Seasonality and multiple maternities: Comparisons between different models

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Abstract. Seasonality of demographic data has been of great interest. The seasonality depends mainly on climatic conditions, and the findings may vary from study to study. Commonly, the studies are based on monthly data. *The population at risk* plays a central role. For births or deaths over short periods, the population at risk is proportional to the lengths of the months. Hence, one must analyse the number of births (deaths) per day. If one studies the seasonality of multiple maternities, the population at risk is the total monthly number of confinements and the number of multiple maternities in a given month must be compared with the monthly number of all maternities. Consequently, one considers the monthly rates of multiple maternities, the monthly number of births is eliminated and one obtains an unaffected seasonality measure of the rates. In general, comparisons between the seasonality of different data sets presuppose standardization of the data to indices with common means, mainly 100.

When seasonal models are applied, one must pay special attention to how well the applied model fits the data. If the goodness of fit is poor, non-significant models obtained can erroneously lead to statements that the seasonality is slight, although the observed seasonal fluctuations are marked. The estimated monthly models chosen are approximately orthogonal and they have little influence on the parameter estimates. Exact orthogonality should be obtained if the data are equidistant, that is, if the months are of equal length (e.g. 30 days), corresponding to 30°. Exactly equidistant data can be observed when circadian rhythms (24 hours) are studied. In this study, we compare seasonal models with models with exact orthogonality.

Keywords. Population at risk, births, deaths, twins, triplets, trigonometric regression, indices, χ^2 test, t test, F test, circadian rhythm, adjusted coefficient of determination, degrees of freedom, ordinary least squares, OLS

Introduction

Seasonality of population data has been of great interest in demographic studies. The seasonality depends mainly on climatic conditions, and hence, the findings may vary from study to study. Commonly, the studies are based on monthly data. *The population at risk* plays a central role. In a study of seasonal variation in the number of births or deaths, the population at risk is the product of the size of the population and the length of the month. Over short periods, the population can be assumed to be constant, and therefore, the population at risk is proportional to the lengths of the months. Hence, for studies of monthly birth and death data one must analyse the number of cases per day.

If one studies the seasonal variation in multiple maternities or in the occurrence of an innate disease, the population at risk is the total number of confinements. Hence, the number of multiple maternities in a given month must be compared with the monthly number of all maternities. Similar attempts should also be applied when one studies stillbirths and the births of males (secondary sex ratio). Hence, one has to consider the monthly rates of multiple maternities, of stillbirths and of males. In the last case, one often considers secondary sex ratio as the rate of males (in %) among all births. If one considers monthly rates, the monthly number of birth data is eliminated and one obtains unaffected seasonality measures of the rates.

The selection of a test for seasonality is not simple because the definition of seasonality is somewhat arbitrary. If one assumes seasonality as "non-flatness" throughout a year, χ^2 test (11 degrees of freedom for monthly data) is an option. But this test is weak since it does not consider the order of the months; it calculates the same test statistic for the data set with high (or low) values occurring in consecutive months and for data in separate months. Consequently, χ^2 tests are appropriate for testing of seasonality strength, but not for model testing ([1], [2]).

Model fit is crucial, but which model is appropriate cannot be fixed a priori. Ultimately, test for seasonality might be integrated into test for clustering, but importantly, the clustering must be cyclic, with a one-year period. In demographic data, the sine curve has been a common feature of studies on seasonal variation (intra-annual fluctuations).

Multiple trigonometric regression models have proved useful for studying all kinds of periodic data, for they are flexible when the data differ from a simple sine curve. Diggle [3] discusses these models under the name of *periodograms* and states that they are good summary descriptions of time series showing a periodic pattern. In its simplest form, the trigonometric regression model is a sine curve and is comparable to the original and the generalized St. Leger models ([4] - [6]). When seasonal models are applied, one must pay special attention to how well the applied model fits the data. A poor fit can erroneously result in a statement that the seasonality is slight, although the observed seasonal fluctuations are marked ([1], [7]). A good knowledge of the seasonal variation during normal years is of fundamental importance for studies of the effects of wars, famines, epidemics or similar privations ([6]), [8]). In this study, we investigate the rate of multiple maternities and compare seasonal (annual) models with circadian models. The rates of multiple maternities are the monthly number of multiple maternities related to the monthly number of all maternities, ignoring the (monthly) number of days. Similar comparisons of the seasonality of births and deaths cannot be performed because the study of births and deaths is based on cases per day. For such objects, a comparison between annual and circadian models cannot be performed.

Methods and Materials

Methods. In earlier papers, Fellman and Eriksson ([5], [6]) gave extensive presentations of seasonal models. They proposed alternative models, but their own studies were mainly based on multiple trigonometric regression models. In many situations, although there are marked seasonal variations, the simple sine curve model does not fit the data. For example, this is the case when the annual data show more than one marked peak, but data showing one peak and one trough may also differ considerably from the sine curve. Such cases could be when the peak or the trough is too larged or when the distance between them differs from one half year. Therefore, multiple trigonometric regression models are noteworthy alternatives.

Following [6], we consider the multiple trigonometric regression model

$$R(t_i) = K + \sum_{m=1}^{M} R_m \sin(mt_i + \alpha_m) + \varepsilon_i = K + \sum_{m=1}^{M} \left(A_m \cos(mt_i) + B_m \sin(mt_i) \right) + \varepsilon_i \ .$$

Hence, the applicable multiple trigonometric regression model is

$$R(t_i) = K + \sum_{m=1}^{M} \left(A_m \sin(mt_i) + B_m \cos(mt_i) \right) + \varepsilon_i \,, \qquad (1)$$

where M is the number of pairs of trigonometric terms, $A_m = R_m \sin \alpha_m$ and $B_m = R_m \cos \alpha_m$. The error terms ε_i are assumed to be independent and homoscedastic. The parameters A_m and B_m $(m = 1, ..., M)$ and K in model (1) are estimated by ordinary least squares (OLS) for the monthly data. With monthly data, one must introduce the restriction $M \leq 5$. The basic parameters α_m and the amplitudes R_m are estimated by the equations

$$\tan(\hat{\alpha}_m) = \frac{\hat{A}_m}{\hat{B}_m} \qquad (2)$$

and

$$\hat{R}_m = \sqrt{\hat{A}_m^2 + \hat{B}_m^2} \,. \qquad (3)$$

The angle should be chosen so that $\sin(\alpha_m)$ and A_m (or alternatively $\cos(\alpha_m)$ and B_m) have the same sign. These restrictions give unique angles α_m. The angles α_m and the amplitudes R_m are estimated from formulae (2) and (3).

The simple regression model, $M = 1$, indicates the sinusoidal model. Good agreement with the sinusoidal model demands that the data pattern has one peak and one trough within a year. In this study, we pay more attention to the model fit and less to the estimates of the individual parameters. Hence, we accept only significant parameter estimates. We assume that the model is compact and optimal when all the non-significant parameter estimates are eliminated and all the remaining parameter estimates are significant.

Fellman and Eriksson [6] observed that the design matrix of the regressor vectors is almost orthogonal, resulting in parameter estimates that are approximately uncorrelated and relatively independent of the number of trigonometric terms in the models. The regressors are $\cos(t)$, $\sin(t)$, $\cos(2t)$, $\sin(2t)$, $\cos(3t)$, $\sin(3t)$, $\cos(4t)$, $\sin(4t)$, $\cos(5t)$ and $\sin(5t)$, and their design matrix is

1	0.965	0.263	0.861	0.508	0.697	0.717	0.483	0.876	0.235	0.972
1	0.714	0.701	0.018	1.000	-0.687	0.726	-0.999	0.037	-0.739	-0.674
1	0.281	0.960	-0.842	0.539	-0.754	-0.656	0.418	-0.908	0.989	0.146

$$X = \begin{bmatrix}
1 & -0.237 & 0.971 & -0.887 & -0.461 & 0.659 & -0.752 & 0.574 & 0.819 & -0.932 & 0.363 \\
1 & -0.692 & 0.722 & -0.042 & -0.999 & 0.750 & 0.661 & -0.996 & 0.084 & 0.629 & -0.777 \\
1 & -0.961 & 0.278 & 0.845 & -0.534 & -0.664 & 0.748 & 0.430 & -0.903 & -0.162 & 0.987 \\
1 & -0.971 & -0.241 & 0.884 & 0.467 & -0.746 & -0.666 & 0.564 & 0.826 & -0.348 & -0.937 \\
1 & -0.714 & -0.701 & 0.018 & 1.000 & 0.687 & -0.726 & -0.999 & 0.037 & 0.739 & 0.674 \\
1 & -0.267 & -0.964 & -0.858 & 0.514 & 0.724 & 0.690 & 0.472 & -0.882 & -0.976 & -0.220 \\
1 & 0.252 & -0.968 & -0.873 & -0.488 & -0.692 & 0.722 & 0.524 & 0.852 & 0.956 & -0.292 \\
1 & 0.703 & -0.711 & -0.012 & -1.000 & -0.720 & -0.694 & -1.000 & 0.024 & -0.686 & 0.728 \\
1 & 0.965 & -0.263 & 0.861 & -0.508 & 0.697 & -0.717 & 0.483 & -0.876 & 0.235 & -0.972
\end{bmatrix} \quad (4)$$

This model is termed the *annual model*. The scalar product shows the estimators are almost orthogonal, as can be seen in the information matrix M, where the diagonal elements are 12 and approximately 6 and the off-diagonal elements are close to zero.

$$M = \begin{bmatrix}
12.000 & 0.038 & 0.047 & -0.025 & 0.038 & -0.049 & 0.052 & -0.047 & -0.016 & -0.057 & -0.003 \\
0.038 & 5.987 & 0.019 & -0.006 & 0.050 & -0.036 & 0.011 & -0.053 & 0.024 & -0.024 & -0.011 \\
0.047 & 0.019 & 6.013 & 0.002 & 0.044 & -0.027 & 0.011 & -0.027 & 0.004 & 0.005 & -0.023 \\
-0.025 & -0.006 & 0.002 & 5.976 & -0.008 & -0.010 & 0.022 & -0.013 & 0.016 & -0.062 & 0.028 \\
0.038 & 0.050 & 0.044 & -0.008 & 6.024 & -0.025 & 0.048 & -0.022 & -0.012 & -0.024 & 0.013 \\
-0.049 & -0.036 & -0.027 & -0.010 & -0.025 & 6.000 & -0.003 & -0.018 & 0.026 & -0.052 & 0.037 \\
0.052 & 0.011 & 0.011 & 0.022 & 0.048 & -0.003 & 6.000 & -0.022 & 0.056 & -0.001 & 0.027 \\
-0.047 & -0.053 & -0.027 & -0.013 & -0.022 & -0.018 & -0.022 & 5.961 & 0.018 & -0.060 & -0.045 \\
-0.016 & 0.024 & 0.004 & 0.016 & -0.012 & 0.026 & 0.056 & 0.018 & 6.039 & -0.092 & 0.098 \\
-0.057 & -0.024 & 0.005 & -0.062 & -0.024 & -0.052 & -0.001 & -0.060 & -0.092 & 5.929 & -0.085 \\
-0.003 & -0.011 & -0.023 & 0.028 & 0.013 & 0.037 & 0.027 & -0.045 & 0.098 & -0.085 & 6.071
\end{bmatrix} \quad (5)$$

Consequently, the model chosen has little influence on the parameter estimates. On the other hand, the error variances of the estimates are based on the unexplained sum of squares and, accordingly, parameter tests are affected by the model chosen. Furthermore, we assume in this study that the error terms are independent and homoscedastic. If this is not the case, the estimates obtained, although unbiased and consistent, will not be efficient. When the data are based on a large number of observations, the monthly rates can be assumed to be asymptotically normal. The whole model can then be tested by the F test and the individual parameters by t tests ([5], [6]). The goodness of fit of the regression model can also be analysed with the coefficients of determination (R^2 or $\overline{R}^2$). Only if there is good agreement between the data and the model will the model-based tests detect the seasonal variation.

Exact orthogonality should be obtained if the data are equidistant, that is, if the months are of equal length (e.g. 30 days), hence corresponding to 30°. Exactly equidistant data can be observed when circadian rhythms (24 hours) are studied. If one splits the day into 12 parts (i.e. two-hour periods), the regression model can be compared with the annual trigonometric regression model given above. This model is named the *circadian model*. Now the design matrix is

$$X=\begin{bmatrix}
1 & 0.966 & 0.259 & 0.866 & 0.500 & 0.707 & 0.707 & 0.500 & 0.866 & 0.259 & 0.966\\
1 & 0.707 & 0.707 & 0.000 & 1.000 & -0.707 & 0.707 & -1.000 & 0.000 & -0.707 & -0.707\\
1 & 0.259 & 0.966 & -0.866 & 0.500 & -0.707 & -0.707 & 0.500 & -0.866 & 0.966 & 0.259\\
1 & -0.259 & 0.966 & -0.866 & -0.500 & 0.707 & -0.707 & 0.500 & 0.866 & -0.966 & 0.259\\
1 & -0.707 & 0.707 & 0.000 & -1.000 & 0.707 & 0.707 & -1.000 & 0.000 & 0.707 & -0.707\\
1 & -0.966 & 0.259 & 0.866 & -0.500 & -0.707 & 0.707 & 0.500 & -0.866 & -0.259 & 0.966\\
1 & -0.966 & -0.259 & 0.866 & 0.500 & -0.707 & -0.707 & 0.500 & 0.866 & -0.259 & -0.966\\
1 & -0.707 & -0.707 & 0.000 & 1.000 & 0.707 & -0.707 & -1.000 & 0.000 & 0.707 & 0.707\\
1 & -0.259 & -0.966 & -0.866 & 0.500 & 0.707 & 0.707 & 0.500 & -0.866 & -0.966 & -0.259\\
1 & 0.259 & -0.966 & -0.866 & -0.500 & -0.707 & 0.707 & 0.500 & 0.866 & 0.966 & -0.259\\
1 & 0.707 & -0.707 & 0.000 & -1.000 & -0.707 & -0.707 & -1.000 & 0.000 & -0.707 & 0.707\\
1 & 0.966 & -0.259 & 0.866 & -0.500 & 0.707 & -0.707 & 0.500 & -0.866 & 0.259 & -0.966
\end{bmatrix} \quad (6)$$

and the orthogonality can be seen in the information matrix

$$N=\begin{bmatrix}
12.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000\\
0.000 & 6.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000\\
0.000 & 0.000 & 6.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000\\
0.000 & 0.000 & 0.000 & 6.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000\\
0.000 & 0.000 & 0.000 & 0.000 & 6.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000\\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 6.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000\\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 6.000 & 0.000 & 0.000 & 0.000 & 0.000\\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 6.000 & 0.000 & 0.000 & 0.000\\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 6.000 & 0.000 & 0.000\\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 6.000 & 0.000\\
0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 6.000
\end{bmatrix} \quad (7)$$

The diagonal elements are exactly 12 and 6 and all off-diagonal elements are exactly zero. On the other hand, the error variance is based on the unexplained sum of squares and, accordingly, parameter tests are affected by the model chosen. Furthermore, we assume also that this model has the same properties as the annual model. When the data are based on a large number of observations, the monthly rates can be assumed to be asymptotically normal. The whole model can then be tested by the F test and the individual parameters by t tests ([5], [6]). The goodness of fit of the regression model can also be analysed with the coefficients of determination (R^2 or $\overline{R}^2$). Still, the model-based tests detect the seasonal variation only if there is good agreement between the data and the model.

Materials. In this study, we apply the annual and circadian models on multiple maternity ratios (MUR) in Switzerland (1876-1890), initially published by Weinberg [9] and twinning rates on the Åland Islands (1750-1949) given by Eriksson [10]. Recently these data sets have been analysed by Fellman [2], where the data sets are reprinted.

Applications

Now we include in our study the detailed processes from the maximal regression model to the optimal model containing only significant parameters. This pathway shows how the included parameter estimates are almost identical irrespectively of the number of regressors of the models. The annual regression models for the Switzerland data are given in Table 1. As a new study, we build circadian regression models based on the approximate data sets where we assume that the year consists of 12 months of equal length (30 days). This attempt yields regression models with identical parameter estimates. The circadian regression models for the Switzerland data are given in Table 2. The corresponding pathways for the Åland data are given in Tables 3 and 4. For the Switzerland MUR data, the optimal annual model is

$$Y_i = 12.390 + 0.780\sin(t_i) - 0.248\cos(t_i) - 0.241\cos(2t_i) - 0.115\cos(4t_i) + 0.197\sin(5t_i)\,, \quad (8)$$

and it differs from the sinusoidal model given earlier ([4], [7]). The optimal circadian model for the Switzerland data is

$$Y_i = 12.393 + 0.778\sin(t_i) - 0.256\cos(t_i) - 0.233\cos(2t_i) - 0.113\cos(4t_i) + 0.200\sin(5t_i)\,. \quad (9)$$

Note the small differences between the two models.

Figure 1

For the Åland twin rate, the optimal annual model is ([4], [7])

$$Y_i = 23.208 - 2.738\sin(t_i) - 2.458\cos(t_i) \quad (10)$$

The optimal circadian model is

$$Y_i = 23.189 - 2.770\sin(t_i) - 2.427\cos(t_i)\,. \quad (11)$$

Note the small differences between the two sinusoidal models.

Figure 2

Discussion

The rates of multiple maternities are the monthly number of multiple maternities related to the monthly number of all maternities, ignoring the (monthly) number of days. Similar comparisons of the seasonality of births and deaths cannot be performed because the study of births and deaths is based on cases per day. Other demographic variables such as sex ratio which is a rate per total number of maternities, could be objects for similar comparisons between annual and circadian models.

Consequently, the model chosen has little influence on the parameter estimates. Exact orthogonality would be obtained if the data are equidistant, that is, if the months are of equal length (e.g. 30 days), corresponding exactly to 30°. Exactly equidistant data can be observed when circadian rhythms (24 hours) are studied. Comparisons of the obtained annual and circadian models show very small discrepancies, arising from the following facts. The monthly rates are obtained as ratios between the twinning maternities and the total maternities in the same month. The models are then applied to these rates. A comparison between the two models shows that the time variables are almost identical. The maximal difference between the two time scales is 4.75 days.

References

[1] Fellman, J. & Eriksson, A. W. (2009). Temporal and Regional Variations in the Seasonality of Births in Åland (Finland), 1653-1950. Biodemography and Social Biology, 55: 103-112.

[2] Fellman (2019). Demographic seasonality. *Twin Research and Human Genetics* accepted

[3] Diggle, P, J. (1996). *Time Series A Biostatistical Introduction*. Oxford Science Publications Clarendon Press, Oxford.

[4] Fellman J. & Eriksson A. W. (1999). Statistical analysis of the seasonal variation in twinning rate. *Twin Research 2*:22-29.

[5] Fellman, J. & Eriksson, A. W. (2000). Statistical analysis of the seasonal variation in demographic data. *Human Biology 72*:851-876.

[6] Fellman, J. & Eriksson, A. W. (2002). Regional, Temporal and Seasonal Variations in Births and Deaths: the Effects of Famines. *Social Biology 48*:86-104.

[7] Fellman, J. (2017). Studies of the seasonal pattern of multiple maternities *Twin Research and Human Genetics 20(3)*:250-256. doi:10.1017/thg.2017.20

[8] Land, K. C. & Cantor, D. (1983). Arima Models of Seasonal Variation in U.S. Birth and Death Rates. *Demography* , *20 (4)*:541-568.

[9] Weinberg, W. (1901). Beiträge zur Physiologie und Pathologie der Mehrlingsgeburten beim Menschen [Contributions in physiology and pathology to the human multiple

maternities]. *Pflügers Archiv für die gesamte Physiologie des Menschen und der Tiere, 88,* 346-430.

[10] Eriksson, A. W. (1973). Human twinning in and around the Åland Islands. *Comment Biol* Suppl. 64, 159 pp. (thesis at the University of Helsinki)

Table 1. The MUR in Switzerland (1876-1890). The set of annual models following the elimination of non-significant regressors in annual models.

	Switzerland	MUR,	1876 -	1890	Annual	models
Intercept	12.390	12.390	12.390	12.390	12.390	12.390
COS1	-0.248	-0.248	-0.248	-0.248	-0.248	-0.248
SIN1	0.780	0.780	0.780	0.780	0.780	0.780
COS2	-0.241	-0.241	-0.241	-0.241	-0.241	-0.241
SIN2	-0.001					
COS3	0.010	0.010				
SIN3	0.025	0.025	0.025	0.025	0.025	
COS4	-0.115	-0.115	-0.115	-0.115	-0.115	-0.115
SIN4	0.019	0.019	0.019			
COS5	-0.022	-0.022	-0.022	-0.022		
SIN5	0.197	0.197	0.197	0.197	0.197	0.197
R^2	0.988	0.994	0.996	0.995	0.995	0.994

Table 2. The MUR in Switzerland (1876-1890). The set of annual models following the elimination of non-significant regressors in circadian models.

	Switzerland	MUR,	1876	- 1890	Circadian	models
Intercept	12.393	12.393	12.393	12.393	12.393	12.393
COS1	-0.256	-0.256	-0.256	-0.256	-0.256	-0.256
SIN1	0.778	0.778	0.778	0.778	0.778	0.778
COS2	-0.233	-0.233	-0.233	-0.233	-0.233	-0.233
SIN2	-0.006					
COS3	0.013	0.013				
SIN3	0.031	0.031	0.031	0.031		
COS4	-0.113	-0.113	-0.113	-0.113	-0.113	-0.113
SIN4	0.021	0.021	0.021			
COS5	-0.032	-0.032	-0.032	-0.032	-0.032	
SIN5	0.2	0.2	0.2	0.2	0.2	0.2
R^2	0.989	0.994	0.995	0.995	0.993	0.992

Table 3. The TWR in Åland (1750-1949). The set of annual models following the elimination of non-significant regressors in annual models.

	Åland	TWR,	1750	-1949	Annual	models			
Intercept	23.203	23.203	23.201	23.199	23.199	23.198	23.200	23.204	23.208
COS1	-2.472	-2.474	-2.475	-2.476	-2.477	-2.474	-2.474	-2.467	-2.458
SIN1	-2.748	-2.749	-2.749	-2.749	-2.751	-2.751	-2.751	-2.746	-2.738
COS2	-0.606	-0.606	-0.61	-0.612	-0.61	-0.608			
SIN2	1.117	1.116	1.115	1.112	1.113	1.111	1.112	1.117	
COS3	-1.096	-1.097	-1.1	-1.1	-1.097	-1.094	-1.093		
SIN3	-0.41	-0.411	-0.411						
COS4	0.138								
SIN4	0.603	0.603	0.598	0.594	0.601				
COS5	0.368	0.367							
SIN5	0.460	0.459	0.454	0.452					
R^2	-0.242	0.373	0.557	0.644	0.692	0.709	0.721	0.672	0.630

Table 4. The TWR in Åland (1750-1949). The set of annual models following the elimination of non-significant regressors in circadian models.

			Åland TWR,		1750-1949		Circadian	model	
Intercept	23.189	23.189	23.189	23.189	23.189	23.189	23.189	23.189	23.189
COS1	-2.427	-2.427	-2.427	-2.427	-2.427	-2.427	-2.427	-2.427	-2.427
SIN1	-2.77	-2.77	-2.77	-2.77	-2.77	-2.77	-2.77	-2.77	-2.770
COS2	-0.647	-0.647	-0.647	-0.647	-0.647	-0.647			
SIN2	1.112	1.112	1.112	1.112	1.112	1.112	1.112	1.112	
COS3	-1.049	-1.049	-1.049	-1.049	-1.049	-1.049	-1.049		
SIN3	-0.453	-0.453	-0.453						
COS4	0.138								
SIN4	0.621	0.621	0.621	0.621	0.621				
COS5	0.379	0.379							
SIN5	0.512	0.512	0.512	0.512					
R^2	-0.215	0.387	0.564	0.644	0.686	0.702	0.711	0.669	0.628

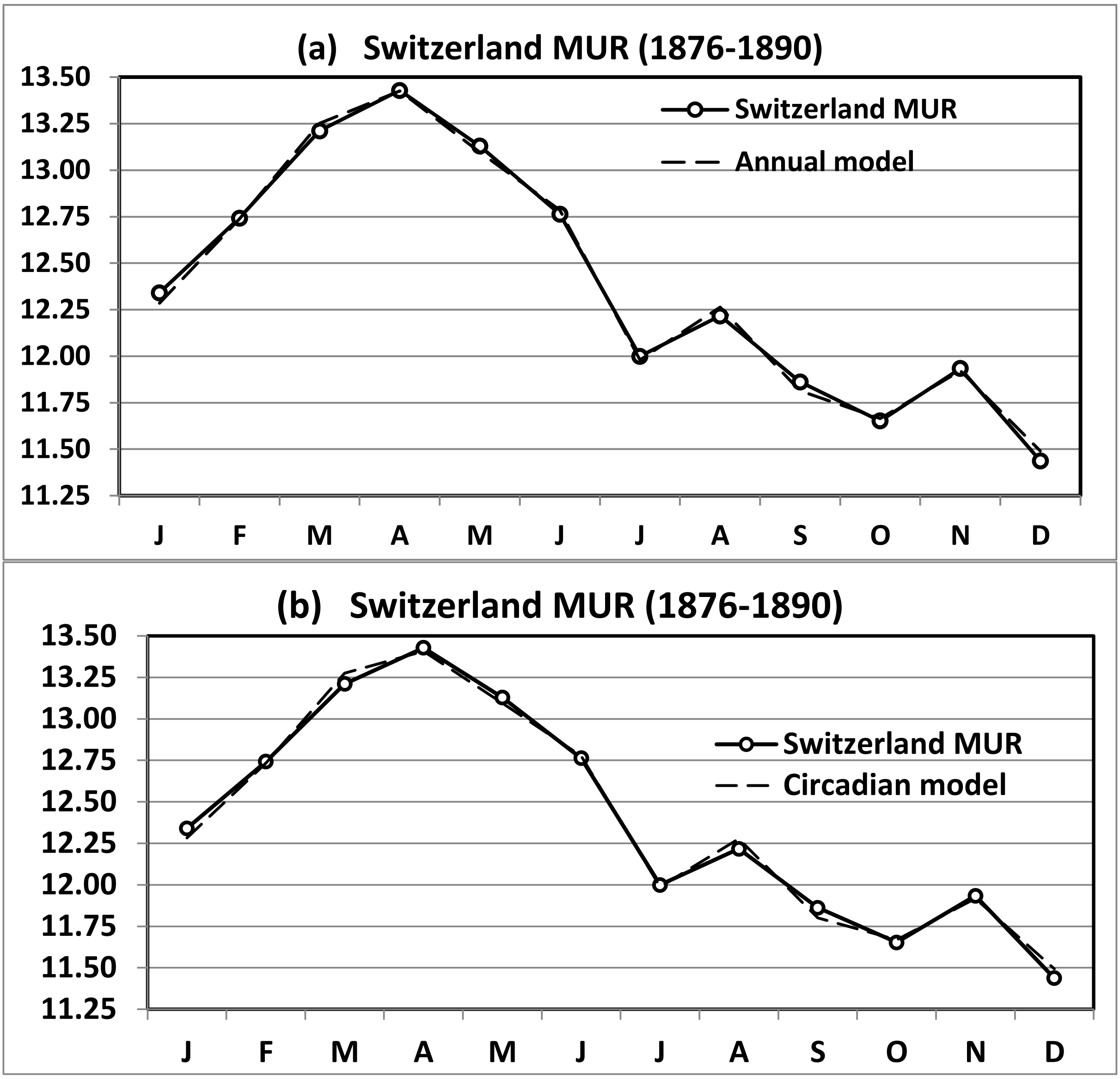
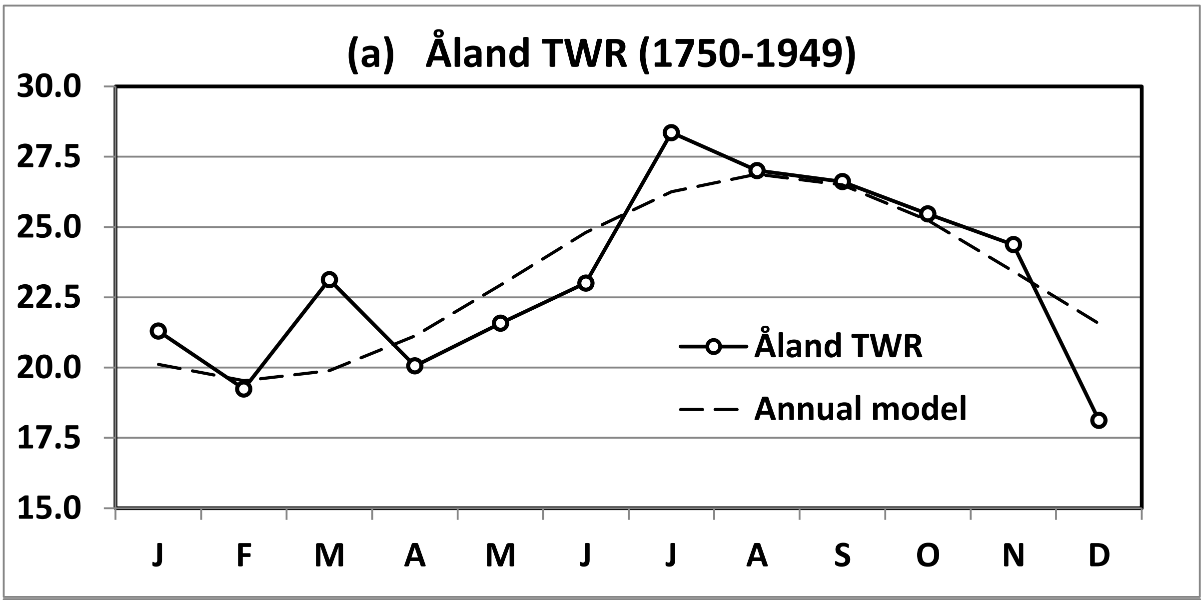


Figure 1. Seasonality of the rate of multiple maternities (MUR) in Switzerland (1876-1890). The obtained optimal model based on the annual models is included in (a) and based on the circadian models is included in (b). Details are discussed in the text.



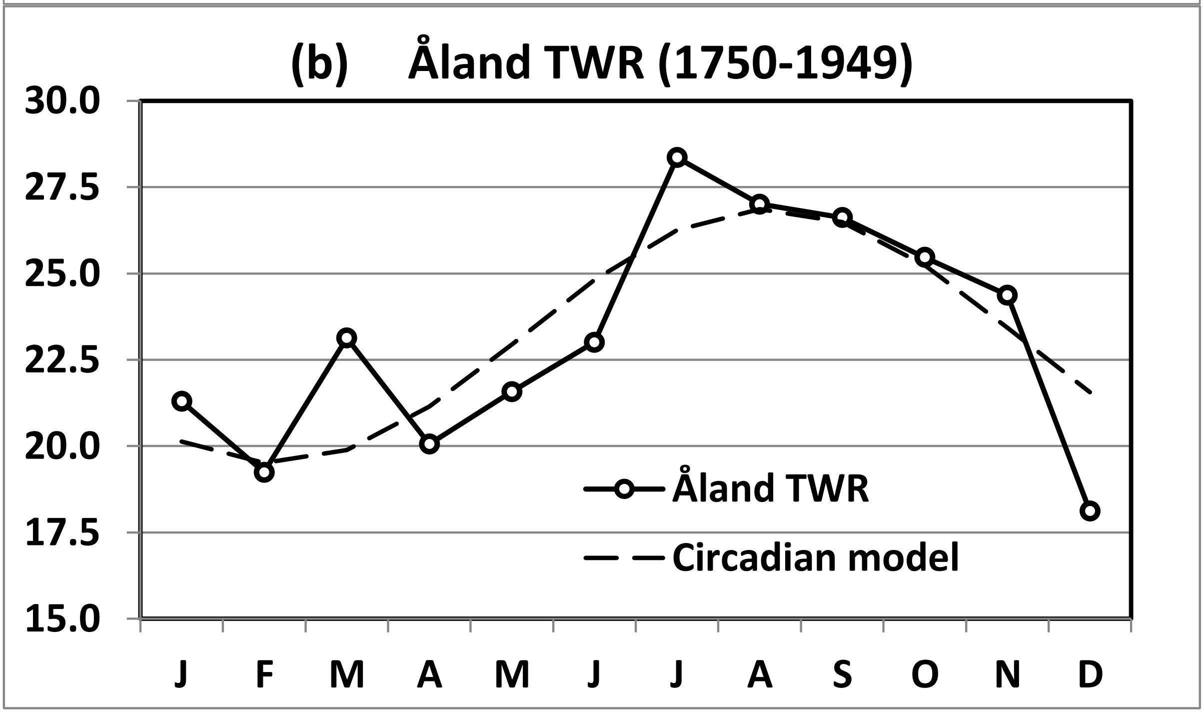


Figure 2. Seasonality of the rate of twin maternities (TWR) in Åland (1750-1949). The obtained sinusoidal model based on the annual models is included in (a) and the obtained sinusoidal model based on the circadian models is included in (b).