

**Modeling and forecasting of realized volatility based on  
high-frequency data: evidence from FTSE-100 index**

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## **Abstract**

As an important parameter, volatility is widely applied in asset pricing, portfolio decisions and value at risk (VaR) etc. A lot of literatures have documented the significant improvements of volatility modeling and forecasting. By using the high-frequency data of FTSE-100 index, and adopting the bi-power test of Barndorff-Nielson and Shephard (2004a, b), we separate the jump components from the realized volatility and model the jump components as well as the continuous components via the application of the newly developed HAR-RV-CJ model.

Our results are in line with Andersen et al. (2007), that the jump components have little prediction power, and we successfully prove that based on the adjusted-R squared and several prediction measurements (RMSE, MAE, etc); HAR-RV-CJ model does perform better than the original HAR-RV model and forecasts more precisely in both the in-sample and out-of-sample predictions.

**Keywords:** Volatility forecasting; High-frequency data; Realized volatility; Bi-power variation; HAR-RV model; HAR-RV-CJ model

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## 1 Introduction

Volatility refers to the price fluctuation over a period of time. In financial economics, volatility is often defined as the standard deviation, although actually volatility and standard deviation are not totally the same. The return volatility is central to financial economics, since in asset pricing, portfolio decisions, and risk management through calculation of value at risk (VaR), the parameter representing the standard deviation is very important. In most of textbooks, the volatility is always constant over time, however, among both finance academics and practitioners, it is widely recognized that the volatility vary importantly over time and volatility itself is not directly observed, it can only be estimated in the context of a model. The two main types of volatility are implied volatility and realized volatility (historical volatility). The implied volatility is often used in option-pricing models. It is the volatility implied by the market price of the option based on an option pricing model. In other words, it is the estimated volatility of a security's price and refers to the market's assessment of future volatility; it is commonly noted as  $\sigma$  in option pricing models. In contrast, realized volatility measures what actually happened in the past. It directly sums the realized log-returns in a given dimension; therefore, more information can be acquired if higher frequency is used. In virtue of the development of information technology, data can be kept in every hour, minute, and even in every second. The extreme high frequency is called ultra high frequency data, which means that every tick is recorded for the calculation of the realized volatility.

However, high frequency data brings microstructure effects such as bid-ask spreads, liquidity ratios, turnover, and asymmetric information that bias the volatility prediction. (Awartani and Corradi 2004) Due to the microstructure effects, the assumption that asset prices in logarithm as a diffusion process becomes less realistic as the time interval decreases. Thus the volatility calculated with very short time intervals is no longer an unbiased and consistent estimator of the daily integrated volatility. Harris (1990), Zhou (1996) and Corsi et al. (2001) found that for return intervals less than a few hours; such a definition is affected by a substantial systematic error. Take the FX

data as an instance, the expectation of daily realized volatility computed with returns at frequencies higher than one hour is systematically larger than the standard deviation of daily returns. Such bias increases with the sampling frequency: at the 1-minute level, it ranges from 30% to about 80% depending on the liquidity of the currency, while at the ultra-high frequency data the estimator is two times larger. (Corsi 2004) Moreover, Mills and Markellos (2008) show that by using the closing prices of Dow-Jones Industrial Average index (DJI), they obtain a volatility estimate of 12.01 percent; however, if the opening prices are used instead, they obtain a lower volatility estimate of around 11.7 percent. This is of course due to the existence of the bid-ask spread as the microstructure effect. Goodhart and O'Hara (1997) have also given a good discussion about the market microstructure.

Therefore, researchers shall find a "trade-off" between accuracy and potential biases caused by the market microstructure effects. So far, a large number of papers have investigated in which level of the sub-interval is the optimal frequency. Andersen and Bollerslev (2001) suggested that in foreign exchange market the optimal frequency is 5-minute. Oomen (2001) found that the best time interval is 25-minute for the volatility prediction of FTSE-100 index. Fang and Wang (2003) stated that in the Shanghai Composite (SSEC) the best interval sample is 10-minute. Bhattacharyya et al. (2009) concluded that for Indian stock markets 30-minute is the optimal frequency. Michiel (2008) argued that in the S&P 100 index, the optimum is more likely to be in the neighborhood of an hour rather than five or 30 minutes. Overall, popular choices in empirical applications are the 5-, 10-, or 30-min intervals, which are believed to strike a balance between the increasing accuracy of higher frequencies and the adverse effects of market microstructure frictions. (Andersen and Bollerslev, 1998; ABDL, 2001; Fleming et al., 2003)

The Heterogeneous Autoregressive model of the Realized Volatility (HAR-RV model) is firstly introduced by Corsi (2004). The basic idea of the model is that agent with different time horizons perceive such as short-term, middle-term and long-term, react to, and cause different types of volatility components which stems from the "Heterogeneous Market Hypothesis" presented by Müller et al. (1993). Corsi (2004)

defines the partial volatility as the volatility generated by a certain market component, and the model is an additive cascade of different partial volatilities. By using high-frequency data of exchange rate of USD/CHF, he compares the HAR-RV model with GARCH (1, 1), J.P. Morgan's RiskMetrics (RM), AR (1), AR (3) and ARFIMA (5, 0.401, 0) models. The author has proved that the HAR-RV model dominates others and successfully achieves the purpose of modeling the long memory behavior of volatility in a simple and parsimonious way.

Forsberg and Ghysels (2007) study the persistence and linear regression properties of various volatility-related processes; they also allow for jumps in the asset return processes and investigate their impact on persistence and linear regression. The models applied are HAR-RV model and MIDAS model. They show that absolute returns have predictive powers for future increments in quadratic variation or realized volatility. Both HAR-RV and MIDAS models involving  $RAV^1$  are relatively robust to the jumps, so there is no need to change the prediction model on days that jumps occur. The Mixed Data Sampling (MIDAS) regressions model is firstly introduced by Ghysel et al. (2004). This MIDAS model involves time series data at different frequencies and has wide applicability in macroeconomics and finance.

Applying related bi-power variation measures, and adopting the jump test of Barndoff-Nielsen and Shephard (2004a, b), Andersen et al.(2007) first develop the HAR-RV-CJ models. In an application to the DM/\$ exchange rate, the S&P 500 market index, and the 30-year U.S. Treasury bond yield, the model comprises the jump components and continuous components of the realized volatility. The authors find that the jump components of realized variance constituted "noise"; that is, it is of no help whatsoever in predicting future realized volatility, only the continuous part has predictive power. They also demonstrate that many of the most significant jumps are readily associated with specific macroeconomic news announcements. This paper does set the stage for a great deal of interesting future studies by separately modeling,

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<sup>1</sup> RAV stands for realized absolute value which is the same as RPV, realized power variance.

forecasting, and pricing the continuous and jump components of the total return variation process.

McAleer and Medeiros (2008) extend the HAR-RV model; they propose a new one that merges long memory and nonlinearities. The new specification is called the Multiple Regime Smooth Transition Heterogeneous Autoregressive (HARST) model. The main advantage of the HARST model is that it can capture both long-range dependence and regime switches (and hence asymmetric effects) in a very simple way. They have successfully shown that when the HAR-RV and HARST models are combined, the forecasting performance improves; and when compared with the alternative latent volatility models (GARCH, EGARCH, GJR-GARCH), the performance of both the HAR and HARST models is far superior.

Chung, Huang and Tseng (2008) incorporate lagged trading volume into the HAR-RV model in order to investigate the role of the trading volume; they term the new forecasting model as the HAR-RV-cum-Volume. The "mixture of distributions hypothesis" provides the theoretical underpinning for the analysis of the link between volume and volatility. (Luu and Martens (2003), Darrat et al. (2003), Holmes and Tomsett (2004), Kalev et al. (2004) and Bauwens et al. (2006) ). Through the use of high-frequency data on the common stocks of IBM, they conclude that with the same regressors, the new model improves the prediction performance of future realized volatility.

Chevallier and Sévi (2009) have been doing an interesting work; they use the high-frequency data from the European CO<sub>2</sub> emissions futures market to test the models. They compare the AR(1)-GARCH(1) model with the HAR-RV model. The authors conclude that (i) the dynamics of realized volatility is well captured using the HAR-RV model with a daily and a weekly component, which outperforms significantly the GARCH specification; (ii) the predictive accuracy of the HAR-RV model outperforms unambiguously other models of conditional volatility based on daily data.

Therefore, it is apparently that the popular realized volatility models have been switched from the standard ARMA family models (including ARIMA, ARFIMA, etc) and ARCH family models (including GARCH, EGARCH, etc) to HAR-RV family

models. (for example, HAR-RV-CJ, HARST, etc) There have been a lot of literatures about this simple but sophisticated HAR-RV model and researchers do find a great improvement by using this model.

The remainder of the article is organized as follows. Section2 provides the theoretical framework of the research; we will introduce several important processes of modeling the continuous as well as the jump parts and show how to separate the jump components by adopting the bi-power test. Section3 is the empirical properties of the high-frequency data; and several stylized facts of the realized volatility series will be introduced too. In section 4, we first present the common methods to model the long-memory persistence and then demonstrate the HAR-RV model, HAR-RV-J model and the HAR-RV-CJ model, in both linear and non-linear forms. The empirical results and in-sample and out-of-sample predictions of the HAR-RV family models are in section 5, we find the HAR-RV-CJ model performs the best. Finally, we conclude in section 6 with several suggestions of future studies.

## 2 Theoretical framework and research methodology

### 2.1 Continuous sample path diffusions

Followed by Andersen et al. (2002a), we first give a univariate risky logarithmic price process  $S(t)$  defined on a complete probability space,  $(\Omega, \mathcal{F}, P)^2$ . The price process evolves in a continuous time over the interval  $[0, T]$ , where  $T$  is a finite integer. The filtration is denoted  $(\mathcal{F}_t)_{t \in [0, T]} \subseteq \mathcal{F}$ , where the information set,  $\mathcal{F}_t$ , contains the full history of the realized values of the asset price and other relevant state variables, and is otherwise assumed to satisfy the usual conditions.

The continuously compounded return over the time interval  $[t-h, t]$  is

$$r(t, h) = S(t) - S(t - h), 0 \leq h \leq t < T. \quad (2.1)$$

Therefore, the cumulative return up to time  $t$ , i.e., the return over the  $[0, t]$  time interval is

$$r(t, h) \equiv r(t, t) = S(t) - S(0) \quad 0 \leq t \leq T. \quad (2.2)$$

Now, we can present the return decomposition form:

$$r(t) \equiv S(t) - S(0) = \mu(t) + M(t) = \mu(t) + M^c(t) + M^j(t), \quad (2.3)$$

where  $\mu(t)$  is a predictable and finite variation process.  $M(t)$  is a local martingale<sup>3</sup> which may be further decomposed into  $M^c(t)$ , a continuous sample path, infinite variation local martingale component, and  $M^j(t)$ , a companion jump martingale. All the parameters are assumed to have initial conditions normalized such that  $\mu(0) \equiv M(0) \equiv M^c(0) \equiv M^j(0) \equiv 0$ , which implies that  $r(t) = S(t)$ .

The most basic continuous-time process used in finance is the Wiener process. The Wiener process works as a mathematical model in modeling the performance of stock price. It is a special form of Markov Stochastic Process; it is also called Brownian motion.

**Definition 1:** A random process  $B_t$ ,  $t \in [0, T]$  is a standard Brownian motion if

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<sup>2</sup> Cont, R. and Tankov, P. (2004) have given a great discussion about the probability space  $(\Omega, \mathcal{F}, P)$ , see *Financial Modelling With Jump Processes* p.28-p.29.

<sup>3</sup> A local martingale is a type of stochastic process, satisfying the localized version of the martingale property.

1.  $B_0 = 0$ .
2.  $B_t$  is continuous.
3. Increments of  $B_t$  are independent. I.e., if  $0 \leq t_0 \leq t_1 \leq \dots \leq t_n$ , then  $B_{t_1} - B_{t_0}, \dots, B_{t_n} - B_{t_{n-1}}$  are independent.
4. If  $0 \leq s \leq t$ ,  $B_t - B_s \sim N(0, t - s)$ .

Thus, the SDE<sup>4</sup> for geometric Brownian motion (GBM) is usually given in this form

$$dS(t) = \mu(t)dt + \sigma(t)dW(t). \quad (2.4)$$

Equation (2.4) is the typical model of continuous-time stochastic volatility where  $W_t$  denotes a standard Brownian motion,  $\mu_t$  is the drift term of finite variation and  $\sigma_t$  is the diffusion term. In other words,  $\mu_t$  is usually regarded as the return rate of the logarithm of stock price,  $\sigma_t$  is the instantaneous volatility (or standard deviation). Consequently,  $\sigma(t)dW(t)$  is a local martingale and  $dS(t)$  follows a special half martingale process. (Protter, 1990)

The volatility function of  $dS(t)$  equals  $\sigma$  for GBM. The constant elasticity of variance (CEV) process of Cox and Ross (1976) permits this function to vary inversely with the level of asset prices; thus

$$dS(t) = \mu(t)dt + \sigma(t)S(t)^{-\beta}dW(t), 0 \leq \beta < 1. \quad (2.5)$$

The above processes i.e., equation (2.4) and (2.5) are nonstationary that have a random walk property; therefore, it is reasonable to postulate a stationary logarithmic price process. The simplest example is the Ornstein-Uhlenbeck (OU) process, whose SDE is

$$dS(t) = \mathcal{K}(\mu - S(t))dt + \sigma(t)dW(t), 0 \leq t \leq T. \quad (2.6)$$

The positive parameter  $\mathcal{K}$  determines the rate at which this process is pulled back towards the mean parameter  $\mu$ . The OU process has been used to model the logarithm of volatility (Scott 1987; Wiggins 1987), because the OU process is a continuous-time extension of the AR (1) process. (Taylor 2005)

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<sup>4</sup> SDE stands for stochastic differential equation.

The OU process can obtain negative values, so it is inappropriate to model positive variables. If we retain the drift function of the OU process and change the volatility function to  $\xi\sqrt{S(t)}$ , we obtain the square-root, or Cox, Ingersoll and Ross (1985) (CIR), process:

$$dS(t) = \mathcal{K}(\mu - S(t))dt + \xi\sqrt{S(t)}dW(t). \quad (2.7)$$

The realizations of the random variables are always positive when the process starts at a positive value. It is necessary to hold the constraint  $2\mathcal{K}\alpha \geq \xi^2$  in order to avoid the sample paths converge to zero. (Taylor 2005)

## 2.2 Announcements arrive and the jump processes

It is widely known that the arrival of important news such as macroeconomic announcements or earnings reports typically induce a discrete jump associated with an immediate revaluation of the asset. (Andersen and Bollerslev (1997) Andersen et al. (2002b)).

For example, below is the plot graph of S&P500 index on the day of 30<sup>th</sup>, June 1999. We can see a big jump after 1 p.m. In fact, the apparent timing of this jump at 13:15 CST (Central Standard Time of the US) or 21:15 (Helsinki time), corresponds just on the time of the 25% increase in the FED (Federal Reserve System) funds rate on that day. The rate hike was accompanied by the statement of the FED as" (it) might not raise rates again in the near term due to conflicting forces in the economy," which gives a positive shock to the stock market obviously. (Andersen et al. 2007)

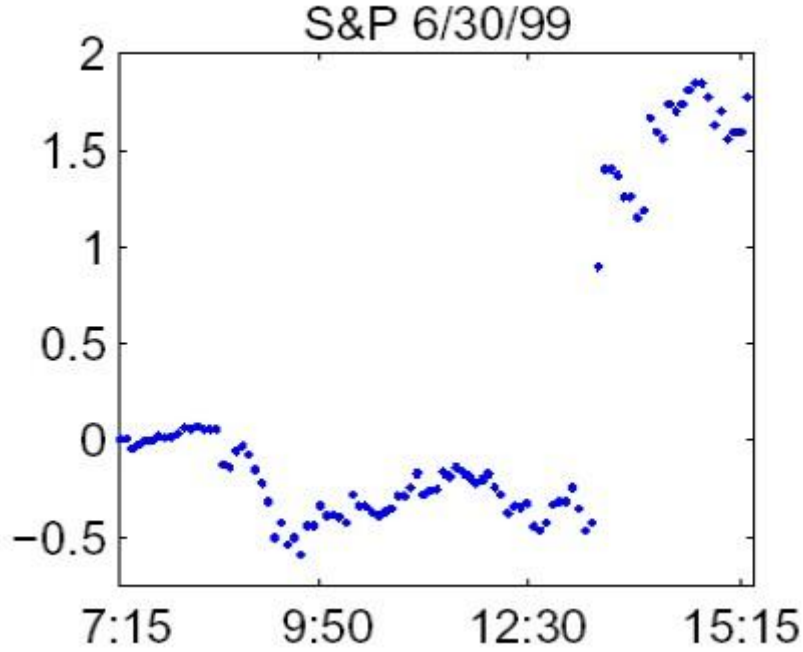


Figure 1 Big jump of S&P 500 index on the day of 30<sup>th</sup> June, 1999

(Source: Andersen et al. 2007)

Therefore, following Merton (1976), we can introduce a model where the asset price has jumps superimposed upon a geometric Brownian motion; that is, one may accommodate the relevant jump features in an arbitrage-free continuous-time logarithmic price process by adding a Poisson jump component with appropriate time variation in the jump intensity and(or) the jump distribution. For example:

$$dS(t) = (\mu(t) - v\lambda(t))dt + \sigma(t)dW(t) + dJ(t). \quad (2.8)$$

Where  $\mu(t)$  is the asset's expected return,  $\lambda(t)$  is the jump intensity (e.g. average number of jumps per year),  $v$  is the average size of the jump as a percentage of the asset price,  $\sigma(t)$  is the return volatility, and  $J(t)$  is related to the Poisson process generating the jumps.

In fact, the Wiener process and the Poisson process described above are the fundamental examples of the Lévy process.

Definition 2: A càdlàg<sup>5</sup> stochastic process  $(X_t)_{t \geq 0}$  on  $(\Omega, \mathcal{F}, P)$  with values in  $\mathbb{R}^d$  such that  $X_0 = 0$  is called a Lévy process if it possesses the following properties:

<sup>5</sup> A càdlàg function is a function defined on the real numbers that is everywhere right-continuous and has left limits everywhere.

1. Independent increments: for every increasing sequence of times  $t_0 \dots t_n$ , the random variables  $\mathbf{X}_{t_0}, \mathbf{X}_{t_1} - \mathbf{X}_{t_0}, \dots, \mathbf{X}_{t_n} - \mathbf{X}_{t_{n-1}}$  are independent.
2. Stationary increments: the law of  $\mathbf{X}_{t+h} - \mathbf{X}_t$  does not depend on  $t$ .
3. Stochastic continuity:  $\forall \varepsilon > 0, \lim_{h \rightarrow 0} \mathbf{P}(|\mathbf{X}_{t+h} - \mathbf{X}_t| \geq \varepsilon) = \mathbf{0}$

The Lévy process is developed by the French mathematician Paul Lévy; it provides the key examples of stochastic processes in continuous time and ingredients for building continuous-time stochastic models.

### 2.3 Semimartingale, quadratic variation and realized variance

Most modern finance theory is based on semimartingales (see, for example, the excellent exposition in Shiryaev (1999, p294-p313.)). Therefore, it is necessary to remind the definition.

Definition 3: Suppose  $S(t)$  is a stochastic process, it is said to be a semimartingale if is decomposable as

$$\mathbf{S}(t) = \boldsymbol{\mu}(t) + \mathbf{M}(t), \boldsymbol{\mu}(\mathbf{0}) = \mathbf{M}(\mathbf{0}) = \mathbf{0}, \quad (2.9)$$

where  $\boldsymbol{\mu}(t)$  refers to a drift term, is a process with locally bounded variation paths(i.e. of bounded variation on any finite subinterval of  $[0, \infty)$ ) and  $\mathbf{M}(t)$  is a local martingale. (Barndorff-Nielson and Shephard, 2002b) For an excellent discussion of probabilistic aspects of this see Protter (1990).

Assume that  $S(t)$  follows the continuous-time semimartingale jump diffusion process:

$$\mathbf{S}(t) = \int_0^t \boldsymbol{\mu}(s) ds + \int_0^t \boldsymbol{\sigma}(s) d\mathbf{W}(s) + \sum_{j=1}^{N(t)} \mathcal{K}(s_j), \quad (2.10)^6$$

where the mean process  $\boldsymbol{\mu}(t)$  is continuous and of finite variation,  $\boldsymbol{\sigma}(t) > 0$  denotes the càdlàg instantaneous volatility and the  $N(t)$  process counts the number of jumps intensity  $\boldsymbol{\lambda}(t)$  and jump size  $\mathcal{K}(s_j)$  where  $\mathcal{K}(t) = \mathbf{S}(t) - \mathbf{S}(t - 1)$  refers to the size of the corresponding discrete jumps in the logarithmic price process. Bollerslev et al. (2009) state that the theory of quadratic variation permits the

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<sup>6</sup> This can be regarded as we incorporate a jump process in equation 2.4. Such as  $dS(t) = \mu(t)dt + \sigma(t)dW(t) + \mathcal{K}(t)dq(t), 0 \leq t \leq T$  where  $dq(t)$  is a counting process which is equal to 1 if there is a jump at time  $t$ , otherwise  $dq(t)=0$ .

derivation of nonparametric volatility measures that grant us to decompose the price variation into two parts, i.e., the continuous and the jump parts:

$$\mathbf{S}(t) = \mathbf{plim} \sum_{j=0}^{n-1} (\mathbf{p}_{\tau_{j+1}} - \mathbf{p}_{\tau_j})^2, \quad (2.11)$$

where  $\boldsymbol{\tau}_0 = \mathbf{0} \leq \boldsymbol{\tau}_1 \leq \dots \leq \boldsymbol{\tau}_t = \mathbf{t}$  denotes a sequence of partitions with  $\sup_j \{\tau_{j+1} - \tau_j\} \rightarrow \mathbf{0}$  for  $n \rightarrow \infty$ , may be expressed as,

$$\mathbf{S}(t) = \int_0^t \boldsymbol{\sigma}^2(\mathbf{s}) d\mathbf{s} + \sum_{j=1}^{N(t)} \mathcal{K}^2(\mathbf{s}_j), \quad (2.12)$$

that is, the integrated variance and the sum of the squared jumps.

Integrated variance (IV) is a measure of the day- $t$  ex post volatility, which is also providing a natural measure of the inherent or notional return variability. This estimator is central to financial economics, whether in asset and derivatives pricing, portfolio selection or risk management. (See, for example, Andersen et al. (2002a))

When the returns of frequency  $M$  ( $M$  is the number of observations within a trading day.) are serially uncorrelated, denote the day  $t$ ,  $j$ th intra-day return by:

$$\mathbf{r}_{i,j}^M = \mathbf{S}_{t,j/M} - \mathbf{S}_{t,(j-1)/M}, \quad (2.13)$$

then via quadratic variation, the daily realized variance (RV) can be defined by the summation of the corresponding  $M$  intra-daily squared returns, namely:

$$\mathbf{RV}_{t,t+1}^M = \sum_{j=1}^M (\mathbf{r}_{t,j}^M)^2. \quad (2.14)$$

Examples of the use of realized variances are given by, for instance, Merton (1980), Poterba and Summers (1986), Schwert (1989), Richardson and Stock (1990), Taylor and Xu (1997) and Christensen and Prabhala (1998). An elegant survey of the literature on this topic, including a discussion of its economic importance, is given by Andersen et al. (2002). See also an excellent contribution by Meddahi (2002).

## 2.4 Separate the jump components

Comte and Renault (1998), Andersen and Bollerslve (1998), ABDL (2001), and Bardorff-Nielson and Shephard (2002a) stated that the realized variance follows directly by the theory of quadratic variation that RV will converge uniformly in

probability to the QV when  $M \rightarrow \infty$ . After that, Bardorff-Nielson and Shephard (2004a,b) presented two empirical processes relating to realized variance are realized power variation (RPV) and realized bi-power variation(BPV).

$$\text{RPV}_{t,t+1}^M = \sum_{i=1}^M |r_{t,i}^M|, \quad (2.15)$$

$$\text{BPV}_{t,t+1}^M = \mu_1^{-2} \sum_{i=2}^M |r_{t,i}^M| |r_{t,i-1}^M|, \quad (2.16)$$

where  $\mu_a \equiv E(|Z|^a)$ ,  $Z \sim N(0,1)$ ,  $a > 0$ ,  $E(|Z|) = \sqrt{2/\pi}$ .

The realized power variation (RPV) is also called the Realized Absolute Value (RAV) which is a summation of the absolute value of every sub-interval return. The realized bi-power variation (BPV) is a summation of the adjacent absolute value of every sub-interval return.  $Z \sim N(0,1)$  means  $Z$  is a random value with the standard normal distribution.

ABDL (1998) and Bardorff-Nielson and Shephard (2002a) proved that for  $M \rightarrow \infty$ ,

$$\lim_{M \rightarrow \infty} \text{RPV}_{t,t+1}^M \rightarrow \int_t^{t+1} \sigma(s) ds \equiv \sigma_{t,t+1}, \quad (2.17)$$

$$\lim_{M \rightarrow \infty} \text{BPV}_{t,t+1}^M \rightarrow \int_t^{t+1} \sigma^2(s) ds \equiv \sigma^2_{t,t+1}. \quad (2.18)$$

Now, combine the equations (2.3) (2.11) and (2.12), we have,

$$\text{RV}_{t,t+1}^M \rightarrow \int_t^{t+1} \sigma^2(s) ds + \sum_{t < s < t+1} \mathcal{K}^2(s), \quad (2.19)$$

This means that the realized variance affords an ex-post measure of the total price variation, including the discontinuous jump part.

Indicated by the equation (2.18),  $\lim_{M \rightarrow \infty} \text{BPV}_{t,t+1}^M \rightarrow \int_t^{t+1} \sigma^2(s) ds \equiv \sigma^2_{t,t+1}$ , we have

$$\text{RV}_{t,t+1}^M - \text{BPV}_{t,t+1}^M \rightarrow \sum_{t < s < t+1} \mathcal{K}^2(s). \quad (2.20)$$

Thus, the jump components can be regarded as the subtraction of RV and BPV; following the suggestion of Barndorff-Nielson and Shephard (2004a), the jump components can be estimated by

$$\check{J}_{t,t+1}^M = \max[\text{RV}_{t,t+1}^M - \text{BPV}_{t,t+1}^M] \quad (2.21)$$

to ensure that all of the daily jump estimates are non-negative.

Above is the discussion of the jump estimates defined by the difference between the realized volatility and the bi-power variation. According to Andersen et al. (2007), the  $\sqrt{\check{J}_{t,t+1}^M}$  series exhibit an unreasonably large number of non-zero small positive values. Therefore, it would be desirable to treat these small jumps as measurement errors, or part of the continuous sample path variation process.

Followed the Bardorff-Nielson and Shephard(2004b), we also define the standardized realized tri-power quantity(TQ), adopted in the bi-power jump test:

$$TQ_{t,t+1}^M = M\mu_{4/3}^{-3} \sum_{i=3}^M (|r_{t,i}^M| |r_{t,i-1}^M| |r_{t,i-2}^M|)^{4/3}, \quad (2.22)$$

where  $E(|Z|^{4/3}) = 2^{2/3} \Gamma(\frac{7}{6}) \Gamma(\frac{1}{2})$ .

Also proved by ABDL (1998) and Bardorff-Nielson and Shephard (2002b), for  $M \rightarrow \infty$ ,

$$\lim_{M \rightarrow \infty} TQ_{t,t+1}^M \rightarrow \int_t^{t+1} \sigma^4(s) ds \equiv \sigma^4_{t,t+1}. \quad (2.23)$$

Thus, following the results from Bardorff-Nielson and Shephard (2004a) and Andersen et al. (2007), the significant jumps and the continuous components are identified as :

$$J_{t,t+1}^\alpha = I(Z_{t,t+1} > \Phi_\alpha)(RV_{t,t+1} - BPV_{t,t+1}), \quad (2.24)$$

$$C_{t,t+1}^\alpha = I(Z_{t,t+1} \leq \Phi_\alpha)RV + I(Z_{t,t+1} > \Phi_\alpha)BPV_{t,t+1}, \quad (2.25)$$

$$Z_{t,t+1} = \sqrt{M} \frac{(RV_{t,t+1} - BPV_{t,t+1})/RV_{t,t+1}}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} + t) \max(1, \frac{TQ_{t,t+1}}{BPV_{t,t+1}^2})}}. \quad (2.26)$$

$$I(x) \text{ is the indicator function: } I_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}. \quad (2.27)$$

$Z_{t,t+1}$  , found by Huang and Tauchen (2005), is very closely approximated by a standard Gaussian distribution. In most papers,  $Z_{t,t+1}$  is used to distinguish the jump, thus, for a significant level  $\alpha$ , and the critical value  $\Phi_\alpha$  which based on the standard Gaussian distribution  $N(0,1)$  . In the case of  $\alpha = 0.999$ ,  $\Phi_\alpha = 3.09$ , if  $Z_{t,t+1} > 3.09$ , a jump is recognized. It is easy to see, the lower the significant level is, the more jump would be identified.

### 3 Empirical properties of the data

#### 3.1 Statistical description of the data

Our data set consists in almost 5 years long (from September 1<sup>st</sup> 2000 to August 31<sup>th</sup> 2005) of the FTSE-100 index, including 1210 trading-days. The intraday interval is 5 minutes from 9:05 to 17:30, 102 observations per day. Thus the number of the total observations is 123420. The data is kindly provided by Olsen & Associates in Zurich, Switzerland. The statistical description of the data is given in Table 1.

The first panel describes the raw data, we shall notice that the statistical properties of  $RV_t$ ,  $BPV_t$ ,  $C_t$  and  $J_t$ , are similar.  $RPV_t$  has the biggest mean, median, max, min, standard deviation and Ljung-Box score, but has the least skewness, kurtosis and Jarque-Bera score. The middle panel shows the data in the squared-root form. We can find a similar situation as mentioned above:  $RPV_t^{1/2}$  is different from others. The Ljung-Box test states that there is no significant autocorrelation in  $J_t^{1/2}$ .

The bottom panel is the logarithmic transformation of realized volatility. Here,  $\text{Log}(J_t+1)$  is quite different from other components. Its skewness is larger than 9, indicating that the distribution is strongly right skewed; the kurtosis is over 130, while others are near to 3, meaning that the jump component explains the asymmetry and leptokurtic of the realized variance.

Moreover, several stylized facts of realized volatilities are: (i) Fat tails: the kurtosis of realized volatilities are much higher than the normal distributions; (ii) Long memory: the autocorrelations of realized volatilities remain very significant at least 6 months; (iii) Distribution property: logarithmic realized volatilities are much closer to normal distributions. These properties of realized volatilities will be discussed in the following sub-chapters.

Table1 Statistical description of realized volatility levels in FTSE-100

	Mean	Median	Max	Min	SD	Skewness	Kurtosis	JB	LB <sub>(10)</sub>
$RV_t$	0.0001	0.0001	0.0032	0.0000	0.0002	7.4080	94.652	434570	2758.6
$BPV_t$	0.0001	0.0001	0.0037	0.0000	0.0002	11.905	233.011	2695886	2199.3
$RPV_t$	0.0622	0.0536	0.3097	0.0198	0.0340	2.0679	10.117	3415.938	7426.7
$C_t$	0.0001	0.0001	0.0032	0.0000	0.0002	9.2941	144.12	1021521	2792.7
$J_t$	0.0000	0.0000	0.0011	0.0000	0.0000	9.5333	130.53	838323	62.229
$RV_t^{1/2}$	0.0091	0.0077	0.0565	0.0025	0.0053	2.4165	13.430	6662.025	5479.6
$BPV_t^{1/2}$	0.0083	0.0071	0.0612	0.0025	0.0048	2.7945	20.222	16528.44	2199.3
$RPV_t^{1/2}$	0.2419	0.2314	0.5565	0.1406	0.0607	1.1164	4.7683	408.9699	8065.1
$C_t^{1/2}$	0.0085	0.0072	0.0565	0.0025	0.0049	2.5810	16.395	10389.74	6464.5
$J_t^{1/2}$	0.0016	0.0000	0.0338	0.0000	0.0036	3.2660	17.776	13158.59	15.843*
$LogRV_t$	-9.6564	-9.7281	-5.7478	-11.947	0.9811	0.4390	3.0780	39.17375	6448.6
$LogBPV_t$	-9.8386	-9.902	-5.5866	-11.961	0.9667	0.4371	3.0474	38.64539	7753.6
$LogRPV_t$	-2.8963	-2.9271	-1.172	-3.9233	0.4724	0.4285	2.8960	37.56548	8250.8
$LogC_t$	-9.7898	-9.865	-5.7478	-11.961	0.9677	0.4397	3.0193	39.00479	7740.7
$Log(J_t+1)$	1.52E-05	0.0000	0.0011	0.0000	6.13E-05	9.5298	130.4322	837029	62.226

Note: 1.  $RV_t$  denotes realized variance;  $BPV_t$  denotes bi-power variance;  $RPV_t$  denotes realized power variance;  $C_t$  and  $J_t$  are continuous and jump components of  $RV_t$  respectively. A significant level  $\alpha = 0.999$  was used in the bi-power jump test. 2. SD stands for standard deviation. JB stands for Jarque-Bera test.  $LB_{(10)}$  stands for the Ljung-Box test for 10<sup>th</sup> lags. \* indicates the LB test is not significant.

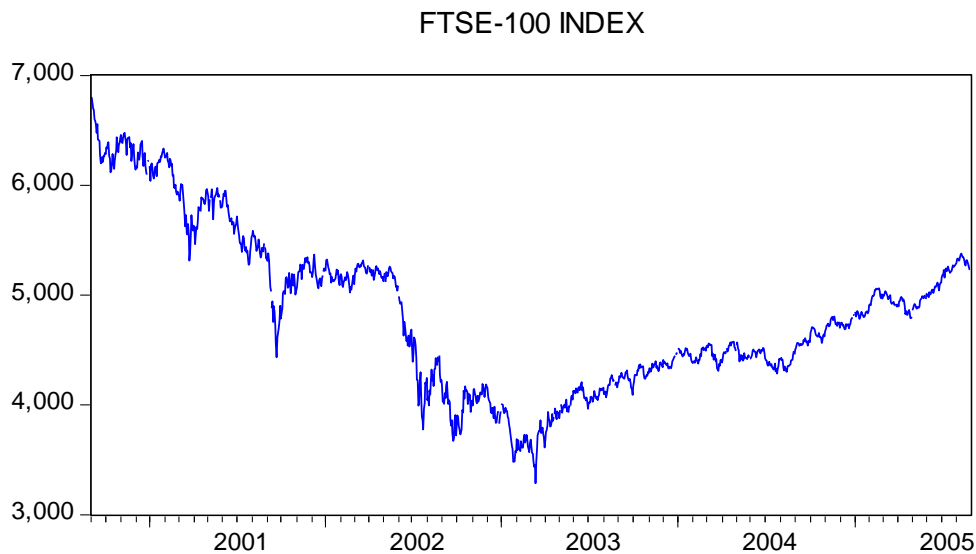


Figure 2 FTSE-100 daily index

### 3.2 Kernel distribution of realized variance and volatility

We first plot the unconditional distribution of realized variance and corresponding continuous and jump components in Figure 3. The distributions of these volatility measures appear strongly right-skewed and have considerably high kurtosis which are confirmed in Table 1. This is the fat-tail property; and it is easy to see that  $RPV_t$  has the mildest distribution while the jump component has the sharpest distribution.

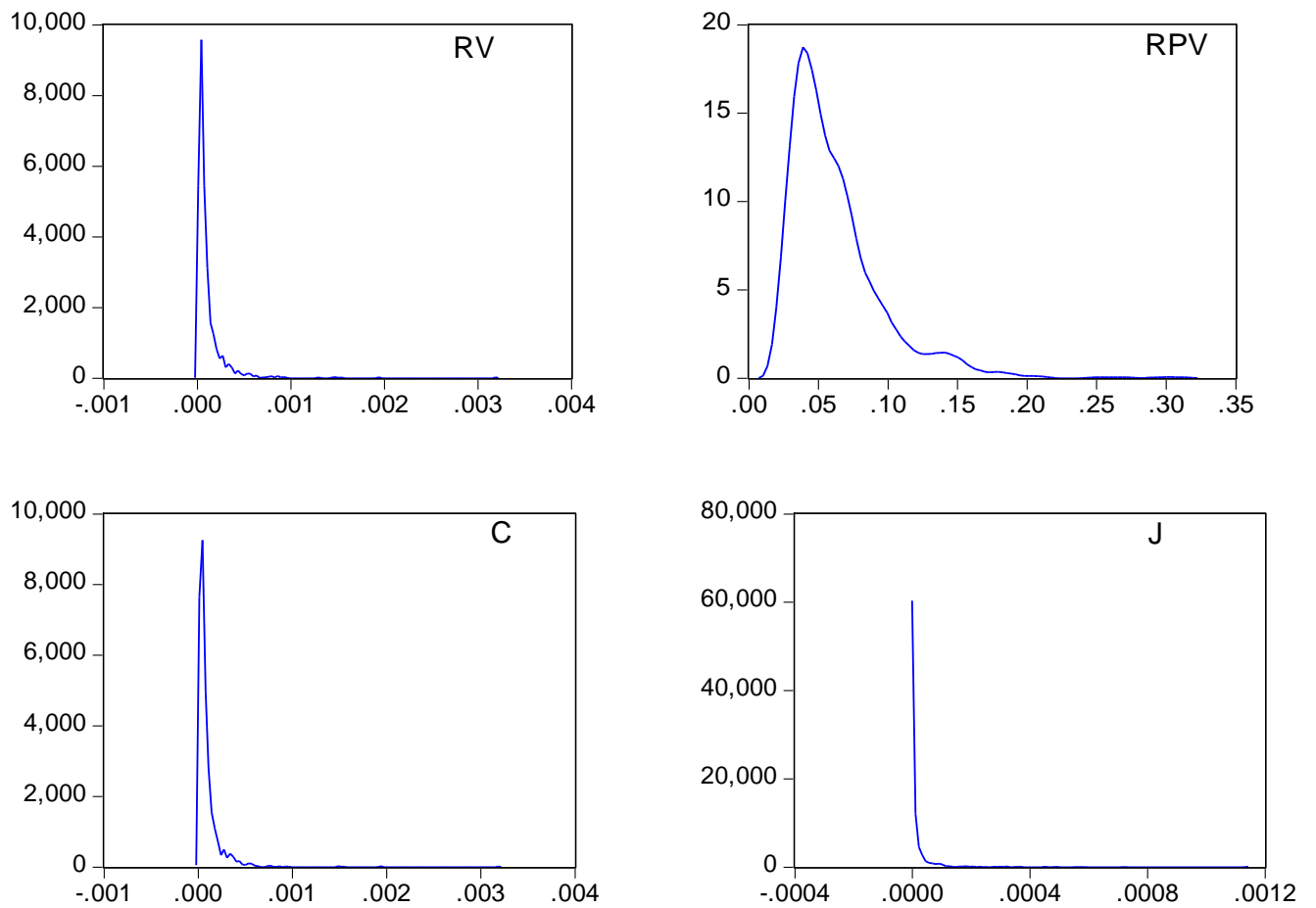


Figure 3 Central kernel density estimates of the distribution for the daily realized volatilities of the FTSE-100

### 3.3 Distribution of the logarithmic transformation of volatility

As popular stochastic volatility model assumes that  $\text{Log}RV_t$  has a normal distribution, so we present the QQ-plots to test the normal distribution of  $\text{Log}RV_t$ ,  $\text{LogRPV}_t$ ,  $\text{Log}C_t$  and  $\text{Log}(J_t+1)$ . We can see that except for the  $\text{Log}(J_t+1)$ , others are quite near to the normal distribution, these are also confirmed by the Jarque-Bera scores in the bottom panel of Table1. Moreover, the logarithmic transformation for the daily realized volatility is closer to normality than other forms of volatility (e.g. square-root transformation). This result is in consistence with previous literate on the modeling of stochastic volatility. (see ABDL (2001, 2003) )

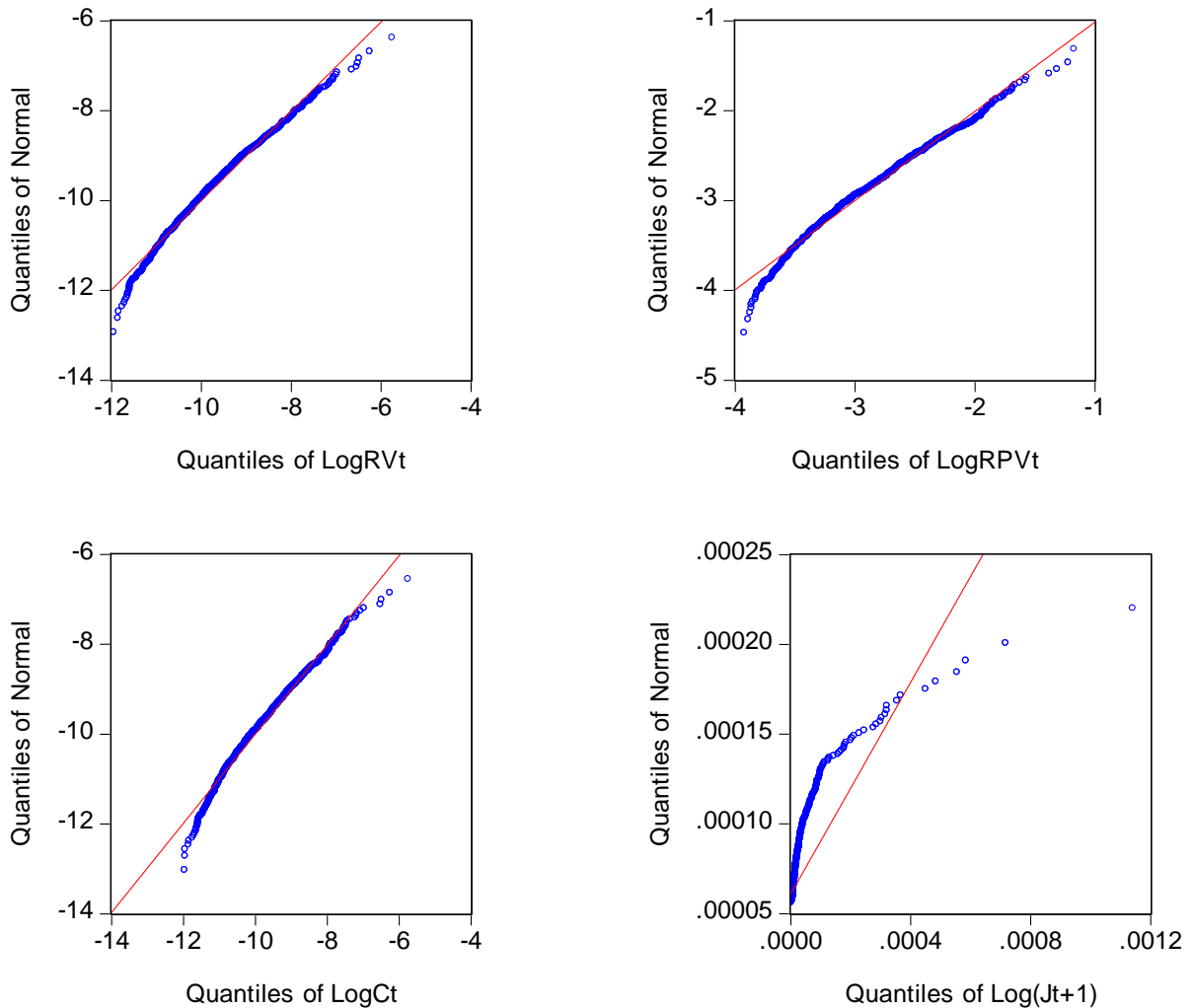


Figure 4 QQ-plots for the daily realized volatility in the logarithmic transformation of the FTSE-100

### 3.4 The autocorrelation of realized volatility

We also test the autocorrelation of  $RV_t$ ,  $RPV_t$ ,  $BPV_t$ ,  $C_t$  and  $J_t$ , and find that  $RPV_t$  is higher autocorrelated than others; and the autocorrelation performances of  $BPV_t$ ,  $RV_t$  and  $C_t$  are quite similar. The autocorrelation coefficients decay as the lags increase, except the series of  $J_t$ . The autocorrelation of the four series are significant for at least 200 lags; this is so called long-memory, one of the stylized properties of realized volatility. Among these parameters,  $RPV_t$  has the biggest persistence and the jump component has little persistence, which is in line with the finding of Forsberg and Ghysels (2007).

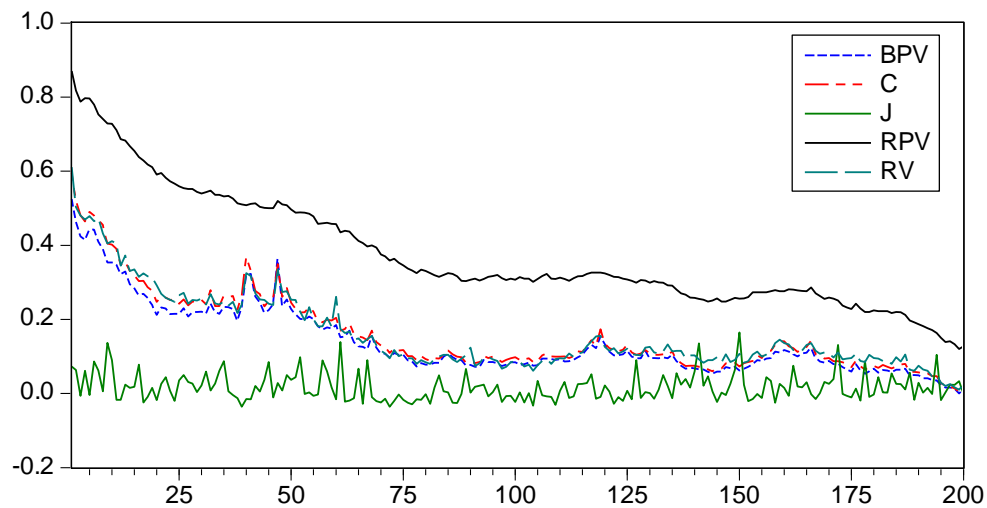


Figure 5 The sample autocorrelation function (SACF) of  $RV_t$ ,  $RPV_t$ ,  $BPV_t$ ,  $C_t$  and  $J_t$ , of the FTSE-100

### 3.5 Daily FTSE-100 index realized variance and jumps

From the following Figure 6, the first panel is the plot of realized variance; it is obvious that the series of  $RV_t$  is stationary. The second panel graphs the Z-statistics, with 0.999 significant level indicated by the horizontal line. The bottom panel graphs the significant jumps corresponding to  $\alpha = 0.999$

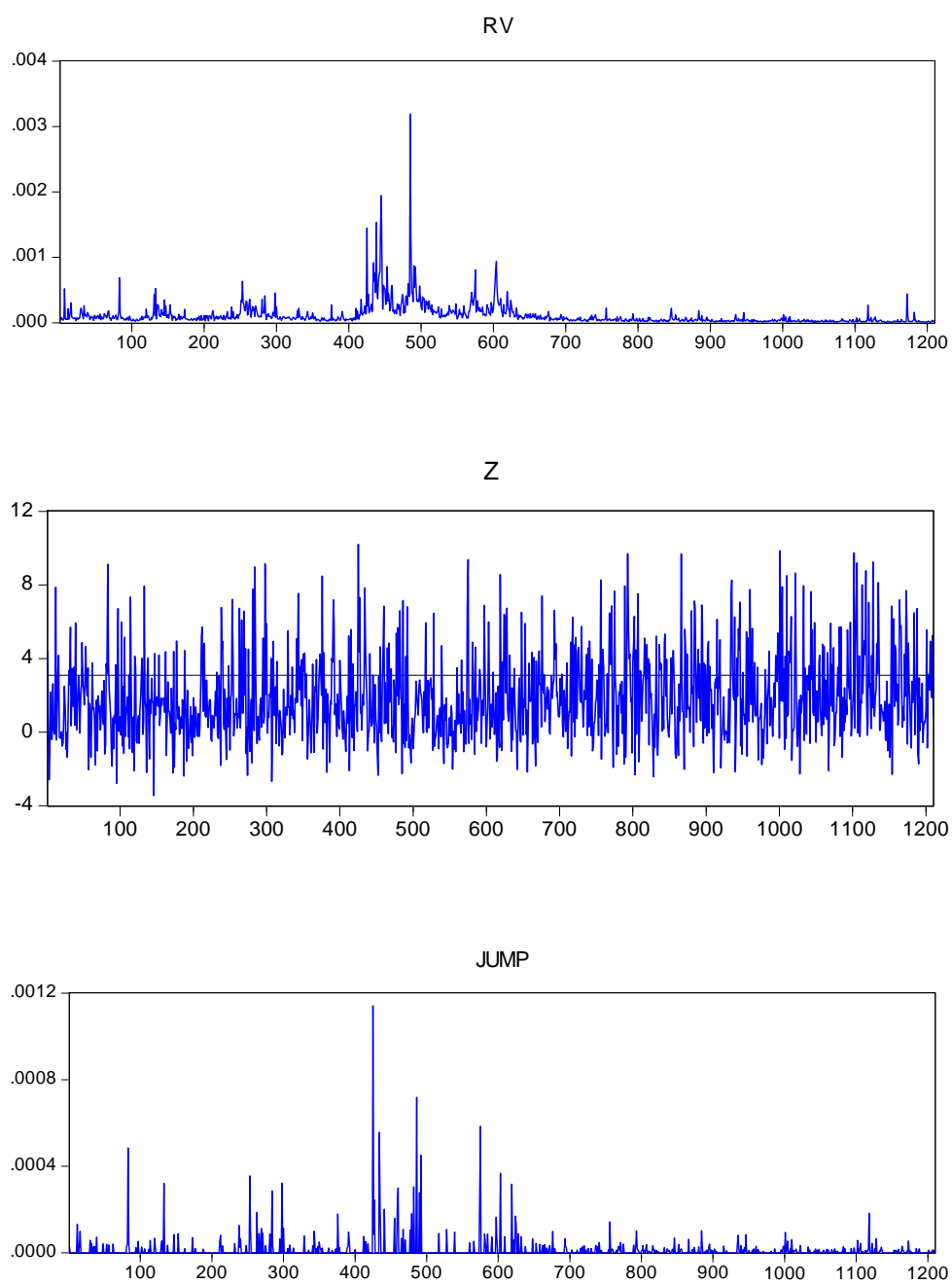


Figure 6 Daily FTSE-100 index realized variance, Z-statistics and Jump

## 4 Methodology

### 4.1 Modeling the long-term persistence.

We first give the definition to long memory. A discrete time series process  $x_t$  with autocorrelation function  $\rho_k$  at lag  $k$ , then according to McLeod and Hipel (1978), the process possesses long memory if the quantity  $\lim_{n \rightarrow \infty} \sum_{k=-n}^n |\rho_k|$  is nonfinite.

A number of articles have argued for the importance of long-memory dependence in financial market volatility. However, standard GARCH and SV (Stochastic Volatility) models are not able to reproduce this feature well. (Corsi, 2004) Therefore, fractional difference operators are usually employed into models in order to capture the long memory, such as ARFIMA (autoregressive fractionally integrated moving average) models of modeling realized volatility or FIGARCH (fractional integrated GARCH) models of modeling return series.

The general ARFIMA (p,d,q) process is given as,

$$\Phi(B)(1 - B)^d x_t = \theta(B)a_t, \quad (4.1)$$

where  $a_t$  is a white noise,  $B$  denotes backshift operator, i.e.,  $Bx_t = x_{t-1}$ .  $\Phi(B)$  and  $\theta(B)$  are lag-polynomials respectively.

This is in the same form of ARIMA model, but note that if  $d$  is non-integer, then  $x_t$  is said to be *fractionally integrated*. It indicates that the autocorrelation function exhibits persistence that is neither consistent with an  $I(1)$  process nor an  $I(0)$  process.

We use the binomial series expansion for any real  $d > -1$ :

$$(1 - B)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k = 1 - dB + \frac{d(d-1)}{2!} B^2 - \frac{d(d-1)(d-2)}{3!} B^3 + \dots \quad (4.2)$$

To demonstrate how to incorporate long-memory behavior, we consider the simple ARFIMA (0,d,0) process

$$(1 - B)^d x_t = (1 - \pi_1 B - \pi_2 B^2 - \dots) x_t = a_t, \quad (4.3)$$

where, using the gamma function, the coefficients are given by

$$\pi_j = -\frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)}.$$

This process can thus be interpreted as an infinite autoregression. Mills and Markellos (2008) state that it is often referred to as fractional white noise, and is the

discrete time analogue of fractional Brownian motion, just as the random walk is the discrete time analogue of Brownian motion.

By inverting the fractional difference operator, we obtain an MA representation:

$$x_t = (1 - B)^{-d} a_t = (1 - \psi_1 B - \psi_2 B^2 - \dots) a_t \quad (4.4)$$

with  $\psi_j = \frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)}$ .

For  $d=0$ ,  $x_t$  is simply white noise and its ACF declines immediately to zero, whereas, for  $d=1$ ,  $x_t$  is a random walk and hence has an ACF that remains (approximately) at unity. For non-integer values of  $d$ , it can be shown that the ACF of  $x_t$  declines hyperbolically to zero (see Figure 7). The autocorrelation function is given by:

$$\rho_k = \frac{\Gamma(1-d)}{\Gamma(d)} \times \frac{\Gamma(k+d)}{\Gamma(k+1-d)} \approx \frac{\Gamma(1-d)}{\Gamma(d)} k^{2d-1} \quad (4.5)$$

for large  $k$ , so that the autocorrelation exhibit a hyperbolic decay, the speed of which depends upon  $d$ . (Mills and Markellos, 2008)

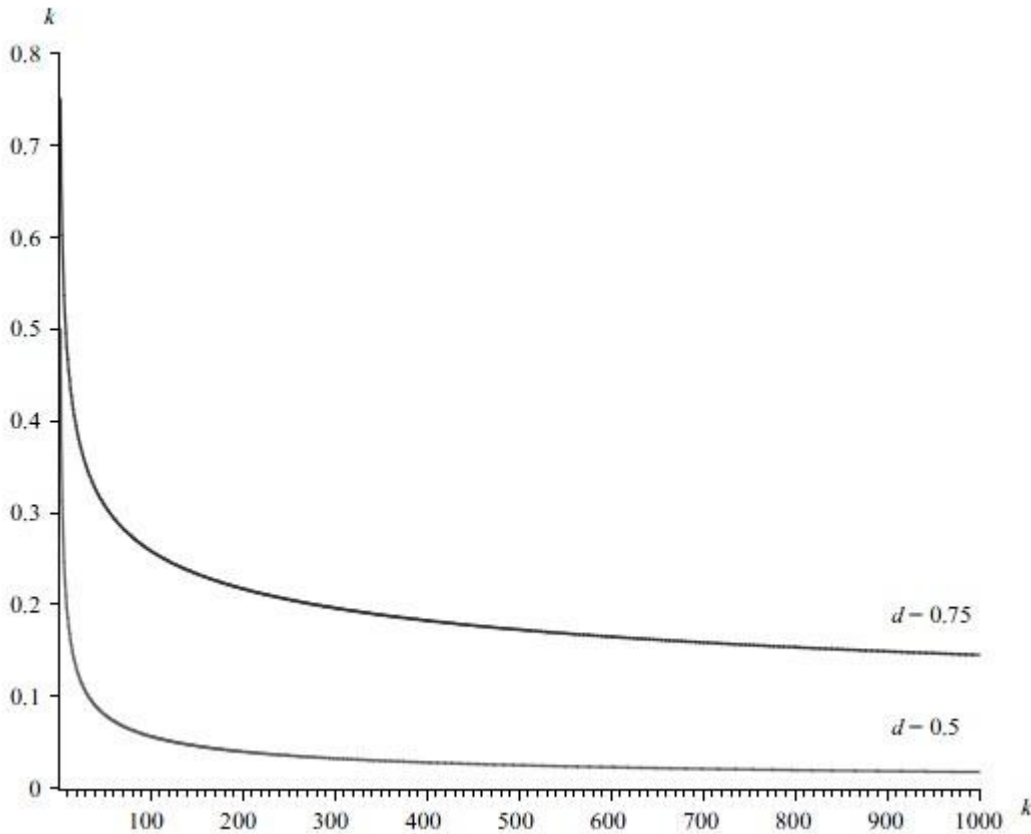


Figure 7 ACFs of ARFIMA(0,d,0) processes with  $d=0.5$  and  $d=0.75$

(Source: Mills and Markellos 2008, p.130)

However, fractional integrated models have some shortcomings; for example, fractional integration is a convenient mathematical form but lacks a clear economic interpretation; the difference operator  $(1-B)^d$  may destroy some useful information and may not be flexible enough to capture the real structure of the data. Moreover, the fractional integrated models are usually complicated and difficult to apply.

## 4.2 The HAR-RV-J model

In 2004, Corsi developed a simple but sophisticated volatility forecasting model which is noted the HAR-RV model:

$$RV_{t,t+H} = H^{-1}(RV_{t,t+1} + RV_{t+1,t+2} + \dots + RV_{t+H-1,t+H}), \quad (4.6)$$

$RV_{t,t+H}$  is the increment of RV from  $t$  to  $t+H$ ; with  $H=1,5,22^7$  indicating 1 day, 5 days (1 week) and 22 days(1 month) average realized volatility.

$$\text{HAR - RV: } RV_{t,t+H} = \alpha_0 + \beta_D RV_{t-1,t} + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \varepsilon_t. \quad (4.7)$$

Let's consider the case  $H=1$  thus,  $RV_{t,t+H} = RV_{t,t+1}$ , implying that the one day ( $H=1$ ) increment of today's RV, is a linear regression of  $\alpha_0$ (constant), yesterday's volatility  $V_{t-1,t}$ , 1 week ago volatility  $V_{t-5,t}$  and 1 month ago volatility  $V_{t-22,t}$ . For the case  $H=5$ , it means, the increment of today's RV is calculated by the average of 5 following days, thus

$$RV_{t,t+5} = \frac{1}{5}(RV_{t,t+1} + RV_{t+1,t+2} + \dots + RV_{t+4,t+5}), \quad (4.8)$$

$RV_{t,t+5}$  is also the dependent variable in equation (4.7).

Of course, realized volatilities over different horizons can be also included as additional explanatory variables on the right side of the equation (4.7); but according to Andersen et al. (2007), the daily, weekly and monthly measures employed here afford a natural economic interpretation.

If we incorporate the jump component as a new additional explanatory variable in the equation 4.8, we can obtain the new model, termed as HAR-RV-J model.

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<sup>7</sup> Note that some authors use 20 as a month lag.

$$RV_{t,t+H} = \alpha_0 + \beta_D RV_{t-1,t} + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_J J_{t-1} + \varepsilon_t, \quad (4.9)^8$$

Practical use of volatility models and forecasts often involves standard deviations as opposed to variances, so that we can get the non-linear model as

$$(RV_{t,t+H})^{1/2} = \alpha_0 + \beta_D RV_{t-1,t}^{1/2} + \beta_W RV_{t-5,t}^{1/2} + \beta_M RV_{t-22,t}^{1/2} + \beta_J J_{t-1}^{1/2} + \varepsilon_t, \quad (4.10)$$

and the logarithmic HAR-RV-J model

$$\log(RV_{t,t+H}) = \alpha_0 + \beta_D \log(RV_{t-1,t}) + \beta_W \log(RV_{t-5,t}) + \beta_M \log(RV_{t-22,t}) + \beta_J \log(J_{t-1} + 1) + \varepsilon_t. \quad (4.11)$$

Note that stated by Andersen et al. (2007), this empirical regularity motivated ABDL (2003) to model the logarithmic realized volatilities, in turn allowing for the use of standard normal distribution theory and related mixture models.

### 4.3 The HAR-RV-CJ model

According to Andersen et al. (2007), we can decompose the realized volatilities that appear as explanatory variables on the right-hand-side into the continuous sample path variability and the jump variation. The new HAR-RV-CJ model can be expressed by

$$RV_{t,t+h} = \alpha_0 + \beta_{CD} C_{t-1} + \beta_{CW} C_{t-5,5} + \beta_{CM} C_{t-22,5} + \beta_{JD} J_{t-1} + \beta_{JW} J_{t-5,t} + \beta_{JM} J_{t-22,t} + \varepsilon_{t,t+h}. \quad (4.12)$$

This model nests the HAR-RV-J model in equation (4.9) for  $\beta_D = \beta_{CD} + \beta_{JD}$ ,  $\beta_W = \beta_{CW} + \beta_{JW}$ ,  $\beta_M = \beta_{CM} + \beta_{JM}$  and  $\beta_J = \beta_{JD}$ .

Of course, we can obtain the non-linear HAR-RV-CJ model as

$$(RV_{t,t+h})^{1/2} = \alpha_0 + \beta_{CD} C_{t-1}^{1/2} + \beta_{CW} (C_{t-5,5})^{1/2} + \beta_{CM} (C_{t-22,5})^{1/2} + \beta_{JD} J_{t-1}^{1/2} + \beta_{JW} (J_{t-5,t})^{1/2} + \beta_{JM} (J_{t-22,t})^{1/2} + \varepsilon_{t,t+h}, \quad (4.13)$$

and the log transformation

$$\log(RV_{t,t+h}) = \alpha_0 + \beta_{CD} \log(C_{t-1}) + \beta_{CW} \log(C_{t-5,5}) + \beta_{CM} \log(C_{t-22,5}) + \beta_{JD} \log(J_{t-1} + 1) + \beta_{JW} \log(J_{t-5,t} + 1) + \beta_{JM} \log(J_{t-22,t} + 1) + \varepsilon_{t,t+h} \quad (4.14)$$

---

<sup>8</sup> Note that this equation is originated from Andersen et al. (2007), they used the jump series from the equation (2.21); but here, we will use the jump series adopted by the bi-power test as shown in the chapter 3.

respectively.

We see, first we have the original HAR-RV models as equation (4.7), and then we add the jump component into it, given the equation (4.9). If we model the jump components and continuous components respectively, we will have the equation (4.12). In the next chapter, we will test the different performances of the models and present the empirical results and both in-sample and out-of-sample predictions.

## 5 Empirical results and prediction

### 5.1 Empirical results

The regression results are presented in Table 2 and 3. Table 2 shows the results of the HAR-RV-J models as equations (4.9) (4.10) and (4.11). It can be seen that after we incorporate the jump component, the fitness level adjusted R-squared increases a little but not very much. In the linear model, the constant values are quite small. The coefficients of the daily, weekly and monthly decrease in turn which confirm the long-memory property of the realized volatility. Note that the coefficients of J are negative which means if the realized volatility is entirely attributable to jumps; it carries no predictive power for the following day's realized volatility. Over the three horizons, adjusted R-squared maximizes at  $h=5$ ; this indicates that the linear model performs the best at the "weekly" horizon. All the coefficients are significant except for the J at weekly and monthly horizons.

Turn to the non-linear models, the adjusted R-squareds increase at different levels. At weekly ( $h=5$ ) horizon, the fitness level of the log transformation arrives at as high as 0.798. All the coefficients are highly significant at 1% level. We have the same result in the different horizons as the linear one, that is the model performs the best at weekly horizon ( $h=5$ ), performs similar at daily ( $h=1$ ) and monthly horizon ( $h=22$ ) respectively.

Overall, when incorporating the jump component, the fitness level increases a little bit. It is worth noticing that Andersen et al. (2007) find a great improvement when they comprise the jump component into the standard HAR-RV model. We think the original HAR-RV models have already performed quite well in our data set; therefore we cannot obtain such a substantial improvement when we incorporate the jump component. Generally, non-linear models perform better than linear models; the log-transformation does better than square-root transformation; we can obtain the highest adjusted R-squared at weekly ( $h=5$ ) horizon by applying the log transformation.

Table 2 Daily, Weekly, and Monthly FTSE-100 HAR-RV-J Regressions

$$RV_{t,t+H} = \alpha_0 + \beta_D RV_{t-1,t} + \beta_W RV_{t-5,t} + \beta_M RV_{t-22,t} + \beta_J J_{t-1} + \varepsilon_t$$

$$(RV_{t,t+H})^{1/2} = \alpha_0 + \beta_D RV_{t-1,t}^{1/2} + \beta_W RV_{t-5,t}^{1/2} + \beta_M RV_{t-22,t}^{1/2} + \beta_J J_{t-1}^{1/2} + \varepsilon_t$$

$$\log(RV_{t,t+H}) = \alpha_0 + \beta_D \log(RV_{t-1,t}) + \beta_W \log(RV_{t-5,t}) + \beta_M \log(RV_{t-22,t}) + \beta_J \log(J_{t-1} + 1) + \varepsilon_t$$

h	RV <sub>t,t+H</sub>			RV <sub>t,t+H</sub> <sup>1/2</sup>			log(RV <sub>t,t+H</sub> )		
	1	5	22	1	5	22	1	5	22
$\alpha_0$	2.52E-05* (5.03E-06)	3.30E-05* (5.65E-06)	4.64E-05* (7.12E-06)	0.001* (0.0002)	0.001* (0.0003)	0.002* (0.0004)	-0.898* (0.174)	-1.128* (0.222)	-1.669* (0.293)
$\beta_D$	0.598* (0.111)	0.406* (0.080)	0.275* (0.071)	0.638* (0.036)	0.555* (0.047)	0.446* (0.055)	0.544* (0.029)	0.504* (0.029)	0.443* (0.031)
$\beta_W$	0.214* (0.050)	0.245* (0.055)	0.175* (0.049)	0.234* (0.029)	0.242* (0.036)	0.201* (0.037)	0.278* (0.025)	0.263* (0.021)	0.229* (0.024)
$\beta_M$	0.048** (0.020)	0.076** (0.368)	0.127** (0.060)	0.051* (0.019)	0.081* (0.028)	0.130** (0.052)	0.079* (0.021)	0.110* (0.023)	0.150* (0.037)
$\beta_J$	-0.455** (0.206)	-0.182 (0.144)	-0.021 (0.112)	-0.224* (0.030)	-0.193* (0.038)	-0.138* (0.042)	0.024* (0.004)	0.026* (0.003)	0.026* (0.003)
$R^2_{HAR-J}$	0.441	0.517	0.427	0.655	0.735	0.646	0.699	0.798	0.755
$R^2_{HAR}$	0.427	0.513	0.428	0.638	0.719	0.637	0.687	0.782	0.738

Note: 1. The table reports the OLS estimates for daily (h=1), weekly (h=5) and monthly (h=22) HAR-RV-J volatility forecast regressions.

2. \* \*\* \*\*\* indicate significant at 1% 5% 10% levels respectively.

3. The stand errors reported in parentheses are based on a Newey-West/Bartlett correction<sup>9</sup>.

4. The last two rows  $R^2_{HAR-J}$  and  $R^2_{HAR}$  are for the HAR-RV-J model and the standard HAR-RV model without the jump component in the right-hand of the equation (4.7).

Turn to Table 3, the models applied here are so called HAR-RV-CJ models, shown as equation (4.12) (4.13) and (4.14). We can have a similar result from the Table 2, i.e., over the three horizon levels, the adjusted R-squared maximizes at the weekly horizon (h=5); non-linear models are better than the linear models; log-transformation is better than the square-root transformation. According to the fitness level- adjusted  $R^2$ ; the HAR-RV-CJ model is better fitted than the HAR-RV-J models.

It is noteworthy that most of the jump coefficients are not significant, which is in line with the conclusion of Andersen et al. (2007). This indicates that jump has no

<sup>9</sup> Wooldridge (2004) discussed why we report the heteroskedasticity-robust error in applied econometrics. See *Introductory Econometrics A Modern Approach* p.275-p.276.

predictive power; in other words, the predictability in the HAR-RV realized regressions is almost exclusively due to the continuous sample path components.

Table 3 Daily, Weekly, and Monthly FTSE-100 HAR-RV-CJ Regressions

$$RV_{t,t+h} = \alpha_0 + \beta_{CD}C_{t-1} + \beta_{CW}C_{t-5,5} + \beta_{CM}C_{t-22,5} + \beta_{JD}J_{t-1} + \beta_{JW}J_{t-5,t} + \beta_{JM}J_{t-22,t} + \varepsilon_{t,t+h}$$

$$(RV_{t,t+h})^{\frac{1}{2}} = \alpha_0 + \beta_{CD}C_{t-1}^{\frac{1}{2}} + \beta_{CW}(C_{t-5,5})^{\frac{1}{2}} + \beta_{CM}(C_{t-22,5})^{\frac{1}{2}} + \beta_{JD}J_{t-1}^{\frac{1}{2}} + \beta_{JW}(J_{t-5,t})^{\frac{1}{2}} + \beta_{JM}(J_{t-22,t})^{1/2} + \varepsilon_{t,t+h}$$

$$\log(RV_{t,t+h}) = \alpha_0 + \beta_{CD}\log(C_{t-1}) + \beta_{CW}\log(C_{t-5,5}) + \beta_{CM}\log(C_{t-22,5}) + \beta_{JD}\log(J_{t-1} + 1) + \beta_{JW}\log(J_{t-5,t} + 1) + \beta_{JM}\log(J_{t-22,t} + 1) + \varepsilon_{t,t+h}$$

h	RV <sub>t,t+h</sub>			RV <sup>1/2</sup> <sub>t,t+h</sub>			log(RV <sub>t,t+h</sub> )		
	1	5	22	1	5	22	1	5	22
$\alpha_0$	2.43E-05*	3.28E-05*	4.60E-05*	0.0009*	0.001*	0.002*	-0.723*	-0.968*	-1.516*
	(4.85E-06)	(5.86E-06)	(7.15E-06)	(0.0002)	(0.0003)	(0.0004)	(0.182)	(0.231)	(0.299)
$\beta_{CD}$	0.563*	0.404*	0.275*	0.610*	0.534*	0.425*	0.544*	0.492*	0.416*
	(0.071)	(0.079)	(0.071)	(0.037)	(0.048)	(0.055)	(0.032)	(0.032)	(0.035)
$\beta_{CW}$	0.284*	0.281*	0.176*	0.303*	0.297*	0.231*	0.302*	0.296*	0.257*
	(0.057)	(0.073)	(0.051)	(0.037)	(0.045)	(0.042)	(0.030)	(0.023)	(0.027)
$\beta_{CM}$	0.070*	0.075*	0.153**	0.060*	0.081**	0.159*	0.067*	0.099*	0.157*
	(0.026)	(0.036)	(0.073)	(0.023)	(0.034)	(0.061)	(0.023)	(0.026)	(0.042)
$\beta_{JD}$	0.172**	0.226**	0.285*	0.048***	0.042	0.053	-0.003	0.0005	0.003
	(0.083)	(0.104)	(0.081)	(0.025)	(0.027)	(0.035)	(0.004)	(0.002)	(0.002)
$\beta_{JW}$	-0.122	0.073	0.172***	-0.046	-0.007	0.012	-0.001	0.001	0.003
	(0.097)	(0.080)	(0.104)	(0.031)	(0.028)	(0.035)	(0.003)	(0.003)	(0.002)
$\beta_{JM}$	-0.133	0.051	-0.006	-0.061**	-0.012	-0.043	0.002	0.001	0.004
	(0.085)	(0.102)	(0.059)	(0.031)	(0.029)	(0.026)	(0.003)	(0.002)	(0.003)
$R^2_{HAR-CJ}$	0.458	0.522	0.431	0.673	0.749	0.658	0.714	0.815	0.776

Note: 1. The table reports the OLS estimates for daily (h=1), weekly (h=5) and monthly (h=22) HAR-RV-J volatility forecast regressions.

2. \* \*\* \*\*\* indicate significant at 1% 5% 10% levels respectively.

3. The stand errors reported in parentheses are based on a Newey-West/Bartlett correction.

## 5.2 Forecasting evaluation measures

The criterion of forecasting evaluation employed here are RMSE (Root Mean Squared Error), MAE (Mean Absolute Error), MAPE (Mean Absolute Percent Error), and TIC (Theil Inequality Coefficient)

These criterion are given as

$$\text{RMSE} = \sqrt{\sum_{t=T+1}^{T+h} (\hat{\sigma}_t - \sigma_t)^2 / h}, \quad (5.1)$$

$$\text{MAE} = \sum_{t=T+1}^{T+h} |\hat{\sigma}_t - \sigma_t| / h, \quad (5.2)$$

$$\text{MAPE} = 100 \sum_{t=T+1}^{T+h} \left| \frac{\hat{\sigma}_t - \sigma_t}{\sigma_t} \right| / h, \quad (5.3)$$

$$\text{TIC} = \frac{\sqrt{\sum_{t=T+1}^{T+h} (\hat{\sigma}_t - \sigma_t)^2 / h}}{\sqrt{\sum_{t=T+1}^{T+h} \hat{\sigma}_t^2 / h} + \sqrt{\sum_{t=T+1}^{T+h} \sigma_t^2 / h}}. \quad (5.4)$$

Note that  $\hat{\sigma}_t$  denotes the estimated value of the realized volatility and  $\sigma_t$  denotes the actual value of the realized volatility;  $h$  denotes the forecast sample, i.e.,  $j=T+1, T+2, \dots, T+h$ .

We will demonstrate both the in-sample and the out-of-sample prediction, the out-of-sample forecasts 210 observations while the first 1000 observations are used to estimate the model parameters.

## 5.3 Prediction results

Table 4 and 5 are our in-sample and out-of-sample prediction results respectively. Different panels present different horizons, i.e., daily ( $h=1$ ), weekly ( $h=5$ ), monthly ( $h=22$ ). When we compare the different values in rows, we find an interesting result; no matter it is in-sample or out-of-sample, the HAR-RV-CJ original model outperforms others according to the RMSE and MAE scores and the log-transformation of the HAR-RV-CJ model outperforms other models according to the MAPE and TIC scores. That is, the HAR-RV-CJ models predict better than both the HAR-RV-J models and the original HAR-RV models. Therefore if we regress the continuous and jump

components of realized volatility into our models respectively, we can obtain a better prediction results.

As shown in Table 3, the jump components have no prediction power; only the continuous parts have prediction power. The HAR-RV-CJ models are good because we get rid of the jump components which can be seen just as noise. This result is in line with Andersen et al. (2007).

Figure 8 and Figure 9 present the in-sample and out-of-sample prediction graphs. It is easy to see that the series fit well in the in-sample prediction, especially on the weekly ( $h=5$ ) horizon. Turn to the out-of-sample graph (Figure 10), it seems that the series do not fit well in the monthly ( $h=22$ ) horizon, but they perform pretty well in the daily ( $h=1$ ) and weekly ( $h=5$ ) horizons which are confirmed by our results.

Table 4 Comparison of the in-sample performance between the models

Horizon	Criterion	HAR-RV	HAR-J(1)	HAR-J(2)	HAR-J(3)	HAR-CJ(1)	HAR-CJ(2)	HAR-CJ(3)
h=1	RMSE	0.000136	0.000135	0.003117	0.540067	<b>0.000132</b>	0.003032	0.525658
	MAE	5.30E-05	5.22E-05	0.001957	0.413375	<b>5.08E-05</b>	0.001901	0.402712
	MAPE	74.33926	72.70833	22.21196	4.392430	69.15025	21.61512	<b>4.277087</b>
	TIC	0.364564	0.358680	0.151373	0.027828	0.351390	0.147039	<b>0.027084</b>
h=5	RMSE	0.0001	9.97E-05	0.002417	0.399539	<b>9.91E-05</b>	0.002354	0.381411
	MAE	4.82E-05	4.74E-05	0.001561	0.307596	<b>4.63E-05</b>	0.001510	0.294104
	MAPE	61.57263	60.74366	16.97598	3.258379	59.27532	16.28659	<b>3.120884</b>
	TIC	0.300710	0.298988	0.119623	0.020604	0.296714	0.116421	<b>0.019668</b>
h=22	RMSE	9.41E-05	9.41E-05	0.002593	0.415981	<b>9.37E-05</b>	0.002547	0.397722
	MAE	5.32E-05	5.33E-05	0.001695	0.315774	<b>5.26E-05</b>	0.001650	0.301748
	MAPE	65.7000	65.74062	17.81379	3.378895	64.62338	17.11173	<b>3.241356</b>
	TIC	0.307754	0.307747	0.130288	0.021476	0.306152	0.127907	<b>0.020533</b>

Note: 1. The table reports the in-sample prediction results for daily ( $h=1$ ), weekly ( $h=5$ ) and monthly ( $h=22$ ) HAR-RV, HAR-RV-J and HAR-RV-CJ models.

2. HAR-RV denotes the standard HAR-RV model, see equation (4.7)

3. HAR-J denotes the HAR-RV-J models; (1), (2), (3) present the equation (4.9) (4.10) and (4.11) respectively.

4. HAR-CJ denotes the HAR-RV-CJ models; (1), (2), (3) present the equation (4.12) (4.13) and (4.14) respectively.

5. The number in bold means it is the smallest one of every row.

Table 5 Comparison of the out-of-sample performance between the models

Horizon	Criterion	HAR-RV	HAR-J(1)	HAR-J(2)	HAR-J(3)	HAR-CJ(1)	HAR-CJ(2)	HAR-CJ(3)
h=1	RMSE	4.55E-05	4.45E-05	0.002159	0.643326	<b>4.37E-05</b>	0.002094	0.612091
	MAE	2.88E-05	2.78E-05	0.001428	0.496536	<b>2.60E-05</b>	0.001357	0.467949
	MAPE	150.1069	144.1828	29.48712	4.730796	132.5931	27.45786	<b>4.473538</b>
	TIC	0.454034	0.448921	0.197630	0.030124	0.449674	0.195146	<b>0.028605</b>
h=5	RMSE	3.54E-05	3.49E-05	0.001533	0.444801	<b>3.43E-05</b>	0.001456	0.412473
	MAE	3.01E-05	2.97E-05	0.001113	0.344620	<b>2.90E-05</b>	0.001043	0.318359
	MAPE	131.5854	130.1031	22.75268	3.230798	127.1368	21.08344	<b>2.990713</b>
	TIC	0.3876	0.384199	0.142163	0.020861	0.381774	0.136910	<b>0.019306</b>
h=22	RMSE	3.90E-05	3.91E-05	0.001525	0.418512	<b>3.81E-05</b>	0.001369	0.368309
	MAE	3.50E-05	3.50E-05	0.001202	0.338309	<b>3.42E-05</b>	0.001072	0.292653
	MAPE	144.0650	144.1467	25.41709	3.146236	140.5200	22.66819	<b>2.721257</b>
	TIC	0.406302	0.406520	0.137661	0.019693	0.400386	0.125491	<b>0.017291</b>

Note: 1. The table reports the out-of-sample prediction results for daily (h=1), weekly (h=5) and monthly (h=22) HAR-RV, HAR-RV-J and HAR-RV-CJ models.

2. HAR-RV denotes the standard HAR-RV model, see equation (4.7)

3. HAR-J denotes the HAR-RV-J models; (1), (2), (3) present the equation (4.9) (4.10) and (4.11) respectively.

4. HAR-CJ denotes the HAR-RV-CJ models; (1), (2), (3) present the equation (4.12) (4.13) and (4.14) respectively.

5. The number in bold means it is the smallest one of every row.

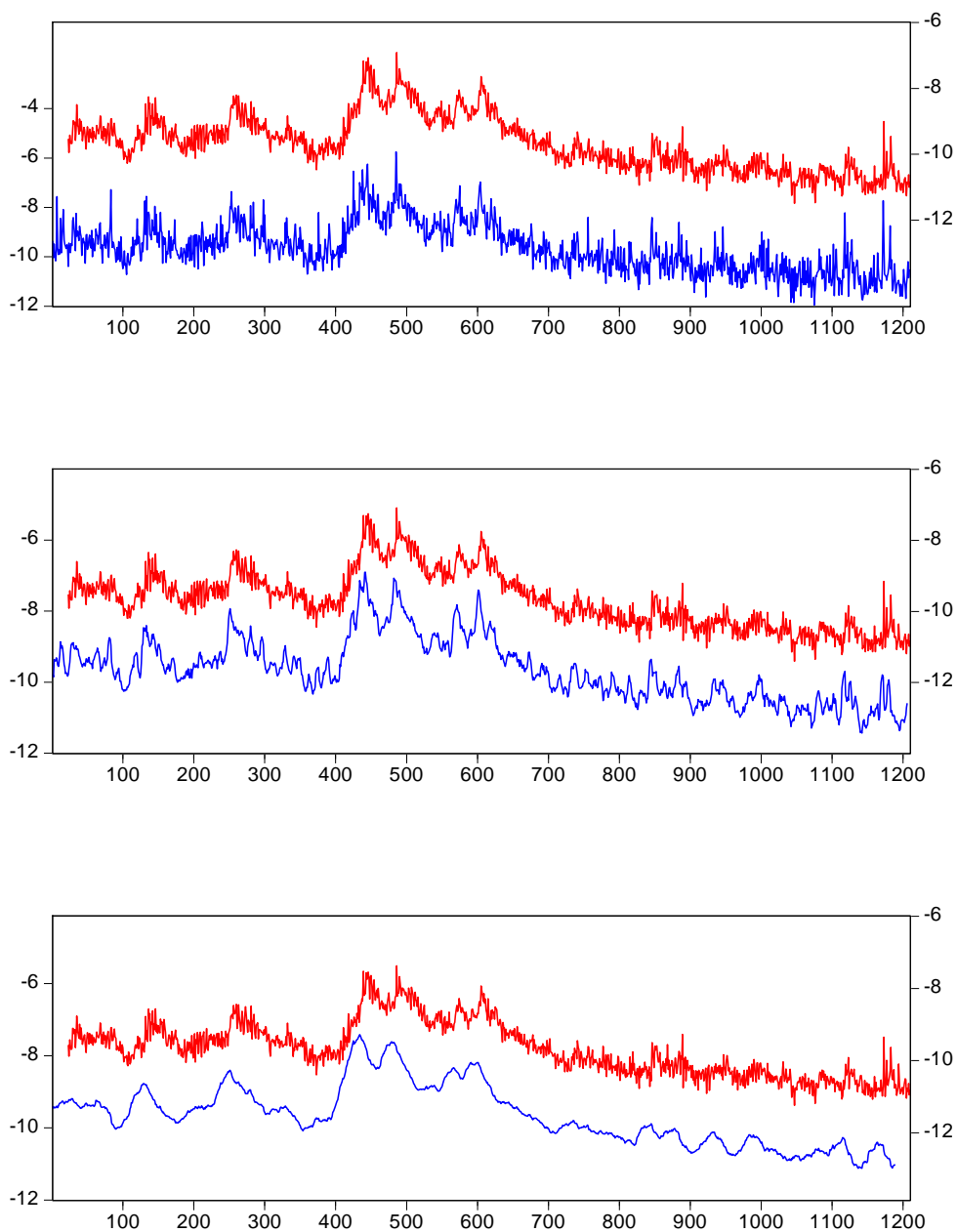


Figure 8 Daily, weekly and monthly FTSE-100 index realized volatility and HAR-RV-CJ in-sample forecasts

Note: The top, middle and bottom panels present daily ( $h=1$ ), weekly ( $h=5$ ) and monthly ( $h=22$ ) realized volatility,  $\log(RV_{t,t+h})$  of the left scale, and the corresponding forecast simulation from the HAR-RV-CJ model (equation 4.14) of the right scale.

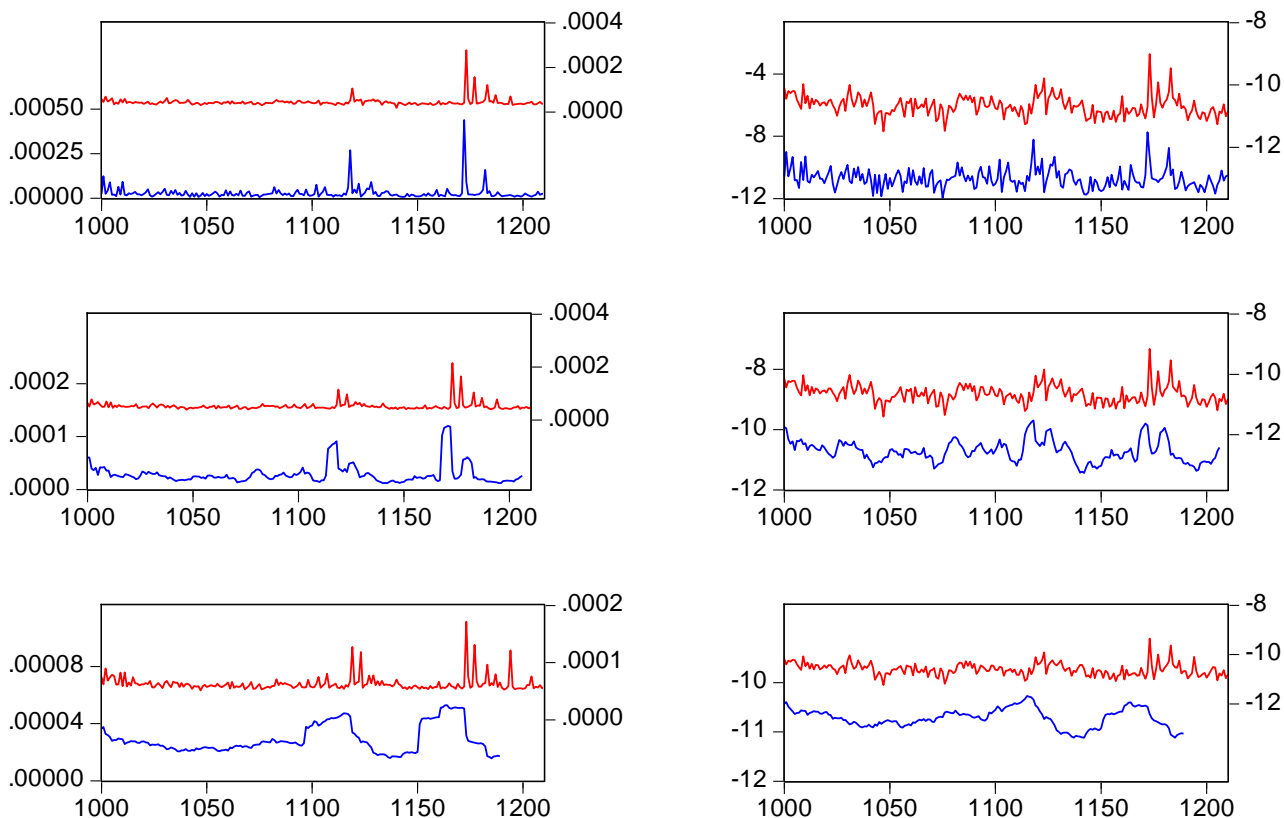


Figure 9 Daily, weekly and monthly FTSE-100 index realized volatility and HAR-RV-CJ out-of-sample forecasts

Note: 1 The top, middle and bottom panels present daily ( $h=1$ ), weekly ( $h=5$ ) and monthly ( $h=22$ ) realized volatilities. The left panels show  $RV_{t,t+h}$  as a regressor (equation 4.12) and the right panels show  $\log(RV_{t,t+h})$  as a regressor (equation 4.14).

2 In every graph, the above one is the predicted series and the below one is the actual series.

## 6 Conclusion

This article first briefly introduces the two main types of volatility—implied volatility and realized volatility. Thanks to the development of information technology, high-frequency data is more widely applied in modeling of realized volatility. The HAR-RV (Heterogeneous Autoregressive model of the Realized Volatility) model was first developed by Corsi (2004), and afterward, there have been tremendous literatures about this simple but sophisticated model. One of popular HAR-RV family models is so called HAR-RV-CJ model developed by Andersen et al. (2007), our research is based on this model.

In the second chapter, we present some continuous sample path diffusions; note that the stock price will react to the announcement of macroeconomic news, and then it is appropriate to model it with jump diffusion processes. Moreover, we give the calculation formula of the realized volatility and two ways to identify the jump component: one is the subtraction of RV and BPV, see equation (2.21); the other one is to separate the jump component by adopting the bi-power jump test, see Barndorff-Nielsen and Shephard (2004a).

In the third chapter, we give the statistical description of our data set and several important properties of the realized volatility series are: (i) the jump component in logarithmic form explains mainly the asymmetry and leptokurtic of the realized variance. (ii) fat tails and leptokurtic: the kurtosis of realized volatilities are much higher than the normal distributions (iii) long memory: the autocorrelation of realized volatilities remain significant for at least 6 months, (iv) logarithmic of realized volatilities are much closer to normal distributions.

We introduce the integrated models—ARFIMA and FIGARCH in the chapter 4; mention that one of the important properties of the volatility model is to model the long-memory property, since the integrated models have shortcomings such as lack of economic meanings and may destroy some vital information; they are not widely used in modeling the realized volatility any longer. After that, we present both linear and non-linear HAR-RV-J model and the HAR-RV-CJ model.

In the chapter 5, we have several remarkable conclusions: *(i)* The HAR-RV-J model performs the best, then the HAR-RV-J model follows. *(ii)* Non-linear models are better than the linear models and logarithmic transformation is better than the square-root transformation. *(iii)* The adjusted R-squared maximizes at the weekly ( $h=5$ ) horizon; but the model predicts better at daily ( $h=1$ ) and monthly ( $h=22$ ) horizons. *(iv)* The coefficients of the jump component in the HAR-RV-CJ models appear statistically insignificant, this indicates that only the continuous part has predictive power.

Several suggestions for further studies are: *(i)* The optimum time interval has always been of importance when using the high-frequency data. There is a new bootstrap method which developed recently by Goncalves and Meddahi (2009), can increase the number of available intraday data each day, without suffering from the "microstructure noise" bias, it seems that we can apply this new method to find the optimum interval. *(ii)* We can incorporate such as regime switch model into the HAR-RV-CJ model to test if we can improve the accuracy. *(iii)* It may be necessary to construct a model like GJR-GARCH can capture the asymmetry.

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