The effects of debt stabilising fiscal rules in a macroeconomic (DSGE) model

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Helsinki
2017
Title of thesis:
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Abstract:
The objective of this study is to investigate the effects of fiscal rules on economy and welfare. The study is conducted by a simulation-based analysis, using a New-Keynesian dynamic stochastic general equilibrium (DSGE) model. The model describes a small and open economy with a public sector and a fraction of non-Ricardian households. The study is counterfactual and delimited to fiscal rules for debt stabilisation, inspired by the EU Stability and Growth Pact. The rules are applied by adjusting either consumption or labour taxation. Three sets of rules are created. First, debt rules where taxes are changed such that the public debt level does not change. Second, rules where taxes are changed such that the debt-to-production ratio is constant. Third, rules where the balance in structural terms is kept constant. The results suggest that when fluctuations in the debt-to-production ratio and the structural balance caused by a technology shock are eliminated, the welfare response is higher using consumption taxation. When public debt is stabilised, the welfare remains higher using labour taxation. Furthermore, the study points towards a procyclical response when using rules providing a fixed debt-to-production ratio compared with rules eliminating fluctuations in the structural balance. However, the difference in the cyclicality in the development of GDP is small when the effects of balanced budget rules are compared with rules eliminating fluctuations in the structural balance.

Keywords:
Fiscal policy, DSGE model, impulse response, fiscal rule, macroeconomic modelling
Acknowledgements

I would like to thank faculty, students and staff of the Economics Department at Hanken School of Economics for support during the process.

I am also grateful for the knowledge and tools I have received from the Economics Department at the Ministry of Finance.

Helsinki, December 2017

Peter Elmgren
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1 Introduction

Fiscal policy is one of the main tools for a government to influence the economy. There are two main benefits of fiscal policy, namely to stabilise the economy and to ensure sound public finances. The latter is to a certain extent controlled by fiscal rules – numerical constraints on expenditures or debt accumulation conditional on economic performance. Because the economic significance of implementing fiscal rules typically is substantial, there are continuous interest and discussions on how such rules should be used.

One approach is to analyse the effects of fiscal rules in macroeconomic models, such as dynamic stochastic general equilibrium (DSGE) models, in order to equip policymakers with insights regarding the consequences of implementing different rules. There is no standard procedure in the literature how fiscal rules should be modelled for example in DSGE models. Therefore, this thesis also investigates how far the actual fiscal rules used in the Stability and Growth Pact of the European Union could be implemented in a fairly standard linear DSGE model. This is an interesting but challenging question since the fiscal rules are getting more and more complex as more weight is put on fiscal rules in the European Union following the debt crisis and the implementation of the currency union.

This study takes a broad perspective and investigates the performance of several types of fiscal rules, first inspired by the literature and later directly by the EU legislation. Three different sets of rules referring to debt are created. First, debt rules where taxes are changed such that the public debt level is kept constant. Second, rules where taxes are changed such that the debt-to-production ratio is constant over time. Third, rules where fluctuations in a balance defined in structural terms are eliminated. The rules are evaluated and discussed based on their effects on economy and welfare, but also based on their ability to communicate their original objectives in the model. The thesis is delimited to rules involving debt. Only consumption and labour taxes are used for adjustment.

This study is conducted based on simulations of fiscal feedback rules similarly as in Leeper (1991). The thesis includes a description of the model in use, which is the New-Keynesian DSGE model of the Finnish Ministry of Finance (also called ”KOOMA”). The model performance is compared to other models used for similar purposes, such as Forni, Monteforte & Sessa (2009). However, there are not many counterfactual studies in the literature where different debt rules are used with taxes.

The result suggests that when eliminating fluctuations in the debt-to-production ratio and the structural balance caused by a technology shock, the welfare response is higher when using consumption taxation than when using labour taxation. However, when public debt is stabilised, the welfare remains higher when labour taxation is used for stabilisation. This is explained by higher volatility in the welfare measure when changing the consumption tax rate than when changing the labour tax rate. The study points towards procyclical responses when rules providing a fixed debt-to-output ratio are compared with rules eliminating fluctuations in the structural balance. In the GDP, no such procyclicality is observed in the simulations when balanced budget rules are compared with rules used to eliminate fluctuations in the structural balance.

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1 The welfare measure is defined in Chapter 4.
2 The terms ”GDP”, ”output” and ”production” are used interchangeably in this thesis.
Instead of using a balanced budget rule, the result suggests the following when the economy is affected by economic growth originated from a positive technology shock. The usage of rules eliminating fluctuations in the structural balance should also be motivated by other than cyclical arguments. Rules providing a constant debt-to-output ratio should be used carefully since these rules affect the economy procyclically.

The finding in this thesis adds to the literature by providing a set of rules for debt stabilisation to be used in counterfactual policy simulations. The study is interesting also from a policymaker perspective, because it discusses the application of different fiscal rules, both in terms of their benefits and their costs. Furthermore, the thesis contributes to the debate on the effects of fiscal rules from a DSGE model perspective.

The thesis is divided into eight chapters. Chapter 2 provides a background to fiscal rules by presenting their legal basis and the Stability and Growth Pact in a nutshell. The chapter provides also a short literature review. Chapter 3 presents the macroeconomic model on which the study is built upon. The methodology Chapter 4 provides background to how the model will be used in the study and how the rules are evaluated. Chapter 5 compares the performance of the model to other models in the literature. Chapter 6 specifies fiscal rules. Chapter 7 presents the results once the rules have been implemented in the model. It also analyses their functionality. The study and the rules are further discussed in Chapter 8. Chapter 9 summarises and concludes the thesis.
2 Background

This chapter provides the background necessary for the study. Firstly, Part 2.1 introduces fiscal policy and fiscal rules. Relevant aspects of the Stability and Growth Pacts are covered in Parts 2.2 and 2.3. Part 2.4 shortly motivates the usage of macroeconomic models in investigating the effects of fiscal rules. Finally, Part 2.5 looks at how fiscal policy and fiscal rules have been used in macroeconomic models in the literature.

2.1 Fiscal policy and fiscal rules

Economic policies may have several different objectives. For instance, the target of an economic policy could be to stabilise the economy, to ensure sound public finances or to promote other targets of a country. For members of the European Union, for example, Article 120 of the Treaty of the Functioning of the European Union (TFEU) obligates the member states to conduct economic policy that contributes to the objectives of the EU. One type of economic policy is fiscal policy, which can be said to take place as a consequence of running a government. In this context the word policy should bring the concept a meaning of running a government with the purpose of achieving certain objectives. These objectives might for example refer to stabilisation of the macro-economy.

There are several reasons to why fiscal policy is of special interest in Europe at the moment. First, a prolonged recession makes especially countercyclical fiscal policy interesting. Second, since the introduction of the Economic and Monetary Union of the European Union (EMU), a sovereign member country has very limited opportunities to conduct an individually tailored monetary policy. Fiscal policy therefore as an important means for economic policy.

It is important to notice that, in the thesis, fiscal policy in itself is not of major interest. Different properties and aspects of fiscal policy, such as the effects of economic stimulation, have been discussed in many studies. Clearly, it may exist stronger and weaker policies with the objective of stimulating, or in other ways changing, the economic performance. Fiscal policy as such remains as a powerful tool since it changes directly either taxes or public expenditures that by definition are part of the gross domestic product. One can therefore conclude that fiscal policy is a potentially strong modifier of the production, but its long term consequences are more debatable. The focus of this study is not on fiscal policy as such but rather on rules within fiscal policy can operate whatever the content of the policy might be. In a report by the International Monetary Fund, a fiscal rule is defined as ”a long-lasting constraint on fiscal policy through numerical limits on budgetary aggregates”.

One could also see fiscal rules as a type of fiscal policy. Once the rules are enforced they change the public economy in order to fulfil certain objectives – fiscal policy. The point made here is that the contents of fiscal policy might change once fiscal rules are enforced. This might be the case for example when the public economy shows large deficits, since the rules are typically created to ensure a healthy performance of the public economy in the long run. The next part gives a short overview of how the rules look like.
2.2 The legal basis for fiscal rules within the EU

The rules for fiscal policy within the EU are determined in the Stability and Growth Pact (SGP) of the European Union. The pact follows the Maastricht Treaty, though it is based on Articles 121 and 126 of the TFEU [European Commission 2017a, European Commission 2017b]. The SGP has been updated several times such as in 2011 when the so-called Six Pack—a pack of more detailed and updated rules to improve the SGP—was created [European Commission 2017b].

The Stability and Growth Pact can be divided into a preventive arm, promoting a sound and sustainable structure of public finances, and into a corrective arm, stating the measures and steps to be taken if a member country finds itself in a situation with too weak or unstable finances. [European Commission 2017a]

The preventive arm is mainly motivated by Article 121 of the TFEU although its implementation is specified further among others in Council Regulation [1466/97] that in turn leaves room for even more specified policies gathered in a code of conduct, see European Commission [2016a], and in a Fiscal Compact, see European Commission [2012]. The Article builds on a statement that fiscal policy should be viewed at with a common concern, that the European Council should formulate broad guidelines for the economic policy of the members of the union and that the European Commission can give warnings to members who do not fulfill these requirements (Council of European Union [2012]).

The corrective arm builds upon Article 126 of the TFEU, where it is stated (i) that excessive deficits are to be avoided, (ii) that the budgets shall be in accordance with budgetary discipline where certain reference values are used, and (iii) that a country not complying with the rules has to take specific correcting measures monitored by the Commission. The Article is specified and implemented for example in Council Regulation [1467/97] which is even further specified in the above mentioned code of conduct (European Commission 2017a). The sanctions for countries not fulfilling the requirements became stricter with the 2011 amendment of SGP (the Six Pack) and additional reporting requirements where taken into use in the Two Pack that builds on Article 136 of the TFEU (European Commission 2017a).

2.3 Fiscal rules as a part of the Stability and Growth Pact

The rules for fiscal policy within the preventive arm of SGP can be divided into two pillars (European Commission 2017a, p. 23). One of the pillars determines a structural balance, which should serve as a base for budget proposals in a near future. The other pillar determines an expenditure aggregate, which is used in a fiscal rule limiting government expenditure growth. The two pillars are discussed in more detail below, but with focus on the first one.

The core of the first pillar, and a central part of the preventive arm, is the medium-term objective or MTO set for the budgetary position of each country. The MTO binds future budgetary proposals to its target. The MTO is calculated in structural terms, meaning the budget position is cyclically adjusted and that large but temporary expenditure or income has

\[\text{medium-term objective} \]

3The legal basis for the corrective arm includes also other regulations that, however, mainly concern enforcement, reporting, monitoring and budgetary surveillance.
to be removed. Therefore, the Commission calculates a structural balance $B_s^t$ as the cyclically adjusted balance $B_a^t$ without large but temporary measures $M_t^t$. Since the cyclically adjusted balance $B_a^t$ should be seen as the balance $B_t$ in terms of GDP $y_t$ subtracted by the output gap $y_{OG}^t$ defined by the Commission, one can summarise the structural balance as

$$B_s^t = \left( \frac{B_t}{y_t} - \varepsilon \cdot y_{OG}^t \right) - M_t^t, \quad (2.1)$$

where the semi-elasticity $\varepsilon^{OG}$ is a measure of the sensitivity of the balance to the output gap and $B_t/y_t - \varepsilon^{OG} y_{OG}^t = B_a^t$. The definition of the structural balance given by (2.1) is central, because the medium-term objective is defined in terms of it. The MTO is a target value for the structural balance (this is a very crucial aspect for understanding the MTO rules presented in subsequent chapters).

The medium-term budgetary objective can be determined in three different ways. The boundary MTO for a country should according to the first pillar be the largest (strictest) of the three different MTOs defined below.

First, it can be defined as a minimum benchmark

$$MTO_{mb}^t = -3 - \varepsilon y_{OG}^t, \quad (2.2)$$

according to which the (general) government cyclically adjusted balance can shrink to three per cent of GDP, but the number is corrected for the fact that different member countries are unequally sensitive to output volatility. The purpose of the safety margin is to ensure that the deficit of a member country does not exceed 3% during a normal business cycle.

Second, a MTO is also defined in terms of implicit liabilities and debt, $MTO_{id}^t$. The idea is to have a budgetary objective where the balance $B_{60}^t$ stabilises the debt on a level corresponding to 60% of GDP. However, the present value of future costs relating to ageing $C_a^t$ are taken into account up to $\alpha = 33\%$ and specific effort measures $F_t$ are taken by countries with a debt in excess of 60% of GDP. This can also be expressed as

$$MTO_{id}^t = B_{60}^t + \alpha C_a^t + F_t. \quad (2.3)$$

Third, an additional MTO is defined for the EMU member states (and for Denmark), such that the deficit has to be less than or equal to 1% of GDP, i.e.

$$MTO_{ema}^t = -1. \quad (2.4)$$

Notice that variables for measures of public sector balance are denoted by calligraphic b. Measures of public sector debt are denoted by the non-calligraphic b. For example is nominal balance denoted by $B_t$ and nominal public debt denoted by $B_i$.

Since the structural balance is defined in proportion to GDP, see equation (2.1), this will also be the case for MTO. Hence, the formulas (2.2), (2.3) and (2.4) should be interpreted as percentages of GDP.

The MTO is defined in terms of surplus meaning that a higher (less negative) MTO indicates a stronger financial position. Since the established bound for MTO is a limit value, the members are allowed to follow a higher medium-term objective if they find it more convenient.

The effort variable $F_t$ gets its value from a linear function $F_t = 2.4 \cdot B_t/y_t - 1.24$, where $B_t$ is the general government level of debt.
Table 1: Adjustment path for countries not obtaining their MTO, % of GDP.

<table>
<thead>
<tr>
<th>Times:</th>
<th>$\frac{d_T}{\gamma_T} \leq 60%$</th>
<th>$\frac{d_T}{\gamma_T} &gt; 60%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceptionally bad, $g_T^Y &lt; 0$ or $y_T^X &lt; -4%$</td>
<td>No adjustment</td>
<td>No adjustment</td>
</tr>
<tr>
<td>Very bad, $-4% \leq y_T^X &lt; -3%$</td>
<td>No adjustment</td>
<td>0.25</td>
</tr>
<tr>
<td>Bad, $-3% \leq y_T^X &lt; -1.5%$</td>
<td>No adj. if $g_T^Y &lt; g_T^P$</td>
<td>0.25 if $g_T^Y &lt; g_T^P$</td>
</tr>
<tr>
<td></td>
<td>0.25 if $g_T^Y &gt; g_T^P$</td>
<td>0.5 if $g_T^Y &gt; g_T^P$</td>
</tr>
<tr>
<td>Normal, $-1.5% \leq y_T^X &lt; 1.5%$</td>
<td>0.5</td>
<td>$&gt; 0.5$</td>
</tr>
<tr>
<td>Good, $y_T^X \geq 1.5%$</td>
<td>$&gt; 0.5$ if $g_T^Y &lt; g_T^P$</td>
<td>$\geq 0.75$ if $g_T^Y &lt; g_T^P$</td>
</tr>
<tr>
<td></td>
<td>$\geq 0.75$ if $g_T^Y &gt; g_T^P$</td>
<td>$\geq 1$ if $g_T^Y &gt; g_T^P$</td>
</tr>
</tbody>
</table>

Define $g_T^Y \equiv \frac{y_T^P - y_T^{P-1}}{y_T^{P-1}}$ as the year-on-year growth rate in production, $y_T^X \equiv \frac{y_T^{OG}}{y_T}$ as the output gap in proportion to production and $g_T^P \equiv \frac{y_T^P - y_T^{P-1}}{y_T^{P-1}}$ as the year-on-year potential growth rate.

If a country is not at its MTO it should adapt its structural balance along a specified adjustment path. Different degrees of fiscal adjustments have to be enforced depending on whether the debt of the country in question is above or below the reference value of 60% of GDP, and on the size of its output gap. Depending on these factors, the fiscal adjustment for the following year should range between zero and at least one per cent of GDP (European Commission, 2017a, p. 38). The adjustments are measured as a reduction in the structural balance in proportion to production. The adjustment path is summarised as percentages of GDP in Table 1.

The second pillar obligates member countries for example to obey an expenditure rule that limits public expenditure growth. The idea is that the member states have to conduct fiscal policies such that they either remain at their MTO or reach it. Further details about the expenditure rules are left out since this study is delimited to rules involving debt and changes in the income side. Only the rules in the first pillar will therefore be discussed in subsequent chapters.

2.4 The need for modelling to simulate fiscal policy

As the EU Commission’s toolkit for fiscal policies is often subject to changes and adjustments, it is of interest to develop tools for evaluating different types of rules. Since fiscal policy ultimately is in the hands of the government of each country, it should also be of

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8 Note that the notation in Table 1 is consistent with earlier notation except for the time index $T$, which here refers to periods on yearly basis.

9 Compare for example the contents of European Commission (2017a) with the contents in European Commission (2016b).
interest on national level to estimate the effects of different policy rules. Clearly, there is a need to also evaluate the rules in a macroeconomic context. A natural way of doing this is to use a macroeconomic model, such as a dynamic stochastic general equilibrium (DSGE) model.

DSGE models are based on microeconomic theory. Though the benefit of using the models has been criticised in many studies, such as in Chari, Kehoe & McGrattan (2009), they are often used as they provide at least some perspective on the economy, as will be seen in the next part.

2.5 Literature review

Since the potential societal gains from conducting good fiscal policy are substantial, many studies investigate how the government behaves or could behave. As will be explained below, in the context of New-Keynesian DSGE models, many studies focus on fiscal policy more generally – not on fiscal policy per se, see the discussion in Part 2.1. Also, it is common to study both fiscal and monetary policy, for example Muscatelli & Tirelli (2005), Colciago et al. (2008), Leeper, Walker & Yang (2013) and Cebi (2012) investigate the interaction between monetary policy and fiscal policy. Furthermore, a significant part of the literature is concerned with more fundamental questions of fiscal policy, such as how consumption should respond to a government consumption shock in a model with both Ricardian and non-Ricardian households, see for example Coenen & Straub (2005) or Iwata (2009). There are in fact very few studies trying to create policy rules inspired for example by the SGP.

There has been a clear interest in fiscal policy already during the years preceding the financial crisis. Many papers make use of DSGE models to study the effects of different constraints on public finances, such as Ratto, Roeger & in’t Veld (2009), Muscatelli & Tirelli (2005), Colciago et al. (2008), Coenen & Straub (2005) and Raila (2006). The discussion has continued during and after the recession with papers focusing on stabilisation properties of fiscal policy (see for example Stähler & Thomas, 2012; Kliem & Kriwoluzky, 2014; Garcia, Restrepo & Tanner, 2011). The fact that many of the papers are written by authors based in Europe reflects, at least to some extent, the view that fiscal policy has become more interesting with the establishment of the EMU. This might be a consequence of member countries no longer being able to respond to asymmetric shocks with monetary policy.

In DSGE models, fiscal policy can be conducted by establishing a constraint on government income or expenditure, usually with fairly simple equations. One early contribution is Leeper (1991), where taxes are determined by feedback from, in this case, the lagged debt level. Similar rules where an objective depends on feedback from other policy variables have been named feedback rules. These rules are to some extent similar to the popular Taylor rule used in monetary policy, even though the policy parameters are not the same.10 Attempts have been made to construct similar rules for fiscal policy. One of them is Kliem & Kriwoluzky (2014), who argue that the most relevant feedback variables for their estimated rule for income tax rate are government debt and worked hours, whereas the rule for capital tax rates should get its feedback from government debt and the level of investment. Another

10The Taylor rule, named after its proposer, see Taylor (1993), determines the nominal interest rate \( r_t \) as a function of a target for real interest rate \( \bar{r}_t \), inflation \( \pi_t \), output gap \( y_{OG}^t \) and an error term following a first-order autoregressive process, i.e. an AR(1) process, according to \( r_t = \bar{r}_t + \phi_1 \pi_t + \phi_2 y_{OG}^t + \varepsilon_t \).
attempt to create Taylor type rules for fiscal policy is by [Kendrick & Amman (2011)] who argue that fiscal policy should be modifiable more frequently to make monetary and fiscal policy more integrated. The paper proposes a constraint for government consumption that gets feedback from output and inflation, i.e. $g_t = \phi_1 y_t + \phi_2 \pi_t + \varepsilon_t$, where $g_t$ is government spending, $y_t$ is production or output, $\pi_t$ is inflation, $\varepsilon_t$ is an error process and the $\phi$s are policy parameters regulating the strength of the rule.

However, as pointed out in [Kliem & Kriwoluzky (2014)], many of the rules are simple ad hoc processes, since there exists no consensus on how fiscal policy should be specified. The lack of consensus may reflect the differences between the public sectors in different countries. For example, the tax rates vary between counties. The lack of consensus may also reflect the dynamics of the economy - a policy may for example work at a time but no longer in the future (the Lucas critique). Furthermore, fiscal policy is more complex than for example monetary policy [Muscatelli & Tirelli (2005)].

Partly due to the lack of consensus on the specification of policy, the literature strives to use simple and transparent rules. As noted in [Iwata (2009)], the simplest rule is just a government budget constraint. If the constraint does not include debt, one will have a balanced budget rule, where the expenditures must equal the incomes (taxes) $\tau_t$ in every period. However, if the constraint does include debt $b_t$, i.e.

$$g_t = \tau_t - b_t,$$

such as in [Garcia et al. (2011)], one will have a procyclical rule since government spending is higher in good times when tax income is higher. As noted in [Garcia et al. (2011)], a similar rule can be countercyclical once government expenditures are determined around a fixed level of tax income $\tau^*$ according to $g_t = \tau^* - b_t$.

Policy can be specified in several ways. A simplistic specification for fiscal policy that is used frequently in the literature [Leeper et al. 2013, Coenen & Straub 2005] is a pure autoregressive process of first-order $AR(1)$ such as

$$g_t = \rho g_{t-1} + \varepsilon_t$$

for government consumption, where $\rho < 1$ is a constant. Here the idea is to make the politics entirely exogenous – it influences the economy only through temporary shocks $\varepsilon_t$. Nevertheless, it is also common to bind the rules to debt and (or) output, such as in [Iwata 2009]. For example, in their search for optimal policy in a fairly rich model, Schmitt-Grohé & Uribe [2005] determine a rule for taxes $\tau_t$ with an autoregressive element, government spending and output according to $\tau_t = \rho \tau_{t-1} + \phi_1 g_t + \phi_2 y_t$. In [Railavo 2006], a basic New-Keynesian model is used to study supply side stabilisation policies and the study uses a debt rule that can be modified to react on government liabilities and debt, both in proportion to production. A similar expression is also used in a more recent paper by [Leeper, Michael & Nora 2010], that tries to fit a detailed real business cycle model to U.S. data. The government spending depends on production and debt according to

$$g_t = -\phi_1 y_t - \phi_2 b_{t-1} + \varepsilon_t.$$ 

11Also [Forni et al. (2009)] underline the problems with the annual perspective within fiscal policy. Since fiscal policy mainly is formed in the annual budgetary proposal of the government, it is generally harder to adapt fiscal policy rapidly to economic fluctuations than it is for monetary policy.
The rule for taxes is similar but with opposite signs. Rules with the same structure are used by Colciago et al. (2008), whereas Muscatelli & Tirelli (2005) make use of output gap (OG) (typically defined as the difference between actual and potential output) such that

\[ g_t = \rho^G g_{t-1} - \phi_1 y_{t-1}^{OG} - \phi_2 b_t \] (2.8)

and \( \tau_t = \rho^\tau \tau_{t-1} + \phi_3 y_{t-1}^{OG} + \phi_4 b_t \), where \( b_t \) is government debt. Forni et al. (2009) use a similar rule for tax on consumption, salaries and capital, but with real debt-to-output ratio as instrument.

The reason why output and debt are among the most popular instruments should be understood from the perspective of stabilising targets. A trade-off between conducting a responsible and a stabilising (or benevolent) fiscal policy is reached once the debt level and the output both are taken into account. The “responsibility” can be assumed to be embedded in the debt level that can be seen as a proxy for the financial performance of the public sector. The output (or a potential output) is a natural measure for economic activity that might be smoothed over time. The strength of the policy can in the models be controlled by parameters.

There are also slightly more complicated attempts to use fiscal policy for stabilisation or balancing purposes. The objective in Medina et al. (2007) is to stabilise the economy of a developing country\(^{12}\) by using fiscal rules. One of their rules includes a structural balance \( b_t^s \). The structural balance is defined as the difference between the balance \( b_t \) and a component with cyclical revenues \( \tau_t^{CR} \), where \( \tau_t^{CR} = \tau_t^{OG} y_t^{OG} \). Output gap is the difference between the actual and the potential output. This means that taxation should be more moderate when the economy performs below its potential (and vice versa). Clearly, the policy is made more countercyclical (instead of procyclical) when structural balance is used as an instrument instead of debt for example in a balanced budget rule. For example, if the economy is in recession (negative output gap), then the cyclical revenues are smaller and the structural balance is hence larger. Less weight is given to the effective debt once the government spending is determined. Another interesting paper is Ratto et al. (2009), where the effects of fiscal stabilisation policies are studied. They use a tax rule where changes in taxes are caused by the size of the difference between the debt-to-GDP ratio and a debt target (which for example could be 60% of GDP) and the change in debt level. The paper uses also other rules similar to those above where output gap is included. In order to specify an output gap, Ratto et al. (2009) use the proportion of the actual capacity utilisation \( \eta_t \) to its moving average steady state level \( \bar{\eta} \) and the proportion of employment \( n_t \) to its moving average steady state level \( \bar{n} \) according to

\[ y_t^{OG} = \left( \frac{\eta_t}{\bar{\eta}} \right)^{1-\alpha^{OG}} \left( \frac{n_t}{\bar{n}} \right)^{\alpha^{OG}}. \] (2.9)

The output gap in the present study will be determined using formula (2.9).

\(^{12}\)More specifically, Medina et al. (2007) consider fluctuations in the Chilean economy caused by the copper price changes.

\(^{13}\)The rule in Medina et al. (2007) includes also an additional component in its cyclical revenues, namely cyclical tax income from businesses relating to copper mining.
The review above shows that rules for fiscal policy are usually very simple. One reason may be a pronounced attractiveness of transparency since there are no standard ways of formulating the rules. Another reason is that fiscal policy in the end is dependent on political decision-making that may be time-inconsistent due to regime changes. Therefore, the processes are better left out of the model or may be included as exogenous shocks. Nevertheless, the rules described above should be seen as an attempt to grasp at least some of the essential features that tend to be present in economic policy. Because the rules themselves are simple, there is much room for analysis. It could be pointed out that the rules presented above are typically not founded on theoretical grounds, meaning that there is usually no theory suggesting the choices and strengths of the instruments. Furthermore, the empirical evidence is often contradictory, see for example in [Kliem & Kriwoluzky](2014).
3 Theoretical foundations – the model

This chapter presents the model in which fiscal rules are created in Chapter 6. Understanding the study does not require full comprehension of the entire model below. However, a reader not familiar with DSGE models may at least focus on the model characteristics and overview presented below in Part 3.1 and the public sector described in 3.3. The block explaining foreign trade is an important part of the model itself, but of less interest in this study and is therefore left to the appendix.

3.1 Model characteristics and overview

The New-Keynesian dynamic stochastic general equilibrium (DSGE) model used in the study describes a small and open economy – small in the sense that it cannot affect foreign prices and open in the sense that agents of the economy trade with foreign countries. The model has many general features standard among DSGE models in the literature, such as external habit motive, both Ricardian and liquidity constrained households (Mankiw, 2000), wage bargaining (Pissarides, 2000) investment adjustment costs and unemployment. The model is calibrated to describe a small country such as Finland. A similar model in the literature is for example Christiano, Eichenbaum & Evans (2005). Also, several parts of the models are similar to the estimated model in Smets & Wouters (2003).

Assume that the economy is populated by a large number of consumers, each supplying one unit of labour. More specifically, as in Galí, López-Salido & Valles (2007), there are two kinds of households – Ricardian households that can save in bonds and rent capital to firms, and liquidity constrained households or "hand-to-mouth" households that consume their entire income in every period and are hence unable to make intertemporal choices. Output is produced through a combination of capital and labour as follows. There is a continuum of domestic intermediate good producing firms manufacturing a differentiated output used as input in the production of (i) final consumption goods, (ii) final investment goods, (iii) government consumption and (iv) export goods. The salaries in the domestic intermediate good sector (which is where the entire labour force is employed) are determined by a bargaining mechanism involving the intermediate firms and the households. Final consumption, investment and export goods are produced by combining domestic intermediate goods with imported intermediate goods. The model is visualised in Figure 13.

\[ \text{Assume that the economy is populated by a large number of consumers, each supplying one unit of labour. More specifically, as in Galí, López-Salido & Valles (2007), there are two kinds of households – Ricardian households that can save in bonds and rent capital to firms, and liquidity constrained households or "hand-to-mouth" households that consume their entire income in every period and are hence unable to make intertemporal choices. Output is produced through a combination of capital and labour as follows. There is a continuum of domestic intermediate good producing firms manufacturing a differentiated output used as input in the production of (i) final consumption goods, (ii) final investment goods, (iii) government consumption and (iv) export goods. The salaries in the domestic intermediate good sector (which is where the entire labour force is employed) are determined by a bargaining mechanism involving the intermediate firms and the households. Final consumption, investment and export goods are produced by combining domestic intermediate goods with imported intermediate goods. The model is visualised in Figure 13.} \]

The following notation is used: real variables (for example real consumption \( c_t \)) are denoted with lower case letters, nominal variables (such as nominal consumption \( C_t \)) are denoted with upper case letters. A nominal value is obtained when a real value is scaled by its price index, i.e. \( C_t = c_t P_t^C \), where \( P_t^C \) is the price index for consumption goods. Steady state variables are denoted as variables in levels but without time index since they are constant (for example \( c \) or \( C \)). Log-linearised variables are denoted with a tilde (such as \( \tilde{c}_t \) or \( \tilde{C}_t \)) and should be interpreted as deviation from its steady state (in per cent), such as \( \tilde{c}_t \equiv \frac{c_t - c}{c} \). One exception is the nominal interest rate, where the upper case letter denotes gross interest rate, i.e. \( R_t = 1 + r_t \).

\[ \text{\textsuperscript{14}} \text{The following notation is used: real variables (for example real consumption } c_t \text{) are denoted with lower} \]

\[ \text{case letters, nominal variables (such as nominal consumption } C_t \text{) are denoted with upper case letters. A} \]

\[ \text{nominal value is obtained when a real value is scaled by its price index, i.e. } C_t = c_t P_t^C, \text{ where } P_t^C \text{ is the} \]

\[ \text{price index for consumption goods. Steady state variables are denoted as variables in levels but without} \]

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\[ \text{tilde (such as } \tilde{c}_t \text{ or } \tilde{C}_t \text{) and should be interpreted as deviation from its steady state (in per} \]

\[ \text{cent), such as } \tilde{c}_t \equiv \frac{c_t - c}{c}. \text{ One} \]

\[ \text{exception is the nominal interest rate, where the upper case letter denotes gross interest rate, i.e. } R_t = 1 + r_t. \]

\[ \text{\textsuperscript{15}} \text{A similar model, named "KOOMA", is used at the Finnish Ministry of Finance as a supportive tool in the production of} \]

\[ \text{macroeconomic forecasts. Since the process to update the KOOMA model is in progress, this study might use} \]

\[ \text{some values and parameters (see the Tables } 3, 4 \text{ and } 5 \text{) that are not part of the model of the} \]

\[ \text{Ministry. The model of the Ministry will be presented in more detail in a forthcoming publication, but} \]

\[ \text{can already now be found online in a linearised format as code for example at } \text{http://vm.fin/en/economic-} \]

\[ \text{forecasts}. \]
In order to be able to use more convenient estimation methods to be more easily applicable, the model is linearised by employing a first-order Taylor approximation. The focus below is, however, not on the linear model, but on the model in its original, non-linear form.

### 3.2 Households

There is a continuum of households of which a share \( \omega^{LC} \) is liquidity constrained and the remaining share \( 1 - \omega^{LC} \) is Ricardian households. This part describes the behaviour of both household types as well as the variables that can be derived from the consumers’ optimisation problem.

#### 3.2.1 Ricardian households

The Ricardian households maximise expected lifetime utility\[16\] subject to a nominal budget constraint, a capital accumulation equation (with adjustment costs) and an employment dynamics equation, i.e.

\[
\max_{c^L_{t+1}, h_t} \left\{ \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[ \epsilon^L_{t+j} \left( c^L_{t+j} - \kappa c^L_{t+j-1} \right)^{1-\sigma} \right] - \epsilon^L_{t+j} \frac{h_{t+j}^{1+\sigma}}{1+\sigma} \right\} \quad \text{s.t.} \quad (3.1)
\]

\[
P^C_t c^L_t (1 + \tau^C_t) + P^I_t i_t + \Psi(v_t) P^I_t k^p_{t-1} (1 + \tau^K_t) + B^H_t + B^F_t = n_t h_t W_t (1 - \tau^W_t) + (1 - n_t) P^C_t b_t + B^K_t P^I_t k^p_{t-1} - \nu_t (1 - \tau^K_t) + \sum_{i} \Gamma(B^F_t) B^F_{t-1},
\]

\[
k_{t}^p - k_{t-1}^p = \delta k_{t-1}^p + \left[ 1 - \Phi(\epsilon^L_{t-1} \frac{i_t}{h_{t-1}}) \right] i_t
\]

(3.3)

and

\[
n_t = (1 - \rho)n_{t-1} + ma_{t-1},
\]

(3.4)

where \( \mathbb{E}_t \) is the expectation operator, \( c^L_{t+1} \) is consumption for the forward-looking households, \( h_t \) are supplied labour in hour, \( i_t \) is investments, \( v_t \) is the capital utilisation rate, \( k^p_t \) is physical capital, \( R^K_t \) is rental rate on capital, \( B^H_t \) is domestic bond holdings, \( B^F_t \) are foreign bond holdings, \( W_t \) is wage, \( n_t \) is the fraction of employed people, \( TR_t \) is a transfer received from the general government, \( D_t \) is dividends, \( R_t \) is the rate of return on capital rented to the firms, \( ma_t \) is the number of matches in the labour market\[17\], \( P^C_t \) is the price index for consumption, \( P^I_t \) is the price index for investment products, \( \epsilon^L_t \) is a preference shock for forward-looking consumers, \( \epsilon^I_t \) is a shock to labour supply and \( \epsilon^L_t \) is an investment specific shock\[18\]. Furthermore, \( \beta \) is the discount factor, \( \kappa \) is a parameter capturing the external habit formation, \( \frac{1}{\sigma} \) is the elasticity of intertemporal substitution (Frisch), \( \frac{1}{\sigma} \) is an elasticity of labour supply, \( \delta \) is the depreciation rate of capital, \( \Gamma(B^F_{t-1}) \) is a country-specific risk

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\[16\] The problem is essentially the same as in Smets & Wouters (2003).

\[17\] See Obstbaum (2012) and Part 3.6 below.

\[18\] The shocks in the model are assumed to follow AR(1) processes of the type \( \epsilon_t = \rho \epsilon_{t-1} + \zeta_t \). Notice that the model could be applied using more shocks than those described in this chapter, where only the most important shocks are mentioned.
physical capital is transformed into effective capital according to $k$ of capital utilisation with respect to rental rate of capital is $\Psi$. The signs of the taxes in budget constraint (3.2) are different depending on which side of the equation they occur. The taxes increases expenditures and decreases incomes.

Let $\Lambda_t$, $\Xi_t$ and $\mu_t$ be the Lagrange multipliers for equations (3.2), (3.3) and (3.4) respectively. Furthermore, assume $\lambda_t$ is the Lagrangian function. As will be explained below, this optimisation problem determines several key variables of the economy.

The first-order conditions following $\frac{\partial \epsilon_t}{\partial c_t}$ yields the marginal utility of wealth

$$\Lambda_t = \frac{\epsilon_t^{cF}(c_t^{F} - \chi c_{t-1}^{FL})^{-\sigma^C}}{P_t^C (1 + \tau_t^C)}.$$  

Combining the first-order conditions for consumption and holdings of domestic bonds yield the Euler equation

$$\frac{\epsilon_t^{cF}}{\epsilon_{t+1}^{cF}} \left( \frac{c_{t+1}^{FL} - \chi c_{t+1}^{FL}}{c_t^{FL} - \chi c_t^{FL}} \right)^{\sigma^C} = \frac{R_t + 1 + \tau_t^C}{\Pi_{t+1} 1 + \tau_{t+1}^C},$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$.

Combining $\frac{\partial \epsilon_t}{\partial B_t}$ and $\frac{\partial \epsilon_t}{\partial b_t}$ gives $R_t = R_t^{F} \Gamma'(B_t^{F})$, i.e. the relationship between domestic and foreign gross interest rates. Note that the risk premium function, $\Gamma(B_t^{F})$, is growing as the domestic real economy holds more foreign debt. In fact it is assumed that $\Gamma(B_t^{F}) = \exp \left( \gamma^{b'}(\frac{B_t^{F}}{P_t^Z n_t b_t K_t L} - b_t^F) \right)$, where $b_t^F$ is the steady state value of foreign debt level as a proportion of the domestic real economy (Schmitt-Grohé & Uribe, 2003). Hence, the nominal net interest rate $r_t^{*}$ will be a function of the foreign interest rate $r_t^{*}$, the holdings of foreign debt $B_t^{F}$ and the size of the domestic real economy $P_t^Z n_t b_t K_t L$. The constant $\gamma^{b'}$ has a small value. The foreign interest rate is assumed to be exogenously given and follows an AR(1) according to $r_t^{*} = \rho^{r'} r_{t-1}^{*} + \epsilon_t^{r'}$.

The Ricardian households own the capital. They bear therefore also the cost of capital utilisation $\Psi(\nu_t)P_t^L k_{t-1}^p(1 + \tau_t^K)$, where $\Psi(\nu_t)$ is a function measuring capital utilisation per unit of physical capital. The capital utilisation cost is increased by a tax on capital $\tau_t^K$. The first-order condition given by $\frac{\partial \epsilon_t}{\partial \nu_t}$ yields $R_t^K = \Psi'(\nu_t)$. Following Christiano et al. (2005), $\nu_t = 1$ in steady state since all capital is assumed to be utilised, $\Psi(1) = 0$ and the elasticity of capital utilisation with respect to rental rate of capital is $\frac{\psi''(1)}{\psi'(1)}$. Therefore, once the first-order condition is log-linearised, the capacity utilisation rate $\eta_t$ will depend only on the rental rate on capital and on the capital utilisation rate according to $\eta_t = R_t^K \nu_t$. Note that physical capital is transformed into effective capital according to $k_t = \nu_t k_t^p$.

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19 As noticed on the next page, the risk premium is increasing in the holdings of foreign bonds. See Schmitt-Grohé & Uribe (2003) for more on debt elastic interest rates.

20 The idea with investment adjustment cost is to impose frictions in the investment process. Clearly, it is more costly to increase than to decrease investments. The function $\Phi$ is assumed to be zero in steady state and when taking the first-order derivative. See Christiano et al. (2005) for more details.

21 A list of all variables, including shocks used in this study, can be found in Tables 3, 4 and 5.
The investments and the real value of capital are determined by the first-order conditions given by $\frac{\partial \ell}{\partial i_t}$ and $\frac{\partial \ell}{\partial \nu_t}$ respectively. Define Tobin’s Q as the ratio between the Lagrange multipliers for capital accumulation and consumer budget constraints, i.e. $Q_t \equiv \frac{\Phi}{\Lambda_t}$. The first-order condition for investments can be modified as

$$1 = Q_t \left[1 - \Phi \left(\frac{\varepsilon^I_t}{\nu_{t+1}}\right) - Q_t \Phi' \left(\frac{\varepsilon^I_t}{\nu_{t+1}}\right) \left(\frac{i_t}{\nu_{t+1}}\right) \varepsilon^I_t\right] + \beta Q_{t+1} \Phi' \left(\frac{\varepsilon^I_{t+1}}{\nu_{t+1}}\right) \left(\frac{i_{t+1}}{\nu_{t+1}}\right)^2 \varepsilon^I_{t+1}.$$ 

The real value of capital given by Tobin’s Q, that also could be interpreted as the ratio between market value of capital and replacement cost, can then be written as

$$Q_t = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[ (1 - \delta) Q_{t+1} + (R^K_{t+1} \nu_{t+1} - \Psi(\nu_{t+1})) (1 + \tau^K_{t+1}) \right].$$

### 3.2.2 Non-Ricardian households

Apart from the wealthier Ricardian households there are also poorer "hand-to-mouth" households. Assume a fraction $\omega^{LC}$ of the households are liquidity constrained, meaning that they can neither save nor lend money and hence not make intertemporal choices regarding consumption – all income is consumed in every period. As in [Ratto et al. 2009], the liquidity constrained households get utility from the same utility function as the Ricardian households, i.e. equation (3.1), but their budget constraint takes the following form

$$P^C_t c^{LC}_t (1 + \tau^C_t) = W_t (1 - \tau^W_t) n_t + TR_t + (1 - n_t) P^C_t b_u t.$$  (3.6)

The total consumption is now given by the shares of forward-looking and liquidity constrained households according to $c_t = \omega^{LC} c^{LC}_t + (1 - \omega^{LC}) c^{FL}_t$.

### 3.3 Public sector

The role of the public sector – that should be interpreted as a general government – is to collect taxes and use them for government spending $g_t$, government investments $i_t^G$, lump sum transfers to households $TR_t$, unemployment benefits $b^U u_t$ and for repaying accumulated debt with interest rate $R_{t-1} B_{t-1}$. The general government receives its revenues from taxes, including a tax on labour $\tau^W_t$, a tax on consumption (value added tax) $\tau^C_t$, a tax referring to the social contribution of employers $\tau^{SC}_t$, a tax on dividends $\tau^D_t$ and a tax on rental capital income $\tau^K_t$. The general government budget constraint (in nominal terms) can thus be written as

$$P_t (n_t w_t h_t) (\tau^W_t + \tau^{SC}_t) + \tau^C_t P_t c_t + \tau^K_t P_t R^K_t k^p_t + B_t$$

$$= P^Z_t g_t + P^G_t i^G_t + TR_t + P^B_t b^U u_t + R_{t-1} B_{t-1}. \quad (3.7)$$

Since the general government in each period will repay its total debt from last period, it typically needs to compensate the budget by taking new loans of approximately similar size.
as the repaid debt. The difference $b_t - b_{t-1}$ could therefore be considered as the real deficit or surplus belonging to period $t$. Solving (3.7) for $B_t$ yields the level of nominal bonds.

The general government spending consists of products and services that are domestically produced. Hence, one can assume that the general government purchases (domestic) intermediate goods and transforms them, with no cost, into a final public consumption good $g_t$.

The public sector variables are in the absence of fiscal rules assumed to be exogenously determined. Hence, all government variables are assumed to follow simple AR(1)-processes, such as $g_t = \rho G g_{t-1} + \varepsilon_t^G$ for government spending, $tr_t = \rho TR tr_{t-1} + \varepsilon_t^{TR}$ for transfers to households, $i_t^C = \rho I^C i_{t-1}^C + \varepsilon_t^{I^C}$ for public investment, $\tau_t^W = \rho W \tau_{t-1}^W + \varepsilon_t^W$ for taxes on labour, $\tau_t^C = \rho C \tau_{t-1}^C + \varepsilon_t^C$ for taxes on consumption, $\tau_t^{SC} = \rho SC \tau_{t-1}^{SC} + \varepsilon_t^{SC}$ for the tax associated with employers’ social contribution fee, $\tau_t^K = \rho K \tau_{t-1}^K + \varepsilon_t^K$ for tax on capital incomes and $\tau_t^D = \rho D \tau_{t-1}^D + \varepsilon_t^D$ for tax on dividends. These variables could be used to conduct fiscal policy simply by using the exogenous shock component $\varepsilon_t$. However, the constraints above are, in accordance with the discussion in Part 2.1, not really fiscal rules since they do not constrain the room for fiscal policy manoeuvre. Also, except for the budget constraint, they are exogenous rules, whereas the interest in this study is to build endogenous, automatic rules.

3.4 Final good sectors

Apart from the general government, there are three final good sectors in the economy, namely a final good sector for consumption, investment and export. These three sectors buy their inputs – differentiated intermediate goods – from domestic and foreign producers, and transform them under perfect competition (with no cost) into a final good that can be consumed, invested or later on exported. The final good sector is modelled similarly as in Adolfson et al. (2007), Christiano et al. (2005), and Smets & Wouters (2003). The production of final consumption and investment goods are technically very similar, and will therefore be treated together here even though they take place separately from each other. Consumers demand consumption and investment services given by $c_t$ and $i_t$ in equations (3.2) and (3.6). The final good producer buys intermediate consumption (investment) goods from both domestic intermediate good producers $z_t^C$ ($z_t^I$) and from importers $m_t^C$ ($m_t^I$). The final good $y_t^C$ (or $y_t^I$) is “produced” according to an aggregation technology where the firm

$$\max_{z_t^C, m_t^C} P_t^C y_t^C - (P_t^Z z_t^C + P_t^{MC} m_t^C) \text{ s.t. } y_t^C = \left(\omega z_t^C + (1 - \omega) m_t^C\right)^{-\frac{1}{\rho}}$$

(3.8)

where $\omega$ is the share of domestic input good and $\rho$ is a parameter determining the substitution between domestic and foreign consumption. Since the firms are assumed to operate under

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24 Also referred to as the “benchmark scenario” in the figures in subsequent chapters.
25 The analysis below focuses on consumer goods even though it may equally well describe the investment goods. In order to derive the similar equations for investment goods, one can substitute each $C$ with $I$ in equations (3.8), (3.9) and (3.10).
perfect competition, one can derive the following conditional factor demands

\[ z_t^C = \omega^C \frac{1}{1 + \rho^C} \left( \frac{P_t^Z}{P_t^C} \right)^{-\frac{1}{1 + \rho^C}} y_t^C \quad \text{and} \quad m_t^C = (1 - \omega^C) \frac{1}{1 + \rho^C} \left( \frac{P_t^{MC}}{P_t^C} \right)^{-\frac{1}{1 + \rho^C}} y_t^C. \]  

(3.9)

Plugging the factor demands (3.9) in the objective function given by (3.8) yields the price index for consumption (investment) goods,

\[ P_t^C = \omega^C - \rho^C P_t^Z + (1 - \omega^C)^{-\rho^C} P_t^{MC}. \]  

(3.10)

The procedure for the export producing firms is similar as for the final consumption and investment producing firms, although not identical, since the export producing firms have some degree of market power. Assume that the export product manufacturer uses a constant elasticity of substitution (CES) production function and minimises its costs according to

\[ \min_{z_t^X, m_t^X} P_t^Z z_t^X + P_t^{MX} m_t^X \quad \text{s.t.} \quad y_t^X = \left( \omega^X z_t^X - \rho^X + (1 - \omega^X) m_t^X - \rho^X \right)^{-\frac{1}{\rho^X}}, \]  

(3.11)

where the parameter interpretation is similar as above. Let \( \lambda_t^X \) be the Lagrange multiplier or the marginal cost of exports. The conditional factor demands, given by the first-order conditions, are

\[ z_t^X = \omega^X \frac{1}{1 + \rho^X} \left( \frac{P_t^Z}{\lambda_t^X} \right)^{-\frac{1}{1 + \rho^X}} y_t^X \quad \text{and} \quad m_t^X = (1 - \omega^X) \frac{1}{1 + \rho^X} \left( \frac{P_t^{MX}}{\lambda_t^X} \right)^{-\frac{1}{1 + \rho^X}} y_t^X, \]  

(3.12)

where

\[ \lambda_t^X = \left[ \omega^X \frac{1}{1 + \rho^X} P_t^Z \frac{P_t^{MX}}{\lambda_t^X} + (1 - \omega^X) \frac{1}{1 + \rho^X} P_t^{MX} \frac{P_t^Z}{\lambda_t^X} \right]^{\frac{1 + \rho^X}{\rho^X}}. \]  

(3.13)

### 3.5 Domestic intermediate good sector – production and pricing

Assume there is a large number of \( n \) firms operating under monopolistic competition producing an intermediate good that is used later in the production of final goods. The producers rent capital and labour services from the households to given prices. In fact, one can assume the intermediate good sector to consist of two separate procedures (two separate stages)\(^{27}\)

See for example Adolphson et al. (2007) and Obstbaum (2012) for a similar specification of the domestic intermediate good sector.

First, assume that in each firm is one worker producing a capital-labour intermediate good \( y_t^{KL} \) according to the Cobb-Douglas production function

\[ y_t^{KL} = \varepsilon_t^a k_t^a h_t^{1-\gamma}, \]  

(3.14)

\(^{27}\)The two procedures includes (i) production of a capital-labour intermediate good and (ii) product differentiation and profit maximisation on a market with price frictions. One can also think about (ii) as being a branch of separate wholesale firms buying their input \( y_t^{KL} \) to the real price \( P_t^{KL} \), which becomes a real marginal cost in terms of consumer price index. The product differentiation takes place at the wholesale level.
where $\varepsilon_t^a$ is an AR(1) shock process to total factor productivity (TFP) and $h_t = \frac{\bar{h}_t}{n_t}$, where $l_t$ is the total amount of work conducted. Equation (3.14) provides also the marginal productivity of labour and the marginal utility of capital, i.e. $mpl_t = \varepsilon_t^a(1 - \gamma) \left( \frac{\bar{h}_t}{n_t} \right)^\gamma$ and $mpk_t = \varepsilon_t^a \gamma \left( \frac{\bar{h}_t}{n_t} \right)^{1-\gamma}$, which will be used later on.

Second, assume the first stage intermediate firm above sells its product at a price corresponding to the nominal marginal cost $MC_t$, to the second stage intermediate good producer. The latter producer will differentiate the product and act on a market with price frictions such as in Calvo (1983). These (latter) firms are able to optimise their prices with probability $1 - \xi$ and will then choose a price path that is optimal from the perspective of the current period – in case there are no possibilities to re-optimise the price in future periods. If a firm is not able to optimise, it will charge the same price as earlier.

If a firm $j$ is able to optimise, it will choose its price $P_t^Z^*$ as to

$$\max_{P_t^Z^*} \left\{ \mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \beta^k \left[ (P_t^Z^* - MC_{t+k})y_{t+k}^Z(j) \right] \right\}, \quad (3.15)$$

where $\Lambda_t$ is a stochastic discount factor (see (3.5)), $MC_t \equiv P_t^Z p_t^{KL}$ is the nominal marginal cost, or the nominal cost charged by the first stage intermediate good producer.

The demand for the differentiated intermediate goods for firm $j$ is $y_t^Z(j) = \left( \frac{P_t^Z}{P_{t+k}^Z} \right)^{-\varphi} y_t^Z$, where $y_t^Z(j) = z_t^C(j) + z_t^I(j) + z_t^G(j) + z_t^X(j)$ captures the entire demand for intermediate goods (including final consumption, investment, government and export goods) and is determined in the final good sectors, see Section 3.4. Once the demand function is substituted into (3.15), one can derive a first-order condition that, after rearranging yields

$$P_t^Z^* = \frac{\varphi^Z}{\varphi^2 - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \beta^k P_{t+k}^Z \varphi^Z y_t^Z MC_t}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \beta^k P_{t+k}^Z \varphi^Z y_t^Z}. \quad (3.16)$$

The New-Keynesian Phillips curve (NKPC) can be derived once the optimality condition following (3.16) is combined with the average pricing outcome of the intermediate firms, which is $P_t^Z = \left( (1 - \xi) P_{t-1}^{Z(1-\varphi^Z)} + \xi P_{t-1}^Z \right)^{1-\varphi^Z}$. Define $\Pi_t^Z \equiv \frac{P_t^Z}{P_{t-1}^Z}$. The log-linearised NKPC will take the following form

$$\Pi_t^Z = \Pi_{t+1}^Z + \frac{(1 - \xi)(1 - \beta \xi)}{\xi} (\varphi^Z + \bar{mc}_t), \quad (3.17)$$

where $\bar{mc}_t$ is the linearised real marginal costs.

### 3.6 Determination of salaries and employment

Wages in the model are determined by a search-and-matching mechanism whereby unemployed labour and vacancies are combined by a matching technology. This part of the model
is based on [Pissarides (2000)] and [Obstbaum (2012)]. The flow of matches \( m_a_t \) will affect the employment dynamics in equation (3.4). Note that unemployment is simply defined as \( u_t = 1 - n_t \), since the entire labour force is normalised to one. The salary is subject to a bargaining procedure whereby workers and firms maximise their surpluses.

Assume that employment opportunities are born as a consequence of a match in the labour market. Matches are “produced” according to the Cobb-Douglas production function

\[
    m_a_t = \sigma^{m_a} u_t^\sigma v_t^{1-\sigma},
\]

where \( \sigma^{m_a} \) is the efficiency in the matching process, \( v_t \) is the stock of vacancies in the beginning of period \( t \) and \( \sigma \) is the elasticity of matching with respect to the stock of unemployed. Define the labour market tightness as \( \theta_t = \frac{u_t}{u_t} \), meaning that the labour market is tighter the higher the employment is or the more vacancies there are. Consequently, one can write the probability that a vacancy will be filled as \( \rho_t^V = \frac{ma_t}{u_t} = \sigma^{m_a} \theta_t^{-\sigma} \) and the probability that an unemployed person finds a job as \( \rho_t^W = \frac{ma_t}{u_t} = \sigma^{m_a} \theta_t^{1-\sigma} \). The inverse of these probabilities can be interpreted as the mean duration of vacancy and unemployment. Furthermore, assume there is a vacancy posting cost \( \kappa_t \). The cost of a vacancy (vacancy posting cost times its duration) will in equilibrium be equal to the expected benefit of that job opportunity. This zero-profit condition is therefore written as \( \frac{\rho_t^V}{\rho_t^W} = E_t \beta_{t,t+1} J_{t+1} \), where \( J_t \) is real surplus of an intermediate firm.

The surplus \( H_t \) for a worker from having a job instead of being unemployed is the salary after taxes subtracted by the dis-utility of work and the unemployment benefit that the person will receive when having no work. Hence,

\[
    H_t = h_t \frac{W_t^*}{P_t^c} (1 - \tau_t^W) - \frac{g(h_t)}{\Lambda_t} - bu_t + E_t \beta_{t,t+1} (1 - \beta_t^{rt+1} + q_t^W) H_{t+1}, \quad (3.18)
\]

where \( g(h_t) = \frac{h_t^{1+\sigma_L}}{1+\sigma_L} \) and \( \frac{g(h_t)}{\Lambda_t} \) is the dis-utility of hours at work (discounted by \( \Lambda_t \) given by equation (3.5)). The expected future worker surplus is subtracted by \( \beta_t^{rt+1} \) that is an exogenous probability that the match in question will terminate in \( t+1 \) and by \( q_t^W \) – the probability that an unemployed person will find a job in the next period.

Let the real surplus \( J_t \) of an intermediate firm\(^{28}\) be

\[
    J_t = P_t^{KL} y_t^{KL} - (1 + \tau_t^{Sc}) \frac{W_t}{P_t^c} h_t - r_t^K k_t + E_t \beta_{t,t+1} (1 - \beta_t^{rt+1}) J_{t+1}, \quad (3.19)
\]

where \( \tau_t^{Sc} \) is the tax associated with social contribution of the employer, \( r_t^K = R_t^K - 1 \) is the net return on capital and \( P_t^{KL} = p_t^{KL} \frac{P_t^c}{P_t} \) is the relative price of the intermediate good in terms of CPI (and therefore a real variable). Notice that \( \frac{\partial h_t}{\partial w_t} \) yields \(-\) when equation (3.14) is inserted in (3.19) – the equilibrium condition for a capital stock of a single firm as \( \frac{P_t^{KL} m P_k}{P_t} = r_t^K \).

The salaries and worked hours are a result of an efficient Nash bargaining process in which the firms and workers maximise their combined surpluses by simultaneously choosing

\(^{28}\) It is assumed that each intermediate firm employ one worker. Hence, \( J_t \) is a measure of the value of one occupied job.
The agents are now optimising with respect to \( h \) for salary determination, i.e. \( \epsilon \) and (A.14)). On an aggregate level one will therefore have equations (3.19) and (3.15)) and the importing and exporting rms (see (A.4), (A.7), (A.11) (3.2), are here defined as the sum of profits from the two intermediate sector rms (given by (3.18) in the equation. Similarly, the employers will maximise \( (3.20) \) with respect to \( w \) by substituting the expression given by (3.19) in the equation. In equilibrium one will therefore have \( \tilde{\eta}_t \frac{\partial H_t}{\partial w_t} J_t = (1 - \tilde{\eta}_t) \frac{\partial J_t}{\partial w_t} H_t \). The equation for new salaries, \( w_t^* \), will follow once the expressions for \( H_t \) and \( J_t \), given by (3.18) and (3.19), are substituted in this equilibrium condition

\[
\frac{W_t^*}{P_t} = \frac{\tilde{\eta}_t}{1 + \tau_{t}^{SC}} \left( P_t^{KL} mp_t - \frac{r_t^K k_t}{h_t} \right) + \frac{1 - \tilde{\eta}_t}{1 - \tau_{t}^{W}} \left( mrs_t(1 + \tau_t^C) + \frac{bu_t}{h_t} + \frac{q_{tW}}{h_t} E_t \beta_{t+1} H_{t+1} \right)
\]

\[
+ \frac{\tilde{\eta}_t}{h_t} E_t \beta_{t+1} (1 - \beta_{t+1}) \left( \frac{1}{1 + \tau_{t}^{SC}} - \frac{1 - \tau_{t+1}^{W}}{(1 - \tau_{t}^{W})(1 + \tau_{t+1}^{SC})} \right).
\]

Notice that the dis-utility of work, that is included in equation \( (3.18) \), has been rewritten in \( (3.21) \) using \( (3.5) \) and the definition of marginal rate of substitution as the ratio between marginal (dis)utility with respect to worked hours and consumption, i.e. \( mrs_t = \frac{M U_{t}^w}{M U_{t}^c} \). Furthermore, the expression for marginal productivity of labour from above is used in \( (3.21) \).

Since only a fraction \( \omega_t^{W} \) of the salaries of the employees are renegotiated in every period, the general real salary level in the economy will be \( w_t = \omega_t^{W} w_t^* + (1 - \omega_t^{W}) w_{t-1} + \varepsilon_t^{W} \), where \( \varepsilon_t^{W} \) is an AR(1) shock process to salaries.

The hours at work, \( h_t \), are determined by the similar procedure as for the wage. However, the agents are now optimising with respect to \( h \) instead of \( w \). Once the equilibrium condition for salary determination, i.e. \( \tilde{\eta}_t (1 - \tau_t^{W}) J_t = (1 - \tilde{\eta}_t)(1 - \tau_t^{SC}) H_{t+1} \), is utilised, one will get

\[
(1 - \tau_t^{W}) P_t^{KL} mp_t = (1 - \tau_t^{SC}) mrs_t (1 + \tau_t^C).
\]

Clearly, the hours, solved for from \( (3.22) \), are not affected by the salary level.

### 3.7 Closing the model

Once the general structure of the model is complete, one can derive a few more relations connecting the different parts of the model together.

The dividends \( D_t \), that are present in the budget constraint of the Ricardian household \( (3.2) \), are here defined as the sum of profits from the two intermediate sector firms (given by equations \( (3.19) \) and \( (3.15) \)) and the importing and exporting firms (see \( (A.4), (A.7), (A.11) \) and \( (A.14) \)). On an aggregate level one will therefore have

\[
D_t = \left[ P_t^{Z} P_t^{KL} y_t^{KL} n_t - \left( h_t (1 + \tau_t^{SC}) W_t^* n_t + P_t^{I} r_t^{K} k_t \right) \right] - V_t + \\
\left[ (P_t^{Z} - P_t^{Z} P_t^{KL}) y_t^{KL} n_t \right] + \left[ (P_t^{X} - \lambda_t^{X}) y_t^{X} n_t \right] + \left[ (P_t^{M} - P_t^{*} \varepsilon_t) m_t n_t \right],
\]

\[
(3.23)
\]

The bargaining strength is determined by an AR(1) process of the form \( \tilde{\eta}_t = \rho \tilde{\eta}_{t-1} + \varepsilon_t^{\eta} \), where \( \varepsilon_t^{\eta} \) is a shock to the bargaining strength of workers.
where \( V_t \equiv P_t^Z \kappa v_t \) is a vacancy posting cost (see Section 3.6) and should be interpreted as "pure waste".

One can also derive an expression for the holdings of foreign bonds. Define the capital adjustment costs \( S_t \equiv \Psi(\nu_t)P_t^I k_t^{n-1} \). Solve the budget constraint (3.2) for domestic holdings of bonds and substitute the expression in the government budget constraint (3.7) to obtain a relation for the holdings of foreign bonds which, after the dividends are substituted by the right-hand side of (3.23), can be written as

\[
B^F_t - R_{t-1}^F = P_t^Z y_t^{KL} n_t - P_t^c c_t - P_t^I i_t - P_t^g g_t - S_t - V_t
+ (P_t^X - \lambda_t^X) y_t^X n_t + (P_t^M - \nu_t^X e_t)m_t n_t \equiv TB_t, \tag{3.24}
\]

telling that the holdings of foreign assets are evolving according to the difference between demand and supply. This is here defined as the trade balance.

Finally, the markets will clear, meaning that \( c_t = y_t^C \), \( i_t = y_t^I \) and \( x_t = y_t^X \). The fact that demand equals supply can also be written as

\[
y_t^{KL} - V_t - S_t = \omega^C \left( \frac{P_t^Z}{P_t^X} \right)^{-\frac{1}{1+\rho^C}} c_t + \omega^I \left( \frac{P_t^Z}{P_t^I} \right)^{-\frac{1}{1+\rho^I}} i_t + \omega^X \left( \frac{P_t^Z}{\lambda_t^X} \right)^{-\frac{1}{1+\rho^X}} x_t + g_t, \tag{3.25}
\]

meaning that value creation in the economy takes place in the domestic intermediate sector, see (3.9) and (3.12).
4 Methodology

This chapter provides a methodological overview. Part 4.1 gives a short note on the model solution, whereas Part 4.2 provides the intuition of how policies are implemented in the model. Part 4.3 discusses the choice of parameters and Part 4.4 presents a welfare measure that is used later to evaluate the rules.

4.1 Model calibration and solution

There are both calibrated and estimated DSGE models. In an estimated model, parameter values are determined based on how observed data fits into the model being estimated. In a calibrated model, numerical methods are used to calculate the parameters either based on theory or observed data. The model used in the present study is calibrated, see for example DeJong & Dave (2011) for more details on calibration of macroeconomic models. Calibration of macroeconomic models is, nevertheless, beyond the scope of this study and will not be handled in further detail.

The DSGE model used in the study can be solved using linear stochastic difference equations. The equation is solved by using the so-called Blanchard-Kahn condition, see Blanchard & Kahn (1980). Again, any additional details should not be necessary from the perspective of this study.

The computer program Matlab with Iris extension is used to manage the model, including solving, simulating and finally plotting the results.

4.2 Creation of model dynamics

The effects of different policies can be studied by exposing the model to exogenous shocks. When the methods described above are applied and the model approaches a solution, it is said to be in equilibrium. There is no movement in the economy and all variables are zero. This follows from the interpretation of the variables being log-linearised. The equilibrium should be interpreted as a state where the economy will find itself in the long run (i.e. there will always be convergence towards the steady state), or as a current state if the economy is not target to shocks.

Conditions where shocks are not affecting the economy are unlikely. In this context the equilibrium is also of less interest. In order to awake the model it needs to be removed from its steady state. Attention is turned to how the variables converge back towards their steady states – the model becomes dynamic. The movements or disturbances are most considerable in the periods following the shock and disappear with time.

Also fiscal policies can be evaluated in the light of shocks. In order to study the impact of a policy regime, the economy has to be removed from its steady state powered by a relevant shock. As noted in Part 2.5 a natural choice within the context of fiscal policy is to directly use a government spending shock, a tax shock or possibly other shocks relating to government

\footnote{More information about the Iris project can at the moment be found online at \url{https://iristoollbocodeplex.com/}}

\footnote{Remember that log-linearised variables (denoted by tilde) measure deviation in per cent from their steady states. In equilibrium the economy is in steady state and the deviation from steady state is zero.}
behaviour, such as in Forni et al. (2009). In such cases the steady state should be interpreted as the benchmark, indicating a development where no policy is used. However, especially in the context of fiscal rules imposing a limitation on fiscal manoeuvre, it might be of interest to compare the economic outcome when rules are used with the outcome where rules are not used, without applying any government shock. The outcomes are then benchmarked against each other – not against the steady state baseline. In such cases the shock can be determined by the economic aspect being emphasised. In Chapter 7, total factor productivity (TFP) shocks are used since they can be assumed to occur frequently and fairly independently of other shocks in the economy.

4.3 Public sector steady state values

This part provides a brief summary of some of the steady state values of the model. A complete list of variables, shocks and parameters of the model is found in Tables 3, 4 and 5.

Public sector expenditure in proportion to GDP is assumed to be around 42% in the study. The public consumption \( g \) is 28% in proportion to GDP. The debt \( b \) is, as already noted, 60% of the production.\(^{31}\) The consumption tax rate is set to 24.5% and the labour tax rate to 30%. The consumption tax rate needs to respond more than income tax rate in order to balance the budget, because earned income is larger than total consumption (see for example Figure 5). It can be noted that the relative sizes of the steady state values for taxes and expenditures are of less importance for the results. The steady state values, however, affect largely the size of the responses in percentage terms.

4.4 Evaluation

The rules are later on in Chapter 7 evaluated using various alternative yardsticks. First, they are evaluated based on how well they transmit their original purpose of the rule into the model. Second, the performances of the rules are evaluated both based on economic implications and welfare effects.

The welfare is determined by the discounted sum of current and future consumer utility from private goods only. The private consumer utility \( cu_t \) is given by equation (3.1), and is linearised to

\[
\tilde{cu}_t = \frac{(c^2(1 - \tau))^{-\sigma c}}{cu} (\tilde{c}_t - \tau \tilde{c}_{t-1}) - \left( \frac{h_t}{cu} \right)^{1+\sigma L} \tilde{h}_t. \tag{4.1}
\]

Since the steady state values of utility \( u \) is an abstract measure, it can hardly be empirically established. The steady state value is arbitrary set to 75.\(^{32}\) The welfare is determined according to

\[
\tilde{we}_t = \tilde{cu}_t + \beta \tilde{we}_{t+1}. \tag{4.2}
\]

---

\(^{31}\)The only motivation for using a steady state debt-to-GDP value of 60% is that the value is used in the Stability and Growth Pact as a limit value and that the current debt ratio for Finland is close to 60%. However, this thesis does not argue that the limit is ideal, see the discussion in Reinhart & Rogoff (2010).

\(^{32}\)The steady state utility value does not affect the shape of the utility response, only its size. When the steady state value is 75, the change in utility has approximately the same size as the change in consumption. The welfare response in the figures below is then scaled such that a 1% TFP shock increases initially welfare by 1.0%.
5 Model performance – a comparison with the literature

A macroeconomic (DSGE) model can be seen as a cocktail of different microeconomic assumptions. Different models might lead to slightly different outcomes and conclusions, depending on used theory, parameter values, calibration methods etc. This chapter compares the behaviour of the model derived in Chapter 3 with the behaviour of the model used in Forni et al. (2009). The comparison is done by analysing the outcome when the model used in the present study is exposed to similar shocks as used in Forni et al. (2009).

The general structure in the two models is similar but there are a few important differences. First, the model in Forni et al. (2009) is estimated whereas the model used in the present study is calibrated. Second, the model used in Forni et al. (2009) includes positive growth rates whereas the growth rates of the model used in the present study are zero in steady state, as noted earlier. Third, in their model the salaries are determined in a standard way, i.e. by the intertemporal problem of the (Ricardian) households. Wages in the model of the present study are determined by a search and matching mechanism as a result of a bargaining process between employers and employees, see Chapter 3.1. Fourth, monetary policy is not included in the model of the present study. Fifth, foreign trade is not included in the model in Forni et al. (2009).

Figures 1 and 2 illustrate the response in the economy to a negative shock in consumption and labour taxation respectively. The figures can be compared to Forni et al. (2009) (see on pages 577 and 575), where similar shocks are used. As in their paper, tax on capital \( \tau^K_t \), tax on labour \( \tau^W_t \) and tax on consumption \( \tau^C_t \) are determined according to the similar rules

\[
\tilde{\tau}^K_t = \rho^{\tau^K} \tilde{\tau}^{K}_{t-1} + \zeta^{\tau^K} (\tilde{B}_t - \tilde{Y}_t - \tilde{P}_t) + \epsilon^{\tau^K}_t, \tag{5.1}
\]

\[
\tilde{\tau}^W_t = \rho^{\tau^W} \tilde{\tau}^{W}_{t-1} + \zeta^{\tau^W} (\tilde{B}_t - \tilde{Y}_t - \tilde{P}_t) + \epsilon^{\tau^W}_t, \tag{5.2}
\]

and

\[
\tilde{\tau}^C_t = \rho^{\tau^C} \tilde{\tau}^{C}_{t-1} + \zeta^{\tau^C} (\tilde{B}_t - \tilde{Y}_t - \tilde{P}_t) + \epsilon^{\tau^C}_t, \tag{5.3}
\]

where \( B \) is the government deficit or debt, \( Y \) is the output level, \( P \) is the price level and the \( \epsilon \) parameters are AR(1) shock components. The rules are used with the values in Table 2.

<table>
<thead>
<tr>
<th>Tax Rule</th>
<th>( \rho^{\tau^K} = 0.96 )</th>
<th>( \zeta^{\tau^K} = 0.02 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour tax rule</td>
<td>( \rho^{\tau^W} = 0.89 )</td>
<td>( \zeta^{\tau^W} = 0.03 )</td>
</tr>
<tr>
<td>Consumption tax rule</td>
<td>( \rho^{\tau^C} = 0.95 )</td>
<td>( \zeta^{\tau^C} = 0.02 )</td>
</tr>
</tbody>
</table>

Table 2: Parameter values used in rules (5.1)–(5.3).

33 The reason for using Forni et al. (2009) for comparison is that their model has many similar elements as the model used in the present study. Also, their model is used for similar purposes. The behaviour of the model used in the present study is also compared to a simpler model used in Leeper et al. (2010).

34 The reason for only comparing consumption and labour tax shocks with the literature is that those are the only taxes used by the rules created in this study, see Chapter 6.
The values are estimated in Forni et al. (2009) and also used in their study.

The small coefficients on $\zeta$ indicate that the estimated effects of debt-to-output ratio on the tax rates are very small. The tax rates have in the empiric not varied much with actual debt or output level. For example, assume a notable 5.7% negative consumption tax shock is applied. If each $\zeta$ equals zero instead of the values in Table 2, the outcome would be affected only marginally. The output would differ with less than 0.01 percentage points. The debt level would be only 0.1–0.2 percentage points higher than using the debt rules above.

How does the model economy react to the tax shocks?

**Consumption tax shock** First, assume the consumption tax rate is changed by a negative shock of 5.7%. The tax rate decreases immediately by around 1.4 percentage points, which is the same as in Forni et al. (2009). The last graph in Figure 1 shows how the shock component evolves with time. The AR coefficient of the shock process is 0.8 giving somewhat less persistent dynamics than in Forni et al. (2009).

As the consumption tax is decreased, the consumption is increased. The liquidity constrained consumption increases more than the Ricardian consumption, since the Ricardian households are able to smooth out their consumption over time. Higher demand for consumption leads to higher production and capacity utilisation as firms produce more. Consequently, people work more. This leads to higher salaries and increasing labour tax incomes for the government. The loss in total taxes will be only slightly less than the loss of consumption taxes. Furthermore, there is a very small increase in capital tax income which is explained by the tax rule – higher debt increases taxes according to the rules.

So far the results regarding the main movements are very similar to the results in Forni et al. (2009). The differences in the responses of the variables discussed above are mainly related to the sizes of the changes – not to the shapes. For example, the model in the present study proposes a stronger positive response in non-Ricardian consumption, but weaker responses in capital utilisation, labour and output. Comparing to Leeper et al. (2010), where a detailed real business cycle (RBC) model is used with a similar policy using a consumption tax shock, the drop in investment and increase in consumption and debt is confirmed. However, and interestingly, in the RBC model the output responds negatively if the tax rate is decreased. This contradicts the results of both DSGE models.

There are also a few differences in the responses of the tax shock compared to the model in Forni et al. (2009). For example, the debt ratio grows more in the model of the present study, since the production responds more slowly to the tax cut. In Figure 1 the initial 1.3% increase in debt should be understood as follows. A 1.3% increase in public debt is 0.8% of GDP since the debt is assumed to be 60% of GDP in steady state. That 0.8% of GDP is mainly caused by a drop in the tax revenues, which is 0.7% of GDP in Figure 1. The drop in the total tax revenues is clearly explained by the consumption tax decrease, that is around 0.8% of GDP\(^{37}\). However, as in Forni et al. (2009), the decrease in total tax revenues

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\(^{35}\)Clearly, from a counterfactual and policymaker perspective, rules with more impact might be interesting. This is why the main rules of this thesis are not empirical, see Chapter 6.

\(^{36}\)Also when taking into account that Leeper et al. (2010) in fact use positive tax shocks.

\(^{37}\)In the model, 0.8% of GDP is close to 6% of the value added tax, which is the size of the shock. In Finnish terms, 0.8% of GDP is roughly 1.7 billion, which is however about 9% of the annual value added taxes collected in Finland (Ministry of Finance, 2017). The difference is due to the fact that not all consumption
is smaller than the decrease in consumption tax revenues, since there is an increase mainly in labour tax incomes. The compensating labour tax income is small, less than 0.4% of GDP, i.e. the total effect of the tax decrease is a clear deficit that increases the debt. Differently from Forni et al. (2009), the compensation effect in the present study is initially non-existent, but emerges as salaries and labour increase. Another important difference concerns inflation. The consumer prices should drop initially creating negative inflation. However, this is not the case in the model since the inflation in Figure 1 is before tax prices.

To conclude, despite several differences in model specification, the effects of a consumption tax shock are very similar between the present model and the model in Forni et al. (2009). Generally, the shapes of the impulse responses are shared but the levels are not.

THE MODEL

Responses (in percent) to a negative consumption tax shock using fiscal rules (5.1)-(5.3).

Figure 1: Example I of model behaviour.

Labour tax shock Second, assume the labour tax rate is changed by a negative shock of 5.3%. The labour tax rate decreases immediately by 1.6%, which again is the same as in Forni et al. (2009). The shock is visible in the last graph in Figure 2. The AR coefficient of the shock is 0.8, again giving somewhat less persistent dynamics than in Forni et al. (2009). is taxed by the (higher) tax rate of 24.5% that is used in the model.
When the labour tax rate is reduced, people are willing to substitute leisure with work. This leads to an increase in worked hours. Since people work more, fewer persons are needed in production. The total effect is positive—labour increases. When people are willing to work more, their salaries are reduced. This reduces the marginal costs for the firms and can be seen as a small drop in inflation. Lower taxes increase directly the spending of the non-Ricardian households. The Ricardian households do not change their consumption behaviour. The purchasing power is increasing despite lower salaries. Therefore, the aggregated demand is increased. There is also a slight push in investments fuelled by the increased demand, visible in Figure 2.

**THE MODEL**

Responses (in per cent) to a negative labour tax shock using fiscal rules (5.1)–(5.3).

Figure 2: Example II of model behaviour.

The big picture of the economy is again supported by the model in Forni et al. (2009), but the levels differ between the models. The differences in output are now smaller than in the previous case with consumption tax shock. However, the capacity utilisation rates do not really agree. The rates are though both very moderate. The inflation discrepancy is smaller than earlier. Also compared to the RBC model in Leeper et al. (2010), the impulse responses are similar. However, the investments in Leeper et al. (2010) do not increase. In their model, this feature is crucially dependent on the fiscal adjustment instrument (Leeper et al. 2010, pp. 11–12).
The public sector behaviour in Figure 2 is also similar as in Forni et al. (2009). Their accumulation of bonds does not respond initially as fast as in the present study, since the output increases directly in their model. The same applies to other revenue measures. Again, the effects can be clarified using numerical values. If the labour tax rate drops initially by 5.3% (that is 1.6 percentage points, since the steady state tax rate is 30%), then the debt increases by 1.1% (that is near 0.7% of GDP). This is mainly caused by the drop in total tax revenues. The drop is 0.7% of GDP. The increase in other tax incomes is marginal. Therefore 0.7% is seen in Figure 2 as an instant loss in labour tax revenues.

To conclude, there are a few differences between the models, especially regarding the sizes of the impulse responses. However, in most cases the models produce responses of similar shape. The rules used in Forni et al. (2009) provide a decent benchmark for the model used in this study. It should be underlined that the effects of the rules in Forni et al. (2009) are very limited and not useful as such in the present study. The reason is that Forni et al. (2009) estimate the behaviour of the public sector based on observed data. This thesis is concerned with implementing fiscal rules in a counterfactual sense. The rules in the present study are used in order to fulfil the requirements of fiscal rules such as they were described in Chapter 2. The study proceeds therefore by specifying more powerful rules, starting with a more careful discussion about the explanatory variables of the rules (the instruments).
6 Implementation of fiscal rules in the model

The estimated rules in the last chapter do not necessarily provide an economically sound policy. This chapter is concerned with the specification of rules that ensure financial sustainability. Attention is paid to how different measures of public debt evolve. Part 6.1 introduces the instruments (i.e. explanatory variables) that are used in the rules presented in Part 6.2.

6.1 Specification of instruments used in fiscal rules

As noted in Part 2.5, public debt $b_t$ and total output of the economy $y_t$ are probably the most useful instruments for policy rules. The reason is simple. The objective of fiscal policy is usually either to stabilise the economy or to ensure financial sustainability of the public sector. The former target relates closely to production, whereas the latter relates to public debt.

An important variable relating to debt is the balance in terms of surplus, here described by $b_t$. Notice that the log-linearised debt $\tilde{b}_t$ should be interpreted as the change in debt level $b_t$ from its steady state level $b$. The balance is defined in Section 3.3 as the change in debt according to $b_t - b_{t-1}$. However, a similar first difference in the log-linearised variable $\tilde{b}_t$ does no longer describe the balance well. The reason is that the balance in that case would be the second difference of debt. The change in the balance in terms of surplus is therefore derived directly from the change in the debt level according to $\Delta b_t = -1.44\tilde{b}_t \equiv -1.44\frac{b_t - b_{t-1}}{b}$. The change in public sector balance is equal to the change in debt scaled by 1.44, since the debt in the model is assumed to be around 144% of the total general government expenditures. This follows from the assumption that debt is 60% of GDP and that the public sector in the model is around 42% in proportion to GDP. Clearly, if debt increases by 1% the balance should show a deficit of around 1.44% of total public spending.

In order to introduce rules defined in structural terms, a potential performance of the economy has to be determined. There are several methods to estimate a potential output. For example, it can be estimated with a production function approach, where variables such as capital utilisation, employment, unemployment and factor productivity are taken into account (Giorno et al., 1995). It can also be estimated with time-series regression methods from trends in the GDP (Giorno et al., 1995). Since potential production cannot be observable, such as other macroeconomic variables, its estimation depends heavily on the used methodology. This is the reason why the potential output should be interpreted only as an indicative variable. Nevertheless, including a potential output enlarges the analytical tractability of the rules. In this study, potential output $y_t^{OG}$ is determined by the production function approach given by equation (2.9) (Ratto et al., 2009). The equation can be rewritten with constant steady state values as

$$y_t^{OG} = \left(\frac{\eta_t}{\eta}\right)^{1-\alpha_{OG}} \left(\frac{n_t}{n}\right)^{\alpha_{OG}}.$$  

(6.1)

Assume that the capacity utilisation rate $\eta_t$ and the employment rate $n_t$ are the same as in Sections 3.2 and 3.6. Assume further that the long run trend values of $\eta$ and $n$ are the same.
as their steady state values.\footnote{The steady state employment level \( n \) is 0.9 in the model, since the steady state unemployment level \( u \) is 0.1 and the total labour force is normalised to one. The steady state capacity utilisation rate \( \eta \) is 1, since all capital is utilised in the steady state.} The actual output is assumed to equal its potential output in steady state, meaning the output gap \( y_t^{OG} \) is zero in steady state since \( y_t - y_t^p \equiv y_t^{OG} \). Because it is problematic to log-linearise around zero, the output gap is linearised around one instead, with the linear form yielding

\[
\tilde{y}_t^{OG} = (1 - \alpha^{OG})\tilde{\eta}_t + \alpha^{OG}\tilde{n}_t.
\]  

(6.2)

\[\text{Ratto et al. (2009) calibrate their value of } \alpha^{OG} \text{ to 0.52. This study uses } \alpha^{OG} = 0.54 \text{ from an earlier version of the paper of Ratto et al. (2009).} \]

Once the output gap is specified, it is possible to define a structural balance inspired by [European Commission]\textit{[2017a]} and equation \( (2.1) \). It is reasonable to assume large but temporary measures \( M_t \) to be zero in this context and the variable is therefore omitted. The balance \( b_t \) as such is a fairly problematic variable in a log-linear context since its steady state value is zero. Equation \( (6.3) \) below defines therefore \( b_t^s \) as a “structural debt” in proportion to GDP. The structural debt in proportion to GDP is here defined in a similar manner as the structural balance in equation \( (2.1) \), i.e. the equation is rewritten as

\[
b_t^s = \frac{b_t}{y_t} + \psi^{OG}y_t^{OG},
\]

(6.3)

meaning the structural debt \( b^s_t \) equals the cyclically adjusted debt. The semi-elasticity of the budget balance to the output gap \( \psi^{OG} \) regulates the impact of the output gap. Note that the sign in front of \( \psi^{OG} \) has been changed since a situation where the economy is below its potential (i.e. a negative output gap) should reduce the debt burden in structural terms. Clearly, \( \psi^{OG} \) in equation \( (6.3) \) has a similar function as \( \varepsilon^{OG} \) in \( (2.1) \). The value corresponding to \( \psi^{OG} \) is determined by the European Commission. The average of the values of the semi-elasticity of the budget balance to the output gap \( \psi^{OG} \) for all 28 member states in the EU is 0.50 (and 0.57 for Finland) [European Commission\textit{[2017a]}]. This means that 50\% of the change in output gap should be taken into account in the determination of the change of structural debt.

It is reasonable to assume that the structural debt \( b^s_t \) in steady state will equal the actual debt \( b_t \) in steady state, which is 0.6. This is due to the assumption that the potential output \( y^p \) is equal to the actual output \( y \) in steady state, meaning that the output gap \( y^{OG} \) should be zero. In order to include the output gap in the log-linearised formula, the expression given by \( (6.1) \) has to be substituted before log-linearisation. The remaining linear expression for structural debt is therefore

\[
\tilde{b}_t^s = \frac{1}{b^s} \left( \frac{b_t}{y_t} (\tilde{b}_t - \tilde{y}_t) + \psi^{OG}((1 - \alpha^{OG})\tilde{\eta}_t - \alpha^{OG}\tilde{n}_t + \alpha^{OG}\tilde{n}_t - 1) \right).
\]

(6.4)

The structural balance in terms of surplus \( \tilde{b}_t^s \) could be defined in similar manner as the (ordinary) balance \( b_t \). The change in the structural debt \( \tilde{b}_t^s \) could consequently be considered as the change in the structural balance \( \tilde{b}_t^s \), when taking into account that the debt is larger
than the public expenditures of the model. Since an increase in the debt level means that
the balance weakens, the relationship can be written as

\[ \hat{b}_t^s = -1.44\hat{b}_t^s, \]  

(6.5)

for the same reason as earlier.

THE MODEL

Responses (in per cent) in the main fiscal policy instruments by a 1\% technology shock without
using fiscal rules.

Figure 3: Instruments for fiscal rules.

The instruments are also presented in Figure 3. The graph illustrates how the variables
respond to a one per cent TFP shock. As the productivity increases, people work less since
the industry can manage with less input. The labour and real salaries decrease. Also, the
return on capital will be lower. The output increases, but with a delay due to frictions in
the economy. Since the potential output increases immediately as the productivity rises, the
output gap turns initially negative but starts to increase with time. At some point the output
gap turns even slightly positive since the production grows with a delay and the technology

\[39\] The only source of movement in the model economy is a one period increase in technology level, which
fades away with time, see the "TFP shock" graph in Figure 3. The smoothing is due to the AR coefficient
being 0.9. The shock produces the dynamics also in Figures 4-8. However, the rules specified below are not
specifically conditional on a technology shock.
has returned closer to its steady state. The slack in employment affects the public income negatively and increases the debt. This development is only temporary, since the boost in production demands soon more employment. This leads to increasing real salaries, resulting in a stronger public balance. Consequently, the balance will initially drop but recovers rapidly as the debt decreases. As shown in Figure 3, the debt in proportion to GDP will decline more than the debt, because production increases and dilutes the proportion. The structural balance (again measured in terms of surpluses) will initially remain stronger than the balance, since it is compensated for the negative output gap. The structural balance tends to be somewhat larger than the balance also in the middle run, reflecting the higher production. Finally, the structural debt can also be compared with the debt in proportion to GDP. Again, because of the negative output gap, the structural debt will initially be somewhat lower than the debt in proportion to GDP.

6.2 Specification of fiscal rules

This part deals with the fundamental question of the thesis, namely how rules for fiscal policy could be specified and implemented in the model provided earlier. First, Section 6.2.1 presents debt stabilisation rules aiming for a smoother debt development. Second, Section 6.2.2 creates rules inspired by the concept of medium-term objective (MTO). The rules are presented and discussed in Chapter 7.

6.2.1 Rules for debt stabilisation and balanced budget

Large changes in the debt level of heavily indebted economies might not be in line with sound public finances as they were described in Chapter 2. It may therefore be desirable to use rules that mitigate debt fluctuations caused by exogenous shocks. The estimated rules derived in Forni et al. (2009) (see Chapter 3) might be too weak to ensure sound public finances. The debt stabilising properties of the rules could therefore be strengthened. This could be done for example by increasing the $\zeta$ parameters in equations (5.1)-(5.3). However, the rules could also be modelled differently and even more simply with only debt as instrument. Assume the change in consumption tax rate $\tilde{\tau}_t^C$ reacts to the change in the level of accumulated debt $\tilde{b}_t$ as well as to the change in the linearised debt $\Delta \tilde{b}_t \equiv \tilde{b}_t - \tilde{b}_{t-1}$ according to

$$\tilde{\tau}_t^C = \rho \tilde{\tau}_{t-1}^C + \phi_1 \tilde{b}_t + \phi_2 \Delta \tilde{b}_t + \epsilon \tilde{\tau}_t^C,$$  

(6.6)

where $\rho^{ac}$ is an autoregressive component providing persistence to the tax rate and $\epsilon^{ac}$ is an exogenous shock component. The $\rho$ coefficients in rules (6.6), (6.7) and (6.11)-(6.13) are set to 0.9.\(^{40}\) Larger $\rho$ values imply more persistence since a larger fraction of the next value in time originates from the current value. This is of course reasonable as decision-making is rigid and policies usually change slowly. The shock component will however not be used, since the more general TFP shock is used in the simulations below.\(^{41}\) The parameters $\phi_1$ and $\phi_2$ are chosen to ensure that the rules are similar to the estimated ones in Forni et al. (2009).\(^{40}\)
and $\phi_2$ are modified in order to give the rule different strengths. The positive signs imply that when the debt increases, the consumption tax has to increase as well in order to reduce indebtedness and vice versa.

Rule (6.6) can also be respecified such that the actual GDP is taken into account. The rule is then similar to the rules in Forni et al. (2009). A positive coefficient for real debt-to-production ratio means in a linearised context that an increase in production should reduce the tax rate, since higher output dilutes the debt-to-output ratio making it less severe. One can therefore write
\[
\tilde{\tau}_t^C = \rho \tilde{\tau}_{t-1}^C - \phi_3 \tilde{y}_t + \phi_4 \tilde{b}_t + \phi_5 \Delta \tilde{b}_t + \varepsilon_t^C. 
\] (6.7)

The rule resembles the tax rules in Leeper et al. (2010), see equation (2.7) and the following discussion, but with the difference that their rule is countercyclical, i.e. the pressure to adjust is stronger during booms.

Rules (6.6) and (6.7) should be considered as debt stabilising when small values on the $\phi$ parameters are used. As the parameter values grow, the rules eliminate debt fluctuation and become either a balanced budget rule or a rule providing a constant debt-to-output ratio. The rules are discussed further in the next chapter.

A balanced budget rule can also be specified differently. Instead of determining taxes by an AR(1) process with instruments powered by parameters, the tax rates can also be solved directly from the public sector budget constraint. Changes in the real economy affect directly the tax rate, i.e. fluctuations in the budget constraint are moved to the tax rate instead of the debt. This happens since the debt variable is removed (or assumed to be zero) in the equations below. The advantage of this alternative method to specify a balanced budget rule is that no parameters are needed.

The rule is based on the public budget constraint in equation (3.7). The expression can be modified such that tax on consumption or tax on labour income, for example, is determined by the constraint. Hence,\n\[
\tau_t = \frac{\text{all expenditures} - \text{other tax incomes} - \text{buffer}}{\text{tax base } \tau}, \number{6.8}
\]
where tax rate $\tau_t$ can be for example consumption tax rate $\tau_t^C$ or income tax rate $\tau_t^W$. The "tax base $\tau$" should be interpreted as the underlying asset, subject or income being taxed, such as consumption or total salaries. The "buffer"-component can be used to make the rule less strict by allowing the tax rate to fluctuate against the change in growth rate in production, for example.

The idea behind rules (6.9) and (6.10) is to keep the debt level almost constant (a semi balanced budget rule). By writing the buffer as $0.6(\tilde{y}_t - \tilde{y}_{t-1})$, the tax rate adjusts in accordance with 60% of the changes in the growth rate.\footnote{The debt could also fluctuate with other values than 60% of the change in growth rate. See also the discussion in footnote 31.} The rule can be seen as less strict than the balanced budget rule, but stricter than the rule providing a constant 60% debt-to-GDP ratio. For example, equation (6.8) can be respecified such that income taxation
changes according to
\[
\tau^W_t = \frac{P^H_t g_t + P^G_t i^G_t + TR_t + P_t b^U_t u - (P_t((n_t w_t h_t)\tau^{SC}_t + \tau^C_t c_t + \tau^D_t d_t + \tau^K_t R^K_t k^p_t))}{P_t w_t n_t h_t} - 0.6(y_t - y_{t-1}) \frac{1}{P_t c_t}.
\] (6.9)

A similar rule can easily be derived for the tax rate on consumption by solving the modified budget constraint for nominal consumption, i.e.
\[
\tau^C_t = \frac{P^H_t g_t + P^G_t i^G_t + TR_t + P_t b^U_t u - (P_t((n_t w_t h_t)(\tau^W_t + \tau^{SC}_t + \tau^D_t d_t + \tau^K_t R^K_t k^p_t)))}{P_t c_t} - 0.6(y_t - y_{t-1}) \frac{1}{P_t c_t}.
\] (6.10)

The rules reach their final form after log-linearisation and are given in the appendix, see the [B] part.

The rules (6.6)-(6.7) are tractable because of their flexibility and simplicity. However, the rules (6.9)-(6.10) can be considered as more “natural”, since the tax rates are solved from the budget constraint and no policy parameters are needed.

**6.2.2 Rules for the structural balance**

The rules presented so far, (6.6), (6.7), (6.9) and (6.10), are simple but fulfill with different degrees of flexibility their objectives to promote a sound public economy. This is illustrated later among the results in Chapter 7. This section targets more specified rules with a stronger connection to the Stability and Growth Pact and especially the MTO.

How could the rules referring to the medium-term objective effectively be implemented in the model? The targets are specified in structural terms, meaning the structural variables specified above have to be used. The rules resemble the specification in Muscatelli & Tirelli (2005) seen in equation (2.8). The levels, such as the three and one percent targets in equations (2.2) and (2.4), are problematic. However, the levels are of less interest in a general equilibrium model, where the variables are measured as changes from their steady states. This means that keeping variables, such as the structural balance, fixed at different levels (for example similarly as in equations (2.2) and (2.4)) is possible only such that the steady state levels of the variables are changed. Focus should instead be turned to the adaptation paths caused by feedback from relevant instruments and not to the levels themselves, since a simulation-based analysis demands dynamics.

Assume that income tax rate, for example, is used to stabilise the structural balance. This can be considered as a measure to fulfil the MTO of a country. The three definitions of MTO can in terms of the model be specified as follows.

Firstly, inspired by equation (2.4), a similar rule can be denoted as
\[
\tilde{\tau}^W_t = \rho^w \tau^w_{t-1} - \psi_1 \tilde{b}^s_t + \varepsilon^w_t,
\] (6.11)
where $\psi_1$ is a constant. A decrease in the structural balance, such as it was defined earlier in Part 6.1, should according to rule (6.11) increase the tax rate in order to lift back the balance.
Secondly, the minimum benchmark MTO takes the actual output gap into account and follows equation (2.2). This means an economy below its potential production must have a structural balance closer to zero than three per cent. A rule similarly as (6.11) can therefore be specified, but with output gap as an additional instrument. The rule is written as

\[
\tilde{\tau}_t^W = \rho \tilde{\tau}_{t-1}^W - \psi_2 \tilde{b}_t^s - \psi_3 y_t^{OG} + \varepsilon_t^W, \tag{6.12}
\]

where the negative sign in front of the output gap indicates that a larger negative output gap should rise the income tax rate in order to reduce the structural deficit. Due to the symmetry brought by the linearity in the model, a positive output gap will force the government to tax less in order to make the finances weaker.\footnote{Though log-linearisation makes it slightly more complicated, rule (6.12) can essentially (and without the shock) be reduced to \(\tilde{\tau}_t^W = \rho \tilde{\tau}_{t-1}^W - (1 - 1.44)\psi_2 (\tilde{b}_t - \tilde{y}_t) - (-1.44)\psi_2 y_t^{OG} \frac{1}{b} + \psi_3 \tilde{y}_t^{OG},\) meaning that the term \((-1.44)\psi_2 y_t^{OG} \frac{1}{b}\) + \(\psi_3\) becomes zero if \(\psi^{OG} = 0.5\) and \(\psi_2 = \frac{5}{6}\psi_3,\) since \(b = 0.6.\) This will assumed in the simulations presented in Figure 7.}

Finally, the third definition of MTO captures the implicit liabilities and debt, given by equation (2.3). The third definition is more challenging. Several aspects need to be taken into account by the rule. Firstly, the rule should stabilise the debt-to-GDP ratio at 60%. For this purpose the debt level \(\tilde{b}_t\) has to be included in the rule. Secondly, the rule should take into account future costs of ageing and it should provide an additional effort if the debt level exceeds 60% of the production. The costs related to ageing \(\alpha C_t^a\) are, however, here assumed to be constant. The costs might change slightly from year to year, but it is reasonable to assume that they are relatively stable in the short and medium run. Furthermore, there might be a trend in the variable due to ageing population and trends are not included in the model. It is therefore obvious that the costs could not be determined endogenously and are therefore left out. Thirdly, the effort variable \(F_t\) should be easier to include. As noted earlier, the effort is given by \(2.4 \frac{\tilde{b}_t}{\tilde{y}_t} - 1.24,\) meaning that \(\tilde{b}_t\) and \(\tilde{y}_t\) should be present in the rule. However, the change in debt level \(\tilde{b}_t\) is already part of the rule (see equation below). The change in output \(\tilde{y}_t\) appears therefore alone in the formula with its parameter. A linear rule capturing these aspects could therefore look like

\[
\tilde{\tau}_t^W = \rho \tilde{\tau}_{t-1}^W - \psi_4 \tilde{b}_t^s + \psi_5 \tilde{y}_t - \psi_6 \tilde{y}_t + \varepsilon_t^W. \tag{6.13}
\]

According to rule (6.13), the income tax rate should increase when the structural balance weakens, since an increased tax rate will through less accumulation of debt affect the structural balance positively. The strength of this policy is controlled by \(\psi_4.\) The next component provides additional adaptation when the debt level deviates from its steady state level of 60% of GDP. Finally, higher output dilutes the debt-to-GDP ratio making the effort variable \(F_t\) smaller – how much is controlled by \(\psi_6.\)

The same rules could also be specified for other variables than income tax rate, for example consumption tax rate \(\tilde{\tau}_t^C.\) Rules (6.11)-(6.13) and their parameters are discussed further in the next chapter.
7 Simulation results

This chapter presents the results when the rules in Part 6.2 are applied in the model provided in Chapter 3. The results of the rules are analysed especially based on their effects on economy and welfare. Comments are also given on how well the original objectives of the rules are transmitted into the model. The policy parameters are when possible chosen such that different strengths are visible in the figures. Notice that the dynamics in all scenarios are caused by a positive TFP shock.

7.1 Results using debt stabilisation and balanced budget rules

The results below are generated by simulations using the rules in Section 6.2.1. The idea in the scenarios is to increase taxes as the balance turns negative – and because of linearity, to decrease taxes when the balance turns positive.

The simulation results using rules (6.6) and (6.7) are presented in Figure 4. The black dotted line corresponds to the benchmark scenario where no changes occur in taxation, i.e. no rules are used. Firstly, the dash dotted (yellow) line corresponds to a medium strong policy using rule (6.6), where $\phi_1 = 0.1$ and $\phi_2 = 5$. The parameters are chosen such that the change in debt burden takes approximately half of its initial size and returns thereafter quickly to its steady state. Secondly, the dashed (blue) line in Figure 4 shows where the policy converges to when the parameter values in (6.6) are large. When the parameters $\phi_1$ and $\phi_2$ are increased enough, the tax rate on consumption adjusts such that the public economy always is in balance. This corresponds to a complete balanced budget rule. Finally, the third rule is given by (6.7) and targets a constant debt-to-output ratio. The rule is presented by the solid (orange) line in Figure 4. Since output is growing, the debt does not have to adjust as much as in the balanced budget rule.

The balanced budget rule forces the balance towards zero by increasing the tax rate on consumption as the economy is hit by a TFP shock. The initial tax increase is near 1.5 percentage points, but after a year the increase is around 0.5 percentage points. The increase in taxation can be seen as a tax shock of the kind in Figure 1, but with a positive sign. Compared with the benchmark (i.e. the no rule case), the following actions can therefore be said to take place among others, when the balanced budget rule is enforced: A higher consumption tax rate decreases consumption for the Ricardian and especially for the non-Ricardian households. More expensive consumption encourages people to substitute consumption with leisure leading to a decrease in worked hours. This explains the additional need for employment seen as a drop in the unemployment in Figure 4.

The public sector debt does not change in absolute terms when applying the balanced budget rule. The TFP shock improves the fundamentals of the ability of the economy to produce. The output gap turns therefore negative, meaning the structural balance shows a surplus. Furthermore, the debt-to-output ratio decreases because of GDP growth. The total tax revenues decline in the benchmark scenario mainly due to a decrease in labour. The

\[\text{Notice that in the balanced budget rule, it is actually enough to increase either of the variables. The reason why } \Delta b_t \text{ is included in rules (6.6) and (6.7) is that the variable can be useful for fine-tuning less strict versions of the rules.}\]
consumption tax rate is high enough to balance the budget but, because of an increase in GDP, total tax revenues in proportion to output decrease slightly.

Applying rule (6.7) provides a debt stabilising and procyclical effect on the economy. Figure 4 illustrates a scenario where the rule is enforced with power enough to keep the debt-to-production ratio constant. The balance turns into deficit because the output is growing. Clearly, the rule behaves very differently compared to the balanced budget rule. Using (6.7) leads to an initial substantial tax increase that, however, already after a year turns into tax cuts as the production increases. This affects the consumption mainly positively.

THE MODEL

Responses (in per cent) to a 1% technology shock using fiscal rules. The dash dotted (yellow) line marks the outcome when rule (6.6) is used with parameter values $\phi_1 = 0.1$ and $\phi_2 = 5$. The dashed (blue) line is also produced by (6.6), but with $\phi_1$ and $\phi_2$ being large. The final solid (orange) line is generated by (6.7) using $\phi_3 = \phi_4 = 1000$ and $\phi_5 = 0$. The benchmark corresponds to a scenario where no rules have been used.

Figure 4: Debt stabilisation rules on consumption taxation.

Compared to the benchmark, rules (6.6) and (6.7) have very different welfare effects. The cost of stabilising the balance by using the first rule is an initial drop in welfare compared to the benchmark. The decrease diminishes after roughly a year when the tax rate returns to its original level. The welfare effect of the second rule (6.7) is more volatile since it depends on output. In this case, the welfare is initially slightly above the benchmark. As
the consumption increases due to tax cuts and the production grows, the welfare rises well above the reference line. The welfare in absolute terms rises in all cases because of the positive TFP shock.

Figure 5 presents the simulation results using rules (6.9) and (6.10). The balance is stabilised as the tax rates on consumption and separately on labour are forced to change with the balance. The results are very similar compared to the balanced budget rule (6.6). However, as illustrated in the figure, there is an initial bend in the balance, which mainly is caused by the change in output growth rate. The idea is that a small deficit can be accepted during times when the output growth is increasing. Similarly, the balance must show a small surplus when the growth slows down. The figure shows reactions in the economy using rules separately specified for the tax rate on labour and consumption. An increase in labour taxation increases salaries and reduces consumption only little. Despite the increase in salaries, the households will on an aggregated level substitute labour with leisure, leading to a drop in worked hours. This leads to an increased demand for workers and a reduction of

THE MODEL

Responses (in per cent) in a few economic variables by a 1% technology shock using rules affecting the labour tax rate according to (6.9) and consumption tax rate according to (6.10). The benchmark corresponds to a scenario where no rules have been used.

Figure 5: Semi balanced budget rules on consumption taxation and labour taxation.

45 A small part of the kink is caused by factors relating to minor imbalances in the calibration.
unemployment. The difference in tax response (see Figure 5) between the two rules depends on differences in the relative sizes of the tax rates.

Rules (6.10) and (6.6) are from a welfare perspective almost identical. Nevertheless, as illustrated in Figure 5, in terms of welfare there is a clear difference between (6.9) and (6.10). This is explained by consumption being lower with the consumption tax rule, making the increase in welfare less than under the alternative regime. This aspect is discussed further in Chapter 8.

7.2 Results using rules for the structural balance

This section presents simulation results using the rules from Section 6.2.2. Three different rules are presented below in Figures 6-8. The rules correspond to equations (6.11), (6.12) and (6.13).

First, Figure 6 illustrates how the fluctuations in the structural balance gradually are reduced as the strength of rule (6.11) is increased by manipulation of $\psi_1$. Fluctuations in the structural balance are eliminated when the rule is strong enough. This is seen as a dash dotted (red) line in the figure. This could be considered as having a constant \(MTO\) as in equation (2.4).

In the benchmark scenario, the structural balance of the public sector is initially in deficit. The reason is that the balance is in deficit and the GDP does not grow yet. However, the output gap is large, which has a positive effect on the structural balance. The economy is initially far away from its potential performance. The structural balance reaches a surplus within a year because the debt decreases, the output increases and the output gap is almost closed.

In order to balance this movement with rule (6.11), the tax rate has initially to be higher, but then lower, than its steady state. The budget itself is not balanced when using the rule, because the tax rate diminishes already after a year. The debt-to-output ratio demonstrates initially an increase, since the debt increases but the output is rigid. This means that keeping the structural balance fixed, lifts the debt-to-production ratio and reduces the balance. Both the debt and the output increase in the medium run by approximately the similar amount. This leads the debt-to-output ratio to evolve fairly smoothly. Clearly, there is a significant difference compared with the balanced budget rule. Rule (6.11) implies flexibility for the debt to fluctuate with the debt-to-GDP ratio and the output gap. This is also seen as a much milder movement in labour tax rate in Figure 6 than in Figure 5. The tax rate rises at most with only about two per cent, as a consequence of the same technology shock as earlier.

The rule leads to moderate changes in the consumption compared to the benchmark. The reduction in the tax rate leads to a somewhat higher consumption for the non-Ricardian households after a couple of years. However, the rule reduces consumption of forward-looking households especially during the first years. Interestingly, the reduction in labour tax rate makes people more willing to work, leading to a decrease in the demand for labour and an increase in unemployment. The welfare shows a clear increase of around 20% compared to

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46 The rule eliminating fluctuation in the structural balance is not concerned with the level at which the structural balance is kept constant.
the benchmark. This is partly explained by the increasing consumption of the non-Ricardian households but also by the increasing growth rate in the Ricardian consumption.

THE MODEL

Responses (in per cent) in a few economic variables to a 1% technology shock using a fiscal policy rule affecting the income tax rate according to (6.11). The solid (yellow) line marks the outcome when rule (6.11) is used with a parameter value \( \psi_1 = 3 \) and the dash dotted (orange) line when \( \psi_1 \) becomes very large. The benchmark corresponds to a scenario where no rules have been used.

Figure 6: Structural balance rule I on labour taxation.

Second, Figure 7 illustrates how the income tax rate changes according to (6.12), i.e. with fluctuations in the structural balance and in the output gap. The output gap is hence affecting the tax rate twice – first in the calculation of the structural debt itself and a second time in the rule. The forces work in different direction and are here assumed to cancel each other out. (The effect of the output gap is eliminated when \( \psi_2 \) and \( \psi_3 \) are chosen such that their quota is equal to the semi-elasticity of the budget balance to the output gap \( \psi^{OG} \), times the size of the public sector per the debt level in steady state. See the discussion in Section 6.2.2 and especially footnote 43). The results presented in Figure 7 demonstrate therefore a fully stabilised debt-to-production ratio when the rule is strong. This is seen as a dash dotted (red) line. The structural balance reflects now the changes in the output gap. The reason is the following one: The output gap is eliminated from the tax rule, meaning that the rule stabilises the debt-to-production ratio. Clearly, a constant debt-to-production
ratio eliminates fluctuations in the structural balance that are not part of the output gap. The structural balance is therefore left purely with the output gap. Hence, the structural balance in Figure 7 is cleaned from the effect caused by changes in the debt-to-production ratio. This is the objective of the rule.

THE MODEL

Responses (in per cent) in a few economic variables by a 1% technology shock using a fiscal policy rule affecting the income tax rate according to (6.12). The solid (yellow) line marks the outcome when rule (6.12) is used with parameter values \( \psi_2 = 3 \) and \( \psi_3 = 3.6 \) whereas the dash dotted (orange) line is produced when \( \psi_2 = 1,000 \) and \( \psi_3 = 1,200 \). The benchmark corresponds to a scenario where no rules have been used.

Figure 7: Structural balance rule II on labour taxation.

There are a couple of economic differences when the debt-to-production ratio is stabilised instead of the structural balance. First, when debt-to-production ratio is stabilised, the balance is initially forced to adjust strongly. The reason is that the debt is increasing while output is lagging behind, leading to a stronger response in the tax rate (compare the tax rates in Figure 6 with Figure 7). Second, the tax rate also remains higher (or it does not decline in the middle run as much as in rule (6.11)), because the debt-to-production ratio does not reduce as much as the surplus in the structural balance. The economic implications are similar to those provided by the previous rule. However, the households reduce their consumption slightly more using rule (6.12), because of the higher taxation. Comparing the
welfare effects of (6.12) with (6.7) suggests that the welfare is slightly higher using the rule affecting consumption taxation than using the rule affecting labour taxation. The reason is that the consumption tax rule (6.7) decreases the tax rate more (after a year). This leads to higher consumption and welfare. The differences between the rules are nevertheless moderate.

Figure 7 illustrates also a weaker parametrisation of the same rule (the solid (yellow) line). The rule is in this scenario not strong enough to fully stabilise the debt-to-production ratio. Therefore, the structural debt does no longer fully depend on the output gap. However, both variables evolve smoother than in the no rule case. An interesting observation is that the structural balance in fact will change its initial sign as the rule gets weaker.

**THE MODEL**

Responses (in per cent) in a few economic variables by a 1% technology shock using a fiscal policy rule affecting the income tax rate according to (6.13). The solid (yellow) line marks the outcome when rule (6.13) is used with parameter values \( \psi_4 = \psi_5 = 3 \) and \( \psi_6 = 1.5 \). whereas the dash dotted (orange) line is produced when \( \psi_4 = \psi_5 = 1000 \) and \( \psi_6 = 500 \). The benchmark corresponds to a scenario where no rules have been used.

Figure 8: Structural balance rule III on labour taxation.

Third, rule (6.13) is presented in Figure 8. With three different instruments, the interpretation gets somewhat more complex. Again, the dash dotted (red) line is generated using very high parameter values on the \( \psi \) parameters. The motivation behind the parameter
values $\psi_4, \psi_5$ and $\psi_6$ is the following one. Initially the coefficient for the structural debt $\psi_4$ is set to a large number such that the structural balance is fully stabilised (for example, the value 1 000 is used in the simulation). Then the coefficients for the debt $\psi_5$ and the output $\psi_6$ are separately determined such that the structural balance captures half of the response in debt and output. For example, if debt is decreased by 0.5\%, then $\psi_5$ is chosen such that the structural balance will increase by around 0.25\%. This gives roughly the values $\psi_4 = 1000, \psi_5 = 1000$ and $\psi_6 = 500$. The structural balance in Figure 8 increases initially since the debt is growing (and the output does not initially grow much). However, the structural balance decreases gradually as the output increases and the debt decreases. Notice that the decrease in the structural balance is faster than the increase in the balance since output is growing.

A lighter parametrisation with the same parameter proportions, such as $\psi_4 = \psi_5 = 3$ and $\psi_6 = 1.5$ is seen in Figure 8 as a (yellow) solid line. Again, the development of the structural balance is very different from the stronger parametrisation and resembles the benchmark scenario. The decreasing effect of output and the increasing effect of debt in the structural balance are though visible.

The third rule can also be seen as a trade-off. On one hand it aims to keep the structural balance fixed. On the other hand it allows for flexibility in the structural balance, both caused by changes in debt and production (see equation (2.3)). Furthermore, the rule can be seen as a mix between (6.11) and (6.12), i.e. as a rule with an objective of both stabilising the debt-to-production ratio and the structural balance. Based on Figure 8 the rule succeeds well in that respect.

The economic implications of the rule are similar as in the previous cases. Since the debt does not influence as much as previously, the initial tax response remains somewhat weaker with a less significant reduction in the non-Ricardian consumption. The welfare effect is slightly higher than the welfare effect using rule (6.12), because the response in tax rate is milder. However, the welfare effect remains lower than the effect of using rule (6.11).

\footnote{The reason why half of the response in debt and output is taken into account can be explained as follows. If the response would be taken 100\% into account, the variable used as instrument would be fully stabilised. The interpretation of MTO as a target value for the structural balance would disappear when debt or output is on a certain level. The interpretation would instead be that how should the structural balance look like such that the underlying problems would not exist. On the other hand, if only little of the response is taken into account, then the structural balance would show only little movement. The impact of the additional instruments would be weak. There is however nothing saying that exactly half of the response is an optimal trade-off.}
8 Discussion

This chapter discusses the study. The effects of fiscal rules are discussed in Part 8.1. Part 8.2 discusses some of the challenges faced in the study and Part 8.3 considers the scope for future research.

8.1 The effects of fiscal rules

The study develops three sets of fiscal rules and simulates them in a DSGE model using a TFP shock. Three different types of fiscal rules can be distinguished, depending on the choice of instruments and parameter strength. The three different types are (i) a balanced budget rule (no debt is accumulated, see equation (6.6)), (ii) a rule balancing the debt-to-output ratio (given both by (6.7) and (6.12)) and (iii) a rule forcing the structural balance toward zero (given by (6.11)). These rules are simulated by adjusting consumption or labour tax rates. Based on the result in the previous chapter, there are clear differences in welfare between stabilising debt, debt-to-output ratio and structural balance. The differences depend partly on the type of tax rate being used in the rules.

How does the welfare evolve when fiscal rules that affect the consumption tax rate are used? Assume consumption taxation is used for adjustment and the economy is hit by a TFP shock. The result shows that the welfare measure is around 30% higher when a rule stabilising the debt-to-production ratio is used compared to when a rule stabilising the debt is used. The reason is that the tax rate will decline when the debt-to-production ratio is stabilised. This increases consumption and thereby consumer utility and welfare. However, if the debt-to-production ratio is stabilised, the balance is constantly in deficit. The indebtedness increases. The debt is on the other hand kept on a constant level with the balanced budget rule. Comparing the welfare effects from applying the two rules with the benchmark welfare (the no rule case), suggests the following: The rule stabilising the debt-to-production ratio increases welfare during the entire simulation period of five years. The welfare under the balanced budget regime diminishes during the first year when taxes are increased and follows the benchmark thereafter. If the structural balance would be stabilised using consumption taxation, the welfare would be almost 10% higher than in the case where the debt-to-output ratio is stabilised. This is due to the negative output gap that is closed after a couple of years when the economy performs close to its potential. The definition and size of the output gap are not unambiguous and should therefore only be seen as indicative (see Part 8.2 below).

Similar patterns are observable when labour taxation is used for adjustment with the same rules as above. Balancing the debt-to-production ratio instead of debt alone increases the welfare response by around 20%. Neutralising the structural balance instead of the debt-to-production ratio increases the welfare response by around 8%. Interestingly, the result indicates that the total welfare response of the TFP shock when balancing either the debt-to-production ratio or the structural balance is a few percentages higher when using consumption taxation rather than when using labour taxation. However, the welfare remains higher when using labour taxation than when using consumption taxation when the balance is stabilised (see for example Figure 5). The reason is that the welfare measure is more sensitive to changes in the consumption tax rate than in the labour tax rate. Consumption
taxation affects consumption more than labour taxation does. Consumption is the most important variable in the welfare measure. Hence, changing consumption taxation affects the welfare of the model more directly than when changing labour taxation. Figures 11-12 display welfare and debt responses when using rules affecting consumption and labour taxation respectively.

The comparison with the models in Forni et al. (2009) and Leeper et al. (2010) in Chapter 3 demonstrates that their models behave in similar manners to responses to tax shocks as the model in this thesis. The finding in Forni et al. (2009) that the effects of changes in consumption and labour taxation are significant and long lasting is supported by the model in this study. The tax changes are of course not as large in Figures 4-8 when caused by the rules (that react on a TFP shock) as in Chapter 5. In Chapter 3, the tax rates themselves are subjected to shocks. The responses should therefore be smaller, as they prove to be.

THE MODEL

Figure 9: Production under fiscal rules on consumption taxation.

Responses (in per cent) in output of a 1% technology shock using different fiscal policy rules affecting the consumption tax rate. The benchmark corresponds to a scenario where no rules have been used.

What is the benefit of using fiscal rules in structural terms? The result points toward higher welfare responses when the structural balance is neutralised, compared to when the debt is kept constant. This should be no surprise since higher welfare is purchased at the cost of new debt. This is in line with the rules. It can be argued that higher production makes it affordable to increase indebtedness. It can also be argued that higher indebtedness
should be accepted during times when the economy is below its potential level and vice versa when the economy is above its potential level. Clearly, one purpose of applying the rules is to provide sustainability and a neutral policy in terms of cyclicalit.

The neutralised cyclicalit is observable in the model result. The study shows the following when the structural balance is kept at zero: The output demonstrates a less procyclical (or in fact a neutral\(^{45}\)) movement compared to the scenario where the debt-to-GDP ratio is kept constant, see Figures 9 and 10. When the rules for the structural balance are applied, the output will initially be higher because the output gap is negative. When the gap is closed, the production is at the same level as when the rule providing a constant debt-to-GDP ratio is used. (The procyclical of the rule providing a constant debt-to-GDP ratio is somewhat weaker when adjusting labour taxation, as illustrated in the figures). The balanced budget rule provides cyclically a very similar response in the GDP as the rule neutralising the structural balance. The higher GDP level afflicted with the structural rule is explained by lower tax rates leading to increased indebtedness.

THE MODEL

Responses (in per cent) in output of a 1\% technology shock using different fiscal policy rules affecting the labour tax rate. The benchmark corresponds to a scenario where no rules have been used.

Figure 10: Production under fiscal rules on labour taxation.

\(^{45}\)Generally, the rule neutralising the structural balance is by definition "neutral", whereas the rule providing a fixed debt-to-GDP ratio is procyclical.
To conclude, the welfare is the highest when applying the structural balance rule, but so is the debt, see Figures 11 and 12. Comparing the rule for the structural balance to the rule with fixed debt-to-GDP ratio, reveals that the welfare remains higher during the entire simulation horizon when using the former rule and that the balance will only initially be in (an even lower) deficit. The balanced budget rule provides the lowest (additional) welfare but it is free of costs in terms of debt accumulation. Based on the result, the following can be concluded assuming the economy is hit by a welfare increasing TFP shock changing public sector balance into a deficit: Using a fiscal rule with constant structural balance instead of a rule with fixed debt-to-GDP ratio shows signs of improved welfare. However, when the rule with a constant structural balance is compared to the balanced budget rule in the same context, it does not show evidence of providing more welfare than the cost of it in terms of additional debt.

**Figure 11:** Welfare and public sector balance under fiscal rules on consumption taxation.

Responses (in per cent) in welfare and debt of a 1% technology shock using different fiscal policy rules affecting the consumption tax rate. The benchmark corresponds to a scenario where no rules have been used.
Responses (in per cent) in welfare and debt of a 1% technology shock using different fiscal policy rules affecting the labour tax rate. The benchmark corresponds to a scenario where no rules have been used.

Figure 12: Welfare and public sector balance under fiscal rules on labour taxation.

8.2 Challenges and criticism

Several aspects should be taken into account when analysing the effects and evaluating the performances of the rules. As the rules get more complicated it makes the interpretation of the result more challenging. There are therefore several issues to be aware of.

The first issue is that changes might be slower in reality than those provided by the rules. One lag in the model is a quarter, i.e. the time span between \( t - 4 \) and \( t \) is one year. The governmental budget is revised only once a year and additionally, reforms and decision-making are often time-consuming. Also, gathering of data could take a quarter in itself. Clearly, the assumption that a government could change its fiscal policy in every quarter based on real time data is far from reality today. However, there are a few reasons why it still might be more convenient to assume that policy can be changed at any point in time and that data is available without delays. Firstly, introducing a four or five quarter delay would make the interpretation of the result even more challenging. The impact of the rules becomes more unclear as the delays are mixed with the rigidities in the model. Secondly, once the effect of a rule is clear, it is afterwards possible to postpone the effects of policy
rigidities by a desired number of periods. Finally, the loss of realism is more moderate when assuming that the governmental budget could have built-in automatic adjustment and that forecasts could be used instead of waiting for data to be recorded.

The second issue relates to the linearity and is likely to be one of the most severe drawbacks of the used method for simulating fiscal rules. Imagine a rule like (6.6). The purpose of the rule is to stabilise debt by increasing taxes when the public economy is in deficit. However, due to linearity, the rule will also decrease taxes if the economy shows a surplus. This might sound unproblematic from the perspective of (in this particular case procyclical) fiscal policy. The problem is, however, that the purpose of the original rule never is to decrease taxes in order to lift back the debt to its 60% limit level. The purpose of the rule is instead only to ensure that the debt does not exceed the level. It would make sense to use a rule that is enforced only when the debt level exceeds or approaches the limit. This is, nevertheless, out of the scope of a fully linear model. When moving into the more complex rules provided by the Commission, this linearity issue becomes an increasing problem as seen in Chapter 7. For example, the MTO concept is a minimum target value of public economy deficit in structural terms — there might be no reason to decrease taxes so that the balance weakens and the economy reaches this minimum deficit limit. However, it is in line with the rule to increase taxes (or in other ways strengthen the balance) once the economy is below the MTO.

How should the linear rules then be interpreted? Though the linearity is a desired feature in DSGE models, helping with the estimation methods and filtering, it has its drawbacks. As argued above, some aspects of the rules cannot be implemented at all. Other aspects can be seen in the simulations only when the result is interpreted carefully. A clear implication of the linearity is therefore that the interpretation becomes more challenging as there are several aspects to keep in mind when analysing for example impulse responses. One way, and maybe the most intuitive way of studying the impulse responses is the following one: Accept that the rules can lead to changes in the debt in both directions and focus instead on the stabilising property of the rule, i.e. what happens with the other variables in the economy when fluctuation is decreased in a policy variable. The rules should therefore be interpreted as forces pushing a variable either directly towards its steady state or towards co-fluctuation with other variables. This corresponds to a real world policy that by some degree of effort (adjusted by either $\phi$ or $\psi$) brings the policy variable toward a desired level. The level can be seen as the limit value specified by the real rule. Another way of studying the impulse responses is to only investigate those parts of the responses where the actual real world rule would have been enforced, i.e. for example where the debt has increased. However, the conclusion of the development over the entire simulation horizon might be incorrect if the policy changes during the path. The gain of this method is to correctly evaluate the economy-wide changes over at least some, although maybe fragmented, period of time. Note that this type of analysis is even more challenging with longer lags in the rules. Finally, there is a third aspect to keep in mind, which does not relate to linearity but to the simulation method itself. The responses in the beginning of the simulation horizon are typically of more

\[49\] Remember that MTO is a lower limit of the balance in structural terms, meaning that clearly, it is more desirable to be above than below the limit. This aspect could not be taken into account by the linear rules.

\[50\] Also, remember that the common objective of the rules (6.6)-(6.13) is to stabilise either debt, debt-to-production ratio or structural debt by using different methods.
interest than in the later part. The reason is that the fluctuations are in all cases reduced after a few periods. It is not necessarily clear to which extent the reduction is caused by normal return towards steady state, by impact of the rule or by indirect effects from the rule being more effective in the initial periods. Hence, the first years of the simulation horizon are the most interesting. Also, it should be kept in mind that the rules will not change the steady state of the model. This means that in the end all the rules lead to the same result, the steady state. The rules can therefore not keep for example the MTO on a -1% level as in the original rule. Instead it has to be assumed that the steady state reflects the target and that deviation from this target symmetrically should be reduced or eliminated according to the rules (6.6)–(6.13).

The third issue to be aware of is the way the output gap has been implemented and used in the model. The present study defines the output gap in a similar manner as Ratto et al. (2009), see equation (6.1). Depending on the capacity utilisation rate and the employment, the output gap is allowed to fluctuate significantly from one period to another. This has consequences on the policy rules (6.11)–(6.13). The method used by the Commission to determine a potential output (that is used for example in the calculation of the MTO) is somewhat more complicated as it includes also other variables and is calculated over a longer period of time (Havik et al., 2014). For this reason the outcome based on the rules presented in this study might be more volatile than it would be with another definition of the output gap. Again, the analysis becomes more challenging. However, this is not likely to be a decisive concern. The implementation of a more volatile definition of the output gap makes the business cycle occur more rapidly. This creates on the other hand opportunities to analyse the impact of an entire business cycle with a simulation horizon of a few years. This may be desirable from other points of views, as discussed above.

It can be concluded that several issues has to be taken into account when analysing the outcome of the rules being implemented in the model. A simulation-based analysis of fiscal rules in a DSGE model is clearly afflicted with several shortcomings. However, as shown in the thesis, there are still many features of implementing the rules that can be analysed.

### 8.3 Suggestions for further research

Studies with fiscal rules remain interesting for at least two reasons. First, there is still much room for improvement in macroeconomic models suitable for fiscal policy simulations. Second, the fiscal rulebook changes often (see European Commission [2017a]). This opens for opportunities to extend the study in several directions. For example, rules in real terms have been used in this study, but they could also be specified in nominal terms. Furthermore, the rules could be used with other taxes such as tax on capital, or with variables from the expenditure side. Additionally, other shocks and different measures of the output gap may have an impact on the result.

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51 As pointed out in a recent paper by Kusi (2017), uncertainty in the output gap measure may have large consequences when designing optimal fiscal policy.
9 Summary and conclusions

The objective of this study is to investigate the effects of fiscal rules on economy and welfare. The study is conducted by a simulation-based analysis, using a New-Keynesian dynamic stochastic general equilibrium (DSGE) model. The model describes a small and open economy with a public sector and a fraction of non-Ricardian households. The underlying purpose is to improve upon the understanding of using different policies in order to ensure a sound and welfare enhancing public economy. This is done in a New-Keynesian DSGE model. Inspired by the literature and the Stability and Growth Pact of the EU, the study develops rules for balancing the public sector budget, including a balanced budget rule, a rule where the debt-to-GDP ratio is constant and a rule where the balance in structural terms is neutralised.

The result suggests that the welfare is higher using consumption taxation when fluctuations in the debt-to-production ratio and the structural balance caused by a technology shock are eliminated. However, the welfare remains higher using rules affecting the labour taxation when fluctuations in public debt caused by a technology shock are eliminated. This is explained by higher welfare volatility of changes in consumption taxation than in labour taxation. The result points towards procyclical responses when rules providing a fixed debt-to-output ratio are compared to rules eliminating fluctuations in the structural balance. However, very limited differences in GDP cyclicality are observed when the responses of the rules eliminating fluctuations in the structural balance are compared with the responses of the balanced budget rules. The study also shows that some features of the original rules are lost mainly due to model linearity. However, the rules created in the study succeed in capturing many of the objectives of the original rules of the Stability and Growth Pact. Also, some aspects of the more complicated rules referring to the medium-term objective are implemented in the model. As the rules get more complicated so do their interpretation.

The following can be concluded for an economy affected by a technology shock. The rules providing a constant debt-to-GDP ratio are procyclical. The rules are therefore perhaps better used for other purposes than as an automatic policy to be fulfilled in every period. For example, the rules can be used as an indicative target level, such as the 60% debt ratio in the Stability and Growth Pact. Using rules neutralising the structural balance instead of rules providing fixed debt-to-output ratio reduces the procyclicality. Automatic rules for the structural balance can therefore be an option if the debt in absolute terms is allowed to grow. However, the study shows that in terms of GDP evolution and with a positive TFP shock, the balanced budget rules are almost as neutral as the rules providing a constant structural balance. Hence, under the assumptions made in this study, the usage of rules providing a constant structural balance should also be motivated by other than cyclical arguments.
10 Svensk sammanfattning

I detta kapitel ges en svensk sammanfattning över avhandlingen med rubriken *Effekterna av skuldstabiliserande fiskala regler i en makroekonomisk modell.*

10.1 Introduktion

Finanspolitik är ett av de huvudsakliga verktygen som stater kan använda sig av för att påverka ekonomin. Syftet med finanspolitiska åtgärder är vanligen att antingen stabilisera ekonomin eller trygga hållbara statsfinansier. För att försäkra sig om att dessa målsättningar uppfylls används exempelvis inom den Europeiska unionen (EU) så kallade fiskala regler – numeriska restriktioner på skuldackumulering och på vissa utgiftsposter. Speciellt inom EU kan betydelsen av fiskala regler sägas ha ökat i och med uppkomsten av valutaunionen samt skuldkrisen på 2010-talet.


Tre olika typer av regler konstrueras och analyseras. För det första används en skuldrregel där den offentliga skulden hålls konstant oberoende av exogena förändringar i ekonomin. Den andra regeln håller skulden i förhållande till BNP konstant. Den tredje regeln balanserar skuld som definierats i strukturella termer, vilket innebär att regeln beaktar den rådande konjunkturen.


Resultaten tyder på att när fiskala regler tillämpas, skiljer sig välfärds effekterna då olika skatter används för att anpassa den offentliga ekonomin vid en TFP-chock. Välfärden är mer känslig för justeringar i konsumtionsbeskattningen eftersom konsumtion påverkas i större grad än vid motsvarande justeringar i inkomstbeskattningen. Studien visar dessutom att
upprätthållandet av en konstant skuldsättningsslutleder till en procyklisk finanspolitik. Dessutom påvisas en närmast neutral BNP-utveckling vid användning av fiskala regler som håller skulden konstant. Slutsatsen är därför att användandet av fiskala regler vid TFP-chockar, vilka eliminerar fluktuationer i den strukturella balansen, framom regler som håller skuldnivån konstant bör kunna motiveras även med andra argument än de relaterade till utjämning av konjunkturer.

10.2 Bakgrund

Medlemmar av Europeiska unionen bör, i enlighet med Fördraget om Europeiska unionens funktionssätt (FEUF), driva ekonomisk politik i linje med unionens målsättningar (Council of European Union 2012). En variant av ekonomisk politik är finanspolitik, vilket innebär att staten justerar intäkter eller utgifter för att påverka ekonomin som helhet. Betydelsen av en lyckad finanspolitik har under de senaste åren ökat eftersom medlemmarna i den Ekonomiska Monetära Unionen (EMU) inte längre självständigt kan använda sig av den andra huvudvarianten av ekonomisk politik, det vill säga penningpolitik. En annan förklaring till att finanspolitiken – och speciellt fiskala regler – är av intresse, är att statsfinanserna i flera EMU-länder uppvisat stora underskott och en snabb ackumulering av nya statsskulder.


Fiskala regler som berör medlemsländerna i EU är samlade i den preventiva och i den korrigerande armen i unionens Stabilitets- och tillväxtpakt (European Commission 2016b). Reglerna inom den preventiva armen kan ytterligare indelas i regler som skall justera uppgörandet av kommande statsbudgeter samt i regler som kopplar de statliga utgifterna till konjunkturindikatorer och den vägen kan begränsa tillväxten i statliga utgifter. Den förstnämnda indelningen är central i denna avhandling eftersom den involverar offentlig skuld.


52Ett centralt koncept i detta sammanhang är fastställandet av en målsättning för statsfinanserna på medellång sikt (eng. Medium-Term Objective, MTO). Denna målsättning fastställs i strukturella termer, vilket bland annat innebär att balansen i statskassan skall sättas i perspektiv till hur väl ekonomin presterar. Därmed inkluderas potentiell produktion och produktionsgap.

10.3 Modellen


Den offentliga sektorn samlar in skatter och använder dem för olika ändamål. Skatteintäkter erhålls av löner, konsumtion, kapital, dividender och företag. Pengarna används för offentlig konsumtion, inkomsttransfereringar, arbetslöshetsunderstöd, offentliga investeringar

53Bortsett från några justeringar sammanfaller modellen i denna studie med Finansministeriets makromodell, även kallad "KOOMA".
och ränteutgifter. Därtöver kan den offentliga sektorn låna pengar. Reglerna som presenteras nedan begränsar alltså denna upplåning på olika sätt genom att justera skatter på konsumtion eller inkomster.

Trots att DSGE-modeller vanligen bygger på liknande antaganden, kan resultaten olika modeller emellan skilja sig åt. Detta kan förklaras bland annat med att man använt sig av olika antaganden vid kalibreringen. För att säkerställa att resultaten som erhålls i denna studie kan appliceras även i andra kontexter, utsätts modellen i studien för liknande fiskala regler som de som används i Forsm et al. (2009). Resultatet av jämförelsen, som presenteras i kapitel 5, är att modellen i denna studie i det stora hela genererar liknande resultat, även om modellerna mer sällan överensstämmer gällande storleken på impulsresponsen.

10.4 Metod


10.5 Implementering av fiskala regler i modellen

Tre typer av fiskala regler utvecklas i studien. Gemensamt för de tre regeltyperna är att samtliga involverar offentlig skuldsättning och påverkar skatter, antingen på konsumtion eller på inkomster.

I den första regeltypen ingår skuldregler. Med skuldregel avses här regler där skattesatsen (på konsumtion eller inkomster) automatiskt justeras så att de offentliga samfundens budget

54Innan modellen kan tas i bruk bör den även kalibreras, vilket inte varit en del av denna studie. Datortprogrammen Matlab och Iris har använts för att lösa modellen, simulera chocker och regler samt skapa figurer.
balanseras. Detta innebär att upplåningen upphör. Regeln kan även användas med mindre vikt, vilket innebär att budgeten kommer att vara nästan balanserad i varje period istället för helt balanserad.

Den andra typen av regler håller skuldsättningsgraden (skuld i förhållande till BNP) konstant över tid. Detta innebär att skulden tillåts växa endast i fall där produktionen växer och vice versa. Eftersom jämväxtsköldviktsättningssnivån är 60 procent, innebär tillämpandet av dessa regler att skuldsättningsgraden hålls konstant vid 60 procent. Detta utgör en av målsättningarna i Stabilitets- och Tillväxtspakten.


10.6 Resultat

Resultaten av simuleringar där de tre kategorierna av fiskala regler tillämpats finns avbildade i figurer 4-8. I samtliga scenarion nedan inträffar en TFP-chock som kommer att öka produktionen. Ökningen sker dock med en fördöjning på grund av friktioner i ekonomin. Av denna orsak kommer produktionsgapet att bli tillfälligt negativt innan produktionen åter stiger mot sin potentiella nivå. Viktigt i denna kontext är att skatteinkomsterna för den offentliga sektorn i initialläget kommer att förändras. I detta fall kommer de att minska, främst eftersom lönesumman till en början minskar på grund av chocken. För att hantera denna förändring i de offentliga finanserna används alltså de tre ovan nämnda regeltyperna för sig.

Vid tillämpning av skuldreglerna höjs skattesatsen tämligen kraftigt i utgångsläget då TFP-chocken inträffar eftersom den offentliga sektorn i annat fall skulle uppvisa ett underskott. De offentliga intäkterna stärks emellertid efter något år då ekonomins prestationsförmåga förbättrats (på grund av chocken). Detta leder till att skatterna sänks för att den offentliga ekonomin skall vara balanserad (det vill säga varken uppvisa ett över- eller ett underskott). I fallet med en TFP-chock kommer beskattningen totalt sätt att skärpas. Detta har en negativ inverkan på konsumtion, produktion och välfärd, i jämförelse med en situation

\[ \text{Den konjunkturkorrigerade balansen visar alltså lättare ett överskott ifall ekonomin befinner sig långt under sin potentiella produktionsnivå (negativt produktionsgap) och vice versa.} \]
där ingen regel används. Å andra sidan kommer den officiella skuldsättningen inte att öka.


10.7 Effekten av fiskala regler

Resultaten tyder på att välfärdsseffekterna är något mer volatila vid användning av konsumtionsskattejusteringar än vid inkomstskattejusteringar. Detta förklaras främst med att konsumtion, som är den viktigaste variabeln vid beräkandet av välfärd, är känsligare för justeringar i konsumtionsbeskattningsen än inkomstbeskattningsen. Då konsumtionsbeskattningsen anpassas efter reglerna kommer TFP-chocken att leda till en ökning i välfärden som är cirka 30 procent större då skuldsättningsgraden stabiliseras än då skulden hålls konstant i absoluta termer. Vid användning av inkomstbeskattningsen är motsvarande skillnad kring 20 procent. Dessutom visar resultaten att användningen av regeln som eliminerar fluktuationer i den strukturella balansen tillsammans med konsumtionsbeskattningsen yttreliga ökar välfärden med nästan 10 procent jämfört med regeln som medför en konstant skuldsättningsgrad. Här är motsvarande siffra för reglerna som fungerar via inkomstbeskattningsen cirka 8 procent. I jämförelse med en situation utan regler är välfärdsseffekterna vid användning av skuldunderläten med inkomstbeskattningsen på motsvarande sätt mindre negativa än vid användning av konsumtionsbeskattningsen. Därmed följe följande slutledning då ekonomin utspisar för en positiv TFP-chock. Ur ett välfärdsperspektiv är det mer önskvärt att tillämpa skuldunderläten med inkomstbeskattningsen och de övriga reglerna (regeln för konstant skuldsättningsgrad samt regeln som eliminerar fluktuationer i den strukturella balansen) med konsumtionsbeskattningsen.

En noggrannare granskning av inverkan på BNP av de olika reglerna finns åskådliggjort i figuren 9 och 10. Figurerna visar att regeln som genererar en konstant skuldsättningsgrad är procyklik jämfört med regeln som eliminerar fluktuationer i den strukturella balansen. Därmed skapar regeln som eliminerar fluktuationer i den strukturella balansen både en högre

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56 Denna regel motsvarar av en konstant målsättning för MTO.

57 Regeln som eliminerar fluktuationer i den strukturella balansen kan anses vara neutral, det vill säga varken kontra- eller procyklik, eftersom den via produktionsgapet beaktar konjunkturläget.

56
välfärds och en mindre procyklisk ekonomisk utveckling, än regeln med konstant skuldsättningsgrad. Kostnaden i form av ytterligare skuldsättning är begränsad främst till det första året.


Studien kan kritiseras på ett flertal punkter. För det första sker förändringar i verkligheten ofta långsammare än de i modellen. Offentligt beslutsfattande och samling av ekonomisk data tar ofta tid varmed det knappast är realistiskt att exempelvis justera skattesatser i varje kvartal. För det andra är reglerna i modellen lineära, vilket exempelvis innebär att över- och underskuldsättning behandlas på samma sätt i modellen, trots att EU-reglerna i själva verket endast skall förhindra överskuldsättning. En tredje utmaning kan finnas i sättet på vilket produktionsgapet är definierat. EU-reglerna för den strukturella balansen bygger på en mindre volatil potentiell produktionsnivå än den som använts i denna studie. Som svar på denna kritik kan konstateras att tidsaspekten i sig är av mindre intresse och att studien istället fokuserar på teoretiska händelseförlopp för en ekonomi som tillämpar gränsvärden för de fiskala reglerna.


10.8 Konklusioner

Appendices

A Further specifications of the model

This part explains some of the more technical parts of the model in Chapter 3.

A.1 Foreign trade

This part explains how the foreign trade is modelled. The structure is reminiscent of Adolfsson et al. (2007). See Figure 13 for a graphical illustration of the model, including the foreign trade part.

A.1.1 Importing sectors

In Chapter 3 it is assumed that an imported differentiated intermediate good \( m^C_t, m^I_t \) or \( m^X_t \) has been bought by the final good sector from a (so far) undefined importer to a given price \( P^{MC}_t, P^{MI}_t \) or \( P^{MX}_t \). How are these quantities and prices determined?

There are three domestic importers\(^{58}\) that buy homogeneous intermediate goods, \( m^C_t, m^I_t \) or \( m^X_t \) from abroad. The importers differentiate these goods before selling them further – on a monopolistic market – to a retailer. The retailer uses an aggregation technology and operates on a perfectly competitive market. Importantly, a share \( \varpi^C, \varpi^I \) or \( \varpi^X \) of these imported intermediate goods are bought directly in euros \( m^{eC}_t, m^{eI}_t \) or \( m^{eX}_t \) (local currency pricing LCP), whereas the rest are bought in a foreign currency (dollars) \( m^{SC}_t, m^{SI}_t \) or \( m^{SX}_t \) (producer currency pricing PCP). The \( \xi \) exchange rate is given by \( e_t \). The retailer sells its modified intermediate imported good to the final good producers at the price \( P^{MC}_t, P^{MI}_t \) or \( P^{MX}_t \) as discussed earlier. Hence, one could see the importing part as to consist of three parallel sectors. Each sector is composed by two types of firms or in total six firm types processing consumption, investment and export goods. Below follows a slightly more technical explanation of the two types of firms, namely the import retailing firms and the importing firms.

**Import retailers** The import retailers buy their input \( m^e_t \) or \( m^8_t \) to a given price \( P^{eC}_t \) or \( P^{8C}_t e_t \) from the importing firms. The retailers “produces” an aggregated import good that can be used in the final good production (see 3.4). Each retailer of intermediate consumption goods faces one the following problem\(^{59}\)

\[
\max_{m^{eC}_t, m^{8C}_t} P^{MC}_t m^C_t - (P^{eC}_t m^{eC}_t + P^{8C}_t e_t m^{8C}_t) \quad \text{s.t.} \quad m^C_t = \left( \varpi^C m^{eC}_t - \varrho^C (1 - \varpi^C) m^{eX}_t \right)^{\frac{1}{\varrho^C}},
\]

\[(A.1)\]

\(^{58}\)These three types of (branches of) firms are completely separated, but treated here together since they operate in an identical manner.

\(^{59}\)The similar problem applies to the corresponding investment and export retailer, just change each \( C \) in (A.1), (A.2) and (A.3) to \( I \) or \( X \) to get the problems for the other sectors.
where $\varphi^C$ is the elasticity of substitution between PCP and LCP goods. This gives the following demand functions for differentiated import goods:

\[
m_t^{EC} = \varphi^C \varphi^{C+1} \left( \frac{p_{t}^{EC}}{p_{t}^{MC}} \right)^{-\frac{1}{\varphi^{C+1}}} m_t^C \quad \text{and} \quad m_t^{SC} = (1 - \varphi^C) \left( \frac{p_{t}^{SC}}{p_{t}^{MC}} \right)^{-\frac{1}{\varphi^{C+1}}} m_t^C. \tag{A.2}
\]

The aggregated price index for the imported intermediate consumption, investment and export goods is determined by a combination of PCP, LCP and oil prices according to

\[
P_{t}^{MC} = \left( \varphi^C p_{t}^{EC} \varphi^C + (1 - \varphi^C) \varphi^C \right) \left( \frac{p_{t}^{SC}}{p_{t}^{C}} \right) \left( \frac{0.1}{\varphi^{C+1}} + o^C P_{t}^{O} \right), \tag{A.3}
\]

where $P_t^O$ is an exogenously determined oil price and $o^C < 1$ is a weight. The oil price is not assumed to affect the investment import price $P_t^M$, i.e. $o^I = 0$.

The aggregated import price index $P_t^M$ is just a weighted sum of $P_t^{MC}$, $P_t^{MI}$ and $P_t^{MX}$, i.e. $P_t^M = \varphi^{PMC} P_t^{MC} + \varphi^{PMI} P_t^{MI} + \varphi^{PMX} P_t^{MX}$, where $\varphi^{PMC} + \varphi^{PMI} + \varphi^{PMX} = 1$. Similarly, the total import $m_t$ is a sum of the imports from three different sectors, $m_t = m_t^C + m_t^I + m_t^X$.

The next section determines the intermediate prices $p_{t}^{EC}$ and $p_{t}^{SC}$.

**Importing firms** The importing firm operates domestically but buys its goods from abroad. In each sector, the importing firm can actually be divided into two separate importing firms, namely (i) one firm importing in euros (LCP) and (ii) another firm importing in dollars (PCP). Both firms differentiate their product and are associated with a certain degree of pricing power. The prices in both firms are subject to frictions. The possibility that firm (i) and (ii) can re-optimise their prices in a certain period is $1 - \xi^{EC}$ and $1 - \xi^{SC}$ respectively.

**Firms buying import goods in euros** If a LCP company (firm (i)) cannot optimise its price, the price changes according to the past inflation $p_{t}^{EC} = p_{t-1}^{EC} \Pi_{t-1}^{EC}$, where $\Pi_{t}^{EC} = \frac{p_{t}^{EC}}{p_{t-1}^{EC}}$.

If it can optimise its price, it is chosen such that the price is optimal given that it can no longer optimise it in the future (Calvo 1983). The maximisation problem for firm $i$ reads

\[
\max_{p_{t}^{EC}(i)} \left\{ \mathbb{E}_t \sum_{k=0}^{\infty} (\xi^{EC})^{k} \Lambda_{t,t+k} b^k \left[ \prod_{j=1}^{n} \Pi_{t+k-j}^{EC} (p_{t+k}^{EC}(i) - p_{t+k}^{EC}(i) m_t^{EC}) \right] \right\}, \tag{A.4}
\]

where $\Lambda_t$ is a stochastic discount factor (see (3.5)) and $p_{t}^{EC}$ is an exogenously given price charged by a foreign seller of the homogeneous intermediate consumer good. Hence the term $p_{t}^{EC}(i)$ should be interpreted as a nominal marginal cost. Once the demand function given by the first expression in (A.2) is substituted (and slightly modified) one can proceed

\(^{60}\)The notation in this section will again be fitted for the consumption sector, but the problem holds equally well for the other two sectors. The corresponding problems for investment and export sector can be derived identically by substituting $C$ with $I$ or $X$ in each equations (A.4), (A.5), (A.6), (A.7), (A.8) and (A.9).
by deriving the first-order condition of (A.4), which yields
\[ P_t^{EC}(i) = -\frac{1}{\delta^C} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\xi^{EC})^k \Lambda_{t,k} \beta_k (\prod_{j=1}^{n} \Pi_{t+k-j}^{EC})^{-\frac{1}{\delta^C+1}} P^*_{t+k} \epsilon_{t+k} P^{MC} \frac{1}{\delta^C+1} m_{t+k}^{EC}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\xi^{EC})^k \Lambda_{t,k} \beta_k (\prod_{j=1}^{n} \Pi_{t+k-j}^{EC})^{-\frac{1}{\delta^C+1}} P^*_{t+k} \epsilon_{t+k} P^{MC} \frac{1}{\delta^C+1} m_{t+k}^{EC}}}. \]  
(A.5)

The general price index for import goods bought in euro \( P_t^{EC} \) can be expressed as a weighted average of optimised and non-optimised prices according to
\[ P_t^{EC} = \left( \xi^{EC} (\Pi_{t-1}^{EC} P_{t-1}^{EC})^{-\delta^C} + (1 - \xi^{EC}) (P_t^{EC}(i))^{-\delta^C} \right)^{-\frac{1}{\delta^C}}, \]  
(A.6)
meaning that a New-Keynesian Phillips curve (NKPC) can be derived once (A.5) is combined with (A.6).

**Firms buying import goods in dollars** Similarly, if a PCP company (firm (ii)) cannot optimise its price, it will change it according to past inflation and exchange rate \( P_t^{SC} = P_{t-1}^{EC} \Pi_{t-1}^{SC} \epsilon_t \), where \( \Pi_t^{SC} = \frac{P_t^{SC}}{P_{t-1}^{SC}} \). However, if it can optimise its price, it will again choose it such that it is optimal given that it can no longer optimise it in the future. The maximisation problem reads this time
\[ \max_{P_t^{SC}(i)} \left\{ \mathbb{E}_t \sum_{k=0}^{\infty} (\xi^{SC})^k \Lambda_{t,k} \beta_k \left[ \prod_{j=1}^{n} \Pi_{t+k-j}^{SC} P_t^{EC}(i) \epsilon_{t+k} - P^*_{t+k} \epsilon_{t+k} m_{t+k}^{SC} \right] \right\}. \]  
(A.7)

When the demand function given by the second expression in (A.2) is substituted (and slightly modified), one can proceed by deriving the first-order condition of (A.7), which yields
\[ P_t^{SC}(i) = -\frac{1}{\delta^C} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\xi^{SC})^k \Lambda_{t,k} \beta_k (\prod_{j=1}^{n} \Pi_{t+k-j}^{EC})^{-\frac{1}{\delta^C+1}} P^*_{t+k} \epsilon_{t+k} P^{MC} \frac{1}{\delta^C+1} m_{t+k}^{SC}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\xi^{SC})^k \Lambda_{t,k} \beta_k (\prod_{j=1}^{n} \Pi_{t+k-j}^{EC})^{-\frac{1}{\delta^C+1}} P^*_{t+k} \epsilon_{t+k} P^{MC} \frac{1}{\delta^C+1} m_{t+k}^{SC}}}. \]  
(A.8)

The general price index for import goods bought in dollars \( P_t^{SC} \) can, similarly as above, be expressed as a weighted average of optimised and non-optimised prices according to
\[ P_t^{SC} \epsilon_t = \left( \xi^{SC} (\Pi_{t-1}^{SC} P_{t-1}^{SC})^{-\delta^C} + (1 - \xi^{SC}) (P_t^{SC}(i))^{-\delta^C} \right)^{-\frac{1}{\delta^C}}, \]  
(A.9)
meaning that another New-Keynesian Phillips curve (NKPC) can be derived once (A.8) is combined with (A.9).

Once the procedure above is conducted also for the investment and export sector, one will end up with in total six Phillips curves.
A.1.2 The exporting sectors

In Section 3.4 it is clarified how the export good is put together from domestic and foreign intermediate goods (see latter part of (3.11)). It is also explained how the factor demands for these intermediate goods are determined (see (3.12)). Nevertheless, it has not been explained how and to what price the final export good is exported abroad, which is the objective of this section.

The exporting sector is similar to the importing firms in Section A.1.1. Assume foreigners demand Finnish goods that are priced in euro $x^{e}_t$ and in dollar $x^{s}_t$ according to

$$x^{e}_t(i) = \left( \frac{P^{e}_t(i)}{P^*_t} \right)^{-\frac{1}{\pi^*_t}} x^*_t \quad \text{and} \quad x^{s}_t(i) = \left( \frac{P^{s}_t(i)}{P^*_t} \right)^{-\frac{1}{\pi^*_t}} x^*_t, \quad \text{(A.10)}$$

where $P^{e}_t(i)$ and $P^{s}_t(i)$ are the euro and dollar prices charged by the Finnish exporter $i$, $P^*_t$ is the foreign dollar price level and $x^*_t$ is total foreign demand for Finnish export goods. The foreign demand is exogenously given by an AR(1) process. Furthermore, let $\chi$ be the fraction of those foreigners who buy their products from Finland in euros, whereas the rest, $1 - \chi$ buy them in dollars. Hence, the aggregated price index for export products can be expressed as $P^X_t = \chi P^{e}_t + (1 - \chi) P^{s}_t$, whereas the total real export can be expressed similarly as $y^X_t = \chi x^{e}_t + (1 - \chi) x^{s}_t$.

Exporting firms The exporting firms buy export goods and differentiate them before selling them abroad. As in Section A.1.1, the exporting companies are divided into two separated firms, namely (i) a euro (PCP) and (ii) a dollar (LCP) exporting firm. Both firms differentiate their product and are associated with a certain degree of pricing power. The prices in both firms are subject to frictions. The possibility that firm (i) and (ii) can re-optimise their prices in a certain period is $1 - \xi^X$ and $1 - \xi^{Xs}$ respectively.

Firms exporting in euros If a PCP company (firm (i)) cannot optimise its price, it will change it according to past inflation $P^{e}_t = P^{e}_{t-1} \Pi^{Xe}_{t-1}$, where $\Pi^{Xe}_{t-1} = \frac{P^{e}_t}{P^{e}_{t-1}}$. If it can optimise its price, it will choose the price such that it is optimal given that the firm can no longer optimise the price in the future (Calvo, 1983). The maximisation problem reads

$$\max_{P^{e}_t(i)} \left\{ E_t \sum_{k=0}^{\infty} (\xi^{Xe})^k \Lambda_{t+k} \left[ \prod_{j=1}^{n} (\Pi^{Xe}_{t+k-j} P^{e}_{t+j} - \lambda^X_t x^{e}_{t+j}) \right] \right\}, \quad \text{(A.11)}$$

where $\Lambda_t$ is a stochastic discount factor (see (3.5)) and $\lambda^X_t$ is the marginal cost given by (3.13), because the exporting firm has the same marginal costs as the final export aggregating firms (perfect competition). When the demand function given by the first expression in (A.10) is

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61 The exogenous foreign dollar price level that emerged in Section A.1.1 is in fact assumed to be the same as the foreign price level in this section, i.e. $P^*_t = P^*_t$.

62 The input of the exporting firms is here also referred to as a "final" export good, even though differentiation still takes place at the exporting firm level.
substituted (and slightly modified) one can derive the first-order condition of (A.11), giving
\[
\mathcal{P}_t^e(i) = -1 \frac{1}{t} \sum_{k=0}^{\infty} \left( \xi \right)^k \lambda_{t+k} \beta^k \left( \prod_{j=1}^{n} \Pi_{t+j-1} \right) \frac{1}{t} \lambda_{t+k} \mathcal{P}_t^e(i) \frac{1}{t} \lambda_{t+k} \lambda^s(i).
\]
(A.12)

The general price index for imported goods bought in euro \( \mathcal{P}_t^e \) can be expressed as a weighted average of optimised and non-optimised prices according to
\[
\mathcal{P}_t^e = \left( \xi \right)^{-t} \left( \prod_{j=1}^{n} \Pi_{t+j-1} \mathcal{P}_t^e(i) \right) + \left( 1 - \xi \right) \left( \mathcal{P}_t^e(i) \right)^{-1},
\]
(A.13)
meaning that a NKPC for euro export prices can be derived once (A.12) is combined with (A.13).

**Firms exporting in dollars** The procedure for a LCP company (firm (ii)) is similar. Absence of price optimisation means \( \mathcal{P}_t^s = \mathcal{P}_t^{s^s} \Pi_{t-1}^X \), where \( \Pi_{t-1}^X = \frac{\tau^e_t}{\tau^s_t} \). Presence of optimisation possibilities means (for the same reason as explained above) that the firm
\[
\max_{\mathcal{P}_t^s} \left\{ -1 \sum_{k=0}^{\infty} \left( \xi \right)^k \lambda_{t+k} \beta^k \left( \prod_{j=1}^{n} \Pi_{t+j-1} \mathcal{P}_t^s(i) \right) - \lambda_{t+k} \lambda^s(i) \right\}.
\]
(A.14)

When the demand function given by the second expression in (A.10) is substituted (and slightly modified) one can derive the first-order condition of (A.14), yielding
\[
\mathcal{P}_t^s(i) = -1 \frac{1}{t} \sum_{k=0}^{\infty} \left( \xi \right)^k \lambda_{t+k} \beta^k \left( \prod_{j=1}^{n} \Pi_{t+j-1} \mathcal{P}_t^s(i) \right) \frac{1}{t} \lambda_{t+k} \mathcal{P}_t^s(i) \frac{1}{t} \lambda^s(i).
\]
(A.15)
The general price index for imported goods bought in dollars \( \mathcal{P}_t^{s^c} \) can again be expressed as a weighted average of optimised and non-optimised prices according to
\[
\mathcal{P}_t^s = \left( \xi \right)^{-t} \left( \prod_{j=1}^{n} \Pi_{t+j-1} \mathcal{P}_t^s(i) \right) + \left( 1 - \xi \right) \left( \mathcal{P}_t^s(i) \right)^{-1},
\]
(A.16)
meaning that yet another NKPC can be derived once (A.15) is combined with (A.16).

**B Log-linearised version of semi balanced budget rules**

The linearised versions of the rules (6.9) and (6.10) become fairly long. Their derivations are straightforward when using first-order Taylor approximation. The rules in their final form is given by
\[
\tau^w_t = \frac{g(\dot{g}_t + \ddot{P}_t^H) + (-g + i^G - TR) \ddot{P}_t + i^G(\dddot{P}_t^G + \ddot{P}_t^G) + TR \ddot{R}_t + ub^j \ddot{u}_t}{g + i^G + TR + b^j u - (mwh) \tau^SC - \tau^C d - \tau^K R^K k^p}.
\]
\[-0.6y(\bar{y} - \bar{y}_{t-1}) + (nwh)\tau^{SC}(\bar{n}tw_t\bar{h}_t) + (nwh)\tau^{SC}\bar{\tau}^{\tau^{SC}} + \tau^C c(\bar{\tau}^C + \bar{c}_t)\]
\[\frac{1}{g + i^G + TR + b^u u - (nwh)\tau^{SC} - \tau^C c - \tau^D d - \tau^K R^K K^p}\]
\[-\frac{\tau^D d(\bar{\tau}^D + \bar{d}_t) + \tau^K R^K K^p(\bar{\tau}^K + \bar{R}^K + \bar{K}^p)}{g + i^G + TR + b^u u - (nwh)\tau^{SC} - \tau^C c - \tau^D d - \tau^K R^K K^p}\]  
\[= \bar{\omega}_t - \bar{n}_t - \bar{h}_t + \varepsilon_t^w, \quad (B.1)\]

and
\[\bar{\tau}^C = \frac{g(\bar{\omega}_t + \bar{P}^H_t) + (-g - i^G - TR)\bar{P}_t + i^G(\bar{\omega}_t + \bar{P}^G_t) + TR\bar{R}_t + ub^u \bar{u}_t}{g + i^G + TR + b^u u + ib - (nwh)(\tau^W + \tau^{SC}) - \tau^D d - \tau^K R^K K^p}\]
\[-\frac{0.6y(\bar{y} - \bar{y}_{t-1}) + (nwh)(\tau^W + \tau^{SC})(\bar{n}_t\bar{w}_t\bar{h}_t) + (nwh)(\tau^W \bar{\tau}^W + \tau^{SC}\bar{\tau}^{SC})}{g + i^G + TR + b^u u + ib - (nwh)(\tau^W + \tau^{SC}) - \tau^D d - \tau^K R^K K^p}\]
\[-\psi\frac{\tau^D D(\bar{\tau}^D \bar{D}_t) + \tau^K R^K K^p(\bar{\tau}^K + \bar{R}^K + \bar{K}^p)}{g + i^G + TR + b^u u + ib - (nwh)(\tau^W + \tau^{SC}) - \tau^D d - \tau^K R^K K^p}\]  
\[-\bar{\omega}_t + \bar{c}_t + \varepsilon_t^C. \quad (B.2)\]
Bibliography


Council Regulation (EC) No (1467/97). of 7 July 1997 on speeding up and clarifying the implementation of the excessive deficit procedure.


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The model in Chapter 3. The numbers without brackets refer to chapters. The numbers in brackets refer to equations. Blue arrows indicate flows of goods or services, whereas orange arrows indicate prices or compensations.

Figure 13: Model overview.
Table 3: Model variables

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<tr>
<th>Variable</th>
<th>Description</th>
<th>Code</th>
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<td>(c_t)</td>
<td>Consumption, private</td>
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<td>(i_t)</td>
<td>Investments</td>
<td>(ip)</td>
</tr>
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<td>(m_t)</td>
<td>Imports</td>
<td>(m)</td>
</tr>
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<td>(t_b)</td>
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<tr>
<td>(y^N_t)</td>
<td>Exports</td>
<td>(x)</td>
</tr>
<tr>
<td>(y^{nit}_t)</td>
<td>Output, (n \cdot y^{nit}_t)</td>
<td>(y^{nit})</td>
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<td>(BH)</td>
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<td>(cp_fl)</td>
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<tr>
<td>(c^{LC}_t)</td>
<td>Consumption, liquidity constrained household</td>
<td>(cp_lc)</td>
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<td>(r^*_t)</td>
<td>Interest rate, foreign</td>
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<td>MRS between consumption and labour</td>
<td>(mrs)</td>
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<td>(y^{nit_roc})</td>
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<tr>
<td>(P^C_t)</td>
<td>Price, final consumption good</td>
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<td>(P^p_t)</td>
<td>Price, final investment good</td>
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<td>(A_t)</td>
<td>Shadow value of consumption</td>
<td>(\text{so_lambda})</td>
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<tr>
<td>(Q_t)</td>
<td>Tobin q, real value of installed capital</td>
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<td>Bargaining strength of workers</td>
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<tr>
<td>(n_t)</td>
<td>Employment, number of workers</td>
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<tr>
<td>(J_t)</td>
<td>Firm surplus</td>
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<td>(n_{thist})</td>
<td>Hours worked, in total</td>
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<td>(h_t)</td>
<td>Hours worked, per person</td>
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<td>(p_t)</td>
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<tr>
<td>(q_t)</td>
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<td>Unemployment</td>
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<tr>
<td>(H_t)</td>
<td>Worker’s surplus</td>
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<tr>
<td>(k_t)</td>
<td>Capital-labour ratio in production</td>
<td>(k)</td>
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<tr>
<td>(z^c_t)</td>
<td>Demand for dom. int. goods in consumption</td>
<td>(\text{oph})</td>
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<td>(z^e_t)</td>
<td>Demand for dom. int. goods in exports</td>
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<td>(z^i_t)</td>
<td>Demand for dom. int. goods in investments</td>
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<td>(\Pi^Z_t)</td>
<td>Inflation, producer price</td>
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<td>Out put of labour and capital intermediate</td>
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<sup>63</sup>Used for solving the model in Matlab.
### Imports

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<td>Demand for import goods in consumption</td>
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<td>$m_t^C$</td>
<td>Demand for import goods in exports</td>
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<td>Demand for import goods in investments</td>
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<td>Import price level</td>
<td>$pm$</td>
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<td>Oil price, $O$</td>
<td>$p_oil$</td>
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<td>$P_t^{MI}$</td>
<td>Price, imported intern. investment good</td>
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<tr>
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### Exports

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<tr>
<td>$x_t^{PC}$</td>
<td>Export volume, PCP</td>
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<td>Foreign $O$-price level, competes with exports</td>
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<td>$\lambda_t^X$</td>
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<tr>
<td>$P_t^{MX}$</td>
<td>Price, final export good</td>
<td>$pjx$</td>
</tr>
<tr>
<td>$P_t^{MX}$</td>
<td>Price, imported intern. export good</td>
<td>$pmex$</td>
</tr>
<tr>
<td>$P_t^e$</td>
<td>Price, LCP for export good</td>
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<tr>
<td>$P_t^{PC}$</td>
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### Public sector & Policy

<table>
<thead>
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<td>$B_t - B_{t-1}$</td>
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<tr>
<td>$i_t^G$</td>
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<td>$b_t^s$</td>
<td>Structural debt</td>
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<td>$\tau_t^C$</td>
<td>Tax on consumption</td>
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<td>$\tau_t^D$</td>
<td>Tax on dividends</td>
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<td>$\tau_t^W$</td>
<td>Tax on earned income [labour tax]</td>
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<tr>
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<td>Tax on investment / investment tax credit</td>
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<td>$TR_t$</td>
<td>Transfers to households</td>
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Table 4: Shock processes used in the study
Table 5: Model parameters

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<tr>
<th>Parameter</th>
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<td>Capital utilization rate</td>
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<td>( \rho^{CFL} )</td>
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<td>Depreciation of capital</td>
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<td>( \delta )</td>
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<tr>
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<td>Discount factor</td>
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<td>( d )</td>
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<td>( D )</td>
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<tr>
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<td>Unemployment benefit</td>
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<td>Unemployment, steady state</td>
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<tr>
<td>( v )</td>
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<td>( y^{PS} )</td>
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### Imports

<table>
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<tr>
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<td>$\rho^e$</td>
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### Exports

<table>
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### Public sector & Policy

<table>
<thead>
<tr>
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<td>Tax on capital, steady state</td>
<td>0.10</td>
</tr>
<tr>
<td>$\rho^rC$</td>
<td>Tax on consumption, AR coefficient</td>
<td>0.90</td>
</tr>
<tr>
<td>$\rho^rD$</td>
<td>Tax on dividends, AR coefficient</td>
<td>0.90</td>
</tr>
<tr>
<td>$\rho^rW$</td>
<td>Tax on earned income, AR coefficient</td>
<td>0.90</td>
</tr>
<tr>
<td>$\rho^rR$</td>
<td>Transfers to households, AR coefficient</td>
<td>0.90</td>
</tr>
<tr>
<td>$TR$</td>
<td>Transfers to households, steady state</td>
<td>0.07</td>
</tr>
</tbody>
</table>