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International stock market comovement in time and scale outlined with a thick pen

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Abstract

We quantify time-varying, bivariate and multivariate comovement between international stock market returns, across various time scales, based on a novel approach of Fryzlewicz and Oh (2011) called thick pen transform. With help of this nonparametric and simple tool, we study 11 countries and examine their comovement with respect to (non-dyadic) time scales/frequencies, development and region. We also consider all possible 2036 different combinations of two or more of these countries. In the two-country case, we make comparisons with cross-correlations, either rolling-window or based on the multi-period returns. We find that in the bivariate set-up with the USA, the BRIC countries, except for Brazil (especially over small time scales), offer diversification benefits, while in the multivariate one, clustering with respect to America or Europe (but not Asia) leads to homogeneous groups. Hence development and region cannot always be considered as ultimate clustering factors. Leave-one-out cross-validation shows a nuanced interplay of time scales, development and region as grouping factors for Brazil, Japan, Hong Kong and Russia. Additionally, we provide an example of a time-scale-dependent portfolio strategy.

Keywords: pen thickness, frequency, volume, thick pen measure of association, cross-spectrum
JEL: C14, E32, G15

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1. Introduction

Stock market comovement is an important topic in finance, primarily due to its relevance for asset allocation, portfolio diversification, risk management and functioning of financial systems. Here we use 'comovement' to label a broadly-understood phenomenon in which two or several entities/time series 'move together'.

We deliberately avoid any reference to a particular quantitative measure, e.g., cross-correlation, regression coefficient, quantiles, tail-dependence coefficient, copula, TailCoR (Ricci and Veredas (2015)) or to a time series model, be it bivariate, multivariate, linear or nonlinear, e.g., uni- and multivariate GARCH, VAR, copula (Rodriguez (2007)), regime switching (Gallo and Otranto (2008)), latent component (Berger and Pozzi (2013)). We also refrain from specifying timing, e.g., pre-, post- or crisis dates, time scales, e.g., short- or long-term, constancy of the quantitative measure over time or its lack, e.g., CCC-GARCH or DCC-GARCH (Engle (2002)), data type, e.g., returns, volatilities, prices or data frequency. Other broad terms employed in a similar way are linkage and interdependence, albeit the latter is used with large rather than small time scales, similarly to integration, which often appears in the context of emerging markets. Nevertheless it seems that there is some degree of consensus over the characterization of contagion, which is an important subclass of comovement. This is done by combining comovement with timing and time scales: contagion is a comovement in a period of crisis, which is short-lived and often considered as atypical (other words used are deviation, change, increase, break, excess); 'transmission of shocks' is a common phrase used then. On the other hand, spillover, often associated with volatility, is considered as 'typical' and sometimes used with large time scales. Large time scales are also linked with the more technical concept of cointegration. The literature on comovement, notably contagion, including definitions and econometric techniques employed, is vast. Here we give an indicative but not exhaustive list of references: Forbes and Rigobon (2002), Kaminsky et al. (2003), Bae et al. (2003), Baur (2004), Dungey et al. (2005).

When examining comovements in international stock markets, factors such as level of development (developed or emerging markets) or geographical location (America, Europe, Asia) are taken into consideration. The USA is often considered as a reference market, typically in a bivariate set-up. More recently a third factor, time scale, has been included in the analysis by several authors. Because of the duality between a time scale and a frequency, small (time) scales are associated with high frequencies while large (time) scales with low frequencies. A natural framework for studying time series with respect to time and scale simultaneously is wavelet analysis (e.g., Percival and Walden (2000)), as it allows a time series to be split into several components, each corresponding to a particular dyadic (i.e., 2 raised to an integer power) frequency band (equivalently, dyadic time scale interval). A collection of wavelet-based applications in finance can be found, for example, in In and Kim (2012). Those dealing with market comovement include the study by Rua and Nunes (2009) who use wavelet squared coherency to measure the extent to which two time series move together over time and across frequencies (see also Madaleno and Pinho (2012)). Gallegati (2012) employs wavelet (contemporaneous) cross-correlations over three smallest time scales ($[2^1, 2^2]$, $[2^2, 2^3]$, $[2^3, 2^4]$ days) to test for market contagion (after filtering out the remaining, long-term component). Lehkonen and Heimonen (2014) assess market comovement by fitting a bivariate DCC GARCH(1,1) model (subject to AR-filtering) to wavelet-decomposed returns of USA and other ten international stock markets. These 11 countries are considered here as well.

In this paper we take a totally new approach in studying comovement from a time-and-scale perspective and apply the so-called thick pen transform (TPT) of Fryzlewicz and Oh (2011), which has a number of very useful properties. In contrast to wavelet analysis, this technique is not restricted to dyadic time scales. It is nonparametric (neither model nor stationarity are imposed), easy to implement and allows for missing values to be present. Based on TPT, Fryzlewicz and Oh (2011) introduce a measure to quantify comovement between two or more time series, thick pen measure of association (TPMA), which is time-varying and visually interpretable. Measure introduced by Baur (2004) shares some of the aspects of the TPT.

Our objective is to use this novel tool to gain new insights into the comovement between 11 international stock markets with respect to level of development, geographical location and (time) scales, and at the same time, demonstrate the usefulness of the TPT in financial applications. Our main result from the bivariate set-up, in which the US market is paired with the remaining 10 countries, is that one of the BRIC (Brazil, Russia, India and China) countries, Brazil, shows higher degree of comovement with the USA compared to the other three and seems to be more similar to Australia and Hong Kong at most time scales (except for the shortest, where it actually is more similar to Canada, UK and Germany). At the largest time scales considered, it is Japan that joins Russia, India and China.

Our main findings from the multivariate analysis in which we examine the role of development and region on market comovement, are that American and European markets show highest degree of comovement and that increasing time trends (peaking around 2010) can be observed. Furthermore, when considering all possible combinations of countries, it turns out that grouping with respect to emerging markets (4 countries) or Asian markets (5 countries) leads to lower degree of comovement compared to other groups of 4 and 5 countries. When five principal groups (developed, emerging, American, European, Asian) are considered in a cross-validation experiment (with group-size adjustment), some expected and unexpected tendencies are uncovered, the latter involve Brazil, Russia, Japan and Hong Kong.

Some of the results based on cross-correlations are similar to those based on the TPMA, but differences exist.

In practical terms, for an US-based investor, short- as well as long-term, Canada, Germany and UK never offer diversification benefits, while China always does. A short-term investor is unlikely to gain benefits by considering Brazil, but a long-term one should consider India, Russia and Japan. Japan, Australia, Hong Kong could, to some extent, be considered useful for the short-term investments, but less so, for the long-term ones. For the international investors (not only US-based), who are interested in time scales of 4-10 days, China might offer some diversification possibilities. Another practical way of using our results is to implement an TPMA-based portfolio strategy and we give an example of such at the end.

The paper is organized as follows. In Section 2, we present the definition of the thick pen transform of Fryzlewicz and Oh (2011) and the thick pen measure of association. In Section 3, we describe the data set, perform bivariate and multivariate comparisons, then all possible comparisons and leave-one-out cross-validation. In the last experiment as well as in the two-market analyses, we provide results based on cross-correlation. To conclude, in Section 3 we provide an example of a portfolio strategy that takes into account short-, intermediate- and long-term holding periods. We summarize our findings and provide a critique of data and methods in Section 4.

2. Methodology

2.1. Thick pen transform

The principal idea behind the TPT can be explained as follows. Imagine plotting a time series $X_1, X_2, \ldots, X_T$ on a piece of paper or using software. This boils down to making a scatterplot of the points (t, X_t) and connecting the points, sequentially, with a line drawn by a pen. Now, imagine repeating the same exercise, but with a thicker pen. And so on. By varying the thickness of the pen, we are able to visualize different features of the data, from high-frequency ones, outlined with small-thickness pens, to low-frequency ones, marked by large-thickness pens. As a result we obtain a time-and-thickness (or time-and-scale) representation of the data, which is not confined to the dyadic scales. Formally, let $X = \{X_t\}_{t=1}^{T}$ be a univariate time series (not necessary stationary) and consider a set of n thickness parameters $\mathcal{T} = \{\tau_1, \tau_2, \ldots, \tau_n\}$, which are positive constants (e.g., positive, increasing integers). For all t and all τ_i define the following random variables

$$\begin{aligned} \tilde{L}_t^{\tau_i}(X) &= \min(X_t, X_{t+1}, \ldots, X_{t+\tau_i}), \\ \tilde{U}_t^{\tau_i}(X) &= \max(X_t, X_{t+1}, \ldots, X_{t+\tau_i}), \end{aligned}$$

which represent the lower and upper boundaries of the area covered be a square pen, respectively. One can view them as the result of applying min/max operator to the overlapping, moving blocks of length τ_i+1. Because at time t we only have observations up to time t available, we will modify the above look-ahead, original definitions of Fryzlewicz and Oh (2011) to the look-back ones, in the following way

$$\begin{aligned} L_t^{\tau_i}(X) &= \min(X_t, X_{t-1}, \ldots, X_{t-\tau_i}), \\ U_t^{\tau_i}(X) &= \max(X_t, X_{t-1}, \ldots, X_{t-\tau_i}) \end{aligned}$$

(if for a given time t the number of lagged observations is smaller than τ_i, then only those are used to calculate the boundaries). We do not impose any finite-moment restrictions on X_t's. The thick pen transform of X is a collection of n pairs of boundaries (hence $2 \cdot n \cdot T$ random variables in total) denoted as

$$TP_{\mathcal{T}}(X) = \left\{ (L_t^{\tau_i}(X), U_t^{\tau_i}(X))_{t=1}^{T} \right\}_{i=1}^{n}. \tag{1}$$

In the top panel of Figure 1 we show the TPT of daily log-returns on S&P500 index between 1997 and 2015 (see Section 3.1 for details) for $n = 4$ thickness

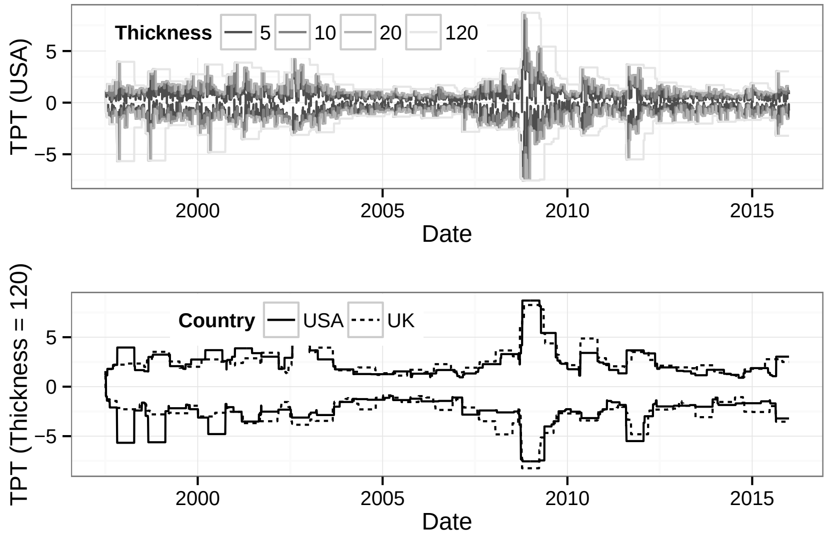

Figure 1: Top: TPT for the (log-returns of) USA for several thickness values. Bottom: TPT for the (log-returns of) USA and TPT for the (log-returns of) UK for the thickness of 120.

values $\{5, 10, 20, 120\}$. Since the resolution of the data is daily, these thickness parameters correspond approximately to physical (time) scales of one week, two weeks, one month and 6 months, respectively, using the trading-day convention. As will be explained in Section 2.1.1, the log-returns have been standardized to a mean-zero and unit-variance sequence. The time series plots of boundaries corresponding to smaller thickness parameters (darker shades of gray) are much rougher compared to the plots of boundaries based on larger thickness parameters (lighter shades of gray). This is in-line with the idea of small thickness parameters capturing the high-frequency portion of the data, while large thickness parameters capturing the low-frequency content. We observe large (in absolute terms) values of the transform, around 2008, 1997, and the beginning of the 2000's. Apart from looking at the transform directly, one can also examine a summary statistic. Fryzlewicz and Oh (2011) suggest the volume and the mean of the pen defined as $V_t^{\tau_i}(X) = U_t^{\tau_i}(X) - L_t^{\tau_i}(X)$ and $M_t^{\tau_i}(X) = (U_t^{\tau_i}(X) + L_t^{\tau_i}(X))/2$, respectively. We use the former in Section 3.5 in a classification experiment.

To conclude this section we emphasize the usefulness of the TPT when it comes to studying financial data. It can be applied to stationary as well as nonstationary time series (without any finite-moment restrictions), it is localized in time and scale and is flexible with respect to the choice of the latter. It is easy to implement and can be computed even if the missing values are present (unless a sequence of NA's exceeds the pen thickness). The last feature is particularly convenient when examining comovement between international stock markets on a daily time resolution.

2.1.1. Thick pen measure of association

To quantify 'comovement' between two or more time series we calculate so-called thick pen measure of association of Fryzlewicz and Oh (2011). Let $X^{(1)} = \{X_t^{(1)}\}_{t=1}^T$ and $X^{(2)} = \{X_t^{(2)}\}_{t=1}^T$ denote two standardized, univariate time series. Since TPMA is intended to measure the overlap between the areas formed by the TPTs of the two time series, it is necessary to put them on the same scale, which can be done by removing their sample means and dividing them by their sample standard deviations (or some other estimators of location and scale, if these are undefined). Let $TP_{\mathcal{T}}(X^{(1)})$ and $TP_{\mathcal{T}}(X^{(2)})$ be their corresponding TPTs for a given set of n thickness parameters $\mathcal{T} = \{\tau_1, \tau_2, \ldots, \tau_n\}$. The TPMA between them, for all t and all τ_i, is defined as

$$\rho_t^{\tau_i}(X^{(1)}, X^{(2)}) = \frac{\min(U_t^{\tau_i}(X^{(1)}), U_t^{\tau_i}(X^{(2)})) - \max(L_t^{\tau_i}(X^{(1)}), L_t^{\tau_i}(X^{(2)}))}{\max(U_t^{\tau_i}(X^{(1)}), U_t^{\tau_i}(X^{(2)})) - \min(L_t^{\tau_i}(X^{(1)}), L_t^{\tau_i}(X^{(2)}))}, \tag{2}$$

which is a bounded random variable, $\rho_t^{\tau_i}(X^{(1)}, X^{(2)}) \in (-1, 1]$ (it does not attain the lower value of -1).

To get a better idea about how TPMA works, focus on the bottom panel of Figure 1, which shows the TPT of USA (solid line) and the TPT of UK (dashed line) for the thickness of 120. We ask ourselves the following question: what is the proportion of the overlap between the areas circumscribed by the solid and dashed lines as we move in time, i.e., what is the value of the TPMA? We start off with a high level of overlap close to 1 (100%), then there is a decrease to about 0.5 as the UK borders fit loosely within the USA borders, then we are briefly back to about 1, down again to about 0.6 and up to 1 prior to year 2000. After 2000, the proportion of the overlap is down to about 0.6, then up again to about 0.8 and so on. To further build intuition about TPMA, consider three examples presented in Figure A.1. Assume that t and τ_i are fixed (suppress the sub- and super-script notation for the moment) and that the vertical range of the grid is from 0 to 6. In the left-most case, the vertical intervals are $[L(X^{(1)}), U(X^{(1)})] = [0, 1]$ and

$[L(X^{(2)}), U(X^{(2)})] = [5, 6]$ and yield TPMA of $-\frac{4}{6}$. The denominator corresponds to the length of their union (or more precisely, the shortest interval containing their union), 6, the numerator to the length of the gap between them, 4, and the minus indicates that they are disjoint. In the middle case, we have $[L(X^{(1)}), U(X^{(1)})] = [0, 3]$, $[L(X^{(2)}), U(X^{(2)})] = [3, 4]$ and TPMA is 0 since the length of the gap is 0. In the last case, $[L(X^{(1)}), U(X^{(1)})] = [0, 4]$, $[L(X^{(2)}), U(X^{(2)})] = [2, 5]$ and TPMA is $\frac{2}{5}$. This value is the ratio of the length of the overlap (hence TPMA is positive) of the two intervals, 2, to the length of their union, 5. Therefore, if for a given pen thickness τ_i and time t, two (or more) time series 'move together', their TPTs will overlap and hence the resulting TPMA will be close to 1. On the other hand, if the time series are out of sync, their TPTs will not overlap and the resulting TPMA will be negative. Nevertheless, it is important to keep in mind that for some thickness values (notably 'large' ones), two independent time series will inevitably yield TPMA close to 1.

In Figure 2 we show TPMA between (standardized) daily log-returns of S&P 500 and of FTSE 100 (see Section 3.1 for details) for the same thickness parameters as before. For the weekly time-scale ($\tau_i = 5$), TPMA oscillates roughly between 0 and 1, and on average, is approximately equal to 0.6. The oscillation range decreases as thickness increases, while the time-averaged TPMA increases. The time-averaged TPMA between $X^{(1)}$ and $X^{(2)}$, for a given thickness τ_i, is

$$\bar{\rho}^{\tau_i}(X^{(1)}, X^{(2)}) = \frac{1}{T} \sum_{t=1}^{T} \rho_t^{\tau_i}(X^{(1)}, X^{(2)}). \tag{3}$$

A plot of $\bar{\rho}^{\tau_i}$ versus τ_i for several values of τ_i is labelled by Fryzlewicz and Oh (2011) as a cross-spectrum.

The smooth lines, which will appear in the upcoming sections, are obtained from a fit of a generalized additive model (GAM) to the data, which includes a cubic regression spline smooth (specifically, its shrinkage version, see the `R` package `mgcv` for details). GAM is a default smoothing method for the `R` package `ggplot2`. This package is used to visualize the results of the current paper.

As already mentioned, Fryzlewicz and Oh (2011) provide a definition of the TPMA for the multivariate case, which boils down to modifying equation (2) accordingly. Namely, given K (standardized) time series $X^{(1)} = \{X_t^{(1)}\}_{t=1}^T$, $X^{(2)} = \{X_t^{(2)}\}_{t=1}^T$, …, $X^{(K)} = \{X_t^{(K)}\}_{t=1}^T$, the TPMA between

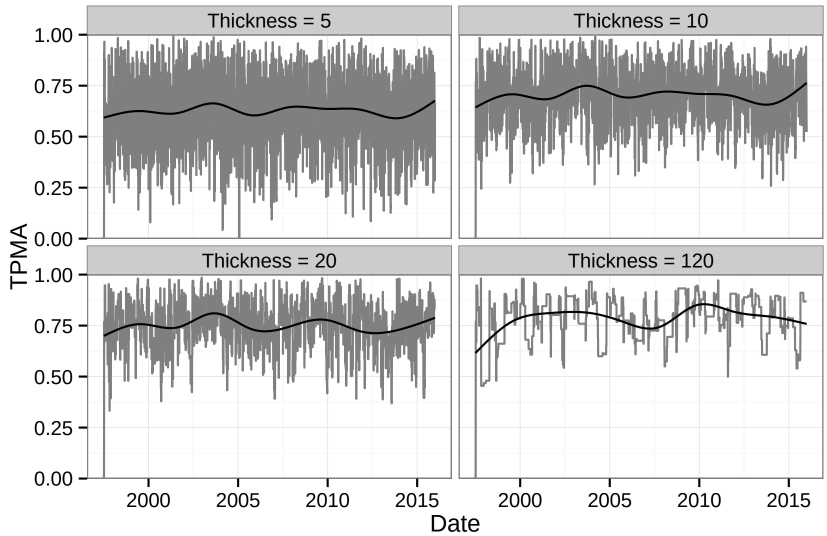

Figure 2: TPMA between the (log-returns of) USA and UK and smooth lines.

them is

$$\rho_t^{\tau_i}(X^{(1)}, \ldots, X^{(K)}) = \frac{\min_k(U_t^{\tau_i}(X^{(k)})) - \max_k(L_t^{\tau_i}(X^{(k)}))}{\max_k(U_t^{\tau_i}(X^{(k)})) - \min_k(L_t^{\tau_i}(X^{(k)})),} \tag{4}$$

for all values of t and τ_i. The properties of this multivariate measure are the same as those of its bivariate counterpart.

For more information about the thick pen transform, thick pen measure of association and more examples, we refer the Reader to Fryzlewicz and Oh (2011).

2.2. Comparison of TPMA with traditional comovement measures

Among the standard comovement measures used in finance, there is no single one that can match all characteristics of TPMA (non-parametric, time-varying, capable of capturing information from different time scales and applicable to bi- as well as multi-variate comparisons), therefore a complete set of parallel analyses to those based on the TPMA presented henceforth, is not attainable. However, in the bivariate case, we can make partial comparisons based on the cross-correlation coefficient. To make correlation time-varying, we combine it with the rolling-window approach on the daily data (at the expense of the frequency perspective) or with the multi-period returns to capture the multi-frequency content (but loosing the time-varying property). Other approaches mentioned in the introduction tend to impose a particular structure on the data-generating mechanism or on the comovement dynamics, or lack the multi-frequency outlook.

3. Empirical results

3.1. Data

The data comprises daily log-returns on adjusted closing prices (in USD) of stock market indices from 11 countries: USA (S&P 500), Canada (S&P 500 TSX), Brazil (IBOVESPA), UK (FTSE 100), Germany (DAX 30), Russia (RTS), Japan (Nikkei 225), Australia (S&P ASX 200), Hong Kong (Hang Seng), India (S&P BSE SENSEX), China (SSE Composite). The same 11 countries were studied in Lehkonen and Heimonen (2014) and are representative in terms of the geographical location (the first three are from America, the next three from Europe and the remaining five from Asia) and the level of development (seven developed and four emerging). The quotes were downloaded from `yahoo` via an `R` routine `get.hist.quotes` and span the period of time from 01/07/1996 to 31/12/2015. The exchange rates were obtained with `getFX` (data obtained in such a way combined with our `R` code allows the results to be reproduced). In Table A.1 we give descriptive statistics of the log-returns, which reflect typical features of such time series (we removed one observation from Brazil on the 15/01/1999 and one from Germany on the 14/12/1998). The number of observations is around 4500 per country and differs from one country to another, which we allow for. This is because (in most cases) we are able to compute the TPT without the need to adjust for the non-synchronous periods (e.g., due to holidays).

3.2. Bivariate comovement: USA and others

We begin our empirical analysis by considering bivariate comparisons in which we pair the log-returns of USA with the log-returns of the other ten countries. We calculate TPMA for $n = 32$ thickness values $1, 2, \ldots, 30$, $60, 120$ and in Figure 3 we show (smoothed) TPMA for selected values of the thickness parameter, 1-5, $10, 15, \ldots, 30, 60, 120$.

For comparison, we calculate rolling-window correlations for several window lengths $5, 10, \ldots, 30$, $60, 120$. Specifically, at a given day t we use $5, 10, \ldots, 30$, $60, 120$ pairs of observations ending on date t (with the missing

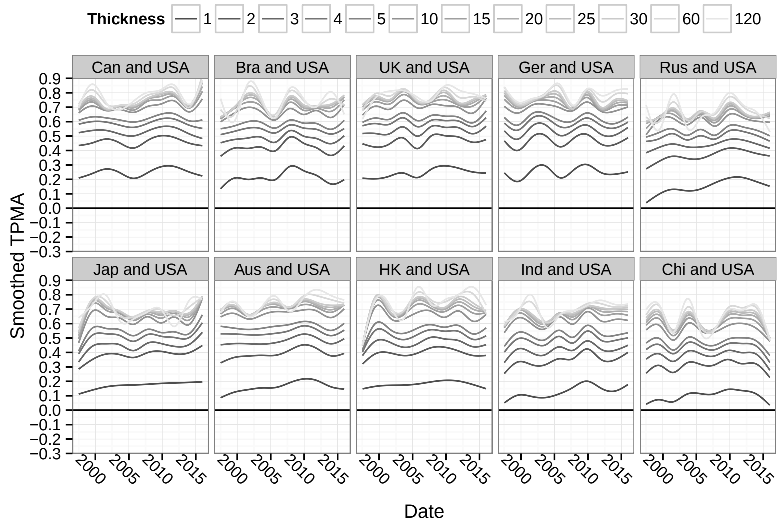

Figure 3: TPMA (smoothed) for the log-returns of USA paired with the log-returns of the other ten countries.

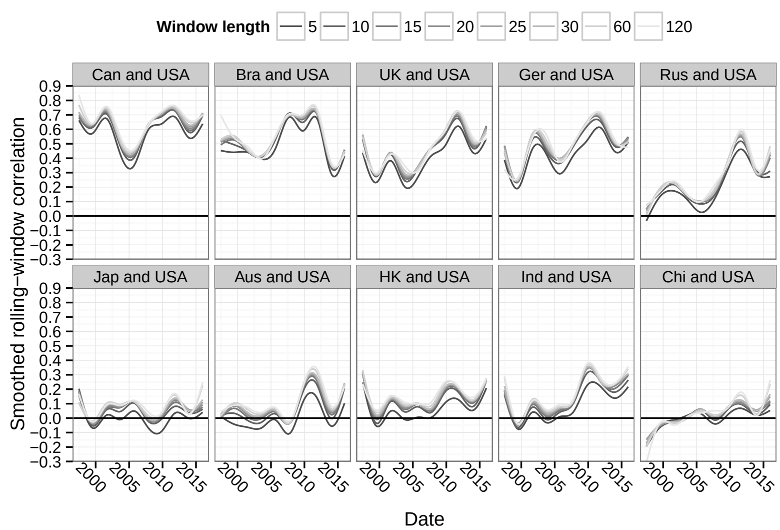

Figure 4: Rolling-window correlation (smoothed) for the log-returns of USA paired with the log-returns of the other ten countries for several values of the window length.

observations removed) to calculate the correlation coefficient for that day. Although we vary the window length, the correlations are only able to capture the comovement on a single time scale (time scale of one day) and hence it is not surprising that the curves depicted in Figure 4, smoothed rolling-window correlations, are similar. Because TPMA has the ability to disentangle comovement scale-wise, the curves in Figure 3 differ much more, especially for the first few thickness values. The largest increase, of about 0.2, is between the thickness of 1 and 2 days, then, an increase by about the same amount between the thickness of 2 and 5 days, and finally a change of about 0.1 for the remaining ones. What can also be gleaned from Figures 3 and 4, is that while the (smoothed) TPMA for the thickness of 1 (corresponding roughly to the daily scale) varies between 0 and 0.3, the (smoothed) rolling-window correlation falls between 0 and 0.8, and of course the latter quantifies the linear relationship, while the former is not limited to such.

TPMA between (the log-returns of) the USA and the following three countries: Russia, India and China is overall smaller compared to that of the USA and the other countries (for all thickness values τ_i). This ranking, which includes the smallest thickness value of 1, can be compared with the rolling-window correlations. Based on Figure 4, USA generally comoves least (correlation oscillating around 0) with Japan, Australia, Hong Kong, India and China (the lower-panels countries). It comoves somewhat more with Russia, then more with the UK and Germany, and finally it comoves most with Canada and Brazil (correlation of about 0.5). This suggest a strong impact of the geographical location on the daily comovement. Returning to the TPMA with $\tau_i = 1$ and Figure 3, we see that the most comoving countries with the USA are Canada, Brazil, UK, Germany and the remaining countries fall between this group and the least-comoving triplet mentioned earlier. In both Figures, 3 and 4, we observe several dips and hikes.

In Figure 5 we present the time-averaged TPMA (3) plotted against thickness $1, 2, \ldots, 30, 60, 120$, i.e., the cross-spectrum, for the same pairs of countries as in Figure 3. To make the viewing of the cross-spectrum for the smaller thickness values better, in the left panel, the scale of the horizontal axis was transformed using log to base 5 function (1 corresponds to 1 week). The right panel shows the original variables. The country labels appear at thickness values 1-5, $10, 15, \ldots, 30, 60, 120$. Other-than-the-mean summary statistics (first quartile Q1, median and third quartile Q3) of the bivariate TPMA can be found in Table A.2.

For comparison, we also calculate correlations of the multi-day log-returns

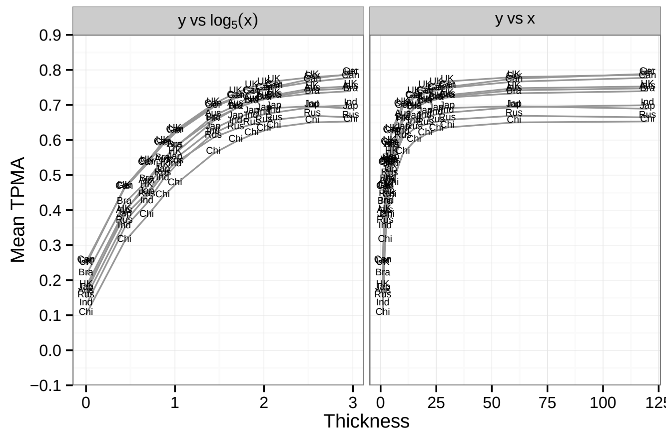

Figure 5: Mean TPMA for the log-returns of USA paired with the log-returns of other ten countries. Left: $\log_5$-transformed x-axis. Right: untransformed.

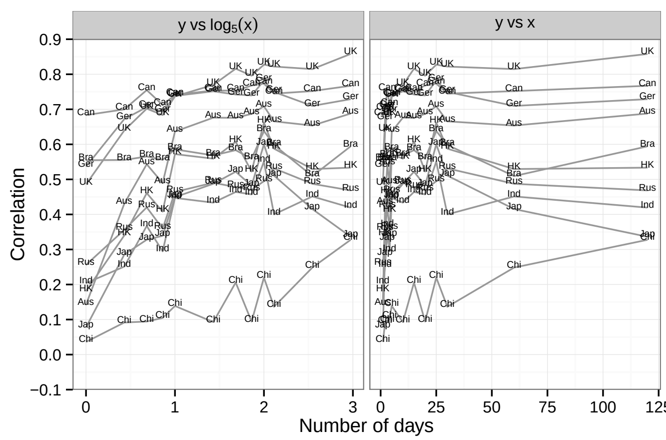

Figure 6: Correlation of the multi-day log-returns of USA paired with the multi-day log-returns of other ten countries. Left: $\log_5$-transformed x-axis. Right: untransformed.

of USA paired with the multi-day log-returns of the other ten countries (multi-day log-returns are obtained by summing the daily log-returns corresponding to a given period). We plot them in Figure 6, using the same lay-out as in Figure 5. Note that one point in Figure 5 is obtained by averaging (over time) many values of the TPMA for a given τ_i, while one point in Figure 6 corresponds to a single correlation obtained from the data recorded at a given frequency (data which will become scarcer and scarcer as we decrease the frequency, i.e., increase the period length).

Based on Figure 5 and using USA as a reference country, Canada, UK and Germany, have similar mean TPMA, for all thickness values, from about 0.3 to about 0.8, which is also the largest compared to other countries. China has the smallest mean TPMA with the USA, which is consistently lower by about 0.2 compared to Canada, UK and Germany. Just above China, we have India, but only for time scales up to a week. For longer time scales, it is Russia that becomes the closest to China. The biggest gap between the mean TPMA of China and its neighbour is about 0.05 and is reported for thickness values from 4 to 10. The cross-spectrum of Japan, starting at about 0.2 and ending at about 0.7, is half-way between that of China and the cross-spectra of Canada, UK and Germany, but eventually it moves closer to those of India, Russia and China. The cross-spectra of Australia and Hong Kong follow that of Japan very closely, with somewhat higher values than Japan's for time scales longer than a week, before they finally move away towards Canada, UK and Germany. Although the mean TPMA of Brazil starts off closer to those of Canada, UK and Germany, at about 0.25, it quickly moves towards the values of Australia and Hong Kong, opening a gap of about 0.05 between itself and the triplet.

Moving onto multi-day correlations and Figure 6, we note that they tend to increase with increasing period length (hence, decreasing frequency) and are somewhat more variable compared to the mean TPMA. The impact of geographical location on the daily correlation, observed earlier in Figure 6, is also visible, including the gap between Russia and UK. China's correlation, ranging from about 0.05 to 0.3, is the smallest for all multi-periods. The largest correlation is between the USA and either Canada (periods up to a week) or the UK (longer ones). Germany's correlation is high (0.75) and similar to that of Canada and the UK for periods between 2-25 days, but drops by about 0.1 for the shortest (similarly to the UK) and the longest (similarly to Canada) periods considered. For the daily data, correlations of the USA with Japan, Australia, Hong Kong, India and Russia are fairly low, but increase to about 0.3-0.6 for the other periods, and to 0.7 in case of Australia. Correlation of the USA and Brazil remains at about 0.55 for all periods.

To summarize, we can say that all but one BRIC countries, Brazil, show lower degree of comovement with the USA, compared to other countries, irrespective of the time scales and hence could offer diversification opportunities for long- as well as short-term investors from the USA. This is particularly true for China, based on both the mean TPMA and the correlation. Also according to both measures, for time scales of several days, there are three comoving groups: Canada, UK, Germany at the upper extreme, China at the lower and the rest in between. At the largest time scales and TPMA-wise, Australia, Brazil and Hong Kong from the middle pack drift away towards the upper end, while Japan, India and Russia towards the lower end. Correlation-wise, two groups seem to emerge, one with the UK, Canada, Germany and Australia and one with Brazil, Hong Kong, Russia, India, Japan and China. Both measures suggest that development is not necessarily a good grouping factor at low frequencies and this is consistent with the results of Lehkonen and Heimonen (2014). Also in line with Lehkonen and Heimonen (2014), we observe high degree of comovement of Canada, UK, Germany and Brazil at small time scales (somewhat smaller for Brazil). The lowest degree of comovement (with the USA) for Japan and Australia at the shortest time scale reported by these authors is replicated by the multi-period correlation, but not by the mean TPMA.

3.3. Multivariate comovement: development and region

We now assess the impact of development and region on markets comovement by calculating multivariate TPMA from equation (4) for the same thickness values as in the previous section. For obvious reasons, a parallel multivariate analysis based on correlation is not possible.

There are seven developed (USA, Can, UK, Ger, Jap, Aus, HK) and four emerging (Bra, Rus, Ind, Chi) markets. As regards region, there are three American (USA, Can, Bra), three European (UK, Ger, Rus) and five Asian (Jap, Aus, HK, Ind, Chi) markets. Due to the differences in size, we can expect that larger groups will produce lower TPMA compared to the smaller ones. We address this issue later in Section 3.4.

In Figure 7 we show smoothed TPMA taking into account grouping with respect to development and in Figure 8 with respect to region, for selected values of the thickness parameter, 1-5, $10, 15, \ldots, 30, 60, 120$. In both figures, the vertical axis spans the interval from -0.25 to 0.75 and has slightly smaller range (equal to 1) than the axis in Figure 3 (equal to 1.2). Bivariate TPMA is larger than the multivariate TPMA, as anticipated, and is corroborated by the summary statistics of Table A.3.

Based on Figure 7, it seems that the two groups are fairly similar, although one has to remember that the first group has more members than the second one. Both groups show, on average, negative multivariate TPMA at the daily time scale, which seems to be increasing with time, reaching a plateau in the developed-markets group aournd 2010 and in the emerging-market group peaking in after 2010. At up-to-one-week time scales, we also note an increase, with the values of the multivariate TPMA between 0 and 0.3. At larger time scales, we observe several lows and highs. When comparing the marginal distributions of the two TPMAs based on quartiles (top two panels of Table A.3) it seems that they are quite similar.

Moving now onto Figure 8, we see that the comovement within the American and the European regions is overall stronger than within the Asian region. Again, we need to keep in mind that the latter group consists of five countries compared to the former groups, with three countries each. At the shortest time scale, there is an increase in TPMA from around 0 by about 0.1 in the American and the European groups, and by the same amount from around -0.2 in the Asian group. Several ups and downs, especially at the largest time scales, can be observed in the European group, and likewise in the American and Asian groups, but are not as prominent. The last three panels of Table A.3 indicate similarity between American and European groups and a lower degree of comovement of the Asian countries.

Figure 9 gives the overall picture of the impact of development and region via the cross-spectra of all five groups. As noted previously, the developed and the emerging groups are similar, and in terms of their average TPMA, fall between the Asian and the non-Asian groups, staying closer to the former than to the latter. The degree of comovement for the American markets is slightly stronger than for the European. It seems that the inclusion of developed, Asian countries in the developed group, weakens this group's degree of comovement. Geographical location is a preferred clustering factor for non-Asian countries. Some increasing time trends can be observed in the multivariate (smoothed) TPMAs. These were not so apparent in the bivariate TPMAs.

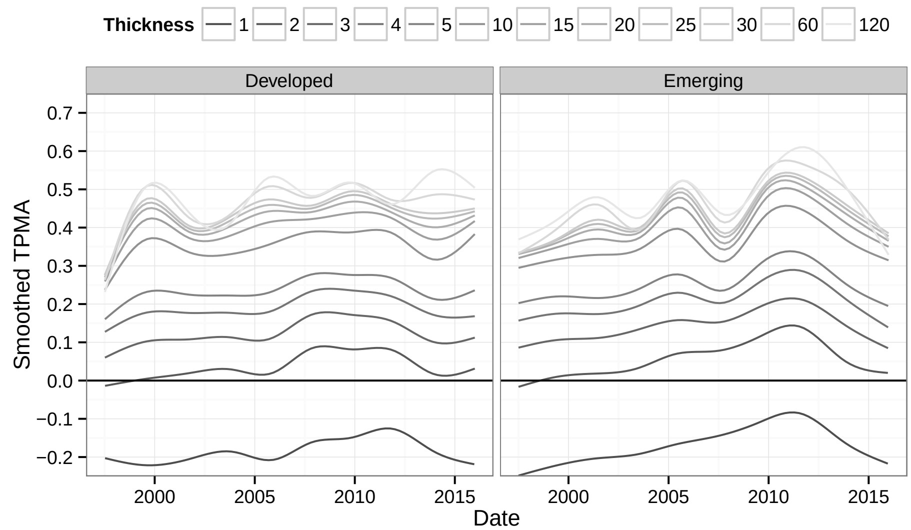

Figure 7: TPMA (smoothed) for developed (USA, Can, UK, Ger, Jap, Aus, HK) and emerging (Bra, Rus, Ind, Chi) markets.

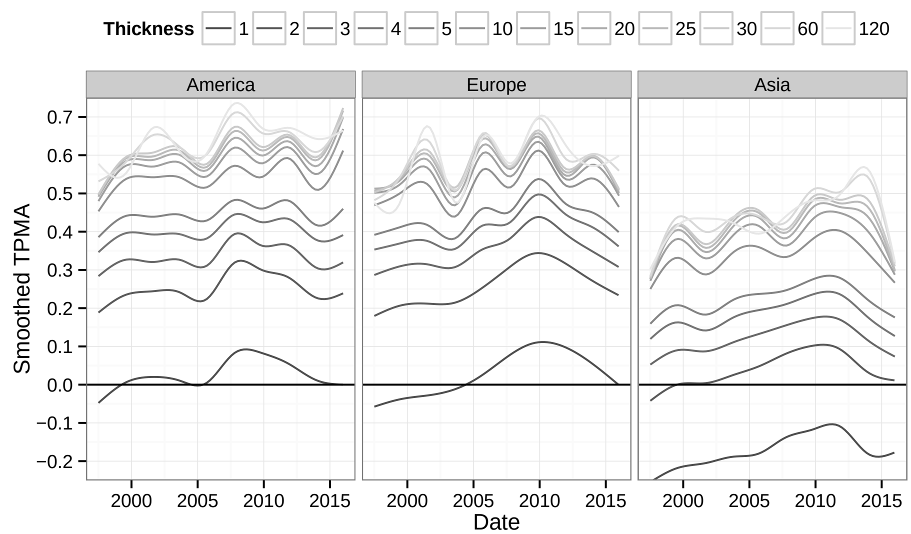

Figure 8: TPMA (smoothed) for American (USA, Can, Bra), European (UK, Ger, Rus) and Asian (Jap, Aus, HK, Ind, Chi) markets.

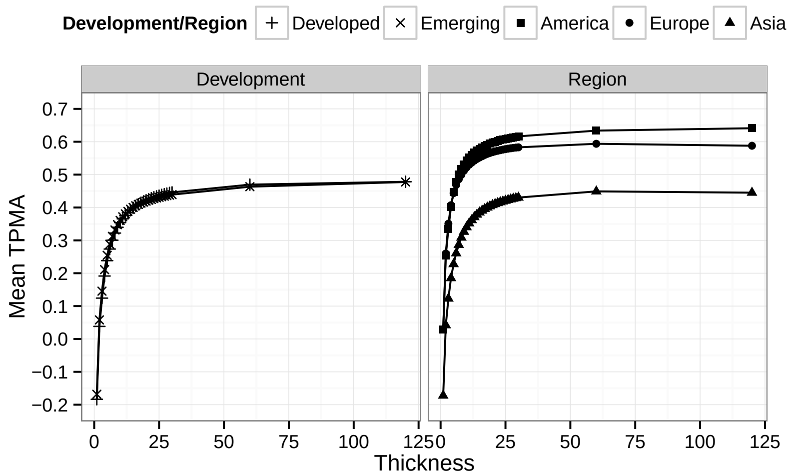

Figure 9: Mean TPMA for the log-returns of countries grouped by development (left) and region (right).

3.4. Bivariate and multivariate comovement: all possible combinations

The objective of this section is to examine the effect of the group size on the TPMA. Because smaller groups will inadvertently lead to smaller values of the TPMA, we wish to consider all group sizes, from 2 to 11 (2036 in total), and for a fixed group size, compare all possible combinations. The number of different combinations of size $K = 2, 3, \ldots, 11$ is determined by the binomial coefficient $\binom{11}{K}$ and is equal to $55, 165, 330, 462$ for the first four, $462, 330, 165, 55$ for the next four, finally, there are 11 different combinations of 10 countries and only one with all 11 countries. The group sizes we examined so far were: 2 (USA paired with other markets), 3 (American and European markets), 4 (emerging markets), 5 (Asian markets) and 7 (developed markets). We would like to determine their ranking, in terms of the mean TPMA, with respect to other groups of the same size and hence assess if factors such as development or geographical location are useful grouping factors.

In Figure 10, we show cross-spectra for all possible combinations of countries, starting with the combinations of 2 countries in the top left and ending with that of 11 in the bottom right, for thickness values $1, 5, 10, 20, 30, 60, 120$. For comparison with the analyses carried out in Sections 3.2 and 3.3, we add black lines corresponding to the cross-spectra from Figures 5 (right panel)

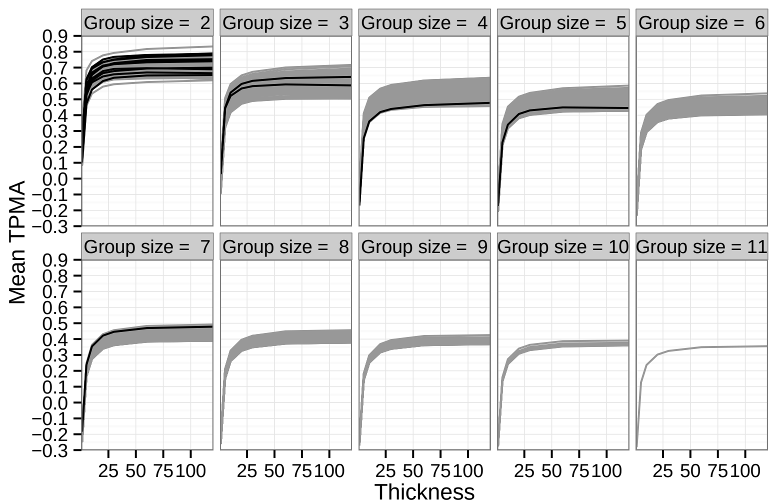

Figure 10: Mean TPMA for the log-returns of all possible combinations of 2, 3, . . ., 11 countries.

and 9. Complete numerical summaries are available upon request.

First of all, we notice that the mean TPMA decreases as the group size increases and likewise the variability among the cross-spectra. For the smallest group size, UK and Germany have the largest mean TPMA for all time scales, while Russia and China the smallest. When examining triplets, the cross-spectrum of the European group is ranked in the middle, with that of the American group just above it. The top triplet in terms of the comovement consists of USA, UK and Germany and the three countries that move together least are USA, Russia and China. Among the quadruples, the BRIC countries are at the lower end of the ranking, just above USA, Russia, India, China that have the smallest mean TPMA. The top ranked are USA, Canada, UK and Germany. Within the quintuples, the Asian countries are also placed at the lower end, above Can, Russia, Japan, India and China. The top of the group is occupied by USA, Canada, UK, Germany and Australia. The top sextuple comprises USA, Canada, UK, Germany, Australia and Hong Kong while the bottom sextuple is made up of Canada, Brazil, Russia, Japan, India and China. Among the septuples, the developed group comes second in the ranking, while the top group has the same countries, except for Japan, which is replaced by Brazil. The least comoving septuple is formed by the BRIC countries combined with USA, Japan and Australia.

We skip the discussion of the remaining K-tuples.

To sum, compared to all possible combinations of the same size, classification with respect to emerging or Asian factors, over all thickness values, does not lead to homogeneous groups. The omnipresence of the spectra of the USA, Canada, UK and Germany among the largest spectra for groups of 2-6 countries (top panels of Figure 10) confirms the close relationship among those countries, over various time scales, while that of USA/Canada, Russia, India and China among the smallest spectra, indicates their low degree of comovement.

3.5. Leave-one-out cross-validation: which of the five groups attracts most and at which time scales?

To further assess the impact of development and region as grouping factors, obviously keeping track of the time scales, we would like to perform a leave-one-out cross-validation. Specifically, we wish to determine, which of the five categories: developed, emerging, American, European or Asian is the dominant one, when it comes to the assignment of a given country at a selected time scale. To overcome the issue of comparing groups of different sizes, we propose to form groups of size 2, by randomly selecting 2 countries from each category. We will use volume of the log-returns as a representation of each country and a Euclidean distance, divided by the square root of the length, as a distance. The procedure (see Section 3.3 of Fryzlewicz and Oh (2011)) is as follows: Calculate volume $V_t^{\tau_i}$ for all 11 countries for thickness values $1, 2, \ldots, 60$. Take the first thickness and select one of the 11 countries. From the remaining countries, form five groups: developed, emerging, American, European or Asian by selecting 2 countries at random with the corresponding labels. Find the centroid of each group by averaging contemporaneously all volumes, i.e., two, from that group. Calculate the distance between the selected country and the five centroids. Assign the country to the closest group. Repeat the process for the other countries. Then continue, in the same manner, for other thickness values. Repeat the whole procedure many times, e.g., 500 times and report the proportion of times a given country, at a given scale was classified to one of the five groups. The results of the above scheme are presented in Figure 11. On the vertical axis we have the proportion, on the horizontal the pen thickness and different symbols represent different groups. Each subplot corresponds to one country.

For comparison, we perform an identical experiment based on multi-period absolute log-returns (to match the volume-based summary in the TPT analysis) and display the corresponding results in Figure 12.

Let's begin our discussion with the TPT-based results and countries that, for more than 50% of the time, are assigned to their correct geographical group for small thickness values and their correct development group for the remaining ones. These are USA, Canada, UK, Germany and Australia. Over longer time scales, the second best choice for the UK and Germany is the American group, European group for Canada, Australia, and to some extend for the USA, which also ends up in the developed group. Brazil starts off as an American country, but then becomes an European country. Hong Kong is also classified as a European country (for all time scales and more than 50% of the time) and likewise Japan, which shows inclination towards Europe at the longest scales. India comes out as an Asian country for short time scales, but ends up in the developed group for the long ones. This is not the case for China, which turns out to be an Asian country for all thickness values. Finally, Russia fails to be classified according to its correct group with respect to either development or region: it begins as a developed country and ends up as an American country. The emerging group almost never attracts any of the countries, and only occasionally serves as the 3rd or 4th alternative for countries such as Russia, Japan, Hong Kong and China.

Somewhat similar conclusions can be drawn from Figure 12, but the picture is (literally) not as clear. Period-dependent allocations for the USA, UK, Russia and Australia are generally in-line with the thickness-dependent selections of Figure 11, and to some extend with those for Germany, India and China. However, Canada seems to end up in the developed group not only at small time scales but at all time scales and Brazil moves between American and European groups. Japan seems to lean towards the developed group and Hong Kong towards developed or Asian, but these tendencies are quite obscured.

In summary and based on the TPT, geographical location is a predominant factor at small time scales for the USA, Canada, Brazil, UK, Germany, Australia, India and China. The level of development determines, at large time scales, correct allocation of all the developed countries except for Japan and Hong Kong. For such time scales, these two countries are incorrectly assigned to the European group, which is also the case for Brazil, while India is incorrectly classified as a developed country. Another puzzle is Russia.

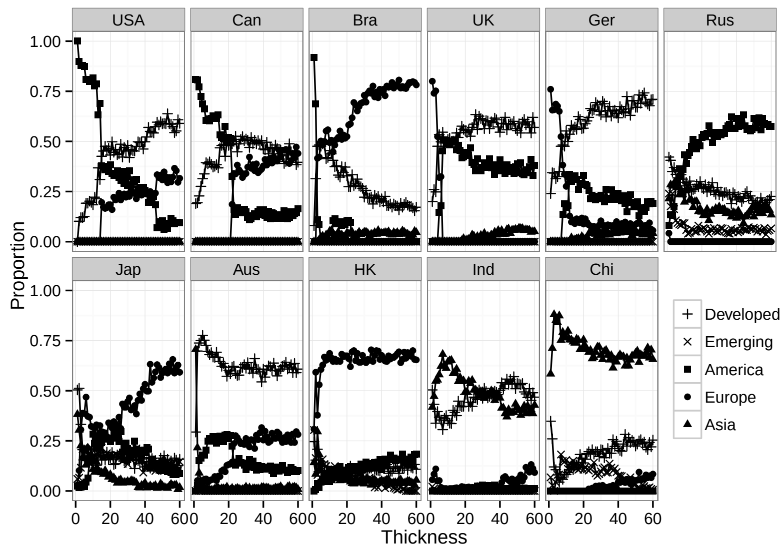

Figure 11: Proportion of times (in 500 replications) a given country was assigned to one of the five groups based on the TPT's volume.

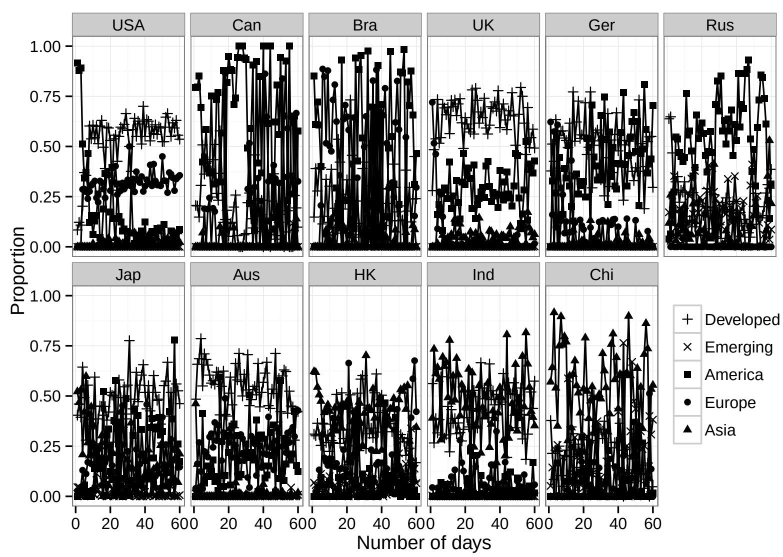

Figure 12: Proportion of times (in 500 replications) a given country was assigned to one of the five groups based on the multi-period absolute returns.

3.6. Example of a time-scale-dependent portfolio strategy

We now provide one way of incorporating our results into a portfolio strategy. We associate a long-term investor with long holding periods (e.g., over one year) and a short-one with short ones (e.g., under one year). Let's assume that we are based in the USA and are looking for another country to diversify our portfolio (two-market portfolio). We employ equal weights and a buy-and-hold strategy (non-overlapping holding periods).

Assuming that a long-term investor concentrates on large time scales when choosing another country for international diversification, our strategy should be based on the TPMA over such time scales. Specifically, for a thickness value matching the length of the holding period (and hence the investor type), we use the average value of the TPMA in the portfolio evaluation period (with or without a gap between the evaluation and formation periods). The length of the evaluation period need not to be equal to the length of the holding period, but the information captured by the values of the TPMA from the evaluation period, should reflect the comovement of the countries over time scales aligned with the length of the holding period. The averaging of the TPMA over evaluation dates makes the ranking rule operable, but other specifications are obviously admissible. Also, TPMA like any other measure of comovement is more about the risk rather than about the risk-and-reward, so one would need to modify such a rule accordingly to take the latter into account.

Our procedure, based on the daily log-returns, is as follows. Let H and E be the lengths of the holding and evaluation periods, respectively, and G the length of the gap between the two (all in days). For a given portfolio formation time t and thickness τ_i equal to the length of the holding period:

1) Calculate the TPMA between the USA, $X^{(1)}$, and other countries, $X^{(2)}$'s (standardized data).
2) Find the mean TPMA over the E-day period ending $G+1$ days before time t, defined as

$$\bar{\rho}^{\tau_i}_{t,G,E}(X^{(1)}, X^{(2)}) = \frac{1}{E} \sum_{j=t-G-E}^{t-G-1} \rho^{\tau_i}_j(X^{(1)}, X^{(2)})$$

(the gap indices are $t-G, \dots, t-1$ for $G > 0$). Using $\bar{\rho}^{\tau_i}_{t,G,E}$, rank the countries from the least- to the most-comoving with the USA.
3) Over the period from t to $t+\tau_i-1$, hold an equal-weight portfolio of the USA and its least-comoving international counterpart.

4) At time $t + \tau_i$, repeat steps 1)-3).

Let's consider a particular scenario in which the holding periods are 3 months, 1 year and 2 years (short-, intermediate- and long-term investments), $H \in \{63, 252, 504\}$, without and with an one-month gap, $G \in \{0, 21\}$, and where the evaluation period consists of 1 month, 6 months and 1 year, $E \in \{21, 126, 252\}$. This particular choice is inspired by the momentum-related studies, predominantly based on monthly data. There the typical choices for E and H are $3, 6, 9, 12$ months, with one month skipped (or not) between E- and H-related dates, but other variants haver recently been considered as well (e.g., Novy-Marx (2012), Gong et al. (2015)). For given values of G and E, the first formation date requires $G + E$ lagged observations available. Hence for the dates corresponding to the beginning of the study period there will be partial overlap of the values of the TPMA for different τ_i's due to the pen thickness exceeding the number of past observations accessible, unless such dates are skipped. If one wished to construct a decision rule based on past multi-period correlations, then the evaluation period would need to be much longer than the holding period to obtain reliable estimates. In addition, it would not be possible to extend such rule to more than two-markets, which is not the case with the TPMA-based strategy.

In our experiment, apart from constructing a two-market portfolio with the USA and its least-comoving match, we also construct a two-market portfolio consisting of the USA and its most-comoving match. The corresponding means, standard deviations and t-statistics are given in Table A.4.

In most cases we observe positive means and hence positive, albeit statistically insignificant, t-statistics. Standard deviations for the least-comoving portfolios are generally smaller, which we would hope for, compared to the most-comoving portfolios. The tendency in the behaviour of standard deviation with respect to the increasing thickness and increasing length of the evaluation period does not seem to be clear-cut, but some level of decrease can be noticed. It seems that when the thickness is large (504) the evaluation period needs to be large as well (for thickness of 504 and E of 21 or 126, standard deviations are higher for the least-comoving portfolios than for the most-comoving portfolios, but when $E = 252$, standard deviations are also what could be expected). The presence or the absence of the one-month gap seems to have no bearing on the results.

There are many configurations in which our strategy could be explored. To begin with, more than two markets and different values of τ_i, E, G could be employed. Additionally, different weights and overlapping holding periods could be considered. We leave that for the future work.

4. Conclusions

In this paper we employ a novel approach of Fryzlewicz and Oh (2011), thick pen transform, to study comovement between international stock market returns. The methodology is easy to implement and leads to a definition of a quantitative measure which is time-dependent, suitable for multivariate as well as bivariate comparisons over different time scales, which do not need to be dyadic. We examine 11 countries that represent different levels of development and several geographical locations: USA, Canada, Brazil, UK, Germany, Russia, Japan, Australia, Hong Kong, India and China.

In bivariate comparisons with the USA, Russia, India and China stand out (mean TPMA of USA and China is always below all the other TPMAs) and Brazil seems to mimic more Australia and Hong Kong, except for the shortest time scales. At the longest time scales, Japan drifts away towards Russia, India and China. UK, Germany and Canada show a high degree of comovement with the USA.

In multivariate comparisons, we observe an increase in (smoothed) TPMA from the end of the 90's towards 2010, with respect to development and region. On average, the developed and emerging groups are similar and fall between Asian and non-Asian groups, the latter two showing high degree of comovement. Compared to groups of the same sizes, BRIC countries and Asian countries come last in the ranking in terms of their average comovement. When matched against five, equally-sized groups of two, most of the countries exhibit scale-dependent preferences, which tend to agree with their geographical location and level of development (USA, Canada, UK, Germany, Australia) or geographical location only (Brazil, India). The situation at large scales is more complicated for Brazil, Russia, Japan and Hong Kong. On the other hand, China is always classified as an Asian country.

The results based on cross-correlations partly coincide with those based on the TPT/TPMA, but discrepancies exist.

To conclude we mention some of the weaknesses of our sample data set and methods. To study carefully whether development and geographical location are useful grouping factors, we would need much larger dataset, preferably all the stock markets. This would be particularly useful in Section 3.5. Regarding the method, one needs to bear in mind that scaling of the data impacts TPT and TPMA. Hence the critique of Forbes and Rigobon (2002), that correlation tends to increase during crisis periods due to changing volatility, could come into play (see also the simulated example therein). There are several scaling-related issues to consider: whether the time series are stationary or not; whether to scale the time series sub-period-wise or for the entire period; what kind of scaling to use (for example, instead of standardizing the data as in the current paper, one can scale the data to an $[0, 1]$ interval by subtracting the smallest observation from it and then dividing by the range); whether the comovement is monitored in real-time or ex-post. All these aspects have to be carefully thought through in the context of a particular application.

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Appendix

Table A.1: Descriptive statistics of log-returns.

Country	mean (in %)	SD (in %)	min	max	skewness	kurtosis	# of obs.
USA	0.0147	1.2572	−0.0947	0.1096	−0.2058	10.7066	4491
Can	0.0130	1.3021	−0.1162	0.1127	−0.7642	12.2214	4644
Bra	−0.0155	2.4946	−0.2102	0.2314	−0.2105	10.4054	4411
UK	0.0039	1.3180	−0.1086	0.1088	−0.2073	9.6755	4802
Ger	0.0132	1.6328	−0.0907	0.1119	−0.1475	6.6040	4628
Rus	0.0071	2.5586	−0.2120	0.2020	−0.4620	11.3927	4640
Jap	−0.0209	1.5630	−0.1111	0.1087	−0.2436	6.5199	4344
Aus	0.0051	1.4267	−0.1286	0.0854	−0.5406	9.2563	4595
HK	−0.0034	1.6880	−0.1473	0.1732	0.0069	12.7450	4450
Ind	0.0092	1.7100	−0.1181	0.1845	−0.0346	9.3203	4343
Chi	0.0248	1.6080	−0.0918	0.0940	−0.2574	7.6089	4646

Table A.2: Descriptive statistics of bivariate TPMA.

		Thickness									
Countries	Stat	1	2	3	4	5	10	20	30	60	120
CanUSA	Q1	0.06	0.30	0.39	0.46	0.50	0.60	0.64	0.66	0.70	0.72
	Median	0.26	0.48	0.55	0.61	0.64	0.71	0.75	0.77	0.78	0.78
	Q3	0.49	0.65	0.70	0.74	0.75	0.81	0.83	0.84	0.85	0.86
BraUSA	Q1	0.02	0.26	0.33	0.41	0.46	0.55	0.60	0.62	0.63	0.63
	Median	0.23	0.43	0.50	0.55	0.59	0.67	0.73	0.74	0.76	0.78
	Q3	0.46	0.60	0.66	0.70	0.72	0.77	0.82	0.83	0.85	0.85
UKUSA	Q1	0.06	0.30	0.40	0.47	0.51	0.61	0.67	0.70	0.72	0.73
	Median	0.25	0.47	0.55	0.60	0.63	0.71	0.76	0.78	0.79	0.80
	Q3	0.46	0.64	0.70	0.73	0.75	0.81	0.84	0.86	0.86	0.88
GerUSA	Q1	0.06	0.29	0.39	0.45	0.50	0.60	0.65	0.69	0.70	0.72
	Median	0.26	0.47	0.55	0.61	0.64	0.71	0.74	0.76	0.78	0.79
	Q3	0.48	0.65	0.70	0.73	0.75	0.80	0.83	0.84	0.85	0.86
RusUSA	Q1	−0.02	0.19	0.27	0.35	0.40	0.50	0.53	0.55	0.56	0.56
	Median	0.16	0.37	0.45	0.51	0.54	0.61	0.66	0.68	0.70	0.68
	Q3	0.37	0.55	0.61	0.65	0.68	0.74	0.76	0.77	0.79	0.78
JapUSA	Q1	0.01	0.21	0.29	0.36	0.41	0.51	0.58	0.60	0.61	0.61
	Median	0.19	0.38	0.45	0.51	0.55	0.63	0.69	0.70	0.71	0.71
	Q3	0.39	0.56	0.62	0.66	0.69	0.76	0.79	0.80	0.79	0.78
AusUSA	Q1	0.00	0.22	0.32	0.40	0.45	0.57	0.63	0.65	0.66	0.67
	Median	0.18	0.40	0.49	0.55	0.59	0.68	0.72	0.74	0.75	0.76
	Q3	0.38	0.57	0.65	0.70	0.72	0.78	0.81	0.81	0.83	0.83
HKUSA	Q1	0.02	0.23	0.31	0.38	0.42	0.54	0.61	0.64	0.66	0.66
	Median	0.19	0.40	0.48	0.54	0.57	0.66	0.72	0.74	0.78	0.78
	Q3	0.40	0.58	0.63	0.67	0.71	0.78	0.83	0.84	0.84	0.86
IndUSA	Q1	−0.06	0.18	0.26	0.34	0.38	0.49	0.56	0.57	0.59	0.61
	Median	0.14	0.35	0.42	0.49	0.53	0.63	0.68	0.70	0.72	0.73
	Q3	0.36	0.53	0.59	0.65	0.68	0.75	0.79	0.81	0.82	0.82
ChiUSA	Q1	−0.09	0.14	0.22	0.29	0.32	0.42	0.50	0.52	0.54	0.52
	Median	0.12	0.30	0.37	0.43	0.47	0.57	0.62	0.65	0.65	0.64
	Q3	0.32	0.48	0.54	0.59	0.62	0.70	0.75	0.76	0.77	0.77

Table A.3: Descriptive statistics of multivariate TPMA.

		Thickness									
Countries	Stat	1	2	3	4	5	10	20	30	60	120
Developed	Q1	−0.31	−0.05	0.04	0.11	0.16	0.28	0.36	0.39	0.41	0.42
	Median	−0.16	0.05	0.13	0.19	0.24	0.35	0.42	0.44	0.47	0.48
	Q3	−0.04	0.14	0.22	0.28	0.32	0.42	0.48	0.51	0.54	0.55
Emerging	Q1	−0.32	−0.05	0.04	0.11	0.15	0.26	0.32	0.33	0.36	0.38
	Median	−0.14	0.06	0.14	0.20	0.25	0.35	0.41	0.43	0.45	0.47
	Q3	0.01	0.18	0.25	0.31	0.35	0.45	0.51	0.54	0.57	0.58
American	Q1	−0.15	0.11	0.20	0.28	0.34	0.45	0.51	0.53	0.55	0.58
	Median	0.05	0.26	0.34	0.41	0.45	0.54	0.60	0.62	0.65	0.66
	Q3	0.22	0.41	0.48	0.53	0.56	0.63	0.69	0.72	0.73	0.72
European	Q1	−0.15	0.12	0.22	0.28	0.33	0.41	0.47	0.48	0.50	0.49
	Median	0.05	0.27	0.35	0.41	0.45	0.53	0.58	0.59	0.61	0.59
	Q3	0.23	0.41	0.49	0.53	0.56	0.64	0.68	0.69	0.70	0.69
Asian	Q1	−0.32	−0.05	0.03	0.10	0.14	0.26	0.33	0.35	0.37	0.37
	Median	−0.15	0.05	0.13	0.19	0.23	0.34	0.40	0.43	0.45	0.44
	Q3	−0.01	0.15	0.23	0.28	0.32	0.42	0.49	0.51	0.53	0.52

Table A.4: Mean (in %'s), standard deviation (in %'s) and t-statistic of the daily log-returns of the equal-weight, two-market, buy-and-hold portfolios consisting of the USA and its least-/most-comoving counterpart for several values of the evaluation (E) and gap (G) lengths and several thickness values.

E	G	Thickness	Portfolio	Mean (in %'s)	SD (in %'s)	t-stat
21	0	63	Least comoving	−0.0051	1.2226	−0.2694
21	0	63	Most comoving	0.0121	1.2531	0.6278
21	0	252	Least comoving	0.0193	1.2457	0.9973
21	0	252	Most comoving	−0.0050	1.3614	−0.2428
21	0	504	Least comoving	−0.0037	1.3915	−0.1720
21	0	504	Most comoving	0.0250	1.1438	1.4319
21	21	63	Least comoving	−0.0039	1.2813	−0.1973
21	21	63	Most comoving	0.0140	1.2280	0.7424
21	21	252	Least comoving	0.0178	1.2480	0.9157
21	21	252	Most comoving	−0.0039	1.3751	−0.1863
21	21	504	Least comoving	−0.0036	1.3912	−0.1674
21	21	504	Most comoving	0.0264	1.1446	1.5074
126	0	63	Least comoving	0.0306	1.2071	1.6172
126	0	63	Most comoving	0.0135	1.2514	0.7009
126	0	252	Least comoving	0.0170	1.2111	0.8952
126	0	252	Most comoving	0.0149	1.3830	0.6963
126	0	504	Least comoving	0.0129	1.1595	0.7038
126	0	504	Most comoving	0.0232	1.1481	1.3035
126	21	63	Least comoving	0.0231	1.1994	1.2262
126	21	63	Most comoving	0.0107	1.2614	0.5519
126	21	252	Least comoving	0.0193	1.2035	1.0196
126	21	252	Most comoving	0.0155	1.3856	0.7200
126	21	504	Least comoving	0.0174	1.1549	0.9514
126	21	504	Most comoving	0.0264	1.1515	1.4758
252	0	63	Least comoving	0.0207	1.2828	1.0154
252	0	63	Most comoving	0.0120	1.2998	0.5906
252	0	252	Least comoving	0.0191	1.1670	1.0254
252	0	252	Most comoving	0.0132	1.3425	0.6295
252	0	504	Least comoving	0.0113	1.1622	0.6096
252	0	504	Most comoving	0.0162	1.2849	0.8026
252	21	63	Least comoving	0.0217	1.2919	1.0528
252	21	63	Most comoving	0.0132	1.3026	0.6473
252	21	252	Least comoving	0.0180	1.1534	0.9749
252	21	252	Most comoving	0.0124	1.3351	0.5914
252	21	504	Least comoving	0.0118	1.1603	0.6320
252	21	504	Most comoving	0.0157	1.2882	0.7766

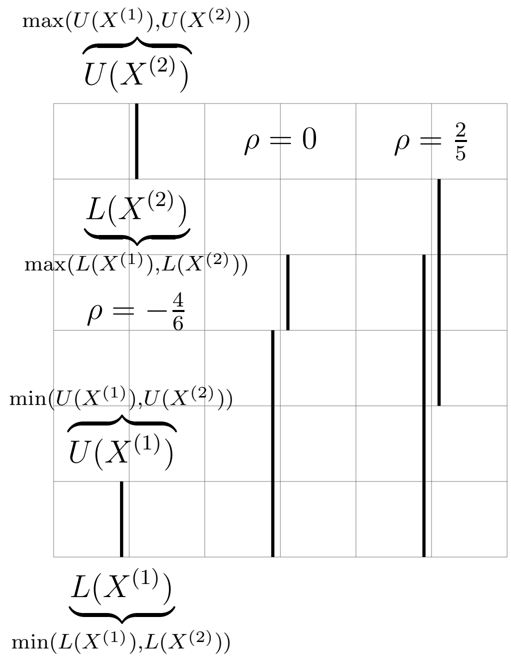

Figure A.1: Three examples of $\rho_t^{\tau_i}(X^{(1)}, X^{(2)}) \in \left\{-\frac{4}{6}, 0, \frac{2}{5}\right\}$ with geometric interpretation (t and τ_i fixed, vertical range of the grid is from 0 to 6). Vertical lines span the lower and upper boundaries of the TPTs of $X^{(1)}$ and $X^{(2)}$.