VOLATILITY SMILE DYNAMICS IN SCENARIO ANALYSIS

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Key words: Volatility Smile, Option Risk Management, Risk Matrix

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Volatility Smile Dynamics in Scenario Analysis

by

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Abstract

The objective of this paper is to suggest a method that accounts for the impact of the volatility smile dynamics when performing scenario analysis for a portfolio consisting of vanilla options. As the volatility smile is documented to change at least with the level of implied at-the-money volatility, a suitable model is here included in the calculation process of the simulated market scenarios. By constructing simple portfolios of index options and comparing the ex ante risk exposure measured using different pricing methods to realized market values, ex post, the improvements of the incorporation of the model are monitored. The analyzed examples in the study generate results that statistically support that the most accurate scenarios are those calculated using the model accounting for the dynamics of the smile. Thus, we show that the differences emanating from the volatility smile are apparent and should be accounted for and that the methodology presented herein is one suitable alternative for doing so.

Comments by Kenneth Högholm, Johan Knif, Lars Nordén, Björn Hanson and Eva Liljeblom are gratefully acknowledged.
1 Introduction

Trading financial instruments, especially derivatives, sets demand on accurate risk management. However, accurate risk management requires the market participant to have the right tools for pricing the instruments as well as a picture of the prevailing risks.

In 1973, Fischer Black and Myron Scholes solved the problem of option pricing and introduced their famous Black & Scholes pricing model (BSM). The BSM is based on the assumption that stock prices follow a geometric Brownian motion and calculates the price of an option from five parameters (strike price, underlying asset price, interest rate, time to maturity and volatility). In addition to the strike price, the level of the underlying asset and the volatility are the components that affect the value of a vanilla option the most. Hence, these components form the two most significant risks of vanilla option trading. The task of keeping track of these two risk parameters is conveniently performed as widely done by practitioners with scenario analysis, but there is still one difficult dimension in the pricing world; the volatility smile.

The BSM assumes that the volatility, $\sigma$, is constant for all strikes and maturities of an option, which obviously is not the case in reality. The volatility is documented to be different for different strikes, $X$, and changing over time, $t$, which implies that the volatility is a function of three different variables, $\sigma(S, X, t)$, where $S$ is the level of the underlying asset. This is the phenomenon often referred to as the volatility smile, due to its graphical form. This smile is documented for example by Duque & Paxon (1994), Taylor & Xu (1994), Gemmill (1996) and Peña et al. (1999) to change as conditions change, usually losing in magnitude as at-the-money (ATM) volatility and time increases and vice versa. Thus, accurate ex ante scenario analysis requires the pricing model to account for these attributes of the smile when calculating the value of the portfolio for different market scenarios.

To date several attempts have been made to capture the structure of the smile and include it in the pricing of options. Amongst others, Derman & Kani (1994), Dupire (1994) and Rubinstein (1994) use lattice models while Stein & Stein (1991), Heston (1993) and Bates (1996) all propose different stochastic volatility models. The impact of liquidity and transaction costs on the smile is examined for example in Longstaff (1995) and Dumas et al. (1996), while Derman (1999) explores different implied volatility function models based on the sticky-rule. Instead of trying to model
what changes, these models focus on what does not change, hence the name. The two main models developed on the stickiness of the smile are the sticky-strike and the sticky-delta models. These build on the assumption that the level of volatility is unchanged for a specific strike or for a specific level of delta, respectively. Sundkvist (2000) develops the sticky-delta model into the sticky-delta smile approximation model (henceforth the SDSA-model), using a Taylor approximation to model the smile.

Providing that the smile does change with market conditions and/or through time, any risk management-method ignoring this phenomenon is likely to generate inefficient risk measures. To explore this topic the SDSA-model is incorporated within this study in the calculation process of the scenario analysis. To investigate the differences established by accounting for the dynamics of the smile, different portfolios of vanilla options are created and the ex ante risk exposure measured using a total of three different pricing methods is compared to realized market returns, ex post. The rest of this study is structured as follows. The scenario analysis and the SDSA-model are examined closer in Chapter 2 and the methodology in Chapter 3. The empirical findings are presented in Chapter 4 while the essay is brought to an end with a summary in Chapter 5.

2 Scenario Analysis and the SDSA-model

As the smile is documented to change upon a change in market conditions, performance of scenario analysis inevitably calls for coverage of this phenomenon. To investigate the possible deviations, a tool for performing scenario analysis as well as a pricing method to cover the smile are needed. This study will utilize scenario analysis in matrix form to visualize the ex ante risks of the created portfolios and the SDSA-model to account for changes in the dynamics of the smile. Although we believe that the ideas presented in this study are suitable for almost any derivatives market, the numerical examples are based on the German DAX-index options market, which is currently one of the largest derivatives markets in the world concerning liquidity.

2.1 Risk management and scenario analysis

The risk management procedure is important to any company involved with trading financial derivatives not only internally (disclosures to management and/or board of
directors), but also externally in the form of involved investors, creditors and regulators. Risk management, besides accurate pricing, includes monitoring the prevailing risks a company or an individual trader/portfolio is exposed to. This requires the participants to have the appropriate knowledge of the actual risks and, of course, the right tools for measuring and keeping track of these. As the risks of vanilla options are rather obvious, it is finding the tool that might cause a few problems.

Following the Basle Accord Amendment in 1993 for calculation of market risk capital using internal models, the Basle Committee on Banking Supervision (1995) have recommended two methods for generating a unified set of risk measures on a daily basis (see for example Alexander [2000]). The first approach is to calculate the Value-at-Risk (VaR) measure that aggregates the several market risks into a single-sum measure (see for example Ju & Pearson [1999], Lopez [1999] and El-Jahel, Perraudin & Sellin [1999] and Boulier et al [1997]). This might be a convenient number to wave in front of investors, but hardly satisfactory from the standpoint of a trader, as a more in-depth view is needed to stay on top of the ever-changing risks. Here the second alternative of quantifying the maximum loss over a set of scenarios for movements in the risk factors, that is scenario analysis, might be more appropriate. See Essay 1 for more on this topic.

![Figure 1](image-url)

**Figure 1**
**The dimensions of the analysis**

Scenario analysis is conducted by calculating the net change in the value of the traded portfolio for changes in two parameters, over its two axes. Optimally, these parameters are represented by the underlying asset and the volatility, as these two are considered the two most significant risks of vanilla option trading. Initially, the value of the portfolio under the prevailing circumstances is calculated, where after the different scenarios of the analysis are simulated, one by one, and a new value of the
The portfolio is calculated. This value is then subtracted from the original value of the portfolio and the net change is perceived.

The obvious weakness of the analysis is that it only provides instantaneous information. This means that the user has to calculate a new matrix when conditions change, that is when the underlying asset moves significantly or when the level of implied volatility changes (this is of course also the situation when using other methods like the VaR). However, scenario analysis is a very flexible risk management tool and the most difficult task is probably finding the appropriate pricing engine for calculating the value of the portfolio for the different scenarios, which as argued above inevitably involves accounting for the volatility smile.

2.2 Modeling the dynamics of the smile using the SDSA-model

It is argued by for example Cox & Ross (1976), Black (1976), Christie (1982) and Koutmos (1996) that volatility increases when the market dives and, accordingly, that it decreases when the market moves upwards. The correlation between spot and volatility is negative. The volatility smile is also documented to flatten as ATM-volatility and maturity increases and vice versa. To account for these dynamics in the scenario analysis process and thus possibly enable a more accurate measure of ex ante risk, a tool for the estimation of the smile and its attributes is needed.

Heuristic rules for describing how the smile changes as the underlying asset moves are often used by market participants, as these are easy to grasp and convenient to use from a practical standpoint. Instead of modeling what changes, emphasis is laid on what does not change, the ‘sticky’ parts. One alternative is to focus on the delta component. The delta is the first derivative of the option price \( c \) with respect to the underlying \( S \), deduced as

\[
\Delta = \Phi \left( \frac{\ln(S / X) + (r + \sigma^2 / 2) t}{\sigma \sqrt{t}} \right) = \Phi(d_1),
\]

for a European call, where \( \Phi(d_1) \) is the standard normal cumulative distribution function of \( d_1 \). The delta of an option is a function of the underlying, strike price, time to maturity, volatility and interest rate, and the delta of call options decreases as strike-prices increase, making delta downward sloping over strikes. As time passes,
the delta of a particular strike approaches one of its boundaries and thus accentuating
the downward slope. These two attributes of the delta correlate with the dynamics of
the observed volatility smile and it is therefore reasonable to believe that the smile can
be described by a function of delta as

$$\sigma(S, X, t) = f(\Delta).$$  \hspace{1cm} (2)

The idea of the model is that the distance of an option to the ATM level
determines its volatility and that this distance can be measured using the delta
component. Delta for an ATM option is approximately 0.5 which means that $\Delta - 0.5$
can be defined as the ‘moneyness’ of any other option in-the- or out-of-the-money.
Derman (1999) uses this analogy to define the sticky-delta model as

$$\sigma(S, X, t) = \sigma_{ATM} + \beta(\Delta - 0.5).$$  \hspace{1cm} (3)

where $\Delta$ is the delta of a call option with a certain strike. However, as the delta of an
ATM call option is not exactly 0.5, formula (3) generates a smile with a positive bias.
To correct for this bias, Sundkvist (2000) suggests the use of the exact ATM delta
instead of the approximation 0.5. This forms

$$\sigma(S, X, t) = \sigma_{ATM} + \beta(\Delta - \Delta_{ATM}).$$  \hspace{1cm} (4)

which gives consistent outputs for ATM volatilities. To be able to estimate the correct
volatility for a certain strike, a volatility index (%) is defined as

$$VI_X = (\sigma_X - \sigma_{ATM}) / \sigma_{ATM},$$  \hspace{1cm} (5)

where the index is always equal to 0 % for ATM options, assuming that the smile
moves with the ATM level. Given a smile, the volatility index is larger than 0 % for
in-the-money calls (out-of-the-money puts) and vice versa. This means that the ATM
volatilities, argued to be the most accurate observations as these options generally are
the most traded contracts, are used for indexing the volatility of other strikes.
Multiplied by the ATM volatility (one plus) the index gives the appropriate level of
implied volatility for the respective strike. For example, if the ATM volatility is 22 %
and an out-of-the-money put has a volatility index of 15 %, its individual volatility is perceived as 1.15 multiplied by 22 % which equals 25.3 %.

As the exact function of the smile is unknown for an individual market, it can be approximated using a polynomial model based on the presented methodology. The volatility index is used as the dependent variable, and exponents up to order \( m \) of the delta for call options on different strikes subtracted by the at-the-money delta, as the independent variables. Higher orders of the independent variables are needed to account for the typical non-linear shape of the smile. This gives the SDSA-model as

\[
VI_X = \sum_{i=1}^{m} \beta_i (\Delta_X - \Delta_{ATM})^i + \epsilon. \tag{6}
\]

Leaving out the intercept will force the polynomial through the ATM volatility, as the model otherwise produces an ATM volatility different from the observed.

### 3 Incorporating the Smile in the Scenario Analysis

If accepting the theories concerning the dynamics of the volatility smile presented in finance literature, risk management accounting for these attributes should be more accurate than other methodologies utilizing a static smile or no smile at all. In the absence of an efficient model to capture these dynamics, the scenario analysis procedure may be rather simplistic (see for example Alexander [2000]). Accurate scenario analysis requires a pricing model that accounts for the anticipated changes to the volatility smile for different market scenarios. Hence, the SDSA-model is incorporated in the pricing process of the analysis to provide individual volatilities for the options included in the constructed portfolios.

#### 3.1 Methodology

Two different portfolios will be created where the first consists of a *long strangle* (long call and put with different strike prices) and the other of a *short straddle* (short call and put with same strike price). See Appendix A for closer detail. The portfolios are set up on 2.6.1999 with expiration on 17.12.1999. To determine the most accurate risk management method, matrixes and scenarios will be calculated using a *static smile*, a *changing smile* and a *constant volatility*. The static smile-method means that
the smile estimated for the set-up day of the portfolio is used for all scenarios during the entire period applying only a parallel shift in the smile with the level of the underlying asset (see also Malz [1999] and Alexander [2000]). On the contrary, the changing smile-approach implies that a new smile is estimated continuously for each corresponding scenario using the SDSA-model, as presented in Figure 2. Finally the constant volatility-approach means that the ATM-volatility is used for all options in the portfolio according to the basic BSM.

**Figure 2**

Simulating the different outcomes of the scenario analysis
When including the calibrated SDSA-model in the analysis, the new ATM-volatility and level of underlying are perceived from the respective scenario of the analysis, and used to estimate the new smile. For instance, if the current underlying asset is priced at 5,500 at close and the level of implied ATM volatility is 25 %, this represents the starting point (0, 0). Then for the scenario representing a 5 % increase in the underlying asset and a 10 % decrease in ATM volatility (5, -10), a smile using these parameters is estimated. In this case with an underlying asset priced at approximately 5,775 and with an ATM volatility of 22.5 %. As this smile is likely to be different than the initial one, the analysis is of course assumed to give different values compared to the one using a static smile. Note however, that a change in the underlying asset does not change the structure of the smile but only the moneyness of the options as the smile is fixed ATM. The smile changes with the level of ATM-implied volatility only.

The strangle and straddle are regarded suitable as they are quite sensitive to changes in the level of implied volatility, which is likely to reflect any differences between the different pricing methodologies. To visualize possible *ex ante* deviations between the three different pricing methodologies, scenario analysis-matrixes for the two portfolios are estimated to cover a time-span of one week. Hence, the estimated scenarios of these matrixes are compared.

Further, to establish which methodology is the most accurate, these matrixes are simulated one week (seven days, Wednesday-Wednesday) forward in time and then compared to the realized portfolio value on that day. This means that only one scenario is analyzed, that is the exact realized scenarios of the underlying asset and
the ATM-volatility. A period of one week is used to avoid possible pricing inconsistencies caused by weekends and holidays. The realized portfolio value is calculated using daily settlement prices provided by the exchange, whereas the theoretical portfolio values used for simulating the matrixes are calculated using the common BSM, provided with volatilities according to the three proposed methods. This procedure is repeated upon comparison every seventh day.

3.2 Data

The time-period used for estimating the coefficients of the SDSA-model is 4.1-21.4.1999 and the data consists of daily observations on the volatility smile, iterated from the midpoint of available market bid/ask spreads on the German DAX-index options market. The options are traded electronically at EUREX™ and are amongst the most liquid index options in the world. The index options are European style.

The empirical evaluation of the different methods is performed during the period 2.6-8.12.1999 (for descriptive statistics see Table 5 or Table E1, Appendix E). Daily DAX-close prices are used as the underlying asset and the three-month EURIBOR as proxy for the risk-free interest rate. The implied ATM-volatility is iterated using the BSM from the call option price closest to the ATM level using the settlement prices.

4 Empirical Findings

Fitting the SDSA-model to the German DAX-index options market according to the methodology discussed previously generated the following results as displayed in Table 1.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>The SDSA-coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta_1 )</td>
</tr>
<tr>
<td>DAX</td>
<td>42.37(^*)</td>
</tr>
</tbody>
</table>

\(^*\) Denotes significance at the 5 %-level.
Orders up to five are used in this study as all contribute to the explanatory power of the SDSA-model, whereas adding higher orders than four proved not to increase the accuracy (the fifth order and higher turn out to be insignificant). The sensitivity of the model to changes in beta values does of course also diminish with higher orders, implying that the important estimates are the betas for orders up to three or four. Here, the constant is left out to force the polynomial through the level of ATM volatility, whilst the $R^2$ statistic represents the fit of the regression model on the market data to which it is calibrated.

To determine the shape of the smiles implied by the SDSA-model, a graph is created in Figure 3 with three different simulated smiles based on the estimated parameters of Table 1. From the figure it is evident that the model is at least capable of capturing one of the documented dynamics of the volatility smiles, which in this case is that the magnitude of the smile increases with lower ATM volatility.

![Figure 3](image)

**Figure 3**

**Hypothetical volatility smiles constructed using the SDSA-model**

Three hypothetical volatility smiles simulated using the beta estimates in Table 1. The smiles are indexed to 100 to enable comparison of the three. The ATM level is 100, the risk-free rate 3% and the maturity 50 days.

The estimates are used to price options in- and out-of-the-money in the two portfolios A and B defined in this study, in comparison to the suggested static smile and constant volatility methods. Thus, the next step is to compare the different methods according to the methodology presented above.
4.1 Portfolio A – the long strangle

The three different scenario analysis-matrixes are estimated covering a time-span of one week to highlight possible deviations when estimating the value of the portfolios using the three different methods for the defined market scenarios. This sort of analysis is appropriate if the portfolio manager wants to realize the portfolio one week from now, and needs to track the *ex ante* risk exposure of this period. This period is of course fictive, meant only to give a picture of the posed problem, and could of course be any given number of days (or fractions of a day). The analysis used in the following examples consists of 49 possible scenarios, covering a ±9% move in the underlying asset and a ±30% change in the implied ATM-volatility. The analyses are shown in tables 2 through 4.

**Table 2**

The analysis calculated using constant volatility across strikes

A change in the underlying asset is viewed horizontally while a change in the implied ATM-volatility is presented vertically. The figures in each scenario represent the net change in the value of the portfolio from scenario (0, 0) on 2.6 to each respective scenario on 9.6.1999.

<table>
<thead>
<tr>
<th></th>
<th>-9 %</th>
<th>-6 %</th>
<th>-3 %</th>
<th>0 %</th>
<th>3 %</th>
<th>6 %</th>
<th>9 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30 %</td>
<td>-466 563</td>
<td>-829 478</td>
<td>-1 078 919</td>
<td>-1 207 950</td>
<td>-1 214 821</td>
<td>-1 102 755</td>
<td>-879 162</td>
</tr>
<tr>
<td>-20 %</td>
<td>-198 237</td>
<td>-515 709</td>
<td>-728 529</td>
<td>-832 763</td>
<td>-828 206</td>
<td>-718 121</td>
<td>-508 649</td>
</tr>
<tr>
<td>-10 %</td>
<td>90 181</td>
<td>-187 889</td>
<td>-369 396</td>
<td>-452 330</td>
<td>-437 401</td>
<td>-327 795</td>
<td>-128 742</td>
</tr>
<tr>
<td>0 %</td>
<td>394 061</td>
<td>150 396</td>
<td>59 708</td>
<td>-68 270</td>
<td>-43 766</td>
<td>66 491</td>
<td>257 949</td>
</tr>
<tr>
<td>10 %</td>
<td>709 933</td>
<td>496 582</td>
<td>366 068</td>
<td>318 337</td>
<td>351 809</td>
<td>463 578</td>
<td>649 622</td>
</tr>
<tr>
<td>20 %</td>
<td>1 035 187</td>
<td>848 803</td>
<td>739 521</td>
<td>706 744</td>
<td>748 710</td>
<td>862 661</td>
<td>1 044 996</td>
</tr>
<tr>
<td>30 %</td>
<td>1 367 846</td>
<td>1 205 681</td>
<td>1 115 522</td>
<td>1 096 413</td>
<td>1 146 496</td>
<td>1 263 157</td>
<td>1 443 137</td>
</tr>
</tbody>
</table>

The number shown in scenario (0, 0) actually represents the *theta* value of each portfolio, as the lapse of time is the only market risk factor affecting the value of the options for this scenario. Further, the long strangle obviously benefits from an increase in market volatility and from large moves in the underlying asset, as clearly visualized by the analysis.

**Table 3**

The analysis calculated using a static smile

A change in the underlying asset is viewed horizontally while a change in the implied ATM-volatility is presented vertically. The figures in each scenario represent the net change in the value of the portfolio from scenario (0, 0) on 2.6 to each respective scenario on 9.6.1999.

<table>
<thead>
<tr>
<th></th>
<th>-9 %</th>
<th>-6 %</th>
<th>-3 %</th>
<th>0 %</th>
<th>3 %</th>
<th>6 %</th>
<th>9 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30 %</td>
<td>-310 520</td>
<td>-706 542</td>
<td>-1 016 264</td>
<td>-1 158 358</td>
<td>-1 175 235</td>
<td>-1 069 106</td>
<td>-846 947</td>
</tr>
<tr>
<td>-20 %</td>
<td>-47 129</td>
<td>-402 602</td>
<td>-682 422</td>
<td>-799 045</td>
<td>-803 826</td>
<td>-699 352</td>
<td>-491 513</td>
</tr>
<tr>
<td>-10 %</td>
<td>235 909</td>
<td>-84 466</td>
<td>-339 466</td>
<td>-434 172</td>
<td>-428 056</td>
<td>-323 849</td>
<td>-126 631</td>
</tr>
<tr>
<td>0 %</td>
<td>534 297</td>
<td>244 373</td>
<td>10 119</td>
<td>-65 419</td>
<td>-49 322</td>
<td>55 669</td>
<td>245 088</td>
</tr>
<tr>
<td>10 %</td>
<td>844 727</td>
<td>581 369</td>
<td>364 603</td>
<td>306 088</td>
<td>331 465</td>
<td>438 044</td>
<td>621 842</td>
</tr>
<tr>
<td>20 %</td>
<td>1 164 661</td>
<td>924 646</td>
<td>722 754</td>
<td>679 571</td>
<td>713 678</td>
<td>822 468</td>
<td>1 002 351</td>
</tr>
<tr>
<td>30 %</td>
<td>1 492 153</td>
<td>1 272 805</td>
<td>1 083 672</td>
<td>1 054 467</td>
<td>1 096 867</td>
<td>1 208 363</td>
<td>1 385 684</td>
</tr>
</tbody>
</table>
It is rather obvious that the risk exposures implied by the analyses for the three different methodologies differ to a certain extent. Even the values for scenario (0, 0) differ compared to the SDSA-values as the smile increases in magnitude over the period due to the lapse of time. The absolute deviation between the analysis utilizing the SDSA-model and the two others can be substantial, at most 168% and 542% versus the static smile and constant volatility analysis, respectively.

Table 4

<table>
<thead>
<tr>
<th>The analysis calculated using a changing smile (SDSA-model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A change in the underlying asset is viewed horizontally while a change in the implied ATM-volatility is presented vertically. The figures in each scenario represent the net change in the value of the portfolio from scenario (0, 0) on 2.6 to each respective scenario on 9.6.1999.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>-9 %</th>
<th>-6 %</th>
<th>-3 %</th>
<th>0 %</th>
<th>3 %</th>
<th>6 %</th>
<th>9 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>-30%</td>
<td>-555 450</td>
<td>-910 326</td>
<td>-1 125 178</td>
<td>-1 194 615</td>
<td>-1 125 350</td>
<td>-931 784</td>
<td>-630 869</td>
</tr>
<tr>
<td>-20%</td>
<td>-317 922</td>
<td>-616 152</td>
<td>-785 346</td>
<td>-824 661</td>
<td>-740 578</td>
<td>-544 632</td>
<td>-251 063</td>
</tr>
<tr>
<td>-10%</td>
<td>-54 015</td>
<td>-303 233</td>
<td>-434 266</td>
<td>-448 352</td>
<td>-351 404</td>
<td>-152 993</td>
<td>134 805</td>
</tr>
<tr>
<td>0%</td>
<td>230 415</td>
<td>23 416</td>
<td>-75 210</td>
<td>-67 622</td>
<td>40 811</td>
<td>241 906</td>
<td>525 300</td>
</tr>
<tr>
<td>10%</td>
<td>530 671</td>
<td>360 276</td>
<td>289 659</td>
<td>316 259</td>
<td>435 153</td>
<td>639 188</td>
<td>919 357</td>
</tr>
<tr>
<td>20%</td>
<td>843 190</td>
<td>704 867</td>
<td>658 851</td>
<td>702 412</td>
<td>830 986</td>
<td>1 038 207</td>
<td>1 316 164</td>
</tr>
<tr>
<td>30%</td>
<td>1 165 304</td>
<td>1 055 406</td>
<td>1 031 304</td>
<td>1 090 208</td>
<td>1 227 848</td>
<td>1 438 480</td>
<td>1 715 088</td>
</tr>
</tbody>
</table>

However, as these deviations are rather random depending on the chosen time-period and the constituents of the portfolio, the interesting comparison is with realized exact market scenarios. To measure the most efficient pricing/risk management method, the estimated risk exposures implied by the analyses are compared to the market value of the portfolio every seventh day. This means that the ex ante risk estimation preciseness of the analysis calculated using the SDSA-model is compared ex post to realized market values. These estimates are recalculated every seventh day as mentioned above. Turn to Appendix B for graphical overview.

It is fairly obvious that the scenarios estimated using the SDSA-model are the ones closest to the realized market values of the portfolio, as these for example imply the lowest standard deviation in the examined sample. Surprisingly, however, the constant volatility method proved to be more accurate than the static smile-approach. This is probably due to the fact that the effect of the smile is partly canceled out as the strangle has one leg on both sides of the ATM-level to a large extent throughout the entire examined period.
To determine whether the deviations are statistically significant, the Wilcoxon matched-pairs signed-rank test is used, employing a significance level of 5%. The Wilcoxon test is considered adequate as it measures the absolute deviations from market prices. In the Wilcoxon test, the differences are ranked and two sums are computed; for the negative and for the positive differences. By ranking the differences, it also accounts for the magnitude of the differences and the systematic errors are included in the deviations, implying that the average deviation is accounted for. As presented in Table C1 (Appendix C), the deviations are statistically significant for all methods.

A complementary testing of the differences between the methods using Bartlett’s test supports the hypothesis of significantly different variances when employing a significance level of 5%. The Bartlett test, on the other hand, tests the variance with regard to the systematic error. The generated chi-square statistic equals 343,792. Further, as the SDSA-analysis and the scenario analysis using a constant ATM-volatility (Vola) is iterated from observed market prices when estimating the corresponding scenarios.
volatility indicate the lowest variance, these two are tested against each other using the *F*-test. This test gives a value of 4.693, which is significant at conventional levels, supporting the hypothesis that the scenarios calculated using the SDSA-model are the most accurate of the three. To further establish these results, the same empirical methodology is applied to the second portfolio.

### 4.2 Portfolio B – the short straddle

The analyses generated for the straddle-portfolio are found in Appendix D. As seen from tables D1 through D3, this portfolio is long theta as the value for scenario (0, 0) is positive. Shorting a straddle also means that the portfolio manager benefits from a sell-off in volatility, which is obvious when evaluating the analysis. The position was also initialized with a bullish view (the portfolio has a positive delta exposure), which generates profits for a limited rise in the DAX-index and vice versa. Again, the estimated scenario analyses using the constant volatility and the static smile method are quite close to each other, concerning the monetary exposures for different market scenarios. However, compared with the analysis estimated with the SDSA-model, the differences are a bit more apparent in this case, too.

![Figure 4](image)

**Figure 4**

*The deviations between the methods and the market*

The deviations of the three different methodologies compared to realized market scenarios for portfolio B presented in graphical form.

The scenarios were again compared to realized market values, which generated the following results as presented in Figure 4. Turn to Appendix E for closer detail. The
differences seem again rather apparent and testing them using the proposed Wilcoxon and Bartlett tests also supports the hypothesis of significantly different values, see Table C2 (Appendix C).

Using the Bartlett test, the chi-square statistic is 350.469. For this portfolio, the SDSA-analysis and the analysis using a static smile indicate the lowest variance. Testing these two against each other using the \textit{F-test} gives a value of 5.740. Both the chi-square and the F-test statistics are again significant at conventional levels. Obviously the results from this portfolio support the hypothesis of the SDSA-model being the most accurate pricing tool of the three, whereas the difference between using a static smile and constant volatilities seem to be random depending on the constituents of the portfolio. However, we feel that for more complex portfolios using a static smile should on average turn out to be more efficient than using constant volatilities, as markets are documented to have volatility smiles.

4.3 Implications

It seems rather obvious when considering the results of this study that the smile changes and that this change is to be modeled. Scenario analysis utilizing the SDSA-model that accounts for the dynamics of the smile in this case proved to be the most accurate method. Whereas this model is by no means perfect as deviations still are apparent and can be replaced with any model proving to be more efficient. However, based on these findings we believe that the analysis based on the SDSA-model is a better alternative than using an analysis based on a static (the present) smile or not using a smile at all.

5 Summary

This study focused on examining the impacts of the volatility smile dynamics concerning option risk management and on presenting a method accounting for this. As argued, risk management requires the participant to have a view of the prevailing risks and a correct pricing method. The risks in this study were monitored using scenario analysis with the common BSM combined with the SDSA methodology as the pricing engine.

Based on the examples, finance literature and the results of the empirical study, it is rather obvious that the smile-phenomenon should be accounted for when
pricing options and performing scenario analysis. The smile changes with the level of
ATM-volatility and time to maturity, thus making any stress- or scenario analysis
testing dependent on the ability of the underlying pricing model to account for this.
Considering the presented arguments and empirical findings, we believe that scenario
analysis based on the SDSA-model is a relatively accurate and efficient tool for
successfully monitoring the monetary risks of vanilla option trading.

Endnotes:

1 See also Essay 4 for more on the characteristics of the smile.
2 See also Essay 5.
3 See also essays 1 and 4.
References


Basle Committee on Banking and Supervision (1995): “An Internal Model Based Approach to Market Risk Capital Requirements”


Appendix A

The constituents of the created portfolios.

Table A1
The constituents of portfolio A (long strangle)
The ATM-level on the day of set-up is 5040 and the ATM-volatility approximately 27.3 %. The interest rate used is 2.6 %. As the contract size is 5 the amount of invested capital equals €2,868,500 while the delta positions is −440. The time to maturity is 198 days on the day of setup.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Strike</th>
<th>Call/Put</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 1999</td>
<td>5000</td>
<td>Put</td>
<td>+1000</td>
</tr>
<tr>
<td>Dec 1999</td>
<td>5500</td>
<td>Call</td>
<td>+1000</td>
</tr>
</tbody>
</table>

Table A2
The constituents of portfolio B (short straddle)
The ATM-level on the day of set-up is 5040 and the ATM-volatility approximately 27.3 %. The interest rate used is 2.6 %. The amount of invested capital equals €-4,120,000 while the delta positions is 1366. The time to maturity is 198 days on the day of setup.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Strike</th>
<th>Call/Put</th>
<th>Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec 1999</td>
<td>5500</td>
<td>Put</td>
<td>-1000</td>
</tr>
<tr>
<td>Dec 1999</td>
<td>5500</td>
<td>Call</td>
<td>-1000</td>
</tr>
</tbody>
</table>
Appendix B
Comparing the methods versus market scenarios for portfolio A.

Figure B1
The deviations between the methods and the market
The deviations of the three different methodologies compared to realized market scenarios for portfolio A presented in graphical form.
Appendix C

The test statistics for the two portfolios.

Table C1

*Z*-scores for Wilcoxon signed-rank test for portfolio A*

The model specified by the label to the left provided estimates closer to the observed prices than the model specified by the label at the top of the column.

<table>
<thead>
<tr>
<th>Strangle</th>
<th>Static</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDSA</td>
<td>4.469*</td>
<td>4.281*</td>
</tr>
<tr>
<td>Constant</td>
<td>4.076*</td>
<td></td>
</tr>
</tbody>
</table>

* denotes significance at the 5 %-level.

Table C2

*Z*-scores for Wilcoxon signed-rank test for portfolio B*

The model specified by the label to the left provided estimates closer to the observed prices than the model specified by the label at the top of the column.

<table>
<thead>
<tr>
<th>Strangle</th>
<th>Static</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDSA</td>
<td>3.940</td>
<td>4.577</td>
</tr>
<tr>
<td>Constant</td>
<td>2.482*</td>
<td></td>
</tr>
</tbody>
</table>

* denotes significance at the 5 %-level.
Appendix D

The analyses for portfolio B.

### Table D1
The analysis calculated using constant volatility across strikes
A change in the underlying asset is viewed horizontally while a change in the implied ATM-volatility is presented vertically. The figures in each scenario represent the net change in the value of the portfolio from scenario (0, 0) on 2.6 to each respective scenario on 9.6.1999.

<table>
<thead>
<tr>
<th></th>
<th>-30 %</th>
<th>-20 %</th>
<th>-10 %</th>
<th>0 %</th>
<th>3 %</th>
<th>6 %</th>
<th>9 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9%</td>
<td>-107 211</td>
<td>-312 784</td>
<td>-549 780</td>
<td>-811 547</td>
<td>1 092 867</td>
<td>1 389 711</td>
<td>1 698 975</td>
</tr>
<tr>
<td>-6%</td>
<td>418 965</td>
<td>156 768</td>
<td>130 183</td>
<td>436 252</td>
<td>-756 921</td>
<td>-1 088 886</td>
<td>-1 429 695</td>
</tr>
<tr>
<td>-3%</td>
<td>852 255</td>
<td>536 599</td>
<td>203 310</td>
<td>142 799</td>
<td>-498 424</td>
<td>-861 234</td>
<td>-1 229 545</td>
</tr>
<tr>
<td>0%</td>
<td>1 179 730</td>
<td>816 941</td>
<td>444 278</td>
<td>64 667</td>
<td>-319 951</td>
<td>-708 234</td>
<td>-1 099 226</td>
</tr>
<tr>
<td>3%</td>
<td>1 390 113</td>
<td>991 358</td>
<td>588 939</td>
<td>184 068</td>
<td>-222 457</td>
<td>-630 090</td>
<td>-1 038 437</td>
</tr>
<tr>
<td>6%</td>
<td>1 477 785</td>
<td>1 057 089</td>
<td>636 208</td>
<td>215 339</td>
<td>-205 380</td>
<td>-625 843</td>
<td>-1 045 964</td>
</tr>
<tr>
<td>9%</td>
<td>1 442 489</td>
<td>1 015 055</td>
<td>587 618</td>
<td>160 322</td>
<td>-266 729</td>
<td>-693 449</td>
<td>-1 119 769</td>
</tr>
</tbody>
</table>

### Table D2
The analysis calculated using a static smile
A change in the underlying asset is viewed horizontally while a change in the implied ATM-volatility is presented vertically. The figures in each scenario represent the net change in the value of the portfolio from scenario (0, 0) on 2.6 to each respective scenario on 9.6.1999.

<table>
<thead>
<tr>
<th></th>
<th>-30 %</th>
<th>-20 %</th>
<th>-10 %</th>
<th>0 %</th>
<th>3 %</th>
<th>6 %</th>
<th>9 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9%</td>
<td>-296 656</td>
<td>-672 039</td>
<td>-900 072</td>
<td>-1 092 867</td>
<td>-1 412 048</td>
<td>-1 689 001</td>
<td></td>
</tr>
<tr>
<td>-6%</td>
<td>259 171</td>
<td>-472 039</td>
<td>-468 886</td>
<td>-204 338</td>
<td>-1 088 886</td>
<td>-1 429 695</td>
<td></td>
</tr>
<tr>
<td>-3%</td>
<td>723 503</td>
<td>441 747</td>
<td>32 003</td>
<td>1 722 087</td>
<td>-498 424</td>
<td>-861 234</td>
<td>-1 390 478</td>
</tr>
<tr>
<td>0%</td>
<td>1 078 288</td>
<td>407 142</td>
<td>141 587</td>
<td>58 829</td>
<td>-294 920</td>
<td>-708 234</td>
<td>-1 229 545</td>
</tr>
<tr>
<td>3%</td>
<td>1 309 807</td>
<td>570 363</td>
<td>407 142</td>
<td>195 911</td>
<td>-180 427</td>
<td>-630 090</td>
<td>-1 099 226</td>
</tr>
<tr>
<td>6%</td>
<td>1 410 263</td>
<td>629 189</td>
<td>570 363</td>
<td>238 416</td>
<td>-152 308</td>
<td>-625 843</td>
<td>-1 038 437</td>
</tr>
<tr>
<td>9%</td>
<td>1 378 437</td>
<td>584 805</td>
<td>629 189</td>
<td>187 988</td>
<td>-208 679</td>
<td>-605 118</td>
<td>-1 001 262</td>
</tr>
</tbody>
</table>

### Table D3
The analysis calculated using a changing smile (SDSA-model)
A change in the underlying asset is viewed horizontally while a change in the implied ATM-volatility is presented vertically. The figures in each scenario represent the net change in the value of the portfolio from scenario (0, 0) on 2.6 to each respective scenario on 9.6.1999.

<table>
<thead>
<tr>
<th></th>
<th>-30 %</th>
<th>-20 %</th>
<th>-10 %</th>
<th>0 %</th>
<th>3 %</th>
<th>6 %</th>
<th>9 %</th>
</tr>
</thead>
<tbody>
<tr>
<td>-9%</td>
<td>-151 276</td>
<td>-672 039</td>
<td>-900 072</td>
<td>-1 092 867</td>
<td>-1 412 048</td>
<td>-1 689 001</td>
<td></td>
</tr>
<tr>
<td>-6%</td>
<td>409 532</td>
<td>-499 801</td>
<td>-468 886</td>
<td>-498 424</td>
<td>-1 088 886</td>
<td>-1 429 695</td>
<td></td>
</tr>
<tr>
<td>-3%</td>
<td>849 630</td>
<td>368 985</td>
<td>332 847</td>
<td>407 142</td>
<td>-498 424</td>
<td>-861 234</td>
<td>-1 390 478</td>
</tr>
<tr>
<td>0%</td>
<td>1 153 172</td>
<td>436 404</td>
<td>236 985</td>
<td>58 829</td>
<td>-294 920</td>
<td>-708 234</td>
<td>-1 229 545</td>
</tr>
<tr>
<td>3%</td>
<td>1 314 155</td>
<td>520 820</td>
<td>236 985</td>
<td>195 911</td>
<td>-180 427</td>
<td>-630 090</td>
<td>-1 099 226</td>
</tr>
<tr>
<td>6%</td>
<td>1 330 265</td>
<td>490 463</td>
<td>236 985</td>
<td>238 416</td>
<td>-152 308</td>
<td>-625 843</td>
<td>-1 038 437</td>
</tr>
<tr>
<td>9%</td>
<td>1 203 991</td>
<td>349 124</td>
<td>236 985</td>
<td>187 988</td>
<td>-208 679</td>
<td>-605 118</td>
<td>-1 001 262</td>
</tr>
</tbody>
</table>
The results for portfolio B when tested against market scenarios.

### Table E1

#### Realized market scenarios

The deviations of the three different methodologies compared to realized market scenarios for portfolio B. The closing price on the DAX-index (Spot) is used as the underlying asset and the ATM-volatility (Vola) is iterated from observed market prices when estimating the corresponding scenarios.

<table>
<thead>
<tr>
<th>Date</th>
<th>Spot</th>
<th>Vola</th>
<th>Market</th>
<th>SDSA</th>
<th>Dev</th>
<th>Static</th>
<th>Dev</th>
<th>Constant</th>
<th>Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/2/1999</td>
<td>5040.34</td>
<td>27.3</td>
<td>-4 120 000.00</td>
<td>-4 220 522.82</td>
<td>2.38 %</td>
<td>-4 220 522.82</td>
<td>2.38 %</td>
<td>-4 463 207.49</td>
<td>7.69 %</td>
</tr>
<tr>
<td>6/9/1999</td>
<td>5250.63</td>
<td>25.3</td>
<td>-3 776 000.00</td>
<td>-3 807 607.05</td>
<td>0.83 %</td>
<td>-3 716 378.72</td>
<td>1.69 %</td>
<td>-3 954 471.16</td>
<td>4.51 %</td>
</tr>
<tr>
<td>6/16/1999</td>
<td>5382.67</td>
<td>24.5</td>
<td>-3 709 000.00</td>
<td>-3 676 252.80</td>
<td>0.89 %</td>
<td>-3 672 047.46</td>
<td>-1.01 %</td>
<td>-3 749 546.46</td>
<td>1.08 %</td>
</tr>
<tr>
<td>6/23/1999</td>
<td>5399.11</td>
<td>24.0</td>
<td>-3 504 500.00</td>
<td>-3 463 783.50</td>
<td>0.76 %</td>
<td>-3 335 440.39</td>
<td>-1.60 %</td>
<td>-3 403 598.62</td>
<td>-2.54 %</td>
</tr>
</tbody>
</table>

Average 5424.61 0.24 -2 836 232.14 -2 863 143.99 0.74 % -2 860 498.52 0.25 % -2 922 698.61 2.19 %

Stddev 269.61 0.03 723 158.49 764 155.31 2.17 % 765 661.55 5.19 % 802 755.04 5.52 %

Min 5019.69 0.16 -4 120 000.00 -4 220 522.82 -4.18 % -4 220 522.82 -12.21 % -4 463 207.49 -11.09 %