Essays on the Macroeconomics of Monetary Union

by

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Foreword

The macroeconomics of EMU is an important and currently relatively intensively studied research topic. Anssi Rantala's doctoral dissertation deals with two sets of issues by adopting a game-theoretic approach to macroeconomic policy making: First, what is the impact of monetary unification on equilibrium unemployment in the presence of strategic wage setting, and second, how does the establishment of a monetary union affect the nominal wage flexibility and thereby unemployment fluctuations around the equilibrium unemployment rate? In the first essay the author shows that a switch from a flexible exchange rate regime to a monetary union will affect macroeconomic performance by falling unemployment if the degree of central bank conservatism in terms of the relative weight of inflation in central bank preferences is sufficiently high. The second essay studies the political economy of labour market flexibility in a monetary union by adopting a simple political economy approach, in which national governments decide on nominal wage flexibility, but face political costs, which increase with the chosen flexibility. Labour market policy coordination tends to increase the chosen level of nominal flexibility under discretionary monetary policy. The third essay provides a theoretical justification for establishing uniqueness of equilibrium in the model applied in the second essay by using the so-called adaptive learning approach of modelling private sector expectations. To conclude, this thesis contributes to the emerging literature about the macroeconomics of EMU in several interesting ways.

This study is part of the research agenda carried out by the Research Unit on Economic Structures and Growth (RUESG). The aim of RUESG is to conduct theoretical and empirical research into important issues in the dynamics of the macro economy, game theory and economic organizations, the financial system as well as problems of labour markets, natural resources, taxation and econometrics.

RUESG was established in the beginning of 1995 and it is now one of the National Centres of Excellence in research. RUESG is financed jointly by the Academy of Finland, the University of Helsinki, the Yrjö Jahnsson Foundation, Bank of Finland and the Nokia Group. This support is gratefully acknowledged.

Helsinki, December 16, 2003

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During the project I had the great opportunity to visit the Research Department of the Bank of Finland in 2000 and again in 2002-03. These two visits proved to be very important to the processing of research ideas, and I actually wrote the greater part of my thesis at the Bank. I would like to thank Head of Department Doctor Juha Tarkka for valuable suggestions concerning the essays and the Research Department for its hospitality and excellent working conditions. In particular, I am indebted to Docent Jouko Vilmunen for his enthusiastic supervision and encouragement during the thesis project.

I was fortunate to be able to spend the academic year 2001-02 at the Institute for International Economic Studies at the University of Stockholm. I thank the Institute for its hospitality and support. A special note of thanks goes to Professor Lars Calmfors for many stimulating discussions about the EMU and labor market issues during my stay.

Professor Mikko Puhakka and Docent Jouko Vilmunen acted as the official examiners of my thesis. They offered numerous constructive comments and suggestions to improve the contents of the manuscript, of which I wish to express my gratitude.

I made the last revisions to the thesis while working at the Pellervo Economic Research Institute. I wish to thank my superiors, Doctors Raija Volk and Vesa Vihriälä for their flexibility and support at the final stages of the project.

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Anssi Rantala

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Chapter 1

Introduction

1 Background

Monetary unification in Europe culminated in the launch of Euro banknotes and coins in January 2002. A common currency among countries has, however, more wide-ranging effects on the economies than the obvious benefits it brings along to travellers and business firms by abolishing exchange commissions and exchange rate uncertainty. The arguments related to the costs of a common currency are typically based on the theory of optimum currency areas put forward by Mundell (1961) about forty years ago. The optimum currency area approach focuses on the impact of asymmetric shocks and the necessary adjustment mechanisms to counteract them. When countries have independent monetary and exchange rate policies, the nominal exchange rate can, at least in principle, be used to restore the labor market equilibrium after an asymmetric shock. If the countries have, however, adopted a common currency, there are basically two alternative mechanisms, which keep unemployment from rising in a country hit by an asymmetric negative shock; either labor mobility between countries has to be considerable, or wages need to be flexible. Needless to say that Europe seems to be a "Mundellian nightmare" with low mobility of labor and inflexible wage structures.

The critics of the theory of optimum currency areas have pointed out that the ability of exchange rate changes to absorb asymmetric shocks may be much weaker than the theory suggests. Moreover, pure exchange rate shocks, that is, exchange rate movements not driven by fundamentals, or even policy-induced exchange rate shocks are often considered to be a source of macroeconomic disturbances. Giving up monetary independence can then be welfare improving, provided that it decreases undesirable exchange rate volatility.

The most often mentioned economic benefits of a common currency are related to increased international integration of financial and product markets. In particular, a

common currency is seen as a stimulus to international trade, which through more efficient resource allocation will increase per capita output and ultimately welfare. De Grauwe (2000) offers a comprehensive treatment of both microeconomic and macroeconomic aspects of monetary integration.

The standard cost-benefit analysis of the impact of monetary unification lacks an important aspect. The formation of a monetary union is a fundamental regime change, which could transform established behavioral relationships in the economy. Focusing on labor markets, one can argue that the monetary regime will affect the constraints faced by the labor unions and policy makers, and thereby the unemployment process may change as well. Calmfors (2001b) provides a thorough survey and an assessment of the literature on the relationship between monetary unification and wage bargaining institutions.

It is instructive to divide the impact of monetary unification on unemployment outcomes through wage setting into two parts. The first part deals with the impact on the equilibrium rate of unemployment. According to a conventional wisdom, money is just a veil in the long run, and hence monetary regime will not affect equilibrium outcomes in the real economy. However, it has been shown by e.g. Cukierman and Lippi (1999) and Soskice and Iversen (2000) that assuming non-atomistic strategically behaving labor unions will break the long-run neutrality so that the monetary policy strategy will affect the real economy.[1] The second part of the question is related to nominal wage flexibility and cyclical sensitivity of unemployment around the equilibrium rate. The incentives for reforms which increase nominal wage flexibility may change with monetary unification, since the agents of the economy realize that the nominal exchange rate can't anymore be used to counteract the impact of asymmetric shocks. This has been studied by Sibert and Sutherland (2000) and Calmfors (2001a).

A careful empirical assessment of the unemployment consequences of the European Economic and Monetary Union, the EMU in short, will have to wait for some time. In order to attain any reliable estimates of the impact, one should use long time series containing several business cycles both before and after the establishment of the EMU. Hence, while waiting for the data to accumulate, at this point theoretical work still provides the only feasible way to conduct research on this topic.

This dissertation consists of three theoretical essays that contribute to the literature on the macroeconomics of monetary union. The essays can be read independently of each other. Since the topics of the essays are related, some repetition is unavoidable. The first essay (Chapter 2) deals with the impact of monetary unification on equilibrium

[1] It is noteworthy that money will still be neutral in a sense that, other things equal, including the central bank's behavior, changes in the nominal money stock will pass on to prices without affecting any real variables. See Soskice and Iversen (2000) for an illustration of this point.

unemployment. The second and the third essays (Chapters 3 and 4, respectively) are related to nominal wage flexibility and unemployment fluctuations around the equilibrium rate in a monetary union. The last two essays complement each other so that the third essay provides a novel theoretical justification for establishing uniqueness of equilibrium in the model applied in the second essay.

All three essays adopt a game-theoretic approach to macroeconomic policy making. The central bank (or banks, as in the first essay) and labor unions are the players of the game in each essay. In the second essay (Chapter 3) there are also national governments involved. In the first and the second essays the main results are driven by strategic interactions either between non-atomistic labor unions and the central bank (or banks), or between national governments and the central bank. The third essay abstracts from strategic behavior and focuses on the implications of adaptive learning on the stability of rational expectations equilibria.

The modelling of central bank preferences adopted in this dissertation deviates from the traditional practice in the literature on monetary policy games in one important respect. The central bank is assumed to target the equilibrium rate of unemployment instead of having an "over-ambitious" unemployment target which produces the celebrated inflation bias result (see Barro and Gordon (1983)).[2] The inflation bias model can be seen as a rather good description of the incentives facing politicians when they are in charge of monetary policy. If the natural rate is too high from the society's point of view due to e.g. imperfections in product and labor markets or distortionary taxation, governments may be tempted to take the soft option and, instead of implementing politically costly structural measures, try to use monetary policy to bring down unemployment. However, when monetary policy is delegated to an independent central bank, there seems no reason whatsoever for the central bank to target anything other than the natural rate (for discussion on this point, see Blinder (1998)).

This monetary policy framework, where the central bank is concerned about keeping inflation, and inflation expectations, low and stable and at the same time striving to limit cyclical swings in resource utilization, has been referred to as constrained or enlightened discretion (see e.g. Bernanke and Mishkin (1997) and Blinder (1998)). It can be said that constrained discretion has become the standard approach to monetary policy in the industrialized countries. In particular, the increasingly popular policy framework known as inflation targeting can also be interpreted as constrained discretion rather than a strict rule.

[2] To be precise, the model utilized in the second and the third essays makes a distinction between the "long-run" and the "short-run" natural rate of unemployment. In the baseline model the central bank targets the long-run natural rate. An extension considers also the short-run natural rate as a target.

Below follows brief summaries of the three essays in the order of their appearance.

2 Strategic Wage Setting and Equilibrium Unemployment in a Monetary Union

The first essay (Chapter 2) investigates the impact of the formation of a monetary union on equilibrium unemployment in the presence of strategic wage setting. According to the natural rate hypothesis developed by Phelps (1967) and Friedman (1968), monetary regime is neutral in the long run, and the equilibrium rate of unemployment is determined only by real factors of the economy. The voluminous literature on monetary policy games initiated by Kydland and Prescott (1977) and Barro and Gordon (1983) combines the natural rate hypothesis with rational expectations and analyzes the optimal behavior of the policy makers with well-known results. Only inflation depends systematically on the conduct of monetary policy, and due to shocks in the economy unemployment fluctuates around an exogenous natural rate. Hence, equilibrium unemployment should not be affected by the establishment of a monetary union, which is just a change of monetary regime.

Recent research by Grüner and Hefeker (1999) and Cukierman and Lippi (2001) suggests, however, that in the presence of non-atomistic labor unions a non-neutrality result of monetary unification emerges due to strategic interaction between labor unions and monetary authorities. In particular, these contributions show that the establishment of a monetary union will increase both unemployment and inflation, provided that national labor unions are "inflation averse". An inflation averse labor union refers to a one which is, in addition to having traditional real wage and unemployment targets, also concerned about inflation *per se*. Inflation aversion moderates wage claims and lowers unemployment under national monetary policy, because unions are willing to compromise over the real wage target in order to reduce inflation. In a monetary union this moderating effect will be smaller and unemployment will rise. This argument, however, hinges on two somewhat questionable assumptions; the existence of both inflation aversion and inflation bias are crucial for the outcome. If either one is missing, monetary unification will have no effect on unemployment.

In this essay, it is shown that in a model with open-economy spillovers, a switch from a flexible exchange rate regime to a monetary union will affect macroeconomic performance, even though national labor unions are not averse to inflation, and there is no inflation bias. In particular, it is shown that unemployment will fall, provided that the degree of central bank conservatism is sufficiently high, whereas with low degrees of conservatism

unemployment increases. Conservatism refers here to the relative weight of inflation in the central bank preferences.

The mechanism behind the non-neutrality result is related to international spillovers created by imperfect substitutability between goods produced in different countries. Due to imperfect substitutability, the labor union's wage claim will affect the real exchange rate, which in turn influences consumer prices and real consumer wages. The strength of the union's ability to affect the real exchange rate depends on the monetary regime, and under a flexible exchange rate also on the preferences of the monetary authorities toward inflation and unemployment. Monetary regime will affect the labor union's perceived real wage–unemployment tradeoff through the real exchange rate channel. In other words, the real consumer wage elasticity of labor demand is endogenous in the model. In this respect the model is similar to Holden's (2003) model with traded and non-traded sectors, where the impact of monetary regime on wage setting is transmitted via labor demand elasticity. Interestingly enough, relative to the flexible exchange rate regime, the inflexibility of the common monetary policy in responding to a country specific wage increase is good news in terms of unemployment if the central bank is conservative, whereas it is bad news if the central bank is fairly liberal.

This essay offers insights for desirable policy choices in a monetary union where labor unions have a strategic position. The analysis suggests that when a monetary union is established, it is possible to attain a lower rate of equilibrium unemployment by delegating monetary policy to a sufficiently conservative central bank. It can be argued that in Europe the creation of the EMU has increased the effective degree of central bank conservatism in most member countries. Hence, one could speculate that the EMU may well have positive effects on equilibrium unemployment in these countries.

3 Labor Market Flexibility and Policy Coordination in a Monetary Union

The second essay (Chapter 3) investigates the political economy of labor market flexibility in a monetary union. Labor market flexibility is often suggested as a remedy for European unemployment problems. Recently the EU Council (2002) urged the member countries on to continue implementing further structural reforms to improve the functioning of the European labor market, and to examine the possibility of introducing more flexible labor contract types into their national law. There seems to be a consensus of opinion that flexible labor markets are especially important in a monetary union, such as the EMU, where some shocks to the economy are very likely asymmetric so that only some regions

or countries of the monetary union area are affected. Labor mobility in the EMU is relatively low, partly due to cultural differences and language barriers, so one can't count much on mobility as a shock absorber mechanism (see OECD (1999)). National fiscal policy could in principle be used as a macroeconomic stabilization tool, but acknowledging that due to political biases fiscal policy tends to err on the lax side has led to a general perception that discretionary fiscal policy should be restricted by rules limiting the size of government deficits.

What are the factors which determine the flexibility of the national labor markets in a monetary union, and what can policy makers do in order to promote flexibility? This essay attempts to give at least partial answers to these questions by adopting a simple political economy approach, in which national governments decide on nominal wage flexibility, but face political costs which are increasing in the chosen level of flexibility. This setup can be seen as a simplification of a complicated process where labor market institutions must be designed and legislation must be enacted in a parliamentary process. In particular, this essay studies the role of international labor market policy coordination in the determination of flexibility. The motive for coordination comes from the fact that governments can attain a strategic position vis-à-vis the central bank by coordinating their actions. In the European Union, there has been efforts to increase coordination in labor market issues. Policy coordination intensified with the 1997 Amsterdam Treaty, which declares that member states shall treat employment "as a matter of common concern, and shall coordinate their action".

It is well established empirically that the unemployment rate is highly persistent (see e.g. Layard, Nickell and Jackman (1991, Ch. 9)). Adding unemployment persistence to a standard Barro and Gordon (1983) natural rate model was first made by Lockwood and Philippopoulos (1994), and further results were derived by Svensson (1997) and Lockwood, Miller and Zhang (1998). They pointed out that in the presence of unemployment persistence discretionary monetary policy leads to a new problem, that is, a stabilization bias emerges. Inflation variability is too high and unemployment variability is too low from the society's point of view. This inefficient stabilization performance will turn out to play an important role in the analysis of flexibility.

The main result of this essay shows that labor market policy coordination between member country governments tends to increase the chosen level of nominal wage flexibility under discretionary monetary policy. The intuition behind this result is simple. The flexibility choice under coordination interacts with the stabilization properties of the economy through the common monetary policy. In particular, coordination internalizes the benefit of reduced stabilization bias of monetary policy, which is brought about by

increased labor market flexibility. Since the impact of coordination on the chosen level of flexibility is related to stabilization properties, it follows that coordination is redundant provided that stabilization of shocks optimal. Therefore, if the central bank has access to a commitment technology, or there is no real persistence in the economy, coordination outcome coincides with the one obtained under national decision making.

As an extension to the basic model the essay considers an alternative unemployment target for the central bank and its implications for labor market flexibility and coordination. As in Svensson (1997) and Røisland (2001), the central bank is assumed to target the short-run natural rate, which results in stabilization bias in the opposite direction than in the baseline model. In this case international coordination of labor market policies is shown to result in less flexible markets than national decision making, since lower flexibility calls for a stronger monetary policy reaction to production shocks, thereby pushing stabilization closer to the optimum. Another extension qualifies some results of Calmfors (2001a), where it was shown that labor market flexibility tends to be higher when a country is inside a monetary union than when it has an independent monetary policy. It turns out that this is not necessarily the case when unemployment persists, since like under policy coordination in a monetary union, under an independent monetary policy the government is in a strategic position and has an incentive to choose more flexible markets in order to improve on stabilization properties of the economy.

4 Adaptive Learning and Multiple Equilibria in a Natural Rate Model with Unemployment Persistence

The formation of the EMU in Europe was a major regime shift in economic policy. Monetary policies were quite different in the member countries before the EMU. The one-size-must-fit-all monetary policy brought about by the EMU is a significant change in the economic environment, and it seems quite plausible to assume that the private sector agents don't immediately know all new parameter values, for example the preferences of the common central bank, which affect the economy. Therefore, rational expectations is a particularly strong assumption in this kind of situation, and some sort of bounded rationality may provide a much more realistic starting point in thinking of the behavior of the agents

The third essay (Chapter 4) demonstrates that adaptive learning approach of modelling private sector expectations can be used as an equilibrium selection mechanism in

a natural rate model augmented with unemployment persistence, which is applied in the second essay (Chapter 3). The starting point in the analysis of adaptive learning in macroeconomics is that some or all agents have imperfect knowledge about the structure of the economy. The agents rely on an econometric learning technology to form expectations and continuously update their estimates on the structure of the economy based on incoming data. Evans and Honkapohja (2001) offer a comprehensive treatment of learning in macroeconomics. Recent years have witnessed a surge of interest in the application of adaptive learning approach on the analysis of monetary policy (see Evans and Honkapohja (2003) for a recent survey of the literature).

Lockwood and Philippopoulos (1994) showed that when unemployment rate is persistent, there are two rational expectations equilibria in a natural rate monetary model. One is associated with low inflation and intuitive comparative statics properties, and the other with high inflation and counter-intuitive comparative statics properties. This essay shows that only one of the two rational expectations equilibria is stable under adaptive learning, and that it is always the low inflation equilibrium with intuitive comparative statics properties, which is learnable.

In the earlier contributions which use the natural rate model with real persistence two solutions to the multiple equilibria problem have been adopted. The first solution is to ignore the "bad" high inflation equilibrium by appealing to counter-intuitive comparative statics properties, and to the fact that the high inflation equilibrium appears only in the infinite horizon model. The second solution treats the two equilibria more equally and considers both cases as being the possible outcomes of the model. This essay proposes a third solution to this problem, namely the use of adaptive learning as a selection tool. Hence, this essay provides a more elegant justification in a theoretical sense for focusing on the low inflation equilibrium than the first approach, where the high inflation equilibrium was ignored mainly because of its unpleasant characteristics. In addition to being an independent contribution, this essay thereby complements the second essay (Chapter 3), where the high inflation equilibrium was ignored in the analysis.

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Chapter 2

Strategic Wage Setting and Equilibrium Unemployment in a Monetary Union

Abstract

This essay investigates the impact of the formation of a monetary union on equilibrium unemployment in a two-country monetary model with strategic wage setting and open-economy spillovers. It is shown that the monetary regime affects the tradeoff between real consumer wages and unemployment faced by the labor unions. Consequently, the equilibrium rate of unemployment is endogenous and depends on the monetary regime. In particular, a switch from a flexible exchange rate regime to a monetary union reduces unemployment, provided that the degree of central bank conservatism is sufficiently high, whereas with low degrees of conservatism unemployment rises. This non-neutrality result of a monetary union does not depend on the existence of an inflation bias, that is, the outcome is unchanged even though the central banks target the equilibrium rate of unemployment.

1 Introduction

This essay investigates the impact of the formation of a monetary union on equilibrium unemployment. The outcome of the analysis seems to be obvious for a well-trained economist. According to the natural rate hypothesis developed by Phelps (1967) and Friedman (1968), monetary policy is neutral in the long run and the equilibrium rate of unemployment is determined only by real factors of the economy. A large literature on monetary policy games initiated by Kydland and Prescott (1977) and Barro and Gordon (1983) combines the natural rate hypothesis with rational expectations and analyzes the optimal behavior of the policy makers with well-known results. Only inflation depends systematically on the conduct of monetary policy, and due to shocks in the economy unemployment fluctuates around an exogenous equilibrium (or natural) rate. Hence, equilibrium unemployment should not be affected by the establishment of a monetary union, which is just a change of monetary regime. Recent research suggests, however, that in the presence of non-atomistic labor unions a non-neutrality result of monetary

policy emerges due to strategic interaction, and the conservatism of the central bank will have a systematic effect on equilibrium unemployment.[1] This has been shown e.g. by Cukierman and Lippi (1999), Lippi (2002), Soskice and Iversen (2000) and Coricelli, Cukierman and Dalmazzo (2000, 2001).

Recent papers by Grüner and Hefeker (1999) and Cukierman and Lippi (2001) analyze the effects of a monetary union on economic performance in a model with strategic wage setting. They show that the establishment of a monetary union will increase both unemployment and inflation, provided that national labor unions are "inflation averse", i.e. in addition to traditional real wage and unemployment targets labor unions are also concerned about inflation *per se*. Inflation aversion moderates wage claims and lowers unemployment, because unions are willing to compromise over the real wage target in order to reduce inflation. In a monetary union each union perceives that its wage demand will have a smaller effect on inflation, which makes unions more aggressive in wage setting. Higher real wages lead to higher unemployment through reduced demand for labor. This argument, however, hinges on the assumed inflation aversion of the labor unions. Absent that, monetary union has no effect on unemployment and inflation.[2] It is noteworthy that the analysis rests on the assumed inflation bias problem of monetary policy. If the central bank targeted the equilibrium rate of unemployment, there would be no inflation bias and so labor unions' incentives for wage restraint because of excess inflation would be absent.

In this essay, it is shown that in a model with open-economy spillovers, a switch from a flexible exchange rate regime to a monetary union will affect macroeconomic performance, even though national labor unions are not averse to inflation and all structural parameters of the model remain unchanged. In particular, it is shown that unemployment will fall, provided that the degree of central bank conservatism is sufficiently high, whereas with low degrees of conservatism unemployment increases. Importantly, and unlike in the previous contributions, the real effects of a monetary union are present independent of the inflation bias problem.

In an earlier version of this essay (Rantala (2001)) it was shown that inflation will increase when a monetary union is established, provided that the central banks both before and after monetary unification have an over-ambitious unemployment target, which

[1] The term "conservatism" will be used to describe the relative weight of inflation deviations from the target in the central bank's objective function. Elsewhere in the literature "independence" is occasionally used interchangeably with conservatism (see e.g. Cukierman and Lippi (1999)). Berger, de Haan and Eijffinger (2001) discusses the difference between the two concepts.

[2] Cukierman and Lippi (2001) show that monetary union will have real effects even without inflation aversion, if there are more than one labor union in each country. But, an inflation bias is needed in order to generate these real effects.

results in inflation bias. These results are not reproduced here, since the purpose of the essay is to highlight the fact that inflation bias is unnecessary for the real effects of a monetary union to emerge in this model setup. Moreover, as monetary policy is in most industrialized countries delegated to an independent central bank, it is difficult to understand why the central bank would target anything other than the equilibrium rate of unemployment.[3,4]

The model framework is similar to a simple two-country monetary model with national labor unions studied in Jensen (1993, 1997). The model focuses on international spillovers via the real exchange rate. Each country is specialized in the production of a traded good, which are imperfect substitutes in consumption. Both goods are consumed in both countries, and their relative demand depends on the relative price between the two goods.

Imperfect substitutability between the two goods creates two policy spillovers. First, in a flexible exchange rate regime national central banks can affect the real exchange rate with their policies, and thus their actions will have repercussions in both countries. This policy spillover implies that monetary policies are interdependent. The central bank behavior is modelled as a non-cooperative simultaneous move game. Once the monetary union is established, the common central bank has only one policy instrument, the area-wide money supply. Therefore, it can't affect the real exchange rate between the two countries. Second, national labor unions choose their nominal wages before monetary policies are set, and their wage setting will also affect the real exchange rate.[5] Obviously, assuming that the labor unions are so large that their actions will affect macroeconomic variables is crucial, since this assumption creates a strategic role for the labor unions in the model. The strength of the labor unions' ability to appreciate the real exchange rate with their nominal wage claims will depend on the monetary regime as well as central bank conservatism. Due to the above mentioned spillovers, monetary regime will in general affect the real wage–unemployment tradeoff faced by the labor unions, i.e. the real consumer wage elasticity of labor demand is endogenous in the model. It follows that the equilibrium rate of unemployment is endogenous as well, and depends, among other things, on the chosen monetary regime. Holden (2003a) shows that monetary regime

[3] For discussion, see e.g. Blinder (1998).

[4] Rantala (2001) also analyzes an alternative starting position for a monetary union, namely an asymmetric credibly fixed exchange rate regime. An asymmetric fix can be thought as a simple and crude description of the Exchange Rate Mechanism (ERM) in Europe (see e.g. Lane (2000)). A similar exercise is conducted in Grüner and Hefeker (1999) and Cukierman and Lippi (2001) without explicitly modelling the exchange rates between the countries. However, realignments of the parity rates were actually quite common in the ERM, and so modelling the ERM as a credible commitment, like in all contributions above, is somewhat problematic.

[5] The real exchange rate motive for wage increases in models without monetary authorities is studied e.g. in Driffill and van der Ploeg (1993) and Rama (1994).

affects the real wage–employment tradeoff in a small open economy with tradable and non-tradable goods sectors.

Although the model specification is closely related to that of Grüner and Hefeker (1999), the mechanism behind the results is completely different. In their model, purchasing power parity is assumed to hold, and therefore economic performance under a flexible exchange rate is solely determined by domestic policy actions. In a monetary union each labor union thus becomes smaller in a sense that their wage claims will have a smaller effect on inflation. As mentioned above, this makes labor unions more aggressive in a monetary union provided that they are inflation averse. In this essay the "size effect" is absent. In contrast, due to the spillover effects, both domestic and foreign wages affect the economic performance in both countries even in a flexible exchange rate regime. The only change in the strategic environment of the labor unions is the replacement of the national central banks by a common central bank. It follows that policy targets as well as policy instruments of the monetary policy authorities will be different in a monetary union. It is thereby quite intuitive that the monetary policy reaction to a unilateral wage increase by one labor union will change as well. The strategic interaction between labor unions and the monetary authorities is thereby transformed and optimal wage setting behavior will be affected. Macroeconomic consequences of a monetary union are *a priori* ambiguous in the model, but as mentioned, they will depend on central bank conservatism.

Monetary union is an extreme form of monetary policy cooperation.[6] In the literature cooperation usually means that central banks set their policy instruments jointly so as to maximize (or minimize) a weighted average of their policy objectives. Rogoff (1985b) showed that monetary policy cooperation can be counterproductive in the presence of domestic credibility problems. In a flexible exchange rate regime monetary policies are constrained by fears of inflationary real exchange rate depreciation. In a cooperative policy regime this real exchange rate effect is internalized. Therefore monetary policies are more expansionary and inflation will be higher. Monetary union differs from a cooperative policy regime in one important respect. A common central bank has only one policy instrument available. This is a crucial difference in a setup with large wage setters. In a cooperative regime, a unilateral wage increase by one national labor union will induce an asymmetric response from the two central banks, as long as the immediate effects of the wage increase in the two economies are asymmetric. However, when a common central bank faces a unilateral wage increase in one member country, its policy will, by definition, be symmetric. Jensen (1993) analyzes monetary policy cooperation in a similar model

[6] For a comprehensive survey on international policy coordination, see Persson and Tabellini (1995).

framework as used in this essay, and shows that in a cooperative regime unemployment is always higher than in a flexible exchange rate regime.[7]

The focus of this essay is on the direct effect of a monetary union on equilibrium unemployment through changes in the strategic interaction between central banks and labor unions. There are at least three additional channels through which equilibrium unemployment might be affected.

First, monetary integration may enhance product market integration and thus also competition via lower transaction costs, the disappearance of the exchange rate risks and easier price comparisons between countries. This has been a common argument in policy debates concerning the effects of the Economic and Monetary Union (EMU) in Europe. Intensified competition in the product markets improves employment in two ways. Labor demand becomes more elastic with respect to wages, and firms' mark-ups are reduced. The effects of increased competition in the product markets in the presence of unionized labor markets has been analyzed by Coricelli, Cukierman and Dalmazzo (2000, 2001). They show that both unemployment and inflation are reduced when competition increases. However, as e.g. Burda (1999) and Calmfors (2001b) point out, it is not clear that increased product market integration will foster competition. On the contrary, integration may help multinational companies to penetrate the markets, and therefore competition can actually decrease.

Second, incentives for labor market reforms can be affected by the establishment of a monetary union.[8] This has been analyzed by Sibert and Sutherland (2000) and Calmfors (2001a). Here, reform refers to any structural measure in the labor markets which reduces the equilibrium rate of unemployment. The government can choose the level of reform before labor unions set their wages and before monetary policy is set. The reform lowers the equilibrium rate of unemployment, but it entails a political cost to the government. This cost can arise for various reasons, e.g. the reform reduces real wages or because workers value labor market institutions intrinsically. The main argument of these papers is intuitive. In a country which is not a member of a monetary union, choosing a low level of reform implies that unemployment is high. In addition, in the presence of an inflation bias, inflation will be high as well. If the government chooses a low level of reform in a monetary union, the first effect is obviously the same. However, the second effect is reduced. High unemployment in one member country has only a small effect on the area-wide average level of unemployment. The common central bank cares about the average level of unemployment, and thus the marginal effect of high unemployment

[7] Monetary policy cooperation with inflation averse labor unions is treated in Jensen (1997).

[8] Saint-Paul (2000) offers a comprehensive analysis and discussion on the political economy of labor market reforms.

in one member country on the inflation bias will be smaller in a monetary union. The government's incentives for labor market reform are thereby reduced in a monetary union and the chosen level of reform will be lower.

Third, labor unions' incentives for coordinated wage bargaining may be affected. The impact of wage bargaining institutions on macroeconomic outcomes is a subject of a voluminous literature.[9] Both theoretical and empirical studies generally point to a direction that a high level of centralization of wage bargaining is associated with lower unemployment than intermediate levels of centralization. The structure of wage bargaining is, however, usually exogenous to the model. Holden (2003b) shows that monetary regime may affect the labor unions' incentives to coordinate their wage setting and thereby affect equilibrium unemployment as well. Provided that a strict central bank disciplines large labor unions so that their wage claims are reduced, a membership in a monetary union may induce a more aggressive wage policy, because labor unions realize that monetary policy will not react as strongly as before, since each union is smaller relative to the central bank. Higher real wages in turn lead to higher unemployment via reduced demand for labor. As the outcome of an uncoordinated wage bargaining becomes worse in a monetary union, the incentives for labor unions to voluntarily coordinate their actions at the national level are stronger. Hence, if the national central bank has a strict policy, e.g. an inflation target, the country's entry into a monetary union may induce large labor unions to coordinate wage setting, which leads to real wage restraint and lower equilibrium unemployment.

The essay is organized as follows. Section 2 presents the model setup. The model is solved and equilibrium unemployment in both monetary regimes is derived in Section 3. Finally, Section 4 analyzes the impact of the formation of a monetary union on equilibrium unemployment. Section 5 studies two special cases where monetary regime is irrelevant for unemployment determination. Section 6 concludes.

2 The Model

2.1 The Basic Structure

The model specification in a flexible exchange rate regime follows that of Jensen (1993, 1997), which is a variant of the standard two-country monetary model studied in Canzoneri and Henderson (1988, 1991). Jensen's model is extended to cover the case of a monetary union.

[9] For a survey see e.g. Calmfors et al. (2001, Ch. 5).

There are two equal sized countries, each specialized in the production of a traded good.[10] All variables are in natural logarithms and foreign variables are denoted with an asterisk. Throughout the essay it is assumed that all structural and policy parameters are equal across countries.

Home and foreign output supplies, denoted by y and y^*, are deterministic functions of employment levels, l and l^*, subject to decreasing returns to scale

$$y = \alpha l, \qquad y^* = \alpha l^*, \tag{1}$$

where $0 < \alpha < 1$. Labor is assumed to be immobile between countries and supplied inelastically. At least from a European perspective, immobile labor seems a reasonable starting point for the analysis, since labor mobility between countries is still quite modest (see e.g. OECD (1999)).

Goods markets are characterized by perfect competition. Profit maximizing firms employ labor up to the point at which real product wages are equal to marginal products of labor

$$w - p = -(1-\alpha)\, l, \qquad w^* - p^* = -(1-\alpha)\, l^*, \tag{2}$$

where nominal wages are denoted by w and w^*, while p and p^* are the product prices of home and foreign countries respectively.[11] Denoting full employment in both countries as $\overline{l}$ and normalizing it to zero it is possible to define the unemployment rate as[12]

$$u = \overline{l} - l = -l, \qquad u^* = \overline{l} - l^* = -l^*. \tag{3}$$

Relative output demand is a function of the real exchange rate, z

$$y - y^* = \delta z, \tag{4}$$

where the parameter $\delta > 0$ can be interpreted as indexing the degree of substitutability between home and foreign goods. A large value of δ implies that the two goods are close

[10] Obviously, a more satisfactory model of a monetary union would be an n-country model where some subset of countries would form a monetary union. This benchmark case is considered for analytical simplicity. Canzoneri and Henderson (1991) present also a three-country version of their model, and Buiter, Corsetti and Pesenti (1995) construct an $(n+1)$-country center-perifery model in order to analyze the collapse of the ERM.

[11] The marginal product of labor is actually $w - p = \ln\alpha - (1-\alpha)\, l$, but the constant is omitted for notational simplicity.

[12] Strictly speaking, this definition of u only approximates the standard definintion $u = \left(\overline{L} - L\right)/\overline{L} = 1 - \left(L/\overline{L}\right)$, where L refers to employment and $\overline{L}$ to labor force. Rewriting this expression as $1-u = L/\overline{L}$, taking logs on both sides and using the approximation $\log(1-u) \approx -u$ and the normalization $\overline{l} = 0$ mentioned in the text, one obtains $u \approx -l$.

substitutes, and therefore a small change in the real exchange rate brings about a large change in relative demand.

The real exchange rate is defined as the relative price of the foreign good in terms of the domestic good, expressed in home country currency

$$z = e + p^* - p, \tag{5}$$

where the nominal exchange rate, denoted by e, is the domestic price of foreign currency. Hence, a decrease in z implies that domestic currency appreciates in real terms. By definition, in a monetary union (the log of) the nominal exchange rate is zero, and the real exchange rate is simply

$$z = p^* - p. \tag{6}$$

Assuming that the underlying preferences over consumption are Cobb-Douglas with a constant share of imports, β, the domestic and foreign consumer price indices (CPIs) are given by

$$\pi = (1-\beta)\, p + \beta\,(e + p^*) = p + \beta z, \tag{7}$$

$$\pi^* = \beta\,(p - e) + (1-\beta)\, p^* = p^* - \beta z. \tag{8}$$

The value of β is restricted to lie in the interval $0 < \beta < \frac{1}{2}$, which is equivalent to assuming a home bias in consumption.[13] It seems that this is a rather mild restriction, as there is a large empirical literature on the home bias in trade showing that people indeed have a strong preference for consumption of their home country goods.[14]

Under flexible exchange rates money demand is assumed to be proportional to nominal income and only domestic residents hold domestic money. The money markets are in equilibrium when nominal money supplies, m and m^*, satisfy simple quantity equations

$$m = p + y, \qquad m^* = p^* + y^*. \tag{9}$$

In a monetary union there is only one currency, which is held by the residents of both countries. Monetary equilibrium requires that union-wide money demand equals union-wide money supply. Following Lane (1996), money demand is proportional to average

[13] This is a common assumption in the open-economy macro literature, see. e.g. Buiter, Corsetti and Pesenti (1995), Lane (1996, 2000) and Zervoyianni (1997).

[14] See e.g. Obstfeld and Rogoff (2000) and references therein.

nominal income of the member countries

$$m^u = \frac{1}{2}(p + y) + \frac{1}{2}(p^* + y^*), \tag{10}$$

where m^u is the union-wide money supply. It is assumed that the common money supply is spread out equally on both member countries, and thus the common central bank can't affect the distribution of money supply across countries.

2.2 Labor Union and Central Bank Preferences

In each country there is a single monopoly union, which sets the nominal wage rate strategically, that is, the union takes into account the following reactions from the monetary authorities. Unions' utility functions are given by

$$U = \gamma(w - \pi) - \frac{1}{2}u^2, \qquad U^* = \gamma(w^* - \pi^*) - \frac{1}{2}u^{*2}, \tag{11}$$

where $w - \pi$ and $w^* - \pi^*$ are the real consumer wages, and u and u^* are unemployment rates in the two countries. The unions' utility depends positively on the real consumer wage and negatively on unemployment, which is consistent with traditional labor union theory (see e.g. Oswald (1985)).[15] This is where the essay departs from much of the recent literature on strategic interaction between labor unions and the central bank. In most studies it is assumed that labor unions care not only about the real wage and unemployment, but also about inflation *per se*.[16]

Why should labor unions be interested in inflation? The standard explanation is that union members have some non-indexed nominal assets or incomes other than their salaries, such as bank deposits or pensions, values of which are eroded by inflation. However, pensioners are usually not labor union members, and it seems unlikely that unions would look after their interests. In many contributions labor unions' inflation aversion is actually driving the results, and therefore a closer look on possible reasons for

[15] The linear-quadratic functional form is widely used in this literature (see e.g. Jensen (1993, 1997), Cukierman and Lippi (1999, 2001), Coricelli, Cukierman and Dalmazzo (2000, 2001) and Berger, de Haan and Eijffinger (2001)). As the utility is linear in the consumer real wage, parallel shifts in labor demand curve do not affect unemployment. Hence, the impact of monetary regime on the equilibrium rate of unemployment comes solely from changes in the slope of the labor demand curve. In other words, the establishment of a monetary union will affect equilibrium unemployment only if the real consumer wage elasticity of the labor demand is affected.

[16] See Cukierman and Lippi (1999) and references therein. However, Soskice and Iversen (2000), Coricelli, Cukierman and Dalmazzo (2000) and Lawler (2002) also abstract from labor union inflation aversion in a closed-economy setting. Holden (2003a) studies a small open-economy case and Coricelli, Cukierman and Dalmazzo (2001) build a two-country monetary union model.

inflation aversion would be desirable.[17]

Monetary policy is discretionary in the model and the loss functions describing policy preferences are of standard quadratic type

$$L = \pi^2 + \lambda \left(u - \overline{u}^f\right)^2, \qquad L^* = \pi^{*2} + \lambda \left(u^* - \overline{u}^f\right)^2, \tag{12}$$

where $0 \leq \lambda < \infty$. Since the model is assumed to be completely symmetric, the policy parameters describing the conservatism of the central bank, λ, and the unemployment target, $\overline{u}^f$, are equal across countries.[18] CPI inflation is defined as the difference between the log price levels in current and previous periods, $\pi - \pi_{-1}$. Without loss of generality, π_{-1} is normalized to zero, and thus CPI and CPI inflation are equivalent in this setup. The inflation target of the central banks is set to zero for analytic convenience.

The parameter λ reflects the relative weight placed by the central bank on unemployment deviations from the target level. λ^{-1} thus measures the Rogoff (1985a) type of conservatism of the central bank.[19] The smaller λ is, the more the central bank cares about inflation deviations and the less about unemployment, i.e. it allows unemployment to deviate more from the target level in order to push inflation closer to zero. Nominal money supplies, m and m^*, are the policy instruments of the central banks.[20]

In a monetary union the common central bank is assumed to care only about the average rates of unemployment and inflation in the area

$$L^u = \left(\frac{\pi + \pi^*}{2}\right)^2 + \lambda \left(\frac{u + u^*}{2} - \overline{u}^u\right)^2, \tag{13}$$

where $\overline{u}^u$ is the unemployment target of the common central bank.[21] It is assumed that conservatism of the central bank, λ, will be unchanged when the monetary union is established.

The aim of the essay is to demonstrate that the formation of a monetary union will

[17] A step into this direction is taken in Berger, Hefeker and Schöb (2002), where inflation aversion is endogenized by assuming that the outside option of the labor union is defined in nominal terms.

[18] The superscript f refers to a flexible exchange rate regime.

[19] It would be conceptually clearer to write the weight parameter in front of the inflation term in the loss function. Then the parameter would have a direct interpretation as the degree of conservatism. The choice made in this essay, however, makes some derivations simpler.

[20] This assumption is, admittedly, at odds with reality, since monetary policy is usually conducted with the short-term nominal interest rate as the instrument. Money supply is, however, a natural choice here as the model is static. Using interest rates as instruments in a static model would be conceptually confusing, as interest rates are intertemporal by definition.

[21] Gros and Hefeker (2002) investigate how the common central bank's choice of policy targets, that is, average monetary union wide welfare losses vs. the sum of national welfare losses, will affect the welfare in member countries in a heterogeneous monetary union.

have real effects independent of the existence of an inflation bias. Therefore, the central bank's unemployment target both under flexible exchange rates and in a monetary union are assumed to be set at an arbitrary level, and it will be shown that the real economy is unaffected by the chosen level. In particular, even though the target would coincide with the equilibrium rate, the real effects will remain.

2.3 Timing of the Model

Events unfold as follows. (i) The two labor unions set their nominal wages simultaneously taking the other union's wage as given. They have full information about the reaction functions of the central banks and take them into account in the wage setting process. In other words, unions act strategically as Stackelberg leaders against the central banks. (ii) Under flexible exchange rates the central banks choose their money supplies simultaneously taking the other central bank's money supply as well as previously set nominal wages as given. In a monetary union the common central bank sets the union-wide money supply. (iii) Production and trade take place and macroeconomic outcomes are realized.

The choice of modelling the game between labor unions and central banks as a Stackelberg game is supported by observations in the real world wage setting and monetary policy practices. Wage contracts are usually negotiated to cover at least a year, whereas monetary policy can be adjusted frequently. It is assumed that the central banks don't have access to a commitment technology. In other words, the essay considers time-consistent discretionary monetary policy. If the central banks could credibly commit to a certain policy rule in advance, it might still be the case that monetary union would affect equilibrium unemployment. All what is needed to generate changes in equilibrium unemployment is that from a labor union's perspective the optimal rule for the common central bank is different from the optimal rules of the national central banks. This conjecture is, however, not investigated further in this essay.

The behavior of the labor unions can be time-inconsistent as well. It may be the case that after the central banks have set their policies, the labor unions find it optimal to renege on the wage contracts and set new, higher or lower, nominal wages. This possibility is ruled out by assuming that nominal wage contracts are legally binding for the whole contract period.

3 Monetary Policy and Wage Setting

In this section the two-stage game between labor unions and central banks is solved in both monetary regimes by backward induction.[22] Starting from the second stage, equilibrium strategies are presented for central banks in a flexible exchange rate regime, and for a common central bank in a monetary union case. Then the wage setting problem at the first stage is solved and equilibrium outcomes are derived for macroeconomic variables.

3.1 Monetary Policy in a Flexible Exchange Rate Regime

The reduced forms for home and foreign rates of unemployment and inflation in a flexible exchange rate regime can be derived from equations (1)-(5) and (7)-(9)[23]

$$u = -m + w, \qquad u^* = -m^* + w^*, \tag{14}$$

$$\pi = (1-\alpha)\, m + \alpha w + \frac{\alpha\beta}{\delta}\left(m - w - m^* + w^*\right), \tag{15}$$

$$\pi^* = (1-\alpha)\, m^* + \alpha w^* - \frac{\alpha\beta}{\delta}\left(m - w - m^* + w^*\right). \tag{16}$$

Equation (14) shows that a domestic wage increase has a direct effect on unemployment in home country, whereas foreign unemployment is unaffected. The home product price is increased (see equation (47) in the Appendix) in order to balance the money market. The real exchange rate appreciates so that the goods markets are in equilibrium (see equation (48) in the Appendix). This reduces domestic consumer prices and increases foreign consumer prices. The total effect on domestic CPI is ambiguous, but the foreign CPI increases, i.e. inflation is exported by a unilateral wage increase.

Next, consider a unilateral domestic monetary expansion. Domestic unemployment falls, and to balance the money market the product price increases (see equation (47) in the Appendix). Goods market equilibrium calls for real exchange rate depreciation. Domestic CPI increases and foreign CPI decreases because of the real exchange rate depreciation.

The home country central bank's problem is to minimize the loss function L in (12) with respect to m, and subject to (14) and (15), taking nominal wages and foreign money

[22] The third stage of the game involves no strategic choices, since product markets are assumed to be competitive.

[23] The reduced forms for the other endogenous variables are presented in the Appendix as equations (47)-(49).

supply m^* as given. The first-order condition becomes

$$\pi = \frac{\lambda}{1-\alpha+\frac{\alpha\beta}{\delta}}\left(u-\overline{u}^f\right), \tag{17}$$

which reveals that whenever the unemployment target is below the equilibrium rate of unemployment, an inflation bias emerges.

Inserting the reduced forms (14) and (15) into the first-order condition and solving it explicitly for m, yields the reaction function of the home central bank

$$m = \frac{\lambda-\Gamma\left(\alpha-\frac{\alpha\beta}{\delta}\right)}{(\lambda+\Gamma^2)}w + \frac{\Gamma\frac{\alpha\beta}{\delta}}{(\lambda+\Gamma^2)}\left(m^*-w^*\right) - \frac{\lambda}{(\lambda+\Gamma^2)}\overline{u}^f, \tag{18}$$

where $\Gamma = 1-\alpha+\alpha\beta\delta^{-1}$. Depending on central bank conservatism, λ^{-1}, the central bank reacts to a domestic wage increase either by expanding or contracting money supply. Cukierman, Rodrigues and Webb (1998) provide empirical evidence on monetary policy responses to wage increases. Their results show that in countries with highly conservative central banks, monetary policy is tightened in response to high wage settlements, whereas in countries with less conservative central banks monetary policy is accommodating.

Due to perfect symmetry of the model, the foreign money supply schedule can be written symmetrically. The two central banks choose their non-cooperative policies simultaneously. The Nash equilibrium policy for the home country is given by[24]

$$m = c_1^f w + c_2^f w^* + c_3^f \overline{u}^f, \quad c_1^f \lessgtr 0, c_2^f \leq 0, -1 < c_3^f \leq 0, \tag{19}$$

where the superscript f refers to the flexible exchange rate regime. The interpretation of c_1^f, c_2^f and c_3^f is straightforward. Home country wage increase induces either negative or positive monetary policy reaction in equilibrium depending on central bank conservatism. Since wage increase in the foreign country doesn't directly affect home country unemployment and is always inflationary, home country money supply is always reduced in equilibrium. Also, it is intuitive that, other things equal, an increase in the unemployment target calls for a reduction in money supply in equilibrium.

Finally, inserting equation (19) into equations (14) and (15) yields the rates of unem-

[24]The explicit forms of c_1^f, c_2^f and c_3^f are given in the Appendix. An analogous equation holds for the foreign country and is given by $m^* = c_1^f w^* + c_2^f w + c_3^f \overline{u}^f$.

ployment and inflation as functions of home and foreign nominal wages[25]

$$u = a_1^f w + a_2^f w^* + a_3^f \overline{u}^f, \quad a_1^f > 0, a_2^f \geq 0, 0 \leq a_3^f < 1, \tag{20}$$

$$\pi = b_1^f w + b_2^f w^* + b_3^f \overline{u}^f, \quad b_1^f \geq 0, b_2^f \geq 0, -1 < b_3^f \leq 0, \tag{21}$$

where, as before, the superscript f refers to the flexible exchange rate regime. These two equations contain the optimal monetary policy responses to unions' nominal wage claims. At the first stage of the game, labor unions rationally anticipate the policy responses and take them into account in wage setting, that is, the labor unions set wages strategically.

The nominal wage elasticity of unemployment, a_1^f, is always positive. The central bank will accommodate the wage increase only partially, since money supply increases are inflationary. If the central bank is conservative enough, it will contract money supply in response to a wage increase, which implies that a_1^f can be greater than unity. Domestic nominal wage increase will thereby always increase domestic unemployment. Less obvious is the fact that a unilateral foreign nominal wage hike will increase unemployment in home country, that is, $a_2^f \geq 0$. However, the explanation is simple and can be seen from the reduced forms in equations (14)-(16). The direct effect of foreign wages on domestic unemployment is nil (see equation (14)), and the direct effect on domestic inflation is positive (see equation (15)). Therefore, as long as the domestic central bank targets both inflation and unemployment deviations, it will reduce money supply in order to push inflation down. Since the domestic nominal wage is assumed to be unaltered, unemployment will go up (see equation (14)). Other things equal, an increase in $\overline{u}^f$ will induce higher unemployment, since monetary policy will be more contractionary.

The nominal wage elasticities of inflation, b_1^f and b_2^f, are both positive, i.e. wage increases are always inflationary. Only in the special case of ultra-conservative central banks with $\lambda = 0$, will b_1^f and b_2^f be zero. As noted above, an increase in $\overline{u}^f$ raises u. But, due to $a_3^f < 1$, $u - \overline{u}^f$ gets smaller and from (17) it follows that the inflation bias problem will be reduced.

3.2 Monetary Policy in a Monetary Union

As in the previous section, first the reduced forms for rates of unemployment and inflation are solved from equations (1)-(4), (6)-(8) and (10). By definition, (the log of) the nominal exchange rate now equals zero, and the real exchange rate is simply expressed as the difference of (the logs of) the foreign and domestic product prices in equation (6). Money demand depends on the average union-wide nominal income. The reduced forms are

[25] The explicit forms of the elasticities are given in the Appendix.

given by[26]

$$u = -m^u + \frac{1}{2}(w + w^*) + \frac{1}{2}\frac{\delta}{\alpha + (1-\alpha)\delta}(w - w^*), \tag{22}$$

$$u^* = -m^u + \frac{1}{2}(w + w^*) - \frac{1}{2}\frac{\delta}{\alpha + (1-\alpha)\delta}(w - w^*), \tag{23}$$

$$\begin{aligned}\pi &= (1-\alpha)m^u + w - \left(\frac{1-\alpha}{2}\right)(w + w^*) \\ &\quad -\frac{1}{2}\left(\frac{2\alpha\beta + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta}\right)(w - w^*),\end{aligned} \tag{24}$$

$$\begin{aligned}\pi^* &= (1-\alpha)m^u + w^* - \left(\frac{1-\alpha}{2}\right)(w + w^*) \\ &\quad +\frac{1}{2}\left(\frac{2\alpha\beta + (1-\alpha)\delta}{\alpha + (1-\alpha)\delta}\right)(w - w^*).\end{aligned} \tag{25}$$

From (22) one sees that in a monetary union unemployment depends directly on both domestic and foreign wages, whereas in a flexible exchange rate regime only domestic wage entered the unemployment equation.[27] This interdependence in a monetary union results from money market equilibrium (10), which can be rewritten as

$$m^u = \frac{1}{2}(p + y) + \frac{1}{2}(p^* + y^*) = \frac{1}{2}(w - u) + \frac{1}{2}(w^* - u^*). \tag{26}$$

In a flexible exchange rate regime a unilateral wage increase doesn't directly affect real variables in the other country, while in a monetary union unemployment (and output) in both countries will in general be affected. From (26) one sees that an increase of w generally affects both u and u^* when m^u and w* are fixed.[28]

As expected, the reduced forms (22)-(25) reveal that an increase in the area-wide money supply, m^u, affects both countries symmetrically. Hence, the common central bank can't affect the real exchange rate, z, which is solely determined by the labor unions (see equation (52) in the Appendix).

The common central bank chooses m^u in order to minimize the loss function (13) subject to (22)-(25). The first-order condition of the common central bank's optimization

[26] The reduced forms for the other endogenous variables are presented in the Appendix as equations (50)-(52).

[27] Unemployment depends indirectly on both home and foreign wages in a flexible exchange rate regime, because monetary policies in both countries are affected by both w and w^*.

[28] From (22) one sees, however, that when $\delta = 1$, unemployment depends directly only on the home country wage.

problem can be written as

$$\frac{\pi+\pi^*}{2} = \frac{\lambda}{(1-\alpha)}\left(\frac{u+u^*}{2} - \overline{u}^u\right), \tag{27}$$

which relates the average rate of inflation to the average rate of unemployment in the monetary union area. Comparing this expression with equation (17), one finds that with a given difference between the average rate of unemployment and the unemployment target, average inflation is higher in a monetary union than under flexible exchange rates. As mentioned in Introduction, a similar result is obtained in monetary policy cooperation case (see e.g. Rogoff (1985b)).

Solving the first-order condition explicitly for m^u yields the reaction function of the common central bank

$$m^u = \frac{\lambda - \alpha(1-\alpha)}{2\left(\lambda + (1-\alpha)^2\right)}(w+w^*) - \frac{\lambda}{\left(\lambda+(1-\alpha)^2\right)}\overline{u}^u. \tag{28}$$

The central bank's reaction to wage increases is symmetric in a sense that the monetary policy reaction to a unilateral wage increase doesn't depend on which country is the source. This characteristic obviously results from the assumed symmetry of the model. If the central bank is conservative enough, it reduces money supply when nominal wages are raised.

Finally, the rates of unemployment and inflation as functions of the nominal wages in both countries and unemployment targets are derived by inserting the reaction function (28) into the reduced forms of u and π[29]

$$u = a_1^u w + a_2^u w^* + a_3^u \overline{u}^u, \quad a_1^u > 0, a_2^u \gtreqless 0, 0 \leq a_3^u < 1, \tag{29}$$

$$\pi = b_1^u w + b_2^u w^* + b_3^u \overline{u}^u, \quad b_1^u > 0, b_2^u \gtreqless 0, -1 < b_3^u \leq 0, \tag{30}$$

where the superscript u refers to the monetary union case. Earlier it was shown that a_2^f is always positive. Now, a_2^u can be negative, i.e. a foreign wage increase may lower unemployment in home country. This may happen if the common central bank is relatively "liberal" (λ is large), or if the goods are close substitutes (δ is large). The difference comes from the fact that the response of the common central bank to a unilateral wage increase by one labor union is symmetric in both countries. It is instructive to consider a case of a relatively liberal central bank, i.e. λ is large. Now, by setting $\delta = 1$ the direct effect of a foreign wage increase on domestic unemployment is nil as in a flexible exchange

[29] The explicit forms for the elasticities are given in the Appendix.

rate regime (see footnote 28 and equations (14) and (22)). The average area-wide unemployment will increase, since foreign unemployment goes up and home unemployment is unaffected. Now, given that the common central bank is fairly liberal, it will increase the area-wide money supply so as to pull down the average unemployment rate. Since the direct effect of a foreign wage increase on domestic unemployment is nil, accommodating monetary policy will lower the domestic rate of unemployment below the initial level and the total effect is thus negative.

The home country nominal wage elasticity of inflation, b_1^u, is always positive in a monetary union. However, the foreign country nominal wage elasticity of inflation, b_2^u, can be negative, if the common central bank is very conservative, i.e. λ is very small. Again, this is caused by the inability of the common central bank to respond asymmetrically to a unilateral wage increase. An increase in foreign wage level increases home inflation less than foreign inflation, provided that $\beta < \frac{1}{2}$ (as assumed in the model). If the central bank is very conservative, it will push the average inflation close to zero by reducing the common money supply. Thereby, the home country inflation can become negative. It is noteworthy that in this model the inflation rates of the member countries of a monetary union need not to be equal, as it is the case e.g. in Grüner and Hefeker (1999) and Cukierman and Lippi (2001).[30]

3.3 Strategic Wage Setting and Equilibrium Unemployment

The labor unions choose their wages in the first stage of the game taking the other union's wage as given. The unions have complete information about the reaction functions of the central banks and they realize that since they are large enough, their wage claims will affect the monetary policy. Hence, the unions set their wages strategically taking into account the following monetary policy responses.

The labor unions maximize their utility functions (11) subject to (20) and (21) in a flexible exchange rate regime, and subject to (29) and (30) in a monetary union. Since the form of the wage setting problem is similar in both monetary regimes, the regime in question is denoted with a superscript i whenever making a distinction between the variables of the two regimes is not necessary. Solving the first-order condition for w^i yields a reaction function of the form

$$w^i = \gamma \frac{(1 - b_1^i)}{(a_1^i)^2} - \frac{a_2^i}{a_1^i} (w^*)^i - \frac{a_3^i}{a_1^i} \overline{u}^i, \tag{31}$$

[30] Even though the model allows inflation differences between the member countries in a monetary union, the equilibrium rate of inflation will be equal since the model is symmetric.

where $i = f, u$. Again, the symmetry of the model implies that the reaction function of the foreign union is of the same form. In Nash equilibrium nominal wages are given by

$$w^i = (w^*)^i = \gamma \frac{(1 - b_1^i)}{a_1^i (a_1^i + a_2^i)} - \frac{a_3^i}{(a_1^i + a_2^i)} \overline{u}^i. \tag{32}$$

Inserting the equilibrium nominal wage (32) into the reduced form of the rate of unemployment, (20) or (29), yields equilibrium unemployment

$$u^i = (u^*)^i = \gamma \frac{(1 - b_1^i)}{a_1^i}. \tag{33}$$

It is noteworthy that equilibrium unemployment is independent of the central banks' unemployment target $\overline{u}^i$.

Once equilibrium unemployment levels in both countries are derived, equilibrium inflation rates can be obtained from equations (17) and (27). Rantala (2001) analyzes the case where $\overline{u}^i = 0$. However, for the purpose of this essay it suffices to notice that equilibrium unemployment doesn't depend on $\overline{u}^i$. It follows that even though the unemployment target $\overline{u}^i$ would coincide with the equilibrium rate of unemployment, which results in the disappearance of inflation bias (see equations (17) and (27)), the real effects brought about by the establishment of a monetary union, which are discussed in the next section, remain unchanged. Moreover, equation (33) reveals the source of unemployment in the model. The equilibrium rate of unemployment is always positive, provided that the labor unions have a real wage target, i.e. $\gamma > 0$.

In order to gain some intuition on the unemployment determination in the model, it is instructive to study the labor union's wage setting in unemployment–real wage space. Employment, and thus also unemployment, is determined from the labor demand curve. From equations (2) and (3) one observes that the slope of the labor demand curve in $(u^i, (w^i - p^i))$ space is $(1 - \alpha)$.[31] However, the labor union's utility depends on the real consumer wage and the rate of unemployment. Contrary to the real product wage, $w^i - p^i$, the real consumer wage, $w^i - \pi^i$, contains also foreign product price as an element, and it turns out to be useful to study the effective labor demand curve for the union in

[31] Since the variables are in logs, the slope coefficient can be directly converted into real product wage elasticity of labor demand, $\eta \equiv -\frac{\partial \log N}{\partial \log W/P} = -\frac{\partial l}{\partial (w-p)} = \frac{\partial u}{\partial (w-p)} = \frac{1}{1-\alpha} \in (1, \infty)$, which in this model depends only on production technology.

$(u^i, (w^i - \pi^i))$ space[32]

$$\begin{aligned}\left(w^i - \pi^i\right) &= \frac{1 - b_1^i}{a_1^i} u^i - \frac{a_1^i b_2^i + (1 - b_1^i)\, a_2^i}{a_1^i} \left(w^*\right)^i \\ &\quad - \frac{a_1^i b_3^i + (1 - b_1^i)\, a_3^i}{a_1^i} \overline{u}^i.\end{aligned} \tag{34}$$

The slope coefficient, $(1 - b_1^i)/a_1^i$, of the effective labor demand curve (34) together with the labor union preference parameter, γ, directly determine the equilibrium rate of unemployment in the model (see equation (33)). Figure 1, where one indifference curve of the labor union is denoted by $U((w - \pi), u; \gamma)$, illustrates the equilibrium.

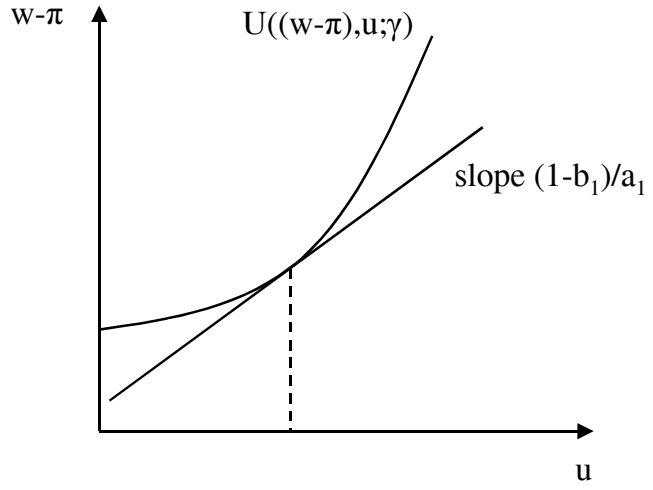

Figure 1: Determination of equilibrium unemployment

The slope coefficient $(1 - b_1^i)/a_1^i$ can be written as a function of the two nominal wage elasticities of unemployment, a_1^i and a_2^i[33]

$$\frac{1 - b_1^i}{a_1^i} = (1 - \alpha) + \frac{\alpha\beta}{\delta} \left(\frac{a_1^i - a_2^i}{a_1^i} \right). \tag{35}$$

This formulation is helpful in unemployment comparisons between the two monetary regimes presented in the next section. Equilibrium unemployment depends on two elasticities. The first one is the nominal wage elasticity of the real exchange rate, which is given by $\partial z / \partial w^i = -\alpha\delta^{-1} (a_1^i - a_2^i) \leq 0$. The stronger effect the wage has on the real exchange rate, the steeper is the effective labor demand curve. The second elasticity is

[32] This expression is obtained from equations (20) and (21) (or from (29) and (30)).

[33] Derivation of this expression can be found in the Appendix.

the home country nominal wage elasticity of unemployment, $\partial u/\partial w^i = a_1^i > 0$. The larger the unemployment loss, the less steep is the effective labor demand curve.

Since a_1^i and a_2^i depend on the monetary regime (and conservatism of the central bank), it is now evident that the slope of the effective labor demand curve the union faces in wage setting is endogenous. In particular, if the slope becomes steeper in a monetary union unemployment will increase, whereas a less steep curve implies reduced unemployment.

The inverse of $(1 - b_1^i)/a_1^i$ is actually the effective real consumer wage elasticity of labor demand

$$\eta_{w-\pi} = -\frac{\partial l}{\partial (w-\pi)} = \frac{\partial u}{\partial (w-\pi)} = \frac{1}{1-\alpha+\frac{\alpha\beta}{\delta}\left(\frac{a_1-a_2}{a_1}\right)}. \tag{36}$$

Therefore, equilibrium unemployment inversely related to $\eta_{w-\pi}$ so that from (33) and (36) one obtains

$$u^i = (u^*)^i = \frac{\gamma}{\eta_{w-\pi}}. \tag{37}$$

4 Equilibrium Unemployment in a Monetary Union

This section analyzes the impact of the formation of a monetary union on equilibrium unemployment. The structural parameters of the economy, as well as central bank and labor union preferences remain the same when a monetary union is established. This essay thereby focuses on the direct effect of a monetary union on economic performance through changes in the strategic environment and the behavior of the labor unions. The degree of central bank conservatism turns out to be a crucial factor in what happens to unemployment in a monetary union.

4.1 Equilibrium Unemployment and Central Bank Conservatism

As discussed in Introduction, unemployment is not affected by central bank conservatism in the standard Barro-Gordon framework. Labor markets are assumed to be atomistic, i.e. it is as Barro and Gordon (1983) put it, "...a game between the policy maker and a large number of private sector agents." It follows that each agent is too small to have any effect on policy maker's behavior, and thus there is no strategic interaction between them. However, abandoning this assumption and introducing non-atomistic labor unions into the model will make the neutrality result disappear.

It is easy to show that equilibrium unemployment goes down in both monetary regimes

when central bank conservatism increases (see the Appendix for derivations).[34] The intuition behind this somewhat surprising result is clear. When λ is small, i.e. conservatism is high, labor unions correctly anticipate tight monetary policy and a large rise in unemployment in response to their wage claims. Conservatism works as a deterrent to wage increases and therefore moderates the labor unions' real wage targets leading to lower unemployment. This can be seen by differentiating the nominal wage elasticities of unemployment with respect to λ in both regimes, which gives

$$\frac{\partial a_1^f}{\partial \lambda} < 0, \qquad \frac{\partial a_1^u}{\partial \lambda} < 0, \tag{38}$$

where an increase in conservatism means that λ becomes smaller.

Strictly speaking, this argument is correct only in the monetary union case, where the nominal wage elasticity of the real exchange rate doesn't depend on central bank conservatism. Under a flexible exchange rate an increase in conservatism strengthens the union's ability to appreciate the real exchange rate, which works into the opposite direction and tends to increase wage demands. Differentiating $(a_1^i - a_2^i)$ (where $i = f, u$) with respect to λ yields

$$\frac{\partial \left(a_1^f - a_2^f\right)}{\partial \lambda} < 0, \qquad \frac{\partial \left(a_1^u - a_2^u\right)}{\partial \lambda} = 0. \tag{39}$$

As shown in the Appendix, the deterrent effect of central bank conservatism dominates the real exchange rate effect, and therefore increased conservatism will decrease unemployment also in a flexible exchange rate regime. Hall and Franzese (1998) and Cukierman and Lippi (1999) provide some empirical evidence supporting the result. They show that when bargaining structure is highly centralized, an increase in central bank conservatism tends to lower unemployment.

[34] The flexible exchange rate case is presented in Jensen (2000), a fact which was kindly brought to the author's notice by Henrik Jensen after the working paper version of this essay (Rantala (2001)) was published.

4.2 The Impact of Monetary Union on Equilibrium Unemployment

The impact of the formation of a monetary union on equilibrium unemployment can be analyzed by studying the following expression

$$sgn\left(u^u - u^f\right) = -sgn\left(\frac{a_2^u}{a_1^u} - \frac{a_2^f}{a_1^f}\right), \tag{40}$$

where the right-hand side is obtained from (35). In the Appendix it is proved that there exists a unique non-negative cut-off point for central bank conservatism, λ, where $u^u = u^f$. This critical value, $\overline{\lambda}$, is given by

$$\begin{aligned} \overline{\lambda} &= \frac{1}{2\delta^2}\left(-A + \sqrt{A^2 + B}\right), \\ A &= (1-\alpha)^2\delta^2 + \alpha(1-\alpha)(3\beta - 1)\delta + 2\alpha^2\beta^2, \\ B &= 4\delta^2\alpha(1-\alpha)^2(1-2\beta)(\alpha\beta + \delta(1-\alpha)) > 0, \end{aligned} \tag{41}$$

where $B > 0$ holds, since $\beta < \frac{1}{2}$. The equilibrium unemployment is lower in a monetary union than under flexible exchange rate regime when central bank conservatism is high enough, i.e. $\lambda \in [0, \overline{\lambda})$. However, when $\lambda \in (\overline{\lambda}, \infty)$ unemployment will be higher in a monetary union. Figure 2 illustrates the result.

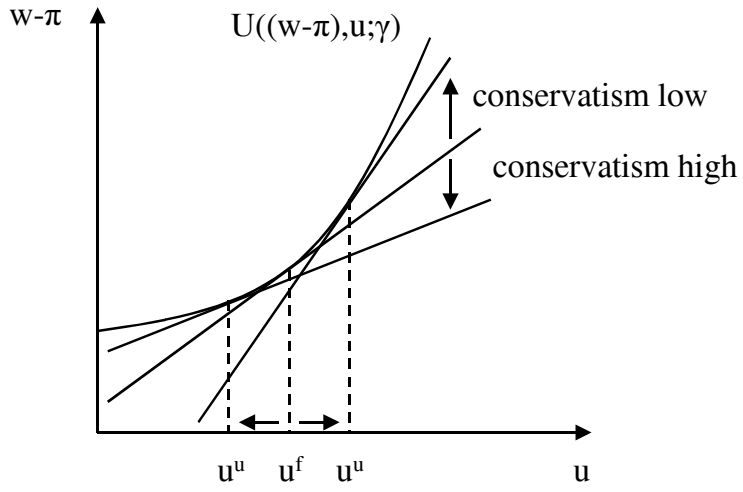

Figure 2: The impact of monetary union on equilibrium unemployment

This result is in sharp contrast with earlier results concerning the impact of monetary union on equilibrium unemployment. Grüner and Hefeker (1999) show that the

establishment of a monetary union will increase unemployment, if national labor unions are inflation averse. However, their result depends crucially on the assumed inflation aversion, and thereby also on the existence of an inflation bias. The above result (41) points out that in this model with open-economy spillovers, monetary union will have real effects even when labor unions are not inflation averse, and without an inflation bias. Interestingly enough, (41) indicates that one can always find a level of conservatism such that monetary union will be beneficial in terms of reduced unemployment.

An intuitive explanation for the above result can be obtained by inspecting the labor union's incentives for wage hikes in the two alternative regimes with different degrees of central bank conservatism. In a monetary union the real exchange rate is solely determined by nominal wages set by the labor unions. The conservatism of the central bank will thereby not affect the unions' ability to appreciate the real exchange rate. As noted in the previous section, an increase in conservatism will improve the labor union's ability to appreciate the real exchange rate in the flexible exchange rate regime (see equation (39)). When conservatism is fairly high, the central banks will give priority to inflation deviations from the target. In the face of a unilateral wage increase, the central bank of the country in question will tighten monetary policy considerably in order to bring inflation down. In the other country the central bank will also tighten its policy, since real exchange rate depreciation is inflationary, but to a lesser extent. The need to react less vigorously results from the assumption of a home bias in consumption, which implies that inflation reaction to a wage hike is stronger in the country in question.[35] Hence, the equilibrium response of the two central banks appreciates the real exchange rate further, and thereby induces more aggressive wage policy than in the monetary union case, which in turn leads to higher unemployment.

On the other hand, when central bank conservatism is low, that is, the focus is on unemployment deviations from the target, the flexible exchange rate regime provides weaker incentives to wage increases than a monetary union. Even though a labor union tried to appreciate the real exchange rate with its wage claim, under a flexible exchange rate the central banks will guarantee with their policies that unemployment rates remain close to the target, which implies that a_1^i and a_2^i are close to zero. Keeping in mind that $\partial z/\partial w^i = -\alpha\delta^{-1}\left(a_1^i - a_2^i\right) \leq 0$, it becomes evident that the real exchange motive for wage hikes is weak when conservatism is low. Irrespective of the degree of conservatism,

[35] To be precise, in a flexible exchange rate regime a unilateral wage increase will increase inflation in home country more than in foreign country provided that the real exchange rate appreciation is not too strong relative to the import share. From (15) and (16) one obtains a condition $\delta > 2\beta$, which limits the substitutability parameter. This restriction is satisfied e.g. when one assumes Cobb-Douglas preferences for consumption, $\delta = 1$, since $\beta < \frac{1}{2}$ in the model. In a monetary union $\beta < \frac{1}{2}$ is alone a sufficient condition for home inflation reaction to exceed foreign reaction (see (24) and (25)).

monetary policy in a monetary union is incapable of affecting the real exchange rate, and so monetary policy reactions can't neutralize the impact of a unilateral wage hike on the real exchange rate. It follows that in a monetary union, the labor unions do have the real exchange motive for pushing up wages also when conservatism is low, and hence their wage policy will be more aggressive than under flexible exchange rates, resulting in higher unemployment.

5 Restoring Neutrality: Two Alternatives

This essay emphasizes that under fairly general conditions the formation of a monetary union will affect the equilibrium rate of unemployment. It was argued that there are basically two critical assumptions, which are needed in order to generate the non-neutrality result presented in the previous section: non-atomistic wage setting and imperfect substitutability between the two goods produced. This section relaxes these two conditions and verifies the emergence of neutrality.

So far, the goods produced in the two countries were assumed to be imperfect substitutes in consumption. This is captured in the model by the substitutability parameter, δ, which describes how the relative price of the goods will affect their relative demand. If δ approaches infinity, the two goods become perfect substitutes in consumption. It is easy to show that in this case the equilibrium rate of unemployment becomes (see the Appendix for derivations)

$$u^f_{|\delta\to\infty} = u^u_{|\delta\to\infty} = \gamma\,(1-\alpha)\,. \tag{42}$$

The neutrality result emerges, because perfect substitutability implies that the real exchange rate becomes exogenous to labor unions. There is effectively only one good in the model. It follows that the purchasing power parity (PPP) has to hold and the real exchange rate can't deviate from zero (in logs), because otherwise the world demand would shift to one country. Hence, the labor unions' incentives to push up nominal wages in order to appreciate the real exchange rate are destroyed. Moreover, comparing (42) with (33) and (35) one immediately sees that unemployment is lower when $\delta \to \infty$. The same neutrality result is obtained in Grüner and Hefeker (1999) and Cukierman and Lippi (2001), when unions are not inflation averse.

The model can easily be modified to show that non-atomistic wage setting is also crucial for the non-neutrality result of the essay. Assuming that there is a continuum of labor union–firm pairs indexed by j and distributed uniformly between $j = 0$ and $j = 1$ in both countries implies, that all unions take the product price and the consumer price

index as given in wage setting. Modifying (2) and (3), labor demand of the representative firm, j, is given by

$$w_j - p = -(1-\alpha)\, l_j = (1-\alpha)\, u_j. \tag{43}$$

A representative union's problem is then

$$\begin{aligned} &\max_{w_j} \left(\gamma (w_j - \pi) - \frac{1}{2} u_j^2 \right), \\ &\text{s.t. (43) and } \pi \text{ given.} \end{aligned} \tag{44}$$

The first-order condition becomes

$$w_j - p = \gamma (1-\alpha)^2 . \tag{45}$$

Inserting the real wage (45) into (43) yields the rate of unemployment in union j

$$u_j = \gamma (1-\alpha) .$$

Aggregation over all labor union–firm pairs gives the corresponding equation for the national rate of unemployment

$$u = \int_{j=0}^{1} u_j dj = \gamma (1-\alpha) , \tag{46}$$

which is clearly independent of the monetary regime. When labor markets are atomistic, the equilibrium rate of unemployment depends only on labor union preferences, γ, and the real product wage elasticity of labor demand (see equation (43) and Footnote 31). As expected, the unemployment outcome is the same as in the infinite substitution case above, that is, (46) coincides with (42).

6 Concluding Remarks

The purpose of the essay is to study the impact of the formation of a monetary union on the equilibrium rate of unemployment. In particular, it is shown that in the presence of strategic interaction between the central bank and the labor unions, changes in monetary regime will have real effects. Monetary regime will affect the labor union's real wage–unemployment tradeoff, i.e. the real consumer wage elasticity of labor demand is endogenous in the model.

The main result of the essay is that the establishment of a monetary union lowers

unemployment, provided that the degree of central bank conservatism is sufficiently high, whereas with low degrees of conservatism unemployment goes up. Interestingly enough, the inflexibility of the common central bank of the monetary union in responding to a country specific wage increase is good news in terms of unemployment if the central bank is conservative, whereas it is bad news if the central bank is fairly liberal. This result indicates that the choice of a monetary regime and the way monetary policy is conducted can be important factors in determining labor market outcomes in countries with highly centralized wage setting. Importantly, and unlike in the previous literature in this field, the results of the essay don't depend on the existence of an inflation bias.

The assumption of a single union in each country is obviously quite restrictive, but it can be justified as the simplest way to model non-atomistic labor markets. Labor is homogenous in the model. Therefore, it makes no sense to consider different levels of centralization of wage bargaining, since unions in this setup can't raise their wages above the average wage rate of the economy.[36] One could follow Lippi (2002) and introduce the level of centralization of wage bargaining into the model by assuming that labor is differentiated in general, and that all types of labor are needed in the production of the single good. Then each union has pricing power and the effects of centralization could be inspected. One likely consequence is that the rate of unemployment would decrease with the number of unions, since each union's ability to appreciate the real exchange rate with its wage claim would be reduced. However, it is unclear how this would affect the relative unemployment performance of a monetary union.

The model belongs to a class of "traditional" open-economy models, which basically are linearized versions of the Mundell-Fleming model. International spillovers operate through the real exchange rate channel. A monetary expansion by one country will have a *beggar-thy-neighbor* effect, since the depreciation of the real exchange rate will shift world demand toward that country. Moreover, inflation will increase in that country and decrease in the other country. Thus the other country will also devalue its currency by expansionary monetary policy and this leads to a situation known as "competitive devaluations". In the traditional literature on international policy coordination this spillover effect creates an incentive to coordinate national monetary policies. However, as men-

[36] There is a voluminous literature on the degree of centralization of wage bargaining and its effects on macroeconomic performance. Calmfors and Driffill (1988) present the well-known "hump-shape" hypothesis, which states that macroeconomic performance is better under a fully decentralized, and on the other hand, under a totally centralized wage bargaining systems than under intermediate levels of centralization, such as industry level bargaining. Open-economy considerations related to the hump-shape hypothesis are studied in Driffill and van der Ploeg (1993), Danthine and Hunt (1994) and Rama (1994). Interactions between the degree centralization of wage bargaining and monetary policy are analyzed e.g. in Cukierman and Lippi (1999), Lippi (2002) and Holden (2003b).

tioned in Introduction, domestic credibility problems may get worse under cooperation.[37]

The analysis in this traditional open-economy framework lacks explicit micro-foundations and the welfare criterion, like the widely used quadratic social welfare function, is *ad hoc*.[38] Recent contributions in the rapidly growing field of "new open-economy macroeconomics" have questioned the *beggar-thy-neighbor* effect in international transmission of shocks from a country to another. The novelty of the approach initiated by Obstfeld and Rogoff (1995, 1996) is in the fully-fledged micro-foundations, which enable the use of the utility of the representative consumer as a welfare criterion in policy evaluation.[39] Corsetti and Pesenti (2001) and Tille (2001) show that instead of being a *beggar-thy-neighbor,* an unanticipated monetary expansion can in terms of welfare actually be a *beggar-thyself* policy. The intuition behind this result is simple. Although a monetary surprise raises output and employment, the associated exchange rate depreciation worsens the terms of trade, thereby reducing the purchasing power of consumers. Then it is possible that consumers are unable to buy enough additional consumption to offset the cost of their increased effort. The main result is that if the substitutability of goods across countries is low, the adverse terms of trade effect dominates and monetary expansion is a *beggar-thyself* policy in terms of welfare. In that case countries have no incentives to boost employment through monetary expansions. This new result brought about by explicit microeconomic underpinnings qualifies the policy implications of the traditional literature by showing that the exchange rate channel may not be as strong as the traditional Mundell-Fleming analysis, which is also applied in this essay, suggests.

The model offers insights for the desirable policy choices in a monetary union. Obviously, one has to be extremely cautious in drawing strong policy implications based on such a simple model. The single union assumption clearly limits the applicability of the model in policy analysis to countries with relatively centralized wage setting. The model seems to suggest that when a monetary union is established, it is possible to attain lower rate of equilibrium unemployment by delegating monetary policy to a sufficiently conservative central bank. In Europe the creation of the EMU has increased the degree of central bank conservatism in most member countries. Thus, based on the analysis, one could speculate that the EMU may well have positive effects on equilibrium unemployment in many member countries. However, it must be stressed, that the deterministic analysis conducted in the essay abstracts completely from stabilization problems in a stochastic

[37] See Persson and Tabellini (2000, Ch. 18) for discussion and analysis on competitive devaluations, credibility problems and proper institutions for sustaining desirable policies.

[38] However, it is well-known that this widely used functional form can be obtained as a second-order Taylor series approximation to the level of expected utility of the representative household in an optimizing model with nominal price rigidities (see e.g. Woodford (2001)).

[39] Lane (2001) surveys the "new open-economy macroeconomics" literature.

setup. Central bank conservatism is a free lunch in the model. The policy implications will probably change, if one takes into account that delegating monetary policy to a central bank which is more conservative than the society will distort stabilization policy. It is quite likely that a tradeoff between low equilibrium unemployment and stabilization properties of the economy emerges, and that one can derive an optimal degree of central bank conservatism, where the weight on unemployment deviations is strictly positive. A thorough investigation of this point is left for future research.

A Appendix

A.1 Derivation of Endogenous Variables and Nominal Wage and Unemployment Target Elasticities

The reduced forms for the other endogenous variables in the flexible exchange rate regime are

$$\begin{aligned} p &= (1-\alpha)\, m + \alpha w, \qquad p^* = (1-\alpha)\, m^* + \alpha w^*, && (47)\\ z &= \frac{\alpha}{\delta}\left(m - w - m^* + w^*\right), && (48)\\ e &= \left(\frac{\alpha}{\delta} + (1-\alpha)\right)(m - m^*) + \left(\alpha - \frac{\alpha}{\delta}\right)(w - w^*). && (49) \end{aligned}$$

The Nash equilibrium nominal wage and unemployment target elasticities of money supply in the flexible exchange rate regime are

$$\begin{aligned} c_1^f &= \frac{\left(\lambda + \Gamma^2\right)\left(\lambda + \Gamma\left(\frac{\alpha\beta}{\delta} - \alpha\right)\right) - \left(\frac{\alpha\beta}{\delta}\Gamma\right)^2}{\Delta} \lesseqgtr 0,\\ c_2^f &= -\frac{\alpha\beta}{\delta}\frac{\Gamma^2}{\Delta} \leq 0,\\ c_3^f &= -\frac{\lambda}{\left(\lambda + \Gamma\left(1-\alpha\right)\right)}\overline{u} \leq 0, \end{aligned}$$

where $\Gamma = 1 - \alpha + \alpha\beta\delta^{-1}$ and $\Delta = \left(\lambda + \Gamma^2\right)^2 - \left(\frac{\alpha\beta}{\delta}\Gamma\right)^2 > 0$.

The nominal wage and unemployment target elasticities of unemployment and inflation in the flexible exchange rate regime are

$$\begin{aligned} a_1^f &= \frac{\Gamma\left(\lambda + \Gamma^2\right)}{\Delta} > 0,\\ a_2^f &= \frac{\alpha\beta}{\delta}\frac{\Gamma^2}{\Delta} > 0,\\ a_3^f &= \frac{\lambda}{\left(\lambda + \Gamma\left(1-\alpha\right)\right)} \geq 0,\\ b_1^f &= \frac{\lambda\left(\lambda + \Gamma^2\right)}{\Delta} \geq 0,\\ b_2^f &= \frac{\alpha\beta}{\delta}\frac{\lambda\Gamma}{\Delta} \geq 0,\\ b_3^f &= -\frac{\lambda\left(1-\alpha\right)}{\left(\lambda + \Gamma\left(1-\alpha\right)\right)} \leq 0. \end{aligned}$$

The reduced forms for the other endogenous variables in the monetary union case are

$$\begin{aligned} p &= w + (1-\alpha)\, m^u - \frac{(1-\alpha)}{2}\,(w + w^*) \\ &\quad + \frac{1}{2}\frac{(1-\alpha)\,\delta}{\alpha + (1-\alpha)\,\delta}\,(w^* - w)\,, \end{aligned} \tag{50}$$

$$\begin{aligned} p^* &= w^* + (1-\alpha)\, m^u - \frac{(1-\alpha)}{2}\,(w + w^*) \\ &\quad - \frac{1}{2}\frac{(1-\alpha)\,\delta}{\alpha + (1-\alpha)\,\delta}\,(w^* - w)\,, \end{aligned} \tag{51}$$

$$z = \frac{\alpha}{\alpha + (1-\alpha)\,\delta}\,(w^* - w)\,. \tag{52}$$

The nominal wage and unemployment target elasticities of unemployment and inflation in the monetary union case are

$$\begin{aligned} a_1^u &= \left(\frac{(1-\alpha)\,(\alpha + 2\,(1-\alpha)\,\delta) + \delta\lambda}{\Omega}\right) > 0, \\ a_2^u &= \left(\frac{\alpha\,(1-\alpha) - \delta\lambda}{\Omega}\right) \lesseqgtr 0, \\ a_3^u &= \frac{\lambda}{\lambda + (1-\alpha)^2} \geq 0, \\ b_1^u &= \frac{\left(\lambda + (1-\alpha)^2\right)(2\alpha\,(1-\beta) + (1-\alpha)\,\delta)}{\Omega} \\ &- \frac{(1-\alpha)^2\,(\alpha + (1-\alpha)\,\delta)}{\Omega} > 0, \\ b_2^u &= \frac{\left(\lambda + (1-\alpha)^2\right)(2\alpha\beta + (1-\alpha)\,\delta)}{\Omega} \\ &- \frac{(1-\alpha)^2\,(\alpha + (1-\alpha)\,\delta)}{\Omega} \lesseqgtr 0, \\ b_3^u &= \frac{-\lambda\,(1-\alpha)}{\left(\lambda + (1-\alpha)^2\right)} \leq 0, \end{aligned}$$

where $\Omega = 2\left(\lambda + (1-\alpha)^2\right)(\alpha + (1-\alpha)\,\delta) > 0$.

A.2 Derivation of Equation (35)

Under a flexible exchange rate, from the reduced forms (14) and (15) one obtains

$$\begin{aligned} a_1^f &= -\frac{\partial m^f}{\partial w} + 1, \\ 1 - b_1^f &= (1-\alpha)\left(1 - \frac{\partial m^f}{\partial w}\right) - \frac{\alpha\beta}{\delta}\left(\frac{\partial m^f}{\partial w} - 1 - \frac{\partial m^{*f}}{\partial w}\right), \end{aligned}$$

and by noting, that $\frac{\partial m^*}{\partial w} = -a_2^f$ (see the Appendix A.1) the slope coefficient (35) can be written as

$$\frac{1 - b_1^f}{a_1^f} = (1-\alpha) + \frac{\alpha\beta}{\delta}\left(\frac{a_1^f - a_2^f}{a_1^f}\right).$$

Similarly, in a monetary union case one obtains

$$\begin{aligned} a_1^u &= -\frac{\partial m^u}{\partial w} + \frac{1}{2} + \frac{\delta}{2\left(\alpha + (1-\alpha)\,\delta\right)}, \\ 1 - b_1^u &= (1-\alpha)\left(-\frac{\partial m^u}{\partial w} + \frac{1}{2} + \frac{\delta}{2\left(\alpha + (1-\alpha)\,\delta\right)}\right) \\ &\quad + \frac{\alpha\beta}{\left(\alpha + (1-\alpha)\,\delta\right)}. \end{aligned}$$

From equations (1), (3) and (4) one gets $z = \frac{\alpha}{\delta}(l - l^*) = -\frac{\alpha}{\delta}(u - u^*)$ and $\frac{\partial z}{\partial w} = -\frac{\alpha}{\delta}(a_1 - a_2)$. From (52) one obtains $\frac{\partial z}{\partial w} = -\frac{\alpha}{\alpha + (1-\alpha)\delta}$, which implies that

$$\frac{\alpha}{\alpha + (1-\alpha)\,\delta} = \frac{\alpha}{\delta}\left(a_1^u - a_2^u\right).$$

Inserting this expression into $1 - b_1^u$ above yields

$$1 - b_1^u = (1-\alpha)\,a_1^u + \frac{\alpha\beta}{\delta}\left(a_1^u - a_2^u\right).$$

Hence, the form of the slope coefficient is similar to the flexible exchange rate case

$$\frac{1 - b_1^u}{a_1^u} = (1-\alpha) + \frac{\alpha\beta}{\delta}\left(\frac{a_1^u - a_2^u}{a_1^u}\right).$$

A.3 Derivation of the Impact of Conservatism on Equilibrium Unemployment

The rates of equilibrium unemployment in both regimes are given by equation (33). Then, equation (35) implies, that $sgn\left(\frac{\partial u}{\partial \lambda}\right) = -sgn\left(\partial\left(\frac{a_2}{a_1}\right)/\partial\lambda\right)$ in both regimes. From Appendix A.1 one obtains $\frac{a_2^f}{a_1^f} = \frac{\left(\frac{\alpha\beta}{\delta}\right)\Gamma}{\lambda+\Gamma^2}$ and $\frac{a_2^u}{a_1^u} = \frac{\alpha(1-\alpha)-\delta\lambda}{(1-\alpha)(\alpha+2(1-\alpha)\delta)+\delta\lambda}$. Differentiation of these expressions with respect to λ yields $\partial\left(\frac{a_2^f}{a_1^f}\right)/\partial\lambda < 0$ and $\partial\left(\frac{a_2^u}{a_1^u}\right)/\partial\lambda = \frac{-\delta((1-\alpha)(2\alpha+2(1-\alpha)\delta))}{((1-\alpha)(\alpha+2(1-\alpha)\delta)+\delta\lambda)^2} < 0$. Hence $\frac{\partial u^f}{\partial\lambda} > 0$ and $\frac{\partial u^u}{\partial\lambda} > 0$.

A.4 Derivation of Equation (41)

Equation (35) implies, that $sgn\left(u^u - u^f\right) = -sgn\left(\frac{a_2^u}{a_1^u} - \frac{a_2^f}{a_1^f}\right)$. The expression in brackets on the right hand side can be written as

$$\begin{aligned}
&\frac{a_2^u}{a_1^u} - \frac{a_2^f}{a_1^f} = \frac{\alpha(1-\alpha)-\delta\lambda}{(1-\alpha)(\alpha+2(1-\alpha)\delta)+\delta\lambda} - \frac{\frac{\alpha\beta}{\delta}\left(1-\alpha+\frac{\alpha\beta}{\delta}\right)}{\lambda+\left(1-\alpha+\frac{\alpha\beta}{\delta}\right)^2} \\
= \;& \delta\frac{\left(-\delta^2\lambda^2+\left(\delta\left(\alpha(1-\alpha)-\delta(1-\alpha)^2\right)-\alpha\beta\left(3\delta(1-\alpha)+2\alpha\beta\right)\right)\lambda\right)}{\left(2\delta(1-\alpha)^2+\alpha(1-\alpha)+\lambda\delta\right)\left(\lambda\delta^2+\delta^2(1-\alpha)^2+2\alpha\beta\delta(1-\alpha)\right)} \\
&+\delta\frac{\alpha(1-\alpha)^2(1-2\beta)\left(\delta(1-\alpha)+\alpha\beta\right)}{\left(2\delta(1-\alpha)^2+\alpha(1-\alpha)+\lambda\delta\right)\left(\lambda\delta^2+\delta^2(1-\alpha)^2+2\alpha\beta\delta(1-\alpha)\right)}.
\end{aligned}$$

The denominator is always positive, and thus the quadratic function of λ in the numerator determines the sign of the expression. Since the constant term in the numerator is positive, and the coefficient in front of the quadratic term is negative, it follows that the quadratic function has only one positive real root. Solving the quadratic equation

$$\begin{aligned}
-\delta^2\lambda^2+\left(\delta\left(\alpha(1-\alpha)-\delta(1-\alpha)^2\right)-\alpha\beta\left(3\delta(1-\alpha)+2\alpha\beta\right)\right)\lambda \\
+\alpha(1-\alpha)^2(1-2\beta)\left(\delta(1-\alpha)+\alpha\beta\right)=0
\end{aligned}$$

yields two real roots, of which only the bigger one is positive

$$\begin{aligned}
\overline{\lambda} &= \frac{1}{2\delta^2}\left(-A+\sqrt{A^2+B}\right), \\
A &= (1-\alpha)^2\delta^2+\alpha(1-\alpha)(3\beta-1)\delta+2\alpha^2\beta^2, \\
B &= 4\delta^2\alpha(1-\alpha)^2(1-2\beta)\left(\alpha\beta+\delta(1-\alpha)\right) > 0.
\end{aligned}$$

Hence, $\left(\frac{a_2^u}{a_1^u} - \frac{a_2^f}{a_1^f}\right) > 0$ when $\lambda \in [0, \overline{\lambda})$ and $\left(\frac{a_2^u}{a_1^u} - \frac{a_2^f}{a_1^f}\right) < 0$ when $\lambda \in (\overline{\lambda}, \infty)$. Since $sgn\left(u^u - u^f\right) = -sgn\left(\frac{a_2^u}{a_1^u} - \frac{a_2^f}{a_1^f}\right)$, it follows that $u^u < u^f$ provided that $\lambda \in [0, \overline{\lambda})$.

A.5 Derivation of Equation (42)

The nominal wage elasticities of unemployment in a flexible exchange rate regime with infinite substitutability between goods, $\delta \to \infty$, can be written as $a^f_{1|\delta\to\infty} = \frac{(1-\alpha)}{\lambda+(1-\alpha)^2}$ and $a^f_{2|\delta\to\infty} = 0$. Unemployment is then given by

$$u^f_{|\delta\to\infty} = \gamma\left((1-\alpha) + \frac{\alpha\beta}{\delta}\left(1 - \frac{a_2^f}{a_1^f}\right)\right) = \gamma\left(1-\alpha\right).$$

In a monetary union the nominal wage elasticities of unemployment are

$$\begin{aligned} a^u_{1|\delta\to\infty} &= \left(\frac{2\left(1-\alpha\right)^2 + \lambda}{2\left(\left(1-\alpha\right)^2 + \lambda\right)\left(1-\alpha\right)}\right), \\ a^u_{2|\delta\to\infty} &= -\frac{\lambda}{2\left(\left(1-\alpha\right)^2 + \lambda\right)\left(1-\alpha\right)}. \end{aligned}$$

It follows that $\frac{a_2^u}{a_1^u} = -\frac{\lambda}{2(1-\alpha)^2+\lambda}$, and unemployment is given by

$$u^u_{|\delta\to\infty} = \gamma\left((1-\alpha) + \frac{\alpha\beta}{\delta}\left(1 - \frac{a_2^u}{a_1^u}\right)\right) = \gamma\left(1-\alpha\right).$$

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Chapter 3

Labor Market Flexibility and Policy Coordination in a Monetary Union

Abstract

This essay investigates the political economy of labor market flexibility in a monetary union. Using a natural rate monetary model augmented with unemployment persistence it is shown that labor market policy coordination between member country governments tends to increase nominal wage flexibility under discretionary monetary policy. The reason is that flexibility choice under coordination interacts with the stabilization properties of the economy through the common monetary policy. In particular, coordination internalizes the benefit of reduced stabilization bias of monetary policy, which is brought about by increased labor market flexibility. However, if the common central bank has a state-dependent unemployment target, the result can be overturned. Policy coordination is irrelevant for flexibility choice if the common central bank has access to a commitment technology or there is no unemployment persistence.

1 Introduction

Labor market flexibility is often suggested as a remedy for European unemployment problems. Recently the EU Council (2002) urged the member countries on to continue implementing further structural reforms to improve the functioning of the European labour market, innovation and competitiveness, and to examine the possibility of introducing more flexible labor contracts into their national law. In their reports, international organizations such as the IMF and the OECD quite seldom miss the opportunity to express their worries about the inflexible labor markets in various European countries (see e.g. IMF (2002) and OECD (2002)). The European Central Bank (ECB) has also continuously advised the member countries of the Economic and Monetary Union (EMU) to continue reforming the labor markets in order to increase flexibility (see e.g. ECB (2002)).

There seems to be a consensus of opinion that flexible labor markets are especially important in a large monetary union, such as the EMU, where some shocks to the economy

are very likely asymmetric so that only some regions or countries of the monetary union area are affected. Labor mobility in the EMU is relatively low, partly due to cultural differences and language barriers, so one can't count much on mobility as a shock absorber mechanism. National fiscal policy could in principle be used as a macroeconomic stabilization tool, but acknowledging that due to political biases fiscal policy tends to err on the lax side has led to a general perception that discretionary fiscal policy should be restricted by rules limiting the size of government deficits. Moreover, the lags in decision making and policy implementation make the correct timing of discretionary fiscal policy extremely difficult. The importance of prudent fiscal policy becomes even more pronounced in a monetary union, where fiscal profligacy of a member country can threaten the stability of the whole area through financial market reactions. In the EMU the Stability and Growth Pact was launched in order to enforce fiscal discipline. However, even without discretionary fiscal policy the government budget still has a stabilization role if the so-called automatic stabilizers are allowed to work, that is, the loss of fiscal revenue and the increase in welfare spending due to a downturn are not counteracted.

Against this background, it is quite natural to ask what factors do determine the flexibility of the national labor markets in a monetary union, and what can policy makers do in order to promote flexibility. This essay attempts to give at least partial answers to these questions by adopting a simple political economy approach, in which national governments decide on nominal wage flexibility and face political costs, which are increasing in the chosen level of flexibility. In particular, the essay investigates the role of labor market policy coordination in the determination of flexibility. Coordination refers to a situation where the flexibility parameters of the member states are chosen so as to minimize joint losses of the national governments, which result from macroeconomic volatility and political costs of making labor markets flexible. In the EU labor market policy coordination intensified with the 1997 Amsterdam Treaty, which declares that member states shall treat employment "as a matter of common concern, and shall coordinate their action". However, in practice the main responsibility of deciding and implementing labor market policy remains with the member states, and so coordination is a somewhat stronger concept in the model than it is in reality in the EU.

Labor market flexibility is a frequently used, but somewhat vague concept. One interpretation is that flexibility determines how fast is the adjustment process of the economy in response to shocks. But, one labor market can be more flexible in responding to a one kind of shock, whereas another labor market can be more flexible when the adjustment is in response to another kind of shock. Therefore, ranking of different labor markets with respect to flexibility can be meaningfully made only in the context of a

particular model of the labor market and with reference to a particular shock (for more discussion on this point, see Pissarides (1997)).

In this essay labor market flexibility is modelled as nominal wage flexibility. Increasing nominal wage flexibility brings about two benefits. First, since nominal wages become more responsive to production shocks, unemployment variability decreases. Second, the need for stabilization policy by the common central bank is reduced, which contributes to lower inflation variability. The degree of nominal wage flexibility is decided by the national government. This can be seen as a simplification of a complicated process where labor market institutions must be designed and legislation must be enacted in a parliamentary process. Changing the institutional setup of the economy is assumed to be so costly and time-consuming that the government sets the flexibility only once and it can't change it afterwards. However, making the labor markets flexible entails political costs, since nominal wage flexibility is not in the interests of labor unions or their leaders. One can argue that the opposition of flexibility arises from distributional conflicts among the workforce. Labor unions are usually considered to be in the hands of senior members, whose jobs are fairly safe. Long-term nominal wage contracting will transform disturbances to labor demand into employment fluctuations, but at the same time it provides more stable and more easily predictable incomes than shorter or more flexible contracts for senior members who are sheltered from unemployment. When the government sets flexibility, it therefore faces a tradeoff between macroeconomic stability and political popularity.

The framework used in the analysis is based on the natural rate model of monetary policy pioneered by Kydland and Prescott (1977) and Barro and Gordon (1983).[1] This approach builds on the idea of a natural rate of unemployment, which was developed by Phelps (1967) and Friedman (1968). They argued on theoretical grounds that the inflation-unemployment tradeoff is a short-run phenomenon, and that it can't be exploited in the long run. Any attempt to lower unemployment below the natural rate by generating inflation will lead to higher inflation without any effect on unemployment. Lucas (1973) derives a neoclassical Phillips curve, or an expectations augmented Phillips curve, where unanticipated changes in money supply generate short-run effects in real economy due to imperfect information on the source of price changes. Fischer (1977), on the other hand, arrives at the very same formulation by modelling labor markets with long-term nominal contracting. The Barro-Gordon model adds an optimizing policy maker into the analysis and points out that the policy maker may have an incentive to exploit the short-run

[1] The standard natural rate model is usually called the Barro-Gordon model, a convention which is somewhat unfair to the original contribution by Kydland and Prescott (1977). For brevity, this usual practice is also followed in this essay.

tradeoff between unexpected inflation and unemployment in order to push unemployment below the natural rate. Provided that the private sector has rational expectations, the government's attempt is anticipated and the result is the celebrated inflation bias without any improvement in the average unemployment rate.

Modelling of monetary policy in this essay deviates from the standard Barro-Gordon model in two respects. First, there is persistence in the actual unemployment rate. Second, the central bank is assumed to target the "long-run" natural rate instead of having an "over-ambitious" unemployment target. Both of these assumptions will affect the equilibrium of the model and are discussed below. Monetary policy is modeled as being discretionary in the model, a view which is in line with the thinking of e.g. Blinder (1998), Bernanke and Mishkin (1997) and King (1997), who are not only prominent academics, but they all are or were at some points of their careers also central bankers. However, a commitment solution for monetary policy is also derived, but it is used only as a reference point, since it yields optimal stabilization of shocks from the society's point of view.

Empirically it is well established that the unemployment rate is highly persistent. Blanchard and Summers (1986) and Layard, Nickell and Jackman (1994) report that when unemployment is regressed on one-period lagged unemployment, the coefficient estimate on the latter is close to unity in annual data for both the UK and the USA. Layard, Nickell and Jackman (1991, Ch. 9) estimates reduced-form employment equations for 19 OECD countries and uses two different measures of persistence. The results indicate that unemployment is highly persistent in most cases.

Unemployment persistence can arise from at least two sources. There may be labor adjustment costs affecting the firms' hiring and firing decisions. Alternatively, the wage setting process itself can work as a source for unemployment dynamics. This essay adopts the second option.[2] In particular, in the spirit of Lindbeck and Snower (1986) and following Blanchard and Summers (1986), it is shown that when the wage setting process is dominated by insiders of the labor market, the effects of production shocks tend to persist, since changes in the number of insiders will lead to changes in the objectives of the labor unions. The insider-outsider approach to labor markets can be applied in many different questions related to employment and unemployment outcomes. A recent survey and assessment of the insider-outsider literature is provided by Lindbeck and Snower (2001).

[2]The second option is advocated e.g. by Blanchard (1991), who writes "I believe–without, I must admit, a tightly argued case at this stage–that, for the issue at hand, the medium-run persistence of unemployment, the dynamics coming from wage bargaining dominate those coming from search, firing, and hiring costs."

When unemployment persistence is incorporated into the Barro-Gordon model the central bank's problem becomes explicitly dynamic in nature. When setting monetary policy, the central bank has to bear in mind that its policy will affect unemployment not only in the current period, but in the future as well. It follows that it becomes more important to stabilize production shocks, because once unemployment rises, it will take long time before it returns to the initial level. Needless to say that this reasoning presupposes that the central bank is forward-looking. Lockwood and Philippopoulos (1994) first analyzed the natural rate model with unemployment persistence under discretionary monetary policy and showed that inflation bias problem is aggravated in the dynamic model and that there are multiple equilibria and indeterminacy problems. Suggested solutions to these problems will be discussed later in the essay. Svensson (1997) and Lockwood, Miller and Zhang (1998) showed that optimal stabilization is increasing in persistence and that discretionary monetary policy leads to a new problem, that is, a stabilization bias emerges. Inflation variability is too high and unemployment variability is too low from the society's point of view. This inefficient stabilization performance will play an important role in the analysis.

The central bank targets the long-run natural rate in the model, which means that the problem of inflation bias disappears, since the central bank's incentives to use monetary policy as a device in lowering the rate of unemployment below the natural rate are destroyed. A justification for rejecting the inflation bias argument is given in the following. First, it must be acknowledged, that the inflation bias model can be seen as a rather good description of the incentives facing politicians when they were in charge of monetary policy. Then, it is quite natural that if the natural rate is too high from the society's point of view, governments may be tempted to use monetary policy to bring down unemployment. As already said, this results in inflation bias, but the average unemployment rate is unchanged. Obviously, structural measures to lower imperfections in product and labor markets were actually needed to combat unemployment, but implementing such measures are assumed to be politically costly, and therefore politicians are inclined to take the soft option. However, when monetary policy is delegated to an independent central bank, there seems no reason whatsoever for the central bank to target anything other than the natural rate. Casual evidence suggests that this is indeed the case. In the 1990s central bank acts were reformed in many countries toward greater independence from political pressures, which was followed by convergence to low inflation.

As an extension to the basic model the essay considers an alternative unemployment target for the central bank and its implications for labor market flexibility and coordination. Due to persistence the short-run natural rate of unemployment differs from the

long-run one. Svensson (1997) argued that the discretionary equilibrium can be improved on by directing the central bank to target the short-run natural rate, and the outcome then would coincide with the commitment solution, which yields optimal stabilization of shocks. However, Røisland (2001) showed that this conjecture is incorrect.[3] The central bank's reaction is the same as in an economy without any real persistence, which means that the central bank stabilizes shocks too little. The short-run natural rate is defined as the unemployment level without inflation surprises and production shocks, that is, it is equal to last period's unemployment multiplied with the persistence parameter of the unemployment process. It follows that the short-run natural rate is endogenous to the central bank. If, however, the central bank misinterprets the changes in the natural rate as arising solely from structural factors and hence fails to recognize the impact of its own actions, it can justify the choice of the short-run natural rate as its target. This type of central bank resembles, but is a milder version of "new conservatism" launched by De Grauwe (2000, Ch. 7). The new conservatism refers to a central bank who systematically estimates the natural unemployment rate to coincide with the observed unemployment rate. According to De Grauwe there is a risk that the ECB may behave like this, and he argues that the policy statements of the ECB often associate the observed level unemployment with structural unemployment.

The modeling strategy of the essay is very similar to that of Calmfors (2001), where the flexibility comparison is made between national monetary policy regime and a membership in a monetary union. Calmfors (2001) allows flexibility to interact with the long-run natural rate and also studies the case where inflation bias is present. However, a notable difference to this essay is that there is no persistence in the economy and coordination of labor market policies is not considered. Another closely related paper by Sibert and Sutherland (2000) studies the same problem as Calmfors (2001), but there flexibility is of more limited nature and can't be interpreted as perfect nominal wage flexibility, because there it affects only the propagation of production shocks to the real economy, leaving inflation surprises still as an effective means to control real activity.[4] The impact of the formation of a monetary union on the need and incentives for flexibility enhancing labor market reform is also discussed and analyzed e.g. in Pissarides (1997), Bean (1998) and Saint-Paul and Bentolila (2000).

This essay identifies at least some of the factors affecting the chosen level of labor mar-

[3] Øistein Røisland kindly informed the author of this fact after a working paper version (Rantala (2003)) of this essay was published.

[4] Sibert and Sutherland's (2000) way of modelling flexibility is consistent with a timing protocol, where wage negotiations occur after the production shock has been realized and observed, but before monetary policy is set in each period.

ket flexibility. In a monetary union the common monetary policy reacts only to common shocks, and therefore the structure of the shocks in the economy is a crucial factor. As expected, the more asymmetric the shocks are, the higher is the chosen level of nominal wage flexibility. Higher marginal political costs in turn discourage flexibility, since the marginal benefit from flexibility is unaltered. Higher persistence leads to prolonged deviations from the natural rate and, quite intuitively, encourages the government to choose more flexible markets.

The monetary union is assumed to be large in the sense that each government is small relative to the central bank so that it will take monetary policy as given. Coordination of national labor market policies, however, brings in a strategic aspect to the model. It is shown that stabilization policy of the central bank is affected by the average flexibility in the monetary union area so that by choosing more flexibility it is possible to improve the stabilization properties of monetary policy. In particular, by choosing more flexibility in a coordinated fashion, the governments can reduce the stabilization bias of monetary policy. Therefore, the marginal benefit from flexibility is higher under coordination, and coordination results in more flexible labor markets than national decision making. But, coordination turns out to be irrelevant for flexibility, if stabilization policy is optimal from the society's point of view. This is the case when there is no persistence in the economy or the central bank has access to a commitment technology.

If the central bank is of new conservative type described above, and stabilizes shocks too little, coordination leads to lower flexibility, thereby pushing the stabilization policy closer to the optimum. The essay discusses also the impact of coordination on social welfare and the government incentives for coordination. According to numerical simulations, it seems that social welfare is higher and government losses are lower under coordination, implying that coordination is welfare improving and that governments do have incentives to coordinate their actions. However, in the new conservatism regime social welfare decreases under coordination, but the reduction of political costs due to lower flexibility reduces government losses. The result is that governments have incentives to coordinate their actions, even though it brings about lower social welfare.

Finally, the essay briefly investigates a situation where a country is not a member of a monetary union, but has an independent monetary policy.[5] Calmfors (2001) shows that the chosen level of labor market flexibility will then be lower than inside a monetary union, because monetary policy can stabilize the country-specific asymmetric part of the production shock, rather than just the common part like in a monetary union. This

[5] Independent monetary policy in this context means that a country can and is willing to use monetary policy as a stabilization tool. Hence, the exchange rate regime can e.g. be a flexible rate regime or a fixed but adjustable rate one.

result is in line with the TINA (there is no alternative) argument discussed in Bean (1998), which states that once policy makers lose monetary policy as a stabilization tool, they will have no alternative but to make labor markets more flexible via reform. It turns out that this is not necessarily the case when unemployment persists. As under policy coordination in a monetary union, under a national monetary policy the government is in a strategic position and has an incentive to choose more flexible markets to improve the stabilization properties of the economy. But, in a large monetary union this effect disappears. Hence, in a monetary union flexibility tends to be more important because monetary policy can no longer stabilize country-specific shocks, but on the other hand, the disappearance of the incentive to increase flexibility for stabilization reasons will lower the chosen level of flexibility. Numerical simulations suggest that the latter effect dominates when persistence is high enough, and so flexibility can actually be lower in a monetary union.

This essay is organized as follows. In Section 2 the basic structure of the model is presented and the related Phillips curves are derived. The common central bank's optimal policy rule under both commitment and discretion is presented in Section 3. Finally, Section 4 analyzes the determination of labor market flexibility assuming first that flexibility decisions are made independently by the national governments. After deriving this benchmark case the focus shifts to policy coordination. Section 5 considers some extensions to the model and studies the impact of coordination on social welfare. Section 6 concludes.

2 The Model

2.1 The Basic Structure

The monetary union consists of a continuum of countries indexed by i and distributed uniformly between $i = 0$ and $i = 1$. Each country contains a continuum of firms indexed by j and distributed uniformly between $j = 0$ and $j = 1$. For simplicity, the firms in all countries produce the same product, so the goods market is perfectly competitive.[6] Throughout the essay, all countries are symmetrical in terms of structural parameters

[6] Alternatively, the product markets in each country could be monopolistically competitive. This assumption would make the structure of the model more complicated, because firms would have pricing power in the markets. Then the central bank could not be assumed to control inflation directly, as it does in this essay. Coricelli, Cukierman and Dalmazzo (2000, 2001) and Soskice and Iversen (2000) consider models where the central bank sets money supply and firms choose prices. Their model setups, however, are static. In this essay the game between the central bank and the labor unions is dynamic, which will become apparent in Section 3. It is unclear how the equilibrium strategies of the three player types would look like in such a dynamic setup. This task is left for future research.

and preferences of the policy makers and labor unions. Only one type of asymmetry is allowed, namely production shocks have a country-specific part.

The model economy evolves over an infinite number of time periods $t = 0, 1, ...$ At $t = 0$, each national government chooses its preferred labor market flexibility, denoted by s_i. Following Calmfors (2001), labor market flexibility is introduced into the model by assuming that in each country there is a flexible sector and an inflexible sector of the economy. Slightly simplifying Calmfors's model, a fraction s_i of the firms is characterized by competitive (spot) labor markets where labor supply and labor demand together determine the equilibrium wage, whereas in a fraction $1 - s_i$ of the firms one period nominal wage contracting prevails and employment is demand determined.[7] After flexibility is determined in all countries, a dynamic game between the common central bank and the private sectors of all countries begins at $t = 1$. Within each time period events unfold as follows. (i) Firm-specific labor unions set nominal wages in the inflexible firms in each country. (ii) Production shocks are realized. (iii) The central bank sets inflation for the whole monetary union area. (iv) Competitive nominal wage is determined in the flexible firms and production occurs in each country.[8]

2.2 The Labor Market

In every period t, firm j's labor demand schedule in country i is given by

$$l^d_{jit} = p_{it} - w_{jit} - \varepsilon_{it}, \tag{1}$$

where l_{jit} is the log of the employment level, p_{it} is the log of the price level, w_{jit} is the log of the nominal wage and ε_{it} is a country i specific production shock, which has zero mean and a variance equal to σ^2_ε.[9] Each firm is assumed to have a pool of immobile workers, whose labor supply schedule is perfectly inelastic with respect to the real wage and is normalized to zero[10]

$$l^s = 0. \tag{2}$$

In the flexible firms characterized by competitive labor markets labor demand equals

[7] In Calmfors (2001) there is a large number of production sectors in the economy, and a large number of firms in each sector. Some sectors are flexible and some inflexible.

[8] It could equally well be assumed that in the flexible firms labor unions still determine wages, but instead of setting nominal wages at stage (i) before production shocks and monetary policy, they would act at stage (iv).

[9] Cobb-Douglas production function with fixed capital stock and α as the labor share yields $(1 - \alpha)^{-1}$ as the (absolute value) of the real wage elasticity of labor demand. As in Lockwood and Philippopoulos (1994) this elasticity is normalized to unity for analytical simplicity.

[10] Complete symmetry across firms, countries and time periods implies: $l^s_{jit} = l^s_{it} = l^s_t = l^s$.

labor supply at every time period, and employment is hence constant at zero, $\widehat{l}_{jit} = 0$, which yields zero unemployment in competitive markets, $\widehat{u} = l^s - \widehat{l}_{jit} = 0$. The spot market nominal wage becomes

$$\widehat{w}_{it} = p_{it} - \varepsilon_{it}, \tag{3}$$

and thereby depends both on the realized production shock and monetary policy reaction, which in turn determines the price level.

In the inflexible firms workers are organized in firm-specific labor unions, who set one-period nominal wages before the production shock is realized and inflation is chosen by the central bank. Firms choose employment after the shock, so employment is demand determined and workers may be off their labor supply schedules. The utility function of a firm-specific labor union is given by

$$U_{jit} = -E_{t-1}\left(l_{jit} - \overline{l}_{jit}\right)^2, \tag{4}$$

where $\overline{l}_{jit}$ is the period t employment target of the labor union and E_{t-1} is a conditional expectations operator. In a standard labor union model the union's employment target, $\overline{l}_{jit}$, is assumed to be constant and quite often below the full employment level. These assumptions bring about a constant positive natural rate of unemployment. Here, the employment target is assumed to be time varying and affected by "insider power". In the spirit of Lindbeck and Snower (1986), and following Blanchard and Summers (1986), it is assumed that the employment target of the union is a weighted (geometric) mean of the currently employed insiders, l_{jit-1}, and the total labor force available to the firm, l^s

$$\overline{l}_{jit} = \rho l_{jit-1} + (1-\rho)\, l^s, \tag{5}$$

where ρ, $(0 \leq \rho < 1)$ is the measure of insider power in wage setting.[11] By setting $\rho = 0$, the model collapses to the standard case with a constant employment target.

The union's optimization problem is to set w_{jit} so as to maximize (4). Maximization yields a nominal wage rule of the form

$$w_{jit} = p^e_{it} - \left(\rho l_{jit-1} + (1-\rho)\, l^s\right), \tag{6}$$

with p^e_{it} being the expectation of the price level. Together with the labor demand schedule (1) this gives

$$l_{jit} = \rho l_{jit-1} + (1-\rho)\, l^s + \left(p_{it} - p^e_{it}\right) - \varepsilon_{it}. \tag{7}$$

[11] As is clear from (2), l^s is actually normalized to zero in the model. This assumption will be utilized in (8).

Defining inflation as $\pi_{it} = p_{it} - p_{it-1}$, from which $\pi^e_{it} = p^e_{it} - p_{it-1}$ follows, and defining the firm-specific unemployment rate at time t to be $u_{jit} = l^s - l_{jit}$, it is possible to derive a firm/union specific neoclassical (expectations augmented) Phillips curve with unemployment persistence

$$u_{jit} = \rho u_{jit-1} - (\pi_{it} - \pi^e_{it}) + \varepsilon_{it}. \tag{8}$$

In this simple setup the parameter capturing the insider power in wage setting is translated directly into the measure of persistence in unemployment rate. The labor union's employment target is in the long run consistent with the total labor force available to the firm, l^s. Thereby unemployment fluctuates around the full employment level and the existence of labor unions doesn't create unemployment in the long run in this model. Hence, flexibility and average unemployment are independent of each other. Increasing nominal wage flexibility doesn't lower average unemployment, but affects the temporal properties of the unemployment rate. The fact that average unemployment is zero in the model is just a normalization. It could equally well be assumed that the long-run employment target of the union is lower than full employment, in which case the average unemployment would be positive in the unionized sector of the economy. Likewise, the competitive markets could be assumed to have some matching frictions, which would cause frictional unemployment. The model setup is chosen because by setting the average unemployment in both flexible and inflexible sectors the same, it is possible to isolate the pure stabilization motive to make the economy more flexible from other possible reasons to promote flexibility.

The country-wide neoclassical Phillips curve is obtained by integrating over the firm specific Phillips curves in both union dominated firms and firms with competitive labor markets with respect to j, which gives

$$\begin{aligned} u_{it} &= \int_0^1 u_{jit} dj = \int_0^{s_i} \widehat{u}_{jit} dj + \int_{s_i}^1 u_{jit} dj \\ &= \rho u_{it-1} - (1 - s_i)(\pi_{it} - \pi^e_{it}) + (1 - s_i)\varepsilon_{it}, \end{aligned} \tag{9}$$

where $u_{it-1} = \int_0^1 u_{jit-1} dj = \int_{s_i}^1 u_{jit-1} dj$. Nominal wage flexibility affects the slope of the Phillips curve and the unemployment effect of the production shock, but persistence of the economy is unaffected. At first it may seem strange that the persistence of the aggregate unemployment is independent of labor market flexibility. However, the intuition behind this outcome is clear. An increase in flexibility will have a "level effect" on unemployment rate so that after, say, a negative production shock u_{it} will not rise as much as with lower

flexibility. But, there is no "speed effect" in increasing flexibility, since unemployment in union dominated firms with long-term nominal contracts will return toward the long-run equilibrium at a constant rate, which is unaffected by flexibility.

The final step in developing the basic structure of the model is to aggregate the country-specific Phillips curves into a monetary union wide one. As mentioned, the production shock consists of a common, monetary union wide, part μ_t and an asymmetric, country specific, part ν_{it}

$$\varepsilon_{it} = \mu_t + \nu_{it}, \tag{10}$$

where the two shocks are independent and symmetrically distributed and have zero means and variances σ_μ^2 and σ_ν^2, respectively.[12] Integrating (9) over i gives

$$\begin{aligned} u_t &= \int_0^1 u_{it} di \\ &= \int_0^1 \left(\rho u_{it-1} - (1-s_i)\left(\pi_{it} - \pi_{it}^e\right) + (1-s_i)\,\mu_t + (1-s_i)\,\nu_{it}\right) di \\ &= \rho u_{t-1} - (1-s)\left(\pi_t - \pi_t^e\right) + (1-s)\,\mu_t, \end{aligned} \tag{11}$$

where $u_{t-1} = \int_0^1 u_{it-1} di$ and $s = \int_0^1 s_i di$. The country-specific shocks, ν_{it}, cancel out, because s_i is decided before the dynamic game between the common central bank and the private sectors of all countries starts, and is not allowed to be state-dependent.[13] Thus $1 - s_i$ is uncorrelated with the realization of the country-specific shock, ν_{it}, and according to the law of large numbers the integral $\int_0^1 (1-s_i)\,\nu_{it} di$ equals zero.

From (11) it is clear that the short-run natural rate of unemployment is different from the long-run natural rate. The former is the rate of unemployment in the absence of an inflation surprise and a production shock, which is obviously state-dependent and equal to ρu_{t-1} at time period t. The latter, the long-run natural rate, is the mean of the unemployment process and normalized here to zero.

[12] With this simple shock structure (equation (10)) the correlation of shocks between the member countries is given by

$$corr\left(\varepsilon_{it}, \varepsilon_{kt}\right) = \frac{cov\left(\varepsilon_{it}, \varepsilon_{kt}\right)}{\sqrt{var\left(\varepsilon_{it}\right)}\sqrt{var\left(\varepsilon_{kt}\right)}} = \frac{cov\left(\mu_t, \mu_t\right)}{\sqrt{var\left(\varepsilon_{it}\right)}\sqrt{var\left(\varepsilon_{kt}\right)}} = \frac{var\left(\mu_t\right)}{var\left(\varepsilon_{it}\right)} = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\nu^2}.$$

[13] Obviously, it may be the case that optimal s_{it} would depend on ν_{it}. This is ruled out in the analysis. Given the interpretation of s_i in the model, it is quite difficult to consider it as being adjustable over the business cycle.

2.3 Government and Central Bank Preferences

The intertemporal social loss function of country i is assumed to be of the standard quadratic form

$$SL_i = E_0 \left(\sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{2} \left((\pi_t - \pi^*)^2 + \lambda u_{it}^2 \right) \right), \tag{12}$$

where β $(0 < \beta < 1)$ is the discount factor, $\lambda > 0$ is the relative weight on unemployment stabilization, and π^* is the inflation target of the society.[14] The government's loss function of country i consists of the social loss function (12) and an additional term κs_i, which is interpreted as a political cost of making the economy more flexible

$$GL_i = E_0 \left(\sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{1}{2} \left((\pi_t - \pi^*)^2 + \lambda u_{it}^2 \right) + \kappa s_i \right) \right) \tag{13}$$

As discussed already in Introduction, a natural explanation for the political cost can be derived from labor union behavior in the model. The senior members of the unions can guarantee stable incomes for themselves by long-term nominal contracting, thereby letting unemployment to absorb shocks which are not stabilized by common monetary policy. By increasing the flexible part of the economy by choosing a higher s_i the government increases wage flexibility in the economy, which contradicts the preferences of the senior members of the labor unions.

The objective function of the common central bank is otherwise identical to the social loss function (12), but has the monetary union wide unemployment rate as an argument instead of a country-specific one

$$V = E_0 \left(\sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{2} \left((\pi_t - \pi^*)^2 + \lambda u_t^2 \right) \right) \tag{14}$$

It is assumed that the common central bank has the same discount rate, the same inflation target, and the same zero unemployment target as the society. Since the unemployment target equals to the long-run natural rate of zero, there is no average inflation bias in the model. For simplicity, the central bank is assumed to have perfect control over the inflation rate π_t. It sets the inflation rate in each period after the productivity shock is realized and observed.

[14] Again, symmetry across countries implies $\pi_i^* = \pi^*$.

3 Monetary Policy

3.1 The Dynamic Game

The equilibrium in the standard Barro-Gordon model is the one-shot Nash equilibrium, which is repeated indefinitely. Here, since unemployment is persistent (see equation (11)), the game between the central bank and the private sector is dynamic instead of being a repeated one. The natural rate model with unemployment dynamics was first analyzed in Lockwood and Philippopoulos (1994). The derivation of optimal monetary policy under different policy regimes in this section follows quite closely that of Svensson (1997).

The stabilization problem of the central bank is a dynamic one, since the extent to which u_t is stabilized after a shock will affect next period's $t+1$ short-run natural rate of unemployment, ρu_t. The state variable of this dynamic game is u_{t-1}, that is , u_{t-1} summarizes the impact of the history of the game on the current short-run natural rate of unemployment, ρu_{t-1}. The fact that the persistence of the natural rate is endogenous in the model is crucial for the outcome.[15] In Barro and Gordon (1983) the natural rate is persistent, but it is exogenous to the central bank. Then the problem is still static in nature and the only change is that the inflation bias is persistent.

The solution concept used in this essay is the Markov-perfect equilibrium, where the current actions of the players depend on the history of the game only through the state variable and trigger strategies are ruled out (see e.g. Fudenberg and Tirole (1991)). Since it is assumed that labor market flexibility is chosen before the dynamic game starts, $s = \int_0^1 s_i di$ is taken to be exogenous to the central bank.

3.2 Commitment as a Reference Point

Even though the purpose of this essay is to focus on the interaction of discretionary (time-consistent) monetary policy and endogenously determined labor market flexibility, it is useful to derive the commitment solution to the dynamic monetary policy game first. The commitment outcome serves as a reference point, because it yields optimal stabilization of production shocks from the society's point of view. It is, however, generally not considered to be a good description of actual monetary policy.

The optimal policy rule under commitment can be derived by utilizing the dynamic programming approach. The present value of losses to the central bank from t onward, given that the precommitment rule is chosen optimally, is denoted by $V^*(u_{t-1})$. Assuming

[15] To be precise, persistence parameter, ρ, is exogenous to the central bank, but u_{t-1} is endogenous.

that $V^*(u_{t-1})$ exists, the Bellman equation characterizing the central bank's optimization problem can be written as

$$V^*(u_{t-1}) = \min_{\pi_t, \pi_t^e} E_{t-1}\left(\frac{1}{2}\left((\pi_t - \pi^*)^2 + \lambda u_t^2\right) + \beta V^*(u_t)\right) \tag{15}$$

subject to (11) and

$$\pi_t^e = E_{t-1}\pi_t. \tag{16}$$

Since the central bank chooses its policy rule before inflation expectations are formed, it can actually choose the expectation π_t^e as well, subject to the requirement that π_t^e is rational. This implies that $\pi_t^e = E_{t-1}\pi_t$ is a constraint on the choice of policy rule by the central bank, not an identity.

The first-order conditions for a minimum with respect to π_t and π_t^e are

$$\begin{aligned}(\pi_t - \pi^*) - \lambda(1-s)u_t - \beta V_u^*(u_t)(1-s) + \phi_t &= 0, &(17)\\ E_{t-1}\left(\lambda(1-s)u_t + \beta V_u^*(u_t)(1-s)\right) - \phi_t &= 0, &(18)\end{aligned}$$

where ϕ_t is the Lagrange multiplier of (16). Eliminating the Lagrange multiplier yields

$$\begin{aligned}&(\pi_t - \pi^*) - \lambda(1-s)u_t - \beta V_u^*(u_t)(1-s) + \\ &E_{t-1}\left(\lambda(1-s)u_t + \beta V_u^*(u_t)(1-s)\right) = 0.\end{aligned} \tag{19}$$

Taking expectations at $t-1$ of (19) gives

$$E_{t-1}\pi_t = \pi^*, \tag{20}$$

which implies that the expected inflation rate equals the target rate.

The central bank's problem is linear-quadratic, so the "guess and verify" technique of solving dynamic programming problems can be applied.[16] In this class of problems the present value of losses to the central bank is a quadratic function of the state variable, u_{t-1}

$$V^*(u_{t-1}) = \gamma_0^* + \gamma_1^* u_{t-1} + \frac{1}{2}\gamma_2^* u_{t-1}^2. \tag{21}$$

Substituting equations (11), (16), (20) and $V_u^*(u_{t-1}) = \gamma_1^* + \gamma_2^* u_{t-1}$ into (19) and solving the equation for π_t gives the precommitment inflation rule

$$\pi_t = \pi^* + b^*\mu_t, \tag{22}$$

[16] See Ljungqvist and Sargent (2000, Ch. 2-4) for more discussion on this point.

where

$$b^* = \frac{(1-s)^2(\lambda + \beta\gamma_2^*)}{\left(1 + (1-s)^2(\lambda + \beta\gamma_2^*)\right)}. \tag{23}$$

Average unemployment in the monetary union will then follow the process

$$u_t = \rho u_{t-1} + (1-s)(1-b^*)\mu_t, \tag{24}$$

and country i unemployment is given by

$$u_{it} = \rho u_{it-1} + (1-s_i)(1-b^*)\mu_t + (1-s_i)\nu_{it}. \tag{25}$$

In order to solve b^*, the coefficient γ_2^* needs to be solved first. The coefficients γ_0^* and γ_1^* are not of interest here, since they don't enter the policy rule. Substituting equations (21)-(24) into the Bellman equation and identifying the coefficient of u_{t-1}^2 results in a Riccati equation in γ_2^*

$$\frac{1}{2}\gamma_2^* = \frac{1}{2}\lambda\rho^2 + \frac{1}{2}\beta\rho^2\gamma_2^*, \tag{26}$$

which gives

$$\gamma_2^* = \frac{\lambda\rho^2}{(1-\beta\rho^2)}. \tag{27}$$

Finally, b^* is solved by inserting (27) into (23), which yields

$$b^* = \frac{(1-s)^2\lambda}{\left(1 + (1-s)^2\lambda - \beta\rho^2\right)}. \tag{28}$$

It is noteworthy that the optimal inflation response to a common component of the production shock, b^*, is increasing with unemployment persistence. In the presence of persistence, a one-time shock to unemployment will affect the unemployment rate also in the future, whereby stabilizing unemployment becomes more important than in a world without persistence. It follows that inflation variability will increase with real persistence, but inflation itself doesn't persist. By setting $\rho = 0$ (and $s = 0$) (28) reduces to a standard static equilibrium in a natural rate model under commitment (see e.g. Persson and Tabellini (2000, Ch. 15)).

3.3 Optimal Monetary Policy under Discretion

In the discretionary regime the monetary authority can't commit to a certain policy rule before inflation expectations are formed, but it re-optimizes in every period after inflation expectations are set and the production shock is realized so as to minimize the

intertemporal loss function. Assuming that $V(u_{t-1})$ exists, the Bellman equation of the central bank's optimization problem can be written as

$$V(u_{t-1}) = E_{t-1} \min_{\pi_t} \left(\frac{1}{2} \left((\pi_t - \pi^*)^2 + \lambda u_t^2 \right) + \beta V(u_t) \right) \tag{29}$$

subject to (11). Since inflation expectations are now given, minimization is carried out only with respect to π_t. The central bank sets inflation after observing the production shock, therefore $\min_{\pi_t}$ is moved inside the expectations operator. Even though the central bank does not take into account the impact of its choice of inflation on inflation expectations, it does pay attention to the fact that current unemployment will have an effect on current expectations of future inflation through $V(u_t)$.

The first-order condition for a minimum in this problem is given by

$$(\pi_t - \pi^*) - \lambda (1 - s) u_t - \beta V_u(u_t) (1 - s) = 0. \tag{30}$$

Again, the value function must be quadratic, since the objective function is quadratic and the constraint is linear. The coefficients of the value function in the discretionary case are written without asterisks

$$V(u_{t-1}) = \gamma_0 + \gamma_1 u_{t-1} + \frac{1}{2} \gamma_2 u_{t-1}^2. \tag{31}$$

The first-order condition becomes

$$(\pi_t - \pi^*) - \lambda (1 - s) u_t - \beta (\gamma_1 + \gamma_2 u_t) (1 - s) = 0. \tag{32}$$

Taking expectations of (32) yields

$$E_{t-1} \pi_t = \pi^* + \beta (1 - s) \gamma_1 + (1 - s) (\lambda + \beta \gamma_2) \rho u_{t-1}. \tag{33}$$

Combining (32) and (33), using (11) and (16) yields the optimal discretionary decision rule

$$\pi_t = a + b \mu_t + c u_{t-1}, \tag{34}$$

where

$$\begin{aligned} a &= \pi^* + \beta\left(1-s\right)\gamma_1, \\ b &= \frac{\left(1-s\right)^2\left(\lambda+\beta\gamma_2\right)}{\left(1+\left(1-s\right)^2\left(\lambda+\beta\gamma_2\right)\right)}, \\ c &= \left(1-s\right)\left(\lambda+\beta\gamma_2\right)\rho. \end{aligned} \tag{35}$$

Average unemployment in the monetary union will follow the process

$$u_t = \rho u_{t-1} + \left(1-s\right)\left(1-b\right)\mu_t, \tag{36}$$

and country i unemployment is given by

$$u_{it} = \rho u_{it-1} + \left(1-s_i\right)\left(1-b\right)\mu_t + \left(1-s_i\right)\nu_{it}. \tag{37}$$

Now, the coefficients γ_1 and γ_2 show up in the policy rule and need to be determined. Substituting equations (34)-(36) into the Bellman equation (29) of the discretionary case and identifying the coefficients of u_{t-1} and u_{t-1}^2 yields two Riccati equations

$$\gamma_1 = \left(1-s\right)^2\left(\lambda+\beta\gamma_2\right)\beta\rho\gamma_1 + \beta\rho\gamma_1, \tag{38}$$

$$\frac{1}{2}\gamma_2 = \frac{1}{2}\left(1-s\right)^2\left(\lambda+\beta\gamma_2\right)^2\rho^2 + \frac{1}{2}\left(\lambda+\beta\gamma_2\right)\rho^2. \tag{39}$$

The system is recursive. Equation (39) determines γ_2, which can be substituted into (38) to identify γ_1. However, equation (39) is quadratic in γ_2 and thus there are in general two solutions for γ_2 with a given set of parameters. Re-organizing the second equation and using c from equation (35) results in

$$\left(1-s\right)\beta\rho c^2 - \left(1-\rho^2\beta\right)c + \lambda\left(1-s\right)\rho = 0, \tag{40}$$

which yields real solutions

$$c = \frac{\left(1-\rho^2\beta\right) \pm \sqrt{\left(1-\rho^2\beta\right)^2 - 4\left(1-s\right)^2\rho^2\beta\lambda}}{2\left(1-s\right)\rho\beta}, \tag{41}$$

if, and only if, the discriminant is non-negative. This constraint gives an upper bound for the weight of unemployment in the central bank's objective function

$$\lambda \leq \frac{\left(1-\rho^2\beta\right)^2}{4\left(1-s\right)^2\beta\rho^2}. \tag{42}$$

Obviously, it is required that (42) is satisfied, since complex solutions make no sense for rates of inflation and unemployment.

Whenever there is a real solution to (40) there seems to be two of them. Lockwood and Philippopoulos (1994) show that the "low inflation" equilibrium which is associated with a lower value of c has plausible comparative statics properties, whereas the "high inflation" equilibrium responds counter-intuitively to changes in parameter values.[17,18] Moreover, the high inflation equilibrium exists only when time horizon is infinite.

In the literature, two approaches to deal with the problem of multiple equilibria in this model has been applied. The first solution is to ignore the "bad equilibrium" with high inflation and to focus on the "good equilibrium". This strategy is followed e.g. by Svensson (1997, 1999), Beetsma and Jensen (1999), Lockwood, Miller and Zhang (1998) and Røisland (2001). The second solution treats the two equilibria more equally and considers them being possible outcomes of the model. This line of reasoning is followed e.g. by Lockwood and Philippopoulos (1994), Lockwood (1997) and Jensen (1999).

This essay adopts the first approach, that is, only the low inflation equilibrium is considered and c becomes

$$c = \frac{(1-\rho^2\beta) - \sqrt{(1-\rho^2\beta)^2 - 4(1-s)^2\rho^2\beta\lambda}}{2(1-s)\rho\beta}. \tag{43}$$

Multiple equilibria would be especially problematic in this essay, since the governments would then be uncertain about the inflation and unemployment processes with a given set of parameter values and would have to base their flexibility decisions on some expectation about the realized equilibrium. The justification for excluding the high inflation equilibrium is novel. By applying the theory of adaptive learning it is shown in Chapter 4 that only the low inflation equilibrium is stable under least-squares learning. That is, if the agents don't know the parameters of the policy rule, but act as econometricians and re-estimate them from existing data in each time period using least-squares estimation technique, they can only learn the low inflation equilibrium. Evans and Honkapohja (2001) provide a comprehensive treatment of learning in macroeconomics.

Now, rearranging the first Riccati equation (38) gives

$$\gamma_1 (1 - \beta((1-s)c + \rho)) = 0, \tag{44}$$

[17] "Low inflation" equilibrium means that with a given $u_{t-1} > 0$ the lower value of c gives lower inflation than the higher value of c. Obviously, when $u_{t-1} < 0$ the opposite happens.

[18] In the "high inflation" equilibrium inflation increases when the weight given to inflation variability in the central bank's objective function increases, and decreases when the real persistence increases or when the central bank cares more about the future.

which holds in general if $\gamma_1 = 0$. The second Riccati equation (39) can be written as

$$\gamma_2 = \frac{\lambda\rho^2 (1-s)^2}{(1-\rho^2\beta)} + \frac{c^2}{(1-\rho^2\beta)}. \tag{45}$$

Substitution of $\gamma_1 = 0$ and (45) into (35) gives the optimal policy rule under discretion

$$\pi_t = \pi^* + b\mu_t + cu_{t-1} \tag{46}$$

where

$$b = \frac{(1-s)^2 \lambda + (1-s)^2 \beta c^2}{\left(1 + (1-s)^2 \lambda - \beta\rho^2 + (1-s)^2 \beta c^2\right)}, \tag{47}$$

and c is given by equation (41). It is easy to see from the optimal policy rule that average inflation bias is absent. Inflation fluctuates around the target rate π^* like in the commitment solution. This is due to the fact that the unemployment target of the central bank is not over-ambitious, but coincides with the long-run natural rate. Since it is obvious from (41) that $c > 0$, it follows that with a given level of s there is a stabilization bias under discretion, that is, $b > b^*$. Discretionary monetary policy overreacts to production shocks causing too much inflation variability and too little unemployment variability relative to the socially optimal stabilization policy. Moreover, $c > 0$ implies that even though there is no average inflation bias in monetary policy, there is a state-dependent inflation bias, that is, inflation responds inefficiently to changes in unemployment. The inefficient stabilization is induced by the fact that under discretion the future state-dependent inflation bias depends on the current unemployment rate. It is therefore important to stabilize unemployment in order to reduce the future state-dependent inflation bias. Quite interestingly, the optimal discretionary policy rule (46) indicates that unlike in the commitment solution, real persistence induces inflation persistence as well. This can be seen by noting that u_{t-1} follows the process (36).

Noting that $\lim_{\rho\to 0}(c) = 0$ (see the Appendix) it becomes evident that (47) and (28) are identical when $\rho = 0$. This is just the well-known fact that in the standard Barro-Gordon framework stabilization is efficient under discretion (see e.g. Persson and Tabellini (2000), Ch. 15).

4 Determination of Labor Market Flexibility

4.1 National Labor Market Policy

When governments choose their preferred labor market flexibility, s_i, they know the monetary policy rule derived in the previous section. However, since there is a large number of countries in the monetary union, each government takes the discretionary monetary policy reaction as given when setting s_i. In other words, inflation variability is exogenous in government's optimization problem. It is easy to show that social welfare is increasing in flexibility, since both inflation and unemployment variability are decreasing in s (see the Appendix).

Since governments are assumed to be able to commit to a certain level of flexibility indefinitely, and the optimal behavior of the central bank is correctly anticipated, it is convenient to express the intertemporal loss function in terms of the endogenous variables and model parameters

$$\begin{aligned} GL_i &= \frac{1}{2(1-\beta)}\left(\left(b^2+\frac{\beta(1-s)^2(1-b)^2}{(1-\beta\rho^2)}c^2\right)\sigma_\mu^2\right. \\ &\quad \left.+\lambda\frac{(1-s_i)^2\left((1-b)^2\sigma_\mu^2+\sigma_\nu^2\right)}{(1-\beta\rho^2)}+2\kappa s_i\right), \end{aligned} \tag{48}$$

which is obtained by inserting the inflation rule (46) into the government's loss function (13) and using the expressions of expected present values of squared future unemployment deviations derived in the Appendix. Parameters b and c are functions of s (and not of s_i), and therefore taken as exogenous. The government faces a tradeoff between unemployment variability (the second term) and political costs (the third term). A higher level of flexibility is needed in order to lower unemployment fluctuations, but it can be attained only through higher political costs.

The first-order condition of the government's problem will be

$$\frac{\partial GL_i}{\partial s_i}=\frac{1}{2(1-\beta)}\left(-\frac{2\lambda(1-s_i)(1-b)^2}{(1-\beta\rho^2)}\sigma_\mu^2-\frac{2\lambda(1-s_i)}{(1-\beta\rho^2)}\sigma_\nu^2+2\kappa\right)=0. \tag{49}$$

The first-order condition determines implicitly the optimal level of labor market flexibility. Solving s_i as a function of s from the first-order condition yields

$$s_i=H(s)=1-\frac{\kappa(1-\beta\rho^2)}{\lambda\left((1-b)^2\sigma_\mu^2+\sigma_\nu^2\right)}, \tag{50}$$

where $H_s > 0$ implies that the flexibility choices of the governments are strategic complements.[19] It follows that multiple equilibria are possible in this model. Since the economic environment of the countries is ex ante identical, that is, the variances of the country specific shocks and the structural parameters are equal across countries, the equilibrium is symmetric, $s_i = s$. Replacing s_i by s in the first-order condition gives

$$\kappa = F(s) = \frac{\lambda(1-s)(1-b)^2}{(1-\beta\rho^2)}\sigma_\mu^2 + \frac{\lambda(1-s)}{(1-\beta\rho^2)}\sigma_\nu^2, \tag{51}$$

where $F(s)$ represents marginal benefits from flexibility at the common level of s. As in Calmfors (2001), the non-uniqueness of equilibria is ruled out by imposing the following condition

$$-\left((1-b)^2 + 2(1-s)(1-b)b_s\right)\sigma_\mu^2 - \sigma_\nu^2 < 0, \tag{52}$$

which implies that $F_s < 0$. The optimal level of labor market flexibility, s^*, solves equation (51) and the equilibrium is illustrated graphically as the intersection of the horizontal line at κ and function $F(s)$ in Figure 1.[20]

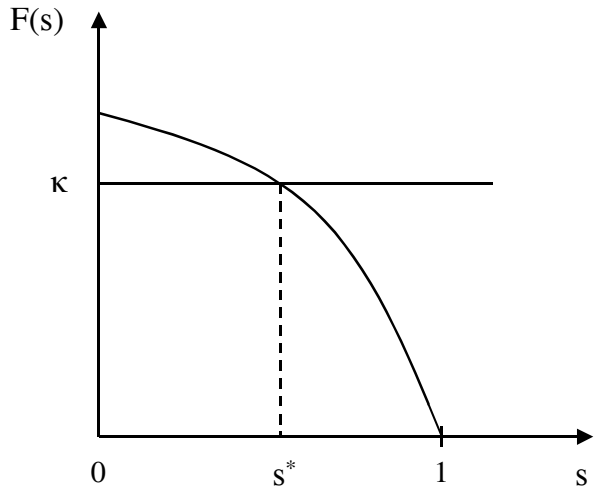

Figure 1: Determination of labor market flexibility under national decision making

This graphical representation can be applied to analyze the comparative statics properties of the equilibrium. First, it is easy to establish that increasing the total variance of production shocks, $\sigma_\mu^2 + \sigma_\nu^2$, so that neither σ_μ^2 nor σ_ν^2 is reduced will shift $F(s)$ upwards while the horizontal line at κ remains unchanged. Other things equal, stronger

[19] $H_s > 0$, because $b_s < 0$ (see the Appendix).

[20] By "optimal flexibility" it is meant that flexibility is optimal from the government's point of view.

disturbances thus bring about more flexible economies. Second, the more asymmetric the shocks, the more flexibility is chosen. This result is also intuitive and follows directly from (51). Raising σ_ν^2 and lowering σ_μ^2 by the same amount so that $\sigma_\mu^2 + \sigma_\nu^2$ is unchanged moves $F(s)$ upwards, since $(1-b)^2$ is always smaller than unity. Third, more persistence tends to increase flexibility, because it leads to prolonged deviations of unemployment from the target level of zero. Numerical analysis is applied to establish the third point, because the impact of ρ on the first term on the RHS of (51) is ambiguous due to the fact that $b_\rho > 0$ (see the Appendix). Numerical simulations, reported in Table 1, indicate that optimal flexibility is positively related to unemployment persistence.

ρ	0.2	0.4	0.6	0.8
$\kappa = 0.08$	0.843	0.862	0.893	0.937
$\kappa = 0.16$	0.668	0.710	0.778	0.871
$\kappa = 0.24$	0.431	0.513	0.638	0.797
$\kappa = 0.32$	0	0.079	0.379	0.695

Other parameter values: λ=0.5, β=0.95, σ_μ^2=0.7 and σ_ν^2=0.3

Table 1 Optimal flexibility, s^*, as persistence varies

Table 1 also shows that a combination of low persistence and high marginal political costs can result in total inflexibility. Graphically this means that given the marginal political cost κ, the function $F(s)$ is below κ at $s = 0$. Another interesting feature of model becomes apparent from Table 1. It has been assumed that the marginal political cost of flexibility, κ, is independent of the insider power in the labor markets, ρ. Then it is fairly obvious that increasing ρ will push optimal flexibility upwards, since the marginal benefit from flexibility is increasing in ρ, whereas the marginal cost of flexibility, κ, is fixed. If this assumption is transformed so that insider power also raises political costs of flexibility, the results can be overturned. The main diagonal of Table 1 from top left corner to bottom right corner demonstrates this point in a special case where $\kappa = 0.4\rho$. Contrary to the basic model, with low values of ρ more persistence decreases flexibility. When ρ is high enough, the impact of ρ on s^* is positive, so that the relation between optimal flexibility and real persistence can be U-shaped.

4.2 International Labor Market Policy Coordination

In the previous section the flexibility decision was left to the hands of the national governments. Since each government is assumed to be small in the sense that it will take monetary union wide variables as given, the choice of s_i is made without taking into account its impact on inflation variability. At first sight it seems that national decision

making leads to a suboptimal level of flexibility, because the inflation variability effect is ignored.

In this section the optimal flexibility is derived assuming that national governments coordinate their policy choices. In this setup where all countries are ex ante symmetrical, the solution is extremely simple. Under coordination, the policy objective is the sum of member country governments' objective functions, that is $GL = \int_0^1 GL_i di$. Symmetry implies $s_i = s$ with all i, and the joint objective becomes

$$\begin{aligned} GL &= \frac{1}{2(1-\beta)}\left(\left(b^2 + \frac{(1-s)^2(1-b)^2}{(1-\beta\rho^2)}\left(\lambda+\beta c^2\right)\right)\sigma_\mu^2 \right. \\ &\left. + \frac{\lambda}{(1-\beta\rho^2)}(1-s)^2\sigma_\nu^2 + 2\kappa s\right), \end{aligned} \tag{53}$$

which is otherwise the same as (48), but s_i is replaced by s and the terms have been reorganized. The flexibility parameter, s, is chosen so as to minimize (53). The term in front of σ_μ^2 can be further simplified by noting that $\frac{(1-s)^2\left(\lambda+\beta c^2\right)}{1-\beta\rho^2} = \frac{b}{1-b}$, which yields

$$GL = \frac{1}{2(1-\beta)}\left(b\sigma_\mu^2 + \frac{\lambda}{(1-\beta\rho^2)}(1-s)^2\sigma_\nu^2 + 2\kappa s\right). \tag{54}$$

The first-order condition under policy coordination will be

$$\frac{\partial GL}{\partial s} = \frac{1}{2(1-\beta)}\left(b_s\sigma_\mu^2 - \frac{2\lambda(1-s)}{(1-\beta\rho^2)}\sigma_\nu^2 + 2\kappa\right) = 0. \tag{55}$$

From (55) one obtains a similar type of expression as under national decision making (equation (51))

$$\kappa = G(s) = -\frac{1}{2}b_s\sigma_\mu^2 + \frac{\lambda(1-s)}{(1-\beta\rho^2)}\sigma_\nu^2. \tag{56}$$

The optimal level of labor market flexibility, s^{**}, solves equation (56) and the equilibrium can be presented graphically as under national policy making. The second-order condition of the optimization problem limits the slope of $G(s)$ to be negative, that is, $G_s < 0$. This implies that multiple equilibria are not possible under policy coordination. Table 2 shows that like under national decision making, optimal flexibility is positively related to persistence.[21]

[21] Also, the main diagonal of the Table from top left corner to bottom right corner reveals that the relation between persistence and optimal flexibility can be U-shaped, provided that political costs and insider power (persistence) are positively related.

ρ	0.2	0.4	0.6	0.8
$\kappa = 0.08$	0.843	0.863	0.894	0.938
$\kappa = 0.16$	0.669	0.714	0.785	0.877
$\kappa = 0.24$	0.439	0.534	0.668	0.820
$\kappa = 0.32$	0	0.276	0.542	0.770

Other parameter values: λ=0.5, β=0.95, σ_{μ}^2=0.7 and σ_{ν}^2=0.3

Table 2 Optimal flexibility, s^{**}, as persistence varies

Comparing (56) with (51) reveals that the optimal flexibility under policy coordination is greater than under national decision making if the following strict inequality holds

$$b_s + \frac{2\lambda (1-s)(1-b)^2}{(1-\beta\rho^2)} < 0. \tag{57}$$

In the Appendix it is shown that (57) indeed holds in the model, provided that unemployment is persistent. The graphical representation of the problem in Figure 2 shows that when this condition holds, the function $G(s)$ will lie above the function $F(s)$ with all values of s between zero and unity.

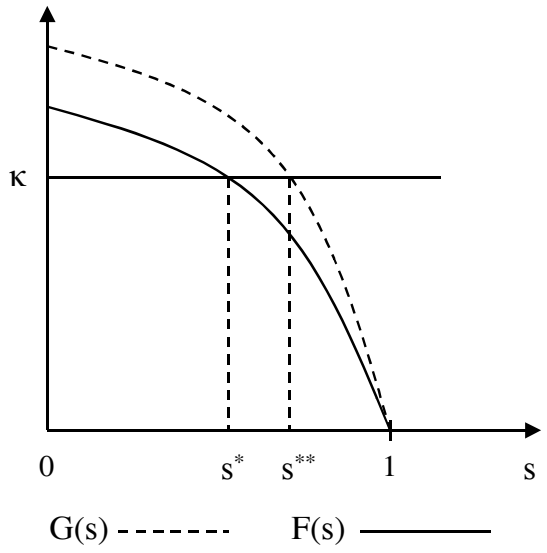

Figure 2: Determination of labor market flexibility under policy coordination

It is noteworthy that in the standard Barro-Gordon framework without unemployment persistence, that is, $\rho = 0$, the above condition doesn't hold, but is equal to zero. The fact that coordination of flexibility choice matters only in case with unemployment persistence and is thus irrelevant in the standard setup deserves a closer look. At first sight,

it seems surprising that even though coordination internalizes the impact of flexibility on inflation variability, the optimal flexibility remains unchanged when $\rho = 0$. However, there is another new effect, which goes to the opposite direction, namely that by increasing flexibility in the monetary union area the stabilization parameter b goes down (see equation (47) with $\rho = c = 0$). This, in turn, reduces optimal flexibility through increased unemployment variability, since $(1 - b)$ goes up (see equation (36)). It turns out that these two effects cancel out in the discretionary monetary policy equilibrium in the model without persistence.

The solution to this "puzzle" is simple. Stabilization of production shocks in the standard Barro-Gordon model is socially optimal in the discretionary equilibrium. It means that under both national decision making and policy coordination the governments know that monetary union wide shocks are stabilized optimally for any level of flexibility. That is, the common central bank's optimal policy minimizes the social loss function (12) with any given s_i. Therefore, the government just balances the marginal loss arising from political costs associated with flexibility and marginal benefits deriving from reduced variability of inflation and unemployment. In other words, the government doesn't have to worry about whether the relative stabilization of inflation with respect to unemployment is optimal or not, and thus there is no interaction between flexibility and stabilization properties.

A similar argument can be applied in the case with unemployment persistence when the central bank has access to a commitment technology. Stabilization is optimal from the society's point of view, which brings about the same type of irrelevance result as in no persistence case.

These observations suggest that the stabilization performance is the key to understanding the impact of policy coordination on the chosen labor market flexibility. As shown in Section 3.3, persistence in the real economy creates a stabilization bias and a state-dependent inflation bias under discretionary monetary policy. Relative to the optimal stabilization, which occurs in the commitment equilibrium, inflation variability is too high ($b > b^*$ and $c > 0$) and unemployment variability is too low relative to the optimum. The social loss function (12) is thus not minimized with a given s_i. This fact gives rise to an incentive to use the choice of flexibility as a tool to improve the stabilization properties of the economy. This can be seen by constructing a measure for the relative stabilization bias under discretion, $\widetilde{b}(\overline{s})$, as

$$\widetilde{b}(\overline{s}) = \frac{b(\overline{s}) - b^*(\overline{s})}{b^*(\overline{s})} = \frac{(1 - b(\overline{s}))\beta c^2}{\lambda}. \tag{58}$$

It is important to notice that in $\widetilde{b}(\overline{s})$, the flexibility parameter, $\overline{s}$, is the equilibrium outcome of the discretionary case also in $b^*(\overline{s})$, although in the model the optimally chosen endogenous s would certainly be different in the two monetary policy regimes.[22] The correct interpretation of $b^*(\overline{s})$ is that it is the unattainable value of the stabilization parameter b which would result in optimal stabilization of production shocks with a given structure of the economy, that is, with a given s. The purpose of (58) is just to show the size of the stabilization bias under discretion as a function of s.

A sufficient condition for the result that by increasing flexibility the stabilization properties of the economy are improved is two-fold. First, the relative stabilization bias, $\widetilde{b}(\overline{s})$, must be decreasing in s. This condition guarantees that b approaches b^* as flexibility is increased. Second, the state-dependent inflation bias parameter, c, must also be decreasing in s, because $c = 0$ under optimal stabilization. It is shown in the Appendix that both conditions, $\widetilde{b}(\overline{s})_s < 0$ and $c_s < 0$, indeed hold in the model. These two conditions together imply that the discretionary inflation rule (46) approaches the optimal rule (22) as s goes up.

Hence, the chosen level of flexibility is higher under policy coordination not because of internalization of inflation variability effect, but because flexibility interacts with stabilization performance so that by choosing more s it is possible to push stabilization closer to the optimum.

5 Extensions and Variations

5.1 State-Dependent Unemployment Target

This section derives the optimal monetary policy rule with state-dependent unemployment target. Svensson (1997) claimed that optimal stabilization can be attained by directing the central bank to target the short-run natural rate of unemployment, that is, $u_t^* = \rho u_{t-1}$ in (14) instead of targeting the long-run natural rate of unemployment, which is normalized to zero in this model. Røisland (2001) showed, however, that in this case the solution is not the commitment solution of the model with unemployment persistence, but the outcome coincides with the solution of the no persistence case. In other words, the monetary policy rule is not optimal from the society's point of view. After having derived the monetary policy rule, the implications of labor market policy coordination in this monetary policy regime will be studied.

[22] As the stabilization parameter b is smaller under commitment, one can conjecture that the optimally chosen s would be higher under commitment than under discretion.

As discussed in Introduction, this regime can be seen to result from a misinterpretation concerning the source of variations of the unemployment rate. From a practical point of view, this kind of monetary policy regime can be questioned, because in reality it is a very demanding task to figure out the short-run natural rate.[23]

In order to work out the optimal monetary policy rule in this case the central bank's problem is restated with the state-dependent unemployment target. Assuming that $V(u_{t-1})$ exists, the Bellman equation can be written as

$$V(u_{t-1}) = E_{t-1} \min_{\pi_t} \left(\frac{1}{2} \left((\pi_t - \pi^*)^2 + \lambda (u_t - \rho u_{t-1})^2 \right) + \beta V(u_t) \right), \tag{59}$$

where minimization is carried out subject to (11). The first-order condition is also sightly modified and is given by

$$(\pi_t - \pi^*) - \lambda (1-s)(u_t - \rho u_{t-1}) - \beta (\gamma_1' + \gamma_2' u_t)(1-s) = 0. \tag{60}$$

Taking expectations of (60) yields

$$E_{t-1}\pi_t = \pi^* + \beta (1-s)\gamma_1' + (1-s)\beta\gamma_2'\rho u_{t-1}. \tag{61}$$

Combining (60) and (61), using (11) and (16) yields an optimal discretionary decision rule

$$\pi_t = a' + b'\mu_t + c' u_{t-1}, \tag{62}$$

where

$$\begin{aligned} a' &= \pi^* + \beta (1-s)\gamma_1', \\ b' &= \frac{(1-s)^2 (\lambda + \beta\gamma_2')}{\left(1 + (1-s)^2 (\lambda + \beta\gamma_2')\right)}, \\ c' &= (1-s)\beta\gamma_2'\rho. \end{aligned} \tag{63}$$

Average unemployment in the monetary union will follow the process

$$u_t = \rho u_{t-1} + (1-s)(1-b')\mu_t, \tag{64}$$

[23] Milton Friedman, for example, responded when he was asked about the natural rate by saying "I don't know what the natural rate is, neither do you, and neither does anyone else" (quoted in King (1999)).

and country i unemployment process is given by

$$u_{it} = \rho u_{it-1} + (1 - s_i)(1 - b')\mu_t + (1 - s_i)\nu_{it}. \tag{65}$$

As before, γ_1' and γ_2' need to be determined. Substituting equations (62)-(64) into the Bellman equation (59) and identifying the coefficients of u_{t-1} and u_{t-1}^2 yields two Riccati equations

$$\gamma_1' = (1 - s)^2 \beta\gamma_2'\beta\rho\gamma_1' + \beta\rho\gamma_1', \tag{66}$$

$$\frac{1}{2}\gamma_2' = \frac{1}{2}(1 - s)^2 (\beta\gamma_2')^2 \rho^2 + \frac{1}{2}\beta\gamma_2'\rho^2. \tag{67}$$

Re-organizing the second equation and using c' from (63) results in

$$(1 - s)\beta\rho c'^2 - \left(1 - \rho^2\beta\right) c' = 0, \tag{68}$$

which yields two real solutions

$$c' = \frac{(1 - \rho^2\beta) \pm \sqrt{(1 - \rho^2\beta)^2}}{2(1 - s)\rho\beta}. \tag{69}$$

As in Section 3.3, the larger root is ignored by appealing to the learning argument put forth in Chapter 4. The smaller root $c' = 0$ is the relevant one, and from (63) it follows that $\gamma_2' = 0$. From (66) and $\beta\rho < 1$ it is thus evident that $\gamma_1' = 0$. The optimal inflation response to a production shock becomes

$$b' = \frac{(1 - s)^2 (\lambda + \beta\gamma_2')}{\left(1 + (1 - s)^2 (\lambda + \beta\gamma_2')\right)} = \frac{(1 - s)^2 \lambda}{1 + (1 - s)^2 \lambda}. \tag{70}$$

When the central bank targets the short-run natural rate, there is neither average inflation bias ($\gamma_1' = 0$) nor state-dependent inflation bias ($\gamma_2' = c' = 0$), but a new stabilization bias problem emerges. As is clear from Section 3.2, the optimal stabilization of production shocks is characterized by b^* in equation (28). The optimal stabilization parameter is increasing in the persistence of the real economy. However, in this case where the central bank targets the short-run natural rate, b' coincides with the optimal stabilization parameter of the economy without any persistence. In other words, the central bank doesn't care about the temporal properties of the unemployment rate, and therefore stabilizes production shocks too little from the society's point of view. The direction of the stabilization bias is altered from under-stabilization of inflation in the

basic setup with unemployment persistence to over-stabilization of inflation.

What about the implications for labor market flexibility? The government's problem is obtained from (48) by setting $c' = 0$

$$GL_i = \frac{1}{2(1-\beta)}\left(b'^2\sigma_\mu^2 + \lambda\frac{(1-s_i)^2\left((1-b')^2\sigma_\mu^2+\sigma_\nu^2\right)}{(1-\beta\rho^2)} + 2\kappa s_i\right). \tag{71}$$

The first-order condition under national decision making is given by

$$\begin{aligned}\frac{\partial GL_i}{\partial s_i} &= \frac{1}{2(1-\beta)}\left(-\frac{2\lambda(1-s_i)(1-b')^2}{(1-\beta\rho^2)}\sigma_\mu^2\right. \\ &\left.-\frac{2\lambda(1-s_i)}{(1-\beta\rho^2)}\sigma_\nu^2 + 2\kappa\right) = 0.\end{aligned} \tag{72}$$

Numerical analysis presented in Table 3 reveals that optimal flexibility, s', under national decision making increases with persistence as in earlier cases.

ρ	0.2	0.4	0.6	0.8
$\kappa = 0.08$	0.843	0.863	0.894	0.937
$\kappa = 0.16$	0.669	0.713	0.782	0.873
$\kappa = 0.24$	0.438	0.531	0.659	0.807
$\kappa = 0.32$	0	0.256	0.509	0.737

Other parameter values: λ=0.5, β=0.95, σ_μ^2=0.7 and σ_ν^2=0.3

Table 3 Optimal flexibility, s', as persistence varies

In order to derive the first-order condition under policy coordination it is useful to write the optimization problem as in the previous section in equation (53)

$$\begin{aligned}GL_i &= \frac{1}{2(1-\beta)}\left(\left(b'^2 + \frac{\lambda(1-s)^2(1-b')^2}{(1-\beta\rho^2)}\right)\sigma_\mu^2\right. \\ &\left.+\lambda\frac{(1-s)^2}{(1-\beta\rho^2)}\sigma_\nu^2 + 2\kappa s\right).\end{aligned} \tag{73}$$

The first-order condition under policy coordination becomes

$$\begin{aligned}\frac{\partial GL}{\partial s} = \frac{1}{2(1-\beta)}&\left(2b'_s\left(b' - \frac{\lambda(1-b')(1-s)^2}{(1-\beta\rho^2)}\right)\sigma_\mu^2\right. \\ &\left.-\frac{2\lambda(1-s)(1-b')^2}{(1-\beta\rho^2)}\sigma_\mu^2 - \frac{2\lambda(1-s)}{(1-\beta\rho^2)}\sigma_\nu^2 + 2\kappa\right) = 0,\end{aligned} \tag{74}$$

with

$$b'_s = -\frac{2\left(1-s\right)\lambda}{\left(1+\left(1-s\right)^2\lambda\right)^2} < 0. \tag{75}$$

Table 4 indicates that, as expected, persistence increases optimal flexibility, s'', under policy coordination.

ρ	0.2	0.4	0.6	0.8
$\kappa = 0.08$	0.843	0.862	0.894	0.937
$\kappa = 0.16$	0.668	0.710	0.780	0.872
$\kappa = 0.24$	0.433	0.518	0.647	0.804
$\kappa = 0.32$	0	0.181	0.468	0.728

Other parameter values: λ=0.5, β=0.95, σ^2_μ=0.7 and σ^2_ν=0.3

Table 4 Optimal flexibility, s'', as persistence varies

Comparing the first-order condition (74) with the one obtained under national decision making (equation (72)) reveals that the sign of the first term inside the brackets in (74) determines the impact of coordination on optimal flexibility. If the term $2b'_s\left(b' - \frac{\lambda(1-b')(1-s)^2}{(1-\beta\rho^2)}\right)$ is positive, then the marginal benefit of choosing more s is smaller under coordination. Keeping in mind that the marginal cost is the same in both regimes, it follows that the optimal s will be lower under coordination. Using (70) to rewrite $(1-b')$ the first term becomes

$$2b'_s\left(b' - \frac{\lambda\left(1-b'\right)\left(1-s\right)^2}{\left(1-\beta\rho^2\right)}\right) = -2b'_s\frac{\beta\rho^2}{\left(1-\beta\rho^2\right)}b' > 0, \tag{76}$$

which verifies that optimal flexibility is indeed lower under policy coordination. The explanation is again related to the implied stabilization properties of the equilibrium monetary policy rule. The central bank's response to production shocks is too small from the society's point of view. In the coordinated equilibrium the governments can affect the stabilization parameter b', and from equations (47) and (70) it is clear that choosing lower s will push b' upwards and stabilization is closer to the optimum. Naturally, when there is no persistence in the real economy, that is $\rho = 0$, the unemployment target of the central bank will be constant at zero, and the model collapses to the standard no persistence case where policy coordination is irrelevant.

5.2 Social Welfare and Government Incentives for Coordination

Social losses defined in (12) are decreasing in labor market flexibility.[24] In the absence of any institutional constraints, it would be possible to attain the first-best solution with zero losses by just setting $s = 1$.[25] Fully flexible labor markets would bring about stable unemployment at zero level, and real wages would be perfectly flexible. Inflation variability would be eliminated as well, since monetary policy is neither needed for nor capable of stabilizing the economy. However, in this model it is assumed that this ideal world can't be reached, because it is not preferred by the labor unions (or their leaders), who have influence over the national government. In order to make labor markets better functioning, the government therefore has to bear increasing political costs.

This section discusses the impact of coordination on social welfare under various monetary policy regimes and studies government incentives for coordination. When coordination is irrelevant for the equilibrium outcome, it immediately follows that social welfare is unchanged under coordinated equilibrium. This applies to both no persistence case and to commitment case treated in earlier sections. Hence, government losses are unaffected as well, and governments have no reason to coordinate their actions.

In the baseline model, that is, when there is persistence in the real economy and the common central bank acts under discretion, coordination leads to more flexible labor markets and stabilization properties of the economy are improved. Both of these effects contribute to reducing the social loss measured by (12). By looking at the government's loss function (13) it becomes clear that even though social loss goes down under coordination, the government may not have incentives to pursue coordination, because political costs are higher. However, numerical analysis in Table 5 suggests that the increase in political costs is not strong enough to overshadow the benefits of flexibility on macroeconomic stability, and so both social losses (SL_i) and government losses (GL_i) are smaller under policy coordination (PC) than under national decision making (ND).[26]

[24] See the Appendix for a proof.

[25] Obviously, political losses defined in (13) would not equal zero with fully flexible labor markets.

[26] Qualitatively similar, but quantitatively weaker, results are obtained with lower political costs ($\kappa = 0.08, 0.16, 0.24$). $\kappa = 0.32$ was chosen in order to make the difference between the two policy regimes as clear as possible.

ρ		0.2	0.4	0.6	0.8
SL_i	ND	3.987	4.000	2.735	1.305
	PC	3.987	2.658	1.523	0.700
GL_i	ND	3.987	4.506	5.160	5.753
	PC	3.987	4.424	4.992	5.628

Other parameter values: κ=0.32, λ=0.5, β=0.95, σ_μ^2=0.7 and σ_ν^2=0.3

Table 5 Social and government losses when persistence varies

When the central bank targets the short-run natural rate, coordination improves stabilization performance, which lowers the social loss, but at the same time flexibility decreases, and both inflation and unemployment variability are increased. The net effect on social welfare is then ambiguous. However, political costs attached to flexibility will go down, which implies that there may be government incentives to coordinate, even though social welfare may decrease.

ρ		0.2	0.4	0.6	0.8
SL_i	ND	3.954	2.710	1.652	0.849
	PC	3.954	3.181	1.908	0.906
GL_i	ND	3.954	4.348	4.910	5.567
	PC	3.954	4.339	4.903	5.565

Other parameter values: κ=0.32, λ=0.5, β=0.95, σ_μ^2=0.7 and σ_ν^2=0.3

Table 6 Social and government losses when persistence varies and the central bank targets the short-run natural rate

Simulation results in Table 6 suggest that in terms of social losses improved stabilization properties under policy coordination are overruled by decreasing flexibility, whereby coordination actually brings about higher social losses. However, as decreased flexibility lowers political costs, government losses under coordination will go down. Hence, governments do have an incentive to coordinate their flexibility choices, even though it entails increased social losses.

The analysis above suggests that if the central bank misinterprets its unemployment target to coincide with the short-run natural rate instead of the long-run one, governments may respond by coordinating their actions although it leads to increased social losses.

5.3 Determination of Flexibility in a Country with Independent Monetary Policy

In Calmfors (2001) it is shown that when the central bank doesn't have an over-ambitious unemployment target, which would result in inflation bias, the chosen level of flexibility

in a monetary union is higher than in a country, which has independent monetary policy. In his study governments don't coordinate their actions in a monetary union. However, as shown in this essay in Section 4.2, when there is no persistence in the economy, as is the case in Calmfors (2001), coordination doesn't matter and thus could be equally well assumed without changing the result. This section extends Calmfors's model to include real persistence and discusses how the results are affected.

Only minor modifications to the previous analysis are needed in order to work out the optimal flexibility level in a country outside a monetary union. The optimization problem of the national central bank is identical to (29), except that monetary union wide variables are replaced by country specific ones. This means that the only change in the central bank's reaction function (46) is that now monetary policy is able to respond to the country specific shock ν_i as well. Therefore, the optimal discretionary monetary policy rule becomes

$$\pi_{it} = \pi^* + b\left(\mu_t + \nu_{it}\right) + cu_{it-1}, \tag{77}$$

where b and c are defined as before. The government's problem is very similar to the one faced inside a monetary union under policy coordination. In both cases the government knows that the choice of flexibility will affect stabilization performance, since b is a function of s. The first-order condition of the government is obtained from (55) by dropping the term containing σ_ν^2, since monetary policy reacts also to the asymmetric shock, and by replacing σ_μ^2 with $\sigma_\mu^2+\sigma_\nu^2$ in the first term inside the brackets. The optimal flexibility, s^{***}, solves

$$\kappa = P\left(s\right) = -\frac{1}{2}b_s\left(\sigma_\mu^2 + \sigma_\nu^2\right). \tag{78}$$

Table 7 shows the optimal flexibility with various parameter values. When a comparison is made between these values and the ones presented in Table 1, it becomes apparent that with low levels of persistence flexibility is higher inside a monetary union, but when ρ is high enough ($\rho \geq 0.6$ in Table 7), flexibility is actually at a lower level in a monetary union than under independent monetary policy. Hence, the result obtained from a static model in Calmfors (2001) is overturned. The explanation is once again related to the stabilization properties of the economy. Even though the need for flexibility is higher in the monetary union due to asymmetric shocks which can't be stabilized with common monetary policy, high persistence brings about severe stabilization bias which can be reduced by choosing more flexibility only under independent monetary policy. In no persistence case analyzed in Calmfors (2001) the latter effect is absent and therefore flexibility is always higher inside a monetary union.

ρ	0.2	0.4	0.6	0.8
$\kappa = 0.08$	0.842	0.862	0.894	0.938
$\kappa = 0.16$	0.654	0.705	0.782	0.878
$\kappa = 0.24$	0	0.481	0.659	0.823
$\kappa = 0.32$	0	0	0.521	0.776

Other parameter values: λ=0.5, β=0.95, σ_μ^2=0.7 and σ_ν^2=0.3

Table 7 Optimal flexibility, s^{***}, as persistence varies

Another interesting comparison can be drawn between a country with independent monetary policy and countries inside a monetary union where flexibility is chosen in a coordinated fashion. When countries consider whether to join or not to join a monetary union, behavioral effects should be taken into account. Hence, this exercise can be justified by noting that in a monetary union governments do have an incentive to coordinate their actions, as was shown in Section 5.2. Numerical examples presented in Table 2 and Table 7 indicate, that flexibility is likely to be higher inside a monetary union. Intuition behind this result is clear, since in both regimes the governments internalize the effect increased flexibility has on stabilization properties, but in a monetary union asymmetric shocks are not stabilized by common monetary policy and therefore more flexibility is called for. However, provided that persistence is high enough (e.g. $\rho = 0.8$), flexibility can be lower inside a monetary union. By looking at (56) and (78) it is clear that this extreme case is possible if $-\frac{1}{2}b_s > \frac{\lambda(1-s)}{(1-\beta\rho^2)}$. It is noteworthy that if this condition holds, it also follows from (56) that some comparative statics properties of the coordination case are counter-intuitive. In particular, the more asymmetric the production shocks are, the less flexibility is chosen in a coordinated equilibrium.

6 Concluding Remarks

This essay studied the determination of labor market flexibility in a large monetary union when national governments decide on the level of nominal wage flexibility independently of each other or in a coordinated fashion, and flexibility entails increasing political costs to the governments. The main result of the essay is that coordination tends to yield more flexible labor markets because flexibility and stabilization properties of the economy are interrelated. More precisely, the stabilization bias of discretionary monetary policy can be alleviated by more flexible labor markets. Each government is too small to be able to affect the average flexibility in the monetary union area, but by coordinating labor market policies the governments can have influence on the common central bank's stabilization policy. Coordination internalizes the positive externality that flexibility generates by

improving the stabilization properties of the economy, and therefore coordination results in more flexible markets.

The essay also analyzed a special case where the central bank misinterprets changes in the short-run natural rate as arising solely from structural factors, thereby stabilizing the production shocks too little. Then coordination is shown to result in less flexible markets, since lower flexibility calls for stronger reaction to production shocks, thereby pushing stabilization closer to the optimum. The essay also digressed from the main subject and briefly looked at the case where a country is outside a monetary union and has an independent monetary policy. The purpose was to examine whether the results of Calmfors (2001) are unaltered when real persistence is introduced. It turned out that if unemployment is persistent enough, flexibility is not necessarily higher inside a monetary union, which is always the case without persistence. The explanation is once again related to the interplay between flexibility and stabilization policy identified in the essay.

Throughout the essay the countries are assumed to be perfectly symmetric in the sense that all structural parameters and preferences are identical. This modelling choice is obviously highly restrictive, but chosen because of analytical simplicity. Moreover, assuming symmetry conforms well with the common practice in the research field on international policy coordination (see e.g. Persson and Tabellini (2000, Ch. 18)). An apparent extension of the model would be to allow some asymmetries, other than country-specific shocks, between countries. It would be interesting to let the persistence parameter, that is, insider power or political costs very across countries. In that case the precise definition of coordination would become important. In the symmetric case there is no difference whether coordination means that all countries must set the same level of flexibility, or that flexibility is allowed the vary across countries. In an asymmetric case this distinction certainly matters and will probably have implications for chosen levels of flexibility under coordination relative to national decision making. Also, it is likely that the desirability of coordination in terms of social welfare and government incentives will be affected as well. Asymmetries in country size is another related issue, which deserves a closer look. In the EMU it is certain that some member countries are so large that their actions do have an effect on the area wide variables. Adding this to the model would be important, but aggregation would certainly become more problematic.

There is at least one more simplifying assumption, which needs to be discussed. A starting point of the analysis of flexibility choice is that governments set flexibility only once and stick to it ever after. In other words, the governments commit to a certain level of flexibility. As was discussed in Introduction, this is quite a plausible assumption provided that flexibility choice has irreversibilities in the form of relatively high costs of rewriting

legislation and reshaping the institutional structure of the economy. It follows from this assumption that the central bank can't behave strategically toward the governments when setting monetary policy, because it takes the flexibility of the economies as given. However, in practice governments resign and new government are appointed. This fact has at least two implications. First, the discount factor of the government may be smaller than that of the society, since the expected life span of the government is only a few years.[27] Second, and probably more important point is that the central bank actually knows that from time to time the flexibility issue is reconsidered in every country in connection with general elections and the formation of the new government. This opens the door for strategic behavior on the common central bank's side. It may be optimal for the central bank to conduct monetary policy so that the governments are forced to choose a relatively high level of flexibility, since inflation and unemployment variability are decreasing in flexibility. So, one is inclined to conjecture that in this case the central bank could act very conservatively and let unemployment to fluctuate a lot in order to compel new governments to choose high flexibility. Obviously, a thorough investigation of this point is outside the scope of this essay and is left for future research.

[27] Same argument could be made about central bankers. King (1997) writes "Central bankers, despite their prudent and austere life-style, do not live for ever. They tend to be judged by the realised value of the loss function during their term of office, and rarely on whether the loss subsequent to their retirement was lower than it would otherwise have been."

A Appendix

A.1 Derivation of $lim_{\rho\to 0}(c)$

It is easy to see from (43) that $lim_{\rho\to 0}(c)$ is of the form $\left(\frac{0}{0}\right)$. Applying L'Hôpital's rule and simplifying yields

$$\lim_{\rho\to 0}(c)=\lim_{\rho\to 0}\frac{\rho\left(\left(1-\beta\rho^2+2\lambda(1-s)^2\right)-\sqrt{\left(1-\rho^2\beta\right)^2-4(1-s)^2\rho^2\beta\lambda}\right)}{(1-s)\sqrt{\left(1-\rho^2\beta\right)^2-4(1-s)^2\rho^2\beta\lambda}}=0.$$

A.2 Derivation of b_s and c_s

Taking the derivative from (47) with respect to s gives

$$\begin{aligned}
b_s &= \frac{\left(-2(1-s)(\lambda+\beta c^2)+2\beta c c_s(1-s)^2\right)\left(1-\beta\rho^2+(1-s)^2(\lambda+\beta c^2)\right)}{\left(1-\beta\rho^2+(1-s)^2(\lambda+\beta c^2)\right)^2}\\
&\quad -\frac{\left(-2(1-s)(\lambda+\beta c^2)+2\beta c c_s(1-s)^2\right)(1-s)^2(\lambda+\beta c^2)}{\left(1-\beta\rho^2+(1-s)^2(\lambda+\beta c^2)\right)^2}\\
&= \frac{-2(1-s)(\lambda+\beta c^2)(1-\beta\rho^2)+2\beta c c_s(1-s)^2(1-\beta\rho^2)}{\left(1-\beta\rho^2+(1-s)^2(\lambda+\beta c^2)\right)^2}\\
&= \frac{-2(1-s)(1-\beta\rho^2)\left((\lambda+\beta c^2)-\beta c c_s(1-s)\right)}{\left(1-\beta\rho^2+(1-s)^2(\lambda+\beta c^2)\right)^2}<0.
\end{aligned}$$

Taking the derivative from (43) with respect to s and simplifying gives

$$\begin{aligned}
c_s &= -\frac{1}{2}\left(1-\beta\rho^2\right)\frac{(1-\beta\rho^2)-\sqrt{\left(1-\rho^2\beta\right)^2-4(1-s)^2\rho^2\beta\lambda}}{\sqrt{\left(1-\rho^2\beta\right)^2-4(1-s)^2\rho^2\beta\lambda}(1-s)^2\rho\beta}\\
&= -\left(1-\beta\rho^2\right)\frac{c}{\sqrt{\left(1-\rho^2\beta\right)^2-4(1-s)^2\rho^2\beta\lambda}(1-s)}<0.
\end{aligned}$$

A.3 Derivation of $\frac{\partial(var(u_i))}{\partial s}$ and $\frac{\partial(var(\pi))}{\partial s}$

From (37) one obtains

$$\begin{aligned}
var(u_{it}) &= var\left(\rho u_{it-1}+(1-s_i)(1-b)\mu_t+(1-s_i)\nu_{it}\right)\\
&= \rho^2 var(u_{it-1})+(1-s_i)^2(1-b)^2\sigma_\mu^2+(1-s_i)^2\sigma_\nu^2,
\end{aligned}$$

assuming $var(u_{it}) = var(u_i)$ with all t and $s_i = s$ gives

$$var(u_i) = \frac{(1-s)^2 (1-b)^2 \sigma_\mu^2 + (1-s)^2 \sigma_\nu^2}{1-\rho^2}.$$

The sign of $\frac{\partial(var(u_i))}{\partial s}$ depends on the sign of $\frac{\partial}{\partial s}\left((1-s)^2 (1-b)^2\right)$

$$\frac{\partial}{\partial s}\left((1-s)^2 (1-b)^2\right) = \frac{-2(1-s)(1-b)((1-b)+b_s(1-s))}{\left(1-\beta\rho^2+(1-s)^2(\lambda+\beta c^2)\right)^2},$$

which is negative provided that

$$-b_s < \frac{1-b}{1-s}.$$

Using the condition $-b_s < \frac{1-b}{2(1-s)}$ from Appendix A.8 it is clear that the condition above holds and

$$\frac{\partial(var(u_i))}{\partial s} < 0.$$

From (46) one obtains

$$\begin{aligned} & var(\pi) = var(b\mu_t + cu_{t-1}) = b^2\sigma_\mu^2 + c^2 var(u) \\ \Rightarrow \quad & \frac{\partial(var(\pi))}{\partial s} = 2bb_s\sigma_\mu^2 + 2cc_s var(u) + \frac{\partial(var(u))}{\partial s}c^2 < 0, \end{aligned}$$

because $b_s < 0$, $c_s < 0$ (see Appendix A.2) and $\frac{\partial(var(u))}{\partial s} < 0$, where the last inequality follows directly from $\frac{\partial(var(u_i))}{\partial s} < 0$.

A.4 Derivation of Expected Present Values of Squared Future Unemployment Deviations for Equation (48)

The expected present value of squared future monetary union wide unemployment deviations is given by

$$E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{t-1}^2\right) = E_0\left(u_0^2 + \beta u_1^2 + \beta^2 u_2^2 + \beta^3 u_3^2 + \beta^4 u_4^2 + ...\right),$$

where

$$\begin{aligned}
\beta E_0\left(u_1^2\right) &= \beta\left(\rho^2 u_0^2 + (1-s)^2(1-b)^2\sigma_\mu^2\right), \\
\beta^2 E_0\left(u_2^2\right) &= \beta^2 E_0\left(\rho^2 u_1^2 + (1-s)^2(1-b)^2\sigma_\mu^2\right) \\
&= \beta^2\left(\rho^4 u_0^2 + \rho^2(1-s)^2(1-b)^2\sigma_\mu^2 + (1-s)^2(1-b)^2\sigma_\mu^2\right) \\
&= \beta^2\rho^4 u_0^2 + \beta^2\left(1+\rho^2\right)(1-s)^2(1-b)^2\sigma_\mu^2, \\
\beta^3 E_0\left(u_{i3}^2\right) &= \beta^3 E_0\left(\rho^2 u_2^2 + (1-s)^2(1-b)^2\sigma_\mu^2\right) \\
&= \beta^3\left(\rho^6 u_0^2 + \rho^4(1-s)^2(1-b)^2\sigma_\mu^2 + \rho^2(1-s)^2(1-b)^2\sigma_\mu^2\right. \\
&\quad \left. + (1-s)^2(1-b)^2\sigma_\mu^2\right) \\
&= \beta^3\rho^6 u_0^2 + \left(1+\rho^2+\rho^4\right)(1-s)^2(1-b)^2\sigma_\mu^2, \\
\beta^4 E_0\left(u_{i4}^2\right) &= \beta^4 E_0\left(\rho^2 u_3^2 + (1-s)^2(1-b)^2\sigma_\mu^2\right) \\
&= \beta^4\left(\rho^8 u_0^2 + \rho^6(1-s)^2(1-b)^2\sigma_\mu^2 + \rho^4(1-s)^2(1-b)^2\sigma_\mu^2\right. \\
&\quad \left. + \rho^2(1-s)^2(1-b)^2\sigma_\mu^2 + (1-s)^2(1-b)^2\sigma_\mu^2\right) \\
&= \beta^4\rho^8 u_0^2 + \beta^4\left(1+\rho^2+\rho^4+\rho^6\right)(1-s)^2(1-b)^2\sigma_\mu^2 \\
&\quad . \\
&\quad . \\
&\quad .
\end{aligned}$$

The first terms of each time period make up a geometric series of the form

$$u_0^2 + \beta\rho^2 u_0^2 + \beta^2\rho^4 u_0^2 + \beta^3\rho^6 u_0^2 + \beta^4\rho^8 u_0^2 + ... = \frac{1}{1-\beta\rho^2}u_0^2.$$

The second term of each time period is of the form

$$\beta^t\left(1 + \rho^2 + \rho^4 + ... + \left(\rho^2\right)^{t-1}\right)(1-s)^2(1-b)^2\sigma_\mu^2.$$

If $\rho^2 \neq 1$ (as is the case here, since $\rho < 1$), the geometric series in the above expression is given by

$$\left(1 + \rho^2 + \rho^4 + ... + \left(\rho^2\right)^{t-1}\right) = \frac{1-\left(\rho^2\right)^t}{1-\rho^2}.$$

Combining the second terms gives

$$\begin{aligned}
& \sum_{t=1}^{\infty}\left(\beta^t\frac{1-\left(\rho^2\right)^t}{1-\rho^2}\left(1-s\right)^2\left(1-b\right)^2\sigma_\mu^2\right) \\
= & \frac{\left(1-s\right)^2\left(1-b\right)^2\sigma_\mu^2}{1-\rho^2}\sum_{t=1}^{\infty}\left(\beta^t\left(1-\left(\rho^2\right)^t\right)\right) \\
= & \frac{\left(1-s\right)^2\left(1-b\right)^2\sigma_\mu^2}{1-\rho^2}\left(\beta+\beta^2+\beta^3+...-\rho^2\beta-\left(\rho^2\beta\right)^2-\left(\rho^2\beta\right)^3-...\right) \\
= & \frac{\left(1-s\right)^2\left(1-b\right)^2\sigma_\mu^2}{1-\rho^2}\left(\frac{\beta}{1-\beta}-\rho^2\beta\left(1+\rho^2\beta+\left(\rho^2\beta\right)^2+...\right)\right) \\
= & \frac{\left(1-s\right)^2\left(1-b\right)^2\sigma_\mu^2}{1-\rho^2}\left(\frac{\beta}{1-\beta}-\frac{\rho^2\beta}{1-\rho^2\beta}\right) \\
= & \frac{\beta\left(1-s\right)^2\left(1-b\right)^2\sigma_\mu^2}{1-\rho^2}\left(\frac{1-\rho^2}{\left(1-\beta\right)\left(1-\rho^2\beta\right)}\right) \\
= & \frac{\beta\left(1-s\right)^2\left(1-b\right)^2\sigma_\mu^2}{\left(1-\beta\right)\left(1-\rho^2\beta\right)}.
\end{aligned}$$

Adding the first and the second terms up and setting $u_0 = 0$, which is only a minor restriction assuming that the discount rate, β, is close to unity, yields

$$E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{t-1}^2\right)=\frac{\beta\left(1-s\right)^2\left(1-b\right)^2\sigma_\mu^2}{\left(1-\beta\right)\left(1-\beta\rho^2\right)}.$$

The derivation of $E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{it}^2\right)$ is almost identical to the one above, but now $\left(1-s\right)^2\times\left(1-b\right)^2\sigma_\mu^2$ is replaced with $\left(1-s_i\right)^2\left(\left(1-b\right)^2\sigma_\mu^2+\sigma_\nu^2\right)$, and u_0 is replaced by u_{i0}, since the variable of interest is the unemployment rate of a country i. The flexibility parameter is thus the country specific one s_i instead of the average s. Another change to the previous derivation is the relevant time index. Now the infinite sum is over the current period unemployment rate, not over the previous period one. The expected present value of squared future country i specific unemployment deviations is given by

$$E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{it}^2\right)=E_0\left(u_{i1}^2+\beta u_{i2}^2+\beta^2u_{i3}^2+\beta^3u_{i4}^2+...\right).$$

Noting that

$$E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{i,t-1}^2\right)=u_{i0}^2+\beta\left(E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{it}^2\right)\right),$$

and solving this expression for $E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{it}^2\right)$ and using the expression for

$E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{t-1}^2\right)$ derived above (keeping in mind that $(1-s)^2(1-b)^2\sigma_\mu^2$ is replaced with $(1-s_i)^2\left((1-b)^2\sigma_\mu^2+\sigma_\nu^2\right)$) yields

$$\begin{aligned}
E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{i,t}^2\right) &= -\frac{u_{i0}^2}{\beta}+\frac{E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{i,t-1}^2\right)}{\beta} \\
&= -\frac{u_{i0}^2}{\beta}+\frac{1}{\beta}\left(\frac{1}{1-\beta\rho^2}u_{i0}^2+\frac{\beta(1-s_i)^2\left((1-b)^2\sigma_\mu^2+\sigma_\nu^2\right)}{(1-\beta)(1-\rho^2\beta)}\right) \\
&= \frac{(1-\beta)\rho^2u_{i0}^2+(1-s_i)^2\left((1-b)^2\sigma_\mu^2+\sigma_\nu^2\right)}{(1-\beta)(1-\beta\rho^2)}.
\end{aligned}$$

Finally, setting $u_{i0}=0$ yields

$$E_0\left(\sum_{t=1}^{\infty}\beta^{t-1}u_{it}^2\right)=\frac{(1-s_i)^2\left((1-b)^2\sigma_\mu^2+\sigma_\nu^2\right)}{(1-\beta)(1-\beta\rho^2)}.$$

A.5 Derivation of b_ρ

Differentiating (47) with respect to ρ and simplifying yields

$$b_\rho=2(1-s)^2\beta\frac{(1-\rho^2\beta)cc_\rho+\rho(\lambda+\beta c^2)}{\left(1+(1-s)^2\lambda-\beta\rho^2+(1-s)^2\beta c^2\right)^2}>0,$$

where c_ρ is given by

$$\begin{aligned}
c_\rho &= \frac{1}{2}\left(1+\rho^2\beta\right)\frac{(1-\rho^2\beta)-\sqrt{(1-\rho^2\beta)^2-4\rho^2\beta\lambda(1-s)^2}}{\sqrt{(1-\rho^2\beta)^2-4\rho^2\beta\lambda(1-s)^2}(1-s)\rho^2\beta} \\
&= \left(1+\rho^2\beta\right)\frac{c}{\sqrt{(1-\rho^2\beta)^2-4\rho^2\beta\lambda(1-s)^2}\rho}>0.
\end{aligned}$$

A.6 Derivation of Condition (57)

In order to show that the condition (57) does actually hold in this model, the expressions for b_s and c_s are needed (see Appendix A.2). Using (47) to rewrite $(1-b)^2$ the second term of (57) becomes

$$\frac{2\lambda(1-s)(1-b)^2}{(1-\beta\rho^2)}=\frac{2\lambda(1-s)(1-\beta\rho^2)}{\left(1+(1-s)^2\lambda-\beta\rho^2+(1-s)^2\beta c^2\right)^2}.$$

Inserting b_s and the above expression into (57) yields

$$\begin{aligned} b_s + \frac{2\lambda(1-s)(1-b)^2}{(1-\beta\rho^2)} &= \\ -\frac{2(1-s)\beta(1-\beta\rho^2)c(c-c_s(1-s))}{\left(1-\beta\rho^2+(1-s)^2(\lambda+\beta c^2)\right)^2} &< 0, \end{aligned}$$

which holds since $c > 0$ and $c_s < 0$, provided that $\rho > 0$.

A.7 Derivation of $lim_{\rho\to 0}(c_s)$

It is easy to see from Appendix A.2 that $lim_{\rho\to 0}(c_s)$ is of the form $\left(\frac{0}{0}\right)$. Applying L'Hôpital's rule gives (after tedious manipulation)

$$\begin{aligned} & lim_{\rho\to 0}(c_s) \\ = & lim_{\rho\to 0}\frac{\frac{\partial}{\partial\rho}\left(-(1-\rho^2\beta)\left((1-\rho^2\beta)-\sqrt{(1-\rho^2\beta)^2-4\rho^2\beta\lambda(1-s)^2}\right)\right)}{\frac{\partial}{\partial\rho}\left(2(1-s)^2\rho\beta\sqrt{(1-\rho^2\beta)^2-4\rho^2\beta\lambda(1-s)^2}\right)} \\ = & lim_{\rho\to 0} - 2\rho\left(\frac{-(1-\beta\rho^2)\sqrt{(1-\rho^2\beta)^2-4\rho^2\beta\lambda(1-s)^2}}{(1-s)^2\left((1-\rho^2\beta)^2-2\rho^2\beta\left(1-\beta\rho^2+4\lambda(1-s)^2\right)\right)}\right. \\ & \left. +\frac{(1-\rho^2\beta)^2+\lambda(1-s)^2(1-3\rho^2\beta)}{(1-s)^2\left((1-\rho^2\beta)^2-2\rho^2\beta\left(1-\beta\rho^2+4\lambda(1-s)^2\right)\right)}\right) \\ = & 0. \end{aligned}$$

A.8 Derivation of Conditions $\widetilde{b}(\overline{s})_s < 0$ and $c_s < 0$

The first condition, $\widetilde{b}(\overline{s})_s < 0$, can be established as follows. The uniqueness of equilibrium condition (52) in the strictest form with $\sigma_\nu^2 = 0$ implies $-b_s < \frac{(1-b)}{2(1-s)}$, and the condition $\widetilde{b}(\overline{s})_s < 0$ is given by

$$\widetilde{b}(\overline{s})_s = \frac{\beta c}{\lambda}(-b_s c + 2c_s(1-b)) < 0.$$

Now, if the above condition holds when $-b_s < \frac{(1-b)}{2(1-s)}$ is inserted into it, $\widetilde{b}(\overline{s})_s < 0$ is shown to be valid. In order to proceed, it is useful to note the following relation between c and c_s, which can be derived (see equation (43) and c_s in Appendix A.2

$$k\frac{c}{(1-s)} + c_s = 0,$$

where

$$k = \frac{1 - \beta\rho^2}{\sqrt{\left(1 - \beta\rho^2\right)^2 - 4\left(1 - s\right)^2 \rho^2 \beta\lambda}} \geq 1.$$

Now, the first condition, $\widetilde{b}\left(\overline{s}\right)_s < 0$, becomes

$$\begin{aligned} \widetilde{b}\left(\overline{s}\right)_s &< \frac{2\beta c\left(1-b\right)}{\lambda}\left(\frac{1}{4}\frac{c}{\left(1-s\right)} + c_s\right) \\ &= \frac{2\beta c\left(1-b\right)}{\lambda}\left(\frac{1}{4} - k\right)\frac{c}{\left(1-s\right)} < 0. \end{aligned}$$

The second part of the sufficient condition for improved stabilization as s increases, that is, $c_s < 0$, is always satisfied, as is evident from Appendix A.2.

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Chapter 4

Adaptive Learning and Multiple Equilibria in a Natural Rate Model with Unemployment Persistence

Abstract

This essay demonstrates that adaptive learning approach of modelling private sector expectations can be used as an equilibrium selection mechanism in a natural rate monetary model with unemployment persistence. In particular, it is shown that only one of the two rational expectations equilibria is stable under least-squares learning, and that it is always the low inflation equilibrium with intuitive comparative statics properties which is learnable. Hence, this essay provides a theoretical justification for focusing on the low inflation equilibrium. Earlier contributions, in which the high inflation equilibrium was ignored mainly because of its unpleasant characteristics, are not theoretically satisfactory.

1 Introduction

Recent years have witnessed a surge of interest in the application of adaptive learning approach on the analysis of monetary policy (see Evans and Honkapohja (2003) for a recent survey of the literature). The starting point in the analysis of adaptive learning in macroeconomics is that some or all agents have imperfect knowledge about the structure of the economy. The agents rely on an econometric learning technology to form expectations and continuously update their estimates on the structure of the economy based on incoming data (see Evans and Honkapohja (2001) for a comprehensive treatment of learning in macroeconomics).

In the literature of learning and monetary policy there are, broadly speaking, two separate classes of papers. The first one looks at the case where the central bank has imperfect knowledge about the functioning of the economy, whereas the private sector has fully rational expectations (see e.g. Sargent (1999) and Cho, Williams and Sargent (2002)). Another, more widely used approach models the private sector being boundedly

rational and the central bank having full information about the structure of the economy (see e.g. Evans and Honkapohja (2002)). However, these two modelling choices are not mutually exclusive, as is demonstrated in Honkapohja and Mitra (2002), where both the central bank and the private sector are learning. Another classification can be made in terms of the macroeconomic framework used in the analysis. Some contributions apply the natural rate model where the Phillips curve is of the neoclassical type, whereas the bulk of the latest literature uses a New-Keynesian framework with forward-looking agents.[1]

The purpose of this essay is to demonstrate that the adaptive learning approach of modelling private sector expectations can be used as an equilibrium selection mechanism in a dynamic monetary policy model of Lockwood and Philippopoulos (1994).[2] Their model is an extension to the classic Kydland and Prescott (1977) and Barro and Gordon (1983) natural rate model.[3] Lockwood and Philippopoulos (1994) show that when unemployment rate is persistent, there are two rational expectations equilibria in the model.[4] One is associated with low inflation and intuitive comparative statics properties, and the other with high inflation and counter-intuitive comparative statics properties. In particular, this essay shows that only one of the two rational expectations equilibria is stable under adaptive learning, and that it is always the low inflation equilibrium with intuitive comparative statics properties which is learnable. Earlier Sargent (1999) has shown that the unique rational expectations equilibrium of the standard Barro-Gordon model is stable under least-squares learning.

The learning approach to expectations formation is especially interesting in a situation where the economy has gone through a regime shift, like the formation of the Economic and Monetary Union (EMU) in Europe. Monetary policies were quite different in the member countries before the EMU, and thus it is quite natural to assume that the agents don't exactly know all parameter values, e.g. the preferences of the common central bank, which affect the economy. After such a dramatic change in the economic environment as the EMU brought about, rational expectations is a particularly strong assumption and some sort of bounded rationality may provide a much more realistic starting point in thinking of the behavior of the agents.

In the earlier contributions which use the natural rate model with real persistence two

[1] Most notable examples of the former approach are Sargent (1999), Cho, Williams and Sargent (2002) and Orphanides and Williams (2003). Evans and Honkapohja (2003) contains a survey of the New-Keynesian strand of the literature.

[2] Rantala (2003) is an almost identical working paper version of this essay.

[3] The standard natural rate model is usually called the Barro-Gordon model, a convention which is somewhat unfair to the original contribution by Kydland and Prescott (1977). For brevity, this usual practice is also followed in this essay.

[4] It is empirically well established that the unemployment rate is highly persistent in most industrialized countries (see e.g. Layard, Nickell and Jackman (1991)).

solutions to the multiple equilibria problem have been adopted. The first solution is to ignore the "bad" high inflation equilibrium by appealing to counter-intuitive comparative statics properties, and to the fact that the high inflation equilibrium appears only in the infinite horizon model. This strategy is followed e.g. by Svensson (1997, 1999), Beetsma and Jensen (1999), Lockwood, Miller and Zhang (1998), Røisland (2001) and also in Chapter 3 of this dissertation. The second solution treats the two equilibria more equally and considers them as being the possible outcomes of the model. This line of reasoning in followed e.g. by Lockwood and Philippopoulos (1994), Lockwood (1997) and Jensen (1999). This essay proposes a third solution to this problem, namely the use of adaptive learning as a selection tool. Hence, this essay provides perhaps a more elegant justification in a theoretical sense for focusing on the low inflation equilibrium than the first approach, where the high inflation equilibrium was ignored mainly because of its unpleasant characteristics.

This essay is organized as follows. Section 2 introduces the model setup. The rational expectations equilibria are solved in Section 3. Expectational stability of the equilibria is studied in Section 4, and 5 analyzes the model under least-squares learning. Section 6 concludes.

2 The Model

The model economy evolves over an infinite number of time periods $t = 1, 2, ...$ The economy consists of a private sector and a central bank. In the private sector there is a continuum of firms producing the same product, so that the goods market is perfectly competitive. In each firm the labor force is organized in a firm-specific labor union, which sets the nominal wage in each time period before the central bank sets inflation.

The assumption of an atomistic labor market is made in order to make inflation exogenous to each individual labor union. This assumption rules out any strategic interactions between the wage setters and the central bank. Assuming that labor union preferences are identical across firms and abstracting from firm-specific disturbances makes it possible to write the model in terms of a single firm-union pair, where the union takes inflation as given. Naturally, one could continue to work with the continuum of firm-union pairs and aggregate over firms later on. The approach adopted here is chosen purely for notational simplicity.[5]

Labor demand determines employment in the model. For simplicity, the labor demand schedule is written directly in terms of unemployment instead of employment and the real

[5] In what follows, the notions "private sector" and "labor union" are used interchangeably.

wage elasticity is set to unity

$$u_t = w_t - p_t, \tag{1}$$

where u_t is the unemployment rate, w_t is the log of the nominal wage and p_t is the log of the price level.

The labor union's loss function is of the form

$$U_t = (u_t - \overline{u}_t)^2, \tag{2}$$

where $\overline{u}_t$ is the time period t unemployment target. The unemployment target of the labor union is assumed to be time varying and affected by "insider power". In the spirit of Lindbeck and Snower (1986), and following the formulation by Blanchard and Summers (1986) with a minor modification (unemployment instead of employment), it is assumed that the unemployment target of the union is a weighted average of the currently unemployed workers, u_{t-1}, and the long-run unemployment target of the labor union, u^0, which is normalized to zero[6]

$$\overline{u}_t = \rho u_{t-1} + (1 - \rho)\, u^0, \tag{3}$$

where $u^0 = 0$ and $0 < \rho < 1$. This formulation captures the insider power in wage setting in a sense that when unemployment is above the long-run target of zero, the currently employed insiders want to set the nominal wage at a level which is consistent with the current unemployment rate u_{t-1}. This wage level is higher than the one consistent with the long-run target of zero unemployment. The parameter ρ is thus a measure of insider power in the model. If the insiders don't have more power than others, ρ equals zero and the unemployment target is constant. In this case the model collapses to the standard Barro-Gordon model. A high ρ implies that insiders dominate wage setting and the unemployment target of the union varies a lot with changes in the number of insiders.

The union's optimization problem is to set w_t so as to minimize the expected value of (2). Carrying out minimization yields a simple nominal wage rule of the form (after inserting (3))

$$w_t = p_t^e + \rho u_{t-1}. \tag{4}$$

Combining the wage rule (4) with the labor demand schedule (1), and using the conventional definitions $\pi_t = p_t - p_{t-1}$ and $\pi_t^e = p_t^e - p_{t-1}$ for inflation and inflationary expectations yields the neoclassical (expectations augmented) Phillips curve with unem-

[6] u^0 could equally well be non-zero. This normalization is made for analytical simplicity only.

ployment persistence

$$u_t = \rho u_{t-1} - (\pi_t - \pi_t^e)\,. \tag{5}$$

In this simple framework the insider power parameter directly determines the persistence of the unemployment rate.[7] The long-run natural rate of unemployment is zero in the model, which results directly from the long-run unemployment target of the labor union being zero.

The intertemporal loss (or value) function of the central bank is of the standard quadratic type

$$V = \sum_{t=1}^{\infty} \delta^{t-1} \frac{1}{2} \left(\lambda u_t^2 + (1-\lambda)\,\pi_t^2\right), \tag{6}$$

where δ $(0 < \delta < 1)$ is the discount factor and λ and $(1-\lambda)$ are the relative weights of unemployment and inflation stabilization, respectively.[8] It is noteworthy that the unemployment target of the central bank is zero, so it coincides with the long-run natural rate. This implies that the central bank doesn't have an over-ambitious unemployment target on average. Hence, there is no average inflation bias in the model. Inflation target of the central bank is, without loss of generality, normalized to zero for analytical convenience. Throughout the essay it is assumed that the central bank doesn't have access to a commitment technology and therefore sets inflation in a discretionary manner.[9]

3 Rational Expectations Equilibria

In this section the model is solved assuming that the private sector has rational expectations. The game between the private sector and the central bank is dynamic. This results from unemployment persistence, because the central bank's decisions today will affect the future through unemployment rate. Obviously, another requirement for the game being dynamic is that the central bank is forward-looking in a sense that it cares about the future, that is, $\delta > 0$. The unemployment rate of the previous time period, u_{t-1}, is the state variable of the dynamic problem, since it summarizes the impact of the history of the game on the current state of the economy.

The solution concept used in the essay is the Markov-perfect equilibrium, where the current actions of the players depend on the history of the game only through the state variable and trigger strategies are ruled out (see e.g. Fudenberg and Tirole (1991)).

[7] See Lockwood, Miller and Zhang (1998) for a formulation where both unemployment and real wage deviations enter the labor union's loss function.

[8] The notation follows that of Lockwood and Philippopoulos (1994) and differs somewhat from the one used in the second essay (Chapter 3).

[9] The commitment solution to the problem is presented e.g. in Svensson (1997).

Although the labor union is setting the nominal wage, equally well it can be said that it chooses inflation expectations. Thereby, the choice variables of the central bank and the labor union are inflation and inflation expectations, respectively.

The linear-quadratic structure of the model implies that the value functions will be quadratic. The labor union has only one target and one instrument. Hence, in a model without uncertainty it can choose inflation expectations in every period so that it will achieve its target. From (2)-(5) it is easy to see that by setting $\pi_t^e = \pi_t$, that is, having rational expectations, yields zero loss in each time period. The central bank's value function from time period t onwards will be of the form

$$V = \frac{\beta}{2} u_{t-1}^2, \tag{7}$$

where $\beta \geq 0$ is the parameter to be solved. Throughout the analysis it is assumed that β is not time dependent.

The dynamic game can now be written as

$$\begin{aligned} \frac{\beta}{2} u_{t-1}^2 &= \min_{\pi_t} \left(\lambda + \delta\beta\right) \frac{u_t^2}{2} + \left(1 - \lambda\right) \frac{\pi_t^2}{2} \quad \text{given (5) and } \pi_t^e \text{ fixed,} & (8) \\ \pi_t^e &= \pi_t. & (9) \end{aligned}$$

The left-hand side of (8) is the present value of losses from t onwards, and the right-hand side is the value of current period losses, plus the losses from $t+1$ onwards evaluated at the optimally chosen level of inflation. Monetary policy is discretionary and thereby the central bank takes inflation expectations as given. Equation (9) is the best response function of the union, which says that rational expectations are optimal.

The first-order condition of the central bank is given by

$$-\left(\lambda + \delta\beta\right) u_t + \left(1 - \lambda\right) \pi_t = 0, \tag{10}$$

which after inserting (5) and solving for π_t yields

$$\pi_t = \frac{\lambda + \delta\beta}{1 + \delta\beta} \pi_t^e + \frac{\lambda + \delta\beta}{1 + \delta\beta} \rho u_{t-1}. \tag{11}$$

This is the best response function of the central bank, which is conditional on fixed β. The rational expectations equilibrium with fixed β is then obtained by setting $\pi_t^e = \pi_t$, which gives

$$\pi_t\left(\beta\right) = \pi_t^e\left(\beta\right) = \frac{\lambda + \delta\beta}{1 - \lambda} \rho u_{t-1}. \tag{12}$$

Since the problem is linear-quadratic, the central bank's and the labor union's actions are linear functions of the state variable.

The next step is to solve β. Inserting (12) into (8) and reorganizing the terms results in the Riccati equation defining β

$$\rho^2\delta^2\beta^2 + \left(\lambda - 1 + (1+\lambda)\,\delta\rho^2\right)\beta + \lambda\rho^2 = 0. \tag{13}$$

This equation is quadratic in β, which implies that there in general are two equilibrium values for β. From (12) it is then obvious that there are multiple rational expectations inflation equilibria in the model. The lower equilibrium value of β corresponds to the low inflation equilibrium, and the higher value corresponds to the high inflation equilibrium.

Solving (8) for β reveals that β is a non-linear function of the optimally chosen rate of inflation π_t and the given inflation expectations π_t^e

$$\beta = \frac{\lambda\left(\rho u_{t-1} + \pi_t^e - \pi_t\right)^2 + (1-\lambda)\,\pi_t^2}{u_{t-1}^2 - \delta\left(\rho u_{t-1} + \pi_t^e - \pi_t\right)^2}, \tag{14}$$

where π_t is determined by (11).[10] Substitution of (14) into (11) and simplifying yields a quadratic equation

$$\begin{aligned} & (1-\lambda)\,\delta\left(\rho u_{t-1} + \pi_t^e\right)\pi_t^2 + \left(u_{t-1}^2 - (1-\lambda)\,\delta\left(\rho u_{t-1} + \pi_t^e\right)^2\right)\pi_t \\ - \quad & \lambda\left(\rho u_{t-1} + \pi_t^e\right)u_{t-1}^2 = 0. \end{aligned} \tag{15}$$

This quadratic function of π_t has two roots, but one of them is always negative irrespective of π_t^e and u_{t-1}. This is inconsistent with (11) and $\beta \geq 0$, so the unconditional reaction function of the central bank is the larger root of (15)[11]

$$\pi_t = \frac{-\left(u_{t-1}^2 - \varphi\left(\pi_t^e\right)\right) + \sqrt{\left(u_{t-1}^2 - \varphi\left(\pi_t^e\right)\right)^2 + 4\lambda\varphi\left(\pi_t^e\right)u_{t-1}^2}}{2\,(1-\lambda)\,\delta\left(\rho u_{t-1} + \pi_t^e\right)}. \tag{16}$$

where $\varphi\left(\pi_t^e\right) = (1-\lambda)\,\delta\left(\rho u_{t-1} + \pi_t^e\right)^2$. It is evident from (16) that the relationship between inflation and inflationary expectations is highly nonlinear.

The two rational expectations equilibria can now be solved by setting $\pi_t^e = \pi_t$ in (16)

[10] (14) is derived from (8) assuming that β is not time varying. The $\min_{\pi_t}$ operator can be removed, since π_t is the optimal inflation chosen by the central bank.

[11] The present value of losses at time period t is non-negative, that is, $\beta \geq 0$. Giving π_t^e and u_{t-1} arbitrary positive values in (11) yields a positive π_t. Since the smaller root of (15) yields negative values for π_t with all possible combinations of π_t^e and u_{t-1}, it is ignored in the analysis.

and simplifying, which yields real solutions

$$\pi_t = \pi_t^e = \frac{1}{2\delta\rho}\left(\left(1-\delta\rho^2\right) \pm \sqrt{\left(1-\delta\rho^2\right)^2 - 4\lambda\left(1-\lambda\right)^{-1}\delta\rho^2}\right) u_{t-1}, \tag{17}$$

if, and only if, the following existence condition holds

$$\frac{\lambda}{1-\lambda} \leq \frac{\left(1-\delta\rho^2\right)^2}{4\delta\rho^2}. \tag{18}$$

Obviously, this condition is assumed to hold, since complex solutions make no sense for rates of inflation and unemployment.

When the unemployment rate of previous time period is above (below) the long-run natural rate of zero, there are two equilibrium levels of inflation, both of which are positive (negative). Hence, there is a state-dependent inflation bias in the model. Naturally, when $u_{t-1} = 0$, inflation is at the target level of zero, since the is no average inflation bias. It is noteworthy that in addition to the multiple equilibria problem, there is also an indeterminacy problem in the model. With given parameter values either one of the two equilibria can emerge as an equilibrium of the dynamic game.

The comparative statics properties for the two equilibria are quite different. At the low inflation equilibrium inflation increases when the central bank puts more weight on unemployment stabilization and less weight on inflation stabilization, that is, when λ becomes bigger. This is intuitive, since the central bank then cares more about persistent unemployment fluctuations, and therefore reacts more strongly to unemployment by creating higher inflation. Moreover, inflation increases when unemployment persistence ρ increases, or when the discount factor δ becomes bigger so that the central bank cares more about the future. An increase in ρ calls for a stronger inflation reaction, because the state-dependent inflation bias in the future gets more serious when unemployment fluctuations become bigger, and so the benefits of higher inflation this period are increased while the costs are unaffected. When the central bank cares more about the future, it pays more attention to persistence in unemployment and therefore reacts more strongly to unemployment deviations. All this is quite intuitive, but it holds only at the low inflation equilibrium. When the economy is at the high inflation equilibrium, changes in the values of these three parameters bring about inflation responses, which are exactly of opposite direction. That is, the comparative statics are counter-intuitive (see the Appendix for derivations).

There exists a unique rational expectations equilibrium in the model provided that

the time horizon of the central bank is finite.[12] In addition, it can be shown that the equilibrium in question approaches the low inflation equilibrium of the infinite horizon model as the terminal period goes further away from the current period (see the Appendix for an illustration of this point).

The two equilibria in (17) can be illustrated graphically in (π_t, π_t^e)-space as intersections of the best response functions of the central bank and the labor union. Before doing that, it is useful to characterize the behavior of the best response function of the central bank (16). First, (16) can be shown to be positive at $\pi_t^e = 0$ when $u_{t-1} > 0$. Second, the slope of (16) can, after tedious manipulations, be shown to be positive with all π_t^e (see the Appendix). These two properties together with the fact that there are exactly two positive inflation equilibria when $u_{t-1} > 0$ guarantee that (16) cuts the 45 degree line twice, first from above and then from below. This is depicted in Figure 1, where the low and high inflation equilibria are denoted by π_- and π_+, respectively.

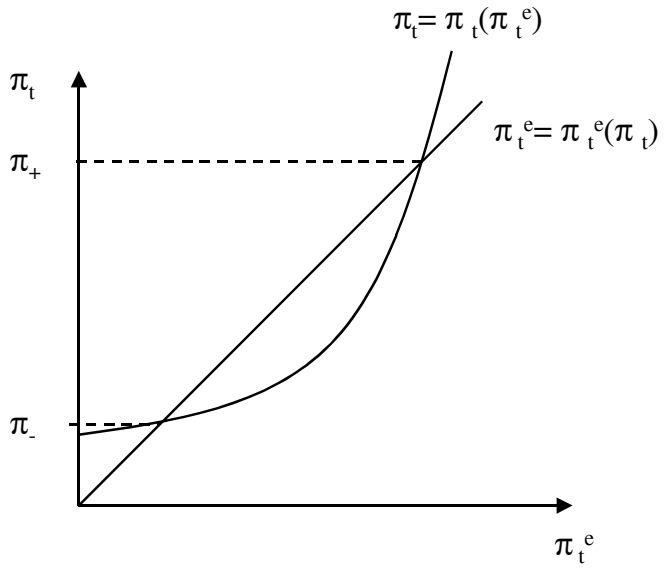

Figure 1: Determination of rational expectations equilibria

The multiplicity of equilibria in the model arises from unemployment persistence, which makes the best response function of the central bank nonlinear in inflationary expectations. In the standard Barro-Gordon model real persistence is absent and the reaction function of the central bank is linear in inflationary expectations with a slope

[12] Lockwood and Philippopoulos (1994) point out that a somewhat weaker condition than a finite horizon suffices to guarantee the uniqueness of equilibrium in the model. Namely, they show that if the inflation variability is eliminated with probability one at some known time period in the future, the high inflation equilibrium disappears.

smaller than unity (see e.g. Sargent (1999)). Therefore, in that case the equilibrium is unique. Moreover, both inflation equilibria in the presence of real persistence yield a higher inflation than in the standard case.[13] This is quite intuitive, since with persistence the incentive to inflate the economy becomes stronger due to the fact that the state-dependent inflation bias in the future can be reduced by choosing a higher inflation today.

4 Expectational Stability of the Equilibria

Rational expectations from the private sector's point of view mean that the agents are endowed with perfect knowledge about the structure of the economy including e.g. the preferences of the central bank and the monetary transmission mechanism. Obviously, this is a very strict requirement which is relaxed in a specific manner in what follows. In particular, the agents are assumed to be boundedly rational in a sense that they don't know the inflation process exactly, but they try to learn it by estimating the process with observed macroeconomic data at each time period. The setting is non-standard, because the system is self-referential in a sense that future data points are affected by the estimates produced today. Analyzing the dynamics of such a system calls for a special technique called stochastic approximation, which is discussed in the next section.

The agents apply the standard least-squares estimation procedure, in which the parameters to be estimated are assumed to be constants over time. However, since the agents are updating their estimates, the true coefficient governing the inflation process is actually changing over time. Their model is thereby dynamically misspecified, but if learning converges the misspecification problem disappears. Throughout the analysis it is assumed that the estimated model is of the same form as the rational expectations equilibria in (17), that is, inflation is assumed to be a constant times last period's unemployment rate (plus some noise in the next section).[14]

This section uses the methodology of Evans and Honkapohja (2001, Ch. 2) to study the learnability of the rational expectations equilibria. The criterion used in the analysis is called expectational stability, or E-stability for short. An equilibrium is said to be stable under learning if it fulfills the E-stability condition. Next section demonstrates that the local convergence of real-time recursive least-squares learning in this model is governed

[13] See Lockwood and Philippopoulos (1994) for an illustration of this point.

[14] In the next section a control error is added for technical reasons to the inflation process. Obviously, the stochastic component could have been introduced into the model already in section 2 without any changes in the main results.

by the very same E-stability condition.[15] The E-stability approach is very useful, since the E-stability of an equilibrium is in most cases technically much easier to establish than least-squares learnability.

The private sector has a perceived law of motion (PLM) of the same form as (17)

$$\pi_t = au_{t-1}, \tag{19}$$

but it doesn't know the correct value of the coefficient in front of u_{t-1}. The parameter a is the private sector's perception of the coefficient. The forecast function used to form expectations is obtained from the PLM

$$\pi_t^e = au_{t-1}. \tag{20}$$

Combining (20) with the best response function of the central bank (16) and reorganizing gives the actual law of motion (ALM) the economy follows when the private sector has the PLM described above

$$\pi_t = \frac{-\left(1-\omega\left(a\right)\right)+\sqrt{\left(1-\omega\left(a\right)\right)^2+4\lambda\omega\left(a\right)}}{2\left(1-\lambda\right)\delta\left(\rho+a\right)}u_{t-1}, \tag{21}$$

where $\omega\left(a\right)=\left(1-\lambda\right)\delta\left(\rho+a\right)^2$. Now, the mapping from the PLM to the ALM takes the form

$$T\left(a\right) = \frac{-\left(1-\omega\left(a\right)\right)+\sqrt{\left(1-\omega\left(a\right)\right)^2+4\lambda\omega\left(a\right)}}{2\left(1-\lambda\right)\delta\left(\rho+a\right)}. \tag{22}$$

Expectational stability is determined by the following differential equation

$$\frac{da}{d\tau} = T\left(a\right)-a, \tag{23}$$

where τ denotes virtual time. The fixed points of (23) give the rational expectations equilibria presented in (17). An equilibrium $\overline{a}$ is said to be E-stable if the fixed point of the differential equation (23) is locally asymptotically stable at that point. Thus the E-stability condition is

$$T'\left(\overline{a}\right) < 1. \tag{24}$$

For the E-stability analysis it is possible to utilize previously derived results concerning the behavior of the best response function of the central bank (16). The expression (22)

[15] This is a standard result in the literature.

is just a special case of (16), where π_t^e is replaced by a and u_{t-1} is set to unity. Hence, it is immediately clear from Figure 1 that the slope coefficient $T'(a)$ is smaller than one at the low inflation equilibrium, $\overline{a}_-$, and greater than one at the high inflation equilibrium, $\overline{a}_+$. The result of the analysis can be summarized as

$$T'(\overline{a}_-) < 1 \Rightarrow \overline{a}_- \text{ is E-stable,} \tag{25}$$

$$T'(\overline{a}_+) > 1 \Rightarrow \overline{a}_+ \text{ is E-unstable.} \tag{26}$$

The low inflation equilibrium with intuitive comparative statics properties is always E-stable, whereas the high inflation equilibrium is never E-stable.

5 Real-Time Adaptive Learning

The purpose of this section is to show that under real-time learning, where the private sector updates the coefficient estimate by running recursive least-squares on actual data, the local convergence to the rational expectations equilibrium is governed by the very same E-stability condition derived in the previous section. The mathematical method which is used in analyzing the convergence of least-squares learning is called the theory of stochastic recursive algorithms (SRA) or stochastic approximation (see Evans and Honkapohja (2001, Ch. 6)).

The basic technique of applying the SRA approach in adaptive learning is presented in Evans and Honkapohja (2001, Ch. 2) and here it is adapted for this particular model. Following Sargent (1999), a disturbance term representing the central bank's imperfect control of inflation is added to the bank's best response function (16). The reason for doing that is mainly technical, since then it is guaranteed that the stochastic approximation tools can be applied. The bank's reaction function becomes

$$\pi_t = \frac{-\left(u_{t-1}^2 - \varphi(\pi_t^e)\right) + \sqrt{\left(u_{t-1}^2 - \varphi(\pi_t^e)\right)^2 + 4\lambda\varphi(\pi_t^e)\,u_{t-1}^2}}{2(1-\lambda)\,\delta(\rho u_{t-1} + \pi_t^e)} + \eta_t, \tag{27}$$

where $\varphi(\pi_t^e) = (1-\lambda)\,\delta(\rho u_{t-1} + \pi_t^e)^2$ and η_t is an independent and identically distributed random term with zero mean.

Assuming rational expectations in this stochastic setup would mean that $\pi_t^e = E_{t-1}\pi_t$, where $E_{t-1}\pi_t$ is obtained from (27), keeping in mind that $E_{t-1}\eta_t = 0$. Solving π_t^e from this expression gives exactly the same functional form as in (17). The inflation process

under rational expectations can be written as

$$\pi_t = \pi_t^e + \eta_t, \tag{28}$$

and hence the rational expectations equilibria are of the same form as (17), but augmented with the control error η_t.

Now, the private sector knows the form of the rational expectations equilibria and updates the coefficient estimate each period after a new data point is observed. The PLM is thus time dependent

$$\pi_t = a_{t-1} u_{t-1} + \eta_t, \tag{29}$$

where a_{t-1} is the coefficient estimate obtained at time period $t-1$,which is used to form inflationary expectations for time period t

$$\pi_t^e = a_{t-1} u_{t-1}. \tag{30}$$

The estimation problem can be written in recursive least-squares (RLS) form

$$\begin{aligned} a_t &= a_{t-1} + t^{-1} R_t^{-1} u_{t-1} \left(\pi_t - a_{t-1} u_{t-1}\right), & (31) \\ R_t &= R_{t-1} + t^{-1} \left(u_{t-1}^2 - R_{t-1}\right), & (32) \end{aligned}$$

where R_t is the moment matrix for the regressors, which in this simple setup without an intercept and with only one regressor is actually a scalar $R_t = t^{-1} \sum_{i=1}^{t} u_{i-1}^2$. The reason for including the stochastic control error η_t to inflation can be seen from (31). Without η_t the model would be deterministic, and u_{t-1} would then approach the equilibrium level of zero asymptotically. But then R_t would be zero at the equilibrium and the estimation problem wouldn't be well defined, since R_t^{-1} enters (31). By introducing a stochastic element into the model, it is guaranteed that R_t is non-zero at the equilibrium.

The ALM determining π_t is now generated by the perceptions prevailing at time period $t-1$

$$\begin{aligned} \pi_t &= \frac{-\left(1-\omega\left(a_{t-1}\right)\right) + \sqrt{\left(1-\omega\left(a_{t-1}\right)\right)^2 + 4\lambda\omega\left(a_{t-1}\right)}}{2\left(1-\lambda\right)\delta\left(\rho + a_{t-1}\right)} + \eta_t \\ &= T\left(a_{t-1}\right) u_{t-1} + \eta_t. \end{aligned} \tag{33}$$

Inserting (33) into (31) yields a stochastic recursive system

$$a_t = a_{t-1} + t^{-1} R_t^{-1} u_{t-1} \left(\left(T\left(a_{t-1}\right) - a_{t-1} \right) u_{t-1} + \eta_t \right), \tag{34}$$

$$R_t = R_{t-1} + t^{-1} \left(u_{t-1}^2 - R_{t-1} \right). \tag{35}$$

Before it is possible to study the convergence of this system by applying results from the stochastic approximation literature, a minor modification is needed. On the right-hand side of (34) there exists R_t, but the SRA approach allows only lagged values of a_t and R_t to appear. This problem is solved by defining a new variable $S_{t-1} = R_t$. The system (34)-(35) becomes

$$a_t = a_{t-1} + t^{-1} S_{t-1}^{-1} u_{t-1} \left(\left(T\left(a_{t-1}\right) - a_{t-1} \right) u_{t-1} + \eta_t \right), \tag{36}$$

$$S_t = S_{t-1} + t^{-1} \left(\frac{t}{t+1} \right) \left(u_t^2 - S_{t-1} \right), \tag{37}$$

which is now in the standard SRA form (see Evans and Honkapohja (2001, Ch. 2)

$$\theta_t = \theta_{t-1} + \gamma_t Q\left(t, \theta_{t-1}, X_t\right), \tag{38}$$

with

$$\theta_t = \begin{pmatrix} a_t \\ S_t \end{pmatrix}, \; X_t = \begin{pmatrix} u_t \\ u_{t-1} \\ \eta_t \end{pmatrix}, \gamma_t = t^{-1} \text{ and} \tag{39}$$

$$Q\left(t, \theta_{t-1}, X_t\right) = \begin{pmatrix} S_{t-1}^{-1} u_{t-1} \left(\left(T\left(a_{t-1}\right) - a_{t-1} \right) u_{t-1} + \eta_t \right) \\ \left(\frac{t}{t+1} \right) \left(u_t^2 - S_{t-1} \right) \end{pmatrix}.$$

The stochastic approximation approach associates an ordinary differential equation (ODE) with the SRA,

$$\frac{d\theta}{d\tau} = h\left(\theta\left(\tau\right)\right), \tag{40}$$

where $h\left(\theta\right)$ is obtained as

$$h\left(\theta\right) = \lim_{t \to \infty} EQ\left(t, \theta, X_t\right). \tag{41}$$

$EQ\left(t, \theta, X_t\right)$ denotes the expectation of $Q\left(\cdot\right)$ for fixed θ. The stochastic approximation results show that the ODE (40) approximates the behavior of the SRA (38) well with large t. Importantly, the limit points of the SRA correspond to locally stable equilibria

of the ODE.[16]

Adapted for the problem at hand, (41) becomes

$$h_a(a,S) = \lim_{t\to\infty} ES^{-1}u_{t-1}\left(\left(T(a)-a\right)u_{t-1}+\eta_t\right), \tag{42}$$

$$h_S(a,S) = \lim_{t\to\infty}\left(\frac{t}{t+1}\right)E\left(u_t^2-S\right), \tag{43}$$

where a and S are fixed. Writing $\lim_{t\to\infty} Eu_t^2 \equiv M$ and noting that $\lim_{t\to\infty}\left(\frac{t}{t+1}\right)=1$ the second equation (43) becomes

$$h_S(a,S) = M - S. \tag{44}$$

The first equation (42) can be rewritten by noting that $E(u_{t-1}\eta_t) = 0$ and $\lim_{t\to\infty} Eu_{t-1}^2 = \lim_{t\to\infty} Eu_t^2 \equiv M$, which gives

$$h_a(a,S) = S^{-1}M\left(T(a)-a\right). \tag{45}$$

Finally, the associated ODE is then

$$\frac{da}{d\tau} = S^{-1}M\left(T(a)-a\right), \tag{46}$$

$$\frac{dS}{d\tau} = M - S. \tag{47}$$

The second equation of the recursive system is globally stable so that S approaches M from any starting point. The stability of the system is thereby governed by the first differential equation, which can be further simplified by noting that $S^{-1}M$ approaches unity

$$\frac{da}{d\tau} = T(a) - a. \tag{48}$$

But, this is exactly the same stability condition as the E-stability condition derived in the previous section. Thus E-stability (E-instability) of an equilibrium implies also (in)stability under least-squares learning.

[16] The technical assumptions required for the convergence conditions to hold include regularity assumptions on Q, conditions on the rate at which $\gamma_t \to 0$ and assumptions on the properties of the stochastic process followed by X_t (see Honkapohja and Evans (2001, Ch. 6) for the precise assumptions).

6 Concluding Remarks

This essay has demonstrated that it is possible to use adaptive learning approach as an equilibrium selection device in a dynamic model of monetary policy by Lockwood and Philippopoulos (1994). When the private sector has imperfect information about the way economy works and try to learn the rational expectations equilibrium, it turns out that the economy always arrives at the low inflation equilibrium of the model. In the earlier literature the high inflation equilibrium was ignored by referring to counter-intuitive comparative statics properties or to the fact that it appeared only in an infinite horizon version of the model. These arguments are obviously quite weak in the theoretical sense. The contribution of this essay is that it perhaps provides a more elegant way based on adaptive learning to exclude the high inflation equilibrium from the analysis.

Obviously, the analysis could be extended to many different directions. It would be interesting to study learning with misspecification in this framework. What would happen if some or all agents try to learn a wrong model? For example, some agents could misinterpret the joint existence of inflation and unemployment as an (average) inflation bias, which would result from an over-ambitious long-run unemployment target of the central bank. Would the model still converge to some restricted perceptions equilibrium (see Evans and Honkapohja (2001, Ch. 3)), which would possibly be different from the rational expectations equilibria? Another extension would be to study the robustness of the learnability results when there exists a stabilization role for monetary policy. A potentially challenging extension is related to the solution technique of the dynamic problem adopted in the essay. The dependence of β on the optimal choice of inflation by the central bank is ignored (see equation (14)). If the central bank would take into account this interdependence, the dynamic problem would obviously become a lot harder to solve. The issues raised in this section call for a closer examination, but are left for future research.

A Appendix

A.1 Comparative Statics Properties of the Rational Expectations Equilibria

The rational expectations equilibria are given by (17). Setting $u_{t-1} = 1$ and dropping time indices yields

$$\pi = \frac{1}{2\delta\rho}\left(\left(1-\delta\rho^2\right) \pm \sqrt{\left(1-\delta\rho^2\right)^2 - 4\lambda\left(1-\lambda\right)^{-1}\delta\rho^2}\right).$$

In what follows the low and high inflation equilibria are denoted by π_- and π_+, respectively.

Now, it is easy to establish the following results concerning the weight of unemployment stabilization in the central bank loss function

$$\begin{aligned}
\frac{\partial \pi_-}{\partial \lambda} &= -\frac{1}{4\delta\rho\sqrt{\left(\left(1-\delta\rho^2\right)^2 - 4\frac{\lambda}{1-\lambda}\delta\rho^2\right)}}\left(-\frac{4}{1-\lambda}\delta\rho^2 - 4\frac{\lambda}{\left(1-\lambda\right)^2}\delta\rho^2\right) > 0, \\
\frac{\partial \pi_+}{\partial \lambda} &= \frac{1}{4\delta\rho\sqrt{\left(\left(1-\delta\rho^2\right)^2 - 4\frac{\lambda}{1-\lambda}\delta\rho^2\right)}}\left(-\frac{4}{1-\lambda}\delta\rho^2 - 4\frac{\lambda}{\left(1-\lambda\right)^2}\delta\rho^2\right) < 0,
\end{aligned}$$

where π_- behaves intuitively and π_+ counter-intuitively with respect to changes in λ.

Next, the comparative statics with respect to ρ are derived

$$\begin{aligned}
\frac{\partial \pi_-}{\partial \rho} &= -\frac{1}{2}\left(1+\delta\rho^2\right)\frac{-1+\delta\rho^2+\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}}{\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}\delta\rho^2} > 0, \\
\frac{\partial \pi_+}{\partial \rho} &= \frac{1}{2}\left(1+\delta\rho^2\right)\frac{-1+\delta\rho^2-\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}}{\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}\delta\rho^2} < 0.
\end{aligned}$$

It is clear that $\frac{\partial \pi_+}{\partial \rho}$ is negative, since $-1+\delta\rho^2 < 0$. In order to establish that $\frac{\partial \pi_-}{\partial \rho} > 0$ one needs to investigate the sign of the numerator of that expression

$$-1+\delta\rho^2+\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)},$$

where the first part $-1+\delta\rho^2$ negative. Now, if the first term squared, that is, $(-1+\delta\rho^2)^2$ is bigger than the expression inside the square root, it follows that the numerator is negative and $\frac{\partial\pi_-}{\partial\rho} > 0$

$$\left(-1+\delta\rho^2\right)^2 - \left(\frac{\lambda - 1 + 2\lambda\delta\rho^2 + 2\delta\rho^2 + \delta^2\rho^4\lambda - \delta^2\rho^4}{\lambda - 1}\right) = 4\frac{\lambda}{1-\lambda}\delta\rho^2 > 0,$$

which verifies that the numerator is negative and $\frac{\partial\pi_-}{\partial\rho} > 0$. π_- behaves intuitively and π_+ counter-intuitively with respect to changes in ρ.

The comparative statics with respect to δ are as follows

$$\begin{aligned}
\frac{\partial\pi_-}{\partial\delta} &= \frac{1}{2}\frac{(\lambda\delta\rho^2 + \lambda - 1 + \delta\rho^2) + (1-\lambda)\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}}{\delta^2\rho\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}(\lambda-1)} > 0, \\
\frac{\partial\pi_+}{\partial\delta} &= -\frac{1}{2}\frac{(\lambda\delta\rho^2 + \lambda - 1 + \delta\rho^2) - (1-\lambda)\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}}{\delta^2\rho\sqrt{\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right)}(\lambda-1)} < 0.
\end{aligned}$$

First, it is useful to examine the expression inside the square root. For real solutions to exist, it must be non-negative. Thus the following inequality need to hold (note that the denominator $\lambda - 1 < 0$)

$$\lambda - 1 + 2\lambda\delta\rho^2 + 2\delta\rho^2 + \delta^2\rho^4\lambda - \delta^2\rho^4 < 0.$$

This restriction limits the term $\lambda\delta\rho^2+\lambda-1+\delta\rho^2$ in the numerators of the both derivatives $\left(\frac{\partial\pi_-}{\partial\delta}$ and $\frac{\partial\pi_+}{\partial\delta}\right)$ to negative values. This can be seen by subtracting the term $\lambda\delta\rho^2 + \lambda - 1 + \delta\rho^2$ from the restriction above, that is $\lambda - 1 + 2\lambda\delta\rho^2 + 2\delta\rho^2 + \delta^2\rho^4\lambda - \delta^2\rho^4$, which yields

$$\begin{aligned}
& \lambda - 1 + 2\lambda\delta\rho^2 + 2\delta\rho^2 + \delta^2\rho^4\lambda - \delta^2\rho^4 - \left(\lambda\delta\rho^2 + \lambda - 1 + \delta\rho^2\right) \\
= \; & \delta\rho^2\left(\lambda - \rho^2\delta + \rho^2\lambda\delta + 1\right) > 0.
\end{aligned}$$

Subtracting a positive term from a negative term equals always something negative, but here the outcome is positive, which implies that the term $\lambda\delta\rho^2 + \lambda - 1 + \delta\rho^2$ is negative provided that the restriction derived above holds. Now, it is clear that $\frac{\partial\pi_+}{\partial\delta} < 0$, since the numerator is negative. To prove that $\frac{\partial\pi_-}{\partial\delta} > 0$ one must study the numerator of that expression more closely. The first term squared is bigger than the term inside the square

root (with $1-\lambda$ moved inside)

$$\begin{aligned} & \left(\lambda\delta\rho^2+\lambda-1+\delta\rho^2\right)^2-(\lambda-1)^2\left(\frac{\lambda-1+2\lambda\delta\rho^2+2\delta\rho^2+\delta^2\rho^4\lambda-\delta^2\rho^4}{\lambda-1}\right) \\ = \; & 4\delta^2\lambda\rho^4>0, \end{aligned}$$

which directly implicates that the numerator is negative, and hence $\frac{\partial\pi_-}{\partial\delta}>0$. Again, π_- behaves intuitively and π_+ counter-intuitively.

A.2 Finite Horizon Case

Assuming that there exists a terminal time period T, the central bank's optimizing problem needs to be modified so that future losses equal zero at T. This implies a terminal condition $\beta_T=0$. The Riccati equation with time subscripts can be derived from (8)-(12) by letting β be time dependent. Then on the left-hand side of (8) β is replaced by β_{t-1}, whereas on the right-hand side β is replaced by β_t. This means that in (10)-(12) β becomes β_t. Now, inserting (12) with β_t into (8) yields a dynamic Riccati equation

$$\beta_{t-1}=(\lambda+\delta\beta_t)\,\rho^2+(\lambda+\delta\beta_t)^2\,\rho^2\,(1-\lambda)^{-1}=f\left(\beta_t\right).$$

The function $f\left(\beta_t\right)$ is an increasing and convex function in the relevant range, that is $\beta_t\in[0,\infty)$

$$\begin{aligned} f'\left(\beta_t\right) &= \delta\rho^2+2\left(\lambda+\delta\beta_t\right)\frac{\rho^2}{1-\lambda}\delta>0, \text{ when } \beta_t\in[0,\infty), \\ f''\left(\beta_t\right) &= 2\delta^2\frac{\rho^2}{1-\lambda}>0. \end{aligned}$$

By drawing a phase diagram (Figure 2) it is possible to investigate the backward iterations from the terminal condition $\beta_T=0$.

The phase diagram makes evident that assuming a finite horizon guarantees a unique equilibrium, as iterating backwards from T makes β_t asymptotically converge to the lower equilibrium value of β_-, which is associated with the low inflation equilibrium. However, it is noteworthy, that the equilibrium is not exactly the low inflation equilibrium of the infinite horizon model, since $\beta_t<\beta_-$ with all $T<\infty$. The two equilibria coincide only when $T\to\infty$, but then also the high inflation equilibrium emerges. The more further away in the future T is, the closer the two equilibria are. In other words, since inflation is positively related to β, inflation approaches gradually zero as t approaches T.

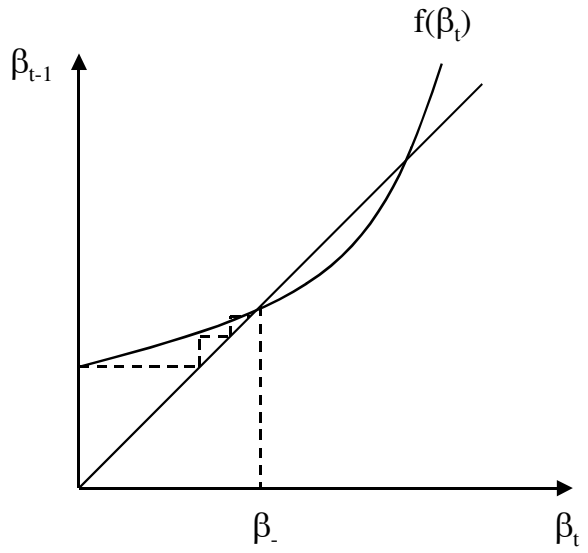

Figure 2: Determination of equilibrium in a finite horizon case

A.3 Behavior of equation (16)

Setting $\pi_t^e = 0$ in (16) yields

$$\begin{aligned}\pi_t(0) &= \frac{-\left(u_{t-1}^2 - \varphi(0)\right) + \sqrt{\left(u_{t-1}^2 - \varphi(0)\right)^2 + 4\lambda\varphi(0)\,u_{t-1}^2}}{2(1-\lambda)\,\delta\rho u_{t-1}} \\ &= \frac{-\left(1-(1-\lambda)\,\delta\rho^2\right) + \sqrt{\left(1-(1-\lambda)\,\delta\rho^2\right)^2 + 4(1-\lambda)\,\lambda\delta\rho^2}}{2(1-\lambda)\,\delta\rho} u_{t-1} > 0.\end{aligned}$$

Taking the derivative of (16) with respect to π_t^e and collecting terms gives

$$\pi_t'(\pi_t^e) = \frac{1}{2}\left(u_{t-1}^2 + \varphi(\pi_t^e)\right) \frac{\left(u_{t-1}{}^2 - \varphi(\pi_t^e)\right) + \sqrt{\left(u_{t-1}^2 - \varphi(\pi_t^e)\right)^2 + 4\lambda\varphi(\pi_t^e)}}{\varphi(\pi_t^e)\sqrt{\left(u_{t-1}^2 - \varphi(\pi_t^e)\right)^2 + 4\lambda\varphi(\pi_t^e)}} > 0.$$

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