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# Substantial Doubt and the Entropy of Auditors' Going Concern Opinions

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**Substantial Doubt and the Entropy of Auditors' Going Concern Modifications**

Abstract

Auditors need to establish a substantial doubt threshold in order to determine the type of audit report to issue, but substantial doubt is not defined in the auditing standards. Auditors are regularly criticized for having high thresholds, which results in too few going concern reports. We apply Shannon entropy from information theory as the criterion to evaluate the informational value of the audit report. Shannon entropy provides a measure of the expected information content associated with the realization of an uncertain event. First, we estimate the client's probability of bankruptcy in our sample. Second, using the distribution of the probability of bankruptcy we calculate the entropy at each point of the probability of bankruptcy. We find that entropy is maximized at the 0.08 probability of bankruptcy.

JEL Classification: G33, G38, M42

Keywords: Bankruptcy, going concern modifications, substantial doubt, Shannon entropy, policy and regulation

Highlights

- The substantial doubt threshold affects the information content of auditor's going concern modifications.
- Shannon entropy suggests that information content is maximized at a decision threshold equal to a bankruptcy probability of 0.08.

## 1. Introduction

It is in the public interest to have audit opinions that are informative with regard to whether a firm is a going concern or not. The issuance of a going concern modification represents the judgment of the auditor in terms of evaluating and deciding what substantial doubt means, as well as at what threshold the evidence becomes sufficient to warrant the inclusion of a going concern modification (Levitan and Knoblett, 1985). The information content of going concern reports, however, is not independent of the technical judgments made by auditors regarding what constitutes a substantial doubt threshold. Determining when there is "substantial doubt" concerning an entity's ability to continue as a going concern is one of the key steps for an auditor (Ashton and Davis, 2002). Our goal is to identify the optimal threshold (cut-off score) for substantial doubt by applying Shannon entropy to a distribution of bankruptcy probabilities so as to derive a point that maximizes the average information content from the partitioning of clean opinions and going concern modifications.

The information content of audit reports has frequently been criticized and it remains an important public policy concern. Recently, the European Commission (2014) amended the Directive on Audits (Directive 2014/56/EU) in order to increase the value of audit reports to external users. In the USA, the Public Company Accounting Oversight Board's (PCAOB) Investor Advisory Group suggested that the auditing standard for going concern considerations should be revised, since auditors have perceivably failed to provide early warning signals in relation to some of the largest US bankruptcies (PCAOB, 2012a).[1] One of the main concerns

[1] According to Francis (2011) and DeFond and Zhang (2014), auditors frequently fail to warn investors (Type II error or false negative) before a client files for bankruptcy, which supports the concerns of the regulators. However, Francis (2011) also finds evidence that "auditors are conservative and over-qualify" (Type I error or false positive) (Francis, 2011).

highlighted by the Investor Advisory Group was the "high hurdle" for going concern opinions (PCAOB, 2012a) as interpreted by auditors with regard to the term "substantial doubt" (AU section 341).

We argue that the cut-off value for substantial doubt should be directly linked to the information conveyed by auditors' going concern modifications. By definition, information is the reduction of uncertainty. That is, in order to have information, two components are needed: an uncertainty that matters (e.g., the uncertainty as to whether a company is financially distressed); and something that resolves the uncertainty (e.g., a communication from the auditor regarding a going concern uncertainty). Specifying an appropriate substantial doubt threshold for auditors to decide whether or not to issue going concern modifications, affects how auditors will assign the population of firms into those issued a standard opinion (which is indicative of less financially distressed firms), and those issued a going concern modification (which is indicative of more financially distressed firms).

To identify this appropriate substantial doubt threshold, we utilize Shannon entropy from information theory to estimate the average information content of the audit report at various hypothetical substantial doubt thresholds. Shannon entropy is an important metric in information theory, since it measures the uncertainty associated with an event and, therefore, also the expected information content associated with the realization of that event. It is predicated on the idea that the more *ex ante* uncertainty concerning whether an event will happen or not, the more information is conveyed by the actual realization of that event. As such, Shannon entropy defines information as the reduction in uncertainty on behalf of users of that information. We apply Shannon entropy to the auditors' binary reporting decision on the entity's ability to continue as a going concern, noting that the substantial doubt threshold that auditors use directly affects users'

*ex ante* uncertainty regarding whether a going concern modification will be issued or not, as well as the average information content associated with the realization of the type of audit report.

We compile a sample of 31,332 firm-year observations covering the period 2003–2015, and we estimate the *ex ante* probability of bankruptcy for these firms. We use the estimated distribution of the firms' probability of bankruptcy and apply Shannon entropy. We find that the average information content of the auditor's going concern opinion is maximized at a substantial doubt threshold of a 0.08 probability of bankruptcy. That is, if the average information content of audit reports is to be maximized, a warning from the auditor is warranted when there is approximately a one-in-twelve chance or higher that the client might go bankrupt.

We also perform additional analyses of the optimal substantial doubt threshold with respect to minimizing misclassification rates and the expected economic costs of misclassification when the costs between the types of misclassifications are not equal. Under these different criteria, the optimal substantial doubt threshold is different from that identified when trying to maximize the average information content of the audit report. If the substantial doubt threshold is at a 0.5 probability of bankruptcy, then the total number of misclassifications is minimized. In particular, at this hypothetical substantial doubt threshold, we estimate that only 13 percent will be misclassified, which of course means that about 87 percent will be correctly classified. However, if the cost of "incorrectly" issuing a standard opinion to a firm that subsequently goes bankrupt is, in relative terms, ten times higher than the cost of "incorrectly" issuing a going concern modification to a firm that subsequently survives, then we estimate that the optimal substantial doubt threshold minimizes the expected economic costs at a bankruptcy probability of 0.1. This is similar to the optimal substantial doubt threshold estimated for maximizing the average information content of an audit report.

The PCAOB is yet to provide a definition of substantial doubt or quantitative guidance on its application, although it has been explicit that "substantial doubt" as it stands in the existing auditing standards is a qualitative evaluation (PCAOB, 2014, p. 3). It is, however, our belief that a discussion about the meaning of "substantial doubt" is not just a semantic quibble, but is in fact far more important, since it effectively determines the point on the probability of bankruptcy continuum that partitions firms with standard opinions from those with going concern modifications. It is difficult to assess the appropriate substantial doubt threshold without invoking some sort of criteria to evaluate the audit opinion in a going concern setting. Maximizing the average information conveyed by auditors' going concern modifications could potentially be a criterion for determining an appropriate substantial doubt threshold. While we acknowledge that determining the criteria used to evaluate and determine an optimal substantial doubt threshold is ultimately a public policy decision to be made by standard setters and regulators, our study aims to inform the promulgation of public policy regarding auditors' responsibility for conveying firms' going concern issues by illustrating how the appropriate threshold for "substantial doubt" is not a trivial semantic matter, but instead has implications concerning the information expected to be conveyed by the audit opinion, as well as the expected misclassification rates. In addition, we contribute to the debate in this regard by using a well-accepted, theoretically grounded approach derived from information theory and applying it to a distribution of the probability of bankruptcy derived from four well-accepted bankruptcy prediction models, rather than relying on normative notions of the appropriateness of the thresholds.

This paper proceeds as follows. In section 2, we discuss the auditors' reporting decisions with respect to going concern modifications as established by the existing standards. We also perform

a short review of regulators' normative discussions regarding the meaning of substantial doubt, as well as reviewing findings from studies that used survey and experimental methods to investigate auditors' and other users' interpretations of the meaning of substantial doubt. In section 3, we introduce the concept of Shannon entropy. In section 4, we discuss our methodology and results with regard to identifying an appropriate substantial threshold that maximizes both entropy and the average information content of audit reports. We also perform additional analyses regarding the expected misclassification rates for various substantial doubt thresholds. Then, in section 5, we discuss the results and limitations of the study. Finally, section 6 concludes the paper.

## 2. Going concern modifications and substantial doubt thresholds

AU section 341 establishes the requirements for the auditor's evaluation of a firm's ability to continue as a going concern, and it requires that the auditor modifies the audit report by including an explanatory paragraph when substantial doubt exists regarding the firm's ability to continue as a going concern (American Institute of Certified Public Accountants [AICPA], 2002).[2] AU section 341 operationalizes the going concern assumption by referencing the entity's ability "to continue to meet its obligations as they become due without substantial disposition of assets outside the ordinary course of business, restructuring of debt, externally forced revisions of its operations, or similar actions" (AU section 341.01). The auditor must assess whether there is "substantial doubt about the entity's ability to continue as a going concern for a reasonable period of time, not to exceed one year beyond the reporting date of financial statements" (AU

[2] See also, section 10A(a)(3) of the Securities Exchange Act of 1934, which requires that the audits of issuers include "an evaluation of whether there is substantial doubt about the ability of the issuer to continue as a going concern during the ensuing fiscal year."

section 341.02). Consequently, the auditing standard effectively specifies a multi-step process for choosing an appropriate audit report in a going concern setting (Davis and Ashton, 2002): 1) establishing a substantial doubt threshold; 2) estimating the probability that a client will not continue as a going concern; 3) comparing the probability of not continuing as a going concern to the substantial doubt threshold; and 4) issuing a modified report if the probability of not continuing as a going concern is greater than the substantial doubt threshold, and issuing a standard report otherwise. In this paper, we are primarily concerned with the first step, that is, what should be considered an appropriate substantial doubt threshold.

The term "substantial doubt" is not defined in the auditing standards issued by the PCAOB. The Financial Accounting Standards Board (FASB) (2009, 2014) noted that "[…] quantitative differences in interpretation of *substantial doubt* exist in practice". Both the PCAOB and the FASB have recognized that a definition of substantial doubt would clarify the meaning of the term and potentially reduce inconsistencies in decisions related to going concerns (FASB, 2014; PCAOB, 2012b). Indeed, the FASB (2014) has recently defined substantial doubt as a threshold where it is *probable* that the going concern assumption is not valid, and where probable may be taken to mean "bankruptcy is likely to occur."[3] In defining substantial doubt, the PCAOB (2012b) acknowledges that one potential approach is to use the criteria found in existing accounting standards, such as "more likely than not," "reasonably possible," and "probable," although it has not yet decided to provide an exact definition of substantial doubt.

Prior studies, by means of surveys or experiments, provide some evidence regarding what auditors and users consider to be an appropriate cut-off value for substantial doubt. Examining

[3] According to the FASB's (2011) ASC master glossary, something is probable if "the future event or events are likely to occur."

auditors such as Asare (1992), Asare and Messier (1992), Davis and Ashton (2002), Knapp et al. (1992), and Ponemon and Raghunandan (1994) find the average probability of bankruptcy associated with substantial doubt to be 0.57, 0.44, 0.39, 0.55, and 0.57, respectively. In all these studies, the auditors were, on average, willing to issue a going concern modified audit report if the firms had a probability that bankruptcy may occur within the following twelve-month period, higher than these cut-off values. In their survey, Ponemon and Raghunandan (1994) also investigate how various users of audit reports interpret the phrase "substantial doubt," with their results suggesting that there are significant differences between groups. In particular, the mean value for bankers was 0.72, for financial analysts it was 0.71, and for judges and legislative staffers it was 0.33. Ponemon and Raghunandan (1994) therefore conclude that users assign much greater weight to significant doubt than auditors intend. Furthermore, Boritz (1991) asserts that substantial doubt exists when the probability of bankruptcy is between 0.50 and 0.70. Consistent with these figures, a report prepared by the Investor Advisory Group of the PCAOB (2012a) assigned a probability of 0.8 or higher to the meaning of the phrase "substantial doubt," which is somewhat similar to the value perceived by investors and bankers in Ponemon and Raghunandan's (1994) study.

The substantial doubt thresholds elicited from the auditors and users of audit reports through surveys and experiments are, however, different from what the observed data indicates, as presented in the review paper by Carson et al. (2013) on actual bankruptcy rates and going concern modified audit reports. Carson et al. (2013) collected 88,359 firm-year observations in the USA for the period 2000–2010. They examined whether the audit report was issued with a going concern modification or not, and whether the company subsequently went bankrupt in the

following year.[4] Over the sample years, 14,055 of the 88,359 firm-year observations (15.91%) received a going concern modified audit opinion, while the remaining 74,304 received standard opinions. Only 396 of the 88,359 observations went bankrupt (0.45%). Of the 396 recorded bankruptcies, only 238 (60.1%) companies had a prior year going concern modified opinion, while the remaining 168 (39.9%) received standard opinions. These rates effectively indicate that if an auditor has issued a going concern modification, there is only a 0.017 probability that the client will actually go bankrupt.[5] This probability might appear low, but the number of going concern modifications outweighs the number of firms actually going bankrupt by a factor of thirty-five, indicating a relatively low threshold for issuing going concern modifications that will result in a large amount of going concern modifications for firms that do not subsequently go bankrupt. Consequently, a going concern modification does not necessarily indicate that there is a high likelihood of bankruptcy.

We argue below that the cut-off value for substantial doubt, is directly linked to the information conveyed by auditors' going concern modifications. Consequently, the substantial doubt threshold that maximizes the average information conveyed by auditors' going concern modifications could potentially be used as a criterion for determining an appropriate substantial doubt threshold.

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[4] "Bankrupt" in the study by Carson et al. (2013) is taken to mean that the firm files for Chapter 7 or Chapter 11 within one year of the audit opinion date. "Surviving" is taken to mean that the firm does not file for Chapter 7 or Chapter 11 within one year of the audit opinion date.

[5] The numbers used in this calculation are taken from Table 2 in Carson et al. (2013, p. 357) and then calculated using Bayes' theorem as: 0.0170 = 0.6010 x 0.0045 / 0.1591. That is, the conditional probability of bankruptcy given a going concern modification is equal to the conditional probability of observing a going concern modification given that a firm went bankrupt multiplied by the unconditional probability of bankruptcy divided by the unconditional probability of observing a going concern report. It is important to note here that the unconditional probability of bankruptcy is *ex post*. Auditors are, however, not charged with predicting actual bankruptcies, and hence their decisions are necessarily based on the assessment of the *ex ante* probability of bankruptcy.

## 3. The entropy of going concern modifications

We introduce entropy as a criterion for evaluating the informational value of the substantial doubt cut-off thresholds of the probability of bankruptcy that could be used in issuing audit opinions. In his seminal paper, Shannon (1948) introduced the notion of quantifying the expected value of the information contained in a message. This has become known as *Shannon entropy*, and the concept has been applied in a wide range of settings.[6] The theoretical underpinning of entropy is that the expected average information content is associated with the realized value of a stochastic variable.

In order to develop a usable measure of the information obtained from observing the occurrence of an event, the first simplification is to ignore any particular features of that event, and to only observe whether or not it happened. The information content of an event can therefore be defined in terms of the probability of the event. Mathematically, Shannon (1948) denoted the entropy $H$ of a discrete stochastic variable $X$ with possible values $\{x_1, .., x_n\}$ and probability $p(x_i)$ $(i=1,\ldots n)$ for $X$ taking a value $x_i$ as:

$$H(X) = \sum_{i=1}^{n} p(x_i) I(x_i) = \sum_{i=1}^{n} p(x_i) log_b \frac{1}{p(x_i)} = -\sum_{i=1}^{n} p(x_i) log_b p(x_i) \quad (1)$$

where $b$ is the base of the logarithm used and $I(x_i)$ denotes the information content of $x_i$.[7] In particular, for $b$=2, the unit of entropy is known as a *bit*.

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[6] Shannon (1948) introduced information theory and the concept of entropy to characterize data flows in communications systems. However, *every* quantity with which a probability distribution can be associated has its own Shannon entropy. Consequently, the concept of entropy has been adopted and utilized across a wide range of disciplines, including the Shannon index, a popular diversity index in the ecological literature (e.g., Hill, 1973); entropy monitoring as a method of assessing anesthetic depth (Bein 2006); Shannon entropy used to analyze the information content in the genome code (Marenduzzo et al., 2006); maximum entropy methods to model wildlife distributions (Baldwin, 2009); as well as entropy-based measures of diversification in economics research (Jacquemin and Berry, 1979; Palepu, 1985).

[7] Entropy as an information measure has some desirable and fundamental properties (Cover and Thomas, 2006):

The entropy $H(X)$ of a stochastic variable $X$ can be interpreted as either the expected uncertainty of the outcome $X$ or the expected amount of information that a realization of $X$ provides (Cover and Thomas, 2006). In Shannon's (1948) entropy measure, they are the same. In general, if a variable $X$ has $2^n$ possible values and all of them have an equal possibility of being the real value of $X$, then the entropy $H(X)$ reaches its maximum $n$, meaning that the outcome of $X$ being a certain value delivers the full $n$ bits of information (Cover and Thomas, 2006). Two necessary conditions are required for the entropy to reach the full $n$ bits of information (Cover and Thomas, 2006):

1. *There are $2^n$ possible values for X to take on.* The variable X must be able to take on a minimum of two values. The more values the stochastic variable can take on, the more information can potentially be conveyed, *ceteris paribus*.
2. *All possible values of X have a uniform probability distribution of being realized*. The possible values of the variable X, irrespective of whether it is two or more, should be equally likely to be observed. That is, the realization of a value of X conveys more information the closer the known finite number of outcomes is to being equally likely to happen.

The Shannon entropy principle is applicable when seeking to understand the average information content of an auditor's going concern report. Again, a necessary simplification that we make here is to ignore any particular features of the event, and we only observe whether a going concern

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1) Information is a non-negative quantity: $I(p) \geqslant 0$.
2) If an event has a probability of 1, we get no information from the occurrence of the event: $I(1)= 0$.
3) If two independent events occur (whose joint probability is the product of their individual probabilities), then the information we get from observing the events is the sum of the two events: $I(p1 * p2) = I(p1) + I(p2)$.
4) The information measure is continuous, as well as a monotonic function of the probability of the event: large (small) changes in probability result in large (small) changes in *information*.

modification was issued or not. We effectively consider the audit report to be a binary classifier that divides this population into two groups and reveals information regarding a firm's relative financial distress: those that are issued a standard opinion are presumably less financially distressed, while those that are given a going concern modification are presumably more distressed.

The more uncertain users are about which audit opinion the auditor is going to issue, the more information is revealed by the auditor when a specific type of audit opinion is issued. In other words, the audit report carries higher entropy if there is more *ex ante* uncertainty associated with the outcome and, therefore, the message that an audit report conveys is, on average, more informative. Thus, at one extreme, if an auditor *always* issues going concern opinions and never a standard opinion (or *vice versa*), there is little additional information conveyed to users by those opinions. Since there is no uncertainty associated with the outcome, learning the actual outcome of the audit report contains no information. In other words, the entropy of the audit report is zero.

At the other extreme, the average information content of an audit report is highest when there is an expectation of equal proportions of standard opinions and going concern modifications being realized. This creates the highest *ex ante* uncertainty about which audit opinion the client is to receive, and when the audit opinion is realized, it reduces the uncertainty and therefore carries the highest average information content. In between these two extremes – that is to say, whenever the proportions of standard opinions and going concern modifications to be realized are expected, although they are not expected to be equal – there exist some uncertainties and the realization of audit reports will carry some information. The more equal (unequal) the expected proportions of the standard opinions and going concern modifications to be realized, the more

(less) information is, on average, conveyed by the actual realization of the audit opinion. The key point is that information is defined as a decrease in uncertainty on the part of the user of the information.

As the "substantial doubt" threshold effectively determines the point on the probability of bankruptcy continuum that partition firms with standard opinions from those with going concern modifications, the insight obtained from Shannon's entropy could be used to estimate the optimal cut-off value for substantial doubt in terms of the *ex ante* probability of bankruptcy in order to maximize the average information content of the auditor's going concern opinion.

An interesting feature of this criterion is that the optimal substantial doubt threshold is necessarily dependent on the shape and location of the distribution of the financially distressed firms. The observed proportions of standard opinions and going concern opinions at a given point in time are a function of 1) the distribution of firms' probability of bankruptcy and 2) the threshold in terms of the probability of bankruptcy at which auditors start issuing going concern modifications. For any given distribution of the probability of bankruptcy, the optimal threshold in terms of the probability of bankruptcy that maximizes the average information content of the audit report is the threshold that results in equal proportions of standard opinions to going concern opinions.

## 4. Sample, methodology, and results

The selection procedure for our empirical sample is detailed in Table 1. The initial sample consists of all post-Sarbanes-Oxley Act of 2002 (SOX) firm-year observations for firms domiciled in the USA obtained from Compustat for the years 2003 to 2015 (77,470 firm-year observations). We choose this sample period because the passage of SOX occurred in 2002, and

the relevant auditing standards have not changed during this period. Following Altman (1968), Ohlson (1980), and Zmijewski (1984), we exclude observations from the financial industries with two-digit SIC codes between 60 and 69 (32,154 firm-year observations). We also exclude observations with insufficient data to calculate our composite measure of the probability of bankruptcy (13,984 firm-year observations). Our final sample consists of 31,332 firm-year observations from 5,146 unique firms over the period 2003–2015.[8]

[Insert Table 1 about here]

In the previous section, we introduced entropy as a criterion for evaluating the informational value of the cut-off thresholds of the probability of bankruptcy that could be used in issuing audit opinions. In order to investigate which probability of bankruptcy cut-off value maximizes entropy and, therefore, the average information content of auditors' going concern reports, our empirical analysis consists of two parts. First, we estimate the client's probability of bankruptcy and then expect the distribution of this sample to be an approximation of the distribution of firms' probabilities of bankruptcy in the population. Second, using that distribution, we calculate the entropy at each point of the probability of bankruptcy if that point was the substantial doubt threshold, and we locate the point where entropy is maximized.

[8] Our sample of 31,332 observations is smaller than the sample of 50,611 observations used by Wu et al. (2010) and the sample of 78,100 observations used by Hillegeist et al. (2004), primarily because our sample period covers 13 years, while Wu et al. (2010) and Hillegeist et al. (2004) cover a sample period of 26 and 20 years, respectively. Our sample shows, on average, 2,410 observations per year, which is slightly higher but comparable to Wu et al.'s (2010) average of 1,945.6 observations per year. Our sample has fewer observations per year than Hillegeist et al.'s (2004) average of 3,905 observations per year.

Estimating the true *ex ante* probability of bankruptcy for audit clients is an elusive concept.[9] To estimate the probability of bankruptcy, we adopt four different methods that rely on either accounting or capital market information, or a combination of the two. The four methods stem from the work of Altman (1968), Ohlson (1980), Zmijewski (1984), and Merton (1974). These methodologies are summarized in Appendix A. From each bankruptcy prediction model, we obtain an estimate of a client's probability of bankruptcy: $PRBANK_A$, $PRBANK_O$, $PRBANK_Z$, and $PRBANK_M$, respectively. In order to arrive at a single estimate of the firms' probability of bankruptcy, we simply use the arithmetic average:[10]

$$PRBANK = \frac{PRBANK_A + PRBANK_M + PRBANK_O + PRBANK_Z}{4}, \quad (2)$$

The advantage of focusing on the arithmetic average as the measure of central tendency is that the measure is sensitive to the idiosyncratic information contained in each of the individual prediction models, especially if one model was to yield a very different prediction to those of the other three models. We therefore believe that the arithmetic average is the most appropriate choice for measuring the central tendency of our bankruptcy measures rather than more robust measures of central tendency that eliminate the idiosyncratic information in each of the probability of bankruptcy measures. We acknowledge that our measure of firms' *ex ante* probability of bankruptcy is not perfect. However, we believe it may serve as a reasonable approximation of the true underlying probability of bankruptcy.

[9] Currently—and in a field that spans accounting, finance, and economics—a range of methods for estimating a firm's probability of bankruptcy exist. They differ in their underlying assumptions, input variables, and use of statistical and numerical techniques. See Bellovary et al. (2007) for a review.

[10] When we run our results with the probability of bankruptcy predictions from each of the four models, the results indicate an even more right skewed probability distribution for all models when compared to the average. This, in turn, would indicate an optimal substantial doubt threshold that is lower than what we report in the main analysis.

Table 2, Panel A, details summary statistics concerning each of the bankruptcy measures, as well as the average of the bankruptcy measures that we use as our approximation of the true underlying probability of bankruptcy. Table 2, Panel B, details the pairwise correlation (both Pearson's correlation coefficient and Spearman's rho) between each of the bankruptcy measures as well as the average bankruptcy.

[Insert Table 2 about here]

Table 2, Panel A, indicates that the four measures do not result in identical estimates of a firm's probability of bankruptcy. In particular, the Merton (1974) bankruptcy model, $PRBANK_M$, results in the lowest probabilities of bankruptcy, on average, and it has the distribution with the highest skewness. In contrast, the Ohlson (1980) bankruptcy model, $PRBANK_O$, results in the highest probabilities of bankruptcy, on average, and it has the distribution with the lowest skewness. The Altman (1968) and Zmijewski (1984) bankruptcy models, $PRBANK_A$ and $PRBANK_Z$, respectively, lie somewhere in between. Our composite estimate of the probability of bankruptcy, *PRBANK*, which is calculated as an average of the four measures, shows an average probability of bankruptcy of 0.1540 and a distributional skewness of about1.8126.

In Table 2, Panel B, $PRBANK_Z$ shows relatively high pairwise correlation (above 0.7) with both $PRBANK_A$ and $PRBANK_O$; however, the remaining pairwise correlations between each of the four bankruptcy probability estimates are somewhat lower (all below 0.6). $PRBANK_M$ exhibits the lowest pairwise correlations with the other measures, most likely because this measure relies on equity information, whereas the others rely more on accounting information. The Spearman correlations are generally a little higher than the Pearson correlations, which suggests that the pairwise monotonic relationships are stronger than the pairwise linear relationships between the

measures. Interestingly, in general, our composite measure, *PRBANK*, shows a higher correlation with each of the individual bankruptcy measures than the correlation *between* each of the four individual bankruptcy measures. This highlights the advantage of focusing on the simple arithmetic average as the measure of the central tendency of the four measures: the measure incorporates the idiosyncratic information contained in each of the individual prediction models, especially when one model yields a very different prediction to those of the other three models.

Figure 1 displays a histogram with the number of observations (across bin sizes of 5 percentage points) of a firm's probability of bankruptcy. The histogram reinforces the information presented in Table 2, Panel A, namely that the distribution of bankruptcy probabilities among the firms is highly skewed to the right. In other words, most firms are financially sound and hence they are not likely to experience any financial difficulties in the foreseeable future.

[Insert Figure 1 about here]

Given the estimated distribution of the probabilities of bankruptcy shown in Table 2 and Figure 1, Figure 2 shows the expected proportion of unmodified versus going concern modified opinions at various hypothetical substantial doubt thresholds in terms of the probability of bankruptcy. The figure clearly demonstrates how the proportion of going concern modifications decreases if the cut-off value for significant doubt increases. For example, if the hypothetical substantial doubt threshold was 0.20, then approximately 25 percent of the sample would receive a going concern modified opinion, compared to only 6 percent at the cut-off value of a 0.5 probability of bankruptcy.

[Insert Figure 2 about here]

Figure 3 shows the entropy at different cut-off values for substantial doubt along the probability of bankruptcy continuum. For the predicted proportions of standard opinions and going concern modified opinions calculated in Figure 2, we use equation (2) to derive the entropy (y-axis) at each point of the various hypothetical substantial doubt thresholds (x-axis).

[Insert Figure 3 about here]

Figure 3 shows that the total entropy (i.e., the information value of both standard and going concern modified opinions) is maximized (value = 1 bit) at a 0.08 probability of bankruptcy, which is a one-in-twelve or higher chance of bankruptcy. At this cut-off, there is an *ex ante* equal possibility that the audit opinion to be realized is standard or modified in relation to going concern issues. This implies that the actual realization of the outcome of the audit report delivers, on average, maximum information to users of the audit report. At higher (lower) cut-offs, both the proportion of going concern modifications (standard opinions) and the entropy of the audit reports decrease. For example, at the substantial doubt threshold of a 0.50 probability of bankruptcy, the total entropy (0.361) is significantly below the maximum, since most audit reports are standard opinions and, thus, only a small proportion of audit opinions convey useful information to users.

The finding that the information value of audit opinions is maximized if the auditors use a 0.08 probability of bankruptcy as the cut-off for substantial doubt is significantly lower than the bankruptcy probability values (ranging from 0.4–0.6) auditors have reported using (Asare, 1992; Asare and Messier, 1992; Davis and Ashton, 2002; Knapp et al., 1992; Ponemon and Raghunandan, 1994). However, audit opinions are regularly regarded as uninformative and

hence regulators are examining means of increasing the information value of audit reports for external users (European Commission, 2014; PCAOB, 2012a). Our finding suggests that a regulation or guidance that serves to lower auditors' substantial doubt threshold for going concern modifications could be useful if the information value is to be increased.

The conclusions derived from Figures 1 to 3 highlight the relationships between the underlying distribution of firms' probability of bankruptcy, substantial doubt thresholds, the degree of uncertainty, and the average information content of realized audit opinions. In the next section, we will focus on how the rate of "misclassifications" and the costs of "misclassifications" affect the results. This will involve the additional analyses: "Misclassifications" and Economic cost of "misclassifications."

In the main analysis presented above, we approximated the threshold that maximizes the average information content of going concern audit reports in accordance with the Shannon entropy principle in our sample. Consistent with the requirements of existing auditing and accounting standards, we focus on firms' *ex ante* probability of bankruptcy. That is, auditors' going concern decisions are inherently probabilistic in nature and, prior to bankruptcy, there is no way of unambiguously knowing which firms will fail. Yet, an appropriate substantial doubt threshold is critical because different thresholds result in different proportions of "misclassifications" of *actual* bankruptcies. While misclassifications cannot be eliminated,[11] they are not without cost, and a large body of auditing literature has focused on misclassifications of *actual* bankruptcies or else highlighted the importance of this (Defond and Zhang, 2014; Francis, 2011). Optimizing the

[11] Auditors' assessment of doubt concerning the appropriateness of the going concern assumption involves assessing the probability of future bankruptcy based on present information, not the prediction of actual bankruptcies. It should therefore be clear that when auditors are judged against a criterion of predicting actual bankruptcies, there will always be a certain number of misclassifications—even if the auditors were correct in assessing the probability of bankruptcy.

substantial doubt threshold that results in the least amount of misclassifications is a different criterion to optimizing the substantial doubt threshold that provides the highest entropy. However, it is important to investigate and contrast how the optimal cut-off value of substantial doubt changes when a different criterion is used.

Using the estimated distribution of the probability of bankruptcy for our sample, as depicted in Table 2 and Figure 1, we now calculate the expected long-term rates of audit opinions that can be labeled as either "correctly classified" or "misclassified" under various hypothetical substantial doubt thresholds. Following the main analysis, we let x be the hypothetical substantial doubt threshold defined in terms of the probability of bankruptcy (PRBANK), which can range from 0 to 1 and is displayed on the x-axis. If an observation's probability of bankruptcy (PRBANK) is greater than the hypothetical substantial doubt threshold (x), then the auditor is deemed to issue a going concern modification. To estimate the number of correctly classified and misclassified audit opinions, we interpret the probability of bankruptcy to yield a persistent long-run rate of bankruptcies, or the "relative frequency" of bankruptcies to non-bankruptcies.[12] We calculate the misclassification rate for the hypothetical substantial doubt thresholds as:

$$\%\, Misclassifed \;=\; \frac{GCM_{MC} + SDN_{MC}}{N} \tag{3}$$

[12] The "classification" of a given hypothetical substantial doubt threshold, say 0.15, is conducted in the following manner. First, there are 10,909 (20,423) observations in our sample that have a probability of bankruptcy equal or above (below) 0.15. These observations would be issued going concern modifications (standard opinions). Second, for the 10,909 observations deemed to be issued with going concern modifications, we find that the average probability of bankruptcy is 0.34934 and, therefore, 3,811 observations (34.93 percent of 10,909) are expected to go bankrupt. They are thus true positives and "correctly classified" (7,098 are false positives and "misclassified"). Similarly, for the 20,423 observations deemed to be issued with a standard opinion, the average probability of bankruptcy is 0.04971 and, therefore, 1,015 observations are expected to go bankrupt. They are hence false negatives and "misclassified" (19,408 are true negatives and "correctly classified"). We conduct similar calculations for all the hypothetical cut-off values.

where $GCM_{MC}$ is the number of going concern modified opinions issued to clients expected to survive, $SDN_{MC}$ is the number of standard opinions issued to clients expected to go bankrupt, and N is the total number of opinions.

Here we consider the optimal substantial doubt threshold to be the threshold where the rate of misclassifications is minimized. Figure 4 presents the results with respect to the expected rates of misclassified audit opinions at various hypothetical values for substantial doubt.

[Insert Figure 4 about here]

As observed in previous studies (e.g., Davis and Ashton, 2002), the total number of misclassifications will be minimized when the cut-off value for substantial doubt is 0.5. At this hypothetical substantial doubt threshold, only 13 percent of all audit reports will be misclassified, which of course means that 87 percent will be correctly classified.

As noted in both Table 2 and Figure 1 above, the distribution of the probability of bankruptcy is right skewed, and the majority of observations have low values for the probability of bankruptcy. This is important in terms of how the expected misclassification rates change in response to a change in the substantial doubt threshold. The long-run rates of misclassification change significantly in response to changes in the lower values of the substantial doubt threshold, simply because that is where most of the observations are located. For example, at a 0.08 probability of bankruptcy threshold, that is, the optimal substantial doubt threshold to maximize entropy, we estimate that 62 percent of the audit reports are correctly classified, while 38 percent are misclassified opinions.

Interestingly, the substantial doubt threshold of a 0.5 probability of bankruptcy, which yields the lowest percentage of misclassifications, is *not* the same point where the rates of misclassification for going concern modified opinions and the rates of misclassified standard opinions are equal, which is at a 0.32 probability of bankruptcy.

The above analysis of the misclassification rates assumes that the relative misclassification costs are equal. As pointed out in the studies by Davis and Ashton (2002) and Carson et al. (2013), this is unlikely to be the case.[13] When the misclassification costs are not equal, the substantial doubt threshold should potentially be adjusted so that the expected cost of misclassifying a going concern as a non-going concern equals the expected cost of misclassifying a non-going concern as a going concern. Next, we investigate how various misclassification cost ratios affect the economic costs at various hypothetical substantial doubt thresholds. Thus, our aim changes from minimizing the number of misclassifications (maximizing accuracy) to minimizing the expected cost of misclassifications. We modify equation (3) presented above and calculate the expected cost of misclassification at each of the hypothetical substantial doubt thresholds along the x-axis:

$$Expected\ Costs\ = \frac{GCM_{MC}*(1-r) + SDN_{MC}*r}{N} \quad (4)$$

where $GCM_{MC}$ is the number of going concern modified opinions issued to clients not expected to go bankrupt, $SDN_{MC}$ is the number of standard opinions issued to clients expected to go bankrupt, and N is the total number of opinions. *r* is the ratio of the cost of the misclassifications indicative of the cost of a misclassified standard opinion divided by the sum of the costs of

[13] As Carson et al. (2013, p. 366) note:
"For example, if an auditor does not issue a GCO and the client later fails, the auditor may incur costs related to litigation and loss of reputation ($C_b$). On the other hand, it is potentially costly for an auditor to issue a GCO when a client does not subsequently fail, because the issuance of a GCO to a subsequently viable company may be perceived as unwarranted and cause the client to become disgruntled and result in the switching of auditors, leading to a loss of audit revenue ($C_a$). It is straightforward to show that an auditor's economic incentive to issue a GCO depends on the ratio of these two costs, i.e., $C_b/C_a$. When the ratio is higher, the auditor is more likely to issue a GCO because the cost of failing to do so is greater."

misclassified going concern modifications and the costs of misclassified standard opinions.[14] The value of *r* will be close to one if the costs between correct and incorrect classifications are much larger for firms that go bankrupt; if *r* is close to zero, then this difference is much greater for firms that survive. Only when *r* is close to 0.5 is the difference in costs about equal for firms that survive and firms that go bankrupt.

To illustrate how misclassification costs affect the optimal substantial doubt threshold, we specify three misclassification cost ratios, namely 0.90, 0.75, and 0.50. The *r* values of 0.90 and 0.75 indicate where the costs between correct and incorrect classifications are about ten and four times larger, respectively, for firms that go bankrupt. That is, the cost of issuing a standard opinion to a firm that subsequently goes bankrupt is higher than the cost of issuing a going concern modification to a firm that subsequently survives. The value of 0.5 is the reference point. It indicates equal misclassification costs and will yield the same optimal substantial doubt threshold results as in Figure 4.

Figure 5 demonstrates the relative expected costs of misclassification for the hypothetical substantial doubt thresholds after adjusting for the three different ratios of misclassification costs. The optimal substantial doubt threshold with respect to misclassifications is the threshold that minimizes the expected costs. When *r*, the cost ratio, is expected to be 0.9 (i.e., the cost of a standard opinion to a firm that subsequently goes bankrupt is ten times higher than issuing a going concern modification to a firm that survives), the optimal substantial doubt threshold that minimizes the expected costs is at a probability of bankruptcy of 0.10. This is relatively close to

[14] We have no reasonable estimates of the actual costs of misclassified going concern modifications and the actual costs of misclassified standard opinions. As such, we can only specify a ratio that is indicative of the relative difference between the costs of misclassified going concern modifications and the costs of misclassified standard opinions.

the threshold of 0.08 that we estimated to yield maximum entropy. Alternatively, when we set *r* to 0.75, the optimal threshold is at a probability of bankruptcy of 0.25.

[Insert Figure 5 about here]

## 5. Discussion and limitations

Our results suggest that the optimal threshold for substantial doubt corresponds to when the firm's probability of bankruptcy is around 0.08. This threshold will result in equal proportions of standard and modified opinions and, consequently, maximum information content in terms of the Shannon entropy of the going concern audit report. The optimal threshold to maximize entropy could be considered to be rather low in comparison to previous research studies showing that auditors interpret "substantial doubt" to be somewhere in the 0.4–0.6 bankruptcy probability range (Asare, 1992; Asare and Messier, 1992; Davis and Ashton, 2002; Knapp et al., 1992; Ponemon and Raghunandan, 1994). However, it is worth noting that such a low threshold is: 1) consistent with the observational data from the study by Carson et al. (2013), who suggest that the number of going concern modifications outweighs the number of firms actually becoming bankrupt by a factor of thirty-five; 2) in line with the observed default rates for *speculative grade* credit ratings;[15] and 3) not necessarily inconsistent with minimizing the expected economic costs of misclassification rates at various substantial doubt thresholds *if* one considers that the relative costs of a misclassified standard opinion might be significantly higher than those of a misclassified going concern modified opinion.

[15] The one-, two-, three-, and four-year cumulative average default rates (%) on speculative grade credit ratings for global non-banks are only 4.21, 7.45, 10.37, and 12.19 percent, respectively (Standard & Poor's, 2012).

In this study, we have focused on the criterion of maximum entropy to derive an optimal substantial doubt threshold, with some additional analyses being conducted regarding misclassification rates and costs. These are, however, just some of the many criteria that could potentially be used to evaluate what the appropriate threshold should be, and we acknowledge that determining the "best" criteria is ultimately a normative decision that rests with standard setters. Yet, it is important to recognize that what constitutes a suitable criterion is not independent of how one decides to conceptualize the purpose of the audit report. For example, if the audit report in the going concern context is viewed simply as a bankruptcy prediction, then a criterion based on minimizing the number of misclassifications is probably quite appropriate, since it speaks directly to the accuracy rate of the audit report with respect to whether or not firms subsequently go bankrupt. In contrast, a criterion such as entropy does not necessarily conceptualize the audit report as a bankruptcy prediction *per se*, but rather as a binary indicator that reveals information about firms' *relative* financial distress.

Our analysis is based on implicit and explicit simplifying assumptions that make our analysis more tractable, although these assumptions are also a source of significant limitations that must be considered when evaluating our findings. First, we assume that all audit reports are independent and, as such, our analysis ignores the specific client situation and the complex client-auditor interaction that give rise to the auditor issuing a going concern modification. Second, we assume in our analysis that if a substantial doubt threshold was defined, then all auditors would be consistent in terms of their application of this threshold. This is highly unlikely, since regulators experience difficulties in defining what substantial doubt should mean. Third, we do not know the true value of firms' *ex ante* probability of bankruptcy and, therefore, our estimates must be considered as approximations. However, we use four existing bankruptcy

models and we believe that the prior use of these measures in the literature indicates that an average value should serve as a reasonable approximation.[16] Yet, the Shannon entropy of a quantity is only as meaningful as the probabilities used to compute it. We acknowledge that each of the bankruptcy models used in the analysis contain identifying assumptions that are implicitly incorporated into our analysis. Furthermore, we use publicly available quantitative information to estimate the firms' probability of bankruptcy, although the auditor is also privy to private and qualitative information. Unlike auditors, we do not have access to such information. These limitations highlight that our analysis is somewhat abstracted from reality, and we note that the specific result we present here should not be given any more weight than is warranted in light of these limitations.

## 6. Conclusion

In this paper, we apply Shannon entropy to evaluate the informational value of going concern audit reports at various hypothetical bankruptcy probability thresholds that auditors could use to evaluate the "substantial doubt" of an entity's ability to continue as a going concern. We investigate what the cut-off value for substantial doubt should be in order to maximize the average information content of the auditor's going concern opinion. We compile a sample of 31,332 firm-year observations covering the period 2003–2015, and we estimate their *ex ante* probability of bankruptcy so we can approximate the distribution of firms' probability of

[16] We conduct a sensitivity analysis as an alternative estimation of a firm's probability of bankruptcy. We estimate a new composite measure using the five bankruptcy models derived from the work of Wu et al. (2010, p. 41). The five models include those of Altman (1968), Ohlson (1980), and Zmijewski (1984), with coefficients as re-estimated on an updated sample of firms by Wu et al. (2010), an updated version of Shumway's (2001) bankruptcy model, and a new comprehensive model by Wu et al. (2010). Our composite bankruptcy measure for the purposes of our sensitivity analysis is the arithmetic average of these five models. Using this measure to approximate the distribution of firms' probability of bankruptcy, we find that the threshold that maximizes the entropy of audit reports in the going concern context is at a 0.148 probability of bankruptcy. Thus, using this alternative measure of the probability of bankruptcy, we find that the optimal threshold is about 7 percentage points higher than what we find in our main analysis.

bankruptcy. Applying the concept of Shannon entropy from information theory, we calculate the substantial doubt threshold value that would maximize the average information content of the auditor's going concern opinion for users to occur when the firm's probability of bankruptcy is around 0.08. That is, the results suggest that auditors should issue going concern opinions when there is about a one-in-twelve chance or higher that the client might go bankrupt.

We believe this paper illustrates that if the auditor's going concern opinion is to act as a red flag for financial statement users, the criteria that determine whether to raise the flag or not are inherently linked to how much information the flag conveys. In other words, the average information content of the auditor's going concern opinion is strongly related to the appropriate cut-off value for substantial doubt. We also believe that this paper highlights that what constitutes a high quality going concern modification is actually a rather nuanced and subtle concept.

The findings, as well as the limitations, of this study provide avenues for future research. First, more factors could be considered and examined in relation to what makes a good audit report in the going concern context. Research related to the informational value of audit reports may have real policy impact: for example, identifying the desirable (undesirable) properties of auditors' going concern opinions and examining how auditors' going concern decisions need to change so as to maximize (minimize) those properties. Another related avenue for future research may be to use Shannon entropy as a criterion for examining whether it would be beneficial for audit reports to contain several categories in order to highlight the severity of financial distress, as well as to determine what the optimal threshold for these different categories would be.

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**Appendix A: Bankruptcy models used to derive the distribution of probabilities of bankruptcy**

The four methods used to estimate the probability of bankruptcy are taken from Altman (1968), Ohlson (1980), Zmijewski (1984), and Merton (1974), as summarized below.

A.1 Altman (1968) model

We follow the approach of Hillegeist et al. (2004) to obtain the probability of bankruptcy based on the Altman (1968) model as follows:[17]

$$PRBANK_A = F(-1.2(WCTA) - 1.4(RETA) - 3.3(EBITTA) - 0.6(V_ETA) - 0.999(STA)) \quad (5)$$

where $F = (1/1\text{-}exp^{(-xB)})$ and, for client *i* and fiscal year-end *t*, *WCTA* = (working capital) / (total assets); *RETA* = (retained earnings) / (total assets); *EBITTA* = (earnings before interest and tax) / (total assets); $V_ETA$ = (market value of equity) / (total liabilities); *STA* = (sales) / (total assets); and $PRBANK_A$ = probability of bankruptcy. The data used to calculate the Altman (1968) model's financial variables were obtained for firms' fiscal year-end from the CRSP/Compustat merged fundamental annual files.

A.2 Ohlson (1980) model

The Ohlson (1980) model is an accounting-based probability model that predicts the probability of bankruptcy as:[18]

$$PRBANK_O = F(-1.32 - 0.407(SIZE) + 6.03(TLTA) - 1.43(WCTA) + 0.0757(CLCA) - 2.379(NITA) - 1.83(FUTL) + 0.285(INTWO) - 1.72(OENEG) - 0.521(CHIN)) \quad (6)$$

[17] Consistent with Hillegeist et al. (2004), we change the signs of the original coefficients so that increases in the outcome are consistent with increases in the probability of bankruptcy. All the coefficients except STA have been multiplied by 100, since, unlike Altman (1968), we define all our variables as ratios. We use the logit link function to transform the outcome of Altman's (1968) multiple discriminate analysis into a probability of bankruptcy measure (Hillegeist et al., 2004). While this transformation is not strictly correct, McFadden (1976) shows that the multiple discriminant analysis and the logit approaches are closely related under normality assumptions (Hillegeist et al., 2004).

[18] Based on Ohlson's (1980, p. 121) Model 1 in Table 4.

where F = $(1/1\text{-}\exp^{(-xB)})$ and, for client *i* and fiscal year-end *t*, *SIZE* = the natural logarithm of total assets in dollars divided by the GNP price level index;[19] *TLTA* = (total liabilities) / (total assets); *WCTA* = (working capital) / (total assets); *CLCA* = (current liabilities) / (current assets); *OENG* = one if total liabilities exceeds total assets, or zero otherwise; *NITA* = (net income) / (total assets); *FUTL* = (cash flow from operations) / (total liabilities); *INTWO* = one if net income was negative for the last two years, or zero otherwise; *CHIN* = $(NI_t\text{-}NI_{t-1})$ / $(|N_{it}|+|NI_{t-1}|)$, where $NI_t$ is the net income for the most recent period; and $PRBANK_O$ = probability of bankruptcy. The data used to calculate the Ohlson (1980) model's financial variables were obtained for firms' fiscal year-end from the CRSP/Compustat merged fundamental annual files.

A.3 Zmijewski (1984) model

Zmijewski (1984) uses the financial ratios of firm performance, leverage, and liquidity to predict bankruptcy:[20]

$$PRBANK_z = N(-4.3-4.5(NITA) + 5.7(TLTA) - 0.004(CACL)) \quad (7)$$

where N(.) is the cumulative normal probability function and, for client *i* and fiscal year-end *t*, *NITA* = (net income) / (total assets); *TLTA* = (total debt) / (total assets); *CACL* = (current assets) / (current liabilities); and $PRBANK_z$ = probability of bankruptcy. The data used to calculate the Zmijewski (1984) model's financial variables were obtained for firms' fiscal year-end from the CRSP/Compustat merged fundamental annual files.

[19] Dollars transformed into 1980 dollars.

[20] Zmijewski (1984) developed numerous models using 40 bankrupt and 40–800 non-bankrupt firms; however, the model based on the 40: 800 proportion of bankrupt to non-bankrupt firms is the model most commonly used by accounting researchers (e.g., Carcello and Neal (2003) and Carcello et al. (1995)).

A.4 Merton (1974) "distance to default" model

We utilize the Merton (1974) corporate bond pricing model to estimate the "distance to default," which is the distance of the asset value of the firm from the face value of its debt scaled by the standard deviation of the firm's assets.

As the market value of equity (E)[21] and the face value of debt (D) [22] are observable, although the market value of assets (V) is not, we follow Crosbie and Bohn (2003), Vassalou and Xing (2004), and Bharath and Shumway (2008) by using the starting values of V = E + D, mean log return of assets (μ), and volatility of assets ($\sigma_V$) to infer V iteratively for each day over the previous year from the Black and Scholes (1973) European call option formula:

$$E = VN(d_1) - e^{-rT}DN(d_2) \quad (8)$$

where $d_1 = \frac{\ln(\frac{V}{D}) + \left(r + \frac{\sigma_V^2}{2}\right)T}{\sigma_V\sqrt{T}}$; $d_2 = d_1 - \sigma_V\sqrt{T} = \frac{\ln(\frac{V}{D}) + \left(\mu - \frac{\sigma_V^2}{2}\right)T}{\sigma_V\sqrt{T}}$; T (= 1 year) is the forecasting horizon; r is the continuously compounded risk free rate;[23] μ is the estimate of the expected annual return of V; and N(.) is the cumulative density function of the standard normal distribution. The data needed to calculate the Ohlson (1980) model's variables were obtained for firms' fiscal year-end from the CRSP/Compustat merged fundamental annual files and the CRSP daily stock files.

[21] The market value of equity (E) is the product of the number of shares outstanding and the daily closing price from CRSP.

[22] We follow prior literature (e.g., Bharath and Shumway, 2008; Crosbie and Bohn, 2003; Vassalou and Xing, 2004) and set the book value of debt equal to "debt in one year" plus half the "long-term debt." Although the amount of the long-term debt that should be included is somewhat arbitrary, prior literature (e.g., Crosbie and Bohn, 2003) has argued that 50 percent is sensible and adequately captures the financing constraints of firms. Consequently, we do the same.

[23] For the risk free rate (r), we use monthly observations of the 1-Year Treasury Bill rate obtained from the Federal Reserve Economic Data.

The values of V thus estimated are used to generate the new values of $\sigma_V$ and $\mu$, which are substituted into $-d_2$ to obtain the Merton distance to default *DD*. *DD* describes how many standard deviations the log of this ratio needs to depart from its mean in order for default to occur. Using the standard normal cumulative density function, we convert the distance to default *DD* into its corresponding implied probability of default (McDonald, 2002, p. 604):

$$PRBANK_M = N(-\textit{Merton DD}) = N(-d_2) \quad (5)$$

**Table 1**
Sample selection.

| | **# obs** |
|---|---|
| Compustat North America 2003–2015 | 77,470 |
| Less: observations in the financial industries (SIC 60–69) | 32,154 |
| Less: missing data to calculate the bankruptcy probabilities | 13,984 |
| **Final sample** | **31,332** |
| **# firms** | **5146** |

**Table 2**
Probability of bankruptcy estimates.

**Panel A: Descriptive statistics (n=31,332)**

| | Average | 1st Quart. | Median | 3rd Quart. | Std. dev. | Skew |
|---|---|---|---|---|---|---|
| PRBANKA | 0.0911 | 0.0059 | 0.0349 | 0.1120 | 0.1519 | 3.4410 |
| PRBANKO | 0.3388 | 0.0692 | 0.2162 | 0.5588 | 0.3219 | 0.8329 |
| PRBANKZ | 0.1337 | 0.0008 | 0.2158 | 0.1150 | 0.2531 | 2.3324 |
| PRBANKM | 0.0523 | 0.0000 | 0.0000 | 0.0013 | 0.1653 | 3.8411 |
| PRBANK | 0.1540 | 0.0252 | 0.0828 | 0.2169 | 0.1823 | 1.8126 |

**Panel B: Correlation matrix**

| | $PRBANK_A$ | $PRBANK_O$ | $PRBANK_Z$ | $PRBANK_M$ | PRBANK |
|---|---|---|---|---|---|
| $PRBANK_A$ | 1.0000 | 0.5795 | 0.7417 | 0.5418 | 0.7096 |
| $PRBANK_O$ | 0.5464 | 1.0000 | 0.8043 | 0.5472 | 0.9631 |
| $PRBANK_Z$ | 0.6913 | 0.7095 | 1.0000 | 0.4466 | 0.8698 |
| $PRBANK_M$ | 0.4422 | 0.3607 | 0.3580 | 1.0000 | 0.6110 |
| PRBANK | 0.7904 | 0.8840 | 0.8862 | 0.6028 | 1.0000 |

Notes to Table 2:

1. $PRBANK_A$, $PRBANK_O$, $PRBANK_Z$, and $PRBANK_M$ are estimates of the probability of bankruptcy using the models of Altman (1968), Ohlson (1980), Zmijewski (1984), and Merton (1974), respectively. PRBANK is a composite measure of the probability of bankruptcy obtained by taking the arithmetic average of the four estimates for a given firm.

2. In Panel B, Pearson's correlation coefficient, r, is depicted in the bottom triangle, while Spearman's rho is depicted in the top triangle.

**Figure 1.** Probability of bankruptcy histogram (n=31,332).

The histogram shows the distribution of the client's estimated probability of bankruptcy, *PRBANK*. *PRBANK* is a composite measure of probability of bankruptcy by taking the arithmetic average of the four estimates for a given firm using the models from Altman (1968), Ohlson (1980), Zmijewski (1984), and Merton (1974). The bin sizes are all 5 percentage points.

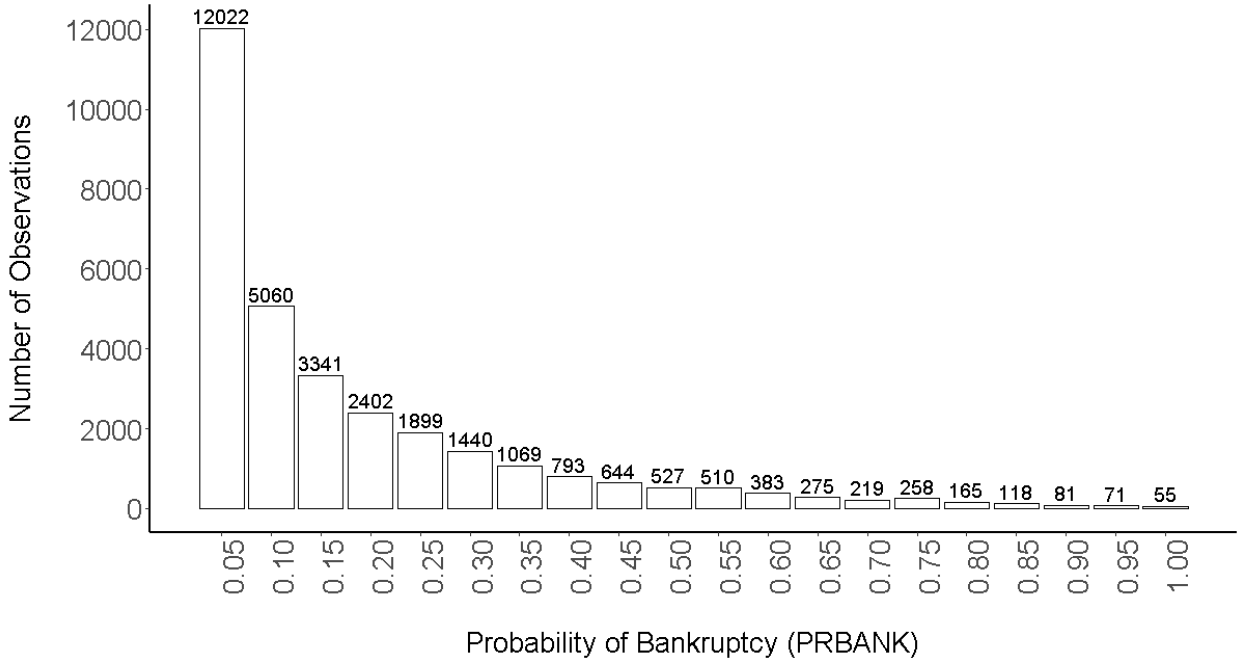

**Figure 2.** Proportion of opinion types at different hypothetical cut-offs (n=31,332).

The expected proportion of standard opinions versus the going concern modified opinions at various hypothetical substantial doubt thresholds in terms of the probability of bankruptcy (*PRBANK)*, given the shape of the distribution of the client's estimated probability of bankruptcy (*PRBANK)*. The expected proportions of standard opinions and going concern modified (GCM) opinions at the different hypothetical substantial doubt thresholds are estimated as follows: Let x be the hypothetical substantial doubt threshold defined in terms of the probability of bankruptcy (*PRBANK*), which can range from 0 to 1 and is displayed on the x-axis. If an observation's probability of bankruptcy (*PRBANK*) is greater than the hypothetical substantial doubt threshold (x), then the auditor is taken to issue a going concern modified opinion, while a standard opinion.

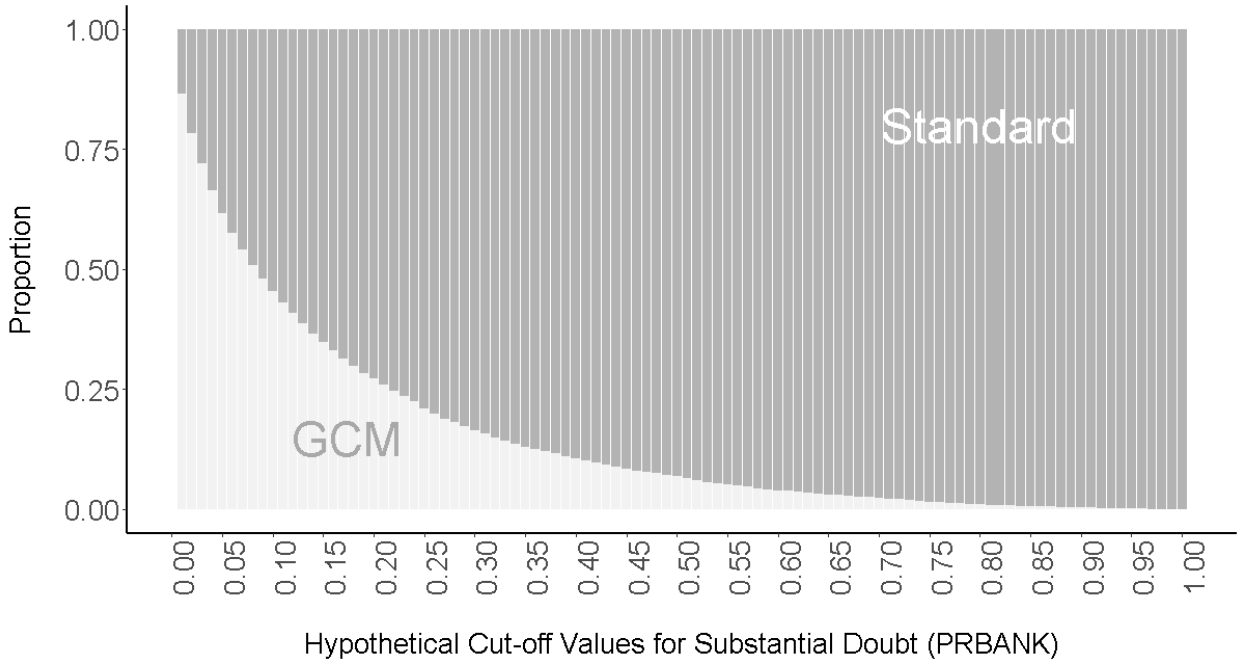

**Figure 3.** Total entropy at different hypothetical cut-offs (n=31,332).

The entropy at different cut-off values for substantial doubt along with the probability of bankruptcy, *PRBANK*. The continuum is estimated by Equation (1) for the set of firms predicted to have standard opinions and those predicted to have going concern opinions.

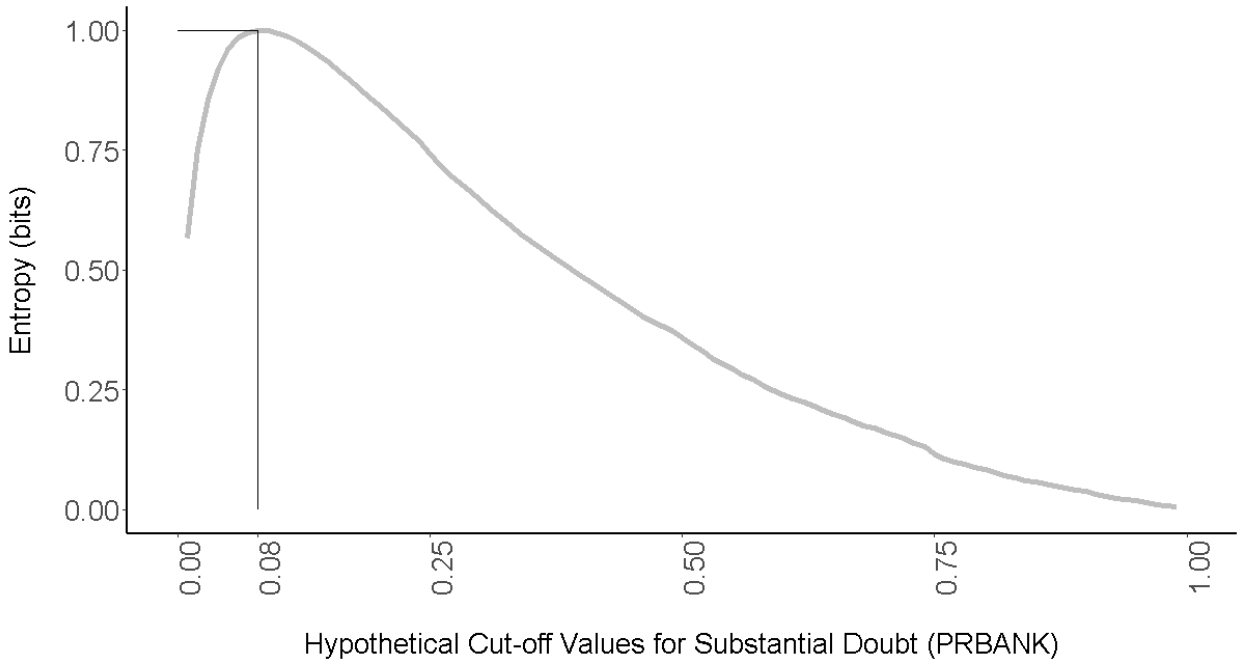

**Figure 4.** Expected rates of misclassified opinions at different hypothetical cut-offs (n=31,332).

The expected long-term rates of audit opinions that can be labeled as "misclassified" under various hypothetical substantial doubt thresholds, given the distribution of the client's estimated probability of bankruptcy, *PRBANK*. The expected proportions of standard opinions and going concern modified opinions that are misclassified at the different hypothetical substantial doubt thresholds are estimated as follows: 1) Let x be the hypothetical substantial doubt threshold defined in terms of the probability of bankruptcy (*PRBANK*), which can range from 0 to 1 and is displayed on the x-axis. If an observation's probability of bankruptcy (*PRBANK*) is greater than the hypothetical substantial doubt threshold (x), then the auditor is taken to issue a going concern modified (GCM) opinion, while a standard opinion is issued if otherwise. 2) In order to estimate the number of audit opinions as misclassifications, we simply interpret the probability of bankruptcy to yield a persistent rate of bankruptcies, or the "relative frequency" of bankruptcies to non-bankruptcies, if there was a sufficiently long-run of trials.

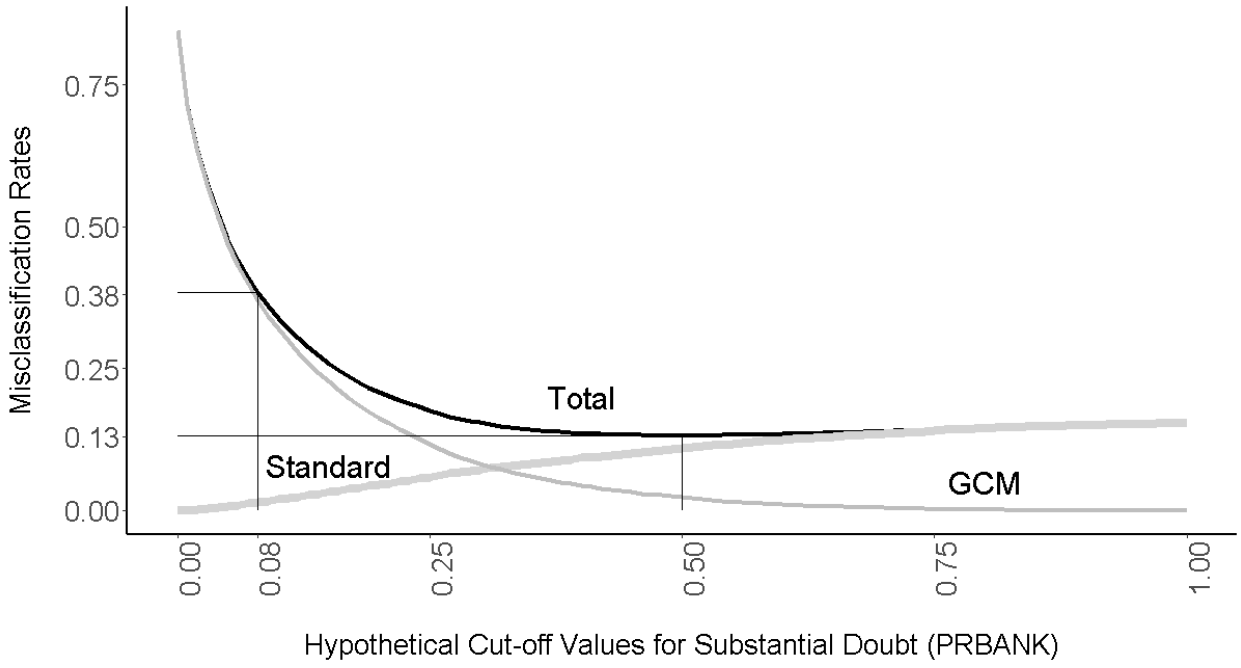

**Figure 5.** Expected economic costs of misclassified opinions at different hypothetical cut-offs under different cost ratios.

The relative expected economic costs of the long-term rates of audit opinions that can be labeled as "misclassified" under various hypothetical substantial doubt thresholds, given the distribution of the client's estimated probability of bankruptcy, *PRBANK*. The expected relative economic costs of opinions that are misclassified at the different hypothetical substantial doubt thresholds are estimated as follows: 1) Let x be the hypothetical substantial doubt threshold defined in terms of the probability of bankruptcy (*PRBANK*), which can range from 0 to 1 and is displayed on the x-axis. If an observation's probability of bankruptcy (*PRBANK*) is greater than the hypothetical substantial doubt threshold (x), then the auditor is taken to issue a going concern modified opinion, and a standard opinion if otherwise. 2) In order to estimate the number of audit opinions as misclassifications, we simply interpret the probability of bankruptcy to yield a persistent rate of bankruptcies, or the "relative frequency" of bankruptcies to non-bankruptcies, if there was a sufficiently long-run of trials. 3) The type of misclassifications are weighted by the ratio, r, of the difference in costs according to whether firms that go bankrupt are misclassified or whether misclassifying the firms that survive. Three values for the r ratio are specified: 0.90, 0.75, and 0.50. The r values of 0.90 and 0.75 indicate where the costs between correct and incorrect classification are about ten and four times larger, respectively, for firms that go bankrupt. The value of 0.5 is the reference point, and it indicates equal misclassification costs.

