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The Partially Cursed and the Analogy-based Expectation Equilibrium *

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Abstract

Recent literature has pointed out a link between the *fully* cursed equilibrium and the analogy-based expectation equilibrium. Yet, the link between the latter and the *partially* cursed equilibrium is not fully understood. This note closes the gap by showing that a partially cursed equilibrium corresponds to a particular analogy-based expectation equilibrium.

KEYWORDS: Analogy Expectations, Bounded Rationality, Curse, Learning

JEL: C72, D82

1 Introduction

In Eyster and Rabin's (2005) *cursed equilibrium*, while having correct conjectures about the marginal distribution of each opponent's type and that of each opponent's action, each player fails to correctly conjecture the extent of correlation between these two. At an extreme, players conjecture no correlation. This corresponds to the fully cursed equilibrium. At another extreme, conjectures are correct. This is the Bayesian-Nash equilibrium. In between, there is a continuum of partially cursed equilibria each of which gives some weight χ to the cursed conjectures. Eyster and Rabin show that "any value of $\chi \in (0, 0.6)$ provides a better fit than does Bayesian Nash equilibrium" in all the experiments that they analyze.

In the *analogy-based expectation equilibrium* (Jehiel and Koessler, 2007) each player understands only the average behavior of their opponents over bundles of states, called the analogy classes, rather than correctly predicting the strategy

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of the opponent state by state as in the Bayesian Nash equilibrium.[1] *After* each round of play, each player observes the others' actions and the analogy class where the underlying state belongs to, but not the exact type. The player adopts the simplest conjecture consistent with her observations and expects that the opponents condition their strategies coarsely on analogy classes rather than on types. The conjectured strategy in the class thus correctly represents the average behavior in the class. Naturally, when analogy classes are not coarser than information sets, the analogy-based expectation equilibrium coincides with the Bayesian Nash equilibrium.

Eyster and Rabin (2005) and Jehiel and Koessler (2007) point out that the analogy-based expectation equilibrium where each player's analogy classes coincide with her information sets corresponds to the fully cursed equilibrium. Yet the connection between the ABEE and the partially cursed equilibrium is not fully understood. In this note the connection is formally established - it is shown that the partially cursed equilibrium corresponds to a particular analogy-based expectation equilibrium (ABEE) of a game where the initial type space is naturally extended.

2 Equilibrium concepts

2.1 The game and the cursed equilibrium

There are N players indexed by $i = 1, ..., N$. An action of player i is a_i and the finite set of actions[2] available to her is A_i. The actions of players other than i are denoted by $a_{-i} \in \times_{j \neq i} A_j$. An action profile is $a \in A = \times_{i=1}^{N} A_i$. Prior to the play of a stage game, nature draws a type profile, $\theta = (\theta_0, \theta_1, ..., \theta_N) \in \Theta_0 \times \Theta_1 \times ... \times \Theta_N$, each type profile with probability $p \in \Delta(\Theta)$. Without loss of generality, we assume that each type profile has a positive probability. The vector of types of players other than i is $\theta_{-i} \in \Theta_{-i} = \times_{j \neq i} \Theta_j$. The outcomes are type and action profile combinations, (a, θ). The payoff depends on the actions and on the type profile: $u_i : A \times \Theta \rightarrow \mathbf{R}$ for $i = 1, ..., N$. These elements construe a simple Bayesian game, $(\Theta_0, (A_i, \Theta_i, u_i)_{i=1}^{N}, p)$.

We can naturally extend the state space since the payoff-irrelevant uncertainty is rich in any environment that a model may try to capture. We will see that, this irrelevant uncertainty can be used to organize observations of opponents' behavior during a learning process thereby leading to partially cursed equilibrium beliefs.

The extended stage game is a static game of incomplete information $(\Omega, \mathcal{B}, q, (A_i; u_i; \Theta_i)_{i=1}^{N}; \widetilde{\theta})$. The underlying exogenous uncertainty is modelled by means of the probability space $(\Omega, \mathcal{B}, q)$ where $\omega \in \Omega = [0, 1] \subset \mathbf{R}$ is an elementary state, $\mathcal{B}$ is the set of Borel sets on [0,1] and q is the Lebesgue measure. Let the type profile be a random variable $\widetilde{\theta}$ on $(\Omega, \mathcal{B}, q)$, that is $\widetilde{\theta} : \Omega \rightarrow \Theta$ and let

[1] See Jehiel (2005) for the seminal contribution in the context of games of almost perfect information.

[2] Independent of the type profile.

$p(\theta) = q(\widetilde{\theta}^{-1}(\theta))$ where $\widetilde{\theta}^{-1}(\theta)$ is the inverse image of θ. The type of i that a state ω is mapped into is denoted by $\widetilde{\theta}_i(\omega)$. For each $\widehat{\theta}_i \in \Theta_i$, we can derive the corresponding information set by $I(\theta_i) = \widetilde{\theta}_i^{-1}(\theta_i)$. The collection of such information sets forms the information partition of i, $\mathcal{P}_i$.

A strategy of player i is a function of her type, $\sigma_i : \Theta_i \to \Delta(A_i)$ and the probability that type θ_i chooses action $a_i \in A_i$ is denoted by $\sigma_i(a_i|\theta_i)$. The strategies of players other than i are denoted by $\sigma_{-i} : \Theta_{-i} \to \times_{j\neq i}\Delta(A_j)$ and a strategy profile is $\sigma : \Theta \to \times_{j=1}^{N}\Delta(A_j)$. The conjecture of i about the strategy of the opponents[3] is denoted by $\widehat{\sigma}_{-i} : \Omega \to \times_{j\neq i}\Delta(A_j)$.

Let us next define the $\chi-$cursed equilibrium introduced in Eyster and Rabin (2005). To formalize the failure to understand correlations between opponents' types and actions, define the *opponents' average strategy* in a set of states $B \subset \Omega$ as follows:

$$\overline{\sigma}_{-i}(a_{-i}|B) = \frac{\sum_{\omega\in B} q(\omega)\sigma_{-i}(a_{-i}|\widetilde{\theta}_{-i}(\omega))}{\sum_{\omega\in B} q(\omega)}. \tag{1}$$

The average strategy may pool together several type profiles and it maps the type-specific strategies into one pooling strategy independent of whichever type is actually drawn from the set of others' types in the image of B, $\widetilde{\theta}_{-i}(B)$.

Definition 1 *A strategy profile σ is a $\chi-$cursed equilibrium if for each i, $\theta_i \in \Theta_i$, $a_i^* \in supp[\sigma_i(\theta_i)]$*

$$\begin{aligned} a_i^* \in \arg\max_{a_i} & \sum_{\theta_{-i}\in\Theta_{-i}} p(\theta_{-i}|\theta_i) \times \\ & \sum_{a_{-i}\in A_{-i}} [\chi\overline{\sigma}_{-i}(a_{-i}|I(\theta_i)) + (1-\chi)\sigma_{-i}(a_{-i}|\theta_{-i})]u_i(a;\theta) \end{aligned} \tag{2}$$

where $\overline{\sigma}_{-i}(a_{-i}|I(\theta_i))$ is given by (1). In particular, if $\chi = 1$, then the equilibrium is fully cursed.

It is straightforward to see that a cursed equilibrium is a Bayesian-Nash equilibrium if $\chi = 0$. Notice yet, that when $\chi > 0$, players put a positive weight on an average strategy in their conjectures. In this average strategy, each player averages over the opponent types *in the player's own information set*. Therefore, when the equilibrium is fully cursed, $\chi = 1$, each player type ignores the correlation between each opponent's private information and actions given their own private information.

[3] The player may be unaware of the opponent's private information partition. Therefore, the conjecture is conditioned on the state rather than on the profile of opponents' types.

2.2 Learning and the analogy-based expectation equilibrium

The analogy-based expectation equilibrium can be regarded as a steady state of *anonymous learning*[4] and thus it is a special case of a conjectural (Battigalli, 1987) or a self-confirming equilibrium[5] (Fudenberg and Levine, 1993; Kreps and Fudenberg, 1995; Dekel et al., 2004). Learning is based upon signals observed after each round of play. In the ABEE, each player observes the other players' actions, a_{-i}, and an analogy class $\alpha_i(\omega)$. The precision of player i's observation of others' types is captured by the collection of such classes, the analogy partition, which partitions the elementary states $\mathcal{A}_i$. An analogy system $(\mathcal{A}_1, ..., \mathcal{A}_N)$ describes the partitions of each player $i = 1, ..., N$. Whereas a player's information partition describes how precisely the player observes information at the interim stage, the analogy partition describes how precisely the player observes the information ex-post when the game is played.

Implicitly, each player retains observations from a large number[6] of preceding rounds. She conjectures that, at a given elementary state, each opponent plays his average strategy of the analogy class where that elementary state belongs to, $\overline{\sigma}_{-i}(\alpha_i(\omega))$. This is the simplest theory consistent with observing the analogy class rather than the precise opponent type[7]. Moreover, this is the only consistent theory where each opponent plays a pooling strategy in each analogy class. In a steady state, the best replies to the average strategy conjectures generate outcomes and perceptions which do not contradict the conjectures.

The analogy-based expectation equilibrium can now be defined as follows

Definition 2 *The triple $(\sigma_i, \widehat{\sigma}_{-i}, \alpha_i)_{i=1}^N$ is an Analogy-based expectation equilibrium if*

1. *for all i, for all $\theta_i \in \Theta_i$, and $a_i^* \in supp[\sigma_i(\theta_i)]$*

$$a_i^* \in \arg\max_{a_i} \sum_{\omega \in \Omega} q(\omega|I(\theta_i)) \sum_{a_{-i} \in A_{-i}} \widehat{\sigma}_{-i}(a_{-i}|\omega) u_i(a; \widetilde{\theta}(\omega)) \qquad (3)$$

2. *for all $\omega \in \Omega$ and for all i, $\widehat{\sigma}_{-i}(a_{-i}|\omega) = \overline{\sigma}_{-i}(a_{-i}|\alpha_i(\omega))$*

We consider only analogy-based expectation equilibria[8] where each player's analogy partition coincides or is finer than the player's information partition, denoted by $\text{ABEE}_{\mathcal{A}_i = \mathcal{P}_i}$ and $\text{ABEE}_{\mathcal{A}_i \leq \mathcal{P}_i}$ respectively.

[4] See Battigalli et al. (1992).

[5] Even partially cursed equilibria and corresponding ABEE are self-confirming, yet only if opponent's actions are not observed.

[6] The sample size is implicitly assumed to be infinite.

[7] Notice that in the ABEE players are not in general assumed to observe the own action-type ex post. This is without loss of generality, since in this paper analogy-partitions will be finer than information partitions. The ABEE implicitly assumes that own payoffs are not used to make inferences about others' strategies. Cursed players may naturally fail to perceive correlations between payoffs and own and others' information and actions. Esponda (2007) and Miettinen (2007) consider the effect of payoff information on cursed steady state beliefs.

[8] The prior distribution is assumed to be known to all. Yet, for average strategy conjectures to emerge in a steady state, players need to know only the distribution of analogy classes,

3 Cursed equilibrium as an ABEE

As easily seen from the definitions (and as acknowledged by Eyster and Rabin (2005) and Jehiel and Koessler (2007)), *when the analogy partitions coincide with the private information partitions, the fully cursed equilibrium and the analogy-based expectation equilibrium coincide.* In the proposition below, I establish even the partially cursed equilibrium as a particular analogy-based expectation equilibrium: the initial type space can be extended to a state space where an equivalent analogy-based expectation equilibrium exists. Notice yet, that the converse does not hold. Namely there generally exist ABEE which are not cursed equilibria.

Proposition 3 *For each $\chi-$cursed equilibrium of the game $(\Theta_0, (A_i, \Theta_i, u_i)_{i=1}^N, p)$ there exists an ABEE of the game $(\Omega, \mathcal{B}, q, (A_i; u_i; \Theta_i)_{i=1}^N; \widetilde{\theta})$ such that the conjectures and the strategies coincide for the $\chi-$cursed equilibrium and the ABEE.*

The proof is relegated to the appendix but the idea is simple. To illustrate (see figure 1), suppose, that there are two players of which one and only one is privately informed. She has two types, θ_1 and θ_2. Suppose that the underlying payoff-irrelevant uncertainty is richer so that the elementary states are the real numbers on a unit interval. Suppose further that the real numbers in $[0, \frac{1}{2})$ are mapped into θ_1 whereas numbers in $[\frac{1}{2}, 1]$ are mapped into θ_2. For a given χ, the $\chi-$cursed equilibrium then corresponds to the ABEE with the following uninformed player's analogy partition of the elementary states: $\{[0, \frac{1-\chi}{2}), [\frac{1-\chi}{2}, \frac{1+\chi}{2}), (\frac{1+\chi}{2}, 1]\}$. The intervals $[0, \frac{1-\chi}{2})$ and $(\frac{1+\chi}{2}, 1]$ keep record of the true strategies of types θ_1 and θ_2, respectively. Their joint probability is $1-\chi$. The interval $[\frac{1-\chi}{2}, \frac{1+\chi}{2})$ bundles together the two types and induces a conjecture that the informed opponent plays the average strategy with probability χ. Thus the partially cursed equilibrium corresponds to an $\text{ABEE}_{\mathcal{A}_i \leq \mathcal{P}_i}$.

Once a player organizes opponent's actions conditioning on a payoff-irrelevant random variable, $\alpha_i(\omega)$, it takes a small step to conjecture that opponents use the average strategy given each class. The observation structure thus leads to cursed beliefs in a natural way. Notice however, that a player must not know her analogy class at the interim stage when she chooses her strategy. If she knew, she would be best-responding either to the correct or to the fully cursed conjectures rather than to the partially cursed ones.

4 Appendix

Proof of proposition 3. Consider the following analogy partition. For each θ_i and each θ_{-i}, there is an analogy class $\alpha_i^\theta \subset \widetilde{\theta}^{-1}(\theta)$ and $\frac{q(\alpha_i^\theta)}{p(\theta)} = 1 - \chi$. On

which they can infer from their observations.

As opposed to the definition in Jehiel and Koessler (2007, p. 5), the coarseness of the partitions is part of the equilibrium description rather than exogenous. This difference is not crucial but rather reflecting the conviction that contradicting observations may induce efforts to track the types and the opponents' behavior more carefully.

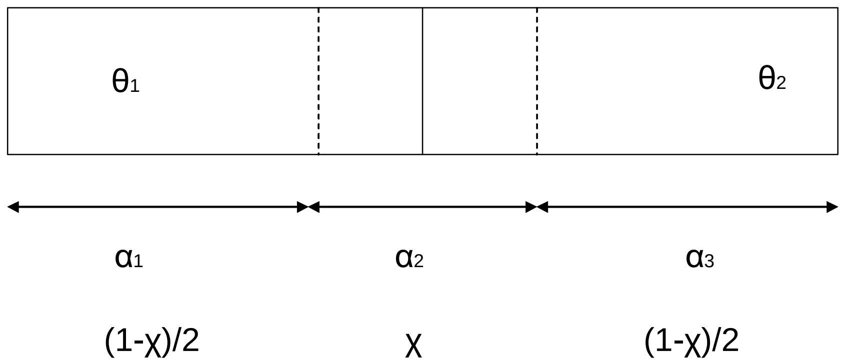

Figure 1: ABEE coinciding with χ-cursed equilibrium.

the other hand, given θ_i, for every θ_{-i}, the states $\widetilde{\theta}^{-1}(\theta)\backslash\alpha_i^\theta$ belong to another class (which also depends on players own type) $\overline{\alpha}_i^{\theta_i}$ so that $\frac{q(\widetilde{\theta}^{-1}(\theta)\backslash\alpha_i^\theta)}{p(\theta)} = \chi$. Notice that this analogy partition is finer than the private information partition of i. Jehiel and Koessler (2007, proposition 2; Eyster and Rabin, 2005 p. 1631) show that when the analogy partition is finer than the private information partition, the analogy based expectation equilibria are equivalent to the Bayesian-Nash equilibria of a virtual game where the payoff of player i when the state is ω and the action profile is a is $\overline{u}_i(a;\omega) = \sum_{\omega\in\Omega} p(\omega'|\alpha_i(\omega))u_i(a;\omega')$ where $\omega \in \Omega$. But in fraction $1-\chi$ of the states that are mapped into θ, there are *only* states that are mapped into θ, and thus $\overline{u}_i(a;\omega) = u_i(a;\theta)$ for these states. On the other hand, in fraction χ of the states that are associated with a given type θ, there are fractions $p(\theta'_{-i}|\theta_i)$ of states associated with types (θ_i, θ'_{-i}) for each $\theta'_{-i} \in \Theta_{-i}$. Thus, the virtual payoff for these states reads $\overline{u}_i(a;\omega) = \sum_{\theta'_{-i}\in\Theta_{-i}} p(\theta'_{-i}|\theta_i)\ u_i(a;(\theta_i,\theta'_{-i}))$. Thus overall, conditional on type θ the virtual payoff for action profile a can be written as $(1-\chi)u_i(a;\theta) + \chi\sum_{\theta'_{-i}\in\Theta_{-i}} p(\theta'_{-i}|\theta_i)\ u_i(a;(\theta_i,\theta'_{-i}))$. But this is exactly the $\chi-$cursed equilibrium virtual game payoff of type θ (Eyster and Rabin, 2005, p. 1631). The Bayesian-Nash equilibria of this game are the $\chi-$cursed equilibria of the original game. ■

References

[1] Battigalli 1987. Comportamento Razionale e d'Equilibria Nio Giochi e Nelle Situazione Sociali. Unpublished undergraduate dissertation, Bocconi University.

[2] Battigalli, P.,Gilli, M., Molinari C., 1992. Learning and Convergence to Equilibrium in Repeated Strategic Interactions: An Introductory Survey. Ricerche Economiche, 46, 335-378.

[3] Dekel, E., Fudenberg, D., Levine, D.K., 2004. Learning to Play Bayesian Games. Games Econ. Behav. 46, 282-303.

[4] Esponda, I., 2007. Behavioral Equilibrium in Economies with Adverse Selection. Am. Econ. Rev 98(4), 1269-91.

[5] Eyster, E., Rabin, M., 2005. Cursed Equilibrium. Econometrica 73, 1623-1672.

[6] Fudenberg, D., Kreps, D.M., 1995. Learning in Extensive Games, I: Self-Confirming Equilibria. Games Econ. Behav. 8, 20-55.

[7] Fudenberg, D., Levine, D.K., 1993. Self-confirming Equilibrium. Econometrica, 61, 523- 546.

[8] Jehiel, P., 2005. Analogy-Based Expectation Equilibrium. J. Econ. Theory, 123, 81-104.

[9] Jehiel, P., Koessler, F., 2007. Revisiting Games of Incomplete Information with Analogy-based Expectations. Games Econ. Behav. 62(2), 533-557.

[10] Miettinen, T., 2009. Paying Attention to Payoffs in Analogy-based Learning. SITE, Stockholm School of Economics, mimeo.