Helsinki Center of Economic Research

Discussion Papers

Forecasting U.S. Macroeconomic and Financial Time Series with Noncausal and Causal AR Models: A Comparison

Markku Lanne
University of Helsinki and HECER

Henri Nyberg
University of Helsinki and HECER

and

Erkka Saarinen
University of Helsinki

Discussion Paper No. 319
March 2011

ISSN 1795-0562

HECER – Helsinki Center of Economic Research, P.O. Box 17 (Arkadiankatu 7), FI-00014 University of Helsinki, FINLAND, Tel +358-9-191-28780, Fax +358-9-191-28781,
E-mail info-hecer@helsinki.fi, Internet www.hecer.fi

HECER
Discussion Paper No. 319

Forecasting U.S. Macroeconomic and Financial Time Series with Noncausal and Causal AR Models: A Comparison*

Abstract

In this paper, we compare the forecasting performance of univariate noncausal and conventional causal autoregressive models for a comprehensive data set consisting of 170 monthly U.S. macroeconomic and financial time series. The noncausal models consistently outperform the causal models in terms of the mean square and mean absolute forecast errors. For a set of 18 quarterly time series, the improvement in forecast accuracy due to allowing for noncausality is found even greater.

JEL Classification: C22, C53, E37, E47

Keywords: Noncausal autoregression, forecast comparison, macroeconomic variables, financial variables.

Markku Lanne

Department of Political and Economic Studies
University of Helsinki
P.O. Box 17
FI-00014
FINLAND

e-mail: markku.lanne@helsinki.fi

Henri Nyberg

Department of Political and Economic Studies
University of Helsinki
P.O. Box 17
FI-00014 University of Helsinki
FINLAND

e-mail: henri.nyberg@helsinki.fi

Erkka Saarinen

Department of Political and Economic Studies
University of Helsinki
P.O. Box 17
FI-00014
FINLAND

e-mail: erkka.saarinen@helsinki.fi

* The authors would like to thank Helinä Laakkonen and Pentti Saikkonen for constructive comments. The authors are responsible for any remaining errors. Financial support from the Academy of Finland and the OP-Pohjola Group Research Foundation is gratefully acknowledged.

1 Introduction

Forecasting is the primary field of application of univariate time series models in economics. Although different kinds of nonlinear models have been introduced in the past few decades, it is the linear causal Gaussian autoregressive (AR) model that remains the prominent model in forecasting in financial and macroeconomic contexts. In this paper, we compare the forecasting performance of this model to corresponding noncausal models that allow for explicit dependence on the future, in contrast to the causal AR model with dependence only on the past. Recently, Lanne and Saikkonen (2011a) have found that many macroeconomic and financial time series exhibit noncausality suggesting that better forecasts can possibly be obtained by allowing for noncausality in the predictive model. In addition, we deviate from the ubiquitous assumption of normality. While this is necessary for identification, as discussed below, the proposed t-distribution for the errors also seems to bring benefits in the form of better fit for a large number of the monthly and quarterly time series we consider.

Noncausal AR models were suggested by Breidt *et al.* (1991), and their properties are studied in the monograph by Rosenblatt (2000). Lanne and Saikkonen (2011b) suggest a new formulation of the noncausal AR model where the dependence on the future is explicitly formulated. Their model can be estimated by the method of maximum likelihood, but forecast computation is not straightforward. This is due to the fact that in the noncausal models, the optimal forecast (in the mean-square sense) is generally nonlinear. However, recently Lanne, Luoto and Saikkonen (2010) have proposed a simulation-based forecasting method for the noncausal autoregressive (AR) model (see Lanne, Luoma and Luoto (2011) for Bayesian approach). It should be pointed out that although forecasts are based on simulations, Lanne *et al.* (2010) show that their method, also employed in this paper, is not computationally burdensome.

In this study, we examine empirically the marginal benefit of allowing for noncausality for forecasting performance. We concentrate on the comprehensive data set of Marcellino, Stock and Watson (2006) consisting of 170 monthly U.S. macroeconomic and financial time series. As many important macroeconomic time series, such as the real GDP and its components, are measured only on a quarterly basis also comparisons of quarterly forecasts are of interest. Thus, in addition to the monthly data, we consider 18 central quarterly U.S. macroeconomic time series mainly related to U.S. output growth, employment and measures of inflation. This latter exercise can also be seen as a robustness check for the findings obtained with the monthly data.

Forecasts are compared based on the usual statistical goodness-of-fit measures and predictive models are selected by using different model selection criteria. In this respect, we closely follow the paper by Marcellino *et al.* (2006). However, they consider only causal AR models and focus on comparing the direct and iterative multiperiod forecasting methods. In the former, a forecast horizon-specific model is employed, whereas in the latter a one-period model is used recursively to obtain the multiperiod forecast. In this paper, we concentrate on iterative forecasts.

Overall, the results suggest the presence of noncausality in many macroeconomic time series in that allowing for noncausality leads to improvements in forecast accuracy. The noncausal model outperforms the causal model in most cases although the differences are not very large in terms of the mean square and mean absolute forecast errors. In particular, at the three and six-month forecast horizons noncausal models yield smaller forecast errors for over 60% of the variables. This improvement is especially strong for variables related to employment, construction, price indices and wages, whereas for interest rates and asset prices, the noncausal model does not seem to bring extra benefit. Causal models also seem to be superior at the one-month forecast horizon.

As far as the quarterly time series are concerned, great improvement in forecast accuracy due to allowing for noncausality is found. For almost all variables and forecast horizons the noncausal model yields superior forecasts, and the differences in forecast accuracy are typically larger than for the monthly series. In many cases, the mean square forecast error is diminished by several percentages, even more than 10%.

The rest of the paper is organized as follows. In Section 2, we introduce the noncausal autoregressive model of Lanne and Saikkonen (2011b) and describe the simulation-based forecasting method of Lanne *et al.* (2010). The monthly and quarterly U.S. data set is presented in Section 3, and the forecasting results are reported in Section 4. Finally, Section 5 concludes.

2 Causal and Noncausal AR Models

2.1 Models

In the previous literature, the causal AR model has commonly been used for forecasting macroeconomic time series. The causal AR(r) model for y_t can be compactly written as

$$\phi(B)\, y_t = \varepsilon_t, \tag{1}$$

where $\phi(B) = 1 - \phi_1 B - \ldots - \phi_r B^r$ and ε_t is an independently and identically distributed ($i.i.d.$) error term with mean zero and variance σ^2. In this expression, B is the usual backshift operator (i.e., $B^k y_t = y_{t-k}$) and the polynomial $\phi(z)$ is assumed to have its zeros outside the unit circle.

The noncausal AR model allows for explicit dependence on the future. In the formulation proposed by Lanne and Saikkonen (2011b), this is achieved by premultiplying the left side of (1) by the polynomial $\varphi(B^{-1}) = 1 - \varphi_1 B^{-1} - \ldots - \varphi_s B^{-s}$ to obtain

$$\varphi(B^{-1})\, \phi(B)\, y_t = \varepsilon_t. \tag{2}$$

For stationarity, we assume that $\varphi(z) \neq 0$ for $|z| \leq 1$. We call model (2) the AR(r,s) model, and the conventional causal AR(r) model is obtained when $s = 0$, so that $\varphi(B^{-1}) = 1$, indicating that y_t depends only on its past values. On the other hand, if $r = 0$, we have a purely noncausal AR(0,s) model with dependence only on future values of y_t.

When the roots of both polynomials $\phi(z)$ and $\varphi(z)$ lie outside the unit circle, y_t in (2) has the following two-sided moving-average presentation,

$$y_t = \sum_{j=-\infty}^{\infty} \psi_j\, \varepsilon_{t-j},$$

where ψ_j is the coefficient of z^j in the Laurent series expansion of $\phi(z)^{-1}\varphi(z^{-1})^{-1} = \psi(z)$. For forecasting purposes, it is useful to write model (2) in the following equivalent form

$$y_t = \phi_1 y_{t-1} + \ldots + \phi_r y_{t-r} + v_t, \tag{3}$$

where

$$v_t = \varphi(B^{-1})^{-1}\varepsilon_t = \sum_{j=0}^{\infty} \beta_j \, \varepsilon_{t+j}. \tag{4}$$

This shows that the noncausality of the AR(r,s) model can also be interpreted as dependence on the future error terms ε_{t+j}, $j \geq 0$.

An important feature of the noncausal AR model is that to distinguish between noncausality and causality the error term ε_t must be non-Gaussian (see, e.g., Lanne and Saikkonen, 2011b). Therefore, throughout the error term in (2) is assumed to be non-Gaussian, and following Lanne and Saikkonen (2011b) and Lanne *et al.* (2010), t-distributed. These authors provide evidence in favor the good fit of the t-distribution in capturing the fat tails in modeling inflation. Although a number of other error distributions could be entertained, also according to our results, the t-distribution seems adequate in most cases. It is mostly some financial time series that are an exception.

Once the error distribution has been specified, the AR(r, s) model can be estimated by the method of maximum likelihood (ML) using numerical methods. Under reasonable regularity conditions, the ML estimator is consistent and asymptotically normally distributed (see Lanne and Saikkonen, 2011b).

2.2 Forecasting in Noncausal AR Models

As Rosenblatt (2000, Corollary 5.4.2) and Lanne *et al.* (2010) have emphasized, the prediction problem in the noncausal AR models is nonlinear, and simulation-based methods are, in general, required in forecasting. In contrast, in the case of causal autoregressions with i.i.d. errors, the prediction problem is linear and (in mean-square sense) optimal linear forecasts can be constructed with explicit formulae without having to resort to simulation-based methods.

Recently, Lanne *et al.* (2010) have proposed a forecasting method for the noncausal AR(r, s) model (2). Their procedure can be described as follows. Let $E_T(\cdot)$ signify the conditional expectation conditional on the observed values $y_1, \ldots, y_T$.

The conditional expectation of representation (3) of the AR(r,s) model yields the mean-square sense optimal forecast of $y_{T+h}, h > 0$,

$$E_T(y_{T+h}) = \phi_1 E_T(y_{T+h-1}) + \ldots + \phi_r E_T(y_{T+h-r}) + E_T(v_{T+h}). \tag{5}$$

Thus, provided forecasts of v_{T+h} are available, multiperiod forecasts at any forecast horizon h can be constructed recursively. Note that in the special case of the causal AR(r) model, (5) reduces to

$$E_T(y_{T+h}) = \phi_1 E_T(y_{T+h-1}) + \ldots + \phi_r E_T(y_{T+h-r}). \tag{6}$$

This corresponds to the iterative multiperiod forecasting approach (see Marcellino *et al.*, 2006) used extensively in the previous studies of causal AR models.

To obtain forecasts for v_{T+h}, Lanne *et al.* (2010) use the approximation

$$v_{T+h} \approx \sum_{j=0}^{M-h} \beta_j \, \varepsilon_{T+h+j}, \tag{7}$$

where the integer M is supposed to be large enough to make the approximation error negligible. Therefore, a close approximation to (5) is

$$E_T(y_{T+h}) \approx \phi_1 E_T(y_{T+h-1}) + \ldots + \phi_r E_T(y_{T+h-r}) + E_T\Big(\sum_{j=0}^{M-h} \beta_j \, \varepsilon_{T+h+j}\Big). \tag{8}$$

Lanne *et al.* (2010) suggest a method for evaluating the last term. Details on the procedure can be found in their paper, but the main idea is to approximate the above-mentioned conditional expectation by simulating N mutually independent realizations from the conditional distribution of $\Big(\varepsilon_{T+1}, \ldots, \varepsilon_{T+M}\Big)$. They also provide simulation evidence that even with relatively small values of the truncation parameter M and the number of simulation replications N, approximation (7) is quite accurate. Based on their simulation results, we set M=50 and N=10000, respectively.

2.3 Model Selection and Forecast Evaluation

In this paper, the forecasts are computed recursively, and the model is respecified at each date. The model selection procedure employed is similar to that proposed by

Lanne and Saikkonen (2011b) and Lanne *et al.* (2010). At first, an adequate causal AR(p) model with a normally distributed error term is specified. To determine the lag order p, three different methods are employed. In addition to the fixed lag p, the Akaike (1974) and Schwarz (1978) information criteria are employed.[1] In model selection, the maximum number of lags is restricted to 12 (i.e., $0 \leq p \leq 12$) and eight (i.e., $0 \leq p \leq 8$) in the case of monthly and quarterly observations, respectively. In forecasting, the AIC and BIC criteria are recomputed at each date. Thus, the selected forecasting model can change at each time when the parameters are re-estimated.

Once the adequate causal model has been specified, we estimate all noncausal AR(r, s) models with the sum of r and s equal to p. Finally, following Breidt *et al.* (1991) and Lanne and Saikkonen (2011b), we select the AR(r,s) model that maximizes the log-likelihood function.

Forecasts $\hat{y}_{T+h} = E_T(y_{T+h})$ from (8) are evaluated by commonly used statistical goodness-of-fit measures, the mean square forecast error and the mean absolute forecast error. If we denote the forecast error by $e_{T+h} = y_{T+h} - \hat{y}_{T+h}$ and the length of the out-of-sample forecasting period by m, the mean squared forecast error (MSFE) can be written as

$$\text{MSFE} = \frac{1}{m} \sum_{i=1}^{m} e_{T+h+i-1}^2, \tag{9}$$

and the mean absolute forecast error (MAFE) as

$$\text{MAFE} = \frac{1}{m} \sum_{i=1}^{m} |e_{T+h+i-1}|, \tag{10}$$

where $i = 1, \ldots, m$. The MSFE and MAFE are computed for each series, forecast model and forecast horizon h from one to 24 months in the case of monthly data. For the quarterly time series, the longest horizon is eight quarters. As in the previous literature, forecasts will be assessed by comparing the MSFE and MAFE of the AR(r,s) model to those of the causal Gaussian AR(p) model.

[1] In both monthly and quarterly time series the fixed lag length is four (i.e., $p = 4$).

3 Data, Estimation and Forecast Samples

Our monthly data set was previously considered by Marcellino *et al.* (2006).[2] It consists of 170 monthly U.S. macroeconomic and financial time series. Most series range from 1959 M1 to 2002 M12, but there are also some variables with shorter sample periods. Following Marcellino *et al.* (2006), we divide the variables into five categories:

A) Income, output, sales and capacity utilization,

B) Employment and unemployment,

C) Construction, inventories and orders,

D) Interest rates and asset prices, and

E) Nominal prices, wages, and money.

Details on the time series are presented in the appendix of Marcellino *et al.* (2006).

Almost all series are subject to transformations, most commonly log-differencing, necessary to guarantee stationarity. We take the transformations and the handling of the large outliers of Marcellino *et al.* (2006) as given. An observation is deemed an outlier if its absolute value exceeds the median of the time series by more than six times. As Marcellino *et al.* (2006) point out, there is disagreement on the order of integration of some variables, especially those belonging to category E. While they regard some of them $I(2)$ processes, we assume all to be $I(1)$.

The quarterly data set comprises 18 central quarterly U.S. macroeconomic time series including the GDP and its components. A detailed description of the data set is given in Table 1. Most series range from the beginning of 1947 to the first or second quarter of 2010, but some variables have shorter sample periods. Thus, our quarterly data set covers a somewhat longer time period than the monthly dataset of Marcellino *et al.* (2006).

We follow Marcellino *et al.* (2006) in selecting the first estimation and out-of-sample forecasting sample periods. In particular, this means that the initial forecast date in the monthly data is 1979 M1. Thus, for most series, out-of-sample forecasts are computed for the period of 1979 M1 to 2002 M12. In other words,

[2] The data is available at Mark Watson's homepage http://www.princeton.edu/∼mwatson/publi.html.

if the forecast horizon is, say, 12 months ($h = 12$), the last forecast is made in December 2001 for December 2002. Similarly in the quarterly data, the first out-of-sample forecasts are made for the first quarter of the year 1979. Furthermore, in estimation, we apply an expansive window approach and the parameters are re-estimated at each date.

4 Forecasting Results

4.1 Monthly Time Series

Tables 2 and 3 summarize the empirical distributions of the relative MSFE (9) and MAFE (10) of the noncausal model in relation to the causal model, respectively. We report the results at the forecast horizons of 1, 3, 6, 9, 12 and 24 months. The forecast comparisons in Tables 2 and 3 involve all variables included in the data set, and the results based on the three model selection criteria mentioned in Section 2.3 are reported.

As an example of the results in Table 2, consider the case where the BIC is used in model selection and the forecast horizon is three months ($h = 3$): the mean relative MSFE is 0.9972 (i.e., 99.72%) indicating that the noncausal AR(r, s) model yields a slight improvement over the causal model. In 10% of the series, the relative MSFE is less than 0.96, while in 10% of the series the relative MSFE exceeds 1.02, and more than 60% of 166 the series have a relative MSFE below unity, indicating the overall superiority of the noncausal model.

A general inspection of the results in Table 2 suggests that when the BIC is used in model selection, the noncausal model appears to forecast somewhat more accurately than the causal model. On the other hand, if the AIC is employed or p is fixed at 4 ($p = 4$), the differences are minor, or even slightly advantageous to the causal model. However, according to Table 3, where the mean absolute forecast error (10) is used in forecast evaluation instead of the MSFE, the results are more favourable to the noncausal model irrespective of the model selection criterion used. The MSFE reacts more strongly to large outliers than the MAFE. Thus, a few large forecast errors in the AR(r, s) model can render the MSFE larger in the noncausal model than in the causal model although in a greater fraction of the series the noncausal AR model outperforms the causal model. The latter is seen by inspecting the fractions of forecasts with the relative MSFE and MAFE less than unity also reported in Tables 2 and 3. This forecast evaluation measure is related to the above-mentioned percentiles of the empirical distribution of the relative MSFE and MAFE in that it gives the percentages of the variables where the noncausal model yields smaller forecast errors than the causal model. For example, in our example case, we can see that the 0.60 percentile is just under unity (i.e. in 60% of the series the relative MSFE is less than unity) but the 0.75 percentile exceeds unity. The exact fraction of the series with the MSFE of the causal model exceeding that of the noncausal model is 0.6145.

The fractions of variables with relative MSFE and MAFE under unity, highlight the substantial differences between the models. Except for the one-month horizon ($h = 1$), the noncausal AR(r,s) model always produces superior forecasts. At forecast horizons longer than one period, the fractions are often higher than 60% and the highest values are obtained for forecast horizons from three to six months. These findings demonstrate how the AR(r,s) model outperforms the causal AR(p) model. A good example of the usefulness of the percentiles and fractions in forecast evaluation can be seen in Table 2 when the forecast horizon is six months and the AIC is used in model selection. In this case, the mean of the relative MSFE statistics is 1.0987, but the median (0.50 percentile) is 0.9982, and for over 60% of the series (Frac=0.6024) the noncausal model forecasts more accurately.

In Table 4, we break down the results of Table 2 to the five categories of time series introduced in Section 3. We report the mean and median (0.50 percentile) of the distribution of the relative MSFEs as well as the fractions introduced above. The results reveal the substantial differences in forecast performance between the different categories of variables.[3] The noncausal model typically outperforms the causal model at all forecast horizons and all categories except for the one-period forecasts and the category including interest rates and asset prices (category D).

[3] The results based on the MAFE leads to similar findings. Details on those results are available upon request.

However, it should be pointed out that also in category D, there are various variables for which allowing for noncausality improves forecasts. As an example, the relative MSFE of the one-period forecasts of the Federal funds rate is 0.85. On the other hand, especially for the exchange rate series, the causal model provides forecasts superior to those of the noncausal model.

The results in Table 5 exclude the series in category D. Not surprisingly, with the exception of the one-month forecasts, the noncausal model is clearly superior. The mean and median values of the MSFE criterion at all multiperiod forecast horizons are less than unity, and the fractions exceed 60%, coming close to 70% in the case of the six-month forecasts.

One reason for why the noncausal model with a t-distributed error term seems to work rather poorly for financial variables might be that the t-distribution does not sufficiently capture their excessive kurtosis. Our estimation results support this view in that for nearly two thirds of the series in category D, the estimate of the degree-of-freedom parameter hits the lower bound of 2.1 set to guarantee finite variance. This suggests that alternative distributions might be preferred for these series. It can also be seen that in almost all financial variables, there is evidence of remaining conditional heteroskedasticity in the residuals of the AR(r, s) model. Thus, even better forecasts could possibly be obtained by taking conditional heteroskedasticity into account in specifying the predictive model. We leave these issues for future research.

As a final check, we restrict ourselves to the series exhibiting noncausality ($s \geq 1$) in the entire sample period, i.e. the cases where the benefit in forecasting due to allowing for noncausality is expected to be greatest. It should be pointed out that in genuine out-of-sample forecasting this of course is not feasible. The main goal of this exercise is just to demonstrate the importance of identifying the variables for which allowing for noncausality is likely to bring gains in forecast accuracy. The results are presented in Table 6 which shows that, except for the interest rates and asset prices (category D), the superiority of the noncausal model over the causal model tends to improve when causal time series are excluded. This shows up as greater fractions than in Table 2. Interestingly, the AIC seems to work a bit better than the BIC in many categories whereas in Tables 2, 4 and 5 the BIC typically produces a predictive model with superior forecasts. One potential explanation might be the tendency of the AIC to yield higher-order models with greater potential to find the correct orders r and s. In other words, as the AIC suggests larger models than the BIC, it is plausible that also noncausality is correctly identified more often when the AIC is used. Therefore, it is likely that a greater fraction of the variables included in Table 6 selected by the AIC are truly noncausal compared with the variables selected by the BIC.

4.2 Quarterly Time Series

In the previous section, we found evidence of the noncausal AR model outperforming the causal AR model in forecasting monthly time series. However, because many interesting macroeconomic variables are measured only on quarterly basis, it is of interest to examine the predictive performance of noncausal and causal AR models also for quarterly time series (see the description of variables in Table 1).

Table 7 presents the values of the relative MSFE statistics. In addition, in the bottom panel, the fractions of the variables with the noncausal model yielding a smaller MSFE than the causal model are also reported. The overall superiority of the noncausal model can be confirmed also in this data set. In fact, the differences in the forecast performance between the noncausal and causal models appear to be even larger than in the case of monthly observations. For example, for the GDP price deflator and employment series, the noncausal AR model yields substantially better forecasts and the improvement is over 10% compared with the causal AR model at almost all forecast horizons and with any model selection criterion.

As in the monthly data, the reported fractions of the variables confirm the finding that, on average, the noncausal AR model forecasts better than the causal AR model. Compared with the results for the monthly data, now also the one-period (one-quarter) forecasts of the noncausal model outperform those of the causal model. On the other hand, at the longest forecast horizon of eight quarters, the causal model seems to work reasonably well.

5 Conclusions

In this paper, we have compared the forecast performance of the noncausal AR model of Lanne and Saikkonen (2011b) to that of the conventional causal AR model. We have examined a comprehensive monthly data set of U.S. macroeconomic and financial time series as well as a more limited collection of quarterly U.S. time series. The main result of the forecast comparison is that the noncausal model tends to yield superior multiperiod forecasts compared to the causal model. The improvement in forecast accuracy is in accordance with the estimation results, suggesting the presence of noncausality. For the quarterly time series, the improvement in forecast accuracy due to allowing for noncausality is greater than for the monthly series. For the former, it also seems to be universal, while for the latter it is confined to multiperiod forecast horizons.

The results suggest that there are some differences between the different categories of time series examined. The noncausal model clearly outperforms the causal model for variables related to output growth, employment, construction, price indices and wages irrespective of the data frequency. On the other hand, the causal AR model, in general, seems to work better among interest rates and asset prices. The neglected conditional heteroskedasticity or inadequacy of the t-distribution specified for the error term are possible reasons for this finding.

References

Akaike, H. (1974). A new look at statistical model identification. *IEEE Transactions on Automatic Control*, 19, 716–723.

Breidt, J., Davis, R.A., Lii, K. S., and M. Rosenblatt (1991). Maximum likelihood estimation for noncausal autoregressive processes. *Journal of Multivariate Analysis* 36, 175–198.

Lanne, M., and P. Saikkonen (2011a). GMM estimation with noncausal instruments. *Oxford Bulletin of Economics and Statistics*, forthcoming.

Lanne, M., and P. Saikkonen (2011b). Noncausal autoregressions for economic time series. *Journal of Time Series Econometrics*, forthcoming.

Lanne, M., Luoma, A., and J. Luoto (2011). Bayesian model selection and forecasting in noncausal autoregressive models. *Journal of Applied Econometrics*, forthcoming.

Lanne, M., Luoto, J., and P. Saikkonen (2010). Optimal forecasting of noncausal autoregressive time series. HECER Discussion Paper No. 286.

Marcellino, M., Stock, J.H., and M.W. Watson (2006). A comparison of direct and iterated AR methods for forecasting macroeconomic time series. *Journal of Econometrics*, 135, 499–526.

Rosenblatt, M. (2000). *Gaussian and Non-Gaussian Linear Time Series and Random Fields*. Springer-Verlag, New York.

Schwarz, G. (1978). Estimating the dimension of a model. *Annals of Statistics*, 6, 461–464.

Table 1: Quarterly U.S. time series.

Series	Trans.	Sample period	Description
gdpdef	ΔLN	1947:Q1–2010:Q1	Gross Domestic Product: Implicit Price Deflator
gdp	ΔLN	1947:Q1–2010:Q1	Gross Domestic Product
gpdi	ΔLN	1947:Q1–2010:Q2	Gross Private Domestic Investment
cpiall	ΔLN	1947:Q1–2010:Q2	Consumer Price Index For All Urban Consumers: All Items
gpsave	ΔLN	1947:Q1–2010:Q1	Gross Private Saving
dpi	ΔLN	1947:Q1–2010:Q1	Disposable Personal Income
pcec	ΔLN	1947:Q1–2010:Q1	Personal Consumption Expenditures
rinvbf	ΔLN	1947:Q1–2010:Q2	Real Gross Private Domestic Investment
cum	ΔLN	1948:Q1–2010:Q2	Capacity Utilization Rate: Manufacturing
ipm	ΔLN	1947:Q1–2010:Q2	Industrial Production Index: Manufacturing
ipt	ΔLN	1947:Q1–2010:Q2	Industrial Production Index: Total
employ	ΔLN	1947:Q1–2010:Q2	Nonfarm Payroll Employment
ruc	Δ	1948:Q1–2010:Q2	Unemployment Rate
hstarts	ΔLN	1959:Q1–2010:Q2	Housing Starts
cut	ΔLN	1967:Q1–2010:Q2	Capacity Utilization Rate: Total
h	ΔLN	1964:Q1–2010:Q2	Indexes of Aggregate Weekly Hours: Total
bopxgs	ΔLN	1960:Q1–2010:Q1	Exports of Goods and Services
rimp	ΔLN	1947:Q1–2010:Q1	Real Imports of Goods and Services

Notes: In the table, quarterly measured U.S time series, their employed abbreviations, transformations and data spans are presented. In the second column showing the employed transformations, Δ denotes the first-order differencing and LN logarithmic transformation. The data are extracted from the Federal Reserve Economic Data (FRED) (Federal Reserve Bank of St. Louis).

Table 2: Distributions of relative MSFEs between the noncausal AR(r,s) model and the causal AR(p) model based on different model selection criteria.

Model Selection	Mean/percentile/fraction	Forecast horizon					
		1	3	6	9	12	24
BIC	Mean	1.0289	0.9972	0.9953	0.9957	0.9959	0.9963
	0.10	0.9563	0.9602	0.9683	0.9690	0.9660	0.9686
	0.25	0.9869	0.9789	0.9872	0.9917	0.9911	0.9939
	0.40	0.9998	0.9939	0.9970	0.9982	0.9991	0.9995
	0.45	1.0010	0.9967	0.9981	0.9992	0.9995	0.9997
	0.50	1.0032	0.9979	0.9991	0.9996	0.9997	0.9998
	0.55	1.0065	0.9988	0.9997	0.9999	1.0000	1.0000
	0.60	1.0090	0.9999	0.9999	1.0001	1.0003	1.0001
	0.75	1.0245	1.0024	1.0014	1.0026	1.0018	1.0010
	0.90	1.0717	1.0195	1.0172	1.0134	1.0142	1.0074
	Frac	0.4096	0.6145	0.6506	0.5783	0.5361	0.5361
AIC	Mean	1.0882	1.0378	1.0987	1.0067	1.0052	0.9829
	0.10	0.9626	0.9428	0.9525	0.9462	0.9355	0.9246
	0.25	0.9877	0.9800	0.9856	0.9853	0.9875	0.9862
	0.40	1.0044	0.9930	0.9944	0.9960	0.9985	0.9974
	0.45	1.0074	0.9956	0.9960	0.9981	0.9993	0.9983
	0.50	1.0119	0.9977	0.9982	0.9991	0.9998	0.9993
	0.55	1.0176	1.0003	0.9990	0.9999	1.0004	0.9997
	0.60	1.0249	1.0031	0.9999	1.0010	1.0017	1.0004
	0.75	1.0537	1.0199	1.0103	1.0093	1.0095	1.0024
	0.90	1.1420	1.1212	1.1126	1.0524	1.0391	1.0215
	Frac	0.3735	0.5482	0.6024	0.5602	0.5181	0.5663
$p = 4$	Mean	1.0528	1.0247	1.0317	1.0217	1.0145	1.0059
	0.10	0.9502	0.9502	0.9568	0.9658	0.9715	0.9776
	0.25	0.9866	0.9758	0.9824	0.9921	0.9946	0.9968
	0.40	0.9980	0.9933	0.9960	0.9979	0.9986	0.9991
	0.45	1.0026	0.9962	0.9983	0.9992	0.9994	0.9995
	0.50	1.0072	0.9981	0.9993	0.9998	0.9999	0.9999
	0.55	1.0134	1.0003	1.0000	1.0003	1.0003	1.0002
	0.60	1.0172	1.0023	1.0009	1.0008	1.0007	1.0006
	0.75	1.0510	1.0110	1.0061	1.0048	1.0035	1.0022
	0.90	1.1569	1.0585	1.0342	1.0300	1.0162	1.0063
	Frac	0.4277	0.5301	0.5542	0.5301	0.5120	0.5241

Notes: Each entry is the indicated summary measure of the distribution of the ratio between the MSFE for the noncausal AR(r, s) model to the MSFE of the causal AR(p) model for the lag selection listed in the first column and the forecast horizon indicated in the column heading. For each cell, the distribution and the summary measure are computed over the 166 series being forecasted. Frac is the percentage of variables with the relative MSFE below unity (i.e., the number of cases for which the AR(r, s) model yields a smaller MSFE than the AR(p) model). Four variables for which the estimation of the AR(r, s) model failed to converge are left out leaving 166 time series in total.

Table 3: Distributions of relative MAFEs of noncausal AR(r,s) and causal AR(p) models based on different model selection criteria.

Model Selection	Mean/percentile/fraction	Forecast horizon					
		1	3	6	9	12	24
BIC	Mean	0.9995	0.9938	0.9940	0.9944	0.9956	0.9970
	0.10	0.9700	0.9751	0.9784	0.9798	0.9774	0.9829
	0.25	0.9909	0.9885	0.9899	0.9924	0.9934	0.9966
	0.40	0.9972	0.9953	0.9968	0.9983	0.9991	0.9993
	0.45	0.9989	0.9972	0.9979	0.9992	0.9996	0.9996
	0.50	1.0000	0.9982	0.9988	0.9996	0.9998	0.9998
	0.55	1.0012	0.9992	0.9997	0.9998	1.0000	0.9999
	0.60	1.0024	0.9996	0.9999	1.0000	1.0001	1.0000
	0.75	1.0075	1.0010	1.0010	1.0009	1.0008	1.0008
	0.90	1.0206	1.0071	1.0063	1.0047	1.0081	1.0040
	Frac	0.5000	0.6446	0.6205	0.5482	0.5843	0.5843
AIC	Mean	1.0070	1.0019	1.0026	0.9984	0.9958	0.9930
	0.10	0.9767	0.9684	0.9779	0.9716	0.9634	0.9586
	0.25	0.9875	0.9860	0.9881	0.9897	0.9888	0.9917
	0.40	0.9988	0.9938	0.9950	0.9968	0.9973	0.9989
	0.45	1.0006	0.9963	0.9958	0.9981	0.9982	0.9994
	0.50	1.0027	0.9981	0.9970	0.9994	0.9991	0.9997
	0.55	1.0050	0.9990	0.9980	0.9998	0.9998	1.0001
	0.60	1.0072	1.0007	0.9992	1.0005	1.0001	1.0004
	0.75	1.0147	1.0096	1.0032	1.0031	1.0028	1.0022
	0.90	1.0397	1.0357	1.0341	1.0220	1.0133	1.0104
	Frac	0.4337	0.5723	0.6386	0.5783	0.5361	0.5361
$p = 4$	Mean	1.0074	0.9973	0.9976	0.9972	0.9987	1.0009
	0.10	0.9740	0.9665	0.9722	0.9744	0.9802	0.9859
	0.25	0.9893	0.9850	0.9900	0.9911	0.9947	0.9977
	0.40	0.9996	0.9955	0.9972	0.9976	0.9990	0.9996
	0.45	1.0011	0.9978	0.9984	0.9986	0.9994	0.9997
	0.50	1.0026	0.9988	0.9992	0.9995	0.9998	0.9999
	0.55	1.0036	0.9998	0.9998	0.9997	1.0000	1.0002
	0.60	1.0064	1.0010	1.0005	1.0002	1.0003	1.0005
	0.75	1.0150	1.0069	1.0028	1.0016	1.0015	1.0012
	0.90	1.0455	1.0191	1.0133	1.0091	1.0066	1.0071
	Frac	0.4217	0.5542	0.5723	0.5482	0.5181	0.5181

Notes: In this table, the MAFE (10) is used instead of the MSFE (9) employed in Table 2. See the notes to Table 2.

Table 4: Forecasting results based on the relative MSFEs, by the category of the series.

Model selection		Forecast horizon					
		1	3	6	9	12	24
(A) *Income, output, sales, capacity utilization (38 series)*							
BIC	Mean	1.0031	0.9965	0.9907	0.9909	0.9917	0.9958
	Median	1.0011	0.9999	0.9997	0.9994	0.9995	1.0001
	Frac	0.4737	0.5263	0.6842	0.5789	0.6316	0.3947
AIC	Mean	1.0176	0.9943	0.9902	0.9912	0.9881	0.9851
	Median	1.0073	0.9990	0.9987	0.9992	0.9992	0.9991
	Frac	0.3947	0.5526	0.6579	0.5263	0.6053	0.6053
$p = 4$	Mean	1.0204	0.9886	0.9898	0.9917	0.9926	0.9972
	Median	1.0167	0.9969	0.9977	0.9994	1.0002	1.0002
	Frac	0.2632	0.5789	0.7368	0.6053	0.4737	0.4737
(B) *Employment and unemployment (23 series)*							
BIC	Mean	1.0283	0.9876	0.9917	0.9934	0.9959	0.9967
	Median	0.9988	0.9848	0.9920	0.9969	0.9990	1.0000
	Frac	0.5217	0.6957	0.6522	0.6957	0.6087	0.5217
AIC	Mean	1.1288	1.0024	0.9913	0.9946	0.9895	0.9852
	Median	1.0033	0.9937	0.9892	0.9971	0.9997	0.9990
	Frac	0.4783	0.6087	0.6957	0.6087	0.5652	0.5652
$p = 4$	Mean	1.0681	0.9771	0.9854	0.9952	0.9999	1.0099
	Median	0.9941	0.9726	0.9719	0.9920	0.9985	0.9998
	Frac	0.6087	0.7391	0.5652	0.6522	0.6087	0.5217
(C) *Construction, inventories and orders (37 series)*							
BIC	Mean	1.0013	0.9953	0.9949	0.9987	0.9962	0.9924
	Median	1.0009	0.9988	0.9991	0.9997	0.9999	0.9997
	Frac	0.3784	0.5676	0.7297	0.6216	0.5405	0.6216
AIC	Mean	1.0507	0.9895	0.9872	0.9915	0.9930	0.9846
	Median	1.0049	0.9904	0.9941	0.9991	0.9995	1.0004
	Frac	0.4865	0.6757	0.6486	0.6486	0.5135	0.4865
$p = 4$	Mean	1.0019	0.9944	0.9902	0.9917	0.9927	0.9918
	Median	0.9926	0.9970	0.9983	0.9986	0.9991	0.9995
	Frac	0.6216	0.5676	0.6486	0.6486	0.5676	0.5676
(D) *Interest rates and asset prices (33 series)*							
BIC	Mean	1.1137	1.0222	1.0063	1.0044	1.0066	1.0107
	Median	1.0161	0.9988	1.0001	1.0002	1.0005	1.0001
	Frac	0.3030	0.6061	0.4848	0.4545	0.3030	0.4545
AIC	Mean	1.2465	1.2001	1.5315	1.0660	1.0620	0.9740
	Median	1.0486	1.0332	1.1011	1.0183	1.0024	0.9995
	Frac	0.2727	0.3939	0.3636	0.3636	0.3636	0.6061
$p = 4$	Mean	1.1869	1.1663	1.1903	1.1310	1.0899	1.0400
	Median	1.0982	1.0146	1.0025	1.0025	1.0008	1.0016
	Frac	0.1515	0.2424	0.3939	0.3030	0.4242	0.3333
(E) *Nominal prices, wages and money (35 series)*							
BIC	Mean	1.0067	0.9825	0.9928	0.9912	0.9903	0.9870
	Median	1.0066	0.9927	0.9949	0.9980	0.9931	0.9987
	Frac	0.4000	0.7143	0.6857	0.5714	0.6000	0.6857
AIC	Mean	1.0284	1.0065	0.9970	0.9919	0.9933	0.9856
	Median	1.0304	0.9996	0.9955	0.9939	0.9971	0.9961
	Frac	0.2571	0.5143	0.6571	0.6571	0.5429	0.5714
$p = 4$	Mean	1.0053	0.9939	1.0019	1.0005	0.9999	0.9955
	Median	0.9986	0.9964	1.0062	1.0010	0.9998	0.9976
	Frac	0.5429	0.5714	0.4000	0.4571	0.5143	0.7143

Notes: The five categories (see Section 3) are the same as in Marcellino *et al.* (2006). In the table, Mean and Median are the mean and median (0.50 percentile) of the distribution of the relative MSFE statistics. See the notes to Table 2.

Table 5: Relative MSFEs between the noncausal AR(r,s) and the causal AR(p) model, excluding interest rates and asset prices.

Model Selection		Forecast horizon					
		1	3	6	9	12	24
BIC	Mean	1.0079	0.9909	0.9926	0.9936	0.9933	0.9927
	Median	1.0012	0.9977	0.9984	0.9993	0.9995	0.9997
	Frac	0.4361	0.6165	0.6917	0.6090	0.5940	0.5564
AIC	Mean	1.0489	0.9976	0.9913	0.9921	0.9911	0.9851
	Median	1.0084	0.9961	0.9959	0.9979	0.9995	0.9992
	Frac	0.3985	0.5865	0.6617	0.6090	0.5564	0.5564
$p = 4$	Mean	1.0195	0.9896	0.9923	0.9946	0.9958	0.9975
	Median	1.0005	0.9961	0.9984	0.9987	0.9998	0.9995
	Frac	0.4962	0.6015	0.5940	0.5865	0.5338	0.5714

Notes: The variables included in category D (interest rates and asset prices) are excluded from this table. Thus, the number of variables is 133. See the notes to Table 2.

Table 6: Relative MSFEs of the noncausal variables when purely causal variables are excluded.

Model selection		Forecast horizon					
		1	3	6	9	12	24
All series							
BIC	Mean	1.0434	1.0041	0.9946	0.9924	0.9928	0.9961
	Median	1.0010	0.9977	0.9981	0.9991	0.9998	0.9998
	Frac	0.4557	0.6076	0.6582	0.5949	0.5190	0.5443
AIC	Mean	1.0879	1.0679	1.1993	1.0126	1.0101	0.9734
	Median	1.0166	0.9967	0.9972	0.9981	0.9995	0.9974
	Frac	0.3797	0.6076	0.6076	0.6076	0.5570	0.6962
$p = 4$	Mean	1.0636	1.0617	1.0678	1.0424	1.0274	1.0109
	Median	1.0069	0.9971	0.9949	0.9968	0.9986	0.9998
	Frac	0.4430	0.5316	0.6076	0.5823	0.5570	0.5316
Excluding interest rates and asset prices (category D)							
BIC	Mean	1.0049	0.9885	0.9914	0.9911	0.9891	0.9893
	Median	1.0009	0.9967	0.9959	0.9989	0.9989	0.9997
	Frac	0.4655	0.6552	0.6897	0.6379	0.5862	0.5517
AIC	Mean	1.0099	0.9869	0.9857	0.9864	0.9856	0.9769
	Median	1.0149	0.9929	0.9929	0.9947	0.9965	0.9972
	Frac	0.3793	0.7414	0.7241	0.6897	0.6552	0.6724
$p = 4$	Mean	0.9935	0.9889	0.9877	0.9873	0.9889	0.9922
	Median	0.9986	0.9932	0.9934	0.9944	0.9986	0.9989
	Frac	0.5517	0.6552	0.6897	0.6897	0.6034	0.6034
(A) *Income, output, sales, capacity utilization*							
BIC	Mean	1.0010	0.9929	0.9864	0.9862	0.9860	0.9903
	Median	0.9998	1.0000	0.9999	0.9996	0.9993	1.0006
	Frac	0.5455	0.4545	0.6364	0.5455	0.6364	0.4545
AIC	Mean	1.0208	0.9849	0.9821	0.9857	0.9801	0.9745
	Median	1.0178	0.9975	0.9951	0.9982	0.9922	0.9974
	Frac	0.2727	0.7273	1.0000	0.5455	0.8182	0.8182
$p = 4$	Mean	1.0151	0.9941	0.9858	0.9855	0.9868	0.9917
	Median	1.0115	0.9971	0.9967	0.9979	1.0005	1.0006
	Frac	0.1818	0.5455	0.9091	0.6364	0.4545	0.3636
(B) *Employment and unemployment*							
BIC	Mean	1.0328	0.9849	0.9968	0.9954	0.9923	0.9913
	Median	1.0289	0.9876	0.9945	0.9985	0.9989	1.0001
	Frac	0.4000	0.7000	0.6000	0.7000	0.7000	0.5000
AIC	Mean	0.9814	0.9695	0.9803	0.9819	0.9748	0.9841
	Median	1.0058	0.9750	0.9896	0.9967	0.9895	0.9964
	Frac	0.5000	0.9000	0.7000	0.8000	0.9000	0.6000
$p = 4$	Mean	0.9653	0.9828	0.9752	0.9739	0.9761	0.9836
	Median	0.9945	0.9846	0.9649	0.9820	0.9934	0.9989
	Frac	0.5000	0.7000	0.7000	0.8000	0.9000	0.6000
(C) *Construction, inventories and orders*							
BIC	Mean	0.9938	0.9914	0.9943	1.0014	0.9935	0.9920
	Median	0.9914	0.9981	0.9978	0.9998	1.0002	1.0006
	Frac	0.5333	0.7333	0.6667	0.6000	0.4667	0.4000
AIC	Mean	0.9904	0.9862	0.9857	0.9951	0.9907	0.9710
	Median	0.9955	0.9816	0.9892	0.9980	0.9993	0.9987
	Frac	0.6000	0.8667	0.6000	0.6667	0.5333	0.6667
$p = 4$	Mean	0.9880	0.9857	0.9839	0.9893	0.9874	0.9899
	Median	0.9892	0.9928	0.9832	0.9891	0.9975	1.0001
	Frac	0.8000	0.7333	0.8000	0.8000	0.5333	0.4667
(D) *Interest rates and asset prices*							
BIC	Mean	1.1482	1.0427	1.0031	0.9972	1.0029	1.0129

Table 6 (continued)

		Forecast horizon					
		1	3	6	9	12	24
	Median	1.0209	1.0030	0.9991	1.0004	1.0014	0.9997
	Frac	0.4286	0.4762	0.5714	0.4762	0.3333	0.5238
AIC	Mean	1.2951	1.2721	1.7463	1.0759	1.0703	0.9578
	Median	1.0486	1.1907	1.1333	1.0330	1.0124	0.9933
	Frac	0.3810	0.2381	0.2857	0.3810	0.2857	0.7619
$p = 4$	Mean	1.2459	1.2494	1.2798	1.1917	1.1307	1.0591
	Median	1.2243	1.0712	1.0035	1.0045	1.0005	1.0023
	Frac	0.1429	0.1905	0.3810	0.2857	0.4286	0.3333
(E) *Nominal prices, wages and money*							
BIC	Mean	1.0018	0.9861	0.9894	0.9847	0.9862	0.9861
	Median	1.0041	0.9937	0.9922	0.9865	0.9916	0.9957
	Frac	0.4091	0.6818	0.7727	0.6818	0.5909	0.7273
AIC	Mean	1.0306	0.9964	0.9899	0.9830	0.9897	0.9789
	Median	1.0283	0.9965	0.9929	0.9873	0.9973	0.9916
	Frac	0.2273	0.5909	0.6818	0.7273	0.5455	0.6364
$p = 4$	Mean	0.9994	0.9913	0.9968	0.9928	0.9967	0.9979
	Median	0.9986	0.9919	0.9999	0.9969	0.9996	0.9964
	Frac	0.5909	0.6364	0.5000	0.5909	0.5909	0.8182

Notes: The mean and median values are calculated based on the variables which are deemed to have a noncausal component ($s \geq 1$) in the final estimated AR(r, s) model when constructing the last forecasts. See the notes to Table 2.

Table 7: Relative MSFEs in quarterly time series.

Variable		Forecast horizon (quarters)							
		1	2	3	4	5	6	7	8
gdpdef	BIC	0.9603	0.8076	0.7963	0.7625	0.7992	0.8011	0.8550	0.8733
	AIC	0.9792	0.8714	0.8221	0.8222	0.8243	0.8096	0.8404	0.9171
	$p = 4$	1.0108	0.8868	0.8098	0.7866	0.8236	0.8292	0.8593	0.9035
gdp	BIC	0.9815	0.9826	0.9868	0.9922	0.9971	0.9958	0.9988	1.0014
	AIC	1.0054	1.0259	1.0617	1.0600	1.0500	0.9983	0.9806	0.9897
	$p = 4$	1.0172	0.9719	0.9903	0.9842	0.9841	0.9777	0.9944	1.0029
gpdi	BIC	1.0136	1.0296	1.0604	1.0141	0.9914	0.9759	0.9981	0.9977
	AIC	1.0033	0.9707	0.9833	0.9611	0.9405	0.9825	1.0230	1.0411
	$p = 4$	1.0093	1.0230	1.0699	1.0371	0.9908	0.9766	1.0010	1.0036
cpiall	BIC	1.0574	0.9218	0.9271	0.9339	0.8956	0.9144	0.9748	0.9510
	AIC	1.1016	0.9422	0.9609	0.9660	0.8936	0.9246	0.9378	0.9573
	$p = 4$	1.2494	0.9987	0.9517	0.9816	0.9274	0.9050	0.9777	0.9739
gpsave	BIC	0.9257	0.9298	0.9702	0.9172	0.9449	0.9966	1.0020	0.9989
	AIC	0.9558	0.9542	0.9134	0.9389	0.9370	0.9961	0.9844	0.9909
	$p = 4$	0.9909	0.9957	1.0409	0.9702	0.9723	0.9913	0.9890	0.9846
dpi	BIC	1.0772	1.0055	1.0510	1.0328	1.0206	1.0254	1.0353	1.0325
	AIC	1.0135	1.0311	1.0665	1.0507	1.0292	1.0182	1.0161	1.0366
	$p = 4$	0.9830	0.9119	0.9365	0.9147	0.9700	0.9605	0.9732	0.9804
pcec	BIC	0.9674	1.0014	0.9792	0.9875	0.9788	0.9904	0.9884	0.9952
	AIC	1.0654	0.9742	1.0003	1.0005	1.0358	0.9642	0.9956	0.9477
	$p = 4$	0.9095	0.9335	0.9293	0.9476	0.9282	0.9262	0.9582	0.9548
rinvbf	BIC	0.9856	0.9667	0.9878	0.9962	1.0024	1.0018	1.0009	1.0003
	AIC	0.9587	0.9404	0.9537	0.9643	0.9929	0.9939	0.9907	0.9990
	$p = 4$	0.9825	0.9646	0.9875	0.9856	0.9941	0.9967	0.9993	1.0000
cum	BIC	1.0319	0.9934	0.9730	0.9788	0.9776	0.9905	0.9935	0.9923
	AIC	0.9908	0.9142	0.9624	0.9393	0.9624	0.9607	1.0050	0.9887
	$p = 4$	1.0284	0.9882	0.9805	0.9803	0.9789	1.0033	1.0021	0.9969
ipm	BIC	1.0011	0.9702	0.9588	0.9579	0.9701	1.0074	1.0226	1.0272
	AIC	0.9801	0.9300	0.9121	0.9238	0.9826	0.9960	0.9919	0.9998
	$p = 4$	0.9993	0.9795	0.9802	0.9636	0.9622	0.9983	1.0136	1.0139
ipt	BIC	0.9989	0.9612	0.9571	0.9665	0.9762	1.0063	1.0072	1.0093
	AIC	1.1280	1.0933	1.1385	1.0245	0.9690	1.0055	0.9418	0.9182
	$p = 4$	0.9513	0.9694	0.9948	0.9789	1.0065	1.0228	1.0155	1.0136
employ	BIC	0.9182	0.8162	0.8477	0.9034	0.9314	1.0092	1.0337	1.0490
	AIC	0.9200	0.8580	0.8396	0.8536	0.9284	0.9748	0.9553	0.9781
	$p = 4$	0.9513	0.8753	0.9152	0.9614	1.0023	1.0312	1.0452	1.0544
ruc	BIC	0.9800	0.9925	1.0061	0.9978	0.9995	0.9906	0.9919	0.9948
	AIC	0.9706	0.8579	0.8619	0.8729	0.8920	0.8961	0.9260	0.9680
	$p = 4$	1.0845	1.0645	1.0079	0.9831	0.9721	0.9661	0.9901	0.9982
hstarts	BIC	0.9930	0.9939	0.9997	1.0014	0.9987	1.0004	1.0004	1.0007
	AIC	0.9896	0.9715	0.9901	0.9871	0.9899	1.0100	1.0054	0.9983
	$p = 4$	0.9886	0.9978	0.9956	0.9873	1.0074	0.9988	1.0126	1.0014
cut	BIC	1.0044	0.9756	0.9787	0.9868	0.9993	1.0044	1.0058	1.0030
	AIC	1.0141	0.9910	1.0110	1.0126	1.0168	1.0193	1.0148	1.0109
	$p = 4$	1.0347	1.0264	1.0454	1.0582	1.0645	1.0484	1.0418	1.0112
h	BIC	0.9478	0.9234	0.9661	0.9778	0.9862	0.9988	1.0054	1.0047
	AIC	0.9484	0.9375	0.9710	0.9766	0.9871	0.9952	0.9949	0.9981
	$p = 4$	0.9179	0.8941	0.9155	0.9264	0.9514	0.9638	0.9828	0.9973
bopxgs	BIC	0.8915	1.0117	1.0067	1.0263	1.0130	1.0050	1.0045	1.0051
	AIC	0.9190	1.0345	1.0456	1.0322	1.0217	1.0096	0.9997	0.9961
	$p = 4$	0.8669	0.9797	1.0193	1.0202	1.0164	1.0139	1.0032	1.0009

Table 7 (continued)

		Forecast horizon (quarters)							
		1	2	3	4	5	6	7	8
rimp	BIC	0.9451	0.9832	0.9899	0.9775	0.9971	1.0010	1.0007	1.0034
	AIC	0.9341	0.9783	0.9901	0.9685	0.9966	1.0023	1.0039	1.0023
	$p = 4$	0.9384	0.9878	0.9791	0.9648	0.9984	1.0078	1.0010	1.0029
Frac	BIC	0.6667	0.7778	0.7778	0.7778	0.8333	0.5000	0.3889	0.3889
	AIC	0.6111	0.7778	0.6667	0.6667	0.7222	0.6667	0.6667	0.7778
	$p = 4$	0.6111	0.8333	0.7222	0.8333	0.7222	0.6667	0.5000	0.4444

Notes: The relative MSFEs between the noncausal AR model and the causal AR model in the quarterly U.S. time series listed in Table 1. See the notes to Table 2.