Statistical arbitrage strategy based on VIX-to-market based signal

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Abstract:

The objective of the paper is to study the relationship between implied volatility and equity index returns. The paper goes through basic finance theory related to pricing models, market hypotheses and dissects what is implied volatility, as well as discusses its importance. After that a trading strategy is formulated based on a signal derived from VIX-S&P500 relationship, with plausible assumptions on why the strategy should work. The study period is from January 1995 to October 2020, including 6488 daily observations for the VIX and S&P500 indexes.

The strategy is based on a signal triggered by certain market sequence and is therefore fully derived from historical price action and with a repeatable process. The signal is first tested with event study methodology, and afterwards hypothetical portfolio returns are constructed to evaluate the performance of a strategy based on this signal with and without transaction costs.

In light of the empirical results of the study, the strategy outperforms the S&P500 index over a 25-year period, with higher returns, lower systematic risk and volatility. The strategy therefore offers some evidence, that excess returns can be made by timing the market by using historical data, even with trading costs accounted for the strategy.

Keywords: implied volatility, S&P500, VIX, signal, statistical arbitrage

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1 INTRODUCTION

Capital markets are an important piece of modern world and economy as it is. Development of markets has allowed governments, companies and other entities to secure financing in the form of debt and equity alike. The development of futures market revolutionized commodities trading and enabled producers to hedge risks related to price swings and therefore contributed to stabler markets and development of supply chains. The first exchange according to Investopedia (2020) goes back as far as 1531 in Antwerp, Belgium. The first stock traded, was supposedly the Dutch East India Company in the Amsterdam Exchange.

Since capital markets have a long history, it is natural that practicians have spent a lot of time honing their ideas and knowledge of the markets and market functioning, with the idea of making a living off it. Market participants with different time horizons, purposes seek to buy and sell for their own gain. Several styles and schools of investing or trading exist from qualitative to quantitative approaches. An investor could choose to make decisions regarding fundamental macroeconomic factors or stock specific fundamentals. This includes investing in certain categories of stocks like value, growth or momentum stocks. For quantitative time-series approaches an investor could use Bayesian or frequentist methods. Then there are also investors that count on passive index strategies. These investors might optimize their process, with diversifying purchases through time by cost averaging as well as minimizing the costs of trading. In the same category, one could classify traders scanning for unusual option volumes and insider buying. These market participants believe they don’t have an edge, so they attempt to follow someone who does. Then there are some market participants that might even consider trading just a casino, where you roll the dice with options expiring tomorrow or next week. Tech bubble of 2000 and investing boom with so called “meme” stocks of 2021 are clear examples of that and irrational speculation tends to resurface from time to time. This speculation has led to sentiment indicators, while styles and schools of serious investing have mostly developed through market theory and research of academics.

Along with making money, the reasoning for research should be clear. Development of better asset pricing should contribute to more stabler market conditions for all the participants, like in the case of commodity futures. Unnecessary price variance or volatility is not desirable as it in traditional finance in quantitative terms stands for risk. In the other hand, one might consider it an important category to focus research on.

Volatility can be seen as the historical variance of an asset, but there is another aspect to it. From the options market, through pricing models, one can derive the future outlook and expected option-implied volatility of the underlying asset. Considering higher market volatility tends to coincide with requirement of higher returns and therefore lower asset prices, one could wonder if an investor could use options to his/her advantage in predicting asset returns as the investors following unusual options volumes attempt to do. The topic of volatility is also relevant, since the finance industry is constantly developing new trading instruments that are more tied to volatility and other derivatives. We have also seen excessive volatility recently, due to the covid crisis, and more recently Ukraine-Russia situation. The share of derivatives trading has been trending upwards in recent years (FIA, 2022). While it may not be relevant here for this paper, it is interesting to note that the development brings a new structural and predictable aspect of markets into play, as options and futures have set expiration dates and the open interest on certain price levels are known beforehand.

1.1 Purpose of the study

The price process of assets is affected by volatility of the process. The purpose of the study is to learn about the implied volatility index VIX, form a strategy and identify if the option market has any predictive power over stock market returns and if implied volatility can be used to time the market.

1.2 Contribution

The motivation and reasoning behind the study is to gain insight on how to successfully use implied volatility as a risk management/trading tool as well as to attempt to capture potential contrarian returns. Referencing P. Giot (2005), there is not much literature about the relationship between implied volatility and forward-looking market returns. Market return predictability in the other hand has been studied extensively in academic literature. In this paper a frequentist study is conducted to study specific scenario of co-movement of implied volatility along with equity index return. The study hopes to offer evidence of whether market derived information can successfully be used as a decision criterion for making trading decisions.

1.3 Scope of the study

The study will first go through the necessary theories on markets and market pricing as well as theories of related anomalies under continuous scrutiny. The paper will also discuss how volatility is in theory connected to the returns and what implied volatility actually stands for. After referring to the several different parts of this work, a strategy is formed based on assumptions of negative correlation between market returns and volatility as well as implied volatility. Also, for the statistical setting an assumption must be made, that option-implied volatility has predictive power over market volatility. These assumptions are backed by scientific literature as well as empirical evidence. The assumptions are used to form transaction signal derived from market data. Basically, it could be argued that if implied volatility predicts volatility and thus negative returns, a sophisticated investor should sell his holdings in a situation, in which the implied volatility is rising. This has not necessarily happened instantaneously in history and could possibly result in a statistical arbitrage opportunity, if the resulting market derived event leads to negative market returns.

After forming the signal, some preliminary event studies will be conducted to test for significant abnormal returns following these events. The application of these signals is tested by constructing strategy portfolio returns and regressing them against the S&P500 index buy and hold portfolio. Transaction costs will also be discussed and considered in the study.

2 THEORETICAL BACKGROUND

Market return predictability and pricing has been studied excessively through the history of markets as is natural, since solving the markets could be a road to riches. Market efficiency, predictability and autocorrelation are intertwined concepts. In an efficient market the market reaction to new relevant information is supposed to be instantaneous and unpredictable, therefore excess returns should be unreachable without an information advantage, also autocorrelation of returns should not exist, since the past information should be priced in. For a practical example, the predictability of returns has been studied in terms of traditional valuation metrics and volatility, see for instance (Bollerslev, Marrone, Xu and Zhou 2014, Kim, Shamsuddin and Lim 2011). To start off, the paper will go through basic models for pricing risk in relation to the expected return and the hypotheses that those models are built upon.

2.1 The Modern Portfolio Theory, the Capital Asset Pricing Model and the Security Market Line

To understand and interpret the dynamic system that the market is one must develop a framework for pricing and a basis for how the markets function. This involves creating a model for data-generating process of the market and evaluating investor motives behind the decisions they make while formulating their portfolios. This is why the modern portfolio theory (MPT) of Harry Markowitz (1952) along with the capital asset pricing model (CAPM) of William Sharpe (1964) and John Lintner (1965) is a great place to start. In MPT the investor is assumed to be risk-averse meaning that when making a decision between investment opportunities with equal return the investor will choose the less-risky alternative. This is called the E-V rule (expected-returns to variance of returns rule). MPT is therefore referred to as mean-variance analysis, since it is a mathematical framework for constructing portfolios of assets that maximize returns given a level of risk. The variance of asset prices is used as a proxy for the risk of the assets. This measure of risk is not all-inclusive in terms of different aspects of business ownership risk, but it is unique for different types of assets. In MPT the main point lies in diversification between all types of financial assets to achieve what is called the mean-variance portfolio, which gives the investors' portfolio an optimal risk-to-reward ratio. Diversification is proven to work, since individual assets do not fully co-vary with each other, so adding different ones into the portfolio reduces the portfolio variance. Nevertheless, Markowitz also notes that the mathematical law of large numbers does not apply for financial assets, since they are too intercorrelated. This implies undiversifiable systematic risk that cannot be diversified away.

The notion of systematic risk is shown by Sharpe (1965) to contribute to the rates of return of an asset. Sharpe also argues that in the market process the efficient frontier of portfolios adjusts to a linear relationship between market risk and return, since the superior (inferior) investment opportunities will correct to the upside (downside), which in relation makes the worse (better) opportunities better (worse). This frontier is a concave function showing the reward to variance ratio that is possible to attain with respective return rates and variance.

The notions of Markowitz's undiversified systematic risk, risk-averse investor and linear risk-to-reward representation of Sharpe are a good place to start thinking about a model for pricing of particular assets instead of a portfolio. Since unsystematic risk should be diversifiable it is logical that the return of an asset should be conditional to its covariance with systemic risk factor and the time-value of money. Capital asset pricing model of Sharpe & Lintner models the returns of an asset as a function of the risk-free rate and covariance with market risk premium. The model is normally expressed in the following form:

$$E(R_i) = R_f + \beta_i (R_M - R_f) \qquad (1)$$

where beta (β) is the covariance with the market risk premium (R_M- R_f) and $E(R_i)$ denotes the expected return of asset i. In regression terms $E(R_i)$ is the dependent variable and (R_M-R_f) is the independent variable. Beta can be considered as a measure of the asset's relative covariance with systematic risk, and it is calculated by estimating the covariance of asset i with respect to the market and divided by market variance:

$$B_i = Cov(R_i, R_m)/Var(R_m) \qquad (2)$$

From CAPM we get the linear expression β_i (R_M- R_f), which represents the slope of the Security Market Line (SML), which is the graph representing rate of return in relation to beta.

Figure 1 - Theoretical Security Market Line

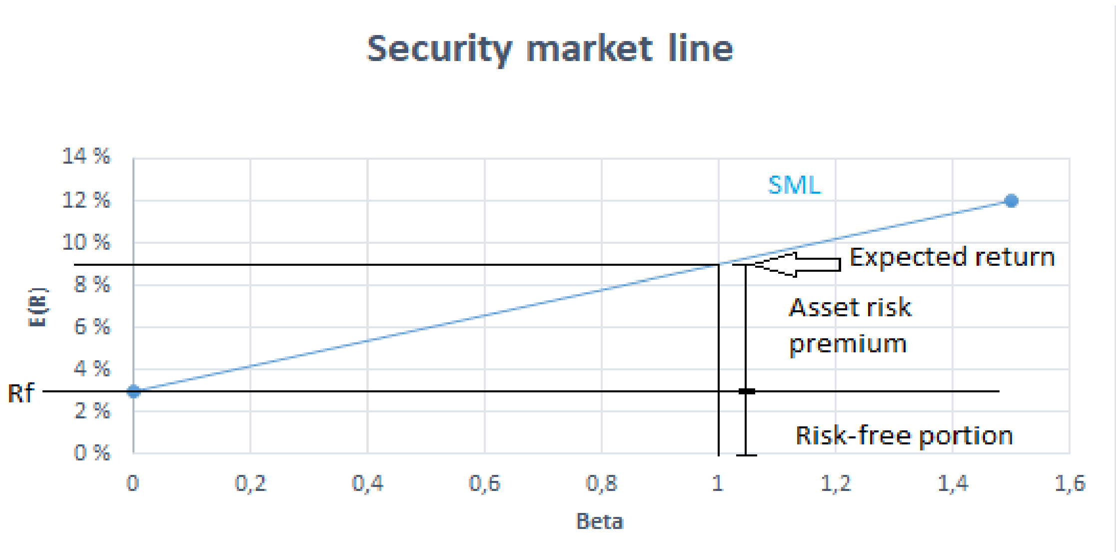


Figure above represents market equilibrium given all assets lie on the SML. Companies above the line are considered undervalued as companies under the line are considered overvalued considering their covariance with the systematic risk factor. In equilibrium all the companies and portfolios converge to this linear equation, if the market is pricing assets correctly. The constant here is 3%, which is the point where the plotted line crosses the y-axis of expected returns. If CAPM holds, then the constant of 3% refers to the risk-free rate as represented earlier.

Considering the convergence of assets expected returns to the return-beta relation implied by the CAPM, the market portfolio should also represent the efficient portfolio. This is due to the fact, that when one diversifies asset specific risk away, then the efficient portfolio simplifies to a market portfolio, which is therefore located in the efficient frontier of Markowitz (1952). No investor should carry excess risks that are diversifiable, and when asset prices converge to CAPM equilibrium, then there is no excess unsystematic risk premium to be compensated for by holding those assets. Obviously, asset prices vary and are not in constant equilibrium. General diversification benefits are only attainable, if unexplained variance of individual assets is not zero or prices are not perfectly correlated Lintner (1965). This logically leads us to discuss the efficient market hypothesis in the next chapter and alternative market hypotheses in chapters further in the text.

2.2 Efficient market hypothesis (EMH)

The baseline for the efficient market hypothesis was set by Maurice Kendall in 1953 (Kendall and Bradford, 1953). Kendall was the one to first introduce a theory that claimed that the security prices are not predictable but rather follow a so-called random walk. This is called the random walk hypothesis. This theory claims that prices are unpredictable.

Kendall was studying wheat prices, so his studies did not incorporate a drift component in the model, but regarding equities the same model holds with a drift component towards the cash flows of the asset. Looking back at the CAPM, these cash flows are represented by the covariance with market risk premium and the risk-free rate. Thus, the cash flows consist of the asset specific premium and a risk-free bond yield. This can also be considered as the earnings yield – earnings/price of an asset. Obviously, the earnings yields, and future cash flows of companies deviate, based on their cost of capital, margins and growth rates. In an efficient market, these premiums are considered to be priced correctly respective to the information that is out there for market participants.

Fama advances Kendall's random walk theory in 1970 by stating different forms of market efficiency, the weak form, the semi-strong form and the strong form. Already in theory, the weakest form of market efficiency reflects all of the available historical information. The semi-strong form assumes the market to reflect the historical information and other publicly available information about the economy and underlying fundamentals of securities, for example splits and earnings.

In the strongest form, efficient market hypothesis states that the markets reflect all information available, historic, public and even non-public information. When all information about the securities in the market is given and priced, an investor cannot get higher expected returns with the same risk level (level of dispersion) as all the information is reflected in the prices. This means that the only way for an investor to gain higher expected returns is by taking more risk (Fama, 1970).

As the efficient market hypothesis is one of the most famous and studied theory, it has also been under scrutiny and criticism. One argument against the EMH is that because investors value securities differently it is impossible to value what the security should be worth under an efficient market. At the same time, one could argue that this is a core reasoning for the whole concept, since EMH is as a concept of unpredictability. This unpredictability implies that there is no way even a sophisticated investor produces more than average returns in the long-term, as anomalies and patterns are priced in after they are revealed to the public. Malkiel (2003) argues that the markets are much more efficient and much less predictable than some academic papers have stated. Malkiel still admits, that in the short run the market is not always perfectly priced, but that in the long run the markets are efficient. This is a controversial statement, since if one can be productive and sophisticated enough to find short-term anomalies for decades, can the markets still be considered efficient? Such quantitative market participants do exist to the authors knowledge (i.e., Renaissance Technologies, Medallion fund to be exact). The discussion around EMH has led to alternative market hypotheses and more complex models, which are discussed further on.

2.3 Fractal and Adaptive market hypothesis

Fractal market hypothesis examines (FMH) the daily randomness and relatively short-term turbulence in the markets, that is mostly witnessed during crashes and crises (Peters 1994, 1989). The alternative investment theory looks at markets as a repeating process with similar sets in it. It investigates the role of liquidity and different investment horizons in pricing of market instruments. During crises, investors supposedly trend towards short-term horizons, reacting to negative price movements and information. This shift in sentiment and convergence of investment horizons is argued to cause illiquidity and market crashes. One could refer to financial crisis of 2008-2009 as an example of such a period. Fractal hypothesis and market theory can also be associated with chaos theory. It can be argued that the baseline for FMH was created by Benoit Mandelbrot, who can be considered the father of fractal geometry. Mandelbrot questioned the normality assumption in commodity prices as early as in 1963.

Adaptive market hypothesis (AMH) was introduced by Andrew Lo in his papers in 2004 and 2005. This is the more known and referred alternative market hypothesis to efficient market theory. Lo starts the paper published in 2004 by considering EMH and its differences in relation to AMH and refers to several market anomalies that are contradictory to EMH. These include overconfidence (Fischoff, Slovic and Lichtenstein 1980, Barber and Odean 2001, Gervais and Odean 2001), overreaction (DeBondt and

Thaler 1985), loss aversion (Kahneman and Tversky 1979, Shefrin and Statman 1985, Odean 1998), herding (Huberman and Regev 2001), psychological accounting (Tversky and Kahneman 1981), miscalibration of probabilities (Lichtenstein, Fischoff and Phillips 1982) and so on. AMH seeks to model economy as a natural ecosystem with evolutionary traits. Abstract description of adaptive market hypothesis refers to market as an ecosystem, where different investor types represent different species. Profitable trader is referred to as survival of the richest by Lo. Lo himself refers to EMH as an application of theories of physics into economics, which do not describe the markets as the economy is a system created by humans and more complex than the laws of physics. He still believes in the traditional risk-to-reward trade-off, but considers it being dependent on the current market environment and argues that it is unlikely to be stable over time.

If one thinks about the statement of non-constant risk-to-reward ratio, the statement can easily be justified intuitively — assuming volatility as a measure of risk — since volatility is time-varying and not a constant. In the other hand, volatility — if measured by VIX — does not have a unit root (Ozair, 2014), so it can be classified as a stationary process as it excerpts mean-reverting behavior. This means that in the long-run volatility stays in a certain range and in terms of return dispersion the risk is in that sense limited. This seems logical since by historical standards valuations matter in longer time frames and in the short-term — one should rely on volatility (Bollerslev et al.2014, Kim et al. 2011). So basically, one could consider the market inefficient in the short-term, but efficient in long term, because of the greater uncertainty associated with long-term returns. This statement obviously only holds if volatility risk is not priced in correctly by the market via the current information it possesses.

In the other hand there are also studies, that find a unit root in the VIX (Chow, Jiang and Li, 2020). Non-stationary VIX relates to the research of Mandelbrot (1963), as he argues for the possibility of infinite variance.

2.4 Multifactor models

CAPM and the assumptions it entails are often considered flawed in the literature. Lintner (1965) pointed out that the diversification benefits wouldn't exist without independent variation in asset returns. Further on, Stephen Ross (1976) derived a pricing theorem called the arbitrage pricing theory (APT) from CAPM and the equilibrium state described by the model. APT is a sort of extension to CAPM, as it breaks the systematic risk into different factors. This set of factors can include macroeconomic or company-specific factors entailing different size groups or categories dependent on valuation for instance. There are two schools of thought about factors. One school considers excess returns by these factors to be anomalies and that therefore the market is inefficient. The other school considers that a factor explaining asset's returns supposedly carries some form of risk premium that is characterized by this asset. If that is the case, then this risk premium should entail excess returns. Therefore, theoretically the excess returns of an asset consist of all the factor risk-premiums of that asset. Finding all the premium enables the statistician to very effectively model the returns.

This brings us to discuss several factor categories that are often referred to in daily discussion of assets currently – value, growth and momentum. These additional factors are considered and evaluated when a regression model is fit into the data and their explanatory power is estimated by statistical means.

Important factor models worth mentioning here are for instance the Fama-French 3-factor model and Carrhart 4-factor model, represented in respective order:

$$\boldsymbol{R_e = \alpha_t + \beta_M RM_t + \beta_{HML} HML_t + \beta_{SMB} SMB_t + \epsilon_t} \tag{3}$$

$$\boldsymbol{R_e = \alpha_t + \beta_M RM_t + \beta_{HML} HML_t + \beta_{SMB} SMB_t + \beta_{UMD} UMD_t + \epsilon_t} \tag{4}$$

These models are cornerstone models in the modern finance to this date. RM obviously stands for a general market factor, while t subscript marks the time or date. HML represents a value factor, which refers to outperformance of value stocks over growth stocks – as is expected by historical standards. SMB represents a size factor, and a positive value means exposure to small size companies measured by capitalization, since small capitalization stocks are expected to generate higher returns by historical standards. UMD in the other hand stands for momentum factor, which is normally calculated by subtracting equally weighted average of highest performing firms from the equally weighted average of lowest performing firms. All of these factors represent systematic risk entailed by the respective categories of stocks. In public discussion HML and SMB have reached sort of a status of given risk factors, even though one might hear someone talking about the small cap anomaly. On the other hand, the UMD factor might be considered an anomaly factor, due to it being dependent only on historical pricing.

2.5 Anomalies and other excess return factors

Multifactor models were the first step from CAPM to quantifying existing market anomalies and excess returns. A paper of Hou, Xue and Zhang (2015) categorize these anomalies into investment, momentum, value-versus-growth, profitability, intangibles and trading frictions. Appendix A lists the anomalies by categories and the academic source, as represented in Hou, Xue and Zhang (2015). This table is not a comprehensive guide to every asset anomaly or excess return found in the markets to date, but it should provide the reader an assessment of different types of phenomena. Factors can be constructed based on anything that categorically systemically deviates from other assets in terms of returns or attributes. The process involves categorizing assets that have highest correlation between each other, ranking them based on this evaluated attribute/factor and then estimating the factor premium via a regression from returns. For a more detailed explanation, one can look up Stambaugh and Yuan (2017).

After you have defined the factor premium, you can use it as a baseline and estimate its explanatory power in terms of other portfolios. To simplify, it is the estimation of joint marginal distributions of risk factors conditional on return distribution. This way a statistician can construct a factor covariance-variance matrix for modelling the stock market process in terms of risk-factors, instead of just mere returns of assets and their covariance to a common factor, the market risk premium.

In a complete market – which is referred to as the Arrow-Debreu market – there is no trading frictions, the pricing is instant and market participants also have perfect information. In the Arrow-Debreu market there is also a price for every asset in every possible state of the market and all the factors are therefore priced correctly. One could consider a market priced correctly by CAPM to be such a market, since it effectively is a market in a general equilibrium. It is good to note, that this complete market described above sounds very much the same as the efficient market described by Fama.

Truthfully, the market can be considered incomplete, since anomalies exist in terms of statistics and statistical proof. If one thinks about the notion of every risk factor being priced correctly constantly, it seems overwhelming. Every factor potentially has a different return distribution, regarding different states of the market. These states could for instance be described with accelerating or decelerating inflation or growth. For example, it seems intuitive that long duration assets do not do well in an inflationary environment. Many market participants seem to have a hard time weighting the probabilities of different states right, so it seems unreasonable to expect markets to price the states and the factor premiums right. From this, it follows that in an incomplete market EMH does not hold up, unless it relies exclusively on the notion that prices are unpredictable, because everything known is priced and it is nearly impossible to be consistently weighting future probabilities and return distributions with more success than the collective of market participants. This means one cannot hope to gain excess returns over a long horizon, as arbitrage opportunities happen randomly and inconsistently. Tail event might wipe away the excess returns of a speculator and eventually, by the law of large numbers the bigger premiums will converge to mean or even a zero. The excess returns can be tied to a certain risk factor and eventually that might disappear, since market participants adapt as they acknowledge this risk factor.

There are studies that argue for the disappearance of registered anomalies, while the market gets more efficient as time goes by (Zaremba, Umutlu & Maydybura 2020). As another example, Fama and French look into the value factor premium in an article called "The Value Premium" in 2020. They themselves originally registered the value anomaly between July 1963 and June 1991 and as the factor has underperformed lately, they test if the factor can still be considered to generate excess returns. Their results are inconclusive, but give some support for a scenario, where registered factor attributed returns can disappear for almost 30-years. In some cases, the factor premium might be tied to a certain state, while that state might be absent for ages like inflation in Japan. The correct pricing of these factor premiums goes beyond this paper, but one could always speculate if the outperformance of growth stocks in 2010s occurred due to low interest rates for instance.

While the pricing of premiums might be too much for this paper, limits of arbitrage is a studied concept in financial literature and important consideration, when forming a strategy. Shleifer and Vishny (1997) write about the potential, issues and realities of statistical arbitrage. They conclude that, while you may have clear opportunities in the markets, agency issues and concentration of capital make anomalies possibly very persistent. There is a famous saying by John Maynard Keynes:" Markets can stay irrational longer, than you can stay solvent". An arbitrageur must deal with idiosyncratic risk as well as undiversifiable risk. The arbitrageur will not get the luxury of the same diversification benefits, while the opportunities are explained by costs. To exploit these opportunities, it requires specific knowledge, managing trading costs and a market segment that has relatively low volatility in relation to returns that are attainable through arbitrage. That's why arbitrage occurs more on liquid currency and bond markets, as mean reversion of mispricing is more dependable Shleifer and Vishny (1997). Regardless of the superior knowledge of the arbitrageur, there is still a risk of following a wrong model and therefore a false signal.

From anomalies and limits of arbitrage the paper is now in a logical place to continue to topics more specific to a strategy, that one can apply and trade on.

2.5.1 De Bondt & Thaler 1985 & 1987. Does the stock market overreact?

The paper has discussed pricing hypotheses, models and anomalies on a general level so far. The next two chapters of the paper will discuss phenomena more explicitly related to the arbitrage strategy that is proposed later, since the strategy attempts to benefit from a short-term overreaction. Baytas & Cakici (1999) refer to market overreaction as systematic overshoot of stock prices, which leads to the mean reversion being predictable from past performance. This obviously violates the efficient market hypothesis, since it implies that the returns are predictable. The idea was introduced to the academic world by De Bondt and Thaler (1985 & 1987) and according to the paper of 1985 itself, it is the first attempt to model and quantify a behavioral anomaly in the financial literature. In the 1985 paper De Bondt and Thaler refer to Kahneman et al. (1982) to establish a baseline that the investor does not always act according to the Bayes law, which leads to misevaluation of probabilities in regard to the examined assets' future market value. Several other papers of market anomalies are also referred to (Basu 1983, De Bondt 1985, Arrow 1982). Investors seem to emphasize recent data and information points rather than the earlier information. In Bayesian terms, the "a priori" is disregarded or its informative value is suppressed.

After constructing a foundation and basis for their work, they form two different hypothesis both in direct contradiction to efficient market hypothesis. The hypotheses are stated as follows:

1. "Extreme movements in stock prices will be followed by subsequent price movements in the opposite direction."

2. “The more extreme the initial price movement, the greater will be the subsequent adjustment.” (De Bondt & Thaler, 1985).

To test for this anomaly, they form a test setting by constructing winner and loser portfolios from CRSP stock data for New York Stock Exchange common stocks. One could also consider these portfolios as positive and negative momentum portfolios. De Bondt & Thaler use data from January 1926 to December 1982 to calculate abnormal returns on 3-year periods of monthly observations. They use equally weighted portfolios of monthly return data to study the CAR and ACAR (or CAAR) formed by using constant mean adjusted model $AR = R - R_E$.[1] They produce 16 non-overlapping event windows and evaluate the winner and loser portfolios for every window, with a hypothesis of the loser portfolio having CAAR above 0 and the winner portfolio having CAAR less than 0.

The results of De Bondt and Thaler are consistent with the overreaction hypothesis, as the winner portfolios lose to market and consequently the loser portfolios turn into winners on average. They conclude that the research in experimental psychology suggests that in violation of Bayes’ rule, most people “overreact” to unexpected and dramatic news events. The question then arises whether such behavior matters at the market level or in a shorter time frame. Hopefully this paper can contribute into answering that question.

2.5.2 Short-term return reversal and overreaction

Short-term return reversals have been studied by for example Jegadeesh and Titman (1995), Sophie Kulp-Tåg (2007) and Blitz, Huij, Lansdorp and Verbeek (2012). In 1995 Jegadeesh and Titman wrote about short-term return reversals that are consistent and follow a market microstructure model. The paper studies asset order-flows and the relationship of market maker inventories, liquidity and the bid-ask spread. Jegadeesh and Titman aim to follow up on earlier studies showing negative serial correlation in the short-term returns. They study if that potential serial correlation is caused by inefficiency in the market or by the bid-ask spread. In regard to this it is good for the reader to understand why this is of importance, thus one needs to examine the interaction in the transaction surface.

A market maker is a financial institution that provides liquidity to the markets. This is done by buying or selling assets when the price process is discrete, and the supply and demand of assets does not match. In this sort of scenario investors are on the market at different times and aren’t buying and selling at the same time, not necessarily because the price is wrong. Market maker can be considered an insurance company, that you pay the bid-ask spread, so that the company provides you the current market price. The portfolio of a market maker is referred to as the inventory, since the market maker does not necessarily aim to take a big view on the direction of the price. Therefore, the market maker tries to diversify the portfolio as well as possible. When a price of certain asset moves it

[1] As a side note, one can check the event study chapter 6.1.1 for definition of these variables and the concept.

alters the optimal portfolio diversification of the market maker, market participant and other participants in the market. In the short-term this can lead to excess risk allocated on a particular asset for the market maker. If the market maker is in the process of providing both the buy and sell side quote for the customer – to improve market liquidity – then the market maker increases the bid-ask-spread to account for the extra risk. The bid-ask-spread can therefore refer to the price of inventory risk, that the market maker is entitled to receive.

Jegadeesh and Titman show that price and volume variance cause changes in the bid-ask-spreads that cause negative autocovariance in short-term returns. To simplify, negative autocovariance is the same as mean-reverting behavior. When a trader is concerned, benefitting from the bid-ask-spread does not really transition into a trading strategy that's making excess returns with less risk, but implies that the trader is a market maker getting paid for providing liquidity. The results of Jegadeesh and Titman therefore support the efficient market hypothesis, but they still point out that the market does not seem to be as resilient to liquidity issues as previously recorded in literature. Mispricing based on liquidity issues can affect prices longer than anticipated. This is an important aspect to understand, when creating a short-term trading strategy. It is also a deduction that supports characteristics emphasized by FMH and can be a considered a limit of arbitrage as discussed by Shleifer and Vishny (1997).

In 2007 Sophie Kulp-Tåg studied overreactions and asymmetric mean-reversion related to the concept. Kulp-Tåg used data from Nordic stock markets of Finland, Denmark, Oslo and Sweden. Daily data was used and Kulp-Tåg employs some of the more sophisticated models in financial literature, such as the asymmetric nonlinear autoregressive model and exponential GARCH – model. Kulp-Tåg notices that the market seems to exhibit certain patterns in terms of price behavior. Asymmetric overreaction is seen in the fact that while investors seem to be more apt on reacting to bad news – price drops happen faster – investors also generally seem to be eager on buying the dip faster, since negative returns revert to mean with a greater magnitude than positive ones. This is obviously related to the momentum anomaly, which is also studied by Jegadeesh and Titman in (1993). Reader can find more studies on momentum in Appendix A.

The studies of Kulp-Tåg nor the ones mentioned from Jegadeesh and Titman do not really justify timing the market on the short-side, but rather on the long-side of exposure. A conventional strategy for short-term reversal is described in Lehmann (1990) as well as Jegadeesh (1990). Lehmann builds a net zero investment portfolio, with 50% of positions short and 50% of positions long and timing a market based on weekly returns, by shorting winners and longing losers. The portfolio seems to generate excess returns and the estimated time frame is 26 weeks, for it is assumed to be such a time frame consisting of enough observations that the estimated error variance converges to its presumed distribution. Jegadeesh (1990) in the other hand uses regression analysis to estimate autocorrelation with return lags of the asset. He finds such autocorrelation on a monthly scale and proceeds to estimate strategies that use this information. Jegadeesh estimates forecasted returns with the estimated negative autocorrelation terms and ranks assets to deciles in terms of expected returns. He then builds equally weighted portfolios to represent the deciles and estimates the abnormal returns generated by these portfolios. The autocorrelation estimates and expected returns are revised after every month. These portfolios seem to create excess returns, so therefore the market can be considered predictable. Jegadeesh offers several explanations for this; size-based adjustments, time-varying market risk and the bid-ask-spread along with thin trading. These results support shorting winners, at least on a monthly scale.

Blitz, Huij, Lansdorp and Verbeek (2012) estimate a short-term residual reversal strategy. They start the paper by considering reports that the conventional strategies proposed by Lehmann (1990) and Jegadeesh (1990) seem to have lost their edge in the recent years. They take the conventional short-term reversal strategy and break it down to its factor exposures. From the factor exposures they conclude, that if they only use residual returns instead of total returns to define the winner and loser stocks, they can then reduce factor exposure to systematic factor risks with positive expectancy. Therefore, they use negatively serial correlated residuals to their advantage. The abnormal returns are estimated with a rolling Fama-French 3-factor model deducting the model estimate from market returns. Portfolios of winners and losers are constructed based on the residual estimates, after that the winners will be shorted, and losers longed. The strategy based on residuals seems to outperform the conventional strategy, with lower volatility and the returns are statistically and economically significant after trading costs. According to Blitz, Huij, Lansdorp and Verbeek the only reasonable explanation for this is that it is a behavioral anomaly, since the results are not consistent with the market microstructure considerations (ie. market maker inventories).

2.6 Properties of volatility

In this chapter the paper discusses and goes through the topic of volatility. The purpose is to mirror the concept against what has been discussed earlier for figuring out how to use volatility as a tool. One can also consider, how the characteristics of volatility reflect on modeling returns and market hypotheses, as well as how one might reach asymmetric risk-to-reward opportunities, by using volatility to his/her advantage. Volatility refers to the dispersion in the price process of an asset. Normally it is measured by standard deviation of first differences, as in returns. It can be measured through various time periods and the values will vary. In theory, volatility represents the quantitative measure of risk and higher periods of volatility are associated with lower equity returns. It can be considered as a driver of short-term moves in asset prices, as it is very hard to often pinpoint exactly why prices move, they just do. Financial data can be relatively noisy. The other side of the same coin is obviously the idea, that returns drive volatility, not the other way around. The lead-lag relationship is discussed further in 2.6.3. Implied volatility is often used as a proxy for volatility, since it conveniently gives an outlook for future volatility.

2.6.1 Implied volatility and option pricing

Implied volatility refers to a mathematical value calculated from option market instruments as mentioned before. It can also be said that implied volatility refers to the volatility parameter set into an option pricing model, that results the current market value of the option.

It represents option-implied expected move/dispersion in the underlying instrument. In option pricing the extrinsic value of an option consists of time until expiry and the implied volatility of the option. The expected movement defines the value of time in regard to this particular asset. The value for implied volatility can be derived from Black-Scholes option pricing model (Black and Scholes 1973). Merton (1973) extended the model to reflect European options on dividend-paying stocks.

$$C(S_t,K,r,tK) = S_t\, e^{-qt}\varphi(d_1) - K\, e^{-rt}\, \varphi(d_2) \tag{5}$$

where C is the call option price, S is the current stock price, K is the option strike price, r is the risk-free interest rate, q is the dividend yield, t is the time to maturity and φ represents a normal cumulative density function. The d term is calculated as:

$$d_1 = \frac{ln\left(^{St}/_{K}\right)+t(r-q+\sigma^2/2)}{\sigma\sqrt{t}} \tag{6}$$

and

$$d_2 = d_1 - \sigma\sqrt{t} = \frac{ln\left(^{St}/_{K}\right)+t(r-q-\sigma^2/2)}{\sigma\sqrt{t}} \tag{7}$$

Volatility implied by several options with different maturities or strike prices can be different for the same underlying asset. The curve drawn in a two-dimensional coordinate system – with y-axis for implied volatility measure and x-axis for option strike price – shows a clear smile, if one uses a group of options with the same underlying asset and expiration date. Bollen and Whaley (2002) refer to this curve as the implied volatility function. When an option moves in and out of the money – the underlying price moves above or under the strike price – it moves along the path of the volatility smile. Implied volatility smile has been present on the markets since the 1987 market crash (Badshah 2010). According to Bollen and Whaley (2002) the reasoning behind the smile is increase in out of the money puts bought to hedge for fat tails in return distributions. The flash crash of -87 made market participants realize the higher-than-realized odds of significant market crashes. As far as the Black-Scholes option pricing model goes, the model output deviates from the smile. This is an important thing to keep in mind, when this paper progresses to the literature review section further on. In terms of option maturities, there exists an implied volatility term structure for every instrument, with an option chain of several maturities. For more information on modelling implied volatility surfaces and term structure see Badshah, 2010.

2.6.2 The VIX (implied volatility index) and volatility risk premium

The implied volatility index VIX is constructed from the option market from options with 23-37 days to 3rd Friday of the month expiration. Only the 3rd Friday S&P500 options are used in calculating the spot values for VIX. Below is the calculation method for the index. (CBOE 2020)

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2 \qquad (8)$$

where

σ is	$\frac{VIX}{100}$ →VIX = σ*100
T	Time to expiration
K	Strike price of the option
F	Forward index level derived from index option prices

$$F = K \quad + e^{RT}(Call\ price - Put\ price) \qquad (9)$$

K_o	First strike below the forward index level F
K_i	is the strike price of i^{th} out of the money option:

- i refers to the number of intervals between K_i and K_o
- A call is used if K_i>K_o
- A put is used if K_i<K_o
- Both put and call if $K_i = K_o$

ΔK_i	Interval between strike prices

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2} \qquad (10)$$

(Note: ΔK for the lowest strike is simply the difference between the lowest strike and the next higher strike. Likewise, ΔK for the highest strike is the difference between the highest strike and the next lower strike.)

R	Risk-free interest rate to expiration
$Q(K_i)$	The midpoint of the bid-ask spread for each option with strike K_i.

The formula above is used for two weekly maturities and the index consists of weighted average of volatility implied by those option chains. The weightings are based on how much the maturity deviates from the 30-day mark, as the first one expires before, and the second maturity expires after. VIX can therefore be considered a market derived number for volatility expectations in the next 30 days. Volatility as a concept represents a lot of different things as risk is often modeled by the variance or standard deviation of the portfolio's returns (Bali, Cakici, Yan and Zhang 2005). In the modern portfolio theory, Markowitz (1952) argues that sophisticated investors choose a risk-return trade-off defined by the perfectly diversified minimum variance portfolio frontier. Lintner (1965) showed that optimum in an equilibrium is never the minimum variance portfolio, but the one with highest possible Sharpe ratio. In any regard, in the academic literature there exists a constant on-going debate about the trade-off between risk and return. Question is, do investors require higher returns for idiosyncratic volatility? See, for example (Bali et al. 2005, Ang, Hodrick, Xing and Zhang 2006).

VIX also relates to a concept called the volatility risk premium (VRP). Excess returns as a reward for higher volatility are referred to as the volatility risk premium (VRP). VRP can be seen as a reward for bearing the volatility risk of an asset. It can be defined as the difference between implied and realized volatility:

$$\textit{VIX-Realized Volatility} = \textit{VRP} \tag{11}$$

This is one simple method of calculating VRP. The issue is that if defined this way, then this is a historical variable. VRP can also be calculated as a forward looking variable, which will be explained further on in the chapter discussing the work of Bollerslev and Zhou (2007).

In a very simple statistical sense volatility could only be defined as a measure of dispersion, but it can also be interpreted as a function of value of time, as is done while calculating the extrinsic option value. The more volatile the underlying price process is, the more time has value since price can travel further.

Another point can be made referring to the leverage effect. One could simplify the concept and interpret volatility as a market derived equity to debt-ratio. In this perspective, the higher returns for higher leverage would seem like logical reasoning behind the argument. In terms of a company balance sheet and cash flow, higher leverage and returns result a profitable company, which has a cost of increased equity volatility. By this logic, one might consider if the returns of companies and sectors with more debt in their balance sheets are suspect to larger impacts caused by market volatility. Further on in the literature review, some information on the subject is provided by Copeland and Copeland (1999). They don't study leverage, but they do study different company specific risk characteristics, as in value and growth factor premiums in relation to the VIX.

Another intriguing fact about the VIX is the shape of the return distribution. It exhibits positive skewness, so the right tail of the distribution is significantly fatter. During volatility spikes the witnessed returns deviate more from 0 in terms of their absolute value. Frequency of negative changes in the VIX is higher, but on the downside, there are big positive changes more often than in a symmetrical normal distribution with 0 mean.

2.6.3 Lead-Lag effect of volatility

The lead-lag effect of VIX and the S&P500 is studied by Chiang (2012). The concept revolves around a theory that there are leaders and laggers in the financial markets. This basically means that certain instruments front-run other instruments and transmit effects that influence returns on the lagging instruments. Chiang finds evidence, that supports the VIX having a stronger pricing effect on the S&P500, than the S&P500 on VIX. Fleming, Ostdiek and Whaley (1995) conclude that VIX tends to both lag and lead stock market returns. The difference between these studies is that the VIX used by Fleming, Ostdiek and Whaley was calculated differently than the current one, even though the concept is the same. Since VIX is theoretically a forward-looking measure, it sounds logical that it would have a pricing effect on the index level. Ozair (2014) disagrees with this notion of causality. He estimates a VAR-model from daily data of S&P500 and the VIX and concludes that the pricing effect of VIX on the equity index is negligible, but that S&P500 has a significant and a persistent effect on VIX.

2.6.4 Leverage effect & negative asymmetric return relation between volatility and equities

The leverage effect refers to a well-established relationship between stock returns and both implied and realized volatility (Figlewski and Wang 2000). Early studies of Black (1976) and Christie (1982) attribute the negative asymmetric return-volatility relationship to leverage and debt-to-equity ratios, as discussed earlier. In the academic literature the effect is modeled by for example Figlewski and Wang (2000) and Bollerslev, Litvinova, Tauchen (2006). Same asymmetry can be witnessed on the index level and is usually even larger than that for individual stocks (Kim and Kon 1994, Tauchen, Zhang, and Liu 1996, Andersen, Bollerslev, Diebold and Ebens 2001). Bollerslev, Litvinova and Tauchen use intraday data and hourly estimates to model the effect and find out that it is more easily detected in shorter time-frame data. In the other hand they refer to Figlewski and Wang (2000) and criticize the leverage effect for being incapable in explaining the persistence of volatility and its full effect on equity returns.

The other explanations for the asymmetric negative relation between returns and volatility are time-varying risk premium or volatility feedback according to Bollerslev, Litvinova and Tauchen (2006). They refer to papers from French, Schwert, and Stambaugh (1987), as well as Campbell and Hentschel (1992) that discuss these effects.

Volatility feedback stands for an event, in which the increase of volatility would increase the rate of return, in turn necessitating an immediate stock-price decline to allow for higher future returns (Bollerslev et al. 2006). According to Hibbert, Daigler and Dupoyet (2008) these are not the primary drivers for the asymmetric volatility-market return relation and they propose a behavioral approach consistent with their results.

Time-varying risk premium seems intuitively to be the answer, as it fits with the interpretation of varying risk-to-reward trade off discussed by Andrew Lo in paper introducing the adaptive market hypothesis (2004). According to Bollerslev et al. (2006) the following researchers: Glosten, Jagannathan and Runkle (1993), Engle and Ng (1993) as well as Nelson (1991) have argued that the empirical relationship between returns and volatility is insignificant, or even negative empirically.

Giot (2005) discusses the implied volatility to returns relationship in his paper 2005. In his paper he finds a connection between excess returns and extended implied volatility observations. More on this paper by Giot later.

2.6.5 Forecastability of volatility and risk-to-reward trade-off

Risk-to-reward trade-off is often considered to be best represented by a reward to variability ratio of some sort. Probably the most intuitive and famous ratio of such was derived by Sharpe in 1966. The Sharpe ratio can be derived using the following equation:

$$\boldsymbol{Sharpe\ ratio} = \frac{\boldsymbol{Rp-Rf}}{\boldsymbol{\sigma p}} \qquad \textbf{(12)}$$

In this equation Rp stands return on portfolio or asset p, while Rf stands for return on risk-free asset and sigma p stands for standard deviation of portfolio or asset p. Later, the results will be evaluated in terms of the Sharpe ratio. As mentioned earlier and argued by Markowitz, an investor should not be able to attain greater returns without a trade-off for higher variance. Consequently, the optimal portfolio allocation should be attainable by maximizing portfolio Sharpe ratio, which makes this ratio very important in finance.

The risk-to-reward is such an important concept for the reason that the predictability of volatility is on solid foundation, even if the predictability of stock returns are not. Hypothetically, if one can predict volatility and if volatility could be used to predict returns via knowledge of a risk-to-reward trade-off. Just plug variance estimates, Sharpe ratio and risk-free rate into the equation and you get your market returns. Obviously, it does not work like this. Returns on instruments derived from volatility are another discussion, which includes futures and options.

Granger and Poon (2003) argue that volatility is definitely predictable, but it does not violate market efficiency, as it is in line with underlying asset and options prices being correct. In some sense this is true, but autocorrelation in volatility could still be interpreted as a form of predictability of returns. Fama was as early as 1965 researching into arguments of Mandelbrot (1963). Fama wanted to determine if large moves follow large moves and small moves follow small moves in the market. This would imply statistical dependence on the time series of market returns — even if the directionality is random — one could argue that this is contradictory to the efficient market hypothesis. The problem with reverting to adaptive market hypothesis instead of EMH is that the hypothesis considers the market such a complex and evolving system that it cannot necessarily be defined with better accuracy. Therefore, the concept of unpredictability is hard to nullify or reject.

In terms of forecasting volatility, it has been shown that VIX itself is a relatively good forecaster of volatility (Huang Kun 2011, Granger and Poon 2003). For reference, Ronnie Söderman (2000) conducts a Granger-causality test to examine the predictive capability of implied volatility on the German DAX index. He concludes that the volatility index can be interpreted as a warning signal predicting big moves. Fleming, Ostdiek and Whaley (1995) also conclude that the VIX appears to be a useful in predicting market volatility, because it imbeds the market expectations. Corrado and Miller (2005) find that the VIX is a much better forecaster than realized volatility, and that it works best as an explanatory variable in a model. They argue that biases in CBOE VIX data have mostly disappeared after 1995.

Glosten, Jagannathan and Runkle (1993) were the ones to introduce the GJR model for estimating and forecasting future volatility. Huang Kun (2011) compares forecasts between models and implied volatility and confirms earlier research that the GJR model is the best estimate of future realized volatility. It is interesting to note, that VIX index is relatively not much worse than GJR. It also seems like VIX works best for a 5-day window, instead of the 30-day window it supposedly measures. In Huangs regression results VIX had a 0.4825 R^2 for a 5-day duration, while only 0.1067 for 30 days. R^2 variable gives us an estimate on how much the model explains of the variance. Therefore, VIX seems to explain relatively large amount in a very short future timeframe. This is essential for the holding periods in this paper are closer to 5 days than to 30, after the signal occurs. The best estimates are still received from a combination of models, especially for longer timeframes of 30 to 60 days. RiskMetrics model developed by JP Morgan and the GJR model seem to work very well together.

3 LITERATURE REVIEW

This section of the paper goes over 3 different studies that have focused on volatility or implied volatility in relation to asset returns and can be considered relevant to the paper.

3.1 Copeland & Copeland (1999) Market timing: Style and Size Rotation using the VIX

In the article Maggie and Thomas Copeland evaluate the return spread between large, small, value and growth stocks in relation to historical and implied future volatility. They apply a strategy to time the market in regard to this spread.

The first strategy deviated in between style factors growth and value, with the idea that rising volatility signals deteriorating market conditions in the eyes of investors. This uncertainty would imply a falling confidence in growth stocks and that investors should rather look for comfort in the form of value stocks. Value stocks are supposedly undervalued and therefore less risky. In the opposite case of regime change to lower estimates for volatility the strategy would shift from value to growth for a momentum strategy. The idea is that the growing confidence would boost the returns in growth stocks. The second strategy of in the paper involves changing the size exposure of the portfolio. Same principal as in the first one, but this time rising volatility implies a change to large-cap stocks as falling volatility is treated by increasing exposure in small-caps. Given that small-caps often have a higher beta than large-caps and market returns have a negative correlation between volatility, then the performance of large-caps with lower betas should be better than the deteriorating performance of small-caps, when volatility increases unexpectedly.

Copeland et al. first reported the contemporaneous relationships between the historical volatility, index return and style/size factors in question. After that they used future volatility estimates in the form of VIX to time the market and regressed the spread between the factor portfolios against changes in the VIX.

3.1.1 Data

The timeframe for the study was different for the style and size strategies as was the data. As mentioned earlier historical volatility as well as estimate for future volatility were used in the calculations.

To measure up the historical volatility, Copeland et al. used daily return observations for S&P 500 index to calculate a monthly volatility measure. Change in the historical volatility level was measured by the difference of volatilities calculated as the estimate of month t minus the estimate of month t-1. Percentage change was calculated by dividing this difference with the value for month t-1. Future volatility was measured by VIX in the form it was calculated back then in between 1986-97, with the option chain for S&P

100.

For the contemporaneous estimates of style-based strategies, the time frame evaluated was May 1981 to September ending in 1997. Six portfolios were constructed of large/medium/small cap growth and the corresponding sizes for value using the Wilshire US Style indexes. For more detailed information one can look at the website of Wilshire associates or the article. To evaluate the association of future returns and the future volatility estimate in regard to style factors, Copeland et al. use daily data from February 17, 1994 until October 20, 1997 for total of 927 observations. They use the BARRA's value and growth indexes, since those indexes actually had futures traded on them, and Wilshire indexes did not.

Timing based on size factor alone was done at the second part of the article. For the contemporaneous part the time frame was the same, but to measure up the differences in between large and small they used Nasdaq and S&P500 indexes as well as Wilshire large and small. For the future returns part, they used S&P500 versus the Value Line futures, where the Value Line-index represents small capitalization stocks in relation to S&P500. Difference is that for the size factor they had the possibility to gather a longer sample than three years. They used data from January 2, 1986 to October 20, 1997. In estimating the 75-day moving average they lost some observations from the start of the data, so the strategy was implemented for 2897 observed dates.

3.1.2 Method

To estimate the contemporaneous relationship between timing and style a linear regression model is used in the paper. First, they use a one factor linear regression to estimate the impacts of change in historical volatility to returns of indexes that describe the corresponding style factor.

$$R_{i,t} = \alpha + \beta var_t + \varepsilon \qquad (13)$$

where $R_{i,t}$ represents the return to the index portfolio of different sizes (large, medium or small cap) at time t. Alpha is the intercept term, while beta is the slope coefficient. Var_t refers to the change in historical volatility at time t. Epsilon stands for a normally distributed error term at time t.

In the second phase, they further the analysis by regressing the same variables to explain spreads between different indexes. To account for this there is a separate index return added to the left side of the equation. They also use different methods to measure historical volatility. The model used is of the following form:

$$R_{m,t} - R_{n,t} = \alpha + \beta var_{k,t} + \varepsilon \qquad (14)$$

where $R_{m,t}$ represents the return to value style portfolio of different sizes while $R_{n,t}$ represents the return of a growth style. Alpha is the intercept term, while beta is the slope coefficient. In this step $Var_{k,t}$ refers to three different measurements of historical volatility: the level, change in the level or percentage change in historical volatility at time t. Regression is estimated three times using these different measurements. Epsilon stands for a normally distributed error term at time t.

Copeland et al. also regress the percentage change in VIX against the long value-short growth portfolio for holding periods of 1 to 20 days. They replace the historical volatility variable with future volatility estimates of VIX. To avoid noisiness in the VIX the percentage change in the VIX is calculated by dividing the spread between the 75-day moving average (75-DMA) and the current value of VIX by the 75-DMA of VIX.

Based on the percentage change in the VIX defined above, they test a trading rule. The trading rule itself is based on the closing price of VIX being "X percent" greater or lower than the 75-DMA. Basically, if VIX is above its 75-DMA the strategy goes long value/large-caps and short growth/small-caps and if below the 75-DMA the strategy goes long growth/small-caps and short value/large-caps. For different trading rules they list daily cumulative and average returns, numbers of roundtrip transactions and number of active dates. The trading rules were defined to have different holding periods after signal triggers (one, two, three and ten days) as well as different thresholds to trigger a signal. The trigger parameter is the percentage change in the VIX described in the paragraph above. Thresholds are 10%, 20%, 30% and so on for every 10% change in the VIX as described earlier until 80% which is the maximum in the sample period. Long growth/small caps transactions happen on crossing thresholds of -10% and -20%. Number of days per scenario is listed in the results as well as daily average and cumulative return.

3.1.3 Results

The contemporaneous results show a clear negative relation with historical volatility and returns on a monthly scale. The effect seems to be higher on small capitalization stocks as well as growth stocks based on the slope coefficients in the regressions results.

In regard to trading according to the future volatility estimates, most of the dates in the sample have a positive percentage change measurement of VIX – meaning VIX closing price above 75-DMA – so the strategy implemented goes mostly long value and large caps. Results show that the strategy creates excess returns based on the volatility signal for holding periods that are at least two days or possibly longer. Returns for rotating between large and small caps seem to be higher than for rotating between value and growth. The increasing deviation from the 75-DMA seems to have a positive effect on the returns of the strategy, so one should not necessarily use the 10% rule, but rules tied to higher thresholds. To conclude, the paper clearly demonstrates effective advantages of timing the size factor. It is good to note that according to this paper this asymmetry is present in return spreads of the biggest indices in the world. The assumptions in formulating the strategy were such, that intuitively they could be derived from time-varying risk aversion. As discussed earlier, Copeland et al consider that investors choose less risky alternatives in times of uncertainty and riskier alternatives on times of confidence. This same subject is discussed in terms of variance risk premium in the next paper.

3.2 Bollerslev and Zhou (2007). Expected Stock Returns and Variance Risk Premia

The article studies the difference between implied and realized variation in the equity markets. Bollerslev and Zhou state that the variance risk premium (VRP) can explain a remarkable portion of the variation in quarterly excess returns on the market portfolio. High premium predicts high future returns and low premium predicts low future returns. According to Bollerslev et al. this dominates another predictor variables that have been considered to work, as in the P/E ratio, default spread, dividend yield and the consumption to wealth ratio. VRP can also be combined with these inferior predictor variables to gain higher explanatory power. A crucial detail behind these statements is that the values for implied and realized variances should be constructed from high-frequency intraday data. Implied variance should be attained from “model-free” process referring to real-time option prices instead of using Black-Scholes option pricing model. Realized variance should be calculated from summation of intraday squared returns to give a better idea of the actual variation instead of using daily or coarser frequency observations.

Bollerslev et al. theorize that VRP is related to the coefficient of relative risk aversion, which varies in between times. When VRP is high, investors presumably allocate wealth to less risky investment alternatives, while it is low the exceedingly risky assets are chosen. This in turn affects expected returns. They also note that empirically it is hard to distinguish if the driving force is just that variance risk premium or the fact that it exhibits covariance with the relative risk aversion as booms and busts occur in the market cycle.

3.2.1 Data

Bollerslev et al. use monthly data of S&P500 returns from January 1990 to January 2005 as their regression’s dependent variable. Data for independent variables comes from several places. For estimate of realized variance they use intraday data to calculate monthly values. Realized volatility is defined as:

$$\boldsymbol{RV} = \sum_{j=1}^{n} \left[p_{t-1+\frac{j}{n}} - p_{t-1+\frac{j-1}{n}}(\Delta)\right]^2$$

$$\rightarrow \textit{Return variation } (t-1,t) \tag{15}$$

where p_t denotes the logarithmic price in time t and n denotes number of observations. The data for equity index returns is provided by the Institute of Financial Markets. Monthly realized variance is calculated via summation of 5-minute squared returns for the S&P500

index within the month.

For quantifying implied volatility, they use the VIX index, as they refer to it as the “model free” industry standard for approximating risk-neutral variance of a fixed 30-day maturity. The options data comes straight from Chicago Board of Options Exchange (CBOE).

The implied volatility from options gives an estimate for variance during period [t, t+1] with the investor information set in moment t, while the realized volatility estimate gives an estimate for time period [t-1, t] in moment t. Variance risk premium is defined as discussed earlier:

$$IV\text{-}RV=VRP \tag{16}$$

In this case the IV of [t, t+1] and RV of [t-1, t] will be used. Since this is the estimation method, the variables are both observable at time t.

Alternative predictor variables are obtained from Standard & Poor’s, as in P/E ratios and dividend yields for the S&P500. Data on the 3-month T-bill, the default spread between Moody’s BAA and AAA corporate bonds, the term spread between 10-year treasury bond and 3-month treasury bill yields as well as a stochastically de-trended risk-free rate are taken from the public website of Federal Reserve of St. Louis. They also use consumption-wealth ratio (CAY), as defined in Lettau and Ludvigson (2001) and download the data from their website. Since Bollerslev et al. have mostly monthly or quarterly observations for some variables, they use the last variable of the respective quarter to represent that quarter to avoid bias. This also allows them to avoid overlap in variables.

3.2.2 Method

First Bollerslev et al. run a regression to predict forward quarterly returns of S&P500, with $\sqrt{IV}$, $\sqrt{RV}$, $\sqrt{IV-RV}$ and other alternative predictor variables listed above as the independent variables on the linear regression model. Since they have predictive variables with autocorrelation and heteroskedasticity, they use Newey-West robust t-statistics based on four lags. They look at the significance of predictor variables as well as adjusted R-squared (R^2) of the regression, since it is a variable that describes how much of the variation is explained by the model. The general form of the model is as follows:

$$R_{it}^{e}=\alpha_i+\beta_i*PredictorVariable_{t-1}+\varepsilon_{it} \tag{17}$$

where R_{it}^{e} is the quarterly excess return. Alpha and beta are as they normally are, with t standing for a time variable and i standing for the specific investment. Predictor variables are lagged to avoid bias in the forecasts. This same model is also used with multiple predictor variables, to study if combination of variables improves forecasts.

After conducting the first regression and obtaining results Bollerslev et al. decided to try and estimate whether this affects factor pricing by estimating two different models. The second regression model in the paper is denoted as follows:

$$R^e_{it}=\alpha_i+\beta_i{*}Factor_t+\varepsilon_{it} \tag{18}$$

where R^e_{it} stands for monthly excess returns on 25 different size and book-to-market sorted portfolios and $Factor_t$ variable refers to market excess return and the VRP. The model is estimated twice, once for market excess returns and once for the VRP.

The third regression model is of the following form:

$$R^e_{mean,i}= \beta_{i,estimate}{*}\lambda +\varepsilon_i \tag{19}$$

where $R^e_{mean,i}$ denotes sample mean returns, $\beta_{i,estimate}$ denotes estimated factor loadings and λ is the factor risk premium. These beta estimates for factor loadings are derived from the 25 portfolios by conducting the second regression model and this regression is done twice, once for market excess as the factor risk premium and a second time for the VRP in factor risk premium. Therefore, it enables to compare the explanatory power of market risk premium and VRP.

3.2.3 Results

The explanatory power of VRP, becomes clear when the first linear regression model is estimated with the predictor variables explaining excess returns for future quarterly returns. The VRP estimated from 5-minute intervals of intraday data provides clearly best explanatory power of 15.14% with a statistically significant t-value. Other variables alone reach explanatory power between -1.63% to +6.32% with varying significance measured by the t-stat. Therefore the 15.14% explanatory power seems dominant**.** Two other valuable variables proposed were the logarithm of P/E ratio and implied volatility alone. Highest explanatory power (27,67%) was achieved with VRP, log of P/E, the term spread (difference between the 10-year and 3-month Treasury yields) and stochastically de-trended risk-free rate (defined as the 1-month Treasury-bill rate minus its trailing twelve-month moving average). It is now good to remark on the volatility smile and implied volatility function discussed earlier. Black-Scholes having a flat implied volatility function could be a possible explanation, why the IV derived from the model fails to explain returns explained by the "model-free" volatility premium.

In the second regression they gain insight if the VRP is a priced risk factor. The regression returns significant t-values for market excess returns, but not for all VRP estimates, thus it suggests that the market is not pricing the VRP factor. Otherwise, it would show as a significant in terms of t-value explaining the returns of the current portfolio returns estimated out of the 25 different ones.

To confirm this, they estimate the third model, where the mean of the portfolio returns is estimated by a constant and a vector of the beta estimates from the second regression plus an error term. This confirms the fact that VRP is not priced, as the λ coefficient gains a significant t-value, while it is defined as market risk premium of Rm-Rf, but while it is defined as VRP the t-value fails to reject the null hypothesis.

Bollerslev et al. (2009) conclude that the empirical results suggest that time-varying risk aversion as well as time-varying volatility risk can be important in interpreting short term variations in expected stock returns. This coincides with the studies of Copeland et al. They concluded that a trading strategy based on assumptions and changes in investor risk aversion can generate excess returns. Further on P.Giot will provide more light on the subject.

3.3 P. Giot(2005).On the relationships between implied volatility indices and stock index returns

The article sets out to show that there is a negative statistically significant relationship between returns of the stock and implied volatility indices (VIX and VXN). According to the article this relationship between stocks and implied volatility is also asymmetric, since negative stock index returns yield bigger changes in VIX than positives do. This asymmetrical response is especially sharp during times of low-volatility, which Giot explains by options traders being reactionary to negative equity returns. This obviously is a different approach and an assumption that the strategy of this paper is based on, since this paper approaches the lead-lag relation as if the option market was the one leading. For Nasdaq 100 and VXN the asymmetric effect is rather weak according to Giot. The dynamics of volatility might be a bit different in the case of Nasdaq as it is for the broader market. Obviously, this might have to do with different factor exposures in the combination of stocks in the index. As discussed earlier, in the article of Bollerslev and Zhou they considered variance risk premium to be a proxy for market risk aversion. This would explain the different behavior in between the relationships of S&P500 and the VIX as well as Nasdaq 100 and VXN, since we can hypothesize that the investors in stocks of Nasdaq have a different risk-aversion profile.

Anyways that is a question and a discussion for another paper. The important part of Giots paper in regard to this one is that he finds evidence of higher future returns when purchases occur during times of higher implied volatility. The subject of research in the first part of the paper is contemporaneous changes in the stock index and implied volatility index. In the second part returns of a trading strategy are evaluated, and this trading strategy aims to test timing the market via certain implied volatility thresholds.

3.3.1 Data

Giot uses log returns of S&P100, VIX, Nasdaq100 and VXN indices. Time-period for S&P100 and the VIX is August 1, 1994, to January 31, 2003. For Nasdaq100 and VXN it is slightly different June 2, 1997, to January 31, 2003. These intervals are divided into three distinct time periods, since the indices exhibited different volatility regimes as well as bull and bear market periods during the sample period. Those periods are for the S&P500 and the VIX: August 1, 1994- May 30, 1997, June2, 1997-March 31, 2000, and April 3, 2000-Janury 31, 2003. For Nasdaq100 and VXN the time periods are January 3, 1995 – May 30, 1997, June 2, 1997 – March 31, 2000, and April 3, 2000 – January 31, 2003. The first period in each index is categorized as low volatility bull market, the second period is high volatility bull market corresponding to the development of dotcom bubble and third period is high volatility bear market.

The data used for initializing the trading algorithm in the forward returns part of the study seems to not be defined in detail for VIX. It is told that the VIX ranks are comprised of a rolling two-year sample, so that for given day t VIX_t is compared with 20 equally spaced percentiles based on rolling two-year history of VIX. Therefore, we assume the date the rolling estimation window for the percentiles starts from two years before the August 1, 1994, which is the first day of the sample period. In the case of VXN_t the percentiles for VXN ranks are defined starting from January 3, 1995, with the two-year rolling percentiles as the VXN data is not available before that date. This is also why the trading strategy for Nasdaq is evaluate between June 2, 1997, and January 31, 2003. The sample is two years shorter than for the S&P100 and VIX.

3.3.2 Method

Giot uses linear regressions with dummy variables to explain the contemporaneous relationships. Returns on the VIX are on the dependent variable and index returns on the independent variables. The constant and beta term of the regression is controlled with a dummy variable to separate dates for negative and positive market returns.

$$rVIX_t = \beta_{-0} D_{-t} + \beta_{+0} D_{+t} + \beta_{-1} (rOEX_t * D_{-t}) + \beta_{+1} (rOEX_t * D_{+t}) + \varepsilon_t \qquad (20)$$

The dummy D_{-t} is 1 if they date is negative and 0 if the date is positive. The reverse is true for D_{+t} and this enables the estimate of negative and positive days separately. r_{VIX_t} obviously stands for log return on VIX on date t and r_{OEX_t} stands for log return on S&P100 on date t. The same is done with Nasdaq100 and VXN. Additionally, Giot adds a quadratic term in the regression to explain size effect in case bigger companies with larger index weights explain more of the variation in the dependent variable. This will not be described in more detail, since Giot disregards it as not very informative variable and it is not important in the context of this paper.

For the forward returns Giot uses a process that evaluates 1, 5, 20 and 60 day forward looking returns depending on the rank of VIX in day t. The process starts from day 0 as the VIX percentile is defined and the intervals start counting registering the produced vector of daily returns as an observation of that respective forward return interval (1, 5, 20 or 60 days). Basically, this means a lot of overlapping observations, so for significance tests the Newey-West standard errors are used to correct for heteroscedasticity and autocorrelation. The rank of VIX is defined as percentile intervals of 5% in regard to historical data. This implies 22 different ranks, with 0 representing below historical lows and 21 representing higher observations than historical high. The model used is of the following form:

$$r60d_t = \delta_1 D_{1t} + \delta 2 D_{2t} + \ldots + \delta_{21} D_{21t} + \varepsilon_\tau \qquad (21)$$

This model is estimated for $r1d_t$, $r5d_t$, $r20d_t$ and $r60d_t$, for where the number stands for the duration of holding period after purchase on time t. The whole term in the left-hand side of the equation stands for returns on the strategy. δ stands for expected return corresponding to the rank of VIX on the starting date of the holding window. It is defined for $r5d_t$ as the log return ln (P_{t+5})-ln (P_t), where P stands for price and t for the time at the time of signal and transaction. Dummy variable D corresponds to 1 or 0 to control for the right VIX percentile at day t estimated by the rolling two-year estimation window.

3.3.3 Results

The results of the first part of Giots study show a clear distinction between coefficients for negative days and positive days, which underlines the asymmetrical relationship. Giot also points out that this asymmetry is weaker in times of high volatility and amplified during times of low volatility. For explanation Giot offers speculation that traders are more eager to bid up volatility in times of low volatility, than they are on times when it's already high compared to historical standards.

In terms of the return-VIX relationship, the longer-term outliers in implied volatility seem to have a connection with higher returns and the high observations of implied volatility seem to coincide with high forward returns on equities, while low observations coincide with low forward returns as written in the article. This characterizes the longer-term relationship between implied volatility and equities. Giot reports the results in a table for every holding period, respective to the corresponding volatility rank the transaction was conducted on. In the scatter plots describing these observations, one can see that this relationship is not linear and if one were to fit a linear model, the error terms would most likely be very heteroskedastic. The correlation between these two variables in contemporaneous time on the other hand shows a very consistent and linear relationship.

The issue in terms of implementing the strategy of Giot is that it offers relatively low number of worthy opportunities, since the statistical relationship between excess returns and VIX numbers only becomes clear on outlier thresholds of VIX. Since VIX is supposedly a mean-reverting process as discussed earlier, these opportunities do not come often.

Giot provides scatter plots in the paper that effectively show heteroskedasticity in between the implied volatility and strategy forward looking holding period returns. To the authors interpretation this visually represents the time-varying risk aversion of market participants.

The results of Giot are contrary to what occurs in this paper, since effectively the numbers produced by Giot provide support to buying equities when the VIX goes higher, as the strategy in this paper responds to higher VIX readings by shorting the market. Obviously, Giot is missing one key restriction imposed in this paper, which is the fact that the market must go up as well, with the VIX, before we can possibly get a signal. Derivation of this signal will be further elaborated in the next chapter.

4 STRATEGY – Signal to transact and Transaction costs

4.1 Signal to transact

In the earlier chapters a foundation and informational edge has been laid for formulating a strategy. Referring to all the literature presented it should be clear that some market inefficiencies exist. Markets are argued to systemically overshoot with some compelling arguments to back that up. Markets also have a tendency to revert back to a conditional mean and investors do misevaluate probabilities, especially if the market returns exhibit leptokurtosis as Mandelbrot (1963) and Fama (1963) argued. Time-varying risk aversion affects the investor psychology and behavior, which could exacerbate moves.

As explained by the leverage effect – volatility and the stock market should have a negative overall correlation. Historically speaking on the daily time frame, this is also true, since the data used has aggregate -0.71 correlation in between the S&P500 and the VIX for the 20-year sample. Regardless of the reasoning behind the negative asymmetric correlation, in an efficient market the effects of prior prices should be priced in as stated by the efficient market hypothesis. Russel and Thaler (1985) argue that this is not necessarily the case if markets have quasi-rational agents and the market participants conducting statistical arbitrage do not have enough pricing power. This is in line with deductions of Shleifer and Vishny (1997) and the theory of limits of arbitrage.

To formulate a trading strategy that exploits this, one should consider certain specifics aspects of the market process. Firstly, transaction costs force a speculator/arbitrageur to focus on best of the opportunities. When you consider the nature of risk and dispersion as a measure of risk — then one should understand that usually the biggest deviations for short-term profits are related to volatility spikes. Positive skewness of the distribution function and clustering are both characteristics of volatility. Since volatility spikes are associated to negative returns, one should focus on risk-aversion, avoiding drawdowns maybe even short-selling to benefit from these anomalies in the short-term, since the drawdowns can be argued to be clustered in terms of volatility.

As discussed earlier the predictive capabilities of implied volatility have been researched and there is compelling evidence that the VIX index is a good forecaster of future volatility. Therefore, if VIX is negatively correlated with the market returns — as discussed on several occasions earlier — then a rising VIX should in some time frame forecast negative stock market returns, unless the movement is priced in instantaneously. This instantaneous pricing is not always the case and obviously the high readings/increase in VIX does not translate into negative returns in the case of outlier observations, as noted in Giot (2005). The lead-lag relation between equities and the option market has also been studied and discussed earlier, so the options pricing a heightened drawdown risk via increased implied volatility should be taken seriously.

The thought process follows a certain guideline, that if there exists a specific scenario, in which the market and VIX returns exhibit positive correlation with the market returns, then it must only be a short-term fluctuation. It means one of the two indexes is inclined to give up some gains in this specific scenario. It can be theorized, that in a scenario, where the option market is pricing a heightened drawdown risk via increased implied volatility, but negative returns have yet to occur, then one should be looking to short the market. The signal is therefore formulated to trigger in scenarios on which the negative correlation between the two indexes (S&P500 and VIX) is broken and they exhibit positive correlation. Therefore, the test setup includes looking for a day or two consecutive days with positive co-movement in the VIX and the S&P 500 (SPX) and then setting the transaction date a day after that or in the close of the chosen date. The purpose is to examine if there are scenarios, where the market fails to price in the volatility risk correctly and investor can benefit from this sequence of events.

The signal for date t can be defined as follows:

$$\begin{gathered} \textbf{\textit{IF } } VIX_{t-1} \,\&\, SPX_{t-1} \,\&\, VIX_{t-2} \,\&\, SPX_{t-2} > 0 \\ \textbf{\textit{Signal}} = 1 \\ \textbf{\textit{ELSE}} \\ \textbf{\textit{No signal}} = 0 \end{gathered} \tag{22}$$

VIX_{t-1} & SPX_{t-1} & VIX_{t-2} & SPX_{t-2} >0 represents the condition for positive co-movement of two days, which happens when returns on dates t-1 and t-2 are positive for both the VIX and the SPX. After receiving a signal, the strategy takes a loan with market portfolio as a collateral and shorts the market for a predetermined duration, so that the investors factor exposure to market risk premium drops to 0. The predetermined duration protects the investor, since it makes the process systematic, as market doesn't necessarily always follow the blueprint and expected outcome. The investor using the strategy is supposedly simultaneously long an index ETF and short futures of the same index, so that he is considered market neutral in terms of equity risk premium. To simplify, the investors strategy is to be permanently long and then hedge the downturns by shorting the futures after a signal. The transaction is conducted instantly after this signal is triggered. Therefore, it is done at cash market close at date t-1 using futures contracts, since that is the timing that the count for day t index returns starts and it is the first day of the predetermined holding period. Strategy is only implementable by using futures contracts, since they are the only available instrument, if the market is not open. For a short-term trade it is also cheaper to use futures, as will be explained further on in the next chapter. The process for constructing the portfolio returns will be discussed in the data section.

4.2 Transaction costs

Obviously, market cannot be deemed inefficient, if the anomalies discovered cannot be benefitted of due to market friction. One could even question, if these anomalies really are anomalies in this case? Traditional way to account for transaction costs in the literature is to calculate costs for selling the position and allocating the money to a proxy for a risk-free asset, which usually is defined as the US 10-year treasury bond. In the current markets this seems to be old fashioned and really a bad way to evaluate market efficiency as market instruments have significantly improved. Also, in regard to this paper, one can argue that the investment horizon in regard to long and short exposure can be described as infinite (long-exposure) and very short (short-exposure). Therefore, it is more fitting to use a shorter duration government bond to evaluate the very short short-selling scenario constructed in here. This will be elaborated further on, when a thorough description of the portfolio return constructing process and strategy is provided.

To invest in an index there are several different possibilities as in investing with your own stock investments and managing the portfolio yourself i.e., creating the index by yourself. This obviously requires a lot of capital to split up to 500 companies in the case of S&P 500. There is also the question of managing the company weightings and so on. Another way to do it is to invest in an ETF that tracks the index and a 3rd way to do it is to invest in stock market futures. In the short-term it is the cheapest to invest in futures, as the transaction cost structure is significantly lower. In the longer term, one could argue that if a sufficient index tracking ETF isn't the cheapest alternative, it's certainly a lot easier than replicating the index by yourself.

If we assume that one owns an index tracking ETF for the long term, then the cheapest way to hedge for short-term market turmoil is to short futures via leveraging your portfolio, so you double up the money you have and go market neutral in terms of equity risk premium. When you're, shorting futures, you're also entitled to implied financing rate, as in the risk-free rate, since you're effectively borrowing for someone that wants to gain long side exposure. In the case of shorting stocks, the person shorting would be doing the borrowing and paying for borrow. This is exactly the opposite in futures, so futures contracts for difference are perfect for this sort of activity.

In this sort of scenario, every time one shorts, one is market neutral, but basically one increases his/her capital, so one gains the same amount if the market drops. After the predetermined holding period one re-adjusts to his/her basic portfolio exposure. Further on we illustrate the process in the data section, where a spreadsheet table elaborates on the steps of the process.

Exact information about ETF and futures trading costs can be found on the website of Chicago Mercantile Exchange.[2]

[2] https://www.cmegroup.com/trading/equity-index/report-the-big-picture-for-the-active-individual-trader.html

5 Data

5.1 Data and the data collection process

Most of the data used is the daily timeframe observations derived from Thomson Reuters Eikon, for SPX and the VIX index for 25 years during 1995 and 2020. The data includes 6488 daily observations for VIX and the S&P500 in between 3.1.1995 – 6.10.2020. SPX and the VIX were chosen for this study, because of the researchers' suspicions of possible short-term mispricing in the equity index following a particular sequence occurring in the time-series.

To evaluate excess returns and use the method of Jensen's alpha — one must also have data and a measure for risk-free returns in the market. To minimize factors of credit-risk and term-risk of a bond — one can consider short-term US government bonds as an appropriate risk-free rate to use in calculation, so therefore 3-month treasury bill rate is used for estimating the risk-free asset.

To conduct analysis of any sort one needs to modify the time-series of daily closing prices to a more suitable form. In the case of VIX and S&P500 this is done by differentiating the time-series to a form of:

$$(p_t - p_{t-1})/p_{t-1} = r_t \tag{23}$$

where t is the variable for time, p is the closing price and r_t stands for the numerical value of percentage return on date t. From values of r_t we can conduct the event study part, but constructing total portfolio returns requires us to use continuously compounded returns or log-returns from the closing price p.

For all the evaluated portfolios and indexes, the continuously compounded total returns are calculated as:

$$R_{TOT} = (\prod (1+r_t)) - 1, \text{ where } t = [1,T] \tag{24}$$

T represents the last observation in the time series. The sign of r_t changes to negative, if we are on the strategy ordered timeframe after a signal occurs. This is because as theorized before, the strategy includes doubling the capital of the portfolio to reduce equity risk premium exposure to 0. This is described in equation 26. Shorting futures is theoretically possible after cash market close and they also enable the leverage for such a maneuver.

The risk-free rate is downloaded via R package quandl and 3-month US treasury bills are used as mentioned before. The data comes in as annualized in a percentage form, so some modifications must be conducted. First, one must turn the rates from percentage into a numerical measure by dividing by 100 and after that convert annual rate into a daily one for every date on the data set. Excess returns are then measured by:

$$r_i - r_f = r_{i,excess} \tag{25}$$

where the i subscript stands for the asset or portfolio time series in question. One could argue for using a 10-year rate instead of the 3-month rate, but in this case, it goes both ways, since when one is shorting futures, then one is entitled to the risk-free rate anyways. To the authors understanding the usage of a smaller rate should increase the excess returns of both the strategy portfolio and the market portfolio and if it were to affect the alpha of estimates, the effect should be negative. The impact of the risk-free rate will be elaborated further here and Jensen's alpha, will be further explained in the method chapter. Excess returns are required to carry out the calculations by the Jensen's method.

Shorting futures earns a right for implied financing rate, because the one shorting is effectively financing the purchasing side of the contract. Therefore, for the dates that the strategy portfolio is short the market via futures one must add the financing rate to the equation:

$$(1-r_t + r_f)*p_{t-1} = p_t \tag{26}$$

The following table illustrates the process of portfolio return construction:

Table 1 – Portfolio return construction

Date	Market excess return	signal	portfolio return	information
t	Rm-Rf	0	Rm-Rf	first observation
t	Rm-Rf	1	Rm-Rf	signal date
t	Rm-Rf	0	-Rm+Rf	signal+1
t	Rm-Rf	0	-Rm+Rf	signal+2
t	Rm-Rf	0	Rm-Rf	return to normal
t	Rm-Rf	0	Rm-Rf	last observation

In the table we have a two-day window after the signal date, that the strategy is implemented in terms of short selling. The duration of the window will be defined after conducting the event study and it will be referred to as the predetermined holding period. The transaction costs will be evaluated by calculating the percentage value for separately for every single transaction conducted by the strategy. This is done by calculating the execution cost plus the bid-ask spread of 0.25 index points and dividing by the contract value. Obviously, the costs are lower for bigger investors and CME reports this as the average through individual investors. According to the CME source the average is 3.54$ for execution and the bid ask spread is 0.25 index points on average, which in dollar terms is 12.5$, since a point in E-mini futures contract is worth 50 dollars. Therefore, the transaction costs in percentage terms per transaction are produced by following equation:

$$(3.54\$+12.5\$)/50*SPX_t = TC_t \tag{27}$$

where SPX_t denotes daily closing price of the S&P500 index on day t and TC_t denotes transaction cost on day t in percentage terms. Evaluating transaction costs this way naturally reflects the authors perception of developments in the markets. The percentage value of transaction costs is a lot higher in the early years of the sample period, because the denominator is a significantly lower number. While the amount of capital tied to a single futures contract grows, the percentage value drops as the denominator increases.

Obviously, this method of estimation is based on costs today, so one could argue that it is a bit lacking, even though it adjusts the costs higher for the older transactions. Spread and execution costs might have changed significantly through the years as markets have developed. On the positive note, this should not be an issue regarding the results, since one could also argue that most likely the big money and big investors —institutions and pension funds — moving the markets have costs that are lower than estimated here.

5.2 Descriptive statistics

The descriptive statistics show nothing disorderly, that should warrant action in terms of methodology, even though the data does not seem to be strictly normally distributed. S&P500 returns seem to be leptokurtic with kurtosis above three. The following table includes the descriptive statistics on collected as well as constructed data.

Table 2 - Descriptive statistics for indexes and portfolio returns

Time series	S&P500 returns	VIX returns	4DH	4DH-TC	1DH	1DH-TC
Mean(%)	0.038253	0.244721	0.038313	0.037724	0.032488	0.031689
Median(%)	0.067183	-0.391773	0.061299	0.060928	0.060638	0.060116
Standard Deviation(%)	1.210376	1.210376	1.210086	1.210112	1.210277	1.210227
Upper quartile(%)	0.590268	3.330867	0.586066	0.586066	0.585425	0.583366
Lower quartile(%)	-0.461983	-3.765246	-0.469825	-0.470739	-0.466217	-0.467761
Min(%)	-11.984050	-29.572650	-11.985002	-11.985002	-11.985002	-11.985002
Max(%)	11.580036	115.597920	11.579045	11.579045	11.579045	11.579045
Kurtosis	10.558825	16.973191	10.572719	10.571201	10.566553	10.568741
Skewness	-0.187805	1.926394	-0.186020	-0.185424	-0.220181	-0.219327

The four columns in the right of VIX returns refer to constructed portfolio returns data. 4DH stands for 4-day-hold, while 4DH-TC is the 4-day-hold minus transaction costs. 1DH stands for 1-day-hold and 1DH-TC stands for the 1-day-hold minus transaction costs.

The decision to report and use these durations will be elaborated on in the results chapter. The data in the table is represented in percentage form, for the rows marked with %. To clarify mean for the S&P500 returns was therefore 0.038%, while maximum move was 11.58%. This means, that for readability, the data was multiplied by 100. Skewness and kurtosis are calculated with the psych package in R from the decimal form data and the results are shown with no other modifications for readability. All of the data is rounded to six decimal points.

As we can see from the table, the constructed portfolios achieve lower mean returns when accounting for trading costs, but also lower variance and higher kurtosis. This implies that the portfolios have a lower tail risk. The higher mean return on the 4DH without accounting for trading costs, seems to suggest that the signal might have value. All of the portfolios are negatively skewed. The 4-day-holding period has a higher skewness than S&P, while the 1-day-holding period does not. The lower skewness and kurtosis in terms of the 1-day-hold implies that the left tail of that distribution is even lower than the 4-day-hold, while returns are more often positive. This might be since the 4-day-hold is neutral during days of positive equity returns, while 1-day-hold has already flipped back to being exposed to equity risk premium. The lower tail risk does not show in the min and max observations though, as they don’t differ very much. Rather it seems that the strategies traded on those dates but consumed the big moves regardless. The results section will discuss these further.

6 METHODOLOGY

This chapter explains the methods used to assess the effectiveness of the signal, as well as the purpose and thought process behind the methods. It is important to note, that the study differs from most of the other studies using the same methods, for it treats the benchmark market index as a single asset. This is why the SML – described in 2.1 – is a relevant and applicable concept here. Therefore, the resulting statistics are head on comparable to benchmark index returns, but it also limits the author in terms of model choice, at least in the case of event study.

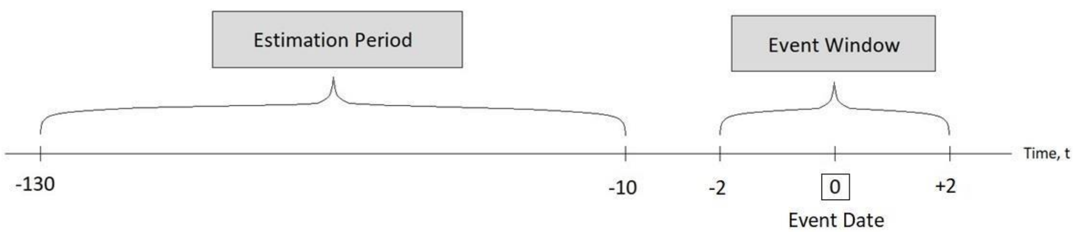
6.1 Method

Two methods are under consideration here. First the event study to identify statistically significant returns and to formulate options for the number of predetermined holding dates after the signal. After the event study results are done, the paper uses that as a basis for constructing the strategy portfolio returns and measures the alpha of the strategy with Jensen's alpha. This two-method procedure eliminates false positives, makes the process more efficient and enables performance evaluation.

6.1.1 Event study

Event studies have a relatively long history in financial research. Maybe one of the most well-known papers on the subject of event studies is the original stock split event study as conducted by (Fama, Fisher, Jensen and Roll 1969) with the purpose of inspecting the process of how the prices of common stocks adjust to the new information that is implicit in the case of a stock split. The basic event study focuses usually on a sample of the abnormal returns of some financial security or securities during a specified time-period. The time-period is chosen around some event that is believed to influence the price of the security/securities (Kothari, Warner 2007, p. 9-10). The event study structure and framework is thoroughly described by (Campbell, Lo and MacKinlay, 1997).

Figure 2 – An example of an event study timeline



Purpose of the event study is often to investigate the informational efficiency of the market and usually the test setting involves a 120-day evaluation window to feed for the model so that it predicts the expected equilibrium returns. Then the model is used to project returns to certain dates around the studied the event. A time frame for [t-3, t+3] or [t-2, t+2] —assuming event date is t — is often used. After that the abnormal values are calculated. The abnormal values or abnormal returns are often referred to as the error term, residual or AR and they can be calculated as follows:

$$\boldsymbol{AR = R - R_e} \qquad \textbf{(28)}$$

R_e denotes the estimated return projected by a model for event window and R denotes the actual observed return. To model or otherwise approximate the R_e one could use several models or just use the market adjusted model:

$$\boldsymbol{AR = R_{it} - R_{tm}} \qquad \textbf{(29)}$$

In the market adjusted model, the researcher subtracts the returns (R_{tm}) of the reference market from the return of the studied market (R_{it}) to gain the abnormal return. The reference market can be a market index, for instance. Another option is to use the comparison period mean adjusted model:

$$\boldsymbol{R_e = \frac{1}{T1-T0} \Sigma_{t\in[T_1-T_0]} R_{it}} \qquad \textbf{(30)}$$

Most common model used is the market model, which is basically a single factor model, such as the CAPM-model:

$$\boldsymbol{AR = R_{it} - (\alpha+\beta^{*}R_{tm})} \qquad \textbf{(31)}$$

Multifactor models such as Carrhart 4-factor model and the Fama-French 3-factor model have their usage as well, since they possibly reduce variance in the abnormal return by explaining more of the variation in the normal return. Multifactor models most benefit the research in such a situation that the researched firms are part of a same sector, segment or otherwise have similar attributes. This is when the variance reduction will be greatest. The implications are also bigger for longer term studies, than for shorter ones. Other than that, the benefits of using a multifactor model are limited. (Campbell, Lo and MacKinlay, 1997). Multifactor models Fama-French 3-factor and the Carrhart 4-factor were described in the chapter multifactor models.

There are also other models, practically anything will do, but as long as the variation in the expected returns is small relative to the movements in the error term/abnormal returns, then the exact specification of the equilibrium model makes little difference to tests of the efficient market hypothesis because, even if the correct model was known, it would only explain small part of the variation. For discussion of models and issues, one can refer to Beaver (1981), Beaver and Landsman (1981) or Kothari and Warner (2007).

It is important to note the gap in between the event and estimation windows, as it exists to reduce the event-induced variance in the estimation window.

Normally it is assumed that the abnormal returns are independent, but sometimes that assumption does not hold. This sort of scenario can occur on the case of clustered events for instance, and this often results in dependency in between the abnormal returns. Several pieces of literature consider the effect off aggregating the abnormal returns over time as well as over different portfolio companies (Kolari and Pynnönen 2010, Kolari and Pynnönen 2005, Kothari and Warner 2007, Lee and Varela 1997). Jaffe (1974) proposes the act of aggregating the returns to averages to fix the clustering issue. Kolari and Pynnönen (2010) in the other hand formulate a new t-test statistics to solve the problem of cross-sectional correlation.

Another reason for using cumulative abnormal returns could also be that the financial data includes so much noise, that the statistical significance of an event is easier to detect by aggregating the abnormal returns across the event window. One would have to also note, that doing so increases the effect of potential cross-sectional correlation in a study with clustered events.

If the event window is from t_1 to t_2, then cumulative abnormal returns (CAR) are denoted as:

$$CAR(t_1, t_2)=\Sigma AR_t \tag{32}$$

One could also cumulate the abnormal returns to evaluate buy and hold strategies during event windows as well. Buy and hold strategies should in theory result less abnormal observations, as if the markets price information instantaneously. In theory there should be no abnormal price movement after day one, since the event should be priced in on the first day of the event window, but in reality, this does not occur. For a scientific reference point one can look up the post-earnings announcement anomaly listed in Appendix A.

To evaluate the effects of an event on a portfolio or a market segment, one could average the abnormal returns and after that cumulate the average abnormal returns into cumulative average abnormal returns:

$$AAR(t_1, t_2)=\Sigma ARt/N \tag{33}$$

$$\rightarrow$$

$$CAAR(t_1, t_2)=\Sigma AARt \tag{34}$$

It is important to note that in context of event study cumulation refers to aggregation through time and averaging is done during the same time period, for example taking a portfolio average during t=1. The reasoning here is the same as with individual cumulative abnormal returns (CARs), but the target of evaluation changes, as before it was a single entity or a company, now it can be a portfolio.

In this paper the event date will be defined via a market signal instead of an external signal or market event, but the methodology will still be used. Technically, as a market index is evaluated, the paper is evaluating CAARs. The idea is to measure a sample of events with date derived from the strategy signal. We are essentially checking if the signal enables statistically significant average abnormal returns for an index portfolio, by timing the market. The purpose of the event study is not only to test the signal and market efficiency, but to also help define the number of dates to hold the position that is taken after the signal. For this, it is convenient to use a simple model to generate the comparison period mean adjusted expected returns.

As defined in the signal section the strategy is to short the market after a sequence of dates on which the negative correlation between the two indexes (S&P500 and VIX) is broken and they exhibit positive correlation. The event study includes a 30-day comparison period and as mentioned before, the thought process follows a guideline, that if the positive correlation is only a short-term fluctuation, then it means one of the two indexes is inclined to give up some gains (losses) in this specific scenario. Therefore, the event date will be set to two days of co-movement. Test setup includes a set of events and as we are evaluating market returns, we do not have a larger index to approximate the market returns – at least not one that would much differ from the index we have or have a better justification than the comparison period mean adjusted returns. This is why the comparison period mean adjusted model mentioned earlier is chosen. The model assumes that the current market returns follow the same distribution as in the evaluation period. To approximate relatively short-term market returns and the reigning volatility regime, the evaluation window for the mean adjusted model can be shorter than one would normally use. The variables estimated will therefore be:

$$AR_t = R_t - E(R_t) \tag{35}$$

$$E(R_t) = \frac{1}{T1-T0} \Sigma_{t \in [T1-T0]} R_{it} \tag{36}$$

To evaluate the consistent significance of the abnormal returns cumulative abnormal returns AAR and cumulative average abnormal returns (CAAR) will be constructed and evaluated. They will be defined as following:

$$AAR(t_1) = 1/N * \Sigma AR_t \tag{37}$$

$$CAAR = 1/N * \Sigma CAR_t \tag{38}$$

For more detailed information about the event-study methodology one could look up the website of professors Schimmer, Levchenko and Müller (2015).[3]

[3] www.eventstudytools.com website provided by three PHDs Shimmer, Levchenko and Müller to assist scholars in empirical work.

6.1.2 Jensen's alpha

Jensen's alpha is another method to evaluate the investment strategy in question. Since treating the index portfolio as a single asset limits our model decision in the event study phase, the author feels like it is necessary to conduct a second evaluation and apply another method to the data. This involves constructing a time series of returns that incorporates the signals and transacts on them. After constructing the portfolio returns, regression is used to evaluate the significance of the statistical properties of portfolio returns. The question mainly is, if the strategy can generate statistically significant alpha of economic value. The model estimated is presented below:

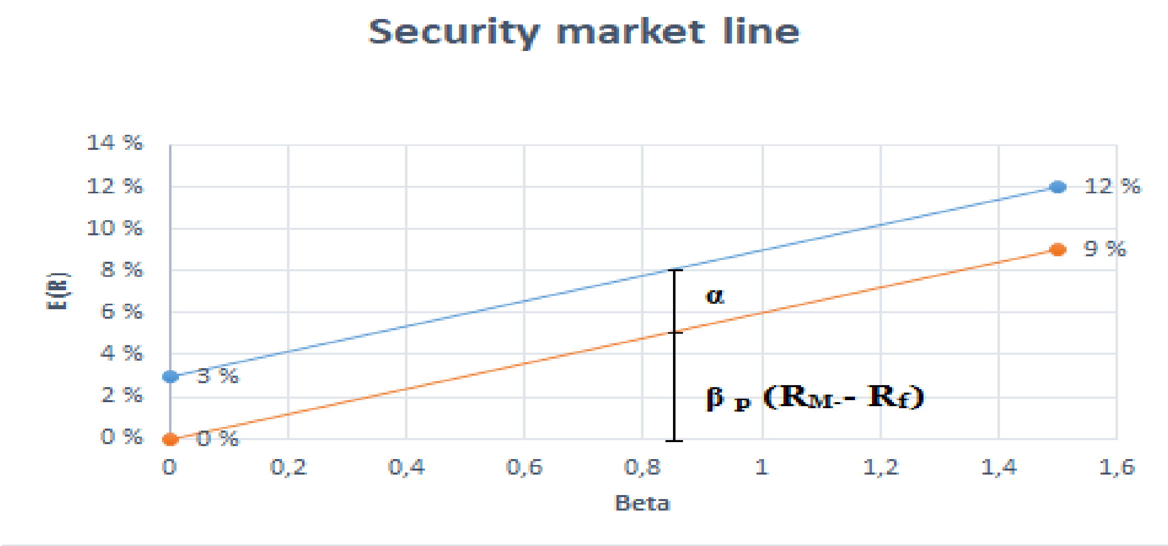
$$R_p - R_f = \alpha_p + \beta_p (R_M - R_f) + \varepsilon_p \tag{39}$$

Jensen's method enables the use of a factor model, so that we can better mirror the results in comparison to the equilibrium state described by CAPM and EMH. The factor model allows for studying the beta and alpha coefficients as well as the residuals. We basically take the CAPM model that isn't constrained to equilibrium and use the position of the security market line to our advantage. When we remove the risk-free rate from the equation and evaluate excess returns, the point where the SML – explained in the chapter about CAPM – crosses y-axis should theoretically drop to 0.

Some practicians argue that a longer dated treasury bond is an appropriate measure of a risk-free asset, but in regard to evaluating alpha further, it shouldn't make a difference, as the risk-free rate is deducted from the equation. Also, as the risk-free rate with a 10-year US treasury bond would most likely be higher, this would imply that equity risk premium is smaller, so smaller changes would be relatively bigger in proportion. This would make potential alpha more significant. As the short position in futures entitles to a financing rate – as discussed in the data section and visualized in table 1 – there would also be larger cash flow from the implied financing rate during active signal dates. This would also amplify the effect. Therefore, this decision to use a 3-month rate should actually be a safe one, in regard to evaluating excess returns and covariance to systemic risk, so that there will be no false conclusions about the results.

When the risk-free rate is set to 0, one can visualize the difference between alpha and market portfolio as below in Figure 3. The blue line above visualizes a portfolio regression estimate from excess returns in the presence of alpha. The orange line below represents the markets beta to returns relation. Both lines are theoretical visualizations, not actual estimates. If the deviation between these lines would be statistically significant, we can then consider that the portfolio is creating reliable excess returns (alpha) in statistical sense. In this visualized case the deviation is around 3%.

Figure 3 – Market SML and portfolio SML with alpha



This is explicitly interesting in this case, since the time series of portfolio returns is formed by buying and selling a market portfolio. If anomalies do not exist, then there should be no excess return generating factors involved in terms of the particular asset distribution of the portfolio. If one were to generate alpha with a strategy based on timing the market, then the generated time series of daily observations would mostly exist above the estimated market SML. This would mean that the observations offer better returns in relation to systematic risk taken.

6.2 Significance testing

After modelling the abnormal returns, the significance of them is then evaluated. To do so one needs to assume that the abnormal returns follow a certain distribution, so the test setup is not without assumptions, unless one uses nonparametric tests. Normally it is assumed that the error terms follow a normal distribution, so if the test statistic can be proven to be statistically different than 0, then the abnormal observations are statistically significant. Different test statistics are used for AR, AAR, CAR, CAAR and so on. If we were to evaluate single day abnormal returns, one could use a simple t-test. In this study, one needs to choose a test study for AAR or CAAR, since a sample of events are tested. A suitable test statistic for AAR and CAAR is the cross-sectional t-test, since it is convenient to estimate. Also, one does not need to worry about event induced variance, for there is no exogenous event registered. There are several events, but there is only one instrument — market index returns — that is evaluated, so in this study one does not need to care about cross-sectional correlation either. Estimated significance tests are denoted as follows:

$$t_{AAR} = \sqrt{N} * \frac{AARt}{Saar} \quad (40)$$

and

$$t_{CAAR} = \sqrt{N} * \frac{CAARt}{Scaar} \quad (41)$$

where t_{AAR} and t_{CAAR} denote the t-values for AAR and CAAR. N is the number of events or observations of AAR in the case of t_{CAAR}.

For regression estimates in the Jensen's alpha, the significance test will be calculated by dividing the coefficient estimates with the standard error of those estimates. The t-values estimated will be compared to t-distribution to get the probability for the null hypothesis. Probabilities under 5% are usually considered significant, but it varies across studies.

6.3 Hypothesis

The study evaluates if this simple market sequence described and derived in the signal chapter results in statistically significant values, violates the efficient market hypothesis and if a sophisticated investor could possibly receive excess returns by implementing strategy based on this signal. This is measured in two ways. The first method of measurement is the traditional event study, with abnormal returns and cumulative abnormal returns. The second method will include constructing a time-series of strategy portfolio returns and using linear regression to compare those returns to benchmark returns, so we can define, if the strategy can create statistically significant alpha. This is called the Jensen's measure or Jensen's alpha. Therefore, the hypotheses will be denoted as:

H_0: AAR=0

H_1: AAR≠0

H_0: CAAR=0

H_1: CAAR≠0

and

H_0: strategy portfolio α=0

H_1: strategy portfolio α≠0

7 RESULTS

The results for the study are presented in this chapter. The following table describes the average abnormal returns for 67 events of a signal occurring. The first signal was always used and overlaps during the event window were discarded to avoid bias. Event window for the event study was set at five days. 5-day window seemed logical and justifiable considering the results of Huang Kun (2011), as per those results the VIX seemed to explain very large amount of volatility in the very short term. Cumulative average abnormal return for the five days is listed as CAAR5.

Table 3 – Results from event study

AAR1	AAR2	AAR3	AAR4	AAR5	CAAR5
-0.3145193	0.0160255	**-0.2614272**	-0.1005345	0.1058648	**-0.5545907**
t-value					
-2.874* **	0.128	**-2.423*** **	-0.803	0.847	**-2.397*** **

Note: The statistically significant values are bolded, and their respective t-values are shown in the 3rd row of the table. *, **, *** denote significance at the 10%, 5% and 2% level of a two-sided test. Results consist of 67 observations for each date, while CAAR thus includes all 335 observations.

The AARs and CAAR are represented in percentage form, so the original values are multiplied by 100. As we can see, the AAR for day one and three is significant in the 2% confidence interval. The cumulative average abnormal is derived from the cumulative abnormal 5-day returns for the 67 events. It is the average of those observations, and the t-value is significant on 2% interval as well. By the estimation method employed, it is clear that the signal has some sort of "value" as it predicts negative returns in the short horizon. It is interesting to note, that the first and third dates after the signal triggers are both negative and significant, but the second date is not. This might be due to negative autocovariance based on the bid-ask spread or market participants trying to "buy-the-dip". Even if the index itself does not have much of a bid-ask spread, the spread affects every single stock in the index. This result fits the mean-reversion after negative returns described by Kulp-Tåg (2007). Based on these abnormals it is fitting to choose a predetermined holding period of 4 days after the signal. Even if the fourth day is not significant in a statistical sense, it is still on average negative and the 0.1% value could be considered to have economical value. One might argue that this is a limitation of the study through fitting the appropriate sample to find results. In the other hand, the information is produced with a proper study and methodology, while the signal clearly has value. Therefore, at this point it gives a basis to follow up on that information with additional research, especially due to the overly simplistic model in the event study. The matter can also be addressed in further studies with out of sample testing. Further on the paper will report returns on predetermined 1-day holding period and 4-day holding period strategies, evaluated by Jensen's method.

The number of signals changes for the strategies depending on the predetermined timeframe. This is because of the process that deals with the overlap. The first signal is always evaluated and the signals occurring during the predetermined holding are discarded. While the predetermined holding time of the short position is one day, then there is no need to discard signals. While the predetermined holding time is 4 days, then some signals are discarded. Also, the event study methodology forces us to discard some signals from the start of the data, when there is no data to estimate the comparison period mean adjusted model, therefore some signals need to be discarded. This same problem does not occur on the Jensen's method, so we do not need to discard signals. Therefore, for the 1-day holding period the strategy receives 87 viable signals, and for the 4-day-holding period the strategy receives 70 signals, when the overlapping signals are removed.

The following table represents the Jensen's method estimates for the 25-year period with different strategies. It is good to note that the Sharpe ratio is calculated from daily average returns and daily standard deviation, so therefore it is not comparable with values from yearly estimates more commonly reported on the internet. The Sharpe ratio is also multiplied by 100 to turn it into a percentage form for the sake of visual appeal. The portfolio Sharpe numbers should be compared to the market Sharpe reported here.

Table 4 – Results from Jensen's method

STRATEGY	ALPHA	T-VALUE	BETA	T-VALUE	R2	TOTAL RETURN	SHARPE
4DH	0.0001114	**1.988****	0.9276	200.348***	0.8609	1233.46%	3.166
4DH-TC	0.0001055	1.882	0.9276	200.345***	0.8609	1183.402%	3.117
1DH	0.00003758	1.285	0.9808	405.972***	0.9621	813.619%	2.684
1DH-TC	0.00002957	1.015	0.9809	407.545***	0.9624	385.182%	2.618
S&P500						642.645%	2.42

Note: The statistically significant t-value for alpha is bolded. *, **, *** denote significance at the 10%, 5% and 2% level of a two-sided t test. The time series of returns each have 6487 observations, thus degrees of freedom is 6485 with intercept and slope being estimated.

As we can see from the table for the 4-day holding period strategy, the alpha is significant on the 5% interval with a t-value of 1.988. The beta of excess index returns over the risk-free rate is significant as is expected and the model has R-squared value of 0.86. Therefore, it seems to explain most of the variance in the strategy portfolio. The alpha is very small, but it can be considered to exist without the transaction costs. Considering the fact that a year has 252 trading days and there was 70 or 87 registered signals, so the days short during a 25-year window amounted to 278 or 87 in total. This means that by quick calculation the 4DH-strategy was only active a bit over 1/25 days during the study period. Active meaning, that the strategy was short the market, long the risk-free asset and neutral on equity risk premium. While non active, the strategy was long the market risk premium. The relative number of days, while the strategy was active is small and this obviously makes it harder to generate higher and statistically significant alpha as such a small number of observations are unlikely to have a major effect on the positioning of the security market line. The days would have to have a major impact in the returns, in comparison to having more signals and therefore days that systematically generate higher returns. Obviously, the amount days for the 1DH was even less being 87 days for 87 signals.

The economic significance of this alpha is relatively small in terms of the size for parameter value of alpha. It is still clear that the strategy manages to reduce systematic risk, generate excess returns and higher risk-adjusted returns in terms of portfolio variance. The beta coefficients are inherently smaller for the strategy portfolios, considering market beta is 1. The standard deviation of the 4DH-TC portfolio was 1.210112%, while the standard deviation of excess S&P500 returns was 1.210376%. The difference is minimal, and the higher Sharpe is mostly due to the higher returns, but the standard deviation still matters. The descriptive statistics in table 2 also pointed to a lower tail risk for strategy portfolios due to higher kurtosis. This seems to also apply in regard to the actual results. What really sticks into ones' eye though, is the impact of compounding interest. In terms of total returns the economic significance is substantial. By applying this strategy, the investor may have had received substantially higher returns for investments in the S&P500. This comes with a lower mean return after transaction costs for the 4DH-TC portfolio, in comparison to being only long the market. While this is the case, the skewness of the portfolio is still lower than the skewness of S&P500, which implies a more symmetrical distribution and possibly even more negative observations in frequency, but not in scale. For the purpose of compounding interest, this is very important, as avoiding drawdowns will evidently lead to higher returns. The same deduction can be made from the results of 1DH strategy, with an opposite perspective. High costs relative to expected returns of trading can eat up the compounded interest.

In general, the 1183% return of the 4DH strategy after transaction costs represented here, would still most likely lose to the returns of a buy-and-hold portfolio of Nasdaq100 or Nasdaq composite during this time. Therefore, the numbers do not seem excessive or unrealistic. In the authors mind, it shows that there might even be room for improvement with additional research.

In terms of the strategy decision, it looks like shorting for one day is a very bad alternative. The strategy loses to the index and the only reason it has a higher Sharpe is the lower standard deviation. For a very risk averse investor, such a strategy might be of value. One could also argue that the minimal benefits of lower variance do not make up for the lower profits, considering the variance reduction occurs on a well-diversified market portfolio that is supposedly on the efficient frontier of portfolios, as described by EMH.

8 DISCUSSION

In retrospect it is good to note that most of the anomalies previously studied in academic literature have a lot longer timeframe to play out than the one in this paper. The dynamics of implied volatility, realized volatility, volatility risk premium and equity index returns are not at all one dimensional and linear and can exhibit heteroscedasticity – as plotted in scatter plots of Giot (2005) – which can be speculated to reflect time-varying risk aversion of investors. Timing the innovations in the market returns to VIX relation is the key to success in generating excess returns with this style of strategy.

To quantify a risk-factor for the excess returns achieved one could look to produce a skewness factor, or a factor that uses some form of volatility risk premium. One could also look to test or alter the signal for a longer-term hedging process in comparison to this one. After all, VIX supposedly provides a 30-day estimate of future volatility. What is to say, that the movements in the VIX are priced correctly for a longer time span? Alternatively, a volatility model GJR could be used or a combination of models. One could also test if the signal provides value during occurrence of negative co-movement. As defined in the signal chapter, the signal now is based on positive co-movement. What if the strategy was to be long risk-free as a default and then short or long dependent only on the signal? Would this provide similar, better or worse results?

A trader could also look to optimize the conditions regarding the signals. Are there other technical conditions that improve the results, or could a trader use just one day of co-movement instead of two with other parameters to narrow the number of signals? Would it be possible to pinpoint factor premiums, that have lower expectancy following the signal, than the general market risk premium? For example, if high leverage factor would be more sensitive to movements in the VIX, could the strategy short bank indexes instead? Copeland et al. (1999) conduct such a study, but more research around different factor premiums and volatility should be conducted. Obviously, this sort of strategy might not be implementable, due to higher trading costs or lack of proper instruments. Further, a trader could also look to attain signals from the developments in VRP, as it is judged to price forward quarter market returns in the paper by Bollerslev and Zhou (2007). There are a lot of different alternatives and research opportunities around the subject.

The market definitely seems to entail a lot of the characteristics emphasized by Mandelbrot (1963) and the FMH. It is still clear that EMH has been a great influencer in developing the methods of modelling the equity market process. Regardless if it is incorrect or correct it is really hard to disregard it. One can see the characteristics of EMH in the fact that statisticians, econometrists and finance professionals are trained to think about the means and reversion to mean in the long run as well as equilibrium states. This is probably also the reason why CAPM has survived through the years and is still held as a major cornerstone in finance literature, even if there is lot of issues with the model and those issues have been pointed out in numerous publications through time. Developing a framework for pricing without using some sort of a mean, expected value or equilibrium would possibly require assuming the market process to be a chaotic system. There would be no conditional variable to measure up against the observations. To the authors inconclusive knowledge of the subject, this sort of mathematic modelling is conducted more in terms of processes in nature, but it has its problems in terms of economics. In a chaotic system, variables are presumed to be predetermined, but random. This means that the trajectory of the processes is dependent on the starting point, but paradoxically random at the same time. The processes are deemed to exhibit similar sets and patterns, such as the co-movement described here, by the signal. Consequently, Mandelbrot published a book "The Fractal Geometry of Nature" in 1982, that is deemed a classic of chaos theory.

Obviously, AMH of Andrew Lo cannot be disregarded either in terms of these results, as the strategy seems to benefit from time-varying risk aversion and investor behavior. One would have to dive deep into psychology to understand this and to possibly define a framework to model it. FMH does not really provide us understanding on the underlying – the investor, human – but that responsibility falls upon the "behavioralists". It is not surprising that FMH has not received that much academic enthusiasm as it merely seeks to model behavior of prices instead of truly understand the underlying causes as behavioral finance does. Another issue is that the models relating to FMH are often nonlinear and require extensive mathematical knowledge to apply. Undergrads might often lack such extensive knowledge. While in the other hand linearity does not really define markets either, as the price process of markets is a process of compound interest. The importance of understanding compounding becomes evident when looking at the results of this paper. A market participant should be very careful to overtrade, but not afraid to trade when an edge is prominent, since in the long run those small victories really do add up.

In regard to market efficiency as defined by EMH, it can be said that the market is probably efficient and unpredictable for most of the market participants, since it takes significant time and effort to get acquainted with equity market function. Using significant time and effort does not guarantee excess returns, even if the person in question manages to find and understand information that could propel thy returns above average. This is because strategies and anomalies might be dependent on the conditional market state. Most anomalies listed on academic papers also take a longer time to reflect on returns and the fact is also that the informational advantage once reached might be lost as market has a tendency to start pricing known anomalies away. This coerces the arbitrageurs to change their approach and find new risk factors that are not priced correctly and adjust to different phases of the market cycle.

Most people in the western civilization are participants in the equity markets through pension funds or other stakes willingly or unwillingly. Therefore, the average investor probably does not reach statistical predictability of excess returns. For a professional this should not be the case. Markets are a fluid system never in equilibrium and therefore a smart, competent profitable trader should be the survival of the richest as referred to by Lo (2004, 2005) and a guardian of some relative form of market efficiency. This sort of trader, while reducing mispricing in the markets, should in one perspective be the guardian of the unaware market participants money. One could think of this as if markets were a totally random system without a direction, how would it fill its purpose of allocating capital efficiently? There must be a mutually beneficial and deterministic process for the markets to fill their purpose. A trader should theoretically ensure, that relatively less sophisticated market participants benefit from markets by buying stakes in companies creating value and benefitting financially from that value. The benefits should be appreciation of the value of their ownership stake, while the deserving company also gains access to more capital to invest until it runs out of efficient investment opportunities.

Sadly, the existence of hedge funds and active management has been on the decline during the recent years as capital flows to passive strategies. It might be good in terms of economic equality and be beneficial for the small investors benefitting from these passive strategies. The question is if this development is good for the market efficiency and allocation of capital? If the void of hedge funds is replaced by sophisticated home offices, it might push the limits of arbitrage back, as capital allocated to arbitrage is not as concentrated. Also, in regard of the future of the industry, it is good to ponder if time-varying risk aversion has a considerable impact on market pricing. For instance, do passive strategies increase systematic risk due to herding behavior? These are clearly current subjects requiring future research.

9 CONCLUSION

In conclusion we can say that the findings of this paper provide arguments to back up all of the discussed hypotheses of market structure, at least in some form. Markets entail characteristics described by EMH, FMH and AMH. While the results would imply clear inefficiency in market pricing, it is unclear if the pricing error is due to mispricing on some risk factor or if the strategy carries some form of risk premium as a factor exposure that the author does not recognize. In this perspective, the study cannot conclude that these results definitely violate the efficient market hypothesis, while the results certainly imply potential for such a dynamic. Clearly the theories of Markowitz (1952) and CAPM of Sharpe (1964) and Lintner (1965) do not explain the results of this study. In the end, the three hypotheses should not be pinpointed against each other in order to find the absolute truth on which of these hypotheses is "best". Neither of these hypotheses are necessarily accurate – in the end, they are assumptions and theories – comparing them to each other for other objectives than better and more accurate asset pricing or higher investment returns is rather immaterial. In the end, they all serve as tools and frameworks for deciphering the mystery of markets.

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APPENDIX A

Category	List of anomalies, topic and source.
Momentum	• Earnings surprise. Foster, Olsen and Shevlin (1984) • Cumulative abnormal stock returns around earnings announcements. Chan, Jegadeesh and Lakonishok (1996) • Price momentum. Jegadeesh Titman (1993) • Revisions in analysts' earnings forecasts. Chan, Jegadeesh and Lakonishok(1996) • Industry Momentum. Moskowitz and Grinblatt (1999)
Value-versus-growth	• Book-to-market equity. Rosenber, Reid and Lanstein(1985) • Reversal. De Bondt and Thaler (1985) • Analysts' earnings forecasts-to-price. Elgers, Lo and Pfeiffer(2001) • Dividend yield. Litzenberger and Ramaswamy(1979) • Net payout yield. Boukdoukh et al.(2007) • Sales growth. Lakonishok, Shleifer and Vishny (1994) • Cash-flow-to price. Lakonishok, Shleifer and Vishny (1994) • Earnings-to-price. Basu (1983) • Market Leverage. Bhandari (1988). • Equity duration. Dechow, Sloan and Soliman (2004) • Long-term growth forecasts of analysts. La Porta (1996)
Investment	• Abnormal corporate investment. Titman,Wei, and Xie (2004) • Investment-to-assets. Cooper, Gulen, and Schill (2008) • Net operating assets. Hirshleifer et al. (2004) • Changes in property, plant, and equipment plus changes in inventory scaled by assets. Lyandres, Sun, and Zhang (2008) • Investment growth. Xing (2008) • Net stock issues. Pontiff and Woodgate (2008) • Composite issuance. Daniel and Titman (2006) • Net external financing. Bradshaw, Richardson, and Sloan (2006) • Inventory growth. Belo and Lin (2011) • Inventory changes. Thomas and Zhang (2002) • Operating accruals. Sloan (1996)

	• Total accruals. Richardson et al. (2005) • Percent operating accruals. Hafzalla, Lundholm, and VanWinkle (2011) • Percent total accruals. Hafzalla, Lundholm, and VanWinkle (2011)
Profitability	• Return on equity. Haugen and Baker (1996) • Return on assets. Balakrishnan, Bartov, and Faurel (2010) • Return on net operating assets. Soliman (2008) • Profit margin. Soliman (2008) • Asset turnover.Soliman (2008) • Capital turnover. Haugen and Baker (1996) • Gross profits-to-assets. Novy-Marx (2013) • *F*-score. Piotroski (2000) • Tax expense surprise. Thomas and Zhang (2011) • Taxable income-to-book income. Green, Hand, and Zhang (2013) • Revenue surprise. Jegadeesh and Livnat (2006) • Number of consecutive quarters with earnings increases. Barth, Elliott, and Finn (1999) • Failure probability. Campbell, Hilscher, and Szilagyi (2008) • *O*-score, Dichev (1998) Campbell, Hilscher, and Szilagyi (2008)
Intangibles	• Organizational capital-to-assets. Eisfeldt and Papanikolaou (2013) • Brand capital-to-assets. Belo, Lin, and Vitorino (2014) • Advertisement expense-to-market. Chan, Lakonishok, and Sougiannis (2001) • R&D-to-sales. Chan, Lakonishok, and Sougiannis (2001) • R&D-to-market. Chan, Lakonishok, and Sougiannis (2001) • R&D capital-to-assets. Li (2011) • Hiring rate. Belo, Lin, and Bazdresch (2014) • Operating leverage, Novy-Marx (2011) • Corporate governance. Gompers, Ishii, and Metrick (2003) • Accrual quality. Francis et al. (2005) • Industries. Fama and French (1997)

Trading Frictions	• The market equity. Banz (1981) • Idiosyncratic volatility. Ang et al. (2006) • Total volatility. Ang et al. (2006) • Systematic volatility. Ang et al. (2006) • Maximum daily return. Bali, Cakici, and Whitelaw (2011) • β Market beta. Frazzini and Pedersen (2014) • Dimson's beta, Dimson (1979) • Short-term reversal, Jegadeesh (1990) • Dispersion of analysts' earnings forecasts. Diether, Malloy, and Scherbina (2002) • Share turnover. Datar, Naik, and Radcliffe (1998) • 1/share price. Miller and Scholes (1982) • Dollar trading volume. Brennan, Chordia, and Subrahmanyam (1998) • Illiquidity as absolute return-to-volume. Amihud (2002)