Tests of a Fama-French Five-Factor Asset Pricing Model in the Nordic Stock Markets

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2017

HANKEN SCHOOL OF ECONOMICS

Department of: Finance and Statistics	**Type of work:** Thesis
Author: Toni Sundqvist	**Date:** 20.03.2017
Title of thesis: Tests of a Fama-French Five-Factor Asset Pricing Model in the Nordic Stock Markets	

Abstract:

This study investigates whether a Fama-French five-factor asset pricing model can explain average returns in the Nordic markets. Further, this study compares the five-factor asset pricing model's performance to that of a CAPM and Fama-French three-factor model. The study rejects all models' descriptions of average returns on the sample. All models perform well in explaining the average returns of portfolios sorted on *size* and *book-to-market* ratio, and portfolios sorted on *size* and *investment*. However, the models have large difficulties in explaining the average returns of portfolios sorted on *size* and *profitability*. While the five-factor model provides a more mean-variance efficient portfolio from its explanatory variables, it fails to improve the intercepts produced in regressions on three factors. Moreover, this study provides more evidence of a disappearing *size effect* and finds that small stocks in the Nordic markets generally have lower *market betas* than big stocks.

Keywords:

Asset pricing, Factor model, Fama-French, Dividend discount model, Nordic

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1 INTRODUCTION

Choosing the right stocks to include in an investment portfolio can, in a sense, be compared to sports betting. A savvy numbers runner knows how team skill, injury and team line-up measures into the odds, and hence will be able to make an educated guess on which team is likely to win a specific game. But if one team is more likely to win, the bookmaker will aim to adjust the odds so that the payout reflects the probabilities for each team winning. In a similar manner, the stock price of a company will always adjust to reflect the expected market value of the stock, leading the pursuit of higher returns to become a complicated analysis of pricing errors, risk, historical performance, and future surprises.

The behavior of stock returns lies right at the heart of any investment management problem. For as long as financial markets have existed, investors have tried to develop strategies and theories in order to predict future returns. Understanding the underlying dynamics of these returns is essential to making good investment choices, much like a professional bettor needs to understand which factors may drive the outcome of a football game. During the 1900s, a plethora of research examining the behavior of stock returns was conducted.

The research carried out in the mid-1900s resulted in many theories that are still widely used in financial practice, the most prominent being the Capital Asset Pricing Model of Sharpe (1964), Lintner (1965) and Mossin (1966), henceforth called the CAPM. While the CAPM is still the most widely accepted description for security pricing, empirical studies have repeatedly found evidence rejecting the common applications of the theory (see Basu 1977, Banz 1981, Rosenberg, Reid & Lanstein 1985, DeBondt, Thaler 1985, Bhandari 1988, Jegadeesh 1990). This contradicting evidence has driven researchers to augment the original CAPM with additional explanatory factors, and to seek completely different factors and explanations for the behavior of stock returns.

In the early 1990s, one of the most influential investigations contesting the CAPM was published by Fama and French (1992). The study rejects the *market beta* associated with the CAPM and instead finds that stock *size* and *book-to-market* (*B/M*) ratio better capture the cross-sectional variation in average stock returns. Soon after, Fama and French (1993) published a study proposing that a three-factor asset pricing model augmenting the CAPM with *size* and *book-to-market* proxies for risk might be a superior description of average returns. The findings of Fama and French (1992, 1993) along with earlier evidence against the CAPM drove people in the finance industry into a long dispute, spurring a huge amount of researchers into investigating the reasons behind these anomalies.

Subsequent analysis of stock returns divided researchers into two groups: those who believed that psychological factors were driving the anomalous behavior of returns, and those who saw the anomalies as unexplained risk. While the dispute between these two beliefs is still in progress, later studies have shown that several of the early anomalies are weak or circumstantial in explaining returns. Recent studies have found additional factors that seem to exhibit a strong relationship with average returns. Novy-Marx (2013) finds that firms with high *profitability* generate significantly higher returns than unprofitable firms. Aharoni, Grundy, and Zeng (2013) find that a statistically significant relation exists between an *investment* proxy and average returns. In the wake of these findings, Fama and French decided to expand their model.

In 2015, Fama and French published a study expanding the three-factor model with two additional factors; *profitability* and *investment*. The study finds that the five-factor model performs better than the three-factor model in explaining average returns for their sample. These results were further reinforced when Fama and French (2016b) found similar results in international model tests.

This thesis adds to the scarce amount of research conducted on the relatively new five-factor model by testing the model on out-of-sample data and by providing more insight into the behavior of small stocks whose returns have been all but neglected in earlier research.

1.1. Research problem and purpose

The first research question of this thesis is:

> Can a five-factor asset pricing model using *market premium*, *size*, *book-to-market ratio*, *profitability* and *investment* risk factors explain average returns in the Nordic markets?

The purpose of this paper is to assess the performance of a Fama and French (2015) five-factor model on out-of-sample data from the Nordic markets. While studies have already tested the model in the North American, European and Asian markets, creating factor portfolios and testing the model in the Nordic markets has been troublesome in the past, due to the lack of a sufficiently large data sample. While it is now possible to obtain Nordic data of sufficient size and for a sufficiently long time-period, these markets are still relatively young and small compared to the big players. As a consequence, the regression portfolios in this analysis is created on 4 x 4 sorts instead of the more commonly used 5 x 5 sorts in order to keep the portfolios well diversified. This matter is further discussed in Chapter 4.

Asset pricing models are simplifications of reality and finding the one "true" asset pricing model that works in every scenario is an exercise in futility. Therefore, in order to assess the performance of a model, results must be compared with alternative models. In addition to testing the explanatory power of a Fama-French five-factor model on Nordic stock data, this paper compares the five-factor model's performance with the performance of a CAPM and a Fama-French three-factor model. Hence the second and more important question this thesis will investigate is:

> Can a five-factor asset pricing model using *market premium*, *size, book-to-market*, *profitability* and *investment* risk factors explain average returns on the Nordic markets better than the CAPM and a three-factor model using *market premium*, *size* and *book-to-market* risk factors?

1.2. Limitations of the study

This study focuses on *size, book-to-market*, *profitability* and *investment*, and their relationship with average stock returns. While previously significant risk proxies from other studies are touched upon, these will not contribute to any part of the analysis made in this study.

The data used in this study is obtained from Thomson Data Stream (TDS) and Worldscope databases. The raw sample contains all available stocks for the target countries with some limitations:

1) The time frame for the study is the 31th of July 1999 to the 30th of June 2015. A suitable benchmark index and risk-free rate were not found for data prior to 1999. Prioritizing the objective to create an accurate analysis, any earlier return data is disregarded in this study.

2) Companies included in the sample are required to be classed as equity, and need to have available data concerning Stock Price, Total Return, Total Assets, Total Liabilities, Shares Outstanding and Operating Income for at least one year of the sample period.

3) The target of the analysis is the Nordic market as a whole. However, Iceland is excluded from the analysis in order to better replicate the chosen benchmark index, and because of the relatively small size of the Icelandic market.

The analysis is conducted on data from the Finnish, Swedish, Norwegian and Danish markets. Factor creation and regressions are made on a joint sample of all the collected data.

1.3. Sequence of the thesis

The sequence of the thesis is as follows. Chapter 2 presents the theoretical framework and empirical findings from research on stock price behavior. In Chapter 3, methods and results from earlier research on the five-factor model are presented. Chapter 4 describes the data used in the analysis and explains the factor and portfolio creation processes of this study. Chapter 5 specifies the tested hypotheses and Chapter 6 presents the results of the study. Finally, Chapter 7 concludes the thesis by summarizing and discussing the findings, and provides suggestions for further research.

2 THEORETICAL FRAMEWORK

2.1. The history of modern portfolio theory

The groundwork for a theory anticipating stock price behavior was laid in the mid-1900s. Up until this point investment theory consisted of more or less common principles e.g. "don't put all your eggs in one basket". Investment professionals knew that in order to invest rationally they must diversify their portfolios, and they must include some insurance against risk in their investments. Harry Markowitz (1952, 1959) consolidated these ideas with mathematics, creating a mean-variance model which would later become the foundation of modern portfolio theory.

Harry Markowitz's research is today a fundamental part of any standard finance education. The Markowitz Portfolio Selection Model illustrates the power of diversification and shows how mean-variance analysis can be used to choose an efficient portfolio from a feasible set of securities in a single period. Assume that a risk averse investor selects a portfolio which maximizes the return for a single period, and has the lowest possible return variance (or standard deviation) to obtain this return. Plotting all efficient portfolios (i.e. portfolios with highest return per variance) for a large set of securities will result in a hyperbolic line (simplified). This hyperbola is called the *efficient frontier* and is illustrated in Figure 1 on the next page. In Figure 1, the rational investor will try to get as far northwest as possible, and will therefore choose a portfolio somewhere along the *efficient frontier* which suits his or her tolerance for risk. The problem with Markowitz's model is that it requires the calculation of a variance-covariance matrix for all included assets; this was no small task in the 1950s. For this reason, Markowitz (1959) proposed that a "single index model" might be a possible solution to simplify the problem. This idea was built upon during the 1960s through several research papers, ultimately resulting in the formulation of the CAPM.

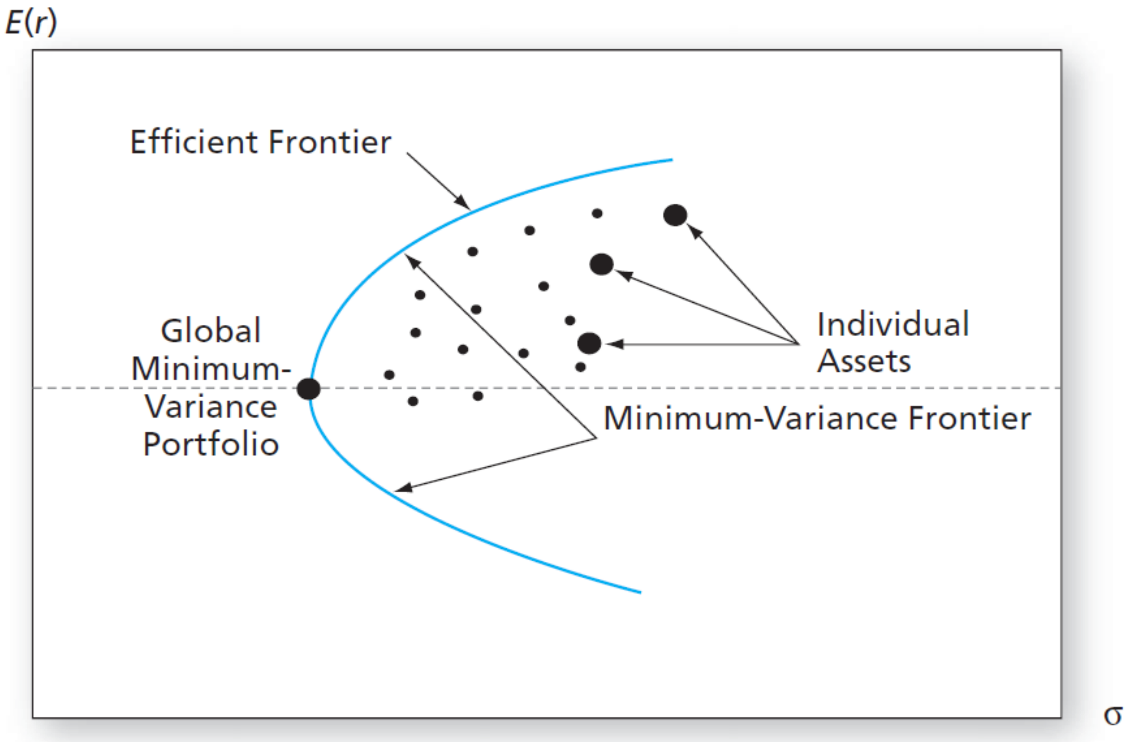

Figure 1 The minimum-variance frontier of risky assets

Source: Bodie, Kane & Marcus (2011)

Sharpe (1964), Lintner (1965) and Mossin (1966) separately conducted research into formulating a new asset pricing model. Albeit the research papers had different perspectives on the problem, they all presented a similar single index model for asset pricing; the CAPM. The CAPM builds on Markowitz's work as a one-period model including a few more assumptions: a) There exists a risk-free interest rate at which all investors can borrow or lend unlimited funds; b) Investors have homogeneous expectations; and c) Markets are in equilibrium. With these assumptions, the efficient frontier is no longer the best investment opportunity for investors. The straight line in Figure 2 on the next page, which has R_f as its intercept and is tangent to T, now represents the mean-variance efficient frontier of the opportunity set. Assuming that investors have homogenious expectations and only care about mean-variance efficiency, they will all invest in a combination of portfolio T and the risk-free asset.

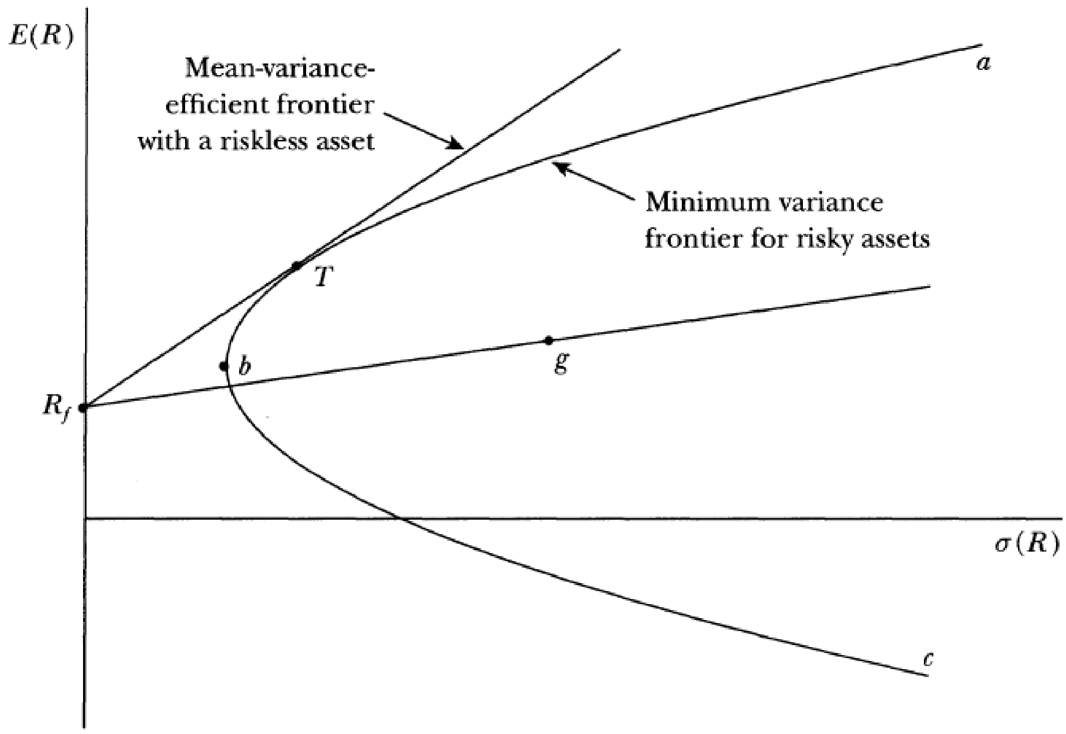

Figure 2 Investment Opportunities

Source: Fama and French (2004)

With homogeneous expectations and market equilibrium, all investors hold the same risky portfolio *T*, which must therefore include all risky assets in the opportunity set. This portfolio is the value-weight *market portfolio* of all risky assets, hereafter referred to as *M*. Based on these assumptions, the expected return of any risky asset can be expressed as a function of R_f, M, and the slope of the straight line $R_f - M$, known as the *security market line*. Hence, the CAPM equation may be expressed as,

$$E(R_i) = R_f + \beta \left[E(R_M) - R_f\right], \tag{1}$$

where $E(R_i)$ is the expected return of risky asset i, R_f is the risk-free rate, $E(R_M)$ is the expected market return, and β equals,

$$\beta = \frac{COV\ (R_i R_M)}{\sigma^2(R_M)}\ , \tag{2}$$

which may be expressed as the sensitivity of asset i's return to variation in the market return (or the covariance between the return of asset i and the market, divided by the variance of the market return), also known as the *market beta* of asset i.

The assumptions of the CAPM have been widely criticized as too strict, and researchers have since the inception of the CAPM tried to create various extensions and alternative models to improve on the CAPM's problems. A well-known alternative model is the zero-beta CAPM by Black (1972), which in effect substitutes the assumption of unlimited lending and borrowing at the risk-free rate for an assumption of unlimited short sales of risky assets. Two other important extensions to the CAPM deserve mention: the intertemporal Capital Asset Pricing Model (ICAPM) by Merton (1973), capturing the multi-period aspects of maximizing return while hedging idiosyncratic risk; and the consumption-based Capital Asset Pricing Model (CCAPM) developed by Rubinstein (1976), Lucas (1978) and Breeden (1979), creating a link between aggregate consumption and stock returns.

Further critique against the CAPM was voiced by researchers who tested the model empirically. Important arguments were made early by Black, Jensen and Scholes (1972) and Black (1972), who found that the *security market line* is flatter than predicted by the standard CAPM. This finding has been repeatedly confirmed and Baker et al. (2011) show that high-beta stocks have actually underperformed low-beta stocks in the U.S. markets since January 1968. A possible explanation for this finding is stated in a recently published paper by Hong and Sraer (2016), who claim that speculative overpricing is more often seen in assets with high *market betas*. However, the empirical findings of CAPM critics were quick opposed by skepticism when Roll (1977) argued that the CAPM is not testable unless an exact composition of the *market portfolio* is known. Since small changes to the composition of the *market portfolio* may have devastating effects on the CAPM, Roll (1977) further claimed that the only testable hypothesis is whether the *market portfolio* is mean-variance efficient.

An important response to the CAPM arrived when Stephen Ross (1976) proposed an alternative asset pricing theory: the arbitrage model of capital asset pricing, or Arbitrage Pricing Theory, hereafter referred to as the APT. Ross' aim was to relax the assumptions of the CAPM by attacking the equilibrium problem from a different perspective. The backbone of the APT is the assumption that arbitrage opportunities cannot persist in well-functioning financial markets. Thus, the APT states that arbitrage can exist momentarily, but in efficient markets the price should adjust quickly as the arbitrage opportunities are exploited. Further, the model assumes that there are n factors of systematic risk causing the variation in asset returns, and enough assets in the market to diversify away idiosyncratic risk.

Using these assumptions, the expected return of an asset can be expressed as a linear function of its sensitivity to the n factors of systematic risk:

$$E(R_i) = R_f + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \ldots + \beta_{in}\lambda_n , \tag{3}$$

where $E(R_i)$ is the expected return of risky asset i, R_f is the risk-free rate, each β_{ij} represents the sensitivity of asset i's return to risk factor j, λ_j is the risk premium for factor j, and n is the number of explanatory risk factors.

Looking at the differences between the CAPM and the APT, the latter is a much more general model. The APT does not need a market portfolio in order to estimate a return-beta relationship, but unlike the CAPM, does not identify any of its risk factors. The APT allows for individual stocks to be mispriced, and thus only applies to well-diversified portfolios unlike the CAPM. Furthermore, the APT can be extended into various multifactor models since the number of factors and the factors themselves are not identified. While the APT relaxes some of the unrealistic CAPM assumptions, the CAPM gains its popularity from its simplicity of use with broad market indices serving as market proxies. While many studies proposed extensions and alternative asset pricing models to the CAPM, other research directly attacked the CAPM's assumptions.

2.2. The efficient market hypothesis

While many theorists in the mid-1900s were trying to create models that predicted future stock prices, the related topic of efficient markets simultaneously enticed experts in the field into some very important research.

The basic assumption to consider regarding efficient markets is that markets should be rational and free of friction. The definition of an efficient market is that security prices in the market always "fully reflect" all available information (Fama 1970). As a result, investors should not be able to generate consistent risk-adjusted abnormal returns based on historical data, since prevailing prices should already reflect all historical data. This implication proposes an enigma for investors who spend time and money to analyze stock data in hopes of reaping superior investment returns. Indeed, this point was stressed by Grossman and Stiglitz (1980), who argued that it is impossible for a competitive economy to always be in equilibrium and to always be perfectly arbitraged when arbitrage is costly.

Samuelson (1965) laid the groundwork for a mathematical hypothesis arguing that stock prices should already reflect all available information and that future prices should fluctuate randomly. A consequence of this is that future price changes should be random and unpredictable; if it could be predicted, the prediction would be part of today's information. This model was built upon by Fama (1970), who distinguished among three levels of market efficiency, depending on what is meant by prices reflecting "all available information". The lowest level of market efficiency is *weak form* efficiency. For a market to be *weak form* efficient, stock prices must reflect all historical price and return data. In other words, investors cannot use historical prices or trends to predict future returns. The second level of market efficiency is the *semi-strong form*. In a *semi-strong* form efficient market, all publicly available information is fully reflected in contemporary stock prices. This includes annual report data, quality of management, earnings forecasts, stock splits etc., in addition to past prices. *Strong form* efficiency is the final level of market efficiency. Stock prices in a *strong-form* efficient market, in addition to all publicly available information, also reflect information only available to company insiders. This implies that future returns are entirely unpredictable through any available information.

In order to test the efficient market hypothesis (EMH), an asset pricing model is needed. The EMH has been widely tested, most often using the CAPM or using different multi-factor models. However, since the EMH is tested using an asset pricing model, the resulting test is a joint test of the efficient market hypothesis and the asset pricing model. Hence, if a test is rejected, the dismissal could be tied to either the EMH or the asset pricing model. Although the notion of continuously efficient markets might seem unrealistic, studies have found varying results when testing the efficient market hypothesis.

Whilst researchers were investigating the EMH and various asset pricing models, a third group of research papers started reporting inconsistent price patterns which could not possibly be explained by the CAPM.

2.3. Anomalies

In the early 1960s, a few years prior to the publication of the CAPM papers, the CRSP and Compustat databases were founded. A decade later, when the databases started to become larger and more available, researchers could finally start making reliable tests on sufficiently large samples of financial data. This is when tests of prevailing asset pricing models and model anomalies became popular areas of investigation.

Theoretical reasoning for anomalies has its foundation in the assumption that markets should be rational and friction-free. It is important to distinguish that the notion of rational markets does not imply that all participants in the market are fully rational investors. The notion is however that since market participants in general strive to benefit from mispriced opportunities, they as a collective are rational and prices will be driven toward their correct values. In rational and friction-free markets, the theoretical assumption is that assets with higher expected returns carry more risk compared to assets with lower expected returns. In order to calculate the expected return an asset pricing model is needed, for which task the CAPM was a natural choice at the time financial data started becoming widely available. In this theoretical framework, as noted by Fama and French (2008), a return pattern which cannot be explained by the chosen asset pricing model is referred to as an anomaly.

One of the early studies contradicting the predictions of the CAPM was made by Basu (1977). He found that stocks with low price/earnings (P/E) ratios yielded significantly higher returns than stocks with high P/E ratios. The results indicated that differences in *market beta* alone could not explain the observed return differences. Another early anomaly was uncovered by Banz (1981), who showed that small stocks had higher returns than could be explained by their *market beta* estimates. Basu (1983) later showed that the *size effect* persists even after controlling for the P/E effect shown in his earlier paper. Rosenberg, Ried, and Lanstein (1985) provided more evidence against the CAPM in their research on average returns for high and low *book-to-market* (*B/M*) firms. They found that firms with high *book-to-market* ratios have significantly higher returns than stocks with low *B/M* ratios. These results were later strengthened when Chan, Hamao and Lakonishok (1991) showed similar results in the Japanese market. Researchers continued to find contradictions against the CAPM in various areas, e.g. long-term return reversals (DeBondt, Thaler 1985), leverage (Bhandari 1988), and momentum (Jegadeesh 1990). Bringing together many of the previously found anomalies, Fama and French (1992) published a study finding that when controlling for *size,* the beta-return relationship in the standard CAPM disappears. The study concluded that the CAPM model simply cannot explain the previous 50 years of average stock returns, and that is if assets are priced rationally the results suggest that stock risks are multidimensional.

Behavioral interpretations contrast the theoretical explanations stating that anomalies represent the risk left unexplained by an asset pricing model. Proponents of behavioral finance believe that some anomalies arise due to violations of the assumptions that markets are rational and friction-free, and are thus unexplainable by theoretical reasoning. According to these views, irrationalities may arise because investors process information incorrectly or because despite a given probability distribution of returns, investors make inconsistent or suboptimal decisions.

An early argument for irrational investor behavior was made by Lakonishok, Shleifer, and Vishny (1994), who revisited the *value effect* found by Rosenberg, Ried, and Lanstein (1985). The authors argued that market participants have consistently overestimated future growth rates of glamour stocks (low *B/M* stocks) relative to value stocks (high *B/M* stocks). The study further found that *value* stocks appear to be no riskier relative to *glamour* stocks by conventional risk measures, and stated that the reason for a significant *value effect* is likely connected to behavioral biases or data snooping. Two additional opinions contesting the notion of rational markets were voiced by Odean (1998) and Barber and Odean (1999), arguing that investors hold losing investments too long and sell winners too early, and that investors trade more than is optimal due to overconfidence.

The second behavioral critique attacks the assumption that markets are friction-free. The critique states that due to trading frictions rational arbitrageurs cannot always fully exploit the mistakes made by irrational investors. In other words, low liquidity, restrictions on short selling, or transaction costs may limit arbitrage and thus prevent rational investors from driving the prices of mispriced assets toward their correct values.

While the CAPM is still today the most well-known method for pricing assets, extensions to the model and alternative models are being proposed continuously. Using the APT framework introduced by Ross (1976), the CAPM may be extended into a multi-factor model. Thus, when a plausible explanation for an anomaly is proposed, a researcher may be able to capture the anomaly by including an additional risk factor into the previously incomplete model. In this manner, the Fama-French three-factor model was created.

2.4. The Fama-French three-factor model

In the wake of the findings rejecting the CAPM, Fama and French (1993) proposed that a three-factor model might be better at explaining the cross-section of average stock returns. The three systematic factors included in their stock pricing model are firm *size, book-to-market* ratio and the *market premium*. Fama and French (1993) suggest that the *size* and *book-to-market* anomalies found in earlier studies may serve as proxies to absorb risk exposure not captured by the CAPM *market beta*, and thus motivate the use of return premiums associated with these factors. They proceed to propose measuring the *size* factor as the return of a well-diversified portfolio of small firms minus the return of a well-diversified portfolio of big firms. Similarly, they propose measuring the *value* factor as a high *book-to-market* portfolio minus a low *book-to-market* portfolio. Naming these factors *SMB* (small minus big) and *HML* (high minus low), they propose the following regression model to explain stock returns:

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + e_{it} \,, \tag{4}$$

where R_{it} is the return of asset i at time t, R_{ft} the risk-free return, $R_{Mt} - R_{ft}$ the excess market return, SMB_t the *size* factor and HML_t the *value* factor. The coefficients β_i, s_i and h_i are the asset's sensitivity to each of the factors, a_i is the intercept and e_{it} is the error term at time t. In accordance with Ross' (1976) arbitrage pricing theory, if these are the relevant risk factors, regression loadings on the factors should absorb the uncertainty and fully explain excess returns, bringing the intercept of the equation to zero.

Fama and French (1993) regress 25 portfolios sorted by *size* and *B/M* on the proposed model and find that the three-factor model performs well in absorbing the CAPM anomalies of their study, bringing most of the anomalous CAPM intercepts close to zero. The authors run further tests on five earnings/price sorted portfolios and five debt/price sorted portfolios yielding somewhat weaker but similar results.

Fama and French (1995) further investigate the *size* and *B/M* effects' relationships with earnings and find that small firms as well as high *B/M* firms generally exhibit lower earnings. The findings are consistent with the thesis that these stocks yield higher returns because they are riskier. In 2006, Fama and French published a paper connecting the factors of their model to financial theory.

2.5. The theoretical intuition behind the Fama and French factors

Fama and French (2006) provide a theoretical explanation to previously found CAPM anomalies by connecting them to the dividend discount model. According to the dividend discount model, the market value of a firm should equal the present value of all expected future dividends,

$$M_t = \sum_{\tau=1}^{\infty} \frac{E(D_{t+\tau})}{(1+r)^{\tau}} , \tag{5}$$

where M_t is the stock price at time t, $E(D_{t+\tau})$ is the expected dividend for the period $t + \tau$, and r is the required rate of return on expected dividends. Following the insights of Miller and Modigliani (1961), the dividend discount model can be adapted to the following valuation formula.

$$M_t = \sum_{\tau=1}^{\infty} \frac{E(Y_{t+\tau} - dB_{t+\tau})}{(1+r)^{\tau}} , \tag{6}$$

where $Y_{t+\tau}$ represents earnings in period $t + \tau$, and $dB_{t+\tau}$ is the change in *book equity* for the period $t + \tau$. Dividing the equation by *book equity*, it follows that increases in market value, and hence future stock returns, should be positively correlated with increases in B/M (or decreases in M/B as it would appear in the adjusted formula). In addition to the connection with *book-to-market* ratio, the formula provides natural explanations for two other variables. Holding all else equal, increases in earnings ($Y_{t+\tau}$) should lead to higher expected stock returns, and increases in *investment* ($dB_{t+\tau}$), should lead to lower expected returns. However, testing these implications on a per share level, Fama and French (2006) find that neither earnings nor *investment* seem to be strong proxies in predicting future stock returns. These somewhat peculiar findings did however not convince all researchers, as was evident in 2013 when two fresh publications reinvestigated the weak variables of Fama and French (2006).

2.6. More evidence on profitability and investment

The connection between *profitability* and stock returns was further investigated by Novy-Marx (2013), finding a strong relationship between *gross profitability* and stock returns. The study claims that despite being similar in philosophy, current earnings, as tested by Fama and French (2006), and *gross profitability* are highly different in terms of both characteristics and covariances. Novy-Marx (2013) claims that *gross profitability* is a better proxy than current earnings because current earnings are reduced by investments that are treated as expenses. Expensed investments, e.g. research and development or advertising, reduce earnings without increasing book value, even though associated with higher future profits, and since the earnings of Equation (6) represent *true economic profitability*, earnings should be measured before such investments are expensed. In addition to a strong *profitability* argument, Novy-Marx (2013) finds a negative correlation between *gross profitability* and *book-to-market*, implying that combining these two factors will significantly improve the performance of investment strategies focusing on either factor.

Further research into the relationship between returns and the variables of Equation (6) was conducted by Aharoni, Grundy and Zeng (2013). The study acknowledges that at the per share level, as investigated by Fama and French (2006), the *investment* factor remains small and insignificant. However, when measured at the firm level, Aharoni, Grundy and Zeng (2013) find that *profitability*, measured as income before extraordinary items, *investment*, measured as growth in Total Assets, and *B/M* ratio are all significantly correlated to returns. The authors argue that the limited success Fama and French (2006) had in finding these relationships is largely related to their measures of *profitability* and *investment*. The argument states that changes in the number of shares are likely to diminish the correlation between returns and *investment* per share, and thence the relationship is stronger when the variables are measured at a firm level. The authors further find that the results regarding *investment* are less robust when considering firms with high *book-to-market* ratios. Concluding their paper, Aharoni, Grundy and Zeng (2013) note that an important question to ask is whether an asset pricing model based on the Miller and Modigliani (1961) framework would outperform the Fama and French (1993) three-factor model in explaining stock returns. After all, even if all variables are significantly related to returns, a model using five interrelated variables does not necessarily provide a significant improvement to a more parsimonious model using only three of the factors.

2.7. A five-factor model

Following the evidence of Novy-Marx (2013) and Aharoni, Grundy and Zeng (2013), Fama and French (2015) decide to test the explanatory power of a five-factor model by including *profitability* and *investment* into their previously successful three-factor model. The study uses *operating profitability* (*OP*) as a measure for a firm's *profitability* and the change in Total Assets as a measure for *investment* (*Inv*), similar to the variable definitions used by Aharoni, Grundy and Zeng (2013). The new explanatory variables are defined in a manner similar to the risk factors of the authors' three-factor model. More specifically, the *profitability* factor is measured as the return of a well-diversified portfolio of stocks with high *operating profitability* minus the return of a well-diversified portfolio of stocks with low *operating profitability*, and is dubbed *RMW* (robust minus weak). Similarly, the *investment* factor is measured as a low-*investment* portfolio minus a high-*investment* portfolio, and is called *CMA* (conservative minus aggressive). Including the new factors to the Fama and French three-factor model results in the following regression:

$$R_{it} - R_{ft} = a_i + \beta_i\left(R_{Mt} - R_{ft}\right) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}\,, \qquad (7)$$

where R_{it} is the return of asset i at time t, R_{ft} the risk-free return, $R_{Mt} - R_{ft}$ the excess market return, SMB_t the *size* factor, HML_t the *value* factor, RMW_t the *profitability* factor, and CMA_t the *investment* factor. The coefficients β_i, s_i, h_i, r_i and c_i are the asset's sensitivity to each of the factors, a_i is the intercept and e_{it} is the error term at time t.

In short, testing the model on U.S. data as well as international samples, Fama and French (2015, 2016b) find evidence supporting that the five-factor model performs better than both the CAPM and the three-factor model in explaining returns on *Size-B/M, Size-OP* and *Size-Inv* sorted portfolios. In the upcoming chapter, methods and results from these two studies are presented in more detail.

3 PREVIOUS RESEARCH

The following chapter presents methodology and relevant findings from previous research papers focusing on similar methods to those used in the empirical part of this thesis. More specifically, the chapter discloses the results from two studies: First presented, the Fama and French (2015) study "A five-factor asset pricing model" is the first publication detailing the authors' five-factor model. The paper tests the model on U.S. data and most closely resembles the methodology used in this thesis. Second, methodology and results from Fama and French (2016b) study "International Tests of a Five-Factor Asset Pricing Model" are detailed. The latter study tests the model on international data samples with a slightly different methodology in creating factor premiums.

3.1. Fama and French (2015): "A five-factor asset pricing model"

"A five-factor asset pricing model" is the first paper presenting the model which often is referred to as the Fama-French five-factor model. The paper focuses on a U.S. sample, including all NYSE, AMEX and NASDAQ stocks on both CRSP and Compustat databases with share codes 10 or 11 and data for *size* and *book-to-market*. The share codes indicate that non-equity stock and foreign listings are removed from the sample. Further, the sample uses monthly returns for a time-period from July 1963 to December 2013, resulting in 606 months of data. The authors use a value-weight portfolio of all sample stocks as the market proxy in their research, and the U.S. one-month Treasury bill as the risk-free rate. NYSE breakpoints are used in order to create factor return premiums and left-hand-side regression portfolios.

Creating their factor returns, Fama and French (2015) use multiple methodologies in order to test how much the results are affected by the way factor premiums are defined. 2 x 3 sorted factor premiums are constructed by distributing the sample into six portfolios sorted independently on *size* and one additional variable. The sample is distributed into portfolios using the NYSE market median as *size* breakpoint, and the 30^{th} and 70^{th} NYSE percentile breakpoints for *book-to-market* (*B/M*), *operating profitability* (*OP*) and *investment* (*Inv*) premiums. Other tested variants of factor returns are created on 2 x 2 sorts on two variables, as well as 2 x 2 x 2 x 2 sorts on *size*, *B/M*, *OP*, and *Inv*, using NYSE median breakpoints for all factors.

When creating left-hand-side regression portfolios, Fama and French (2015) conduct tests using two different methods in order to distill more information regarding the five-factor model's robustness. The first collection of left-hand-side portfolios is constructed by sorting the sample into three sets of 25 portfolios, using NYSE quintile breakpoints on *size* and one other variable. Thus, the portfolios are sorted on *Size-B/M, Size-OP* and *Size-Inv* and distributed into three sets of 25 portfolios. The second collection of tests is made on three sets of 32 portfolios, sorting the sample by *size* and pairs of two other variables. The sorts are defined as 2 x 4 x 4, using the NYSE median as *size* breakpoint and NYSE quartiles for the two other variables. In this manner the sample is distributed into *Size-B/M-OP, Size-B/M-Inv* and *Size-OP-Inv* sorted portfolios resulting in three sets of 32 portfolios.

Fama and French (2015) present regression details and model comparisons of the best-performing three-, four-, and five-factor models, using all created combinations of left-hand-side portfolios and risk factors. Model comparisons are made by interpreting Gibbons, Ross, and Shanken (1989) GRS statistics, average absolute intercepts and two additional metrics derived from alpha values and return deviations. The study finds that the GRS test rejects all suggested models' descriptions of returns. However, the five-factor model along with a four-factor model dropping the *HML* factor performs best, both in regard to GRS statistics and alpha metrics on the tested sample. These results are robust regardless of how the portfolios and factors are created. In most tests, the standard three-factor model along with a few additional models follow closely behind the five-factor model in regard to all metrics except the GRS statistic. However, in tests on 32 *Size*-OP-*Inv* sorted portfolios the five-factor model and the four-factor model dropping the *HML* factor perform exceptionally well compared to other models.

Aside from finding that the five-factor model outperforms the three-factor model in tests on their U.S. sample, Fama and French (2015) report certain key findings. By conducting spanning regressions on the five explanatory variables of the sample, the authors find that the *HML* factor becomes redundant in explaining returns after the inclusion of *profitability* and *investment* factors. Furthermore, the authors find that the model's performance is robust to the way its factors are defined. The authors find however, that the five-factor model has trouble capturing average returns on firms which produce large negative *RMW* and *CMA* slopes, i.e. "stocks whose returns behave like those of firms that invest a lot despite low profitability" (Fama and French 2015).

3.2. Fama and French (2016b): "International Tests of a Five-Factor Asset Pricing Model"

In the research paper "International Tests of a Five-Factor Asset Pricing Model", Fama and French test their previously published five-factor model on out-of-sample data. Fama and French (2016b) conduct tests on four regional samples: North America, Europe, Japan and Asia Pacific. The study is conducted on data from Bloomberg, Datastream and Worldscope databases, and uses monthly returns for a time-period from July 1990 to October 2015. Similar to the U.S. study, Fama and French (2016b) construct their market factors from value-weight portfolios of full regional samples, and use a one-month U.S. Treasury bill as the risk-free rate. The methodology for creating factor returns and left-hand-side regression portfolios is similar to that of their 2015 paper with some changes.

Fama and French (2016b) create right-hand-side (RHS) factor returns in a similar way for every region in order to keep their results comparable. The RHS explanatory returns of the study are made solely on 2 x 3 sorted portfolios, since the authors had previously found that different methods in creating the factors led to similar regression results. However, the method differs from the U.S. paper in regard to breakpoints when constructing the factor returns. Fama and French (2016b) use the extreme 10th percentiles from each regional sample when constructing the *size* factor in lieu of using the NYSE median breakpoint as done in their 2015 paper. Breakpoints for all other RHS factors are the 30th and 70th percentiles as in the U.S. study. In other words, all factors are created from samples which drop all stocks lying in the middle 80% of the sample *size* distribution.

The left-hand-side regression portfolios of the Fama and French (2016b) international paper are similarly to the Fama and French (2015) study created using two styles of sorts. Tests in this paper are conducted on three sets of 25 five portfolios created from 5 x 5 sorts, as well as three sets of 32 portfolios created from 2 x 4 x 4 sorts. However, the breakpoints when distributing stocks into these portfolios differ. When sorting into 25 portfolios, *size* breakpoints for the regions in the international paper are the 3rd, 7th, 13th, and 25th percentiles of the region's aggregate *market capitalization*. The authors report that these breakpoints correspond roughly to the NYSE quintile breakpoints used in their earlier studies. Breakpoints for the other variables are obtained by distributing the biggest 10% of stocks in a region into five groups containing a similar number of stocks. When sorting into 32 portfolios, the middle 80% of *size* is dropped from the region samples creating one *size* group consisting of the largest 10% of stocks, and one group consisting of the smallest 10%. Other variables are sorted into four groups using quartile breakpoints from the remaining 20% of stocks.

In their international paper, Fama and French (2016b) use a similar methodology to Fama and French (2015) U.S. study when comparing model performance. The GRS statistic of Gibbons, Ross, and Shanken (1989) is used to test if a model can fully explain average returns for a region, and a combination of the GRS statistic and alpha metrics is used to compare model performance between three-, four-, and five-factor models.

The authors find that for the North America, Europe, and Asia Pacific samples the GRS test rejects all models descriptions of average returns. However, all models pass the GRS test for every set of portfolios created from the Japanese sample. Fama and French (2016b) report that the relationship between *B/M* and average return is so strong in the Japanese sample that intercepts from five-factor regressions are highly dependent on whether a sort involves *B/M*. The authors conclude that there is not enough return variation in the Japanese sample to challenge any of the considered models.

Further, the study finds that the five-factor model generally shows the strongest performance regarding all metrics, followed by four factor models that drop the *HML* or *RMW* factor. The three-factor model shows the weakest performance in almost every test and metric, and is especially weak in tests on sets where *profitability* is one of the sorting variables.

Fama and French (2016a) find that the results from model comparisons generally line up with intercepts found in factor spanning tests. More specifically, when the authors found a weak intercept in the factor spanning tests for a region, models including the weak factor generally performed worse. Results from factor spanning tests in the study differ widely depending on region. The authors find that all five factors are important for describing average returns in the North American sample for the time-period. In the European and Asia Pacific samples, the *Mkt* (i.e. *market premium*), *HML* and *RMW* factors are significant, while the remaining factors are found insignificant. In the Japanese sample, only *HML* and *RMW* factors are significant. In contrast to Fama and French (2015), the findings of Fama and French (2016b) find that the *HML* factor is important for describing 1990-2015 average returns in all regions.

Fama and French (2016b) also report that global versions of both a three-factor model and a five-factor model were tested but not presented in their tables. Conclusions from these tests were that both models failed to explain international returns. In order to test global versions of the five-factor model's risk premiums, 20 regressions were estimated where each regional factor (five factors per region) was regressed on all five global factors. The authors report that five out of 20 intercepts were more than three standard errors from zero, with an additional seven intercepts more than two standard errors from zero.

The next chapter presents the data sample used in the empirical part of this thesis, and details the methods used when creating risk factors and regression portfolios for the analysis.

4 DATA

4.1. The sample

The data sample used in the analysis consists of monthly Price and Total Return data downloaded from Thomson Datastream, hereafter abbreviated TDS, and accounting data from Worldscope database. The collected accounting data includes Total Assets, Total Liabilities, Shares Outstanding, and Operating Income, where Operating Income is defined as "Net Sales minus Operating Expenses", and Operating Expenses is defined as the "sum of all expenses related to operations" (Thomson Reuters 2013). Data was collected for all available active and dead stocks in the Finnish, Swedish, Norwegian, and Danish markets from 31.12.1997 to 30.06.2016. The raw downloaded sample consisted of 3434 stocks quoted in the countries' local currencies.

4.2. Filtering the sample

The downloaded sample included a large amount of stocks which were already dead at the beginning of the research period, as well as some missing data types and data errors which ought to be removed. The filtering process in this study was conducted in a manner similar the guidelines proposed by Ince and Porter (2006) in order to make the sample applicable for analysis.

The process started with downloading static constituent list data from TDS (i.e. information about the stocks included in a list). Data was downloaded for all available stocks on the Finnish, Swedish, Norwegian and Danish markets. Initially data for Iceland was also considered for the analysis, but due to the Icelandic sample consisting of only 31 stocks after the filtering process, these stocks were removed in order to instead replicate the countries represented in the chosen benchmark index, MSCI Nordic Countries Index. The constituent lists and other variables making up the raw unfiltered sample are presented in Appendix C Table C1. Next, the sample was filtered by screening the static stock information variables from TDS. This process is denoted Step 1 in Panel A of Table 1 on the next page. The variables screened were 'TYPE', 'GEOG', 'ECUR' and 'TIME' (see Appendix C Table C1). All stocks with an instrument type other than 'EQ' were first removed from the sample. This filter removed most of the investment companies and other non-equity companies whose returns stem from financing or from the performance of other companies. Subsequently, all stocks with a geographical code or currency other than expected for the country being screened were deleted. This screen removed foreign listings, i.e. stocks quoted in other currencies than the rest of a market sample. Foreign listings are companies registered in another country than the screened exchange, and were removed since they may follow different laws and regulations than domestic listings. Furthermore, in an expanded study foreign listings may show up as duplicates in several markets. Finally, the 'TIME' variable shows the timestamp of the last data update for a stock. Stocks whose data had not been updated since 31.12.1997 were removed from the sample. After the Step 1 filters, time series data for the remaining sample was downloaded.

Table 1 Sample and Filtering the Data

Number of stocks in each regional sample and each step of the filtering process. RAW indicates the original downloaded sample. Step 1 shows the number of stocks left after filtering the static variables instrument type, geographical code, currency, and timestamp for the last data update. Step 2 shows the stocks left after filtering stocks with missing data. Step 3 shows the remaining stocks after double-checking for dead stocks, and Step 4 shows the final sample after removing multiple instances of stocks. Panel A shows the number of stocks in each step of the filtering process and Panel B shows the number of stocks in each region of the final sample.

Panel A: Number of stocks in filtering stages

Constituent List	RAW	Step 1	Step 2	Step 3	Step 4
Active DK	138	132	128	127	122
Active FN	134	128	125	121	113
Active NW	165	156	153	153	150
Active SD	312	291	273	269	246
Dead DK	392	238	189	189	171
Dead FN	278	148	91	91	85
Dead NW	604	332	290	290	277
Dead SD	1411	565	378	378	356
Total	3434	1990	1627	1618	1520

Panel B: Number of stocks in final sample

Data	Nr of Stocks	Percentage
Denmark	293	19.28 %
Finland	198	13.03 %
Norway	427	28.09 %
Sweden	602	39.61 %
Total Active Stocks	631	41.51 %
Total Dead Stocks	889	58.49 %
Total Stocks	1520	100.00 %

In Steps 2 to 4 the aim was to further remove stocks whose time-series data was non-applicable for analysis. In step 2, stocks for which the download request failed to obtain time-series data for one or several variables were removed. Step 3 double-checked the data for variables that were stationary during the whole sample period, and removed stocks for which such data was found. In the same step the data was corrected for trailing ends, which is a phenomenon known to TDS data. That is, when a stock is delisted, TDS time-series data will display the last known number for every future data point even if the stock is dead. Such trailing ends were deleted from the sample. Finally, in Step 4 the sample was checked for multiple classes of stock by sorting the sample by name and comparing adjacent stocks' Shares Outstanding. For matching columns, the stock which had the largest number of return observations was saved while the other was removed from the sample. The remaining stocks composed the final research sample of 1520 stocks.

As a final step of the screening process the sample return matrix was manually cleaned of the most extreme observations which were very likely data errors, although this is hard to prove without comparison data from another database. Other possible sources for these extreme observations are merger or acquisition situations where stocks' accounting data are joined, and the individual stocks' price changes are booked using arcane methods. Worldscope and TDS databases might also fail to book these events simultaneously or similarly. The sample was cleaned of any monthly returns above 300% which had negative returns of over 100% within three months surrounding the extreme observation. Some observations in the sample with a monthly return of over 1000% were observed and removed along with surrounding months regardless of the surrounding months' returns. Any negative monthly returns of more than -100% occurring right before a stock is delisted were removed. The manual screens removed around 50 extreme observations along with returns for the months surrounding the observations. In the sorting part presented later, the stocks with removed values are dropped from the sorts for those years which have empty data. After screening the material manually, the country samples were converted into euros by dividing all variables except Shares Outstanding by their relevant exchange rates. The data was compiled into one large sample and tested for validity.

A simple correlation test was conducted to check the rough validity of the final sample. The sample was tested by creating a value-weight index for the whole sample, and testing its correlation against the benchmark index used in the regressions of this study. The sample index was weighted using Stock Price multiplied by Shares Outstanding, adjusting the stock weights in the index at the end of June each year. Observations for the market benchmark, MSCI Nordic Countries Index, were downloaded and converted into euros, after which returns for the sample index were compared against returns of the benchmark; a correlation of 0.975 was found between the final sample and MSCI Nordic Countries Index. An important insight is that a correlation test does not reveal problems which might exist in the smaller stock groups. Such problems are however very hard to test for without a suitable small cap comparison index. Still, a high correlation indicated that the sample was roughly accurate to describe the Nordic markets during the sample period, and was thus declared sufficient for analysis. The next section discusses the process of constructing factor premiums for the analysis.

4.3. Constructing the Fama French Factors

The factors used in the analysis were constructed in a manner similar to the process described in Fama and French (2015), but relying solely on 2 x 3 sorts for creating the factors. This approach was chosen because it is the most common method in addition to the fact that Fama and French found no differences in model performance when testing different sorting methods in their 2015 research paper.

4.3.1. Variable definitions

This subsection defines the variables needed in the factor creation process.

Market capitalization or *market cap* was used as a measure of *size* for each stock and was calculated by multiplying the Price (P) at 31st of December each year with Shares Outstanding at 31st of December for the same year. The Price data was obtained from TDS and Shares Outstanding from Worldscope.

Book equity was calculated as yearly Total Assets minus Total Liabilities from Worldscope.

Book-to-market ratio (*B/M*) was calculated from the previous two variables by dividing *book equity* by *market cap*.

Operating profitability (*OP*) was calculated by dividing Operating Income by *book equity*. This shortcut approach was chosen due to different accounting practices leading to scarce Worldscope data for a singular calculated approach from either EBIT or Net Sales. Also, this measure was chosen over unadjusted earnings numbers like EBIT because e.g. EBIT may place stocks into wrong portfolios as a result of one-time expenses not related to operations. In addition to these facts, Novy-Marx (2013) also argued against using measures which may contain expensed investments.

Finally, *investment* (*Inv*) was calculated as Total Assets$_{t-1}$ minus Total Assets$_{t-2}$, and dividing the result by Total Assets$_{t-2}$.

Monthly returns for stocks, *market premium* and risk-free rate were all calculated as arithmetic returns from the assets' TDS Total Return indices (RI), i.e. RI_t minus RI_{t-1} divided by RI_{t-1}. Using returns for the Total Return index implies that dividends are reinvested.

In the next subsection the sorting process for creating the factor portfolios is presented.

4.3.2. The sorting process and factor construction

The next step in the analysis was to create sorted portfolios from which the Fama and French factor return series could be calculated. Since the first downloaded data point was at 31st of December 1997 and the *investment* variable is calculated from two years of accounting data, the first available year of accounting data used in the sorting process was at the end of fiscal year 1998. The portfolios were sorted at the end of June each year, and therefore the first available return observation in the final analysis is the return of July 1999, sorted according to accounting data at the end of fiscal year 1998. Thus, the time period for the actual analysis is July 1999 to June 2015, or 192 months of return data.

The sorting process begun by defining yearly breakpoints for *size, book-to-market*, *operating profitability* and *investment* variables. The yearly sample *market cap* median was used as the breakpoint for *size*. This is akin to the approach used by Fama and French (2015) as opposed to the authors' earlier approach of using 10th and 90th percentiles as breakpoints for *size*. The former method was chosen in order to retain a larger part of the sample within the sorting portfolios from which the factors are created. For all other factors, yearly sample 30th and 70th percentiles were used as breakpoints in the sorting method.

Having obtained the breakpoints, the stocks in the sample were independently distributed for every year into six *Size-B/M* portfolios, six *Size-OP* portfolios and six *Size-Inv* portfolios created from the intersections of the yearly breakpoints. The portfolios were thereafter value-weighted according to their *market cap* and monthly returns were calculated for each of the 18 portfolios. After calculating the sorted portfolio returns, the actual factor returns were calculated.

Factor returns were calculated from sorted portfolio returns obtained in the previous step. From the six *Size-B/M* sorted portfolios, the *value factor* (*HML*) was calculated by subtracting the monthly mean returns of the two portfolios with low *book-to-market* ratio (70th percentile) from the monthly mean returns of the two portfolios with high *B/M*. The neutral (mid 40%) portfolios were ignored when calculating the factors with 30th and 70th percentile breakpoints. *Profitability* (*RMW*) and *investment* (*CMA*) factors were calculated in a similar manner from their corresponding sorted portfolio returns, keeping in mind that *CMA* is calculated as low *investment* minus high *investment* in contrast to the other factors.

The *size* factor was calculated as a mean from all three 2 x 3 sorts. From the *Size-B/M* sorted portfolios, the mean of all big portfolios' returns were subtracted from the mean of all small portfolios' returns. This process was repeated for the two other sorts and thus, three partial *SMB* factors were created. The mean of these factors comprised the final *SMB* factor used in the regressions. Table 2 presents the breakpoints and components, as well as the calculations used in the factor creation process.

Table 2 Construction of *size, B/M, OP,* and *Inv* factors

Independent sorts are used to distribute stocks into two *size* groups, and three *book-to-market* (*B/M*), *operating profitability* (*OP*), and *investment* (*Inv*) groups. The resulting groups are the components used when creating the risk factors, and are labeled with two letters. The first letter describes the *size* group, small (*S*) or big (*B*). The second letter describes the *B/M* group, high (*H*), neutral (*N*), or low (*L*), the *OP* group, robust (*R*), neutral (*N*), or weak (*W*), or the *Inv* group, conservative (*C*), neutral (*N*), or aggressive (*A*). Stocks in each component are value-weighted to calculate the component's monthly returns.

Breakpoints	Factors and their components
Size : Yearly sample median	$SMB_{B/M} = (SH + SN + SL) / 3 - (BH + BN + BL) / 3$
	$SMB_{OP} = (SR + SN + SW) / 3 - (BR + BN + BW) / 3$
	$SMB_{Inv} = (SC + SN + SA) / 3 - (BC + BN + BA) / 3$
	$SMB = (SMB_{B/M} + SMB_{OP} + SMB_{Inv}) / 3$
B/M: Yearly sample 30th and 70th percentiles	$HML = (SH + BH) / 2 - (SL + BL) / 2 = [(SH + BH) - (SL + BL)] / 2$
OP: Yearly sample 30th and 70th percentiles	$RMW = (SR + BR) / 2 - (SW + BW) / 2 = [(SR + BR) - (SW + BW)] / 2$
Inv : Yearly sample 30th and 70th percentiles	$HML = (SC + BC) / 2 - (SA + BA) / 2 = [(SC + BC) - (SA + BA)] / 2$

4.3.2.1. Additional cleaning of extreme sorting values

During the calculation of sorted portfolio returns, many irrational characteristics-based values were observed, and additional filters had to be introduced in order to control certain extreme characteristics. All following filters remove a specific stock for only those years in which unsatisfactory characteristics are found.

In the yearly sorting process, stocks with a Price of less than 1€ on any month of a given year were excluded from the yearly sort. This procedure possibly introduces a small survivorship bias since stocks are excluded the year in which they are delisted, but was done since the prices behaved erratically and were prone to data errors in the months leading up to the delisting. A secondary reason for this filter was to further reduce rounding errors stemming from the fact that TDS reports data with a precision of only two decimals.

Stocks with a *market cap* of less than 1 million euros were also excluded from yearly sorts. The *market cap* threshold of 1 million euros was introduced since it is the minimum *market cap* requirement for listing on a Nasdaq Nordic market (NASDAQ 2017), and smaller stocks are not likely to follow common risk patterns. As a *size* comparison it is worth mentioning that a stock must have publicly traded shares worth at least 40 million dollars to list on the New York Stock Exchange, illustrating the *size* difference between a small stock in the Nordic markets versus a small stock on the New York Stock Exchange (NYSE 2017).

Negative *book-to-market* values were removed since these not only were likely troubled companies or data errors, but also because a negative *book equity* value switches the sign for the *profitability* factor. *Profitability* was filtered very roughly by removing companies with more than plus-minus three times *book equity* from a yearly sort. This roughly corresponds removing observations where a company would report a return on equity of more than ±300%. A company was similarly excluded from a yearly sort if an *investment* value of less than -0.5 or more than 1.0 was observed. This would imply that the company in question lost half its assets, or more than doubled its assets in the given year, which seems very unlikely during normal recurring circumstances.

The filters applied in the sorting phase further removed approximately 100 stocks per year from the sample. The next section describes the left-hand-side portfolios used in the regressions, as well as the market benchmark and risk-free rate of the analysis.

4.4. Regression portfolios and other data used in the regression analysis

Regressions were run on three sets of 16 left-hand-side regression portfolios. Monthly returns for the portfolios were calculated in a manner similar to constructing the factor portfolios; value-weight portfolios were created from independent 4 x 4 sorts with 25^{th}, 50^{th} and 75^{th} yearly sample percentiles as breakpoints for both sorting variables. In this fashion, the whole sample was divided into 16 portfolios for each sort, sorting once by *Size-B/M,* once by *Size-OP* and finally by *Size-Inv*. Thus, regressions were run on 48 portfolios for each of the three models, testing the CAPM, a three-factor model using *SMB* and *HML* in addition to the *market premium*, and a five-factor model adding *RMW* and *CMA* factors to the three-factor model. The choice of using 4 x 4 sorts instead of more commonly used 5 x 5 sorts was made to keep the regression portfolios more diversified.

Table 3 Average number of stocks in regression portfolios and yearly sorts

Average number of stocks for portfolios formed on *size* and *B/M, size* and *OP, size* and *Inv,* and average yearly number of stocks in the distributed sample; July 1999 – June 2015. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Similarly, stocks are allocated independently into four *B/M* groups using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-B/M* portfolios. In the sort for June of year t, *B* is *book equity* at the end of fiscal year t – 1 and *M* is *market cap* at the end of December year t – 1, calculated as Price multiplied by Common Shares Outstanding at the end of December year t – 1. *Size-OP* and *Size-Inv* portfolios are formed in a similar way, but the second sorting variable is *operating profitability* or *investment*. *Operating profitability*, *OP,* in the sort for June of year t is Operating Income at the end of fiscal year t – 1 divided by *book equity* at the end of fiscal year t – 1. *Investment*, *Inv,* is the change in Total Assets from the end of fiscal year t – 2 to the end of fiscal year t – 1, divided by t – 2 Total Assets. The table shows the average yearly number of stocks in each group, as well as the total yearly number of stocks distributed.

Panel A: Size-B/M Portfolios

	Low	2	3	High
Small	18	26	41	59
2	32	38	36	38
3	43	39	33	28
Big	50	41	34	19

Panel B: Size-OP Portfolios

	Low	2	3	High
Small	61	37	27	19
2	40	38	35	31
3	28	33	40	43
Big	15	36	42	51

Panel C: Size-Inv Portfolios

	Low	2	3	High
Small	47	35	30	32
2	39	34	34	37
3	28	34	38	43
Big	29	41	42	32

Panel D: Yearly number of stocks

Year	Number of Stocks
1999	616
2000	633
2001	581
2002	625
2003	535
2004	577
2005	587
2006	591
2007	597
2008	631
2009	539
2010	585
2011	586
2012	539
2013	524
2014	516
2015	515
Mean:	575.12

Panels A through C in Table 3 show the average number of stocks in each of the regression portfolios. The aim was to keep at least 20-30 stocks in each portfolio for sufficient diversification of idiosyncratic risk; this target is met for all except four portfolios in the sample. It is evident from Panel A that high *B/M* companies are often smaller companies, while low *B/M* is tilted towards bigger companies. A similar phenomenon can be observed in Panel B, where low *operating profitability* is strongly tilted towards smaller companies, and high *operating profitability* is more often found in stocks with higher *market capitalizations*. Panel D shows the total number of stocks in the yearly sorts.

In order to run model regressions on these portfolios, proxies for market returns and risk-free returns are needed. MSCI Nordic Countries Index was used as the market benchmark of this study. This benchmark was chosen since it was the only available index which had data for the whole time-period and was constructed from stocks in the targeted markets. More specifically, the index is constructed from large- and mid-cap companies across different industries in the target markets and has 70 constituents at the time of writing (MSCI 2017). The index was quoted in USD and thus had to be converted into EUR for the analysis. The TDS Total Return index (RI) for a 1-month Euribor rate was used as a proxy for the risk-free rate. Monthly returns of the chosen risk-free rate were subtracted from monthly returns of the market benchmark and regression portfolios for the regression analysis. The upcoming section presents descriptive statistics for variables used in the empirical part of this study.

4.5. Descriptive statistics

This section discloses descriptive statistics for the regression portfolios and explanatory factors used in the regressions. First, regression portfolio statistics are presented, showing mean returns and standard deviations of the portfolios. Second, the right-hand-side explanatory factors are discussed, again looking at mean returns and standard deviations.

4.5.1. Regression portfolios

The goal of this study is to investigate how well the targeted regression models can explain average excess returns on portfolios with large differences in constituent *size, B/M, profitability* and *investment*. In this subsection the average return patterns between such portfolios are examined. The return distributions of this study reveal that big portfolios tend to benefit from high *book-to-market* ratios and low *investment* ratios, while small portfolios show a weak relationship between returns and the same ratios. Conversely, small portfolios tend to benefit from high *profitability* while the effect is weak on big portfolios, with the exception of one portfolio. This portfolio, positioned in the largest *size* quartile and lowest *profitability* quartile of the study, shows the highest average return of all portfolios in the *Size-OP* sorted sub-sample. A detailed discussion of average portfolio returns for the sample follows.

Table 4 Average excess returns and standard deviations of regression portfolios

Average monthly percent excess returns and standard deviations for value-weight portfolios formed on *size* and *B/M, size* and *OP, size* and Inv; July 1999 – June 2015, 192 months. At the end of June each year t, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are similarly allocated independently to four *B/M* groups, four *OP* groups, and four *Inv* groups using sample quartile breakpoints. The intersections of the sorts produce 16 value-weight portfolios for each sorted pair of variables. The table shows average monthly returns in excess to the one-month Euribor rate.

	Excess return					Standard deviation			
Panel A: Size-B/M Portfolios									
	Low	2	3	High		Low	2	3	High
Small	0.62	0.45	0.38	0.36	Small	8.06	5.33	4.15	4.44
2	0.09	0.65	0.53	0.42	2	7.53	5.52	4.77	5.21
3	0.43	0.88	0.82	0.81	3	6.72	5.40	5.41	5.69
Big	0.27	0.65	0.95	1.03	Big	7.08	5.39	5.78	6.42
Panel B: Size-OP Portfolios									
	Low	2	3	High		Low	2	3	High
Small	-0.09	0.48	1.04	0.90	Small	5.61	3.94	4.04	5.19
2	-0.30	0.47	0.79	0.92	2	7.05	4.73	5.00	5.10
3	0.23	0.84	0.75	0.85	3	6.96	5.22	5.17	5.96
Big	1.02	0.51	0.36	0.61	Big	9.88	5.86	6.37	7.06
Panel C: Size-Inv Portfolios									
	Low	2	3	High		Low	2	3	High
Small	0.47	0.38	0.60	0.21	Small	5.32	4.21	4.56	5.73
2	0.30	0.76	0.59	0.07	2	6.26	4.66	4.75	6.34
3	0.86	0.99	0.72	0.41	3	5.88	4.96	5.57	6.42
Big	1.08	0.92	0.13	0.38	Big	7.74	5.57	6.75	7.18

Panel A of Table 4 shows average monthly excess returns and the standard deviations of 16 value-weight portfolios, sorted independently into four *size* groups and four *B/M* groups (4 x 4 sorts). The portfolio returns of Panel A tell a very different story compared to the data observed by Fama and French (2015) in the U.S. markets. The two highest *B/M* columns show an inverse *size* distribution, with bigger portfolios showing higher returns than small portfolios, while the two lower *B/M* columns do not seem to follow any clear pattern. The *value effect* is strong in the biggest row of stocks and weak in the three smaller rows, the smallest row indicating a peculiar reverse *value* premium. However, disregarding the smallest row, a sharp return rise between the first and second *B/M* quartile can be observed. Comparing with Appendix B Table B1 this return rise is observed in conjunction with the portfolios' average *B/M* rising from approximately 0.23 to 0.53. Looking at the standard deviations in Panel A it is also evident that portfolios in the lowest *B/M* column have higher standard deviations than portfolios in other columns. Fama and French have found the lowest *B/M* column to often be a problem in their regression models (Fama and French 1993, 2015, 2016b). The relatively higher standard deviations of the top left portfolio (8.06) and bottom right portfolio (6.42) can also possibly be connected to the fact that these portfolios contain a lower number of stocks. However, unless the stocks in these portfolios are all highly correlated, the observation indicates that the stocks in these portfolios are generally more volatile.

In Panel B of Table 4 average returns and standard deviations of *Size-OP* sorted portfolios are presented. The return patterns of the portfolios in Panel B are very different to Panel A regarding the relationship between *size* and portfolio average return. The two highest *OP*-columns indicate a normal linear *size effect*, with smaller stocks generally yielding higher average returns. No clear patterns are seen in the second *OP*-column, but the lowest *OP*-column indicates a return premium on bigger stocks, reversing the *size effect* seen in the higher *profitability* columns. The biggest portfolio of the lowest *OP*-column stands out in particular, with the highest average monthly return in the sort (1.02 percent) and an abnormally high standard deviation (9.88). A strong *profitability effect* can be observed in the smaller rows, while the effect is reversed in the biggest row of stocks. If the abnormal return of the bottom-left portfolio is disregarded, no clear *profitability effect* can be discerned in the biggest row of stocks. A sharp decline in the returns can be observed between columns one, two and three in the two smallest rows and between columns one and two in the third row. Again, a comparison with Appendix B Table B1 shows that this decline happens when *profitability*, which can be roughly compared to return on equity, approaches zero. The exception is the biggest portfolio in the first column, which has negative average *profitability* but the highest return of all the portfolios in the sort.

Panel C of Table 4 displays average returns and standard deviations of *Size-Inv* sorted portfolios. The return patterns of Panel C resemble those of Panel A in that the *size effect* is mostly reversed and the *investment effect* is stronger in bigger portfolios. Portfolios in the two lowest *investment* quartiles indicate a relatively strong inverse *size effect* while no strong patterns are seen in the higher *investment* quartiles. A strong *investment* premium can be observed for bigger stocks while the patterns are weak in the smaller rows. Fama and French (2016b) reported a sharp drop in returns in their highest *investment* column, which is also seen in the three smallest rows of Panel C. The drop happens between columns two and three on the last row, which indicates that bigger stocks are penalized more for investing aggressively in this sample.

The next subsection describes average return patterns of the study's explanatory regression variables.

4.5.2. Risk factors

The following subsection presents summary statistics for the right-hand-side factor returns. The mean return and standard deviation of the *market premium* is similar to the same statistics of the average left-hand-side regression portfolio. The *size* premium is slightly negative for the sample but not significantly distinguishable from zero. *HML*, *RMW* and *CMA* factors are relatively strong but weaker and more volatile than what has been observed on other markets by Fama and French (2015, 2016b). *Size* components of the *HML* and *CMA* factors show stronger premiums for big stocks while the components of *RMW* indicate that the *profitability effect* is stronger for small stocks. The correlation matrix shows quite strong relationships between all the factors of the three-factor model (i.e. $R_M - R_f$, *SMB* and *HML*), and a strong correlation between the two factors, *RMW* and *CMA*, added in the expanded five-factor model. The reason for these two correlated factor groups is logical, since *RMW* and *CMA* are values stemming from the results over a fiscal year, while all other factors describe the situation on the last day of a fiscal year.

Panel A of Table 5 on the next page shows average returns, standard deviations and t-statistics for the factor returns. The average equity premium ($R_M - R_f$) is 0.55 percent per month with a standard deviation of 6.70 ($t = 1.17$). These statistics resemble the average left-hand-side regression portfolio which has a calculated average return of 0.57 percent per month and a standard deviation of 5.79. Fama and French (2016b) observe similar means with slightly lower standard deviations in the European and North American markets, and a slightly higher mean with a similar standard deviation in the Asian Pacific market.

The average return of the *SMB* premium shown in Table 5 Panel A is negative for the sample (mean = -0.20) but not significantly different from zero ($t = -0.88$). Fama and French (2015, 2016b) along with other research has found that the *size effect* has been disappearing in more recent years. Similar evidence regarding the *size effect* is found in this study. There is however a difference in methodology, since this study uses, as previously mentioned, the 50th percentile sample *size* breakpoint when constructing the *SMB* factor, while Fama and French have used extreme 10th percentile sample *size* breakpoints (smallest ten percent and biggest ten percent) when creating the factors in most of their published work. The *SMB* standard deviation is similar to what is found in by Fama and French (2015, 2016b).

Table 5 Summary statistics for monthly factor returns

$R_M - R_f$ is the monthly return on the MSCI Nordic Countries Index minus the one-month Euribor rate. At the end of June each year, stocks are allocated to two *size* groups using the sample median breakpoint. Stocks are also independently distributed into three *book-to-market* (*B/M*), *operating profitability* (*OP*), and *investment* (*Inv*) groups, using 30th and 70th sample percentile breakpoints. In Panel A, the *B/M* factor, *HML*, uses six value-weight (VW) portfolios from the intersections of the *size* and *B/M* sorts, and the *profitability* and *investment* factors, *RMW* and *CMA*, use VW portfolios from the intersections of the *size* and *OP* or *Inv* sorts. HML_b is the average return on the portfolio of big high *B/M* stocks minus the average return on the portfolio of big low *B/M* stocks, HML_s is constructed similarly from portfolios of small stocks, *HML* is the average of HML_s and HML_b, and HML_{s-b} is the difference between them. RMW_s, RMW_b, *RMW*, and RMW_{s-b} and CMA_s, CMA_b, *CMA*, and CMA_{s-b} are defined similarly, but using high and low *OP* or *Inv* instead of *B/M*. *SMB* is the average return on the nine portfolios of small stocks minus the average return on the nine portfolios of big stocks. Panel A and B of the table shows average monthly returns (Mean), standard deviations of monthly returns (Std dev.) and t-statistics for the average returns. Panel C shows the correlations between each factor. Bolded t-statistics indicate significance at the 5% level.

Panel A: Averages, standard deviations, and t-statistics for monthly returns

	$R_M - R_f$	*SMB*	*HML*	*RMW*	*CMA*
Mean	0.55	-0.20	0.46	0.55	0.43
Std dev.	6.70	3.21	4.34	4.31	3.81
t-Statistic	1.17	-0.88	1.51	1.82	1.60

Panel B: Small and Big components of factor returns

	HML_s	HML_b	HML_{s-b}
Mean	0.19	0.73	-0.53
Std dev.	4.74	5.52	5.51
t-Statistic	0.59	1.88	-1.38

	RMW_s	RMW_b	RMW_{s-b}
Mean	1.08	0.01	1.07
Std dev.	3.44	6.56	5.94
t-Statistic	**4.50**	0.03	**2.57**

	CMA_s	CMA_b	CMA_{s-b}
Mean	0.21	0.64	-0.43
Std dev.	3.01	6.04	5.73
t-Statistic	1.01	1.52	-1.07

Panel C: Correlations between different factors

	$R_M - R_f$	*SMB*	*HML*	*RMW*	*CMA*
$R_M - R_f$	1.00	-0.54	-0.48	-0.13	-0.19
SMB	-0.54	1.00	0.39	0.11	-0.09
HML	-0.48	0.39	1.00	0.08	0.23
RMW	-0.13	0.11	0.08	1.00	-0.54
CMA	-0.19	-0.09	0.23	-0.54	1.00

The *HML*, *RMW*, and *CMA* premiums found in this study are generally weaker and have a higher standard deviation compared to Fama and French (2015, 2016b) premiums. There are likely several reasons for these differences. Currency fluctuations, smaller market size, a focus on smaller stocks, and exposure to unknown macro-economic variables may all adversely affect the factor returns. The currency fluctuations alone may also dilute portfolio returns, resulting in weaker factor premiums.

The *value* premium for the Nordic market is positive (t = 1.51) but not significantly different from zero, with a mean monthly return of 0.46 percent and a standard deviation of 4.34 percent. As a comparison, Fama and French (2016b) found large *value* premiums (average *HML* returns) in all sample markets, with only North America having a *value* premium which was less than two standard errors from zero (t = 1.13). The standard deviation of this thesis' *HML* factor is slightly higher than reported by Fama and French (2015, 2016b), who reported standard deviations around three percent. The *profitability* premium (average *RMW* return) is quite strong (mean = 0.55) for the sample with a t-value of 1.82. The standard deviation of 4.31 is again slightly higher than seen in previous research papers. The *investment* premium has a mean return of 0.43 percent (t = 1.60) with a standard deviation of 3.81.

Panel B of Table 5 presents summary statistics for the small and big components of the *HML*, *RMW* and *CMA* factor premiums. *HML* and *CMA* premiums are larger for big stocks. This observation lines up with the average return patterns seen in Table 4, where strong and linear premiums are present for big stocks in both cases. *Value* premiums and *investment* premiums for small stocks are weak with t-values for HML_s and CMA_s of 0.59 and 1.01 respectively while the corresponding premiums are larger but not two standard errors from zero for big stocks (t = 1.88 and 1.52 respectively). These statistics contradict earlier evidence by Fama and French (2015, 2016b), who have consistently found stronger premiums associated with small factor components. The *profitability* premium is contrastingly very strong for small stocks and non-existent for bigger stocks (t-values for RMW_s and RMW_b are 4.50 and 0.03). The reason for the weak RMW_b component is strongly associated with the abnormally high return of the bottom-left portfolio in Table 4 Panel B. For all of the three premiums a considerably higher standard deviation is found for big stocks.

Panel C of Table 5 shows the correlation matrix for each set of factors. Most of the estimates are more than two standard errors from zero and the *market premium* has a negative correlation with all other factors. Negative correlations with the *market premium* suggest that diversified investment strategies focusing on any factor with a positive mean return should be able to produce higher returns without added variance compared to the overall market. Further, a large negative correlation between the *SMB* factor and the *market premium* is observed, implying that small stocks generally have lower *market betas* than big stocks. This result goes directly against the evidence found by Fama and French (2015), and can be confirmed by comparing *market betas* from

CAPM regressions found in Appendix A Tables A1, A3 and A5. The connection between the *market beta* and the correlation coefficient stems from the fact that value-weight portfolios of big stocks tend to closely track the market (since the market index is also value-weighted). As a result, the *SMB* premium is strongly tied to the difference between returns on small-stock portfolios and the *market premium*.

Further examination of Panel C of Table 5 shows high correlations between the *market premium*, the *size* factor (*SMB*) and the v*alue* factor (*HML*). Correlations with the *market premium* are negative while the correlation between *SMB* and *HML* is strongly positive. *RMW* and *CMA* factors show large negative correlation with each other but have low correlation with the remaining factors. These results seem logical since the *RMW* and *CMA* factors are tied to the previous year income result (assuming a constant debt/equity ratio), while the remaining factors are based on the situation at the end of the previous fiscal year (*book equity* and *market cap*). The high negative correlation also suggests that an investment strategy focused solely on *operating profitability* is improved by controlling for *investment*. Next, factor spanning tests for the risk factors are presented to further detail possibly redundant factors and to discuss multicollinearity between risk factors.

4.5.2.1. Factor spanning regressions

Factor spanning regressions are a means to test if an explanatory factor can be explained by a combination of other explanatory factors. Spanning tests are performed by regressing returns of one factor against the returns of all other factors, and analyzing the intercepts from that regression.

Table 6 on the next page shows regressions for the five-factor model's explanatory variables, where four factors explain returns on the fifth. In the $R_M - R_f$ regressions, the intercept is strongly positive at 0.94 percent per month (t = 2.56). Regressions to explain *RMW* and *CMA* factors are also both strongly positive and more than three standard errors from zero at 0.83% (t = 3.36) and 0.63% (t = 3.06) per month respectively. However, regressions to explain *SMB* and *HML* factors both show insignificant intercepts, with intercepts of 0.04% (t = 0.21) and 0.37% (t = 1.38) respectively. In accordance with Huberman and Kandel (1987), these results suggest that removing either the *SMB* or *HML* factor would not hurt the mean-variance-efficient tangency portfolio produced by combining the remaining four factors. Implications from these results are that because of multicollinearity between factor returns, the *HML* and *SMB* factors do not help the five-factor model's description of average returns over the period.

Table 6 Factor spanning regressions on five factors

Spanning regressions using four factors to explain average returns on the fifth; July 1999 – June 2015, 192 months. $R_M - R_f$ is the value-weight return on the MSCI Nordic Markets Index minus the one-month Euribor rate. *SMB* is the *size* factor; *HML* is the *value* factor; *RMW* is the *profitability* factor; and *CMA* is the *investment* factor. The factors are constructed using individual sorts of stocks into two *size* groups and three *B/M* groups, three *OP* groups, or three *Inv* groups. *Int* is the regression intercept. Bolded t-statistics indicate significance at the 5% level.

	Int	$R_M - R_f$	*SMB*	*HML*	*RMW*	*CMA*	R^2
$R_M - R_f$							
Coefficient	0.94		-0.96	-0.32	-0.36	-0.55	0.44
t-Statistic	**2.56**		-7.79	-3.35	-3.57	-4.60	
SMB							
Coefficient	0.04	-0.24		0.17	-0.12	-0.28	0.38
t-Statistic	0.21	-7.79		3.54	-2.22	-4.59	
HML							
Coefficient	0.37	-0.17	0.35		0.19	0.35	0.31
t-Statistic	1.38	-3.35	3.54		2.54	4.00	
RMW							
Coefficient	0.83	-0.17	-0.21	0.17		-0.72	0.37
t-Statistic	**3.36**	-3.57	-2.22	2.54		-10.60	
CMA							
Coefficient	0.63	-0.17	-0.35	0.21	-0.50		0.45
t-Statistic	**3.06**	-4.60	-4.59	4.00	-10.60		

While not presented in the tables of this thesis, similar spanning regressions were conducted by removing redundant factors one at a time. Factor spanning regressions after removing the *SMB* factor did not result in a significant *HML* factor (t = 1.48), nor did removing the *HML* factor eliminate the redundancy of the *SMB* factor (t = 0.56). Removing both *SMB* and *HML* factors will by definition create a model with no insignificant intercepts. However, factor spanning tests on the factors of the Fama and French three-factor model resulted in regressions where only the *HML* factor showed a significant intercept ($R_M - R_f$: t = 1.60; *SMB*: t = -0.71). These results prove without doubt a significant overlap between the risk factors of the Fama and French three- and five-factor models for the studied sample.

While spanning tests are sample specific regarding both time-period and stock composition, the results call into question the number of relevant risk factors in a model, as well as the robustness over time of specific risk factors. The problems found in the spanning tests on this sample appear to mainly stem from collinearity between $R_M - R_f$, *SMB*, and *HML* factors. Reasons for the low intercepts found in these tests sample may possibly be tied to return dilution stemming from currency fluctuations within the portfolios. Alternatively, there might just not be enough return variation connected to *size* and *value* in the sample.

The next chapter declares the hypotheses of the study and presents the GRS regressions that test if a model completely captures the return patterns in the sample.

5 HYPOTHESES AND TESTING METHODS

As described in Section 1.1, the main research problem of this study is to assess the performance of the Fama-French five-factor model on Nordic stock data. This problem is investigated by comparing the performance of a CAPM, the Fama- French three-factor model and the Fama-French five-factor model on sample data from the Nordic markets. The first task was to test if the different models can explain the average excess returns in regressions of the sorted portfolios presented in Table 4 on page 31. This was done by analyzing the obtained alpha values from 48 regressions of the portfolios' returns on the factor returns of the different asset pricing models. The regression formulas used are variations of Equations (4) and (7) on pages 13 and 16, including the relevant explanatory variables for each model. If a model completely captures expected returns, the intercept should be indistinguishable from zero. Hence, the first hypothesis is:

H_0 = The regression alpha is not significantly different from zero

H_1 = The regression alpha is significantly different from zero

To obtain a more absolute answer to the question if a tested model can provide a complete explanation of return patterns, GRS *f*-tests were conducted on results obtained from the first hypothesis' portfolio regressions. The GRS statistic is used to test if the alpha values from regressions are jointly indistinguishable from zero. The second hypothesis is:

H_0 = The regression alphas are jointly indistinguishable from zero

H_1 = The regression alphas are jointly distinguishable from zero

Finally, average individual regression alphas and joint GRS regression *f*-values were used together in order to compare the performance between the tested models. Testing these hypotheses should provide answers to both research questions presented in Section 1.1.

5.1. The GRS regression equation

The GRS test was developed by Gibbons, Ross and Shanken (1989), and serves as a test of mean-variance efficiency between a left-hand-side collection of assets or portfolios and a right-hand-side model or portfolio. The following regression defines the GRS test:

$$fGRS = \frac{T}{N} \times \frac{T-N-L}{T-L-1} \times \frac{\hat{\alpha}' \times \hat{\Sigma}^{-1} \times \hat{\alpha}}{1 + \bar{\mu}' \times \hat{\Omega}^{-1} \times \bar{\mu}} \sim F(N, T-N-L)\,, \quad (8)$$

where $\hat{\alpha}$ is a $N \times 1$ vector of estimated intercepts, $\hat{\Sigma}$ an unbiased estimate of the residual covariance matrix, $\bar{\mu}$ a $L \times 1$ vector of the factor portfolios' sample means, and $\hat{\Omega}$ an unbiased estimate of the factor portfolios' covariance matrix.

The GRS test is used in this study to determine whether the alpha values from individual model regressions are jointly non-significant, and hence to find out if a model completely captures the sample return variation. The main numerator, $\hat{\alpha}' \times \hat{\Sigma}^{-1} \times \hat{\alpha}$, is defined as the difference between the max squared Sharpe ratio one can construct with the combination of the left-hand-side (LHS) and right-hand-side (RHS) returns, and the max one can construct with only RHS factor returns (Fama and French 2016a). Thus, as intercepts from individual regressions approach zero, the GRS statistic will also approach zero. However, since the GRS statistic derives its results from comparing the optimal LHS and RHS portfolios, the resulting statistic is not strictly comparable between models. A model containing a higher number of explanatory variables will generally have a more efficient optimal RHS portfolio, which leads to a larger denominator in the GRS equation. The larger denominator works as leverage for the numerator, and thus allows higher alpha values compared to a model with less RHS factors. For this reason, the GRS results of this study are interpreted with caution and in conjunction with average absolute alpha values for model comparison purposes.

In the next chapter, results from the study are detailed. First, a performance summary is presented for each model and comparisons between the models are made. Subsequently, the intercepts of the individual portfolio regressions are interpreted in a more detailed analysis.

6 REGRESSIONS AND MODEL COMPARISON TESTS

6.1. Model performance summary

A set of several summary metrics were deployed in order to compare the performance of the asset pricing models. GRS statistics and average alpha values were used as the main two metrics in order to determine how good the different asset pricing models performed in explaining portfolio returns. In addition to these metrics, average absolute alpha spread was added for a more complete picture of the alpha results. Furthermore, different models' explanatory power was measured using adjusted R-squared and average Akaike information criterion for the sets of regressions.

Table 7 Summary statistics for model comparison tests

Summary statistics for tests of CAPM, three- and five-factor models; July 1999 – June 2015, 192 months. The table tests the ability of CAPM, three- and five-factor models to explain monthly excess returns on 16 *Size-B/M* portfolios (Panel A), 16 *Size-OP* portfolios (Panel B), 16 *Size-Inv* portfolios (Panel C) and a joint sample of all 48 portfolios (Panel D). For each panel, the table shows the tested model, the GRS statistic testing whether the expected values of all 16 or 48 intercept estimates are zero, the p-value for the GRS statistic, the average absolute value of the intercepts, *Avg* $|\alpha|$, the average absolute deviation from the average intercept, the average adjusted R^2, and the average Akaike information criterion, *Avg AIC*, from the model regressions. Bolded GRS-statistics indicate significance at the 5% level.

Panel A: Size-B/M Portfolios

	fGRS	*pGRS*	*Avg* $\vert\alpha\vert$	*Avg* $\vert\alpha - \bar{\alpha}\vert$	*Avg Adj* R^2	*Avg AIC*
CAPM	1.27	0.22	0.32	0.22	0.53	1132
3-Factor Model	0.95	0.51	0.18	0.13	0.75	1006
5-Factor Model	1.12	0.34	0.26	0.14	0.78	972

Panel B: Size-OP Portfolios

	fGRS	*pGRS*	*Avg* $\vert\alpha\vert$	*Avg* $\vert\alpha - \bar{\alpha}\vert$	*Avg Adj* R^2	*Avg AIC*
CAPM	**3.86**	**0.00**	0.41	0.33	0.55	1121
3-Factor Model	**3.38**	**0.00**	0.35	0.30	0.72	1026
5-Factor Model	**2.74**	**0.00**	0.38	0.25	0.79	959

Panel C: Size-Inv Portfolios

	fGRS	*pGRS*	*Avg* $\vert\alpha\vert$	*Avg* $\vert\alpha - \bar{\alpha}\vert$	*Avg Adj* R^2	*Avg AIC*
CAPM	**1.89**	**0.02**	0.33	0.29	0.56	1116
3-Factor Model	1.50	0.10	0.25	0.23	0.72	1029
5-Factor Model	1.52	0.10	0.27	0.15	0.79	960

Panel D: ALL Portfolios

	fGRS	*pGRS*	*Avg* $\vert\alpha\vert$	*Avg* $\vert\alpha - \bar{\alpha}\vert$	*Avg Adj* R^2	*Avg AIC*
CAPM	**2.43**	**0.00**	0.35	0.28	0.55	1123
3-Factor Model	**2.31**	**0.00**	0.26	0.22	0.73	1020
5-Factor Model	**1.95**	**0.00**	0.30	0.18	0.79	964

Table 7 presents summarized results from the model comparison tests. Panels A through C display summaries for regressions of the sorted sets of 16 left-hand-side portfolios, while Panel D includes all 48 regressions in a total summary. The GRS test in Panel D clearly rejects the second hypothesis for all models tested. Furthermore, the GRS test in Panel D suggests that the five-factor model is the model closest to a complete description of asset returns. However, for reasons discussed in the previous chapter, GRS *f*-values between models cannot be strictly compared. Instead, the *fGRS* is mainly interpreted as a test to reject or accept a model's explanation of returns on a set of left-hand portfolios. As previously mentioned, the *fGRS* in this study is used in comparisons between models only in combination with a comparison of average alpha values. Fama and French (2016a) instead use the numerator of a GRS regression as a comparison value when choosing which factors to include in a model.

Let us turn to a more detailed analysis of the metrics shown in Table 7. In every panel of the table, explanatory power measured by average adjusted R-squared and the average Akaike information criterion clearly improves with the inclusion of more factors. This is expected due to the method used to create the factors, the essence being that adding a risk factor whose returns are calculated from return differences within the sample, and sorting the sample by the same factor, will automatically increase almost any explanatory power metric of a regression. This idea is further discussed in the next section. However, for this reason, metrics for explanatory power are of trivial value in providing additional help to compare these types of asset pricing models.

A few interesting results can be discerned from the GRS *f*-values and average absolute alpha values of Table 7. First, as mentioned previously, Panel D of Table 7 shows that in a joint *fGRS* test, the five-factor model is closest to a complete explanation of returns (*fGRS* = 1.95). However, when comparing the absolute the average alpha values of Panel D, the three-factor model shows a lower average absolute alpha (0.26) compared to the five-factor model (0.30). Tests on separate characteristics sorted sets of portfolios reveal that regarding both *fGRS* and average absolute alpha values, the three-factor model actually outperforms the five-factor model in two out of three sets of GRS tests. In tests of *Size-B/M* sorted portfolios, the three-factor model (*fGRS* = 0.95, $Avg|\alpha|$ = 0.18) tops the five-factor model (*fGRS* = 1.12, $Avg|\alpha|$ = 0.26) in both metrics. Similar results are found when observing the summary metrics of *Size-Inv* sorted portfolios, with three-factor *fGRS* and average absolute alpha of 1.50 and 0.25 respectively compared to 1.52 and 0.27 respectively for the five-factor model. Tests of *Size-OP* sorted portfolios show a lower average absolute alpha for the three-factor model (0.35 versus 0.38 for the 5-factor model) but a higher *fGRS* value (3.38 versus 2.74 for the five-factor model). The CAPM model shows the highest *fGRS* values and average alpha values for every set of portfolios tested.

To further clarify the comparison of alpha values between the three-factor and five-factor models, Tables 8 through 10 in the next section confirm that for every sorted set of portfolios, non-significant alpha values are more often found in three-factor regressions than in five-factor regressions. The average alpha spread around the mean alpha value is slightly lower in the five-factor regressions.

In order to provide more insight into the performance of models using different combinations of explanatory variables, a full model performance summary of all possible factor combinations using the *market premium* and a minimum of two additional factors is included in Appendix A Table A7. The results from this table show that stronger GRS statistics are often found in models including the *RMW* and *CMA* factors. This observation lines up with the factor spanning results showing that *RMW* and *CMA* are the most important factors improving the mean-variance efficiency of the optimal five-factor RHS portfolio. Models including the *SMB* factor generally show weaker GRS statistics compared to other models using the same number of factors, further demonstrating the connection between GRS statistics and results from the factor spanning tests. Moreover, models that include the second sorting factor (in addition to the *size* factor) of a portfolio set as an explanatory variable generally show superior GRS performance in tests of that specific set. This result lines up with the findings of Daniel and Titman (2012), who claim that forming portfolios based on characteristics will also diversify away sources of variation not related to the characteristic. Regarding alpha values, the table shows that higher average absolute alpha values are often seen in models including the *RMW* factor. This observation indicates that while the *RMW* factor improves the mean-variance efficiency of the optimal RHS portfolio, the factor has limited success in improving the different models' explanations of excess returns.

Conclusions from the performance comparison tests are as follows. A GRS test on the joint set of all tested portfolios clearly rejects all tested models as complete descriptions of average returns. The three-factor model elicits the lowest average absolute alpha values of the three tested models throughout all tests and shows a higher *fGRS* value compared to the five-factor model in one of three tests on sorted sets of portfolios, as well as in the test on a joint set of all portfolios. Considering all the evidence in Table 7 supplemented by evidence regarding individual regression alphas in the next section, it is clear that the three-factor model shows the strongest performance out of the three models for the sample. The next section provides a more thorough examination of alpha values from individual left-hand-side portfolio regressions.

6.2. Regression details

Individual regression intercepts, their corresponding t-values, and R-squared values are next presented in order to provide a more detailed picture of model performance. Significant alpha patterns between models are compared and further analyzed by looking at regression slopes in Appendix A and characteristics data in Appendix B. Looking explicitly at the number of significant alpha values (5% confidence ratio), the CAPM performs slightly worse than the three-factor model, and the five-factor model shows the worst performance of the three tested models. R-squared values from the regressions improve in models using a higher number of explanatory factors.

Prior to presenting the regression details, a cautionary connection between R-squared values and the risk factors must be addressed. In an OLS regression, R-squared values will always increase with the inclusion of more factors. This is inherent to the way OLS regressions are built, but the effect is much stronger and becomes true for almost any explanatory power metric when the regressions' explanatory factors are created from return differences in the data itself. Consider the inclusion of a *size* factor to the normal CAPM regression on *Size-B/M* sorted portfolios in Table 8 on the next page. The CAPM regresses portfolio returns on a *market premium* which quite closely resembles the returns on big diversified portfolios (row 4 in Table 8) as a result of the market index being value-weighted (and constructed from mid and large cap stocks in this case). For this reason the CAPM will generally show higher R-squared values in big portfolio regressions. Including the *size* factor will attempt to help explain anomalous returns on smaller portfolios which the CAPM cannot explain, and is defined as the mean monthly return difference between small and large portfolio returns (resembling small index minus market index). In other words the *size* factor can be defined as a subtraction between a subset of the data and the existing *market premium*. The method to construct the factor will hence automatically bring the augmented model's explanatory power in small portfolio regressions closer to the R-squared values found in big portfolio regressions, boosting the average explanatory power more than it would for regressions of randomly picked portfolio sets. This same phenomenon is apparent in all other risk factors which are included as sorting variables and simultaneously used as explanatory variables created from return differences in the sample data.

Table 8 Regressions for 16 value-weight *Size-B/M* portfolios

Regressions for 16 value-weight *Size-B/M* portfolios; July 1999 – June 2015, 192 months. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are independently allocated to four *B/M* groups, again using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-B/M* portfolios. The left-hand-side (LHS) variables in each set of 16 regressions are the monthly excess returns on the 16 *Size-B/M* portfolios. The right-hand-side (RHS) variables are $R_M - R_f$ for the CAPM, $R_M - R_f$, the *size* factor, *SMB*, and the value factor, *HML* for the three-factor model, and $R_M - R_f$, *SMB*, *HML*, the *profitability* factor, *RMW*, and the *investment* factor, *CMA* for the five-factor model. The factors are constructed using independent 2 x 3 sorts on *size* and each of *B/M, OP,* and *Inv.* Panel A of the table shows intercepts, intercept t-statistics and R^2 values for the CAPM regressions. Panel B and Panel C shows the same information for three- and five-factor regressions respectively. Bolded t-statistics indicate significance at the 5% level. The five-factor regression equation is,

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$$

CAPM

	α Low	2	3	High	$t(\alpha)$ Low	2	3	High	R^2 Low	2	3	High
Small	0.17	0.15	0.17	0.15	0.41	0.55	0.76	0.59	0.45	0.49	0.38	0.32
2	-0.39	0.32	0.27	0.13	-1.17	1.18	1.07	0.49	0.61	0.51	0.44	0.44
3	-0.04	0.55	0.50	0.51	-0.15	**2.22**	1.88	1.65	0.73	0.58	0.52	0.42
Big	-0.27	0.28	0.59	0.64	-1.51	1.36	**2.23**	**2.10**	0.87	0.70	0.57	0.54

3-Factor Model

	α Low	2	3	High	$t(\alpha)$ Low	2	3	High	R^2 Low	2	3	High
Small	0.61	0.17	0.12	-0.04	1.85	0.80	0.64	-0.23	0.68	0.68	0.58	0.71
2	-0.03	0.25	0.11	-0.17	-0.12	1.28	0.60	-1.16	0.79	0.76	0.73	0.84
3	0.09	0.40	0.30	0.19	0.41	**1.97**	1.44	0.86	0.78	0.73	0.72	0.71
Big	-0.03	0.05	0.17	0.11	-0.24	0.29	0.90	0.53	0.94	0.77	0.79	0.78

5-Factor Model

	α Low	2	3	High	$t(\alpha)$ Low	2	3	High	R^2 Low	2	3	High
Small	0.60	0.36	0.22	0.09	1.78	1.70	1.17	0.52	0.69	0.72	0.62	0.74
2	0.12	0.45	0.18	-0.04	0.58	**2.73**	1.09	-0.29	0.86	0.84	0.77	0.87
3	0.26	0.46	0.44	0.35	1.22	**2.36**	**2.17**	1.70	0.83	0.77	0.75	0.76
Big	0.02	0.18	0.22	0.18	0.12	1.02	1.16	0.87	0.94	0.81	0.80	0.82

Table 8 shows alpha values along with their corresponding t-values and R-squared values from regressions on 16 *Size-B/M* sorted portfolios. Considering the t-values in the table, all tested models perform surprisingly well in explaining average returns across the sample. The CAPM regressions find significant alpha values in three portfolio regressions, one in the third *size* quartile and two in the largest *size* quartile. The three-factor regressions improve on all of these problematic portfolios, leaving only one significant alpha in the third *size* quartile (t = 1.97). The five-factor regressions find larger intercept t-values on most portfolios compared to the three-factor regressions, of which three are more than two standard errors from zero. Looking at the characteristics of the problematic portfolio apparent in all models (comparing with Appendix B) reveals no exceptional characteristics which might shed light into the reasons behind the significant alpha value. Further, the regression slopes in Appendix A Table A1 show no peculiarities in the CAPM and three-factor regressions for the portfolio in question. However, the five-factor regressions show unusually high *RMW* and *CMA* slopes for the portfolio, indicating that the portfolio's returns behave like a portfolio with lower *investment* and higher *profitability* than expected. This observation is interesting considering that the portfolio in question actually shows slightly higher mean *investment* compared to other portfolios in the second *B/M* column, when comparing the portfolios' characteristics in Appendix B. An additional observation from the regression slopes in Appendix A Tables A1 and A2 is that in both three- and five-factor regressions, *HML* slopes increase with *size* except for portfolios in the extreme low *B/M* row. The value-weight average characteristics of Appendix B on the other hand show that mean *book-to-market* value decreases with *size*. Fama and French (2015) note however that one should not expect univariate characteristics and multivariate regression slopes connected to the characteristic to line up. The slopes estimate marginal effects holding constant all other explanatory variables, and the characteristics are measured with lags relative to returns when pricing should be forward looking (Fama and French 2015). Fama and French (2015) found the top-left *Size-B/M* sorted portfolio to be a large problem in their U.S. paper, and similarly Fama and French (2016b) found that the whole lowest *B/M* quintile destroys the three-factor model's performance in their international paper. While the top-left portfolio in Table 8 does show a high alpha value in the three-factor regressions compared to the CAPM regressions, the value is insignificant and the low *B/M* column as a whole does not pose a problem for any of the regression models.

Table 9 Regressions for 16 value-weight *Size-OP* portfolios

Regressions for 16 value-weight *Size-OP* portfolios; July 1999 – June 2015, 192 months. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are independently allocated to four *OP* groups, again using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-OP* portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 *Size-OP* portfolios. The RHS variables are $R_M - R_f$ for the CAPM, $R_M - R_f$, *SMB*, and *HML* for the three-factor model, and $R_M - R_f$, *SMB*, *HML*, *RMW*, and *CMA* for the five-factor model. The factors are constructed using independent 2 x 3 sorts on *size* and each of *B/M, OP,* and *Inv*. Panel A of the table shows intercepts, intercept t-statistics and R^2 values for the CAPM regressions. Panel B and Panel C shows the same information for three- and five-factor regressions respectively. Bolded t-statistics indicate significance at the 5% level.

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$$

CAPM

	α				$t(\alpha)$				R^2			
	Low	2	3	High	Low	2	3	High	Low	2	3	High
Small	-0.40	0.28	0.84	0.62	-1.36	1.26	**3.67**	**2.25**	0.45	0.38	0.36	0.42
2	-0.73	0.19	0.50	0.61	**-2.16**	0.82	**2.02**	**2.50**	0.54	0.53	0.51	0.54
3	-0.19	0.51	0.43	0.45	-0.58	**2.12**	1.78	1.88	0.54	0.57	0.56	0.67
Big	0.48	0.11	-0.11	0.07	0.93	0.48	-0.54	0.37	0.45	0.68	0.80	0.86

3-Factor Model

	α				$t(\alpha)$				R^2			
	Low	2	3	High	Low	2	3	High	Low	2	3	High
Small	-0.31	0.18	0.67	0.54	-1.37	1.09	**3.92**	**2.49**	0.68	0.65	0.65	0.66
2	-0.67	0.09	0.40	0.58	**-2.56**	0.58	**2.50**	**3.34**	0.73	0.79	0.81	0.78
3	-0.23	0.43	0.23	0.35	-0.77	**2.26**	1.19	1.76	0.63	0.74	0.74	0.79
Big	0.43	0.00	-0.32	0.22	0.84	-0.01	-1.75	1.24	0.47	0.70	0.84	0.88

5-Factor Model

	α				$t(\alpha)$				R^2			
	Low	2	3	High	Low	2	3	High	Low	2	3	High
Small	-0.14	0.36	0.68	0.61	-0.72	**2.23**	**3.81**	**2.75**	0.78	0.71	0.65	0.67
2	-0.27	0.26	0.44	0.52	-1.76	1.95	**2.73**	**2.96**	0.92	0.86	0.81	0.78
3	0.03	0.56	0.28	0.43	0.15	**3.09**	1.48	**2.13**	0.80	0.78	0.75	0.80
Big	0.76	0.40	-0.18	0.09	**2.75**	**2.20**	-0.99	0.60	0.86	0.83	0.85	0.91

Table 9 shows regression details for portfolios sorted on *size* and *profitability*. The GRS tests of Table 7 reject every model's description of returns in *Size-OP* sorted portfolio regressions. From the same table, it is also clear that the five-factor model outperforms the CAPM as well as three-factor model in the GRS test on the set of portfolios portrayed in Table 9, while the three-factor model has a slight edge on average absolute alpha values.

Examining the intercepts in Table 9, it is clear that these portfolios pose big problems for all of the tested asset pricing models. The CAPM and the three-factor model elicits significant alpha values in regressions for the same six out of 16 left-hand-side portfolios. The five-factor model rectifies one of the six troublesome portfolios, but introduces four additional significant regression alphas, bringing the total to nine out of 16 significant alpha values. The most problematic portfolios for all models are situated in the top right corner of Table 9, that is portfolios of smaller stocks with high *profitability*. The highest intercept value is found in the three-factor regression of the portfolio on row 1, column 3 (t = 3.92). In regard to intercept values this same portfolio is also the most problematic portfolio for the CAPM (t = 3.67) and the five-factor model (t = 3.81). Observing characteristics data in Appendix B for the problematic portfolios of the top right quadrant of Table 9 reveals that these portfolios show lower *book-to-market* values compared to other portfolios in the same *size* row, and higher *B/M* compared to other portfolios in the same *profitability* column. The five-factor regression slopes in Appendix A Table A4 further reveal that *RMW* and *CMA* slopes are weak and mainly non-significant for the quadrant, indicating that these factors contribute marginally to the five-factor model's explanation of the problematic portfolios' returns. The three-factor slopes in Table A3 of Appendix A show no distinctive properties in these portfolio regressions.

Considering the statistics in Table 9, it might seem surprising that the five-factor model has by far the strongest GRS statistic in tests of this subset of portfolios. The prime reason for this outcome stems from the main denominator of the GRS test, i.e. the optimal portfolio created from right-hand-side explanatory variables. When alpha values are further from zero, i.e. when the models show weaker alpha values, the effect from a "stronger" optimal right-hand-side portfolio is more prominent. This denominator is the same regardless of the alpha values in the numerator (and regardless of sorting variables), and as mentioned in Section 5.1, can be instead thought of as a leverage-ratio for the numerator. For the *Size-OP* sorted regressions of Table 9, the leverage from the denominator is simply strong enough to push the five-factor model ahead of the pack, even if the overall significant alpha values are worse in the five-factor regressions.

Table 10 Regressions for 16 value-weight *Size-Inv* portfolios

Regressions for 16 value-weight *Size-Inv* portfolios; July 1999 – June 2015, 192 months. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are independently allocated to four *Inv* groups, again using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-Inv* portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 *Size-Inv* portfolios. The RHS variables are $R_M - R_f$ for the CAPM, $R_M - R_f$, *SMB*, and *HML* for the three-factor model, and $R_M - R_f$, *SMB*, *HML*, *RMW*, and *CMA* for the five-factor model. The factors are constructed using independent 2 x 3 sorts on *size* and each of *B/M, OP,* and *Inv*. Panel A of the table shows intercepts, intercept t-statistics and R^2 values for the CAPM regressions. Panel B and Panel C shows the same information for three- and five-factor regressions respectively. Bolded t-statistics indicate significance at the 5% level.

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$$

CAPM

	α				$t(\alpha)$				R^2			
	Low	2	3	High	Low	2	3	High	Low	2	3	High
Small	0.17	0.19	0.36	-0.10	0.61	0.78	1.45	-0.32	0.46	0.29	0.40	0.44
2	-0.09	0.48	0.32	-0.31	-0.30	**2.09**	1.31	-1.04	0.56	0.51	0.48	0.54
3	0.52	0.68	0.38	-0.03	1.79	**2.95**	1.45	-0.12	0.52	0.56	0.57	0.70
Big	0.63	0.51	-0.36	-0.16	1.64	**2.86**	-1.64	-0.78	0.51	0.79	0.79	0.84

3-Factor Model

	α				$t(\alpha)$				R^2			
	Low	2	3	High	Low	2	3	High	Low	2	3	High
Small	0.14	0.07	0.34	0.00	0.62	0.36	1.71	-0.02	0.64	0.62	0.63	0.67
2	-0.10	0.34	0.20	-0.22	-0.44	**2.07**	1.25	-1.07	0.75	0.75	0.78	0.79
3	0.45	0.49	0.23	-0.08	1.68	**2.65**	1.16	-0.39	0.59	0.73	0.75	0.80
Big	0.52	0.38	-0.36	-0.08	1.40	**2.26**	-1.64	-0.41	0.56	0.82	0.79	0.85

5-Factor Model

	α				$t(\alpha)$				R^2			
	Low	2	3	High	Low	2	3	High	Low	2	3	High
Small	0.13	0.19	0.52	0.30	0.68	1.03	**2.64**	1.32	0.76	0.67	0.66	0.71
2	-0.06	0.36	0.33	0.18	-0.40	**2.33**	**2.12**	1.02	0.88	0.81	0.80	0.86
3	0.57	0.53	0.30	0.14	**2.65**	**2.99**	1.51	0.67	0.76	0.77	0.78	0.82
Big	0.37	0.30	0.03	0.06	1.79	**2.00**	0.16	0.34	0.87	0.87	0.84	0.90

Regression details from *Size-Inv* sorted portfolio regressions are presented in Table 10. The regressions of Table 10 show a similar distribution of significant alpha values to those encountered in Tables 8 and 9. The CAPM and the three-factor model have trouble explaining the same three portfolios found in rows 2-4 of column 2 in Table 10. The five-factor model does not manage to solve these problematic portfolios and further introduces three additional significant alpha values. The highest t-value is found in the five-factor regressions on the portfolio in row 3, column 2 (α = 0.53, t = 2.99). The same portfolio is the most problematic portfolio for every tested model, with t-values of 2.95 and 2.65 in the CAPM and three-factor model regressions respectively. Portfolio characteristics in Appendix B show no abnormal characteristics for the portfolio in question, while the slopes of Appendix A tables A5 and A6 show slightly higher *HML* loadings compared to adjacent portfolio regressions, indicating that none of the risk factors are able to absorb the abnormal return variation of this portfolio.

The following conclusions are drawn from this section. Most of the portfolios showing significant alpha values are problematic for all presented models. Similar problems between CAPM and three-factor regressions are likely a result of high correlation between the models' risk factors. Further, the *RMW* and *CMA* factors do not generally succeed in absorbing the abnormal return patterns of those portfolios posing problems for the CAPM and three-factor model. Regression slopes and portfolio characteristics of these problematic portfolios show no clear differences compared to those from regressions of non-problematic portfolios, indicating that these return abnormalities are not connected to the risk factors included in the tested models.

The next chapter concludes the thesis with a summary and discussion of the most important findings and provides suggestions for future research.

7 CONCLUSIONS

This thesis investigated whether a Fama-French five-factor asset pricing model can explain average returns on a sample of Nordic stock data. Further, this study compared the performance of the CAPM, the Fama-French three-factor model, and the Fama-French five-factor model in order to find out which model best describes the returns of 48 sample portfolios with large differences in *size, book-to-market*, *operating profitability* and *investment*. To measure the performance of each model, GRS statistics and model regression intercepts were analyzed.

The GRS tests of this study rejected all models' explanations of average returns on a joint sample of all test portfolios. Joint-sample GRS tests further showed that the five-factor model is closest to a complete description of average returns. GRS tests on separate sets of characteristic-sorted portfolios showed that portfolios sorted on *size* and *book-to-market* ratio pose no problems for any of the tested models. Furthermore, only the CAPM was rejected in GRS tests on *Size-Inv* sorted portfolios. However, all models failed the GRS test to explain average returns of the set of portfolios sorted on *size* and *profitability*.

Further examination of results from regressions of separate sets of portfolios revealed that the three-factor model outperforms the five-factor model regarding average absolute intercepts in all tests, and regarding GRS statistics in two out of three sets of characteristic-sorted portfolios. A more detailed analysis found that five-factor regressions result in the highest number of significant intercepts for every set of portfolios. Studying all the results, while taking into consideration that GRS statistics favor models with a higher number of explanatory factors, the evidence states that for this sample the five-factor model fails to improve on the three-factor model's description of average returns. These conclusions widely contrast the findings of Fama and French (2015, 2016b).

7.1. Discussion

While the five-factor model tested in this thesis was constructed following the methodology of Fama and French (2015), the Nordic data sample provided a very different base for factor creation. Fama and French (2016b) used *size* groupings that roughly correspond NYSE breakpoints, but the breakpoints of this thesis were calculated by dividing the sample into *size* groups containing a similar number of stocks. More specifically, Fama and French (2016b) smallest regional *size* breakpoint was the 75th sample *size* percentile, by which one can approximate that three of this thesis' four *size* groups lie in the smallest *size* quintile of Fama and French (2015, 2016b) research. Some may argue that this discrepancy reduces comparability between the research results, but since evidence has found repeatedly that similarly constructed risk premiums are sample specific both regarding region and time-period, observing a populace of smaller stocks should fall into a similar category of differences. On the contrary, this research provides much needed insight into the behavior of small stocks, whose return patterns have oft been overlooked in studies of asset pricing models and CAPM anomalies.

Another important point for discussion concerns the validity of the conclusions drawn from the model comparison tests. While the analysis of average OLS regression intercepts conclusively supported the three-factor model, the GRS tests supported the five-factor model in portfolio sets where all models failed the GRS test. As the second hypothesis of this thesis states, the intent was to assess which model most closely describes excess returns by testing if regression alpha values are jointly indistinguishable from zero. While the GRS statistic is tremendously useful in compiling individual regression results into one explicit number accepting or rejecting a model, the incomparability between different model's GRS statistics made the results slightly harder to interpret. In hindsight, presenting the GRS numerator in the performance comparison table would have provided a more comparable number (the difference in Sharpe value between the optimal LHS and RHS portfolios), which would have improved the reader's ease to observe the results. However, considering all presented alpha metrics it is quite clear that the GRS *f-values* supporting the five-factor model were encountered only due to a higher GRS denominator, and it is highly unlikely that observing the numerators would show a lower value for the five-factor model in any tested set of portfolios.

When initiating this research, the expectation was to find results supporting the five-factor model as a superior description of average returns. Instead, the results contrasted those from earlier research and showed that results from similar asset pricing tests may differ widely depending on the observed sample. In addition to contrasting model comparison results, the studied effects were weaker and more volatile compared to earlier research by Fama and French (2015, 2016b). One possible explanation for these findings is inefficient pricing of small stocks. Since smaller stock groups have less following and are largely off-limits for big mutual funds and institutional investors, it is plausible that many of the observed portfolios may include pricing inefficiencies which weaken the CAPM anomalies observed by the pricing models. A second possible explanation is that currency fluctuations or exposure to unexplained macro-economic risk dilutes the factor premiums and portfolio returns. Currency fluctuations are unaccounted for in the tested models which may contribute to collinearity between the risk factors and adversely affect the returns of both risk factors and left-hand-side portfolios. Theoretical explanations could state that these currency fluctuations are a source of risk not included in the models and thus call for an augmented model. Regardless of the reason for these discrepancies, the results from this study provide more evidence that the *size, book-to-market*, *profitability* and *investment effect*s and their associated risk premiums are dependent on the sample population and time-period.

Correlations between the risk factors of this thesis provide some additional interesting implications regarding strategies for investing. First, the thesis found lower *market betas* associated with small stock groups, which goes against earlier evidence found by Fama and French (2015). This result, stemming from a high negative correlation between the *SMB* and $R_M - R_f$ factors, indicates that small stocks may serve as good risk-hedges in some diversified strategies despite a slightly lower return on small stocks. Second, all observed risk factors had a negative correlation with the *market premium*, and the *RMW* and *CMA* factors showed very low correlation with the other factors as well as a high negative correlation between themselves. Further, the *RMW* and *CMA* factors along with the *HML* factor displayed the highest mean returns along with the $R_M - R_f$ factor. Considering these results, diversified investment strategies focusing on *profitability* and *investment* (*RMW* and *CMA*) in addition to asset *value* (*HML*), seem to produce exceptionally mean-variance efficient returns on the tested sample. These implications line up with the strong right-hand-side optimal portfolio described in the five-factor GRS tests, and with the redundancies found in the factor-spanning regressions of this study.

Adhering to the conclusions and discussion of results, one may suggest that the sample provided some limitations for the conducted research. The Nordic market sample is still too small for a study of return patterns found in larger stock groups, thus necessitating a focus on smaller stocks in order to obtain sufficient portfolio diversification. Moreover, dividing the sample into *size* groups containing a similar number of stocks produces portfolios whose constituents may possess very large differences in *size*. Value-weight returns of such portfolios may depend heavily on the returns of the portfolios' biggest stocks. Using equal-weight returns for the left-hand-side portfolios could provide more return diversification and further insights into the return patterns of the sample.

7.2. Suggestions for further research

The evidence found in this study calls for further research of asset pricing models based on the Miller and Modigliani (1961) framework. This thesis found more evidence of a weak *size effect* on the Nordic markets, and all the studied effects were weaker and more volatile compared to earlier evidence found by Fama and French (2015, 2016b). Further research on similar asset pricing models may provide perspective by dropping the *SMB* factor and sorting portfolios according to one or several of the remaining factors. Future studies might instead use industry portfolios or conduct separate tests on small-, mid-, and large-cap portfolios in order to preserve some separation of different stock types. Research including such separations would however need to be conducted on a bigger sample.

Further, since the weak effects found in this thesis may be connected to currency risk, the evidence calls for further research on the effects currency fluctuations may have on factor returns. Alternatively, research on a European sample may be conducted on a sample of those countries that use the euro as their currency and have sufficiently developed equity markets. Fama and French (2016b) reported that a plausible assumption of market integration was important when they chose which countries to include in a region, which should also hold for a sample of developed markets in the Eurozone.

Finally, the weak model performance found in regressions of portfolios sorted on *size* and *operating profitability* along with the high average absolute alpha values found in regressions including the *RMW* factor demand a more detailed study of the *profitability effect* in the Nordic markets. Fama and French (2016a) found that *cash profitability* performs better than *operating profitability* in a five-factor model. Testing this finding and further investigating the *profitability effect* in the Nordic markets may be able to shed more light into the reasons behind the relatively higher intercepts this study found in regressions including the *RMW* factor. After all, it is not inconceivable that a suboptimal proxy for *profitability* was the primary issue holding back the five-factor model's performance in the tests of this study.

SVENSK SAMMANFATTNING

INLEDNING

Sedan finansiella marknaders begynnelse har investerare och ekonomiforskare strävat till att skapa en bättre förståelse för aktieavkastningar. En uppsjö av forskning under andra halvan av 1900-talet gav upphov till många teorier och tillgångsprissättningsmodeller vilka än idag används flitigt inom den finansiella branschen. Den mest kända teorin för prissättning av tillgångar utvecklades av Sharpe (1964), Lintner (1965) och Mossin (1966) och kallas the Capital Asset Pricing Model, hädanefter förkortat CAPM.

Trots att CAPM är den mest använda och utlärda teorin för tillgångsprissättning har ett mångtal empiriska studier funnit resultat som förkastar modellens praktiska tillämpning. Banz (1981) upptäcker att små aktier historiskt sett haft högre avkastningar än vad som kan förklaras av CAPM-modellen. Likaså finner Rosenberg, Ried och Lanstein (1985) att aktier med hög *book-to-market*-kvot (bokfört eget kapital genom totalt *börsvärde*) avkastat signifikant bättre än aktier med låg *book-to-market*-kvot. Resultaten från dessa och andra dylika studier under andra halvan av 1900-talet drev vissa forskare till att bygga ut CAPM-modellen med flera förklarande faktorer medan andra sökte alternativa förklaringar för aktieavkastningar.

En av de mest inflytelserika undersökningar som bestred CAPM-modellen publicerades av Fama och French (1992). Studien förkastar *marknadsbetat* associerat med CAPM och finner istället att aktiers *storlek* och *book-to*-market-kvot bättre förklarar tvärsnittsvariationen i genomsnittliga aktieavkastningar. Ett år senare publicerade Fama och French (1993) en studie som föreslår att en trefaktormodell som bygger ut CAPM med riskfaktorer för *storlek* och *book-to-market* är potentiellt bättre än CAPM på att förklara aktieavkastningar. Dessa rön tillsammans med tidigare kritiska forskningsresultat mot CAPM skapade en lång dispyt mellan ekonomiforskare som snart flockades till att undersöka orsakerna bakom dessa anomalier.

Påföljande undersökningar delade upp forskare i två grupper. Vissa ansåg att anomalierna var bevis på att psykologiska faktorer spelar in på prissättningen av aktier, medan andra betraktade anomalierna som oförklarade källor av risk i CAPM modellen. Medan forskare än idag är oense om orsakerna till dessa anomalier, har nyare forskning visat att flera av anomalierna är svaga eller beroende på tidsperiod och omständighet.

Nyare studier har emellertid upptäckt andra faktorer som verkar ha en stark anknytning till aktieavkastningar. Novy-Marx (2013) visar att företag med hög *lönsamhet* genererar betydligt högre avkastningar än mindre lönsamma företag. Dylikt upptäcker Aharoni, Grundy och Zeng (2013) en signifikant relation mellan företags *investeringsmängd* och medelavkastningar. I kölvattnet av dessa upptäckter byggde Fama och French ut sin trefaktormodell.

År 2015 publicerade Fama och French en forskningsstudie som bygger ut deras trefaktormodell med två ytterligare riskfaktorer: *lönsamhet* och *investeringsmängd*. Resultaten från Fama och French (2015) studie visar att femfaktormodellen förbättrar trefaktormodellens förklaring till portföljavkastningar för deras sampel. Dessa resultat förstärktes ytterligare när Fama och French (2016b) fann liknande resultat i internationella test av modellen.

Denna avhandling bidrar till den föga mängd undersökning som gjorts angående Fama-French-femfaktormodellen genom att testa modellen på ett nordiskt datasampel. Utöver detta ger avhandlingen en bättre inblick i beteendet av små företags aktiekurser vars avkastningar blivit så gott som försummade i tidigare forskning angående prissättningsmodeller.

Problemområde och syfte

Avhandlingens första forskningsfråga är:

> Kan en femfaktormodell med riskfaktorerna *marknadspremium, storlek*, *book-to-market*, *lönsamhet* och *investeringsmängd* förklara medelavkastningar på Nordiska marknaden?

Avhandlingens syfte är att bedöma Fama och Frenchs (2015) femfaktormodells prestationsförmåga på ett nordiskt datasampel. Tidigare studier har testat modellen på nordamerikanska, europeiska och asiatiska datasampel, men inga omfattande test av modellen har gjorts på nordiska sampel. Tidigare har det varit problematiskt att skapa regionala riskfaktorer och att testa prissättningsmodeller på nordiska marknader på grund av databrist. Medan det idag är möjligt att skapa ett tillräckligt brett sampel över en tillräckligt lång tidsperiod, är den nordiska marknaden ännu relativt ung och liten jämfört med de stora aktörerna. På grund av detta lägger avhandlingen mer fokus på mindre företag och avviker lite från den metodik som använts i tidigare forskning på större marknader.

Tillgångsprissättningsmodeller är förenklingar av verkligheten och det är meningslöst att försöka hitta *en* modell som felfritt kan förklara aktieavkastningar. Av denna orsak måste flera modeller jämföras för att skapa en meningsfull insyn om en modells prestationsförmåga. Utöver att testa femfaktormodellens förmåga att förklara avkastningar på den nordiska marknaden jämförs i denna avhandling modellens prestation med CAPM-modellens och Fama-French-trefaktormodellens resultat på samma sampel. Således lyder avhandlingens andra forskningsfråga:

> Kan en femfaktormodell med riskfaktorerna *marknadspremium*, *storlek*, *book-to-market*, *lönsamhet* och *investeringsmängd* förklara medelavkastningar på Nordiska marknaden bättre än en CAPM-modell och en trefaktormodell med riskfaktorerna *marknadspremium*, *storlek* och *book-to-market*?

Avgränsningar och datasampel

Avhandlingen fokuserar på faktorerna *storlek*, *book-to-market*, *lönsamhet* och *investeringsmängd*. Studiens datasampel hämtades från databaserna Thomson Data Stream (TDS) och Worldscope. Det obearbetade samplet består av alla befintliga aktier för målmarknaderna med vissa begränsningar:

1) Undersökningens tidsperiod är 31 juli 1999 till 30 juni 2015.

2) För att tas med i samplet måste företag vara klassade som ”equity” och ha tillgänglig data angående aktiepris, totalavkastning, totala tillgångar, totala skulder, antal utestående stamaktier och driftsintäkter för minst ett helt år av tidsperioden.

3) Undersökningens målmarknad är den nordiska marknaden i dess helhet. Island uteslöts dock från undersökningen för att bättre replikera det valda jämförelseindexet samt på grund av den isländska marknadens ringa storlek.

Undersökningen utfördes på sampeldata från finska, svenska, norska och danska marknaderna. Samplet filtrerades för att ta bort potentiella datafel, multipla aktieklasser av samma företag och extremobservationer som sannolikt härstammar från företagsköp och andra specialsituationer. Samtliga data konverterades till euro och slogs ihop till ett slutligt sampel bestående av 1520 aktier. Riskfaktorer skapades och regressionsanalyser kördes på det hopslagna slutliga samplet.

VARIABLER, SORTERINGAR OCH REGRESSIONSMODELLER

I avhandlingens empiriska del prövas och jämförs som tidigare nämnt tre olika prissättningsmodellers förmåga att förklara portföljavkastningar på ett nordiskt datasampel. Förklaringsförmågan jämfördes genom att köra regressionsanalyser av ett flertal portföljer med stora skillnader i *storlek*, *book-to-market*, *lönsamhet*, och *investeringsmängd* på de olika modellerna. Följande kapitel förklarar kort regressionsmodellerna, variablerna, riskpremierna och testportföljerna som används i avhandlingens empiriska del.

Den fullständiga femfaktormodellens regressionsform är,

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it} ,$$

där R_{it} är avkastningen på tillgång i vid tidpunkt t, R_{ft} är riskfria räntan, $R_{Mt} - R_{ft}$ *marknadspremien*, SMB_t *storleksfaktorn*, HML_t *book-to-market-faktorn*, RMW_t *lönsamhetsfaktorn* och CMA_t *investeringsfaktorn*. Koefficienterna β_i, s_i, h_i, r_i och c_i är tillgångens känslighet till var och en av faktorerna, a_i är interceptet och e_{it} är feltermen vid tidpunkt t.

CAPM och trefaktormodellens regressionsformer är delar av denna modell, där trefaktormodellens regressionsform kan skapas genom att ta bort *lönsamhets-* och *investeringsfaktorn* samt deras koefficienter ur femfaktormodellen. På liknande sätt kan CAPM-modellens regressionsform skapas genom att ytterligare ta bort *storleks-* och *book-to-market-faktorn* ur trefaktormodellens regressionsform.

Variabler

Börsvärdet på en aktie användes som mätvärde för *storlek* i avhandlingens empiriska del. *Börsvärdet* räknades ut genom att multiplicera aktiepriset den 31 december varje år med antal utestående stamaktier.

Eget kapital räknades ut som skillnaden i totala tillgångar och totala skulder.

Book-to-market-kvoten räknades ut som *eget kapital* dividerat med *börsvärdet*.

Rörelsemarginalen användes som mätvärde för *lönsamhet* och räknades ut som *driftsintäkter* dividerat med *eget kapital*.

Investeringsmängd räknades ut som ändringen i totala tillgångar dividerat med totala tillgångar föregående år.

Riskpremier

I detta avsnitt förklaras hur modellernas riskpremier, dvs. regressionernas oberoende variabler, är skapade. Faktorerna presenteras i ordning från vänster till höger i regressionsmodellen ovan med undantag för *storleksfaktorn* som presenteras till sist. Alla avkastningar för analysens portföljer och riskpremier är månadsavkastningar utöver avkastningen på 1-månads euriborränta för samma månad. Avkastningarna räknar med att eventuella dividender återinvesteras.

Marknadspremien $(R_M - R_f)$ som användes i analysen är den månatliga avkastningen (utöver 1-mån euribor) på MSCI Nordic Markets Index. Detta index valdes för att det bäst representerar målmarknaderna och för att inga andra lämpliga index med data från 1999 kunde hittas.

Resterande faktorpremier räknades ut som avkastningsskillnader mellan portföljer med stora skillnader i de variabler som faktorerna mäter. För att skapa dessa faktorer gjordes tre uppsättningar av sorterade portföljer. I slutet av juni varje år delades datasamplets aktier in i två storleksgrupper med sampelmedianens *storlek* som brytpunkt. Aktierna delades även in i tre grupper enligt *book-to-market*, *lönsamhet* och *investeringsmängd* med 30e och 70e percentilerna av dessa variabler som brytpunkter. Genom att distribuera aktierna i portföljer enligt skärningspunkterna från *storlek-* och *book-to-market*-sorteringarna skapades sex stycken portföljer. På samma sätt skapades sex portföljer från sorteringar enligt *storlek* och *lönsamhet* samt *storlek* och *investeringsmängd* för totalt tre uppsättningar av sex portföljer. Datasamplets aktier omfördelades årligen enligt samma princip. Portföljernas värdeviktade avkastningar utgör grund för konstruktionen av femfaktormodellens faktorpremier.

Book-to-market-premien (HML) räknades ut som månadsavkastningen på en portfölj av aktier med hög *book-to*-market-kvot minus månadsavkastningen på en portfölj av aktier med låg *book-to-market*-kvot. Från sorteringen enligt *storlek* och *book-to-market* räknades medeltal av månadsavkastningarna för de två portföljer som har hög *book-to-market*-kvot och de två portföljer som har låg *book-to-market*-kvot. Skillnaden mellan dessa två utgör det månatliga *HML*-premiet. På liknande sätt skapades månatliga avkastningspremier för *lönsamhet* (RMW) och *investeringsmängd* (CMA). *Storlekspremien* (SMB) räknades ut som medeltalet av månadsavkastningarna för alla ”små” portföljer (sammanlagt nio portföljer) minus medeltalet för alla ”stora” portföljer.

Testportföljer

Portföljerna som tillgångsprissättningsmodellerna testas på skapades från sorteringar på ett liknande sätt som riskfaktorerna. Kvartilbrytpunkter för storlek och var och en av de övriga variablerna användes för att dela upp samplets aktier i tre uppsättningar av 16 portföljer (4 x 4 sorteringar). Portföljernas värdeviktade månatliga avkastningar fungerar som beroende variabler i regressionsanalysen. Således kördes sammanlagt 48 regressionsanalyser per modell (16 portföljer och tre uppsättningar) för att testa varje modells förmåga att förklara testportföljernas avkastningar.

HYPOTESER OCH TESTMETODER

För att bedöma Fama och Frenchs (2015) femfaktormodells prestation jämfördes CAPM, trefaktormodellen och femfaktormodellen i regressionsanalyser på 48 portföljer med stora skillnader i *storlek*, *book-to-market*, *lönsamhet* och *investeringsmängd*. Den första uppgiften var att ta reda på om de olika modellerna kan förklara portföljernas avkastningar. För att besvara frågan jämfördes intercepten från samtliga regressionsanalyser mellan modellerna. Om en modell fullständigt förklarar avkastningarna från en regression borde interceptet vara oskiljbart från noll. Den första hypotesen är:

H_0 = Regressionens intercept avviker inte signifikant från noll

H_1 = Regressionens intercept avviker signifikant från noll

För att få ett klarare svar på frågan om en testad modell kan förklara avkastningsmönstren i samplet användes GRS-testet utvecklat av Gibbons, Ross och Shanken (1989). GRS testet jämför sharpekvoten (avkastning dividerat med standardavvikelse) mellan den optimala kompositionen (kompositionen med högsta möjliga sharpekvoten) av testportföljerna och den optimala kompositionen av modellernas förklarande faktorer. GRS-testet svarar på frågan om regressionernas interceptvärden gemensamt skiljer från noll. Således är den andra hypotesen:

H_0 = Regressionernas intercept är gemensamt oskiljbara från noll

H_1 = Regressionernas intercept är gemensamt skiljbara från noll

Slutligen jämfördes modellernas GRS-resultat samt intercepten från modellernas portföljregressioner för att ta reda på vilken modell presterat bäst i att förklara testportföljernas avkastningar. Resultaten från dessa test torde tillhandahålla svar på de två forskningsfrågor som presenterades i inledningen.

RESULTAT FRÅN TIDIGARE FORSKNING

I detta kapitel presenteras de viktigaste resultaten från tidigare undersökningar angående femfaktormodellen.

Fama och Frenchs (2015) studie ”A five-factor asset pricing model” utgör premiären för forskarnas femfaktormodell. I studien testar Fama och French modellen på ett amerikanskt sampel över tidsperioden juli 1963 till december 2013. Metoderna som används i studien motsvarar i hög grad den metodik som används i denna avhandling med undantag för datasamplet. Det amerikanska samplet är dock betydligt större och undersökningen görs över en längre tidsperiod. Den största skillnaden uppstår i storleksgrupperingarna. Enligt vad Fama och French (2016b) rapporterar angående brytpunkterna kan man anta att ca 80 % av det amerikanska samplet tillhör den största storlekskvartilen (största 25 %) av det nordiska samplet som undersöks i denna avhandling. Fama och French (2015) använder NYSE (New York Stock Exchange) brytpunkter vid skapandet av både riskpremier och testportföljer. Riskpremierna skapas enligt den metodik som förklarades i föregående kapitel förutom att forskarna bygger *marknadspremiet* från sitt sampel och mäter avkastningar utöver 1-månadsräntan på amerikanska statsskuldväxlar. Testportföljerna skapas från 5 x 5 sorteringar för sammanlagt tre uppsättningar av 25 testportföljer.

Fama och French (2015) använder GRS-test och intercept från regressioner för att jämföra olika kombinationer av tre-, fyr- och femfaktormodeller. Forskarna upptäcker att GRS-testen förkastar alla modellers beskrivningar av portföljavkastningar. Femfaktormodellen och en fyrfaktormodell som lämnar bort *HML*-faktorn presterar generellt bäst i testen enligt alla mätvärden. I de flesta testen följer dock trefaktormodellen och några andra modeller tätt i hälarna på femfaktormodellen. Utöver dessa resultat observerar forskarna att *HML*-faktorn blir överflödig i modeller där *lönsamhet* och *investeringsmängd* är inkluderade som förklarande faktorer.

I studien ”International Tests of a Five-Factor Asset Pricing Model” testar Fama och French (2016b) femfaktormodellen på internationella datasampel över tidsperioden juli 1990 till oktober 2015. Forskarna utför skilda test på fyra regionala sampel: Nordamerika, Europa, Japan och Asien-Stillahavet. Riskfaktorer skapas skilt för varje region enligt samma principer som i Fama och Frenchs tidigare studier. Forskarna använder dock motsvarigheter till NYSE brytpunkter vid storleksgrupperingarna för att bättre kunna jämföra resultaten med tidigare studier. Studiens GRS-test förkastar alla modellers beskrivningar av portföljavkastningar förutom i testen på det japanska samplet. Forskarna konstaterar angående detta att det inte finns tillräckligt med variation i det japanska samplets portföljavkastningar för att utmana någon av modellerna. Studien påvisar även stora skillnader i de regionala riskfaktorerna och noterar att en modell som använder globala faktorer är avsevärt sämre på att förklara portföljavkastningar. Vidare visar studien att femfaktormodellen generellt presterar starkast enligt alla mätvärden. Trefaktormodellen presterar speciellt svagt när det gäller att förklara avkastningarna på testportföljer där *lönsamhet* är en av sorteringsvariablerna.

RESULTATREDOVISNING

Faktoreffekter

Denna studie visar annorlunda avkastningsmönster mellan portföljer med stora skillnader i *storlek*, *book-to-market*, *lönsamhet* och *investeringsmängd* jämfört med belägg från tidigare forskning på större marknader. De studerade effekterna är svagare och mer volatila än vad som noterats i tidigare studier av Fama och French (2015, 2016b). Denna upptäkt kan möjligen vara en följd av att valutafluktuationer eller exponering för oförklarade makroekonomiska risker späder ut samplets faktorpremier.

Samplets *storlekseffekt* är svag och aningen negativ, vilket indikerar att avkastningar på diversifierade portföljer av stora aktier är jämbördiga med avkastningar på diversifierade portföljer av små aktier. Emellertid finner studien att portföljer av små aktier har lägre *marknadsbetan* än portföljer av stora aktier, vilket indikerar att dylika portföljer kan verka som risksäkringar i strategier som fokuserar på större aktier. Angående storlek ger avhandlingen också en större inblick i små aktiers avkastningar genom att gruppera aktierna i portföljer enligt samplets kvartilbrytpunkter istället för NYSE brytpunkter som använts i tidigare studier. Resultaten stöder tidigare resultat som påvisat en avtagande *storlekseffekt* i nyare studier och föreslår att modellernas prestation inte hade försämrats om *storleksfaktorn* utelämnats i test på det Nordiska samplet.

Book-to-market-, *lönsamhets-* samt *investeringseffekten* är alla relativt starka i samplet men ingen avviker mer än två standardfel från noll. Dessa effekter är även mer volatila i det nordiska samplet jämfört med rön från tidigare undersökningar. Jämförelser av effekterna mellan olika storleksgrupper visar att *book-to-market-effekten* och *investeringseffekten* är starkare i portföljer av stora aktier medan *lönsamhetseffekten* är synnerligen stark i portföljer av små aktier.

Höga korrelationer observeras mellan *marknadspremiet*, *storlekspremiet* och *book-to-market-premiet*. *SMB* och *HML* faktorerna har hög negativ korrelation med marknaden och hög positiv korrelation sinsemellan. Omfattningstest av faktorerna visar att *marknadspremiets* och *storleksfaktorns* avkastningar kan förklaras genom en kombination av *HML*-faktorn och den återstående faktorn. Detta resultat styrker konstaterandet att *storleksfaktorn* är överflödig i prissättningsmodellerna för det nordiska samplet. Vidare tyder resultatet på att investeringsstrategier som fokuserar på aktier med hög *book-to-market*-kvot generellt presterar bättre än ordinära diversifieringsstrategier.

Dylikt observeras en hög negativ korrelation mellan samplets *lönsamhetspremie* och *investeringspremie* medan korrelationen mellan dessa faktorer och övriga premier är svag. Dessa två grupper av korrelerade faktorer uppstår möjligen till följd av att *lönsamhets*- och *investeringspremiet* räknas utgående från resultaten över föregående räkenskapsperiod medan resterande faktorer beskriver situationen vid slutet av räkenskapsperioden. Övergripande tyder resultaten från faktorkorrelationerna på att investeringsstrategier som fokuserar på *lönsamhet* och *investeringsmängd* utöver *book-to-market*-kvoten torde prestera synnerligen starkt på samplet utifrån ett ”avkastning-per-varians-perspektiv”.

Modelltest

Avhandlingens primära analys bedömer och jämför CAPM-modellens, trefaktormodellens och femfaktormodellens förmåga att förklara avkastningar på nordiska testportföljer. Samtliga modeller klarar tämligen bra av att förklara avkastningar på portföljuppsättningar sorterade enligt *storlek* och *book-to-market* samt uppsättningar sorterade enligt *storlek* ock *investeringsmängd*. Däremot har alla modeller stora problem när det kommer till att förklara avkastningar på portföljuppsättningen som är sorterad enligt *storlek* och *lönsamhet*.

Studiens GRS-test förkastar tydligt alla modellers beskrivningar av portföljavkastningar och visar att femfaktormodellen är närmast en komplett förklaring till avkastningar vid ett sammanslaget GRS-test av alla 48 portföljers avkastningar. Vid jämförelser av enskilda portföljregressioner kommer det dock fram att ett högre antal intercept som avviker signifikant från noll förekommer i femfaktorregressioner än i regressioner på de andra modellerna. Resultaten från regressionstest av sorterade portföljuppsättningar visar att trefaktormodellen presterar bättre än femfaktormodellen i alla test angående antalet signifikanta intercept och angående interceptens medelavvikelse från noll. I GRS-test presterar trefaktormodellen bättre än femfaktormodellen på två av tre sorterade portföljuppsättningar.

Avkastningarna på portföljer sorterade enligt *storlek* och *lönsamhet* alstrar problem för alla modeller. Fastän femfaktormodellen klarar sig bättre i GRS-testet av portföljuppsättningen finnes flera signifikanta intercept i femfaktorregressionerna jämfört med regressioner på de andra modellerna. Denna diskrepans uppstår på grund av att intercepten generellt är mycket högre i alla modellregressioner gjorda av denna portföljuppsättning. När alla modeller presterar dåligt stöder GRS-testet ofta den modell vars beroende variabler (riskfaktorer) kan skapa en starkare optimalportfölj. En närmare analys finner att modeller som inkluderar avhandlingens *lönsamhetsfaktor* skapar högre intercept och resulterar i en sämre förmåga att förklara avkastningar.

En granskning av alla resultat visar att femfaktormodellen klarar sig relativt bra i GRS-testen på grund av modellens starka optimalportfölj men är sämre än trefaktormodellen på att förklara portföljavkastningar. Således kan slutsatsen dras att trefaktormodellen presterar bäst av de testade modellerna i att förklara portföljavkastningar på avhandlingens nordiska sampel.

AVSLUTNING

I denna avhandling undersöktes om Fama och Frenchs femfaktormodell kan förklara avkastningsmönster på ett nordiskt sampel. Vidare bedömdes och jämfördes tre prissättningsmodeller för att ta reda på vilken modell bäst beskriver avkastningarna på 48 testportföljer med stora skillnader i *storlek*, *book-to-market*-kvot, *lönsamhet* och *investeringsmängd*. För att bedöma modellernas prestation analyserades resultat från GRS-test samt intercept från portföljregressioner.

Avhandlingens GRS-test förkastade alla modeller som kompletta förklaringar på samplets portföljavkastningar. Vidare stödde GRS-testet femfaktormodellen som den närmast kompletta förklaringen av avkastningar. En närmare analys av resultaten visade emellertid att femfaktormodellen hade svårt att förklara individuella portföljavkastningar på samplet. Trefaktormodellen producerade bättre interceptvärden i avhandlingens alla test och presterade bättre än femfaktormodellen på två av tre portföljuppsättningar. En analys av alla resultat ledde till slutsatsen att femfaktormodellen misslyckas förbättra trefaktormodellens förklaring av portföljavkastningar på avhandlingens sampel. Dessa resultat bestrider tidigare forskningsresultat av Fama och French (2015, 2016b).

Resultaten från studien kräver ytterligare forskning angående *lönsamhetseffekten* och angående valutafluktuationers påverkan på prissättningsmodeller. Valutafluktuationer kan mycket väl ha bidragit till svagare faktoreffekter i samplet. Närmare undersökningar kring ämnet kan bidra till en bättre förståelse för hur valutafluktuationer påverkar prissättningsmodellers förklaringsförmåga. Avhandlingens *lönsamhetseffekt* var stark och torde fungera bra i investeringsstrategier, men presterade dåligt när det gällde att förklara avkastningar. Vidare forskning krävs kring detta resultat och angående alternativa mätvärden för *lönsamhet* för att bättre förstå femfaktormodellens svaga prestation i denna avhandling.

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APPENDIX A REGRESSION TABLES

Table A1 CAPM and Three Factor Regressions for *Size-B/M* Sorted Portfolios

CAPM and three-factor regressions for 16 value-weight *Size-B/M* portfolios; July 1999 – June 2015, 192 months. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are independently allocated to four *B/M* groups, again using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-B/M* portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 *Size-B/M* portfolios. The RHS variables are $R_M - R_f$ for the CAPM, and $R_M - R_f$, *SMB*, and *HML* for the three-factor model. The factors are constructed using independent 2 x 3 sorts on *size* and each of *B/M, OP,* and *Inv.* Panel A shows CAPM intercepts, slopes for $R_M - R_f$ and t-statistics for these coefficients. Panel B shows three-factor intercepts, slopes for $R_M - R_f$, *SMB*, and *HML*, and t-statistics for these coefficients. Bolded t-statistics indicate significance at the 5% level.

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + e_{it}$$

Panel A: CAPM

		Coefficient					t-Statistic			
		Low	2	3	High		Low	2	3	High
a	Small	0.17	0.15	0.17	0.15	Small	0.41	0.55	0.76	0.59
	2	-0.39	0.32	0.27	0.13	2	-1.17	1.18	1.07	0.49
	3	-0.04	0.55	0.50	0.51	3	-0.15	**2.22**	1.88	1.65
	Big	-0.27	0.28	0.59	0.64	Big	-1.51	1.36	**2.23**	**2.10**

		Low	2	3	High		Low	2	3	High
$R_M - R_f$	Small	0.81	0.56	0.38	0.37	Small	12.94	14.04	11.04	9.69
	2	0.88	0.59	0.47	0.52	2	17.61	14.64	12.61	12.54
	3	0.86	0.62	0.58	0.55	3	23.35	16.81	14.82	12.05
	Big	0.99	0.68	0.66	0.71	Big	37.45	21.83	16.52	15.54

Panel B: 3-Factor Model

		Low	2	3	High		Low	2	3	High
a	Small	0.61	0.17	0.12	-0.04	Small	1.85	0.80	0.64	-0.23
	2	-0.03	0.25	0.11	-0.17	2	-0.12	1.28	0.60	-1.16
	3	0.09	0.40	0.30	0.19	3	0.41	**1.97**	1.44	0.86
	Big	-0.03	0.05	0.17	0.11	Big	-0.24	0.29	0.90	0.53

		Low	2	3	High		Low	2	3	High
$R_M - R_f$	Small	0.97	0.79	0.58	0.69	Small	15.88	19.61	16.27	21.49
	2	1.02	0.89	0.77	0.90	2	22.41	24.98	23.10	32.54
	3	0.96	0.85	0.86	0.90	3	22.85	22.76	22.30	22.23
	Big	0.80	0.80	0.90	0.95	Big	33.94	23.51	25.32	23.76

		Low	2	3	High		Low	2	3	High
SMB	Small	1.31	0.84	0.63	0.79	Small	10.77	10.57	8.83	12.28
	2	1.13	0.93	0.75	0.81	2	12.51	13.10	11.40	14.83
	3	0.57	0.58	0.63	0.70	3	6.81	7.79	8.19	8.61
	Big	-0.26	0.06	0.13	-0.08	Big	-5.48	0.93	1.80	-1.01

		Low	2	3	High		Low	2	3	High
HML	Small	-0.57	0.03	0.14	0.38	Small	-6.62	0.61	2.69	8.29
	2	-0.47	0.20	0.32	0.56	2	-7.32	3.98	6.93	14.38
	3	-0.15	0.29	0.37	0.56	3	-2.57	5.39	6.79	9.84
	Big	-0.40	0.37	0.68	0.83	Big	-12.07	7.59	13.59	14.83

Table A2 Five Factor Regressions for *Size-B/M* Sorted Portfolios

Five-factor regressions for 16 value-weight *Size-B/M* portfolios; July 1999 – June 2015, 192 months. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are independently allocated to four *B/M* groups, again using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-B/M* portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 *Size-B/M* portfolios. The RHS variables are $R_M - R_f$, *SMB*, *HML*, *RMW*, and *CMA*, constructed using independent 2 x 3 sorts on *size* and each of *B/M, OP,* and *Inv*. The table shows five-factor intercepts, slopes for $R_M - R_f$, *SMB*, *HML*, *RMW*, and *CMA*, and t-statistics for these coefficients. Bolded t-statistics indicate significance at the 5% level.

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$$

		Coefficient				t-Statistic				
		Low	2	3	High		Low	2	3	High
a	Small	0.60	0.36	0.22	0.09	Small	1.78	1.70	1.17	0.52
	2	0.12	0.45	0.18	-0.04	2	0.58	**2.73**	1.09	-0.29
	3	0.26	0.46	0.44	0.35	3	1.22	**2.36**	**2.17**	1.70
	Big	0.02	0.18	0.22	0.18	Big	0.12	1.02	1.16	0.87
		Low	2	3	High		Low	2	3	High
$R_M - R_f$	Small	0.99	0.76	0.58	0.68	Small	15.50	19.14	16.07	21.32
	2	1.02	0.87	0.77	0.88	2	25.48	27.67	23.94	33.71
	3	0.94	0.86	0.85	0.89	3	23.78	23.32	22.21	22.47
	Big	0.79	0.79	0.90	0.95	Big	32.17	23.56	24.77	24.51
		Low	2	3	High		Low	2	3	High
SMB	Small	1.37	0.84	0.66	0.80	Small	10.86	10.64	9.21	12.63
	2	1.22	0.95	0.80	0.82	2	15.30	15.27	12.56	15.87
	3	0.60	0.64	0.65	0.72	3	7.67	8.72	8.57	9.15
	Big	-0.26	0.08	0.16	-0.02	Big	-5.24	1.22	2.21	-0.22
		Low	2	3	High		Low	2	3	High
HML	Small	-0.60	0.05	0.13	0.38	Small	-6.79	0.94	2.64	8.67
	2	-0.50	0.21	0.31	0.57	2	-8.91	4.75	6.93	15.54
	3	-0.16	0.26	0.37	0.57	3	-2.82	5.15	7.02	10.37
	Big	-0.40	0.37	0.67	0.81	Big	-11.59	7.89	13.26	14.96
		Low	2	3	High		Low	2	3	High
RMW	Small	-0.08	-0.28	-0.19	-0.21	Small	-0.80	-4.74	-3.52	-4.55
	2	-0.35	-0.35	-0.18	-0.22	2	-5.94	-7.57	-3.91	-5.71
	3	-0.31	-0.17	-0.24	-0.29	3	-5.21	-3.19	-4.27	-4.90
	Big	-0.07	-0.22	-0.12	-0.19	Big	-1.94	-4.46	-2.31	-3.30
		Low	2	3	High		Low	2	3	High
CMA	Small	0.16	-0.06	0.04	-0.01	Small	1.42	-0.83	0.60	-0.18
	2	0.16	0.00	0.09	-0.01	2	2.29	-0.03	1.63	-0.18
	3	0.05	0.12	0.01	0.01	3	0.65	1.84	0.16	0.08
	Big	-0.01	0.01	0.06	0.14	Big	-0.26	0.17	1.00	1.96

Table A3 CAPM and Three Factor Regressions for *Size-OP* Sorted Portfolios

CAPM and three-factor regressions for 16 value-weight *Size-OP* portfolios; July 1999 – June 2015, 192 months. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are independently allocated to four *OP* groups, again using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-OP* portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 *Size-OP* portfolios. The RHS variables are $R_M - R_f$ for the CAPM, and $R_M - R_f$, *SMB*, and *HML* for the three-factor model. The factors are constructed using independent 2 x 3 sorts on *size* and each of *B/M, OP,* and *Inv.* Panel A shows CAPM intercepts, slopes for $R_M - R_f$ and t-statistics for these coefficients. Panel B shows three-factor intercepts, slopes for $R_M - R_f$, *SMB*, and *HML*, and t-statistics for these coefficients. Bolded t-statistics indicate significance at the 5% level.

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + e_{it}$$

Panel A: CAPM

		Coefficient					t-Statistic			
		Low	2	3	High		Low	2	3	High
a	Small	-0.40	0.28	0.84	0.62	Small	-1.36	1.26	**3.67**	**2.25**
	2	-0.73	0.19	0.50	0.61	2	**-2.16**	0.82	**2.02**	**2.50**
	3	-0.19	0.51	0.43	0.45	3	-0.58	**2.12**	1.78	1.88
	Big	0.48	0.11	-0.11	0.07	Big	0.93	0.48	-0.54	0.37
		Low	2	3	High		Low	2	3	High
$R_M - R_f$	Small	0.56	0.36	0.36	0.50	Small	12.90	11.15	10.67	12.18
	2	0.78	0.51	0.53	0.56	2	15.48	15.05	14.36	15.45
	3	0.77	0.59	0.58	0.73	3	15.51	16.29	16.14	20.37
	Big	1.00	0.72	0.85	0.98	Big	12.97	20.81	28.04	35.12

Panel B: 3-Factor Model

		Coefficient					t-Statistic			
		Low	2	3	High		Low	2	3	High
a	Small	-0.31	0.18	0.67	0.54	Small	-1.37	1.09	**3.92**	**2.49**
	2	-0.67	0.09	0.40	0.58	2	**-2.56**	0.58	**2.50**	**3.34**
	3	-0.23	0.43	0.23	0.35	3	-0.77	**2.26**	1.19	1.76
	Big	0.43	0.00	-0.32	0.22	Big	0.84	-0.01	-1.75	1.24
		Low	2	3	High		Low	2	3	High
$R_M - R_f$	Small	0.81	0.60	0.61	0.79	Small	19.19	19.17	19.16	19.57
	2	1.07	0.78	0.84	0.82	2	22.00	27.05	28.45	25.67
	3	0.99	0.83	0.83	0.96	3	17.41	23.52	23.38	26.19
	Big	0.87	0.73	0.98	0.88	Big	9.10	16.95	28.93	26.91
		Low	2	3	High		Low	2	3	High
SMB	Small	1.01	0.65	0.60	0.84	Small	12.08	10.55	9.42	10.52
	2	1.13	0.78	0.89	0.87	2	11.68	13.58	15.27	13.61
	3	0.70	0.71	0.52	0.63	3	6.20	10.11	7.39	8.61
	Big	-0.51	-0.17	0.10	-0.08	Big	-2.70	-1.98	1.44	-1.29
		Low	2	3	High		Low	2	3	High
HML	Small	-0.05	0.21	0.31	0.21	Small	-0.85	4.75	6.91	3.75
	2	0.01	0.22	0.25	0.13	2	0.21	5.47	5.95	2.84
	3	0.13	0.20	0.37	0.22	3	1.59	3.95	7.38	4.27
	Big	0.02	0.16	0.34	-0.24	Big	0.16	2.69	7.10	-5.29

Table A4 Five Factor Regressions for *Size-OP* Sorted Portfolios

Five-factor regressions for 16 value-weight *Size-OP* portfolios; July 1999 – June 2015, 192 months. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are independently allocated to four *OP* groups, again using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-OP* portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 *Size-OP* portfolios. The RHS variables are $R_M - R_f$, *SMB*, *HML*, *RMW*, and *CMA*, constructed using independent 2 x 3 sorts on *size* and each of *B/M, OP,* and *Inv*. The table shows five-factor intercepts, slopes for $R_M - R_f$, *SMB*, *HML*, *RMW*, and *CMA*, and t-statistics for these coefficients. Bolded t-statistics indicate significance at the 5% level.

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$$

		Coefficient				t-Statistic			
		Low	2	3	High	Low	2	3	High
a	Small	-0.14	0.36	0.68	0.61	-0.72	**2.23**	**3.81**	**2.75**
	2	-0.27	0.26	0.44	0.52	-1.76	1.95	**2.73**	**2.96**
	3	0.03	0.56	0.28	0.43	0.15	**3.09**	1.48	**2.13**
	Big	0.76	0.40	-0.18	0.09	**2.75**	**2.20**	-0.99	0.60
$R_M - R_f$	Small	0.80	0.57	0.62	0.77	21.41	18.87	18.27	18.29
	2	1.03	0.76	0.83	0.84	35.62	30.09	27.20	25.15
	3	0.98	0.82	0.83	0.95	22.25	23.70	22.56	24.87
	Big	0.92	0.67	0.95	0.89	17.57	19.68	27.00	29.73
SMB	Small	1.07	0.64	0.62	0.83	14.46	10.69	9.20	9.90
	2	1.18	0.80	0.91	0.92	20.63	15.84	14.89	13.77
	3	0.81	0.73	0.54	0.63	9.27	10.66	7.46	8.31
	Big	-0.17	-0.20	0.05	-0.13	-1.63	-2.92	0.72	-2.16
HML	Small	-0.06	0.23	0.30	0.22	-1.25	5.45	6.49	3.81
	2	0.02	0.23	0.25	0.10	0.61	6.52	5.76	2.19
	3	0.10	0.20	0.36	0.23	1.62	4.14	7.15	4.27
	Big	-0.11	0.21	0.37	-0.23	-1.53	4.43	7.63	-5.66
RMW	Small	-0.35	-0.25	-0.03	-0.10	-6.28	-5.66	-0.59	-1.66
	2	-0.70	-0.29	-0.09	0.02	-16.41	-7.72	-1.95	0.37
	3	-0.57	-0.23	-0.12	-0.13	-8.79	-4.60	-2.21	-2.28
	Big	-0.98	-0.58	-0.15	0.26	-12.77	-11.54	-2.83	5.80
CMA	Small	0.10	-0.08	0.04	-0.04	1.57	-1.41	0.65	-0.60
	2	0.02	-0.01	0.02	0.13	0.38	-0.14	0.28	2.15
	3	0.19	0.01	0.04	-0.02	2.49	0.22	0.56	-0.25
	Big	0.75	-0.17	-0.15	-0.07	8.08	-2.84	-2.38	-1.36

Table A5 CAPM and Three Factor Regressions for *Size-Inv* Sorted Portfolios

CAPM and three-factor regressions for 16 value-weight *Size-Inv* portfolios; July 1999 – June 2015, 192 months. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are independently allocated to four *Inv* groups, again using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-Inv* portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 *Size-Inv* portfolios. The RHS variables are $R_M - R_f$ for the CAPM, and $R_M - R_f$, *SMB*, and *HML* for the three-factor model. The factors are constructed using independent 2 x 3 sorts on *size* and each of *B/M, OP,* and *Inv*. Panel A shows CAPM intercepts, slopes for $R_M - R_f$ and t-statistics for these coefficients. Panel B shows three-factor intercepts, slopes for $R_M - R_f$, *SMB*, and *HML*, and t-statistics for these coefficients. Bolded t-statistics indicate significance at the 5% level.

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + e_{it}$$

Panel A: CAPM

		Coefficient				t-Statistic			
		Low	2	3	High	Low	2	3	High
a	Small	0.17	0.19	0.36	-0.10	0.61	0.78	1.45	-0.32
	2	-0.09	0.48	0.32	-0.31	-0.30	**2.09**	1.31	-1.04
	3	0.52	0.68	0.38	-0.03	1.79	**2.95**	1.45	-0.12
	Big	0.63	0.51	-0.36	-0.16	1.64	**2.86**	-1.64	-0.78
$R_M - R_f$	Small	0.54	0.34	0.43	0.57	13.17	9.18	11.62	12.48
	2	0.70	0.50	0.49	0.70	16.15	14.38	13.65	15.52
	3	0.63	0.56	0.63	0.80	14.68	16.14	16.29	21.58
	Big	0.83	0.74	0.90	0.99	14.48	27.85	27.62	33.02

Panel B: 3-Factor Model

		Coefficient				t-Statistic			
		Low	2	3	High	Low	2	3	High
a	Small	0.14	0.07	0.34	0.00	0.62	0.36	1.71	-0.02
	2	-0.10	0.34	0.20	-0.22	-0.44	**2.07**	1.25	-1.07
	3	0.45	0.49	0.23	-0.08	1.68	**2.65**	1.16	-0.39
	Big	0.52	0.38	-0.36	-0.08	1.40	**2.26**	-1.64	-0.41
$R_M - R_f$	Small	0.78	0.61	0.66	0.81	18.41	17.66	17.91	18.37
	2	0.99	0.76	0.79	0.99	23.80	24.71	26.50	25.65
	3	0.82	0.79	0.90	1.02	16.30	22.75	24.24	26.35
	Big	0.70	0.84	0.88	0.97	10.17	26.68	21.35	25.80
SMB	Small	0.80	0.74	0.77	1.03	9.45	10.69	10.53	11.70
	2	0.98	0.70	0.83	1.18	11.86	11.34	14.05	15.34
	3	0.53	0.49	0.71	0.68	5.36	7.06	9.62	8.87
	Big	-0.64	0.12	-0.08	0.06	-4.70	1.88	-0.97	0.78
HML	Small	0.11	0.27	0.10	-0.05	1.90	5.44	1.95	-0.87
	2	0.10	0.28	0.26	-0.04	1.76	6.51	6.28	-0.71
	3	0.15	0.34	0.29	0.14	2.15	7.02	5.57	2.59
	Big	0.12	0.22	0.00	-0.11	1.24	4.85	0.06	-2.10

Table A6 Five Factor Regressions for *Size-Inv* Sorted Portfolios

Five-factor regressions for 16 value-weight *Size-Inv* portfolios; July 1999 – June 2015, 192 months. At the end of June each year, stocks are distributed into four *size* groups using sample quartile breakpoints. Stocks are independently allocated to four *Inv* groups, again using sample quartile breakpoints. The intersections of the two sorts produce 16 *Size-Inv* portfolios. The LHS variables in each set of 16 regressions are the monthly excess returns on the 16 *Size-Inv* portfolios. The RHS variables are $R_M - R_f$, *SMB*, *HML*, *RMW*, and *CMA*, constructed using independent 2 x 3 sorts on *size* and each of *B/M, OP,* and *Inv*. The table shows five-factor intercepts, slopes for $R_M - R_f$, *SMB*, *HML*, *RMW*, and *CMA*, and t-statistics for these coefficients. Bolded t-statistics indicate significance at the 5% level.

$$R_{it} - R_{ft} = a_i + \beta_i(R_{Mt} - R_{ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it}$$

	Coefficient					t-Statistic				
		Low	2	3	High		Low	2	3	High
α	Small	0.13	0.19	0.52	0.30	Small	0.68	1.03	**2.64**	1.32
	2	-0.06	0.36	0.33	0.18	2	-0.40	**2.33**	**2.12**	1.02
	3	0.57	0.53	0.30	0.14	3	**2.65**	**2.99**	1.51	0.67
	Big	0.37	0.30	0.03	0.06	Big	1.79	**2.00**	0.16	0.34
		Low	2	3	High		Low	2	3	High
$R_M - R_f$	Small	0.82	0.60	0.63	0.75	Small	21.99	17.44	16.78	17.10
	2	1.02	0.78	0.76	0.91	2	33.60	26.98	25.75	27.12
	3	0.83	0.80	0.90	0.98	3	20.25	23.61	24.34	25.23
	Big	0.82	0.88	0.79	0.91	Big	20.91	30.61	20.99	27.58
		Low	2	3	High		Low	2	3	High
SMB	Small	0.94	0.76	0.74	0.94	Small	12.70	11.04	9.97	10.80
	2	1.14	0.77	0.82	1.08	2	18.84	13.48	13.89	16.19
	3	0.67	0.55	0.76	0.64	3	8.17	8.18	10.27	8.29
	Big	-0.26	0.24	-0.20	-0.11	Big	-3.29	4.13	-2.70	-1.67
		Low	2	3	High		Low	2	3	High
HML	Small	0.05	0.27	0.13	0.02	Small	0.92	5.61	2.54	0.25
	2	0.03	0.25	0.28	0.04	2	0.73	6.16	6.73	0.90
	3	0.10	0.32	0.28	0.18	3	1.74	6.74	5.36	3.34
	Big	-0.07	0.15	0.09	-0.02	Big	-1.32	3.85	1.80	-0.48
		Low	2	3	High		Low	2	3	High
RMW	Small	-0.18	-0.21	-0.23	-0.35	Small	-3.33	-4.14	-4.24	-5.46
	2	-0.27	-0.13	-0.19	-0.49	2	-6.08	-2.97	-4.44	-9.85
	3	-0.38	-0.15	-0.16	-0.28	3	-6.19	-2.96	-2.98	-4.81
	Big	-0.31	-0.04	-0.45	0.01	Big	-5.28	-0.96	-8.00	0.23
		Low	2	3	High		Low	2	3	High
CMA	Small	0.34	0.01	-0.13	-0.31	Small	5.16	0.14	-1.92	-3.98
	2	0.38	0.19	-0.06	-0.35	2	6.99	3.63	-1.09	-5.86
	3	0.29	0.14	0.09	-0.16	3	3.97	2.34	1.41	-2.39
	Big	0.97	0.30	-0.40	-0.44	Big	13.96	5.98	-5.99	-7.55

Table A7 Full summary statistics for model comparison tests

Summary statistics for tests of CAPM, three-, four-, and five-factor models; July 1999 – June 2015, 192 months. The table tests the ability of CAPM, three-, four- and five-factor models to explain monthly excess returns on 16 *Size-B/M* portfolios (Panel A), 16 *Size-OP* portfolios (Panel B), 16 *Size-Inv* portfolios (Panel C) and a joint sample of all 48 portfolios (Panel D). For each panel, the table shows the tested model, the GRS statistic testing whether the expected values of all 16 or 48 intercept estimates are zero, the *p*-value of the GRS statistic, the average absolute value of the intercepts, $Avg\ |\alpha|$, the average adjusted R^2, and the average Akaike information criterion, *Avg AIC*, from the model regressions. Bolded GRS-statistics indicate significance at the 5% level.

Panel A: Size-B/M Portfolios

	fGRS	*pGRS*	*Avg* $\|\alpha\|$	*Avg Adj* R^2	*Avg AIC*
SMB RMW	1.56	0.08	0.45	0.70	1041
CAPM	1.27	0.22	0.32	0.53	1132
SMB HML RMW	1.25	0.23	0.29	0.78	973
SMB CMA	1.23	0.25	0.26	0.69	1048
SMB RMW CMA	1.22	0.25	0.35	0.71	1036
SMB HML RMW CMA	1.12	0.34	0.26	0.78	972
RMW CMA	1.08	0.38	0.42	0.57	1114
SMB HML CMA	1.01	0.45	0.14	0.76	991
HML RMW	0.99	0.47	0.22	0.66	1056
HML CMA	0.99	0.47	0.13	0.64	1072
HML RMW CMA	0.97	0.50	0.28	0.67	1051
SMB HML	0.95	0.51	0.18	0.75	1006

Panel B: Size-OP Portfolios

	fGRS	*pGRS*	*Avg* $\|\alpha\|$	*Avg Adj* R^2	*Avg AIC*
CAPM	**3.86**	**0.00**	0.41	0.55	1121
SMB CMA	**3.81**	**0.00**	0.41	0.73	1018
SMB RMW	**3.68**	**0.00**	0.50	0.76	987
HML CMA	**3.50**	**0.00**	0.34	0.62	1083
SMB HML CMA	**3.48**	**0.00**	0.35	0.75	1001
SMB HML	**3.38**	**0.00**	0.35	0.72	1026
HML RMW	**3.32**	**0.00**	0.37	0.66	1056
SMB HML RMW	**3.26**	**0.00**	0.41	0.79	964
RMW CMA	**2.90**	**0.00**	0.45	0.62	1075
SMB RMW CMA	**2.87**	**0.00**	0.42	0.77	981
HML RMW CMA	**2.76**	**0.00**	0.39	0.67	1045
SMB HML RMW CMA	**2.74**	**0.00**	0.38	0.79	959

Panel C: Size-Inv Portfolios

	fGRS	*pGRS*	*Avg* $\|\alpha\|$	*Avg Adj* R^2	*Avg AIC*
SMB RMW	**2.40**	**0.00**	0.46	0.75	1004
SMB HML RMW	**2.13**	**0.01**	0.36	0.77	987
HML RMW	**2.01**	**0.01**	0.31	0.64	1071
CAPM	**1.89**	**0.02**	0.33	0.56	1116
SMB CMA	**1.73**	**0.04**	0.23	0.76	995
SMB RMW CMA	1.55	0.09	0.32	0.78	974
SMB HML RMW CMA	1.52	0.10	0.27	0.79	960
RMW CMA	1.52	0.10	0.38	0.63	1067
SMB HML	1.50	0.10	0.25	0.72	1029
HML CMA	1.46	0.12	0.19	0.63	1069
SMB HML CMA	1.44	0.13	0.19	0.77	984
HML RMW CMA	1.38	0.16	0.29	0.67	1045

Panel D: ALL Portfolios

	fGRS	*pGRS*	*Avg* $\|\alpha\|$	*Avg Adj* R^2	*Avg AIC*
CAPM	**2.43**	**0.00**	0.35	0.55	1123
SMB CMA	**2.36**	**0.00**	0.30	0.73	1020
SMB RMW	**2.36**	**0.00**	0.47	0.74	1011
SMB HML	**2.31**	**0.00**	0.26	0.73	1020
SMB HML CMA	**2.25**	**0.00**	0.23	0.76	992
SMB HML RMW	**2.21**	**0.00**	0.35	0.78	975
HML CMA	**2.21**	**0.00**	0.22	0.63	1075
HML RMW	**2.20**	**0.00**	0.30	0.66	1061
SMB RMW CMA	**2.00**	**0.00**	0.36	0.75	997
RMW CMA	**2.00**	**0.00**	0.42	0.61	1085
SMB HML RMW CMA	**1.95**	**0.00**	0.30	0.79	964
HML RMW CMA	**1.93**	**0.00**	0.32	0.67	1047

APPENDIX B CHARACTERISTICS OF STOCKS IN SORTED PORTFOLIOS

Table B1 Average *size, B/M, OP* and *Inv* of stocks in portfolios sorted on *Size-B/M, Size-OP,* and *Size-Inv*

Time-series averages of market cap (Size), book-to-market ratios (*B/M*), profitability (*OP*), and investment (*Inv*) for portfolios formed on *size* and *B/M, size* and *OP,* and *size* and *Inv*. In the sort for June of year t, *B* is *book equity* at the end of fiscal year t – 1 and *M* is market cap at the end of December year t – 1, calculated as Price multiplied by common shares outstanding at the end of December year t – 1. *Size-OP* and *Size-Inv* portfolios are formed in a similar way, but the second sorting variable is *operating profitability* or Investment. *operating profitability*, *OP,* in the sort for June of year t is Operating Income at the end of fiscal year t – 1 divided by *book equity* at the end of fiscal year t – 1. *Investment*, *Inv,* is the change in Total Assets from the end of fiscal year t – 2 to the end of fiscal year t – 1, divided by t – 2 Total Assets. Each of the ratios for a portfolio for a given year is the value-weight average of the ratios for the firms in the portfolio. The table shows the time-series averages of the ratios for the 16 portfolio formation years 1999 – 2015.

		Size-BM Sorted Portfolios				*Size-OP Sorted Portfolios*				*Size-Inv Sorted Portfolios*			
		Low	2	3	High	Low	2	3	High	Low	2	3	High
Size (M€)	Small	29	29	29	26	27	28	29	29	27	28	29	28
	2	109	108	99	102	101	101	108	107	103	102	106	106
	3	445	422	426	436	410	440	422	446	423	429	444	429
	Big	37209	15471	12154	6668	6266	10553	15733	43836	13122	14994	25461	29488
		Low	2	3	High	Low	2	3	High	Low	2	3	High
B/M	Small	0.23	0.54	0.88	1.75	1.10	1.22	0.93	0.66	1.10	1.13	1.03	0.84
	2	0.25	0.52	0.87	1.74	0.96	1.09	0.84	0.51	0.94	0.91	0.85	0.79
	3	0.24	0.52	0.86	1.63	0.95	0.95	0.73	0.43	0.82	0.77	0.72	0.65
	Big	0.19	0.52	0.84	1.36	0.80	0.71	0.62	0.29	0.56	0.56	0.47	0.46
		Low	2	3	High	Low	2	3	High	Low	2	3	High
OP	Small	-0.11	0.04	0.03	0.01	-0.25	0.08	0.17	0.37	-0.14	0.06	0.08	0.07
	2	0.09	0.12	0.09	0.06	-0.19	0.08	0.17	0.34	-0.02	0.12	0.16	0.12
	3	0.19	0.16	0.12	0.08	-0.16	0.09	0.18	0.34	0.05	0.16	0.17	0.18
	Big	0.34	0.25	0.16	0.10	-0.07	0.08	0.18	0.42	0.15	0.24	0.26	0.29
		Low	2	3	High	Low	2	3	High	Low	2	3	High
Inv	Small	0.13	0.08	0.06	0.03	0.01	0.07	0.09	0.17	-0.15	0.01	0.11	0.37
	2	0.13	0.09	0.08	0.07	0.05	0.09	0.09	0.14	-0.14	0.01	0.11	0.38
	3	0.17	0.11	0.10	0.08	0.07	0.11	0.13	0.16	-0.12	0.01	0.10	0.37
	Big	0.11	0.09	0.07	0.09	0.02	0.06	0.10	0.13	-0.11	0.02	0.11	0.32

APPENDIX C STOCKS, INDICES AND DATA VARIABLES

Table C1 Downloaded constituent lists, indices and stock variables

Names, symbols and databases for downloaded variables used in the analysis. Panel A shows constituent list name, symbol and database of the lists naming all stocks used in the analysis of this thesis. Panel B shows variable name, symbol and database of the benchmark index, risk-free rate, and exchange rates of this study. Panel C shows the variable description, symbol, time density and database of the data variables downloaded for each instrument.

Panel A: Constituent lists included in original sample

Constituent List Name	Symbol	Database
OMX Helsinki	LHEXINDX	TDS
OMX Copenhagen	LCOSEASH	TDS
Oslo Exchange All-Share	LOSLOASH	TDS
OMX Stockholm	LSWSEALI	TDS
Dead Stocks DK	DEADDK	TDS
Dead Stocks FN	DEADFN	TDS
Dead Stocks NW	DEADNW	TDS
Dead Stocks SD	DEADSD	TDS

Panel B: Benchmark, risk-free rate, and exchange rates

Variable Name	Symbol	Database
MSCI Nordic	MSNORD$	MSCI
EBF Euribor 1M Delayed	EIBOR1M-IR	EBF
Swedish Krona To Euro	SDECBSP	ECB
Norwegian Krone To Euro	NWECBSP	ECB
Danish Krone To Euro	DKECBSP	ECB
US $ To Euro	USECBSP	ECB

Panel C: Stock variables downloaded from TDS and Worldscope

Variable description	Symbol	Time Density	Database
Type Of Instrument	TYPE	Static	TDS
Geographic Group	GEOG	Static	TDS
Currency - Earnings	ECUR	Static	TDS
Time - Latest Value	TIME	Static	TDS
Price (Adjusted - Default)	P	Monthly	TDS
Total Return Index	RI	Monthly	TDS
Common Shares Outstanding	WC05301	Yearly	Worldscope
Total Assets	WC02999	Yearly	Worldscope
Total Liabilities	WC03351	Yearly	Worldscope
Operating Income	WC01250	Yearly	Worldscope