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## Estimate for the Bulk Viscosity of Strongly Coupled Quark Matter Using Perturbative QCD and Holography

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Modern hydrodynamic simulations of core-collapse supernovae and neutron-star mergers require knowledge not only of the equilibrium properties of strongly interacting matter, but also of the system's response to perturbations, encoded in various transport coefficients. Using perturbative and holographic tools, we derive here an improved weak-coupling and a new strong-coupling result for the most important transport coefficient of unpaired quark matter, its bulk viscosity. These results are combined in a simple analytic pocket formula for the quantity that is rooted in perturbative quantum chromodynamics at high densities but takes into account nonperturbative holographic input at neutron-star densities, where the system is strongly coupled. This expression can be used in the modeling of unpaired quark matter at astrophysically relevant temperatures and densities.

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**Introduction**—During the last ten years, neutron stars (NSs) and their binary mergers—observable through both electromagnetic and gravitational waves (GW) [1,2]—have established themselves as the leading laboratory for dense quantum chromodynamics (QCD) matter. While the

observable properties of single quiescent NSs and even the inspiral parts of NS mergers are mostly determined by the equation of state (EoS) of the constituent matter, the ringdown phase of a NS merger constitutes a considerably more complicated out-of-equilibrium system. In preparation for the eventual observation of a ringdown GW signal, extensive hydrodynamic simulations of NS mergers are currently being carried out, with one crucial challenge being to correctly account for energy dissipation and transport in NS matter [3].

Among the different transport coefficients, the bulk viscosity  $\zeta$ , which quantifies energy dissipation during a rapid compression or expansion of matter, stands out as particularly important [4–12]. For isolated NSs, it affects the emission of continuous GWs [13], expected to be detectable in next-generation GW observatories such as the Einstein Telescope [14] and Cosmic Explorer [15], and determines the maximal rotation frequencies of pulsars in a temperature-dependent fashion, giving rise to the so-called  $r$ -mode stability window in the 1–100 keV range [16–18] (for a review of NS oscillatory modes, see [19]). In NS

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mergers, the bulk viscosity on the other hand provides damping for density oscillations, affecting both the inspiral [20] and post-merger dynamics, of which the latter involves temperatures up to tens of MeVs. The bulk viscosity may indeed leave a detectable imprint on the post-merger GW waveform [21–26], the magnitude of which is however still under discussion [27].

The dominant contribution to the bulk viscosity comes about when weak interactions cannot keep pace with the compression rate, leading to deviations from beta equilibrium and a nonequilibrium contribution to the pressure, against which work can be done. This effect peaks when the timescales of macroscopic oscillations and microscopic flavor-changing rates match. In the nuclear matter phase, the value of  $\zeta$  depends on multiple factors, such as whether direct Urca processes are allowed or if hyperons or Cooper pairing between nucleons are present, each affecting in particular the temperature scale where  $\zeta$  reaches its maximal value (see Ref. [28] for a review).

The first milliseconds of a binary NS merger are known to involve baryon densities up to several nuclear saturation densities  $n_{\text{sat}} \approx 0.16/\text{fm}^3$  as well as temperatures up to several tens of MeVs (see, e.g., [29]). Such conditions may also lead to the creation of deconfined QM [30–34], the transport properties of which differ significantly from those of nuclear matter. While the value of the QM bulk viscosity is expected to strongly depend on the presence and details of quark pairing, differences between various partially paired configurations are expected to be smaller than between quark and nuclear matter [28]. This makes the bulk viscosity an interesting quantity for tracking the possible creation of QM during mergers.

Despite the phenomenological importance of the bulk viscosity, our ability to predict its behavior remains limited owing to the unavailability of controlled first-principles quantum-field-theory methods at NS densities. The leading first-principles tools include perturbative QCD (pQCD), available only at very high densities (see, e.g., [35–37]), and holography, which describes the strong-coupling limit of a class of QCD-like theories [38–42]. For QM, leading-order perturbative results for several transport coefficients were derived some thirty years ago [43,44] and improved to next-to-leading order (NLO) later [45,46], whereas at strong coupling, the shear viscosity and the electrical and thermal conductivities were first evaluated only recently in two holographic models [47,48]. For the bulk viscosity, only the minuscule purely QCD contribution has been considered in recent literature [47,49], but for the dominant contribution stemming from an interplay between the electroweak and strong sectors, no strong-coupling prediction is currently available at all.

In this work, we derive state-of-the-art results for the thermodynamic response of QM to a change in its flavor content, thus providing novel predictions for the bulk viscosity. We do so using both perturbative and holographic

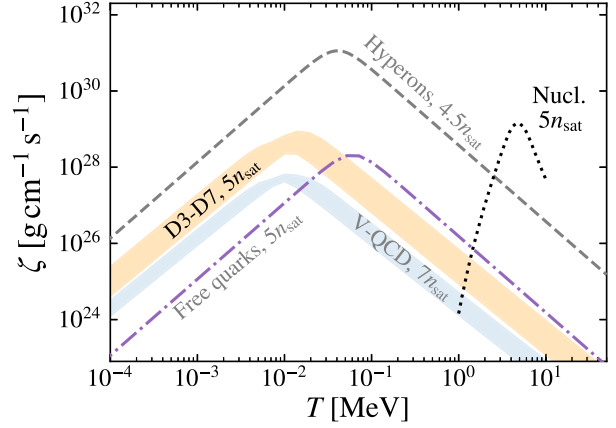


FIG. 1. The bulk viscosity  $\zeta$  of NS matter, evaluated at rotation frequency  $\omega = 2\pi \times 1$  kHz and given as a function of  $T$  for a baryon density  $n_B \approx 5n_{\text{sat}}$ . The uncertainty bands of the holographic results are assessed via their matching to QCD: The D3-D7 result is matched to pQCD quark densities including their uncertainty bands, while the uncertainty of the V-QCD result is estimated by varying the parameters of the model within limits set by lattice-QCD results. Finally, nuclear and hyperonic matter results (labeled Nucl. and Hyperons) from Refs. [24,50] are shown for comparison. Note that for technical reasons, the V-QCD result is shown for  $7n_{\text{sat}}$  and the hyperonic one for  $4.5n_{\text{sat}}$ . We observe that our QM results always peak within the  $r$ -mode stability window 1–100 keV, but are strongly suppressed at the  $O(10$  MeV) temperatures involved in NS mergers. This may, however, be related to the absence of quark pairing in our setup (see [51] for a counterexample in the color-flavor-locked case).

methods, and in particular derive the first strong-coupling predictions for the quantity. Our results are applicable for unpaired QM and serve as a starting point for any partially unpaired phase [28].

The main result of our work is shown in Fig. 1, where we display the bulk viscosity of NS matter as a function of temperature for a baryon density of roughly  $5n_{\text{sat}}$ . For QM, we include results corresponding to the free-theory limit, evaluated at a fixed strange quark mass ( $m_s = 93.4$  MeV), as well as our two holographic models, D3-D7 and V-QCD, but not the pQCD result, which is not under quantitative control at intermediate densities. For the confined phase, we display results corresponding to both nuclear [24] and hyperonic [50] matter. As we discuss in detail in the remaining sections of this Letter, our results paint a consistent picture of the behavior of the QM bulk viscosity that displays a stark qualitative difference to that witnessed in the confined phases of QCD. Furthermore, we observe that for astrophysically relevant densities and temperatures, nearly all temperature dependence in the QM result originates from the flavor-changing interactions. For our D3-D7 computation, this leads to a simple analytic result for  $\zeta$ , given in Eq. (5) below, that we suggest for use as an approximation for the bulk viscosity of unpaired QM in future phenomenological applications.

*Setup*—For unpaired three-flavor QM in the neutrino-transparent regime, the leading contribution to the bulk viscosity arises from  $W$ -boson exchange in the process  $u + d \leftrightarrow u + s$ . Outside beta equilibrium, i.e., when the  $d$  and  $s$  quark chemical potentials differ  $\mu_d \neq \mu_s$ , the quark densities  $n_d$  and  $n_s$  change with rates proportional to an electroweak rate  $\lambda_1$  [52–54], so that

$$\frac{dn_d}{dt} = -\frac{dn_s}{dt} \approx \lambda_1(\mu_s - \mu_d). \quad (1)$$

Neglecting quark masses, the leading low- $T$  contribution to the rate becomes [54,55]

$$\lambda_1 = \left(1 + \sigma \log \frac{\Lambda}{T}\right)^4 \frac{64}{5\pi^3} G_F^2 \sin^2 \theta_c \cos^2 \theta_c \mu_d^5 T^2, \quad (2)$$

where  $G_F$  is the Fermi constant and  $\theta_c$  the Cabibbo angle. The quartic prefactor on the right-hand side represents the only known  $\mathcal{O}(\alpha_s)$  correction to the rate, which is moreover logarithmically enhanced at low temperatures as it originates from a so-called non-Fermi-liquid (nFL) contribution to the specific heat of QM [57] (see also [58,59]). As discussed in detail around Fig. 5 of the Supplemental Material [60], this correction allows us to gauge the importance of the (partially unknown)  $\mathcal{O}(\alpha_s)$  corrections to the rate: for  $\sigma = 0$ , the result reduces to the leading-order rate, while for  $\sigma \equiv 4\alpha_s/(9\pi)$  and  $\Lambda \approx 0.158 \sqrt{\alpha_s} \sqrt{\mu_u^2 + \mu_d^2 + \mu_s^2}$  one recovers the result derived in [57].

While the unknown QCD corrections to the rate may be sizable, we note that the qualitative behavior of the rate likely remains the same at strong coupling: In holography, the QCD contribution to the rate, replacing the leading-order multiplicative factor  $\mu_d^5 T^2$  above, is available from the convolution of two flavor-current correlators. For these correlators, calculations at nonzero quark densities in the D3-D7 model show a linear dependence on the temperature at low frequencies [61–63], consistent with the formula we use. Furthermore, the normalization of the correlators depends on the number of colors and flavors but not on the 't Hooft coupling, thus keeping the rate constant in the strong-coupling limit.

A study of energy dissipation during a compression-decompression cycle near beta equilibrium connects  $\zeta$  to various susceptibilities  $\chi_{ij} \equiv \partial^2 p / \partial \mu_i \partial \mu_j$  and reaction rates (see Supplemental Material Sec. A [60]) [64]. If we only take into account the  $u + d \leftrightarrow u + s$  process [46], this leads to

$$\zeta = \frac{\lambda_1 A_1^2}{\omega^2 + (\lambda_1 C_1)^2}, \quad (3)$$

where the coefficients  $A_1$  and  $C_1$ , determined by various susceptibilities and quark densities, are found in

Eqs. (33) and (34) of the Supplemental Material [60], and  $\omega$  denotes the angular frequency of density oscillations (see [19] for discussion) [65].

The combination of susceptibilities appearing in Eq. (33) vanishes if the  $d$  and  $s$  quarks are degenerate in mass—a fact most easily verified if (33) is given in terms of the inverse susceptibility matrix (see Supplemental Material [60] for details). This implies that a nonzero strange quark mass must be implemented in both the weak- and strong-coupling setups, which we briefly introduce below.

*Methods*—In this section, we review our perturbative and holographic determinations of the susceptibilities that enter Eq. (3). In both calculations, we treat electrons as non-interacting and (numerically) solve the corresponding chemical potential  $\mu_e$  from the charge neutrality condition  $2n_u/3 - n_d/3 - n_s/3 = n_e = T^2 \mu_e/3 - \mu_e^3/(3\pi^2)$ . Together with the beta-equilibrium conditions  $\mu_s = \mu_d$ ,  $\mu_u = \mu_d - \mu_e$ , this allows us to obtain  $\zeta$  in terms of  $\mu_d$ ,  $T$ ,  $\omega$ . Finally, our results will depend on the parameter  $X \equiv \bar{\Lambda}/(2\mu_d)$  which parametrizes our results' dependence on the unphysical renormalization scale  $\bar{\Lambda}$  in the  $\overline{\text{MS}}$  scheme. It appears directly in our pQCD results and indirectly in the D3-D7 ones, where it enters through the high-density matching of the model to pQCD.

*Perturbative QCD:* For vanishing quark masses, the perturbative pressure of deconfined unpaired QCD matter is known up to order  $\alpha_s^{5/2}$  at nonzero temperatures and densities [66,67] and up to partial  $\mathcal{O}(\alpha_s^3)$  in the  $T = 0$  limit [37,68,69]. Up to the highest fully known order  $\mathcal{O}(\alpha_s^{5/2})$ , the result can be split into two distinct terms corresponding to contributions from the hard and soft momentum scales, which for  $\mu \gg T$  are of order  $\mu$  and  $\alpha_s^{1/2} \mu$ , respectively. We treat the additional mass-dependent contribution to the pressure  $p_m$  within the mass-expansion scheme of [70], where  $m_s$  is formally treated as a quantity of  $\mathcal{O}(\alpha_s^{1/2} \mu)$  and the light quark masses are neglected. This mass expansion is performed to  $\mathcal{O}(m_s^4)$  and up to a combined  $\mathcal{O}(\alpha_s^{5/2})$  (for the full mass-dependence at  $T = 0$ , see [35]). For the value of the  $s$  quark mass, we use the physical  $\overline{\text{MS}}$  renormalized value  $m_s \approx 93.4$  MeV [71]. We have confirmed that additionally including nonzero  $m_u$  and  $m_d$  terms would lead to a vanishingly small effect, while the chemical potentials realized in NSs are not large enough to allow for heavier quarks. For the soft contribution, evaluated in the massless limit, we furthermore use an analytic small- $T/\mu$  expansion derived in [67] that is valid for  $T \lesssim 100$  MeV. Mass corrections to this result start at  $\mathcal{O}(\alpha_s^3)$  and can therefore be neglected.

The perturbative pressure described above can be readily differentiated to obtain predictions for the coefficients  $A_1$ ,  $C_1$  and eventually for  $\zeta$  as functions of the three quark chemical potentials and the renormalization scale parameter  $X$ . The results constitute lengthy closed-form expressions in terms of standard special functions and their

derivatives, allowing for inexpensive evaluation of the necessary quantities.

**Holography:** The D3-D7 model [72] is the holographic dual of  $\mathcal{N} = 4$  SU( $N_c$ ) super Yang-Mills theory with  $N_f$  copies of  $\mathcal{N} = 2$  hypermultiplets in the quenched approximation  $N_f/N_c \ll 1$ . It consists of  $N_f$  probe D7-branes embedded in the AdS<sub>5</sub> × S<sup>5</sup> spacetime, while baryon charge is introduced by turning on an electric field on the D7-branes [73,74] and temperature by modifying the geometry to that of a black brane. Following [75], we extrapolate the model to the physically relevant  $N_c = N_f = 3$  and fix  $\alpha_s \approx 0.285$  so that the pressure matches the Stefan-Boltzmann value at high density, extending the model's validity towards higher densities. Although the field content of the model differs from that of QCD, we note that the thermodynamic coefficients  $A_1$  and  $C_1$ , obtained through chemical-potential derivatives of the pressure, are highly insensitive to the additional fields in the D3-D7 model.

At vanishing temperature, the pressure of the D3-D7 model takes the simple form [75,76]

$$p = \frac{1}{4\pi^2} \sum_{i=u,d,s} (\mu_i^2 - M_i^2)^2, \quad (4)$$

where  $M_i$  are the constituent quark masses that we fix by equating quark densities with pQCD at  $\mu_d = 1$  GeV and varying  $X \in [1/2, 2]$ . Doing so, we obtain  $M_u \in (522.5, 434.6)$  MeV,  $M_d \in (526.4, 435.9)$  MeV, and  $M_s \in (541.8, 450.1)$  MeV, within this interval in  $X$ . In what follows, in addition to estimating uncertainties by matching to pQCD at different values of  $X$ , we also vary this matching density within  $\mu_d \in [1, 2]$  GeV. At  $T \neq 0$ , we finally compute the pressure numerically, following methods introduced in [73,74].

The other holographic model we use is V-QCD [77], which is a bottom-up model tuned to reproduce QCD physics as closely as possible (see, e.g., the reviews [40,41,78]). It combines the improved holographic QCD model for pure Yang-Mills theory [79,80] to a description of flavors introduced via tachyonic brane actions [81–83], featuring, e.g., a running  $\alpha_s$ , as reviewed in the Supplemental Material [60]. Given that quarks are treated as unquenched ( $N_f/N_c \sim 1$ ) in V-QCD, the model should capture their physics more realistically than the D3-D7 model. Indeed, V-QCD by construction agrees with various qualitative properties of QCD (such as confinement and asymptotic freedom), and its parameters are fitted to data, including lattice results for the pressure [84,85] and baryon number susceptibilities [85] at  $\mu = 0$ . The model is consistent with all known astrophysical observations in the NS-matter regime [86,87], but eventually becomes inconsistent with pQCD at high densities [88].

In this paper, we otherwise follow the treatment of the above V-QCD papers but relax the assumption of exact

chiral symmetry in the QM phase by turning on a nonzero strange quark mass, thus extending the prescription of [89]. The corresponding mass parameter of the model is fixed by demanding that the masses of kaons and  $\eta$  mesons are well reproduced in the vacuum (see Supplemental Material [60] and Refs. [90–93] for details). We find that this procedure underpredicts the dependencies of quark number susceptibilities on the strange quark mass at zero  $\mu$  and high  $T$ , where the results can be benchmarked against lattice data [94]. This leads us to expect that this model similarly underpredicts the effects of the strange quark mass in physical quantities at high densities.

Finally, we quantify the underlying uncertainty of our results by allowing the V-QCD parameters vary within limits set by the lattice QCD fit in the chirally symmetric phase [85,95], but otherwise follow the computational strategy of [96] in determining the quantities appearing in Eq. (3). In both holographic setups, the variation procedure we perform thus corresponds to the uncertainties associated with the respective matching procedures.

**Results**—Our main result for the bulk viscosity of unpaired QM is displayed in Fig. 1. It highlights a qualitative contrast between the behavior of  $\zeta$  in the confined and deconfined phases of QCD, with the more suppressed QM results peaking at lower temperatures, and in addition demonstrates the important effect of interaction corrections in the latter case. Consistently with our expectations for quantities that vanish in the degenerate-mass limit, V-QCD appears to predict somewhat lower values for  $\zeta$  than our other methods, but nevertheless retains the same qualitative features.

A closer inspection of our results reveals a number of interesting further findings. Explicit calculations show that in all three approaches, the bulk viscosity is insensitive to the  $T$  dependence originating from the coefficients  $A_1$  and  $C_1$  of Eq. (3). As demonstrated in Fig. 4 of Supplemental Material [60], to a good accuracy we can indeed set  $T = 0$  in these functions and only keep the  $T$  dependence of the electroweak rate  $\lambda_1$  in Eq. (2). Another universal characteristic that all our results exhibit is an approximate quartic dependence on the strange quark mass, which has been noted before in [56].

While the full  $\zeta$  depends on the rate  $\lambda_1$ , we may construct physical features of the bulk viscosity that are sensitive only to QCD input. For example, the peak value of the viscosity,  $\zeta_{\text{peak}} \equiv \zeta(T_{\text{peak}})$ , and its rescaled zero-frequency limit  $\lambda_1 \zeta(\omega = 0)$  that corresponds to the dc bulk viscosity entering the Israel-Stewart theory [12,97,98] are completely insensitive to the electroweak rate and can be fully extracted from the coefficients  $A_1$  and  $C_1$  in Eqs. (33)–(34). These two quantities are shown in Fig. 2, where we observe a good agreement between our pQCD and D3-D7 results for densities where both predictions are available, while V-QCD again appears to underestimate the quantities (see discussion in Supplemental Material [60]).

Setting  $T = 0$  in  $A_1$  and  $C_1$ , we find that the D3-D7 calculation leads to a remarkably simple analytic formula as a function of  $\mu_d$

$$\zeta = \frac{4\lambda_1\mu_d^6(M_s^2 - M_d^2)^2}{K_d^2 K_s^2 \omega^2 + \pi^4 \lambda_1^2 (K_d + K_s)^2}, \quad (5)$$

where we have defined  $K_i \equiv 3\mu_d^2 - M_i^2$ . We stress that for the  $M_i$  in this formula, one should use the constituent quark-mass ranges listed below Eq. (4), leading to the uncertainty ranges visible in Fig. 1. To express this as a function of  $n_B$  for the small temperatures of relevance to BNS mergers, one can further use the  $T = 0$  pressure in Eq. (4) to numerically relate  $n_B$  to  $\mu_d$  in beta equilibrium.

Returning finally to the bulk viscosity itself, we note that it is straightforward to compare our NNLO pQCD results to lower perturbative orders, as shown in Fig. 2 of the Supplemental Material [60]. We find that the difference between the NLO and NNLO results is non-negligible even at  $40n_{\text{sat}}$ , and that the results diverge rapidly at lower densities, making extrapolation to the NS realm impossible. While the naive free quark expression can, in principle, be extrapolated to low densities, it completely fails to take into account the effects of interactions, which become

increasingly important much before the hadronic phase is eventually reached. For phenomenological purposes, the compact D3-D7 bulk viscosity of Eq. (5) is on the other hand appealing as it is rooted in pQCD but takes into account the strongly coupled nature of the theory at low densities. To this end, despite its limitations discussed above, we recommend the use of this result in the modeling of dense unpaired QM at astrophysically relevant densities and temperatures, and similarly expect the V-QCD result to provide a reasonable lower bound for the bulk viscosity.

An important limitation of our present approach is finally related to the fact that the pairing channel and the magnitude of the superconducting gap in low- and moderate-density QM remains unknown (though see [99] for a recent model-independent study bounding the gap at high densities). To obtain estimates for the bulk viscosity in various pairing channels, corrections to both the electroweak rate in Eq. (2) and to the thermodynamic functions entering through Eqs. (33)–(34) should be separately considered. While the latter are expected to be subleading, the former may be substantial given that the contribution of gapped quark modes to the reaction rate is exponentially suppressed. While the detailed evaluation of these corrections is left for future work, we note that the electroweak rate receives  $O(\alpha_s)$  QCD corrections even in the unpaired phase, some of which are presently known [57]. Their effect is studied in Fig. 5 of the Supplemental Material [60], where we observe that, in agreement with the  $\lambda_1$  independence of  $\zeta_{\text{peak}}$ , they primarily simply shift the peak of the viscosity to lower temperatures.

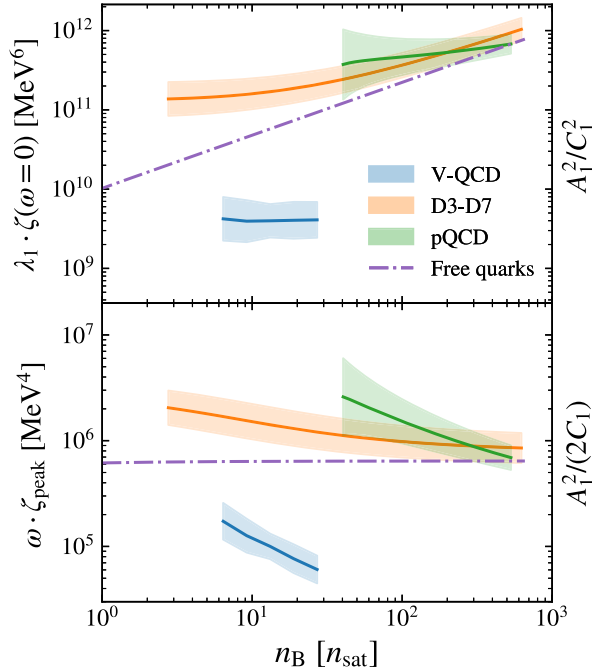


FIG. 2. A comparison of the values of two quantities characterizing the bulk viscosity: its zero-frequency limit and peak value,  $\zeta(\omega = 0)$  and  $\zeta_{\text{peak}}$ . These quantities are multiplied by different factors so that they depend only on  $A_1$  and  $C_1$  in Eqs. (33) and (34) and are independent of the oscillation frequency  $\omega$  and the electroweak rate  $\lambda_1$ , as indicated by the expressions on the right vertical axes. The error bars in these panels capture the variation of model parameters in the different models as described in the main text.

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