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Tough Negotiations:
Bilateral Bargaining with Durable Commitments *

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Abstract

We offer a tractable model of tough negotiations and delayed agreement. The setting is an infinite horizon bilateral bargaining game in which negotiators can make strategic commitments to durable offers. Commitments decay stochastically, but uncommitted negotiators can make new commitments. The game's unique Markov Perfect equilibrium outcome takes the form of a war of attrition: Negotiators initially commit to incompatible offers, but agreement occurs once a negotiator's commitment decays. If commitments decay more quickly, the terms of the agreement become more equal. In expectation, more patient, committed, and less risk averse negotiators obtain a larger fraction of the surplus.

KEYWORDS: Bargaining, Commitment, Disagreement, War of attrition

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1 Introduction

Thomas Schelling (1956, 1960, 1966) pioneered the analysis of bilateral bargaining as a game of strategic commitment. By committing to an irrevocable bargaining position, a negotiator aims to force concessions from the opponent. According to Schelling (1960, Appendix B), there is no *a priori* reason to expect that the outcome of such a struggle for dominance will be symmetric or even efficient. In this paper we formalize Schelling's assertion with the help of an infinite horizon complete information bargaining game. The model yields specific predictions about the ultimate division of surplus as well as the extent of delay in reaching agreement.

For concreteness, consider the case of bargaining between a trade union and the management of a firm. Schelling (1956, p.286) makes the following observation:

> "...it has not been uncommon for union officials to stir up excitement and determination on the part of the membership during or prior to a wage negotiation. If the union is going to insist on \$2 and expects the management to counter with \$1.60, an effort is made to persuade the membership not only that the management could pay \$2 but even perhaps that the negotiators themselves are incompetent if they fail to obtain close to \$2. The purpose – or, rather, a plausible purpose suggested by our analysis – is to make clear to the management that the negotiators could not accept less than \$2 *even if they wished to* because they no longer control the members or because they would lose their own positions if they tried."

That is, if the trade union's leader makes an explicit public proposal, and then immediately backs down from it, the leadership position is jeopardized. Hence, the public statement commits the leader, at least for some time.

As an empirical illustration, consider the 2004-5 National Hockey League lock-out (Podnieks, 2005; Staudohar, 2005). The lockout lasted 310 days and entailed the cancellation of 1230 league matches that season. Revenue losses were approximately 2 billion US dollars, out of which about half would have been players' wages. The conflict started when NHL, led by Commissioner Gary Bettman, attempted to convince players to accept a new salary structure. The players' association, NHLPA, under executive director Bob Goodenow, considered the proposal to involve a salary cap, which the association had vowed never to accept. Eventually, however, as negotiations for the 2005-6 season started, the union caved in and signed a new agreement. Bob Goodenow whose hardline stance against a salary cap contributed to the costly impasse, resigned from his position five days after the agreement was ratified.

Of course, Schelling's argument is not limited to labor negotiations. For example, strategic public statements often play a central role in territorial conflicts, as state leaders are usually reluctant to give up a publicly announced territorial claim. Such statements have severely limited the scope for subsequent negotiations over contested territories between Israel and Palestine, between India and Pakistan, as well as between several other nation-pairs (Huth, 1996).

Formal modeling of strategic commitment in bilateral bargaining starts with Crawford (1982, Section 4). He considers a two-period model.[1] In the first period, both sides make commitments; in the second period, they negotiate subject to the constraints that the commitments impose. If commitments are likely to decay before negotiation starts, each side has an incentive to demand the whole surplus in the hope that the opponent's commitment will decay. Hence, there is conflict with positive probability.[2] On the other hand, if commitments are unlikely to decay, there are other equilibria too, many of which are efficient. For example, both sides may commit to an even split of the surplus. The lesson, it seemed, was that a positive probability of conflict is inevitable if commitments are not very credible, but might be avoidable otherwise.

Ellingsen and Miettinen (2008), henceforth E-M, reconsider Crawford's analysis, showing that the efficient equilibria do not survive iterated elimination of weakly dominated strategies (IEWDS). The only iteratively weakly undominated outcome is that in which each side demands the whole surplus. Moreover, if there is a small cost of making commitments this is the unique iteratively *strictly* dominant outcome. The central point of E-M is that, in simultaneous-move models of bilateral bargaining, compatible-commitment equilibria are artificial. If one party, say A, believes that the opponent B will make a certain acceptable offer, then it does not make sense for A to propose exactly the same outcome. Either B is in fact making the offer that A expects, in which case A can simply agree to it, or B is making some different offer, in which

[1]Other two-period models of strategic commitment in bargaining include Muthoo (1996), Ellingsen (1997), and Güth, Ritzberger and van Damme (2004).

[2]As shown by Li (2011) this argument extends to the case of an infinite bargaining horizon under some additional assumptions.

case being flexible could be strictly better (if B's actual offer is more generous) and never worse, as A is still free to reject the offer.

As E-M acknowledge, their result is nonetheless problematic. Instead of a weak prediction, that virtually anything may happen, one obtains the strong but apparently unrealistic prediction that negotiations almost never succeed. In the present paper, we demonstrate that the apparent lack of realism in E-M is only due to the assumption that negotiation ends after two periods. Instead of a one-period deadline, we here allow negotiation to continue indefinitely, assuming for simplicity that commitments decay stochastically at a constant rate. Even if the negotiators can make new commitments once their previous commitment has decayed, they will only make such a new commitment if, at the time, the opponent's offer is unacceptable. But rational negotiators will not make offers that are rejected by a flexible opponent. In fact, our model's unique Markov Perfect equilibrium outcome has the property that both players arrive at the negotiation table committed to proposals that will be immediately accepted by an uncommitted opponent. Consequently, there is an initial phase of disagreement followed by agreement to the offer that happens to last longest. That is, the model proposes an explanation for why negotiations often take the form of a prolonged war of attrition.

In the Markov Perfect equilibrium, the exponential decay of commitments translates into an exponential duration of disagreement, which incidentally happens to fit well the duration of actual labor conflicts; see for example Kiefer (1988). Depending on the model's parameters, the average duration of conflicts as well as the ultimate division of surplus can take any value. In the limit where offers decay infinitely fast, there is immediate agreement to an even split. In the opposite limit, with infinitely slow decay of offers, both sides demand all the surplus and disagree forever.

If negotiators differ with respect to their ability to commit, their patience, or their risk aversion, these differences are reflected in the proposals. Relatively impatient, risk-averse, and poorly committed negotiators receive a smaller share of the surplus. The model is also extended to allow for correlated decay of commitments, for example due to external mediation. While faster decay improves efficiency, and is always welcomed by the weaker negotiator, the stronger negotiator may prefer slower uncorrelated decay of commitments.

The model also admits equilibria that are not Markov perfect, including equilibria that rely on non-stationary strategies. When commitments decay fast, we show that all subgame perfect equilibria converge to the immediate agreement on an even split. In this sense, the model confirms that the result of Rubinstein (1982), who also focuses on the case of short-lived offers, does not hinge on his alternating moves assumption. On the other hand, as commitments decay more slowly, the set of subgame perfect equilibria expands. In the limit, as decay is infinitely slow, any feasible outcome can be supported. Since we think that most of these non-stationary equilibria require unrealistically coordinated expectations concerning what should happen after a minor deviation from the equilibrium path, we instead emphasize the Markov perfect equilibrium, at least for negotiations that don't have many parallels or precedents that may have shaped the negotiators' expectations.

Before presenting the model, let us briefly survey some related research. Previous infinite horizon bargaining models with complete information and fully rational negotiators have provided two separate explanations for delayed agreement.[3]

First, the bargaining game may have many equilibria, some (but not all) of which are inefficient. Examples include Fernandez and Glazer (1991), Haller and Holden (1990), van Damme et al (1990), Perry and Reny (1993), and Li (2011). However, these models typically produce a large set of equilibria, and without further refinement they lack specific predictions. When refinements are imposed, efficient outcomes tend to be selected. For example, when negotiators have a weak preference for simple strategies, Binmore, Piccione, and Samuelson (1998) show that immediate agreement is the only evolutionarily stable outcome in the model of van Damme et al (1990).[4] Generally, it is fair to say that none of these models predicts delay.

Second, and more closely related, Li (2007) shows how reference-dependent preferences may give rise to substantial delays. Specifically, Li assumes that a negotiator is unwilling to accept any offer that yields a smaller net present value than any previous offer that she has turned down. Along the equilibrium path, negotiators gradually build up commitment by rejecting the opponent's offers.[5] Anticipating this, each side initially makes meager offers. The

[3]There is also a literature that explains disagreement or delay as a consequence of biased beliefs; see for example Yildiz (2004), Ali (2006), and Bénabou and Tirole (2009).

[4]Incidentally, the approach of Binmore, Piccione, and Samuelson (1998) tends to select stationary strategies.

[5]Previously, Compte and Jehiel (2004) considered the case in which termination of negotiations without agreement yields some history-dependent payoff, for example an arbitration outcome (instead of a constant payoff of zero to each player as is normally assumed). In that case too, there may be delay and non-constant offers.

main difference relative to our model is that players do not commit to own offers, but instead commits to be consistent – in net present value terms – with previous rejections. Fershtman and Seidman (1993) previously explored the consequences of assuming that players are unwilling to accept any share smaller than those that they have previously turned down. In that case, without Li's net present value assumption, there is no delay if the horizon is indefinite.[6] We see the two approaches as complementary. In reality, negotiators can build commitment both by announcing own proposals, as in our model, and by publicly rejecting the opponent's proposals. It is an empirical issue whether negotiators primarily become committed to their own nominal proposals, as we assume, or to net present values associated with rejections of the opponent's proposals, as Li assumes.

Our model is also related to Kambe (1999) and Wolitzky (2012), who consider infinite horizon bargaining games in which negotiators publicly announce proposals to which they may become committed. (In Wolitzky's model, the commitment is again to a net present value, as in Li, 2007.) However, Kambe and Wolitzky both assume that the negotiator has private information concerning whether the commitment sticks or not. An important feature of their analysis is thus the possibility of signaling private information through rejection of the opponent's offers, just as in Abreu and Gul (2000) and Compte and Jehiel (2002).[7] We agree that private information about valuations is sometimes important, but in many cases it seems to us that negotiators have roughly similar information concerning the credibility of the commitments. In particular, information tends to be quite symmetric when commitment is related to the reactions of third parties, as in Schelling's case of the committed trade union leader. Indeed, we doubt that Bob Goodenow had better information than Gary Bettman concerning the willingness of NHLPA's many members to fight the salary cap.

This latter point is more general. While we think that asymmetric information helps to

[6]Fershtman and Seidmann (1993) also explore what happens when negotiators face a deadline. Then there is a positive probability of delay. In a different setting, with randomness in the speed of communication, Ma and Manove (1993) show that deadlines may induce not only delays, but also offers that are rejected with positive probability and disagreements. In a sense,random delay introduces asymmetric information about valuations; when making an offer, the proposer does not know what the responder's valuation will be when the offer arrives.

[7]We refer to these papers for a more extensive discussion of other literature on delayed agreement under conditions of incomplete information.

explain some disagreements, we dispute the conventional wisdom that asymmetric information is essential for understanding disagreement among rational parties who can write binding contracts.[8] Kennan and Wilson (1993, p.101) perhaps put the conventional view most starkly: "The hypothesis that private information is an underlying source of conflict is currently the only one based on the usual test of rationality, namely relentless maximizing behavior." By providing a complete information model of disagreement, we show that Schelling (1956, page 287) needs not appeal to asymmetric information when he claims that commitment tactics "all run the risk of establishing an immovable position that goes beyond the ability of the other to concede, and thereby provoke the likelihood of stalemate or breakdown." In a fully dynamic model, the fundamental logic of disagreement proposed by Crawford (1982, Section 4) continues to be valid even when there is only a small probability that commitments decay.

The paper is organized as follows. Section 2 introduces the basic model. Section 3 derives and characterizes the model's solutions. Section 4 relaxes several of the assumptions. In particular, it investigates the effect of asymmetric commitment opportunities and discount rates, as well as correlated decay of commitments. (An Appendix considers the role of asymmetries in risk aversion.) Section 5 concludes and briefly discusses the impact of a variety of other possible changes to the model's assumptions.

2 Model

There are two negotiators, henceforth called players. Players are indexed $i = A, B$ and bargain over a fixed surplus of size 1. For simplicity, we initially assume that players have identical preferences and technologies, relegating a complete characterization of asymmetric cases to Section 4.

The size of the surplus and the rationality of the players are common knowledge. A player's utility is assumed to be linear in the player's share of the surplus (but as shown in the Appendix the major insights extend to the case of risk aversion). Players are impatient, with per period discount factor δ.

[8]Fearon (1995) and Powell (2004,2006) demonstrate that disagreements in general, and war in particular, can also arise when parties are unable to commit to contracts.

2.1 Timing and actions

The bargaining game, call it G^{∞}, has infinite horizon. In each period t, actions are taken in two stages – the proposal stage and the response stage. At the proposal stage, players can make long-lived offers unless they have already done so; at the response stage, they accept or reject offers. If the players arrive at the response stage without any offer on the table, each player has an equal chance of being picked to make a short-lived offer that the opponent may either accept or reject. In between the two stages, no new action can be taken, but time passes and commitment to offers may decay.

The assumption that each period is divided into a proposal stage and a response stage closely matches Crawford (1982) and E-M. Indeed, the basic assumption of this literature is that negotiators always have the opportunity to make unilateral commitments before they sit down to engage in bilateral talks. We believe that this is also realistic. Certainly, in the NHL-example above, both Gary Bettman and Bob Goodenow made unilateral commitments before sitting down to negotiate in the same room. While our argument can in principle be simplified by only admitting long-run offers, as argued in Section 5, both realism and connection to the earlier literature speaks in favor of retaining the two-stage formulation.

For simplicity, we assume that no time passes between the end of one period and the start of the next.[9]

2.1.1 *Period 1*

The first period is special, since players have not been able to make previous commitments. In period 1, the available actions and payoffs are the following.

(a) The proposal stage. Each player i chooses either to make a specific proposal for how to split the surplus or to wait and remain uncommitted. Specific proposals are fully characterized either by the amount offered to the opponent or by the amount demanded by the proposer. Our formulas are shorter in the former case. Thus, we let $s_1^i \in [0, 1]$ denote an offer made by player i to player j in period 1. Denote the waiting action by w, and let the set of proposal

[9]Two remarks are in order concerning the timing. (i) It is difficult to imagine how a commitment could break down without any time elapsing. (ii) It is straightforward to generalize the model to the case in which time passes between periods, but it complicates the analysis without generating new insight.

stage actions be denoted $S = [0,1] \cup \{w\}$.

The set of randomized offers is the set of probability distributions on S. Let $\mathcal{P}$ denote the set of all probability distributions on S. Let ϕ_1^i denote a randomized offer of player i, and let $p^i(s)$ denote the associated probability (density) that player i takes the action s.

As in E-M, we assume that it is costly to make specific proposals. Initially, we assume that any offer $s^i \in [0,1]$ entails a small positive cost c to player i, whereas w is free. Subsequently, we will simplify by assuming that c is of second order magnitude (lexicographically small).[10] In what follows, when we say that player i has made a commitment, we mean that i has made a proposal $s^i \in [0,1]$.

(b) Delay. Some time passes between the making of an offer and the opportunity to respond. During this time, a player's commitment can potentially decay. Specifically, if player i makes an offer $s^i \in [0,1]$, the offer survives until the response stage with probability $q \leq 1$. With probability $1-q$, player i instead enters the response stage with $s^i = w$.

(c) The response stage. Each player now observes the offers made at the proposal stage; if a player made a mixed offer, only the realization is observed by the opponent. If only player i is committed, player j either (i) accepts the offer, getting a share s_1^i while leaving a share $1 - s_1^i$ to player i, or (ii) rejects the offer. After a rejection, Period 1's negotiation is over, and players must wait for bargaining to resume next period. If both players are committed, the outcome depends on whether the commitments are compatible or not. If $s_1^A + s_1^B < 1$, offers are incompatible, there is no agreement in period 1, and bargaining resumes next period. If $s_1^A + s_1^B \geq 1$, one of the players is randomly picked to make an accept or reject decision. If player i is picked and accepts, player i gets the share s_1^j and player j gets the remainder. If player i is picked and rejects, bargaining resumes next period. Finally, if no player has a surviving offer, i.e., if $s^A = s^B = w$, each player has an even chance of being picked to make a short-lived take-it-or-leave-it offer. Let such short-lived offers be denoted $\widetilde{s}_1^i$. It does not matter whether

[10] Some readers have objected that the cost of making a commitment cannot be very large. When a trade union negotiator makes a public statement, that costs nothing at all. We agree. That is precisely why we think that it is justified to make the cost lexicographically small.

From a technical point of view, the cost c is merely the simplest way to express the benefit of flexibility over a compatible commitment. Our main insights can be obtained through the alternative but more cumbersome route of trembling-hand perfection. Flexibility is optimal against all the opponent's firm commitments; a firm commitment is only optimal against one of them.

it is costly to make a short-lived offer, as long as the cost is sufficiently small; for simplicity we assume that it is costless. Let randomized short-lived offers be denoted $\underline{\phi}_1^i$, and let $\underline{P}$ denote the set of all probability distributions on $[0,1]$. If the offer is accepted, the game ends and players get the corresponding payoffs. If the offer is rejected, the game continues to the next period where both players will be initially uncommitted.[11]

Observe that the set of opponent offers that player i can choose to accept is $A(s^i) = \{s^j \in [0,1] \mid s^i + s^j \geq 1\}$. Formally, player i's response is thus a function $z^i : A(s^i) \to \{Y, N\}$, where Y denotes acceptance and N denotes rejection. A mixed response is a function $\rho^i : A(s^i) \to [0,1]$ yielding for each offer s^j the probability that player i accepts it.

2.1.2 *Period t*

Suppose negotiations did not end before period t. Suppose moreover that in the beginning of period $t-1$ player i was committed to the offer $s_{t-1}^i \in [0,1]$. Then, unless the commitment subsequently decayed, player i remains committed to s_{t-1}^i at the proposal stage of period t. That is, $s_t^i = s_{t-1}^i$.[12]

If instead player i is flexible at the beginning of period t, a new offer $s^i \in S$ is made.[13]

As before, any offer decays with probability $1-q$ before the response stage, implying that offers decay exponentially. In the following, we refer to s_t^i as player i's offer at period t's commitment stage and $s_{t+}^i \in \{s_t^i, w\}$ as player i's (surviving) offer at the beginning of period t's response stage.

Period t's response stage is analogous with the response stage of period 1.

[11] The latter assumption is of minor importance. In the main case of interest, infinite horizon bargaining with brief time intervals, the specification of what happens in the (w, w) case is altogether irrelevant.

[12] We think it is psychologically plausible that a player who is already committed cannot subsequently change the offer to become less generous. The trade union leader who publicly insisted on a wage w yesterday would find it hard, after a day's negotiation impasse, to increase the demand above w. That said, our main results would still hold if we allowed players to adjust their committed offers downward (this deviation is not attractive in the MPE that we derive).

[13] As mentioned in Section 5, we could alternatively have conducted the analysis under the assumption that offers decay permanently, with qualitatively similar results. We could also put other restrictions on the set of feasible commitments following a commitment's decay.

2.2 Histories and strategies

For simplicity, we assume that any randomization is only privately observable, and that players condition their strategies only on the public history (if at all). The public history of the game comprises the actual offers and responses as well as the observed decays of previous commitments.

Let $\mathcal{H}$ be the set of all possible finite histories. Players have perfect recall, and subject to the constraints imposed by current commitments, they can condition their actions on all previous events. Allowing mixed strategies, a commitment strategy for player i is a function $\phi^i : \mathcal{H} \to \mathcal{P}$. Similarly a short-run offer strategy is a function $\underline{\phi}^i : \mathcal{H} \to \underline{\mathcal{P}}$. Of course, a flexible player in the position to respond to the opponent's offer may condition the response on whether the offer is a commitment or a short-run offer; these represent different histories. Therefore, we do not need separate notation for responses to long-run and short-run offers. Let the response strategy be denoted $\rho^i : \mathcal{H} \to [0, 1]$. Accordingly, a complete strategy for player i can be written $\sigma_i = (\phi^i, \underline{\phi}^i, \rho^i)$. Let the set of feasible strategies for a player be denoted Σ.

Let $\mathcal{G}$ be the set of subgames of G^∞(including G^∞). Let player i's expected payoff in subgame g be denoted $U^i_g(\sigma_i, \sigma_j)$.

2.3 Remarks

The model is essentially an extension of E-M to the multi-period case, but there are slight differences in exposition and technical detail. We now find it more natural to speak about "offers" than "demands". The formulation "I offer s" suggests that the proposer will get *exactly* $1 - s$ if the proposal succeeds, whereas the seemingly similar formulation "I demand $1 - s$" could be interpreted as saying that the proposer will get *at least* $1 - s$ if the proposal succeeds. Whereas E-M assumed that any excess surplus would go to the flexible player in case less than all the surplus is demanded by a committed opponent, our new formulation gets to the same outcome more directly.

Another change is that we here explicitly model the negotiators' opportunity to reject the opponent's proposal, thereby making the model fully non-cooperative. Assuming some sharing rule for the case of compatible commitments or mutual flexibility might seem reasonable as a first pass, but it begs the question where exactly to stop invoking cooperative behavior.

Finally, we are now explicit about the delay that occurs between offers and responses within a period. If there were no delay between the two stages, it would be difficult to justify the assumption $q < 1$ in the single period case.

3 Analysis

After a brief recapitulation of the single-period analysis of E-M, we go on to characterize the unique Markov Perfect equilibrium outcome of the infinite bargaining game.[14] We then describe the set of non-stationary subgame perfect equilibrium outcomes. In Section 4, we generalize the Markov Perfect equilibrium analysis with respect both to player asymmetries and the process of commitment decay.

3.1 One period

Let us start by analyzing a single-period bargaining game, call it G^1. Suppose the cost of making a specific proposal is $c \in (0, \delta(1-q)^2/2)$.

Proposition 1 *There is a unique subgame perfect Nash equilibrium of G^1. At the proposal stage, each player i commits to the offer $s^i = 0$. In case both commitments decay and player i is chosen to be the proposer, player i makes the offer $s^i = 0$. If only player i's commitment decays, or if both commitments decay and player i is chosen to be responder, player i accepts any offer $s^j \in [0,1]$. Since the commitments are incompatible, there is disagreement with probability q^2.*

The result corresponds closely to part (i) of Proposition 3 of E-M, but since the model is slightly different, the Appendix provides a modified proof.

Proposition 1 says that each player offers the smallest amount that a flexible opponent is willing to accept. The existence of such a bad equilibrium is unsurprising; the Demand Game of Nash (1953), henceforth NDG, also has similarly inefficient equilibria. The surprise is that no other equilibria exist. In the NDG there is a continuum of efficient equilibria, so why are there no efficient equilibria here? As indicated above, a key difference is the cost of making offers. The efficient equilibria in the NDG have the property that players make exactly compatible

[14]The case of a long but finite bargaining game is quite complicated, so we leave it aside.

demands. In our model, compatible offers cannot arise in equilibrium because a player is better off by deviating to being flexible and accepting the opponent's offer.[15]

We note in passing that a similar refinement of the set of equilibria can be obtained in other ways too, for example through trembling-hand perfection. The general lesson is that when players have the option of being flexible, equilibria in which they make exactly compatible offers are not robust.

3.2 Infinitely many periods

Let us now consider the infinite horizon game, G^∞. From now on, we assume that the commitment cost c is lexicographically small.

3.2.1 Markov Perfect equilibria

Stationary strategies, also called Markov strategies, depend only on the current state. (See Maskin and Tirole, 2001, for a rigorous definition, justification, and analysis of Markov strategies in games with observable actions.) In our setting, the state is defined by the surviving offers. Since only flexible players can make a commitment, a stationary long-run offer strategy ϕ_t^i only depends on known features of *the opponent's* current commitment status $s^j_{(t-1)+}$. Acceptance decisions depend on the pair of current commitments s_{t+}. (When players make long-run offers in period 1, apply the convention $s_{0+} = (w, w)$.)

Let us use hats to indicate that strategies are stationary. Thus, player i's stationary long-run offer is a function $\widehat{\phi}^i : S \to \mathcal{P}$. Analogously, stationary short-run offers (for the case that both players are flexible) are written $\widehat{\underline{\phi}}^i \in \underline{\mathcal{P}}$, and responses $\widehat{\rho}^i : S^2 \to [0, 1]$. Let the set of stationary strategies be denoted $\widehat{\Sigma}$. A Markov Perfect equilibrium (MPE) is a subgame perfect equilibrium in Markov strategies.

Definition 1 *A strategy profile* $\widetilde{\sigma} \in \Sigma^2$ *forms a Markov Perfect equilibrium of* G^∞ *if and only if (i)* $\widetilde{\sigma} \in \widehat{\Sigma}^2$,*and (ii)* $U_g^i(\widetilde{\sigma}^i, \widetilde{\sigma}^j) \geq U_g^i(\sigma^i, \widetilde{\sigma}^j)$ *for all subgames* $g \in \mathcal{G}$ *and strategies* $\sigma^i \in \Sigma, i = A, B$.

[15] If we introduce a cost of making demands in the NDG, together with a rule that unclaimed surplus goes to the player making no demands, then this modified NDG would also have a unique and inefficient equilibrium; see Section 5.

As it turns out, the model has a unique Markov Perfect equilibrium outcome. While the proof would be more compact if we started by positing the equilibrium, we think that it is more instructive instead to provide the steps by which the equilibrium can be discovered.

To establish the result, the following notation is useful: For any strategy profile σ, and pair of commitments (s^i, s^j), let $V^i(s^i, s^j \mid \sigma)$ denote player i's expected payoff at the end of the current period's offer stage. The analysis proceeds in six steps.

Step 1: Response behavior. Consider any MPE strategy profile $\widehat{\sigma}$. We begin by studying the response behavior of player i when facing a durable or non-durable offer s^j. By rejecting the offer, player i obtains the continuation payoff

$$R^i(s^i, s^j; \widehat{\sigma}) = \begin{cases} V^i(s^i, s^j \mid \widehat{\sigma}) & \text{if both offers are durable;} \\ V^i(\widehat{\phi}^i(s^j), s^j \mid \widehat{\sigma}) & \text{if only } j \text{ has a durable offer;} \\ V^i(\widehat{\phi}^i(w), \widehat{\phi}^j(w) \mid \widehat{\sigma}) & \text{if no offer is durable.} \end{cases}$$

Markov perfection implies that any offer s^j must be accepted if $s^j > R^i$ and rejected if $s^j < R^i$. (Otherwise, player i has a profitable deviation in some subgame.) Hence, let us refer to R^i as player i's acceptance threshold.

Step 2. In an MPE, any non-durable equilibrium offer by player j satisfies

$$s^j = V^i(\widehat{\phi}^i(w), \widehat{\phi}^j(w) \mid \widehat{\sigma}) := \widehat{s}^j_n,$$

and any durable equilibrium offer by player j satisfies

$$s^j = \min_{s^k} V^i(\widehat{\phi}^i(s^k), s^k \mid \widehat{\sigma}) := \widehat{s}^j.$$

Since the set of offers is bounded, $\widehat{s}^j$ is well-defined. The claims are proved by contradiction. Consider first the case that both players are flexible at the response stage, and player j is picked to make a non-durable offer. (i) Consider any higher offer $\widehat{s}^j_n + \varepsilon$. By Step 1, this offer will certainly be accepted, but so will the lower offer $\widehat{s}^j_n + \varepsilon/2$, which is therefore preferable. (ii) Consider next any lower offer $\widehat{s}^j_n - \varepsilon$. By Step 1, this offer will be rejected, because player i expects to get $\widehat{s}^j_n$ in the continuation. Since the continuation itself must form an MPE of the ensuing subgame, and since a rejection means that there cannot be agreement before a full period has passed, player j can get at most $\delta(1 - \widehat{s}^j_n)$ following the rejection. As it would have been possible to obtain a payoff arbitrarily close to $1-\widehat{s}_n^j$ by offering slightly more than $\widehat{s}_n^j$, the lower offer is suboptimal for all $\delta<1$.

Consider now the case of a durable offer by player j. If player j expects player i to be flexible at the period's response stage, the logic is analogous to the case of non-durable offers above, but with the following additional step: Suppose a flexible player i rejects the durable offer s^j. Then in an MPE, i's continuation payoff $V^i(\widehat{\phi}^i(s^j), s^j \mid \widehat{\sigma})$ is independent of s^j. To see this, suppose to the contrary that $V^i(\widehat{\phi}^i(s_1^j), s_1^j \mid \widehat{\sigma}) > V^i(\widehat{\phi}^i(s_2^j), s_2^j \mid \widehat{\sigma})$ for some $s_1^j \neq s_2^j$. Since both s_1^j and s_2^j are rejected, the difference in continuation payoff must be caused by $\widehat{\phi}^i(s_1^j) \neq \widehat{\phi}^i(s_2^j)$. But since these counter-commitments are both incompatible with the offer they react to (otherwise, the offer should be accepted), each only matters along a path where there is a loophole in j's commitment. Thus the optimal counter-offer must be independent of the initial commitment; that is, $\widehat{\phi}^i(s_2^j)$ cannot be part of an MPE. As in the case of short-run offers, then, it is never optimal to commit to any offer that is rejected by a flexible opponent.

Step 3: There is no MPE in mixed strategies.

We need to show that any offer $s^j = R^i$ is accepted with probability 1. Suppose to the contrary that player i rejects $s^j = R^i$ with probability p. Since player i is only willing to randomize if this continuation yields R^i, the continuation can give player j at most $\delta(1-R^i)$. Since j can get a payoff arbitrarily close to $1-R^i$ by offering i slightly more than R^i, the conclusion follows. Thus, we may confine attention to pure strategies, $\phi^i : S \to S$ and $\rho^i : S \to \{0,1\}$.

Step 4. In an MPE, durable offers must be incompatible, i.e., $\widehat{s}^j < 1-\widehat{s}^i$.

Suppose to the contrary that there is an MPE with $\widehat{s}^j \geq 1-\widehat{s}^i$. Since we have proved that the offers coincide with the opponent's acceptance threshold, the condition implies $V^i(\widehat{s}^i, \widehat{s}^j \mid \widehat{\sigma})$ $\geq 1-$ $V^j(\widehat{s}^j, \widehat{s}^i \mid \widehat{\sigma})$. If only one player becomes flexible, the opponent's offer is accepted with probability 1, by previous steps. If both players become flexible simultaneously, stationarity implies that if the (random) proposer's offer is rejected, each player i will commit anew to $\widehat{s}^i$. Therefore, each short-run offer $\widehat{s}^i$ also corresponds to the opponent's acceptance threshold $V^j(\widehat{s}^j, \widehat{s}^i \mid \widehat{\sigma})$. We are now ready to prove that player j is better off by being flexible than sticking to the supposed offer $\widehat{s}^j \geq 1-\widehat{s}^i$. The fixed offer $\widehat{s}^j = V^i$ yields an expected payoff to player j

$$\delta(q^2\overline{V}^j + q(1-q)V^j + (1-q)q(1-V^i) + (1-q)^2\overline{V}) = \delta\overline{V}^j,$$

where $\overline{V}^j = (V^j + 1 - V^i)/2$ is player j's expected payoff when both players are committed as well as when both are flexible, due to the random proposer assumption, and arguments of the functions V^i and V^j are dropped for brevity. If instead player j is flexible, the expected payoff is

$$\delta(qV^j + (1-q)\overline{V}^j).$$

A few lines of algebra shows that the latter payoff is at least as large as the former whenever $V^i \geq 1 - V^j$, which is precisely what we have assumed. Since the flexible strategy also saves the (lexicographically small) commitment cost, the deviation pays.

Step 5. Constructing an equilibrium. The above steps imply that the first-period strategy pair $(\widehat{\phi}^i(w), \widehat{\phi}^j(w))$ must entail either incompatible commitment, $s^A + s^B < 1$, or at least one player remaining flexible. Let us now show that there is an MPE in which the players start out making incompatible commitments, $s_S^A + s_S^B < 1$ and subsequently follow the Markov strategies

$$\rho^i = \begin{cases} 0 & \text{if } s^j < s_S^j; \\ 1 & \text{otherwise,} \end{cases}$$

and, whenever flexible at the proposal stage,

$$\phi^i = \begin{cases} w & \text{if } s_{(t-1)+}^j > \overline{s}^j; \\ s_S^i & \text{otherwise,} \end{cases}$$

where $\overline{s}^j$ solves the equation $\delta(q\overline{s}^j + (1-q)/2) = s_S^j$.[16] That is, at the response stage, a flexible player i constantly rejects all offers below the threshold s_S^j, and accepts all offers at or above it; at the proposal stage, the flexible player makes a new incompatible offer s_S^i if the opponent was flexible or committed to an unacceptable offer in the previous period, or if by error some offer equal to or only slightly better than s_S^j was rejected, but remains flexible if (again by error) a sufficiently good offer was rejected in the previous period. Let V^i denote player i's equilibrium payoff associated with this strategy profile. Thus, $s_S^B = V^A$ and $s_S^A = V^B$. Since the acceptance

[16]The left-hand side of the last equation is the payoff from remaining flexible and accepting $\overline{s}^j$ at the response stage. The right-hand side is the expected payoff associated with making a new commitment.

thresholds are stationary,

$$V^A = \delta \left[q^2 V^A + q(1-q)(1-V^B) + (1-q)qV^A + (1-q)^2 \frac{1-V^B+V^A}{2} \right], \tag{1}$$

where the first term in the bracket corresponds to the case that both commitments stick, the second term corresponds to the case that only player A's commitment sticks, the third term corresponds to the case that only player B's commitment sticks, and the fourth term corresponds to the case that neither player's commitment sticks. Analogously,

$$V^B = \delta \left[q^2 V^B + q(1-q)(1-V^A) + (1-q)qV^B + (1-q)^2 \frac{1-V^A+V^B}{2} \right]. \tag{2}$$

The unique solution to these two equations yields the equilibrium payoffs

$$V_S^A = V_S^B = \frac{\delta\,(1-q^2)}{2(1-\delta q^2)}. \tag{3}$$

It is easily checked that no player has an incentive to unilaterally defect from the posited equilibrium path by deviating at the proposal stage: A lower offer is always rejected, and so merely delays the agreement without any compensating gain; a higher offer is accepted in exactly the same states as the equilibrium offer; and staying flexible instead of committing yields player i the payoff $\delta(qV_S^i + (1-q)\overline{V}^i)$ which is easily checked to be less than V_S^i when the payoffs are as in (3).

Note also that there cannot be any Markov Perfect equilibrium with incompatible offers that differ from those we have identified. Since the stationary acceptance strategy cannot vary across periods, and since all offers – also that made in the first period – must equal the flexible opponent's smallest acceptance threshold, offers must be the same in all periods.

Step 6. Ruling out other MPE candidates. Having characterized a unique MPE outcome with incompatible commitments, we finally need to rule out equilibria in which one or both players are flexible. (i) Let us first check whether there is any MPE in which exactly one player, say player i, starts out being flexible. Recall that the potential gain from flexibility is the ability to accept a committed opponent's offer. Now, what limits player j's ability to succeed with a low offer is player i's ability to reject and subsequently to make a counter-commitment (rejecting without making such a commitment only delays agreement without improving i's payoff). That is, by Step 2, $s^j = \widehat{s}^j$. Viewed from the beginning of period 1, the associated payoff for player i in case j's commitment sticks is thus $\delta V^i(\widehat{\phi}^i(\widehat{s}^j), \widehat{s}^j \mid \widehat{\sigma})$. But then player i

is better off by deviating and committing to $s^j = \widehat{\phi}^i(\widehat{s}^j)$ immediately, obtaining the expected payoff $V^i(\widehat{\phi}^i(\widehat{s}^j), \widehat{s}^j \mid \widehat{\sigma})$ in case j's commitment sticks. (ii) The case of both players starting out flexible is ruled out in similar fashion.

Hence, in the unique Markov Perfect equilibrium outcome, players start out by making incompatible offers.

Let us now investigate the equilibrium payoffs, characterized by (3), as the period length tends to zero. To do so, note that $\delta = e^{-rt}$ and $q = e^{-kt}$, and take the limit as t tends to $0+$. After an application of L'Hôpital's rule, we have

$$V_S^A = V_S^B = \frac{k}{2k+r}.$$

Proposition 2 *The symmetric game G^∞ has a unique Markov Perfect equilibrium outcome. In the limit as period length goes to zero, both players immediately make the offer $k/(2k+r)$, and the first player to have a decaying commitment accepts the opponent's offer.*

Since the sum of offers is smaller than 1, there is always some conflict. Since offers are accepted by the first player who has a loophole, the duration of the conflict is driven entirely by the rate of decay of commitments. To compute the expected conflict duration, note that at any time t, the probability that players have reached agreement is $1 - q^2$, which in the continuous time limit equals $1 - e^{-2kt}$. The rate at which players reach agreement is given by the derivative, $2ke^{-2kt}$. Thus, conflict duration is given by the exponential distribution.[17] The expected conflict duration is,[18]

$$D(k) = \int_0^\infty t \cdot 2ke^{-2kt} dt = \frac{1}{2k}.$$

For example, if t and k are measured on a yearly scale, and the instantaneous rate of decay corresponds to one breakdown per year ($k = 1$), then the expected duration of conflict is half a year.

[17] Interestingly, durations of actual labor conflicts are often claimed to be approximately exponentially distributed; see for example Kiefer (1988).

[18] Integrate by parts and use L'Hôpital's rule to get the second equality. (Of course, the equality is a well known property of the exponential distribution.)

3.2.2 Non-stationary subgame perfect equilibria

Let us now consider whether there are efficient non-stationary subgame perfect equilibria, with or without commitment on the equilibrium path.

Consider first the possibility of an efficient non-stationary equilibrium with commitment. One player – say player B – initially chooses to be flexible, whereas the other player, player A, initially commits. Efficiency dictates that B immediately agrees to A's offer. In case there are several such equilibria, we focus on the equilibrium that maximizes player A's payoff, call this payoff $\overline{V}$.

The offer $\underline{V} = 1 - \overline{V}$, must satisfy two conditions. First, it must be sufficiently large to keep player B from deviating and making an own commitment. Call this the *lower bound condition*: $\underline{V} \geq V_L$. Second, the offer $\underline{V}$ must be sufficiently small to induce a rejection by player B whenever player A deviates and offers $b < \underline{V}$. Call this the *upper bound condition*: $\underline{V} \leq V_U$. We have found an efficient equilibrium if these bounds are consistent, i.e., if $V_U \geq V_L$. Let us now derive the two bounds.

Step (i): Deriving the lower bound. The lower bound condition is found by computing the expected payoff, V_L, that player B can ensure herself by deviating to the smallest offer that player A is willing to accept in case A's commitment decays. In order to minimize B's incentive to deviate, continuation equilibria induced by A's rejection of an offer by B should be as favorable as possible to A, thus raising the offer that B has to make. Since the best offer that B will ever accept in equilibrium is V_L, it follows that if A rejects B's (deviating) offer, A's best policy is to renew the commitment to V_L.

Letting V_H denote the expected payoff of player A in case of rejection and renewal, as evaluated from the beginning of the next period, it follows that the optimal deviation by player B is to offer exactly V_H. Thus, V_L is given by

$$\begin{aligned} V_L &= \delta \left[q^2 V_L + q(1-q)(1-V_H) + (1-q)qV_L + (1-q)^2 \frac{V_L + (1-(1-V_L))}{2} \right] \\ &= \delta[q(1-q)(1-V_H) + (1-q(1-q))V_L]. \end{aligned} \tag{4}$$

The first term on the right hand side of the first equation is B's payoff in case both players' commitments stick. The second term is B's payoff in case only A has a loophole, the third term is B's payoff in case only B has a loophole, and the fourth term is B's expected payoff in case both players have loopholes. In the latter case, each player is proposer with probability 1/2, and the proposer offers exactly the opponent's expected continuation payoff.

Analogously, we have

$$\begin{aligned} V_H &= \delta\left[q^2V_H + q(1-q)(1-V_L) + (1-q)qV_H + (1-q)^2\frac{1-V_L+(1-V_L)}{2}\right] \\ &= \delta[qV_H + (1-q)(1-V_L)]. \end{aligned} \quad (5)$$

For (δ, q) inside the unit square, the unique solution to this pair of equations is

$$\begin{aligned} \widetilde{V}_L &= \frac{\delta q(1-q)}{1-\delta q^2}, \\ \widetilde{V}_H &= \frac{\delta(1-q)}{1-\delta q^2}. \end{aligned}$$

Note that the lower bound $\widetilde{V}_L$ is strictly smaller than the stationary payoff V_S^B.

Step (ii): Deriving the upper bound. Consider a deviation to a more aggressive commitment than $\overline{V} = 1 - V_U$ by player A. To prevent such a deviation, it must be profitable for player B to reject A's proposal, and propose the continuation equilibrium that is best for player B – namely, that in which B eventually gets $1 - \widetilde{V}_L$. Player B's maximum expected payoff to rejection is

$$V_U = q^2\delta V_U + q(1-q)\delta(1-\widetilde{V}_L) + (1-q)q\delta V_U + (1-q)^2\delta(1-\widetilde{V}_L),$$

yielding the solution

$$\widetilde{V}_U = \frac{\delta(1-q)}{1-\delta q^2}.$$

Note that $\widetilde{V}_U = \widetilde{V}_L/q > \widetilde{V}_L$. Thus, there are generally multiple efficient equilibria.

However, we are interested in the outcome as the time between periods approaches zero, in which case q approaches 1. Recalling that $\delta = e^{-rt}$ and $q = e^{-kt}$, and taking the limit as t tends to 0+, we see that if only one player commits, there is a unique pair of equilibrium payoffs. The flexible player obtains a payoff

$$\lim_{t\to 0+} \widetilde{V}_L = \lim_{t\to 0+} \widetilde{V}_U = \frac{k}{2k+r}.$$

and the committed player obtains the remainder, $(k+r)/(2k+r)$.

With this result in hand, it immediately follows that all efficient outcomes that yield a more even payoff distribution can also be sustained in a subgame perfect equilibrium: Let players make exactly compatible commitments that yield each player more than $k/(2k+r)$. Because a player who deviates from this strategy by waiting may be "punished" in case the opponent's offer decays and both are flexible at the response stage (by selection of the worst continuation equilibrium from then onwards), it is not worth attempting to save the lexicographically small commitment cost.

Proposition 3 *In the continuous time limit, any efficient outcome yielding each player a payoff of at least $k/(2k+r)$ can be sustained in a (non-stationary) subgame perfect equilibrium of G^∞.*

As the rate of decay goes to infinity, the difference between the stationary and the non-stationary outcomes vanishes; there is immediate agreement on the equal split in both cases. However, for sufficiently slow decay rates, non-stationary strategies admit virtually any outcome.

4 Extension: Asymmetries and correlation

To what extent do our previous results depend on our assumptions that players have identical utility functions and that commitments decay independently? As we shall now show, the results generalize naturally. (In the expressions below, superscripts are used according to the conventions $i \in \{A, B\}$ and $j \neq i$.)

Let the discount factor of player i be denoted δ^i. We retain the assumption that decay rates are constant. Let q^i be the probability that player i's commitment sticks if only player i commits. If both players are committed, let p^I be the probability that both commitments survive this period, and let p^i be the probability that only player i's commitment survives. If decay rates are independent of whether the opponent commits, we thus have $p^i = q^i(1-q^j)$. However, here we do not assume independence.

For brevity, we only consider the stationary equilibrium of the general game.

Generalizing (1) yields the two equations

$$V^i = \delta^i \left[p^I V^i + p^i(1-V^j) + p^j V^i + (1-p^I-p^i-p^j)\frac{V^i+(1-V^j)}{2} \right], \tag{6}$$

which have the solution

$$V_*^i = \frac{\delta^i \left(1 + p^i - p^j - p^I\right)(1 - \delta^j)}{2 + 2\delta^j \delta^i p^I - (\delta^i + \delta^j)(1 + p^I) - (\delta^j - \delta^i)(p^i - p^j)}. \tag{7}$$

To obtain the payoffs in the continuous time case, insert $\delta^i = e^{-r^i t}$, $p^I = e^{-(k^I + k^i + k^j)t}$, $p^i = e^{-(k^I + k^i)t}(1 - e^{k^j t})$. Taking the limit as t tends to 0+ yields

$$V_*^i = \frac{r^j \left(2k^j + k^I\right)}{2r^i r^j + (r^i + r^j)k^I + 2(r^i k^j + r^j k^i)}. \tag{8}$$

Inspection reveals that player i's expected payoff, V^i, is an increasing function of the opponent's decay rate, k^j, and the opponent's discount rate r^j, while it is a decreasing function of the own rates of decay and discount, k^i and r^i. In other words, a player benefits from increases in own patience and strength of commitment, and suffers from increases in an opponent's patience and sterngth of commitment.

A corollary of this result is that players who can affect their own decay rate have an incentive to reduce it. In the case that the own decay rate can be costlessly reduced, the unique equilibrium choice of decay rate is 0; if it is costly to reduce the decay rate, other outcomes are possible.

The impact of correlated decay is slightly more subtle. Differentiation of V^i with respect to k^I yields an expression which is positive if and only if $r^i > k^j - k^i$. Thus, a greater probability of joint decay always improves the payoff for player i if i is relatively weakly committed ($k^i \geq k^j$), but not necessarily otherwise. When the own commitment is stronger, it may be better to wait for a larger expected share than to increase the probability of settling early with a smaller expected share. The result thus suggests that it will be the weaker parties who most enthusiastically welcome the presence of mediators as well as of external pressure for flexibility in negotiations.

Note finally that when decay rates go to infinity we obtain Rubinstein's result that payoffs are determined by the relative patience $r^i/(r^i + r^j)$.

Of course, these characterization results are only of interest if a stationary equilibrium continues to exist in the general case. To derive an existence condition, we must consider players' incentive to deviate from the posited equilibrium strategies. Since we have already characterized the optimal commitment strategies, the only relevant one-step deviation is for one of the players to refrain from making a commitment, instead waiting in order to agree to the opponent's proposal.[19] For player i, making an own commitment is then strictly preferable to staying flexible if and only if

$$V^i > \delta^i \left[q^j V^i + (1 - q^j) \frac{V^i + (1 - V^j)}{2} \right].$$

Inserting for V^i from (6) and simplifying, the condition becomes

$$\left(p^i - p^j + q^j - p^I\right)(1 - V^i - V^j) > 0.$$

Thus the inefficient stationary commitment equilibrium (with $V_*^i + V_*^j < 1$) exists if $p^i - p^j + q^j - p^I > 0$ for $i = A, B$. If we assume that $p^j = q^j(1 - q^i)$ (so that decay rates are independent), then $p^I = q^i q^j$, and the condition becomes $q^j - q^j q^j > 0$, which is trivially satisfied. When $p^i = p^j$, it is only when a player can powerfully reduce the opponent's commitment power by refraining to commit (and thereby bring q^j below p^I in the formula above) that the incompatible commitment equilibrium fails to exist.

Finally, in Appendix B we consider the case of non-linear utility. In the stationary equilibrium, a player whose risk-aversion increases will make a more generous offer and receive a less generous offer.

5 Final remarks

The paper integrates two major strands of bargaining literature. The first strand, associated with Schelling (1956, 1960) and Crawford (1982) focuses on strategic commitments. That is, it emphasizes the role of inflexibility. The second strand, associated with Ståhl (1972) and Rubinstein (1982) focuses on the opportunity for continued negotiation in the absence of agreement. That is, it emphasizes the role of flexibility.

By admitting strategic commitment as well as opportunities for continued negotiation, the model illuminates how equilibrium outcomes vary with the available commitment technology. When commitments decay swiftly, the model's predictions closely resemble the efficient and relatively symmetric outcome derived by Rubinstein. When commitments decay slowly, predictions instead resemble the inefficient and asymmetric outcomes emphasized by Schelling and Crawford.

[19] As usual, if there is no profitable one-step deviation, there is no profitable many-step deviation.

Let us end by highlighting the role of some critical assumptions.

First, let us reiterate that the cost of making offers is crucial for getting rid of compatible-commitment equilibria. However, as shown in Ellingsen and Miettinen (2008) for the one-shot case, such equilibria can also be eliminated through equilibrium refinement. It would be interesting to see what equilibrium refinement would be required to derive the same outcome without commitment costs.

Second, our cost-of-commitment function is rather special. Suppose realistically that less generous commitments entail higher costs, greater decay, or both. Under what conditions on the cost function would the model's main results survive? Our conjecture is that there is a large class of convex cost functions under which there exists a unique Markov Perfect equilibrium with essentially the same features as here.[20]

Third, we assume that a player whose commitment has decayed may commit anew. However, it is quite straightforward to compute the Markov Perfect equilibrium of the modified model in which the players cannot make a second commitment. While there are some quantitative changes, the qualitative features of the results remain the same. Specifically, a player whose commitment decays must either accept the opponent's proposal immediately or wait for the opponent to become uncommitted too, after which there is an expected equal split. In Markov Perfect equilibrium, offers are still incompatible, but set at the level that a flexible opponent will immediately accept. Since the flexible player's continuation payoff following (an off-equilibrium) rejection is now somewhat less attractive, equilibrium offers will be less generous, but in general still positive. However, the fast and slow decay limits are the same as before, with payoffs of 1/2 and 0 respectively.

Fourth, we have assumed that negotiations can go on indefinitely. What if there is a final deadline after which the gain from trade evaporates? The model with a deadline obviously poses some new challenges. It must be solved by backward induction. Preliminary investigations indicate that much of the agreement may take place at the deadline rather than at the first moment that one of the players becomes flexible, but we leave this question for further research.

Finally, a referee has asked whether it is possible to obtain all our results in a simpler

[20]This conjecture is easy to prove in the one-shot case. However, it is also easy to prove that there are cost functions such that all equilibria are asymmetric. In particular, if it is cheap to obtain perfect commitment, the model has no symmetric equilibrium.

model. The answer is affirmative. Consider for example the following dynamic version of the Nash Demand Game: In every period, any uncommitted player can make a demand. Demands are durable in the same way as are the offers above. Whenever demands are incompatible, there is no deal that period. If demands are compatible, the largest demand is implemented (i.e., any surplus goes to the most generous player), with randomization if there is a tie. There is no cost of making a demand of zero, but a lexicographically small cost of making any other demand. Due to the cost of making positive demands, it is easy to see that this model also lacks compatible-commitment equilibria. Instead, as one might check using analogous arguments to those we advanced above, when the period length goes to zero the unique Markov Perfect equilibrium outcome is characterized as in Proposition 2.[21]

6 Appendix A: Proof of Proposition 1.

It is trivial that no player i has an incentive to deviate from the proposed equilibrium: Rejection of $s^j = 0$ yields the payoff 0. Since this is identical to the payoff under acceptance, there is nothing to gain by rejecting at the response stage. Consider the offer stage. Given $s^j = 0$, any own offer $s^i \in [0, 1]$ yields an expected payoff $\delta[(1 - s^i)(1 - q)q + (1/2)(1 - q)^2] - c$, where the first term is the payoff in case the own commitment sticks and the opponent's commitment decays, and the second term is the expected payoff in case both commitments decay (in which case the random proposer will make an offer of 0). The expression is maximized by the offer $s^i = 0$. The remaining strategy, w, yields an expected payoff $\delta(1/2)[(1 - q)q + (1 - q)^2]$, which is smaller than $\delta[(1 - q)q + (1/2)(1 - q)^2] - c$ under our condition on c.

Let us next prove that G^1 has no other subgame perfect equilibrium. We do this by showing that if player j plays optimally at the response stage, all offers $s^i \neq 0$ are iteratively strictly dominated. Observe first that player i strictly prefers w to any offer $s^i \in [1/2, 1]$: The latter

[21] In Appendix C, we sketch the analysis of the simplified model. We think that simplicity has considerable merit. However, we also sympathize with the objection – raised against previous versions of our work – that one should perhaps not be allowed the luxury of just assuming agreement or specific splits at any node of a non-cooperative bargaining game, especially if that game already requires explicit acceptance and rejection in order to reach agreement at other nodes. The assumption that compatible offers entail (a specific) immediate agreement constitutes such a simplification.

commitment strategy gives player i a payoff $\delta[q(1-s^i)+(1-q)/2]-c < \delta/2$ when player j chooses w. It gives at most $\delta[qs_j+(1-q)/2]-c < \delta[qs_j+(1-q)/2]$ when player j chooses a compatible commitment $s_j \in [1-s_i, 1]$. Finally it gives at most $\delta[q(1-q)(1-s_i+s_j)+(1-q)^2/2]-c$ when player j chooses an incompatible offer. Given that $(1-s_i) < 1/2$, this is smaller than the payoff to remaining flexible $\delta[qs_j+(1-q)/2]$.

After these strategies are eliminated, for any offer $s^i \in (0, 1/2)$, there exists some $\epsilon > 0$ such that s^i is strictly dominated by the mixed strategy $\phi^i = (p^i(\epsilon) = 1-s^i, p^i(w) = s^i)$: If player j plays w, player i's payoff to the pure strategy s^i is $\delta[q(1-s^i)+(1-q)/2]-c$; whereas under the mixed strategy ϕ^i he gets $\delta[q(1-s^i)(1-\epsilon)+((1-q)(1-s^i)+s^i)/2)]-(1-s^i)c$, which is greater for sufficiently small ϵ. If player j plays $s^j \in [0, 1/2)$, then the payoff to a pure commitment strategy $s^i \in (0, 1/2)$ is $\delta[q(1-q)(1-s^i)+(1-q)qs^j+(1-q)^2/2]-c$, whereas the payoff to the mixed strategy is $\delta[q(1-q)(1-s^i)(1-\epsilon)+ (1-q+qs^i)qs^j+(1+q-qs^i)(1-q)/2]-(1-s^i)c$, which is again greater for sufficiently small ϵ.

The only remaining proposal strategies are 0 and w. Could there be a subgame perfect equilibrium in which player i plays w with probability $p^i(w) > 0$ and offers 0 otherwise? No: given our assumption on c, the unique best response by player i to any mixture over w and 1 by j is $s^i = 0$.

7 Appendix B: Risk Aversion

Now, let the players hold non-linear utility functions, so that $u^i(s^j)$ is player i's utility from consuming the share (accepting the offer) of the surplus s^j.

It is straightforward to check that our construction of the stationary equilibrium does not depend on the risk neutrality assumption. Thus the optimal commitment offers now satisfy $u^A(s^B) = V^A$ and $u^B(s^A) = V^B$. Correspondingly

$$V^A = \delta\left[q^2V^A + q(1-q)u^A(1-s^A) + (1-q)qu^A(s^B) + (1-q)^2\frac{u^A(1-s^A)+u^A(s^B)}{2}\right] \quad (9)$$

and

$$V^B = \delta\left[q^2V^B + q(1-q)u^B(1-s^B) + (1-q)qu^B(s^A) + (1-q)^2\frac{u^B(1-s^B)+u^B(s^A)}{2}\right]. \quad (10)$$

To study the effect of risk-aversion on optimal commitments, let us substitute $V^A = u^A(s^B)$ and $V^B = u^B(s^A)$ into the system and solve for $u^A(s^B)$ and $u^B(s^B)$, respectively. This yields

$$u^A(s^B) = Ku^A(1 - s^A), \tag{11}$$

$$u^B(s^A) = Ku^B(1 - s^B), \tag{12}$$

where

$$K = \frac{\delta(1-q)}{1 - \delta(1-q^2)/2}.$$

Differentiation yields

$$\frac{\partial u^A(s^B)}{\partial s^B} ds^B = -K \frac{\partial u^A(1 - s^A)}{\partial s^A} ds^A \tag{13}$$

and

$$\frac{\partial u^B(s^A)}{\partial s^A} ds^A = -K \frac{\partial u^B(1 - s^B)}{\partial s^B} ds^B. \tag{14}$$

Dividing and rearranging, we have

$$\frac{\frac{\partial u^A(s^B)}{\partial s^B}}{\frac{\partial u^A(1-s^A)}{\partial s^A}} \frac{ds^B}{ds^A} = \frac{\frac{\partial u^B(s^A)}{\partial s^A}}{\frac{\partial u^B(1-s^B)}{\partial s^B}} \frac{ds^A}{ds^B}.$$

Suppose we make u_A more concave. Since we are free to make affine transformations, let the new utility function have the same value as the old function in the two points $1 - s^A$ and s^B. Greater concavity then implies that the new function is above the old function in the interval $\left[s^B, 1 - s^A\right]$, and thus that the derivative at s^B goes up and that the derivative at $1 - s^A$ goes down. Thus, the equality only continues to hold if ds^B/ds^A goes down. From each of the equations (13) and (14), we see that s^A and s^B move in opposite directions. Thus, we must have s^B down and s^A up. In other words, if player A becomes more risk-averse, A's offer becomes more generous and B's offer becomes less generous.

8 Appendix C: Simplified game.

While our results are easy to describe and interpret, the model set-up is relatively complex. We could have derived almost exactly the same results in a less complex setting, by invoking short-cuts: Instead of fully describing the extensive form of the game, some outcomes might be assumed directly.

Specifically, consider the following simplification, which is the Nash Demand Game alluded to in the concluding section, but couched in the terms of offers (as elsewhere in the paper) instead of demands: Long-run offers are formulated and they decay as in the original model but (i) two compatible offers always share the surplus as suggested by the more generous offer (in the NDG, this means greediest demand implemented exactly); (ii) no short-run offers are formulated, but a unilaterally uncommitted player can reject the opponent's long-run offer and formulate a new long-run offer at the subsequent offer stage; (iii) If both players make the same offer, it is random which offer gets to count, (iv) It is costless to make the offer 1 (demand 0); all other offers carry a lexicographically small cost.

The most controversial short-cut is (i), since it directly assumes agreement as well as a particular sharing rule.

It is straightforward to see that the argument of Proposition 1 goes through in this variant of game G^1. Thus players $i = 1, 2$ commit to offer $s^i = 0$ in the simplified game as well.

The MPE of this variant of game G^∞ also coincides with that of the original game. To see this, notice that the responder's continuation payoff $R^i(s^i, s^j|\hat{\sigma})$ coincides with that defined before, except that instances involving the flexible strategy w are replaced by instances involving the strategy 1. The optimal durable offer by i is constructed as before and it coincides with

$$min_{s^k} V^j(\hat{\phi}^j(s^k), s^k|\hat{\sigma}).$$

The steps 2 to 6 go through as before. The only difference is that we can immediately replace the continuation payoff for j in the case that both are flexible, $(1 - V^i + V^j)/2$, by 1/2. Since in the symmetric case we have shown that the equilibrium is symmetric, and thus $V^A = V^B$, this does not impact the equilibrium payoffs, (3). At step (iv) the compatible commitments equilibrium is ruled out by considering a deviation (to offering 1, i.e., demanding 0) by the player who receives the larger offer.

Concerning non-stationary SPE, we can solve for the greatest threat in a similar manner as before. As it turns out

$$V_H = V_L = V_U = V_S = \frac{\delta(1 - q^2)}{2(1 - \delta q^2)}.$$

The difference from the previous model arises because two flexible players will here each, by assumption, get expected payoff 1/2. Every efficient sharing yielding both players more than V_S can thus be sustained in a non-stationary SPE through the threat of playing the MPE.

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