Economics Letters 246 (2025) 112091

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

A note on indeterminacy of unemployment equilibrium ☆

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ARTICLE INFO

JEL classification:
J64
E24
J30

Keywords:
Search
Matching
Wages
Multiple equilibria

ABSTRACT

According to Mortensen (1989), increasing returns to scale in matching may cause multiple equilibria if collective bargaining or efficiency wage is applied. I show that a unique equilibrium exists, also if the Nash bargaining solution is used, if productivity is constant.

1. Introduction

The possible existence of multiple unemployment equilibria has been one of the pertinent themes in search and matching models of the labor market. Agents meet in a decentralized market characterized by frictions, and they may make choices for example about reservation cost (Diamond, 1982), search intensity (Pissarides, 1986; Howitt and McAfee, 1987) or productivity (Burdett and Smith, 2002). Usually, firms decide on job creation. Multiple equilibria may exist due to externalities and feedbacks, and typically, multiplicity of equilibria is associated with a matching function which has increasing returns to scale with respect to unemployment and vacancies. An example of this is Pissarides (1986), where in a high (low) employment equilibrium workers search for jobs at a high (low) intensity, and firms create a large (small) number of jobs. However, increasing returns to scale is only a necessary condition for multiple equilibria, and Pissarides does not solve whether multiple equilibria *do* exist for some parameter values.

Mortensen (1989) studies whether multiple equilibria can emerge in a model where firms create jobs, and wages are determined by collective bargaining (an insider-outsider model), by an efficiency model or by "market clearing". The matching function may have various returns to scale, and productivity may depend on aggregate employment. Mortensen states (on p. 368) that if wages are determined by collective bargaining or by an efficiency wage model, then "...multiple employment equilibria are possible given any one of the following circumstances: (1) External economies in the production process that cause labor productivity to increase with aggregate employment. (2) Scale economies in the process by which unemployed workers and vacant jobs are matched in the sense that a doubling of both more than doubles the matching rate".

I show that Mortensen's statement is not correct regarding item 2. The model turns out to have a unique equilibrium for all the wage models considered if the marginal product of labor is independent of aggregate employment. The result holds also for the Nash bargaining solution.

Section 2 presents Mortensen's model and replicates his analysis. The uniqueness result is derived in Section 3, and Section 4 presents some concluding remarks. Some results are derived in the online Appendix.

2. Mortensen's model and analysis

Time is continuous and extends to infinity, and the common rate of time preference is r. There is a large number l of infinitely living workers, of whom u are unemployed at any moment of time, and $e = l-u$ are employed. There is a large number of firms which may maintain jobs that are filled with one worker or vacant. The number of vacancies is v at any moment of time. Vacancies and unemployed workers meet in pairs in order to form matches. The aggregate rate of matching is $m(u,v)$, and Mortensen assumes that $m(0,v) = m(u,0) = 0$, $m(u,v)$ increases in u and v, $m(u,v)/u$ decreases in u, and $m(u,v)/v$ decreases in v. Mortensen specifies $m(u,v) = u^k m(1,v/u)$ where $k \in (0,2)$. In the sequel I denote $m(u,v) = m$, and I use IRS (DRS) for increasing (decreasing) returns to scale.

☆ This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

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https://doi.org/10.1016/j.econlet.2024.112091
Received 7 July 2024; Received in revised form 23 November 2024; Accepted 23 November 2024
Available online 29 November 2024
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The marginal product of labor is f, and the marginal revenue product of labor is y. In the cases of market clearing wage and collective bargaining $y = f$. In the efficiency wage model $y < f$ because of monitoring cost. Mortensen assumes that f may depend on aggregate employment. I present only the case where f is constant, in order to highlight the properties of the matching function.

The asset pricing equations for the agents are

$$rU = b + \frac{m}{u}(W - U) + \dot{U}, \quad (1)$$

$$rW = w - \delta(W - U) + \dot{W}, \quad (2)$$

$$rV = -c + \frac{m}{v}(J - V) + \dot{V}, \quad (3)$$

$$rJ = y - w - \delta(J - V) + \dot{J}. \quad (4)$$

The expected value of unemployment is U. An unemployed worker enjoys leisure b and meets a vacancy at rate m/u. A meeting results in employment, and a worker's expected value increases to W. The value of unemployment may evolve in time, this is denoted by $\dot{U}$. An employed worker receives wage w. The match dissolves at rate δ, and the worker returns to unemployment. Also the value of employment may evolve in time, this is denoted by $\dot{W}$.

The expected value of a vacant job is V. A vacancy pays flow cost c and meets a worker at rate m/v, and the expected value of the job changes to J. The firm earns y and pays wage w. When the match dissolves, the firm may post a vacancy again. The time derivatives of V and J are $\dot{V}$ and $\dot{J}$. Mortensen considers a steady state equilibrium where $\dot{U} = \dot{W} = \dot{V} = \dot{J} = 0$. Free entry and exit of vacancies gives $V = 0$.

Mortensen solves the equilibrium of the model in (J, e)-space. Each firm takes J and e as given when it decides whether to post a vacancy.[1] The values of J and e are determined endogenously in the end, and Mortensen considers whether multiple unemployment equilibria can emerge.

Employment and the value of a filled job adjust according to

$$\dot{e} = \eta(J, e)(l - e) - \delta e, \quad (5)$$

$$\dot{J} = (r + \delta)J - \pi(J, e), \quad (6)$$

where $\eta(J, e) = m/u$, $\pi(J, e) = y - w(J, e)$, and where (6) is given by (4). In a steady state equilibrium $\dot{e} = \dot{J} = 0$. Mortensen does not present the slopes of these curves explicitly, but I provide them here. When $\dot{e} = 0$, (5) gives

$$\frac{dJ}{de} = \frac{\eta - (l - e)\eta_e + \delta}{(l - e)\eta_J}. \quad (7)$$

Expression η_J denotes the partial derivative of m/u with respect to J when e is constant and v adjusts to a change in J such that $V = 0$. Expression η_e denotes the partial derivative of m/u with respect to e when J is constant and v adjusts to a change in e such that $V = 0$. We have $\eta_J > 0$, $\eta_e < 0$ for IRS, $\eta_e > 0$ for DRS, and $dJ/de > 0$ on curve $\dot{e} = 0$ (See the online Appendix). Curve $\dot{e} = 0$ begins at $(J, e) = (0, 0)$ because if $J = 0$, there are no vacancies, and then $e = 0$. Also, curve $\dot{e} = 0$ asymptotes l as $J \to \infty$.

The slope of curve $\dot{J} = 0$ is

$$\frac{dJ}{de} = \frac{\pi_e}{r + \delta - \pi_J}, \quad (8)$$

where π_e and π_J depend on the wage determination model. Because curve $\dot{e} = 0$ is increasing, a necessary condition for multiple equilibria is that also curve $\dot{J} = 0$ is increasing. Fig. 1 illustrates the cases where curve $\dot{J} = 0$ is decreasing or increasing.

The market clearing wage makes workers indifferent between employment and unemployment, thus $w = b$. Then $\pi_J = \pi_e = 0$, and a unique equilibrium exists because curve $\dot{J} = 0$ is horizontal. Next I consider the outcomes of collective bargaining and an efficiency wage model. I replicate Mortensen's analysis until a point where he concludes that a unique equilibrium exists if the matching function has non-increasing returns to scale, but multiple equilibria may exist if returns to scale are increasing.

[1] The standard assumption, used for example in Pissarides (2000), is that each firm takes unemployment and the number of vacancies as given.

2.1. Collective bargaining

In the collective bargaining model it is assumed that all workers belong to a union which maximizes the expected lifetime utility of a currently employed member by choosing worker's share θ of the match surplus. The union solves problem

$$rW = \max_{\theta \in [0,1]} \{w - \delta(W - U)\} \quad (9)$$

where $\delta(W - U)$ is a currently employed worker's expected loss of wealth due to becoming unemployed. This gives (See the online Appendix)

$$\pi = (\delta J)^{1/2}(f - b + J\eta(J, e))^{1/2} - J\eta(J, e), \quad (10)$$

and

$$\pi_e = \left((1/2)\left(1 - \theta^*\right) - 1\right) J\eta_e \quad (11)$$

where $\theta^* \in [0, 1]$ solves problem (9). Then $\pi_e > (<)\ 0$ for IRS (DRS). Mortensen argues (but does not show explicitly) that $\pi_J < 0$, and then $dJ/de > (<)\ 0$ on $\dot{J} = 0$ for IRS (DRS), illustrated in Fig. 1. He concludes that multiple equilibria may exist if returns to scale are increasing because then curve $\dot{J} = 0$ may cross curve $\dot{e} = 0$ more than once. Mortensen's analysis of the possibility of multiple equilibria ends here.

2.2. Efficiency wage

Mortensen applies the model of Shapiro and Stiglitz (1984) where a worker can collect wage and leisure utility simultaneously by shirking. The employer monitors the worker at frequency λ, at cost a every time when checking the effort. Then $y = f - a\lambda$. The worker is fired if caught shirking. In equilibrium a worker's expected cost of shirking equals the value of leisure: $\lambda(W - U) = b$. The firm solves problem

$$rJ = \max_{\lambda}(f - a\lambda - w - \delta J). \quad (12)$$

This gives (See the online Appendix)

$$\pi = f - b - 2(ab)^{1/2}(r + \delta + \eta(J, e))^{1/2}. \quad (13)$$

Then $\pi_J < 0$ because $\eta_J > 0$. Also, $\pi_e > (<)\ 0$ if $\eta_e < (>)\ 0$. Then $dJ/de > (<)\ 0$ on $\dot{J} = 0$ for IRS (DRS), shown in Fig. 1. As in the case of collective bargaining, Mortensen concludes that non-increasing returns to scale guarantees uniqueness of equilibrium while increasing returns to scale does not.

3. Uniqueness of equilibrium

Mortensen (1989) concluded that multiple equilibria may exist if (i) wages are determined by collective bargaining or by the efficiency wage model, and (ii) the marginal product of labor is independent of aggregate employment, and (iii) the matching function has increasing returns to scale. I show that this claim is wrong: a unique equilibrium exists. The problem in Mortensen's analysis – which is correct per se – is that it is not carried out far enough. I continue the analysis from the point where Mortensen ended his. The strategy to prove uniqueness is as follows: I solve the steady state value of J as a function of employment. Then I show that it is impossible that there exist two steady state equilibria (e_1, J_1) and (e_2, J_2) where $e_2 > e_1$ and $J_2 > J_1$, given the parameter values.

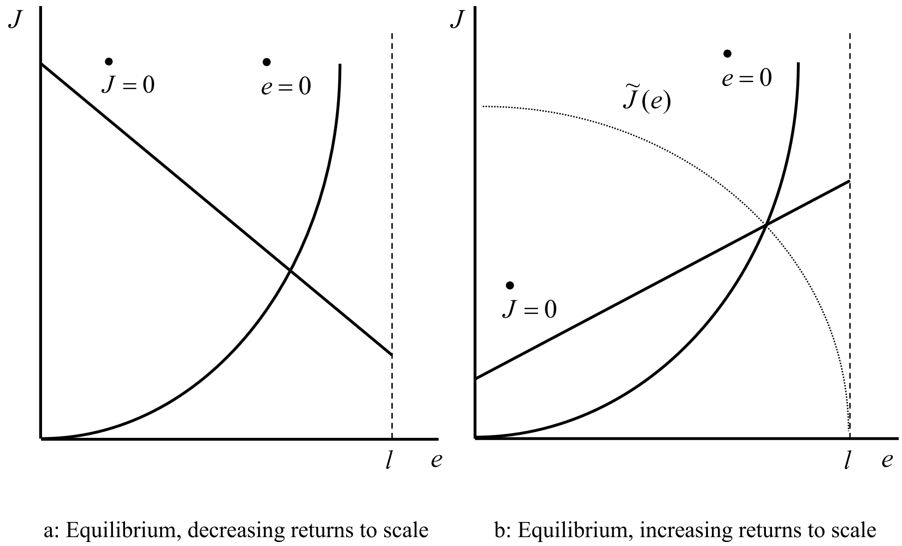

Fig. 1. Equilibrium when the matching function has decreasing or increasing return to scale.

3.1. Collective bargaining

Eqs. (4) and (10) give

$$J = \frac{(\delta J)^{1/2}\left(f - b + J\eta(J,e)\right)^{1/2} - J\eta(J,e)}{r+\delta}. \tag{14}$$

This gives

$$J = \frac{\delta(f-b)}{(r+\delta)^2 + (2r+\delta)\eta(J,e) + (\eta(J,e))^2} \tag{15}$$

on curve $\dot{J}=0$, and

$$\frac{dJ}{de} = -\frac{(2r+\delta+2\eta)J\eta_e}{(r+\delta)^2 + (2r+\delta)\eta + \eta^2 + (2r+\delta+2\eta)J\eta_J}. \tag{16}$$

Curve $\dot{J}=0$ is increasing (decreasing) for IRS (DRS) because $\eta_J > 0$, and $\eta_e < (>)\ 0$ for IRS (DRS).

Let $J(0)$ and $J(l)$ denote the values of J given by (15) when $e=0$ and $e=l$, respectively. When $e=0$, then $u=l$, and η is finite. Then $J(0)>0$. When $e=l$, then $u=0$. Then $J(l) \leq \delta(f-b)/(r+\delta)^2$. Because of the properties of curve $\dot{e}=0$, a steady state equilibrium exists (See Fig. 1).

When $\dot{e}=0$, (5) gives $\eta(J,e) = \delta e/(l-e)$. Using this in (15) yields

$$\tilde{J}(e) = \frac{\delta(f-b)}{(r+\delta)^2 + (2r+\delta)\dfrac{\delta e}{l-e} + \left(\dfrac{\delta e}{l-e}\right)^2}. \tag{17}$$

A steady state equilibrium satisfies Eq. (17), and it is located also on curve $\dot{e}=0$ which is upward-sloping. Because $d\tilde{J}(e)/de < 0$ (See Fig. 1), it is impossible that there exist two steady state equilibria (e_1, J_1) and (e_2, J_2) where $e_2 > e_1$ and $J_2 > J_1$, given the parameter values.[2] We end up with

Claim 1. *A unique equilibrium exists if wages are determined by collective bargaining.*

Because curve $\dot{J}=0$ cuts curve $\dot{e}=0$ from the above, the equilibrium is a saddle point as Mortensen (1989) argues.

3.2. Efficiency wage

Using (4) and (13) results in

$$J = \frac{f - b - 2(ab)^{1/2}\left(r+\delta+\eta(J,e)\right)^{1/2}}{r+\delta} \tag{18}$$

on curve $\dot{J}=0$ which is increasing (decreasing) for IRS (DRS). Curve $\dot{J}=0$ begins above the origin, and $J(l)$ is finite, thus a steady state equilibrium exists. Using $\eta(J,e) = \delta e/(l-e)$ gives

$$\tilde{J}(e) = \frac{f - b - 2(ab)^{1/2}\left(r+\delta+\dfrac{\delta e}{l-e}\right)^{1/2}}{r+\delta} \tag{19}$$

in a steady state equilibrium. Because $d\tilde{J}(e)/de < 0$, following the logic in the above yields

Claim 2. *A unique equilibrium exists if wages are determined by the efficiency wage model.*

Again, curve $\dot{J}=0$ cuts curve $\dot{e}=0$ from the above, thus the equilibrium is a saddle point.

3.3. Nash bargaining solution

Mortensen does not consider the Nash bargaining solution. I include it here because it is commonly used in the labor market literature. As in the collective bargaining model, $y=f$. The Nash bargaining solution gives $W-U = \theta(W-U+J)$, and using (1)–(4) yields $w = b + \theta(f - b + (v/u)c)$. Eq. (3) and $m/u = \eta(J,e)$ give $(v/u)c = J\eta(J,e)$, thus $w = b + \theta(f - b + J\eta(J,e))$. Then

$$\pi = (1-\theta)(f-b) - \theta J\eta(J,e). \tag{20}$$

We have $\pi_e = -\theta J\eta_e > (<)\ 0$ for IRS (DRS), and $\pi_J = -\theta\eta - \theta J\eta_J < 0$. Then, by (8), curve $\dot{J}=0$ is increasing (decreasing) for IRS (DRS). Using (4) gives

$$J = \frac{(1-\theta)(f-b)}{r+\delta+\theta\eta(J,e)}. \tag{21}$$

Curve $\dot{J}=0$ begins above the origin, and $J(l)$ is finite, thus a steady state equilibrium exists. Substituting $\delta e/(l-e)$ for $\eta(J,e)$ yields

$$\tilde{J}(e) = \frac{(1-\theta)(f-b)}{r+\delta+\dfrac{\theta\delta e}{l-e}} \tag{22}$$

where the right-hand side decreases in e, and we have

Claim 3. *A unique equilibrium exists if wages are determined by the Nash bargaining solution.*

As before, the equilibrium is a saddle point.

[2] This type of method is used in Burdett and Smith (2002) in addressing the possibility of multiple equilibria in a related model.

4. Conclusion

I showed that increasing returns to scale in matching cannot be a cause of multiple equilibria in the model of Mortensen (1989), contrary to the author's claim. Although increasing returns to scale is a necessary condition for multiplicity of equilibria when the marginal product of labor is independent of aggregate employment and wages are determined by collective bargaining or by an efficiency wage model, it turns out that, after all, no multiple equilibria exist. This holds also if wages are determined by the Nash bargaining solution. It follows that multiple equilibria can emerge only if the marginal product of labor increases in aggregate employment. This can be seen in Eqs. (17), (19) and (22), because $\tilde{J}(e)$ increases in e only if f increases in e. This is, however, a non-standard assumption in the literature.

The result suggests that multiplicity of unemployment equilibrium requires – given constant marginal product of labor – that firms' and workers' choices are interdependent, like in Pissarides (1986), Howitt and McAfee (1987) and Burdett and Smith (2002). This interdependence does not exist in Mortensen (1989), and this rules out multiple equilibria even in the presence of increasing returns to scale in matching.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

I thank the anonymous referee for valuable comments.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2024.112091.

Data availability

No data was used for the research described in the article.

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