



Momentum and the Fama-French Five-Factor Model: International Evidence

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Abstract: <p>This study contributes to the literature on the implications of adding the momentum factor to the Fama-French Five-factor model. In comparison with the predominantly U.S. samples used in the literature (Fama & French 2016b; 2018), the novel contribution of the study is to provide out-of-sample evidence of the recent Six-factor model using a sample consisting of data from the four regions of North America, Europe, Japan and Asia Pacific.</p> <p>As such, this study bundles together recent research in international asset pricing, the Five- and the recent Six-factor model to extend on the literature. Given the international nature, this study also contributes towards the literature on the momentum anomaly. Research has both insinuated that momentum be regarded as a major and inherent market property and declared momentum to be a geographical irregularity that persists in some markets but not in others.</p> <p>The empirical framework of this study measures the efficacy of the novel Six-factor model in describing international average returns using a methodology consisting of tests of the RHS factor through mean-variance spanning and a series of LHS tests of individual and joint comparison of alpha on various portfolio sorts, including sorts on size-momentum. In using Fama & French's factor definitions and using the seven regionally diversified portfolio sorts, it is the first comprehensive international study of the Six-factor model to the best of this author's knowledge.</p> <p>In the formal GRS F-test, the results demonstrate that the Five-and the Six-factor models are often both rejected, or both not rejected, but the tendency of over-rejection is not as apparent as previous research would suggest. On several portfolio sorts, both models pass the GRS F-test. Most notably, the results demonstrate that the Six-factor model is better equipped at pricing momentum than the Five-factor model. The factor spanning regressions confirm momentum is an important variable in describing North American, European and Asian Pacific returns. The Six-factor model noticeably reduces the average absolute intercepts produced by regressions on portfolios possessing the momentum characteristic. At the same time, the model does not sacrifice meaningful accuracy in describing portfolios not sorted on momentum, and even marginally reduces the average absolute intercept on multiple sorts. These results conform with previous literature emphasizing the momentum factor in regions where it is found strong. However, the results also confirm the literature that Japan is an outlier with regards which suggests that momentum perhaps is not a universal pricing error and does not need to be taken into consideration everywhere.</p> <p>Thus, assuming that the Five-factor model is a good performer, this study concludes that the Six-factor model is to be regarded as a viable alternative to the Five-factor model.</p>	
Keywords: Asset pricing, Fama-French, five-factor model, six-factor model, momentum	

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Sammandrag: Denna studie bidrar till den akademiska forskningen med insikter i att lägga till momentumfaktorn till femfaktormodellen. I jämförelse med huvudparten av forskningen i ämnet som är baserat på data i främsta hand från USA (Fama & French, 2016b; 2018) så bidrar den här studien med out-of-sample-bevis på sexfaktormodellen utförd på ett internationellt sampel bestående av de fyra regionerna Nordamerika, Europa, Japan och Asien-Stillahavsregionen. Studien bidrar också till den akademiska litteraturen genom att anknyta färsk forskning inom tillgångsprissättning, fem- och sexfaktormodellen. Med tanke på arbetets internationella karaktär bidrar studien även till forskning som utförts kring momentumanomalin. Den forskning som utförts inom ämnet har påvisat att momentumeffekten är en väsentlig marknadsegenskap, men även låtit antyda att anomalin verkar vara geografiskt oregelbunden och uppvisar sig inte på alla marknader. Avhandlingens empiriska ramverk mäter den nya sexfaktormodellens effektivitet i att beskriva internationell medelaktieavkastning genom medelvarians-spanning test av högerledsfaktorn och analys av intercept från regressioner på diverse testportföljer, inklusive portföljer sorterade på momentum. Genom användning av Fama & French faktordefinitioner och sju regionalt diversifierade portföljsorteringar, är den här den första internationellt omfattande studien kring sexfaktormodellen enligt denna författares bästa vetenskap. Studiens resultat påvisar att både femfaktormodellen och sexfaktormodellen samtidigt förkastas eller inte förkastas av det formella GRS F-testet. Samtliga modeller har tenderat att förkastas av testet i tidigare forskning, men i den här studien klarar båda modellerna av GRS F-testet på åtskilliga portföljsorteringar. Det mest anmärkningsvärda resultatet från denna studie är att till skillnad från femfaktormodellen så klarar sexfaktormodellen väl av att prissätta momentumeffekten. Faktor-spanning testen påvisar att momentumfaktorn är viktig för att beskriva medelavkastningar i Nordamerika, Europa och i Asien-Stillahavsområdet. Sexfaktormodellen minskar märkbart intercepten från regressionerna som utförts på momentum-portföljerna från dessa tre regioner. På de resterande portföljsorteringarna minskar inte precisionen nämnvärt utan modellen minskar marginellt på intercepten på flera portföljsorteringar. Studiens resultat överensstämmer med tidigare forskning som betonat att momentumfaktorn är viktig i de områden där effekten påfunnits. Resultatet bekräftar även den tidigare forskning som konstaterat Japan sticka ur mängden. Det här innebär att momentumanomalin inte nödvändigtvis är universell och behövs således inte beaktas överallt. Förutsatt att femfaktormodellen också visar god prestanda så drar studien således slutsatsen att sexfaktormodellen kan betraktas som ett gott alternativ till femfaktormodellen.	
Nyckelord: Tillgångsprissättning, Fama-French, femfaktormodellen, sexfaktormodellen, momentumeffekten	

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1 INTRODUCTION

Factor models have emerged in response to the empirical failures of the Capital Asset Pricing model. Over 300 effects anomalous under the CAPM have been discovered through empirical exercise to date (Harvey, Liu & Zhu, 2016). These effects fuel the search for new and more accurate but still parsimonious asset pricing models that shrink the list of anomalous effects to practical levels. Herein, came the Fama & French's (1993) Three-factor model which was quickly adopted for its real-world usefulness. However, many effects nevertheless remain anomalous under the Three-factor model. In response to these shortcomings, Fama & French's (2015) Five-factor model added the investment and profitability factors to the Three-factor model. Recent evidence reveals that the model improves the description of average returns, but still some prominent effects remain anomalous (Fama & French, 2016b).

One such notable anomaly, momentum, had earlier prompted the Carhart (1997) model, which augments the Three-factor model with a momentum factor to target the anomaly. Like the Three-factor model, the Carhart model enjoys wide employment, perhaps due to the body of considerable empirical evidence suggesting that momentum not only persists but remains strong and exhibits across diverse markets and even across asset classes. However, some have insinuated that the effect is a geographical irregularity due to its absence in some markets, predominantly the case of Japan.

In the search for new parsimonious models, prominent researchers (Lewellen, Nagel & Shanken, 2010; Harvey, Liu & Zhu, 2016; Fama & French, 2018) consider that factors with a strong theoretical grounding should be prioritised. Therefore, despite the empirical evidence behind the strong momentum effect, many researchers employ momentum factors with caution due to the absence of consensus behind the cause responsible for the noted anomaly. Others argue that this should not frustrate adoption as empirical evidence has shown momentum is an important market characteristic that is not yet fully understood.

Nonetheless, this presents a problem since neither the Three- nor the Five-factor model can price momentum, and the Carhart model does not house the additional properties of the Five-factor model. For these reasons, prominent authors called for tests of a Six-factor model that adds the momentum factor. Fama & French (2016b, 2018) heeded these calls and incorporated tests of the Six-factor model, finding that the Six-factor model can price momentum on a U.S. sample.

Despite the novel international evidence on the Five-factor model, few authors have studied the Six-factor model on a non-U.S. sample. In this context, this study adds to the literature by testing the Six-factor model on an international dataset, thereby providing out-of-sample evidence of the model's ability to describe returns, and whether it amounts to a viable alternative to the Five-factor model. Indeed, using a regionally diversified sample over a period of 1.11.1990 to 31.7.2019, this thesis investigates whether the Six-factor model can describe average returns in the four regions of North America, Europe, Japan, and Asia Pacific.

The results indicate that the momentum factor constitutes an important factor in describing returns in North America, Europe, and Asia Pacific, but is trivial in Japan. Asset pricing tests further show that the Six-factor model offers relief to the task of pricing momentum in the three former regions. Therefore, this study concluded that the Six-factor model is a viable alternative to the Five-factor model in North America, Europe, and Asia Pacific.

1.1 Purpose

The purpose of the study is to investigate the implications of adding momentum to the Fama-French Five-factor model, and to compare the resulting Six-factor model's efficacy on an international dataset.

1.2 Contribution

Earlier research (Fama & French, 2015; 2016a; 2016b) compared the Five-factor model revealing it prevails against the Three-factor model both on U.S. and international samples. However, given the strong empirical evidence behind the momentum factor, Fama & French's theory-dominant reasoning of factor addition has been criticised. Asness (2014) and Blitz, Hanauer, Vidojevic & Van Vliet (2018) criticise the absence of proper reasoning for leaving out momentum. Fama & French (2016b) heeded their calls and tested a Six-factor model on U.S. Size-Mom portfolios, and Fama & French (2018) extended the Six-factor model onto a comprehensive set of U.S. portfolio sorts. Due to the emphasis on various firm fundamentals, asset pricing literature is largely conducted on U.S. samples and virtually every model is first thoroughly cemented using U.S. data. However, this also highlights the need for out-of-sample evidence. The primary contribution of this thesis is to test the Six-factor model out-of-sample on international data and compare its performance to the Five-factor model.

As international studies have become more commonplace in the literature, important insights of the various factors have extended from the international plane, including the momentum factor, WML (Fama & French, 2012; Asness, Moskowitz & Pedersen, 2013). Given the Five-factor model's inability to price momentum (Fama & French, 2016b), this study extends the literature with four sets of regionally diversified portfolios sorted on Size and Momentum to judge the Six-factor model's ability to price momentum. Subsequently, the assessment of whether the results demonstrates an improvement over the Five-factor model is one of the cornerstones of this thesis.

Previous research has both suggested that the momentum effect might constitute a geographical irregularity and, paradoxically, emphasised its importance internationally (Asness, Moskowitz & Pedersen, 2013). Using a regionally diversified sample consisting of the regions of North America, Europe, Japan, and Asia Pacific, thereby incorporating regions on both sides of this discussion, this thesis ensures this topic is extended upon. Using both RHS and LHS approaches to model comparison, this study will extend the literature on WML's viability in the four regions. In particular, this thesis morphs the studies of Fama & French (2016a), who compare the Five-factor model with the Three-factor model on the four regions and Fama & French (2016b, 2018), who initially tested the Six-factor model more comprehensively on a U.S. sample. Fama & French (2015; 2016a) expected future work to both introduce additional factors, and for parsimony, identify redundant factors. This study extends this topic to include WML.

Previous literature has presented opinions on theoretical versus empirically motivated factor inclusions. As momentum still lacks sound consensus, this study contributes to the empirical vetting process of WML's inclusion into asset pricing models.

1.3 Limitations of the study

The sample studied in this thesis has been obtained from the Kenneth French Data Library. As such, it does not construct its own factors or create its own test portfolios. Therefore, this study trusts the portfolios and factor definitions are appropriate for use in reaching the study's research objectives.

The choice to use regionally diversified factors and portfolios also constitutes a limitation, given the assumption made that the markets are at least to some degree integrated. Further, like Fama & French (2016a), this study takes the perspective of an American investor and uses the U.S. one-month Treasury bill to proxy the risk-free rate across all four regions.

Most of the previous research on this subject has been conducted on U.S. samples typically run longer than this study. The sample period of this study runs from 1.11.1990 to 31.7.2019. This is the earliest available starting point of data from the database that ensures all regions run the same sample period. Although similar sample periods have not proven excessively problematic in previous literature, some of the studies this study compares with run on sample periods roughly twice this size.

1.4 Structure of the thesis

This thesis is composed of eight themed chapters. Chapter 2 introduces important theoretical concepts related to risk and return, anomalies, and asset pricing models. Chapter 3 presents a literature review of relevant papers to build a frame of reference. It covers previous studies on Fama & French and momentum models, most notably the Five-factor model and the momentum augmented Six-factor model. The purpose is further for this chapter to aid in the formulation of appropriate research hypotheses and execution of the following empirical analysis. Chapter 4 formulates hypothesis out of the research questions and presents the empirical methodology of this study. Chapter 5 presents the sample of the study and analyses the data by presenting descriptive statistics. Chapter 6 presents the results of the empirical study, which then Chapter 7 will discuss in detail and mould into inferences. Lastly, Chapter 8 states the study's conclusions and proposes suggestions for future research.

2 THEORETICAL BACKGROUND

This chapter introduces the most relevant theoretical frameworks of asset pricing. Fundamental theories are introduced relating to risk, return and efficient markets, followed by an introduction to the CAPM and market anomalies. Next, common asset pricing factors, the empirical evidence behind them and theories on what causes anomalies are introduced. The last two subchapters introduce asset pricing models. In the first, the Fama & French models are introduced in chronological order. In the second, the literature behind momentum models are introduced and the Six-factor model presented.

2.1 Risk, Return and Efficient markets

In theory, an efficient market is one wherein price can and will reflect available information fully (Fama, 1970) – therein asserting that the investor should not be able to consistently beat the market. This theory is conditioned on the following fundamental assumptions: investor rationality, arbitrage, and independent deviation from rationality (Jain, 2012) – which will now be elaborated in turn. Investor rationality implies that investors are willing to adapt to new information as soon as it is made available, and as they are rational, they cannot systematically make over- or underestimations of asset characteristics and prospects given information possessed. The arbitrage condition, in turn, maintains that despite irrationality the market can remain efficient as rational investors rid the market of mispricing through arbitrage trading, the simultaneous buying and selling of discrete substitute securities to earn a riskless profit. Finally, independent deviation from rationality is the offsetting of investors excess optimism and pessimism making the market rational on average as irrationalities form a double-sided margin around the correct price (Jain, 2012).

The Efficient Markets Hypothesis (hereafter EMH) comprises three tiers of efficiency – the weak, semi-strong and strong (Fama, 1970). The tiers reflect the assumed information that make up market prices. The weak form reflects all historical information, where prices are unpredictable and run along a “random-walk”, defined as a price series where all adjustments represent random departures from earlier prices (Malkiel, 2003). The semi-strong form incorporates all publicly available information, and prices adjust according to new public information. The strong form of the hypothesis includes all information no matter the level of privacy and publicity. This is the most stringent form as it portrays an informationally efficient market that accounts for all information, past and present, including information obtained through monopolistic

access (Fama, 1970). Grossman & Stiglitz (1980) argued that some inefficiency is a necessary dynamic to facilitate the correction of prices as otherwise there would be no incentive driving data collection if prices strictly reflect all available information. This was also an implicit belief of Fama's (1970) discussion on the absence of proper methodologies for its measurement. At least, trading costs make the extreme form of the EMH unrealistic. A more economically sensible interpretation of the EMH was given by Jensen (1978), allowing prices to reflect information until the marginal benefits of acting upon it no longer exceeded the marginal costs (Fama, 1991). The EMH can be observed from the rational and the behavioural point of view as finance theory has questioned the efficiency of the rational 'risk-based view' whereby informed investors offset irrationality (Shiller, 2003). Therefore, there exist variables beyond risk and return, to which the markets react to, and the 'behavioural view' observes these deviations from theoretical assumptions of frictionless markets and rational investors. Malkiel (2003) pointed out that investors collectively making mistakes is an inherent property of stock markets, resulting in persistent mispricing in the short term. Shiller (2003) proposed that the markets reflect irrational human behaviour, hence certain factors may be explained by skilful exploitation of biased beliefs. Indeed, behavioural finance, the study of finance from the perspective of sociology and psychology, offers new perspectives, often paradoxical, to the theory of efficient markets (Shiller, 2003).

Because the future is unknown, ideal investment strategies merely frame the best result given available information as no investment portfolio is exactly quantifiable. On these grounds Markowitz (1952) divided the portfolio selection process into two stages: moulding observation into beliefs, and transliteration of beliefs into portfolios. In Markowitz's Modern Portfolio theory (MPT) diversification is a central tenet. Investors are risk averse and simultaneously consider expected return desirable and variance undesirable (Markowitz, 1952). The cornerstone of MPT is that holding expected return constant, investors should minimise variance, and holding variance constant, they should maximise expected return (Elton & Gruber, 1997). This assumes that investors only consider variance and mean return, which they estimate over a single period (Elton & Gruber, 1997). It also assumed that risk is controlled through diversification, and market imperfections are ignored (Markowitz, 1952). Diversification reduces variance for a level of expected return as the investor considers how assets co-move with one another. This means one can create more efficient portfolios than if the interaction between the assets were ignored (Elton & Gruber, 1997). Accordingly, MPT shows that

rational investors choose mean-variance efficient portfolios that offer the least variance for the level of return.

As the MPT inputs are all estimation and require effort to estimate, simpler frameworks were sought after (Elton & Gruber, 1997). This led to the earliest asset pricing models emerged, prominently the Single-Index model of Sharpe (1964), which required fewer estimates (Elton & Gruber, 1997). Sharpe's model eventually evolved into the CAPM. While the tools have changed, this process is still relevant today. This thesis will focus and refer to equilibrium asset pricing models (Fama, 1991) born out of MPT and the CAPM. Equilibrium pricing covers factor models from the theories of capital asset pricing, its augmentations, and the literature on arbitrage pricing.

2.2 The Capital Asset Pricing Model

The Capital Asset Pricing model (CAPM), contrived by Treynor (1962), Sharpe (1964), Lintner (1965a & 1965b) and Mossin (1966) can be credited for the first precise definition of risk and expected return. The CAPM is a single-factor model built upon the MPT and adds two further assumptions (see section 2.1 for MPT assumptions); that investors are able to borrow and lend at the risk-free rate of interest unlimitedly, and that investors have homogeneous expectations (Sharpe, 1964). The CAPM states that there is a linear relationship between expected return of an asset and its covariance with market portfolio (Huberman & Wang, 2005), meaning risk and return are determined by market beta exposure alone. The CAPM can price individual assets, and if it holds, assets are priced appropriately when the estimation equals the present value of the future discounted cash flows. While the model was ground-breaking at the time, and motivated through empirical evidence by Black, Jensen & Scholes (1972), Fama & Macbeth (1973) and Blume & Friend (1973) of the model, Basu (1977), Banz (1981), De Bondt & Thaler (1985) and Fama & French (1992; 1993) offer a particularly dim view of its practical use. The CAPM equation can be summarized as follows:

$$r_{jt} - r_{ft} = \alpha_i + \beta_i^M (r_{Mt} - r_{ft}) + \varepsilon_{jt} \quad (1)$$

Where r_{jt} is the return of security j in month t , r_{Mt} is the market portfolio return, r_{ft} is the risk-free rate, α_i the intercept, β_i^M the sensitivity of the expected asset excess return to the expected market excess return, and ε_{jt} the error term.

The CAPM has been criticised for being too restrictive in its assumptions, which Sharpe (1964) already forecasted in his seminal paper. The assumption that investors may unlimitedly borrow and lend at the risk-free rate of return is not realistic. As studies have

revealed, arbitrage may not be free of market friction and involves some risk on the part of the investor (Shleifer & Summers, 1990). However, whether this matters for its real-world implications is still a topic for researchers to confirm, as asset pricing models are approximations rather than true stories of return. Some criticism also targets the empirical applicability of the model, perhaps most notably criticism for its limited empirical success (Fama & French, 2004). Fama & French (2004) criticize the model's ability to capture stock returns and the model's validity in the applications, the main source of criticism being the market risk factor in explaining stock returns. The CAPM is still popular due to its ease of deployment but is nevertheless not an exemplary model in its capacity to capture variation in equity return.

2.3 Anomalies

Since the beginnings of asset pricing, researchers have established inconsistencies with rational pricing theory. These so-called market anomalies can be defined as statistically significant return patterns that deviate from the conventional rational pricing logic. Anomalies have been featured in earlier works that precede modern asset pricing, and to a large degree developed from investing strategies, such as “value investing” by Graham & Dodd (1934). While asset pricing got its start with the frameworks provided by Treynor (1962), Sharpe (1964), Lintner (1965a; 1965b) and Mossin (1966) summarizing the theoretical framework of Markowitz (1952) into a more practical one, however, technological advancements with databases allowed for new tools, the ability to dig deeper with larger samples to conduct in-depth studies. With the applicability of the pricing framework, patterns in returns that the CAPM was unable to explain became known as anomalies (Fama & French, 1996). As the CAPM provided the tool that subsequently sparked the study of market anomalies, it has also resulted in a flood of anomaly discoveries, including Fama & French's (1992; 1993) size and value. Harvey, Liu & Zhu (2016) document that at least 316 anomalous effects have been discovered through empirical asset pricing to date. However, most anomalous patterns discovered through empirical exercise are not robust or dependable out-of-sample (Malkiel, 2003; Linnainmaa & Roberts, 2018).

Since anomalies have tended to disappear sometime after reaching fame, or not show up out-of-sample (Harvey, Liu & Zhu, 2016), it is not completely clear whether risk adjusted mispricing exists or if they are random statistical departures only perceived to be persistent (Schwert, 2003). Moreover, some anomalies may not in fact be violating rational pricing logic simply because they violate the CAPM. The joint hypothesis

problem (Fama, 1991) expresses that the testing of market efficiency must be tested jointly with some equilibrium asset pricing model due to the impossibility of testing abnormal returns without a comparison of expected return predicted by asset pricing models to real return. As a result, anomalies may reflect either market inefficiency or model inaccuracy. While market anomalies are deviations from the suggested price an asset should be given a certain general criterion, it is ambiguous whether they are a result of market inefficiency or a bad model. Hence, the other possibility that the theoretical model employed itself was inadequate must be considered (Fama & French, 1991). In sum, this means that some return patterns anomalous under the CAPM might represent real risk characteristics that the CAPM is unable to explain (Malkiel, 2003).

Therefore, equally significant as the subject of finding anomalies is the subject of trying to find an economic explanation to understand their existence. Evidence also shows that while the theoretical intuition of the CAPM is imperative, the model itself is often inaccurate in describing average returns to make it useful in practice (Fama & French, 1993). The empirical failures of the CAPM have as a result led to factor models emerging (Fama & French, 2018). Researchers have tried to link anomalies that seem to stand the test of time to economic theory. Regardless of what causes them, they have sparked a search for a new more accurate parsimonious asset pricing model that augments the CAPM with factors that proxy established risk premiums. Thereby they attempt to shrink the list of anomalies to a more practical level to acquire more useful models. At the same time, in exchange for parsimony, this search is a task that should balance accuracy with simplicity and restraint. Lewellen, Nagel & Shanken (2010) argue factors derived from economic theory should take a priority, and those discovered from empirical exercise need to suffer through higher levels of empirical scrutiny before acceptance.

Nowadays, there are some contrasting schools on the subject of economic theory behind anomalies, but most prominent are the rational risk-based view and the behavioural view (Asness, Frazzini, Israel & Moskowitz, 2014). The risk-based view ties factors to traditional risk-based theory and is often characterised by the assumptions of rational investors and efficient markets. Under this interpretation, a factor premium implies there is a compensation for taking on additional risk and provided that risk perceptions remain unchanged, the premium will persist (Asness et al, 2014). The behavioural view does not require the investors to be rational or the markets efficient. De Bondt & Thaler (1985; 1987) were among the first to show evidence of investor overreaction. Therefore, on the contrary, the behavioural view ties to behavioural finance and sees anomalies as

violations of rational theory caused by investor irrationality. Irrational behaviour in turn may account for improper access to information or the lack of interest in evaluating a correct market price. The risk-based view is older and more established, and most asset pricing factors that have achieved general acceptance have at some point been tied to theory relating risk and return (Shiller, 2003).

2.4 Common asset pricing factors

Asset pricing factors can be summarized as drivers of return and can generally be divided into “style factors” and “macroeconomic factors” (Zhang, Hopkins, Satchell & Schwob, 2009). “Style factors” proxy anomalous effects believed to represent either compensation for risk or systematic deviation from rational theory, i.e. behavioural finance. Macroeconomic factors are generally influential macroeconomic metrics that affect security prices broadly. In line with the capital market theory, equity returns are solely compensation for systematic risk, suggesting the selection of a set of macroeconomic state variables (Zhang et al, 2009). Factors are often grouped according to how they have come about as most factors are the result of empirical exercise. However, their basis often lies in anomalies that the CAPM was unable to price, such as the size and value. More generally accepted factors have been related to finance theory where they have been proposed as compensations for risk. An academic perspective often favours the former where, as covered in Chapter 2.3, factors derived in theory should take a priority (Lewellen, Nagel & Shanken, 2010). The more practical industry perspective generally accepts more factors through empirical evidence. Factors have a practical side in the development of investment strategies. Research has allowed for new tools that have materialized into new funds that manage allocations according to a set of factors that proxy known anomaly characteristics. Because of their value in the industry (Asness et al, 2014), representatives of the academic world as well as the industry are joined in keeping factor definitions tested.

This subchapter presents some of the most well-established “style factors” which proxy effects that have been empirically vetted and linked to various theories, although not all of them have been met with unanimity. The first four consist of the four variables that Fama & French (1993; 2015) incorporated into their two asset pricing models augmenting the CAPM, and the last is the momentum factor.

2.4.1 Size & Value

The size factor Small-Minus-Big, SMB, captures the difference in return of an index of only small companies versus an index with only large companies with similar book-to-market equity (Fama & French, 1993). Fama & French (1992; 1993) justified its inclusion by referencing empirical evidence that smaller firms often earn higher risk-adjusted returns than larger ones. Empirical evidence has shown the size effect to exist both in the U.S. and internationally (Van Dijk, 2011).

As for its theoretical motivation, Banz (1981) suggested size originates from the theory of mergers, where large firms may pay a premium for small firm stocks. Chan & Chen (1991) cite financial distress as a possible origin, suggesting the size premium amounts to compensation for holding cyclically sensitive and distressed companies. Dichev (1998) developed a theory linking insolvency risk to higher return. Some researchers have suggested that size compensates for liquidity risk (Van Dijk, 2011). Fama & French (1995) speculate that size may be explained as a risk premium for earnings persistence after demonstrating a link between size factors in earnings and returns. Banz (1981) reference market inefficiency by suggesting that some investors have inferior information towards smaller stocks, resulting in a natural aversion. The Merton model provides intuition for perceived risks by investors. This theory suggests that investors' appeal to hold stock is impacted by their self-perception regarding information about the firm. This implies that investors would demand a premium for owning smaller firms they believe they have an informational disadvantage in owning (Merton, 1987). An explanation from Shleifer & Vishny (1997) highlights the limits of the arbitrage condition which requires investors to be rational and not limited by market adversities. They propose that the pool of investors with the information and the means of conducting arbitrage is limited. Increased difficulty to perform arbitrage positions on small firms make them more prone to mispricing, and therefore they provide a premium in relation to large stocks.

Recently, researchers have speculated whether the size premium has diminished due to its time-variation and the recently observed small premiums (Asness, Frazzini, Israel, Moskowitz & Pedersen, 2018; Van Dijk, 2011). One possible reason is that investor overreaction to other quality measures drowns out the size premium when all metrics are controlled for. The premium reappears when controlling for quality or junk of a firm (Asness et al, 2018).

A vast body of literature connects book-to-market ratio to average stock returns in most markets around the world (Asness, Moskowitz & Pedersen, 2013). This has come to be called the value effect, indicating that equities with high book-to-market ratio tend to achieve better average returns than those with a low ratio. The value factor High-Minus-Low, HML, captures the difference in return between an index of value stocks and an index of growth stocks, this time controlling for similar size (Fama & French, 1993). Like with the size premium, the literature documents value as related to economic fundamentals (Fama & French, 1992; 1993). The general interpretation is that certain companies are less popular among investors and therein sell at lower prices relative to their book value or earnings. The value effect implies that “value stocks” with high book-to-market have outperformed “growth stocks” with low book-to-market. While the firm fundamental approach has been known since Graham & Dodd (1934) coined value investing, the mysteries of the premium were uncovered by the research of De Bondt & Thaler (1985), Rosenberg, Reid & Lanstein (1985), Fama & French (1992) and Lakonishok, Schleifer & Vishny (1994).

Chen & Zhang (1998) offer a risk-based factor explanation which infers that increased risk of distress represents a premium for value stocks. Characteristics such as a decline in operational fundamentals and high leverage can be connected to distress risk (Griffin & Lemmon, 2002). Risk based explanations also suggest the value premium is an answer to the increased risk for value-minus-growth strategies during market downturns (Petkova & Zhang, 2005). Zhang (2005) motivated this claim by costly reversibility caused by value firms disproportionately wanting to scale down during downturns, but since reductions are more costly than expansions, they end up disproportionately affected.

The value premium has also been tied to behavioural explanations. Lakonishok, Schleifer & Vishny (1994) suggest optimism drives the overpricing of “glamour” stocks, those with low B/M, yet, evidence shows value stocks are no riskier. They consequently tied it to possible irrational behaviour on the part of the market actors. La Porta, Lakonishok, Schleifer & Vishny (1997) find that a sizeable portion of the return spread of value and growth stocks can be explained by systematic positive earnings surprises for value stocks.

2.4.2 Profitability & Investment

The profitability factor Robust-Minus-Weak, RMW, captures the effect where profitable companies have achieved a better return. Earlier evidence illustrates a relation between

expected profitability and future stock returns to mispricing (Fama & French, 2006). The profitability premium has been explained through the behavioural view, suggesting it may be related to overpricing with systematic frictions limiting the efficiency of the market to diversify away the anomaly. Lam, Wang & Wei (2016) suggest the profitability anomaly is explained by both behavioural and risk-based reasoning, as macroeconomic risk would account for part of it, and misvaluation based on investor sentiment for a part of the remaining premium. As for the factor, Fama & French (2015) use operating profitability less interest expenses, whereas Novy-Marx (2013) use gross profitability. Subsequent studies have incorporated multiple alternative proxies, including cash profitability and ROE to proxy profitability (Fama & French, 2018; Barillas & Shanken, 2018)

The investment factor Conservative-Minus-Aggressive, CMA, captures the effect where companies with large investments have achieved smaller returns. Like profitability, investment is often associated with mispricing (Fama & French, 2006). Risk-based explanations exist for investment that point toward macroeconomic and business cyclicity. At the centre of the theory is the documented tendency of investment going up as the risk profile of projects are low. As per the systematic risk and return relationship, the return achieved by such investing firms should decrease (Berk, Green & Naik, 1999). The behavioural view also offers intuition for investment, namely that the negative investment anomaly would be interlinked with investment in overvalued equity. Titman, Wei & Xie (2004) argue that the anomaly could be linked to investors' sluggish reaction to deal with empire-building managers. As for the factor proxy, Fama & French (2006) use asset growth on a per share basis, but Aharoni, Grundy & Zheng (2013) suggest this be measured at a firm level. Fama & French (2015) adopted asset growth through total assets on a firm level.

As a response to most papers investigating anomaly characteristics as isolated effects, Fama & French (2006) show their factor composition conforms with valuation theory. Although Fama & French's initial motivation stemmed from empirical evidence, Fama & French (2015) did not directly link them to any specific risk- nor behavioural justifications. Instead, they related the factors to finance theory, with derivations from Modigliani & Miller (1961) theory and the Discount dividend model. The theoretical aspects of the Three-factor model were connected to financial theory in Fama & French (2006), applying the DDM to relate size and value to return. Fama & French (2015) further link profitability and investment. As per the DDM, a share's market value is the

discounted value of expected dividends per share. The following equation reads if at time, t , the stocks of two companies have the same expected dividends but different price, the lower priced stock has a higher expected return and is riskier in the long run (Fama & French, 2015):

$$m_t = \sum_{\tau=1}^{\infty} E(d_{t+\tau}) / (1+r)^\tau \quad (2)$$

Where m_t is the share price at time t , $E(d_{t+\tau})$ the expected DPS for the period $t + \tau$, and r is roughly the long-run average expected return, or IRR of the expected dividends (Fama & French, 2015). Through some manipulating, Fama & French (2015) extract the relations of return and profitability, investment and Book-to-market by applying the theories of Modigliani & Miller (1961) who demonstrated that the time t total market value of the stock implied by (2) becomes:

$$M_t = \sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau}) / (1+r)^\tau \quad (3)$$

Where $Y_{t+\tau}$ is total equity earnings for the period $t + \tau$, and $dB_{t+\tau}$ represents the change in total book equity (Fama & French, 2015). This becomes the following equation when dividing by t book equity:

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau}) / (1+r)^\tau}{B_t} \quad (4)$$

With (4), Fama & French connected that (1) a lower value M_t or a higher book-to-market ratio indicates higher expected return, that (2) profitability, higher expected earnings, indicates higher expected return, and that (3) investment, higher expected growth in book-to-market, indicates lower expected return (Fama & French, 2015).

2.4.3 Momentum

The momentum factors Up Minus Down, UMD, or Winners Minus Losers, WML, (henceforth used interchangeably) capture the effect coined by Jegadeesh & Titman (1993) where past return information would have predicting power on future returns, as a historical excess return spread of the “winners” and the “losers”. This study focuses on price momentum and uses it synonymously with momentum. The momentum anomaly refers to the empirical evidence that overperforming equities continue to overperform in

the near future, and vice versa for those that have underperformed (Jegadeesh & Titman, 1993). Momentum seems to exhibit the strongest prevalence in shorter horizons of a month or less but has also been documented for horizons lasting upwards of a year (Moskowitz & Grinblatt, 1999). De Bondt & Thaler (1985, 1987) first documented return reversals over longer horizons, with Jegadeesh & Titman (1993) documenting it both over medium and longer horizons. The anomaly has since persisted (Asness, Moskowitz & Pedersen, 2013) and researchers have mapped some dependence between types of momentum (Moskowitz & Grinblatt, 1999) and momentum across markets and asset classes (Asness, Moskowitz & Pedersen, 2013). Previous literature had long argued momentum is a geographical irregularity, with Japan often cited as a notable exception due to the absence of a momentum effect (Asness, 2011; Fama & French, 2012). Rouwenhorst (1998) discovered evidence of momentum effects across multiple markets and their tendency to correlate with U.S. momentum returns. While momentum was also found to relate negatively with size, momentum was also found to exist among big firms. Asness, Moskowitz & Pedersen (2013) demonstrate that momentum effects exist across several markets and asset classes. Further, they discover a linkage between value and momentum that may be especially important internationally and speculate there may even exist a global momentum factor.

Exponents of the risk-based view have found it hard to justify momentum. Herein, Johnson (2002) explains momentum by suggesting past returns imply higher growth rate risk, which prompts higher future required returns. Momentum could also constitute compensation for risk, possibly in cash flow risk or discount rate risks, or it could constitute shared-economic risk (Asness et al, 2014). Another rational risk-based theory of Holden & Subrahmanyam (2002) is that new information settles slowly, and momentum constitutes compensation for uncertainty.

Arguments against a systematic risk-based view were already implied in Jegadeesh & Titman (1993), as estimated market betas went in the opposite direction than if the beta risk was compensated for. Exponents of the behavioural view suggest momentum should be regarded as a symptom of under- or overreaction to earnings events and seek explanations for either reaction (Asness et al, 2014). Chan, Jegadeesh & Lakonishok (1996) suggest that medium horizon momentum is partly explained by earnings event underreactions as a gradual market response to new information. Barberis, Schleifer & Vishny (1998) suggest conservatism where investors attach improper proportions to new

information causes momentum by virtue of underreaction. Hong & Stein (1999) suggest there is a group of investors creating volume who are informed with a delay.

Despite extensive research on the topic, and the effect having stood the test of time, Subrahmanyam (2018) suggests there is little general agreement as to a discernible cause of momentum. To work on this problem Subrahmanyam (2018) suggests research should take an alternative approach where tests should focus on ruling out alternative explanations instead of testing specific theories.

Carhart (1997) created a four-factor model by adding momentum to the Fama-French Three-factor model, and through comparison provided evidence implying that momentum is a valuable addition. The Carhart model quickly grew popular and momentum has since joined size and value as standard risk factors for practical applications. Because of this, Fama & French's (2015) decision to leave out momentum has been a topic of discussion. Fama & French (2008) earlier commented that momentum may fragment the EMH rationale. A fundamental problem is the difficulty to view momentum as a priced risk. However, it bears to ask whether Fama & French's risk-based reasoning is adequate given the direct violation of the EMH that momentum presents. As momentum remains and arguably the longer the stay the more reason there is to not ignore it even if it is not yet fully understood, assuming that a higher level of empirical vetting can substitute theory as Lewellen, Nagel & Shanken (2010) puts it. Asness et al (2014) state that regardless of whether momentum be explained through behavioural or rational explanations, theory is likely to provide consensus on it at some point.

2.5 Equilibrium asset pricing models

Asset pricing models attempt to describe the returns of assets such as stocks. Each asset pricing model developed to date involves risk as the key element (Erdinc, 2018). Models generally adopt a set of useful variables that add accuracy in the pricing process by eliminating the mispricing of risks. The selection of an adequate asset pricing model is critical as the anomalous effects are to be benchmarked relative to these models given the bad model hazard (Fama, 1991). Most anomalies have been discovered in tests conducted relative to the CAPM. But some anomalies might be rewards for taking on a specific risk that the model itself is unable to explain. This offers credibility to the task of investigating different factor models. Further models that have achieved acceptance by both academia and practice. The most common such factor model is the Fama-French

Three-factor model, but other models such as the Carhart Four-factor model are also widely used. There is no true model, rather all models simplify the description of returns (Fama & French, 2016a). Moreover, for the model to remain parsimonious, of importance is to achieve accuracy with as few factors added as possible. Many of the factors build upon the empirical theories of Arbitrage Pricing Theory and have been benchmarked with a previously vetted asset pricing model. With intuition from the literature stemming from the search for persistent anomalous effect and from the Arbitrage Pricing Theory, APT, of Ross (1976) researchers have motivated the addition of factors to these models.

Some argue that due to the availability and frequency of data used in finance, it is possible to find statistically significant relationships beyond the economically sound, and the reuse of data sets without proper probing may lead to data snooping (Lewellen, Nagel & Shanken, 2010; Harvey, Liu & Zhu, 2016). To counter these two problems, Lo & MacKinlay (1990) advise that the addition of factors should be handled with care and the model be justified through empirical testing. Harvey, Liu & Zhu (2016) suggest empirical research add some further scrutiny in the drawing of inferences. One could infer from this that it may be wise to adhere to the commonly used asset pricing models, rather than constantly deviating towards new trends.

2.5.1 Multifactor models

While the CAPM is derived from finance theory, and asset pricing factor additions have been studied and related to theory, and practical anomaly capture has become one of the most contentious issues in finance today (Foye, 2018), acceptance of the subsequent multifactor models generally extends through empirical observation. For practical reasons, the world can be made easier if multiple dimensions of return characteristics are reduced to a handful of factor betas. The Arbitrage Pricing Theory (APT) presented by Ross (1976) assumes arbitrage opportunities will not persist. The fundamental contribution is that the stochastic characteristics of asset returns follow a factor structure (Huberman & Wang, 2005). If assets do not offer arbitrage, the APT theorizes that expected returns may be expressed as linear relationship of the loading of several systematic factors (Elton & Gruber, 1997). APT generally loosens the assumptions of the CAPM and is more practically applicable since it does not require measuring the market portfolio and allows the targeting of effects anomalous under the CAPM (Dhrymes, Friend & Gultekin, 1984). APT does not prescribe the factors, rather it is additive and various models can be created by applying its intuition. Moreover, since it is not limited

to specific factors, CAPM is sometimes stated as a special single-factor case of an APT model. Returns follow a “factor intensity structure” when expressed in (5):

$$r_j = a_j + b_{j1}F_1 + b_{j2}F_2 + \dots + b_{jn}F_n + \varepsilon_j \quad (5)$$

Where a_j is a constant, F_n is a systematic factor, b_{jn} is a factor loading of the j th asset to factor n , and ε_j is the idiosyncratic random shock of the risky asset with mean zero. (Sarpong, 2020)

Idiosyncratic shocks are assumed neither correlated across securities nor correlated with the factors. The relationship in (6) between expected return and factor loadings is formed, following a factor structure (Sarpong, 2020):

$$E(r_j) = r_f + b_{j1}RP_1 + b_{j2}RP_2 + \dots + b_{jn}RP_n \quad (6)$$

Where $E(r_j)$ is the expected return of the asset j , r_f the risk-free rate, RP_n the risk premium of the factor, and b_{jn} the factor loading of the j th asset to RP_n . (Sarpong, 2020)

With models based in APT, academic and industry actors alike have been able to empirically identify anomalous effects, and discount others, from benchmarking to an asset pricing model. At least 316 factors have been tested most of which recently (Harvey, Liu & Zhu, 2016), with Cochrane (2011) calling this the “zoo of new factors”. Harvey, Liu & Zhu (2016) point out that the sheer number of factors is evidence that most factors could be refuted as a consequence of data mining. They argue that factors derived from theory should have a lower hurdle than factors discovered from empirical exercise. Cochrane (2011) emphasizes that to keep asset pricing models parsimonious, it is vital to dissect which variables possess unique information about average returns and which are subsumed by other variables. Fama & French generally favour risk-based factor explanations. The works of Fama & French (1996; 2006; 2015), linking factors to theory, is an approach to establish factors of sound credibility in rational theory. In contrast, other authors favour other interpretations such as behavioural interpretations guided by mispricing (see Lakonishok, Schleifer & Vishny, 1994; La Porta, 1996; La Porta et al, 1997).

2.5.2 Fama – French factor models

With the CAPM critique in mind, and to put the “zoo” of anomalies in order, Fama & French (1992) studied a wide range of factors and in Fama & French (1993) published the Three-factor model with two new factors they argued would capture an additional

dimension of systematic risk over the CAPM's market beta. The Fama & French mission can be narrowed down to a few key steps. First, to identify factors with explanatory power and unique properties. Second, to identify factors that produce spreads among securities that are easily tradable, hence the practice of factor and portfolio sorting with breakpoints. Third, to identify variables that forecast returns in presence of other factors. Fourth, compare asset pricing models and make judgments on the number of necessary variables and whether they are unique and independent.

Fama & French's (1993) Three-factor model augments the CAPM with the additional factors SMB and HML, which proxy the size and value effects respectively. Fama & French (1996) document that the model explains several regularities that are anomalous under the CAPM, including firm size, book-to-market, past sales growth, long-run reversals, cash-flow-to-price as well as earnings-to-price. The Fama-French Three-factor model is composed of value-weighted (hereafter VW) excess market returns, size-, and book-to-market-related portfolio return spreads. Fama & French suggested the size factor be measured by taking the return of a portfolio of a diversified set of small firm stocks minus the return of a portfolio of a diversified set of bigger firm stocks (Fama & French, 1996), and the value factor be measured by taking the return of a portfolio of well-diversified high B/M firms minus the return of a portfolio of well-diversified low B/M firms (Fama & French, 1996). The size-factor SMB and the value factor HML augment the CAPM to form the following equity return regression formula:

$$r_{jt} - r_{ft} = \alpha_i + \beta_i^M (r_{Mt} - r_{ft}) + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + \varepsilon_{jt} \quad (7)$$

In the equation, r_{jt} is the return of security j in month t , r_{Mt} is the market portfolio return, r_{ft} is the risk-free rate, the appropriate betas and the size and value factors, and last is the error term ε_{jt} .

The Fama-French Three-factor model quickly became a standard tool due to its success in out-predicting the CAPM (Fama & French, 1993). However, as illustrated by the vast empirical evidence of the literature (see e.g. Harvey, Liu & Zhu, 2016), there are additional factors that potentially aid in the description of the cross-section of equity returns. Taking the insights from these researchers in mind, Fama & French (2015) presented a Five-factor model, which further augments the Three-factor model with the profitability, RMW, and investment, CMA, variables. These two factor additions were supported by the intuition of multiple studies (see e.g. Novy-Marx, 2013; Aharoni, Grundy & Zheng, 2013). Previously, Fama & French (2006) had discovered that earnings

should be related to higher expected returns and investment to lower expected returns. Unfortunately, they were not yet able to recommend these factors to be used for predicting the future return of equities. Novy-Marx (2013) documented a strong relationship of a gross profitability factor to average return. They speculated this was due to reductions in current earnings when certain investments, such as R&D, are seen as expenses. Titman, Wei & Xie (2004), Anderson & Garcia-Feijoo (2006) and Aharoni, Grundy & Zheng (2013) document a negative relationship between growth in investment and average returns.

Fama & French (2015) deemed RMW and CMA appropriate candidates due to evidence showing the Three-factor model tended to overlook variation in average return related to these two characteristics, and its inadequacy to appropriately model expected return. However, to justify their addition and the creation of the Five-factor model, they took the earlier approach of Fama & French (2006), where they apply the Dividend discount model to develop on profitability and investment and related size and value factors to economic theory. Aharoni, Grundy & Zheng (2013) show that Fama & French's (2006) predictions are validated once the variables are measured at the firm level. As discussed in Chapter 2.4.2, Fama & French (2015) utilize the DDM cultivated with intuition from Modigliani & Miller (1961) to justify the addition of the profitability and investment factors. Fama & French do not define the SMB and HML factors differently to their earlier work, although other studies through tests of factor redundancy have tried redefining the factors (see e.g. Asness, 2014; Fama & French, 2016a). Equation (8) exhibits the Fama-French Five-factor model:

$$r_{jt} - r_{ft} = \alpha_i + \beta_i^M (r_{Mt} - r_{ft}) + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + \beta_i^{RMW} RMW_t + \beta_i^{CMA} CMA_t + \varepsilon_{jt} \quad (8)$$

In the equation, r_{jt} is the return of security j in month t , r_{Mt} is the market portfolio return, r_{ft} is the risk-free rate, the appropriate betas and the size, value, profitability and investment factors, and last is the error term ε_{jt} .

Fama & French (2015) found the Five-factor model to provide a better description of return than the Three-factor model, although, as expected, it still did not fully explain the cross section of returns. On their U.S. sample, the Five-factor model could describe patterns in returns on portfolios formed on size, book-to-market, profitability, and investment adequately. The Five-factor model further gained international merit when Fama & French (2016a) compared it to the Three-factor models on the four developed regions of North America, Europe, Japan and Asia Pacific, providing mostly sound

evidence in favour of the Five-factor model. Fama & French (2016b) further documents that, unlike the Three-factor model, the Five-factor model could explain average returns on portfolios sorted on stock beta, volatility and net share issues.

Beyond the above, emerging studies have provided further positive feedback of its ability to describe returns out-of-sample. Studies on country-specific portfolios and factors have also been conducted, and cover countries from most continents. While most studies cover developed countries, Lin (2017), Foye (2018) and Foye & Valentincic (2020) have employed the Fama-French Five-factor model on different developing markets. Using a Chinese sample, Lin (2017) found the Five-factor model to consistently outperform the Three-factor model. Foye (2018) found the Five-factor model to outperform the Three-factor model in Eastern Europe and Latin America. Foye & Valentincic (2020) conclude that profitability and investment add to the explanation of Indonesian returns.

Even though the Five-factor model has been demonstrated as an improved alternative to the Three-factor model, it has not been conclusively vetted yet. Indeed, the model has sparked debate on parsimony. Most notable is that while it might provide better results compared with a Three-factor model, it is not necessarily better than its alternative combinations, such as a four-factor model that has dropped a variable (see Cakici, 2015; Hou, Xue & Zhang, 2015). Ignoring the market risk factor, the Five-factor model doubles the number of factors and interaction between the variables thereby increases the difficulty of describing average returns. If a four-factor model is equally adept at capturing the cross-section of average returns as the Five-factor model, it may be the case that the four-factor model is superior. For parsimony, such a model should be preferred. Because of this, factor redundancy tests are generally conducted in the literature.

Much of the critique of the Five-factor model is also intertwined, and authors have suggested solutions that could tackle multiple problems at once. Several studies have taken the opposite approach and suggested various factor additions with varying results. Racicot & Rentz (2017) added the liquidity factor but concluded that the market factor was the only consistently dependable one. Roy & Santhakumar (2018) investigated human capital by adding labour income growth to the Five-factor model and found this model to work well. Walkshäusl (2016) found all but the market factor become insignificant with the addition of a misvaluation factor. A functional concern is that the addition of yet another factor adds difficulty to controlling for robustness. Fama & French (2018) caution that more factors often go hand in hand with the multiple comparisons problem. Nonetheless, Fama & French (2016a) stated that they expected

future research to augment the Five-factor model with additional variables, hinting to the momentum variable, which continues to be relevant in the literature. The omission of momentum is perhaps the most notable critique regarding the factor composition (see e.g. Asness, 2014). This critique implies the momentum factor is a motivated addition, and alike the Carhart Four-factor model, a Six-factor model should be tested.

2.6 A Six-factor model

Jegadeesh & Titman's (1993) seminal work on the momentum anomaly prompted Carhart (1997), to develop the factor Up Minus Down, UMD. Carhart also augmented the Three-factor model with the UMD factor to create the Carhart Four-factor model in response to the Three-factor model's inability to explain momentum:

$$r_{jt} - r_{ft} = \alpha_i + \beta_i^M(r_{Mt} - r_{ft}) + \beta_i^{SMB}SMB_t + \beta_i^{HML}HML_t + \beta_i^{UMD}UMD_t + \varepsilon_{jt} \quad (9)$$

In the equation, r_{jt} is the return of security j in month t , r_{Mt} is the market portfolio return, r_{ft} is the risk-free rate, the factors & betas of size, value and momentum, and last is the error term ε_{jt} . Momentum is commonly abbreviated WML, Winners Minus Losers.

Like the Three-factor model, the Carhart model enjoys empirical motivation and WML has become a common addition in asset pricing literature. When the Three-factor model was created, it was commonly believed that the momentum effect would go away over time. However, one reason for WML's continued popularity relates to the empirical evidence that the momentum effect persists, and where it is found it is often found to be strong (Asness, Moskowitz & Pedersen, 2013). Moreover, reasons that have held back the WML's acceptance are perhaps no longer relevant, as suggested by Asness et al (2014). They dedicate their paper to refuting commonly cited reasons for leaving out momentum: (1) that momentum returns are negligible and inconsistent, (2) that momentum is likely to disappear, (3) that momentum is not universal and only exists among small cap stocks, (4) that momentum is not useful as a factor in portfolio construction or that different measures govern the results, (5) that long-only investors cannot take advantage of momentum, (6) that momentum is limited by trading costs and taxes, (7) and that momentum is wholly absent of theory. Asness et al (2014) end by stating that the empirical evidence is in favour of including momentum, and not including it is a question of caution on the part of individual researchers, perhaps stemming from the lack of consensus on the theoretical cause of the factor. Another reason for the Carhart model's continued use is the considerable evidence that neither the Three- nor Five-factor model can price momentum (Fama & French, 2016b).

In part due to the above, studies have proposed a sixth factor should be tested (Asness, 2014; Blitz et al, 2018). WML was initially left out when creating the Five-factor model due to a lack of an academic consensus and risk-based reasoning, as straying away from theory might enable data dredging (Fama & French, 2018). Asness et al (2014) do not believe this to be a problem and criticise the strict limitation to risk-based factors. Similarly, Blitz et al (2018) argue that the literature remaining conflicted on whether theoretical motivation should focus on risk or mispricing is a problem for consensus. Although some researchers consider empirical motivation to be enough, Asness et al (2014) propose that the consistent empirical evidence implies there are theoretical explanations behind WML, researchers just do not have the tools to understand it yet.

Heeding to these calls, Fama & French (2016b) augment the Five-factor model with WML to test a Six-factor model on U.S. Size-Mom portfolios. They find that the model alleviates at least in part the Five-factor model's inability to price momentum. Further, Fama & French (2018), include the Six-factor model, including several variations, in tests on a comprehensive U.S. dataset. Barillas & Shanken (2018) in testing ten common new factors also find that a six-factor model which includes momentum performs well, albeit this model used different factor proxies for some of the effects than Fama & French. Below is the equation for the Fama & French Six-factor model:

$$r_{jt} - r_{ft} = \alpha_i + \beta_i^M(r_{Mt} - r_{ft}) + \beta_i^{SMB}SMB_t + \beta_i^{HML}HML_t + \beta_i^{RMW}RMW_t + \beta_i^{CMA}CMA_t + \beta_i^{WML}WML_t + \varepsilon_{jt} \quad (10)$$

In the equation, r_{jt} is the return of security j in month t , r_{Mt} is the market portfolio return, r_{ft} is the risk-free rate, the SMB_t , HML_t , RMW_t , CMA_t and WML_t are the Fama & French factor proxies for size, value, profitability, investment and momentum, with their respective slope coefficients in front, and last is the error term ε_{jt} .

3 EMPIRICAL LITERATURE

This chapter presents important literature relating to the research topic. It starts off by presenting recent international literature on momentum and momentum models. To substantiate the case for adding a sixth factor, further empirical evidence revealing a mostly positive response of the Five-factor model is presented. This query has included (1) the comparison of the model to the Three-factor model, (2) whether it shortens the list of anomalies, and (3) factor redundancy. Within this literature review are several international studies to tangent this study's empirical purpose. In the previous chapter, research building on the case that ultimately led to Fama & French (2018) testing with a sixth factor were presented. Towards the end of this literature review, studies aiding in this study's research objective of testing a Six-factor model on the international plane will be presented. By chance, the subsequent topics follow a mostly chronological order. It starts by covering two major research papers related to the international presence of momentum and the subsequent performance of the Carhart model internationally, then it presents Fama & French's seminal presentation of the Five-factor model, with the subsequent papers by Fama & French and Cakici testing the model in various dimensions. The last two papers in this chapter incorporate tests of a Six-factor model adding momentum on known problematic anomaly portfolios on a U.S. dataset.

3.1 "Size, value, and momentum in international stock returns" by Eugene F. Fama & Kenneth R. French (2012)

In this study Fama & French examine international stock returns to provide clarity on three issues. First, following seminal research they investigate size, value and momentum patterns from developed markets. Second, they compare the performance of the Fama-French Three-factor model and the Carhart Four-factor model internationally on size-value and size-momentum portfolios. Third, they investigate whether asset pricing can be considered integrated across regions.

3.1.1 *Data & Method*

The sample period consists of monthly data from November 1989 until March 2011, a total of 245 months. 23 countries are grouped into the four regions of North America, Europe, Asia Pacific, and Japan to attain sufficiently large coverage. The international stock returns and accounting data used were taken from Bloomberg, Datastream and Worldscope. These four regions have since been used for much of the subsequent international asset pricing studies, by Fama & French and other authors. Due to heavy

focus on large stocks in the literature, Fama & French's sample considers all size groups and incorporates regional data.

The authors test the CAPM, the Three- and the Carhart model on 5x5 and 4x5 sorted Size-B/M and Size-Momentum portfolios with both regional and global factors. Incorporating regional factors and regional portfolios provides evidence as to how the models perform in different markets and incorporating global factors and global portfolios provides evidence of market integration. To test and compare the models, the authors use the GRS F-statistic and a set of summary statistics including the average absolute intercept, the average intercept standard error, the maximum Sharpe ratio of excess returns, $SR(a)$, and the average regression coefficient of determination R^2 .

3.1.2 Contribution

Fama & French find empirical evidence supporting the existence of the momentum in all regions except Japan, and the premiums decrease with size.

Fama & French demonstrate that when regional factors are used, the Carhart model performs as well or better against the Three-factor model on all four regional sets of Size-B/M portfolios. The Carhart model still produces moderately large average absolute intercepts on Size-Mom portfolios. The problems centre in the sets with heavy tilts toward winners and losers, but Fama & French state that empirical evidence shows these problems would rarely apply in applications. Therefore, the Carhart model may still prove useful to describe the momentum effect.

In testing market integration, the Carhart model is found to possess characteristics that aid the explanation of global Size-B/M and Size-Mom portfolios. However, the rejection of the GRS F-statistic and the large average absolute intercepts suggest the global CAPM, Three-factor and Carhart models do poorly for regional Size-B/M and Size-Momentum portfolios, implying failure in pricing integration. Subsequently, the authors do not recommend the employment of global models to explain regional portfolio returns and point instead to regional models.

3.2 “Value and Momentum Everywhere” by Clifford S. Asness, Tobias J. Moskowitz & Lasse H. Pedersen (2013)

Value and momentum have often been studied separately or in isolation rather than jointly, especially when specific assets or non-U.S. datasets are studied. By studying value and momentum across eight diverse markets, Asness, Moskowitz & Pedersen

uncover evidence of a connection between the anomalies. The focus is on the interaction of momentum and value strategies, but the study offers a favourable view of momentum internationally, going as far as suggesting there may exist a common global factor structure regarding value and momentum. Asness, Moskowitz & Pedersen adjudge momentum a universal factor that should be included in factor models. They investigate variation in value and momentum premia across markets and asset classes, correlation of returns, economic drivers, and benchmark models for portfolios of global securities.

3.2.1 Data & Method

Asness, Moskowitz & Pedersen's study is comprehensive, and studies eight different classes of assets calculated into returns from five types of data. All samples run until July 2011 but have different start dates due to the diverse set of databases used. First, global individual stocks from the U.S., U.K., continental Europe and Japan and run from January 1972 for U.S. & U.K., or January 1974 for Europe & Japan. Second, global equity indices of 18 developed countries run from January 1978. Third, spot exchange rates of 10 currencies run from January 1979. Fourth, global government bonds of 10 countries run from January 1982. Fifth, 27 different commodity futures run from January 1972. Asness, Moskowitz & Pedersen do not attempt coming up with superior predictors of returns, rather they use the simplest and most standard measures for value and momentum for each asset class. The authors construct 48 value and momentum test portfolios, and value and momentum factors for each asset class. Other test assets such as Size-B/M and Size-Mom portfolios were also included.

The authors begin by examining the performance of value and momentum separately and jointly in each market and asset class, and subsequently also study comovement. Then the authors conduct a series of asset pricing tests. They conduct informal tests such as measuring the performance of value and momentum from a market on others. They then conduct formal asset pricing tests, with the market, value, and momentum factors as inputs in a three-factor model used to estimate of each market and asset class, then comparing the results to actual average return. This analysis is extended to a Global three factor model to conduct formal tests and compare pricing models. This model is compared with the CAPM, the Carhart model and a six-factor model with two additional bond return factors. To infer the economic value of value and momentum, the authors run cross-sectional and time-series tests. Statistics used to compare are correlation, the GRS F-statistic, the average absolute alpha, the cross-sectional R^2 , and the Sharpe ratio.

3.2.2 Contribution

Asness, Moskowitz & Pedersen find consistent and universal value and momentum return premia across the eight different markets and asset classes, including novelties to the literature. Confirming earlier studies, momentum is not a significant premium in Japan. However, they find a strong common factor structure among their studied returns. Asness, Moskowitz & Pedersen not only find comovement subject to all markets studied, but value and momentum strategies comove across asset classes as well. Value strategies were found to positively correlate with other value strategies, and momentum strategies were found to positively correlate with other momentum strategies. The authors found a negative correlation between value and momentum, even across asset classes, implying there exists a global factor structure. Cakici, Fabozzi & Tan (2013) confirm the negative relationship extends to emerging markets. Asness, Moskowitz & Pedersen's global three-factor model performs better than the other models in the initial tests and performs well on other test assets as well.

Asness, Moskowitz & Pedersen suggest that value and momentum are best used jointly as separate variables in a model. They state that momentum should be viewed in the context of the combination of value and momentum, even in Japan. Although momentum does not work well in Japan, value works exceptionally well in Japan, which is an implication they believe the literature should confront. The authors link funding liquidity risk as a reason behind the relationship between value and momentum. They state that their results indicate that global risk factors exist for which both value and momentum compensate. They speculate that momentum represents investors clustering around popular assets that have appreciated, and value represents the contrarian position. Asness, Moskowitz & Pedersen state that the comprehensive findings of momentum should pacify concerns about data mining even if it does not conform with common rational theory, as empirical evidence clearly favours momentum and rational theory alone is unfit to explain other asset classes than equity.

3.3 “A five-factor asset pricing model” by Eugene F. Fama & Kenneth R. French (2015)

Fama & French present a model that adds two additional factors to the Three-factor model. The Five-factor model combines factors representing size, value, profitability and investment. Following Fama & French (2006) where SMB and HML were related to rational risk-based theory, here they follow suit with RMW and CMA, meaning the Five-factor model proposed is presented as rationally motivated.

3.3.1 Data & Method

The sample includes all U.S. equities available from CRSP and Compustat that meet criteria of available information. The sample runs from July 1963 until December 2013 covering 606-months. The primary factors are constructed through 2x3 sorts, but they also test 2x2 and 2x2x2x2 sorts in order to compare. The market is proxied by a VW portfolio of all stocks, using the U.S. one-month Treasury bill as the risk-free rate.

The definitions of SMB and HML are the same as earlier research, and the two factor additions, profitability, RMW, and investment, CMA, use empirically motivated proxies. RMW and CMA are constructed like HML, but with a second sort on either operating profitability or investment. RMW is formed as the portfolio return on a well-diversified set of stocks with high operating profitability minus portfolio return on a well-diversified set of stocks with low operating profitability. CMA is formed by the portfolio return of a well-diversified set of low-investment stocks minus portfolio return of a well-diversified set of high-investment stocks. For their LHS portfolio creation process, Fama & French use two distinct sorting methods. They create 25 portfolios per pair of 5x5 sorts of Size-B/M, Size-Profitability, and Size-Investment and 32 portfolios per pair of 2x4x4 sorts of Size-B/M-Profitability, Size-B/M-Investment, and Size-Profitability-Investment. The 5x5 portfolios use NYSE quintile breakpoints on size and the additional characteristics, whereas the 2x4x4 use the NYSE median for its size breakpoint and NYSE quartiles for the two additional characteristics.

The authors test the Five-factor model against the Three-factor model and for this purpose they conduct two distinct approaches. Testing whether a factor can be ruled redundant or a useful addition, Fama & French conduct factor spanning regressions where each factor, in turn, is put on the LHS to be explained by the four remaining factors on the RHS. In testing and comparing model performance they use the LHS method with test assets. They employ the GRS F-test and summary statistics based on analysis of average absolute alpha. Fama & French show relative performance for three-, four- and five-factor model compositions for all sets of LHS portfolios.

3.3.2 Contribution

The key finding of the paper is that the Five-factor model performs well in the U.S. and provides a better description of average returns than the Three-factor model with regards to the test portfolios. Despite that the GRS F-test was found to reject the return description of the models in their comparison, the Five-factor model is concluded a

decent model as unexplained average return for individual portfolios is consistently close to zero. In fact, the Five-factor model outperforms the Three-factor model on all metrics and is deemed robust regardless of the factor or portfolio construction process used. However, a four-factor model without HML performed similarly to the Five-factor model on the performance measures. Factor spanning regressions imply HML is redundant. Fama & French suggest it is made redundant by RMW and CMA and propose that alternative value factor definitions may be needed. Fama & French still favour the Five-factor model, stating that addition and subtraction of factors have their difficulties. Other problems were also documented. A major asset pricing problem are portfolios of small stocks with negative exposure to RMW and CMA. The Five-factor model fails to capture low average returns on small stock portfolios with negative exposures to RMW and CMA, whose returns behave alike companies that invest in spite of low profitability. In their concluding remarks, Fama & French discuss controlling for additional variables. Hinting at the addition of momentum they note the problem that correlations among variables may cause poor diversification of portfolios used in factor construction. Conversely, if variables were to be dropped, hinting at HML, the factors would need to be reconstructed due to the possibly detrimental nature of controlling for unused characteristics.

3.4 “International tests of a five-factor asset pricing model” by Eugene F. Fama & Kenneth R. French (2016a)

Fama & French test the Five-factor model out-of-sample on international stock returns. This is one of the two studies conducted to acquire further evidence to their findings in Fama & French (2015), the other study Fama & French (2016b) testing the model on various anomalous effects known to cause problems with the Three-factor model.

3.4.1 Data & Method

The sample includes equities from the 23 developed markets grouped into the four regions of North America, Europe, Japan and Asia Pacific as in Fama & French (2012), taken from Bloomberg, Datastream and Worldscope. The sample period runs monthly from July 1990 to December 2015. This is shorter than Fama & French (2015) due to the availability of data in international studies.

The study is conducted similarly to Fama & French (2015) for the results to remain comparable. Consequently, the LHS portfolios are generated akin to Fama & French (2015), although the factor creation process is limited to only 2x3. The factor construction method differs as instead of NYSE breakpoints they adopt the top 90% for

and bottom 10% percentiles for big and small stocks respectively. For B/M, OP and Inv, the breakpoints are at the 30th and 70th percentiles of the big stock variable corresponding to the region. The LHS portfolio breakpoints differ too. For the 5x5 sorts, the breakpoints for size are at the 3rd, 7th, 13th and 25th percentiles of aggregate market capitalization, and the breakpoints for B/M, OP and Inv here are the same as the 2x3 sorts bar the use of quintile breakpoints for the big stocks. For 2x4x4 sorts, with size defined as the upper 90% and bottom 10% of market capitalization, and quartiles for the remaining characteristics. Despite the international sample, the authors again employ the U.S. one-month Treasury bill as the risk-free rate.

Fama & French (2016a) conduct tests of model comparison on test assets, and factor spanning regressions are also consulted. To test and compare model performance on test assets, the GRS F-statistic and summary statistics are employed. The summary statistics used for this study were slightly changed from Fama & French (2015), which is tackled more in depth in the next chapter. The evaluation criteria are the average absolute intercepts, the ratio of unexplained dispersion of LHS average returns relative to the total dispersion of LHS average returns, a measure that tells about the intercept dispersion relative to the dispersion of LHS average returns, and the average regression R^2 .

3.4.2 Contribution

While investigating return premia, Fama & French find that for small stocks in North America, Europe, and Asia Pacific average equity returns increase with B/M and profitability but decrease with investment. In Japan, average equity returns increase with B/M, but there is no strong relation to profitability and investment.

Fama & French's factor spanning regressions return different results from one region to another. Only HML and RMW prove important in all regions. SMB is only important in North America. CMA proves itself redundant in Europe and Japan. Contrary to Fama & French (2015), factor spanning tests here reveal a different story about HML, as all five factors contribute unique information in North America. Fama & French chose to drop Japan from the LHS test result discussion comparison since the lack of variation in the sample means the results unlikely poses much of a threat to the Three-factor model. The Five-factor model performs well internationally and outperforms the Three-factor model in all three remaining regions on all metrics considered. With the factor spanning results in mind, Fama & French conduct LHS tests with four-factor models dropping variables in a similar fashion to Fama & French (2015). Indeed, the Five-factor model performance

is sometimes tied with two four-factor models that drop HML or CMA. However, for North America, Europe and Asia Pacific, the Five-factor model outperforms four-factor models that drop HML or RMW, whereas a four-factor model that drops CMA achieves similar performance to the Five-factor model in Europe. Thus, the problem of redundant factors seems to carry over into the LHS tests. Furthermore, the Three-factor model consistently underperforms all the models considered. Fama & French also test global models and comment on market integration. They find that global Three- and Five-factor models perform poorly on the regional test assets and recommend the use of local factors. However, even local models seem subpar in Asia Pacific, the region initially hypothesized to be least integrated. The authors reaffirm the finding of Fama & French (2015), that the Five-factor model tends to not capture the lower average returns of smaller stocks whose returns behave like those of low profitability firms that invest aggressively. In their concluding remarks, Fama & French comment that they expect the literature to add factors, such as momentum, and pinpoint redundant factors.

3.5 “The Five-Factor Fama-French Model: International Evidence” by Nusret Cakici (2015)

Alike Fama & French (2016a), Cakici tests the Five-factor model on an international sample, compares the model with several asset pricing models and tests the extent of integration between the markets. The study is significant in that it includes additional model variations to which it compares the Five-factor model, and sheds additional light on the factor redundancy discussion. Compared to Fama & French (2016a), Cakici both confirms and contradicts observed trends in the former.

3.5.1 Data & Method

Cakici uses data from July 1992 to December 2014 of 23 developed stock markets grouped into the regions of North America, Europe, Japan, Asia Pacific, and Global. Cakici constructs 2x3 sorted factors and forms 25 Size-B/M, 25 Size-gross profitability, and 25 Size-Inv portfolios through 5x5 sorts. The paper investigates seven asset pricing models, the Five-factor model and six models testing multiple Fama & French factor variations; three-factor models that combine MKT & SMB with HML, RMW or CMA, and four-factor models that combine MKT & SMB with pairs of HML, RMW and CMA. To evaluate model performance Cakici uses the GRS F-test statistic and summary statistics; the average absolute intercept, the standard error of the intercepts, the average coefficient of determination R^2 , and the unexplained squared Sharpe ratio, $SR(a)$. This

approach is similar to that of Fama & French (2012). Cakici further conducts factor spanning tests to evaluate possible factor redundancy.

3.5.2 Contribution

The factor spanning tests reaffirm Fama & French's (2016a) results that HML is not redundant in most regions. HML is a strong contributor in North America, Europe, Japan, and Asia Pacific. Like Fama & French (2016a), the results show HML to be marginally significant in North America, contrary to Fama & French (2015). However, in contradiction with Fama & French's result that CMA is important in North America and Asia Pacific but redundant in Europe and Japan, Cakici finds CMA important in Europe and redundant in North America, Japan and Asia Pacific.

Cakici demonstrates that the Five-factor model holds similarly on North American, European and Global Markets as it does on a U.S. data set. When comparing it to the Three-factor model, the GRS F-test suggests that the Five-factor model outperforms the Three-factor model overall, in line with (Fama & French (2016a)). However, in contrast with the general sentiment of Fama & French (2016a), when extending the comparison to all six comparison models, Cakici states that the Five-factor model is rarely the clear winner. In fact, the results emphasize the performance of various four-factor models, confirming the conclusions made in earlier papers that nested four-factor models often perform similarly to the Five-factor model. While the edge in favour of the Five-factor model over the Three-factor model carries over to the summary statistics, Cakici does not deem this substantial enough. As the results of Japan and Asia Pacific are consistently worse, Cakici questions the applicability of the Five-factor model. Further, Cakici's results suggest the markets are not fully integrated.

3.6 “Dissecting Anomalies with a Five-Factor Model” by Eugene F. Fama & Kenneth R. French (2016b)

In the previous papers, LHS portfolios were finer sorts on the characteristics used in factor construction. In this study, Fama & French consider anomalies the model does not target and that have been demonstrated problematic for the Three-factor model. Fama & French investigate whether the Five-factor model is able to describe the average returns of portfolios sorted on these assorted anomalies.

3.6.1 Data & Method

The sample consists of 618 months of U.S. equity data from July 1963 until December 2014. Fama & French form portfolios sorted on size and each anomaly characteristic of

interest in turn; 25 Size-Market Beta, 35 Size-Net Share Issues, 25 Size-Return Variance, 25 Size-Residual Variance, 25 Size Accrual, and 25 Size-Momentum portfolios. Fama & French form six factors, the Fama-French factors and WML. In LHS tests, Fama & French test the Five-factor model against the Three-factor model, three four-factor models and CAPM. The Six-factor model is tested on the Size-Mom portfolios. The authors employ similar evaluation metrics as Fama & French (2016a); the GRS F-test statistic, the average absolute alpha, the ratio of unexplained dispersion of LHS average returns relative to the total dispersion of LHS average returns, the intercept dispersion relative to the dispersion of LHS average returns, and the average regression R^2 .

3.6.2 Contribution

The Five-factor model outperforms the Three-factor model on all portfolio sorts and Fama & French conclude that the list of anomalies shrinks with the Five-factor model in comparison to the Three-factor model. The authors suggest this is partially explained by anomalous returns becoming less anomalous, and partially by anomaly returns sharing factor exposures implying they describe the same effect. The Five-factor model reduces the average returns of anomalies left unexplained except for accruals and momentum, which remain problematic. However, all models not incorporating WML are unable to describe the returns of Size-Mom portfolios. These findings are consistent with the literature in that neither the Three- nor the Five-factor model can price momentum. Fama & French show that the Six-factor model offers relief to these problems. Adding WML improves model performance, while there are still unexplained momentum returns among small stocks. These results contrast Fama & French's own custom not to include momentum, the results here are treated as a hint for the task of augmenting the model to six on an international plane.

3.7 “Choosing factors” by Eugene F. Fama & Kenneth R. French (2018)

Fama & French (2018) investigate nested models on a similarly comprehensive U.S. sample as Fama & French (2015). Heeding to popular demand, they incorporate the Six-factor model and variations into their tests.

3.7.1 Data & Method

The sample consists of U.S. equity data running 636 months from July 1963 until June 2016 which they split into adjacent pairs of 318 months. The authors compare two types of factors, excess return factors and spread factors. Fama & French dedicate the methodology to comparing two approaches to compare relative model performance, the

LHS and RHS approaches. They use factor spanning regression and utilizes GRS F-tests on the spanning regressions to test whether multiple factors add to a base model's explanation of expected return. The research hypothesis is that this methodology is particularly useful in choosing among nested models. Additionally, they investigate a new performance metric by Barillas & Shanken (2016) for the RHS approach. Barillas & Shanken's performance metric lies in the assumption that models should be compared based on the max squared Sharpe ratio for intercepts produced from regression of LHS returns on the factors. Studying several possible six factor model compositions, they look at the marginal contribution of factors to the model's squared Sharpe. In LHS tests they form 5x5 portfolios on ME and BE/ME, OP, CP, Inv and Mom, as well as problematic anomaly portfolios sorted on 5x5 ME and beta, accruals, variance of daily returns and residuals, and 5x7 ME and net share issuance. To compare they apply the summary statistics of Fama & French (2016a; 2016b) as well as the max mean squared Sharpe.

3.7.2 Contribution

Based on economically large intercepts and t-statistics corresponding to significance at the 1% level, Fama & French's factor spanning regressions reveal that the momentum factor adds to the Five-factor model. The factor also provides marginal contribution in all three model dimensions considered. On Barillas & Shanken's metric, Fama & French find a six-factor model combining a set of small stock spread proxies for market, size, value, profitability, investment, and momentum factors dominates the other models. They also compared several possible six-factor model compositions from different factor proxies. However, the base model is also a decent performer, therefore they do not recommend a switch of factors. Fama & French also introduce a cash profitability factor, RMWc, with this paper. Fama & French's results imply that this factor dominates operating profitability on Barillas & Shanken's approach, but on other metrics there is not much of a difference.

Fama & French conclude that the RHS approach is useful for choosing among nested models and particularly in the decision regarding an additional factor. In their concluding remarks they emphasize that factors should be linked to theory to limit the data dredging of including more and more variables into asset pricing models. They caution that the absence of theoretical footing models incentivises such data fishing.

4 METHODOLOGY

This chapter presents the research questions, sets the hypotheses to answer them, and presents the methodologies to be employed in Chapter 6.

4.1 Research questions & hypotheses

The purpose of this thesis is to investigate the implications of adding momentum to the Five-factor model, and to compare the resulting Six-factor model's efficacy on an international dataset. This can be developed into three distinct but broadly interlinked research questions. As the theoretical framework and literature review illustrate, the research topic is driven by the underlying presumption that WML comprises valuable characteristics for the description of return. Thereby follows the first research question:

Does WML provide enough unique and useful properties to be considered a valuable addition to the Five-factor model in North America, Europe, Japan, and Asia Pacific?

Furthermore, as discussed in Chapter 2, the efficacy of a model often embodies two further interwoven implications, the model's performance in describing average returns and the relative performance to an established model. A model should describe various asset characteristics accurately to be deemed a practical tool. As the literature suggests, a presumptive benefit of adding WML should arise from the ability to mitigate momentum. The second research question follows:

Can the Six-factor model describe average returns in North America, Europe, Japan, and Asia Pacific?

Of particular concern for the Six-factor model's practical viability is its relative performance to the Five-factor model. The third research question is as follows:

Can the Six-factor model describe average returns better than the Five-factor model in North America, Europe, Japan, and Asia Pacific?

Turning now to the formulation of appropriate testable hypotheses, as demonstrated in the literature review, a typical strategy used to judge asset pricing models is through the intercepts produced from time-series regression on excess return. This is the LHS approach which customarily tests a model on test portfolios sorted on different asset characteristics. The alternative, the RHS approach, uses mean-variance factor spanning to test whether the RHS factors can price each other.

Due to contradicting evidence as to whether momentum exhibits across all markets, it is motivated to investigate whether WML is important in the various markets considered. Mean-variance spanning tests are employed to investigate whether WML's contribution is mostly unique. A factor that is explained by the exposures to other factors of an asset pricing model does not lend useful properties to the model. To investigate whether WML is a valuable addition, four factor spanning regressions are run corresponding to each of the four international regions of this study. Mean-variance spanning is introduced more in depth in Chapter 4.3. If WML returns economically significant intercepts, WML is not redundant and is considered to be a valuable addition. The first statistical hypothesis reads as following:

$$H_0: \alpha = 0$$

$$H_1: \alpha \neq 0$$

The question of whether the Six-factor model can explain average returns in the four regions can be extended to LHS tests. The formal GRS F-statistic tests the hypothesis that the regression intercepts for all test portfolios are jointly equal to zero. This formal test of power is used to accept or reject the model's complete description of return. Therefore, it contributes to the research question of whether the Six-factor model provides an adequate description of returns. Statistical hypothesis 2:

$$H_0: \alpha_i = \dots = \alpha_n = 0$$

$$H_1: \alpha_i \text{ different than zero, for some } i = 1, \dots, n$$

The GRS F-test is an absolute test of model performance. As discussed later in Chapter 4.4.1, GRS F-test statistics are not suitable for strict comparison and tend to over-reject. To assess model performance, an asset pricing model must be compared to an alternative model, here the Five-factor model. Therefore, this study employs an analysis of average absolute alpha to offer performance measures which can be compared. This study increases the power of these inferences with the employment of additional supplementary summary statistics, introduced in Chapter 4.4.2. However, as shown in Fama & French (2015; 2016a), these results can be at odds with the former test of power. The GRS has a tendency to over-reject on multiple sets that in analysis of summary statistics yield economically insignificant average absolute alpha (Goyal, Zhongzhi & Sahn-Wook, 2018).

Lastly, the study complements the discussion by reporting the number of significant alphas for each set of LHS test assets. This study aims to discuss improvements in the individual regression alphas from within the sets of test assets. Therefore, a third statistical hypothesis is set. In individual tests of alpha, if a model is to completely capture expected return, the intercept should be indistinguishable from zero:

$$H_0: \alpha_i = 0$$

$$H_1: \alpha_i \neq 0$$

The number of significant alphas presents additional information regarding which model produces fewer individual regression intercepts that are non-zero. However, the focus of the analysis of individual alphas is to provide further insights on and discuss the location of problems in the models' ability to describe the return of the various LHS test portfolios.

4.2 Empirical framework

This thesis tests a version of the momentum augmented six-factor models tested in the above-noted literature (Asness, 2014; Fama & French 2016b; 2018). The Six-factor model is then compared against the conventional Five-factor model. The Five-factor model to be tested is denoted as following (see Chapter 2.5.2 for further detail):

$$r_{jt} - r_{ft} = \alpha_i + \beta_i^M (r_{Mt} - r_{ft}) + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + \beta_i^{RMW} RMW_t + \beta_i^{CMA} CMA_t + \varepsilon_{jt} \quad (11)$$

In the equation, r_{jt} is the return of security j in month t , r_{Mt} is the market portfolio return, r_{ft} is the risk-free rate, the appropriate factors & betas and the size, value, profitability and investment factors, and last is the error term ε_{jt} . The Six-factor model tested in this thesis is denoted as following (see Chapter 2.6 for further details):

$$r_{jt} - r_{ft} = \alpha_i + \beta_i^M (r_{Mt} - r_{ft}) + \beta_i^{SMB} SMB_t + \beta_i^{HML} HML_t + \beta_i^{RMW} RMW_t + \beta_i^{CMA} CMA_t + \beta_i^{WML} WML_t + \varepsilon_{jt} \quad (12)$$

In the equation, r_{jt} is the return of security j in month t , r_{Mt} is the market portfolio return, r_{ft} is the risk-free rate, the Kenneth French factors size, value, profitability, investment and momentum factors are the standard factor definitions of Fama & French international literature (Kenneth French Data Library, 2020). Last is the error term ε_{jt} .

The approach to empirical research adopted for this study uses the output of regression analysis with ordinary least squares as the basis for the estimation and tests of

significance. This study uses a threefold methodology consisting of (1) factor spanning, (2) the GRS F-test and (3) a set of evaluation metrics with footing in the analysis of average absolute alpha. Further, as an additional metric, individual alphas within the test assets generated with each model are studied and reported to discuss the source of the improvements, or deterioration, resulting from the employment of the new model.

To test model performance, the study first applies the output of the factor spanning regression to infer whether the momentum factor adds to the description of average returns in each region. Second, an analysis of LHS portfolio intercepts is employed, with the formal GRS F-test to test whether the models completely explain average returns, and inferences of relative performance made from the comparison of average absolute alphas supplemented with additional summary statistics. The methodology is inspired by Fama & French's works on the Five- and the Six-factor models covered in the previous chapter, which this study closely follows and aims to extend upon. Other models, such as the CAPM, the Three-factor model and the Carhart model, are left out as they are both nested on the two models being compared.

The individual and joint comparison of alpha require output from time-series regression on large sets of LHS portfolios. Lewellen, Nagel & Shanken (2010) caution that such asset pricing tests can mislead strong explanatory power. In essence, with enough data any model can be rejected by getting the standard errors low enough to estimate the alphas. Harvey, Liu & Zhu (2016) demonstrated this sentiment as they re-evaluated the statistical significance of previous research and found that a lot of research within the immediate field of finance is likely to return false out-of-sample. Herein, Lewellen, Nagel & Shanken (2010) offer a few suggestions for improving the empirical tests that may be useful for the purposes of this thesis project. First, a model should be tested on different portfolio sorts and characteristics. Second, the level of confidence in the output from test statistics should be reported and the bar set appropriately. Harvey, Liu & Zhu (2016) state that the OLS regression may overstate the significance of the cross-sectional asset pricing tests, thus recommending not using weak t-statistic cut-offs.

4.3 Mean-variance spanning tests

Fama & French (2018) state that RHS tests are powerful in the choice between nested models, particularly in the choice of whether a factor should be added. This thesis employs a series of factor spanning tests to justify the use of the Six-factor model in international asset pricing tests and to assess how the contending factors contribute to

the explanation of average return in four international regions. In their seminal paper, Huberman and Kandel (1987) presented the mean-variance spanning approach. Mean-variance spanning (hereafter factor spanning) regresses returns of one factor on other factors. Put differently, factor spanning tests whether an explanatory variable can be explained by the combination of the remaining explanatory variables. Factor spanning tests are conducted to test the viability of a model's factor composition. This is done by putting one factor on the LHS and regressing it with the rest of the factors. Based on the regression output, one can make inferences about which factors provide unique and economically significant properties that make them important in describing returns. The regression intercept provides the average return that would be left unexplained by the exposures of these factors. Therefore, the test is also useful in revealing any explanatory factors which can be explained by the combination of the other factors making it a redundant factor in the description of returns. The logic follows that if average return is captured by exposures to other factors, then that factor is fruitless in the model's explanation of average returns (Fama & French, 2016b). Dropping such a factor from the model should then be considered.

The following formula illustrates a test of factor spanning, with the momentum variable on the LHS:

$$WML_t = \alpha_s + \beta_s^m(r_{Mt} - r_{ft}) + \beta_s^{smb}SMB_t + \beta_s^{hml}HML_t + \beta_s^{rmw}RMW_t + \beta_s^{cma}CMA_t + \varepsilon_t \quad (13)$$

where α_s is the regression intercept of the spanning regression, the β_s are the slope coefficients for the respective factors, $r_{Mt} - r_{ft}$, SMB_t , HML_t , RMW_t , CMA_t represent the remaining five excess return factors on the RHS trying to describe the excess returns of the sixth. ε_t denotes the regression residuals.

Following the framework of Huberman & Kandel (1987), this study seeks to make judgements about whether WML improves the mean-variance-efficient tangency portfolio produced with the Fama & French asset pricing factors, in the four regions of North America, Europe, Japan and Asia Pacific. The empirical study starts with the factor spanning tests, as these results are used to justify the utilization of a Six-factor model in the international asset pricing tests. Further, the factor spanning tests allow this study to discuss and contribute to the literature on the redundancy of factors.

As factor spanning tests are definitive within a sample, a redundant factor captured by the other factors can be dropped from the model. However, on the macro-level, factor

spanning inferences are sample specific (Fama & French, 2016a), meaning that factor redundancy needs to be confirmed through testing numerous samples over different sample periods, hence the growing literature testing asset pricing models on different samples. Here, this study extends the literature on the Six-factor model to the international plane.

4.4 Summary asset pricing tests

That factor spanning tests imply the factor is important does not mean it helps describe all LHS portfolio returns with nontrivial loadings on that factor (Fama & French, 2016a). Therefore, the factor spanning tests are not enough to answer the question of how a Six-factor model will perform on the different test portfolios. In this study, from the output of time-series regression on excess return of test portfolios, the Six-factor model's ability to describe international test assets is assessed and subsequently compared to the Five-factor model. Formal tests will be conducted, and informal summary statistics will be calculated based on output from these regressions. Multiple portfolio characteristics are employed, including the Fama & French sorts on Size and each of the following, B/M, Operating profitability, and Investment. A novel contribution for this study is the inclusion of four sets of regional portfolios sorted on Size and Momentum. While the primary target of this analysis concerns these 5x5 sorted portfolios, the study further complements the analysis by showing results of how the models perform on triple sorted portfolios sorted on Size and combinations of B/M, Operating profitability, and Investment. As a result, the study uses portfolios of two different sorting methodologies. The sample will be presented more in depth in Chapter 5.

4.4.1 The GRS F-test

If expected return is completely captured by an asset pricing model, the intercept from regression of an asset's excess returns on the factor returns should be indistinguishable from zero (Fama & French, 2015). With test assets and factors, the Gibbons, Ross & Shanken (1989) F-test (or GRS) tests this hypothesis. It tests mean-variance efficiency between an RHS model and a set of LHS portfolios and can be employed to obtain an absolute answer to whether an asset pricing model provides a complete description of return patterns (Erdinc, 2017). In asset pricing literature, GRS F-statistics are used to either accept or reject a model in its ability to explain test asset returns. Furthermore, because the GRS F-statistics are generated from the comparison of optimal LHS and RHS portfolios, it is not useful for strict comparison of two models (Erdinc, 2018).

The GRS F-test is computed from the portfolio regression output. Given a certain asset pricing model the GRS will evaluate a whole portfolio set. If the GRS F-test is run on a single portfolio, i.e. number of regressions are equal to 1, then the GRS tests that one portfolio as a significance test of alpha, thus essentially making it comparable to the normal t-test. However, when the asset pricing model is tested on multiple portfolios comprising multiple alphas the GRS will assume the F distribution (Gibbons et al, 1989). Given a set of multiple alphas generated from regression outputs for each corresponding portfolio, the GRS tests whether these alphas are jointly indistinguishable from zero (Erdinc, 2017). Econometrically, the GRS is the following function:

$$\frac{T - N - K}{N} (1 + E[f]' \Omega^{-1} E[f])^{-1} \alpha' \Sigma^{-1} \alpha \sim F_{(N, T-N-K)} \quad (14)$$

Where T equals the number of observations, K equals the number of explanatory variables, N equals the number of portfolios tested and equal to the number of estimated individual alphas in a set of LHS portfolios. $E[f]$ is a $K \times 1$ vector of factor mean values, and thus $E[f]'$ is its $1 \times K$ transpose. Ω^{-1} is the inverse of the Ω covariance matrix of the K explanatory variables. Σ^{-1} is the inverse of the regression residual covariance matrix Σ of N error terms. Additionally, the probability of attaining a GRS F-statistic larger than the one observed if the true intercepts are all zero is to be calculated and be denominated $p(GRS)$.

As the GRS F-test tests whether a model is true, not whether they have differing values or whether one model performs better than another, the GRS typically rejects with high confidence, as shown in earlier literature (Barillas & Shanken, 2018). Since previous research suggests most models are rejected by the GRS F-test, the statistic alone is seldom used to make inferences about a new factor addition. As common to the literature, this study faces bad model problems as any model only approximates the pricing process, and consequently likely to be rejected in tests of power (Fama & French, 2012). Models may also fail due to improper factor construction or due to the difficulty of capturing anomaly patterns with factors of simple sorts (Fama & French, 2012). The GRS F-test tests the absolute fit of an asset pricing model, and since most are rejected, the statistic is often treated as a complementary statistic in asset pricing tests.

4.4.2 Comparison of average absolute alpha with summary statistics

This study is focused upon the relative improvements in the descriptions of average returns provided by the Six-factor model in comparison to the Five-factor model.

Therefore, it ought to determine which model provides a better story of average returns while still acknowledging that the models remain imperfect.

Individual and joint assessment of alpha will be conducted to make relative inferences of model performance from the regression output of LHS test portfolios formed on different characteristics. The joint approach employs metrics to investigate the pricing errors generated by each asset pricing model for each set of 25 or 32 portfolios using a set of informal summary statistics with basis in comparison of average absolute alpha. Following the methodology of Fama & French (2015; 2016a), a range of summary statistics are employed to strengthen the inferences made from the comparison of average absolute alphas. Therefore, this study is able to test which model provides the better description when regressed on returns on portfolios formed on diverse characteristics while still acknowledging that the models remain imperfect.

The methodology of the empirical study relies heavily on Fama & French's (2015; 2016a; 2016b) approach of comparing average absolute alphas complemented with three summary statistics computed from return deviation and alpha. If A is set to indicate an average value, the average absolute intercept is denoted $A|a_i|$ and is the simplest of the summary statistics. The average absolute alpha gives a comprehensive picture of the performance of each model on each set of 25 and 32 portfolios. The reference point concerning its dispersion is zero because the asset pricing hypothesis states that the true intercepts are zero. Therefore, the better model at describing return should provide a lower average absolute intercept than the model it is compared to.

The next two summary statistics concern measures of dispersion of the intercepts produced by an asset pricing model. In Fama & French (2015), dispersion of the LHS average excess returns is measured relative to the simple average of all LHS average excess returns. However, from an asset pricing perspective, Fama & French (2016b) argue the VW market average excess return is the logical reference point for measuring the dispersion of LHS average excess returns. This definition was employed in Fama & French (2016a) and will be employed in this study as well.

Let R_i denote the time-series average excess return on LHS portfolio i , and let R denote the cross-section average, then let r_i be defined as the difference between the average cross-section and the average return on the VW market, so that $r_i = R - R_i$. Thus, the first of the three additional summary statistics is given as the average absolute intercept over the average absolute value of r_i , $A|a_i|/A|r_i|$ (Fama & French, 2016b). This metric

tells the range which the $|a_i|$ are in respect to $|r_i|$ for the different sets of LHS portfolios and factor definitions, and expresses the proportion of the dispersion in average excess returns that a model leaves unexplained. The second summary statistic is the average squared intercept over the average squared r_i , Aa_i^2/Ar_i^2 . It is also a ratio of intercept dispersion produced by a model relative to the dispersion of LHS average returns (Fama & French, 2016a). In other words, the first tells the proportion of average excess return dispersion the model leaves unexplained, and the second tells the proportion of LHS expected return variance left unexplained. Fama & French's (2015) variant of the second metric, $Aa_i^2/A\mu_i^2$, adjusts the numerator and denominator for measurement error. In Fama & French (2015), Aa_i^2 is the difference between the squared estimates of the regression intercept and its standard error. Similarly, their version of $A\mu_i^2$ is the average of the difference between portfolio realized deviations r_i^2 and the squares of their standard errors. In both ratios, the numerator measures model intercept dispersion for a set of LHS portfolios and the denominator is a measure of dispersion of LHS expected returns. Fama & French (2016b) noted that for international studies this double adjustment can produce extreme ratios. They speculate that because international studies work with a shorter sample period, the issue would lie in measurement error in the estimates of measurement error. Because of this issue, Fama & French (2016a; 2016b) do not double adjust their ratios, but rather they include an additional metric estimating the proportion of the dispersion of the intercepts that can be traced back to sampling error. This study takes this approach as well. The intercept estimate is the true intercept plus an estimation error, $a_i = \alpha_i + \varepsilon_i$. As α_i is a constant the expected value of a_i^2 is $E(a_i^2) = \alpha_i^2 + E(\varepsilon_i^2)$, and averaging over LHS assets it becomes $AE(a_i^2) = A\alpha_i^2 + AE(\varepsilon_i^2)$. Since the expected value of ε_i , $E(\varepsilon_i)$ is a_i variance due to estimation error (Fama & French, 2016b). The estimate for $E(\varepsilon_i^2)$ used in this study is the squared sample standard error of a_i , $s^2(a_i)$. Given the average of the squared sample standard errors of the intercepts is denoted $As^2(a_i)$ and Aa_i^2 is the sample estimate of $AE(a_i^2)$, the third summary statistic, $As^2(a_i)/Aa_i^2$, measures the proportion of unexplained average returns dispersion attributable to sampling error (Fama & French, 2016a; 2016b).

To interpret the complementary dispersion values, the respective values for $|a_i|/|r_i|$ and Aa_i^2/Ar_i^2 are better the lower they are, as it implies intercept dispersion is proportionately low to that of the LHS average returns. Conversely, a large value of $As^2(a_i)/Aa_i^2$ would be better as it implies a large portion the dispersion of the intercepts

is sampling error rather than dispersion of the true intercepts. This is akin to a coefficient of determination of dispersion (Fama & French, 2016b). Consequently, integrating these summary statistics establishes a more holistic approach to the making of inferences based in average absolute alpha, thereby increasing the power of these inferences.

Last, this study shows two additional statistics. First is the number of significant alphas among each set of 25 or 32 portfolios. Analysis of individual alphas within sets of test portfolios is a common approach to compare models. Moreover, the analysis of individual alphas extends upon the comparison of average absolute alphas by locating the improvements. If the hypothesis reads that for a model completely describing returns, the alphas should be indistinguishable from zero, then a better model should produce fewer significant alphas within the sets of test portfolios. This study treats the number of significant alphas as an additional summary statistic for the results discussion. The second additional statistic is the average coefficient of determination, AR^2 . The coefficient of determination is used to measure the fit of a model for a given sample. The standard coefficient is invariably additive and will increase with the incorporation of additional factors to the regression, while the adjusted statistic does not increase the value invariably as the factors are added and therefore offers a better comparison. Therefore, this study calculates the statistic from the coefficient adjusted for degrees of freedom, Adjusted R^2 . The coefficient of determination will be used to offer additional detail into the fit of the same set of regressions between the regions, and will be interpreted as a model diagnostic toward the assumption made of the sample, that the diversified regions are sufficient proxies for their respective markets.

5 DATA

5.1 Sample selection

As data availability presents a hurdle in international studies, data for this study was collected from Kenneth French Data Library. The sample period consists of monthly data from 1.11.1990 to 31.07.2019, running from the earliest date from Kenneth French's database considering all data requirements. As the study extends upon recent literature, it covers regional data of the four international regions of North America, Europe, Asia-Pacific (excl. Japan) and Japan. The need for a regionally diversified sample stems from a data constraint with international samples as opposed to the more than half a century worth of reliable data frequently used in U.S. asset pricing tests. As a result, regionally diversified LHS portfolios are used to increase the power of the tests, as the subsequently improved regression fits enhance the precision of the intercepts (Fama & French, 2016a). Regional factors are also preferred due to evidence suggesting regional models fare better in explaining diversified returns than global models. This implies that there is not enough market integration between various countries (Griffin, 2002) or regions (Fama & French, 2012; 2015; 2016a; Cakici, 2015). Due to these caveats, this study restricts its empirical analysis to testing only regional factors.

To diversify internationally, the four regions comprise of 23 developed and stable countries grouped by approximate location and resemblance. The first region, North America, consists of Canada and the United States. The second region, Europe, consists of Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. The third region, Asia Pacific, consists of Australia, Hong Kong, New Zealand, and Singapore; and the fourth region, Japan, is its own region (Fama & French, 2012).

In commenting on the regions, Fama & French (2016a) state that parsimony is essential for the power of the tests, but likewise the selection considers the assumption that the countries make up an integrated market. Because some markets are more problematic than others, only larger and stable economies are considered. That the United States and Canada are close to one market is a fair assumption. Similarly, as the countries of Europe are either part of the European Union or participate in its open market provisions, thus integration is arguably a fair assumption in this case as well. As Fama & French (2016a) note, it is Asia Pacific where the assumption of integration proves the most controversial in its applicability. However, it makes for a sufficient proxy for asset pricing literature. At the time of writing their paper, Fama & French (2016a) noted that North America,

Europe, Japan and Asia Pacific accounted for 48%, 30%, 18% and 4% of the global market capitalization respectively. Scepticism over the assumption of market integration made here for any of the groups, such as Asia-Pacific, would likely be revealed in the comparable size of the average absolute alpha and the regression R^2 . Except for Japan, regional diversification limits the ability to capture country-specific risks meaning the factors do not necessarily constitute the most adequate factors for any single country.

Fama & French (2015) found evidence that some factor construction methods may be more attractive for characteristics such as isolating factor premium estimates. However, Fama & French (2015; 2016a) recommend 2x3 sorts citing precedence and the flexibility in accommodating factor additions and subtraction as a major benefit. Further, Fama & French (2015) find little evidence that the other sorts used to construct the factors, 2x2 or 2x2x2x2, lead to different regression results. As a result, this study uses the 2x3 sorted factor sets for each region available from the Kenneth French Data Library (2020).

The market factor, MKT-RF, is the regional VW market portfolio minus the U.S. one-month T-bill rate. The five style factors are the four remaining factors of the Five-factor model, that is, SMB, HML, RMW and CMA, and momentum, WML. At the end of each June, SMB, HML, RMW and CMA are constructed from six VW portfolios formed on Size-Book-to-market, six VW portfolios formed on Size-Operating profitability and six VW portfolios formed on Size-Investment. The breakpoints for size are the top 90% and bottom 10% stocks by market capitalization for each region, and the breakpoints for book-to-market equity, operating profitability and investment are the 30th and 70th percentiles in each region of the respective ratios for the big stocks (Fama & French, 2016a). Stocks are first sorted into two size groups, then respectively assigned into three groups of book-to-market equity, operating profitability, and investment. SMB is the difference in average return on nine “small stock” portfolios and nine “big stock” portfolios. HML is the difference in average return on two “value” portfolios and two “growth” portfolios. RMW is the difference in average return on two “robust” operating profitability portfolios and two “weak” operating profitability portfolios. CMA is the difference in average return on two “conservative” investment portfolios and two “aggressive” investment portfolios. (Kenneth French Data Library, 2020)

The portfolios used to construct WML are formed monthly. The factor is constructed from portfolios formed on Size-Lagged momentum. The lagged momentum return is the stock’s cumulative return for month $t-12$ to $t-2$ for portfolios formed at the end of month $t-1$. This produces six VW portfolios for each region, SL, SN, SW, BL, BN, BW. “S”/”B”

indicate size, and “L”/”N”/”W” indicate “losers”, “neutral” and “winners” with breakpoints at the 30th and 70th percentiles of the lagged momentum return of the big stocks (Kenneth French Data Library, 2020). WML is accordingly the equal-weighted average of the difference of the returns of two “winner” portfolios and two “loser” portfolios (Kenneth French Data Library, 2020).

The test portfolios used in this study include 5x5 sorts on Size and Book-to-market, Size and Profitability, Size and Investment and Size and Momentum. Breakpoints for size use the 3rd, 7th, 13th and 25th percentiles of the regional aggregate market capitalization. The breakpoints for B/M, OP, Inv and Lagged momentum use the 20%, 40%, 60th and 80th percentiles of the region’s big stocks (Kenneth French Data Library, 2020). Fama & French (1995) demonstrate that B/M, investment, and profitability are correlated, and as a result, portfolios from Size–B/M, Size-OP and Size-Inv sorts do not isolate the characteristics in average returns. Fama & French (2015; 2016a) include triple sorted portfolios that sort the variables jointly to disentangle the average return dimensions (Fama & French, 2015). This study shows additional results of portfolios from 2x4x4 sorts on Size-B/M-Profitability, Size-B/M-Investment and Size-Profitability-Investment. Breakpoints for size use the top 90% and bottom 10% of the regional market capitalization. The stocks are first assigned into “Small” or “Big”, and then according to four groups each of two of the following characteristics B/M, Inv or OP with quartile breakpoints (Kenneth French Data Library, 2020).

The international research returns of the Kenneth French Data Library are updated renditions of the returns used in Fama & French (2012) (Kenneth French Data Library, 2020). With the employment of this set of data, this study ensures the model logic can carry over the discussion of earlier studies, primarily Fama & French (2012; 2015; 2016a; 2016b; 2018). The paper closely follows the methodology of Fama & French (2016a) and much of their restrictions and logic carry over. All return data to be used in this study are in U.S. dollars and include dividends and capital gains, and not continuously compounded (Kenneth French Data Library, 2020). Further, the risk-free rate is the U.S. one-month Treasury bill rate for all four regions in this study, and the monthly excess returns are returns in excess of this rate. Consequently, in conforming to earlier literature with the employment of this set of data, this study assumes the view of an American investor.

5.2 Descriptive statistics

This section presents summary descriptive statistics over the sample. It investigates the average monthly premiums for each factor, the associated descriptive statistics of this sample, as well as a correlation analysis between the same factors between the regions. Further, it investigates the test assets to uncover the tilts within the portfolio sorts to discuss the return distribution with reference to the anomalous effects they represent.

Table 1 Descriptive statistics of the factor returns

	MKT-RF	SMB	HML	RMW	CMA	WML	MKT-RF	SMB	HML	RMW	CMA	WML
N=345	North America						Europe					
Mean	0.73	0.15	0.16	0.32	0.22	0.57	0.51	0.04	0.30	0.38	0.18	0.90
Std Dev	4.19	2.75	3.26	2.40	2.64	4.72	4.79	2.14	2.38	1.56	1.79	3.91
t-Mean	3.23	0.98	0.93	2.48	1.53	2.25	1.99	0.37	2.31	4.52	1.82	4.30
Kurtosis	1.69	4.96	5.24	10.05	5.54	8.70	1.75	1.01	3.26	0.94	3.82	7.93
Skewness	-0.75	0.35	0.66	0.25	1.04	-0.15	-0.57	-0.03	0.35	-0.25	0.44	-1.24
	Japan						Asia Pacific					
Mean	0.08	0.11	0.28	0.13	0.05	0.08	0.73	-0.16	0.60	0.25	0.35	0.82
Std Dev	5.43	3.19	2.90	2.14	2.37	4.37	5.76	2.90	2.95	2.70	2.41	4.32
t-Mean	0.27	0.65	1.80	1.10	0.36	0.36	2.36	-1.02	3.81	1.74	2.70	3.54
Kurtosis	0.62	1.86	2.03	1.89	4.24	2.89	2.63	2.79	11.20	3.29	1.94	19.89
Skewness	0.21	0.09	-0.13	-0.01	-0.69	-0.46	-0.38	0.44	1.49	-0.18	0.04	-2.94

Table 1 presents the descriptive statistics for returns of the six factors from November 1990 until July 2019 constructed for North America, Europe, Japan and Asia Pacific. Presented are the factor excess return means, the standard deviation, the kurtosis, the skewness and the t-Mean, the ratio of mean to its standard error, are presented.

In Table 1, the first row under each region shows the average monthly premiums achieved over the sample period in each of the markets. Average equity premiums are 0.73% (t-Mean=3.23), 0.51% (t-Mean=1.99), 0.08% (t-Mean=0.27) and 0.73% (t-Mean=2.36) per month in North America, Europe, Japan, and Asia Pacific respectively. Equity premiums are large and significant in North America, Europe and Asia Pacific, but small and not significant in Japan. Average size premiums are 0.15% (t-Mean=0.98), 0.04% (t-Mean=0.37), 0.11% (t-Mean=0.65) and -0.16% (t-Mean=-1.02) per month for North America, Europe, Japan and Asia Pacific. These premiums are not significant in any region, although highest in North America and lowest in Asia Pacific. Average value premiums are 0.16% (t-Mean=0.93), 0.30% (t-Mean=2.31), 0.28% (t-Mean=1.80) and 0.60% (t-Mean=3.81) per month for North America, Europe, Japan and Asia Pacific. Hence, the value premium is not significant in North America, but large and significant in Europe and Asia Pacific, and moderate albeit significant only at the 10% level in Japan. Average profitability premiums are 0.32% (t-Mean=2.48), 0.38% (t-Mean=4.52), 0.13% (t-Mean=1.10) and 0.25% (t-Mean=1.74) per month in North America, Europe, Japan,

and Asia Pacific. Profitability premiums are high and significant in North America and Europe, moderate albeit significant only at the 10% level in Asia Pacific, and not significant in Japan. Average investment premiums are 0.22% (t-Mean=1.53), 0.18% (t-Mean=1.82), 0.05% (t-Mean=0.36) and 0.35% (t-Mean=2.70) per month in North America, Europe, Japan and Asia Pacific. Investment premiums are high and significant in Asia Pacific, moderately high albeit only significant at the 10% level in Europe, moderate but not significant in North America and not significant in Japan. Average momentum premiums are 0.57% (t-Mean=2.25), 0.90% (t-Mean=4.30), 0.08% (t-Mean=0.36) and 0.82% (t-Mean=3.54) per month in North America, Europe, Japan and Asia Pacific. Momentum premiums have been large and significant and outscored all other factor premiums during this sample period in North America, Europe, Asia Pacific. On the other hand, Japanese momentum premiums are small and not significant.

Looking at all factor premiums Japan seems to be an outlier due to its premiums having achieved noticeably lower averages compared to the other regions on all factors except value. Further, premiums in Asia Pacific behave differently to North America and Europe. Kurtosis is positive across the board and high for some sets, suggesting that extreme realisations are fairly prevalent, especially in regard to WML. WML produces consistent negative skewness in all regions and is the most dramatically skewed metric. Arnott, Harvey, Kalesnik & Linnainmaa (2019) suggest that this is caused by its sensitivity to crashes. Across the four regions, the other factors skew in both directions.

Table 2 Descriptive statistics of regional factor correlation

MKT-RF	North America	Europe	Asia Pacific	Japan	SMB	North America	Europe	Asia Pacific	Japan
North America	1.00				North America	1.00			
Europe	0.80	1.00			Europe	0.31	1.00		
Asia Pacific	0.73	0.75	1.00		Asia Pacific	0.23	0.45	1.00	
Japan	0.45	0.50	0.49	1.00	Japan	0.08	0.33	0.16	1.00

HML	North America	Europe	Asia Pacific	Japan	RMW	North America	Europe	Asia Pacific	Japan
North America	1.00				North America	1.00			
Europe	0.61	1.00			Europe	0.22	1.00		
Asia Pacific	0.20	0.21	1.00		Asia Pacific	0.17	0.08	1.00	
Japan	0.45	0.40	0.11	1.00	Japan	0.00	0.11	0.06	1.00

CMA	North America	Europe	Asia Pacific	Japan	WML	North America	Europe	Asia Pacific	Japan
North America	1.00				North America	1.00			
Europe	0.56	1.00			Europe	0.73	1.00		
Asia Pacific	0.32	0.31	1.00		Asia Pacific	0.47	0.45	1.00	
Japan	0.23	0.38	0.23	1.00	Japan	0.46	0.43	0.31	1.00

Table 2 shows simple factor correlation tests of each factor between the regions. Table 2 reveals that equity premiums are the most correlated, in a range of 0.45-0.80. The

strongest equity premium correlation is between North America and Europe, but correlations are also strong between North America and Asia Pacific and between Europe and Asia Pacific. Japanese premiums correlate less with all three regions. Equity premiums are also the single most correlated premium for each region, the only exception being a draw with the WML correlation between North America and Japan.

There are two interesting findings related to momentum. First, momentum premiums are the next highest correlated overall. Second, ignoring the market factor, momentum premiums are the most correlated for all six regional pairings among the remaining factors, and only ties once with SMB between Europe and Asia Pacific. The WML momentum factor correlations are in the range of 0.31-0.73, lowest between Japan and Asia Pacific and highest between North America and Europe. SMB correlations are in the range of 0.08-0.45, lowest between North America and Japan, and highest between Europe and Asia Pacific. HML factor correlations are in the range of 0.11-0.61, lowest between Japan and Asia Pacific, and highest between North America and Europe. RMW premiums are the least correlated across the board, in a range of 0.00-0.22. CMA premiums correlate in the range of 0.23-0.56, low between Japan and North America and between Japan and Asia Pacific and highest between North America and Europe.

Some general implications from the correlation tests are that the factor correlations between the North American and European data sets are quite similar. As noted in earlier research by Fama & French (2016a), the high correlation between North American and European equity premiums is intriguing as the markets account for much of the market capitalization. In this sample, Japanese premia are the least correlated with premiums of the same characteristic in the other regions, and the gap to Asia Pacific is quite telling as well. Hence the lower Japanese premiums and the contrasting Asian Pacific premiums seen in Table 1 seem to carry over into the simple correlations of Table 2. The sets of regional factors can be assumed different to the other regions in this study.

Table 3 compares the monthly excess return for the 5x5 sorted portfolios. It sheds light on how B/M, OP, Inv and Mom average return spreads vary in regard to Size. Fama & French (2016a) note that size quantiles differ in their actual size between the four regions. The biggest size quantiles are comparable between North America and Europe, but the Japanese quintile is half the size and the Asia Pacific quintile only a third the size of the North American quantile. As a result, actual size should be noted when comparing quantiles. Individual quantiles are better compared within the same region.

Table 3 Average monthly excess returns from regional 5x5 sorts**Size-B/M portfolios**

		North America					Europe					
		Low B/M	2	3	4	High B/M	Low B/M	2	3	4	High B/M	
Small		0.43	0.67	1.01	0.87	1.16	Small	-0.03	0.34	0.43	0.56	0.74
2		0.53	0.71	0.85	0.86	0.89	2	0.27	0.44	0.53	0.66	0.76
3		0.92	0.75	0.86	0.81	0.93	3	0.39	0.56	0.52	0.54	0.73
4		0.92	0.82	0.87	0.82	0.87	4	0.55	0.53	0.58	0.56	0.66
Big		0.73	0.71	0.67	0.69	0.67	Big	0.39	0.56	0.55	0.64	0.50
		Japan					Asia Pacific					
		Low B/M	2	3	4	High B/M	Low B/M	2	3	4	High B/M	
Small		0.27	0.31	0.39	0.35	0.48	Small	0.64	0.33	0.61	0.97	1.36
2		0.25	-0.07	0.06	0.24	0.20	2	-0.05	0.21	0.32	0.56	0.85
3		-0.16	0.00	0.03	0.09	0.26	3	0.16	0.35	0.68	0.67	0.75
4		-0.17	0.04	0.10	0.16	0.18	4	0.66	0.77	0.56	0.92	0.96
Big		-0.05	0.07	0.09	0.22	0.40	Big	0.60	0.79	0.81	0.79	1.01

Size-OP portfolios

		North America					Europe					
		Low OP	2	3	4	High OP	Low OP	2	3	4	High OP	
Small		0.85	1.11	1.02	1.09	1.09	Small	0.13	0.65	0.73	0.88	0.75
2		0.53	0.88	0.97	1.08	1.14	2	0.25	0.56	0.63	0.72	0.95
3		0.67	0.86	0.87	0.98	1.09	3	0.27	0.53	0.76	0.63	0.78
4		0.64	0.91	0.93	0.89	1.01	4	0.22	0.57	0.71	0.70	0.72
Big		0.35	0.60	0.67	0.82	0.76	Big	0.16	0.54	0.57	0.48	0.62
		Japan					Asia Pacific					
		Low OP	2	3	4	High OP	Low OP	2	3	4	High OP	
Small		0.29	0.42	0.39	0.35	0.64	Small	0.71	1.22	1.06	1.14	1.11
2		0.05	0.12	0.24	0.22	0.24	2	0.18	0.52	0.67	0.74	0.69
3		0.00	0.07	0.12	0.07	0.15	3	0.15	0.70	0.60	0.82	0.84
4		-0.10	0.06	0.16	0.22	0.05	4	0.49	0.78	0.75	0.86	0.95
Big		0.00	0.04	0.17	0.10	0.11	Big	0.59	0.82	0.91	0.85	0.67

Size-Inv portfolios

		North America					Europe					
		Low Inv	2	3	4	High Inv	Low Inv	2	3	4	High Inv	
Small		1.21	1.14	1.05	0.99	0.52	Small	0.51	0.67	0.67	0.64	0.18
2		0.94	0.96	0.92	0.91	0.46	2	0.56	0.73	0.73	0.61	0.36
3		0.94	0.93	0.96	0.90	0.62	3	0.63	0.63	0.67	0.49	0.35
4		0.94	0.97	0.92	0.93	0.65	4	0.62	0.62	0.65	0.65	0.42
Big		0.77	0.68	0.69	0.71	0.65	Big	0.54	0.59	0.51	0.45	0.46
		Japan					Asia Pacific					
		Low Inv	2	3	4	High Inv	Low Inv	2	3	4	High Inv	
Small		0.35	0.34	0.47	0.37	0.45	Small	1.03	1.14	1.20	1.04	0.51
2		0.19	0.14	0.19	0.25	0.05	2	0.54	0.86	0.59	0.58	0.02
3		0.13	0.13	0.08	0.04	0.01	3	0.57	0.97	0.65	0.77	0.13
4		0.10	0.09	0.17	-0.01	0.07	4	0.60	0.78	0.99	0.87	0.53
Big		0.09	0.01	-0.04	0.12	0.03	Big	0.85	0.74	0.78	0.80	0.54

Size-Mom portfolios

		North America					Europe					
		Low Mom	2	3	4	High Mom	Low Mom	2	3	4	High Mom	
Small		0.30	0.88	1.11	1.32	1.60	Small	-0.39	0.33	0.59	0.94	1.55
2		0.48	0.87	0.87	0.95	1.33	2	-0.16	0.38	0.65	0.91	1.29
3		0.49	0.82	0.88	0.93	1.16	3	0.05	0.43	0.61	0.82	1.10
4		0.54	0.81	0.88	0.88	1.18	4	0.18	0.51	0.59	0.73	1.03
Big		0.48	0.67	0.63	0.76	1.01	Big	0.15	0.46	0.60	0.63	0.69
		Japan					Asia Pacific					
		Low Mom	2	3	4	High Mom	Low Mom	2	3	4	High Mom	
Small		0.36	0.49	0.44	0.54	0.38	Small	0.19	0.78	1.12	1.61	1.57
2		0.10	0.15	0.22	0.22	0.25	2	-0.52	0.61	0.75	0.94	1.15
3		0.13	-0.01	0.10	0.18	0.13	3	-0.21	0.50	0.74	1.08	0.95
4		0.11	0.14	0.11	0.02	0.21	4	0.25	0.65	0.75	0.93	0.95
Big		0.09	-0.07	-0.10	0.09	0.12	Big	0.80	0.66	0.90	0.89	0.97

For Size-B/M portfolios, Table 3 reveals that there is a positive relationship between returns and B/M. The value effect is strong in Europe and Japan and strongest in Asia Pacific across all size quintiles. Although value grows invisible in North America and smaller in Europe and Asia Pacific in the bigger size quintiles, Japan experiences a

gradually stronger spread with larger size quintiles. Size-B/M returns mostly agree with the prediction that small stocks display a stronger value effect (Fama & French, 2015). However, in the lowest B/M quintile in North America and Europe, there is a reversed size effect as returns increase with size.

For sorts on Size-OP, average returns increase with profitability in each size quintile for North America, Europe, Japan, and Asia Pacific. Thus, there are signs of a profitability effect, where highly profitable companies have tended to yield higher returns than those with low profitability (Fama & French, 2015). However, the effect is most noticeable between the lowest profitability quintiles, especially in Europe and Asia Pacific, and from there on increases gradually. Furthermore, both the returns and the profitability spread decrease with size across nearly all profitability quintiles in all four regions. In Japan, the profitability spreads are generally smaller than in the other regions.

Within the Size-Inv portfolios, average returns decrease with investment in North America, Europe, and Asia Pacific, but the major drop happens between the two largest investment quintiles. In Japan, the investment effect is at best ambiguous and only exhibits in the middle of the pack. For the other regions, investment spreads decrease with size quintiles. There is also a strong size effect as smaller size quintiles yield higher returns and then decrease gradually across the four first size quintiles, but the highest investment quantile is an exception and a reverse size effect is true in North American and European high investment quintiles.

Size-Mom portfolios reveal that average returns gradually increase with momentum across all size quintiles in North America, Europe, and Asia Pacific. In Japan there is no discernible momentum effect. Moreover, momentum spreads are the highest of all four portfolio sorts. While high across all size quintiles, still they decrease gradually with size. However, compared to Size-Inv portfolios, there is an opposite relation between size and the premium as the low momentum quintile experiences an increase in returns with size in North America, Europe and Asia Pacific. Japan displays a conventional size effect across all momentum quintiles as returns grow smaller with larger size quintiles.

Table 4 presents the average excess return for the 2x4x4 sorted portfolios. These portfolios allow to control for an additional dimension, but from a cursory overview, Table 4 seems to conform to the results expected following Table 3.

Table 4 Average monthly excess returns from regional 2x4x4 sorts**Size-B/M-OP portfolios**

	Small				Big			
	Low B/M	2	3	High B/M	Low B/M	2	3	High B/M
North America								
Low OP	0.36	0.53	0.64	0.90	0.89	0.43	0.43	0.55
2	0.54	0.75	0.88	1.02	0.56	0.77	0.76	0.85
3	0.89	0.91	1.02	1.07	0.73	0.71	0.77	0.74
High OP	0.93	1.07	1.07	1.18	0.81	0.71	0.84	0.76
Europe								
Low OP	-0.56	-0.23	0.16	0.22	0.05	0.01	0.44	0.40
2	-0.19	0.26	0.63	0.76	0.28	0.60	0.58	0.69
3	0.25	0.58	0.83	1.03	0.47	0.58	0.63	0.80
High OP	0.63	0.80	1.17	1.07	0.53	0.57	0.74	0.98
Japan								
Low OP	-0.19	-0.09	0.22	0.39	-0.14	-0.03	-0.07	0.15
2	-0.23	0.12	0.20	0.38	-0.02	0.06	0.11	0.23
3	0.04	0.25	0.36	0.44	0.03	0.07	0.33	0.47
High OP	0.14	0.36	0.49	0.60	-0.04	0.28	0.43	0.43
Asia Pacific								
Low OP	-0.40	-0.15	0.46	1.29	-0.48	0.20	0.53	0.84
2	-0.12	0.00	0.38	0.85	0.47	0.83	0.90	1.25
3	-0.01	0.48	0.91	1.29	0.95	0.83	0.97	1.13
High OP	0.53	1.02	1.27	1.42	0.71	0.74	0.88	1.71

Size-B/M-Inv portfolios

	Small				Big			
	Low B/M	2	3	High B/M	Low B/M	2	3	High B/M
North America								
Low Inv	0.91	1.00	1.04	1.26	0.76	0.71	0.84	0.81
2	0.98	0.93	0.96	1.15	0.75	0.67	0.72	0.82
3	0.82	0.97	1.13	0.86	0.64	0.69	0.83	0.78
High Inv	0.43	0.74	0.74	0.78	1.00	0.78	0.55	0.38
Europe								
Low Inv	0.22	0.60	0.76	0.69	0.38	0.61	0.65	0.59
2	0.48	0.60	0.82	0.87	0.44	0.72	0.67	0.77
3	0.51	0.63	0.75	0.82	0.44	0.51	0.55	0.67
High Inv	0.14	0.29	0.46	0.57	0.51	0.33	0.58	0.39
Japan								
Low Inv	0.07	0.20	0.29	0.51	-0.09	0.02	0.07	0.35
2	0.05	0.21	0.30	0.38	-0.13	0.14	0.19	0.25
3	-0.09	0.18	0.29	0.40	-0.25	0.11	0.22	0.20
High Inv	0.12	0.22	0.28	0.57	-0.01	0.06	0.17	0.39
Asia Pacific								
Low Inv	0.07	0.37	0.94	1.10	0.75	0.68	0.78	1.09
2	0.46	0.85	1.00	1.40	0.98	0.62	0.90	0.99
3	0.46	0.62	0.86	0.99	0.55	0.82	0.78	0.98
High Inv	-0.09	0.28	0.44	0.59	0.45	0.74	0.60	0.92

Size-OP-Inv portfolios

	Small				Big			
	Low OP	2	3	High OP	Low OP	2	3	High OP
North America								
Low Inv	0.86	1.00	0.99	1.32	0.72	0.77	0.80	0.75
2	0.94	0.96	1.00	0.99	0.64	0.75	0.75	0.74
3	0.49	0.99	0.97	1.01	0.80	0.70	0.68	0.72
High Inv	0.21	0.39	0.72	0.95	0.22	0.76	0.73	1.22
Europe								
Low Inv	0.00	0.62	0.76	0.85	0.33	0.53	0.67	0.69
2	0.11	0.65	0.74	0.88	0.43	0.55	0.72	0.66
3	0.00	0.50	0.69	0.84	0.34	0.49	0.49	0.59
High Inv	-0.51	0.14	0.35	0.63	0.05	0.57	0.54	0.57
Japan								
Low Inv	0.17	0.25	0.35	0.31	-0.06	0.16	0.12	0.23
2	0.18	0.18	0.28	0.29	-0.03	0.07	0.17	0.10
3	0.14	0.17	0.22	0.19	-0.13	0.17	0.20	-0.05
High Inv	0.14	0.06	0.22	0.28	0.16	-0.08	0.10	0.12
Asia Pacific								
Low Inv	0.19	0.45	1.04	0.74	0.73	1.01	0.69	0.99
2	0.31	0.76	1.03	1.21	0.81	0.96	0.76	1.02
3	-0.01	0.22	0.81	0.92	0.63	0.73	1.11	0.53
High Inv	-0.60	-0.03	0.12	0.55	0.41	0.70	0.75	0.38

Within the Size-B/M-OP portfolios the small size group quantiles see average returns increase with both profitability and value in all four regions. The big size group quantiles exhibit the profitability effect throughout the B/M quartiles in North America, Europe, and Japan, and Asia Pacific. An outlier is the “Low-Low” portfolio in North America. In both Japanese small and big group quantiles, the spreads expose noticeably smaller value and profitability effects in comparison to the other regions.

For Size-B/M-Inv portfolios, the small size group reaffirms the value effect as average returns increase with B/M, and this is true for all regions. In the big size group quantiles, there is a value effect in all regions except for the high investment quartiles in North America and Europe, where returns decrease with higher B/M. There is a visible size effect in all regions except for the higher B/M quartiles in Europe and Asia Pacific. Small group quantiles in North America, Europe, and Asia Pacific reveal that average returns generally decrease with investment. In these regions, the effect carries over somewhat to the higher value quartiles of the big size group quantiles. In Japan, the investment quartiles remain roughly the same for all B/M quartiles, big or small.

Both investment and profitability effects are visible in the small size group quantiles of all four regional Size-OP-Inv portfolios. In North America, Europe, and Asia Pacific the profitability effect is large, and the spreads diminish with the rate of investment. For the big size group, controlling for the other characteristic the effects are weaker and inconsistent, but the profitability effect is visible in Europe. Small firms that generously invest in spite of low profitability show disproportionately poor performance in North America, Europe, and Asia Pacific, with large negative returns in Europe and Asia Pacific. These returns presented a problem for the Five-factor model in Fama & French (2015; 2016a).

6 RESULTS

This chapter presents the regression details and test results. The chapter starts by presenting the factor spanning results. Next, the results from the formal test of Gibbons, Ross and Shanken (1989) are presented. This absolute test is then followed by presenting the results of the comparison of summary statistics based on average absolute alpha to measure relative performance. Asset pricing tests on 5x5 sorted portfolio returns are the main source of this study's inferences, with special attention paid to portfolios sorted on Size and Momentum. Results from the 2x4x4 portfolio sorts are presented in a separate section "Additional results". The results are discussed in greater detail, formed into inferences, and subsequently discussed in Chapter 7.

This study heeds the cautions put forward by Lewellen, Nagel & Shanken (2010) and Harvey, Liu & Zhu (2016) who cumulatively highlight the importance of setting appropriate criteria for establishing significance to mitigate the acceptance of redundant factors or the subsequent acceptance of less efficiently parsimonious models. Although not explicitly stated, Fama & French (2016a) seem to be using a slightly higher cut-off point for their t-statistics. For the formal tests, this study applies a critical t-statistic of 1.96 with regard to the degrees of freedom. This corresponds to a 5% level of significance. Further attention is paid to statistics that reach a higher level, a 1% level of significance. The possibility of making inferences from results adhering to lower thresholds is ignored.

6.1 Factor spanning test results

Table 5 shows the intercepts and slopes of four factor spanning regressions. These regressions represent each of the four regional WML factors regressed on the remaining five factors.

Table 5 WML factor spanning regression results

	Coefficient						t-statistic						R^2
	Int	Mkt	SMB	HML	RMW	CMA	Int	Mkt	SMB	HML	RMW	CMA	
North America													
<i>WML</i>	0.67	-0.21	0.31	-0.68	0.13	0.36	2.70	-3.18	3.23	-5.53	1.02	2.30	0.15
Europe													
<i>WML</i>	0.68	-0.12	0.11	-0.25	0.81	0.28	3.36	-2.64	1.29	-2.14	5.58	1.98	0.24
Japan													
<i>WML</i>	0.13	-0.12	-0.04	-0.33	0.42	0.04	0.60	-2.60	-0.55	-3.47	2.81	0.29	0.15
Asia Pacific													
<i>WML</i>	1.08	-0.07	0.08	-0.52	-0.05	0.37	4.62	-1.44	1.09	-5.25	-0.43	3.39	0.16

According to the framework of Huberman & Kandel (1987), intercepts whose t-statistics exceed the critical cut-off value are determined to be important for describing average returns for a region, and t-statistics below this value can be determined as trivial, or redundant, in describing average returns.

The factor spanning regressions reveal that the momentum factor is not rendered redundant in North America, Europe, and Asia Pacific. Indeed, WML is an important factor for describing the average returns in all regions except Japan. WML provides economically large and statistically significant intercepts at the 1% level of significance in North America (0.67, $t=2.70$) Europe (0.68, $t=3.36$) and Asia Pacific (1.08, $t=4.62$). Despite the magnitude of the intercepts measured in the other regions, the factor spanning results show that WML is redundant in Japan as the regression produces economically small and insignificant intercepts (0.13, $t=0.60$). This implies that there is not enough momentum variation in the sample, consistent with the Table 1 and Table 3 descriptive statistics suggesting there is no discernible momentum effect in Japan. Since these results suggest leaving WML out does not harm the mean-variance-efficient tangency portfolio produced by the remaining Fama & French asset pricing factors, therefore the results suggest it is trivial and dropping it in Japan should be considered. The consistent HML slopes of the regions indicate negative correlation with WML, North America (-0.68, $t=-5.53$), Europe (-0.25, $t=-2.14$), Japan (-0.33, $t=-3.47$) and Asia Pacific (-0.52, $t=-5.25$).

The coefficient of determination, R^2 , of the factor spanning regressions are low compared to factor spanning tests conducted on the other factors on a similar sample in Fama & French's (2016a) Five-factor spanning tests. This implies that WML is to a large degree independent of the remaining explanatory variables.

6.2 Summary asset pricing statistics

Table 6 shows the assorted summary statistics generated from regression on 5x5 portfolio sorts using the Five- and Six-factor models. Results of the formal GRS F-test are represented by columns GRS and $p(\text{GRS})$. The next four columns contain the informal summary statistics, the average absolute alpha and its supplementary dispersion metrics. $N. \alpha_i^*$ represents the number of significant alphas generated in each set of 25 portfolios. Last, the average adjusted coefficient of determination is represented by AR^2 .

Table 6 Summary asset pricing statistics for portfolios of 5x5 sorts

	GRS	$p(GRS)$	$Al a_i$	$ \frac{Al a_i}{Al r_i} $	$\frac{Aa_i^2}{Ar_i^2}$	$\frac{As^2(a_i)}{Aa_i^2}$	$N. a_i^*$	AR^2	GRS	$p(GRS)$	$Al a_i$	$ \frac{Al a_i}{Al r_i} $	$\frac{Aa_i^2}{Ar_i^2}$	$\frac{As^2(a_i)}{Aa_i^2}$	$N. a_i^*$	AR^2
Size-B/M	North America								Europe							
Five-factor model	3.05	0.02	0.11	0.97	0.96	0.31	6	0.93	1.94	0.11	0.08	0.71	0.44	0.45	6	0.94
Six-factor model	2.79	0.03	0.09	0.80	0.73	0.41	3	0.93	2.00	0.10	0.08	0.69	0.43	0.47	5	0.94
	Japan								Asia Pacific							
Five-factor model	1.27	0.34	0.11	0.77	0.57	0.61	3	0.92	2.25	0.07	0.16	0.69	0.52	0.37	2	0.89
Six-factor model	1.25	0.35	0.11	0.76	0.55	0.63	3	0.93	1.92	0.12	0.13	0.58	0.39	0.52	2	0.89
Size-OP	North America								Europe							
Five-factor model	2.03	0.10	0.08	0.52	0.27	0.54	4	0.93	4.56	0.00	0.11	0.66	0.42	0.25	8	0.95
Six-factor model	2.01	0.10	0.08	0.51	0.28	0.54	5	0.93	4.41	0.01	0.12	0.70	0.45	0.23	9	0.95
	Japan								Asia Pacific							
Five-factor model	1.28	0.34	0.10	0.86	0.67	0.49	2	0.93	2.69	0.04	0.18	0.98	0.93	0.26	7	0.89
Six-factor model	1.27	0.34	0.10	0.85	0.66	0.49	3	0.93	3.01	0.02	0.17	0.93	0.84	0.30	7	0.90
Size-Inv	North America								Europe							
Five-factor model	2.66	0.04	0.10	0.66	0.63	0.27	6	0.93	0.96	0.56	0.05	0.49	0.25	1.11	2	0.95
Six-factor model	2.66	0.04	0.10	0.62	0.55	0.31	5	0.93	1.03	0.50	0.06	0.58	0.31	0.92	2	0.95
	Japan								Asia Pacific							
Five-factor model	1.20	0.38	0.10	0.87	0.72	0.52	3	0.94	4.19	0.01	0.23	1.09	1.11	0.20	8	0.89
Six-factor model	1.19	0.39	0.10	0.86	0.70	0.53	2	0.94	3.75	0.01	0.22	1.00	0.97	0.24	8	0.89
Size-Mom	North America								Europe							
Five-factor model	3.70	0.01	0.26	1.16	1.33	0.15	10	0.84	4.94	0.00	0.25	0.79	0.70	0.10	11	0.88
Six-factor model	3.35	0.02	0.12	0.54	0.40	0.21	5	0.92	4.38	0.01	0.17	0.53	0.33	0.11	9	0.93
	Japan								Asia Pacific							
Five-factor model	1.45	0.26	0.11	0.89	0.78	0.85	5	0.87	5.83	0.00	0.45	1.41	1.59	0.07	14	0.85
Six-factor model	1.43	0.26	0.09	0.77	0.68	0.55	5	0.92	4.78	0.00	0.28	0.87	0.69	0.12	12	0.89

Among the 5x5 sorted portfolios, the GRS statistic either rejects, or does not reject, both Five- and Six-factor models in each set. The GRS rejects and does not reject both models in eight sets each. Both models are also awarded similar statistics within a narrow margin in most of the sets. This implies that the absolute test determines that both models are, or are not, close to the complete descriptions of the test asset returns. Although the GRS F-test is an absolute test and cannot be used to conclude relative performance, across all test assets the GRS statistics show the Six-factor model is marginally closer to a complete description of returns, although in Europe and Asia Pacific the Five-factor statistic is lower for two sets and one set respectively.

Starting with the Size-B/M portfolios, the GRS rejects ($p\text{-value} < 0.05$) both models only in North America and does not reject in Europe, Japan, and Asia Pacific. The Six-factor model is closer to not be rejected in North America, and further away from being rejected in Asia Pacific. Although the difference is small, the Five-factor GRS statistic is lower in Europe and further away from rejection. Both models provide similar GRS statistics for Japanese Size-B/M portfolios.

For Size-OP portfolios, the GRS statistic is high and rejects both models in Europe and Asia Pacific, with both models rejected with high significance ($p\text{-value} < 0.01$) in the former region. The North American and Japanese GRS statistics are smaller and do not reject either model. While the statistics are similar between the models, the Six-factor model provides a lower GRS. The North American GRS statistics are also closer to rejecting both models than the Japanese.

The GRS rejects both models' description of Size-Inv portfolios in North America and Asia Pacific, with the Five-factor model rejected with high significance (p-value <0.01) in the latter region. The GRS does not reject in Europe and Japan where the statistics are small. Although rejected, the Asian Pacific set generates a lower GRS statistic for the Six-factor model.

For Size-Mom portfolios, the GRS F-test rejects in North America, Europe, and Asia Pacific. Both models are rejected with high significance (p-value <0.01) in Europe and Asia Pacific. In North America and Europe, the Six-factor model is closer not to be rejected. Interestingly, the GRS does not reject either model on Size-Mom portfolios in Japan. Further, in Japan, the models provide similar GRS statistics. This particularity is discussed in the next chapter.

Moving on to the informal analysis of average absolute alpha, both models achieve similar average absolute alphas on Size-OP and Size-Inv portfolios, but the Six-factor model produces marginally lower alphas on Size-B/M portfolios. Size-Mom portfolios stand out here as the Six-factor model provides noticeably lower average absolute alphas in three regions. At the same time, the Six-factor model does not sacrifice much accuracy in explaining portfolios not sorted on momentum in all four regions.

Skipping right to the Size-Mom portfolios, indeed the Six-factor model veritably shines. The average absolute alphas are significantly reduced in North America, Europe, and Asia Pacific, and slightly reduced in Japan. In North America, Europe and Asia Pacific, the average absolute alphas and all summary statistics particularly disfavour the Five-factor model which produces high average absolute alphas. In particular, the Five-factor $A|a_i|/A|r_i|$ and Aa_i^2/Ar_i^2 in North America and Asia Pacific are greater than one implying that the intercepts are more disperse than the average returns. The values of the Six-factor model are significantly better in all four regions, and the $As^2(a_i)/Aa_i^2$ is made higher across both models in North America, Europe and Asia Pacific.

Although the Japanese Size-Mom average absolute alphas, $A|a_i|/A|r_i|$ and Aa_i^2/Ar_i^2 favour the Six-factor model, the Six-factor $As^2(a_i)/Aa_i^2$ is smaller. This is another interesting particularity and suggests that although there is less intercept dispersion relative to the dispersion of LHS average returns, there is also more dispersion attributable to dispersion in the true intercepts and proportionally less to sampling error.

For Size-B/M portfolios, the average absolute alphas produced by the Five- and the Six-factor models are similar in Japan and Europe, whereas the Six-factor alphas are lower in North America and Asia Pacific. Nonetheless, the Six-factor model outperforms the Five-factor model on Size-B/M portfolios as it achieves lower $|a_i|/|r_i|$ and Aa_i^2/Ar_i^2 as well as higher $As^2(a_i)/Aa_i^2$ in all four regions. However, the substantial relative improvements can only be found in North America and Asia Pacific.

Size-OP portfolios are the most inconclusive set with the smallest marginal benefit in favour of either model. On all metrics, the Six-factor model underperforms the Five-factor model in Europe but outperforms in Asia Pacific. There is a visible difference in average absolute alphas, $|a_i|/|r_i|$, Aa_i^2/Ar_i^2 and $As^2(a_i)/Aa_i^2$ only in these two regions. The results are very similar for both models in North America and Japan, amounting to a tie, if not a negligible edge for the Six-factor model in the latter region.

Size-Inv portfolios favour the Six-factor model in North America, Japan, and Asia Pacific. Although the Six-factor average absolute alphas are only marginally dissimilar between these regions, there is visible improvement in $|a_i|/|r_i|$, Aa_i^2/Ar_i^2 and $As^2(a_i)/Aa_i^2$ revealing a negligible to marginal improvement in favour of the Six-factor model in the three regions. In Europe, the Five-factor model comparatively performs better in describing Size-Inv returns on all metrics.

In North America, the average absolute alphas are consistently low for both models across all sets of LHS portfolios except for a large Five-factor alpha from Size-Mom portfolios. The lower Six-factor average absolute alpha on Size-Mom portfolios is the defining element in North America. For Europe, both models achieve similar average absolute alphas on Size-B/M portfolios, but Five-factor alphas from Size-OP and Size-Inv portfolios are marginally lower. Without considering Size-Mom, Europe proves the most disputable for the Six-factor model's relative performance. However, the Six-factor alpha from Size-Mom portfolios are considerably lower than the large Five-factor alpha and proves to be the major improvement in Europe as well. For Japan, the Six-factor average absolute alphas are small for both models and all sets of LHS portfolios and offers a marginal improvement for the test assets not sorted on momentum. Lastly, for Asia Pacific, the Six-factor model performs better on all four sets of portfolios. Still, what sets it apart is again the reduction in alpha achieved from Size-Mom portfolios. On Asian Pacific Size-Mom portfolios the Five-factor model produces the highest average absolute

alpha. However, both Five- and Six-factor alphas are consistently larger in Asia Pacific than those of any other region.

Therefore, in three of the regions, the Six-factor model largely outperforms the Five-factor model, whereas the model is controversial in Japan. This is further delved upon in Chapter 7.

Proceeding to the last two statistics, starting with $N. \alpha_i^*$, the results reveal that the Six-factor model reduces the number of significant alphas within the sets of Size-Mom portfolios in North America, Europe, and Asia Pacific. The Five-factor model produces 10, 11 and 14 significant alphas whereas the Six-factor model produces 5, 9 and 12 significant alphas respectively for North America, Europe, and Asia Pacific. In Japan, both models produce 5 significant alphas. For the remaining sets of test assets, both models generate the same number of significant alphas in five out of twelve sets. The Six-factor model produces three respectively one fewer significant alpha on Size-B/M portfolios in North America and Europe, and one fewer each for Size-Inv portfolios in North America and Japan. In contrast, the Six-factor model generates one more significant alpha on Size-OP portfolios in North America, Europe, and Japan and Size-Inv portfolios in Japan.

Lastly, the average adjusted R^2 coefficients are consistently high and between 0.84-0.95 for the entire sample. For the regressions on test assets not sorted on momentum, the difference in average adjusted R^2 between the models is minor. However, the Six-factor model provides a noticeable improvement to the coefficient for Size-Mom portfolios. The Five- and Six-factor model regressions generated 0.84 and 0.92, 0.88 and 0.93, 0.87 and 0.92, and 0.85 and 0.89 in North America, Europe, Japan, and Asia Pacific respectively. Furthermore, the average adjusted R^2 metrics are consistently lower for the Asia Pacific region compared to the others.

6.3 Additional results

Table 7 below shows the additional results of portfolios sorted on Size-B/M-OP, Size-B/M-Inv and Size-OP-Inv. These consist of the same set of statistics as Table 6, but the results were generated from sets of 32 portfolios of 2x4x4 sorts.

Table 7 Summary asset pricing statistics for portfolios of 2x4x4 sorts

	<i>GRS</i>	<i>p</i> (<i>GRS</i>)	$\overline{Al\ a_i}$	$\frac{\overline{Al\ a_i}}{\overline{Al\ r_i}}$	$\frac{\overline{A\alpha_i^2}}{\overline{A\sigma_i^2}}$	$\frac{\overline{As^2(a_i)}}{\overline{A\sigma_i^2}}$	$N.\ a_i^*$	AR^2	<i>GRS</i>	<i>p</i> (<i>GRS</i>)	$\overline{Al\ a_i}$	$\frac{\overline{Al\ a_i}}{\overline{Al\ r_i}}$	$\frac{\overline{A\alpha_i^2}}{\overline{A\sigma_i^2}}$	$\frac{\overline{As^2(a_i)}}{\overline{A\sigma_i^2}}$	$N.\ a_i^*$	AR^2
Size-B/M-OP	North America								Europe							
Five-factor model	0.92	0.60	0.10	0.61	0.46	0.91	2	0.86	2.09	0.12	0.15	0.49	0.24	0.41	9	0.87
Six-factor model	0.80	0.70	0.10	0.59	0.45	0.95	2	0.86	2.16	0.11	0.15	0.50	0.27	0.38	9	0.87
	Japan								Asia Pacific							
Five-factor model	0.81	0.69	0.10	0.51	0.34	1.04	1	0.88	1.69	0.21	0.22	0.58	0.40	0.53	6	0.79
Six-factor model	0.83	0.68	0.09	0.51	0.33	1.08	0	0.88	1.79	0.18	0.22	0.58	0.39	0.57	5	0.79
Size-B/M-Inv	North America								Europe							
Five-factor model	1.59	0.24	0.10	0.72	0.52	0.53	4	0.89	1.66	0.21	0.09	0.66	0.39	0.80	4	0.90
Six-factor model	1.44	0.29	0.09	0.66	0.45	0.61	3	0.89	1.82	0.17	0.10	0.68	0.42	0.75	3	0.90
	Japan								Asia Pacific							
Five-factor model	0.74	0.75	0.07	0.48	0.25	1.47	1	0.90	1.78	0.18	0.20	0.83	0.65	0.45	8	0.84
Six-factor model	0.73	0.76	0.07	0.48	0.24	1.52	1	0.90	1.86	0.16	0.20	0.82	0.67	0.46	6	0.84
Size-OP-Inv	North America								Europe							
Five-factor model	2.39	0.08	0.12	0.69	0.49	0.46	5	0.88	2.15	0.11	0.11	0.51	0.29	0.38	9	0.90
Six-factor model	2.37	0.09	0.12	0.70	0.46	0.48	4	0.88	2.08	0.12	0.12	0.53	0.32	0.37	8	0.90
	Japan								Asia Pacific							
Five-factor model	0.71	0.77	0.08	0.86	0.81	1.21	0	0.90	2.83	0.05	0.21	0.67	0.45	0.50	5	0.82
Six-factor model	0.70	0.78	0.07	0.83	0.73	1.32	0	0.90	3.07	0.04	0.22	0.70	0.53	0.45	7	0.82

In all four regions, the GRS does not reject either model on Size-B/M-OP and Size-B/M-Inv portfolios. In fact, the GRS only rejects once for the Six-factor model on Size-OP-Inv portfolios in Asia Pacific. This is also the only set in this study where the GRS F-statistic determines one model to be sufficiently close to the complete description of returns but not the other.

The comparison of average absolute alphas indicates that the margin between the models yields tighter results for 2x4x4 portfolios than the 5x5 portfolios. Indeed, both models perform similarly on Size-B/M-OP, Size-B/M-Inv and Size-OP-Inv portfolios on all metrics.

In North America, the average absolute alphas are consistently low for both models across all sets of LHS portfolios. The informal summary statistics show the Six-factor alphas generate a marginal improvement on Size-B/M-OP and Size-B/M-Inv portfolios. In Europe, although both models achieve similar performance, the Five-factor model performs marginally better on all three sets. In Asia Pacific, the alphas achieved by both models are again consistently larger than those for any of the other regions. The summary statistics show that both models perform similarly on Size-B/M-OP and Size-B/M-Inv portfolios, but the Size-OP-Inv portfolio results favour the Five-factor model on all three metrics. As for Japan, the informal summary statistics show a marginal improvement for the Six-factor model in all three sets.

In sum, Size-B/M-OP portfolios are the most inconclusive set of the 2x4x4 sorted portfolios. The North American and Japanese average absolute alphas are smaller relative to the European and Asian Pacific for both models. The summary statistics favour the Six-factor model in North America and Japan, whereas the European values

favour the Five-factor model. For Size-B/M-Inv portfolios, the Six-factor model again performs better than the Five-factor model in North America and Japan, and the Five-factor model performs better in Europe. For the Size-OP-Inv portfolios, the Six-factor model provides a slight edge over the Five-factor model in Japan, but the Five-factor model marginally edges out the Six-factor model in Europe, and Asia Pacific.

Turning now to the tests of individual alpha, the Six-factor model produces fewer significant alphas from Size-B/M-OP in Japan and Asia Pacific. Both models produce the same number of significant alphas on North American and European Size-B/M-OP portfolios. For Size-B/M-Inv portfolios, the Six-factor model produces fewer significant alphas in North America, Europe, and Asia Pacific, but both models produce one significant alpha each in Japan. For Size-OP-Inv portfolios, the Six-factor model produces fewer significant alphas in North America and Europe, neither model produces any significant alphas in Japan, and the Five-factor model produces fewer significant alphas in Asia Pacific. Both models produce comparatively fewer significant alphas in Japan than in the other regions.

The average adjusted R^2 statistics from regressions on 2x4x4 test assets confirm the results of the 5x5 test assets not sorted on momentum. First, the coefficient of determination is high across all sets, and both models generate the same average adjusted R^2 for both models. Second, the coefficient for Asia Pacific is higher than in the other regions.

7 DISCUSSION

This chapter discusses the results presented in Chapter 6 and relates it to recent literature and theories associated with the findings. The answers to the research hypotheses are summarized and presented along with the concluding remarks and recommendations for future research in Chapter 8.

7.1 Discussion of factor spanning results

The factor spanning results suggest that WML adds to the Five-factor model internationally, providing large economically significant intercepts in both North America and Europe, and an even stronger intercept in Asia Pacific. In North America, Europe and Asia Pacific, the t-statistics correspond to a 1% level of significance. At the same time, WML is redundant in Japan and does not help in the description of Japanese average returns. This implies that the WML could be dropped from the Japanese model as it does not contribute to the mean-variance efficient tangency portfolio produced by combining the remaining Fama & French asset pricing factors. This finding conforms with the literature that has demonstrated that momentum fails in Japan (Fama & French, 2012). At the same time, these findings emphasize earlier findings pointing to the fact that momentum generally is an important factor in markets where it persists. For this reason, while the factor can be dropped in Japan, the factor spanning tests infer that WML is an important factor and a motivated inclusion to the Five-factor model. This is in line with the factor spanning tests of Fama & French (2018) who found WML to produce an economically large intercept and a very high t-statistic using a long U.S. sample.

While the factor spanning results do not alone prove WML's contribution to the description of all LHS portfolio returns, the results of the LHS comparison of average absolute alpha generally confirm what the factor spanning regressions had already presented. The LHS tests infer that the model is quite successful in improving the explanation of test assets, with the major benefit arising from the ability to price momentum.

The results of the factor spanning tests identified yet another redundant factor, WML in Japan. Fama & French's (2016a) international Five-factor spanning tests revealed that the factors' respective importance varies from region to region. Only the HML and RMW factors were important everywhere. This casts some doubt towards the use of models as large as the Five- and Six-factor model since the results imply that models dropping one

or multiple variables could perform as well with less complication. In the name of parsimony, such a model should be preferred.

Lastly, the WML factor spanning regressions produce low values of R^2 . Asness (2014) argues that coming up with uncorrelated factors producing high average returns constitutes exemplary investment intuition. For this same reason, Fama & French (2018) were inclined to leave out momentum due to it being largely independent of other factors. While this conforms with the goal of describing cross-section of returns using portfolios not formed on momentum, Asness (2014) argues this is unjustified for practical uses as in regions where momentum is important in describing average returns, the intercepts are economically very large. In particular, WML is motivated if the intention is to form the best portfolio for investment.

7.2 Discussion of the GRS-F test

In similar tests, the GRS F-test has tended to easily reject each of the models, and with high degrees of significance as well. These studies have often concluded that the models compared do not completely capture the sample return variation. In this study, as the Table 6 and Table 7 results show, the GRS F-test rejects only around half the time on 5x5 portfolios, which means the GRS F-test is less consistent in rejecting models on the various portfolio sorts compared with earlier studies. Indeed, on several portfolio sorts both models pass the GRS F-test, especially the additional results of the 2x4x4 sorts.

However, in this study, when the GRS F-test rejects or does not reject the hypothesis, the results involve both models. Asian Pacific Size-OP-Inv portfolios are the only exception to the rule where only one model is rejected by the GRS F-test, and in this case, it was the Six-factor model. Hence, on all sets of 5x5 portfolios, inferences of the GRS F-test both accept and reject the second statistical hypothesis for both models. Overall, the results show that the GRS does not over-reject to as high confidence as previous research and where the test does not reject, the results shown here imply that the Five- and Six-factor models are similarly close to a complete description of equity returns. Unfortunately, except for the one set of portfolios in the additional results, inferences from this absolute test do not directly answer the question of whether one model explains return and the other does not. The absence of a clear contrast between the models in the absolute test implies this study is affected by the same dilemma of previous research in that formal tests often lack power in deciding between models.

Nonetheless, more informative formal judgements can be made on a regional basis from the results in Table 6 and Table 7. Out of the seven sorts, the GRS F-test does not reject either model four times on the North American sorts, five times on the European sorts and three times for the Asian Pacific sorts. A notable finding is that the GRS F-test does not reject either model in Japan on any of the portfolio sorts, coherent with Fama & French (2016a) who report similarly. These results indicate that both models are close to a complete description of return and that they work in describing Japanese equity returns. However, an interesting caveat is that the Size-Mom GRS statistic is not visibly impacted despite the redundancy of the WML factor. These Size-Mom portfolio results are discussed in depth in the next subchapter.

Based on the results presented in Table 6 and Table 7, formal judgements can also be made from model performance on the different portfolio sorts. Again, the GRS F-test implies that both models are close to the complete description of expected returns on multiple sets. From Table 7, the results suggest that the GRS F-test had an easier time in rejecting the 5x5 portfolio sorts than it does the 2x4x4 portfolio sorts. The results show that the GRS F-test only rejects once in the 2x4x4 sorts. None of the Size-B/M-OP and Size-B/M-Inv portfolios are rejected by the GRS. Out of the four regions, Size-B/M and Size-Inv portfolios are rejected only once in North America, Size-OP is rejected twice in Europe and Asia Pacific. Interesting to this thesis is that Size-Mom portfolios are rejected in North America, Europe, and Asia Pacific, but not in Japan. These results contrast both the findings from the factor spanning tests, in which the WML factor was deemed a trivial factor in explaining Japanese returns, and the informal analysis of absolute average alpha. As highlighted by Barillas & Shanken (2016), an issue where RHS results and test asset pricing results may not reach the same conclusions may arise, therefore making interpretation harder. Barillas & Shanken (2016) suggest that when factor pricing and test asset performance contradict, it is the former evidence that decides which model is superior since a comprehensive comparison of two models should test both the ability in pricing assets and factors. From within the same sample, LHS assets do not invalidate factor spanning inferences (Fama & French, 2016a). Fama & French (2018) reaffirm the usefulness of the factor spanning approach in the comparison of nested models, in particular when the choice is whether to add a factor or not. Although Barillas & Shanken (2016) do not present a complete methodology, they suggested a new performance metric focusing on RHS factors, which was briefly covered in Chapter 3.7.

Thus, as the GRS does not contradict in the absolute assessment of both models in Japan, the informal asset pricing tests are consulted to examine the specifics. As presented in the next subchapter, the differences are at best inconclusive regarding Japan, and the Six-factor model does not hurt accuracy on portfolios not sorted on momentum. A comparison of the two results suggests that while these RHS and LHS results contrast, here emphasizing Japan, they are not contradicting in the sense implied by Barillas & Shanken (2016). Therefore, although the Size-Mom GRS statistics are distinctive, they do not present a barrier to making relative inferences based the following informal comparisons of the remaining set and relating these to the factor spanning evidence.

A reflection on the GRS F-test does not seem to over-reject the models as consistently as previous research is offered by listing two potential caveats. First, previous research is conducted on datasets much larger than that of this study, which is typically associated with the tendency of the GRS F-test to over-reject. Secondly, Barillas & Shanken (2018) state that high GRS p-values may tell less about an asset pricing model's adequacy and more about imprecision in estimating alphas produced by a model. Furthermore, previous research has highlighted the occasional failure of the GRS to reject despite the large average absolute alpha produced by the model in question (Goyal, Zhongzhi & Sahn-Wook, 2018).

Lastly, since the results reveal the GRS F-test typically rejects or not rejects both models, this study uses caution and chooses to, like Fama & French (2016a), treat the Table 6 GRS F-test results as an additional summary statistic conjointly with the informal summary statistics of the average alpha. Consequently, switching to an informal interpretation, that is, which model is closer to the complete representation of those not rejected, and which model is closer to not be rejected among those rejected. To summarize, among the unrejected sets, 11 out of the 19 sets indicate the Five-factor model is closer to rejection and therefore the Six-factor model represents the closer representation to a complete description of expected returns. Among the rejected sets, in 6 out of 8 sets the Six-factor model was closer to not be rejected in all but two sets, again implying that the Six-factor model is closer. Across these sets, the Six-factor model is a closer representation to the complete description of average returns. However, as the GRS is not a test of relative power, this does not change the inferences already made in relation to the formal inferences to Statistical hypothesis 2 from Chapter 4.1. To make relative inferences, this study moves to the comparisons of summary statistics related to the average absolute alpha.

7.3 Discussion of the summary statistics

The major edge for the Six-factor model provided by the results of the informal analysis is that it provides a decidedly better description of average return concerning Size-Mom portfolios in North America, Europe, and Asia Pacific, which reaffirms the factor spanning inference by mapping that the factor possesses a distinct useful property. Based on average absolute alpha and the dispersion metrics, the Six-factor model corrects the Five-factor model's inherent inability to price momentum. The high Size-Mom Five-factor intercepts are greatly reduced with the Six-factor model, down to levels more in line with the other portfolio sorts in North America and Europe.

Looking at the Size-Mom regression details, Appendix 7 shows the 25 intercepts and the respective t-statistics produced by the Five-factor model of each region. These details show that Size-Mom portfolios are problematic for the Five-factor model primarily due to the model's inability to accurately describe return in the microcap quantile and both outer tail quantiles representing the "winners" and the "losers". Problems with momentum manifest in the outer tails, which is consistent with earlier literature on the Three- and Five-factor models' inability to price momentum, the literature on the "winners" and "losers", and evidence that momentum is observed more often in smaller companies than large ones (Rouwenhorst, 1998). At the same time, Japanese alphas do not exhibit this imprecision in the outer tails. Appendix 8 shows the Six-factor model alphas from regressions of the same Size-Mom portfolios. The Six-factor alphas show that the Six-factor model is useful for tackling these characteristics specifically. However, these effects are not universal, as Japan was found to be an outlier. Furthermore, while the improvements in alphas are major, these details show that problems with microcaps are not completely eradicated with the addition of WML, as Fama & French (2016b) also noted regarding their U.S. sample. In Appendix 8, small "winner" portfolios still measure a large significant intercept in North America, Europe, and Asia Pacific.

On the six sets of portfolios not sorted on momentum, the Six-factor summary statistics reveal the average absolute alphas are overall only marginally different to the Five-factor model, and often not convincingly different at all. The biggest single improvement of the Six-factor model among these portfolios is a reduction in Size-B/M average absolute alpha in Asia Pacific. Appendix 1 & 2 show the Size-B/M regressions for the Five- and the Six-factor model respectively. These tables show that the intercepts in both outer tails of the B/M quantiles see consistent reduction in the alpha across the size quantiles when the Six-factor model is employed.

Any reduction in average absolute alpha was for the most part confirmed by improvements in the proportional LHS average return dispersion left unexplained to total dispersion in the LHS average returns, and the proportion of the average return dispersion attributable to sampling error. The only exception was the Size-Mom portfolios in Japan, where the Six-factor average absolute alpha was lower, but a larger proportion of the average return dispersion was attributable to dispersion in the true intercepts and proportionally less to sampling error. Moreover, unlike for the other regions, the Size-Mom alphas were already quite small with the Five-factor model, and the number of significant alphas for the Japanese set were equally many, comparably few to the other regions, and exhibited in the same five portfolios.

Overall, the summary statistics of these results reveal a tilt in favour of the Six-factor model. Of the remaining 24 sets on portfolios not sorted on momentum, 14 favour the Six-factor model and 6 favour the Five-factor model on all summary statistics, at least negligibly. Among the portfolios not sorted on momentum, the most consistent tilt in favour of the Six-factor model concerns the Size-B/M portfolios, where all summary statistics favour the model. In North America, portfolios not sorted on momentum reveal marginally better to better performance for Six-factor average absolute alphas and the dispersion metrics in four sets, Size-B/M, Size-Inv, Size-B/M-OP and Size-B/M-Inv, and shows unconvincingly different performance on the remaining Size-OP and Size-OP-Inv portfolios. While the Six-factor model outperforms the Five-factor model on these metrics, on 5x5 sorts both models were only not rejected by the GRS F-test on Size-OP portfolios. In Europe, the portfolios not sorted on momentum show a marginal improvement on the Size-B/M portfolios, but marginally worse performance on Size-OP, Size-Inv, Size-B/M-OP, Size-B/M-Inv and Size-OP-Inv. In general, Europe is the only region where the Six-factor model sacrifices accuracy in multiple sets although the differences are marginal. At the same time, the GRS only rejected both models of Size-OP and Size-Mom portfolios. In Japan, the Six-factor model can be associated with a negligible to marginal improvements on all six portfolio sorts not sorted on momentum. In Asia Pacific, the Six-factor model offers the biggest improvement on portfolios not sorted on momentum as all 5x5 sorts marginally favour the Six-factor model on all metrics. But at the same time, the models achieve unconvincingly different performance between the models on Size-B/M-OP and Size-B/M-Inv, and marginally worse performance on Size-OP-Inv, where the GRS F-test had also rejected the Six-factor model, but not the Five-factor model.

The 2x4x4 portfolios, the Six-factor model provided for most regions a marginal improvement or unconvincingly different description of returns, the exception being the Size-OP-Inv portfolios where the alphas and dispersion statistics marginally tilt towards a Five-factor model in two of the regions each. However, in this set, the Six-factor model provides clear benefits over the Five-factor model in Japan. The Six-factor model is not universally better in all markets, but it is also only marginally a worse description in those sets where it did not provide better results than the Five-factor model.

As the average absolute alphas are close in each set of LHS portfolios other than Size-Mom, the task of conjuring conclusions from these tests is made harder than for the former two. However, even where the Six-factor model performs worse, the difference is trivial given its favourable properties in pricing momentum in three of the regions. An important implication here is that it does not seem like the addition of momentum is detrimental for the description of average returns in comparison to the Five-factor model. Making a concerted inference of the entire sample with all four regions together, tests of average absolute alpha reveal that the Six-factor model generates a lower alpha in 21 out of the 28 comparisons, 17 out of 28 show tilts in favour of the Six-factor model on all metrics. Naturally, some of these improvements are unconvincingly small. The primary challenge for the model turned out to be the Size-OP-Inv portfolios, where the alphas marginally favour the Five-factor model in all regions except Japan. For Japan, where there is no visible momentum effect, in the name of parsimony, one should prefer the Five-factor model over the Six-factor model. This conclusion is made since all tests of LHS assets convey only unconvincingly different to marginally better on all portfolios not sorted on momentum, and the evidence from factor spanning tests that WML is redundant in Japan.

These results suggest that both models are compelling alternatives internationally. When Size-Mom portfolios are considered, the contributions of a Six-factor model are obvious, as unlike the Five-factor model, it provides comparatively decent performance in pricing momentum. Thus, following recent literature that proposes momentum is an important market attribute and therefore WML should be considered in markets where the effect persists, the Six-factor model can be viewed as an attractive alternative to the Five-factor model that may as well provide a better description of average returns. Furthermore, save for Size-OP-Inv, among the remaining six sets of the LHS portfolios, conjointly across the regions, there is no set where the Six-factor model does not provide a marginally better or not a convincingly different result than the Five-factor model.

However, both models achieve for the most part economically insignificant average absolute alphas suggesting that they are both adequate models, but overall, the Six-factor alphas show marginal to major improvements across the LHS sets not sorted on momentum. Therefore, this study concludes that overall, the Six-factor model provides a better description of returns than the Five-factor model on the regions of North America, Europe, and Asia Pacific. However, a comment must be offered in the case of Europe, where the Six-factor model does sacrifice some negligible to marginal accuracy in describing the returns of some of the LHS assets not sorted on momentum. First, the factor spanning results produce an economically large and significant intercept in Europe. Second, the momentum effect is strong resulting in large pricing errors which the Six-factor model is demonstrated able to mitigate. Therefore, this study concludes the Six-factor model performs better in Europe as well.

Lastly, across all seven sets of portfolios, the average adjusted R^2 is also consistently lower in Asia Pacific. Although the coefficient cannot be strictly compared across samples, one can speculate as to how this occurs. Indeed, the average absolute alphas are also visibly higher in Asia Pacific for both models and across all sets compared to the other regions. In fact, Table 6 and Table 7 confirm that only Size-B/M and Size-Inv portfolios produce economically smaller average absolute alphas, both models produce average absolute alphas of 0.20 or more across all other portfolio sorts. Further, this idiosyncrasy is visible throughout the Appendices 1-14. Therefore, a likely explanation concerns the limits of the approach of using regionally diversified data from several developed markets for this international study. The data chapter covered the assumption made that each region is formed of countries with reasonably similar characteristics. However, the greatest concern relayed by Fama & French (2012) concerns the Asian Pacific region. As the most ambitious exercise regarding the assumption of integration, if the region actually consists of markets comparably further apart, this may lead to a greater degree of unexplained variance in the sample as neither LHS portfolios nor the factors formations are able to efficiently capture the variation. Consequently, the results shown here suggest that regional models perform somewhat inadequately in Asia Pacific.

8 CONCLUSIONS

This thesis investigated the Six-factor model from an international perspective, whether the momentum factor, WML, contributes to the story of returns in the four regions of North America, Europe, Japan, and Asia Pacific. This thesis also investigated whether the model is able to explain international returns and how this performance compares to the Five-factor model. This chapter summarizes the inferences made, presents the conclusions, and discusses suggestions for future research.

In North America, Europe, and Asia Pacific the factor spanning tests confirmed that WML is an important addition to the Five-factor model, as each of the three regressions produced economically large and significant intercepts. However, factor spanning tests also demonstrate that WML is a redundant factor in Japan as its returns were captured by its exposures to the remaining factors.

Although the GRS F-test rejected both models on North American, European, and Asian Pacific Size-Mom portfolios, the informal analysis of alpha confirmed WML to be important to the explanation of momentum. The Six-factor model provided a better description of Size-Mom portfolio returns on all summary statistics considered in North America, Europe, and Asia Pacific. Average absolute alphas were reduced to levels close to those of other portfolio sorts, which were, for the most part, found economically small across both models. Accordingly, the number of individual significant alphas saw the single biggest reduction with the employment of the Six-factor model. These results indicate that the WML addition increases the model's ability to price momentum. Indeed, the Six-factor model did not sacrifice accuracy in the return descriptions of the remaining portfolio sorts. On portfolios not sorted on momentum, both models passed the GRS F-test on multiple portfolio sorts. The GRS also did not over-reject as much as shown in some previous studies. The analysis of average absolute alpha with summary statistics confirmed that the Six-factor model provided an equal or marginal improvement in the description of returns on most test assets in the four regions. Although Europe saw a marginally higher alpha with the Six-factor model on portfolios not sorted on momentum, the model provided a major improvement in the description of momentum. This confirms the spanning tests in that WML possesses distinct useful properties when added to the Five-factor model.

The LHS tests further confirm the factor spanning results in that Japan remains an outlier. Although the GRS passed both models on all seven portfolio sorts, both models

produce economically smaller average absolute alphas to begin with on Size-Mom portfolios, unlike for other regions where the Five-factor model produced very high Five-factor alphas. This was somewhat unsurprising given the evidence from the Japanese factor spanning regression, and the implication from the descriptive statistics in Chapter 5.3 that there is no visible momentum effect in Japan. Fama & French's (2016a) comprehensive Five-factor spanning tests revealed that the importance of the Fama-French factors varies a lot between the regions. This thesis identified yet another redundant factor in Japan. As the results of this study confirmed that WML does not improve the description of Japanese returns, this study concludes that the Five-factor model should be preferred in Japan. However, based on the factor spanning tests of this study and by Fama & French (2016a), the only significant factors in Japan are RMW and HML. This suggests that a model which drops perhaps even multiple factors likely could provide an equal or better description of Japanese returns. Thus, for parsimony, perhaps neither the Five- nor the Six-factor model is the most efficient choice in describing Japanese returns.

In reference to these results, this study concludes that the Six-factor model is a viable alternative that was found to regularly outperform the Five-factor model in North America, Europe, and Asia Pacific. In particular, the Six-factor model proves a useful tool in mitigating the pricing error produced by the momentum anomaly.

This study seems to confirm both views on momentum covered throughout this thesis. The strong momentum effect in North America, Europe and Asia Pacific, the economically significant intercepts from factor spanning regressions, and the ability of the Six-factor model to price momentum indicate that WML houses useful properties that contribute to the description of returns. These results extend the literature suggesting that WML constitutes an important addition in markets where the momentum effect reveals itself clearly. Simultaneously, the descriptive statistics illustrated that there are no visible signs of momentum in Japan and the empirical evidence from the empirical study demonstrate that WML is trivial in Japan, thus further extending on the literature suggesting that momentum is not universal.

Lastly, the theoretical sections of this study covered that explanations of the anomalies often take a risk-based or a behavioural view, and that some researchers hesitate to recommend factors that lack theoretical motivation whereas others believe a strong vetting process is an appropriate alternative until theoretical consensus can be asserted. The momentum effect remains empirically motivated and has not disappeared over time,

in fact, it has been found strong across markets. This study contributes to the empirical evidence and the vetting process of the WML factor, showing that, alike Asness, Moskowitz & Pedersen (2013) and Asness et al (2014) suggest, given the magnitude of the effect, momentum models should be considered at least as viable alternatives.

8.1 Suggestions for further research

Fama & French (2016a) stated they expect further studies to not only add factors, but also identify redundant ones. This study limited its factor spanning tests to putting WML on the LHS to further investigate the multicollinearity between the factors. It would be interesting for future research to regress each of the factors on the remaining five with WML as one of the explanatory variables to investigate whether WML makes any of the other factors redundant internationally, and if so, in which regions? This would extend the topic on factor redundancy.

Given the model's ability to price momentum, it would be interesting to see the topic of whether either the Five- or the Six-factor model help shrink the list of known problematic anomalies be extended to international samples.

The results suggest the Six-factor model could benefit from being investigated on country-based data. Specifically, the Asian Pacific regional data used in this study produced higher average absolute alphas than the other regions across all portfolio sorts. This indicates that regional factors do a poor job of describing returns of the Asian Pacific countries. Most recently, Foye & Valentincic (2020) included a Six-factor model in tests of the Five-factor model in Indonesia. This type of study could be extended to any of the Asian Pacific countries, or any other country for that matter. Further, as this study limited the framework to only regional factors, the topic of global models could be revisited with the Six-factor model, especially since studies (Asness, Moskowitz & Pedersen, 2013) have highlighted the importance of treating value and momentum together in international studies.

A popular topic concerns the different factor proxies that exist for each characteristic. In response to Fama & French's (2015) conclusion that RMW and CMA made HML redundant in the U.S., Asness (2014) recommended further studies should use the more updated HMLm factor of Asness & Frazzini (2013). HMLm differs as book-to-market employs in the denominator the most recent monthly stock price (Barillas & Shanken, 2018), rather than the six-month old upon portfolio formation. Indeed, Asness (2014) and Barillas & Shanken (2018) confirm that when the HMLm factor is added alongside

WML, the value factor is no longer redundant. Furthermore, Fama & French (2016b) demonstrate that accruals remain a problematic anomaly with the Five-factor model. Ball, Gerakos, Linnainmaa & Nikolaev (2015) propose operating profitability be substituted with a cash profitability factor, CP, to better target average returns associated with accruals. Fama & French (2018) found that models that adopt cash profitability factors dominate models that use operating profitability in the U.S. Therefore, it would be advisable for future studies to consider using HMLm and, or, CP. Likewise it would be interesting to see whether the benefits of these two factors extend to international samples.

Barillas & Shanken (2018) compared different compositions of 10 current asset pricing factors found in the literature. Studying a U.S. sample, their findings favour a six-factor model consisting of MKT, SMB, HMLm, the investment, IA, and profitability, ROE, factors of Hou, Xue & Zhang (2016), and WML. It would be interesting to investigate how various six factor models composed of different factor definitions perform internationally against the Six-factor model studied here.

Lastly, even if the model requires further vetting, the possible economic significance in the form of both academic and practical uses for the Six-factor model are many. Even if a model only proxies a yet unknown more efficient parsimonious model, a model that captures anomalous effects can be practical in the real-world applications. Fama & French (2015) noted the Five-factor model likely represents a better choice in many practical applications. For example, on the topic of socially responsible and ethical investing, Blitz & Fabozzi (2017) find evidence that sin stocks no longer display positive abnormal returns once the Five-factor model is used. Asness, Frazzini & Pedersen (2019) employ the Five- and Six-factor models to investigate the abnormal returns of Quality minus Junk, QMJ, finding that the latter model reduces the alpha across all U.S. and Global portfolios considered. Given the empirical evidence behind momentum, one can assume there are practical uses for the Six-factor model where the Five-factor model is considered.

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Appendix 1 Size-B/M Five-factor alphas and slopes

North America										Europe													
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha								
Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High				
Small	-0.21	0.00	0.29	0.11	0.33	Small	-1.61	-0.04	3.28	1.53	4.80	Small	-0.18	0.03	0.08	0.00	0.12	Small	-1.98	0.45	1.17	-0.01	2.16
2	-0.15	-0.06	-0.04	-0.04	-0.10	2	-1.60	-0.72	-0.60	-0.64	-1.89	2	0.00	0.01	-0.04	-0.05	0.06	2	-0.06	0.08	-0.67	-0.78	0.99
3	0.20	-0.09	-0.05	-0.16	-0.05	3	2.11	-0.97	-0.72	-2.11	-0.74	3	0.18	0.04	-0.17	-0.18	-0.01	3	2.16	0.53	-2.35	-2.53	-0.18
4	0.30	0.05	-0.02	-0.12	-0.07	4	2.90	0.67	-0.26	-1.48	-1.01	4	0.28	-0.01	-0.08	-0.13	-0.10	4	3.40	-0.07	-1.12	-1.66	-1.22
Big	0.06	0.02	-0.07	-0.01	-0.19	Big	1.04	0.26	-1.00	-0.14	-2.15	Big	0.10	-0.04	0.02	0.07	-0.02	Big	1.43	-0.72	0.38	0.90	-0.27
MKT					MKT					MKT					MKT								
Small	1.01	0.97	0.93	0.88	0.91	Small	29.09	33.79	39.58	47.23	49.43	Small	0.99	0.98	0.94	0.95	0.91	Small	47.03	55.83	64.00	71.66	70.65
2	1.06	1.02	1.02	1.00	1.04	2	41.81	49.39	57.65	59.18	71.43	2	1.06	1.03	0.99	0.98	0.99	2	62.02	69.29	71.44	73.18	76.59
3	1.06	1.06	1.05	1.04	1.06	3	41.76	44.86	54.14	52.29	57.37	3	1.06	1.05	1.03	1.01	1.05	3	54.25	62.07	60.16	61.50	62.83
4	1.05	1.03	1.04	1.05	1.08	4	37.66	47.59	51.12	47.32	56.51	4	1.03	1.02	1.02	1.03	1.05	4	53.85	54.67	61.31	55.63	57.06
Big	0.99	0.97	1.00	0.95	1.14	Big	64.70	55.21	55.72	54.13	49.46	Big	0.93	1.00	1.03	1.00	1.03	Big	56.34	70.92	73.40	58.06	53.89
SMB					SMB					SMB					SMB								
Small	1.28	1.13	1.15	1.01	0.98	Small	25.88	27.58	34.40	37.81	37.27	Small	1.06	1.03	0.96	0.96	0.96	Small	26.63	30.87	34.55	38.33	39.19
2	0.94	1.04	0.91	0.75	0.83	2	25.79	35.13	36.16	31.22	39.94	2	0.99	0.90	0.92	0.88	0.88	2	30.74	31.75	34.99	34.77	35.81
3	0.82	0.73	0.59	0.54	0.54	3	22.55	21.75	21.31	19.20	20.35	3	0.73	0.73	0.68	0.66	0.65	3	19.51	22.70	20.99	21.35	20.46
4	0.45	0.37	0.35	0.26	0.31	4	11.25	12.03	11.94	8.30	11.30	4	0.35	0.41	0.35	0.41	0.43	4	9.70	11.58	11.05	11.73	12.37
Big	-0.29	-0.19	-0.13	-0.16	-0.17	Big	-13.04	-7.65	-5.01	-6.25	-5.01	Big	-0.31	-0.18	-0.21	-0.20	-0.23	Big	-10.01	-6.72	-7.83	-6.00	-6.40
HML					HML					HML					HML								
Small	-0.63	-0.29	-0.14	0.11	0.41	Small	-9.74	-5.44	-3.17	3.29	11.95	Small	-0.58	-0.35	-0.19	0.04	0.25	Small	-10.93	-7.85	-5.08	1.21	7.79
2	-0.60	-0.35	-0.07	0.28	0.49	2	-12.70	-9.05	-2.04	8.80	17.99	2	-0.53	-0.23	0.03	0.32	0.44	2	-12.40	-6.20	0.94	9.62	13.69
3	-0.58	-0.07	0.11	0.34	0.48	3	-12.32	-1.62	2.92	9.36	13.96	3	-0.62	-0.15	0.15	0.38	0.52	3	-12.64	-3.62	3.54	9.17	12.46
4	-0.65	-0.01	0.12	0.35	0.53	4	-12.61	-0.29	3.27	8.58	14.88	4	-0.47	-0.08	0.14	0.38	0.68	4	-9.78	-1.79	3.33	8.21	14.69
Big	-0.46	-0.14	0.21	0.43	0.82	Big	-16.19	-4.28	6.25	13.26	19.18	Big	-0.62	-0.25	0.01	0.32	0.80	Big	-14.89	-7.06	0.30	7.33	16.70
RMW					RMW					RMW					RMW								
Small	-0.60	-0.36	-0.31	-0.16	-0.13	Small	-9.38	-6.75	-7.21	-4.50	-3.83	Small	-0.49	-0.28	-0.24	0.00	-0.01	Small	-7.39	-5.11	-5.24	0.07	-0.31
2	-0.25	-0.21	0.02	0.07	0.06	2	-5.22	-5.43	0.66	2.32	2.14	2	-0.26	-0.13	0.01	0.16	-0.01	2	-4.82	-2.88	0.19	3.88	-0.22
3	-0.15	0.09	0.19	0.21	0.11	3	-3.10	2.05	5.27	5.84	3.08	3	-0.32	0.02	0.19	0.18	0.00	3	-5.22	0.40	3.51	3.55	0.09
4	-0.30	0.04	0.17	0.23	0.07	4	-5.82	0.98	4.48	5.69	1.89	4	-0.24	0.00	0.15	0.09	0.01	4	-3.91	0.07	2.94	1.59	0.12
Big	0.19	0.06	-0.01	-0.09	-0.14	Big	6.76	1.80	-0.38	-2.68	-3.23	Big	0.00	0.28	0.01	-0.04	-0.41	Big	-0.04	0.64	0.37	-0.79	-6.74
CMA					CMA					CMA					CMA								
Small	0.05	-0.15	0.01	0.00	-0.01	Small	0.59	-2.19	0.11	-0.09	-0.23	Small	-0.26	-0.16	-0.11	0.10	0.20	Small	-4.05	-2.95	-2.50	2.35	5.16
2	-0.26	-0.02	0.10	-0.01	0.08	2	-4.29	-0.42	2.33	-0.23	2.37	2	-0.29	-0.05	0.04	0.04	0.20	2	-5.47	-1.09	0.98	0.93	4.94
3	-0.14	-0.27	-0.06	0.02	0.08	3	-2.29	-4.78	-1.25	0.51	1.80	3	-0.36	-0.04	0.10	-0.01	0.12	3	-6.05	-0.83	1.91	-0.20	2.35
4	-0.05	-0.20	0.05	0.02	0.05	4	-0.78	-3.91	0.94	0.46	1.05	4	-0.29	0.12	0.13	0.03	-0.03	4	-4.90	2.18	2.46	0.49	-0.60
Big	0.03	0.11	0.00	-0.10	-0.19	Big	0.72	2.53	0.04	-2.31	-3.48	Big	0.04	0.36	0.04	-0.06	-0.48	Big	0.87	8.40	0.95	-1.04	-8.11

Japan										Asia Pacific													
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha								
Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High	Low	2	3	4	High				
Small	0.13	0.16	0.19	0.16	0.19	Small	0.81	1.37	1.78	2.12	2.63	Small	0.23	-0.02	0.07	0.43	0.70	Small	1.13	-0.11	0.60	3.75	6.39
2	0.14	-0.20	-0.15	0.04	-0.09	2	0.88	-2.17	-1.68	0.55	-1.75	2	-0.25	-0.18	-0.19	-0.16	-0.02	2	-1.79	-1.52	-1.58	-1.43	-0.15
3	-0.23	-0.08	-0.15	-0.13	-0.02	3	-1.74	-0.80	-1.80	-1.80	-0.37	3	0.02	-0.23	-0.10	-0.10	-0.16	3	0.15	-1.63	-0.77	-0.76	-1.16
4	-0.19	-0.02	-0.04	-0.04	-0.11	4	-1.61	-0.26	-0.39	-0.47	-1.35	4	0.16	0.05	-0.24	-0.02	-0.13	4	1.10	0.37	-1.83	-1.33	-0.94
Big	0.07	0.04	-0.03	0.03	0.12	Big	0.82	0.46	-0.40	0.37	0.88	Big	0.19	0.02	-0.04	-0.17	-0.07	Big	1.67	0.18	-0.34	-1.61	-0.50
MKT					MKT					MKT					MKT								
Small	1.12	0.97	0.99	0.94	0.97	Small	34.51	41.00	44.56	61.39	64.77	Small	1.11	1.10	1.06	1.00	0.93	Small	27.17	39.13	44.17	43.85	43.44
2	1.09	1.01	0.96	0.98	1.01	2	34.53	54.53	54.03	71.77	99.00	2	0.96	1.07	0.99	0.99	0.98	2	34.43	44.92	42.32	44.07	46.45
3	1.06	0.98	0.96	0.93	1.01	3	40.28	50.46	56.74	63.81	78.30	3	0.98	1.04	1.09	1.02	0.97	3	31.33	36.94	43.29	38.86	35.55
4	1.05	0.95	0.92	0.95	1.05	4	43.93	48.14	48.78	54.27	63.61	4	0.99	1.02	1.03	1.04	1.10	4	33.86	36.05	39.61	41.25	39.32
Big	1.01	0.95	0.99	0.99	1.14	Big	61.09	54.34	57.38	52.10	40.10	Big	0.98	1.00	1.03	0.98	1.02	Big	44.19	57.77	50.10	47.00	36.04
SMB					SMB					SMB					SMB								
Small	1.41	1.16	1.09	1.00	0.99	Small	27.97	31.37	31.63	41.82	42.59	Small	1.23	1.06	1.10	1.12	1.26	Small	18.08	22.66	27.35	29.40	34.91
2	1.23	0.97	0.93	0.82	0.82	2	24.94	33.60	33.67	38.67	51.95	2	0.77	0.80	0.79	0.82	0.94	2	16.45	20.14	20.23	21.96	26.51
3	0.91	0.65	0.66	0.61	0.65	3	22.21	21.43	25.05	26.96	32.40	3	0.55	0.56	0.65	0.55	0.60	3	10.40	11.87	15.34	12.51	13.02
4	0.49	0.36	0.31	0.30	0.43	4	13.18	11.68	10.60	11.09	16.81	4	0.37	0.22	0.29	0.28	0.19	4	7.56	4.66	6.67	6.73	4.08
Big	-0.19	-0.26	-0.17	-0.07	-0.07	Big	-7.25	-9.71	-6.28	-2.40	-1.58	Big	-0.16	-0.14	-0.08	-0.27	-0.40	Big	-4.21	-4.87	-2.46	-7.74	-8.39
HML					HML					HML					HML								
Small	-0.36	-0.20	-0.05	0.06	0.31	Small	-5.34	-4.04	-1.04	1.79	9.81	Small	-0.78	-0.51	-0.19	-0.06	0.26	Small	-8.99	-8.47	-3.78	-1.30	5.61
2	-0.55	-0.23	0.05	0.13	0.43	2	-8.25	-6.04	1.47	4.65	20.18	2	-0.46	-0.33	-0.15	0.26	0.62	2	-7.70	-6.39	-2.94	5.44	13.73
3	-0.48	-0.21	0.07	0.25	0.49	3	-8.70	-5.27	2.06	8.33	18.12	3	-0.54	-0.18	-0.03	0.19	0.64	3	-8.08	-3.01	-0.61	3.30	10.92
4	-0.42	-0.14	0.10	0.31	0.55	4	-8.38	-3.30	2.51	8.53	15.99	4	-0.25	-0.15	0.00	0.24	0.63	4	-4.07	-2.55	0.06	4.50	10.48
Big	-0.61	-0.06	0.22	0.42	0.77	Big	-17.77	-1.66	6.13	10.57	12.93	Big	-0.46	-0.03	0.04	0.28	0.71	Big	-9.59	-0.85	1.02	6.27	11.74
RMW					RMW					RMW					RMW								
Small	-0.04	0.00	0.07	-0.06	0.06	Small	-0.41	-0.02	0.89	-1.21	1.17	Small	-0.43										

Appendix 2 Size-B/M Six-factor alphas and slopes

North America					Europe						
Coefficient					Coefficient						
Alpha					Alpha						
Low	2	3	4	High	Low	2	3	4	High		
Small	-0.20	0.01	0.27	0.12	0.31	Small	-1.49	0.10	3.00	1.68	4.48
2	-0.07	-0.04	-0.02	-0.01	-0.10	2	-0.81	-0.50	-0.27	-0.16	-1.82
3	0.17	-0.07	-0.02	-0.11	-0.04	3	1.76	-0.78	-0.28	-1.54	-0.50
4	0.27	0.08	-0.02	-0.09	-0.04	4	2.60	0.96	-0.20	-1.07	-0.49
Big	0.07	0.02	-0.04	0.00	-0.14	Big	1.23	0.33	-0.62	0.03	-1.59
MKT					MKT						
Low	2	3	4	High	Low	2	3	4	High		
Small	1.01	0.96	0.93	0.88	0.91	Small	28.52	33.14	39.43	46.37	49.26
2	1.04	1.01	1.01	0.99	1.04	2	42.16	48.50	56.77	58.61	70.25
3	1.07	1.05	1.04	1.02	1.06	3	41.82	44.03	53.48	52.02	56.40
4	1.06	1.02	1.04	1.04	1.06	4	37.59	46.76	50.25	46.63	56.04
Big	0.99	0.97	0.99	0.95	1.13	Big	63.61	54.26	54.94	53.17	49.13
SMB					SMB						
Low	2	3	4	High	Low	2	3	4	High		
Small	1.29	1.14	1.14	1.02	0.97	Small	25.59	27.32	33.68	37.46	36.53
2	0.97	1.04	0.92	0.76	0.83	2	27.61	34.90	36.22	31.77	39.34
3	0.80	0.74	0.61	0.56	0.55	3	21.95	21.65	21.77	20.06	20.36
4	0.44	0.38	0.35	0.28	0.32	4	10.79	12.24	11.82	8.76	11.95
Big	-0.28	-0.19	-0.12	-0.15	-0.14	Big	-12.62	-7.43	-4.52	-5.96	-4.35
HML					HML						
Low	2	3	4	High	Low	2	3	4	High		
Small	-0.64	-0.30	-0.12	0.10	0.43	Small	-9.52	-5.47	-2.54	2.82	12.07
2	-0.68	-0.36	-0.09	0.24	0.48	2	-14.41	-9.09	-2.61	7.60	17.12
3	-0.55	-0.09	0.07	0.30	0.46	3	-11.20	-1.91	1.95	7.99	12.94
4	-0.62	-0.04	0.12	0.32	0.49	4	-11.57	-0.85	3.00	7.49	13.46
Big	-0.47	-0.14	0.18	0.79	0.77	Big	-15.93	-4.24	5.29	12.37	17.61
RMW					RMW						
Low	2	3	4	High	Low	2	3	4	High		
Small	-0.60	-0.35	-0.32	-0.15	-0.13	Small	-9.32	-6.68	-7.32	-4.44	-3.95
2	-0.23	-0.20	0.03	0.88	0.06	2	-5.13	-5.35	0.79	2.53	2.15
3	-0.15	0.09	0.19	0.22	0.11	3	-3.25	2.12	5.50	6.20	3.17
4	-0.31	0.04	0.17	0.24	0.07	4	-5.94	1.09	4.49	5.89	2.12
Big	0.19	0.06	-0.01	-0.09	-0.13	Big	6.84	1.82	-0.24	-2.62	-3.07
CMA					CMA						
Low	2	3	4	High	Low	2	3	4	High		
Small	0.06	-0.14	-0.01	0.00	-0.02	Small	0.67	-2.06	-0.11	0.05	-0.47
2	-0.22	-0.01	0.11	0.01	0.08	2	-3.74	-0.24	2.60	0.18	2.38
3	-0.16	-0.26	-0.04	0.05	0.09	3	-2.58	-4.60	-0.88	1.02	1.99
4	-0.07	-0.19	0.05	0.04	0.07	4	-1.01	-3.65	0.99	0.81	1.51
Big	0.03	0.11	0.02	-0.09	-0.16	Big	0.89	2.57	0.37	-2.14	-3.03
WML					WML						
Low	2	3	4	High	Low	2	3	4	High		
Small	-0.02	-0.02	0.03	-0.02	0.03	Small	-0.69	-0.91	1.77	-1.15	1.99
2	-0.12	-0.02	-0.03	-0.05	0.00	2	-5.81	-1.43	-2.26	-3.31	-0.28
3	0.05	-0.02	-0.05	-0.07	-0.02	3	2.35	-1.24	-3.05	-4.10	-1.59
4	0.04	-0.04	-0.01	-0.05	-0.06	4	1.93	-1.99	-0.43	-2.85	-3.59
Big	-0.02	-0.01	-0.04	-0.02	-0.07	Big	-1.39	-0.53	-2.61	-1.19	-4.02
Japan					Asia Pacific						
Coefficient					Coefficient						
Alpha					Alpha						
Low	2	3	4	High	Low	2	3	4	High		
Small	0.14	0.17	0.20	0.16	0.19	Small	0.21	-0.01	0.03	0.41	0.62
2	0.13	-0.19	-0.14	0.04	-0.09	2	-0.24	-0.07	-0.14	-0.19	-0.03
3	-0.22	-0.07	-0.14	-0.12	-0.03	3	0.07	-0.22	-0.11	-0.06	-0.11
4	-0.19	-0.02	-0.03	-0.03	-0.10	4	0.17	0.11	-0.10	0.06	-0.05
Big	0.06	0.04	-0.02	0.03	0.12	Big	0.14	-0.01	0.03	-0.13	0.00
MKT					MKT						
Low	2	3	4	High	Low	2	3	4	High		
Small	1.11	0.96	0.99	0.94	0.97	Small	1.11	1.10	1.06	1.00	0.94
2	1.10	1.00	0.95	0.97	1.01	2	0.96	1.06	0.99	0.99	0.98
3	1.06	0.98	0.95	0.93	1.02	3	0.98	1.04	1.09	1.02	0.97
4	1.05	0.95	0.91	0.94	1.04	4	0.99	1.01	1.02	1.03	1.10
Big	1.01	0.95	0.98	0.99	1.15	Big	0.98	1.00	1.02	0.97	1.01
SMB					SMB						
Low	2	3	4	High	Low	2	3	4	High		
Small	1.40	1.15	1.09	1.00	0.99	Small	1.23	1.07	1.10	1.12	1.25
2	1.23	0.97	0.92	0.82	0.82	2	0.77	0.81	0.80	0.82	0.93
3	0.91	0.65	0.66	0.61	0.65	3	0.55	0.56	0.65	0.55	0.60
4	0.49	0.36	0.31	0.30	0.43	4	0.37	0.22	0.30	0.29	0.20
Big	-0.19	-0.27	-0.17	-0.30	-0.07	Big	-0.16	-0.14	-0.08	-1.45	-0.39
HML					HML						
Low	2	3	4	High	Low	2	3	4	High		
Small	-0.38	-0.23	-0.05	0.06	0.31	Small	-0.77	-0.51	-0.18	-0.05	0.30
2	-0.52	-0.26	0.03	0.12	0.43	2	-0.47	-0.38	-0.17	0.27	0.63
3	-0.49	-0.23	0.05	0.24	0.50	3	-0.57	-0.19	-0.03	0.17	0.62
4	-0.42	-0.15	0.09	0.29	0.54	4	-0.26	-0.18	-0.07	0.21	0.59
Big	-0.61	-0.07	0.19	0.42	0.78	Big	-0.43	-0.02	0.01	0.26	0.67
RMW					RMW						
Low	2	3	4	High	Low	2	3	4	High		
Small	-0.01	0.03	0.07	-0.06	0.05	Small	-0.43	-0.31	-0.04	-0.10	-0.13
2	0.23	-0.02	0.08	-0.06	-0.06	2	-0.28	-0.17	0.01	0.15	-0.10
3	0.07	-0.06	0.05	0.00	-0.09	3	-0.21	0.13	0.29	0.12	-0.06
4	0.06	-0.05	-0.02	0.04	0.00	4	0.04	0.14	0.28	0.27	-0.17
Big	-0.02	0.00	0.04	-0.02	-0.14	Big	-0.02	0.22	0.05	-0.19	-0.44
CMA					CMA						
Low	2	3	4	High	Low	2	3	4	High		
Small	-0.01	0.12	0.00	-0.04	0.07	Small	0.46	0.28	0.16	0.13	0.12
2	0.13	0.13	0.10	0.05	0.06	2	-0.08	-0.01	0.00	-0.18	-0.13
3	0.03	-0.07	0.12	0.10	0.06	3	-0.31	-0.04	0.09	-0.06	-0.19
4	-0.14	0.03	0.05	0.16	0.13	4	-0.07	0.20	0.12	0.03	-0.03
Big	0.05	0.09	0.02	-0.06	-0.17	Big	-0.16	-0.05	0.13	0.24	-0.10
WML					WML						
Low	2	3	4	High	Low	2	3	4	High		
Small	-0.07	-0.08	-0.01	0.00	0.02	Small	0.02	-0.01	0.04	0.02	0.07
2	0.07	-0.07	-0.08	-0.03	0.01	2	-0.02	-0.11	-0.05	0.03	0.01
3	-0.02	-0.04	-0.07	-0.05	0.02	3	-0.04	-0.01	0.01	-0.04	-0.05
4	0.00	-0.05	-0.04	-0.08	-0.04	4	-0.01	-0.05	-0.14	-0.07	-0.08
Big	0.01	-0.04	-0.10	0.02	0.03	Big	0.05	0.02	-0.06	-0.03	-0.07

Appendix 3 Size-OP Five-factor alphas and slopes

North America													Europe												
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
MKT					MKT					MKT					MKT										
Small	0.21	0.28	0.10	0.14	0.13	Small	2.32	3.80	1.40	1.66	1.38	Small	-0.16	0.13	0.17	0.30	0.19	Small	-2.43	2.26	-2.94	5.11	2.56		
2	-0.11	-0.03	-0.01	0.05	-0.03	2	-2.03	-0.45	-0.13	0.63	-0.38	2	-0.09	-0.01	-0.05	0.00	0.32	2	-1.44	-0.23	-0.89	0.08	4.98		
3	0.05	0.00	-0.07	-0.09	-0.03	3	0.54	-0.03	-0.95	-1.28	-0.32	3	-0.12	-0.06	0.08	-0.06	0.09	3	-1.73	-0.87	1.23	-0.78	1.24		
4	0.05	0.10	0.06	-0.10	0.03	4	0.48	1.50	0.73	-1.42	0.36	4	-0.09	-0.05	0.01	0.05	0.11	4	-0.99	-0.66	0.13	0.64	1.53		
Big	-0.10	-0.02	-0.05	0.16	-0.03	Big	-1.24	-0.28	-0.74	2.70	-0.49	Big	0.08	0.19	0.08	-0.25	0.01	Big	0.92	2.94	1.14	-4.02	0.16		
SMB					SMB					SMB					SMB										
Small	0.94	0.87	0.92	0.94	1.00	Small	38.61	43.88	47.75	40.71	40.20	Small	0.97	0.90	0.95	0.95	0.95	Small	63.13	66.17	71.91	71.00	56.80		
2	1.05	0.99	1.01	1.03	1.12	2	71.18	57.95	55.46	52.99	52.77	2	1.02	0.97	1.01	1.01	1.05	2	71.99	67.29	73.38	72.93	71.40		
3	1.10	1.01	0.99	1.08	1.13	3	49.00	50.39	48.47	56.00	50.04	3	1.07	1.03	1.00	1.03	1.05	3	65.69	66.71	65.04	61.23	60.22		
4	1.11	1.02	1.00	1.05	1.06	4	38.73	56.39	48.20	53.74	45.05	4	1.01	1.00	1.05	1.04	1.05	4	49.36	58.62	63.11	58.15	60.81		
Big	1.07	1.05	1.03	0.93	0.96	Big	48.66	53.50	59.41	57.26	62.38	Big	1.03	0.99	1.02	1.03	0.94	Big	52.40	66.29	62.10	72.69	63.82		
HML					HML					HML					HML										
Small	-0.16	0.36	0.30	0.32	0.31	Small	-3.46	9.65	8.36	7.38	6.80	Small	-0.22	0.07	0.03	0.06	-0.05	Small	-5.66	2.10	0.94	1.66	-1.17		
2	-0.23	0.21	0.15	0.25	0.25	2	-8.38	6.79	4.31	6.97	6.27	2	-0.09	0.13	0.18	0.18	-0.01	2	-2.53	3.55	-5.19	5.20	-0.37		
3	-0.17	0.10	0.21	0.27	0.18	3	-3.96	2.61	5.57	7.54	4.23	3	-0.09	0.14	0.20	0.15	0.08	3	-2.24	3.71	5.30	3.58	1.88		
4	-0.18	0.16	0.14	0.16	0.05	4	-3.34	4.76	3.75	4.39	1.06	4	0.04	0.27	0.20	0.10	0.00	4	0.70	6.25	4.80	2.23	0.07		
Big	0.26	0.21	0.13	-0.02	-0.28	Big	6.45	5.71	4.14	-0.75	-9.69	Big	0.13	0.10	-0.10	0.10	-0.21	Big	2.61	2.63	-2.39	2.70	-5.67		
RMW					RMW					RMW					RMW										
Small	-0.61	0.05	0.23	0.27	0.27	Small	-13.61	1.29	6.34	6.36	5.87	Small	-0.47	-0.06	0.04	0.10	0.14	Small	-9.82	-1.40	0.93	2.31	2.68		
2	-0.65	0.09	0.24	0.42	0.66	2	-23.99	2.74	7.25	11.79	16.74	2	-0.46	-0.05	0.15	0.27	0.18	2	-10.44	-1.20	3.58	6.29	3.89		
3	-0.66	0.05	0.34	0.55	0.59	3	-15.90	1.47	8.97	15.52	14.07	3	-0.41	-0.04	0.23	0.24	0.28	3	-8.00	-0.77	4.76	4.55	5.13		
4	-0.72	-0.04	0.22	0.40	0.41	4	-13.59	-1.21	5.69	11.04	9.35	4	-0.62	0.01	0.20	0.19	0.19	4	-9.61	0.24	3.74	3.34	3.59		
Big	-0.77	-0.37	-0.06	0.07	0.40	Big	-19.04	-10.19	-1.80	2.18	14.15	Big	-1.14	-0.46	-0.06	0.47	0.51	Big	-18.49	-9.76	-1.20	10.67	11.11		
CMA					CMA					CMA					CMA										
Small	0.05	-0.07	-0.07	-0.14	-0.30	Small	0.85	-1.38	-1.63	-2.59	-5.07	Small	-0.05	0.10	0.05	0.02	-0.06	Small	-1.00	2.39	1.33	0.59	-1.12		
2	-0.01	0.07	0.06	-0.14	-0.16	2	-0.29	1.62	1.48	-3.02	-3.23	2	-0.12	0.12	0.10	0.03	-0.09	2	-2.71	2.69	2.33	0.78	-1.93		
3	-0.15	0.04	-0.02	-0.14	-0.17	3	-2.75	0.90	-0.51	-2.97	-3.11	3	0.00	0.04	-0.07	-0.05	-0.12	3	-0.01	0.81	-1.59	-0.91	-2.30		
4	-0.03	0.00	0.02	0.07	-0.03	4	-0.41	-0.05	0.50	1.58	-0.62	4	0.01	0.03	0.05	-0.03	-0.12	4	0.11	0.51	0.92	-0.47	-2.36		
Big	-0.37	-0.12	-0.05	-0.04	0.10	Big	-7.04	-2.66	-1.12	-0.93	2.81	Big	-0.25	-0.01	0.17	-0.03	0.02	Big	-4.10	-0.27	3.43	-0.65	0.38		
Japan													Europe												
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
MKT					MKT					MKT					MKT										
Small	0.09	0.20	0.15	0.14	0.38	Small	1.20	3.03	1.92	1.67	3.48	Small	0.40	0.68	0.28	0.38	0.42	Small	3.01	5.23	2.65	3.20	3.49		
2	-0.13	-0.10	0.04	-0.03	-0.02	2	-1.93	-1.56	0.42	-0.32	-0.24	2	-0.20	-0.27	-0.08	-0.07	0.09	2	-1.65	-2.16	-0.69	-0.57	0.81		
3	-0.16	-0.11	-0.07	-0.13	-0.04	3	-1.91	-1.43	-0.86	-1.34	-0.34	3	-0.29	-0.21	-0.16	0.00	0.13	3	-2.16	-1.42	-1.29	0.00	1.06		
4	-0.19	-0.10	0.02	0.03	-0.11	4	-1.93	-1.07	0.17	0.36	-1.08	4	-0.24	-0.02	-0.04	-0.11	0.22	4	-1.55	-0.18	-0.22	-0.94	1.64		
Big	0.11	0.07	0.12	-0.02	-0.03	Big	0.90	0.68	1.48	-0.27	-0.36	Big	0.00	0.11	0.10	-0.06	0.02	Big	0.04	0.95	0.91	-0.61	0.20		
SMB					SMB					SMB					SMB										
Small	1.01	0.92	0.95	0.95	1.05	Small	65.07	67.17	60.24	56.63	47.53	Small	1.09	0.88	0.92	0.89	0.98	Small	42.23	34.41	43.50	38.32	41.70		
2	1.02	0.98	0.95	1.00	1.07	2	75.66	75.81	46.75	55.19	54.24	2	0.99	0.97	0.95	1.02	1.04	2	41.81	39.93	43.23	40.25	45.78		
3	1.02	0.92	0.96	0.99	1.05	3	60.41	57.23	57.12	49.11	44.38	3	1.00	1.03	1.04	1.03	1.04	3	38.65	35.31	43.10	42.66	41.89		
4	1.04	0.94	0.91	0.99	1.03	4	51.92	47.33	50.37	51.30	49.76	4	1.12	1.02	0.99	1.05	1.01	4	36.15	37.92	31.98	44.73	37.80		
Big	1.12	0.98	0.96	0.96	1.01	Big	43.92	49.34	57.97	58.95	65.05	Big	1.01	0.97	1.00	1.01	0.98	Big	44.42	44.58	48.44	55.29	47.54		
HML					HML					HML					HML										
Small	1.10	1.00	1.05	1.06	1.23	Small	45.48	47.07	43.04	40.81	36.07	Small	1.32	1.15	1.01	1.09	0.98	Small	30.62	26.87	28.54	27.97	25.04		
2	0.93	0.82	0.84	0.95	1.06	2	44.34	41.01	26.78	33.57	34.49	2	1.04	0.80	0.76	0.84	0.75	2	26.22	19.59	20.73	19.71	19.83		
3	0.72	0.67	0.63	0.62	0.80	3	27.72	26.78	24.05	19.94	21.88	3	0.75	0.54	0.54	0.47	0.56	3	17.20	11.11	13.30	11.58	13.41		
4	0.43	0.33	0.32	0.38	0.43	4	13.79	10.82	11.31	12.64	13.28	4	0.35	0.16	0.20	0.37	0.32	4	6.68	3.60	3.83	9.27	7.25		
Big	-0.09	-0.23	-0.24	-0.21	-0.15	Big	-2.25	-7.50	-9.43	-8.50	-6.16	Big	-0.35	-0.39	-0.12	-0.03	-0.08	Big	-9.09	-10.76	-3.38	-1.06	-2.21		
RMW					RMW					RMW					RMW										
Small	0.06	0.15	0.15	0.06	0.07	Small	1.93	5.37	4.63	1.69	1.60	Small	-0.43	0.07	0.33	0.35	0.10	Small	-7.85	1.33	7.32	6.98	2.06		
2	0.12	0.19	0.11	0.10	0.06	2	4.22	6.86	2.54	2.61	1.38	2	-0.02	0.47	0.26	0.26	-0.13	2	-0.48	9.06	5.54	4.85	-2.73		
3	0.14	0.15	0.14	0.15	-0.02	3	4.12	4.46	3.96	3.48	-0.49	3	-0.05	0.43	0.13	0.05	0.03	3	-0.96	6.96	2.47	0.90	0.54		
4	0.03	0.15	0.11	0.19	0.02	4	0.76	3.70	2.81	4.65	0.46	4	0.16	0.18	0.03	0.15	-0.06	4	2.44	3.08	0.42	3.07	-1.14		
Big	-0.19	-0.03	-0.01	0.13	-0.06	Big	-3.49	-0.71	-0.18	3.69	-1.73	Big	0.14	0.17	0.13	-0.01	-0.29	Big	2.78	3.62	2.98	-0.35	-6.52		
RMW					RMW					RMW					RMW										
Small	-0.23	-0.05	0.06	0.07	0.24	Small	-4.37	-1.09	1.22	1.22	3.29	Small	-0.51	-0.07	0.13	0.29	0.24	Small	-8.00	-1.08	2.57	4.93	4.07		
2	-0.36	-0.05	-0.03	0.29	0.33	2	-7.92	-1.22	-0.51	4.74	5.03	2	-0.55	-0.06	0.27	0.27	0.20	2	-9.42	-0.9					

Appendix 4 Size-OP Six-factor alphas and slopes

North America													Europe												
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	0.19	0.30	0.11	0.18	0.16		2.01	4.03	1.53	2.06	1.65		-0.18	0.13	0.17	0.28	0.18		-2.67	2.09	2.97	4.76	2.36		
	-0.11	-0.02	0.00	0.07	0.02		-2.05	-0.29	0.03	0.93	0.24		-0.13	-0.02	-0.04	0.04	0.34		-2.05	-0.30	-0.73	0.62	5.27		
	0.06	-0.10	-0.04	-0.06	-0.01		0.68	-0.36	-0.56	-0.86	-0.16		-0.14	-0.04	0.09	-0.03	0.10		-1.94	-0.64	1.31	-0.47	1.29		
	0.05	0.03	0.07	-0.06	0.00		0.46	1.45	0.90	-0.88	0.03		-0.04	-0.02	0.03	0.10	0.11		-0.48	-0.24	0.42	1.33	1.49		
Big					Big					Big					Big										
	-0.07	0.04	-0.03	0.15	-0.05		-0.83	0.61	-0.52	2.48	-0.92		0.16	0.18	0.11	-0.25	-0.01		1.86	2.72	1.49	-3.98	-0.17		
MKT					MKT					MKT					MKT										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	0.95	0.86	0.92	0.93	0.99		38.56	43.08	46.88	40.04	39.45		0.97	0.90	0.94	0.95	0.96		62.85	65.55	71.04	70.64	56.33		
	1.05	0.98	1.00	1.02	1.11		70.10	56.94	54.48	52.12	52.61		1.02	0.97	1.00	1.00	1.04		72.83	66.58	72.49	72.63	70.69		
	1.10	1.02	0.98	1.07	1.12		48.12	50.34	47.74	55.29	49.14		1.08	1.02	1.00	1.03	1.05		65.29	65.91	64.23	60.54	59.48		
	1.11	1.02	0.99	1.04	1.07		38.12	55.53	47.32	53.37	45.06		1.00	0.99	1.05	1.03	1.05		48.95	58.07	62.39	58.19	60.12		
Big					Big					Big					Big										
	1.06	1.03	1.03	0.93	0.97		47.97	54.53	58.39	56.73	62.67		1.02	0.99	1.02	1.03	0.94		53.07	65.74	61.46	71.88	63.65		
SMB					SMB					SMB					SMB										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	1.12	0.92	0.98	1.04	1.09		31.69	32.11	35.13	31.11	30.29		1.05	0.92	1.01	0.95	0.95		35.98	35.81	40.20	37.61	29.85		
	0.89	0.80	0.88	0.95	0.98		41.47	32.53	33.53	33.96	32.73		0.93	0.87	0.92	0.89	0.98		35.21	31.85	35.10	34.31	35.23		
	0.65	0.58	0.57	0.66	0.81		19.92	19.96	19.33	24.01	24.82		0.71	0.64	0.69	0.67	0.77		22.97	21.93	23.73	20.92	23.00		
	0.29	0.32	0.34	0.40	0.49		6.97	12.19	11.19	14.23	14.45		0.42	0.37	0.39	0.42	0.40		10.97	11.49	14.35	12.73	12.05		
Big					Big					Big					Big										
	-0.30	-0.22	-0.22	-0.20	-0.19		-9.41	-8.01	-8.56	-8.43	-8.47		-0.21	-0.26	-0.23	-2.48	-0.26		-5.87	-9.07	-7.41	-4.82	-9.20		
HML					HML					HML					HML										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	-0.13	0.34	0.29	0.28	0.29		-2.75	8.76	7.71	6.34	5.99		-0.21	0.07	0.03	0.06	-0.05		-5.45	2.16	0.88	1.83	-1.06		
	-0.23	0.20	0.13	0.23	0.20		-7.93	6.20	3.83	6.11	4.92		-0.08	0.13	0.18	0.17	-0.02		-2.16	3.57	5.06	4.88	-0.59		
	-0.18	0.12	0.18	0.24	0.16		-4.09	3.17	4.61	6.47	3.75		-0.09	0.14	0.20	0.14	0.08		-2.07	3.55	5.20	3.37	1.82		
	-0.18	0.16	0.13	0.12	0.08		-3.17	4.62	3.23	3.18	1.68		0.02	0.26	0.19	0.08	0.00		0.39	5.97	4.59	1.82	0.07		
Big					Big					Big					Big										
	0.23	0.14	0.12	0.10	-0.25		5.42	3.97	3.55	-0.32	-8.53		0.10	0.10	-0.11	0.10	-0.20		2.09	2.72	-2.62	2.70	-5.44		
RMW					RMW					RMW					RMW										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	-0.62	0.05	0.23	0.28	0.28		-13.76	1.39	6.39	6.56	5.99		-0.50	-0.07	0.04	0.08	0.13		-9.87	-1.54	1.02	1.77	2.32		
	-0.65	0.09	0.24	0.43	0.67		-23.94	2.79	7.30	11.95	17.38		-0.51	-0.06	0.17	0.31	0.21		-11.12	-1.27	3.67	6.97	4.30		
	-0.66	0.05	0.35	0.56	0.59		-15.82	1.35	9.18	15.83	14.11		-0.43	-0.02	0.24	0.27	0.29		-8.05	-0.39	4.72	4.86	5.02		
	-0.72	-0.04	0.22	0.41	0.40		-13.56	-1.22	5.76	11.46	9.27		-0.56	0.05	0.22	0.25	0.19		-8.48	0.91	4.06	4.38	3.41		
Big					Big					Big					Big										
	-0.76	-0.36	-0.06	0.06	0.40		-19.05	-10.38	-1.72	2.10	14.13		-1.05	-0.47	-0.03	0.47	0.49		-16.86	-9.62	-0.57	10.15	10.14		
CMA					CMA					CMA					CMA										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	0.03	-0.05	-0.07	-0.12	-0.29		0.60	-1.16	-1.49	-2.25	-4.81		-0.05	0.10	0.06	0.02	-0.06		-1.16	2.30	1.37	0.42	-1.20		
	-0.01	0.07	0.07	-0.13	-0.14		-0.32	1.74	1.60	-2.75	-2.75		-0.13	0.12	0.10	0.05	-0.08		-3.10	2.62	2.40	1.11	-1.72		
	-0.14	0.03	-0.01	-0.12	-0.16		-2.60	0.62	-0.18	-2.62	-2.95		-0.01	0.04	-0.07	-0.04	-0.12		-0.15	0.93	-1.52	-0.73	-2.24		
	-0.03	0.00	0.03	0.10	-0.05		-0.41	-0.08	0.65	2.08	-0.90		0.03	0.04	0.06	0.00	-0.13		0.41	0.76	1.09	-0.06	-2.35		
Big					Big					Big					Big										
	-0.35	-0.09	-0.04	-0.04	0.09		-6.70	-2.01	-0.94	-1.10	2.45		-0.21	-0.02	0.18	-0.03	0.01		-3.67	-0.37	3.64	-0.66	0.18		
WML					WML					WML					WML										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	0.04	-0.03	-0.02	-0.05	-0.04		2.01	-1.77	-1.04	-2.77	-1.90		0.03	0.01	-0.01	0.02	0.02		1.57	0.72	-0.45	1.54	0.86		
	0.00	-0.01	-0.02	-0.03	-0.07		0.27	-1.05	-1.05	-2.09	-4.32		0.05	0.01	-0.01	-0.05	-0.03		-3.38	0.42	-0.83	-3.04	-1.94		
	-0.02	0.04	-0.04	-0.05	-0.02		-1.01	2.27	-2.66	-2.91	-1.07		0.03	-0.02	-0.01	-0.03	-0.01		1.35	-1.19	-0.58	-1.72	-0.40		
	0.00	0.00	-0.02	-0.06	0.04		0.08	0.24	-1.26	-3.83	2.29		-0.07	-0.05	-0.03	-0.08	0.00		-2.78	-2.35	-1.63	-3.87	0.08		
Big					Big					Big					Big										
	-0.05	-0.09	-0.02	0.02	0.04		-2.87	-6.23	-1.43	1.38	2.95		-0.11	0.02	-0.04	0.00	0.03		-5.18	0.94	-2.01	0.17	1.83		

Japan													Asia Pacific												
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	0.09	0.20	0.15	0.14	0.38		1.24	3.02	1.91	1.72	3.49		0.31	0.64	0.30	0.35	0.44		2.29	4.74	2.69	2.88	3.61		
	-0.12	-0.10	0.05	-0.03	-0.02		-1.88	-1.56	0.46	-0.32	-0.16		-0.25	-0.25	-0.05	0.10	0.11		-2.07	-1.94	-0.45	0.79	0.94		
	-0.16	-0.11	-0.07	-0.13	-0.04		-1.89	-1.37	-0.80	-1.29	-0.33		-0.35	-0.06	-0.10	0.00	0.16		-2.55	-0.38	-0.76	0.02	1.20		
	-0.19	-0.09	0.03	0.03	-0.10		-1.95	-0.99	0.29	0.34	-0.99		-0.12	0.04	0.07	-0.02	0.24		-0.78	0.30	0.45	-0.19	1.69		
Big					Big					Big					Big										
	0.11	0.07	0.12	-0.02	-0.03		0.87	0.77	1.50	-0.22	-0.41		0.10	0.16	0.07	0.02	-0.05		0.83	1.41	0.61	0.18	-0.51		
MKT					MKT					MKT					MKT										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	1.01	0.92	0.95	0.95	1.05		64.30	66.40	59.61	55.96	46.95		1.09	0.89	0.92	0.90	0.97		42.70	34.46	43.28	38.27	41.49		
	1.02	0.98	0.95	1.00	1.06		74.87	74.99	46.16	54.59	53.75		0.99	0.97	0.94	1.01	1.04		41.99	39.72	43.01	41.47	45.55		
	1.01	0.92	0.95	0.98	1.05		59.68	56.63	56.50	48.52	43.84		1.01	1.02	1.04	1.03	1.04		38.80	35.75	43.02	42.46	41.68		
	1.04	0.93	0.90	0.99	1.02		51.48	47.00	50.20	50.88	49.43		1.11	1.01	0.98	1.05	1.01		36.24	37.80	31.98	44.89	37.61		
Big					Big</																				

Appendix 5 Size-INV Five-factor alphas and slopes

North America										Europe													
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha								
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High
Small					Small					Small					Small								
	0.39	0.33	0.27	0.23	-0.15		3.78	4.49	3.83	3.23	-1.67		0.01	0.09	0.12	0.13	-0.09		0.08	1.59	1.83	2.12	-1.15
2	-0.06	0.06	0.01	0.01	-0.19	2	-1.00	0.77	0.19	0.10	-2.78	2	-0.09	0.07	0.12	0.00	0.01	2	-1.42	0.08	1.97	0.04	0.17
3	-0.09	-0.03	0.04	0.03	0.01	3	-1.16	-0.39	0.61	0.40	0.12	3	-0.08	-0.04	0.02	-0.06	-0.03	3	-1.03	-0.63	0.23	-0.81	-0.37
4	-0.02	0.05	0.05	0.12	0.03	4	-0.20	0.74	0.74	1.54	0.24	4	-0.05	-0.01	0.07	0.06	0.06	4	-0.63	-0.18	0.96	0.79	0.71
Big					Big					Big					Big								
	-0.03	-0.11	-0.03	0.10	0.13		-0.60	-1.97	-0.55	1.39	1.55		-0.02	-0.01	0.00	0.02	-0.03		-0.27	-0.12	0.00	0.29	-0.38
MKT					MKT					MKT					MKT								
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High
Small					Small					Small					Small								
	1.00	0.86	0.85	0.90	0.96		36.80	43.87	45.07	46.70	39.30		0.98	0.88	0.87	0.93	1.01		66.49	65.99	59.77	63.97	56.92
2	1.12	0.96	0.94	1.02	1.05	2	70.94	49.38	49.61	58.80	58.35	2	1.05	0.98	0.93	0.99	1.05	2	69.03	66.49	64.39	73.04	75.60
3	1.14	1.00	1.00	1.04	1.05	3	53.29	55.33	52.45	53.88	44.04	3	1.10	1.00	0.96	1.00	1.10	3	61.35	62.21	57.76	60.45	64.74
4	1.11	1.02	1.00	1.01	1.08	4	53.36	56.58	51.56	47.50	38.21	4	1.04	1.02	0.96	1.02	1.11	4	54.70	63.95	60.61	60.77	58.90
Big					Big					Big					Big								
	1.01	0.97	0.98	1.00	1.04		68.04	65.86	68.03	52.66	47.45		1.01	0.98	1.03	1.01	0.97		65.11	68.31	74.32	67.86	58.31
SMB					SMB					SMB					SMB								
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High
Small					Small					Small					Small								
	1.22	0.97	0.92	0.90	1.15		31.40	34.56	33.90	32.81	32.80		1.03	0.85	0.88	0.93	1.09		36.85	33.33	33.68	33.99	32.36
2	0.92	0.80	0.83	0.92	0.91	2	40.66	28.63	30.61	36.81	35.50	2	0.94	1.03	0.82	0.87	0.98	2	32.50	36.85	29.95	33.71	37.23
3	0.56	0.52	0.57	0.65	0.81	3	18.16	19.98	20.91	23.64	23.67	3	0.72	0.65	0.62	0.66	0.79	3	21.11	21.34	19.77	21.09	24.69
4	0.33	0.20	0.33	0.40	0.50	4	11.27	7.90	12.08	13.23	12.35	4	0.43	0.38	0.34	0.34	0.46	4	12.02	12.50	11.30	10.76	12.94
Big					Big					Big					Big								
	-0.10	-0.17	-0.24	-0.26	-0.22		-4.53	-8.24	-11.65	-9.55	-7.18		-0.16	-0.19	-0.21	-0.19	-0.19		-5.58	-6.93	-7.99	-6.87	-6.16
HML					HML					HML					HML								
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High
Small					Small					Small					Small								
	-0.08	0.14	0.11	0.25	0.03		-1.67	3.77	3.26	7.12	0.66		-0.06	0.10	0.16	0.00	-0.30		-1.71	2.91	4.29	-0.09	-6.78
2	-0.01	0.06	0.15	0.14	-0.07	2	-0.32	1.59	4.21	4.31	-2.11	2	0.10	-0.06	0.18	0.16	-0.17	2	2.63	-1.71	5.06	4.61	-4.83
3	0.16	0.18	0.15	0.17	-0.13	3	4.13	5.24	4.30	4.77	-2.86	3	0.16	0.23	0.26	0.11	-0.19	3	3.61	5.71	6.26	2.56	-4.46
4	0.13	0.18	0.20	0.01	-0.14	4	3.29	5.34	5.42	0.19	-2.67	4	0.16	0.11	0.18	0.20	-0.14	4	3.44	2.83	4.54	4.79	-2.95
Big					Big					Big					Big								
	-0.05	0.00	0.03	0.00	-0.02		-1.83	0.08	1.04	-0.09	-0.39		-0.14	0.00	-0.09	0.00	0.17		-3.56	-0.11	-2.59	0.00	4.13
RMW					RMW					RMW					RMW								
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High
Small					Small					Small					Small								
	-0.43	-0.08	-0.09	-0.12	-0.27		-8.58	-2.12	-2.59	-3.46	-5.93		-0.20	0.03	0.07	-0.04	-0.34		-4.22	0.78	1.60	-0.97	-6.16
2	-0.15	0.00	0.10	0.07	-0.28	2	-5.14	-0.11	2.81	2.32	-8.29	2	-0.04	-0.20	0.08	0.09	-0.26	2	-0.88	-4.22	1.75	2.04	-6.00
3	0.11	0.23	0.21	0.16	-0.34	3	2.82	6.92	5.87	4.38	-7.70	3	0.07	0.05	0.13	0.01	-0.17	3	1.22	1.06	2.45	0.11	-3.21
4	0.03	0.19	0.18	0.08	-0.29	4	0.66	5.58	5.02	2.02	-5.58	4	0.05	0.00	0.03	0.08	-0.18	4	0.80	0.05	0.67	1.46	-3.09
Big					Big					Big					Big								
	-0.05	0.10	0.01	-0.03	-0.05		-1.89	3.70	0.41	-0.90	-1.20		-0.08	0.09	-0.03	0.01	0.17		-1.72	1.94	-0.78	0.29	3.25
CMA					CMA					CMA					CMA								
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High
Small					Small					Small					Small								
	0.36	0.18	0.13	-0.14	-0.52		5.53	3.88	2.81	-3.14	-9.01		0.32	0.27	-0.02	0.04	-0.46		6.97	6.55	-0.40	0.79	-8.32
2	0.45	0.37	0.17	-0.12	-0.67	2	11.97	8.01	3.68	-2.96	-15.71	2	0.36	0.32	0.07	-0.12	-0.47	2	7.80	6.97	1.57	-2.80	-11.07
3	0.26	0.23	0.06	-0.27	-0.68	3	5.12	5.44	1.41	-5.99	-11.95	3	0.26	0.23	0.06	-0.13	-0.57	3	4.74	4.62	1.15	-2.55	-11.02
4	0.34	0.27	0.01	-0.08	-0.57	4	6.92	6.21	0.26	-1.61	-8.47	4	0.32	0.34	0.06	-0.18	-0.67	4	5.47	6.85	1.22	-3.51	-11.56
Big					Big					Big					Big								
	0.48	0.35	0.12	-0.33	-0.82		13.55	10.07	3.45	-7.24	-15.86		0.68	0.39	0.16	-0.47	-0.66		14.31	8.81	3.76	-10.39	-13.01
Japan										Asia Pacific													
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha								
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High
Small					Small					Small					Small								
	0.11	0.13	0.27	0.13	0.27		1.43	1.77	2.75	1.38	2.74		0.59	0.61	0.58	0.40	-0.04		4.59	5.36	4.05	3.29	-0.29
2	-0.03	-0.06	-0.02	0.05	-0.16	2	-0.47	-0.83	-0.24	0.53	-1.96	2	-0.07	0.13	-0.17	-0.22	-0.35	2	-0.64	1.12	-1.34	-1.66	-2.89
3	-0.06	-0.07	-0.12	-0.14	-0.13	3	-0.85	-0.78	-1.57	-1.41	-1.40	3	-0.33	0.18	-0.19	0.09	-0.35	3	-2.39	1.30	-1.52	0.66	-2.78
4	-0.05	-0.09	0.01	-0.16	-0.05	4	-0.47	-0.97	0.16	-1.72	-0.51	4	-0.41	-0.14	0.16	0.11	-0.01	4	-3.35	-1.09	1.20	0.69	-0.06
Big					Big					Big					Big								
	0.03	-0.12	-0.12	0.05	0.01		0.38	-1.44	-1.43	0.63	0.10		-0.19	-0.14	0.09	0.11	0.20		-1.62	-1.32	0.93	1.01	1.42
MKT					MKT					MKT					MKT								
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High
Small					Small					Small					Small								
	1.07	0.92	0.92	0.97	1.04		68.33	61.72	45.12	51.58	51.87		1.02	0.91	0.91	0.97	1.09		39.97	40.81	32.37	30.90	39.41
2	1.07	0.96	0.92	0.99	1.07	2	78.60	67.35	57.77	50.67	63.06	2	0.99	0.97	0.91	1.03	1.03	2	45.79	41.23	36.25	39.91	43.38
3	1.06	0.95	0.94	0.94	1.05	3	69.62	55.28	58.05	46.62	54.86	3	1.05	0.98	1.00	1.02	1.07	3	38.39	35.86	40.01	36.96	43.54
4	1.08	0.91	0.90	0.97	1.06	4	52.39	50.35	48.57	51.26	53.09	4	1.04	0.98	1.01	1.04	1.09	4	43.62	37.96	39.68	34.31	32.73
Big					Big					Big					Big								
	1.01	1.01	0.97	0.96	1.05		60.07	60.37	54.86	59.60	62.10		0.99	1.05	0.97	0.96	0.99		43.23	49.31	53.89	44.66	36.06
SMB					SMB					SMB					SMB								
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High
Small					Small					Small					Small								
	1.11	0.98	1.00	1.08	1.26		45.60	42.37	31.56	36.83	49.51		1.27	0.99	1.14	1.09	1.35		29.70	26.63	24.31	27.34	28.98
2	0.95	0.85	0.81	0.91	1.03	2	44.90	38.57	32.68	29.95	39.08	2	0.88	0.78	0.70	0.91	0.99	2	24.53	19.93	16.60	21.23	24.76
3	0.72	0.64	0.61	0.70	0.79	3	30.46	24.14	24.21	22.46	26.37	3	0.62	0.51	0.48	0.54	0.79	3	13.43	11.30	11.40	11.67	19.18
4	0.																						

Appendix 6 Size-INV Six-factor alphas and slopes

North America												Europe																																									
Coefficient Alpha						t-statistic Alpha						Coefficient Alpha						t-statistic Alpha																																			
		Low	2	3	4	High			Low	2	3	4	High			Low	2	3	4	High			Low	2	3	4	High																										
MKT						MKT						MKT						MKT																																			
Small	0.34	0.31	0.26	0.26	-0.13	Small	3.38	4.14	3.61	3.67	-1.40	Small	-0.02	0.07	0.11	0.14	-0.08	Small	-0.24	1.26	1.71	2.12	-1.07	Small	-1.65	1.07	2.49	0.71	0.18	Small	-0.05	-0.01	0.04	0.04	0.01	Small	-1.15	-0.20	0.69	-0.46	-0.39	Small	-0.33	0.33	1.14	1.15	0.89	Small	-0.49	-0.06	0.67	0.49	-1.14
2	-0.06	0.06	0.03	0.02	-0.14	2	-0.95	0.80	0.38	0.25	-2.12	2	-0.11	0.06	0.16	0.04	0.01	2	68.64	69.70	64.04	73.15	74.71	2	60.79	61.64	57.28	59.81	64.02	2	54.05	63.56	59.86	60.15	58.16	2	64.69	67.48	74.46	67.04	59.72	2	1.12	0.96	0.94	1.02	1.03						
3	1.12	1.00	1.00	1.03	1.05	3	53.03	54.40	51.63	52.92	43.29	3	1.10	1.00	0.95	1.00	1.10	3	12.14	12.80	11.36	10.94	12.99	3	1.24	1.07	0.91	0.67	-3.11	3	1.94	0.87	0.97	2.01	-2.64	3	1.26	1.79	2.55	3.07	-5.71	3	0.72	0.64	0.61	0.70	0.79						
4	0.28	1.11	0.61	1.35	0.11	4	52.78	55.78	50.93	47.05	37.79	4	0.44	0.36	0.63	0.67	0.79	4	3.23	2.51	4.39	4.54	-3.05	4	1.24	0.87	0.97	2.01	-2.64	4	1.94	0.87	0.97	2.01	-2.64	4	0.39	0.38	0.34	0.33	0.45												
Big	-0.01	-0.10	-0.03	0.12	0.12	Big	67.33	64.75	66.94	51.83	46.85	Big	-0.17	-0.19	-0.20	-2.15	-0.20	Big	-5.66	-6.88	-7.86	-6.77	-6.59	Big	-2.01	1.94	0.33	0.61	1.97	Big	-0.05	0.10	0.01	-0.03	-0.05																		
SMB						SMB						SMB						SMB																																			
Small	1.01	0.87	0.86	0.89	0.95	Small	37.09	43.87	44.65	46.04	38.55	Small	0.98	0.89	0.87	0.92	1.01	Small	66.30	65.75	59.18	63.20	56.23	Small	68.64	69.70	64.04	73.15	74.71	Small	60.79	61.64	57.28	59.81	64.02	Small	54.05	63.56	59.86	60.15	58.16	Small	64.69	67.48	74.46	67.04	59.72						
2	1.12	0.96	0.94	1.02	1.03	2	69.76	48.54	48.72	57.78	58.66	2	1.05	0.96	0.93	0.99	1.05	2	12.14	12.80	11.36	10.94	12.99	2	1.24	1.07	0.91	0.67	-3.11	2	1.94	0.87	0.97	2.01	-2.64	2	1.94	0.87	0.97	2.01	-2.64	2	0.39	0.38	0.34	0.33	0.45						
3	1.12	1.00	1.00	1.03	1.05	3	53.03	54.40	51.63	52.92	43.29	3	1.10	1.00	0.95	1.00	1.10	3	12.14	12.80	11.36	10.94	12.99	3	1.24	1.07	0.91	0.67	-3.11	3	1.94	0.87	0.97	2.01	-2.64	3	1.94	0.87	0.97	2.01	-2.64	3	0.39	0.38	0.34	0.33	0.45						
4	1.10	1.01	1.00	1.02	1.09	4	52.78	55.78	50.93	47.05	37.79	4	0.44	0.36	0.63	0.67	0.79	4	3.23	2.51	4.39	4.54	-3.05	4	1.24	0.87	0.97	2.01	-2.64	4	1.94	0.87	0.97	2.01	-2.64	4	0.39	0.38	0.34	0.33	0.45												
Big	1.01	0.96	0.98	0.99	1.04	Big	67.33	64.75	66.94	51.83	46.85	Big	-0.17	-0.19	-0.20	-2.15	-0.20	Big	-5.66	-6.88	-7.86	-6.77	-6.59	Big	-2.01	1.94	0.33	0.61	1.97	Big	-0.05	0.10	0.01	-0.03	-0.05																		
HML						HML						HML						HML																																			
Small	-0.04	0.16	0.13	0.22	0.01	Small	-0.82	4.27	3.47	6.04	0.13	Small	-0.06	0.10	0.16	0.00	-0.31	Small	-1.49	3.10	4.32	-0.11	-6.76	Small	2.78	4.71	4.74	4.23	-4.80	Small	3.67	5.44	5.97	2.34	-4.40	Small	3.23	2.51	4.39	4.54	-3.05	Small	-3.39	-0.15	-3.16	-0.13	4.69						
2	-0.01	0.05	0.13	0.13	-0.12	2	-0.40	1.43	3.65	3.83	-3.53	2	0.11	0.16	0.17	0.14	-0.17	2	2.78	4.71	4.74	4.23	-4.80	2	3.67	5.44	5.97	2.34	-4.40	2	3.23	2.51	4.39	4.54	-3.05	2	-3.39	-0.15	-3.16	-0.13	4.69												
3	0.12	0.16	0.15	0.16	-0.13	3	2.87	4.50	4.14	4.22	-8.21	3	0.17	0.22	0.25	0.10	-0.19	3	2.78	4.71	4.74	4.23	-4.80	3	3.67	5.44	5.97	2.34	-4.40	3	3.23	2.51	4.39	4.54	-3.05	3	-3.39	-0.15	-3.16	-0.13	4.69												
4	0.09	0.15	0.20	0.02	-0.13	4	2.25	4.41	5.42	0.54	-2.31	4	0.16	0.10	0.18	0.19	-0.15	4	3.23	2.51	4.39	4.54	-3.05	4	3.67	5.44	5.97	2.34	-4.40	4	3.23	2.51	4.39	4.54	-3.05	4	-3.39	-0.15	-3.16	-0.13	4.69												
Big	-0.08	-0.01	0.03	0.12	-0.01	Big	-2.63	-0.30	0.98	-0.75	-0.17	Big	-0.13	-0.01	-0.11	0.00	0.19	Big	-3.39	-0.15	-3.16	-0.13	4.69	Big	-3.39	-0.15	-3.16	-0.13	4.69	Big	-3.39	-0.15	-3.16	-0.13	4.69	Big	-3.39	-0.15	-3.16	-0.13	4.69												
RMW						RMW						RMW						RMW																																			
Small	-0.44	-0.08	-0.09	-0.12	-0.26	Small	-8.80	-2.25	-2.65	-3.33	-5.84	Small	-0.22	0.01	0.07	-0.04	-0.34	Small	-4.58	0.25	1.37	-0.86	-5.79	Small	-1.26	1.79	2.55	3.07	-5.71	Small	1.94	0.87	0.97	2.01	-2.64	Small	1.94	0.87	0.97	2.01	-2.64												
2	-0.15	0.00	0.10	0.08	-0.27	2	-5.11	-0.10	2.88	2.37	-8.29	2	-0.06	0.08	0.12	0.13	-0.26	2	1.26	1.79	2.55	3.07	-5.71	2	1.94	0.87	0.97	2.01	-2.64	2	1.94	0.87	0.97	2.01	-2.64																		
3	0.12	0.24	0.21	0.16	-0.34	3	3.11	7.04	5.84	4.45	-7.66	3	0.06	0.09	0.17	0.04	-0.17	3	1.26	1.79	2.55	3.07	-5.71	3	1.94	0.87	0.97	2.01	-2.64	3	1.94	0.87	0.97	2.01	-2.64																		
4	0.03	0.19	0.18	0.08	-0.29	4	0.86	5.76	4.96	1.95	-5.62	4	0.08	0.05	0.05	0.11	-0.16	4	1.26	1.79	2.55	3.07	-5.71	4	1.94	0.87	0.97	2.01	-2.64	4	1.94	0.87	0.97	2.01	-2.64																		
Big	-0.05	0.10	0.01	-0.03	-0.05	Big	-1.74	3.77	0.41	-0.78	-1.24	Big	-0.10	0.09	0.01	0.03	0.10	Big	1.26	1.79	2.55	3.07	-5.71	Big	1.94	0.87	0.97	2.01	-2.64	Big	1.94	0.87	0.97	2.01	-2.64																		
CMA						CMA						CMA						CMA																																			
Small	0.33	0.17	0.12	-0.13	-0.51	Small	5.20	3.60	2.64	-2.78	-8.75	Small	0.31	0.26	-0.02	0.04	-0.45	Small	6.76	6.35	-0.46	0.81	-8.22	Small	7.61	6.25	1.89	-2.43	-10.98	Small	4.63	4.88	1.42	-2.34	-10.96	Small	5.63	7.19	1.34	-3.28	-11.38	Small	14.10	8.78	4.21	-10.21	-13.70						
2	0.45	0.37	0.17	-0.12	-0.64	2	11.90	7.98	3.83	-2.81	-15.52	2	0.36	0.26	0.08	-0.10	-0.47	2	7.61	6.25	1.89	-2.43	-10.98	2	4.63	4.88	1.42	-2.34	-10.96	2	5.63	7.19	1.34	-3.28	-11.38	2	14.10	8.78	4.21	-10.21	-13.70												
3	0.29	0.24	0.06	-0.27	-0.68	3	5.71	5.64	1.38	-5.80	-11.81	3	0.26	0.24	0.07	-0.12	-0.57	3	4.63	4.88	1.42	-2.34	-10.96	3	5.63	7.19	1.34	-3.28	-11.38	3	14.10	8.78	4.21	-10.21	-13.70																		
4	0.36	0.28	0.01	-0.09	-0.58	4	7.38	6.53	0.16	-1.75	-8.50	4	0.33	0.35	0.07	-0.17	-0.66	4	5.63	7.19	1.34	-3.28	-11.38	4	14.10	8.78	4.21	-10.21	-13.70																								
Big	0.49	0.36	0.12	-0.31	-0.83	Big	13.97	10.16	3.43	-6.94	-15.82	Big	0.67	0.39	0.18	-0.47	-0.69	Big	14.10	8.78	4.21	-10.21	-13.70	Big	14.10	8.78	4.21	-10.21	-13.70																								
WML						WML						WML						WML																																			
Small	0.06	0.04	0.02	-0.05	-0.04	Small	2.77	2.24	1.21	-2.98	-1.78	Small	0.03	0.03	0.01	0.00	-0.01	Small	1.82	1.72	0.52	-0.21	-0.33	Small	2.143	0.58	-2.94	-3.74	-0.07	Small	0.74	-2.32	-2.59	-1.92	0.16	Small	-1.65	-2.85	-1.12	-2.08	-1.08	Small	1.27	-0.30	-3.77	-1.16	4.20						
2	0.00	-0.01	-0.02	-0.01	-0.07	2	-0.32	-0.33	-1.35	-1.02	-5.01	2	0.03	0.01	-0.05	-0.06	0.00	2	1.43	0.58	-2.94	-3.74	-0.07	2	0.74	-2.32	-2.59	-1.92	0.16	2	-1.65	-2.85	-1.12	-2.08	-1.08	2	1.27	-0.30	-3.77	-1.16	4.20												
3	-0.07	-0.03	0.00	-0.02	-0.01	3	-4.12	-1.84	0.10	-1.24	-0.28	3	0.02	-0.04	-0.05	-0.04	0.00	3	0.74	-2.32	-2.59	-1.92	0.16	3	-1.65	-2.85	-1.12	-2.08	-1.08	3	1.27	-0.30	-3.77	-1.16	4.20																		
4	-0.06	-0.04	0.01	0.02	0.02	4	-3.29	-2.59	0.83	1.24	0.86	4	-0.04	-0.05	-0.02	-0.04	-0.02	4	-1.65	-2.85	-1.12	-2.08	-1.08	4	1.27	-0.30	-3.77	-1.16	4.20																								
Big	-0.04	-0.02	0.00	-0.04	0.01	Big	-3.00	-1.28	-0.04	-2.32	0.71	Big	0.02	-0.01	-0.06	-0.02	0.08	Big	1.27	-0.30	-3.77	-1.16	4.20	Big	1.27	-0.30	-3.77	-1.16	4.20																								
MKT						MKT						MKT						MKT																																			
Small	0.34	0.31	0.26	0.26	-0.13	Small	3.38	4.14	3.61	3.67	-1.40	Small	-0.02	0.07	0.11	0.14	-0.08	Small	-0.24	1.26	1.71	2.12	-1.07	Small	-1.65	1.07	2.49	0.71	0.18	Small	-0.05	-0.01	0.04	0.04	0.01	Small	-0.33	0.33	1.14	1.15													

Appendix 7 Size-MOM Five-factor alphas and slopes

North America													Europe												
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	-0.53	0.05	0.35	0.61	0.87		-2.98	0.61	4.55	6.21	5.51		-0.63	-0.17	0.02	0.38	1.05		-4.96	-2.10	0.30	5.09	7.85		
2	-0.51	-0.09	-0.07	0.08	0.50	2	-2.72	-1.07	-0.96	0.97	3.11	2	-0.54	-0.20	-0.01	0.30	0.71	2	-3.65	-2.47	-0.11	4.08	6.12		
3	-0.46	-0.14	-0.02	0.03	0.33	3	-2.38	-1.38	-0.26	0.30	2.02	3	-0.32	-0.15	-0.04	0.12	0.45	3	-2.09	-1.64	-0.48	1.58	3.36		
4	-0.39	-0.09	0.04	0.05	0.40	4	-1.92	-0.93	0.49	0.57	2.19	4	-0.14	-0.05	-0.07	0.05	0.40	4	-0.80	-0.53	-0.83	0.55	2.95		
Big					Big					Big					Big										
	-0.26	-0.09	-0.06	0.00	0.39		-1.41	-0.90	-0.81	0.03	2.15		-0.15	-0.11	0.00	-0.02	0.14		-0.85	-1.08	0.06	-0.17	0.90		
MKT																									
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	1.15	0.92	0.84	0.83	0.95		24.16	41.09	40.98	31.73	22.60		1.06	0.89	0.85	0.86	0.96		36.58	48.86	53.76	50.38	31.20		
2	1.31	1.03	0.98	0.92	1.03	2	26.43	45.14	48.81	41.12	24.28	2	1.15	0.98	0.95	0.93	1.05	2	33.83	51.84	61.27	55.88	39.34		
3	1.30	1.08	0.99	0.95	1.01	3	25.25	40.71	48.39	41.55	23.43	3	1.17	1.01	0.98	0.98	1.06	3	33.25	49.15	58.96	54.97	34.41		
4	1.31	1.06	0.97	0.96	0.98	4	24.46	42.92	49.28	41.85	20.35	4	1.15	1.04	0.97	0.98	1.03	4	28.25	49.84	53.08	48.93	33.18		
Big					Big					Big					Big										
	1.18	1.02	0.91	0.94	0.92		23.70	39.49	46.78	40.15	18.83		1.20	1.03	0.95	0.94	0.95		28.58	42.66	63.82	44.95	26.09		
SMB																									
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	0.97	0.80	0.85	0.92	1.11		14.22	24.95	29.00	24.82	18.52		1.01	0.83	0.82	0.85	1.08		18.43	24.04	27.17	26.26	18.51		
2	0.79	0.74	0.77	0.85	1.16	2	11.06	22.69	26.86	26.65	19.04	2	0.87	0.84	0.78	0.79	1.05	2	13.48	23.38	26.50	24.92	20.58		
3	0.46	0.46	0.49	0.68	0.97	3	6.31	12.19	16.63	20.68	15.79	3	0.60	0.64	0.63	0.66	0.83	3	9.01	16.32	19.92	19.63	14.17		
4	0.17	0.21	0.23	0.33	0.69	4	2.18	5.84	8.16	9.97	9.96	4	0.25	0.32	0.38	0.42	0.56	4	3.19	8.03	10.95	11.07	9.45		
Big					Big					Big					Big										
	-0.24	-0.21	-0.21	-0.12	0.23		-3.37	-5.78	-7.45	-3.49	3.24		-0.26	-0.14	-0.21	-0.18	-0.05		-3.22	-2.98	-7.51	-4.54	-0.76		
HML																									
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	0.47	0.30	0.26	0.06	-0.22		5.30	7.24	6.80	1.30	-2.80		-0.02	0.11	0.13	0.02	-0.22		-0.27	2.41	3.29	0.35	-2.90		
2	0.43	0.28	0.22	0.03	-0.43	2	4.60	6.65	6.04	0.77	-5.45	2	0.16	0.25	0.23	0.01	-0.21	2	1.86	5.23	5.87	0.34	-3.15		
3	0.51	0.33	0.27	0.02	-0.50	3	5.32	6.65	7.08	0.52	-6.21	3	0.27	0.20	0.17	0.10	-0.16	3	3.10	3.93	4.04	2.27	-2.03		
4	0.53	0.31	0.20	0.00	-0.45	4	5.28	6.68	5.32	-0.11	-4.97	4	0.23	0.17	0.21	0.11	-0.11	4	2.26	3.14	4.52	2.23	-1.44		
Big					Big					Big					Big										
	0.42	0.16	0.12	-0.13	-0.40		4.59	3.29	3.31	-3.05	-4.39		0.09	0.16	0.10	-0.01	-0.19		0.88	2.70	2.69	-0.23	-2.10		
RMW																									
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	-0.41	-0.03	-0.04	-0.17	-0.30		-4.61	-0.77	-1.11	-3.47	-3.91		-0.76	-0.08	0.07	0.12	0.02		-8.39	-1.35	1.44	2.17	0.21		
2	-0.18	0.17	0.25	0.13	-0.13	2	-2.01	3.96	6.75	3.11	-1.61	2	-0.64	-0.05	0.18	0.17	0.12	2	-6.03	-0.87	3.61	3.34	1.45		
3	-0.14	0.23	0.22	0.22	-0.03	3	-1.51	4.64	5.91	5.28	-0.34	3	-0.70	-0.06	0.13	0.32	0.27	3	-6.34	-0.89	2.57	5.75	2.76		
4	-0.20	0.15	0.22	0.21	0.01	4	-2.00	3.39	6.09	4.93	0.16	4	-0.76	-0.11	0.16	0.32	0.29	4	-5.98	-1.66	2.85	4.99	3.03		
Big					Big					Big					Big										
	-0.22	-0.01	0.08	0.20	-0.02		-2.43	-0.15	2.33	4.54	-0.27		-0.78	-0.01	0.15	0.39	0.38		-5.91	-0.18	3.16	5.96	3.27		
CMA																									
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	-0.43	0.02	0.00	0.08	0.02		-3.80	0.34	0.09	1.24	0.21		-0.32	0.03	0.16	0.19	0.12		-3.60	0.51	3.27	3.56	1.23		
2	-0.44	0.00	0.00	0.13	0.13	2	-3.69	-0.07	0.02	2.46	1.25	2	-0.29	-0.05	0.04	0.21	0.07	2	-2.81	-0.91	0.84	4.10	0.88		
3	-0.48	-0.10	-0.04	0.16	0.20	3	-3.93	-1.65	-0.83	2.91	1.99	3	-0.37	-0.03	0.10	0.11	0.08	3	-3.43	-0.55	2.00	1.95	0.83		
4	-0.37	0.00	-0.01	0.09	0.13	4	-2.88	-0.08	-0.27	1.61	1.16	4	-0.32	0.01	0.10	0.06	0.02	4	-2.58	0.12	1.71	0.99	0.22		
Big					Big					Big					Big										
	-0.38	0.09	0.03	0.19	-0.05		-3.19	1.43	0.62	3.48	-0.44		-0.20	0.03	0.19	0.14	-0.12		-1.56	0.38	4.17	2.25	-1.07		

Japan													Asia Pacific												
Coefficient Alpha					t-statistic Alpha					Coefficient Alpha					t-statistic Alpha										
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	0.08	0.26	0.23	0.34	0.17		0.55	2.57	2.76	3.38	1.00		-0.44	0.28	0.70	1.15	1.07		-2.89	2.17	6.21	7.89	6.40		
2	-0.16	-0.07	0.02	0.02	0.10	2	-1.02	-0.76	0.22	0.23	0.68	2	-1.30	-0.10	0.08	0.45	0.76	2	-7.63	-0.84	0.63	3.75	4.35		
3	-0.09	-0.21	-0.08	0.03	-0.02	3	-0.58	-2.26	-0.92	0.32	-0.12	3	-0.99	-0.28	0.08	0.55	0.49	3	-4.92	-2.20	0.67	4.30	2.63		
4	-0.06	-0.03	-0.05	-0.13	0.07	4	-0.38	-0.29	-0.53	-1.27	0.44	4	-0.74	-0.27	0.00	0.23	0.35	4	-3.56	-1.90	0.01	1.84	1.79		
Big					Big					Big					Big										
	0.01	-0.13	-0.23	-0.03	0.07		0.05	-1.09	-2.45	-0.28	0.42		-0.18	-0.25	0.05	0.08	0.43		-0.74	-1.57	0.46	0.62	1.96		
MKT																									
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	1.11	0.92	0.86	0.84	1.02		35.31	45.39	49.83	40.26	29.45		1.09	0.89	0.88	0.90	1.13		36.61	35.02	39.76	31.59	34.55		
2	1.15	0.95	0.89	0.91	1.01	2	35.55	54.40	51.17	46.68	34.04	2	1.13	0.96	0.90	0.88	1.04	2	33.85	39.48	37.87	37.36	30.19		
3	1.11	0.97	0.86	0.88	0.97	3	34.86	49.97	48.63	44.88	31.84	3	1.14	0.96	0.91	0.94	1.06	3	28.76	37.84	36.93	37.43	28.85		
4	1.13	0.97	0.93	0.90	0.98	4	32.15	45.24	48.34	43.55	30.72	4	1.16	1.06	0.95	0.93	1.15	4	28.38	37.93	39.44	38.31	29.55		
Big					Big					Big					Big										
	1.15	0.98	1.00	0.97	0.99		27.51	40.77	52.91	48.55	28.36		1.07	1.02	1.00	0.99	0.99		22.13	33.11	47.32	40.27	22.86		
SMB																									
	Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		
Small					Small					Small					Small										
	1.16	0.92	0.92	0.91	1.11		23.89	29.30	34.23	28.09	20.64		1.18	0.96	0.89	1.06	1.21		23.71	22.48	24.11	22.09	22.01		
2	0.93	0.79	0.77	0.79	0.86	2	18.54	29.05	28.44	26.27	18.82	2	0.87	0.80	0.70	0.81	1.03	2	15.51	19.66	17.62	20.50	18.01		
3	0.64	0.57	0.59	0.60	0.70	3	12.96	19.06	21.52	19.71	14.71	3	0.55	0.47	0.46	0.51	0.81	3	8.35	10.92	11.21	12.12	13.18		
4	0.37	0.35	0.34	0.34	0.47	4	6.80	10.40	10.47	10.52	9.44	4	0.20	0.24	0.22	0.21	0.52	4	2.97	5.12	5.56	5.18	8.00		
Big					Big					Big															

Appendix 8 Size-MOM Six-factor alphas and slopes

North America												Europe																	
Coefficient Alpha						t-statistic Alpha						Coefficient Alpha						t-statistic Alpha											
Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High							
MKT						MKT						MKT						MKT											
Small	-0.14	0.16	0.34	0.50	0.59	Small	-1.34	2.10	4.42	5.53	4.92	Small	-0.30	-0.06	0.01	0.26	0.79	Small	-3.66	-0.79	0.45	3.93	7.09	Small	-1.46	-0.91	0.13	2.83	5.16
2	-0.05	0.06	-0.05	-0.01	0.13	2	-0.66	0.84	-0.66	-0.18	1.52	2	-0.11	-0.07	0.01	0.19	0.44	2	1.15	0.23	-0.01	-0.03	1.36	2	3.08	1.86	-0.58	-1.26	0.53
3	-0.01	0.04	0.00	-0.09	-0.03	3	-0.12	0.57	-0.02	-1.20	-0.35	3	0.10	0.02	0.00	0.00	0.13	3	3.70	1.42	-0.01	-3.69	-3.20	3	4.30	5.96	52.41	57.16	55.49
4	0.08	0.08	0.05	-0.06	-0.01	4	0.71	1.09	0.72	-0.82	-0.07	4	0.33	0.14	-0.05	-0.10	0.05	4	53.09	51.71	53.30	57.45	39.19	4	61.58	57.53	60.52	63.13	56.33
Big	0.16	0.08	-0.06	-0.17	-0.05	Big	1.63	1.14	-0.79	-2.60	-0.57	Big	0.36	0.12	0.00	-0.24	-0.30	Big	54.03	56.42	58.50	63.25	51.04	Big	43.03	59.96	52.41	57.16	55.49
SMB						SMB						SMB						SMB											
Small	1.03	0.88	0.84	0.86	1.04	Small	36.26	44.10	40.42	35.72	32.53	Small	1.00	0.87	0.86	0.88	1.00	Small	53.09	51.71	53.30	57.45	39.19	Small	61.58	57.53	60.52	63.13	56.33
2	1.17	0.99	0.97	0.95	1.15	2	53.05	55.12	47.97	46.09	51.37	2	1.07	0.96	0.94	0.95	1.10	2	28.71	27.56	26.56	27.13	27.16	2	54.03	56.42	58.50	63.25	51.04
3	1.16	1.02	0.99	0.99	1.12	3	42.82	51.13	47.52	48.95	45.08	3	1.09	0.98	0.98	1.00	1.12	3	17.65	20.30	20.23	21.55	18.79	3	43.03	59.96	52.41	57.16	55.49
4	1.17	1.01	0.97	0.99	1.11	4	40.83	53.17	48.41	48.68	39.97	4	1.06	1.01	0.97	1.01	1.09	4	7.00	11.03	11.02	11.97	13.41	4	49.10	53.16	63.16	66.94	48.93
Big	1.04	0.97	0.91	1.00	1.05	Big	38.82	48.70	46.02	58.72	47.14	Big	1.11	0.99	0.95	0.98	1.03	Big	-3.98	-2.80	-7.52	-7.87	-3.18	Big	53.09	51.71	53.30	57.45	39.19
HML						HML						HML						HML											
Small	1.15	0.85	0.84	0.87	0.98	Small	28.27	29.45	28.45	25.37	21.54	Small	1.07	0.85	0.82	0.83	1.04	Small	30.06	26.79	27.04	28.67	21.48	Small	28.71	27.56	26.56	27.13	27.16
2	0.99	0.81	0.78	0.81	0.99	2	31.41	31.63	26.91	27.43	30.95	2	0.94	0.86	0.78	0.77	1.00	2	17.65	20.30	20.23	21.55	18.79	2	7.00	11.03	11.02	11.97	13.41
3	0.67	0.54	0.50	0.63	0.81	3	17.23	19.02	16.69	21.64	22.72	3	0.67	0.66	0.64	0.64	0.78	3	4.30	5.96	52.41	57.16	55.49	3	53.09	51.71	53.30	57.45	39.19
4	0.38	0.28	0.24	0.28	0.50	4	9.24	10.35	8.33	9.45	12.69	4	0.33	0.35	0.38	0.40	0.50	4	-3.98	-2.80	-7.52	-7.87	-3.18	4	49.10	53.16	63.16	66.94	48.93
Big	-0.04	-0.14	-0.21	-0.19	0.02	Big	-1.17	-4.77	-7.31	-7.98	0.77	Big	-0.17	-0.10	-0.21	-2.01	-0.13	Big	-2.12	4.32	2.90	2.66	-2.20	Big	-3.21	1.43	3.17	2.97	0.13
RMW						RMW						RMW						RMW											
Small	0.07	0.19	0.26	0.17	0.07	Small	1.32	5.03	6.66	3.77	1.12	Small	-0.14	0.07	0.14	0.06	-0.13	Small	-2.93	1.68	3.36	1.47	-2.04	Small	2.04	4.77	5.68	1.44	-2.30
2	-0.03	0.13	0.20	0.13	-0.06	2	-0.82	3.87	5.24	3.27	-1.32	2	0.00	0.20	0.22	0.05	-0.11	2	0.04	4.77	5.68	1.44	-2.30	2	2.34	3.27	3.75	3.71	-0.71
3	0.06	0.15	0.25	0.14	-0.13	3	1.08	3.80	6.34	3.61	-2.79	3	0.12	0.14	0.16	0.15	-0.04	3	4.91	2.30	4.34	3.75	0.36	3	1.74	1.71	2.72	1.88	-0.60
4	0.06	0.14	0.18	0.11	-0.04	4	1.04	3.86	4.65	2.78	-0.68	4	0.06	0.10	0.20	0.17	0.02	4	-1.74	1.71	2.72	1.88	-0.60	4	0.91	2.30	4.34	3.75	0.36
Big	-0.01	-0.02	0.12	0.10	0.05	Big	-0.17	-0.43	3.14	1.14	1.16	Big	-0.10	0.08	0.10	0.07	-0.03	Big	-2.12	4.32	2.90	2.66	-2.20	Big	-2.93	1.68	3.36	1.47	-2.04
CMA						CMA						CMA						CMA											
Small	-0.33	-0.01	-0.04	-0.19	-0.36	Small	-6.43	-0.33	-1.14	-4.26	-6.12	Small	-0.37	0.05	0.06	-0.02	-0.28	Small	-6.11	0.96	1.15	-0.37	-3.41	Small	-2.29	2.09	3.84	0.89	-3.19
2	-0.10	0.19	0.25	0.11	-0.20	2	-2.48	5.98	6.88	2.95	-4.80	2	-0.13	0.11	0.19	0.04	-0.20	2	-2.94	2.48	3.24	3.36	-1.64	2	-2.42	2.07	3.11	2.45	-2.00
3	-0.06	0.26	0.23	0.20	-0.09	3	-1.21	7.16	6.01	5.49	-2.07	3	-0.19	0.14	0.18	0.17	-0.12	3	-2.42	2.07	3.11	2.45	-2.00	3	-2.12	4.32	2.90	2.66	-2.20
4	-0.11	0.18	0.23	0.19	-0.06	4	-2.14	5.36	6.18	5.05	-1.21	4	-0.19	0.11	0.19	0.14	-0.13	4	-2.12	4.32	2.90	2.66	-2.20	4	-2.12	4.32	2.90	2.66	-2.20
Big	-0.14	0.02	0.08	0.17	-0.11	Big	-2.92	0.68	2.33	5.36	-2.61	Big	-0.16	0.26	0.14	0.13	-0.15	Big	-2.12	4.32	2.90	2.66	-2.20	Big	-2.12	4.32	2.90	2.66	-2.20
WML						WML						WML						WML											
Small	-0.22	0.08	0.00	0.02	-0.13	Small	-3.27	1.59	0.02	0.31	-1.75	Small	-0.19	0.07	0.16	0.14	0.01	Small	-3.21	1.43	3.17	2.97	0.13	Small	-2.15	0.09	0.98	3.57	-0.68
2	-0.19	0.08	0.01	0.08	-0.07	2	-3.66	1.81	0.26	1.63	-1.38	2	-0.11	0.00	0.05	0.16	-0.04	2	-2.15	0.09	0.98	3.57	-0.68	2	-3.15	0.65	2.29	1.14	-0.83
3	-0.24	-0.01	-0.03	0.10	0.01	3	-3.76	-0.15	-0.63	2.01	0.17	3	-0.19	0.03	0.12	0.06	-0.06	3	-1.64	1.65	1.84	0.00	-2.09	3	-2.17	4.10	1.18	1.88	-0.60
4	-0.12	0.08	0.00	0.03	-0.08	4	-1.73	1.87	-0.07	0.57	-1.30	4	-0.12	0.09	0.10	0.00	-0.13	4	-1.64	1.65	1.84	0.00	-2.09	4	-2.17	4.10	1.18	1.88	-0.60
Big	-0.15	0.18	0.03	0.10	-0.29	Big	-2.30	3.85	0.62	2.60	-5.47	Big	0.02	0.12	0.19	0.05	-0.30	Big	-2.17	4.10	1.18	1.88	-0.60	Big	-2.17	4.10	1.18	1.88	-0.60
WML						WML						WML						WML											
Small	-0.58	-0.16	0.01	0.16	0.42	Small	-25.44	-9.74	0.51	8.36	16.33	Small	-0.48	-0.16	0.01	0.17	0.37	Small	-21.88	-8.19	0.78	9.32	12.60	Small	-31.31	-10.52	-1.29	9.24	17.60
2	-0.68	-0.22	0.03	0.14	0.55	2	-37.82	-15.26	-1.99	8.56	30.42	2	-0.63	-0.20	-0.02	0.16	0.40	2	-31.31	-10.52	-1.29	9.24	17.60	2	-26.61	-12.09	-2.63	9.99	18.61
3	-0.67	-0.27	-0.03	0.17	0.53	3	-30.38	-16.58	-1.59	10.56	26.60	3	-0.62	-0.24	-0.05	0.18	0.47	3	-26.61	-12.09	-2.63	9.99	18.61	3	-24.49	-14.01	-1.30	10.48	22.81
4	-0.69	-0.24	-0.03	0.17	0.60	4	-29.85	-15.93	-1.58	10.09	26.81	4	-0.70	-0.27	-0.03	0.21	0.52	4	-24.49	-14.01	-1.30	10.48	22.81	4	-29.40	-15.66	0.41	19.19	26.44
Big	-0.63	-0.26	0.00	0.25	0.66	Big	-29.23	-15.92	-0.07	18.22	36.26	Big	-0.77	-0.34	0.01	0.32	0.65	Big	-29.40	-15.66	0.41	19.19	26.44	Big	-29.40	-15.66	0.41	19.19	26.44

Japan												Asia Pacific																	
Coefficient Alpha						t-statistic Alpha						Coefficient Alpha						t-statistic Alpha											
Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High		Low	2	3	4	High							
MKT						MKT						MKT						MKT											
Small	0.15	0.28	0.23	0.32	0.12	Small	1.34	2.96	2.72	3.49	0.81	Small	-0.02	0.45	0.63	0.97	0.63	Small	-0.19	3.50	5.50	6.71	4.45	Small	-6.22	0.75	1.18	2.18	1.94
2	-0.09	-0.04	0.01	-0.01	0.04	2	-0.91	-0.52	0.15	-0.09	0.82	2	-0.69	0.09	0.02	0.25	0.28	2	-6.22	0.75	1.18	2.18	1.94	2	3.25	-0.52	1.00	3.24	-0.41
3	-0.01	-0.19	-0.08	0.00	-0.08	3	-0.14	-2.27	-0.91	0.01	-0.38	3	-0.33	-0.06	0.13	0.41	-0.06	3	-0.39	-0.05	0.37	0.47	-0.74	3	4.24	2.22	0.87	-2.26	-1.95
4	0.02	0.01	-0.04	-0.15	0.00	4	0.19	0.07	-0.44	-1.68	0.01	4	-0.06	-0.01	0.05	0.06	-0.13	4	-0.39	-0.05	0.37	0.47	-0.74	4	4.24	2.22	0.87	-2.26	-1.95
Big	0.11	-0.08	-0.22	-0.06	-0.01	Big	0.93	-0.92	-2.39	-0.78	-0.15	Big	0.69	0.26	0.10	-0.24	-0.30	Big	4.24	2.22	0.87	-2.26	-1.95	Big	4.24	2.22	0.87	-2.26	-1.95
SMB						SMB						SMB						SMB											
Small	-0.48	-0.16	0.03	0.21	0.35	Small	-17.63	-6.81	1.34	9.68	9.41	Small	1.06	0.88	0.88	0.91	1.16	Small	44.18	35.90	40.07	32.96	42.66	Small	51.59	41.29	38.02	40.35	38.42
2	-0.57	-0.20	0.0																										

Appendix 9 Size-B/M-OP Five-factor alphas and slopes

										North America										Europe																			
										Alpha					t-statistic					Alpha					t-statistic														
										Small		Big			Small		Big			Small		Big			Small		Big												
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High				
Low	-0.23	-0.04	-0.04	0.04	Low	0.45	-0.08	-0.08	-0.22	Low	-0.61	-0.20	-0.22	0.20	Low	2.17	-0.56	-0.62	-2.58	Low	-0.38	-0.13	0.30	-0.23	Low	0.25	-0.17	0.15	0.07	Low	-2.61	-3.42	-2.50	-1.59	Low	1.10	2.40	1.00	0.33
2	-1.13	-0.13	-0.02	0.09	2	0.05	0.05	0.07	0.03	2	-0.94	-1.20	-0.55	1.00	2	0.24	0.48	0.48	0.37	2	-0.25	-0.13	0.01	0.08	2	0.03	0.24	0.04	-0.09	2	-2.13	-1.62	0.23	1.06	2	-2.13	-1.62	0.23	1.06
3	0.02	-0.01	0.10	0.08	3	0.17	-0.05	0.05	-0.11	3	0.17	-0.10	1.39	0.90	3	0.80	-0.63	-0.32	-0.84	3	-0.09	-0.05	0.04	0.22	3	0.03	-0.01	-0.11	-0.04	3	-1.15	-0.88	0.71	2.48	3	0.31	-0.31	-0.13	-0.22
High	-0.08	0.06	0.05	0.10	High	0.04	-0.13	-0.05	-0.37	High	-1.52	1.09	0.02	0.60	High	0.63	-1.20	-0.31	-1.43	High	0.12	0.02	0.30	0.30	High	0.06	-0.30	-0.06	0.15	High	2.63	0.40	3.51	1.60	High	0.88	-0.59	-1.92	0.26
										MKT					MKT					MKT					MKT														
										Small					Big					Small					Big														
Low	1.11	1.12	1.10	1.14	Low	1.22	1.02	0.99	1.12	Low	27.28	22.20	21.92	20.55	Low	22.01	38.04	29.71	48.86	Low	1.06	1.07	0.93	1.05	Low	0.98	1.07	1.01	0.99	Low	32.08	31.97	33.76	32.27	Low	18.27	27.13	39.78	49.35
2	1.07	1.10	0.99	1.01	2	1.09	1.02	1.08	1.02	2	29.74	38.11	60.72	40.90	2	19.89	36.34	44.45	47.37	2	1.00	0.95	0.97	1.00	2	0.96	0.97	1.03	1.04	2	36.55	50.94	68.86	56.21	2	25.66	42.42	55.91	44.06
3	1.07	0.96	0.94	0.99	3	1.07	0.96	0.92	0.99	3	36.04	36.04	51.70	40.65	3	23.87	40.61	38.32	29.36	3	1.03	1.03	1.00	1.03	3	1.03	1.01	0.99	1.08	3	58.66	80.31	79.70	48.63	3	39.79	51.79	41.86	33.94
High	1.03	1.00	0.99	1.14	High	0.98	1.00	0.99	1.20	High	72.34	70.38	50.01	24.84	High	62.41	35.95	25.07	17.26	High	1.03	1.00	1.01	0.99	High	0.94	1.06	0.93	1.13	High	100.34	77.70	82.20	24.07	High	57.70	41.74	26.44	20.34
										SMB					SMB					SMB					SMB														
Low	1.28	1.21	1.13	1.07	Low	0.92	0.12	-0.10	-0.11	Low	21.66	16.77	15.82	13.60	Low	0.35	2.32	-4.01	-2.92	Low	1.16	1.14	1.09	1.12	Low	-0.05	-0.03	-0.08	0.02	Low	18.40	17.81	10.23	16.40	Low	-0.51	-0.39	-1.64	-0.60
2	1.06	0.78	0.80	0.84	2	-0.26	-0.02	-0.05	0.04	2	20.54	18.81	34.19	23.80	2	-3.27	-0.83	-1.54	1.36	2	0.80	0.83	0.83	0.87	2	-0.17	-0.18	-0.10	-0.05	2	15.44	23.89	28.83	25.72	2	-2.42	-4.10	-2.88	-1.11
3	0.94	0.76	0.76	0.81	3	-0.21	-0.12	0.28	0.00	3	22.47	31.48	29.07	23.14	3	-4.83	-4.14	-0.84	-0.07	3	0.01	0.83	0.83	0.83	3	-0.07	-0.09	-0.12	-0.06	3	27.13	36.27	36.09	22.54	3	-1.44	-2.40	-2.66	-1.04
High	0.91	0.83	0.87	0.94	High	-0.16	-0.13	0.02	0.59	High	44.71	40.47	30.68	14.45	High	-7.17	-3.33	0.40	5.87	High	0.90	0.89	0.89	0.81	High	-0.22	0.00	-0.10	0.19	High	16.23	36.28	24.31	10.27	High	-7.01	-0.03	-1.41	-1.87
										HML					HML					HML					HML														
										Small					Big					Small					Big														
Low	-1.00	-0.59	0.01	0.58	Low	-0.75	-0.10	0.09	0.61	Low	-13.32	-6.29	2.07	5.59	Low	-7.26	-1.49	1.47	14.31	Low	-0.89	-0.33	0.30	0.24	Low	-0.94	-0.27	0.04	0.50	Low	-10.69	-3.94	4.33	2.96	Low	-7.29	-2.73	0.71	9.95
2	-0.49	-0.01	0.33	0.55	2	-0.51	-0.10	0.38	0.67	2	-7.30	-0.10	10.85	11.97	2	-4.96	-1.85	8.45	16.79	2	-0.67	-0.31	0.21	0.26	2	-0.30	-0.16	-0.01	0.03	2	-7.75	-3.44	-0.46	-1.32	2	-2.51	-3.60	-1.18	0.42
3	-0.38	0.18	0.46	0.62	3	-0.38	0.16	0.28	0.68	3	6.92	13.71	13.71	13.77	3	6.96	4.05	6.25	10.80	3	-0.40	0.13	0.45	0.56	3	-0.50	-0.12	0.38	0.62	3	-8.98	4.21	14.83	10.81	3	-7.72	-2.55	6.37	7.96
High	-0.01	0.42	0.58	0.75	High	-0.39	0.02	0.42	0.94	High	-0.47	15.90	15.70	8.86	High	-13.33	0.43	5.65	7.23	High	-0.21	0.32	0.64	0.50	High	-0.41	0.14	0.43	0.51	High	-8.35	0.89	13.12	4.85	High	-10.03	2.23	4.56	3.68
										RMW					RMW					RMW					RMW														
										Small					Big					Small					Big														
Low	-0.83	-1.14	-0.86	-0.61	Low	-0.71	-0.65	-0.46	-0.34	Low	-11.07	-12.29	-9.33	-5.99	Low	-7.01	-9.71	-7.52	-8.08	Low	-1.12	-0.51	-0.33	-0.63	Low	-0.99	-0.86	-0.64	-0.73	Low	-10.76	-4.78	-3.83	-6.13	Low	-5.89	-6.93	-8.09	-11.55
2	-0.46	-0.12	-0.04	0.18	2	-0.21	-0.08	-0.03	0.02	2	6.83	-2.34	-1.57	-1.33	2	-2.07	-1.59	-0.69	0.42	2	-0.23	0.09	0.28	0.18	2	-0.30	-0.17	0.27	0.29	2	-4.15	-2.41	7.43	2.81	2	-2.51	-3.60	-3.68	-3.09
3	0.00	0.20	0.13	0.18	3	0.13	0.14	0.01	0.14	3	1.80	6.60	3.83	3.90	3	2.33	3.73	-0.29	2.24	3	0.07	0.07	0.32	0.39	3	0.28	0.18	0.18	0.28	3	2.07	7.93	6.37	0.23	High	5.50	0.68	4.32	1.94
High	-0.38	0.38	0.30	0.12	High	0.33	0.28	0.48	0.12	High	16.80	14.58	9.77	1.38	High	11.92	5.54	6.44	3.28	High	0.07	0.32	0.39	0.09	High	0.28	0.77	0.50	0.32	High	2.07	7.93	6.37	0.23	High	5.50	0.68	4.32	1.94
										CMA					CMA					CMA					CMA														
										Small					Big					Small					Big														
Low	0.10	0.20	-0.02	-0.13	Low	-0.46	-0.16	-0.24	-0.08	Low	1.88	1.66	-0.26	-0.97	Low	-3.52	-1.87	-2.08	-1.43	Low	-0.52	-0.41	-0.12	0.19	Low	-0.20	0.22	0.06	0.30	Low	-5.15	-3.95	-1.39	1.82	Low	-1.24	1.79	0.72	-4.78
2	-0.21	0.03	0.10	0.00	2	-0.43	0.03	0.01	-0.16	2	-2.43	0.40	0.29	0.00	2	-4.42	1.21	-0.17	-3.18	2	-0.19	0.07	0.17	0.21	2	0.20	0.03	0.08	-0.24	2	-3.88	1.31	2.76	4.08	2	-1.75	4.69	5.18	-0.28
3	-0.03	0.08	0.04	-0.05	3	-0.11	0.00	0.07	-0.15	3	-0.49	1.92	0.88	-0.79	3	-1.55	-0.68	1.19	-1.86	3	-0.15	0.15	0.10	0.17	3	-0.08	0.28	0.06	-0.03	3	-2.81	4.08	2.79	2.64	3	-1.00	4.75	0.81	-0.28
High	-0.07	-0.14	-0.16	-0.21	High	0.14	0.12	-0.29	-0.57	High	-2.17	-2.26	-3.30	-1.91	High	3.66	1.82	-3.11	-3.41	High	-0.07	0.05	-0.10	0.32	High	0.06	-0.08	-0.09	-0.17	High	-2.22	1.26	-1.68	2.51	High	1.29	-1.05	-0.76	-0.99
										Japan					Japan					Japan					Japan														
										Alpha					t-statistic					Alpha					t-statistic														
										Small		Big			Small		Big			Small		Big			Small		Big												
Low	-0.20	-0.18	0.00	0.09	Low	0.38	0.03	-0.10	-0.04	Low	-0.97	-1.67	0.04	0.94	Low	1.52	0.21	-0.83	-0.31	Low	-0.40	-0.25	-0.12	0.72	Low	-0.04	-0.07	0.04	-0.20	Low	-2.05	-1.02	-0.38	2.04	Low	-0.11	-0.28	0.27	0.72
2	-0.28	-0.06	-0.01	0.08	2	0.10	0.03	0.02	-0.01	2	-2.01	-0.60	-0.18	0.04	2	1.63	0.62	0.14	-0.04	2	-0.13	-0.44	-0.09	0.22	2	0.28	0.41	0.01	-0.11	2	-0.57	-1.17	-0.54	1.30	2	-0.88	2.37	1.66	-1.66
3	-0.02	0.04	0.08	0.08	3	-0.02	0.02	0.11	0.11	3	-0.21	0.15	1.16	0.84	3	0.48	0.17	1.11	0.72	3	0.12	-0.23	0.34	0.34	3	0.12	-0.12	-0.12	0.01	3	0.12	-2.23	2.39	2.07	3	2.16	-1.08	-0.80	1.85
High	-0.04	0.08	0.10	0.22	High	-0.09	0.10	0.19	-0.05	High	-0.31	1.15	1.35	1.31	High	-1.04	-0.92	1.34	-0.22	High	-0.10	0.13	0.35	0.35	High	0.08	-0.19	-0.13	0.16	High	-1.26	1.53	2.30	2.07	High	0.81	-1.96	-0.80	1.03
										MKT					MKT					MKT					MKT														
										Small					Big					Small					Big														
Low	1.12	1.01	1.03	1.11	Low	1.02	1.03	1.04	1.1																														

Appendix 11 Size-B/M-INV Five-factor alphas and slopes

				North America												Japan												Europe												Asia Pacific																			
				Coefficient Alpha				t-Statistic Alpha				Coefficient Alpha				t-Statistic Alpha				Coefficient Alpha				t-Statistic Alpha				Coefficient Alpha				t-Statistic Alpha																											
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High								
Low	0.02	0.08	0.02	0.19	Low	-0.04	-0.07	-0.01	0.01	-0.08	2	1.03	0.23	0.56	0.17	1.38	Low	-0.37	-0.70	-0.28	-0.78	Low	-0.15	-0.02	0.13	0.17	2	0.03	0.08	0.13	0.17	2	0.48	-1.25	2.10	1.97	2	0.48	-1.25	2.10	1.97	2	0.48	-1.25	2.10	1.97	2	0.48	-1.25	2.10	1.97								
2	0.08	0.00	0.05	0.17	2	-0.10	-0.11	-0.08	-0.08	2	1.03	0.23	0.56	0.17	1.38	2	-0.37	-0.70	-0.28	-0.78	2	0.03	0.08	0.13	0.17	2	0.03	0.08	0.13	0.17	2	0.48	-1.25	2.10	1.97	2	0.48	-1.25	2.10	1.97	2	0.48	-1.25	2.10	1.97	2	0.48	-1.25	2.10	1.97	2	0.48	-1.25	2.10	1.97				
3	-0.06	0.03	0.21	-0.05	3	0.04	-0.04	0.13	-0.07	3	-0.81	0.44	3.51	-0.47	3	0.33	-0.39	1.37	-0.54	3	-0.04	-0.14	-0.20	-0.04	3	0.04	0.14	-0.20	-0.04	3	0.69	0.03	0.54	1.90	3	0.69	0.03	0.54	1.90	3	0.69	0.03	0.54	1.90	3	0.69	0.03	0.54	1.90	3	0.69	0.03	0.54	1.90	3	0.69	0.03	0.54	1.90
High	-0.27	-0.12	-0.15	-0.02	High	0.41	0.12	-0.19	-0.25	High	3.86	-1.43	-1.74	-0.17	High	3.57	1.00	1.60	-2.50	High	-0.08	0.14	-0.20	-0.04	High	0.16	0.16	-0.20	-0.04	High	0.16	0.16	-0.20	-0.04	High	-1.08	-1.93	-2.46	-4.36	High	-1.08	-1.93	-2.46	-4.36	High	-1.08	-1.93	-2.46	-4.36	High	-1.08	-1.93	-2.46	-4.36					
				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big											
Low	1.14	1.06	1.09	1.14	Low	1.00	0.96	1.02	1.08	Low	41.44	39.93	45.68	31.21	Low	33.91	36.85	41.59	41.72	Low	1.03	1.08	1.04	1.06	Low	0.91	1.05	1.02	1.09	Low	45.03	51.50	53.87	37.12	Low	34.85	39.49	44.06	41.55	Low	1.03	1.08	1.04	1.06	Low	0.91	1.05	1.02	1.09	Low	45.03	51.50	53.87	37.12	Low	34.85	39.49	44.06	41.55
2	0.89	0.98	0.93	1.03	2	0.99	0.98	0.97	1.06	2	38.88	56.48	58.27	37.70	2	33.85	35.59	38.79	42.99	2	0.97	0.98	0.97	0.97	2	0.92	1.05	1.02	1.08	2	59.99	71.30	67.01	48.45	2	35.10	52.45	44.27	41.67	2	0.97	0.98	0.97	0.97	2	0.92	1.05	1.02	1.08	2	59.99	71.30	67.01	48.45	2	35.10	52.45	44.27	41.67
3	1.01	0.96	0.95	0.97	3	0.99	0.99	0.97	1.15	3	54.48	60.99	60.39	34.73	3	33.82	39.34	37.93	45.99	3	0.99	0.96	0.92	0.94	3	0.96	1.01	0.99	1.01	3	72.94	74.87	59.47	47.52	3	38.80	39.54	44.34	40.96	3	1.01	0.97	0.92	0.94	3	0.96	1.01	0.99	1.01	3	72.94	74.87	59.47	47.52	3	38.80	39.54	44.34	40.96
High	1.05	1.06	1.01	0.97	High	1.05	1.09	1.04	1.03	High	55.57	46.89	44.76	35.99	High	34.68	31.45	33.09	34.56	High	1.09	0.99	1.01	1.01	High	0.98	1.03	0.99	0.99	High	67.65	60.09	55.36	33.81	High	48.04	43.73	37.22	30.80	High	1.05	1.09	1.04	1.03	High	55.57	46.89	44.76	35.99	High	34.68	31.45	33.09	34.56					
				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big											
Low	0.20	1.02	1.02	1.00	Low	-0.22	0.01	-0.02	0.07	Low	32.08	37.48	29.73	10.45	Low	-5.45	0.16	-1.52	1.30	Low	0.05	0.88	0.02	1.00	Low	-0.02	0.82	0.16	-0.02	Low	21.77	22.13	26.17	18.72	Low	-4.84	2.38	-1.22	0.60	Low	0.05	0.88	0.02	1.00	Low	-0.02	0.82	0.16	-0.02	Low	21.77	22.13	26.17	18.72	Low	-4.84	2.38	-1.22	0.60
2	0.07	0.65	0.72	0.82	2	-0.19	-0.11	-0.08	0.01	2	29.88	36.91	31.83	20.99	2	-5.15	-2.83	-0.76	0.91	2	0.82	0.87	0.84	0.82	2	-0.15	-0.14	-0.14	0.13	2	26.72	33.15	30.43	21.72	2	-2.66	3.25	-2.27	2.70	2	0.82	0.87	0.84	0.82	2	-0.15	-0.14	-0.14	0.13	2	26.72	33.15	30.43	21.72	2	-2.66	3.25	-2.27	2.70
3	0.05	0.84	0.80	0.93	3	-0.31	-0.06	-0.08	-0.06	3	35.61	36.07	35.72	23.24	3	-3.78	-1.77	-2.24	1.37	3	0.86	0.83	0.86	0.83	3	-0.18	-0.19	-0.06	-0.01	3	33.47	34.29	28.07	21.77	3	-3.88	3.00	-1.52	2.70	3	0.86	0.83	0.86	0.83	3	-0.18	-0.19	-0.06	-0.01	3	33.47	34.29	28.07	21.77	3	-3.88	3.00	-1.52	2.70
High	1.09	0.98	0.93	0.87	High	-0.06	-0.06	0.02	-0.06	High	40.20	30.35	28.75	15.95	High	-1.45	-1.32	0.38	-1.52	High	1.01	0.98	0.86	1.02	High	-0.14	-0.02	-0.13	-0.15	High	33.12	33.12	24.13	19.15	High	-3.32	-0.54	-2.60	-2.45	High	1.01	0.98	0.86	1.02	High	-0.14	-0.02	-0.13	-0.15	High	33.12	33.12	24.13	19.15	High	-3.32	-0.54	-2.60	-2.45
				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big											
Low	-0.57	-0.26	0.14	0.59	Low	-0.39	-0.18	0.13	0.54	Low	-11.44	-5.35	3.15	8.65	Low	-7.58	-3.78	2.75	11.33	Low	-0.42	-0.09	0.24	0.46	Low	-0.55	-0.21	-0.02	0.48	Low	-7.34	-1.63	4.88	6.39	Low	-6.87	-3.38	-3.11	-0.41	Low	-7.34	-1.63	4.88	6.39	Low	-6.87	-3.38	-3.11	-0.41	Low	-7.34	-1.63	4.88	6.39					
2	-0.32	0.18	0.29	0.55	2	-0.38	0.00	0.33	0.64	2	-8.45	5.46	9.93	10.78	2	-8.09	1.24	7.03	13.83	2	-0.46	0.11	0.29	0.41	2	-0.45	-0.23	0.16	0.59	2	-6.38	3.10	8.01	8.08	2	-6.87	-4.52	2.69	9.09	2	-0.46	0.11	0.29	0.41	2	-0.45	-0.23	0.16	0.59	2	-6.38	3.10	8.01	8.08	2	-6.87	-4.52	2.69	9.09
3	-0.27	0.21	0.51	0.59	3	-0.37	-0.01	0.36	0.79	3	-7.86	7.03	17.38	11.31	3	-7.02	-0.25	7.59	13.11	3	-0.15	0.20	0.45	0.52	3	-0.73	-0.21	0.17	0.97	3	-4.47	6.15	11.54	10.44	3	-4.73	-3.31	6.62	15.61	3	-0.15	0.20	0.45	0.52	3	-0.73	-0.21	0.17	0.97	3	-4.47	6.15	11.54	10.44	3	-4.73	-3.31	6.62	15.61
High	-0.40	0.26	0.61	0.71	High	-0.45	0.20	0.33	0.76	High	-11.48	6.23	14.45	10.07	High	-7.97	3.28	5.60	13.85	High	-0.52	0.07	0.44	0.45	High	-0.28	0.01	0.41	0.89	High	-12.91	1.65	0.36	6.33	High	-5.05	0.18	6.10	10.94	High	-0.52	0.07	0.44	0.45	High	-12.91	1.65	0.36	6.33	High	-5.05	0.18	6.10	10.94					
				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big											
Low	-0.34	-0.37	-0.16	-0.16	Low	0.12	-0.04	-0.03	-0.13	Low	-6.66	-7.53	-5.70	-2.44	Low	2.33	-0.78	-0.60	-2.67	Low	-0.32	-0.07	0.01	-0.18	Low	0.10	0.22	0.22	-0.20	Low	-4.41	-1.05	0.20	-1.98	Low	1.25	2.67	-3.85	-2.39	Low	-0.32	-0.07	0.01	-0.18	Low	0.10	0.22	0.22	-0.20	Low	-4.41	-1.05	0.20	-1.98					
2	0.04	0.12	0.04	0.04	2	0.35	0.09	-0.03	0.07	2	1.08	3.74	1.52	0.05	2	7.38	1.67	-0.58	1.50	2	-0.13	0.13	0.05	-0.05	2	-0.02	0.22	0.02	-0.25	2	-2.48	3.03	1.03	-0.38	2	-2.48	3.03	1.03	-0.38	2	-0.13	0.13	0.05	-0.05	2	-0.02	0.22	0.02	-0.25	2	-2.48	3.03	1.03	-0.38					
3	0.13	0.25	0.15	0.04	3	0.02	0.08	-0.03	-0.28	3	3.78	8.51	5.15	-0.71	3	0.33	1.80	-0.69	-4.70	3	0.01	0.13	0.20	-0.05	3	-0.19	-0.03	0.14	0.04	3	0.16	3.12	4.06	-0.78	3	-2.43	3.46	-0.31	-3.08	3	0.01	0.13	0.20	-0.05	3	-0.19	-0.03	0.14	0.04	3	0.16	3.12	4.06	-0.78					
High	-0.19	0.05	0.06	-0.09	High	0.17	0.04	0.06	-0.22	High	-5.40	1.18	1.43	-1.27	High	3.01	0.66	1.11	-3.01	High	-0.46	-0.16	0.09	-0.14	High	0.18	0.02	0.19	-0.35	High	-7.16	-3.04	1.47	-1.53	High	2.68	0.25	2.32	-3.44	High	-0.46	-0.16	0.09	-0.14	High	0.18	0.02	0.19	-0.35	High	-7.16	-3.04	1.47	-1.53					
				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big				Small				Big											
Low	0.50	0.69	0.51	0.21	Low	0.68	0.52	0.46	0.20	Low	7.50	11.04	8.89	2.97	Low	8.60	8.64	7.66	3.20	Low	0.22	0.42	0.28	0.41	Low	0.78	0.62	0.40	0.21	Low	3.18	6.77	4.80	4.65	Low	0.65	0.78	0.66	0.38	Low	0.22	0.42	0.28	0.41	Low	0.78	0.62	0.40	0.21										
2	0.35	0.62	0.29	0.06	2	0.49	0.20	-0.01	0.00	2	6.72	6.20	7.27	0.91	2	8.13	3.04	-0.13	0.32	2	0.22	0.20	0.28	0.39	2	0.42	0.37	0.32	-0.15	2	4.48	7.08	6.19	6.37	2	0.42	0.37	0.32	-0.15	2	4.48	7.08	6.19	6.37															
3	0.03	0.00	-0.09	-0.06	3	-0.11	-0.05	-0.01	-0.11	3	7.79	0.12	-2.32	0.92	3	-1.58	-0.88	-3.15	-1.88	3	-0.18	0.01	0.02	0.03	3	0.01	0.32																																

Appendix 12 Size-B/M-INV Six-factor alphas and slopes

			Coefficient			North America			I-statistic			Coefficient			Europe			I-statistic				
			Alpha			Alpha			Alpha			Alpha			Alpha			Alpha				
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
0.64	0.08	0.02	0.21	0.02	-0.02	0.22	0.02	-0.02	0.23	0.02	-0.02	0.24	0.02	-0.02	0.25	0.02	-0.02	0.26	0.02	-0.02		
2	0.10	0.02	0.05	0.17	3	0.08	-0.07	-0.02	0.08	3	0.07	0.10	0.20	3	0.06	0.06	0.06	0.20	3	0.05	0.05	
3	0.05	0.04	0.21	0.17	3	0.06	-0.05	0.16	0.04	3	0.01	-0.51	1.61	0.30	3	0.09	0.11	0.11	0.30	3	0.08	0.08
High	-0.25	-0.09	-0.12	0.20	High	0.38	0.12	-0.18	-0.20	High	-0.52	-1.02	-1.44	-0.21	High	-0.32	-0.99	-1.49	-1.82	High	-0.27	-0.81
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
1.14	1.06	1.09	1.14	Low	0.90	0.66	1.01	1.06	Low	0.67	0.72	0.76	0.47	Low	2.43	2.68	2.93	1.06	Low	3.54	3.79	
2	0.99	0.67	0.93	1.03	2	0.69	0.97	1.06	2	47.41	56.81	67.43	37.10	2	3.81	4.03	3.88	41.99	2	3.81	4.03	
3	1.01	0.96	0.93	0.93	3	0.68	0.99	0.97	1.04	3	50.50	58.33	69.34	34.03	3	3.42	3.89	3.63	34.76	3	3.42	3.89
High	1.04	1.05	1.01	0.90	High	1.00	1.03	1.04	1.01	High	54.71	60.21	43.90	24.85	High	43.58	41.44	32.47	33.90	High	44.90	43.34
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
1.00	1.03	1.02	1.01	Low	-0.19	0.01	-0.04	0.10	Low	-0.87	-1.21	2.02	7.96	Low	-8.44	-3.57	1.83	9.70	Low	-0.43	0.26	
2	0.98	0.67	0.72	1.02	2	-0.18	-0.09	-0.02	0.11	2	3.24	3.85	3.67	3.14	2	-1.84	-1.86	-0.64	0.32	2	-1.84	-1.86
3	0.95	0.84	0.81	0.94	3	-0.23	-0.06	-0.07	-0.05	3	35.23	35.06	35.29	23.24	3	-7.43	-3.89	-1.92	-0.98	3	-7.43	-3.89
High	1.10	1.09	0.94	0.90	High	-0.28	-0.06	-0.09	-0.05	High	40.22	30.99	28.77	15.74	High	17.70	-4.50	-0.49	-1.07	High	17.70	-4.50
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
-0.58	-0.24	0.14	0.57	Low	-0.45	-0.18	0.09	0.47	Low	-0.87	-1.21	2.02	7.96	Low	-8.44	-3.57	1.83	9.70	Low	-0.43	0.26	
2	-0.34	0.15	0.30	0.50	2	-0.40	0.02	0.32	0.64	2	4.34	4.42	6.02	10.40	2	-8.16	-0.64	6.55	13.20	2	-8.16	-0.64
3	0.04	0.01	-0.08	-0.05	3	-0.09	-0.06	-0.10	-0.00	3	3.81	8.02	6.14	-0.61	3	6.45	1.35	-0.50	-4.62	3	6.45	1.35
High	-0.43	0.22	0.38	0.50	High	-0.42	0.20	0.32	0.72	High	-1.70	5.22	13.34	4.34	High	7.11	3.15	5.18	12.63	High	-1.70	5.22
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
-0.34	-0.37	-0.16	-0.16	Low	0.13	-0.04	-0.02	-0.11	Low	-0.63	-1.01	-3.67	-2.37	Low	2.37	-0.78	-0.44	-2.49	Low	-0.63	-1.01	
2	0.13	0.26	0.15	-0.03	2	-0.05	0.08	-0.03	-0.28	2	3.24	3.85	3.67	3.14	2	-1.84	-1.86	-0.64	0.32	2	-1.84	-1.86
3	0.13	0.26	0.15	-0.03	3	-0.04	0.02	-0.04	-0.04	3	-0.80	-1.79	-0.72	-1.66	3	1.89	0.85	-1.72	-1.58	3	-0.80	-1.79
High	-0.18	0.06	-0.06	-0.09	High	0.16	0.04	0.07	-0.21	High	-3.30	1.54	-1.21	1.21	High	3.01	0.66	1.14	-3.89	High	-3.30	1.54
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
0.64	0.08	0.02	0.21	0.02	-0.02	0.22	0.02	-0.02	0.23	0.02	-0.02	0.24	0.02	-0.02	0.25	0.02	-0.02	0.26	0.02	-0.02		
2	0.10	0.02	0.05	0.17	3	0.08	-0.07	-0.02	0.08	3	0.07	0.10	0.20	0.20	3	0.06	0.06	0.06	0.20	3	0.05	0.05
3	0.05	0.04	0.21	0.17	3	0.06	-0.05	0.16	0.04	3	0.01	-0.51	1.61	0.30	3	0.09	0.11	0.11	0.30	3	0.08	0.08
High	-0.25	-0.09	-0.12	0.20	High	0.38	0.12	-0.18	-0.20	High	-0.52	-1.02	-1.44	-0.21	High	-0.32	-0.99	-1.49	-1.82	High	-0.27	-0.81
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
1.00	1.03	1.02	1.01	Low	-0.19	0.01	-0.04	0.10	Low	-0.87	-1.21	2.02	7.96	Low	-8.44	-3.57	1.83	9.70	Low	-0.43	0.26	
2	0.99	0.67	0.93	1.03	2	0.69	0.97	1.06	2	47.41	56.81	67.43	37.10	2	3.81	4.03	3.88	41.99	2	3.81	4.03	
3	1.01	0.96	0.93	0.93	3	0.68	0.99	0.97	1.04	3	50.50	58.33	69.34	34.03	3	3.42	3.89	3.63	34.76	3	3.42	3.89
High	1.04	1.05	1.01	0.90	High	1.00	1.03	1.04	1.01	High	54.71	60.21	43.90	24.85	High	43.58	41.44	32.47	33.90	High	44.90	43.34
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
1.00	1.03	1.02	1.01	Low	-0.19	0.01	-0.04	0.10	Low	-0.87	-1.21	2.02	7.96	Low	-8.44	-3.57	1.83	9.70	Low	-0.43	0.26	
2	0.98	0.67	0.72	1.02	2	-0.18	-0.09	-0.02	0.11	2	3.24	3.85	3.67	3.14	2	-1.84	-1.86	-0.64	0.32	2	-1.84	-1.86
3	0.95	0.84	0.81	0.94	3	-0.23	-0.06	-0.07	-0.05	3	35.23	35.06	35.29	23.24	3	-7.43	-3.89	-1.92	-0.98	3	-7.43	-3.89
High	1.10	1.09	0.94	0.90	High	-0.28	-0.06	-0.09	-0.05	High	40.22	30.99	28.77	15.74	High	17.70	-4.50	-0.49	-1.07	High	17.70	-4.50
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
-0.58	-0.24	0.14	0.57	Low	-0.45	-0.18	0.09	0.47	Low	-0.87	-1.21	2.02	7.96	Low	-8.44	-3.57	1.83	9.70	Low	-0.43	0.26	
2	-0.34	0.15	0.30	0.50	2	-0.40	0.02	0.32	0.64	2	4.34	4.42	6.02	10.40	2	-8.16	-0.64	6.55	13.20	2	-8.16	-0.64
3	0.04	0.01	-0.08	-0.05	3	-0.09	-0.06	-0.10	-0.00	3	3.81	8.02	6.14	-0.61	3	6.45	1.35	-0.50	-4.62	3	6.45	1.35
High	-0.43	0.22	0.38	0.50	High	-0.42	0.20	0.32	0.72	High	-1.70	5.22	13.34	4.34	High	7.11	3.15	5.18	12.63	High	-1.70	5.22
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
-0.34	-0.37	-0.16	-0.16	Low	0.13	-0.04	-0.02	-0.11	Low	-0.63	-1.01	-3.67	-2.37	Low	2.37	-0.78	-0.44	-2.49	Low	-0.63	-1.01	
2	0.13	0.26	0.15	-0.03	2	-0.05	0.08	-0.03	-0.28	2	3.24	3.85	3.67	3.14	2	-1.84	-1.86	-0.64	0.32	2	-1.84	-1.86
3	0.13	0.26	0.15	-0.03	3	-0.04	0.02	-0.04	-0.04	3	-0.80	-1.79	-0.72	-1.66	3	1.89	0.85	-1.72	-1.58	3	-0.80	-1.79
High	-0.18	0.06	-0.06	-0.09	High	0.16	0.04	0.07	-0.21	High	-3.30	1.54	-1.21	1.21	High	3.01	0.66	1.14	-3.89	High	-3.30	1.54
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
0.64	0.08	0.02	0.21	0.02	-0.02	0.22	0.02	-0.02	0.23	0.02	-0.02	0.24	0.02	-0.02	0.25	0.02	-0.02	0.26	0.02	-0.02		
2	0.10	0.02	0.05	0.17	3	0.08	-0.07	-0.02	0.08	3	0.07	0.10	0.20	0.20	3	0.06	0.06	0.06	0.20	3	0.05	0.05
3	0.05	0.04	0.21	0.17	3	0.06	-0.05	0.16	0.04	3	0.01	-0.51	1.61	0.30	3	0.09	0.11	0.11	0.30	3	0.08	0.08
High	-0.25	-0.09	-0.12	0.20	High	0.38	0.12	-0.18	-0.20	High	-0.52	-1.02	-1.44	-0.21	High	-0.32	-0.99	-1.49	-1.82	High	-0.27	-0.81
Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High	Low	Small	High		
1.00	1.03	1.02	1.01	Low	-0.19	0.01	-0.04	0.10	Low	-0.87	-1.21	2.02	7.96	Low	-8.44	-3.57	1.83	9.70	Low	-0.43	0.26	
2	0.99	0.67	0.93	1.03	2	0.69	0.97	1.06	2	47.41	56.81	67.43	37.10</									

Appendix 13 Size-OP-INV Five-factor alphas and slopes

			North America						Japan						Europe																
			Coefficient Alpha			Statistic Alpha			Coefficient Alpha			Statistic Alpha			Coefficient Alpha			Statistic Alpha													
Low	2	High	Low	2	High	Low	2	High	Low	2	High	Low	2	High	Low	2	High	Low	2	High	Low	2	High								
0.08	0.00	0.01	0.26	Low	0.04	-0.06	0.10	-0.15	Low	0.51	0.02	0.06	2.93	Low	0.37	-0.73	0.95	-1.92	Low	-0.17	0.03	0.05	0.05	Low	-0.04	-0.04	0.03	-0.01			
0.28	0.08	0.11	-0.03	2	-0.16	-0.02	-0.04	-0.10	2	1.39	1.04	1.54	-0.23	2	-1.30	-0.23	0.51	-1.25	2	-0.34	0.11	0.05	0.14	2	0.14	0.05	-0.01	-0.04			
3	-0.24	0.27	0.10	-0.01	3	0.19	0.04	0.01	0.01	3	-1.13	3.58	1.1	0.14	3	-0.31	-0.01	0.04	0.15	3	-0.31	-0.01	0.04	0.15	3	-0.31	-0.01	0.04	0.15		
High	-0.24	-0.36	-0.15	-0.04	High	-0.11	0.07	0.06	0.47	High	-1.32	-3.85	-2.01	-0.64	High	-0.08	0.57	0.39	3.31	High	-0.55	-0.20	-0.15	0.17	High	-0.08	0.15	-0.12	0.06		
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
1.21	1.14	1.03	1.07	Low	1.11	1.06	0.87	0.07	Low	39.40	47.79	44.33	45.34	Low	39.76	47.68	32.05	48.24	Low	1.07	1.04	1.03	1.06	Low	1.02	1.06	0.96	1.00			
2	1.16	0.86	0.92	2	1.01	1.05	0.95	0.07	2	30.63	49.55	54.06	63.92	2	32.10	39.43	43.13	43.30	2	0.96	0.93	0.92	0.97	2	0.88	0.98	1.01	0.96			
3	1.09	1.08	1.05	1.07	High	0.96	1.07	1.04	1.05	High	21.21	10.62	51.49	60.09	High	33.59	30.79	27.17	27.58	High	1.05	1.04	1.04	1.04	High	1.06	0.98	1.04	0.99		
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
0.63	0.31	0.26	0.28	Low	0.07	0.06	0.14	-0.18	Low	-8.53	7.01	5.51	6.51	Low	1.31	1.46	2.79	4.55	Low	-0.42	0.10	0.16	0.22	Low	0.17	-0.08	-0.07	-0.33			
2	-0.46	0.09	0.27	0.28	2	0.32	0.29	0.17	-0.11	2	-4.70	2.44	8.68	9.73	2	5.12	6.76	4.10	4.76	2	0.05	0.11	0.19	0.18	2	0.01	0.02	0.05	0.03		
3	-0.41	0.04	0.22	0.26	3	0.57	0.14	0.18	-0.26	3	-3.96	1.17	6.73	9.23	3	5.79	5.71	4.14	5.03	3	0.17	0.28	0.22	0.14	3	-0.31	0.18	-0.12	-0.16		
High	-0.69	0.13	0.20	0.29	High	0.04	0.25	0.16	-0.18	High	-7.59	2.57	6.20	5.24	High	0.83	3.51	-2.22	-2.48	High	-0.44	-0.09	0.06	0.09	High	0.04	0.22	0.24	-0.02		
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
-1.00	-0.19	0.05	0.25	Low	-0.61	-0.16	0.02	0.34	Low	-13.57	-4.27	1.15	5.89	Low	-11.93	-3.91	0.47	8.87	Low	-0.88	-0.19	0.11	0.30	Low	-0.70	-0.20	0.12	0.50			
2	-0.99	-0.15	0.13	0.36	2	-0.19	-0.12	0.13	0.34	2	-10.21	-4.01	4.21	12.67	2	-3.13	-3.38	3.04	8.58	2	-0.50	-0.16	0.10	0.26	2	-0.99	-0.20	0.32	0.52		
3	-0.68	-0.26	0.06	0.43	3	-0.62	-0.19	0.08	0.21	3	-6.55	-7.05	1.76	15.32	3	-9.69	-3.73	1.87	4.09	3	-0.53	-0.15	0.18	0.29	3	-0.78	-0.16	0.11	0.36		
High	-0.88	-0.29	0.08	0.42	High	-0.64	0.09	0.12	0.54	High	-9.82	-5.90	2.10	12.83	High	-12.14	0.43	1.74	7.69	High	-0.82	-0.33	-0.05	0.01	High	-0.85	0.01	0.42	0.42		
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
0.65	0.25	0.25	0.20	Low	0.17	0.46	0.67	0.49	Low	6.93	4.44	4.16	3.37	Low	1.15	8.74	4.25	9.81	Low	0.42	0.38	0.35	0.26	Low	0.34	0.71	0.65	0.60			
2	0.30	0.28	0.20	0.12	2	0.15	0.01	0.20	0.29	2	2.40	5.81	5.08	3.39	2	1.92	0.19	3.73	5.38	2	0.13	0.39	0.32	0.25	2	0.08	0.31	0.46	0.11		
3	0.02	0.08	0.01	-0.15	3	-0.20	-0.16	-0.23	-0.04	3	0.17	1.78	12.18	-1.14	3	-2.39	-2.41	-4.20	-0.65	3	-0.54	-0.02	0.00	-0.08	3	-0.43	-0.28	-0.02	0.08		
High	-0.37	-0.47	-0.40	-0.53	High	-0.23	-0.64	-0.48	-0.75	High	-3.22	-7.48	-8.22	-2.42	High	-10.71	-7.74	-5.34	-8.33	High	-0.62	-0.45	-0.44	-0.40	High	-0.58	-0.77	-0.63	-0.77		
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
0.03	0.03	0.12	0.05	Low	-0.05	0.17	-0.04	-0.06	Low	-0.25	0.12	1.42	0.81	Low	-0.52	1.29	-0.33	-0.47	Low	-0.02	-0.03	0.16	0.44	Low	-0.29	-0.05	-0.27	0.09			
2	0.02	-0.01	0.03	0.07	2	0.02	-0.04	-0.03	-0.06	2	0.28	0.51	0.51	0.85	2	-0.58	-0.34	-0.44	-0.47	2	0.02	0.19	0.24	0.43	2	0.06	0.47	-0.24	0.21		
3	-0.03	-0.04	0.02	-0.07	3	-0.14	0.12	0.07	-0.20	3	-0.33	-0.51	0.25	0.81	3	0.04	-0.47	0.63	1.40	3	0.04	-0.47	0.63	1.40	3	-0.15	-0.08	0.17	-0.17		
High	-0.01	-0.10	0.05	0.07	High	0.36	-0.18	0.04	0.06	High	-0.06	-1.10	0.47	0.74	High	1.69	-1.33	0.33	0.32	High	-0.25	-0.18	-0.53	-0.11	High	-0.22	0.16	0.36	-0.07		
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
1.18	1.00	1.02	1.05	Low	1.10	1.01	0.99	0.98	Low	53.92	61.45	61.04	49.95	Low	44.51	38.20	41.75	38.61	Low	1.13	0.98	0.94	1.08	Low	1.06	1.00	1.02	0.94			
2	0.68	0.90	0.95	0.97	2	0.99	0.95	1.00	0.94	2	50.37	62.63	72.47	55.83	2	36.77	41.34	42.96	37.51	2	1.18	1.00	0.92	0.93	2	1.01	0.99	1.02	1.01		
3	1.01	0.94	0.92	0.97	3	1.04	0.95	0.95	0.98	3	48.94	63.47	55.99	67.87	3	38.32	39.23	44.21	33.08	3	1.08	1.08	0.93	0.96	3	1.03	0.98	0.92	1.07		
High	1.12	0.98	1.01	1.09	High	1.14	1.03	0.92	1.06	High	38.03	51.55	49.10	58.09	High	26.41	38.58	39.53	47.09	High	1.32	1.03	1.06	1.08	High	0.99	0.99	1.12	0.97		
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
0.91	0.81	0.88	0.88	Low	0.11	-0.08	0.08	0.03	Low	30.91	36.07	31.34	37.04	Low	2.92	-2.06	2.18	0.87	Low	1.33	1.13	1.01	1.01	Low	-0.06	-0.12	0.07	0.10			
2	0.93	0.81	0.86	0.84	2	0.02	0.05	0.02	0.01	2	30.27	36.00	42.15	31.37	2	0.58	-0.95	0.48	0.19	2	1.14	1.10	0.74	0.64	2	-0.16	-0.11	0.13	0.11		
3	0.97	0.81	0.79	0.86	3	-0.04	-0.12	-0.09	-0.16	3	30.27	35.27	30.91	32.92	3	-0.85	-3.29	2.78	-3.39	3	1.48	1.00	0.76	0.74	3	-0.27	-0.18	-0.11	-0.08		
High	1.12	0.94	0.96	1.11	High	-0.11	0.04	-0.26	-0.02	High	23.33	31.91	30.04	37.87	High	-1.69	0.91	-7.27	-6.62	High	1.12	1.16	1.00	1.01	High	-0.09	-0.13	0.16	0.13		
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
0.02	0.16	0.18	0.12	Low	-0.20	-0.19	0.00	0.15	Low	0.52	4.82	5.15	27.92	Low	-1.39	-3.37	0.02	2.87	Low	-0.60	0.10	0.13	0.07	Low	0.28	0.16	-0.20	-0.04			
2	0.04	0.16	0.19	0.07	2	0.08	0.10	0.20	0.05	2	0.95	5.19	6.89	1.92	2	0.95	2.17	4.74	0.88	2	-0.70	0.13	0.30	0.40	2	0.32	0.04	0.17	-0.28		
3	0.13	0.24	0.18	0.19	3	0.17	0.13	0.15	0.14	3	3.10	7.93	9.32	3.42	3	3.10	6.83	3.24	3.01	3	-0.73	0.23	0.27	0.27	3	0.48	0.38	0.22	-0.08		
High	-0.01	0.05	-0.01	-0.09	High	-0.26	0.05	0.15	-0.14	High	-0.22	1.25	-0.15	-2.22	High	-2.86	0.84	3.82	3.01	High	-0.97	-0.28	0.19	0.14	High	-0.17	0.13	-0.29	-0.08		
Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High	Low	2	3	High
0.15	0.01	0.01	0.11	Low	-0.33	-0.30	0.32	0.33	Low	-3.84	-2.82	0.25	2.14	Low	-6.38	-3.45	3.61	10.60	Low	-0.64	-0.56	0.06	0.37	Low	-0.92	0.23	0.38	0.38			
2	-0.27	-0.23	0.00	0.11	2	-0.72	-0.03	0.32	0.32	2	-5.71	-1.99	1.97	2.14	2	-4.64	-4.04	2.82	5.45	2	-0.45	-0.29	0.02	0.22	2	-0.88	-0.13	0.29	0.38		
3	-0.26	-0.19	-0.06	0.22	3	-0.81	-0.26	0.20	0.42	3	-6.52	-3.21	-1.00	3.87	3	-8.94	-3.20	2.82	4.32	3	-0.82	-0.25	0.03	0.23	3	-0.76	-0.05	0.43	0.46		
High	-0.42	-0.20	-0.01	0.30	High	-1.21	-0.01	0.02	0.35	High	-8.08	-3.81	-0.12	4.70	High	-8.44	-0.11	0.32	4.75	High	-0.51	-0.41	0.03	0.28	High	-0.78	-0.21	0.03	0.24</		

