University-firm competition in basic research and university funding policy

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Abstract
We characterize equilibrium investments in basic research by the commercial and university sectors contingent on public funding of the university. We find that firms invest in basic research despite the opportunities for free riding and we present conditions under which firms even have incentives to augment the public funding to the university. We characterize the socially optimal volume of public funding for the university sector. Finally, we compare total investments in a mixed duopoly with those of duopolies composed of two universities as well as two profit-maximizing firms.

1 | INTRODUCTION

It has been estimated that developed countries spend approximately 20% of their R&D investments on basic research (Gersbach, Schneider, & Schneller, 2013). According to a traditional view, basic research is more important for productivity growth than other types of R&D (shown by, e.g., Griliches, 1986). An influential stream of research has therefore, emphasized the structural need for public policies to support investments in basic research. Classical studies such as Nelson (1959) and Arrow (1962) establish how features such as appropriability problems, uncertainty and long delays associated with basic research lead to underinvestments in basic research, thereby justifying a central role for the government to promote such investments. Stephan (1996) provides an extensive survey of the literature on the topic of how competitive markets provide poor incentives for the production of knowledge as this could be a public good that cannot be appropriated. As Rosenberg (1990) insightfully discusses, the possibility of free-riding on scientific knowledge created by other parties raises...
the question: Why do commercial firms invest in basic research at all? This question attracts attention to the interaction between the public and private for-profit, investments in basic research.

In this study, we revisit the question of how to characterize the socially optimal volume of public funding for basic research. The novel feature is that our characterization relies on the strategic interaction between the university sector, operating subject to the principle of scientific commons, and the commercial sector able to enjoy the knowledge spillovers from the university sector. We examine basic research in its role as knowledge serving as an input for subsequent development activities. Formally, we design an analytical model capturing the interaction between the university sector, funded by the government (and possibly via donations or licensing revenues from the commercial sector), and the commercial sector operating with the objective of profit maximization. In the public sector we take into account that the decisions regarding investments are made by universities operating with funding acquired from the government. Further, we also take into account that raising public funds to finance research by the university sector tends to cause costly distortions to the economy.

We initially present the equilibrium investments by the commercial and university sectors in a mixed duopoly contingent on the public funding of the university. In particular, we find that the commercial sector invests in basic research despite the opportunities for free riding and despite the fact that we do not explicitly account for the role of basic research as facilitating subsequent development activities (analyzed in Cohen & Levinthal, 1989). Further, we establish that the firm has an incentive to augment the university’s public funding with a donation if the firm’s profit motive exceeds the public funding to the university.

Next, we characterize the socially optimal volume of the funding to the university sector and we focus, in particular, on how this depends on the following factors: the commercial profit motive for basic research, the social value of the innovation and the costs of research. Interestingly, the socially optimal volume of public university funding exceeds (falls short of) the profit motive when this profit motive is sufficiently weak (strong). We compare the total investment in a mixed duopoly with a public university competing against a for-profit firm with that of a duopoly composed of two universities. We find that the socially optimal funding to a sector with two universities exceeds the socially optimal funding to a mixed duopoly. Further, we show that the total investment in the presence of optimal funding policy under university duopoly exceeds that under mixed duopoly unless the profit is very close to total welfare, that is unless consumer surplus associated with the innovation is very small. Finally, we compare the total investment in a mixed duopoly with that of a duopoly with two profit-maximizing firms. In that respect we find that the investment in basic research with a mixed duopoly cannot exceed that with a for-profit duopoly unless the public funding to the university exceeds the private profit motive.

De Fraja (2016) has earlier characterized optimal public funding for research from two different perspectives: How should research resources be allocated among different institutions differentiated by their reputation and capacity given that these institutions have informational advantages relative to the policymaker? And, how should funding of basic research be balanced against funding of applied research? Ottaviani (2019) examines alternative allocation mechanisms for research funding across different fields of study. Gersbach, Schetter, and Schneider (2015) address the question of how much to invest in basic research from the perspective of long-term economic growth. In contrast to these studies we characterize the volume of optimal public funding of basic research with a perspective focusing on research competition between the university sector and the commercial sector. Within

1We apply a conventional model of R&D competition that can be viewed as a static representation of the classical models of R&D races (e.g., the single stage game in Harris & Vickers, 1987).
such a framework our characterization of optimal policy internalizes the potential crowding-out effects, whereby expanded university research induces a reduction of research by the commercial sector.

Salter and Martin (2001) argue that there is often “considerable mutual interaction between public and private research activities.” The strategic investment interaction between the private and public sectors has earlier been analyzed by focusing on a distribution of roles such that the government concentrates on basic research, whereas the private sector gives priority to applied research. Gersbach, Schneider, and Schneller (2010) and Lacetera (2009) are examples of such studies. With such a distribution of roles imposed, the returns to the firms associated with basic research activities show up as an increased productivity with respect to applied research or development activities. Furthermore, Czarnetzki and Thorwarth (2012) argue that investments to acquire basic research capabilities are essential component for a firm’s success in the high-tech sector to a much higher extent than in low-tech companies. Our model does not impose any asymmetric role distribution on the organizations active in different types of research, and it does not distinguish between research and development. Our approach is motivated by our goal to understand how the structural differences between universities and firms shape the resulting investment equilibrium and on how the volume of public funding to universities can be designed to improve the performance of this equilibrium in research investments.

Our study focuses on universities exclusively in their roles as research institutions, and we limit attention to funding policies in the form of performance-based funding for scientific success. In this respect our analysis is linked to Galasso, Mitchell, and Virag (2018), who develop a model of grand innovation prizes (GIP), which do not preclude the winner from also obtaining patent rights. They demonstrate that patents and prizes are complements for a specified category of innovations, implying that GIPs tend to be preferable to patent races or prizes that require the technologies to be placed in the public domain. Our study does not explore the links to patent policy, because we focus exclusively on basic research. Their study, however, provides a rationale for why we structure the payments from government to university as we do.

Our study proceeds as follows. Section 2 presents the model of competition between the university sector and the commercial sector with a characterization of the investment equilibrium. It also explores the donation incentives of firms as well as the licensing royalties as instruments funded by firms to transfer resources for research to universities. In Section 3 we characterize the socially optimal volume of public funding to the university sector. Section 4 conducts a comparison between the investment in basic research in a mixed duopoly, consisting of a university and firm, and that in a duopoly composed of two universities. Section 5 develops a duopoly model with two profit-maximizing firms and compares the resulting equilibrium research intensities with that of the mixed duopoly. Section 6 presents concluding comments.

2 | INNOVATION COMPETITION BETWEEN THE UNIVERSITY AND THE FIRM

2.1 | The mixed duopoly model

We focus on competition in basic research between a profit-maximizing firm and a university. Conditional on success with respect to basic research—irrespective of whether that success is
a result of the firm’s basic research or of that undertaken by the university—the for-profit firm subsequently continues with development to convert the innovation into a commercial new product or service, generating an expected monopoly profit denoted by $\pi$. Even though we do not formally model the stage whereby the firm invests to develop a successful basic research into a commercial product, we interpret the outcome of these activities to be included in the expected monopoly profit.\(^3\)

The university is an organization governed by its charter and by funding guidelines which require it to use all its available funds toward basic research. In addition, the university is required to disseminate publicly the results of its (successful) research.\(^4\) This means that new knowledge created through the basic research undertaken by the university freely spills over to the commercial sector. Rosenberg (1990) made reference to this feature by saying that “scientific knowledge is “on the shelf,” costlessly available to all comers.”

We focus on the university exclusively in its role as a producer of new scientific knowledge. Thus, we disregard all aspects associated with the university as an important institution for productivity-enhancing education. In our model, a welfare-maximizing policymaker makes the decision regarding the volume of funding for the university. We assume that the funding is performance-based (as per Galasso et al., 2018). More precisely, the policymaker reimburses the university $V$ for a successful basic innovation, which satisfies the criterion of scientific novelty.\(^5\) The novelty criterion means that $V$ is not paid to the university unless it is the only player to achieve success. The policymaker can only detect whether the research of the university has been successful in a way meeting the novelty criterion, but the policymaker does not interfere in matters regarding the organization and implementation of research. In other words, the conduct of research is delegated to the university subject to the monetary incentives designed by the policymaker. Further, we assume that the social value of the scientific innovation is $W$. Of course, the policy instrument $V$ is restricted by $W$ so that $V < W$.

The university and the firm simultaneously decide how much to invest in basic research. Basic research is inherently uncertain. Following the seminal models of R&D competition (Harris & Vickers, 1985, 1987, as well as Grossman & Shapiro, 1987) we assume that the firm and university determine success probabilities, denoted by $r_F$ and $r_U$ ($0 \leq r_F \leq 1$, $0 \leq r_U \leq 1$), respectively. Formally, the firm’s investment in basic research is the solution to the optimization problem

$$\max_{r_F, r_U} \phi(r_F, r_U) = r_F \pi + (1 - r_F) r_U \pi - \frac{c}{2} (r_F)^2.$$  \hspace{1cm} (1)

The second term in this objective function captures the disclosure requirement imposed on the university. It means that the firm can benefit from a successful innovation achieved by the

\(^3\)We discuss the reasons for this formalization as well as the limitations thereof in Section 6.

\(^4\)Many government or nonprofit foundation financiers of basic research require that the results of the research in question are published or otherwise publicly disseminated, and that primary data are shared with the research community. For example, the National Science Foundation, the EU (the so-called “S-Plan”) and the Gates Foundation impose this type of requirement.

\(^5\)In this model we disregard research funding of an ex ante lump-sum character, as we want to emphasize the differences in research incentives between universities and firms with an emphasis on the novelty criterion characterizing scientific innovations. This feature is present in the university funding system of some countries, for example, Finland and the United Kingdom, where the past research performance of a university determines some components of its funding.
university, as the university is required to make scientific research results publicly available. This feature is a key distinction between the university and the firm as a producer of basic research. The third term is the cost to the firm of conducting research. It is strictly increasing and convex as a function of the success probability.

Formally, in this study we predominantly focus on duopolistic sector-level competition between the university sector and the commercial sector. Of course, this perspective might underestimate the knowledge transfer generated by an innovation in the university sector. In a wider perspective knowledge created by the university might also intensify competition within the corporate sector if this knowledge is widely disseminated for free among competing firms. This aspect is incorporated in the way we cover for-profit competition in Section 5.

Given the monetary incentive $V$ for research success, the university determines its investment to solve the optimization problem

$$\max_U \psi(r_U, r_F) = r_U(1 - r_F)V - \frac{c}{2}(r_U)^2.$$  \hspace{1cm} (2)

The university's objective function captures the idea that the investment in basic research is determined to maximize the surplus to the university.\(^6\) Further, the objective function (2) incorporates the novelty criterion. More precisely, the first term in (2) captures the feature that the monetary reward $V$ does not accumulate to the university unless it is the only player to achieve research success.

For reasons of transparency, we focus on the configuration where the firm and the university have equal costs. This means that the cost parameter $c$ is common across the objective functions (1) and (2).\(^7\) We focus on symmetric costs it so to highlight the particular reward mechanisms associated with the different types of research institutions. We make the following assumption regarding the parameters of the model.

**Assumption 1.**

(a) The social return of a successful innovation exceeds the private one. Formally, $W > \pi \geq 0$.

(b) The innovation project is so challenging that it is excessively costly to achieve certain success. Formally, $c > 2W$.

Assumption 1(a) is conventional and it simply means that there is consumer surplus in addition to the benefits flowing to the producing sector. Assumption 1(b) captures that uncertainty is a generic feature of significant research challenges. It means that even resources at the level of twice the social value of the innovation are insufficient to guarantee success. Assumption 1(b) together with the requirement that $V < W$, introduced earlier, imply that $c > 2V$.

The reaction functions associated with (1) and (2) are given by $r_F = \frac{V - r_U}{c}(1 - r_U)$ and $r_U = \frac{V}{c}(1 - r_F)$, respectively. These reaction functions mean that the investments in research

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\(^6\)Universities are often multidivision organizations. Inevitably, there are some divisions which generate a surplus, whereas others incur deficits. University management often tend to encourage surplus-generating divisions to cross-subsidize loss-making activities to increase activity while achieving a balanced budget.

\(^7\)Our model can be extended to asymmetric costs in a straightforward way. However, this study does not primarily focus on aspects associated with the implications of cost asymmetries.
are strategic substitutes. Solving the system of equations determined by the reaction functions we find that the equilibrium with respect to the investments in basic research is given by

\[ r_F^* = \frac{(c - V)\pi}{c^2 - \pi V} \] (3)

and

\[ r_U^* = \frac{(c - \pi)V}{c^2 - \pi V} \] (4)

respectively. The numerators and denominators in both (3) and (4) are positive based on the parameter restrictions we have imposed in Assumption 1 together with \( V < W \).

In light of Assumption 1 we can conclude that not only the university, but also the firm will invest in basic research. The feature that the private firm does invest in basic research (\( r_F^* > 0 \)) despite the option of free-riding on the basic research conducted by the university is very interesting. No matter how much the university invests, with the convex costs of research there is always a private value for the firm of having a “second kick at the can.” This seems to be a general feature whenever there are decreasing returns to the research investments.\(^8\)

Comparing (3) with (4) we can also draw another interesting conclusion. Through the commitment of resources to the university sector the policymaker can determine whether the university sector plays a bigger role than the commercial sector as far as basic research in concerned. Formally, based on a comparison of (3) with (4) we see that \( r_U^* > r_F^* \) if and only if \( V > \pi \). In other words, by deciding on the budget for the university sector the policymaker in effect determines the distribution of market shares between the university and the commercial sectors as far as basic research in concerned.

We can summarize our findings according to the following result.

**Result 1.** Competition regarding basic research between a university and a firm yields an equilibrium given by (3) and (4). In particular, the commercially oriented firm invests in basic research (\( r_F^* > 0 \)). Further, by its control of the resources allocated to the university (\( V \)) the policymaker determines whether the university is dominant in the market for basic research. Formally, \( r_U^* > r_F^* \) if and only if \( V > \pi \).

Result 1 is very interesting in light of Rosenberg’s (1990) general arguments for why firms (may or may not) conduct basic research. Our model could be viewed as an analytical formalization of some of these arguments. In particular, our model highlights that a commercial firm engaged in competition with a university has strategic incentives to invest in basic research whenever there are profit opportunities in the product market despite the presence of the option of free-riding from successful university research and the absence of research as a mechanism to enhance productivity in development activities. However, through the public university funding (\( V \)) the policymakers may cause crowding-out effects whereby the firm’s research investment is reduced or even eliminated as a consequence of increased public university funding. We address this issue in Section 3.

\(^8\)Under these circumstances it is also socially optimal with some degree of duplicated research effort.
2.2 | Are there incentives for donations?

We now extend the mixed duopoly model in the previous section by opening the opportunity for the firm to contribute to the funding of the university through a donation. The donation extends the resources for independent research by the university without giving the firm any influence to interfere in the research decisions of the university in any way whatsoever. Further, the presence of a donation does not affect the system regarding novelty and disclosure requirements subject to which the university operates.\(^9\)

Formally, the firm’s combination of investment in basic research and donation to the university is the solution to the optimization problem

\[
\max_{r_F, n_{UF}, r_U} \phi (r_F, n_{UF}, r_U) = r_F \pi + (1 - r_F)(n_U + n_{UF})\pi - \frac{c}{2}(r_F^2 + (n_U + n_{UF})^2 - n_U^2),
\]

where \(n_{UF}\) measures how the donation enhances the success probability of the research undertaken by the university. The cost to the firm of a donation which raises the university’s success probability from \(n_U\) to \(n_U + n_{UF}\) is \(\frac{c}{2}((n_U + n_{UF})^2 - n_U^2)\). Formally, this captures the feature that the donation is pooled with the university’s own resources to fund the operations of one laboratory rather than to run a separate laboratory based on the donation.

Facing the monetary incentive \(V\), the university now determines its investment to solve the optimization problem

\[
\max_{n_U} \psi (n_U, r_F, n_{UF}) = (n_U + n_{UF})(1 - r_F)V - \frac{c}{2}(n_U)^2.
\]

The objective function (6) incorporates the novelty criterion precisely like (2). From the university’s perspective the donation increases its probability of success without any cost. In Appendix A we derive the following equilibrium combination of investments in basic research under the condition that \(\pi \geq V\):

\[
r_F^{**} = \frac{\pi}{c + \pi}, \quad r_{UF}^{**} = \frac{\pi - V}{c + \pi}, \quad r_U^{**} = \frac{V}{c + \pi}.
\]

Clearly, the magnitude of \(\pi\) relative to \(V\) is decisively important for the incentives of the firm to support the university with a donation. If \(V\) exceeds \(\pi\) it is optimal for the firm to execute no donation, and under such circumstances the equilibrium investments in basic research are characterized by (3) and (4). Otherwise it is in the firm’s interest to direct resources to support the university’s research with donations. We summarize this finding with the following result.

**Result 2.** If the firm’s profit motive exceeds the monetary reward offered by the government to the university (\(\pi > V\)) the firm has an incentive to augment the university’s public funding with a donation and the equilibrium configuration of research

\(^9\)In the presence of licensing a university could also offer a royalty-free license of research results to a firm sponsoring the associated research project. Under such circumstances there could be a tradeoff for the university between accepting ex ante funding, analyzed in this section, versus ex post licensing revenues, analyzed in the next section.
investments is then given by (7). Otherwise the firm undertakes no donation and under such circumstances the investment equilibrium is given by (3) and (4).

The firm benefits equally well from successful research by the university as from its own successful research due to the disclosure requirement imposed on the university. This feature combined with the decreasing returns to the research investment (the convex cost function) means that the firm has an incentive to equalize the success probabilities across its own laboratory and that of the university. This insight is formally captured by (7), because it holds true that $r_F^{**} = r_U^{**} + r_U^{**} = \frac{\pi}{c + \pi}$ whenever $\pi \geq V$. However, in the opposite case with $\pi < V$ the firm cannot afford to achieve this equalization of the success probabilities. In that case the firm actually has no incentive to make any donation. To modify the success probabilities from the equilibrium configuration given by (3) and (4). Generally, (7) shows that $V$ decreases the donations to the university.

Our analysis is conducted under the assumption that the university sector is mandated to disseminate publicly the results of successful research. If instead there were imperfect spillovers from the university to the firm, such imperfections would increase the firm’s incentive to invest in basic research and decrease its incentives to donate.

It should be noted that our analysis above assumes that the firm transforming a successful basic innovation into a product or service will enjoy monopoly rents regardless of whether the firm or the university succeeds in basic research. With the public disclosure requirement imposed on university research, it could be that a university success induces more intense product market competition, which would diminish the incentive of the firm to donate. In such a scenario, however, the social value of successful university research would be likely to increase, which, in turn, would induce the policymaker to increase $V$.

### 2.3 University licensing of research results

In this section, we examine the effects of allowing the university sector to license the results of its successful research as opposed to making these results available to firms for free (as examined in the benchmark model in Section 2.1). The Bayh-Dole Act of 1980 allowed the U.S. universities to engage in licensing with the results from federally funded research. We will demonstrate that there is a theoretical basis for the Bayh-Dole Act to stimulate research—as its proponents originally argued. However, we find that increased licensing rates may either increase or decrease the research investments of the for-profit sector.

Modifying the benchmark model of a mixed duopoly presented in Section 2.1 to incorporate university licensing, the payoff for the firm changes to

$$\phi_L(n_L, n_{UL}) = n_L\pi + (1 - n_L)n_{UL}(1 - \alpha)\pi - \frac{c}{2}(n_L)^2,$$

where the subscript $L$ refers to the case of university licensing and where $n_{UL}$ and $n_L$ refers to the research undertaken by the university and the firm, respectively. Further, the parameter $\alpha$ captures the licensing rate ($0 \leq \alpha \leq 1$). In the presence of licensing the objective function of the university changes to

$$\psi_L(n_L, n_{UL}) = n_{UL}(1 - n_L)(V + \alpha\pi) - \frac{c}{2}(n_{UL})^2.$$


Differentiating (8) and (9), respectively, and solving the resulting system of equations defined by the first-order conditions yields the equilibrium investments,

\[ r^*_{L} = \frac{\pi (c - (1 - \alpha)(V + \pi \alpha))}{c^2 - \pi (1 - \alpha)(V + \pi \alpha)}, \quad r^*_{UL} = \frac{(c - \pi)(V + \pi \alpha)}{c^2 - \pi (1 - \alpha)(V + \pi \alpha)}. \]  

(10)

In light of this investment equilibrium we can formulate the following result on the effect of licensing rates on investments (the proof is found in Appendix B).

**Result 3.** A higher licensing rate always increases the university’s research investment, whereas it increases the for-profit’s research investment only if \( V > \pi (1 - 2\alpha) \). Furthermore, total research investment increases with higher licensing rates.

Licensing revenues clearly increase the value of research to the university thereby providing incentives for the university to expand its research activities. The effects of increased licensing rates on the research investment of the firm are much less obvious. On the one hand, a higher licensing rate makes it more costly to enjoy the spillovers from successful university research, it tends to stimulate the firm’s research investment. This happens for sufficiently low licensing rates (\( 2\alpha < 1 \)). Such low licensing rates could be the outcome under circumstances where the firm has sufficiently strong bargaining power relative to the university. On the other hand, with sufficiently strong bargaining power of the university, a marginal increase in the licensing rate stimulates university research to such an extent that the firm finds it optimal to cut back on its own investment. In other words, with a sufficiently strong bargaining power of the university, the expansion of university research induced by a marginal increase in the licensing fee causes a crowding out effect on the firm’s research.

The effects of the Bayh-Dole Act have been empirically studied (see, e.g., Jensen & Thursby, 2001 and Thursby & Thursby, 2002). These studies tend to generally find that universities increased their licensing activity, and that they increased reliance of industry on federally funded university research. These findings are consistent with our model.

Overall, donations and licensing are alternative instruments to stimulate university research. We next compare these instruments. Comparing the total research investments with donations (Section 2.2) to total investments in the presence of licensing, straightforward calculations reveal the following result (the proof can be found in Appendix C).

**Result 4.** The total research investments with university licensing exceeds those with a system of donations if \( V > g(\pi, \alpha) = \frac{c(c - \pi)}{c^2 - (1 - \alpha)\pi c - c\pi^2 - \alpha \pi^2 - \pi} \).

According to Result 4 the public funding has to exceed a threshold, which depends positively on the firm’s investment incentives (\( \pi \)), for university licensing to generate higher research investments than the system with donations. In particular, as shown in Appendix C, the system with university licensing always yields higher investments than that with donations if \( V > \pi \) independently of \( \alpha \). By Result 3 we know that \( \frac{\partial g(\pi, \alpha)}{\partial \alpha} < 0 \), meaning that a higher value of \( \alpha \) lowers the threshold of \( V \), above which licensing generates higher research investments than donations. In particular, for \( \alpha = 1 \) the investments with university licensing always exceed those with donations even for arbitrarily small levels of public funding. This feature seems intuitive because a very high licensing rate shifts the research incentives associated with profits from the firm to the university.
It should be observed that the timing of the dispersal of the resources to the university is different under donations than under the licensing case. Donations accumulate ex ante, before the research activity, whereas licensing revenues reach the university ex post and only when the innovation is successfully commercialized. Thus, from the perspective of the university it would be riskier to rely on the cashflow from the licensing revenue stream.

Our model of licensing could be extended in a number of dimensions, in particular to oligopoly markets to facilitate a distinction between exclusive licenses and nonexclusive multiple licenses, or to mechanisms for the endogenous determination of licensing terms. The performance of a system with licensing could also be compared with the outcome associated with various ways of organizing research cooperation between the firm and university in line the approach with endogenous spillovers developed by Cabon-Dhersin and Lahmandi-Ayed (2011). In the next section we turn to the problem of the policymaker in charge of deciding the volume of public funding ($V$).

### 3 | OPTIMAL FUNDING POLICY

The policymaker determines the funding to the university and its volume contingent on research success to maximize social welfare. This volume of funding to universities, $V$, is the instrument whereby the policymaker attempts to affect the research investments of the universities. Ultimately, the policymaker cares about the benefits to consumers or citizens of a scientific innovation irrespectively of whether the innovation has its roots in the activities of the university or the firm. Further, the volume of funding to the university is determined to take into account not only the actual costs to the public purse, but also the distortion associated with public funds. Our formalized analysis of the optimal funding volume focuses on the benchmark model with no donations and no licensing as presented in Section 2.1.

The following objective function formally captures these considerations:

$$\max_V \Gamma(V) = \left( r_U^s + r_F^s - r_U^s r_F^s \right) W - r_U^s (1 - r_F^s) \Phi(V). \tag{11}$$

The first term in (11) captures that the social value of a successful innovation is $W$ independently of whether the innovation is conducted in the university sector or in the commercial sector. The costs of raising public funds to finance successful and novel research by the university is captured by the strictly increasing and convex function $\Phi(V)$. We will focus on a linear representation $\Phi(V) = \gamma V$ such that the parameter $\gamma > 1$ measures the distortion cost associated with public finance.

An extensive literature in public economics has presented assessments of the marginal costs of public funds. For example, the classical study by Browning (1976) estimates that the marginal costs of funds raised by taxes on labor income in the United States clearly exceeds one (in the range $1.09$–$1.16$ per dollar of tax revenue). On the theoretical side, Allgood and Snow (1998) analytically derive characterizations of the marginal welfare costs associated with wage taxes imposed on a heterogeneous population. In his model focusing on the gains from redistribution induced by
taxation, Sandmo (1998) emphasizes that each tax instrument in a comprehensive tax system is associated with different marginal costs of funds. In this study, we make no specific assumption regarding the size of the marginal costs of public funds except that it exceeds one ($\gamma > 1$).

The objective function $\Gamma(V)$ is concave if $c$ is sufficiently large. As we show in Appendix D, Assumption 1 is a sufficient condition to guarantee that $\Gamma(V)$ is concave. Under such circumstances the sufficient and necessary first-order condition associated with the optimization problem (11) is given by

$$\begin{vmatrix} \frac{\partial r_U^*}{\partial V} (1 - r_F^*) - \frac{\partial r_F^*}{\partial V} r_U^* \end{vmatrix} (W - \gamma V) - \gamma r_U^* (1 - r_F^*) + \frac{\partial r_F^*}{\partial V} W = 0.$$ 

The optimality condition (12) incorporates interesting effects operating in different directions. In particular, in light of (3) and (4) an increase in $V$ induces an increase in the research investment of the university $\frac{\partial r_U^*}{\partial V} (1 - r_F^*) = \frac{(c - \pi) c^2}{|c^2 - \pi V|^2} > 0$, whereas it induces a decrease in the research investment of the firm $\frac{\partial r_F^*}{\partial V} = -\frac{(c - \pi) c \pi}{|c^2 - \pi V|^2} < 0$. These properties mean that public funding of the university not only stimulates research conducted by the university, but it also has a crowding-out effect on research conducted by the firm. The condition (12) incorporates both of these effects.

Assume that $c > 2W$ so that we can guarantee that the first-order order condition (12) solves the maximization problem (11). Under this condition the socially optimal volume of funding for the university is given by

$$ V = \hat{V} = \frac{cW (c - 2\pi)}{2\gamma c (c - \pi) - \pi W}. \quad (13) $$

In the following result we characterize the socially optimal funding policy.

**Result 5.** The socially optimal volume of funding for the university is given by (13) and it satisfies the properties (a)–(b) below:

(a) $0 < \hat{V} < W$.
(b) $\hat{V} > (\leq) \pi$ if and only if $\pi < (\geq) \hat{\pi} = \frac{W}{2c\gamma + W}c$.

For the formal proof of Result 5 we refer to Appendix D. Result 5(a) means that the socially optimal volume of funding is feasible. Result 5(b) provides a comparison between the socially optimal volume of university funding and the strength of the firm’s profit motive. According to Result 3(b) the socially optimal volume of university funding exceeds the firm’s profit as long as this profit lies below the threshold $\hat{\pi} = \frac{W}{2c\gamma + W}c$. Thus, the limitation of our analysis to the case with no donations translates into an assumption that the profit lies below this threshold. Conversely, the socially optimal funding to the university falls short of the firm’s profit if this profit exceeds the threshold in question. In particular, it holds true that $\hat{V} = \pi$ precisely when $\pi = \hat{\pi}$.

From the perspective of social welfare it does not matter whether the university or the firm is successful with respect to basic research. What matters is that society can benefit from an innovation in basic innovation, and that this innovation is translated into products and services available to consumers. However, when designing the program of university funding the policymaker anticipates the consequences not only for the research conducted by the university sector, but also the implications for the research conducted by the commercial sector. In
particular, the policymaker internalizes the crowding-out effects of university research on commercial research. As (3) shows, the firm’s investment in research is significantly determined by the profit motive. The commercial investments in basic research will be low when the profit opportunities are weak. Under precisely those circumstances there are strong social incentives in favor of university funding. Conversely, in the presence of strong commercial incentives for research investments the commercial sector will increase its contributions to research and the policymaker can reduce its role as a financier of research. Result 5(b) captures precisely this intuition. Result 5(b) essentially formalizes the idea that there is a threshold with respect to the profit motive such that the socially optimal volume of university funding exceeds (falls short of) the firm’s achievable profit if and only if that profit falls short of (exceeds) that threshold. The restriction of our analysis of the optimal funding volume to the case with no donations is justified only when the socially optimal volume of university funding exceeds the firm’s achievable profit.

In the next result we characterize the socially optimal volume of university funding (13) by exploring the comparative statics properties with respect to the commercial profit motive, the social value of the innovation and the costs of research. The formal proof is presented in Appendix E.

Result 6. The socially optimal funding of universities satisfies the following comparative statics properties:

(a) The socially optimal funding is reduced in response to a stronger commercial profit motive. Formally, $\frac{\partial V}{\partial \pi} < 0$.

(b) The socially optimal funding is expanded in response to a higher social value of the innovation. Formally, $\frac{\partial V}{\partial W} > 0$.

(c) The socially optimal funding is expanded in response to higher costs of research. Formally, $\frac{\partial V}{\partial c} > 0$.

Result 6(a) formally captures the idea that stronger commercial profit incentives reduce the need for public funding directed to the university sector. The reason is that stronger commercial profit incentives will induce the commercial sector to expand its research investments. Result 6(a) implies a very interesting structural relationship between the intensity of competition in product markets and optimal university funding. As intensified competition reduces the achievable commercial profits it calls for an expansion of the socially optimal volume of university funding. Consequently, competition policy successful in inducing intensified competition also calls for adjustments, more precisely expansions, of university funding.\(^{11}\)

Result 6(b) formalizes the natural feature that the socially optimal volume of university funding increases as a function of the social value of the innovation. Finally, Result 6(c) means that more challenging, and thereby more costly, innovation calls for a higher volume of university funding. Intuitively, one source behind this result is that higher innovation costs reduce the investments by the commercial sector, analogous to Result 6(a), and that, in turn, induces an expansion of the socially optimal university funding.

\(^{11}\)It should be emphasized that so far our model has focused on duopoly competition between the university sector and the commercial sector. The model has not formally incorporated intra-industry competition in the product market associated with the innovation. In that respect our reference to intensity of competition has been incomplete. In the next section we also incorporate the dimension of intra-industry competition in the commercial sector, and therefore, the objective function of the firm will be modified accordingly.
4 | THE PERFORMANCE OF PUBLICLY FUNDED BASIC RESEARCH: MIXED VS UNIVERSITY DUOPOLY

In this section, we compare the investments in basic research undertaken in a mixed duopoly with those of a duopoly consisting of two publicly funded universities. We undertake this comparison in recognition of the fact that the industry configuration affects the socially optimal funding of research conducted in universities. Initially, we characterize the optimal funding policy with an industry consisting of two publicly funded universities, and we compare this volume of funding with that of mixed duopoly. Subsequently, we explore the relationship between the total research investments under the two industry configurations in the presence of the optimal funding policies.

4.1 | Optimal funding policy with a university duopoly

Consider a research sector consisting to two publicly funded universities, A and B. These universities operate subject to the disclosure and novelty rules described in the previous sections. Facing a funding policy characterized by \( V \) the universities make their research investments to solve the following optimization problems:

\[
\max_{r_A^U} \psi_A^U(r_A, r_B) = r_A(1 - r_B)V - \frac{c}{2}(r_A)^2, \tag{14}
\]

\[
\max_{r_B^U} \psi_B^U(r_B, r_A) = r_B(1 - r_A)V - \frac{c}{2}(r_B)^2. \tag{15}
\]

Straightforward solution of these optimization problems establishes that the symmetric equilibrium investment is given by

\[
r_A^U = r_B^U = \frac{V}{c + V}, \tag{16}
\]

where the superscript \( U \) denotes the equilibrium research in the university duopoly. With a university duopoly the policymaker designs the funding policy to maximize social welfare according to

\[
\max_V \Gamma(V) = \left( r_A^U + r_B^U - r_A^U r_B^U \right) W - \left( r_A^U (1 - r_B^U) + r_B^U (1 - r_A^U) \right) \gamma V. \tag{17}
\]

According to (17), the policymaker designs its funding policy in a way which internalizes that the universities respond in line with the investment equilibrium (16). In (17) the first term captures the expected value of the benefit to society of the innovation. This benefit is realized with the probability that at least one university is successful \( r_A^U + r_B^U - r_A^U r_B^U \). The second term measures the social funding cost, inclusive of the distortion created by raising public funds, of the bonus for successful research. In light of the novelty criterion this bonus is paid out to only one university. In particular, the objective function (17) incorporates the possibility that society can benefit from a successful innovation for which neither university can claim novelty.
In Appendix F we calculate the socially optimal funding volume and we compare it with that of a mixed duopoly as given by (13). From Appendix F we can see that the socially optimal funding volume is given by

\[ V = \hat{V}^{U} = \frac{W}{2\gamma}, \]  

(18)

implying that \( \hat{V} < \hat{V}^{U} \). Consequently, we can formulate the following conclusion.

**Result 7.** The socially optimal funding volume with a university duopoly is given by \( V = \hat{V}^{U} = \frac{W}{2\gamma} \) and this volume exceeds the optimal funding for a mixed duopoly.

It can be shown that the total research investments and success probabilities increase as a function of public university funding in the mixed duopoly as well as in the university duopoly. Intuitively, the research-enhancing effect of public funding is stronger in a university duopoly because in the university duopoly there is very intense rivalry between the universities to win the research race and overcome the novelty threshold. Thus, \( V \) increases the research intensity of both research suppliers. In contrast, as can be seen from (3), in the mixed duopoly the private firm decreases its investment in response to an increase in \( V \) as a reflection of the fact that with higher \( V \) the private firm can benefit from stronger free-riding effects.

### 4.2 Total research investments: Mixed vs university duopoly

In this section, we will formally compare the total research investments in basic research under mixed duopoly with that under university duopoly. We will conduct this comparison taking into account that the optimal public funding volume under university duopoly exceeds that under mixed duopoly, as shown in Result 7.

We let \( R^{U}(V) = \frac{2V}{c + V} \) denote the sum of the success probabilities in the configuration with a university duopoly. Clearly, \( R^{U}(V) \) is strictly increasing as a function of \( V \) for any \( V \). Further, with \( R(V) = \frac{c(V + \pi) - 2\pi V}{c^2 - \pi^2} \) denoting the sum of the success probabilities in the mixed duopoly we find that \( R(\hat{V}) > R^{U}(\hat{V}) \) if and only if \( \hat{V} > \pi \). According to Result 5(b) this applies for \( \pi < \hat{\pi} \).

If \( \pi > \hat{\pi} \) the firm has incentives to support the university with a donation. By (7) the sum of success probabilities is given by \( r^{*}_{F} + r^{**}_{UP} + r^{**}_{U} = \frac{\pi}{c + \pi} \). Under such circumstances it holds true that \( R^{U}(\hat{V}_{U}) = \frac{2\hat{V}_{U}}{c + \hat{V}_{U}} = \frac{2W}{2\pi + W} > \frac{\pi}{c + \pi} \). The last inequality is valid if \( W > \gamma\pi \). This means that \( W > \gamma\pi \) is a sufficient condition for the sum of success probabilities under university duopoly to exceed that under mixed duopoly also when \( \pi > \hat{\pi} \).

As there is a strictly monotonic relationship between the sum of the success probabilities in a duopolistic industry and the associated total investment in the industry we can combine the two properties established above to draw the following conclusion.

---

12 Analytically, \( \frac{\partial R^{U}}{\partial V} = \frac{2\pi}{(c + V)^2} \) and \( \frac{\partial R^{MIX}}{\partial V} = \frac{c(e - \pi)^2}{(c^2 - \pi^2)^2} \), where \( R^{U} = r^{U}_{A} + r^{U}_{B} \) and \( R^{MIX} = r^{F}_{F} + r^{*}_{U} \).
Result 8. The total investment under university duopoly exceed that under mixed duopoly whenever $W > \gamma \pi$.

The condition $W > \gamma \pi$ captures the idea the innovation yields a sufficient amount of consumer surplus on top of the profit to the commercial sector. In light of realistic estimates of $\gamma$, this imposes very modest requirements. In fact, if the $W > \gamma \pi$ condition were violated the incentives of the policymaker to support the university would be only marginally stronger than the profit incentives, and under such circumstances the basic research investments conducted by the university duopoly would no longer exceed those to the mixed duopoly.

5 | THE PERFORMANCE OF PUBLICLY FUNDED BASIC RESEARCH: MIXED VS FOR-PROFIT DUOPOLY

How does a mixed duopoly with a public university competing against a for-profit firm perform compared with a duopoly composed of two profit-maximizing firms? In this section, we will characterize the potential benefits of publicly funded basic research by comparing the total research investments conducted by a mixed duopoly with those of two competing firms. For that reason we initially formulate a symmetric for-profit duopoly model to serve as a benchmark for the comparison.

Consider two identical profit-maximizing firms, $A$ and $B$, belonging to the same industry. These firms are engaged in competition with respect to their investments in basic research. The firms anticipate that the effects on profits of successful research depend on whether the rival succeeds or not. Formally, the two firms determine the research investments to solve the optimization problems

$$\max_i \phi(r_i, r_j) = r_i(1 - r_j)\pi + r_i r_j \lambda \pi - \frac{c}{2}(r_i)^2, \quad i, j \in \{A, B\}, \quad i \neq j,$$

where, as before, $\pi$ denotes the monopoly profit achievable if the firm is the only one to succeed in research and $\lambda, \lambda \in [0, \frac{1}{2}]$, is a measure of the intensity of competition in the relevant product market. The case $\lambda = \frac{1}{2}$ captures a configuration where the firms collude, whereas $\lambda = 0$ means so that the intra-industry product market competition is so intense that it erodes profits completely.

The reaction function for firm $i$ ($i = A, B$) associated with (19) is given by $r_i = \frac{\pi}{c + (1 - \lambda)\pi}$. Solving the system of equations determined by the reaction functions for each firm we find the for-profit investment equilibrium to be given by

$$r_i^* = \frac{\pi}{c + (1 - \lambda)\pi}, \quad i = A, B.$$  (20)

In particular, from (20) we can infer that intensified intra-industry competition (lower $\lambda$) reduces the research investments.

Based on (3), (4), and (20) we can compare total research investments in the mixed duopoly with a public university with that in a duopoly composed of two profit-maximizing firms.
Substitution and simplification shows that the mixed duopoly will invest more in basic research than the for-profit duopoly if

\[
\frac{c(\pi + V) - 2\pi V}{c^2 - \pi V} > \frac{2\pi}{c + (1 - \lambda)^2}. \tag{21}
\]

At first sight this comparison might seem very simple. On a general level, and independently of how the public funding of the university is determined, we can draw an interesting conclusion in the following result.

**Result 9.** The investment in basic research with a mixed duopoly cannot exceed that with a for-profit duopoly unless the public funding to the university sufficiently exceeds the private profit motive \((V > \pi)\).

For the proof of Result 9 we refer to Appendix G. It should be emphasized that Result 9 formulates a necessary condition for the mixed duopoly to invest more in basic research than the private duopoly and this holds true independently of the intensity of intra-industry competition \((\lambda, \lambda \in [0, \frac{1}{2}])\). We can intuitively understand Result 9 in light of the feature that university research has a crowding-out effect on private research, as emphasized in the previous section. To expand total research in the mixed duopoly to exceed that in a for-profit duopoly it therefore, seems very natural that the public funding of universities must be sufficiently significant.

As we show in Appendix G, the LHS of (21) is strictly increasing in \(V\), whereas the RHS is independent of \(V\). Further, the RHS of (21) is strictly increasing in \(\lambda\). This means that intensified intra-industry competition (lower \(\lambda\)) relaxes the requirement on \(V\) for the mixed duopoly to invest more in basic research than the for-profit duopoly.

In the presence of socially optimal university funding the condition (21) is translated into

\[
\frac{c(\pi + \hat{V}(\pi)) - 2\pi \hat{V}(\pi)}{c^2 - \pi \hat{V}(\pi)} > \frac{2\pi}{c + (1 - \lambda)^2},
\]

where \(\hat{V}(\pi)\) is given in (13) and with further characterizations reported in Results 5 and 6. Substitution of the socially optimal policy into (3) and (4) makes it possible to formulate the following result.

**Result 10.** In the presence of the socially optimal volume of university funding, the mixed duopoly invests more (less) in basic research than the for-profit duopoly if \(\gamma < (>) \hat{\gamma}\), where \(\gamma \hat{=} \frac{W(c^2 - (2 + \lambda)\pi c + \pi(1 + 3\lambda))}{2\pi c (c - (1 - \lambda)^2)}\) is strictly increasing as a function of the intensity of product market competition.

For the formal proof of Result 10 we refer to Appendix H.

Result 10 captures the intuitive idea that the mixed duopoly invests more in basic research than the for-profit duopoly precisely when the costs of raising public funds for university research are not excessively high, more precisely below the threshold \(\hat{\gamma}\) defined in Result 10. It is interesting to observe that this threshold \(\hat{\gamma}\) depends on the intensity of intra-industry competition \(\lambda\). More precisely, intensified intra-industry competition (lower \(\lambda\)) raises the distortion cost threshold, above which a for-profit duopoly invests more in research than a mixed duopoly. A
higher threshold means that intensified product market competition makes it more likely that the mixed duopoly would outperform the for-profit duopoly with respect to the research investments. Intensified product market competition reduces the incentive for the for-profit duopoly to invest in research, which, in turn, reduces the \( V \) at which the mixed duopoly's research would match that of the for-profit duopoly. The public funding could then reach such a reduced \( V \) for an extended spectrum of distortion factors. For example, for \( \lambda = 0 \) this threshold is simply \( \frac{W(c - \pi)}{2c\pi} \).

At the other extreme, with \( \lambda = \frac{1}{2} \) this threshold is \( \hat{V} = \frac{W(2c^2 - 5\pi c + 5\pi^2)}{2c^2(2c - \pi)} \).

The socially optimal public funding of the university depends on the distortion of raising public funds, captured by the parameter \( \gamma \). Further, by Result 9, the mixed duopoly invests more than the for-profit duopoly when \( V > \pi \). A sufficiently small distortion of raising public funds makes it more likely that the university invests more than the private firm.

The socially optimal \( V \) depends on the social value of the innovation (\( W \)) and on the achievable private profit (\( \pi \)), and, in particular, on the ratio \( \frac{W}{\pi} \). It may happen that the for-profit duopoly invests more in basic research than the mixed duopoly if the ratio \( \frac{W}{\pi} \) is sufficiently small. We formalize this conclusion in the following Corollary, where we find that it is not always the case that the equilibrium investment in a basic research in a mixed duopoly exceeds that of a for-profit duopoly.

**Corollary 1.** If \( W \leq 2\pi \) the for-profit duopoly has higher equilibrium research investments than the mixed duopoly.

We refer to Appendix I for the formal proof of Corollary 1.

The condition \( W \leq 2\pi \) does not specify unreasonable parameter configurations. For example, the condition holds true if we focus on innovation for a new product, for which there is linear inverse demand, and if we measure the social value of the innovation by total surplus defined as the sum of industry profit and consumer surplus. Generally, sufficiently soft product market competition (higher profits) combined with sufficiently high distortion costs of public funding induce a for-profit duopoly to implement higher investments in research than in a mixed duopoly with a university supported by socially optimal public funding.

6 | CONCLUDING COMMENTS

This study demonstrates how the structural differences between universities and firms shape the equilibrium investments in basic research. We show that the commercial sector invests in basic research despite the opportunities for free riding and despite the fact that we do not account for the role of basic research as facilitating subsequent development activities. We also characterize the conditions under which the commercial sector would find it optimal to support the university with donations. Alternatively, the university can extend its research investment through licensing royalties from the firm and we find that this possibility increases the total research investment. Subsequently, we explore how the volume of public funding to universities affects the performance of the investment equilibrium. Most importantly, we characterize the socially optimal volume of the funding to the university sector with a particular emphasis on how this depends on the commercial profit motive for basic research, on the social value of the innovation and on the costs of research. We find that the optimal public funding of the university sector exceeds (falls short of) the profit motive when this profit motive is sufficiently weak (strong).
We compare the total equilibrium investment in a mixed duopoly with those of duopolies composed of two universities as well as two profit-maximizing firms. We find that the socially optimal volume of funding with a university duopoly exceeds that associated with a mixed duopoly. Further, in the presence of optimal funding policy the total investment under university duopoly exceeds that under mixed duopoly if the innovation yields a sufficient amount of consumer surplus on top of the profits. Finally, we demonstrate that the mixed duopoly invests more than the for-profit duopoly as long as the distortion of raising public funds is not excessively high and the welfare created by the innovation is sufficiently high compared with the private profit to the firm. In particular, we find that intensified product market competition raises the distortion cost threshold, below which a mixed duopoly invests more in research than a for-profit duopoly.

In a recent study, Arora, Belenzon, and Patacconi (2018) establish empirically that large firms seem to have decreased the propensity to publish research results in academic journals. Such a finding is not inconsistent with the conclusions of our study. A decreased propensity to publish in academic journals may indicate a greater tendency to keep research results a secret rather than to publish them. For that reason an observation with decreased publication activity on behalf of firms is not equivalent to a decrease in basic research investments.

Our model can be extended in a number of ways. Overall, it is designed to analyze the interaction between the investments in basic research between a university and a commercial firm with a particular emphasis on the feature that the university sector is mandated to disseminate publicly the results of successful research, whereas the firm is not required to reveal such results. It would be important to extend our model to a framework highlighting another central component of the interaction between the university and the firm. Namely, the investments in basic research by the university may affect not only the volume but also the nature of the firm's investments. This feature could be captured within a framework where the firm also decides on how to allocate its investments between basic research and development activities, whereby successful research is transformed into commercial products and services. It seems plausible that expanded investments in basic research by the university could shift the investment incentives of the firm from highly risky basic research to less risky development activities. However, such a shift could also, at least partly, be offset by the likely feature that a firm's investments in basic research could enhance its productivity associated with the development investments whereby successful basic research is commercialized.

Our study could be also be extended to capture how the interaction between the university and commercial sectors is influenced by intra-industry competition between firms in a more detailed way that deepens the analysis reported in Section 5. Finally, future studies could also explore the effects of cost differences between the university sector and the commercial sector for optimal public funding policy, with a particular focus on considerations associated with the potential role of policy as an instrument foster cost efficiency in the university sector.

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APPENDIX A: DERIVATION OF THE RESEARCH EQUILIBRIUM (7)

Differentiation of the objective functions (5) and (6) yields the following equation system of first-order conditions:

\[
\frac{\partial \phi}{\partial r_F} = (1 - (r_U + r_{UF}))\pi - cr_F = 0, \quad (A1)
\]

\[
\frac{\partial \phi}{\partial r_{UF}} = (1-r_F)\pi - c(r_U + r_{UF}) = 0, \quad (A2)
\]

\[
\frac{\partial \Gamma}{\partial r_U} = (1-r_F)V - cr_U = 0. \quad (A3)
\]

Solution of the system of equations defined by (A1)–(A3) leads to the equilibrium research investments given by (7).

APPENDIX B: PROOF OF RESULT 3

Note that when $\alpha = 0$ then $r_L^* = r^*$ (derived in Section 2.1.1) and $r_{UL}^* = r_U$. Differentiating the equilibrium research investments in the presence of licensing with respect to the licensing rate $\alpha$ yields

\[
\frac{\partial r_{UL}^*}{\partial \alpha} = \frac{\pi(c^2 - (V + \pi\alpha)^2)(c - \pi)}{(c^2 - \pi(1 - \alpha)(V + \pi\alpha))^2}
\]

and

\[
\frac{\partial r_L^*}{\partial \alpha} = \frac{c\pi(V - \pi(1 - 2\alpha))(c - \pi)}{(c^2 - \pi(1 - \alpha)(V + \pi\alpha))^2}.
\]

The denominators of both the above expressions are positive. The factor in the numerator of both expressions $c - (\pi + \gamma)$ is positive by the second-order condition. The factor $c^2 - (V + \pi\alpha)^2$ in the numerator of $\frac{\partial r_{UL}^*}{\partial \alpha}$ is also positive by the second-order condition, implying that $\frac{\partial r_{UL}^*}{\partial \alpha} > 0$. The derivative $\frac{\partial r_L^*}{\partial \alpha}$ is either positive or negative, and the condition for it being positive is given in Result 3.

For the effect of royalty rates on the total research investment the two effects above are added according to

\[
\frac{\partial (r_{UL}^* + r_L^*)}{\partial \alpha} = \frac{\pi(c - \pi)(c + V + (2\alpha - 1)\pi) - (V + \pi\alpha)^2}{(c^2 - \pi(1 - \alpha)(V + \pi\alpha))^2}
\]
Since \( c > V + \alpha \pi \) it follows that
\[
\frac{\partial (r_{UL}^* + r_L^*)}{\partial \alpha} > \frac{\pi (c - \pi) ((c + V + (2\alpha - 1)\pi) - (V + \pi \alpha))}{(c^2 - \pi (1 - \alpha)(V + \pi \alpha))^2} = \frac{\pi (c - \pi)(c + (\alpha - 1)\pi)}{(c^2 - \pi (1 - \alpha)(V + \pi \alpha))^2} > 0.
\]

\[\Box\]

**APPENDIX C: PROOF OF RESULT 4**

By (10) and (7) the sum of research investments under licensing exceeds that associated with donations if \( r_{UL}^* + r_L^* - (r_{UL}^* + r_F^* + r_{UL}^*) > 0 \). By substitution this is equivalent to the condition
\[
h(V, \pi, \alpha) = [c^2 - (1 - \alpha)c\pi - \alpha\pi^2] \alpha\pi + [c^2 - (1 - \alpha)c\pi - \alpha\pi^2]V - c\pi (c - \pi) > 0.
\]

By (C1) the difference between the investments under licensing and donations is strictly increasing as a function of \( V \) because
\[
\frac{\partial (r_{UL}^* + r_L^* - (r_F^* + r_{UL}^* - r_{UL}^*))}{\partial V} = c^2 - \pi ((1 - \alpha)c + \alpha \pi) > 0.
\]

Further, by Result 3 this same difference is also strictly increasing as a function of \( \alpha \). Further, with \( h(V, \pi, \alpha) \) defined in (C1) it holds true that \( h(V, \pi, 0) < 0 \) if \( V < \pi \), whereas \( h(V, \pi, 1) > 0 \). Finally, we observe that \( h(V, \pi, 0) = 0 \) if \( V = \pi \). Combination of all these properties implies that the sum of research investments under licensing exceeds that with donations if
\[
V > g(\pi, \alpha) = \left( \frac{c(c - \pi)}{c^2 - (1 - \alpha)c\pi - \alpha\pi^2} - \alpha \right) \pi.
\]

In particular, the condition (C3) is satisfied when \( V > \pi \) if \( \alpha = 0 \), whereas it is satisfied for all positive values of \( V \) when \( \alpha = 1 \). By Result 3 we know that \( \frac{\partial g(\pi, \alpha)}{\partial \alpha} < 0 \), meaning that a higher value of \( \alpha \) lowers the threshold of \( V \), above which licensing generates higher research investments than donations. Sufficient conditions for university licensing to always generate higher research investments than the system with donations is that either \( V > \pi \) (independently of alpha) or \( \alpha = 1 \) (independently of \( V \)).

\[\Box\]

**APPENDIX D: PROOF OF RESULT 5**

**Concavity of \( \Gamma(V) \):**

Differentiation of the objective function \( \Gamma(V) \) in (8) yields
\[
\begin{split}
\Gamma'(V) &= \left( \frac{\partial r_{UL}^*}{\partial V} (1 - r_F^*) - \frac{\partial r_F^*}{\partial V} r_{UL}^* \right) (W - \gamma V) - \gamma r_{UL}^* (1 - r_F^*) + \frac{\partial r_{UL}^*}{\partial V} W, \\
\end{split}
\]
where \( \frac{\partial r_{UL}^*}{\partial V} = \frac{(c - \pi)c^2}{|c^2 - \pi V|^2} > 0 \) and \( \frac{\partial r_F^*}{\partial V} = -\frac{(c - \pi)c\pi}{|c^2 - \pi V|^2} < 0 \). Differentiating these equilibrium success probabilities once more we find that \( \frac{\partial^2 r_{UL}^*}{\partial V^2} = \frac{2\pi (c - \pi)c^2}{|c^2 - \pi V|^3} > 0 \) and \( \frac{\partial^2 r_F^*}{\partial V^2} = -\frac{2\pi^2 (c - \pi)c}{|c^2 - \pi V|^3} < 0 \), respectively. Inserting these into the second-order derivative we find that
\[
\Gamma''(V) = -\frac{2c^2(c - \pi)}{[c^2 - \pi V]^4}(\pi^2 W (c - V) + c(c - \pi)[\gamma c^2 - 2\pi (W - \gamma V)]) < 0.
\]

The sign of this second-order derivative follows from Assumption 1 in combination with the parameter restriction that \(\gamma > 1\).

Result 5(a): The feasibility of \(\hat{V}(0 < \hat{V} < W)\) follows directly from an investigation of the inequalities in question.

Result 5(b): Consider the equation

\[
\frac{cW(c - 2\pi)}{2\gamma(c - \pi) - \pi W} = \pi.
\]

\((A1)\) a quadratic equation with respect to \(\pi\). The equation has two solutions \(\pi = c\) and \(\pi = \frac{W}{2\gamma + W}c\), out of which only the latter one is feasible in light of Assumption 1. Furthermore, the LHS of (B.1) is strictly decreasing as a function of \(\pi\), whereas the RHS is strictly increasing. The combination of these findings facilitates the conclusion formulated in Result 5(b).

\(\square\)

**APPENDIX E: PROOF OF RESULT 6**

(a) Differentiation of (13) with respect to \(\pi\) yields

\[
\frac{\partial \hat{V}}{\partial \pi} = -\frac{c^2W(2\gamma W - W)}{[2\gamma c - \pi W]^2} < 0.
\]

(b) Differentiation of (13) with respect to \(W\) yields

\[
\frac{\partial \hat{V}}{\partial W} = \frac{2\gamma^2(c - 2\pi)(c - \pi)}{[2\gamma c - \pi W]^2} > 0.
\]

(c) Differentiation of (13) with respect to \(c\) yields

\[
\frac{\partial \hat{V}}{\partial c} = \frac{2W\pi(c - \pi)(4\gamma W - W)}{[2\gamma c - \pi W]^2} > 0.
\]

**APPENDIX F: PROOF OF RESULT 7**

Due to symmetry \(r_A^U = r_B^U \neq r_U^U = \frac{V}{c + V}\) the objective function (17) can be written according to

\[
\max_V \Gamma(V) = r_U^U (2 - r_U^U) W - 2r_U^U (1 - r_U^U)\gamma V,
\]

which yields the first-order condition

\[
\frac{\partial r_U^U}{\partial V} [(2 - r_U^U) W - 2(1 - r_U^U)\gamma V] - \left[ r_U^U \frac{\partial r_U^U}{\partial V} W + 2\gamma r_U^U \left(1 - r_U^U\right) - \frac{\partial r_U^U}{\partial V}\right] = 0, \quad (F.1)
\]
where \( \frac{\partial r^U}{\partial V} = \frac{c}{(c + V)^2} > 0 \). Substitution shows that (F.1) can be simplified to a linear equation with the solution \( V = \hat{V}^U = \frac{W}{2\gamma} \).

As for the relationship between \( \hat{V} \) and \( \hat{V}^U \) direct substitution shows that the inequality \( \hat{V} < \hat{V}^U \) is equivalent to \( W < 2\gamma c \). The latter inequality follows directly from Assumption 1 (b).

\[ \square \]

**APPENDIX G: PROOF OF RESULT 9**

The LHS of (21) is a strictly increasing function of \( V \) because

\[
\frac{\partial}{\partial V} \left\{ \frac{c(\pi + V) - 2\pi V}{c^2 - \pi V} \right\} = \frac{c(c - \pi)^2}{[c^2 - \pi V]^2} > 0,
\]

whereas the RHS of (21) is independent of \( V \).

Further, by substitution of \( V = \pi \) we find that the inequality (18) is reversed because

\[
\frac{2\pi}{c + \pi} > \frac{2\pi}{c + (1 - \lambda)\pi}
\]

for all feasible values of \( \lambda \). Combined with the finding that the LHS of (21) is strictly increasing as a function of \( V \) we can draw the conclusion that the inequality cannot hold unless \( V > \pi \).

This means that the investment in basic research with a mixed duopoly cannot exceed that with a for-profit duopoly unless \( V > \pi \) by a sufficient margin.

\[ \square \]

**APPENDIX H: PROOF OF RESULT 10**

Substitution of the socially optimal volume of university funding (given by (10)) into the equilibrium investments (3) and (4) shows that the mixed duopoly will generate total investments in basic research

\[
r^u_0(\hat{V}) + r^u_r(\hat{V}) = \frac{Wc + \pi(2\gamma c - 3W)}{2(\gamma c^2 - \pi W)}.
\]

Based on straightforward differentiation it can be verified that this is a strictly decreasing function of \( \gamma \). Based on (9) the for-profit duopoly invests the volume \( \frac{2\pi}{c + (1 - \lambda)\pi} \) in basic research.

Thus, the investment in a for-profit duopoly is independent of \( \gamma \). Finally, we observe that the total investment in a mixed duopoly is identical to that in a for-profit duopoly under the condition

\[
\frac{Wc + \pi(2\gamma c - 3W)}{2(\gamma c^2 - \pi W)} = \frac{2\pi}{c + (1 - \lambda)\pi}.
\]

This condition is satisfied if and only if
\[ \gamma = \hat{\gamma} = \frac{W(c^2 - (2 + \lambda)\pi c + \pi^2(1 + 3\lambda))}{2c\pi(c - (1 - \lambda)\pi)}. \]

Consequently, it must hold true that the mixed duopoly invests more in basic research than the for-profit duopoly if and only if \( \gamma < \hat{\gamma} \). Furthermore, this threshold is strictly decreasing as a function of \( \lambda \), because \( \frac{\partial \hat{\gamma}}{\partial \lambda} = \frac{-W(c^2 - 3\pi c + 2\pi^2)}{c(c - (1 - \lambda)\pi)^2} < 0 \) as long as \( c > 2\pi \). This means that intensified product market competition (lower \( \lambda \)) raises the threshold \( \hat{\gamma} \). □

**APPENDIX I: PROOF OF COROLLARY 1**

Let \( W = \alpha \pi \). Under such circumstances the threshold for the distortion parameter is given by \( \hat{\gamma} = \frac{\alpha}{2} \cdot \frac{(c^2 - (2 + \lambda)\pi c + \pi^2(1 + 3\lambda))}{2c(c - (1 - \lambda)\pi)}. \) The constraint \( \alpha \leq 2 \) implies that \( \hat{\gamma} < 1 \) if and only if \( c^2 - (2 + \lambda)\pi c + \pi^2(1 + 3\lambda) < c\pi(c - (1 - \lambda)\pi) \). This condition can be rewritten as \(-(1 + \lambda)c + \pi(1 + 3\lambda) < 0\), which always holds by Assumption 1(b) for all \( \lambda \in [0, \frac{1}{2}] \). But, the property that \( \hat{\gamma} < 1 \) means that the mixed duopoly induces lower total investments than the for-profit duopoly for all feasible values (\( \gamma > 1 \)) of the distortion parameter. □