddev</td>
<td>7.58%</td>
</tr>
</tbody>
</table>

As visualized in Figure 5, it is apparent that the pricing errors of the SRM_{INV} are located for out-of-the-money calls. This is mostly due to the fact that these options are valued close to zero, which means that small errors in monetary terms cause large percentage deviations. The largest error for the SRM_{INV} (85.02%) occurs for a call option with a moneyness of 12% and a maturity of 31 days, which is observed to be valued at €0.4 whilst the SRM priced the call at €2.67. However, as these deviations can be considered minor, we feel that the SRM_{INV} is a fairly efficient pricing tool.

To graphically highlight the pricing capability of the SRM_{INV}, the average estimates from Table 1 for the inversely calculated parameters are used to draw the volatility surface generated by the model, which gives the graph as presented in Figure 6.
6 Summary and Conclusions

The purpose of this study was to investigate the accuracy of the SRM on real market data in the form of the DAX index options market. Two different techniques of
parameter estimation were used, the method of moments and implicit estimation by numerical inversion.

The results indicate that the SRM with parameters estimated by inversion using the available prices the preceding day, is the most accurate pricing method of the three (SRM\textsubscript{INV}, SRM\textsubscript{MoM} and the BSM) this study. Surprisingly, the same model with parameters estimated using a rolling window and the MoM proved to be inferior to the benchmark BSM. Thus the importance of stable and correct estimation of the parameters becomes increasingly evident. It is also difficult, \textit{a priori}, to determine the whether the estimated parameters are the most accurate. This might demand numerous simulations and experimentation depending on the market and its conditions. However, as indicated by the graphs in Appendix B the estimates herein are quite constant in time and considering the empirical findings regarding pricing performance, it would seem that the method of inversely calculated parameters using the LMM as suggested in this study, is a useful methodology concerning parameter estimation.

Estimating these parameters consumes only a few seconds of time using a 700 MHz personal computer but as the pricing of approximately 2,500 options takes about one and a half minute, the model is not an alternative considering live trading, at least not when dealing with efficient markets. However, as a risk management tool for scenario analysis purposes (see essays 1 and 3), smile-benchmark and pricing tool when setting up longer-term strategies, this model is by all means highly competitive.

\textbf{Endnotes:}

1 The Levenberg-Marquardt method is a Nonlinear Least Squares implementation used in this study to minimize the absolute error regarding the respective problems. For more info see for example Moré (1977).
2 Under the assumption of stochastic volatility it is impossible to take a risk-free position, and hence the risk premium represents the price an investor is willing to pay for the volatility risk.
3 The MoM is used for example in Scott (1987), Wiggins (1987) and Chesney & Scott (1989) to estimate continuous volatility processes.
4 A similar approach can be found in for example Corrado & Su (1997) where observations the preceding day are used to price options on the following day.
5 Note that the \textit{Constant Proportional Risk Parameter} is linked to the more common \textit{Constant Relative Risk Aversion} parameter \(q\), so that \(q = 1 - \gamma\). Hence, \(\gamma > 1\) indicates risk loving, \(\gamma < 1\) risk aversion, \(\gamma = 1\) risk neutrality and \(\gamma = 1\) log utility.
6 As the German DAX-index options market is currently one of the largest markets in the world concerning liquidity, these options should be ideal for the aims of this study.
7 According to Lewis (2000) this value should represents a fairly ‘true’ estimate. Similar results are also indicated in for example Friend & Blume (1975).
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Appendix A.

Figure A1  
The DAX index and the implied volatility
A graphical view of the spot index and implied volatility during the observed time-period. The overall trend for the index was obviously bullish, whilst the implied volatility seemed to drop accordingly.
Appendix B.

Figure B1
The estimated parameters using the MoM
A graphical view of the stability in time of the estimated parameters using the MoM

Figure B2
The estimated parameters using inversion
A graphical view of the stability in time of the estimated parameters using inversion