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Combined Lagrange Multiplier Test for ARCH in Vector Autoregressive Models*

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Abstract

In this paper we propose a combined Lagrange multiplier (LM) test for autoregressive conditional heteroskedastic (ARCH) errors in vector autoregressive (VAR) models by following a suggestion in Dufour et al. (2010) of replacing an exact Monte Carlo (MC) test by a bootstrap MC test when the model includes lags. The test circumvents the problem of high dimensionality in multivariate tests for ARCH in VAR models. It is computationally simple since it only requires computing univariate statistics. The bootstrap MC test is shown to be asymptotically exact. Monte Carlo simulations show that the test has good finite-sample properties. The test is robust against a non-normal error distribution, while other multivariate LM tests for ARCH suffer from size distortion. We present two financial applications of multivariate LM tests for ARCH to credit default swap (CDS) prices and Euribor interest rates. The results indicate that the errors are skewed and heavy-tailed, and that there are significant ARCH effects.

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1 Introduction

The Lagrange multiplier (LM) test for autoregressive conditional heteroskedasticity (ARCH) of Engle (1982) is widely used as a specification test in univariate time series models. It is easy to compute from an auxiliary regression involving the squared least squares (LS) residuals. The LM statistic is asymptotically distributed as χ^2 under the null hypothesis. Besides, there are other less frequently used LM tests for ARCH such as the one-sided test of Lee and King (1993), and Portmanteau tests for ARCH such as the test of McLeod and Li (1983).

Testing for ARCH in multivariate time series models is routinely done equation-by-equation by applying univariate tests (e.g. the LM test of Engle 1982) to the individual equations. However, as emphasised by Dufour et al. (2010), univariate statistics from individual equations are not independent if the errors are contemporaneously correlated. The application of univariate tests in multivariate models leads to problems with combining the outcomes of the tests. Multivariate LM tests for ARCH are available but have not been much used. General multivariate GARCH models have $O(n^4)$ parameters, where n is the dimension of the process (Engle and Kroner 1995, p. 126). The multivariate generalisation of the LM test (see e.g. Lütkepohl 2006) requires estimating a large number of parameters in the auxiliary regression. Another multivariate LM tests for ARCH is the test of constant error covariance matrix of Eklund and Teräsvirta (2007). In addition, there are multivariate portmanteau tests (Ling and Li 1997, Duchesne and Lalancette 2003, and Dufour et al. 2010) and kernel-based tests (Duchesne and Lalancette 2003) for ARCH. In this paper we will focus on LM tests and not study these tests.

The Monte Carlo (MC) test technique was introduced by Dwass (1957) in statistics. MC tests make it possible to conduct exact inference in small samples based on pivotal test statistics. MC tests were introduced in econometrics by Dufour (2006) and applied to specification testing by Dufour et al. (2004, 2010). MC test techniques deliver exact finite-sample specification tests in regression models with exogenous regressors. Dufour et al. (2004) study MC tests for heteroskedasticity and ARCH in linear regression models with fixed or exogenous regressors. Dufour et al. (2010) propose combined equation-by-equation tests for error autocorrelation (AC) and ARCH errors in multivariate linear regression models with exogenous regressors. Gonzalez and Teräsvirta (2006) apply MC test techniques to testing linearity against smooth transition models.

In this paper we propose a combined equation-by-equation LM test for ARCH errors in vector autoregressive (VAR) models by following a suggestion in Dufour et al. (2010) of replacing an exact MC test by a bootstrap MC test when the model includes lags. The test circumvents the problem of high dimensionality in multivariate tests for ARCH in VAR models. It is computationally simple since it only requires computing n univariate statistics. The bootstrap MC test is shown to be asymptotically exact. Our combined LM test for ARCH extends the combined LM test for ARCH of Dufour et al. (2010) in regression models with exogenous regressors to VAR models, and the univariate bootstrap LM test for ARCH proposed by Gel and Chen (2012) to the multivariate case.

The LM tests for ARCH are evaluated in simulation experiments. The results show that the combined test has good finite-sample properties. Besides volatility clustering, perhaps the most important stylised facts of financial time series are skewed and heavy-tailed distributions. The combined LM test is robust against a non-normal error distribution, while the other multivariate LM tests for ARCH suffer from size distortion. The test of constant error covariance matrix (Eklund and Teräsvirta 2007) outperforms the combined test in terms of power when the errors are normal, but becomes unreliable when the errors are skewed and heavy-tailed. We present two financial applications of the multivariate LM tests for ARCH to credit default swap (CDS) prices and Euribor interest rates. The results indicate that the errors are skewed and heavy-tailed, and that there are significant ARCH effects.

The paper is organised as follows. The combined equation-by-equation LM test for ARCH in VAR models is described in Section 2. Section 3 outlines the framework of MC specification tests based on standardised multivariate LS residuals and introduces the bootstrap algorithm. The results of simulation experiments investigating the finite-sample properties of the multivariate LM tests for ARCH are reported in Section 4. The tests are applied to CDS prices and Euribor interest rates in Section 5. Section 6 concludes.

2 Multivariate LM Tests for ARCH

The observations on the $n \times 1$ vector $\mathbf{y}_t = (y_{1t}, \dots, y_{nt})'$ are assumed to be generated by an n-variate VAR model

$$\mathbf{y}_t = \mathbf{\Pi}_1 \mathbf{y}_{t-1} + \cdots + \mathbf{\Pi}_p \mathbf{y}_{t-p} + \mathbf{u}_t, \quad t = 1, \dots, T, \tag{1}$$

where $\mathbf{\Pi}_1, \dots, \mathbf{\Pi}_p$ are $n \times n$ parameter matrices. The null hypothesis is that the errors $\{\mathbf{u}_t\}$ are IID$(\mathbf{0}, \mathbf{\Omega})$ against the alternative hypothesis that they are conditionally heteroskedastic:

$$\mathbf{u}_t = \mathbf{H}_t^{1/2} \varepsilon_t, \tag{2}$$

where $\mathbf{H}_t = \mathrm{E}(\mathbf{u}_t \mathbf{u}_t' | \mathcal{F}_{t-1})$ is the conditional covariance matrix of the errors $\mathbf{u}_t$, $\mathcal{F}_{t-1} = \sigma(\varepsilon_{t-1}, \varepsilon_{t-2}, \dots)$ is the σ-field generated by $\{\varepsilon_{t-1}, \varepsilon_{t-2}, \dots\}$ and $\{\varepsilon_t\}$ is a sequence of IID$(\mathbf{0}, \mathbf{I}_n)$ random variables.

2.1 Combined LM Test

Following Dufour et al. (2010), we present the combined equation-by-equation LM test for ARCH in VAR models. The LM test for ARCH of order h (Engle 1982) in equation i is a test of $H_0 : b_1 = \cdots = b_h = 0$ against $H_1 : b_j \neq 0$ for at least one $j \in \{1, \dots, h\}$ in the auxiliary regression

$$\widehat{u}_{it}^2 = b_0 + b_1 \widehat{u}_{i,t-1}^2 + \cdots + b_h \widehat{u}_{i,t-h}^2 + e_{it},$$

where $\widehat{\mathbf{u}}_i = (\widehat{u}_{i1}, \dots, \widehat{u}_{iT})'$, $i = 1, \dots, n$, are the LS residuals from model (1). The test statistic has the form

$$LM_i = TR_i^2, \tag{3}$$

where R_i^2 is the coefficient of determination in the auxiliary regression for equation i. The combined LM statistic is given by (Dufour et al. 2010)

$$\widetilde{LM} = 1 - \min_{1 \leq i \leq n} (p(LM_i)), \tag{4}$$

where $p(LM_i)$ is the p-value of the LM_i statistic. The p-value may be derived from the asymptotic distribution of LM_i, which is $\chi^2(h)$.

The combined LM test rejects the null hypothesis of no ARCH effect if at least one of the individual LM_i statistics is significant (Dufour et al. 2010). The combined test is closely related to a Bonferroni-type testing procedure, but different from the Bonferroni bound the bootstrap MC test technique produces a simulated joint p-value. The bootstrap algorithm for simulating the p-value of the combined LM test for ARCH in VAR models is outlined in Section 3.

2.2 Other Tests

The other tests considered in the simulations are the multivariate generalisation of the LM test (see e.g. Lütkepohl 2006, sect. 16.5) and a test of constant error covariance matrix (Eklund and Teräsvirta 2007).

Multivariate LM Test

The multivariate LM test for ARCH of order h is a generalisation of the univariate test and is based on the auxiliary regression

$$\mathrm{vech}(\widehat{\mathbf{u}}_t \widehat{\mathbf{u}}_t') = \mathbf{b}_0 + \mathbf{B}_1 \mathrm{vech}(\widehat{\mathbf{u}}_{t-1} \widehat{\mathbf{u}}_{t-1}') + \cdots + \mathbf{B}_h \mathrm{vech}(\widehat{\mathbf{u}}_{t-h} \widehat{\mathbf{u}}_{t-h}') + \mathbf{e}_t, \tag{5}$$

where $\mathbf{b}_0$ is a $\frac{1}{2}n(n+1)$-dimensional parameter vector and $\mathbf{B}_1, \ldots, \mathbf{B}_h$ are $\frac{1}{2}n(n+1) \times \frac{1}{2}n(n+1)$ parameter matrices. The operator vech is the half-vectorisation operator. The null hypothesis is $H_0 : \mathbf{B}_1 = \cdots = \mathbf{B}_h = \mathbf{0}$ against $H_1 : \mathbf{B}_j \neq \mathbf{0}$ for at least one $j \in \{1, \ldots, h\}$. The multivariate LM statistic is of the form

$$MLM = \frac{1}{2} Tn(n+1) - T\mathrm{tr}(\widehat{\mathbf{\Omega}}_{\mathrm{vech}} \widehat{\mathbf{\Omega}}^{-1}), \tag{6}$$

where $\widehat{\mathbf{\Omega}}_{\text{vech}}$ is the estimator of the error covariance matrix from the auxiliary model (5) and $\widehat{\mathbf{\Omega}} = T^{-1}\sum_{t=1}^{T}\widehat{\mathbf{u}}_t\widehat{\mathbf{u}}_t'$ is the estimator of the error covariance matrix from the VAR model (1) (see e.g. Lütkepohl 2006, sect. 16.5). The MLM statistic is asymptotically distributed as $\chi^2(n^2(n+1)^2h/4)$ under the null hypothesis.

LM Test of Constant Error Covariance Matrix

Eklund and Teräsvirta (2007) propose an LM test of constant error covariance matrix which may be viewed as a multivariate diagnostic test for ARCH if the alternative is a constant conditional correlation autoregressive conditional heteroskedasticity (CCC-ARCH) process of order h:

$$\mathbf{H}_t = \mathbf{D}_t\mathbf{P}\mathbf{D}_t,$$

where

$$\mathbf{D}_t = \text{diag}(h_{1t}^{1/2}, \ldots, h_{nt}^{1/2}) \tag{7}$$

is a diagonal matrix of conditional standard deviations of the errors $\{\mathbf{u}_t\}$ and $\mathbf{P} = (\rho_{ij}),\ i,j = 1,\ldots,n$, is a positive definite matrix of conditional correlations, i.e. $\rho_{ij} = 1$ for $i = j$. The conditional variance $\mathbf{h}_t = (h_{1t},\ldots,h_{nt})'$ is assumed to follow a CCC-ARCH(h) process:

$$\mathbf{h}_t = \mathbf{a}_0 + \sum_{j=1}^{h}\mathbf{A}_j\boldsymbol{u}_{t-j}^{(2)}, \tag{8}$$

where $\mathbf{a}_0 = (a_{01},\ldots,a_{0n})'$ is an n-dimensional vector of positive constants, $\mathbf{A}_1,\ldots,\mathbf{A}_h$ are $n \times n$ diagonal matrices and $\boldsymbol{u}_t^{(2)} = (u_{1t}^2,\ldots,u_{nt}^2)'$. The idea behind the test is that under the alternative hypothesis the error variances are time-varying, whereas the correlations are constant over time. The restriction of constant correlations decreases the dimensions of the null hypothesis from $O(n^4)$ to $O(n)$. The null hypothesis is $H_0 : \text{diag}(\mathbf{A}_1) = \cdots = \text{diag}(\mathbf{A}_h) = \mathbf{0}$ against $H_1 : \text{diag}(\mathbf{A}_j) \neq \mathbf{0}$ for at least one $j \in \{1,\ldots,h\}$. The LM statistic has the form

$$LM_{CCC} = T\mathbf{s}(\widehat{\boldsymbol{\theta}})'\mathbf{I}(\widehat{\boldsymbol{\theta}})^{-1}\mathbf{s}(\widehat{\boldsymbol{\theta}}), \tag{9}$$

where $\mathbf{s}(\widehat{\boldsymbol{\theta}})$ and $\mathbf{I}(\widehat{\boldsymbol{\theta}})$ are the score vector and information matrix, respectively, estimated under the null hypothesis (see Eklund and Teräsvirta 2007 for details). The LM_{CCC} statistic is asymptotically distributed as $\chi^2(nh)$ under the null hypothesis.

2.3 Comparison of Tests

Under the null hypothesis the conditional variances and covariances of the errors are constant. Under the alternative hypothesis the conditional variances of the errors are time-varying in all tests. The tests differ in what they assume about the conditional covariances of the errors under the alternative. The combined LM test is based on univariate tests for ARCH. Implicit in the test is therefore an assumption that the conditional covariances of the errors are constant. In the multivariate LM test the conditional covariances of the errors are time-varying under the alternative. Since $\mathbf{B}_j$, $j = 1, \ldots, n$, are $\frac{1}{2}n(n+1) \times \frac{1}{2}n(n+1)$ parameter matrices, the degrees of freedom in the asymptotic χ^2-distribution of the MLM statistic of order h are $n^2(n+1)^2h/4$. In the LM_{CCC} test the conditional correlations are constant and thus the conditional covariances are proportional to the square root of the product of the conditional variances. Since $\mathbf{A}_j$, $j = 1, \ldots, n$, are diagonal matrices, the degrees of freedom in the asymptotic χ^2-distribution of the LM_{CCC} statistic are nh. Taking $n = 5$ and $h = 2$ from our empirical application in Section 5.2 as an example would result in $n^2(n+1)^2h/4 = 450$ degrees of freedom in the asymptotic distribution of the MLM statistic. The asymptotic distribution of the LM_{CCC} statistic has $nh = 10$ degrees of freedom.

3 Monte Carlo Test Technique and Bootstrap Algorithm

In this section we present the MC test technique before outlining the bootstrap algorithm for the combined LM test for ARCH errors in VAR models. For a general treatment and proofs, see Dufour (2006); for a treatment of specification tests, see Dufour et al. (2010).

Dufour et al. (2010) develop a framework for MC tests which employs standardised multivariate residuals from the multivariate linear regression model

$$\mathbf{Y} = \mathbf{X}\mathbf{B} + \mathbf{U}, \tag{10}$$

where $\mathbf{Y} = (\mathbf{y}_1, \dots, \mathbf{y}_n)$ is a $T \times n$ matrix, $\mathbf{X}$ is a $T \times k$ matrix of full column rank, $\mathbf{B}$ is a $k \times n$ parameter matrix and $\mathbf{U} = (\mathbf{u}_1, \dots, \mathbf{u}_n)$ is a $T \times n$ matrix of errors. The regressors are assumed to be exogenous, i.e. $\mathbf{X}$ is taken as fixed for statistical analysis. Test statistics are based on Cholesky-standardised multivariate residuals

$$\widetilde{\mathbf{W}} = \widehat{\mathbf{U}}\, \mathbf{S}_{\widehat{\mathbf{U}}}^{-1}, \tag{11}$$

where $\mathbf{S}_{\widehat{\mathbf{U}}}$ is the Cholesky factor of $T^{-1}\widehat{\mathbf{U}}'\widehat{\mathbf{U}}$, i.e. $\mathbf{S}_{\widehat{\mathbf{U}}}$ is the (unique) upper triangular matrix such that

$$\widehat{\mathbf{\Omega}} = \mathbf{S}_{\widehat{\mathbf{U}}}'\mathbf{S}_{\widehat{\mathbf{U}}}, \quad \widehat{\mathbf{\Omega}}^{-1} = (T^{-1}\widehat{\mathbf{U}}'\widehat{\mathbf{U}})^{-1} = \mathbf{S}_{\widehat{\mathbf{U}}}^{-1}(\mathbf{S}_{\widehat{\mathbf{U}}}^{-1})', \tag{12}$$

and $\widehat{\mathbf{U}} = (\widehat{\mathbf{u}}_1, \dots, \widehat{\mathbf{u}}_n)$ are the LS residuals from (10). We use the notation as in Dufour et al. (2010):

$$\widetilde{\mathbf{W}} = (\widetilde{\mathbf{w}}_1, \dots, \widetilde{\mathbf{w}}_n) = (\widetilde{\mathbf{w}}_1, \dots, \widetilde{\mathbf{w}}_T)' \quad \text{and} \quad \widetilde{\mathbf{w}}_i = (\widetilde{w}_{i1}, \dots, \widetilde{w}_{iT})'.$$

The standardised residuals are

$$\widetilde{\mathbf{w}}_t = (\mathbf{S}_{\widehat{\mathbf{U}}}^{-1})'\widehat{\mathbf{u}}_t.$$

The framework delivers exact specification tests in finite samples (Dufour et al. 2010, Theorem 1).

The VAR model (1) defines a pure autoregression which can be written in the linear regression form (10) with $\mathbf{X}_t = (\mathbf{y}_{t-1}', \dots, \mathbf{y}_{t-p}')$ a typical row of $\mathbf{X}$ and $\mathbf{B} = (\mathbf{\Pi}_1, \dots, \mathbf{\Pi}_p)'$ an $np \times n$ parameter matrix.

Standardised versions of the LM_i statistics in (3) are obtained by replacing the residuals $\widehat{u}_{it}$ by the standardised residuals $\widetilde{w}_{it}$. The combined $\widetilde{LM}$ statistic is then given by (4). Similarly, standardised versions of the MLM statistic in (6) and LM_{CCC} statistic in (9) are obtained by replacing the residuals $\widehat{\mathbf{u}}_t$ by the standardised residuals $\widetilde{\mathbf{w}}_t$. Because the VAR model contains lags of the dependent variable, the distributions of test statistics based on Cholesky-standardised multivariate residuals are not free of nuisance parameters. To circumvent the problem, we use a parametric bootstrap. The bootstrap combined $\widetilde{LM}^*$ test is summarised in the following algorithm. The algorithm is a modification of the algorithm in Dufour et al. (2010) to autoregressions. The LS estimator $\widehat{\mathbf{B}}$ of $\mathbf{B}$ under the null hypothesis is used in a parametric bootstrap in step 3. The same algorithm is used with the bootstrap MLM and LM_{CCC} tests, which we denote by MLM^* and LM^*_{CCC}, respectively.

Algorithm 1 (Bootstrap combined LM test for ARCH)

1. From the data, compute $\widetilde{LM}$ *in (4) and denote it* $\widetilde{LM}^{(0)}$.

2. Obtain N *draws from*

$$\mathbf{W}_1, \dots, \mathbf{W}_T \sim \mathit{NID}(\mathbf{0}, \mathbf{I}_n)$$

and denote the drawn variates $\mathbf{W}^{(j)}$, $j = 1, \dots, N$.

3. For each draw j*, conditional on the matrix* $\mathbf{X}$*, the Cholesky factor* $\mathbf{S}_{\widehat{\mathbf{U}}}$ *of the residuals* $\widehat{\mathbf{U}}$ *and the LS estimator* $\widehat{\mathbf{B}}$ *of* $\mathbf{B}$*, construct a bootstrap replication*

$$\mathbf{Y}^{(j)*} = \mathbf{X}\widehat{\mathbf{B}} + \mathbf{W}^{(j)}\mathbf{S}_{\widehat{\mathbf{U}}}, \quad j = 1, \dots, N.$$

Regress $\mathbf{Y}^{(j)*}$ *on* $\mathbf{X}$ *and obtain the associated residual matrix* $\widehat{\mathbf{U}}^{(j)*}$*, covariance matrix* $\widehat{\mathbf{\Omega}}^{(j)*} = T^{-1}\widehat{\mathbf{U}}^{(j)*\prime}\widehat{\mathbf{U}}^{(j)*}$ *and Cholesky factor* $\mathbf{S}^{(j)*}_{\widehat{\mathbf{U}}}$*. Obtain the simulated standardised residuals*

$$\widetilde{\mathbf{W}}^{(j)*} = \widehat{\mathbf{U}}^{(j)*}(\mathbf{S}^{(j)*}_{\widehat{\mathbf{U}}})^{-1} = (\widetilde{\mathbf{w}}^{(j)*}_1, \dots, \widetilde{\mathbf{w}}^{(j)*}_n),$$

where $\widetilde{\mathbf{w}}^{(j)*}_i = (\widetilde{w}^{(j)*}_{i1}, \dots, \widetilde{w}^{(j)*}_{iT})'$, $i = 1, \dots, n$.

4. Compute the bootstrap LM statistic for equation i *and MC draw* j *using (3),* *denoting it* $\widetilde{LM}_i^{(j)*}$*. Compute* $\widetilde{LM}^{(j)*} = 1 - \min_{1\leq i\leq n}(p(\widetilde{LM}_i^{(j)*}))$ *using (4) as in step 1.*

5. Given $\widetilde{LM}^{(j)*}$*,* $j = 1, \ldots, N$*, compute the number of simulated values greater than or equal to* $\widetilde{LM}^{(0)}$*. The bootstrap p-value is*

$$\widehat{p}^*(\widetilde{LM}) = \frac{N\widehat{G}_N(\widetilde{LM}^{(0)}) + 1}{N+1},$$

where

$$\widehat{G}_N(\widetilde{LM}^{(0)}) = \frac{1}{N}\sum_{j=1}^{N} I(\widetilde{LM}^{(j)*} > \widetilde{LM}^{(0)}),$$

and $I(A)$ *is the indicator function taking the value* 1 *if* A *is true and* 0 *otherwise.*

The null hypothesis is rejected at the significance level α if the simulated p-value $\widehat{p}^*(\widetilde{LM}) \leq \alpha$.

The asymptotic validity of $\widetilde{LM}^*$ depends on the ability of the bootstrap to mimic the asymptotic distribution of $\widetilde{LM}$ under the null hypothesis. We make the following additional assumption about the errors.

Assumption 2 *In addition to* $\{\mathbf{u}_t\} \sim IID(\mathbf{0}, \mathbf{\Omega})$*, the errors have finite fourth moments:*

$$E|u_{it}|^4 < \infty, \quad i = 1, \ldots, n.$$

Proposition 1 states the asymptotic validity of the bootstrap combined $\widetilde{LM}^*$ test for ARCH errors in VAR models.

Proposition 1 *Under Assumption 2 and under* H_0*, as* $T \to \infty$*,*

$$\sup_{0<c<\infty} \left| P^*(\widetilde{LM}^* \leq c) - P(\widetilde{LM} \leq c) \right| \to 0,$$

where P^* *denotes the bootstrap probability measure.*

Proof. By Lemma 2 of Gel and Chen (2012),

$$\sup_{0<c<\infty} |\mathrm{P}^*(LM_i^* \leq c) - \mathrm{P}(LM_i \leq c)| \to 0, \quad i = 1, \ldots, n.$$

Gel and Chen prover their result for bootstrap errors obtained by resampling the residuals, but generating the bootstrap errors parametrically as in Algorithm 1 does not alter the asymptotics. The continuity of $f(\cdot) = 1 - \min_{1\leq i\leq n}(p(LM_i))$ in (4) follows from a composition of continuous functions. Finally, the weak convergence result follows by applying the continuous mapping theorem (CMT) to $f(\cdot)$. ■

An issue in finite samples concerns nuisance parameters in the error distribution of $\mathrm{vec}(\mathbf{W}_1, \ldots, \mathbf{W}_T)$. Dufour et al. (2010) use Student-t errors in the distribution of $\mathrm{vec}(\mathbf{W}_1, \ldots, \mathbf{W}_T)$.

Remark 3 *Dufour et al. (2010) extend the MC test technique to the case of nuisance parameters in the distribution of vec*$(\mathbf{W}_1, \ldots, \mathbf{W}_T)$ *by maximising the p-value over the nuisance parameter space (denoted maximised MC procedure, MMC procedure). Dufour (2006) shows that the MMC procedure has level* α *when the regressors in model (10) are exogenous.*

Remark 4 *Proposition 1 also holds if the errors in the distribution of vec*$(\mathbf{W}_1, \ldots, \mathbf{W}_T)$ *are skew-t* $skT(0, 1; \lambda, v)$, $v \geq 5$.

In the simulations in Section 4 and the empirical example of credit default swap (CDS) prices in Section 5.1 we use skew-t $\mathrm{skT}(0, 1; \lambda, v)$ errors in the resampling scheme of the bootstrap tests.

4 Simulations

We conduct Monte Carlo simulations of the size and power of the multivariate LM tests for ARCH in finite samples. The tests considered are the bootstrap combined $\widetilde{LM}^*$ test, asymptotic and bootstrap MLM and MLM^* tests, and asymptotic and bootstrap LM_{CCC} and LM^*_{CCC} tests.

The model for the conditional mean is a VAR(2) model

$$\mathbf{y}_t = \mathbf{\Pi}_1\mathbf{y}_{t-1} + \mathbf{\Pi}_2\mathbf{y}_{t-2} + \mathbf{u}_t, \quad t = 1, \ldots, T,$$

with $\mathbf{\Pi}_1 = \mathrm{diag}(0.5)$ and $\mathbf{\Pi}_2 = \mathrm{diag}(0.3)$. The dimensions are $n = 2$ and 5. The series lengths are $T = 200$, 400 and 800. The number of Monte Carlo replications is 20000 for $n = 2$ and 10000 for $n = 5$. The error distribution of $\mathrm{vec}(\mathbf{W}_1, \ldots, \mathbf{W}_T)$ in the resampling scheme of the bootstrap tests is independent normal $\mathrm{NID}(0, 1)$. The number of replications in the bootstrap tests is $N = 499$.[1] We present the results in the form of p-value discrepancy plots, as recommended by Davidson and MacKinnon (1998). The plots show the empirical rejection probabilities (ERPs) as a function of the nominal level α.

4.1 Size

In the simulations for size, the errors $\{\mathbf{u}_t\}$ are independent normally distributed, $u_{it} \sim \mathrm{NID}(0, 1)$, or independent skew-$t$ distributed, $u_{it} \sim \mathrm{skT}(0, 1; \lambda, v)$, $i = 1, \ldots, n$, where λ is the skewness parameter and v is the degrees of freedom parameter. The value of λ is $\lambda = -0.5$ and the values of v are $v = 12$, 5 and 2. Notice that $v = 5$ is the smallest value of the degrees of freedom for which the errors have finite fourth moments. For example, with the choice of parameters $\lambda = -0.5$ and $v = 5$, the skewness is -1.840 and the excess kurtosis is 12.650. The variance of the errors is not finite when $v = 2$. In the bivariate models ($n = 2$) the errors are correlated with covariance matrix

$$\mathbf{\Omega} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}.$$

In the models with $n = 5$ the covariance matrix has the form of a Toeplitz matrix with elements ρ^i on the ith subdiagonal and superdiagonal.

The bootstrap combined $\widetilde{LM}^*$ test and the bootstrap MLM^* and LM^*_{CCC} tests of orders $h = 2$, 5 and 10 have size close to the nominal level in bivariate models ($n = 2$) with normal errors, as is seen from Figure 1. The asymptotic MLM test is slightly oversized at the nominal significance levels 1% and 5%. The asymptotic LM_{CCC} test is undersized. The size distortions of the asymptotic MLM and LM_{CCC} tests increase with h. The size distortions decrease with the series length T, as is seen by comparing Figure 1 with Figure 2 which shows the results of the tests of

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[1] All estimations and numerical calaculations are done using code written in R, version 2.15.2.

orders $h = 2$ and 5 for $T = 400$, and Figure 3 which shows the results of the tests of orders $h = 2$ and 5 for $T = 800$. In order to save space, in what follows we mainly focus on the results for $T = 200$. The size of the tests does not depend on the correlation ρ, as is seen by comparing the size distortions when $\rho = 0$ in Figure 1 and $\rho = 0.9$ in Figure 4. This result is not surprising since the tests are based on Cholesky-standardised residuals which robustify the tests against error correlation. We also investigated the effect of the correlation on the size of the tests based on non-standardised residuals. The results (not reported) show that the bootstrap combined $\widetilde{LM}^*$ test is slightly oversized and the LM^*_{CCC} test is slightly undersized when $\rho = 0.9$; otherwise the size distortions are smaller when $\rho = 0.9$ than when $\rho = 0$. Figure 5 demonstrates the effect of the dimensions. The size distortions are larger when the dimensions increase to $n = 5$.

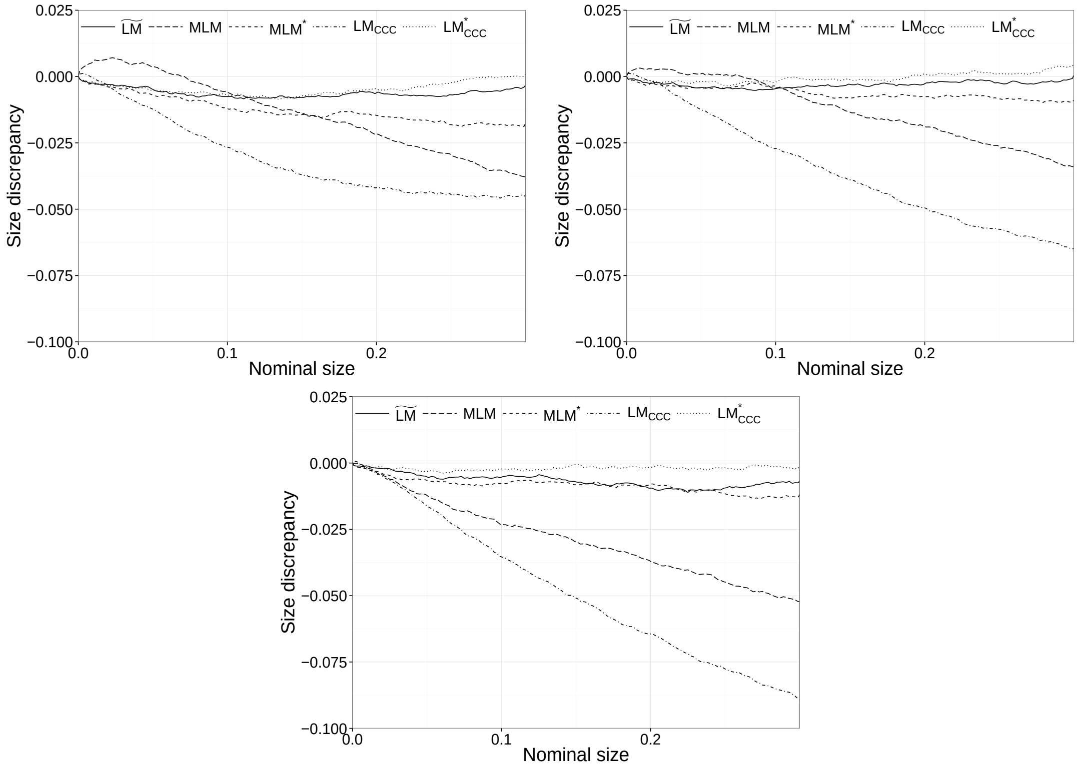

Figure 1: Empirical size discrepancy of the multivariate LM tests for ARCH of orders $h = 2$, 5 and 10 when $n = 2$, $T = 200$ and normal errors with $\rho = 0$.

The size distortions are larger when the errors are skewed and heavy-tailed than when the errors are normal. Figure 6 shows the size discrepancies when the errors are $\mathrm{skT}(0, 1; -0.5, 12)$. The size of the bootstrap combined $\widetilde{LM}^*$ test is close to the nominal level. The simulated size of $\widetilde{LM}^*$ at the nominal 5% level is 4.6% when

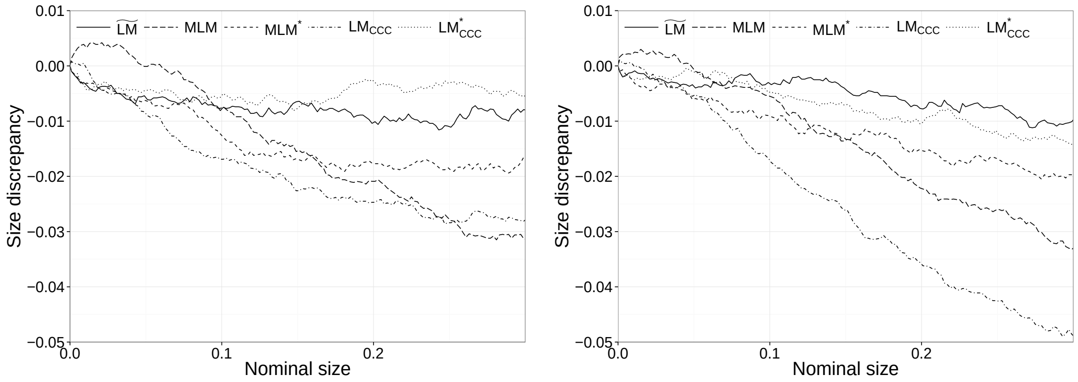

Figure 2: Empirical size discrepancy of the multivariate LM tests for ARCH of orders $h = 2$ and 5 when $n = 2$, $T = 400$ and normal errors with $\rho = 0$.

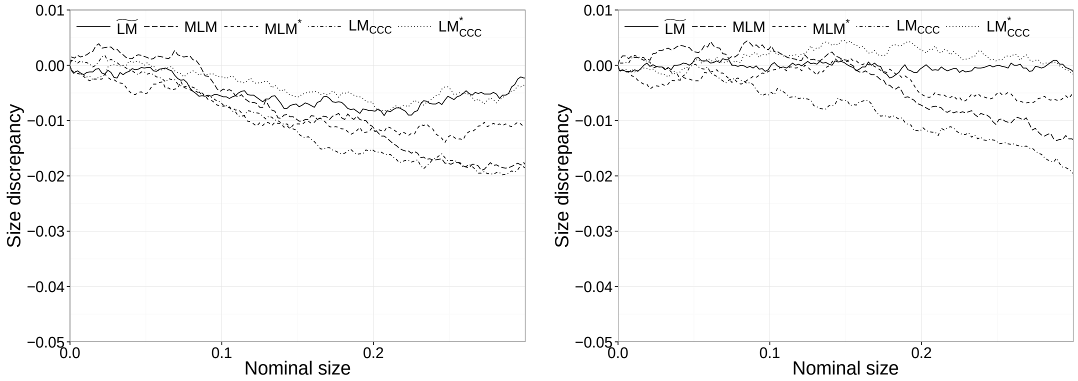

Figure 3: Empirical size discrepancy of the multivariate LM tests for ARCH of orders $h = 2$ and 5 when $n = 2$, $T = 800$ and normal errors with $\rho = 0$.

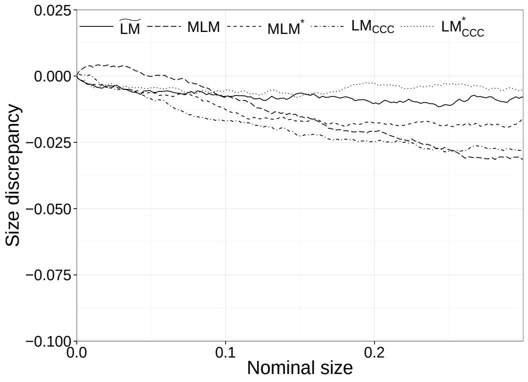

Figure 4: Empirical size discrepancy of the multivariate LM tests for ARCH of order $h = 2$ when $n = 2$, $T = 200$ and normal errors with $\rho = 0.9$.

$T = 200$, 5.1% when $T = 400$ and 5.0% when $T = 800$. The MLM and MLM^* tests are slightly oversized, but the LM_{CCC} and LM^*_{CCC} tests are severely oversized with skew-t errors. The size distortions of MLM and LM_{CCC} increase with T. The simulated size of MLM is 6.7% when $T = 200$, 7.0% when $T = 400$ and 7.0% when $T = 800$. The simulated size of LM_{CCC} is 7.4% when $T = 200$, 9.9% when $T = 400$

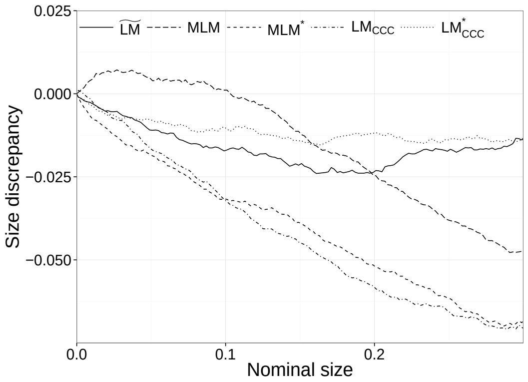

Figure 5: Empirical size discrepancy of the multivariate LM tests for ARCH of order $h = 2$ when $n = 5$, $T = 200$ and normal errors with $\rho = 0$.

and 11.2% when $T = 800$. Bootstrapping the MLM and LM_{CCC} tests does not correct the size distortions. The simulated size of the bootstrap MLM^* test is 5.8% when $T = 200$, 6.2% when $T = 400$ and 6.6% when $T = 800$. The simulated size of the bootstrap LM^*_{CCC} test is 8.9% when $T = 200$, 10.6% when $T = 400$ and 11.5% when $T = 800$. The size distortions are larger when the errors are $\mathrm{skT}(0, 1; -0.5, 5)$ and $\mathrm{skT}(0, 1; -0.5, 2)$; otherwise the results are similar and are therefore not shown.

In principle, skew-t $\mathrm{skT}(0, 1; \lambda, v)$ errors could be used in the resampling scheme of the bootstrap tests (see Remark 4). Table 1 shows the simulated size of the tests at the nominal 5% level when $T = 200$, and the errors are independent normal $\mathrm{NID}(0, 1)$, and skew-t $\mathrm{skT}(0, 1; 0, v)$, $\lambda = -0.5$, $v = 12$, 5 and 2. The error distribution of $\mathrm{vec}(\mathbf{W}_1, \ldots, \mathbf{W}_T)$ in the resampling scheme of the bootstrap tests is independent normal $\mathrm{NID}(0, 1)$ and skew-t $\mathrm{skT}(0, 1; 0, v)$, $v = 12$, 5 and 2. We found that the results do not materially depend on the skewness; the parameter λ is therefore set to zero. The bootstrap combined $\widetilde{LM}^*$ test with normal errors in the resampling scheme is well sized for all values of the degrees of freedom v. The asymptotic MLM and LM_{CCC} tests are oversized. The bootstrap MLM^* and LM^*_{CCC} tests with normal errors in the resampling scheme are also oversized. The bootstrap MLM^* and LM^*_{CCC} tests with skew-t $\mathrm{skT}(0, 1; 0, v)$ errors in the resampling scheme have size close to the nominal level provided the degrees of freedom in the errors and the resampling scheme are the same. This poses a problem in practice because the number of degrees of freedom is unknown and an estimate $\widehat{v}$ must be used in place of the unknown v.

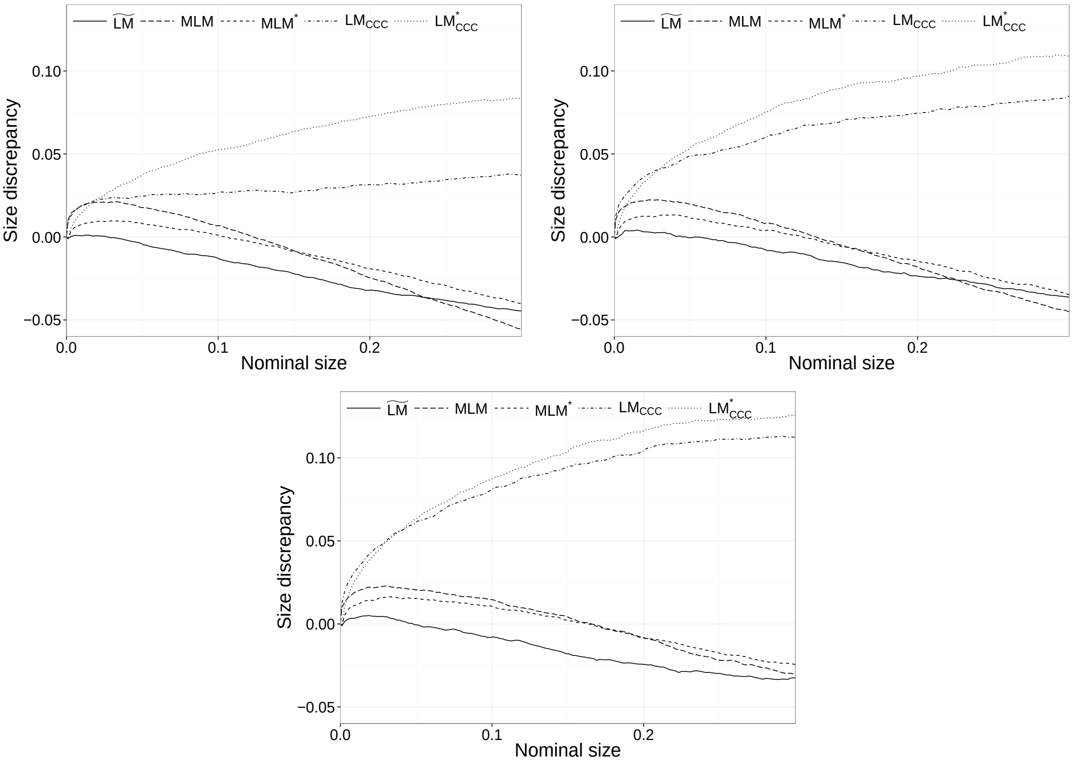

Figure 6: Empirical size discrepancy of the multivariate LM tests for ARCH of order $h = 2$ when $n = 2$, $T = 200$, 400 and 800 and $\mathrm{skT}(0, 1; -0.5, 12)$ errors with $\rho = 0$.

Table 1: Simulated size of the multivariate LM tests for ARCH of order $h = 2$ when $n = 2$ and $T = 200$ with skewed and heavy-tailed errors. The error distribution of $\mathrm{vec}(\mathbf{W}_1, \ldots, \mathbf{W}_T)$ in the resampling scheme of the bootstrap tests is independent normal $\mathrm{NID}(0,1)$ and skew-t $\mathrm{skT}(0,1;0,v)$, $v = 12$, 5 and 2. The nominal level is $\alpha = 0.05$.

Errors	$\widetilde{LM}^*$				MLM	MLM^*				LM_{CCC}	LM^*_{CCC}			
	N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)
$\mathrm{NID}(0,1)$	0.046	0.044	0.031	0.031	0.054	0.044	0.025	0.008	0.000	0.038	0.046	0.017	0.002	0.000
$\mathrm{skT}(0,1;-0.5,12)$	0.046	0.046	0.034	0.036	0.067	0.058	0.042	0.016	0.001	0.074	0.089	0.043	0.010	0.000
$\mathrm{skT}(0,1;-0.5,5)$	0.048	0.049	0.043	0.041	0.092	0.085	0.069	0.036	0.010	0.136	0.154	0.099	0.041	0.004
$\mathrm{skT}(0,1;-0.5,2)$	0.044	0.043	0.040	0.040	0.099	0.095	0.085	0.070	0.033	0.195	0.211	0.158	0.103	0.035

Outliers frequently occur in time series with conditional heteroskedasticity. Following Tsay et al. (2000), we consider additive outliers (AO) and innovational outliers (IO) in bivariate models with outlier parameters $\omega = (3.5, 3.5)'$ and $\omega = (8, 8)'$, respectively. We concentrate on the case where there is a simultaneous outlier in both series at $T = 101$. All tests are oversized in the presence of AO in Figure 7; IO have a larger effect on the size of the bootstrap combined $\widetilde{LM}^*$ test than on the other tests.

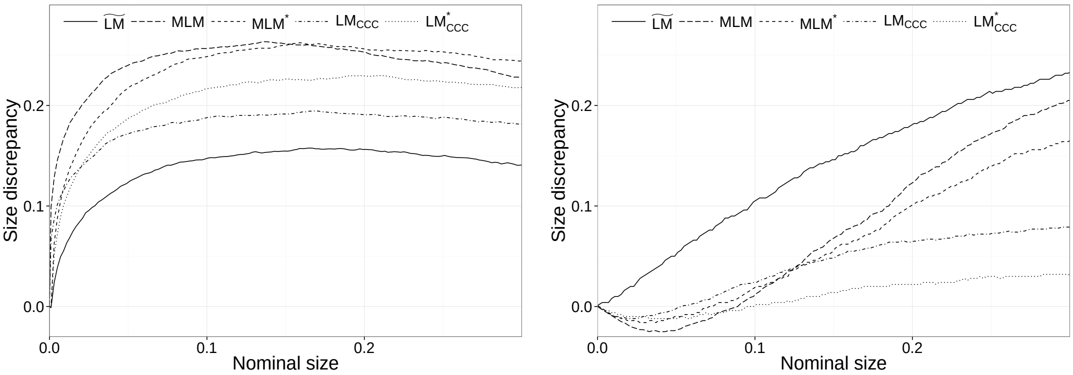

Figure 7: Empirical size discrepancy of the multivariate LM tests for ARCH of order $h = 2$ when $n = 2$, $T = 200$, normal errors with $\rho = 0$ and additive outliers (AO) with outlier parameter $\omega = (3.5, 3.5)'$, and innovational outliers (IO) with outlier parameter $\omega = (8, 8)'$.

The results on size of the tests can be summarised as follows. The bootstrap combined $\widetilde{LM}^*$ test, and the bootstrap MLM^* and LM^*_{CCC} tests have good size properties if the errors are normal. The asymptotic MLM test is slightly oversized at the nominal significance levels 1% and 5%. In contrast, the asymptotic LM_{CCC} test is undersized. The bootstrap combined $\widetilde{LM}^*$ test is robust against a non-normal error distribution. The bootstrap MLM^* and LM^*_{CCC} tests suffer from size distortions with skewed and heavy-tailed errors. The LM tests for ARCH are not robust against outliers.

4.2 Power

In the power simulations $\mathbf{u}_t = \mathbf{H}_t^{1/2}\varepsilon_t$, where $\{\varepsilon_t\}$ is as $\{\mathbf{u}_t\}$ before independent normally distributed, $\varepsilon_{it} \sim \mathrm{NID}(0, 1)$, or independent skew-$t$ distributed, $\varepsilon_{it} \sim \mathrm{skT}(0, 1; \lambda, v)$, $i = 1, \ldots, n$. We conduct power simulations in bivariate models

Table 2: DGPs in the simulations for power.

CCC-GARCH$(1,1)$	ECCC-GARCH$(1,1)$	BEKK
Constant		
$(0.02, 0.02)'$	$(0.02, 0.02)'$	$(0.02, 0.02)'$
ARCH parameters		
$\begin{pmatrix} 0.08 & 0 \\ 0 & 0.08 \end{pmatrix}$	$\begin{pmatrix} 0.08 & 0.001 \\ 0.004 & 0.08 \end{pmatrix}$	$\begin{pmatrix} \sqrt{0.08} & 0 \\ 0 & \sqrt{0.08} \end{pmatrix}$
GARCH parameters		
$\begin{pmatrix} 0.9 & 0 \\ 0 & 0.9 \end{pmatrix}$	$\begin{pmatrix} 0.9 & 0.004 \\ 0.02 & 0.9 \end{pmatrix}$	$\begin{pmatrix} \sqrt{0.9} & 0 \\ 0 & \sqrt{0.9} \end{pmatrix}$
Conditional correlation		
0, 0.25, 0.5, 0.75, 0.9		—

($n = 2$) using CCC-GARCH, extended CCC-GARCH (ECCC-GARCH) and BEKK models to define the data-generation process (DGP) under the alternative. For definitions of the models, see e.g. Silvennoinen and Teräsvirta (2009). The parameters are contained in Table 2. In addition, we consider a CCC-GARCH model with $n = 5$, where the parameters on the diagonals are as in the model with $n = 2$.

Size-power curves against $h = 2$ when $n = 2$, $T = 200$ and normal errors are presented in Figures 8–13. The power functions of the asymptotic and bootstrap tests are indistinguishable from each other because the tests are size-adjusted. The order of power dominance is $LM_{CCC} > \widetilde{LM}^* > MLM$ both when the DGP is a CCC-GARCH model in Figure 8 and when the DGP is an ECCC-GARCH model in Figure 9 with $\rho = 0$. The power dominance is reversed when $\rho = 0.9$: $MLM > LM_{CCC} > \widetilde{LM}^*$. The power of MLM in increasing in ρ, and when ρ is large MLM is more powerful than LM_{CCC}. The tests are about equally powerful when the alternative is a BEKK-GARCH model in Figure 10. The power differences are larger when the DGP is a CCC-GARCH model and the dimensions increase to $n = 5$ in Figure 11. Figure 12 demonstrates the situation when $n = 2$ series follow a CCC-GARCH process and the remaining 3 series are NID$(0, 1)$. The bootstrap combined $\widetilde{LM}^*$ test and the LM_{CCC} test are about equally powerful, but the MLM test suffers a loss in power compared to the situation in Figure 11 where all $n = 5$ series follow a CCC-GARCH process. Size-power curves against $h = 2$ when $n = 2$,

$T = 200$ and skew-t skT$(0, 1; -0.5, 12)$ errors are presented in Figure 13. The tests are equally powerful with normal and skew-t errors.

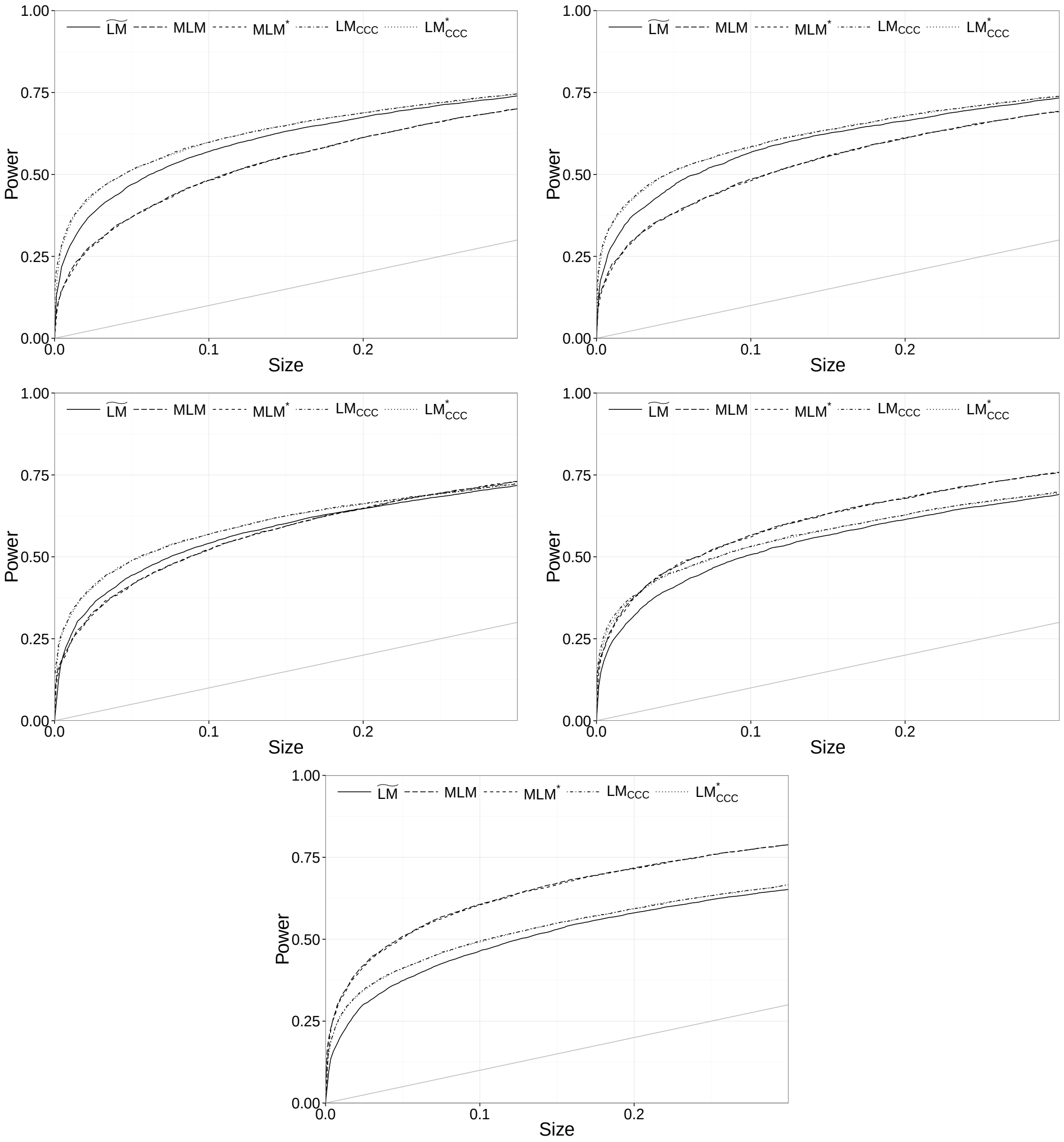

Figure 8: Size-power curves of the multivariate LM tests for ARCH against $h = 2$ when $n = 2$, $T = 200$ and normal errors. The errors are generated from a CCC-GARCH process with conditional correlation $\rho = 0, 0.25, 0.5, 0.75$ and 0.9.

The results from the power simulations can be summarised as follows. The LM_{CCC} test is in general the most powerful test for ARCH followed by the bootstrap combined $\widetilde{LM}^*$ test. The MLM test has lower power than the other multivariate tests except when the errors are highly correlated, in which case MLM is more powerful than $\widetilde{LM}^*$ and LM_{CCC}.

A general conclusion from our size and power simulations is that the bootstrap combined $\widetilde{LM}^*$ test has good size properties both when the errors are normal and

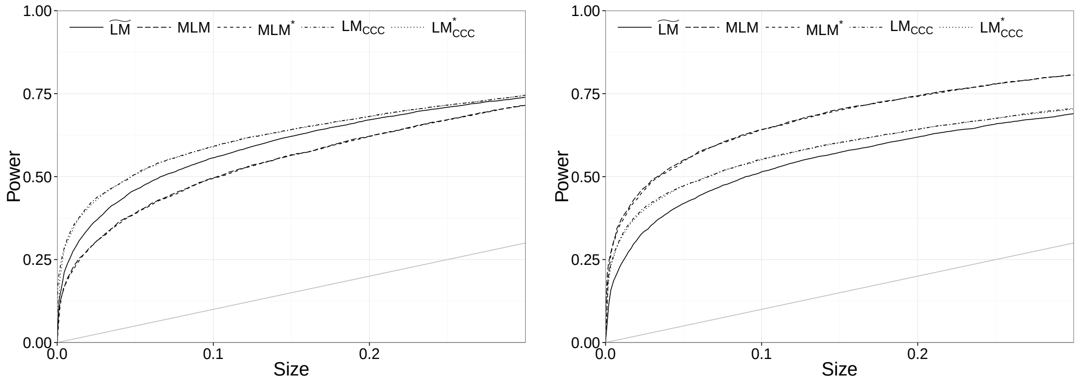

Figure 9: Size-power curves of the multivariate LM tests for ARCH against $h = 2$ when $n = 2$, $T = 200$ and normal errors. The errors are generated from an ECCC-GARCH process with conditional correlation $\rho = 0$ and 0.9.

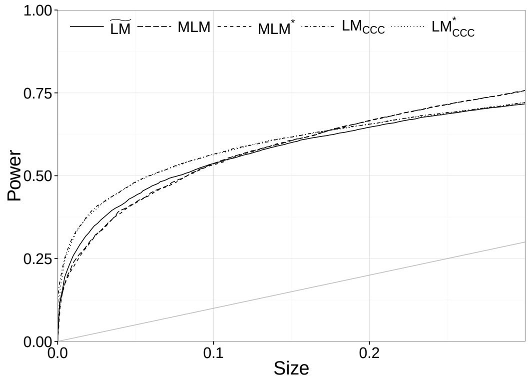

Figure 10: Size-power curve of the multivariate LM tests for ARCH against $h = 2$ when $n = 2$, $T = 200$ and normal errors. The errors are generated from a BEKK-GARCH process.

when the errors are skewed and heavy-tailed. The LM_{CCC} test is the most powerful test for ARCH, but is not robust against a non-normal error distribution. Another general conclusion is that the tests for ARCH are not robust against outliers. If outliers are detected, it may be a good idea to model them before testing for ARCH.

5 Empirical Examples

5.1 Credit Default Swap Prices

In our first empirical example we apply the multivariate LM tests for ARCH to VAR models estimated on credit default swap (CDS) prices data. A CDS is a credit derivative which provides a bondholder with protection against the risk of default by the company. If a default occurs, the holder is compensated for the loss by

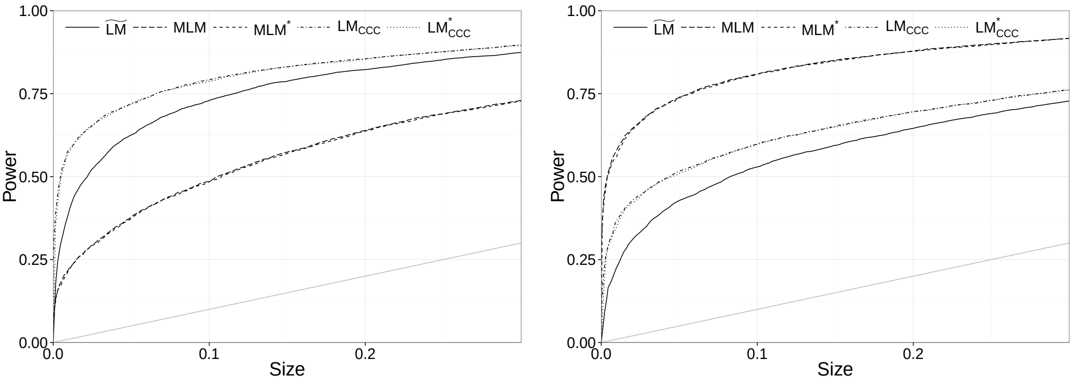

Figure 11: Size-power curves of the multivariate LM tests for ARCH against $h = 2$ when $n = 5$, $T = 200$ and normal errors. The errors are generated from a CCC-GARCH process with conditional correlation $\rho = 0$ and 0.9.

an amount which equals the difference between the par value of the bond and its market value after the default. The CDS price is the annualised fee (expressed as a percentage of the principal) paid by the protection buyer. We denote by p_t^{CDS} the CDS price and p_t^{CS} the credit spread on a risky bond over the risk-free rate. The basis is the difference between the CDS price and bond spread:

$$s_t = p_t^{\text{CDS}} - p_t^{\text{CS}}.$$

If the two markets price credit risk equally in the long run, prices should be equal, so that the basis $s_t = 0$. The vector $\mathbf{y}_t$ with the value 1 appended is $\mathbf{y}_t = (p_t^{\text{CDS}}, p_t^{\text{CS}}, 1)'$. The non-arbitrage relation is tested as an equilibrium relation in a cointegrated VAR model for $\mathbf{y}_t$ (see Blanco et al. 2005, and Ahlgren and Catani 2014).

We take a subsample of the companies in Table 1 of Blanco et al. The companies in our subsample are Bank of America, Citigroup, Goldman Sachs, Barclays Bank and Vodafone, the first three of which are US and the remaining two European companies. We use 5-year maturity CDS prices and credit spreads from Datastream. The data are daily observations from 1 January 2009 to 31 January 2012, and the number of daily observations is $T = 804$. Based on the Schwarz (SC) and Hannan–Quinn (HC) information criteria, we select lag length $p = 2$ for Bank of America, $p = 3$ for Citigroup, Goldman Sachs and Vodafone, and $p = 4$ for Barclays Bank.

Figure 14 shows plots of the standardised residuals $\widetilde{\mathbf{w}}_t$ from the VAR models. We observe that the bond and CDS markets share periods of high volatility. This

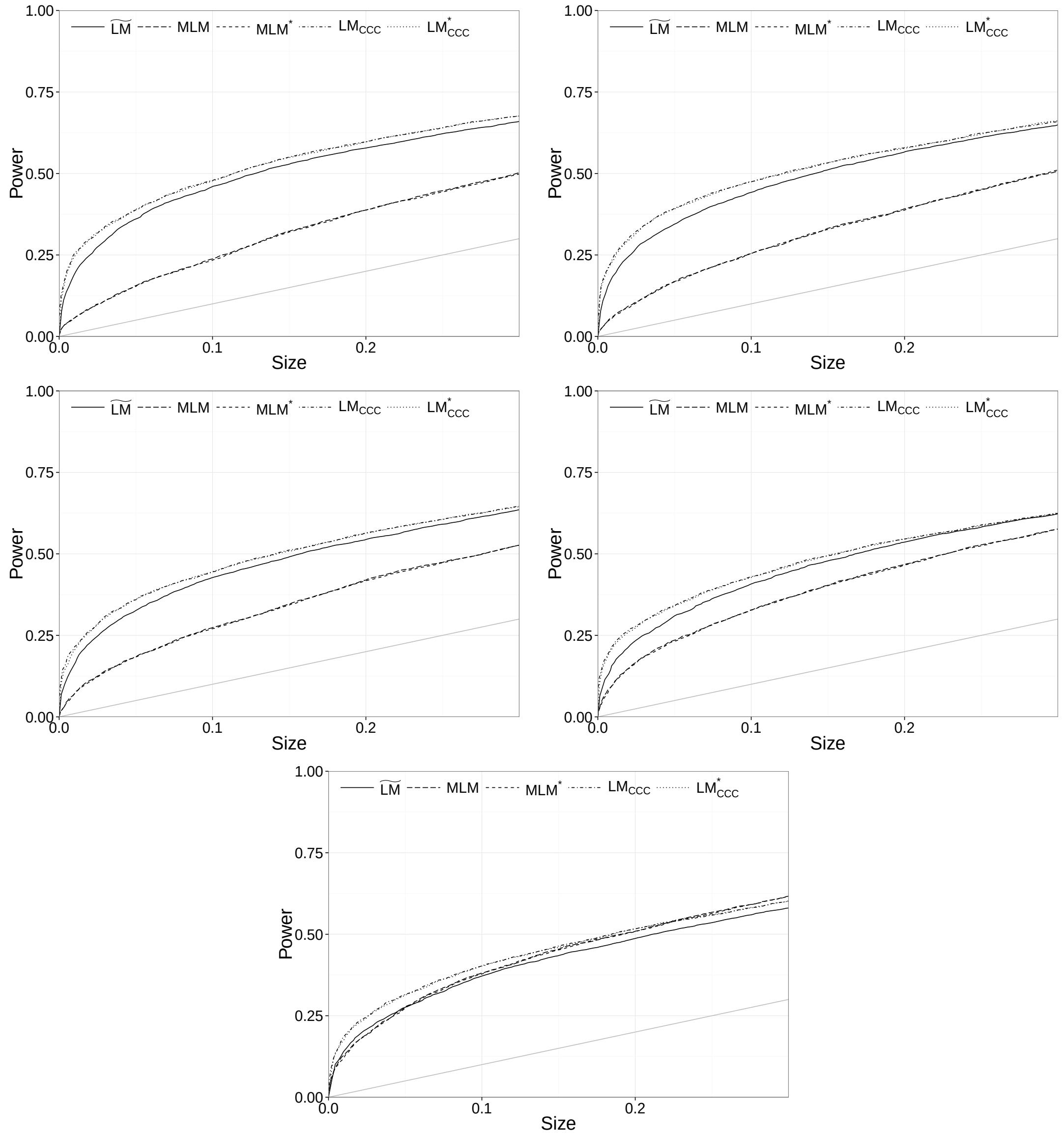

Figure 12: Size-power curves of the multivariate LM tests for ARCH against $h = 2$ when $n = 5$, $T = 200$ and normal errors. The errors are generated from a CCC-GARCH process with $n = 2$, conditional correlation $\rho = 0, 0.25, 0.5, 0.75$ and 0.9, and the remaining 3 series are independent normal $\mathrm{N}(0, 1)$.

suggests that multivariate tests for ARCH will be more powerful than univariate tests. Large outliers is another major feature of the CDS prices data, and there is evidence that large outliers coincide for the two series. For all companies there are some extremely large standardised residuals ($\widetilde{w}_{it} > 8$) in the beginning of the sample period, corresponding to the financial crisis in the first half of 2009. We include dummy variables in the VAR models to account for outliers which are deemed not to belong to high-volatility clusters.[2]

Table 3 reports the parameter estimates from GARCH$(1, 1)$ models with skew-t

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[2] The dummy variables are listed in the notes to Table 4.

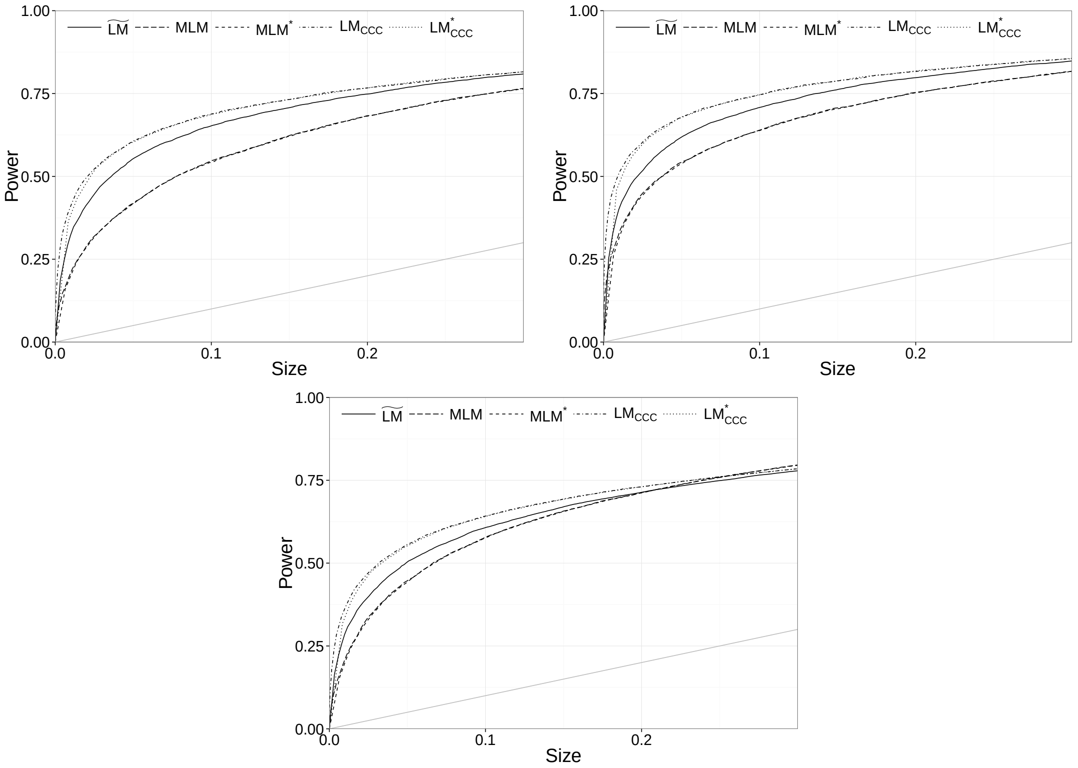

Figure 13: Size-power curves of the multivariate LM tests for ARCH against $h = 2$ when $n = 2$, $T = 200$ and $\mathrm{skT}(0, 1; -0.5, 12)$ errors. The errors are generated from CCC-GARCH, ECCC-GARCH and BEKK-GARCH processes with conditional correlation $\rho = 0$.

$\mathrm{skT}(0, 1; \lambda, v)$ errors fitted to the standardised residuals $\widetilde{\mathbf{w}}_t$ from the VAR models. The skewness parameter $\widehat{\lambda}$ is about 1 for all series. The degrees of freedom parameter $\widehat{v}$ varies between 2 and 4. The estimates support skewed and heavy-tailed errors of the VAR models. If the estimates of degrees of freedom are credible, $\mathrm{skT}(0, 1; \lambda, v)$ instead of $\mathrm{NID}(0, 1)$ errors should be used in the resampling scheme of the bootstrap tests.

The results of LM tests for ARCH of orders $h = 2$, 5 and 10 are reported in Table 4, which shows the p-values of the asymptotic univariate LM_{CDS} and LM_{CS}

Table 3: Parameter estimates from GARCH(1, 1) models with skew-t $\mathrm{skT}(0, 1; \lambda, v)$ errors fitted to the standardised residuals $\widetilde{\mathbf{w}}_t$ from the VAR models for the CDS prices data. The skewness parameter is λ and the degrees of freedom parameter is v.

	Bank of America		Citigroup		Goldman Sachs		Barclays Bank		Vodafone	
	p_t^{CDS}	p_t^{CS}	p_t^{CDS}	p_t^{CS}	p_t^{CDS}	p_t^{CS}	p_t^{CDS}	p_t^{CS}	p_t^{CDS}	p_t^{CS}
$\hat{\lambda}$	1.112	1.032	1.064	1.024	1.072	0.985	1.012	0.934	1.056	0.946
$\hat{v}$	3.455	2.147	3.632	2.361	2.908	2.148	4.314	2.401	2.079	3.215

tests, bootstrap combined $\widetilde{LM}^*$ test, asymptotic and bootstrap MLM and MLM^* tests, and asymptotic and bootstrap LM_{CCC} and LM_{CCC}^* tests. In addition to the full sample period of $T = 804$ observations, we divide the data into 2 sub-periods of $T = 402$ observations and 4 sub-periods of $T = 201$ observations. The error distribution of $\text{vec}(\mathbf{W}_1, \ldots, \mathbf{W}_T)$ in the resampling scheme of the bootstrap tests is independent normal $\text{NID}(0, 1)$ and skew-t $\text{skT}(0, 1; 0, v)$, $v = 12$, 5 and 2. We found that the results do not materially depend on the skewness; the parameter λ is therefore set to zero. In the resampling scheme of the bootstrap tests we use $N = 999$. Hence, the smallest possible p-value of the bootstrap tests equals 0.001. The results for the full sample period and normal errors in the resampling scheme of the bootstrap tests show that all multivariate tests are significant at the 1% level. In fact, almost all p-values are 0.000 or 0.001. The p-values of the bootstrap tests with $\text{skT}(0, 1; 0, v)$ errors in the resampling scheme are larger than the p-values with $\text{NID}(0, 1)$ errors. Focusing on the results with $v = 5$, which is the smallest value of the degrees of freedom for which the errors have finite fourth moments, we find that all tests are significant at the 5% level, and all tests are significant at the 1% level except $\widetilde{LM}^*$ of $h = 2$ for Bank of America. In the first sub-period of $T = 402$ observations, the bootstrap tests with normal errors are significant at the 5% and 1% levels, with the exceptions of MLM^* of $h = 5$ and 10 for Citigroup, and $h = 5$ for Goldman Sachs. The p-values of the bootstrap tests with $\text{skT}(0, 1; 0, 5)$ errors are larger than the p-values with $\text{NID}(0, 1)$ errors. We find that all tests are significant at the 5% level, with the exception of $\widetilde{LM}^*$ of $h = 10$ for Citigroup and MLM^* of $h = 5$ for Goldman Sachs. In the second sub-period, the bootstrap tests are significant at the 5% level, with the exceptions of $\widetilde{LM}^*$ of $h = 2$ for Bank of America and $h = 5$ for Vodafone, and MLM^* of $h = 2$, 5 and 10 for Vodafone. The rejection of the null hypothesis of no ARCH effect for all companies in the full sample, and all companies in the sub-periods except Vodafone in sub-period 2 suggest that the p-values represent power, and that the power is close to 1 for the series lengths $T = 804$ and 402.

The sub-periods of $T = 201$ observations reveal some interesting differences in the outcomes of the tests. The bootstrap combined $\widetilde{LM}^*$ test with normal errors in the resampling scheme detects ARCH effects in about half the cases. The MLM and MLM^*, and LM_{CCC} and LM^*_{CCC} tests find more evidence for ARCH. Taking Bank of America as an example, and if we use the significance level 5%, $\widetilde{LM}^*$ rejects $h = 5$ and 10 in sub-period 1, $h = 2$, 5 and 10 in sub-period 2, $h = 2$ in sub-period 3 and no value of h in sub-period 4. The MLM and MLM^* tests reject $h = 5$ and 10 in sub-period 1, and $h = 2$, 5 and 10 in sub-periods 2, 3 and 4. The LM_{CCC} and LM^*_{CCC} tests reject all values of h in all sub-periods.

It is also interesting to compare the outcomes of the bootstrap tests with $\mathrm{NID}(0,1)$ and $\mathrm{skT}(0,1;0,v)$ errors in the resampling scheme. The results for the bootstrap combined $\widetilde{LM}^*$ test reveal that in only few cases do the tests lead to conflicting outcomes. Continuing with the example of Bank of America, the p-values of the test of $h = 2$ with $\mathrm{NID}(0,1)$ errors and $\mathrm{skT}(0,1;0,v)$ errors with $v = 12$, 5 and 2 in sub-period 1 are 0.357, 0.325, 0.235 and 0.125, and in sub-period 3 are 0.020, 0.027, 0.039 and 0.031. Thus inferences about ARCH based on the bootstrap combined $\widetilde{LM}^*$ test do not depend on the errors in the resampling scheme. Conflicting outcomes are recorded for the bootstrap MLM^* and LM^*_{CCC} tests. The p-values of the former in sub-period 1 are 0.113, 0.125, 0.128 and 0.144, and in sub-period 3 are 0.009, 0.031, 0.065 and 0.098. The p-values of latter in sub-period 1 are 0.018, 0.046, 0.092 and 0.170, and in sub-period 3 are 0.001, 0.001, 0.006 and 0.035. Consequently, a very different picture of the evidence for ARCH may emerge from the bootstrap MLM^* and LM^*_{CCC} tests depending on the errors in the resampling scheme.

The fact that the standardised residuals $\widetilde{\mathbf{w}}_t$ from the VAR models are skewed and heavy-tailed should be taken into account when interpreting the results of the bootstrap tests with $\mathrm{NID}(0,1)$ errors in the resampling scheme. Our simulations in Section 4 show that the bootstrap combined $\widetilde{LM}^*$ test is robust against a non-normal error distribution, whereas the asymptotic and bootstrap MLM and MLM^* tests, and LM_{CCC} and LM^*_{CCC} tests in particular are oversized. The findings of more rejections for MLM^* and LM^*_{CCC} with normal errors in the resampling scheme should be qualified. For the degrees of freedom in Table 3, the empirical size of

MLM and MLM^* with NID$(0,1)$ errors in the resampling scheme is estimated to be at least 9%, and LM_{CCC} and LM^*_{CCC} between 15% and 20%. Fewer rejections are recorded for the bootstrap MLM^* and LM^*_{CCC} tests with skT$(0,1;0,5)$ errors in the resampling scheme.

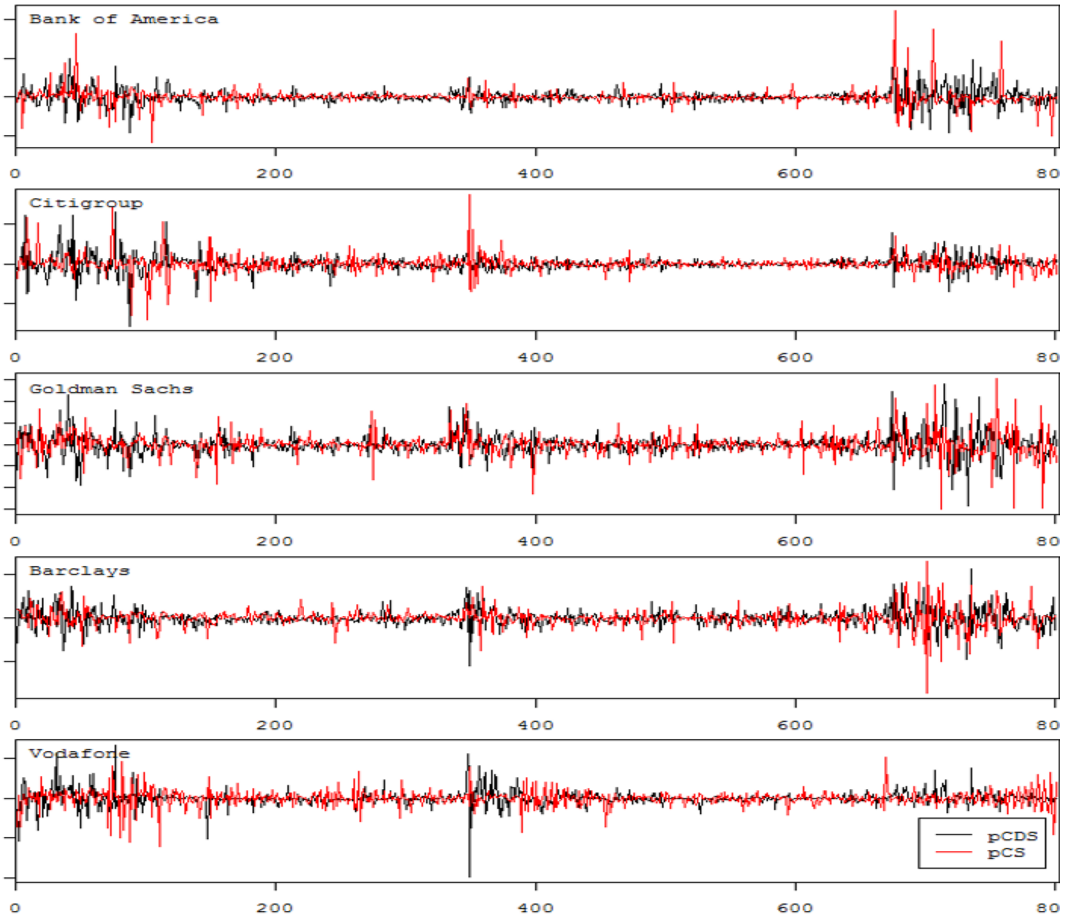

Figure 14: Standardised residuals $\widetilde{\mathbf{w}}_t$ from the VAR models for the CDS prices data.

Table 4: LM tests for ARCH in the VAR models for the CDS prices data. The number of observations is $T = 804$ in the full sample, $T = 402$ in the 2 sub-periods and $T = 201$ in the 4 sub-periods. The error distribution of $\text{vec}(\mathbf{W}_1, \dots, \mathbf{W}_T)$ in the resampling scheme of the bootstrap tests is independent normal NID$(0, 1)$ and skew-*t* skT$(0, 1; 0, v)$, $v = 12$, 5 and 2. The table reports the *p*-values of the tests. Notes: The lag length of the VAR model is $p = 2$ for Bank of America, $p = 3$ for Citigroup, $p = 3$ for Goldman Sachs, $p = 4$ for Barclays Bank and $p = 3$ for Vodafone. The VAR models contain dummy variables taking the value 1 for the date in question and 0 otherwise: 25 February 2009, 10 April 2009 and 8 June 2009 for Citigroup, 9 April 2009 for Goldman Sachs, 6 February 2009 and 4 June 2009 for Barcalys Bank, and 8 June 2009 for Vodafone.

h	LM_{CDS}	LM_{CS}	$\widetilde{LM}^*$				MLM	MLM^*				LM_{CCC}	LM^*_{CCC}			
			N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)
$T = 804$																
Bank of America																
2	0.000	0.004	0.002	0.008	0.023	0.027	0.000	0.001	0.001	0.001	0.005	0.000	0.001	0.001	0.002	0.023
5	0.000	0.028	0.001	0.001	0.004	0.012	0.000	0.001	0.001	0.001	0.013	0.000	0.001	0.001	0.001	0.037
10	0.000	0.001	0.001	0.001	0.002	0.016	0.000	0.001	0.001	0.001	0.011	0.000	0.001	0.001	0.001	0.039
Citigroup																
2	0.000	0.001	0.001	0.001	0.005	0.008	0.000	0.001	0.001	0.003	0.017	0.000	0.001	0.001	0.001	0.020
5	0.000	0.002	0.001	0.001	0.006	0.015	0.000	0.001	0.001	0.004	0.019	0.000	0.001	0.001	0.001	0.035
10	0.000	0.007	0.001	0.002	0.007	0.029	0.000	0.001	0.001	0.002	0.044	0.000	0.001	0.001	0.002	0.073
Goldman Sachs																
2	0.000	0.000	0.001	0.001	0.004	0.003	0.000	0.001	0.001	0.007	0.020	0.000	0.001	0.001	0.001	0.019
5	0.000	0.000	0.001	0.002	0.004	0.017	0.000	0.001	0.001	0.008	0.027	0.000	0.001	0.001	0.002	0.044
10	0.000	0.000	0.001	0.001	0.002	0.011	0.000	0.001	0.001	0.004	0.036	0.000	0.001	0.001	0.001	0.048
Barclays Bank																
2	0.000	0.000	0.001	0.001	0.001	0.007	0.000	0.001	0.001	0.001	0.008	0.000	0.001	0.001	0.001	0.010
5	0.000	0.000	0.001	0.001	0.004	0.007	0.000	0.001	0.001	0.001	0.007	0.000	0.001	0.001	0.001	0.015
10	0.000	0.000	0.001	0.001	0.002	0.010	0.000	0.001	0.001	0.001	0.009	0.000	0.001	0.001	0.001	0.023
Vodafone																
2	0.000	0.000	0.001	0.001	0.002	0.011	0.000	0.001	0.001	0.006	0.021	0.000	0.001	0.001	0.001	0.019
5	0.000	0.000	0.001	0.001	0.002	0.018	0.000	0.001	0.001	0.006	0.039	0.000	0.001	0.001	0.001	0.031
10	0.000	0.000	0.001	0.001	0.001	0.012	0.000	0.001	0.001	0.006	0.041	0.000	0.001	0.001	0.001	0.051

h	LM_{CDS}	LM_{CS}	$\widetilde{LM}^*$				MLM	MLM^*				LM_{CCC}	LM^*_{CCC}			
			N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)
$T = 402$, Sub-period 1																
Bank of America																
2	0.001	0.239	0.003	0.011	0.023	0.027	0.000	0.001	0.002	0.014	0.055	0.000	0.001	0.001	0.009	0.050
5	0.000	0.225	0.001	0.001	0.004	0.009	0.000	0.001	0.001	0.002	0.023	0.000	0.001	0.001	0.005	0.046
10	0.000	0.002	0.001	0.001	0.001	0.011	0.000	0.001	0.001	0.002	0.026	0.000	0.001	0.001	0.001	0.065
Citigroup																
2	0.000	0.066	0.003	0.006	0.019	0.032	0.000	0.004	0.001	0.005	0.030	0.000	0.001	0.001	0.004	0.046
5	0.002	0.222	0.001	0.017	0.036	0.054	0.000	0.010	0.002	0.012	0.066	0.000	0.001	0.002	0.007	0.080
10	0.009	0.459	0.001	0.043	0.059	0.110	0.000	0.030	0.003	0.020	0.126	0.000	0.002	0.001	0.012	0.166
Goldman Sachs																
2	0.308	0.002	0.008	0.015	0.019	0.031	0.003	0.007	0.021	0.049	0.083	0.000	0.001	0.003	0.010	0.070
5	0.002	0.009	0.003	0.011	0.029	0.052	0.008	0.013	0.038	0.091	0.139	0.000	0.001	0.001	0.008	0.118
10	0.000	0.081	0.001	0.001	0.001	0.012	0.000	0.001	0.002	0.021	0.099	0.000	0.001	0.001	0.002	0.111
Barclays Bank																
2	0.000	0.000	0.001	0.001	0.001	0.002	0.000	0.001	0.001	0.001	0.008	0.000	0.001	0.001	0.001	0.008
5	0.000	0.000	0.001	0.001	0.001	0.004	0.000	0.001	0.001	0.004	0.036	0.000	0.001	0.001	0.001	0.033
10	0.000	0.000	0.001	0.001	0.001	0.004	0.000	0.001	0.001	0.001	0.025	0.000	0.001	0.001	0.002	0.055
Vodafone																
2	0.000	0.000	0.002	0.005	0.010	0.019	0.000	0.001	0.003	0.015	0.049	0.000	0.001	0.001	0.004	0.022
5	0.001	0.000	0.003	0.008	0.020	0.037	0.000	0.001	0.006	0.023	0.081	0.000	0.001	0.001	0.003	0.067
10	0.006	0.000	0.001	0.003	0.005	0.028	0.000	0.001	0.004	0.031	0.114	0.000	0.001	0.001	0.006	0.087

h	LM_{CDS}	LM_{CS}	$\widetilde{LM}^*$				MLM	MLM^*				LM_{CCC}	LM^*_{CCC}			
			N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)
$T = 402$, Sub-period 2																
Bank of America																
2	0.168	0.112	0.196	0.176	0.155	0.096	0.000	0.001	0.001	0.001	0.013	0.000	0.001	0.001	0.005	0.053
5	0.000	0.385	0.001	0.006	0.015	0.024	0.000	0.001	0.001	0.001	0.014	0.000	0.001	0.001	0.003	0.044
10	0.000	0.101	0.001	0.001	0.007	0.018	0.000	0.001	0.001	0.001	0.008	0.000	0.001	0.001	0.002	0.040
Citigroup																
2	0.000	0.040	0.001	0.003	0.011	0.026	0.000	0.001	0.001	0.006	0.034	0.000	0.001	0.002	0.003	0.041
5	0.000	0.019	0.001	0.001	0.002	0.011	0.000	0.001	0.001	0.002	0.047	0.000	0.001	0.001	0.001	0.048
10	0.000	0.009	0.001	0.001	0.001	0.016	0.000	0.001	0.001	0.004	0.050	0.000	0.001	0.001	0.001	0.071
Goldman Sachs																
2	0.000	0.000	0.001	0.001	0.002	0.011	0.000	0.001	0.001	0.003	0.035	0.000	0.001	0.001	0.001	0.026
5	0.000	0.000	0.001	0.003	0.004	0.022	0.000	0.001	0.001	0.004	0.031	0.000	0.001	0.001	0.001	0.043
10	0.000	0.000	0.001	0.001	0.001	0.010	0.000	0.001	0.001	0.002	0.023	0.000	0.001	0.001	0.001	0.047
Barclays Bank																
2	0.295	0.000	0.001	0.002	0.004	0.009	0.000	0.001	0.001	0.002	0.004	0.000	0.001	0.001	0.001	0.018
5	0.000	0.000	0.001	0.001	0.002	0.008	0.000	0.001	0.001	0.001	0.010	0.000	0.001	0.001	0.001	0.026
10	0.000	0.000	0.001	0.001	0.001	0.015	0.000	0.001	0.001	0.001	0.002	0.000	0.001	0.001	0.001	0.035
Vodafone																
2	0.072	0.012	0.029	0.037	0.047	0.037	0.084	0.097	0.108	0.141	0.107	0.000	0.001	0.001	0.009	0.055
5	0.094	0.049	0.100	0.114	0.108	0.106	0.543	0.503	0.461	0.371	0.248	0.000	0.001	0.002	0.013	0.111
10	0.002	0.141	0.006	0.024	0.046	0.083	0.096	0.070	0.121	0.167	0.204	0.000	0.001	0.001	0.011	0.150

h	LM_{CDS}	LM_{CS}	$\widetilde{LM}^*$				MLM	MLM^*				LM_{CCC}	LM^*_{CCC}			
			N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)
$T = 201$, Sub-period 1																
Bank of America																
2	0.227	0.621	0.357	0.325	0.235	0.125	0.098	0.113	0.125	0.128	0.144	0.015	0.018	0.046	0.092	0.170
5	0.001	0.715	0.005	0.009	0.018	0.046	0.000	0.001	0.001	0.008	0.066	0.000	0.001	0.002	0.025	0.142
10	0.002	0.251	0.008	0.007	0.029	0.067	0.000	0.001	0.001	0.008	0.084	0.000	0.001	0.001	0.010	0.158
Citigroup																
2	0.135	0.841	0.222	0.205	0.171	0.089	0.000	0.002	0.005	0.021	0.049	0.004	0.013	0.023	0.082	0.137
5	0.411	0.928	0.604	0.558	0.441	0.250	0.012	0.018	0.031	0.083	0.158	0.005	0.010	0.035	0.151	0.319
10	0.792	0.943	0.954	0.918	0.774	0.433	0.532	0.480	0.426	0.371	0.313	0.003	0.004	0.044	0.224	0.449
Goldman Sachs																
2	0.992	0.068	0.111	0.125	0.115	0.086	0.573	0.555	0.449	0.377	0.235	0.004	0.008	0.030	0.087	0.143
5	0.152	0.101	0.154	0.183	0.150	0.117	0.497	0.458	0.450	0.353	0.253	0.000	0.001	0.015	0.062	0.211
10	0.000	0.331	0.001	0.001	0.005	0.021	0.030	0.029	0.058	0.118	0.204	0.000	0.001	0.001	0.019	0.164
Barclays Bank																
2	0.000	0.000	0.001	0.001	0.006	0.013	0.000	0.001	0.003	0.013	0.036	0.000	0.001	0.001	0.003	0.026
5	0.001	0.001	0.005	0.007	0.020	0.033	0.000	0.001	0.005	0.027	0.072	0.000	0.001	0.001	0.008	0.057
10	0.000	0.010	0.001	0.002	0.006	0.016	0.000	0.001	0.001	0.004	0.053	0.000	0.001	0.001	0.007	0.109
Vodafone																
2	0.891	0.002	0.006	0.015	0.027	0.020	0.007	0.016	0.039	0.062	0.104	0.000	0.001	0.005	0.018	0.068
5	0.611	0.009	0.028	0.023	0.045	0.059	0.000	0.002	0.008	0.035	0.105	0.000	0.001	0.005	0.030	0.157
10	0.847	0.000	0.001	0.004	0.011	0.036	0.000	0.005	0.007	0.049	0.123	0.000	0.001	0.002	0.020	0.159

h	LM_{CDS}	LM_{CS}	$\widetilde{LM}^*$				MLM	MLM^*				LM_{CCC}	LM^*_{CCC}			
			N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)
$T = 201$, Sub-period 2																
Bank of America																
2	0.000	0.044	0.001	0.001	0.003	0.008	0.000	0.001	0.001	0.009	0.040	0.000	0.001	0.001	0.006	0.031
5	0.000	0.007	0.001	0.004	0.009	0.016	0.000	0.001	0.001	0.003	0.024	0.000	0.001	0.002	0.005	0.054
10	0.000	0.085	0.001	0.001	0.001	0.012	0.000	0.001	0.001	0.003	0.008	0.000	0.001	0.001	0.007	0.084
Citigroup																
2	0.333	0.737	0.551	0.446	0.357	0.160	0.014	0.033	0.039	0.063	0.101	0.020	0.022	0.041	0.099	0.189
5	0.103	0.803	0.167	0.174	0.139	0.121	0.004	0.009	0.020	0.055	0.116	0.000	0.001	0.009	0.027	0.164
10	0.280	0.980	0.427	0.406	0.348	0.241	0.000	0.001	0.002	0.021	0.094	0.000	0.001	0.012	0.081	0.339
Goldman Sachs																
2	0.053	0.003	0.010	0.007	0.020	0.028	0.000	0.001	0.001	0.014	0.056	0.000	0.001	0.001	0.004	0.034
5	0.001	0.037	0.001	0.007	0.016	0.053	0.000	0.001	0.001	0.005	0.064	0.000	0.001	0.001	0.003	0.081
10	0.000	0.221	0.002	0.006	0.010	0.033	0.000	0.001	0.002	0.011	0.053	0.000	0.001	0.001	0.005	0.108
Barclays Bank																
2	0.000	0.000	0.001	0.001	0.001	0.002	0.000	0.001	0.001	0.001	0.002	0.000	0.001	0.001	0.001	0.005
5	0.000	0.000	0.001	0.001	0.001	0.006	0.000	0.001	0.001	0.001	0.003	0.000	0.001	0.001	0.001	0.011
10	0.000	0.001	0.001	0.001	0.001	0.007	0.000	0.001	0.001	0.001	0.002	0.000	0.001	0.001	0.001	0.021
Vodafone																
2	0.000	0.000	0.001	0.001	0.002	0.004	0.000	0.000	0.001	0.001	0.007	0.000	0.001	0.001	0.001	0.006
5	0.000	0.000	0.001	0.002	0.004	0.013	0.000	0.001	0.001	0.004	0.029	0.000	0.001	0.001	0.001	0.011
10	0.000	0.008	0.001	0.003	0.010	0.041	0.000	0.001	0.001	0.002	0.036	0.000	0.001	0.001	0.001	0.032

h	LM_{CDS}	LM_{CS}	$\widetilde{LM}^*$				MLM	MLM^*				LM_{CCC}	LM^*_{CCC}			
			N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)
$T = 201$, Sub-period 3																
Bank of America																
2	0.119	0.013	0.020	0.027	0.039	0.031	0.005	0.009	0.031	0.065	0.098	0.000	0.001	0.001	0.006	0.035
5	0.148	0.065	0.100	0.109	0.111	0.101	0.020	0.032	0.043	0.102	0.166	0.000	0.001	0.002	0.008	0.090
10	0.431	0.342	0.566	0.502	0.367	0.251	0.020	0.021	0.045	0.100	0.194	0.000	0.001	0.002	0.027	0.227
Citigroup																
2	0.103	0.015	0.031	0.034	0.055	0.056	0.005	0.014	0.027	0.060	0.107	0.000	0.001	0.005	0.021	0.094
5	0.340	0.036	0.064	0.075	0.082	0.095	0.049	0.044	0.066	0.114	0.182	0.000	0.001	0.007	0.039	0.207
10	0.029	0.237	0.057	0.067	0.089	0.110	0.013	0.010	0.030	0.076	0.185	0.000	0.001	0.010	0.084	0.318
Goldman Sachs																
2	0.136	0.567	0.241	0.187	0.153	0.086	0.025	0.035	0.054	0.100	0.113	0.063	0.050	0.099	0.185	0.231
5	0.386	0.379	0.612	0.533	0.416	0.238	0.093	0.100	0.136	0.160	0.187	0.023	0.033	0.087	0.206	0.373
10	0.131	0.015	0.026	0.042	0.066	0.089	0.156	0.121	0.162	0.192	0.253	0.000	0.001	0.010	0.076	0.313
Barclays Bank																
2	0.746	0.008	0.015	0.025	0.033	0.044	0.039	0.052	0.073	0.092	0.127	0.000	0.001	0.003	0.020	0.084
5	0.724	0.070	0.120	0.142	0.132	0.094	0.630	0.586	0.484	0.418	0.277	0.000	0.001	0.016	0.062	0.217
10	0.959	0.369	0.575	0.496	0.446	0.270	0.522	0.467	0.449	0.367	0.321	0.008	0.007	0.063	0.247	0.512
Vodafone																
2	0.093	0.292	0.157	0.158	0.122	0.096	0.563	0.507	0.487	0.346	0.254	0.003	0.004	0.026	0.069	0.154
5	0.404	0.258	0.421	0.388	0.299	0.186	0.373	0.356	0.288	0.281	0.257	0.002	0.001	0.020	0.111	0.268
10	0.471	0.708	0.689	0.618	0.518	0.289	0.591	0.565	0.491	0.422	0.325	0.013	0.015	0.083	0.289	0.527

h	LM_{CDS}	LM_{CS}	$\widetilde{LM}^*$				MLM	MLM^*				LM_{CCC}	LM^*_{CCC}			
			N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)		N	skT(12)	skT(5)	skT(2)
							$T = 201$, Sub-period 4									
							Bank of America									
2	0.955	0.475	0.711	0.678	0.539	0.232	0.000	0.001	0.001	0.002	0.024	0.002	0.004	0.017	0.070	0.153
5	0.264	0.882	0.438	0.383	0.295	0.180	0.000	0.001	0.001	0.009	0.050	0.000	0.001	0.012	0.059	0.195
10	0.027	0.804	0.061	0.064	0.072	0.094	0.000	0.001	0.001	0.006	0.052	0.000	0.001	0.003	0.010	0.139
							Citigroup									
2	0.016	0.743	0.043	0.038	0.054	0.068	0.001	0.005	0.017	0.056	0.093	0.000	0.002	0.003	0.033	0.114
5	0.001	0.649	0.008	0.008	0.023	0.033	0.001	0.008	0.019	0.048	0.122	0.000	0.001	0.002	0.022	0.114
10	0.001	0.517	0.002	0.007	0.014	0.052	0.001	0.001	0.005	0.042	0.136	0.000	0.001	0.001	0.015	0.173
							Goldman Sachs									
2	0.012	0.136	0.025	0.031	0.033	0.043	0.004	0.017	0.015	0.058	0.109	0.000	0.002	0.004	0.016	0.082
5	0.086	0.122	0.160	0.151	0.134	0.132	0.002	0.002	0.022	0.064	0.146	0.000	0.001	0.003	0.029	0.176
10	0.004	0.439	0.009	0.013	0.033	0.078	0.000	0.001	0.006	0.033	0.116	0.000	0.001	0.002	0.018	0.202
							Barclays Bank									
2	0.995	0.009	0.014	0.023	0.030	0.042	0.000	0.001	0.001	0.002	0.019	0.000	0.001	0.001	0.012	0.063
5	0.000	0.024	0.001	0.001	0.005	0.031	0.000	0.001	0.001	0.001	0.021	0.000	0.001	0.001	0.003	0.071
10	0.001	0.009	0.004	0.007	0.022	0.055	0.000	0.001	0.001	0.001	0.019	0.000	0.001	0.001	0.003	0.095
							Vodafone									
2	0.786	0.079	0.123	0.129	0.108	0.090	0.824	0.778	0.715	0.520	0.270	0.000	0.001	0.005	0.025	0.095
5	0.773	0.324	0.544	0.440	0.367	0.193	0.999	0.999	0.989	0.903	0.550	0.000	0.001	0.008	0.048	0.168
10	0.344	0.715	0.553	0.492	0.394	0.224	0.965	0.959	0.891	0.725	0.437	0.000	0.001	0.001	0.016	0.181

5.2 Euribor Interest Rates

Our second empirical example uses Euribor interest rates data (see e.g. Ahlgren and Antell 2013). The data consist of $T = 172$ monthly observations from December 1998 to March 2013 on the 1, 3, 6, 9 and 12 month Euribor interest rates. All interest rates are nominal and annualised. The Data were retrieved from `www.euribor-ebf.eu`. We fit a VAR model with lag length $p = 3$ to the interest rates data. Figure 15 graphs correlograms of the squares and cross products of the standardised residuals $\widetilde{\mathbf{w}}_t$. The correlograms show significant correlations in the squares and cross products. The significant cross correlations suggest that multivariate tests for ARCH will be more powerful than univariate tests.

The results of LM tests for ARCH of orders $h = 2$ and 12 are reported in Table 5, which shows the p-values of the asymptotic univariate LM tests, bootstrap combined $\widetilde{LM}^*$ test, asymptotic and bootstrap MLM and MLM^* tests, and asymptotic and bootstrap LM_{CCC} and LM^*_{CCC} tests. The error distribution of $\mathrm{vec}(\mathbf{W}_1, \ldots, \mathbf{W}_T)$ in the resampling scheme of the bootstrap tests is independent normal $\mathrm{NID}(0, 1)$. In the resampling scheme of the bootstrap tests we use $N = 999$. Hence, the smallest possible p-value of the bootstrap tests equals 0.001. The p-values of the multivariate tests are all 0.000 or 0.001. The results for the individual LM tests show that for $h = 2$, 4 out of 5 tests are significant at the 5% level and 3 out of 5 tests at the 1% level, and for $h = 12$, 4 out of 5 tests are significant at the 5% level, and 3 out of 5 tests at the 1% level.

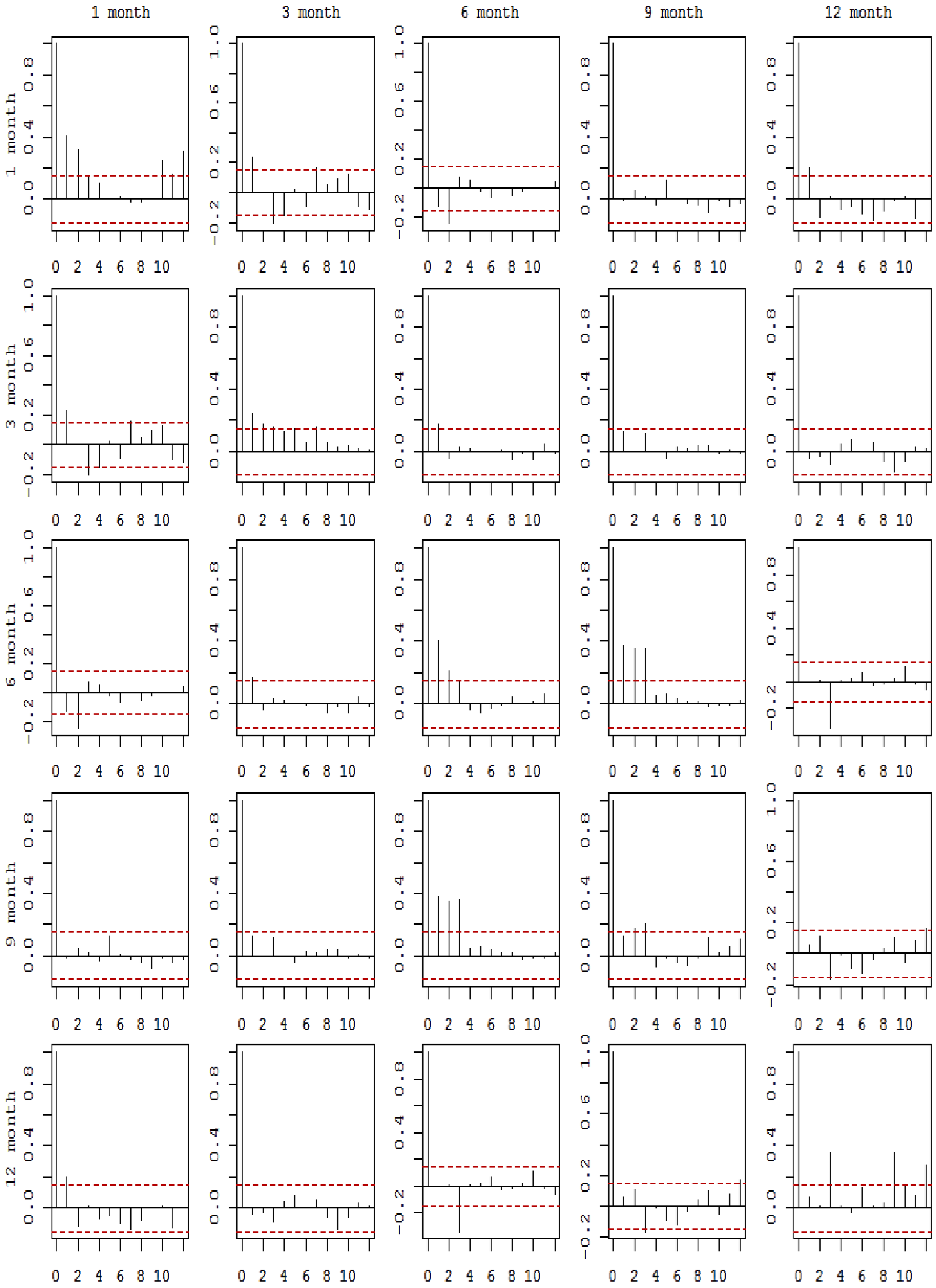

Figure 15: Correlograms of the squares and cross products of the standardised residuals $\widetilde{\mathbf{w}}_t$ from the VAR model for the Euribor interest rates data.

Table 5: LM tests for ARCH in the VAR model for the Euribor interest rates data. The number of observations is $T = 172$. The error distribution of $\text{vec}(\mathbf{W}_1, \ldots, \mathbf{W}_T)$ in the resampling scheme of the bootstrap tests is independent normal NID$(0, 1)$. The table reports the p-values of the tests. Notes: The lag length of the VAR model is $p = 3$. The MLM test cannot be computed because the number of parameters in the auxiliary regression exceeds the number of observations.

h	LM_1	LM_3	LM_6	LM_9	LM_{12}	$\widetilde{LM}^*$	MLM	MLM^*	LM_{CCC}	LM^*_{CCC}
2	0.000	0.002	0.000	0.031	0.677	0.001	—	—	0.000	0.001
12	0.000	0.111	0.001	0.048	0.000	0.001	—	—	0.000	0.001

6 Conclusions

In this article we propose a combined LM test for ARCH errors in VAR models by following a suggestion in Dufour et al. (2010) of replacing an exact MC test by a bootstrap MC test when the model includes lags. The test circumvents the problem of high dimensionality in multivariate tests for ARCH in VAR models. It is computationally simple since it only requires computing univariate statistics. We show that the bootstrap MC test is asymptotically valid. Monte Carlo simulations show that the test has good finite-sample properties, and is robust against skewed and heavy-tailed errors. We present two financial applications of multivariate LM tests for ARCH to CDS prices and Euribor interest rates. The results show that the errors are skewed and heavy-tailed, and that there are significant ARCH effects.

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